# Computer Algebra Independent Integration Tests

Summer 2023 edition

3-Logarithms/62-3.3-u-a+b-log-c-d+e-x-^n-^p

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# Contents

# CHAPTER 1

# INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ 547 ]. This is test number [ 62 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 99.27 ( 543 ) | 0.73 ( 4 ) |
| Mathematica | 99.27 ( 543 ) | 0.73 ( 4 ) |
| Maple | 62.89 ( 344 ) | 37.11 ( 203 ) |
| Maxima | 40.59 ( 222 ) | 59.41 ( 325 ) |
| Fricas | 40.40 ( 221 ) | 59.60 ( 326 ) |
| Giac | 39.49 ( 216 ) | 60.51 ( 331 ) |
| Mupad | 38.21 ( 209 ) | 61.79 ( 338 ) |
| Sympy | 30.35 ( 166 ) | 69.65 ( 381 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

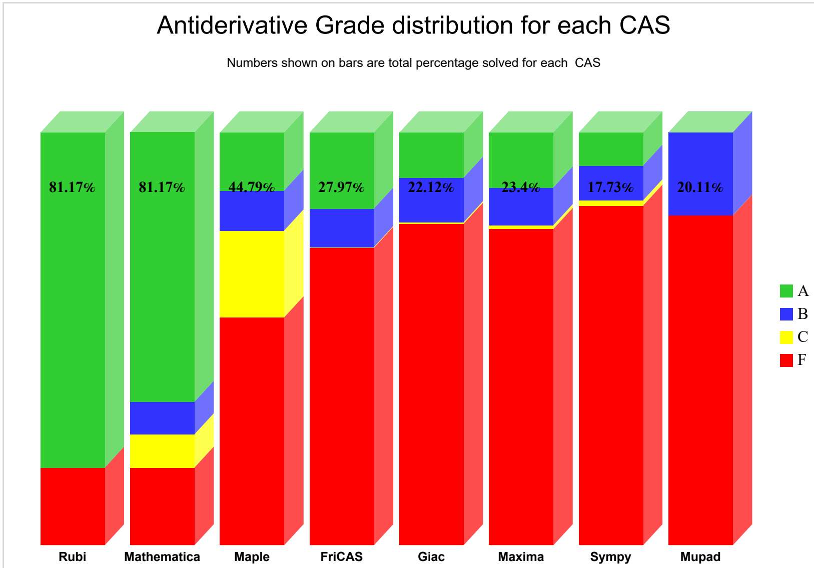
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

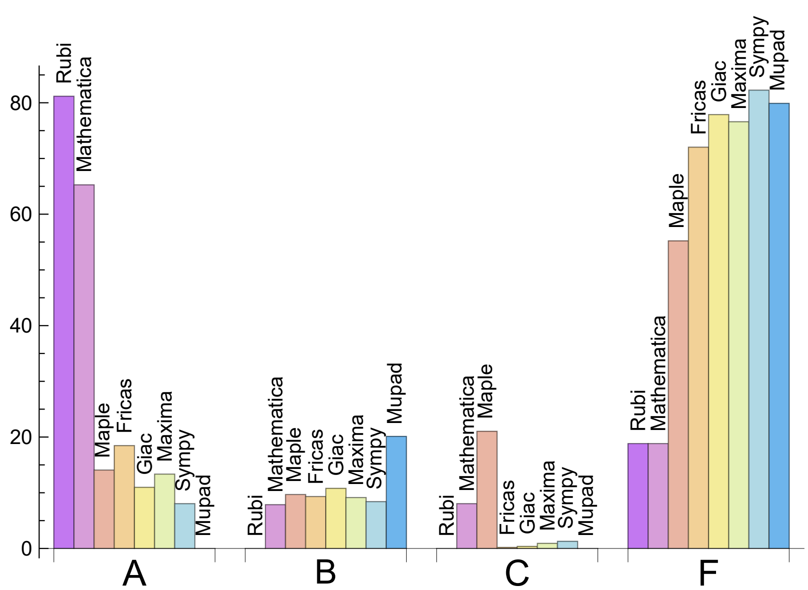
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 81.170 | 0.000 | 0.000 | 18.830 |
| Mathematica | 65.265 | 7.861 | 8.044 | 18.830 |
| Fricas | 18.464 | 9.324 | 0.183 | 72.029 |
| Maple | 14.077 | 9.689 | 21.024 | 55.210 |
| Maxima | 13.346 | 9.141 | 0.914 | 76.600 |
| Giac | 10.969 | 10.786 | 0.366 | 77.879 |
| Sympy | 8.044 | 8.410 | 1.280 | 82.267 |
| Mupad | 0.000 | 20.110 | 0.000 | 79.890 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the grades of the CAS systems.




The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

| System | Number failed | Percentage normal failure | Percentage time-out failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 4 | 100.00 | 0.00 | 0.00 |
| Mathematica | 4 | 100.00 | 0.00 | 0.00 |
| Maple | 203 | 100.00 | 0.00 | 0.00 |
| Fricas | 326 | 73.01 | 0.00 | 26.99 |
| Maxima | 325 | 79.69 | 0.00 | 20.31 |
| Giac | 331 | 98.79 | 0.00 | 1.21 |
| Mupad | 338 | 0.00 | 100.00 | 0.00 |
| Sympy | 381 | 56.43 | 38.85 | 4.72 |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) |
|---|---|
| Rubi | 0.30 |
| Fricas | 0.30 |
| Giac | 0.42 |
| Mathematica | 0.84 |
| Maxima | 1.19 |
| Mupad | 1.34 |
| Maple | 2.96 |
| Sympy | 8.45 |

Table 1.5: Time performance for each CAS

| System | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|
| Mupad | 133.53 | 1.27 | 31.00 | 1.07 |
| Maxima | 205.23 | 3.25 | 85.50 | 1.20 |
| Rubi | 220.89 | 1.00 | 156.00 | 1.00 |
| Sympy | 257.43 | 2.13 | 44.00 | 1.13 |
| Fricas | 268.55 | 2.14 | 95.00 | 1.46 |
| Mathematica | 309.49 | 1.27 | 164.00 | 1.06 |
| Maple | 434.49 | 2.31 | 121.00 | 1.24 |
| Giac | 524.56 | 2.75 | 41.50 | 1.09 |

Table 1.6: Leaf size performance for each CAS

## 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the $y$ axis is the percentage solved which Rubi itself needed the number of rules given the $x$ axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

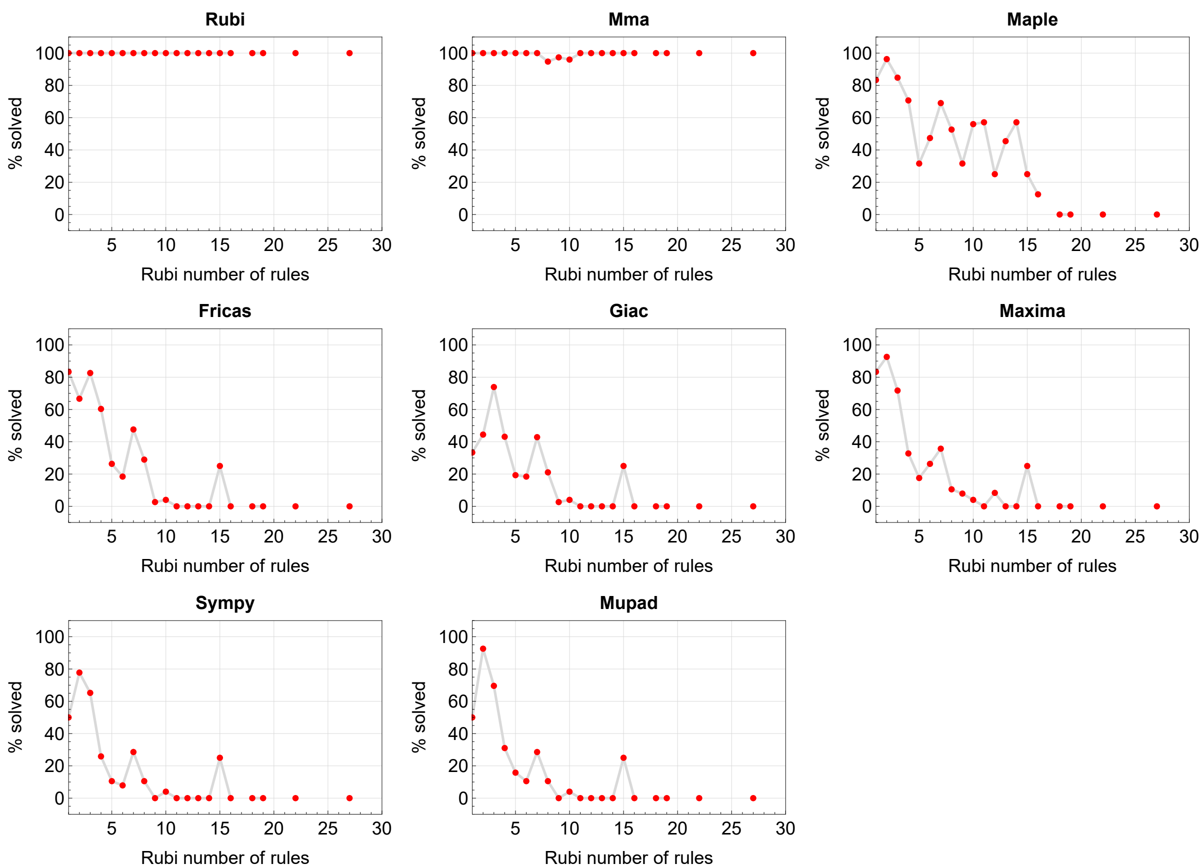



Figure 1.1: Solving statistics per number of Rubi rules used

## 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

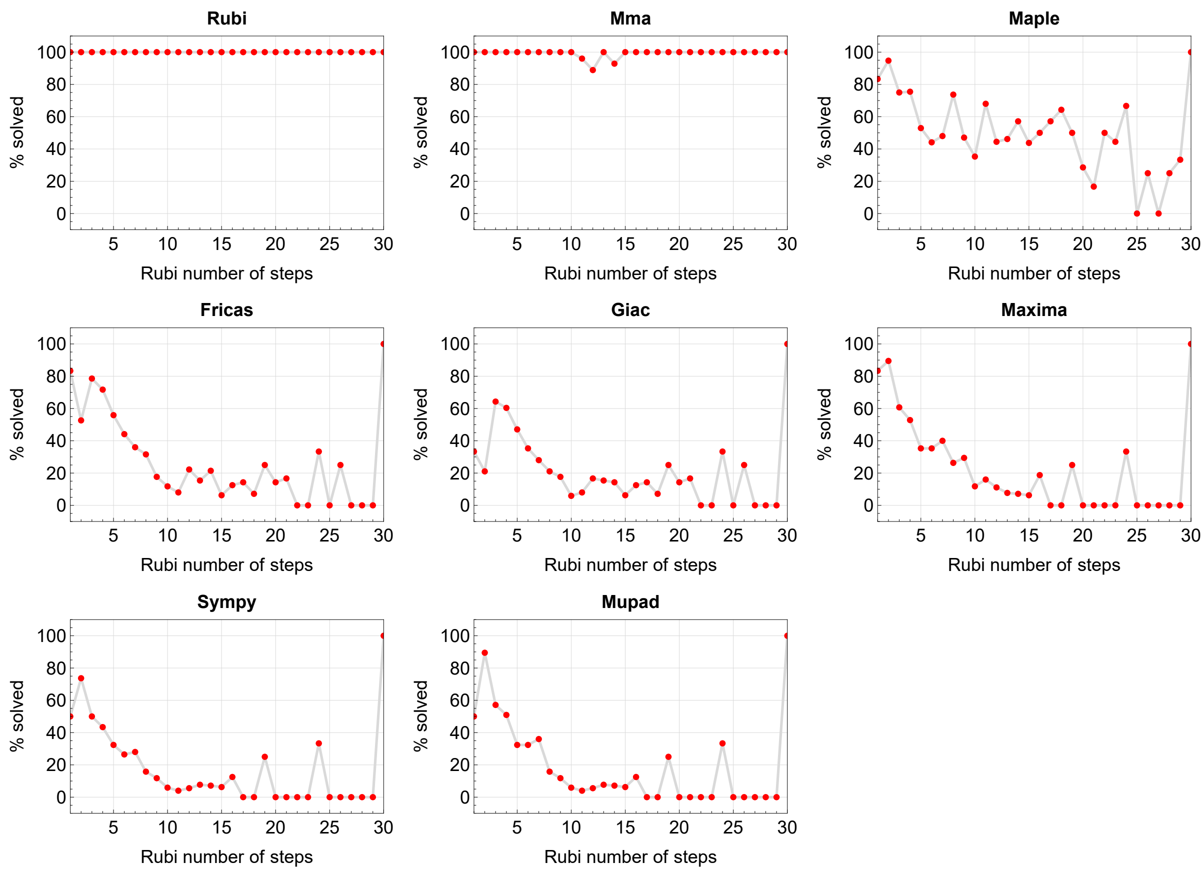



Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

## 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

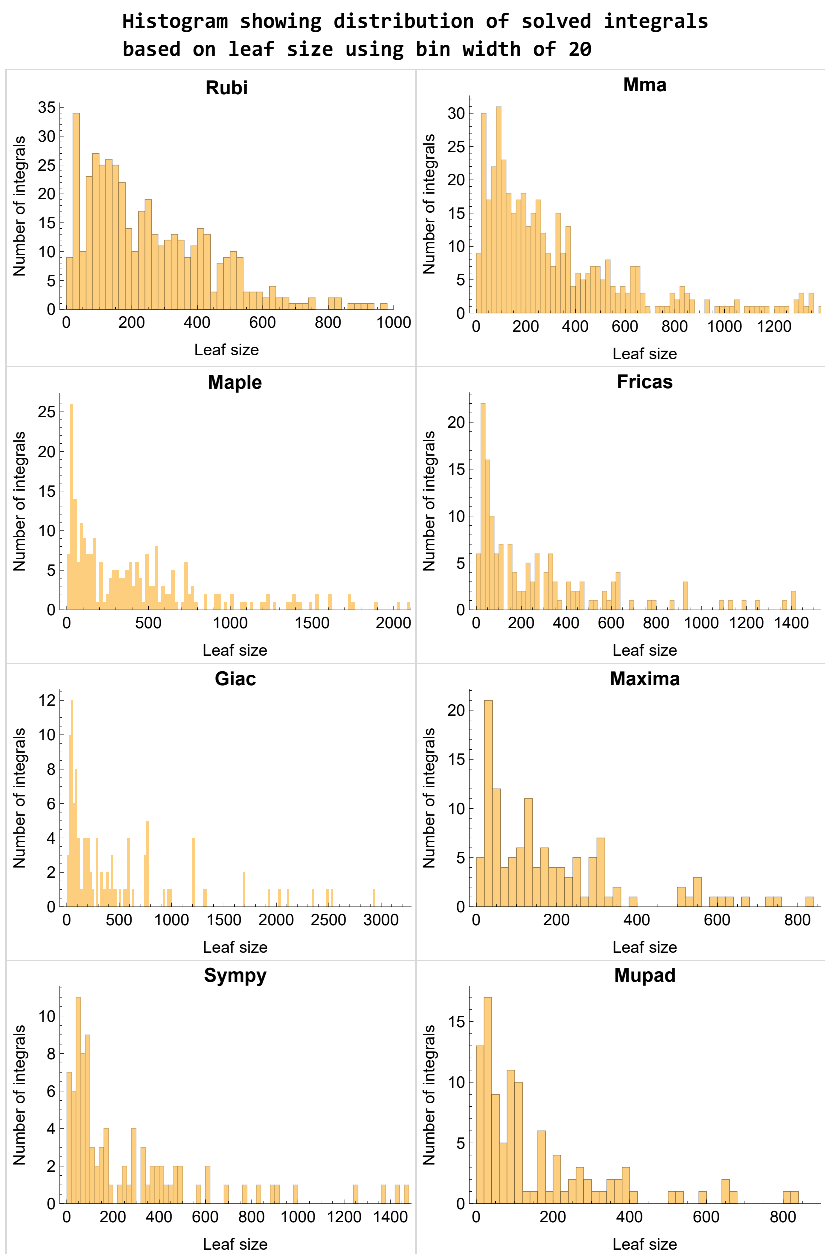



Figure 1.3: Solved integrals based on leaf size distribution

## 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

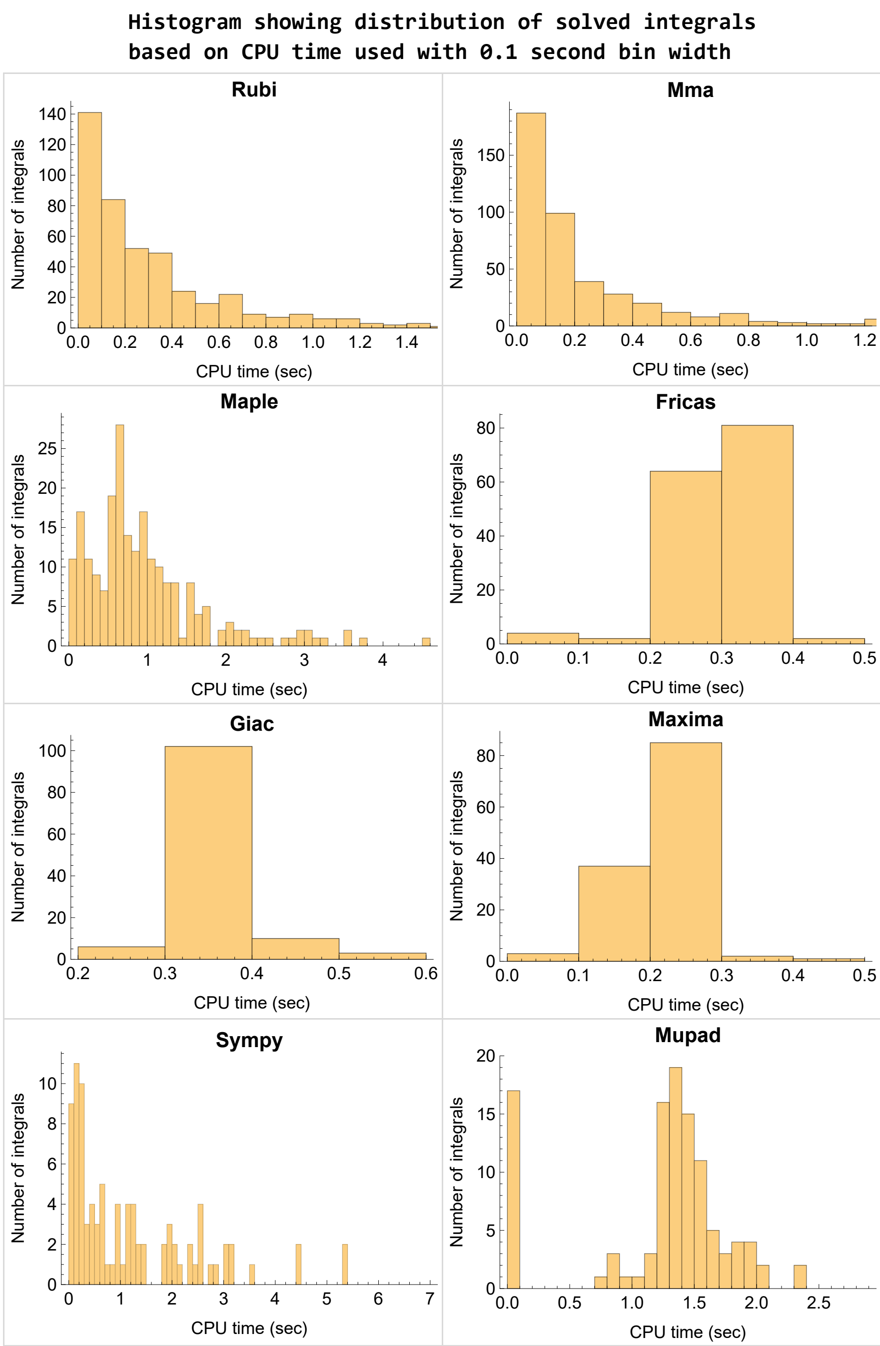



Figure 1.4: Solved integrals histogram based on CPU time used

## 1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

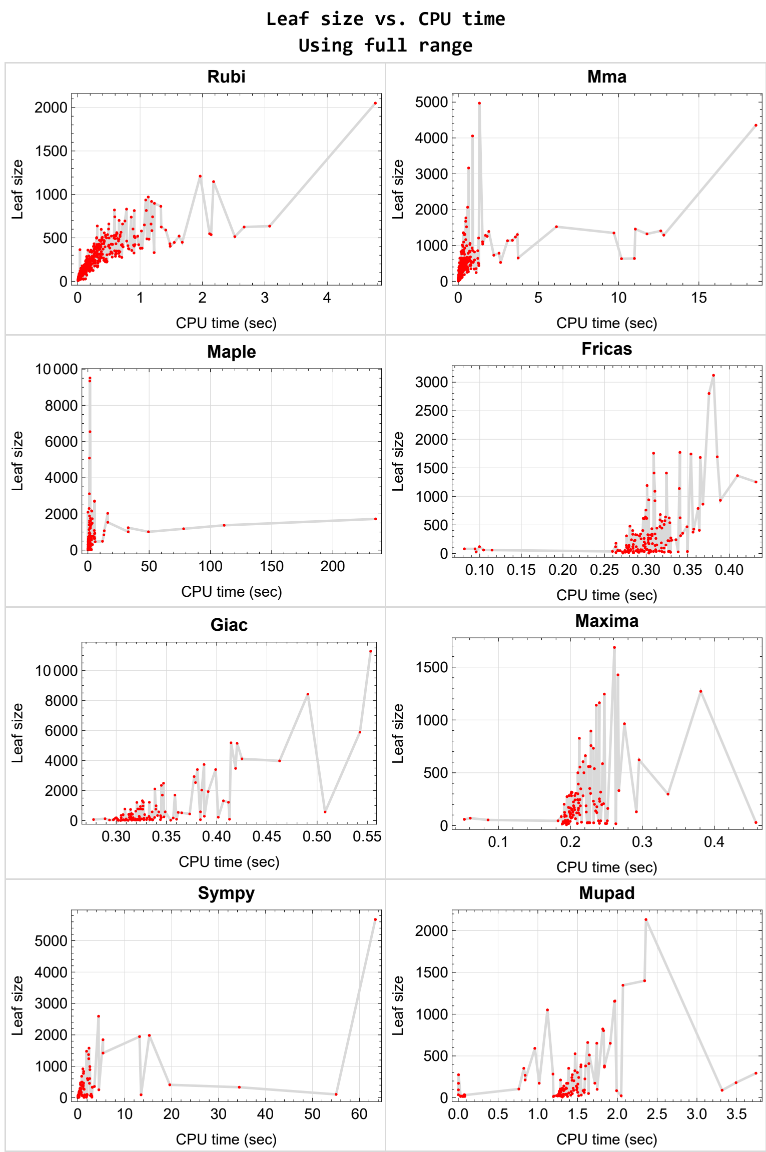



Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{92, 93, 98, 99, 103, 104, 108, 109, 110, 114, 115, 116, 120, 121, 122, 127, 132, 137, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 174, 196, 197, 209, 214, 215, 216, 234, 235, 236, 237, 238, 239, 240, 241, 332, 346, 350, 375, 376, 377, 395, 396, 399, 400, 448, 449, 453, 454, 458, 459, 463, 464, 468, 469, 473, 477, 481, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 507, 508, 509, 510, 511, 512, 513, 517, 540, 541, 542, 543, 544, 545, 546, 547}

## 1.10 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

**Rubi** {}

**Mathematica** {278, 522}

**Maple** {21, 22, 23, 40, 48, 49, 50, 51, 56, 57, 62, 63, 86, 88, 89, 90, 91, 94, 95, 96, 97, 100, 101, 102, 217, 218, 219, 220, 221, 222, 223, 226, 227, 231, 232, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 306, 322, 331, 343, 344, 349, 358, 359, 360, 361, 362, 363, 364, 365, 366, 382, 383, 384, 385, 386, 387, 388, 389}

**Maxima** Verification phase not currently implemented.

**Fricas** Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.14 Important notes about some of the results

### Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

## Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

## Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

## Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

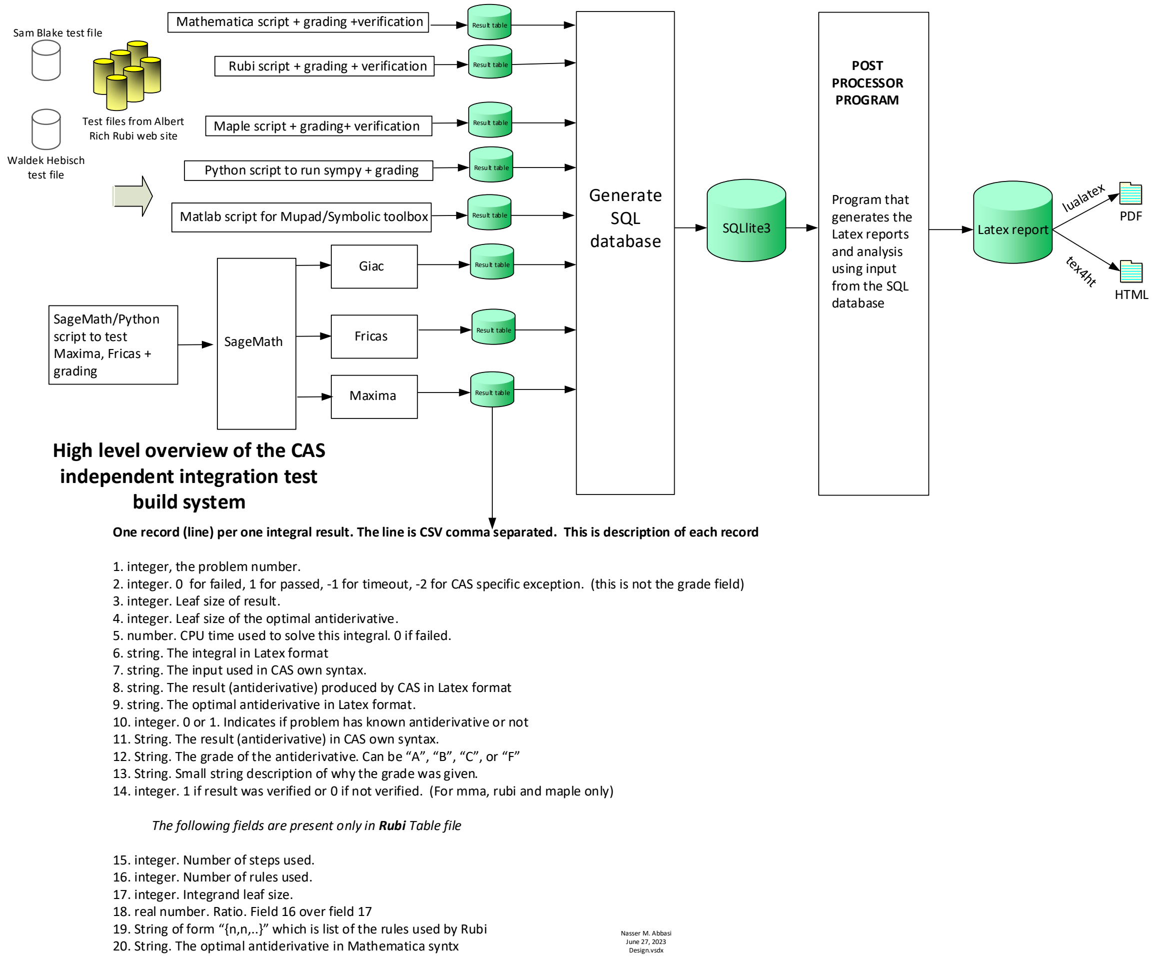
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



# CHAPTER 2

## DETAILED SUMMARY TABLES OF RESULTS

## 2.1 List of integrals sorted by grade for each CAS



### Rubi

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 100, 101, 102, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 128, 129, 130, 131, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 162, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 373, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 450, 451, 452, 455, 456, 457, 460, 461, 462, 465, 466, 467, 470, 471, 472, 474, 475, 476, 478, 479, 480, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 506, 514, 515, 516, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

**B grade** { }

**C grade** { }

**F normal fail** { 370, 371, 372, 374 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Mma

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 96, 97, 100, 101, 102, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 130, 131, 134, 135, 136, 138, 139, 140, 141, 145, 146, 147, 148, 149, 162, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 193, 194, 195, 198, 199, 200, 202, 204, 205, 206, 207, 210, 211, 212, 213, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 268, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 300, 302, 303, 304, 305, 306, 307, 308, 309, 331, 333, 334, 335, 336, 337, 338, 344, 345, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 374, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 433, 434, 435, 436, 437, 440, 441, 444, 445, 446, 447, 451, 452, 455, 456, 457, 460, 461, 462, 465, 466, 467, 470, 471, 472, 475, 476, 478, 479, 480, 482, 483, 484, 485, 506, 514, 515, 516, 518, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 534, 539 }

**B grade** { 56, 57, 62, 63, 94, 95, 128, 129, 133, 225, 229, 230, 231, 339, 340, 341, 342, 343, 373, 394, 397, 398, 432, 438, 439, 442, 443, 450, 474, 489, 490, 491, 492, 493, 494, 495, 531, 532, 533, 535, 536, 537, 538 }

**C grade** { 142, 143, 144, 150, 151, 201, 203, 208, 266, 267, 269, 270, 293, 294, 295, 296, 297, 298, 299, 301, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 486, 487, 488 }

**F normal fail** { 191, 192, 276, 520 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Maple

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 19, 20, 33, 38, 39, 41, 47, 64, 65, 66, 67, 68, 69, 70, 72, 75, 76, 77, 78, 81, 82, 83, 84, 85, 87, 140, 175, 176, 177, 178, 179, 184, 185, 186, 193, 194, 195, 213, 252, 279, 280, 281, 282, 303, 307, 309, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 345, 354, 379, 380, 381, 402, 403, 407, 422, 423, 425, 444, 484 }

**B grade** { 17, 18, 35, 36, 37, 42, 43, 44, 45, 46, 52, 53, 54, 55, 60, 61, 71, 73, 74, 79, 80, 180, 181, 182, 183, 187, 188, 191, 192, 304, 305, 308, 351, 352, 353, 355, 356, 357, 404, 405, 406, 420, 421, 426, 427, 428, 429, 430, 431, 435, 436, 437, 441 }

**C grade** { 21, 22, 23, 40, 48, 49, 50, 51, 56, 57, 62, 63, 86, 88, 89, 90, 91, 94, 95, 96, 97, 100, 101, 102, 204, 205, 206, 207, 208, 217, 218, 219, 220, 221, 222, 223, 226, 227, 231, 232, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 306, 322, 331, 343, 344, 349, 358, 359, 360, 361, 362, 363, 364, 365, 366, 382, 383, 384, 385, 386, 387, 388, 389 }

**F normal fail** { 9, 10, 11, 12, 13, 14, 15, 16, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 58, 59, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 128, 129, 130, 131, 133, 134, 135, 136, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 162, 170, 171, 172, 173, 189, 190, 198, 199, 200, 201, 202, 203, 210, 211, 212, 224, 225, 228, 229, 230, 233, 275, 276, 277, 278, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 323, 324, 325, 326, 327, 328, 329, 330, 347, 348, 367, 368, 369, 370, 371, 372, 373, 374, 378, 390, 391, 392, 393, 394, 397, 398, 401, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 424, 432, 433, 434, 438, 439, 440, 442, 443, 445, 446, 447, 450, 451, 452, 455, 456, 457, 460, 461, 462, 465, 466, 467, 470, 471, 472, 474, 475, 476, 478, 479, 480, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 506, 514, 515, 516, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Fricas

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 22, 31, 33, 34, 38, 39, 41, 64, 65, 66, 67, 68, 69, 70, 71, 72, 75, 76, 81, 82, 85, 87, 88, 89, 90, 91, 94, 95, 96, 97, 138, 139, 140, 141, 173, 175, 176, 177, 178, 179, 183, 184, 185, 186, 191, 192, 193, 194, 195, 212, 213, 252, 268, 279, 281, 303, 304, 307, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 379, 380, 381, 402, 403, 407, 408, 409, 418, 422, 423, 425, 444, 445, 446, 447, 450, 451, 452, 483, 484, 485, 516 }

**B grade** { 17, 18, 19, 23, 35, 36, 37, 42, 43, 44, 45, 46, 47, 52, 53, 54, 55, 60, 61, 100, 101, 102, 142, 143, 144, 187, 305, 308, 404, 405, 406, 410, 420, 421, 426, 427, 428, 429, 430, 431, 435, 436, 437, 441, 455, 456, 457, 482, 486, 487, 488 }

**C grade** { 16 }

**F normal fail** { 32, 40, 48, 49, 50, 51, 56, 57, 58, 59, 62, 63, 73, 74, 77, 78, 79, 80, 83, 84, 86, 145, 146, 147, 148, 149, 150, 151, 162, 170, 171, 172, 180, 181, 182, 188, 189, 190, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 306, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 343, 344, 345, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 378, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 401, 419, 424, 432, 433, 434, 438, 439, 440, 442, 443, 489, 490, 491, 492, 493, 494, 495, 506, 514, 515, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { 9, 10, 11, 12, 13, 14, 15, 24, 25, 26, 27, 28, 29, 30, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 156, 157, 158, 159, 160, 161, 411, 412, 413, 414, 415, 416, 417, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 500, 501, 502, 503, 504, 505 }

## Maxima

**A grade** { 3, 4, 5, 6, 7, 8, 13, 14, 15, 16, 20, 32, 37, 38, 39, 41, 42, 46, 64, 65, 66, 67, 68, 69, 70, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 195, 213, 252, 268, 303, 304, 307, 308, 345, 358, 359, 360, 361, 363, 364, 365, 366, 379, 380, 381, 402, 403, 406, 407, 419, 421, 422, 423, 425, 426, 429, 430, 431, 444 }

**B grade** { 1, 2, 17, 18, 19, 35, 36, 43, 44, 45, 47, 52, 53, 54, 55, 60, 61, 71, 75, 175, 176, 177, 178, 179, 183, 184, 185, 186, 187, 188, 189, 190, 279, 280, 305, 337, 338, 339, 340, 341, 342, 404, 405, 420, 427, 428, 435, 436, 437, 441 }

**C grade** { 9, 10, 11, 12, 309 }

**F normal fail** { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 34, 40, 48, 49, 50, 51, 56, 57, 58, 59, 62, 63, 72, 88, 89, 90, 91, 94, 95, 96, 97, 100, 101, 102, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 128, 129, 130, 131, 133, 134, 135, 136, 162, 180, 181, 182, 191, 192, 193, 194, 210, 211, 212, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 306, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334, 335, 336, 343, 344, 362, 367, 368, 369, 370, 371, 372, 373, 374, 378, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 401, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 424, 432, 433, 434, 438, 439, 440, 442, 443, 445, 446, 447, 450, 451, 452, 455, 456, 457, 460, 461, 462, 465, 466, 467, 470, 471, 472, 474, 475, 476, 478, 479, 480, 506, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { 31, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 169, 170, 171, 172, 173, 174, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 329, 330, 331, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 377, 418, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 513, 514, 515, 516, 517 }

## Giac

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 33, 39, 41, 42, 64, 65, 66, 67, 68, 69, 70, 76, 82, 85, 88, 89, 90, 91, 140, 141, 142, 175, 176, 177, 178, 179, 184, 185, 186, 195, 213, 252, 268, 307, 308, 381, 402, 403, 407, 408, 423, 425, 426, 444, 445, 446, 447, 484, 485, 486 }

**B grade** { 17, 18, 19, 22, 23, 35, 36, 37, 38, 43, 44, 45, 46, 47, 52, 53, 54, 55, 60, 61, 87, 94, 95, 96, 97, 100, 101, 102, 183, 187, 303, 304, 305, 379, 380, 404, 405, 406, 409, 410, 420, 421, 422, 427, 428, 429, 430, 431, 435, 436, 437, 441, 450, 451, 452, 455, 456, 457, 487 }

**C grade** { 11, 12 }

**F normal fail** { 9, 10, 13, 14, 15, 16, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 40, 48, 49, 50, 51, 56, 57, 58, 59, 62, 63, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 83, 84, 86, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 128, 129, 130, 131, 133, 134, 135, 136, 138, 139, 143, 144, 145, 146, 147, 148, 149, 150, 151, 162, 170, 171, 172, 173, 180, 181, 182, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 306, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 378, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 401, 411, 412, 413, 414, 415, 416, 417, 418, 419, 424, 432, 433, 434, 438, 439, 440, 442, 443, 460, 461, 462, 465, 466, 467, 470, 471, 472, 474, 475, 476, 478, 479, 480, 482, 483, 488, 489, 490, 491, 492, 493, 494, 495, 506, 514, 515, 516, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { 169, 209, 215, 513 }

## Mupad

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 33, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 52, 53, 54, 55, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 175, 176, 177, 178, 179, 183, 184, 185, 186, 187, 195, 213, 252, 268, 279, 281, 303, 304, 305, 307, 308, 337, 338, 379, 380, 381, 402, 403, 404, 405, 406, 407, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 431, 435, 436, 437, 441, 444 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 40, 48, 49, 50, 51, 56, 57, 58, 59, 62, 63, 86, 88, 89, 90, 91, 94, 95, 96, 97, 100, 101, 102, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 128, 129, 130, 131, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 162, 170, 171, 172, 173, 180, 181, 182, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 306, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 378, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 401, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 424, 432, 433, 434, 438, 439, 440, 442, 443, 445, 446, 447, 450, 451, 452, 455, 456, 457, 460, 461, 462, 465, 466, 467, 470, 471, 472, 474, 475, 476, 478, 479, 480, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 506, 514, 515, 516, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

**F(-2) exception fail** { }

## Sympy

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 10, 16, 20, 38, 39, 64, 65, 66, 67, 68, 69, 70, 76, 79, 80, 82, 83, 84, 85, 87, 176, 177, 178, 179, 186, 195, 303, 379, 380, 381, 402, 403, 407, 422, 423, 444 }

**B grade** { 11, 12, 13, 17, 18, 19, 35, 36, 37, 41, 42, 44, 45, 46, 47, 52, 53, 54, 55, 60, 61, 175, 183, 184, 185, 187, 252, 304, 305, 307, 404, 405, 406, 420, 421, 425, 426, 427, 428, 429, 430, 431, 435, 436, 437, 441 }

**C grade** { 72, 73, 74, 75, 77, 78, 81 }

**F normal fail** { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 40, 48, 49, 50, 51, 56, 57, 58, 59, 62, 63, 71, 86, 88, 89, 90, 91, 94, 95, 96, 97, 100, 101, 102, 105, 106, 107, 111, 112, 113, 117, 118, 119, 123, 124, 125, 126, 128, 129, 130, 131, 133, 134, 135, 136, 138, 139, 140, 141, 142, 145, 146, 147, 148, 170, 171, 172, 173, 180, 181, 182, 188, 189, 190, 191, 192, 193, 194, 200, 201, 205, 206, 212, 213, 217, 218, 219, 220, 221, 224, 225, 226, 227, 229, 230, 231, 242, 243, 244, 245, 246, 248, 249, 250, 254, 255, 259, 275, 276, 277, 278, 279, 280, 281, 282, 306, 309, 313, 329, 330, 331, 337, 338, 343, 344, 345, 351, 352, 353, 354, 374, 382, 383, 384, 385, 408, 409, 410, 412, 413, 414, 415, 416, 418, 419, 424, 432, 433, 434, 438, 439, 440, 442, 443, 445, 446, 447, 450, 451, 452, 455, 456, 457, 460, 461, 462, 465, 466, 467, 470, 471, 472, 474, 475, 476, 478, 479, 480, 482, 483, 484, 485, 486, 489, 490, 491, 492, 493, 515, 516, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539 }

**F(-1) timedout fail** { 9, 14, 15, 93, 114, 115, 116, 120, 121, 122, 137, 143, 144, 150, 151, 165, 198, 199, 204, 207, 208, 209, 210, 211, 215, 216, 222, 223, 228, 232, 233, 237, 247, 251, 253, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 308, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334, 339, 340, 341, 342, 347, 348, 349, 350, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 378, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 411, 417, 469, 481, 487, 488, 494, 495, 509, 514, 518 }

**F(-2) exception fail** { 43, 149, 162, 164, 166, 168, 169, 202, 203, 335, 336, 377, 401, 506, 508, 510, 512, 513 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 74 | 87 | 188 | 78 | 88 | 88 | 119 |
| N.S. | 1 | 1.00 | 0.91 | 1.07 | 2.32 | 0.96 | 1.09 | 1.09 | 1.47 |
| time (sec) | N/A | 0.024 | 0.007 | 0.132 | 0.204 | 0.278 | 0.121 | 0.413 | 1.494 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 57 | 67 | 125 | 60 | 68 | 68 | 88 |
| N.S. | 1 | 1.00 | 0.93 | 1.10 | 2.05 | 0.98 | 1.11 | 1.11 | 1.44 |
| time (sec) | N/A | 0.018 | 0.005 | 0.125 | 0.192 | 0.295 | 0.105 | 0.277 | 1.451 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 40 | 47 | 71 | 42 | 46 | 48 | 57 |
| N.S. | 1 | 1.00 | 0.98 | 1.15 | 1.73 | 1.02 | 1.12 | 1.17 | 1.39 |
| time (sec) | N/A | 0.013 | 0.004 | 0.119 | 0.201 | 0.289 | 0.084 | 0.307 | 1.389 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 21 | 26 | 31 | 25 | 26 | 31 | 25 |
| N.S. | 1 | 1.00 | 1.00 | 1.24 | 1.48 | 1.19 | 1.24 | 1.48 | 1.19 |
| time (sec) | N/A | 0.006 | 0.004 | 0.080 | 0.190 | 0.272 | 0.070 | 0.301 | 0.067 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 22 | 17 | 16 | 12 | 16 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 1.47 | 1.13 | 1.07 | 0.80 | 1.07 | 1.00 |
| time (sec) | N/A | 0.008 | 0.008 | 0.253 | 0.263 | 0.264 | 0.244 | 0.302 | 1.373 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 36 | 43 | 20 | 47 | 29 | 37 | 36 |
| N.S. | 1 | 1.00 | 1.00 | 1.19 | 0.56 | 1.31 | 0.81 | 1.03 | 1.00 |
| time (sec) | N/A | 0.011 | 0.010 | 0.290 | 0.250 | 0.281 | 0.277 | 0.293 | 1.357 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 47 | 64 | 21 | 67 | 48 | 58 | 64 |
| N.S. | 1 | 1.00 | 0.75 | 1.02 | 0.33 | 1.06 | 0.76 | 0.92 | 1.02 |
| time (sec) | N/A | 0.017 | 0.012 | 0.282 | 0.240 | 0.290 | 0.273 | 0.344 | 1.501 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 57 | 87 | 20 | 90 | 71 | 78 | 67 |
| N.S. | 1 | 1.00 | 0.67 | 1.02 | 0.24 | 1.06 | 0.84 | 0.92 | 0.79 |
| time (sec) | N/A | 0.022 | 0.018 | 0.289 | 0.249 | 0.284 | 0.300 | 0.316 | 1.269 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | C | F(-2) | F(-1) | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 75 | 0 | 78 | 0 | 0 | 0 | 96 |
| N.S. | 1 | 1.00 | 0.77 | 0.00 | 0.80 | 0.00 | 0.00 | 0.00 | 0.98 |
| time (sec) | N/A | 0.035 | 0.011 | 0.000 | 0.203 | 0.000 | 0.000 | 0.000 | 1.320 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | C | F(-2) | A | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 63 | 0 | 65 | 0 | 105 | 0 | 82 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.88 | 0.00 | 1.42 | 0.00 | 1.11 |
| time (sec) | N/A | 0.027 | 0.009 | 0.000 | 0.207 | 0.000 | 55.014 | 0.000 | 1.274 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | C | F(-2) | B | C | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 50 | 0 | 49 | 0 | 90 | 52 | 46 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.98 | 0.00 | 1.80 | 1.04 | 0.92 |
| time (sec) | N/A | 0.021 | 0.007 | 0.000 | 0.193 | 0.000 | 0.902 | 0.307 | 1.281 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | C | F(-2) | B | C | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 0 | 25 | 0 | 63 | 25 | 45 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 1.00 | 0.00 | 2.52 | 1.00 | 1.80 |
| time (sec) | N/A | 0.016 | 0.003 | 0.000 | 0.192 | 0.000 | 0.960 | 0.354 | 1.263 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F(-2) | B | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 58 | 0 | 45 | 0 | 92 | 0 | 67 |
| N.S. | 1 | 1.00 | 1.18 | 0.00 | 0.92 | 0.00 | 1.88 | 0.00 | 1.37 |
| time (sec) | N/A | 0.023 | 0.068 | 0.000 | 0.239 | 0.000 | 13.486 | 0.000 | 1.290 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F(-2) | F(-1) | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 72 | 0 | 45 | 0 | 0 | 0 | 113 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.58 | 0.00 | 0.00 | 0.00 | 1.47 |
| time (sec) | N/A | 0.027 | 0.063 | 0.000 | 0.242 | 0.000 | 0.000 | 0.000 | 1.309 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F(-2) | F(-1) | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 85 | 0 | 45 | 0 | 0 | 0 | 140 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.45 | 0.00 | 0.00 | 0.00 | 1.39 |
| time (sec) | N/A | 0.036 | 0.066 | 0.000 | 0.251 | 0.000 | 0.000 | 0.000 | 1.326 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | C | A | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 45 | 0 | 53 | 27 | 54 | 0 | 45 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 1.18 | 0.60 | 1.20 | 0.00 | 1.00 |
| time (sec) | N/A | 0.021 | 0.023 | 0.000 | 0.086 | 0.096 | 3.005 | 0.000 | 1.264 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 112 | 525 | 500 | 614 | 495 | 758 | 275 |
| N.S. | 1 | 1.00 | 0.85 | 4.01 | 3.82 | 4.69 | 3.78 | 5.79 | 2.10 |
| time (sec) | N/A | 0.052 | 0.036 | 1.359 | 0.210 | 0.297 | 0.903 | 0.339 | 1.433 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 85 | 322 | 282 | 324 | 294 | 399 | 172 |
| N.S. | 1 | 1.00 | 0.86 | 3.25 | 2.85 | 3.27 | 2.97 | 4.03 | 1.74 |
| time (sec) | N/A | 0.041 | 0.010 | 0.616 | 0.218 | 0.289 | 0.499 | 0.307 | 1.377 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 59 | 111 | 131 | 140 | 146 | 174 | 94 |
| N.S. | 1 | 1.00 | 0.91 | 1.71 | 2.02 | 2.15 | 2.25 | 2.68 | 1.45 |
| time (sec) | N/A | 0.026 | 0.007 | 0.348 | 0.198 | 0.297 | 0.265 | 0.299 | 1.297 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 29 | 36 | 40 | 40 | 41 | 45 | 35 |
| N.S. | 1 | 1.00 | 1.00 | 1.24 | 1.38 | 1.38 | 1.41 | 1.55 | 1.21 |
| time (sec) | N/A | 0.011 | 0.005 | 0.221 | 0.195 | 0.295 | 0.143 | 0.323 | 1.286 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 63 | 309 | 0 | 46 | 0 | 49 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 4.90 | 0.00 | 0.73 | 0.00 | 0.78 | 0.00 |
| time (sec) | N/A | 0.039 | 0.070 | 0.868 | 0.000 | 0.268 | 0.000 | 0.297 | 0.000 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 123 | 456 | 0 | 117 | 0 | 286 | 0 |
| N.S. | 1 | 1.00 | 1.28 | 4.75 | 0.00 | 1.22 | 0.00 | 2.98 | 0.00 |
| time (sec) | N/A | 0.045 | 0.044 | 0.979 | 0.000 | 0.288 | 0.000 | 0.388 | 0.000 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 135 | 144 | 734 | 0 | 263 | 0 | 1218 | 0 |
| N.S. | 1 | 1.00 | 1.07 | 5.44 | 0.00 | 1.95 | 0.00 | 9.02 | 0.00 |
| time (sec) | N/A | 0.062 | 0.061 | 1.144 | 0.000 | 0.298 | 0.000 | 0.412 | 0.000 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 152 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.102 | 0.166 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 127 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.077 | 0.038 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 106 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.062 | 0.033 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 80 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.049 | 0.013 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 139 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.070 | 0.090 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 163 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.084 | 0.118 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 192 | 203 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.108 | 0.145 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 103 | 0 | 0 | 59 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.57 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.043 | 0.077 | 0.000 | 0.000 | 0.115 | 0.000 | 0.000 | 0.000 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 88 | 0 | 59 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.67 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.047 | 0.057 | 0.000 | 0.053 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 26 | 0 | 22 | 0 | 22 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 1.30 | 0.00 | 1.10 | 0.00 | 1.10 | 1.00 |
| time (sec) | N/A | 0.034 | 0.023 | 1.503 | 0.000 | 0.322 | 0.000 | 0.324 | 1.249 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 42 | 0 | 0 | 27 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.64 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.053 | 0.099 | 0.000 | 0.000 | 0.339 | 0.000 | 0.000 | 0.000 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 178 | 315 | 589 | 393 | 471 | 568 | 1209 | 526 |
| N.S. | 1 | 1.00 | 1.77 | 3.31 | 2.21 | 2.65 | 3.19 | 6.79 | 2.96 |
| time (sec) | N/A | 0.071 | 0.203 | 1.615 | 0.211 | 0.317 | 2.168 | 0.321 | 1.469 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 226 | 428 | 284 | 340 | 410 | 770 | 352 |
| N.S. | 1 | 1.00 | 1.52 | 2.87 | 1.91 | 2.28 | 2.75 | 5.17 | 2.36 |
| time (sec) | N/A | 0.049 | 0.139 | 1.074 | 0.198 | 0.293 | 1.164 | 0.322 | 0.824 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 120 | 150 | 265 | 187 | 221 | 252 | 424 | 212 |
| N.S. | 1 | 1.00 | 1.25 | 2.21 | 1.56 | 1.84 | 2.10 | 3.53 | 1.77 |
| time (sec) | N/A | 0.038 | 0.099 | 0.853 | 0.214 | 0.286 | 0.665 | 0.314 | 0.840 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 91 | 95 | 98 | 102 | 119 | 134 | 183 | 104 |
| N.S. | 1 | 1.00 | 1.04 | 1.08 | 1.12 | 1.31 | 1.47 | 2.01 | 1.14 |
| time (sec) | N/A | 0.027 | 0.033 | 0.352 | 0.192 | 0.284 | 0.388 | 0.314 | 0.761 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 29 | 36 | 40 | 40 | 41 | 45 | 35 |
| N.S. | 1 | 1.00 | 1.00 | 1.24 | 1.38 | 1.38 | 1.41 | 1.55 | 1.21 |
| time (sec) | N/A | 0.011 | 0.005 | 0.061 | 0.200 | 0.297 | 0.150 | 0.311 | 0.001 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 62 | 217 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 3.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.039 | 0.011 | 0.702 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 57 | 127 | 85 | 95 | 333 | 88 | 84 |
| N.S. | 1 | 1.00 | 0.77 | 1.72 | 1.15 | 1.28 | 4.50 | 1.19 | 1.14 |
| time (sec) | N/A | 0.023 | 0.046 | 0.810 | 0.187 | 0.299 | 3.116 | 0.301 | 1.325 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 83 | 283 | 167 | 274 | 1945 | 201 | 173 |
| N.S. | 1 | 1.00 | 0.74 | 2.53 | 1.49 | 2.45 | 17.37 | 1.79 | 1.54 |
| time (sec) | N/A | 0.050 | 0.066 | 1.076 | 0.199 | 0.303 | 13.160 | 0.333 | 1.017 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 141 | 110 | 455 | 301 | 507 | 0 | 401 | 283 |
| N.S. | 1 | 1.00 | 0.78 | 3.23 | 2.13 | 3.60 | 0.00 | 2.84 | 2.01 |
| time (sec) | N/A | 0.058 | 0.090 | 1.608 | 0.210 | 0.322 | 0.000 | 0.313 | 1.190 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 365 | 365 | 360 | 1343 | 827 | 1190 | 1241 | 2345 | 1051 |
| N.S. | 1 | 1.00 | 0.99 | 3.68 | 2.27 | 3.26 | 3.40 | 6.42 | 2.88 |
| time (sec) | N/A | 0.340 | 0.174 | 2.201 | 0.212 | 0.301 | 2.349 | 0.345 | 1.121 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 287 | 247 | 850 | 554 | 760 | 774 | 1315 | 591 |
| N.S. | 1 | 1.00 | 0.86 | 2.96 | 1.93 | 2.65 | 2.70 | 4.58 | 2.06 |
| time (sec) | N/A | 0.271 | 0.098 | 1.477 | 0.213 | 0.300 | 1.226 | 0.407 | 0.962 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 186 | 144 | 459 | 314 | 401 | 394 | 583 | 268 |
| N.S. | 1 | 1.00 | 0.77 | 2.47 | 1.69 | 2.16 | 2.12 | 3.13 | 1.44 |
| time (sec) | N/A | 0.114 | 0.052 | 0.612 | 0.202 | 0.284 | 0.690 | 0.384 | 0.840 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 59 | 111 | 131 | 140 | 146 | 174 | 94 |
| N.S. | 1 | 1.00 | 0.91 | 1.71 | 2.02 | 2.15 | 2.25 | 2.68 | 1.45 |
| time (sec) | N/A | 0.029 | 0.006 | 0.144 | 0.203 | 0.276 | 0.265 | 0.357 | 0.001 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 194 | 737 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.75 | 6.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.086 | 0.111 | 1.270 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 126 | 537 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 4.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.065 | 0.057 | 0.907 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 202 | 202 | 204 | 661 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 3.27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.224 | 0.123 | 1.346 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 317 | 317 | 302 | 755 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 2.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.366 | 0.214 | 2.187 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 598 | 598 | 475 | 2719 | 1687 | 2802 | 2594 | 5182 | 2133 |
| N.S. | 1 | 1.00 | 0.79 | 4.55 | 2.82 | 4.69 | 4.34 | 8.67 | 3.57 |
| time (sec) | N/A | 0.377 | 0.266 | 5.422 | 0.261 | 0.376 | 4.445 | 0.414 | 2.359 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 432 | 432 | 333 | 1733 | 1140 | 1771 | 1578 | 2932 | 1157 |
| N.S. | 1 | 1.00 | 0.77 | 4.01 | 2.64 | 4.10 | 3.65 | 6.79 | 2.68 |
| time (sec) | N/A | 0.263 | 0.149 | 3.551 | 0.236 | 0.341 | 2.407 | 0.378 | 1.969 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 265 | 265 | 201 | 906 | 662 | 923 | 836 | 1321 | 511 |
| N.S. | 1 | 1.00 | 0.76 | 3.42 | 2.50 | 3.48 | 3.15 | 4.98 | 1.93 |
| time (sec) | N/A | 0.149 | 0.080 | 1.740 | 0.221 | 0.311 | 1.270 | 0.326 | 1.647 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 85 | 322 | 282 | 324 | 294 | 399 | 172 |
| N.S. | 1 | 1.00 | 0.86 | 3.25 | 2.85 | 3.27 | 2.97 | 4.03 | 1.74 |
| time (sec) | N/A | 0.038 | 0.011 | 0.257 | 0.201 | 0.305 | 0.499 | 0.328 | 0.002 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 335 | 1396 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.12 | 8.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.122 | 0.184 | 1.988 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 190 | 190 | 410 | 1268 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.16 | 6.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.109 | 0.246 | 2.022 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 342 | 342 | 620 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.380 | 0.474 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 564 | 564 | 843 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.49 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.694 | 0.705 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 340 | 340 | 258 | 1493 | 1163 | 1756 | 1372 | 2488 | 823 |
| N.S. | 1 | 1.00 | 0.76 | 4.39 | 3.42 | 5.16 | 4.04 | 7.32 | 2.42 |
| time (sec) | N/A | 0.196 | 0.143 | 3.524 | 0.240 | 0.309 | 2.313 | 0.347 | 1.816 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 112 | 525 | 500 | 614 | 495 | 758 | 275 |
| N.S. | 1 | 1.00 | 0.85 | 4.01 | 3.82 | 4.69 | 3.78 | 5.79 | 2.10 |
| time (sec) | N/A | 0.051 | 0.015 | 0.589 | 0.219 | 0.301 | 0.900 | 0.326 | 0.003 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 205 | 503 | 2172 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.45 | 10.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.159 | 0.175 | 3.017 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 248 | 248 | 531 | 2156 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.14 | 8.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.164 | 0.457 | 3.224 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 19 | 24 | 23 | 22 | 24 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 1.26 | 1.21 | 1.16 | 1.26 | 1.21 | 1.21 |
| time (sec) | N/A | 0.004 | 0.005 | 0.076 | 0.191 | 0.275 | 0.067 | 0.316 | 0.070 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 37 | 36 | 40 | 27 | 36 | 42 | 44 | 48 |
| N.S. | 1 | 1.00 | 0.97 | 1.08 | 0.73 | 0.97 | 1.14 | 1.19 | 1.30 |
| time (sec) | N/A | 0.010 | 0.002 | 0.117 | 0.196 | 0.260 | 0.085 | 0.310 | 1.433 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 51 | 55 | 37 | 51 | 63 | 62 | 73 |
| N.S. | 1 | 1.00 | 0.93 | 1.00 | 0.67 | 0.93 | 1.15 | 1.13 | 1.33 |
| time (sec) | N/A | 0.013 | 0.007 | 0.120 | 0.190 | 0.274 | 0.104 | 0.330 | 1.419 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 32 | 34 | 30 | 36 | 34 | 31 |
| N.S. | 1 | 1.00 | 1.00 | 1.28 | 1.36 | 1.20 | 1.44 | 1.36 | 1.24 |
| time (sec) | N/A | 0.011 | 0.007 | 0.080 | 0.191 | 0.276 | 0.107 | 0.319 | 0.083 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 48 | 52 | 38 | 48 | 63 | 65 | 94 |
| N.S. | 1 | 1.00 | 0.98 | 1.06 | 0.78 | 0.98 | 1.29 | 1.33 | 1.92 |
| time (sec) | N/A | 0.018 | 0.006 | 0.139 | 0.192 | 0.269 | 0.135 | 0.333 | 1.488 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 67 | 71 | 51 | 67 | 95 | 91 | 138 |
| N.S. | 1 | 1.00 | 0.92 | 0.97 | 0.70 | 0.92 | 1.30 | 1.25 | 1.89 |
| time (sec) | N/A | 0.027 | 0.007 | 0.143 | 0.196 | 0.281 | 0.158 | 0.315 | 1.484 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 24 | 32 | 35 | 32 | 36 | 39 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 1.33 | 1.46 | 1.33 | 1.50 | 1.62 | 1.21 |
| time (sec) | N/A | 0.006 | 0.006 | 0.226 | 0.197 | 0.285 | 0.141 | 0.323 | 0.070 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 24 | 54 | 102 | 27 | 0 | 0 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 2.25 | 4.25 | 1.12 | 0.00 | 0.00 | 0.96 |
| time (sec) | N/A | 0.018 | 0.006 | 1.079 | 0.204 | 0.279 | 0.000 | 0.000 | 1.513 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | C | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 16 | 0 | 14 | 17 | 0 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 0.00 | 0.93 | 1.13 | 0.00 | 1.00 |
| time (sec) | N/A | 0.014 | 0.004 | 0.173 | 0.000 | 0.284 | 3.014 | 0.000 | 0.082 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | C | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 33 | 20 | 0 | 87 | 0 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 2.06 | 1.25 | 0.00 | 5.44 | 0.00 | 1.12 |
| time (sec) | N/A | 0.014 | 0.003 | 0.099 | 0.210 | 0.000 | 1.483 | 0.000 | 0.035 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | C | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 33 | 20 | 0 | 87 | 0 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 2.06 | 1.25 | 0.00 | 5.44 | 0.00 | 1.12 |
| time (sec) | N/A | 0.011 | 0.003 | 0.107 | 0.189 | 0.000 | 1.472 | 0.000 | 0.031 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | C | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 8 | 8 | 8 | 9 | 19 | 7 | 10 | 0 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 1.12 | 2.38 | 0.88 | 1.25 | 0.00 | 2.25 |
| time (sec) | N/A | 0.005 | 0.003 | 0.096 | 0.195 | 0.303 | 1.341 | 0.000 | 0.031 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 10 | 9 | 8 | 8 | 7 | 8 | 8 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.80 | 0.80 | 0.70 | 0.80 | 0.80 |
| time (sec) | N/A | 0.005 | 0.002 | 0.132 | 0.195 | 0.272 | 0.041 | 0.304 | 1.308 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | C | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 17 | 19 | 0 | 60 | 0 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 0.95 | 0.00 | 3.00 | 0.00 | 0.80 |
| time (sec) | N/A | 0.014 | 0.003 | 0.095 | 0.197 | 0.000 | 1.942 | 0.000 | 0.030 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | C | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 19 | 20 | 0 | 102 | 0 | 18 |
| N.S. | 1 | 1.00 | 1.08 | 0.76 | 0.80 | 0.00 | 4.08 | 0.00 | 0.72 |
| time (sec) | N/A | 0.015 | 0.003 | 0.117 | 0.197 | 0.000 | 1.967 | 0.000 | 0.033 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 22 | 40 | 27 | 0 | 94 | 0 | 25 |
| N.S. | 1 | 1.00 | 1.05 | 1.90 | 1.29 | 0.00 | 4.48 | 0.00 | 1.19 |
| time (sec) | N/A | 0.016 | 0.003 | 0.207 | 0.224 | 0.000 | 1.992 | 0.000 | 1.235 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 22 | 40 | 27 | 0 | 94 | 0 | 25 |
| N.S. | 1 | 1.00 | 1.05 | 1.90 | 1.29 | 0.00 | 4.48 | 0.00 | 1.19 |
| time (sec) | N/A | 0.016 | 0.003 | 0.174 | 0.225 | 0.000 | 2.013 | 0.000 | 0.075 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 15 | 26 | 13 | 15 | 0 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.86 | 0.93 | 1.07 | 0.00 | 1.00 |
| time (sec) | N/A | 0.015 | 0.003 | 0.179 | 0.232 | 0.285 | 1.844 | 0.000 | 0.059 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 16 | 15 | 15 | 16 | 14 | 14 | 14 |
| N.S. | 1 | 1.00 | 0.94 | 0.88 | 0.88 | 0.94 | 0.82 | 0.82 | 0.82 |
| time (sec) | N/A | 0.008 | 0.002 | 0.105 | 0.189 | 0.279 | 0.048 | 0.312 | 1.196 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 27 | 24 | 26 | 0 | 66 | 0 | 23 |
| N.S. | 1 | 1.00 | 1.04 | 0.92 | 1.00 | 0.00 | 2.54 | 0.00 | 0.88 |
| time (sec) | N/A | 0.016 | 0.003 | 0.217 | 0.223 | 0.000 | 2.717 | 0.000 | 1.293 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 34 | 26 | 27 | 0 | 109 | 0 | 25 |
| N.S. | 1 | 1.00 | 1.10 | 0.84 | 0.87 | 0.00 | 3.52 | 0.00 | 0.81 |
| time (sec) | N/A | 0.016 | 0.003 | 0.181 | 0.227 | 0.000 | 2.857 | 0.000 | 1.263 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 187 | 163 | 172 | 131 | 179 | 173 | 342 | 116 |
| N.S. | 1 | 1.00 | 0.87 | 0.92 | 0.70 | 0.96 | 0.93 | 1.83 | 0.62 |
| time (sec) | N/A | 0.122 | 0.037 | 0.444 | 0.198 | 0.264 | 0.765 | 0.305 | 1.354 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 177 | 172 | 483 | 150 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 2.73 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.186 | 0.061 | 0.385 | 0.206 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 285 | 285 | 260 | 261 | 215 | 341 | 269 | 626 | 172 |
| N.S. | 1 | 1.00 | 0.91 | 0.92 | 0.75 | 1.20 | 0.94 | 2.20 | 0.60 |
| time (sec) | N/A | 0.143 | 0.052 | 0.831 | 0.211 | 0.311 | 1.374 | 0.319 | 1.361 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 266 | 9346 | 0 | 305 | 0 | 582 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 31.26 | 0.00 | 1.02 | 0.00 | 1.95 | 0.00 |
| time (sec) | N/A | 0.304 | 0.625 | 1.542 | 0.000 | 0.292 | 0.000 | 0.349 | 0.000 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 219 | 197 | 1889 | 0 | 192 | 0 | 337 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 8.63 | 0.00 | 0.88 | 0.00 | 1.54 | 0.00 |
| time (sec) | N/A | 0.203 | 0.168 | 1.340 | 0.000 | 0.303 | 0.000 | 0.341 | 0.000 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 126 | 937 | 0 | 105 | 0 | 159 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 6.74 | 0.00 | 0.76 | 0.00 | 1.14 | 0.00 |
| time (sec) | N/A | 0.115 | 0.068 | 0.962 | 0.000 | 0.277 | 0.000 | 0.328 | 0.000 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 63 | 309 | 0 | 46 | 0 | 49 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 4.90 | 0.00 | 0.73 | 0.00 | 0.78 | 0.00 |
| time (sec) | N/A | 0.035 | 0.010 | 0.098 | 0.000 | 0.278 | 0.000 | 0.318 | 0.000 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 26 | 31 | 20 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.29 | 0.83 | 1.08 | 1.08 |
| time (sec) | N/A | 0.025 | 0.160 | 0.045 | 0.263 | 0.284 | 1.183 | 0.326 | 1.208 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 26 | 55 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 2.29 | 0.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.024 | 0.354 | 0.013 | 0.266 | 0.317 | 0.000 | 0.320 | 1.237 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | F | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 339 | 339 | 1674 | 9517 | 0 | 681 | 0 | 3473 | 0 |
| N.S. | 1 | 1.00 | 4.94 | 28.07 | 0.00 | 2.01 | 0.00 | 10.24 | 0.00 |
| time (sec) | N/A | 0.533 | 0.463 | 1.702 | 0.000 | 0.317 | 0.000 | 0.419 | 0.000 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | F | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 259 | 259 | 1015 | 5089 | 0 | 433 | 0 | 2031 | 0 |
| N.S. | 1 | 1.00 | 3.92 | 19.65 | 0.00 | 1.67 | 0.00 | 7.84 | 0.00 |
| time (sec) | N/A | 0.350 | 0.251 | 1.273 | 0.000 | 0.307 | 0.000 | 0.385 | 0.000 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 177 | 208 | 2300 | 0 | 239 | 0 | 973 | 0 |
| N.S. | 1 | 1.00 | 1.18 | 12.99 | 0.00 | 1.35 | 0.00 | 5.50 | 0.00 |
| time (sec) | N/A | 0.171 | 0.138 | 1.008 | 0.000 | 0.282 | 0.000 | 0.327 | 0.000 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 123 | 456 | 0 | 117 | 0 | 286 | 0 |
| N.S. | 1 | 1.00 | 1.28 | 4.75 | 0.00 | 1.22 | 0.00 | 2.98 | 0.00 |
| time (sec) | N/A | 0.045 | 0.030 | 0.223 | 0.000 | 0.263 | 0.000 | 0.323 | 0.000 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 188 | 63 | 22 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 7.83 | 2.62 | 0.92 | 1.08 | 1.08 |
| time (sec) | N/A | 0.023 | 0.232 | 0.015 | 0.278 | 0.297 | 2.767 | 0.370 | 1.283 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 289 | 104 | 24 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 12.04 | 4.33 | 1.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.022 | 2.107 | 0.033 | 0.290 | 0.292 | 11.143 | 0.338 | 1.299 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 351 | 6545 | 0 | 1090 | 0 | 8422 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 18.65 | 0.00 | 3.11 | 0.00 | 23.99 | 0.00 |
| time (sec) | N/A | 0.592 | 0.794 | 1.727 | 0.000 | 0.311 | 0.000 | 0.491 | 0.000 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 261 | 261 | 256 | 3114 | 0 | 588 | 0 | 4112 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 11.93 | 0.00 | 2.25 | 0.00 | 15.75 | 0.00 |
| time (sec) | N/A | 0.248 | 0.211 | 1.247 | 0.000 | 0.317 | 0.000 | 0.425 | 0.000 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 135 | 144 | 734 | 0 | 263 | 0 | 1218 | 0 |
| N.S. | 1 | 1.00 | 1.07 | 5.44 | 0.00 | 1.95 | 0.00 | 9.02 | 0.00 |
| time (sec) | N/A | 0.056 | 0.056 | 0.451 | 0.000 | 0.312 | 0.000 | 0.333 | 0.000 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 703 | 95 | 22 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 29.29 | 3.96 | 0.92 | 1.08 | 1.08 |
| time (sec) | N/A | 0.022 | 0.344 | 0.023 | 0.299 | 0.296 | 6.422 | 0.328 | 1.278 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 934 | 153 | 24 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 38.92 | 6.38 | 1.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.023 | 2.719 | 0.026 | 0.302 | 0.313 | 35.271 | 0.346 | 1.292 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 404 | 404 | 374 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.494 | 0.398 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 255 | 255 | 235 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.240 | 0.207 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 106 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.062 | 0.028 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 0 | 22 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.85 | 1.00 | 1.00 |
| time (sec) | N/A | 0.031 | 3.579 | 0.119 | 0.586 | 0.000 | 0.537 | 0.354 | 1.266 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 0 | 24 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.92 | 1.00 | 1.00 |
| time (sec) | N/A | 0.068 | 0.300 | 0.146 | 0.595 | 0.000 | 1.871 | 0.339 | 1.381 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 0 | 24 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.92 | 1.00 | 1.00 |
| time (sec) | N/A | 0.145 | 0.317 | 0.163 | 0.598 | 0.000 | 8.446 | 0.357 | 1.371 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 526 | 526 | 446 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.581 | 0.789 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 330 | 330 | 282 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.297 | 0.309 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 127 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.074 | 0.093 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 0 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.040 | 2.519 | 0.121 | 0.613 | 0.000 | 0.000 | 0.463 | 1.363 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 0 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.071 | 0.534 | 0.124 | 0.626 | 0.000 | 0.000 | 0.386 | 1.414 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 0 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.145 | 0.532 | 0.122 | 0.654 | 0.000 | 0.000 | 0.424 | 1.473 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 660 | 660 | 511 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.719 | 1.247 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 413 | 413 | 326 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.363 | 0.496 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 152 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.092 | 0.117 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 0 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.041 | 2.293 | 0.118 | 0.618 | 0.000 | 0.000 | 0.581 | 1.339 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 0 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.079 | 7.919 | 0.117 | 0.646 | 0.000 | 0.000 | 0.448 | 1.405 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 0 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.156 | 5.384 | 0.115 | 0.666 | 0.000 | 0.000 | 0.886 | 1.487 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 383 | 383 | 331 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.515 | 0.315 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 283 | 283 | 252 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.366 | 0.178 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 164 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.189 | 0.107 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 80 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.050 | 0.009 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 0 | 24 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.92 | 1.00 | 1.00 |
| time (sec) | N/A | 0.035 | 0.067 | 0.150 | 0.601 | 0.000 | 1.266 | 0.375 | 1.367 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 422 | 422 | 1281 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 3.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.917 | 1.696 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 325 | 325 | 828 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.614 | 0.696 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 220 | 338 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.54 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.278 | 0.492 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 139 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.067 | 0.059 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 0 | 24 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.92 | 1.00 | 1.00 |
| time (sec) | N/A | 0.041 | 0.205 | 0.124 | 0.620 | 0.000 | 5.792 | 0.459 | 1.539 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 520 | 520 | 1523 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.625 | 6.118 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 421 | 421 | 527 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.977 | 2.637 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 311 | 311 | 353 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.14 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.397 | 1.110 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 163 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.078 | 0.120 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 0 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.039 | 0.416 | 0.145 | 0.635 | 0.000 | 0.000 | 0.577 | 1.691 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 163 | 137 | 0 | 0 | 538 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.00 | 3.30 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.107 | 0.144 | 0.000 | 0.000 | 0.328 | 0.000 | 0.000 | 0.000 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 118 | 0 | 0 | 311 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 2.36 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.056 | 0.090 | 0.000 | 0.000 | 0.341 | 0.000 | 0.000 | 0.000 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 83 | 113 | 0 | 185 | 0 | 110 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.16 | 0.00 | 1.91 | 0.00 | 1.13 | 0.00 |
| time (sec) | N/A | 0.039 | 0.056 | 0.862 | 0.000 | 0.318 | 0.000 | 0.319 | 0.000 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 80 | 0 | 0 | 224 | 0 | 101 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 0.00 | 0.00 | 2.77 | 0.00 | 1.25 | 0.00 |
| time (sec) | N/A | 0.037 | 0.110 | 0.000 | 0.000 | 0.326 | 0.000 | 0.334 | 0.000 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F | A | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 114 | 85 | 0 | 0 | 425 | 0 | 164 | 0 |
| N.S. | 1 | 1.00 | 0.75 | 0.00 | 0.00 | 3.73 | 0.00 | 1.44 | 0.00 |
| time (sec) | N/A | 0.051 | 0.031 | 0.000 | 0.000 | 0.357 | 0.000 | 0.331 | 0.000 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 78 | 0 | 0 | 789 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.54 | 0.00 | 0.00 | 5.44 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.064 | 0.033 | 0.000 | 0.000 | 0.362 | 0.000 | 0.000 | 0.000 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 176 | 78 | 0 | 0 | 1252 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.44 | 0.00 | 0.00 | 7.11 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.092 | 0.035 | 0.000 | 0.000 | 0.432 | 0.000 | 0.000 | 0.000 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 590 | 590 | 854 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.411 | 1.213 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 510 | 510 | 680 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.909 | 0.756 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 418 | 418 | 566 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.35 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.700 | 0.687 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 312 | 312 | 424 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.36 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.512 | 0.422 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 423 | 423 | 557 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.713 | 0.792 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 503 | 503 | 639 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.920 | 1.216 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 583 | 583 | 728 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.189 | 2.215 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 204 | 26 | 22 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 7.85 | 1.00 | 0.85 | 1.00 | 1.00 |
| time (sec) | N/A | 0.028 | 0.439 | 0.158 | 0.315 | 0.291 | 21.145 | 0.375 | 1.223 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 174 | 26 | 22 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 6.69 | 1.00 | 0.85 | 1.00 | 1.00 |
| time (sec) | N/A | 0.025 | 0.126 | 0.151 | 0.311 | 0.271 | 0.700 | 0.329 | 1.258 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 174 | 39 | 24 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 6.69 | 1.50 | 0.92 | 1.00 | 1.00 |
| time (sec) | N/A | 0.030 | 0.434 | 0.129 | 0.314 | 0.294 | 1.845 | 0.394 | 1.210 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 166 | 63 | 24 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 6.38 | 2.42 | 0.92 | 1.00 | 1.00 |
| time (sec) | N/A | 0.030 | 0.715 | 0.119 | 0.317 | 0.296 | 7.008 | 0.383 | 1.336 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 24 | 26 | 0 | 26 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.07 | 0.86 | 0.93 | 0.00 | 0.93 | 0.93 | 0.93 |
| time (sec) | N/A | 0.171 | 0.867 | 0.144 | 0.649 | 0.000 | 2.660 | 0.727 | 1.349 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 24 | 26 | 0 | 26 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.07 | 0.86 | 0.93 | 0.00 | 0.93 | 0.93 | 0.93 |
| time (sec) | N/A | 0.167 | 0.881 | 0.147 | 0.656 | 0.000 | 1.236 | 1.638 | 1.342 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 24 | 26 | 0 | 26 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.07 | 0.86 | 0.93 | 0.00 | 0.93 | 0.93 | 0.93 |
| time (sec) | N/A | 0.168 | 1.492 | 0.122 | 0.692 | 0.000 | 9.710 | 1.013 | 1.358 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 24 | 26 | 0 | 26 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.07 | 0.86 | 0.93 | 0.00 | 0.93 | 0.93 | 0.93 |
| time (sec) | N/A | 0.036 | 3.035 | 0.123 | 0.648 | 0.000 | 0.979 | 0.344 | 1.340 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 24 | 26 | 0 | 27 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.07 | 0.86 | 0.93 | 0.00 | 0.96 | 0.93 | 0.93 |
| time (sec) | N/A | 0.037 | 0.145 | 0.122 | 0.652 | 0.000 | 2.830 | 0.370 | 1.366 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 24 | 26 | 0 | 27 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.07 | 0.86 | 0.93 | 0.00 | 0.96 | 0.93 | 0.93 |
| time (sec) | N/A | 0.038 | 0.074 | 0.122 | 0.666 | 0.000 | 26.944 | 0.434 | 1.485 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 81 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.037 | 0.045 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 26 | 26 | 20 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.08 | 0.83 | 1.08 | 1.08 |
| time (sec) | N/A | 0.019 | 0.208 | 0.121 | 0.291 | 0.308 | 31.020 | 0.339 | 1.204 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 131 | 46 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 5.46 | 1.92 | 0.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.019 | 5.811 | 0.006 | 0.313 | 0.302 | 0.000 | 0.416 | 1.307 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 48 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.85 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.037 | 9.498 | 0.148 | 0.672 | 0.302 | 0.000 | 1.856 | 1.409 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 26 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.032 | 0.047 | 0.129 | 0.659 | 0.307 | 0.000 | 0.780 | 1.314 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 26 | 24 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.92 | 1.00 | 1.00 |
| time (sec) | N/A | 0.032 | 7.629 | 0.125 | 0.630 | 0.315 | 4.596 | 0.387 | 1.353 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 62 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.38 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.037 | 8.524 | 0.127 | 0.655 | 0.363 | 0.000 | 0.447 | 1.628 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-2) | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 0 | 26 | 0 | 0 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 0.00 | 1.08 | 0.00 | 0.00 | 1.08 |
| time (sec) | N/A | 0.018 | 0.256 | 0.151 | 0.000 | 0.343 | 0.000 | 0.000 | 1.285 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 474 | 474 | 343 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.383 | 0.991 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 348 | 348 | 262 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.261 | 0.244 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 225 | 181 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.145 | 0.093 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 103 | 0 | 0 | 60 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.58 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.039 | 0.043 | 0.000 | 0.000 | 0.105 | 0.000 | 0.000 | 0.000 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 0 | 26 | 20 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 0.00 | 1.08 | 0.83 | 1.08 | 1.08 |
| time (sec) | N/A | 0.020 | 0.147 | 0.127 | 0.000 | 0.285 | 1.873 | 0.435 | 1.284 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 315 | 315 | 589 | 554 | 757 | 478 | 682 | 572 | 661 |
| N.S. | 1 | 1.00 | 1.87 | 1.76 | 2.40 | 1.52 | 2.17 | 1.82 | 2.10 |
| time (sec) | N/A | 0.333 | 0.338 | 0.895 | 0.229 | 0.281 | 0.844 | 0.508 | 1.627 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 244 | 244 | 375 | 356 | 539 | 308 | 427 | 369 | 393 |
| N.S. | 1 | 1.00 | 1.54 | 1.46 | 2.21 | 1.26 | 1.75 | 1.51 | 1.61 |
| time (sec) | N/A | 0.275 | 0.194 | 0.649 | 0.233 | 0.276 | 0.589 | 0.307 | 1.543 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 157 | 214 | 202 | 351 | 170 | 226 | 209 | 208 |
| N.S. | 1 | 1.00 | 1.36 | 1.29 | 2.24 | 1.08 | 1.44 | 1.33 | 1.32 |
| time (sec) | N/A | 0.186 | 0.102 | 0.648 | 0.227 | 0.303 | 0.402 | 0.313 | 1.430 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 66 | 92 | 201 | 71 | 85 | 92 | 100 |
| N.S. | 1 | 1.00 | 0.84 | 1.16 | 2.54 | 0.90 | 1.08 | 1.16 | 1.27 |
| time (sec) | N/A | 0.091 | 0.038 | 0.566 | 0.233 | 0.301 | 0.217 | 0.298 | 1.746 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 27 | 35 | 101 | 34 | 31 | 35 | 31 |
| N.S. | 1 | 1.00 | 1.00 | 1.30 | 3.74 | 1.26 | 1.15 | 1.30 | 1.15 |
| time (sec) | N/A | 0.025 | 0.004 | 0.504 | 0.213 | 0.298 | 0.067 | 0.306 | 1.413 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 91 | 177 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 2.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.125 | 0.047 | 0.968 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 151 | 141 | 317 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 2.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.219 | 0.104 | 1.072 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 250 | 226 | 510 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 2.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.350 | 0.151 | 1.198 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 579 | 579 | 374 | 1135 | 1427 | 939 | 1479 | 1213 | 1346 |
| N.S. | 1 | 1.00 | 0.65 | 1.96 | 2.46 | 1.62 | 2.55 | 2.09 | 2.32 |
| time (sec) | N/A | 1.021 | 0.367 | 0.918 | 0.266 | 0.303 | 1.884 | 0.327 | 2.070 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 464 | 464 | 267 | 721 | 964 | 606 | 918 | 769 | 803 |
| N.S. | 1 | 1.00 | 0.58 | 1.55 | 2.08 | 1.31 | 1.98 | 1.66 | 1.73 |
| time (sec) | N/A | 0.654 | 0.208 | 0.679 | 0.275 | 0.298 | 1.100 | 0.309 | 1.826 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 238 | 238 | 171 | 397 | 586 | 336 | 473 | 420 | 408 |
| N.S. | 1 | 1.00 | 0.72 | 1.67 | 2.46 | 1.41 | 1.99 | 1.76 | 1.71 |
| time (sec) | N/A | 0.350 | 0.108 | 0.659 | 0.245 | 0.284 | 0.602 | 0.316 | 1.642 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 113 | 89 | 176 | 304 | 141 | 175 | 178 | 163 |
| N.S. | 1 | 1.00 | 0.79 | 1.56 | 2.69 | 1.25 | 1.55 | 1.58 | 1.44 |
| time (sec) | N/A | 0.135 | 0.043 | 0.575 | 0.228 | 0.279 | 0.338 | 0.315 | 1.503 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 27 | 54 | 128 | 53 | 51 | 59 | 50 |
| N.S. | 1 | 1.00 | 1.00 | 2.00 | 4.74 | 1.96 | 1.89 | 2.19 | 1.85 |
| time (sec) | N/A | 0.041 | 0.005 | 0.503 | 0.222 | 0.266 | 0.092 | 0.384 | 1.637 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 189 | 334 | 331 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.33 | 2.35 | 2.33 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.203 | 0.172 | 0.942 | 0.267 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 273 | 273 | 360 | 0 | 622 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.32 | 0.00 | 2.28 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.381 | 0.397 | 0.000 | 0.295 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 485 | 485 | 664 | 0 | 1271 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.37 | 0.00 | 2.62 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.660 | 0.680 | 0.000 | 0.381 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | B | F | A | F | F | F(-1) |
| verified | N/A | Yes | N/A | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 230 | 230 | 0 | 567 | 0 | 401 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 2.47 | 0.00 | 1.74 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.465 | 0.000 | 3.017 | 0.000 | 0.296 | 0.000 | 0.000 | 0.000 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | B | F | A | F | F | F(-1) |
| verified | N/A | Yes | N/A | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 177 | 0 | 361 | 0 | 260 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 2.04 | 0.00 | 1.47 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.323 | 0.000 | 2.457 | 0.000 | 0.305 | 0.000 | 0.000 | 0.000 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 112 | 200 | 0 | 149 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 1.61 | 0.00 | 1.20 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.247 | 0.238 | 2.124 | 0.000 | 0.313 | 0.000 | 0.000 | 0.000 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 69 | 88 | 0 | 70 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 1.24 | 0.00 | 0.99 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.145 | 0.112 | 1.548 | 0.000 | 0.300 | 0.000 | 0.000 | 0.000 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 23 | 24 | 29 | 24 | 17 | 24 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 1.04 | 1.26 | 1.04 | 0.74 | 1.04 | 1.00 |
| time (sec) | N/A | 0.047 | 0.038 | 0.524 | 0.458 | 0.290 | 0.087 | 0.309 | 2.048 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 34 | 32 | 34 | 68 | 99 | 34 | 34 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 1.06 | 2.12 | 3.09 | 1.06 | 1.06 |
| time (sec) | N/A | 0.159 | 0.126 | 0.822 | 0.254 | 0.276 | 2.611 | 0.368 | 1.330 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 34 | 32 | 34 | 122 | 180 | 34 | 34 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 1.06 | 3.81 | 5.62 | 1.06 | 1.06 |
| time (sec) | N/A | 0.203 | 0.160 | 0.618 | 0.248 | 0.282 | 4.979 | 0.370 | 1.493 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 485 | 485 | 818 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.143 | 0.723 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 417 | 417 | 644 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.54 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.895 | 0.423 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 349 | 349 | 534 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.53 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.658 | 0.332 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 256 | 256 | 281 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.456 | 0.234 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 340 | 340 | 526 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.676 | 0.383 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 406 | 406 | 608 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.905 | 0.509 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 381 | 381 | 556 | 416 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.46 | 1.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.032 | 0.593 | 1.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 323 | 323 | 534 | 256 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.65 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.614 | 0.232 | 0.899 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 242 | 242 | 369 | 155 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.52 | 0.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.419 | 0.146 | 1.023 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 316 | 316 | 547 | 243 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.73 | 0.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.617 | 0.250 | 0.901 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 372 | 372 | 568 | 370 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.53 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.822 | 0.395 | 0.928 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 34 | 32 | 34 | 35 | 0 | 0 | 34 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 1.06 | 1.09 | 0.00 | 0.00 | 1.06 |
| time (sec) | N/A | 0.073 | 0.172 | 0.236 | 0.254 | 0.312 | 0.000 | 0.000 | 1.630 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 305 | 305 | 242 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.469 | 1.113 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 210 | 210 | 185 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.324 | 0.375 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 114 | 0 | 0 | 118 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 0.00 | 0.00 | 1.03 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.192 | 0.233 | 0.000 | 0.000 | 0.100 | 0.000 | 0.000 | 0.000 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 31 | 32 | 32 | 41 | 0 | 32 | 31 |
| N.S. | 1 | 1.00 | 1.00 | 1.03 | 1.03 | 1.32 | 0.00 | 1.03 | 1.00 |
| time (sec) | N/A | 0.054 | 0.010 | 0.680 | 0.215 | 0.293 | 0.000 | 0.320 | 1.546 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 34 | 32 | 34 | 43 | 37 | 34 | 34 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 1.06 | 1.34 | 1.16 | 1.06 | 1.06 |
| time (sec) | N/A | 0.087 | 0.190 | 0.282 | 0.280 | 0.285 | 154.494 | 0.349 | 1.454 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 34 | 32 | 34 | 68 | 0 | 0 | 34 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 1.06 | 2.12 | 0.00 | 0.00 | 1.06 |
| time (sec) | N/A | 0.083 | 0.509 | 0.247 | 0.313 | 0.317 | 0.000 | 0.000 | 1.497 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 34 | 32 | 34 | 92 | 0 | 34 | 34 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 1.06 | 2.88 | 0.00 | 1.06 | 1.06 |
| time (sec) | N/A | 0.083 | 0.580 | 0.237 | 0.344 | 0.310 | 0.000 | 0.383 | 1.498 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 402 | 402 | 379 | 1208 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 3.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.253 | 0.352 | 2.030 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 241 | 241 | 224 | 721 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 2.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.159 | 0.172 | 1.244 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 110 | 394 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 3.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.097 | 0.073 | 0.778 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 62 | 217 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 3.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.034 | 0.003 | 0.566 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 155 | 111 | 378 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 2.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.140 | 0.046 | 1.118 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 252 | 252 | 196 | 494 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 1.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.181 | 0.115 | 1.724 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 402 | 402 | 311 | 771 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 1.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.272 | 0.202 | 3.169 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 469 | 469 | 876 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.424 | 0.328 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 215 | 215 | 460 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.14 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.191 | 0.239 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 194 | 737 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.75 | 6.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.078 | 0.042 | 0.732 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 264 | 264 | 353 | 1427 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.34 | 5.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.251 | 0.175 | 2.905 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 427 | 427 | 630 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.48 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.342 | 0.350 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 660 | 660 | 1521 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.499 | 0.511 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 308 | 308 | 799 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.263 | 0.254 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 335 | 1396 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.12 | 8.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.127 | 0.072 | 1.056 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 372 | 372 | 599 | 2696 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.61 | 7.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.359 | 0.256 | 5.395 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 602 | 602 | 1025 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.494 | 0.724 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 31 | 29 | 31 | 37 | 24 | 31 | 31 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 1.07 | 1.28 | 0.83 | 1.07 | 1.07 |
| time (sec) | N/A | 0.120 | 0.099 | 0.046 | 0.294 | 0.281 | 2.601 | 0.336 | 1.313 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 26 | 31 | 20 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.29 | 0.83 | 1.08 | 1.08 |
| time (sec) | N/A | 0.025 | 0.007 | 0.010 | 0.260 | 0.318 | 1.232 | 0.442 | 1.260 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 33 | 31 | 33 | 61 | 26 | 33 | 33 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 1.06 | 1.97 | 0.84 | 1.06 | 1.06 |
| time (sec) | N/A | 0.129 | 0.531 | 0.244 | 0.271 | 0.290 | 2.778 | 0.323 | 1.283 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 33 | 31 | 33 | 111 | 0 | 33 | 33 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 1.06 | 3.58 | 0.00 | 1.06 | 1.06 |
| time (sec) | N/A | 0.158 | 1.502 | 0.034 | 0.279 | 0.308 | 0.000 | 0.331 | 1.256 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 31 | 29 | 223 | 69 | 26 | 31 | 31 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 7.69 | 2.38 | 0.90 | 1.07 | 1.07 |
| time (sec) | N/A | 0.137 | 0.495 | 0.014 | 0.331 | 0.309 | 7.959 | 0.323 | 1.429 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 188 | 63 | 22 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 7.83 | 2.62 | 0.92 | 1.08 | 1.08 |
| time (sec) | N/A | 0.025 | 0.030 | 0.013 | 0.282 | 0.296 | 2.785 | 0.327 | 1.200 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 33 | 31 | 488 | 118 | 27 | 33 | 33 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 15.74 | 3.81 | 0.87 | 1.06 | 1.06 |
| time (sec) | N/A | 0.123 | 4.729 | 0.033 | 0.294 | 0.334 | 8.398 | 0.378 | 1.335 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 33 | 31 | 723 | 203 | 29 | 33 | 33 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 23.32 | 6.55 | 0.94 | 1.06 | 1.06 |
| time (sec) | N/A | 0.156 | 9.877 | 122.391 | 0.314 | 0.332 | 42.429 | 0.416 | 1.323 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 281 | 241 | 507 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.189 | 0.157 | 0.935 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 170 | 388 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 2.14 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.136 | 0.088 | 0.805 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 95 | 284 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 2.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.090 | 0.051 | 0.799 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 62 | 217 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 3.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.033 | 0.003 | 0.571 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 85 | 276 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 2.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.103 | 0.028 | 0.644 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 141 | 338 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 2.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.136 | 0.057 | 0.813 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 250 | 208 | 429 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.186 | 0.141 | 0.965 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 265 | 265 | 220 | 550 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 2.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.193 | 0.179 | 0.945 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 186 | 153 | 435 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 2.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.150 | 0.088 | 0.836 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 114 | 311 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 2.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.117 | 0.060 | 0.795 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 57 | 127 | 85 | 95 | 333 | 88 | 84 |
| N.S. | 1 | 1.00 | 0.77 | 1.72 | 1.15 | 1.28 | 4.50 | 1.19 | 1.14 |
| time (sec) | N/A | 0.022 | 0.034 | 0.625 | 0.200 | 0.318 | 3.122 | 0.314 | 1.569 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 152 | 354 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.147 | 0.069 | 0.735 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 240 | 240 | 199 | 437 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.176 | 0.120 | 1.124 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 335 | 335 | 269 | 549 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 1.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.225 | 0.248 | 1.658 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 397 | 397 | 331 | 549 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.372 | 0.191 | 1.695 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 278 | 278 | 243 | 431 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 1.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.252 | 0.106 | 0.938 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 203 | 172 | 356 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.137 | 0.028 | 0.682 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 245 | 245 | 224 | 415 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 1.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.227 | 0.060 | 0.663 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 331 | 331 | 279 | 499 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 1.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.279 | 0.108 | 1.200 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 369 | 369 | 339 | 606 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.295 | 0.197 | 1.168 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 276 | 276 | 263 | 491 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 1.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.246 | 0.087 | 0.832 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 239 | 239 | 184 | 405 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 1.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.129 | 0.034 | 0.750 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 290 | 290 | 280 | 492 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 1.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.232 | 0.121 | 1.156 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 388 | 388 | 350 | 598 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 1.54 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.279 | 0.232 | 1.752 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 417 | 417 | 530 | 624 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.27 | 1.50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.356 | 0.926 | 1.568 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 344 | 344 | 455 | 496 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.32 | 1.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.312 | 0.728 | 1.177 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 165 | 969 | 130 | 373 | 0 | 194 | 366 |
| N.S. | 1 | 1.00 | 1.19 | 6.97 | 0.94 | 2.68 | 0.00 | 1.40 | 2.63 |
| time (sec) | N/A | 0.058 | 0.111 | 1.362 | 0.292 | 0.356 | 0.000 | 0.317 | 1.837 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 383 | 383 | 521 | 556 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.36 | 1.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.323 | 0.817 | 0.910 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 460 | 460 | 596 | 656 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.30 | 1.43 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.369 | 1.013 | 2.913 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 534 | 534 | 433 | 1619 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 3.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.680 | 0.467 | 1.317 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 491 | 491 | 382 | 1521 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 3.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.599 | 0.369 | 1.306 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 503 | 503 | 407 | 1406 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 2.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.304 | 0.549 | 1.313 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 560 | 560 | 475 | 1619 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 2.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.626 | 0.456 | 2.247 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 326 | 275 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.277 | 0.142 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 506 | 506 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.395 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 278 | 278 | 307 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.321 | 0.023 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 510 | 510 | 631 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.24 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.448 | 10.176 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 27 | 20 | 120 | 21 | 0 | 0 | 19 |
| N.S. | 1 | 1.00 | 1.12 | 0.83 | 5.00 | 0.88 | 0.00 | 0.00 | 0.79 |
| time (sec) | N/A | 0.023 | 0.006 | 0.642 | 0.209 | 0.329 | 0.000 | 0.000 | 1.321 |

| Problem 280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 81 | 62 | 119 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.93 | 1.48 | 2.83 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.036 | 0.013 | 0.621 | 0.200 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 82 | 44 | 0 | 43 | 0 | 0 | 43 |
| N.S. | 1 | 1.00 | 2.00 | 1.07 | 0.00 | 1.05 | 0.00 | 0.00 | 1.05 |
| time (sec) | N/A | 0.040 | 0.025 | 0.665 | 0.000 | 0.316 | 0.000 | 0.000 | 1.368 |

| Problem 282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 80 | 85 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.70 | 1.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.044 | 0.020 | 0.649 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 371 | 371 | 376 | 153 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 0.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.407 | 0.175 | 0.647 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 292 | 292 | 297 | 77 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.177 | 0.028 | 0.556 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 324 | 324 | 330 | 106 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.273 | 0.051 | 0.667 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 414 | 414 | 405 | 186 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.323 | 0.121 | 0.728 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 416 | 416 | 403 | 148 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.36 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.498 | 0.227 | 0.580 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 383 | 383 | 369 | 127 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.305 | 0.094 | 0.573 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 359 | 359 | 297 | 86 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.24 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.212 | 0.052 | 0.563 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 359 | 359 | 294 | 94 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.163 | 0.048 | 0.566 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 398 | 398 | 378 | 124 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 0.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.338 | 0.099 | 0.638 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 423 | 423 | 371 | 150 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.35 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.307 | 0.146 | 0.629 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 498 | 498 | 480 | 175 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.35 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.583 | 0.151 | 0.753 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 401 | 401 | 383 | 85 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.329 | 0.045 | 0.596 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 433 | 433 | 416 | 114 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.418 | 0.063 | 0.651 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 530 | 530 | 484 | 163 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.446 | 0.160 | 0.641 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 473 | 473 | 348 | 102 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 0.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.348 | 0.055 | 0.611 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 537 | 537 | 506 | 158 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.479 | 0.129 | 0.657 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 521 | 521 | 458 | 145 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.541 | 0.148 | 0.656 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 497 | 497 | 464 | 94 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.388 | 0.195 | 0.573 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 497 | 497 | 359 | 112 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 0.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.303 | 0.066 | 0.582 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 536 | 536 | 536 | 132 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.611 | 0.579 | 0.654 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 91 | 95 | 98 | 102 | 117 | 134 | 183 | 104 |
| N.S. | 1 | 1.00 | 1.04 | 1.08 | 1.12 | 1.29 | 1.47 | 2.01 | 1.14 |
| time (sec) | N/A | 0.048 | 0.040 | 0.335 | 0.200 | 0.299 | 0.636 | 0.300 | 1.282 |

| Problem 304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 120 | 150 | 265 | 187 | 219 | 252 | 424 | 212 |
| N.S. | 1 | 1.00 | 1.25 | 2.21 | 1.56 | 1.82 | 2.10 | 3.53 | 1.77 |
| time (sec) | N/A | 0.083 | 0.099 | 0.457 | 0.222 | 0.328 | 4.497 | 0.312 | 1.285 |

| Problem 305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 226 | 428 | 284 | 336 | 410 | 770 | 352 |
| N.S. | 1 | 1.00 | 1.52 | 2.87 | 1.91 | 2.26 | 2.75 | 5.17 | 2.36 |
| time (sec) | N/A | 0.084 | 0.142 | 0.721 | 0.206 | 0.307 | 19.603 | 0.325 | 1.388 |

| Problem 306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 62 | 217 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 3.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.071 | 0.011 | 0.242 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 57 | 127 | 86 | 95 | 333 | 88 | 84 |
| N.S. | 1 | 1.00 | 0.77 | 1.72 | 1.16 | 1.28 | 4.50 | 1.19 | 1.14 |
| time (sec) | N/A | 0.059 | 0.045 | 0.351 | 0.192 | 0.322 | 34.401 | 0.360 | 1.990 |

| Problem 308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 83 | 283 | 169 | 272 | 0 | 201 | 173 |
| N.S. | 1 | 1.00 | 0.74 | 2.53 | 1.51 | 2.43 | 0.00 | 1.79 | 1.54 |
| time (sec) | N/A | 0.083 | 0.061 | 0.612 | 0.205 | 0.330 | 0.000 | 0.345 | 1.717 |

| Problem 309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 247 | 247 | 220 | 298 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.89 | 1.21 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.248 | 0.126 | 0.362 | 0.336 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 831 | 831 | 862 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.783 | 0.887 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 499 | 499 | 637 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.502 | 0.485 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 317 | 317 | 464 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.268 | 0.318 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 397 | 397 | 576 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.438 | 0.162 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 551 | 551 | 811 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.47 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.630 | 0.351 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 701 | 701 | 821 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.663 | 1.007 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 447 | 447 | 623 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.420 | 0.687 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 371 | 371 | 485 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.266 | 0.475 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 461 | 461 | 668 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.426 | 0.529 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 694 | 694 | 930 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.757 | 0.863 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 936 | 936 | 1254 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.090 | 1.802 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 739 | 739 | 1103 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.49 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.858 | 1.512 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 430 | 430 | 590 | 1231 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.37 | 2.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.386 | 0.493 | 1.542 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 814 | 814 | 1209 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.49 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.907 | 1.267 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 970 | 970 | 1391 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.43 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.131 | 1.903 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 897 | 897 | 1237 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.230 | 3.561 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 815 | 815 | 1132 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.099 | 3.069 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 821 | 821 | 1143 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.589 | 3.375 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 919 | 919 | 1304 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.42 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.184 | 3.683 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 477 | 477 | 754 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.393 | 0.388 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 347 | 347 | 488 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.239 | 0.253 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 229 | 178 | 392 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 1.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.127 | 0.088 | 0.746 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 19 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 0.86 | 1.09 | 1.09 |
| time (sec) | N/A | 0.082 | 0.394 | 0.150 | 0.276 | 0.300 | 21.008 | 0.316 | 1.271 |

| Problem 333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 27 | 0 | 33 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 0.00 | 1.22 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.090 | 0.019 | 2.548 | 0.000 | 0.350 | 0.000 | 0.000 | 0.000 |

| Problem 334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 27 | 0 | 33 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 0.00 | 1.22 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.118 | 0.003 | 2.303 | 0.000 | 0.319 | 0.000 | 0.000 | 0.000 |

| Problem 335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 31 | 30 | 0 | 35 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 1.07 | 0.00 | 1.25 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.089 | 0.018 | 2.878 | 0.000 | 0.328 | 0.000 | 0.000 | 0.000 |

| Problem 336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 31 | 30 | 0 | 35 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 1.07 | 0.00 | 1.25 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.126 | 0.003 | 2.796 | 0.000 | 0.306 | 0.000 | 0.000 | 0.000 |

| Problem 337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 27 | 20 | 120 | 21 | 0 | 0 | 19 |
| N.S. | 1 | 1.00 | 1.12 | 0.83 | 5.00 | 0.88 | 0.00 | 0.00 | 0.79 |
| time (sec) | N/A | 0.022 | 0.005 | 0.713 | 0.199 | 0.304 | 0.000 | 0.000 | 1.284 |

| Problem 338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 27 | 20 | 120 | 21 | 0 | 0 | 19 |
| N.S. | 1 | 1.00 | 1.12 | 0.83 | 5.00 | 0.88 | 0.00 | 0.00 | 0.79 |
| time (sec) | N/A | 0.091 | 0.002 | 0.831 | 0.211 | 0.291 | 0.000 | 0.000 | 1.203 |

| Problem 339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 37 | 252 | 24 | 246 | 34 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 6.81 | 0.65 | 6.65 | 0.92 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.018 | 0.143 | 0.925 | 0.213 | 0.327 | 0.000 | 0.000 | 0.000 |

| Problem 340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 252 | 24 | 246 | 34 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 9.33 | 0.89 | 9.11 | 1.26 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.052 | 0.122 | 0.938 | 0.210 | 0.327 | 0.000 | 0.000 | 0.000 |

| Problem 341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 259 | 24 | 243 | 34 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 9.59 | 0.89 | 9.00 | 1.26 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.014 | 0.199 | 0.921 | 0.210 | 0.312 | 0.000 | 0.000 | 0.000 |

| Problem 342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 259 | 24 | 243 | 34 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 9.59 | 0.89 | 9.00 | 1.26 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.089 | 0.135 | 0.955 | 0.206 | 0.309 | 0.000 | 0.000 | 0.000 |

| Problem 343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 238 | 238 | 494 | 1756 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.08 | 7.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.255 | 0.144 | 2.022 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 292 | 761 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.74 | 4.53 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.178 | 0.086 | 1.280 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 98 | 156 | 123 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 1.61 | 1.27 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.089 | 0.020 | 0.700 | 0.237 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 26 | 26 | 19 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.08 | 0.79 | 1.08 | 1.08 |
| time (sec) | N/A | 0.075 | 0.423 | 0.154 | 0.262 | 0.307 | 0.976 | 0.304 | 1.168 |

| Problem 347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 500 | 500 | 993 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.462 | 0.522 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 372 | 372 | 655 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.268 | 0.371 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 349 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 243 | 243 | 194 | 616 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 2.53 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.150 | 0.157 | 1.289 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 350 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 25 | 27 | 27 | 0 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.08 | 0.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.122 | 0.620 | 0.623 | 0.257 | 0.290 | 0.000 | 0.338 | 1.200 |

| Problem 351 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 286 | 286 | 464 | 766 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.62 | 2.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.293 | 0.458 | 1.212 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 352 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 234 | 434 | 543 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.85 | 2.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.242 | 0.221 | 1.101 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 353 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 193 | 210 | 361 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.09 | 1.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.124 | 0.095 | 1.053 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 354 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 153 | 144 | 169 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.095 | 0.051 | 1.094 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 355 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 204 | 227 | 370 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 1.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.195 | 0.160 | 1.066 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 356 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 251 | 251 | 255 | 552 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 2.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.257 | 0.265 | 1.178 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 357 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 308 | 308 | 311 | 782 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 2.54 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.338 | 0.320 | 1.155 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 358 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 232 | 232 | 221 | 1180 | 231 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 5.09 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.148 | 0.144 | 78.149 | 0.232 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 359 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 197 | 1012 | 207 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 5.19 | 1.06 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.123 | 0.131 | 32.940 | 0.246 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 360 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 164 | 843 | 178 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.04 | 5.34 | 1.13 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.095 | 0.101 | 13.040 | 0.237 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 361 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 116 | 657 | 139 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.17 | 6.64 | 1.40 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.064 | 0.077 | 5.247 | 0.235 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 362 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 128 | 756 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.45 | 8.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.052 | 0.073 | 4.598 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 363 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 111 | 709 | 162 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.09 | 6.95 | 1.59 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.052 | 0.098 | 5.356 | 0.252 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 364 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 204 | 901 | 198 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.31 | 5.78 | 1.27 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.090 | 0.104 | 5.392 | 0.243 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 365 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 193 | 240 | 1070 | 229 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.24 | 5.54 | 1.19 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.127 | 0.114 | 13.378 | 0.241 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 366 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 230 | 230 | 273 | 1237 | 253 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.19 | 5.38 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.172 | 0.118 | 32.955 | 0.240 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 367 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 705 | 863 | 976 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.22 | 1.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.332 | 0.287 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 368 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 602 | 602 | 825 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.37 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.886 | 0.170 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 369 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 309 | 309 | 457 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.48 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.321 | 0.178 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 370 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 823 | 0 | 823 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.232 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 371 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 607 | 0 | 513 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.471 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 372 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 939 | 0 | 781 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.00 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.619 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 373 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 522 | 522 | 1163 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.615 | 0.465 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 374 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | F | F | F | F | F | F(-1) |
| verified | N/A | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 519 | 0 | 519 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.071 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 375 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 23 | 25 | 25 | 20 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 0.87 | 1.09 | 1.09 |
| time (sec) | N/A | 0.008 | 0.049 | 0.038 | 0.302 | 0.308 | 2.757 | 0.323 | 1.213 |

| Problem 376 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 23 | 108 | 45 | 22 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 4.70 | 1.96 | 0.96 | 1.09 | 1.09 |
| time (sec) | N/A | 0.008 | 0.346 | 0.033 | 0.384 | 0.297 | 31.823 | 0.311 | 1.235 |

| Problem 377 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 23 | 0 | 25 | 0 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 0.00 | 1.09 | 0.00 | 1.09 | 1.09 |
| time (sec) | N/A | 0.007 | 0.066 | 0.049 | 0.000 | 0.296 | 0.000 | 0.452 | 1.305 |

| Problem 378 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 364 | 364 | 394 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.035 | 0.043 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 379 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 258 | 258 | 205 | 354 | 274 | 329 | 384 | 741 | 323 |
| N.S. | 1 | 1.00 | 0.79 | 1.37 | 1.06 | 1.28 | 1.49 | 2.87 | 1.25 |
| time (sec) | N/A | 0.213 | 0.128 | 0.655 | 0.195 | 0.287 | 1.175 | 0.314 | 1.466 |

| Problem 380 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 196 | 152 | 284 | 224 | 256 | 296 | 467 | 203 |
| N.S. | 1 | 1.00 | 0.78 | 1.45 | 1.14 | 1.31 | 1.51 | 2.38 | 1.04 |
| time (sec) | N/A | 0.182 | 0.079 | 0.565 | 0.204 | 0.294 | 0.646 | 0.315 | 1.453 |

| Problem 381 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 110 | 76 | 116 | 165 | 156 | 189 | 209 | 102 |
| N.S. | 1 | 1.00 | 0.69 | 1.05 | 1.50 | 1.42 | 1.72 | 1.90 | 0.93 |
| time (sec) | N/A | 0.124 | 0.017 | 0.399 | 0.211 | 0.285 | 0.299 | 0.338 | 1.343 |

| Problem 382 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 227 | 626 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.44 | 3.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.166 | 0.074 | 0.757 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 383 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 180 | 517 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.88 | 5.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.133 | 0.025 | 0.373 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 384 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 251 | 589 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.61 | 3.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.220 | 0.086 | 0.389 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 385 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 234 | 327 | 642 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.40 | 2.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.322 | 0.117 | 0.441 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 386 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 742 | 742 | 605 | 2094 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 2.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.594 | 0.715 | 0.065 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 387 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 558 | 558 | 492 | 1724 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 3.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.426 | 0.501 | 234.845 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 388 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 397 | 397 | 341 | 1372 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 3.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.305 | 0.329 | 111.258 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 389 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 232 | 232 | 329 | 1012 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.42 | 4.36 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.205 | 0.139 | 49.443 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 390 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 637 | 637 | 605 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.314 | 0.191 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 391 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 270 | 476 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.233 | 0.159 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 392 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 421 | 421 | 765 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.320 | 0.213 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 393 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 1210 | 1210 | 2067 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.963 | 0.582 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 394 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 649 | 649 | 1355 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.068 | 0.318 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 395 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 36 | 34 | 168 | 93 | 0 | 36 | 36 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 4.94 | 2.74 | 0.00 | 1.06 | 1.06 |
| time (sec) | N/A | 0.025 | 0.664 | 0.098 | 0.464 | 0.360 | 0.000 | 0.367 | 1.881 |

| Problem 396 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 36 | 34 | 207 | 93 | 0 | 36 | 36 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 6.09 | 2.74 | 0.00 | 1.06 | 1.06 |
| time (sec) | N/A | 0.027 | 0.814 | 0.187 | 0.515 | 0.325 | 0.000 | 0.505 | 2.328 |

| Problem 397 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 2050 | 2050 | 4971 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.42 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 4.770 | 1.318 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 398 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 1147 | 1147 | 3163 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.178 | 0.645 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 399 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 36 | 34 | 292 | 135 | 0 | 36 | 36 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 8.59 | 3.97 | 0.00 | 1.06 | 1.06 |
| time (sec) | N/A | 0.025 | 1.454 | 0.108 | 0.513 | 0.309 | 0.000 | 0.601 | 2.791 |

| Problem 400 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 36 | 34 | 335 | 135 | 0 | 36 | 36 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 9.85 | 3.97 | 0.00 | 1.06 | 1.06 |
| time (sec) | N/A | 0.027 | 2.144 | 0.120 | 0.558 | 0.316 | 0.000 | 0.594 | 2.745 |

| Problem 401 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 62 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.058 | 0.105 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 402 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 43 | 54 | 99 | 46 | 56 | 90 | 63 |
| N.S. | 1 | 1.00 | 0.47 | 0.59 | 1.08 | 0.50 | 0.61 | 0.98 | 0.68 |
| time (sec) | N/A | 0.067 | 0.070 | 0.577 | 0.204 | 0.288 | 0.135 | 0.324 | 1.583 |

| Problem 403 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 58 | 70 | 159 | 61 | 75 | 113 | 92 |
| N.S. | 1 | 1.00 | 0.57 | 0.69 | 1.56 | 0.60 | 0.74 | 1.11 | 0.90 |
| time (sec) | N/A | 0.077 | 0.089 | 0.569 | 0.202 | 0.295 | 0.156 | 0.325 | 1.586 |

| Problem 404 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 132 | 642 | 559 | 1409 | 609 | 1697 | 380 |
| N.S. | 1 | 1.00 | 0.82 | 4.01 | 3.49 | 8.81 | 3.81 | 10.61 | 2.38 |
| time (sec) | N/A | 0.142 | 0.051 | 5.581 | 0.227 | 0.324 | 2.560 | 0.346 | 1.838 |

| Problem 405 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 121 | 100 | 386 | 317 | 639 | 360 | 772 | 242 |
| N.S. | 1 | 1.00 | 0.83 | 3.19 | 2.62 | 5.28 | 2.98 | 6.38 | 2.00 |
| time (sec) | N/A | 0.103 | 0.011 | 1.512 | 0.242 | 0.323 | 1.138 | 0.321 | 1.513 |

| Problem 406 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 69 | 165 | 148 | 231 | 178 | 283 | 111 |
| N.S. | 1 | 1.00 | 0.88 | 2.12 | 1.90 | 2.96 | 2.28 | 3.63 | 1.42 |
| time (sec) | N/A | 0.069 | 0.009 | 0.417 | 0.211 | 0.331 | 0.516 | 0.333 | 1.361 |

| Problem 407 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 34 | 42 | 45 | 50 | 53 | 59 | 41 |
| N.S. | 1 | 1.00 | 1.00 | 1.24 | 1.32 | 1.47 | 1.56 | 1.74 | 1.21 |
| time (sec) | N/A | 0.023 | 0.007 | 0.083 | 0.195 | 0.304 | 0.219 | 0.337 | 1.320 |

| Problem 408 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 83 | 0 | 0 | 65 | 0 | 78 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.78 | 0.00 | 0.94 | 0.00 |
| time (sec) | N/A | 0.092 | 0.094 | 0.000 | 0.000 | 0.311 | 0.000 | 0.335 | 0.000 |

| Problem 409 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 163 | 0 | 0 | 171 | 0 | 582 | 0 |
| N.S. | 1 | 1.00 | 1.33 | 0.00 | 0.00 | 1.39 | 0.00 | 4.73 | 0.00 |
| time (sec) | N/A | 0.116 | 0.076 | 0.000 | 0.000 | 0.299 | 0.000 | 0.336 | 0.000 |

| Problem 410 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 189 | 0 | 0 | 444 | 0 | 3401 | 0 |
| N.S. | 1 | 1.00 | 1.12 | 0.00 | 0.00 | 2.63 | 0.00 | 20.12 | 0.00 |
| time (sec) | N/A | 0.159 | 0.120 | 0.000 | 0.000 | 0.313 | 0.000 | 0.381 | 0.000 |

| Problem 411 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 219 | 190 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.260 | 0.221 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 412 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 176 | 160 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.193 | 0.054 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 413 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 134 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.153 | 0.044 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 414 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 104 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.124 | 0.017 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 415 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 181 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.175 | 0.171 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 416 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 194 | 211 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.216 | 0.200 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 417 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 237 | 237 | 272 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.278 | 0.279 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 418 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 131 | 0 | 0 | 80 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.61 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.100 | 0.143 | 0.000 | 0.000 | 0.094 | 0.000 | 0.000 | 0.000 |

| Problem 419 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 109 | 0 | 70 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.64 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.101 | 0.147 | 0.000 | 0.061 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 420 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 232 | 466 | 304 | 405 | 457 | 987 | 370 |
| N.S. | 1 | 1.00 | 1.47 | 2.95 | 1.92 | 2.56 | 2.89 | 6.25 | 2.34 |
| time (sec) | N/A | 0.117 | 0.210 | 5.982 | 0.201 | 0.364 | 2.503 | 0.340 | 1.541 |

| Problem 421 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 128 | 156 | 290 | 202 | 268 | 286 | 544 | 225 |
| N.S. | 1 | 1.00 | 1.22 | 2.27 | 1.58 | 2.09 | 2.23 | 4.25 | 1.76 |
| time (sec) | N/A | 0.089 | 0.127 | 1.970 | 0.191 | 0.305 | 1.229 | 0.362 | 1.592 |

| Problem 422 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 113 | 173 | 112 | 148 | 156 | 236 | 113 |
| N.S. | 1 | 1.00 | 1.15 | 1.77 | 1.14 | 1.51 | 1.59 | 2.41 | 1.15 |
| time (sec) | N/A | 0.058 | 0.038 | 0.438 | 0.200 | 0.324 | 0.587 | 0.323 | 1.444 |

| Problem 423 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 34 | 42 | 45 | 50 | 53 | 59 | 41 |
| N.S. | 1 | 1.00 | 1.00 | 1.24 | 1.32 | 1.47 | 1.56 | 1.74 | 1.21 |
| time (sec) | N/A | 0.021 | 0.007 | 0.102 | 0.183 | 0.305 | 0.208 | 0.322 | 1.347 |

| Problem 424 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 67 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.080 | 0.012 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 425 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 69 | 139 | 90 | 113 | 357 | 96 | 89 |
| N.S. | 1 | 1.00 | 0.86 | 1.74 | 1.12 | 1.41 | 4.46 | 1.20 | 1.11 |
| time (sec) | N/A | 0.054 | 0.075 | 1.336 | 0.194 | 0.279 | 3.558 | 0.321 | 3.316 |

| Problem 426 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 88 | 305 | 172 | 310 | 1984 | 221 | 180 |
| N.S. | 1 | 1.00 | 0.74 | 2.56 | 1.45 | 2.61 | 16.67 | 1.86 | 1.51 |
| time (sec) | N/A | 0.099 | 0.094 | 3.759 | 0.197 | 0.307 | 15.248 | 0.402 | 3.492 |

| Problem 427 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 115 | 487 | 306 | 563 | 5673 | 443 | 293 |
| N.S. | 1 | 1.00 | 0.77 | 3.27 | 2.05 | 3.78 | 38.07 | 2.97 | 1.97 |
| time (sec) | N/A | 0.124 | 0.123 | 11.925 | 0.208 | 0.319 | 63.372 | 0.373 | 3.742 |

| Problem 428 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 409 | 409 | 400 | 1537 | 895 | 1742 | 1421 | 3738 | 1154 |
| N.S. | 1 | 1.00 | 0.98 | 3.76 | 2.19 | 4.26 | 3.47 | 9.14 | 2.82 |
| time (sec) | N/A | 0.714 | 0.196 | 16.244 | 0.228 | 0.354 | 5.391 | 0.387 | 1.964 |

| Problem 429 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 323 | 323 | 277 | 939 | 605 | 1137 | 894 | 2106 | 652 |
| N.S. | 1 | 1.00 | 0.86 | 2.91 | 1.87 | 3.52 | 2.77 | 6.52 | 2.02 |
| time (sec) | N/A | 0.553 | 0.110 | 5.678 | 0.218 | 0.340 | 2.594 | 0.338 | 1.741 |

| Problem 430 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 211 | 211 | 164 | 540 | 348 | 622 | 466 | 939 | 302 |
| N.S. | 1 | 1.00 | 0.78 | 2.56 | 1.65 | 2.95 | 2.21 | 4.45 | 1.43 |
| time (sec) | N/A | 0.280 | 0.059 | 1.533 | 0.214 | 0.328 | 1.244 | 0.324 | 1.497 |

| Problem 431 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 69 | 165 | 148 | 231 | 178 | 283 | 111 |
| N.S. | 1 | 1.00 | 0.88 | 2.12 | 1.90 | 2.96 | 2.28 | 3.63 | 1.42 |
| time (sec) | N/A | 0.063 | 0.013 | 0.423 | 0.202 | 0.287 | 0.470 | 0.315 | 1.309 |

| Problem 432 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 324 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.178 | 0.141 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 433 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 200 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.141 | 0.181 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 434 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 222 | 222 | 316 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.42 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.466 | 0.314 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 435 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 492 | 492 | 378 | 2034 | 1245 | 3121 | 1846 | 5146 | 1400 |
| N.S. | 1 | 1.00 | 0.77 | 4.13 | 2.53 | 6.34 | 3.75 | 10.46 | 2.85 |
| time (sec) | N/A | 0.638 | 0.193 | 16.190 | 0.247 | 0.381 | 5.381 | 0.420 | 2.342 |

| Problem 436 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 306 | 306 | 231 | 1084 | 732 | 1692 | 991 | 2532 | 651 |
| N.S. | 1 | 1.00 | 0.75 | 3.54 | 2.39 | 5.53 | 3.24 | 8.27 | 2.13 |
| time (sec) | N/A | 0.369 | 0.093 | 5.224 | 0.232 | 0.385 | 2.585 | 0.379 | 1.911 |

| Problem 437 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 121 | 100 | 386 | 317 | 639 | 360 | 772 | 242 |
| N.S. | 1 | 1.00 | 0.83 | 3.19 | 2.62 | 5.28 | 2.98 | 6.38 | 2.00 |
| time (sec) | N/A | 0.094 | 0.018 | 1.511 | 0.201 | 0.299 | 1.020 | 0.324 | 1.428 |

| Problem 438 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 177 | 646 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 3.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.274 | 0.228 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 439 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 209 | 209 | 444 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.252 | 0.343 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 440 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 376 | 376 | 660 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.814 | 0.555 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 441 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 132 | 642 | 559 | 1409 | 609 | 1697 | 380 |
| N.S. | 1 | 1.00 | 0.82 | 4.01 | 3.49 | 8.81 | 3.81 | 10.61 | 2.38 |
| time (sec) | N/A | 0.137 | 0.034 | 5.170 | 0.225 | 0.309 | 2.070 | 0.358 | 1.600 |

| Problem 442 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 231 | 1095 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 4.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.355 | 0.335 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 443 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 274 | 274 | 1301 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 4.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.380 | 0.380 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 444 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 29 | 41 | 40 | 42 | 48 | 53 | 36 |
| N.S. | 1 | 1.00 | 1.00 | 1.41 | 1.38 | 1.45 | 1.66 | 1.83 | 1.24 |
| time (sec) | N/A | 0.018 | 0.006 | 0.086 | 0.194 | 0.312 | 0.207 | 0.328 | 0.077 |

| Problem 445 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 279 | 279 | 252 | 0 | 0 | 243 | 0 | 517 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 0.00 | 0.00 | 0.87 | 0.00 | 1.85 | 0.00 |
| time (sec) | N/A | 0.553 | 0.550 | 0.000 | 0.000 | 0.303 | 0.000 | 0.365 | 0.000 |

| Problem 446 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 164 | 0 | 0 | 140 | 0 | 248 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 0.00 | 0.00 | 0.78 | 0.00 | 1.39 | 0.00 |
| time (sec) | N/A | 0.297 | 0.117 | 0.000 | 0.000 | 0.311 | 0.000 | 0.348 | 0.000 |

| Problem 447 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 83 | 0 | 0 | 65 | 0 | 78 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.78 | 0.00 | 0.94 | 0.00 |
| time (sec) | N/A | 0.082 | 0.036 | 0.000 | 0.000 | 0.294 | 0.000 | 0.333 | 0.000 |

| Problem 448 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 28 | 30 | 35 | 24 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 1.07 | 1.25 | 0.86 | 1.07 | 1.07 |
| time (sec) | N/A | 0.047 | 0.207 | 0.157 | 0.856 | 0.294 | 1.503 | 0.348 | 1.282 |

| Problem 449 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 28 | 30 | 59 | 26 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 1.07 | 2.11 | 0.93 | 1.07 | 1.07 |
| time (sec) | N/A | 0.044 | 0.478 | 0.188 | 0.858 | 0.303 | 3.772 | 0.347 | 1.244 |

| Problem 450 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 326 | 1310 | 0 | 0 | 573 | 0 | 3975 | 0 |
| N.S. | 1 | 1.00 | 4.02 | 0.00 | 0.00 | 1.76 | 0.00 | 12.19 | 0.00 |
| time (sec) | N/A | 0.902 | 0.413 | 0.000 | 0.000 | 0.323 | 0.000 | 0.463 | 0.000 |

| Problem 451 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 224 | 269 | 0 | 0 | 328 | 0 | 1930 | 0 |
| N.S. | 1 | 1.00 | 1.20 | 0.00 | 0.00 | 1.46 | 0.00 | 8.62 | 0.00 |
| time (sec) | N/A | 0.444 | 0.221 | 0.000 | 0.000 | 0.302 | 0.000 | 0.392 | 0.000 |

| Problem 452 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 163 | 0 | 0 | 171 | 0 | 582 | 0 |
| N.S. | 1 | 1.00 | 1.33 | 0.00 | 0.00 | 1.39 | 0.00 | 4.73 | 0.00 |
| time (sec) | N/A | 0.116 | 0.074 | 0.000 | 0.000 | 0.308 | 0.000 | 0.342 | 0.000 |

| Problem 453 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 28 | 267 | 71 | 26 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 9.54 | 2.54 | 0.93 | 1.07 | 1.07 |
| time (sec) | N/A | 0.045 | 0.667 | 0.181 | 1.153 | 0.333 | 5.011 | 0.323 | 1.244 |

| Problem 454 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 28 | 406 | 112 | 27 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 14.50 | 4.00 | 0.96 | 1.07 | 1.07 |
| time (sec) | N/A | 0.044 | 8.431 | 0.177 | 1.136 | 0.318 | 31.297 | 0.359 | 1.270 |

| Problem 455 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 432 | 432 | 438 | 0 | 0 | 1682 | 0 | 5889 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 0.00 | 0.00 | 3.89 | 0.00 | 13.63 | 0.00 |
| time (sec) | N/A | 1.478 | 1.281 | 0.000 | 0.000 | 0.365 | 0.000 | 0.543 | 0.000 |

| Problem 456 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 322 | 322 | 322 | 0 | 0 | 931 | 0 | 11278 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 2.89 | 0.00 | 35.02 | 0.00 |
| time (sec) | N/A | 0.659 | 0.378 | 0.000 | 0.000 | 0.389 | 0.000 | 0.553 | 0.000 |

| Problem 457 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 189 | 0 | 0 | 444 | 0 | 3401 | 0 |
| N.S. | 1 | 1.00 | 1.12 | 0.00 | 0.00 | 2.63 | 0.00 | 20.12 | 0.00 |
| time (sec) | N/A | 0.156 | 0.117 | 0.000 | 0.000 | 0.319 | 0.000 | 0.399 | 0.000 |

| Problem 458 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 28 | 1116 | 107 | 26 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 39.86 | 3.82 | 0.93 | 1.07 | 1.07 |
| time (sec) | N/A | 0.045 | 0.553 | 0.173 | 1.570 | 0.319 | 17.311 | 0.348 | 1.298 |

| Problem 459 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 28 | 1486 | 165 | 27 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 53.07 | 5.89 | 0.96 | 1.07 | 1.07 |
| time (sec) | N/A | 0.043 | 23.057 | 0.177 | 1.579 | 0.314 | 135.869 | 0.398 | 1.336 |

| Problem 460 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 488 | 488 | 458 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.109 | 0.487 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 461 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 311 | 311 | 298 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.607 | 0.264 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 462 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 134 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.161 | 0.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 463 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 0 | 26 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 0.00 | 0.87 | 1.00 | 1.00 |
| time (sec) | N/A | 0.067 | 4.541 | 0.161 | 11.956 | 0.000 | 0.608 | 0.429 | 1.360 |

| Problem 464 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 0 | 27 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 0.00 | 0.90 | 1.00 | 1.00 |
| time (sec) | N/A | 0.100 | 0.257 | 0.164 | 10.690 | 0.000 | 2.502 | 0.402 | 1.416 |

| Problem 465 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 625 | 625 | 545 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.339 | 0.936 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 466 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 396 | 396 | 348 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.727 | 0.386 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 467 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 176 | 160 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.193 | 0.134 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 468 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 0 | 26 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 0.00 | 0.87 | 1.00 | 1.00 |
| time (sec) | N/A | 0.081 | 2.657 | 0.163 | 10.799 | 0.000 | 52.802 | 0.538 | 1.449 |

| Problem 469 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 0 | 0 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.113 | 1.276 | 0.181 | 10.701 | 0.000 | 0.000 | 0.520 | 1.387 |

| Problem 470 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 355 | 355 | 315 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.867 | 0.257 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 471 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 229 | 208 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.470 | 0.146 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 472 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 104 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.129 | 0.021 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 473 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 0 | 27 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 0.00 | 0.90 | 1.00 | 1.00 |
| time (sec) | N/A | 0.068 | 0.079 | 0.191 | 10.492 | 0.000 | 1.499 | 0.490 | 1.351 |

| Problem 474 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 404 | 404 | 1040 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.483 | 1.522 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 475 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 275 | 435 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.707 | 0.727 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 476 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 181 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.188 | 0.106 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 477 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 0 | 27 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 0.00 | 0.90 | 1.00 | 1.00 |
| time (sec) | N/A | 0.091 | 0.323 | 0.191 | 10.750 | 0.000 | 24.357 | 0.511 | 1.565 |

| Problem 478 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 514 | 514 | 652 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.518 | 3.733 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 479 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 380 | 380 | 491 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.005 | 1.274 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 480 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 194 | 211 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.217 | 0.224 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 481 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 0 | 0 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.081 | 0.568 | 0.185 | 12.525 | 0.000 | 0.000 | 0.835 | 1.702 |

| Problem 482 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 171 | 153 | 0 | 0 | 624 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 3.65 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.185 | 0.247 | 0.000 | 0.000 | 0.341 | 0.000 | 0.000 | 0.000 |

| Problem 483 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 124 | 0 | 0 | 353 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 2.54 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.137 | 0.127 | 0.000 | 0.000 | 0.344 | 0.000 | 0.000 | 0.000 |

| Problem 484 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 89 | 118 | 0 | 201 | 0 | 124 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.15 | 0.00 | 1.95 | 0.00 | 1.20 | 0.00 |
| time (sec) | N/A | 0.110 | 0.182 | 0.652 | 0.000 | 0.298 | 0.000 | 0.289 | 0.000 |

| Problem 485 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 84 | 0 | 0 | 240 | 0 | 110 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 2.79 | 0.00 | 1.28 | 0.00 |
| time (sec) | N/A | 0.091 | 0.113 | 0.000 | 0.000 | 0.336 | 0.000 | 0.310 | 0.000 |

| Problem 486 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F | A | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 120 | 91 | 0 | 0 | 467 | 0 | 184 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 0.00 | 0.00 | 3.89 | 0.00 | 1.53 | 0.00 |
| time (sec) | N/A | 0.129 | 0.118 | 0.000 | 0.000 | 0.349 | 0.000 | 0.305 | 0.000 |

| Problem 487 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F(-1) | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 152 | 91 | 0 | 0 | 863 | 0 | 324 | 0 |
| N.S. | 1 | 1.00 | 0.60 | 0.00 | 0.00 | 5.68 | 0.00 | 2.13 | 0.00 |
| time (sec) | N/A | 0.137 | 0.063 | 0.000 | 0.000 | 0.368 | 0.000 | 0.344 | 0.000 |

| Problem 488 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 184 | 91 | 0 | 0 | 1362 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.49 | 0.00 | 0.00 | 7.40 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.204 | 0.070 | 0.000 | 0.000 | 0.410 | 0.000 | 0.000 | 0.000 |

| Problem 489 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 635 | 635 | 4354 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 6.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.076 | 18.558 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 490 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 547 | 547 | 1323 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.42 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.115 | 11.765 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 491 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 447 | 447 | 1407 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 3.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.548 | 12.625 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 492 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 330 | 330 | 785 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.225 | 2.540 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 493 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 449 | 449 | 1289 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.676 | 12.812 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 494 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 537 | 537 | 1349 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.138 | 9.704 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 495 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 625 | 625 | 1457 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.667 | 11.041 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 496 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 30 | 26 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.00 | 0.87 | 1.00 | 1.00 |
| time (sec) | N/A | 0.058 | 0.524 | 0.191 | 0.935 | 0.302 | 33.766 | 0.399 | 1.242 |

| Problem 497 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 30 | 26 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.00 | 0.87 | 1.00 | 1.00 |
| time (sec) | N/A | 0.053 | 0.161 | 0.158 | 0.938 | 0.291 | 0.933 | 0.376 | 1.221 |

| Problem 498 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 43 | 27 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.43 | 0.90 | 1.00 | 1.00 |
| time (sec) | N/A | 0.055 | 0.519 | 0.179 | 0.940 | 0.312 | 2.781 | 0.360 | 1.308 |

| Problem 499 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 67 | 27 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 2.23 | 0.90 | 1.00 | 1.00 |
| time (sec) | N/A | 0.061 | 0.799 | 0.187 | 0.930 | 0.313 | 15.332 | 0.488 | 1.222 |

| Problem 500 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 34 | 28 | 30 | 0 | 29 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.06 | 0.88 | 0.94 | 0.00 | 0.91 | 0.94 | 0.94 |
| time (sec) | N/A | 0.227 | 1.135 | 0.180 | 11.059 | 0.000 | 3.559 | 1.014 | 1.335 |

| Problem 501 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 34 | 28 | 30 | 0 | 29 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.06 | 0.88 | 0.94 | 0.00 | 0.91 | 0.94 | 0.94 |
| time (sec) | N/A | 0.208 | 1.201 | 0.162 | 11.047 | 0.000 | 1.381 | 2.238 | 1.329 |

| Problem 502 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 34 | 28 | 30 | 0 | 29 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.06 | 0.88 | 0.94 | 0.00 | 0.91 | 0.94 | 0.94 |
| time (sec) | N/A | 0.214 | 1.711 | 0.161 | 11.013 | 0.000 | 17.419 | 1.412 | 1.365 |

| Problem 503 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 34 | 28 | 30 | 0 | 29 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.06 | 0.88 | 0.94 | 0.00 | 0.91 | 0.94 | 0.94 |
| time (sec) | N/A | 0.072 | 3.996 | 0.176 | 11.081 | 0.000 | 1.061 | 0.369 | 1.314 |

| Problem 504 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 34 | 28 | 30 | 0 | 31 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.06 | 0.88 | 0.94 | 0.00 | 0.97 | 0.94 | 0.94 |
| time (sec) | N/A | 0.076 | 0.207 | 0.168 | 11.028 | 0.000 | 3.851 | 0.354 | 1.344 |

| Problem 505 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 34 | 28 | 30 | 0 | 31 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.06 | 0.88 | 0.94 | 0.00 | 0.97 | 0.94 | 0.94 |
| time (sec) | N/A | 0.079 | 0.090 | 0.188 | 11.197 | 0.000 | 51.955 | 0.418 | 1.403 |

| Problem 506 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 86 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.070 | 0.080 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 507 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 28 | 30 | 30 | 24 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 1.07 | 1.07 | 0.86 | 1.07 | 1.07 |
| time (sec) | N/A | 0.040 | 0.234 | 0.195 | 0.930 | 0.273 | 13.569 | 0.352 | 1.182 |

| Problem 508 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 28 | 176 | 54 | 0 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 6.29 | 1.93 | 0.00 | 1.07 | 1.07 |
| time (sec) | N/A | 0.035 | 1.998 | 0.198 | 1.204 | 0.307 | 0.000 | 0.440 | 1.251 |

| Problem 509 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 56 | 0 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.87 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.069 | 15.262 | 0.204 | 11.313 | 0.296 | 0.000 | 3.232 | 1.326 |

| Problem 510 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 30 | 0 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.069 | 0.056 | 0.177 | 11.183 | 0.305 | 0.000 | 0.989 | 1.255 |

| Problem 511 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 30 | 27 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.00 | 0.90 | 1.00 | 1.00 |
| time (sec) | N/A | 0.063 | 10.606 | 0.207 | 10.975 | 0.332 | 5.938 | 0.348 | 1.273 |

| Problem 512 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 74 | 0 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 2.47 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.074 | 13.559 | 0.197 | 11.257 | 0.360 | 0.000 | 0.424 | 1.563 |

| Problem 513 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-2) | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 28 | 0 | 30 | 0 | 0 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 0.00 | 1.07 | 0.00 | 0.00 | 1.07 |
| time (sec) | N/A | 0.036 | 0.322 | 0.285 | 0.000 | 0.340 | 0.000 | 0.000 | 1.267 |

| Problem 514 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 432 | 432 | 326 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.613 | 0.606 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 515 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 281 | 227 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.366 | 0.178 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 516 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 131 | 0 | 0 | 80 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.61 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.103 | 0.097 | 0.000 | 0.000 | 0.082 | 0.000 | 0.000 | 0.000 |

| Problem 517 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 28 | 0 | 30 | 24 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 0.00 | 1.07 | 0.86 | 1.07 | 1.07 |
| time (sec) | N/A | 0.042 | 0.233 | 0.213 | 0.000 | 0.351 | 6.080 | 0.554 | 1.302 |

| Problem 518 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 249 | 249 | 190 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.353 | 0.095 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 519 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 335 | 335 | 284 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.540 | 0.144 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 520 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 515 | 515 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.678 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 521 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 287 | 257 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.669 | 0.129 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 522 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 519 | 519 | 637 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.961 | 10.981 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 523 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 427 | 427 | 386 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.536 | 0.581 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 524 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 258 | 258 | 231 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.355 | 0.189 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 525 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 120 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.221 | 0.073 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 526 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 67 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.079 | 0.003 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 527 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 165 | 117 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.317 | 0.045 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 528 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 268 | 268 | 225 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.414 | 0.134 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 529 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 425 | 425 | 363 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.559 | 0.261 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 530 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 519 | 519 | 927 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.908 | 0.405 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 531 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 240 | 240 | 852 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 3.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.426 | 0.233 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 532 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 324 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.180 | 0.046 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 533 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 288 | 288 | 652 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.605 | 0.176 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 534 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 463 | 463 | 654 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.785 | 0.408 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 535 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 742 | 742 | 4056 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 5.47 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.200 | 0.889 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 536 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 349 | 349 | 1769 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 5.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.623 | 0.475 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 537 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 177 | 646 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 3.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.272 | 0.098 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 538 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 410 | 410 | 1350 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 3.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.805 | 0.311 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 539 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 659 | 659 | 1057 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.175 | 0.733 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 540 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 35 | 33 | 35 | 41 | 27 | 35 | 35 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 1.06 | 1.24 | 0.82 | 1.06 | 1.06 |
| time (sec) | N/A | 0.169 | 0.135 | 0.180 | 1.550 | 0.309 | 3.554 | 0.465 | 1.344 |

| Problem 541 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 28 | 30 | 35 | 24 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 1.07 | 1.25 | 0.86 | 1.07 | 1.07 |
| time (sec) | N/A | 0.047 | 0.011 | 0.026 | 0.905 | 0.295 | 1.485 | 0.295 | 1.224 |

| Problem 542 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 37 | 35 | 37 | 65 | 29 | 37 | 37 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 1.06 | 1.86 | 0.83 | 1.06 | 1.06 |
| time (sec) | N/A | 0.194 | 0.614 | 0.330 | 0.884 | 0.343 | 3.728 | 0.291 | 1.241 |

| Problem 543 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 37 | 35 | 37 | 115 | 31 | 37 | 37 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 1.06 | 3.29 | 0.89 | 1.06 | 1.06 |
| time (sec) | N/A | 0.224 | 1.966 | 0.309 | 0.912 | 0.290 | 12.189 | 0.310 | 1.262 |

| Problem 544 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 35 | 33 | 302 | 77 | 29 | 35 | 35 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 9.15 | 2.33 | 0.88 | 1.06 | 1.06 |
| time (sec) | N/A | 0.194 | 1.360 | 0.171 | 2.060 | 0.295 | 16.904 | 0.306 | 1.479 |

| Problem 545 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 28 | 267 | 71 | 26 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 9.54 | 2.54 | 0.93 | 1.07 | 1.07 |
| time (sec) | N/A | 0.045 | 0.094 | 0.026 | 1.204 | 0.296 | 5.208 | 0.290 | 1.275 |

| Problem 546 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 37 | 35 | 701 | 126 | 31 | 37 | 37 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 20.03 | 3.60 | 0.89 | 1.06 | 1.06 |
| time (sec) | N/A | 0.192 | 13.246 | 0.319 | 1.269 | 0.330 | 16.727 | 0.375 | 1.347 |

| Problem 547 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 37 | 35 | 1039 | 211 | 32 | 37 | 37 |
| N.S. | 1 | 1.00 | 1.06 | 1.00 | 29.69 | 6.03 | 0.91 | 1.06 | 1.06 |
| time (sec) | N/A | 0.220 | 19.109 | 0.323 | 1.272 | 0.330 | 112.691 | 0.446 | 1.419 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [397] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 5 | 3 | 1.00 | 10 | 0.300 |
| 2 | A | 4 | 3 | 1.00 | 10 | 0.300 |
| 3 | A | 3 | 3 | 1.00 | 10 | 0.300 |
| 4 | A | 2 | 2 | 1.00 | 8 | 0.250 |
| 5 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 6 | A | 3 | 3 | 1.00 | 10 | 0.300 |
| 7 | A | 4 | 3 | 1.00 | 10 | 0.300 |
| 8 | A | 5 | 3 | 1.00 | 10 | 0.300 |
| 9 | A | 7 | 5 | 1.00 | 12 | 0.417 |
| 10 | A | 6 | 5 | 1.00 | 12 | 0.417 |
| 11 | A | 5 | 5 | 1.00 | 12 | 0.417 |
| 12 | A | 4 | 4 | 1.00 | 12 | 0.333 |
| 13 | A | 5 | 5 | 1.00 | 12 | 0.417 |
| 14 | A | 6 | 5 | 1.00 | 12 | 0.417 |
| 15 | A | 7 | 5 | 1.00 | 12 | 0.417 |
| 16 | A | 3 | 3 | 1.00 | 10 | 0.300 |
| 17 | A | 6 | 3 | 1.00 | 16 | 0.188 |
| 18 | A | 5 | 3 | 1.00 | 16 | 0.188 |
| 19 | A | 4 | 3 | 1.00 | 16 | 0.188 |
| 20 | A | 3 | 2 | 1.00 | 14 | 0.143 |
| 21 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 22 | A | 4 | 4 | 1.00 | 16 | 0.250 |
| 23 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 24 | A | 7 | 5 | 1.00 | 18 | 0.278 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 25 | A | 6 | 5 | 1.00 | 18 | 0.278 |
| 26 | A | 5 | 5 | 1.00 | 18 | 0.278 |
| 27 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 28 | A | 5 | 5 | 1.00 | 18 | 0.278 |
| 29 | A | 6 | 5 | 1.00 | 18 | 0.278 |
| 30 | A | 7 | 5 | 1.00 | 18 | 0.278 |
| 31 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 32 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 33 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 34 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 35 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 36 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 37 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 38 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 39 | A | 3 | 2 | 1.00 | 14 | 0.143 |
| 40 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 41 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 42 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 43 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 44 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 45 | A | 8 | 7 | 1.00 | 24 | 0.292 |
| 46 | A | 9 | 7 | 1.00 | 22 | 0.318 |
| 47 | A | 4 | 3 | 1.00 | 16 | 0.188 |
| 48 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 49 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 50 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 51 | A | 11 | 9 | 1.00 | 24 | 0.375 |
| 52 | A | 19 | 7 | 1.00 | 24 | 0.292 |
| 53 | A | 15 | 7 | 1.00 | 24 | 0.292 |
| 54 | A | 11 | 7 | 1.00 | 22 | 0.318 |
| 55 | A | 5 | 3 | 1.00 | 16 | 0.188 |
| 56 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 57 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 58 | A | 9 | 9 | 1.00 | 24 | 0.375 |
| 59 | A | 16 | 12 | 1.00 | 24 | 0.500 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 60 | A | 13 | 7 | 1.00 | 22 | 0.318 |
| 61 | A | 6 | 3 | 1.00 | 16 | 0.188 |
| 62 | A | 6 | 5 | 1.00 | 24 | 0.208 |
| 63 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 64 | A | 2 | 2 | 1.00 | 6 | 0.333 |
| 65 | A | 3 | 3 | 1.00 | 8 | 0.375 |
| 66 | A | 4 | 3 | 1.00 | 8 | 0.375 |
| 67 | A | 3 | 3 | 1.00 | 9 | 0.333 |
| 68 | A | 4 | 4 | 1.00 | 11 | 0.364 |
| 69 | A | 5 | 4 | 1.00 | 11 | 0.364 |
| 70 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 71 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 72 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 73 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 74 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 75 | A | 1 | 1 | 1.00 | 10 | 0.100 |
| 76 | A | 1 | 1 | 1.00 | 8 | 0.125 |
| 77 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 78 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 79 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 80 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 81 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 82 | A | 1 | 1 | 1.00 | 12 | 0.083 |
| 83 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 84 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 85 | A | 7 | 7 | 1.00 | 16 | 0.438 |
| 86 | A | 11 | 9 | 1.00 | 16 | 0.562 |
| 87 | A | 14 | 7 | 1.00 | 16 | 0.438 |
| 88 | A | 14 | 6 | 1.00 | 24 | 0.250 |
| 89 | A | 11 | 6 | 1.00 | 24 | 0.250 |
| 90 | A | 8 | 6 | 1.00 | 22 | 0.273 |
| 91 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 92 | N/A | 0 | 0 | 1.00 | 24 | 0.000 |
| 93 | N/A | 0 | 0 | 1.00 | 24 | 0.000 |
| 94 | A | 26 | 7 | 1.00 | 24 | 0.292 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 95 | A | 20 | 7 | 1.00 | 24 | 0.292 |
| 96 | A | 12 | 7 | 1.00 | 22 | 0.318 |
| 97 | A | 4 | 4 | 1.00 | 16 | 0.250 |
| 98 | N/A | 0 | 0 | 1.00 | 24 | 0.000 |
| 99 | N/A | 0 | 0 | 1.00 | 24 | 0.000 |
| 100 | A | 33 | 7 | 1.00 | 24 | 0.292 |
| 101 | A | 17 | 8 | 1.00 | 22 | 0.364 |
| 102 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 103 | N/A | 0 | 0 | 1.00 | 24 | 0.000 |
| 104 | N/A | 0 | 0 | 1.00 | 24 | 0.000 |
| 105 | A | 17 | 9 | 1.00 | 26 | 0.346 |
| 106 | A | 12 | 9 | 1.00 | 24 | 0.375 |
| 107 | A | 5 | 5 | 1.00 | 18 | 0.278 |
| 108 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 109 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 110 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 111 | A | 20 | 9 | 1.00 | 26 | 0.346 |
| 112 | A | 14 | 9 | 1.00 | 24 | 0.375 |
| 113 | A | 6 | 5 | 1.00 | 18 | 0.278 |
| 114 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 115 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 116 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 117 | A | 23 | 9 | 1.00 | 26 | 0.346 |
| 118 | A | 16 | 9 | 1.00 | 24 | 0.375 |
| 119 | A | 7 | 5 | 1.00 | 18 | 0.278 |
| 120 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 121 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 122 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 123 | A | 18 | 7 | 1.00 | 26 | 0.269 |
| 124 | A | 14 | 7 | 1.00 | 26 | 0.269 |
| 125 | A | 10 | 7 | 1.00 | 24 | 0.292 |
| 126 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 127 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 128 | A | 33 | 8 | 1.00 | 26 | 0.308 |
| 129 | A | 25 | 8 | 1.00 | 26 | 0.308 |
| 130 | A | 15 | 8 | 1.00 | 24 | 0.333 |
| 131 | A | 5 | 5 | 1.00 | 18 | 0.278 |
| 132 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 133 | A | 59 | 8 | 1.00 | 26 | 0.308 |
| 134 | A | 41 | 8 | 1.00 | 26 | 0.308 |
| 135 | A | 21 | 9 | 1.00 | 24 | 0.375 |
| 136 | A | 6 | 5 | 1.00 | 18 | 0.278 |
| 137 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 138 | A | 6 | 4 | 1.00 | 24 | 0.167 |
| 139 | A | 5 | 4 | 1.00 | 24 | 0.167 |
| 140 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 141 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 142 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 143 | A | 5 | 4 | 1.00 | 24 | 0.167 |
| 144 | A | 6 | 4 | 1.00 | 24 | 0.167 |
| 145 | A | 28 | 15 | 1.00 | 26 | 0.577 |
| 146 | A | 21 | 15 | 1.00 | 26 | 0.577 |
| 147 | A | 15 | 15 | 1.00 | 26 | 0.577 |
| 148 | A | 10 | 12 | 1.00 | 26 | 0.462 |
| 149 | A | 14 | 14 | 1.00 | 26 | 0.538 |
| 150 | A | 19 | 15 | 1.00 | 26 | 0.577 |
| 151 | A | 25 | 15 | 1.00 | 26 | 0.577 |
| 152 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 153 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 154 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 155 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 156 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 157 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 158 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 159 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 160 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 161 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 162 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 163 | N/A | 0 | 0 | 1.00 | 24 | 0.000 |
| 164 | N/A | 0 | 0 | 1.00 | 24 | 0.000 |
| 165 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 166 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 167 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 168 | N/A | 0 | 0 | 1.00 | 26 | 0.000 |
| 169 | N/A | 0 | 0 | 1.00 | 24 | 0.000 |
| 170 | A | 14 | 6 | 1.00 | 24 | 0.250 |
| 171 | A | 11 | 6 | 1.00 | 24 | 0.250 |
| 172 | A | 8 | 6 | 1.00 | 22 | 0.273 |
| 173 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 174 | N/A | 0 | 0 | 1.00 | 24 | 0.000 |
| 175 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 176 | A | 8 | 6 | 1.00 | 30 | 0.200 |
| 177 | A | 7 | 6 | 1.00 | 30 | 0.200 |
| 178 | A | 6 | 5 | 1.00 | 28 | 0.179 |
| 179 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 180 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 181 | A | 7 | 7 | 1.00 | 30 | 0.233 |
| 182 | A | 11 | 9 | 1.00 | 30 | 0.300 |
| 183 | A | 30 | 15 | 1.00 | 32 | 0.469 |
| 184 | A | 24 | 15 | 1.00 | 32 | 0.469 |
| 185 | A | 16 | 10 | 1.00 | 32 | 0.312 |
| 186 | A | 8 | 7 | 1.00 | 30 | 0.233 |
| 187 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 188 | A | 5 | 5 | 1.00 | 32 | 0.156 |
| 189 | A | 9 | 9 | 1.00 | 32 | 0.281 |
| 190 | A | 16 | 12 | 1.00 | 32 | 0.375 |
| 191 | A | 14 | 8 | 1.00 | 32 | 0.250 |
| 192 | A | 12 | 8 | 1.00 | 32 | 0.250 |
| 193 | A | 10 | 8 | 1.00 | 32 | 0.250 |
| 194 | A | 8 | 7 | 1.00 | 30 | 0.233 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 195 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 196 | N/A | 0 | 0 | 1.00 | 32 | 0.000 |
| 197 | N/A | 0 | 0 | 1.00 | 32 | 0.000 |
| 198 | A | 27 | 14 | 1.00 | 31 | 0.452 |
| 199 | A | 20 | 14 | 1.00 | 31 | 0.452 |
| 200 | A | 14 | 14 | 1.00 | 31 | 0.452 |
| 201 | A | 9 | 11 | 1.00 | 31 | 0.355 |
| 202 | A | 13 | 13 | 1.00 | 31 | 0.419 |
| 203 | A | 18 | 14 | 1.00 | 31 | 0.452 |
| 204 | A | 20 | 14 | 1.00 | 23 | 0.609 |
| 205 | A | 14 | 14 | 1.00 | 23 | 0.609 |
| 206 | A | 9 | 11 | 1.00 | 23 | 0.478 |
| 207 | A | 13 | 13 | 1.00 | 23 | 0.565 |
| 208 | A | 18 | 14 | 1.00 | 23 | 0.609 |
| 209 | N/A | 0 | 0 | 1.00 | 32 | 0.000 |
| 210 | A | 12 | 8 | 1.00 | 32 | 0.250 |
| 211 | A | 10 | 8 | 1.00 | 32 | 0.250 |
| 212 | A | 8 | 7 | 1.00 | 30 | 0.233 |
| 213 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 214 | N/A | 0 | 0 | 1.00 | 32 | 0.000 |
| 215 | N/A | 0 | 0 | 1.00 | 32 | 0.000 |
| 216 | N/A | 0 | 0 | 1.00 | 32 | 0.000 |
| 217 | A | 14 | 8 | 1.00 | 29 | 0.276 |
| 218 | A | 11 | 8 | 1.00 | 29 | 0.276 |
| 219 | A | 8 | 6 | 1.00 | 27 | 0.222 |
| 220 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 221 | A | 8 | 4 | 1.00 | 29 | 0.138 |
| 222 | A | 12 | 7 | 1.00 | 29 | 0.241 |
| 223 | A | 15 | 8 | 1.00 | 29 | 0.276 |
| 224 | A | 19 | 12 | 1.00 | 31 | 0.387 |
| 225 | A | 10 | 8 | 1.00 | 29 | 0.276 |
| 226 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 227 | A | 10 | 5 | 1.00 | 31 | 0.161 |
| 228 | A | 14 | 9 | 1.00 | 31 | 0.290 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 229 | A | 23 | 13 | 1.00 | 31 | 0.419 |
| 230 | A | 12 | 9 | 1.00 | 29 | 0.310 |
| 231 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 232 | A | 12 | 6 | 1.00 | 31 | 0.194 |
| 233 | A | 17 | 7 | 1.00 | 31 | 0.226 |
| 234 | N/A | 0 | 0 | 1.00 | 29 | 0.000 |
| 235 | N/A | 0 | 0 | 1.00 | 24 | 0.000 |
| 236 | N/A | 0 | 0 | 1.00 | 31 | 0.000 |
| 237 | N/A | 0 | 0 | 1.00 | 31 | 0.000 |
| 238 | N/A | 0 | 0 | 1.00 | 29 | 0.000 |
| 239 | N/A | 0 | 0 | 1.00 | 24 | 0.000 |
| 240 | N/A | 0 | 0 | 1.00 | 31 | 0.000 |
| 241 | N/A | 0 | 0 | 1.00 | 31 | 0.000 |
| 242 | A | 14 | 8 | 1.00 | 25 | 0.320 |
| 243 | A | 11 | 8 | 1.00 | 25 | 0.320 |
| 244 | A | 8 | 7 | 1.00 | 23 | 0.304 |
| 245 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 246 | A | 7 | 8 | 1.00 | 25 | 0.320 |
| 247 | A | 11 | 10 | 1.00 | 25 | 0.400 |
| 248 | A | 14 | 10 | 1.00 | 25 | 0.400 |
| 249 | A | 15 | 10 | 1.00 | 25 | 0.400 |
| 250 | A | 12 | 10 | 1.00 | 25 | 0.400 |
| 251 | A | 9 | 8 | 1.00 | 23 | 0.348 |
| 252 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 253 | A | 11 | 9 | 1.00 | 25 | 0.360 |
| 254 | A | 15 | 10 | 1.00 | 25 | 0.400 |
| 255 | A | 18 | 10 | 1.00 | 25 | 0.400 |
| 256 | A | 16 | 8 | 1.00 | 27 | 0.296 |
| 257 | A | 13 | 8 | 1.00 | 27 | 0.296 |
| 258 | A | 8 | 5 | 1.00 | 25 | 0.200 |
| 259 | A | 12 | 10 | 1.00 | 27 | 0.370 |
| 260 | A | 15 | 9 | 1.00 | 27 | 0.333 |
| 261 | A | 16 | 11 | 1.00 | 27 | 0.407 |
| 262 | A | 13 | 9 | 1.00 | 27 | 0.333 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 263 | A | 8 | 4 | 1.00 | 24 | 0.167 |
| 264 | A | 14 | 11 | 1.00 | 27 | 0.407 |
| 265 | A | 17 | 12 | 1.00 | 27 | 0.444 |
| 266 | A | 19 | 13 | 1.00 | 27 | 0.482 |
| 267 | A | 16 | 12 | 1.00 | 27 | 0.444 |
| 268 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 269 | A | 18 | 13 | 1.00 | 27 | 0.482 |
| 270 | A | 21 | 14 | 1.00 | 27 | 0.518 |
| 271 | A | 31 | 13 | 1.00 | 27 | 0.482 |
| 272 | A | 28 | 10 | 1.00 | 27 | 0.370 |
| 273 | A | 18 | 7 | 1.00 | 24 | 0.292 |
| 274 | A | 32 | 12 | 1.00 | 27 | 0.444 |
| 275 | A | 10 | 8 | 1.00 | 26 | 0.308 |
| 276 | A | 11 | 9 | 1.00 | 26 | 0.346 |
| 277 | A | 9 | 7 | 1.00 | 34 | 0.206 |
| 278 | A | 11 | 9 | 1.00 | 34 | 0.265 |
| 279 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 280 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 281 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 282 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 283 | A | 16 | 8 | 1.00 | 19 | 0.421 |
| 284 | A | 11 | 5 | 1.00 | 19 | 0.263 |
| 285 | A | 15 | 10 | 1.00 | 19 | 0.526 |
| 286 | A | 18 | 9 | 1.00 | 19 | 0.474 |
| 287 | A | 16 | 13 | 1.00 | 19 | 0.684 |
| 288 | A | 15 | 14 | 1.00 | 19 | 0.737 |
| 289 | A | 11 | 10 | 1.00 | 17 | 0.588 |
| 290 | A | 11 | 4 | 1.00 | 16 | 0.250 |
| 291 | A | 17 | 14 | 1.00 | 19 | 0.737 |
| 292 | A | 16 | 14 | 1.00 | 19 | 0.737 |
| 293 | A | 23 | 8 | 1.00 | 19 | 0.421 |
| 294 | A | 18 | 5 | 1.00 | 19 | 0.263 |
| 295 | A | 22 | 10 | 1.00 | 19 | 0.526 |
| 296 | A | 23 | 10 | 1.00 | 19 | 0.526 |
| 297 | A | 18 | 7 | 1.00 | 17 | 0.412 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 298 | A | 23 | 10 | 1.00 | 19 | 0.526 |
| 299 | A | 22 | 14 | 1.00 | 19 | 0.737 |
| 300 | A | 18 | 11 | 1.00 | 19 | 0.579 |
| 301 | A | 18 | 4 | 1.00 | 16 | 0.250 |
| 302 | A | 24 | 16 | 1.00 | 19 | 0.842 |
| 303 | A | 4 | 3 | 1.00 | 23 | 0.130 |
| 304 | A | 4 | 3 | 1.00 | 27 | 0.111 |
| 305 | A | 4 | 3 | 1.00 | 27 | 0.111 |
| 306 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 307 | A | 5 | 4 | 1.00 | 27 | 0.148 |
| 308 | A | 4 | 3 | 1.00 | 27 | 0.111 |
| 309 | A | 12 | 6 | 1.00 | 16 | 0.375 |
| 310 | A | 28 | 19 | 1.00 | 29 | 0.655 |
| 311 | A | 21 | 12 | 1.00 | 29 | 0.414 |
| 312 | A | 10 | 5 | 1.00 | 27 | 0.185 |
| 313 | A | 16 | 5 | 1.00 | 29 | 0.172 |
| 314 | A | 23 | 12 | 1.00 | 29 | 0.414 |
| 315 | A | 23 | 16 | 1.00 | 29 | 0.552 |
| 316 | A | 16 | 9 | 1.00 | 29 | 0.310 |
| 317 | A | 10 | 5 | 1.00 | 26 | 0.192 |
| 318 | A | 15 | 9 | 1.00 | 29 | 0.310 |
| 319 | A | 26 | 18 | 1.00 | 29 | 0.621 |
| 320 | A | 34 | 18 | 1.00 | 29 | 0.621 |
| 321 | A | 25 | 12 | 1.00 | 29 | 0.414 |
| 322 | A | 13 | 7 | 1.00 | 27 | 0.259 |
| 323 | A | 29 | 12 | 1.00 | 29 | 0.414 |
| 324 | A | 36 | 18 | 1.00 | 29 | 0.621 |
| 325 | A | 36 | 13 | 1.00 | 29 | 0.448 |
| 326 | A | 32 | 10 | 1.00 | 29 | 0.345 |
| 327 | A | 20 | 9 | 1.00 | 26 | 0.346 |
| 328 | A | 35 | 11 | 1.00 | 29 | 0.379 |
| 329 | A | 12 | 6 | 1.00 | 22 | 0.273 |
| 330 | A | 10 | 5 | 1.00 | 22 | 0.227 |
| 331 | A | 8 | 4 | 1.00 | 20 | 0.200 |
| 332 | N/A | 0 | 0 | 1.00 | 22 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 333 | A | 4 | 4 | 1.00 | 32 | 0.125 |
| 334 | A | 5 | 5 | 1.00 | 36 | 0.139 |
| 335 | A | 4 | 4 | 1.00 | 38 | 0.105 |
| 336 | A | 5 | 5 | 1.00 | 38 | 0.132 |
| 337 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 338 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 339 | A | 1 | 1 | 1.00 | 38 | 0.026 |
| 340 | A | 2 | 2 | 1.00 | 39 | 0.051 |
| 341 | A | 1 | 1 | 1.00 | 34 | 0.029 |
| 342 | A | 3 | 3 | 1.00 | 35 | 0.086 |
| 343 | A | 13 | 7 | 1.00 | 24 | 0.292 |
| 344 | A | 11 | 6 | 1.00 | 24 | 0.250 |
| 345 | A | 8 | 9 | 1.00 | 22 | 0.409 |
| 346 | N/A | 0 | 0 | 1.00 | 24 | 0.000 |
| 347 | A | 12 | 6 | 1.00 | 25 | 0.240 |
| 348 | A | 10 | 5 | 1.00 | 25 | 0.200 |
| 349 | A | 8 | 4 | 1.00 | 23 | 0.174 |
| 350 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 351 | A | 10 | 5 | 1.00 | 18 | 0.278 |
| 352 | A | 9 | 4 | 1.00 | 18 | 0.222 |
| 353 | A | 6 | 3 | 1.00 | 16 | 0.188 |
| 354 | A | 6 | 3 | 1.00 | 15 | 0.200 |
| 355 | A | 9 | 4 | 1.00 | 18 | 0.222 |
| 356 | A | 10 | 5 | 1.00 | 18 | 0.278 |
| 357 | A | 11 | 5 | 1.00 | 18 | 0.278 |
| 358 | A | 11 | 7 | 1.00 | 24 | 0.292 |
| 359 | A | 10 | 7 | 1.00 | 24 | 0.292 |
| 360 | A | 9 | 7 | 1.00 | 22 | 0.318 |
| 361 | A | 8 | 6 | 1.00 | 21 | 0.286 |
| 362 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 363 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 364 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 365 | A | 9 | 6 | 1.00 | 24 | 0.250 |
| 366 | A | 11 | 6 | 1.00 | 24 | 0.250 |
| 367 | A | 52 | 19 | 1.22 | 26 | 0.731 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 368 | A | 38 | 16 | 1.00 | 24 | 0.667 |
| 369 | A | 17 | 12 | 1.00 | 23 | 0.522 |
| 370 | F | 0 | 0 | N/A | 0.000 | N/A |
| 371 | F | 0 | 0 | N/A | 0.000 | N/A |
| 372 | F | 0 | 0 | N/A | 0.000 | N/A |
| 373 | A | 28 | 13 | 1.00 | 23 | 0.565 |
| 374 | F | 0 | 0 | N/A | 0.000 | N/A |
| 375 | N/A | 0 | 0 | 1.00 | 23 | 0.000 |
| 376 | N/A | 0 | 0 | 1.00 | 23 | 0.000 |
| 377 | N/A | 0 | 0 | 1.00 | 23 | 0.000 |
| 378 | A | 1 | 1 | 1.00 | 16 | 0.062 |
| 379 | A | 7 | 7 | 1.00 | 32 | 0.219 |
| 380 | A | 7 | 7 | 1.00 | 30 | 0.233 |
| 381 | A | 6 | 5 | 1.00 | 29 | 0.172 |
| 382 | A | 6 | 6 | 1.00 | 32 | 0.188 |
| 383 | A | 4 | 4 | 1.00 | 32 | 0.125 |
| 384 | A | 7 | 7 | 1.00 | 32 | 0.219 |
| 385 | A | 11 | 9 | 1.00 | 32 | 0.281 |
| 386 | A | 35 | 9 | 1.00 | 32 | 0.281 |
| 387 | A | 29 | 9 | 1.00 | 32 | 0.281 |
| 388 | A | 23 | 9 | 1.00 | 30 | 0.300 |
| 389 | A | 17 | 8 | 1.00 | 29 | 0.276 |
| 390 | A | 13 | 5 | 1.00 | 32 | 0.156 |
| 391 | A | 15 | 9 | 1.00 | 32 | 0.281 |
| 392 | A | 23 | 11 | 1.00 | 32 | 0.344 |
| 393 | A | 73 | 27 | 1.00 | 32 | 0.844 |
| 394 | A | 41 | 19 | 1.00 | 31 | 0.613 |
| 395 | N/A | 0 | 0 | 1.00 | 34 | 0.000 |
| 396 | N/A | 0 | 0 | 1.00 | 34 | 0.000 |
| 397 | A | 148 | 32 | 1.00 | 32 | 1.000 |
| 398 | A | 64 | 22 | 1.00 | 31 | 0.710 |
| 399 | N/A | 0 | 0 | 1.00 | 34 | 0.000 |
| 400 | N/A | 0 | 0 | 1.00 | 34 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 401 | A | 3 | 3 | 1.00 | 40 | 0.075 |
| 402 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 403 | A | 4 | 4 | 1.00 | 32 | 0.125 |
| 404 | A | 7 | 4 | 1.00 | 20 | 0.200 |
| 405 | A | 6 | 4 | 1.00 | 20 | 0.200 |
| 406 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 407 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 408 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 409 | A | 5 | 5 | 1.00 | 20 | 0.250 |
| 410 | A | 6 | 5 | 1.00 | 20 | 0.250 |
| 411 | A | 8 | 6 | 1.00 | 22 | 0.273 |
| 412 | A | 7 | 6 | 1.00 | 22 | 0.273 |
| 413 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 414 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 415 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 416 | A | 7 | 6 | 1.00 | 22 | 0.273 |
| 417 | A | 8 | 6 | 1.00 | 22 | 0.273 |
| 418 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 419 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 420 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 421 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 422 | A | 4 | 3 | 1.00 | 24 | 0.125 |
| 423 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 424 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 425 | A | 5 | 4 | 1.00 | 26 | 0.154 |
| 426 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 427 | A | 4 | 3 | 1.00 | 26 | 0.115 |
| 428 | A | 7 | 7 | 1.00 | 28 | 0.250 |
| 429 | A | 9 | 8 | 1.00 | 28 | 0.286 |
| 430 | A | 10 | 8 | 1.00 | 26 | 0.308 |
| 431 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 432 | A | 5 | 5 | 1.00 | 28 | 0.179 |
| 433 | A | 5 | 5 | 1.00 | 28 | 0.179 |
| 434 | A | 8 | 8 | 1.00 | 28 | 0.286 |
| 435 | A | 16 | 8 | 1.00 | 28 | 0.286 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 436 | A | 12 | 8 | 1.00 | 26 | 0.308 |
| 437 | A | 6 | 4 | 1.00 | 20 | 0.200 |
| 438 | A | 6 | 6 | 1.00 | 28 | 0.214 |
| 439 | A | 6 | 6 | 1.00 | 28 | 0.214 |
| 440 | A | 10 | 10 | 1.00 | 28 | 0.357 |
| 441 | A | 7 | 4 | 1.00 | 20 | 0.200 |
| 442 | A | 7 | 6 | 1.00 | 28 | 0.214 |
| 443 | A | 7 | 7 | 1.00 | 28 | 0.250 |
| 444 | A | 3 | 3 | 1.00 | 14 | 0.214 |
| 445 | A | 12 | 7 | 1.00 | 28 | 0.250 |
| 446 | A | 9 | 7 | 1.00 | 26 | 0.269 |
| 447 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 448 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 449 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 450 | A | 21 | 8 | 1.00 | 28 | 0.286 |
| 451 | A | 13 | 8 | 1.00 | 26 | 0.308 |
| 452 | A | 5 | 5 | 1.00 | 20 | 0.250 |
| 453 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 454 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 455 | A | 34 | 8 | 1.00 | 28 | 0.286 |
| 456 | A | 18 | 9 | 1.00 | 26 | 0.346 |
| 457 | A | 6 | 5 | 1.00 | 20 | 0.250 |
| 458 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 459 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 460 | A | 18 | 10 | 1.00 | 30 | 0.333 |
| 461 | A | 13 | 10 | 1.00 | 28 | 0.357 |
| 462 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 463 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 464 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 465 | A | 21 | 10 | 1.00 | 30 | 0.333 |
| 466 | A | 15 | 10 | 1.00 | 28 | 0.357 |
| 467 | A | 7 | 6 | 1.00 | 22 | 0.273 |
| 468 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 469 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 470 | A | 15 | 8 | 1.00 | 30 | 0.267 |
| 471 | A | 11 | 8 | 1.00 | 28 | 0.286 |
| 472 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 473 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 474 | A | 26 | 9 | 1.00 | 30 | 0.300 |
| 475 | A | 16 | 9 | 1.00 | 28 | 0.321 |
| 476 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 477 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 478 | A | 42 | 9 | 1.00 | 30 | 0.300 |
| 479 | A | 22 | 10 | 1.00 | 28 | 0.357 |
| 480 | A | 7 | 6 | 1.00 | 22 | 0.273 |
| 481 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 482 | A | 7 | 5 | 1.00 | 28 | 0.179 |
| 483 | A | 6 | 5 | 1.00 | 28 | 0.179 |
| 484 | A | 5 | 5 | 1.00 | 28 | 0.179 |
| 485 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 486 | A | 5 | 5 | 1.00 | 28 | 0.179 |
| 487 | A | 6 | 5 | 1.00 | 28 | 0.179 |
| 488 | A | 7 | 5 | 1.00 | 28 | 0.179 |
| 489 | A | 29 | 16 | 1.00 | 30 | 0.533 |
| 490 | A | 22 | 16 | 1.00 | 30 | 0.533 |
| 491 | A | 16 | 16 | 1.00 | 30 | 0.533 |
| 492 | A | 11 | 13 | 1.00 | 30 | 0.433 |
| 493 | A | 15 | 15 | 1.00 | 30 | 0.500 |
| 494 | A | 20 | 16 | 1.00 | 30 | 0.533 |
| 495 | A | 26 | 16 | 1.00 | 30 | 0.533 |
| 496 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 497 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 498 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 499 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 500 | N/A | 0 | 0 | 1.00 | 32 | 0.000 |
| 501 | N/A | 0 | 0 | 1.00 | 32 | 0.000 |
| 502 | N/A | 0 | 0 | 1.00 | 32 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 503 | N/A | 0 | 0 | 1.00 | 32 | 0.000 |
| 504 | N/A | 0 | 0 | 1.00 | 32 | 0.000 |
| 505 | N/A | 0 | 0 | 1.00 | 32 | 0.000 |
| 506 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 507 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 508 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 509 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 510 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 511 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 512 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 513 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 514 | A | 12 | 7 | 1.00 | 28 | 0.250 |
| 515 | A | 9 | 7 | 1.00 | 26 | 0.269 |
| 516 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 517 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 518 | A | 9 | 5 | 1.00 | 28 | 0.179 |
| 519 | A | 11 | 9 | 1.00 | 30 | 0.300 |
| 520 | A | 12 | 10 | 1.00 | 30 | 0.333 |
| 521 | A | 10 | 8 | 1.00 | 38 | 0.210 |
| 522 | A | 12 | 10 | 1.00 | 38 | 0.263 |
| 523 | A | 15 | 9 | 1.00 | 33 | 0.273 |
| 524 | A | 12 | 9 | 1.00 | 33 | 0.273 |
| 525 | A | 9 | 7 | 1.00 | 31 | 0.226 |
| 526 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 527 | A | 9 | 5 | 1.00 | 33 | 0.152 |
| 528 | A | 13 | 8 | 1.00 | 33 | 0.242 |
| 529 | A | 16 | 9 | 1.00 | 33 | 0.273 |
| 530 | A | 20 | 13 | 1.00 | 35 | 0.371 |
| 531 | A | 11 | 9 | 1.00 | 33 | 0.273 |
| 532 | A | 5 | 5 | 1.00 | 28 | 0.179 |
| 533 | A | 11 | 6 | 1.00 | 35 | 0.171 |
| 534 | A | 15 | 10 | 1.00 | 35 | 0.286 |
| 535 | A | 24 | 14 | 1.00 | 35 | 0.400 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 536 | A | 13 | 10 | 1.00 | 33 | 0.303 |
| 537 | A | 6 | 6 | 1.00 | 28 | 0.214 |
| 538 | A | 13 | 7 | 1.00 | 35 | 0.200 |
| 539 | A | 18 | 8 | 1.00 | 35 | 0.229 |
| 540 | N/A | 0 | 0 | 1.00 | 33 | 0.000 |
| 541 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 542 | N/A | 0 | 0 | 1.00 | 35 | 0.000 |
| 543 | N/A | 0 | 0 | 1.00 | 35 | 0.000 |
| 544 | N/A | 0 | 0 | 1.00 | 33 | 0.000 |
| 545 | N/A | 0 | 0 | 1.00 | 28 | 0.000 |
| 546 | N/A | 0 | 0 | 1.00 | 35 | 0.000 |
| 547 | N/A | 0 | 0 | 1.00 | 35 | 0.000 |

# CHAPTER 3

## LISTING OF INTEGRALS

# 3.1 $\int \log^4(c(d+ex))\,dx$



## Optimal result

Integrand size = 10, antiderivative size = 81

$$\int \log^4(c(d+ex))\,dx = 24x-\frac{24(d+ex)\log(c(d+ex))}{e}+\frac{12(d+ex)\log^2(c(d+ex))}{e}-\frac{4(d+ex)\log^3(c(d+ex))}{e}+\frac{(d+ex)\log^4(c(d+ex))}{e}$$

```
[Out] 24*x-24*(e*x+d)*ln(c*(e*x+d))/e+12*(e*x+d)*ln(c*(e*x+d))^2/e-4*(e*x+d)*ln(c
*(e*x+d))^3/e+(e*x+d)*ln(c*(e*x+d))^4/e
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2436, 2333, 2332}

$$\int \log^4(c(d+ex))\,dx = \frac{(d+ex)\log^4(c(d+ex))}{e}-\frac{4(d+ex)\log^3(c(d+ex))}{e}+\frac{12(d+ex)\log^2(c(d+ex))}{e}-\frac{24(d+ex)\log(c(d+ex))}{e}+24x$$

```
[In] Int[Log[c*(d + e*x)]^4,x]
```

```
[Out] 24*x - (24*(d + e*x)*Log[c*(d + e*x)])/e + (12*(d + e*x)*Log[c*(d + e*x)]^2
)/e - (4*(d + e*x)*Log[c*(d + e*x)]^3)/e + ((d + e*x)*Log[c*(d + e*x)]^4)/e
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \log^4(cx)\,dx, x, d+ex\right)}{e}\\
&= \frac{(d+ex)\log^4(c(d+ex))}{e} - \frac{4\text{Subst}\left(\int \log^3(cx)\,dx, x, d+ex\right)}{e}\\
&= -\frac{4(d+ex)\log^3(c(d+ex))}{e} + \frac{(d+ex)\log^4(c(d+ex))}{e} + \frac{12\text{Subst}\left(\int \log^2(cx)\,dx, x, d+ex\right)}{e}\\
&= \frac{12(d+ex)\log^2(c(d+ex))}{e} - \frac{4(d+ex)\log^3(c(d+ex))}{e}\\
&\quad + \frac{(d+ex)\log^4(c(d+ex))}{e} - \frac{24\text{Subst}\left(\int \log(cx)\,dx, x, d+ex\right)}{e}\\
&= 24x - \frac{24(d+ex)\log(c(d+ex))}{e} + \frac{12(d+ex)\log^2(c(d+ex))}{e}\\
&\quad - \frac{4(d+ex)\log^3(c(d+ex))}{e} + \frac{(d+ex)\log^4(c(d+ex))}{e}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \log^4(c(d+ex))\,dx = \frac{24ex - 24(d+ex)\log(c(d+ex)) + 12(d+ex)\log^2(c(d+ex)) - 4(d+ex)\log^3(c(d+ex)) + (d+ex)\log^4}{e}$$

```
[In] Integrate[Log[c*(d + e*x)]^4,x]
```

```
[Out] (24*e*x - 24*(d + e*x)*Log[c*(d + e*x)] + 12*(d + e*x)*Log[c*(d + e*x)]^2 -
 4*(d + e*x)*Log[c*(d + e*x)]^3 + (d + e*x)*Log[c*(d + e*x)]^4)/e
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

| method | result |
|---|---|
| risch | $\frac{(ex+d)\ln(c(ex+d))^4}{e}-\frac{4(ex+d)\ln(c(ex+d))^3}{e}+\frac{12(ex+d)\ln(c(ex+d))^2}{e}-24x\ln\left(c(ex+d)\right)+24x-$ |
| derivativedivides | $\frac{\ln(cex+cd)^4(cex+cd)-4(cex+cd)\ln(cex+cd)^3+12(cex+cd)\ln(cex+cd)^2-24(cex+cd)\ln(cex+cd)+24cex+24cd}{ce}$ |
| default | $\frac{\ln(cex+cd)^4(cex+cd)-4(cex+cd)\ln(cex+cd)^3+12(cex+cd)\ln(cex+cd)^2-24(cex+cd)\ln(cex+cd)+24cex+24cd}{ce}$ |
| norman | $x\ln\left(c(ex+d)\right)^4+\frac{d\ln(c(ex+d))^4}{e}+24x-24x\ln\left(c(ex+d)\right)+12x\ln\left(c(ex+d)\right)^2-4x\ln$ |
| parallelrisch | $\frac{x\ln(c(ex+d))^4e-4x\ln(c(ex+d))^3e+\ln(c(ex+d))^4d+12x\ln(c(ex+d))^2e-4\ln(c(ex+d))^3d-24\ln(c(ex+d))xe+12\ln(c(e}{e}$ |

```
[In] int(ln(c*(e*x+d))^4,x,method=_RETURNVERBOSE)
```

```
[Out] (e*x+d)*ln(c*(e*x+d))^4/e-4*(e*x+d)*ln(c*(e*x+d))^3/e+12*(e*x+d)*ln(c*(e*x+
d))^2/e-24*x*ln(c*(e*x+d))+24*x-24*d/e*ln(e*x+d)
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \log^4(c(d+ex))\,dx = \frac{(ex+d)\log\left(cex+cd\right)^4-4\,(ex+d)\log\left(cex+cd\right)^3+12\,(ex+d)\log\left(cex+cd\right)^2+24\,ex-24\,(ex+d)\log}{e}$$

```
[In] integrate(log(c*(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] ((e*x + d)*log(c*e*x + c*d)^4 - 4*(e*x + d)*log(c*e*x + c*d)^3 + 12*(e*x +
d)*log(c*e*x + c*d)^2 + 24*e*x - 24*(e*x + d)*log(c*e*x + c*d))/e
```

## Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \log^4(c(d+ex))\,dx = 24e\left(-\frac{d\log\left(d+ex\right)}{e^2}+\frac{x}{e}\right)-24x\log\left(c(d+ex)\right) + \frac{(-4d-4ex)\log\left(c(d+ex)\right)^3}{e}+\frac{(d+ex)\log\left(c(d+ex)\right)^4}{e} + \frac{(12d+12ex)\log\left(c(d+ex)\right)^2}{e}$$

```
[In] integrate(ln(c*(e*x+d))**4,x)
```

```
[Out] 24*e*(-d*log(d + e*x)/e**2 + x/e) - 24*x*log(c*(d + e*x)) + (-4*d - 4*e*x)*
log(c*(d + e*x))**3/e + (d + e*x)*log(c*(d + e*x))**4/e + (12*d + 12*e*x)*l
og(c*(d + e*x))**2/e
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(81) = 162$.

Time = 0.20 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.32

$$\int \log^4(c(d+ex))\,dx = -4\,e\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log\left((ex+d)c\right)^3 + x\log\left((ex+d)c\right)^4 + \left(e\left(\frac{4\left(d\log(ex+d)^3 + 3\,d\log(ex+d)^2 - 6\,ex + 6\,d\log(ex+d)\right)\log\left((ex+d)c\right)}{e^3} - \frac{d\log(ex+d)^4 + 4\right.\right.$$

```
[In] integrate(log(c*(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -4*e*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)*c)^3 + x*log((e*x + d)*c)^4 +
 (e*(4*(d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*l
og((e*x + d)*c)/e^3 - (d*log(e*x + d)^4 + 4*d*log(e*x + d)^3 + 12*d*log(e*x
 + d)^2 - 24*e*x + 24*d*log(e*x + d))/e^3) - 6*(d*log(e*x + d)^2 - 2*e*x +
2*d*log(e*x + d))*log((e*x + d)*c)^2/e^2)*e
```

## Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \log^4(c(d+ex))\,dx = \frac{(ex+d)\log\left((ex+d)c\right)^4}{e} - \frac{4\,(ex+d)\log\left((ex+d)c\right)^3}{e} + \frac{12\,(ex+d)\log\left((ex+d)c\right)^2}{e} - \frac{24\,(ex+d)\log\left((ex+d)c\right)}{e} + \frac{24\,(ex+d)}{e}$$

```
[In] integrate(log(c*(e*x+d))^4,x, algorithm="giac")
```

```
[Out] (e*x + d)*log((e*x + d)*c)^4/e - 4*(e*x + d)*log((e*x + d)*c)^3/e + 12*(e*x
 + d)*log((e*x + d)*c)^2/e - 24*(e*x + d)*log((e*x + d)*c)/e + 24*(e*x + d)
/e
```

## Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.47

$$\int \log^4(c(d+ex))\,dx = 24\,x-24\,x\,\ln\left(c\,d+c\,e\,x\right)+12\,x\ln\left(c\,d+c\,e\,x\right)^2 -4\,x\ln\left(c\,d+c\,e\,x\right)^3+x\ln\left(c\,d+c\,e\,x\right)^4+\frac{12\,d\ln\left(c\,d+c\,e\,x\right)^2}{e} -\frac{4\,d\ln\left(c\,d+c\,e\,x\right)^3}{e}+\frac{d\ln\left(c\,d+c\,e\,x\right)^4}{e}-\frac{24\,d\,\ln\left(d+e\,x\right)}{e}$$

```
[In] int(log(c*(d + e*x))^4,x)
```

```
[Out] 24*x - 24*x*log(c*d + c*e*x) + 12*x*log(c*d + c*e*x)^2 - 4*x*log(c*d + c*e*
x)^3 + x*log(c*d + c*e*x)^4 + (12*d*log(c*d + c*e*x)^2)/e - (4*d*log(c*d +
c*e*x)^3)/e + (d*log(c*d + c*e*x)^4)/e - (24*d*log(d + e*x))/e
```

# 3.2 $\int \log^3(c(d+ex))\,dx$



## Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \log^3(c(d+ex))\,dx = -6x+\frac{6(d+ex)\log(c(d+ex))}{e} - \frac{3(d+ex)\log^2(c(d+ex))}{e}+\frac{(d+ex)\log^3(c(d+ex))}{e}$$

```
[Out] -6*x+6*(e*x+d)*ln(c*(e*x+d))/e-3*(e*x+d)*ln(c*(e*x+d))^2/e+(e*x+d)*ln(c*(e*
x+d))^3/e
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2436, 2333, 2332}

$$\int \log^3(c(d+ex))\,dx = \frac{(d+ex)\log^3(c(d+ex))}{e}-\frac{3(d+ex)\log^2(c(d+ex))}{e} + \frac{6(d+ex)\log(c(d+ex))}{e}-6x$$

```
[In] Int[Log[c*(d + e*x)]^3,x]
```

```
[Out] -6*x + (6*(d + e*x)*Log[c*(d + e*x)])/e - (3*(d + e*x)*Log[c*(d + e*x)]^2)/
e + ((d + e*x)*Log[c*(d + e*x)]^3)/e
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \log^3(cx)\,dx, x, d+ex\right)}{e} \\
&= \frac{(d+ex)\log^3(c(d+ex))}{e} - \frac{3\text{Subst}\left(\int \log^2(cx)\,dx, x, d+ex\right)}{e} \\
&= -\frac{3(d+ex)\log^2(c(d+ex))}{e} + \frac{(d+ex)\log^3(c(d+ex))}{e} + \frac{6\text{Subst}\left(\int \log(cx)\,dx, x, d+ex\right)}{e} \\
&= -6x + \frac{6(d+ex)\log(c(d+ex))}{e} - \frac{3(d+ex)\log^2(c(d+ex))}{e} + \frac{(d+ex)\log^3(c(d+ex))}{e}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$
\int \log^3(c(d+ex))\,dx = \frac{-6ex + 6(d+ex)\log(c(d+ex)) - 3(d+ex)\log^2(c(d+ex)) + (d+ex)\log^3(c(d+ex))}{e}
$$

```
[In] Integrate[Log[c*(d + e*x)]^3,x]
[Out] (-6*e*x + 6*(d + e*x)*Log[c*(d + e*x)] - 3*(d + e*x)*Log[c*(d + e*x)]^2 + (
d + e*x)*Log[c*(d + e*x)]^3)/e
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| risch | $\frac{(ex+d)\ln(c(ex+d))^3}{e}-\frac{3(ex+d)\ln(c(ex+d))^2}{e}+6x\ln(c(ex+d))-6x+\frac{6d\ln(ex+d)}{e}$ |
| derivativedivides | $\frac{(cex+cd)\ln(cex+cd)^3-3(cex+cd)\ln(cex+cd)^2+6(cex+cd)\ln(cex+cd)-6cex-6cd}{ce}$ |
| default | $\frac{(cex+cd)\ln(cex+cd)^3-3(cex+cd)\ln(cex+cd)^2+6(cex+cd)\ln(cex+cd)-6cex-6cd}{ce}$ |
| norman | $x\ln(c(ex+d))^3+\frac{d\ln(c(ex+d))^3}{e}-6x+6x\ln(c(ex+d))-3x\ln(c(ex+d))^2+$ $\frac{6d\ln(c(ex+}{e}$ |
| parallelrisch | $\frac{x\ln(c(ex+d))^3e-3x\ln(c(ex+d))^2e+\ln(c(ex+d))^3d+6\ln(c(ex+d))xe-3\ln(c(ex+d))^2d-6ex+6d\ln(c(ex+d))+6d}{e}$ |

```
[In] int(ln(c*(e*x+d))^3,x,method=_RETURNVERBOSE)
[Out] (e*x+d)*ln(c*(e*x+d))^3/e-3*(e*x+d)*ln(c*(e*x+d))^2/e+6*x*ln(c*(e*x+d))-6*x
+6*d/e*ln(e*x+d)
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \log^3(c(d+ex))\,dx = \frac{(ex+d)\log(cex+cd)^3-3(ex+d)\log(cex+cd)^2-6ex+6(ex+d)\log(cex+cd)}{e}$$

```
[In] integrate(log(c*(e*x+d))^3,x, algorithm="fricas")
[Out] ((e*x + d)*log(c*e*x + c*d)^3 - 3*(e*x + d)*log(c*e*x + c*d)^2 - 6*e*x + 6*
(e*x + d)*log(c*e*x + c*d))/e
```

## Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \log^3(c(d+ex))\,dx = -6e\left(-\frac{d\log(d+ex)}{e^2}+\frac{x}{e}\right)+6x\log(c(d+ex)) + \frac{(-3d-3ex)\log(c(d+ex))^2}{e}+\frac{(d+ex)\log(c(d+ex))^3}{e}$$

```
[In] integrate(ln(c*(e*x+d))**3,x)
[Out] -6*e*(-d*log(d + e*x)/e**2 + x/e) + 6*x*log(c*(d + e*x)) + (-3*d - 3*e*x)*l
og(c*(d + e*x))**2/e + (d + e*x)*log(c*(d + e*x))**3/e
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(61) = 122$.

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int \log^3(c(d+ex))\,dx = -3\,e\left(\frac{x}{e} - \frac{d\log\left(ex+d\right)}{e^2}\right)\log\left((ex+d)c\right)^2 + x\log\left((ex+d)c\right)^3 - e\left(\frac{3\left(d\log\left(ex+d\right)^2 - 2\,ex + 2\,d\log\left(ex+d\right)\right)\log\left((ex+d)c\right)}{e^2} - \frac{d\log\left(ex+d\right)^3 + 3\,d\log\left(ex+d\right)^2 - 6}{e^2}\right.$$

```
[In] integrate(log(c*(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -3*e*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)*c)^2 + x*log((e*x + d)*c)^3 -
 e*(3*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*log((e*x + d)*c)/e^2 -
(d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))/e^2)
```

## Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \log^3(c(d+ex))\,dx = \frac{(ex+d)\log\left((ex+d)c\right)^3}{e} - \frac{3\,(ex+d)\log\left((ex+d)c\right)^2}{e} + \frac{6\,(ex+d)\log\left((ex+d)c\right)}{e} - \frac{6\,(ex+d)}{e}$$

```
[In] integrate(log(c*(e*x+d))^3,x, algorithm="giac")
```

```
[Out] (e*x + d)*log((e*x + d)*c)^3/e - 3*(e*x + d)*log((e*x + d)*c)^2/e + 6*(e*x
+ d)*log((e*x + d)*c)/e - 6*(e*x + d)/e
```

## Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int \log^3(c(d+ex))\,dx = 6\,x\,\ln\left(c\,d+c\,e\,x\right) - 6\,x - 3\,x\ln\left(c\,d+c\,e\,x\right)^2 + x\ln\left(c\,d+c\,e\,x\right)^3 - \frac{3\,d\ln\left(c\,d+c\,e\,x\right)^2}{e} + \frac{d\ln\left(c\,d+c\,e\,x\right)^3}{e} + \frac{6\,d\,\ln\left(d+e\,x\right)}{e}$$

```
[In] int(log(c*(d + e*x))^3,x)
```

```
[Out] 6*x*log(c*d + c*e*x) - 6*x - 3*x*log(c*d + c*e*x)^2 + x*log(c*d + c*e*x)^3
- (3*d*log(c*d + c*e*x)^2)/e + (d*log(c*d + c*e*x)^3)/e + (6*d*log(d + e*x)
)/e
```

# 3.3 $\int \log^2(c(d+ex))\,dx$



## Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \log^2(c(d+ex))\,dx = 2x - \frac{2(d+ex)\log(c(d+ex))}{e} + \frac{(d+ex)\log^2(c(d+ex))}{e}$$

```
[Out] 2*x-2*(e*x+d)*ln(c*(e*x+d))/e+(e*x+d)*ln(c*(e*x+d))^2/e
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2436, 2333, 2332}

$$\int \log^2(c(d+ex))\,dx = \frac{(d+ex)\log^2(c(d+ex))}{e} - \frac{2(d+ex)\log(c(d+ex))}{e} + 2x$$

```
[In] Int[Log[c*(d + e*x)]^2,x]
[Out] 2*x - (2*(d + e*x)*Log[c*(d + e*x)])/e + ((d + e*x)*Log[c*(d + e*x)]^2)/e
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \log^2(cx)\,dx, x, d+ex\right)}{e} \\
&= \frac{(d+ex)\log^2(c(d+ex))}{e} - \frac{2\text{Subst}\left(\int \log(cx)\,dx, x, d+ex\right)}{e} \\
&= 2x - \frac{2(d+ex)\log(c(d+ex))}{e} + \frac{(d+ex)\log^2(c(d+ex))}{e}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \log^2(c(d+ex))\,dx = \frac{2ex - 2(d+ex)\log(c(d+ex)) + (d+ex)\log^2(c(d+ex))}{e}$$

```
[In] Integrate[Log[c*(d + e*x)]^2,x]
[Out] (2*e*x - 2*(d + e*x)*Log[c*(d + e*x)] + (d + e*x)*Log[c*(d + e*x)]^2)/e
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

| method | result | size |
|---|---|---|
| risch | $\frac{(ex+d)\ln(c(ex+d))^2}{e} - 2x\ln\left(c(ex+d)\right) + 2x - \frac{2d\ln(ex+d)}{e}$ | 47 |
| derivativedivides | $\frac{(cex+cd)\ln(cex+cd)^2 - 2(cex+cd)\ln(cex+cd) + 2cex + 2cd}{ce}$ | 57 |
| default | $\frac{(cex+cd)\ln(cex+cd)^2 - 2(cex+cd)\ln(cex+cd) + 2cex + 2cd}{ce}$ | 57 |
| norman | $x\ln\left(c(ex+d)\right)^2 + \frac{d\ln(c(ex+d))^2}{e} + 2x - 2x\ln\left(c(ex+d)\right) - \frac{2d\ln(c(ex+d))}{e}$ | 57 |
| parallelrisch | $\frac{x\ln(c(ex+d))^2 e - 2\ln(c(ex+d))xe + \ln(c(ex+d))^2 d + 2ex - 2d\ln(c(ex+d)) - 2d}{e}$ | 61 |

```
[In] int(ln(c*(e*x+d))^2,x,method=_RETURNVERBOSE)
[Out] (e*x+d)*ln(c*(e*x+d))^2/e-2*x*ln(c*(e*x+d))+2*x-2*d/e*ln(e*x+d)
```

## Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \log^2(c(d+ex))\,dx = \frac{(ex+d)\log(cex+cd)^2 + 2\,ex - 2\,(ex+d)\log(cex+cd)}{e}$$

```
[In] integrate(log(c*(e*x+d))^2,x, algorithm="fricas")
[Out] ((e*x + d)*log(c*e*x + c*d)^2 + 2*e*x - 2*(e*x + d)*log(c*e*x + c*d))/e
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \log^2(c(d+ex))\,dx = 2e\left(-\frac{d\log(d+ex)}{e^2}+\frac{x}{e}\right) - 2x\log(c(d+ex)) + \frac{(d+ex)\log(c(d+ex))^2}{e}$$

```
[In] integrate(ln(c*(e*x+d))**2,x)
[Out] 2*e*(-d*log(d + e*x)/e**2 + x/e) - 2*x*log(c*(d + e*x)) + (d + e*x)*log(c*(
d + e*x))**2/e
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.73

$$\int \log^2(c(d+ex))\,dx = -2\,e\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log((ex+d)c) + x\log((ex+d)c)^2 - \frac{d\log(ex+d)^2 - 2\,ex + 2\,d\log(ex+d)}{e}$$

```
[In] integrate(log(c*(e*x+d))^2,x, algorithm="maxima")
[Out] -2*e*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)*c) + x*log((e*x + d)*c)^2 - (
d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))/e
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \log^2(c(d+ex))\,dx = \frac{(ex+d)\log\left((ex+d)c\right)^2}{e} - \frac{2\,(ex+d)\log\left((ex+d)c\right)}{e} + \frac{2\,(ex+d)}{e}$$

```
[In] integrate(log(c*(e*x+d))^2,x, algorithm="giac")
```

```
[Out] (e*x + d)*log((e*x + d)*c)^2/e - 2*(e*x + d)*log((e*x + d)*c)/e + 2*(e*x +
d)/e
```

## Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \log^2(c(d+ex))\,dx = 2\,x - 2\,x\,\ln\left(c\,d + c\,e\,x\right) + x\ln\left(c\,d + c\,e\,x\right)^2 + \frac{d\ln\left(c\,d + c\,e\,x\right)^2}{e} - \frac{2\,d\,\ln\left(d + e\,x\right)}{e}$$

```
[In] int(log(c*(d + e*x))^2,x)
```

```
[Out] 2*x - 2*x*log(c*d + c*e*x) + x*log(c*d + c*e*x)^2 + (d*log(c*d + c*e*x)^2)/
e - (2*d*log(d + e*x))/e
```

# 3.4 $\int \log(c(d+ex))\,dx$



## Optimal result

Integrand size = 8, antiderivative size = 21

$$\int \log(c(d+ex))\,dx = -x + \frac{(d+ex)\log(c(d+ex))}{e}$$

```
[Out] -x+(e*x+d)*ln(c*(e*x+d))/e
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2436, 2332}

$$\int \log(c(d+ex))\,dx = \frac{(d+ex)\log(c(d+ex))}{e} - x$$

```
[In] Int[Log[c*(d + e*x)],x]
```

```
[Out] -x + ((d + e*x)*Log[c*(d + e*x)])/e
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}(\int \log(cx)\,dx, x, d+ex)}{e} \\
&= -x + \frac{(d+ex)\log(c(d+ex))}{e}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$
\int \log(c(d+ex))\,dx = -x + \frac{(d+ex)\log(c(d+ex))}{e}
$$

```
[In] Integrate[Log[c*(d + e*x)],x]
[Out] -x + ((d + e*x)*Log[c*(d + e*x)])/e
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

| method | result | size |
|---|---|---|
| risch | $x\ln\left(c(ex+d)\right) - x + \frac{d\ln(ex+d)}{e}$ | 26 |
| norman | $x\ln\left(c(ex+d)\right) + \frac{d\ln(c(ex+d))}{e} - x$ | 28 |
| parallelrisch | $\frac{\ln(c(ex+d))xe-ex+d\ln(c(ex+d))+d}{e}$ | 32 |
| parts | $x\ln\left(c(ex+d)\right) - e\left(\frac{x}{e} - \frac{d\ln(ex+d)}{e^2}\right)$ | 33 |
| derivativedivides | $\frac{(cex+cd)\ln(cex+cd)-cex-cd}{ce}$ | 36 |
| default | $\frac{(cex+cd)\ln(cex+cd)-cex-cd}{ce}$ | 36 |

```
[In] int(ln(c*(e*x+d)),x,method=_RETURNVERBOSE)
[Out] x*ln(c*(e*x+d))-x+d/e*ln(e*x+d)
```

## Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \log(c(d+ex))\,dx = -\frac{ex - (ex+d)\log(cex+cd)}{e}$$

```
[In] integrate(log(c*(e*x+d)),x, algorithm="fricas")
[Out] -(e*x - (e*x + d)*log(c*e*x + c*d))/e
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \log(c(d+ex))\,dx = -e\left(-\frac{d\log(d+ex)}{e^2} + \frac{x}{e}\right) + x\log(c(d+ex))$$

```
[In] integrate(ln(c*(e*x+d)),x)
[Out] -e*(-d*log(d + e*x)/e**2 + x/e) + x*log(c*(d + e*x))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \log(c(d+ex))\,dx = \frac{(ex+d)c\log((ex+d)c) - (ex+d)c}{ce}$$

```
[In] integrate(log(c*(e*x+d)),x, algorithm="maxima")
[Out] ((e*x + d)*c*log((e*x + d)*c) - (e*x + d)*c)/(c*e)
```

## Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \log(c(d+ex))\,dx = \frac{(ex+d)c\log((ex+d)c) - (ex+d)c}{ce}$$

```
[In] integrate(log(c*(e*x+d)),x, algorithm="giac")
[Out] ((e*x + d)*c*log((e*x + d)*c) - (e*x + d)*c)/(c*e)
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \log(c(d+ex))\,dx = x\,\ln\left(c\,(d+e\,x)\right) - x + \frac{d\,\ln\left(d+e\,x\right)}{e}$$

```
[In] int(log(c*(d + e*x)),x)
[Out] x*log(c*(d + e*x)) - x + (d*log(d + e*x))/e
```

# 3.5 $\int \frac{1}{\log(c(d+ex))} dx$



## Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \frac{1}{\log(c(d+ex))} dx = \frac{\text{LogIntegral}(c(d+ex))}{ce}$$

```
[Out] Li(c*(e*x+d))/c/e
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2436, 2335}

$$\int \frac{1}{\log(c(d+ex))} dx = \frac{\text{LogIntegral}(c(d+ex))}{ce}$$

```
[In] Int[Log[c*(d + e*x)]^(-1),x]
```

```
[Out] LogIntegral[c*(d + e*x)]/(c*e)
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, d+ex\right)}{e}$$
$$= \frac{\text{li}(c(d+ex))}{ce}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(c(d+ex))}\,dx = \frac{\text{LogIntegral}(c(d+ex))}{ce}$$

```
[In] Integrate[Log[c*(d + e*x)]^(-1),x]
[Out] LogIntegral[c*(d + e*x)]/(c*e)
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\text{Ei}_1(-\ln(cex+cd))}{ce}$ | 22 |
| default | $-\frac{\text{Ei}_1(-\ln(cex+cd))}{ce}$ | 22 |
| risch | $-\frac{\text{Ei}_1(-\ln(cex+cd))}{ce}$ | 22 |

```
[In] int(1/ln(c*(e*x+d)),x,method=_RETURNVERBOSE)
[Out] -1/c/e*Ei(1,-ln(c*e*x+c*d))
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1}{\log(c(d+ex))}\,dx = \frac{\text{log\_integral}\,(cex+cd)}{ce}$$

```
[In] integrate(1/log(c*(e*x+d)),x, algorithm="fricas")
[Out] log_integral(c*e*x + c*d)/(c*e)
```

## Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{1}{\log(c(d+ex))}\,dx = \frac{\operatorname{li}(cd+cex)}{ce}$$

```
[In] integrate(1/ln(c*(e*x+d)),x)
[Out] li(c*d + c*e*x)/(c*e)
```

## Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{\log(c(d+ex))}\,dx = \frac{\operatorname{Ei}(\log(cex+cd))}{ce}$$

```
[In] integrate(1/log(c*(e*x+d)),x, algorithm="maxima")
[Out] Ei(log(c*e*x + c*d))/(c*e)
```

## Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1}{\log(c(d+ex))}\,dx = \frac{\operatorname{Ei}(\log((ex+d)c))}{ce}$$

```
[In] integrate(1/log(c*(e*x+d)),x, algorithm="giac")
[Out] Ei(log((e*x + d)*c))/(c*e)
```

## Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(c(d+ex))}\,dx = \frac{\operatorname{logint}(c\,(d+e\,x))}{c\,e}$$

```
[In] int(1/log(c*(d + e*x)),x)
[Out] logint(c*(d + e*x))/(c*e)
```

# 3.6 $\int \frac{1}{\log^2(c(d+ex))} \, dx$



## Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\log^2(c(d+ex))} \, dx = -\frac{d+ex}{e\log(c(d+ex))} + \frac{\text{LogIntegral}(c(d+ex))}{ce}$$

```
[Out] Li(c*(e*x+d))/c/e+(-e*x-d)/e/ln(c*(e*x+d))
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2436, 2334, 2335}

$$\int \frac{1}{\log^2(c(d+ex))} \, dx = \frac{\text{LogIntegral}(c(d+ex))}{ce} - \frac{d+ex}{e\log(c(d+ex))}$$

```
[In] Int[Log[c*(d + e*x)]^(-2),x]
```

```
[Out] -((d + e*x)/(e*Log[c*(d + e*x)])) + LogIntegral[c*(d + e*x)]/(c*e)
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, d+ex\right)}{e} \\ &= -\frac{d+ex}{e\log(c(d+ex))} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, d+ex\right)}{e} \\ &= -\frac{d+ex}{e\log(c(d+ex))} + \frac{\text{li}(c(d+ex))}{ce}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log^2(c(d+ex))}\,dx = -\frac{d+ex}{e\log(c(d+ex))} + \frac{\text{LogIntegral}(c(d+ex))}{ce}$$

```
[In] Integrate[Log[c*(d + e*x)]^(-2),x]
[Out] -((d + e*x)/(e*Log[c*(d + e*x)])) + LogIntegral[c*(d + e*x)]/(c*e)
```

## Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

| method | result | size |
|---|---|---|
| risch | $-\frac{ex+d}{\ln(c(ex+d))e} - \frac{\text{Ei}_1(-\ln(cex+cd))}{ce}$ | 43 |
| derivativedivides | $\frac{-\frac{cex+cd}{\ln(cex+cd)} - \text{Ei}_1(-\ln(cex+cd))}{ce}$ | 45 |
| default | $\frac{-\frac{cex+cd}{\ln(cex+cd)} - \text{Ei}_1(-\ln(cex+cd))}{ce}$ | 45 |

```
[In] int(1/ln(c*(e*x+d))^2,x,method=_RETURNVERBOSE)
[Out] -1/ln(c*(e*x+d))/e*(e*x+d)-1/c/e*Ei(1,-ln(c*e*x+c*d))
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int \frac{1}{\log^2(c(d+ex))}\,dx = -\frac{cex + cd - \log(cex + cd)\,\mathrm{log\_integral}(cex + cd)}{ce\log(cex + cd)}$$

```
[In] integrate(1/log(c*(e*x+d))^2,x, algorithm="fricas")
[Out] -(c*e*x + c*d - log(c*e*x + c*d)*log_integral(c*e*x + c*d))/(c*e*log(c*e*x
+ c*d))
```

## Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{1}{\log^2(c(d+ex))}\,dx = \frac{-d - ex}{e\log(c(d+ex))} + \frac{\mathrm{li}(cd + cex)}{ce}$$

```
[In] integrate(1/ln(c*(e*x+d))**2,x)
[Out] (-d - e*x)/(e*log(c*(d + e*x))) + li(c*d + c*e*x)/(c*e)
```

## Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{1}{\log^2(c(d+ex))}\,dx = \frac{\Gamma(-1, -\log(cex + cd))}{ce}$$

```
[In] integrate(1/log(c*(e*x+d))^2,x, algorithm="maxima")
[Out] gamma(-1, -log(c*e*x + c*d))/(c*e)
```

## Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{1}{\log^2(c(d+ex))}\,dx = \frac{\text{Ei}(\log((ex+d)c))}{ce} - \frac{ex+d}{e\log((ex+d)c)}$$

```
[In] integrate(1/log(c*(e*x+d))^2,x, algorithm="giac")
[Out] Ei(log((e*x + d)*c))/(c*e) - (e*x + d)/(e*log((e*x + d)*c))
```

## Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log^2(c(d+ex))}\,dx = \frac{\text{logint}(c\,(d+e\,x))}{c\,e} - \frac{d+e\,x}{e\,\ln\left(c\,\left(d+e\,x\right)\right)}$$

```
[In] int(1/log(c*(d + e*x))^2,x)
[Out] logint(c*(d + e*x))/(c*e) - (d + e*x)/(e*log(c*(d + e*x)))
```

## 3.7 $\int \frac{1}{\log^3(c(d+ex))} dx$



### Optimal result

Integrand size = 10, antiderivative size = 63

$$\int \frac{1}{\log^3(c(d+ex))} dx = -\frac{d+ex}{2e\log^2(c(d+ex))} - \frac{d+ex}{2e\log(c(d+ex))} + \frac{\text{LogIntegral}(c(d+ex))}{2ce}$$

```
[Out] 1/2*Li(c*(e*x+d))/c/e+1/2*(-e*x-d)/e/ln(c*(e*x+d))^2+1/2*(-e*x-d)/e/ln(c*(e
*x+d))
```

### Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2436, 2334, 2335}

$$\int \frac{1}{\log^3(c(d+ex))} dx = \frac{\text{LogIntegral}(c(d+ex))}{2ce} - \frac{d+ex}{2e\log^2(c(d+ex))} - \frac{d+ex}{2e\log(c(d+ex))}$$

```
[In] Int[Log[c*(d + e*x)]^(-3),x]
```

```
[Out] -1/2*(d + e*x)/(e*Log[c*(d + e*x)]^2) - (d + e*x)/(2*e*Log[c*(d + e*x)]) +
LogIntegral[c*(d + e*x)]/(2*c*e)
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\log^3(cx)}\,dx, x, d+ex\right)}{e}\\
&= -\frac{d+ex}{2e\log^2(c(d+ex))} + \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, d+ex\right)}{2e}\\
&= -\frac{d+ex}{2e\log^2(c(d+ex))} - \frac{d+ex}{2e\log(c(d+ex))} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, d+ex\right)}{2e}\\
&= -\frac{d+ex}{2e\log^2(c(d+ex))} - \frac{d+ex}{2e\log(c(d+ex))} + \frac{\text{li}(c(d+ex))}{2ce}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

$$\int \frac{1}{\log^3(c(d+ex))}\,dx = \frac{-\frac{(d+ex)(1+\log(c(d+ex)))}{\log^2(c(d+ex))} + \frac{\text{LogIntegral}(c(d+ex))}{c}}{2e}$$

```
[In] Integrate[Log[c*(d + e*x)]^(-3),x]
[Out] (-(((d + e*x)*(1 + Log[c*(d + e*x)]))/Log[c*(d + e*x)]^2) + LogIntegral[c*(
d + e*x)]/c)/(2*e)
```

## Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

| method | result | size |
|---|---|---|
| risch | $-\frac{\ln(c(ex+d))xe+d\ln(c(ex+d))+ex+d}{2e\ln(c(ex+d))^2}-\frac{\mathrm{Ei}_1(-\ln(cex+cd))}{2ce}$ | 64 |
| derivativedivides | $\frac{-\frac{cex+cd}{2\ln(cex+cd)^2}-\frac{cex+cd}{2\ln(cex+cd)}-\frac{\mathrm{Ei}_1(-\ln(cex+cd))}{2}}{ce}$ | 66 |
| default | $\frac{-\frac{cex+cd}{2\ln(cex+cd)^2}-\frac{cex+cd}{2\ln(cex+cd)}-\frac{\mathrm{Ei}_1(-\ln(cex+cd))}{2}}{ce}$ | 66 |

```
[In] int(1/ln(c*(e*x+d))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(ln(c*(e*x+d))*x*e+d*ln(c*(e*x+d))+e*x+d)/e/ln(c*(e*x+d))^2-1/2/c/e*Ei
(1,-ln(c*e*x+c*d))
```

## Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{1}{\log^3(c(d+ex))}\,dx$$
$$= -\frac{cex - \log\left(cex+cd\right)^2 \log\_integral\left(cex+cd\right) + cd + \left(cex+cd\right)\log\left(cex+cd\right)}{2\,ce\log\left(cex+cd\right)^2}$$

```
[In] integrate(1/log(c*(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(c*e*x - log(c*e*x + c*d)^2*log_integral(c*e*x + c*d) + c*d + (c*e*x +
 c*d)*log(c*e*x + c*d))/(c*e*log(c*e*x + c*d)^2)
```

## Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{1}{\log^3(c(d+ex))}\,dx = \frac{-d-ex+(-d-ex)\log\left(c(d+ex)\right)}{2e\log\left(c\left(d+ex\right)\right)^2} + \frac{\mathrm{li}\left(cd+cex\right)}{2ce}$$

```
[In] integrate(1/ln(c*(e*x+d))**3,x)
```

```
[Out] (-d - e*x + (-d - e*x)*log(c*(d + e*x)))/(2*e*log(c*(d + e*x))**2) + li(c*d
 + c*e*x)/(2*c*e)
```

## Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.33

$$\int \frac{1}{\log^3(c(d+ex))}\,dx = -\frac{\Gamma(-2, -\log(cex+cd))}{ce}$$

```
[In] integrate(1/log(c*(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -gamma(-2, -log(c*e*x + c*d))/(c*e)
```

## Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{\log^3(c(d+ex))}\,dx = \frac{\operatorname{Ei}(\log((ex+d)c))}{2\,ce} - \frac{ex+d}{2\,e\log((ex+d)c)} - \frac{ex+d}{2\,e\log((ex+d)c)^2}$$

```
[In] integrate(1/log(c*(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/2*Ei(log((e*x + d)*c))/(c*e) - 1/2*(e*x + d)/(e*log((e*x + d)*c)) - 1/2*(
e*x + d)/(e*log((e*x + d)*c)^2)
```

## Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{1}{\log^3(c(d+ex))}\,dx = \frac{\operatorname{logint}(c\,(d+e\,x))}{2\,c\,e} - \frac{\frac{c\,d}{2} + \ln(c\,(d+e\,x))\,\left(\frac{c\,d}{2} + \frac{c\,e\,x}{2}\right) + \frac{c\,e\,x}{2}}{c\,e\ln(c\,(d+e\,x))^2}$$

```
[In] int(1/log(c*(d + e*x))^3,x)
```

```
[Out] logint(c*(d + e*x))/(2*c*e) - ((c*d)/2 + log(c*(d + e*x))*((c*d)/2 + (c*e*x
)/2) + (c*e*x)/2)/(c*e*log(c*(d + e*x))^2)
```

# 3.8 $\int \frac{1}{\log^4(c(d+ex))}\,dx$



## Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = -\frac{d+ex}{3e\log^3(c(d+ex))} - \frac{d+ex}{6e\log^2(c(d+ex))} - \frac{d+ex}{6e\log(c(d+ex))} + \frac{\text{LogIntegral}(c(d+ex))}{6ce}$$

```
[Out] 1/6*Li(c*(e*x+d))/c/e+1/3*(-e*x-d)/e/ln(c*(e*x+d))^3+1/6*(-e*x-d)/e/ln(c*(e
*x+d))^2+1/6*(-e*x-d)/e/ln(c*(e*x+d))
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2436, 2334, 2335}

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = \frac{\text{LogIntegral}(c(d+ex))}{6ce} - \frac{d+ex}{3e\log^3(c(d+ex))} - \frac{d+ex}{6e\log^2(c(d+ex))} - \frac{d+ex}{6e\log(c(d+ex))}$$

```
[In] Int[Log[c*(d + e*x)]^(-4),x]
```

```
[Out] -1/3*(d + e*x)/(e*Log[c*(d + e*x)]^3) - (d + e*x)/(6*e*Log[c*(d + e*x)]^2)
- (d + e*x)/(6*e*Log[c*(d + e*x)]) + LogIntegral[c*(d + e*x)]/(6*c*e)
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
```

```
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\log^4(cx)}\,dx, x, d+ex\right)}{e}\\
&= -\frac{d+ex}{3e\log^3(c(d+ex))} + \frac{\text{Subst}\left(\int \frac{1}{\log^3(cx)}\,dx, x, d+ex\right)}{3e}\\
&= -\frac{d+ex}{3e\log^3(c(d+ex))} - \frac{d+ex}{6e\log^2(c(d+ex))} + \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, d+ex\right)}{6e}\\
&= -\frac{d+ex}{3e\log^3(c(d+ex))} - \frac{d+ex}{6e\log^2(c(d+ex))} - \frac{d+ex}{6e\log(c(d+ex))} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, d+ex\right)}{6e}\\
&= -\frac{d+ex}{3e\log^3(c(d+ex))} - \frac{d+ex}{6e\log^2(c(d+ex))} - \frac{d+ex}{6e\log(c(d+ex))} + \frac{\text{li}(c(d+ex))}{6ce}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$
\int \frac{1}{\log^4(c(d+ex))}\,dx = \frac{-\frac{(d+ex)(2+\log(c(d+ex))+\log^2(c(d+ex)))}{\log^3(c(d+ex))} + \frac{\text{LogIntegral}(c(d+ex))}{c}}{6e}
$$

```
[In] Integrate[Log[c*(d + e*x)]^(-4),x]
```

```
[Out] (-(((d + e*x)*(2 + Log[c*(d + e*x)] + Log[c*(d + e*x)]^2))/Log[c*(d + e*x)]
^3) + LogIntegral[c*(d + e*x)]/c)/(6*e)
```

## Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{cex+cd}{3\ln(cex+cd)^3}-\frac{cex+cd}{6\ln(cex+cd)^2}-\frac{cex+cd}{6\ln(cex+cd)}-\frac{\mathrm{Ei}_1(-\ln(cex+cd))}{6}}{ce}$ | 87 |
| default | $\frac{-\frac{cex+cd}{3\ln(cex+cd)^3}-\frac{cex+cd}{6\ln(cex+cd)^2}-\frac{cex+cd}{6\ln(cex+cd)}-\frac{\mathrm{Ei}_1(-\ln(cex+cd))}{6}}{ce}$ | 87 |
| risch | $-\frac{x\ln(c(ex+d))^2e+\ln(c(ex+d))^2d+\ln(c(ex+d))xe+d\ln(c(ex+d))+2ex+2d}{6e\ln(c(ex+d))^3}-\frac{\mathrm{Ei}_1(-\ln(cex+cd))}{6ce}$ | 92 |

[In] int(1/ln(c*(e*x+d))^4,x,method=_RETURNVERBOSE)

[Out] 1/c/e*(-1/3*(c*e*x+c*d)/ln(c*e*x+c*d)^3-1/6*(c*e*x+c*d)/ln(c*e*x+c*d)^2-1/6
*(c*e*x+c*d)/ln(c*e*x+c*d)-1/6*Ei(1,-ln(c*e*x+c*d)))

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = \frac{\log(cex+cd)^3\,\text{log\_integral}(cex+cd)-2\,cex-(cex+cd)\log(cex+cd)^2-2\,cd-(cex+cd)\log(cex-}{6\,ce\log(cex+cd)^3}$$

[In] integrate(1/log(c*(e*x+d))^4,x, algorithm="fricas")

[Out] 1/6*(log(c*e*x + c*d)^3*log_integral(c*e*x + c*d) - 2*c*e*x - (c*e*x + c*d)
*log(c*e*x + c*d)^2 - 2*c*d - (c*e*x + c*d)*log(c*e*x + c*d))/(c*e*log(c*e*
x + c*d)^3)

## Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = \frac{-d-ex+\left(-\frac{d}{2}-\frac{ex}{2}\right)\log(c(d+ex))^2+\left(-\frac{d}{2}-\frac{ex}{2}\right)\log(c(d+ex))}{3e\log(c(d+ex))^3} + \frac{\mathrm{li}(cd+cex)}{6ce}$$

[In] integrate(1/ln(c*(e*x+d))**4,x)

[Out] (-d - e*x + (-d/2 - e*x/2)*log(c*(d + e*x))**2 + (-d/2 - e*x/2)*log(c*(d +
e*x)))/(3*e*log(c*(d + e*x))**3) + li(c*d + c*e*x)/(6*c*e)

## Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.24

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = \frac{\Gamma(-3, -\log(cex+cd))}{ce}$$

```
[In] integrate(1/log(c*(e*x+d))^4,x, algorithm="maxima")
[Out] gamma(-3, -log(c*e*x + c*d))/(c*e)
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = \frac{\mathrm{Ei}(\log((ex+d)c))}{6\,ce} - \frac{ex+d}{6\,e\log((ex+d)c)} - \frac{ex+d}{6\,e\log((ex+d)c)^2} - \frac{ex+d}{3\,e\log((ex+d)c)^3}$$

```
[In] integrate(1/log(c*(e*x+d))^4,x, algorithm="giac")
[Out] 1/6*Ei(log((e*x + d)*c))/(c*e) - 1/6*(e*x + d)/(e*log((e*x + d)*c)) - 1/6*(
e*x + d)/(e*log((e*x + d)*c)^2) - 1/3*(e*x + d)/(e*log((e*x + d)*c)^3)
```

## Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.79

$$\int \frac{1}{\log^4(c(d+ex))}\,dx = -\frac{(d+e\,x)\left(\frac{1}{6\,\ln(c\,(d+e\,x))}+\frac{1}{6\ln(c\,(d+e\,x))^2}+\frac{1}{3\ln(c\,(d+e\,x))^3}\right)}{e} - \frac{\mathrm{expint}(-\ln(c\,(d+e\,x)))}{6\,c\,e}$$

```
[In] int(1/log(c*(d + e*x))^4,x)
[Out] - ((d + e*x)*(1/(6*log(c*(d + e*x))) + 1/(6*log(c*(d + e*x))^2) + 1/(3*log(
c*(d + e*x))^3)))/e - expint(-log(c*(d + e*x)))/(6*c*e)
```

# 3.9 $\int \log^{\frac{5}{2}}(c(d+ex))\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 98

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx = -\frac{15\sqrt{\pi}\,\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{8ce} + \frac{15(d+ex)\sqrt{\log(c(d+ex))}}{4e} - \frac{5(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{2e} + \frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))}{e}$$

```
[Out] -5/2*(e*x+d)*ln(c*(e*x+d))^(3/2)/e+(e*x+d)*ln(c*(e*x+d))^(5/2)/e-15/8*erfi(
ln(c*(e*x+d))^(1/2))*Pi^(1/2)/c/e+15/4*(e*x+d)*ln(c*(e*x+d))^(1/2)/e
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2436, 2333, 2336, 2211, 2235}

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx = -\frac{15\sqrt{\pi}\,\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{8ce} + \frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))}{e} - \frac{5(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{2e} + \frac{15(d+ex)\sqrt{\log(c(d+ex))}}{4e}$$

```
[In] Int[Log[c*(d + e*x)]^(5/2),x]
```

```
[Out] (-15*Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/(8*c*e) + (15*(d + e*x)*Sqrt[Lo
g[c*(d + e*x)]])/(4*e) - (5*(d + e*x)*Log[c*(d + e*x)]^(3/2))/(2*e) + ((d +
 e*x)*Log[c*(d + e*x)]^(5/2))/e
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \log^{\frac{5}{2}}(cx)\,dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))}{e} - \frac{5\text{Subst}\left(\int \log^{\frac{3}{2}}(cx)\,dx, x, d+ex\right)}{2e}$$

$$= -\frac{5(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{2e} + \frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))}{e} + \frac{15\text{Subst}\left(\int \sqrt{\log(cx)}\,dx, x, d+ex\right)}{4e}$$

$$= \frac{15(d+ex)\sqrt{\log(c(d+ex))}}{4e} - \frac{5(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{2e} + \frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))}{e} - \frac{15\text{Subst}\left(\int \frac{1}{\sqrt{\log(cx)}}\,dx, x, d+ex\right)}{8e}$$

$$
\begin{aligned}
&= \frac{15(d+ex)\sqrt{\log(c(d+ex))}}{4e} - \frac{5(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{2e} \\
&\quad + \frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))}{e} - \frac{15\text{Subst}\left(\int \frac{e^x}{\sqrt{x}}\,dx, x, \log(c(d+ex))\right)}{8ce} \\
&= \frac{15(d+ex)\sqrt{\log(c(d+ex))}}{4e} - \frac{5(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{2e} \\
&\quad + \frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))}{e} - \frac{15\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\log(c(d+ex))}\right)}{4ce} \\
&= -\frac{15\sqrt{\pi}\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{8ce} + \frac{15(d+ex)\sqrt{\log(c(d+ex))}}{4e} \\
&\quad - \frac{5(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{2e} + \frac{(d+ex)\log^{\frac{5}{2}}(c(d+ex))}{e}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$
\int \log^{\frac{5}{2}}(c(d+ex))\,dx = \frac{-15\sqrt{\pi}\text{erfi}\left(\sqrt{\log(c(d+ex))}\right) + 2c(d+ex)\sqrt{\log(c(d+ex))}\left(15 - 10\log(c(d+ex)) + 4\log^2(c(d+ex))\right)}{8ce}
$$

```
[In] Integrate[Log[c*(d + e*x)]^(5/2),x]
[Out] (-15*Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]] + 2*c*(d + e*x)*Sqrt[Log[c*(d +
e*x)]]*(15 - 10*Log[c*(d + e*x)] + 4*Log[c*(d + e*x)]^2))/(8*c*e)
```

## Maple [F]

$$
\int \ln\left(c(ex+d)\right)^{\frac{5}{2}}\,dx
$$

```
[In] int(ln(c*(e*x+d))^(5/2),x)
[Out] int(ln(c*(e*x+d))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(log(c*(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

## Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx = \frac{2\,(cex+cd)\left(4\,\log\left(cex+cd\right)^{\frac{5}{2}}-10\,\log\left(cex+cd\right)^{\frac{3}{2}}+15\,\sqrt{\log\left(cex+cd\right)}\right)+15i\,\sqrt{\pi}\,\mathrm{erf}\left(i\,\sqrt{\log\left(cex+cd\right.}\right.}{8\,ce}$$

```
[In] integrate(log(c*(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(c*e*x + c*d)*(4*log(c*e*x + c*d)^(5/2) - 10*log(c*e*x + c*d)^(3/2)
+ 15*sqrt(log(c*e*x + c*d))) + 15*I*sqrt(pi)*erf(I*sqrt(log(c*e*x + c*d))))
/(c*e)
```

## Giac [F]

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx = \int \log\left((ex+d)c\right)^{\frac{5}{2}}\,dx$$

[In] integrate(log(c*(e*x+d))^(5/2),x, algorithm="giac")

[Out] integrate(log((e*x + d)*c)^(5/2), x)

## Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int \log^{\frac{5}{2}}(c(d+ex))\,dx = \frac{\ln\left(c\left(d+e\,x\right)\right)^{5/2}\left(\frac{15\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-\ln(c\,(d+e\,x))}\right)}{8}+c\left(d+e\,x\right)\left(\frac{15\sqrt{-\ln(c\,(d+e\,x))}}{4}+\frac{5\,(-\ln(c\,(d+e\,x)))^{3/2}}{2}+(-\ln\left(c\left(d\right.\right.\right.\right.}{c\,e\left(-\ln\left(c\,\left(d+e\,x\right)\right)\right)^{5/2}}$$

[In] int(log(c*(d + e*x))^(5/2),x)

[Out] (log(c*(d + e*x))^(5/2)*((15*pi^(1/2)*erfc((-log(c*(d + e*x)))^(1/2)))/8 + c*(d + e*x)*((15*(-log(c*(d + e*x)))^(1/2))/4 + (5*(-log(c*(d + e*x)))^(3/2))/2 + (-log(c*(d + e*x)))^(5/2))))/(c*e*(-log(c*(d + e*x)))^(5/2))

# 3.10 $\int \log^{\frac{3}{2}}(c(d+ex))\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 74

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx = \frac{3\sqrt{\pi}\,\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{4ce} - \frac{3(d+ex)\sqrt{\log(c(d+ex))}}{2e} + \frac{(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{e}$$

```
[Out] (e*x+d)*ln(c*(e*x+d))^(3/2)/e+3/4*erfi(ln(c*(e*x+d))^(1/2))*Pi^(1/2)/c/e-3/
2*(e*x+d)*ln(c*(e*x+d))^(1/2)/e
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2436, 2333, 2336, 2211, 2235}

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx = \frac{3\sqrt{\pi}\,\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{4ce} + \frac{(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{e} - \frac{3(d+ex)\sqrt{\log(c(d+ex))}}{2e}$$

```
[In] Int[Log[c*(d + e*x)]^(3/2),x]
```

```
[Out] (3*Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/(4*c*e) - (3*(d + e*x)*Sqrt[Log[c
*(d + e*x)]])/(2*e) + ((d + e*x)*Log[c*(d + e*x)]^(3/2))/e
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \log^{\frac{3}{2}}(cx)\,dx, x, d+ex\right)}{e} \\
&= \frac{(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{e} - \frac{3\text{Subst}\left(\int \sqrt{\log(cx)}\,dx, x, d+ex\right)}{2e} \\
&= -\frac{3(d+ex)\sqrt{\log(c(d+ex))}}{2e} + \frac{(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{e} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{\log(cx)}}\,dx, x, d+ex\right)}{4e} \\
&= -\frac{3(d+ex)\sqrt{\log(c(d+ex))}}{2e} + \frac{(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{e} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{e^x}{\sqrt{x}}\,dx, x, \log(c(d+ex))\right)}{4ce} \\
&= -\frac{3(d+ex)\sqrt{\log(c(d+ex))}}{2e} + \frac{(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{e} \\
&\quad + \frac{3\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\log(c(d+ex))}\right)}{2ce}
\end{aligned}
$$

$$= \frac{3\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{4ce} - \frac{3(d+ex)\sqrt{\log(c(d+ex))}}{2e} + \frac{(d+ex)\log^{\frac{3}{2}}(c(d+ex))}{e}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx$$
$$= \frac{3\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\log(c(d+ex))}\right) + 2c(d+ex)\sqrt{\log(c(d+ex))}(-3+2\log(c(d+ex)))}{4ce}$$

```
[In] Integrate[Log[c*(d + e*x)]^(3/2),x]
[Out] (3*Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]] + 2*c*(d + e*x)*Sqrt[Log[c*(d + e*
x)]]*(-3 + 2*Log[c*(d + e*x)]))/(4*c*e)
```

## Maple [F]

$$\int \ln\left(c(ex+d)\right)^{\frac{3}{2}}dx$$

```
[In] int(ln(c*(e*x+d))^(3/2),x)
[Out] int(ln(c*(e*x+d))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(log(c*(e*x+d))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [A] (verification not implemented)

Time = 55.01 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.42

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx$$
$$= \begin{cases} \tilde{\infty}x & \text{for } c=0 \\ x\log(cd)^{\frac{3}{2}} & \text{for } e=0 \\ \frac{\left(-\sqrt{-\log(cd+cex)}(cd+cex)\left(\log(cd+cex)-\frac{3}{2}\right)+\frac{3\sqrt{\pi}\operatorname{erfc}\left(\sqrt{-\log(cd+cex)}\right)}{4}\right)\log(cd+cex)^{\frac{3}{2}}}{ce(-\log(cd+cex))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(e*x+d))**(3/2),x)
```

```
[Out] Piecewise((zoo*x, Eq(c, 0)), (x*log(c*d)**(3/2), Eq(e, 0)), ((-sqrt(-log(c*
d + c*e*x))*(c*d + c*e*x)*(log(c*d + c*e*x) - 3/2) + 3*sqrt(pi)*erfc(sqrt(-
log(c*d + c*e*x)))/4)*log(c*d + c*e*x)**(3/2)/(c*e*(-log(c*d + c*e*x))**(3/
2)), True))
```

## Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx$$
$$= \frac{2(cex+cd)\left(2\log(cex+cd)^{\frac{3}{2}}-3\sqrt{\log(cex+cd)}\right)-3i\sqrt{\pi}\operatorname{erf}\left(i\sqrt{\log(cex+cd)}\right)}{4\,ce}$$

```
[In] integrate(log(c*(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(c*e*x + c*d)*(2*log(c*e*x + c*d)^(3/2) - 3*sqrt(log(c*e*x + c*d)))
- 3*I*sqrt(pi)*erf(I*sqrt(log(c*e*x + c*d))))/(c*e)
```

## Giac [F]

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx = \int \log\left((ex+d)c\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(log(c*(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(log((e*x + d)*c)^(3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \log^{\frac{3}{2}}(c(d+ex))\,dx$$
$$= \frac{\ln\left(c\left(d+e\,x\right)\right)^{3/2}\left(\frac{3\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-\ln(c\,(d+e\,x))}\right)}{4}+c\left(\frac{3\sqrt{-\ln(c\,(d+e\,x))}}{2}+\left(-\ln\left(c\left(d+e\,x\right)\right)\right)^{3/2}\right)\left(d+e\,x\right)\right)}{c\,e\left(-\ln\left(c\,\left(d+e\,x\right)\right)\right)^{3/2}}$$

```
[In] int(log(c*(d + e*x))^(3/2),x)
[Out] (log(c*(d + e*x))^(3/2)*((3*pi^(1/2)*erfc((-log(c*(d + e*x)))^(1/2)))/4 + c
*((3*(-log(c*(d + e*x)))^(1/2))/2 + (-log(c*(d + e*x)))^(3/2))*(d + e*x)))/
(c*e*(-log(c*(d + e*x)))^(3/2))
```

# 3.11 $\int \sqrt{\log(c(d+ex))}\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 50

$$\int \sqrt{\log(c(d+ex))}\,dx = -\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{2ce}+\frac{(d+ex)\sqrt{\log(c(d+ex))}}{e}$$

```
[Out] -1/2*erfi(ln(c*(e*x+d))^(1/2))*Pi^(1/2)/c/e+(e*x+d)*ln(c*(e*x+d))^(1/2)/e
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2436, 2333, 2336, 2211, 2235}

$$\int \sqrt{\log(c(d+ex))}\,dx = \frac{(d+ex)\sqrt{\log(c(d+ex))}}{e}-\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{2ce}$$

```
[In] Int[Sqrt[Log[c*(d + e*x)]],x]
```

```
[Out] -1/2*(Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/(c*e) + ((d + e*x)*Sqrt[Log[c*
(d + e*x)]])/e
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{\log(cx)}\,dx, x, d+ex\right)}{e}\\
&= \frac{(d+ex)\sqrt{\log(c(d+ex))}}{e} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\log(cx)}}\,dx, x, d+ex\right)}{2e}\\
&= \frac{(d+ex)\sqrt{\log(c(d+ex))}}{e} - \frac{\text{Subst}\left(\int \frac{e^x}{\sqrt{x}}\,dx, x, \log(c(d+ex))\right)}{2ce}\\
&= \frac{(d+ex)\sqrt{\log(c(d+ex))}}{e} - \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\log(c(d+ex))}\right)}{ce}\\
&= -\frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{2ce} + \frac{(d+ex)\sqrt{\log(c(d+ex))}}{e}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \sqrt{\log(c(d+ex))}\,dx = -\frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{2ce} + \frac{(d+ex)\sqrt{\log(c(d+ex))}}{e}$$

```
[In] Integrate[Sqrt[Log[c*(d + e*x)]],x]
[Out] -1/2*(Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/(c*e) + ((d + e*x)*Sqrt[Log[c*
(d + e*x)]])/e
```

## Maple [F]

$$\int \sqrt{\ln\left(c\left(ex+d\right)\right)}dx$$

```
[In] int(ln(c*(e*x+d))^(1/2),x)
[Out] int(ln(c*(e*x+d))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\log(c(d+ex))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(log(c*(e*x+d))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(41) = 82.

Time = 0.90 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\int \sqrt{\log(c(d+ex))}\,dx = \begin{cases} \tilde{\infty}x & \text{for}\ c=0 \\ x\sqrt{\log\left(cd\right)} & \text{for}\ e=0 \\ \frac{\left(\sqrt{-\log\left(cd+cex\right)}(cd+cex)+\frac{\sqrt{\pi}\,\text{erfc}\left(\sqrt{-\log\left(cd+cex\right)}\right)}{2}\right)\sqrt{\log\left(cd+cex\right)}}{ce\sqrt{-\log\left(cd+cex\right)}} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(e*x+d))**(1/2),x)
[Out] Piecewise((zoo*x, Eq(c, 0)), (x*sqrt(log(c*d)), Eq(e, 0)), ((sqrt(-log(c*d
+ c*e*x))*(c*d + c*e*x) + sqrt(pi)*erfc(sqrt(-log(c*d + c*e*x)))/2)*sqrt(lo
g(c*d + c*e*x))/(c*e*sqrt(-log(c*d + c*e*x))), True))
```

## Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \sqrt{\log(c(d+ex))}\,dx = -\frac{-i\sqrt{\pi}\,\mathrm{erf}\left(i\sqrt{\log(cex+cd)}\right) - 2\,(cex+cd)\sqrt{\log(cex+cd)}}{2\,ce}$$

```
[In] integrate(log(c*(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*sqrt(pi)*erf(I*sqrt(log(c*e*x + c*d))) - 2*(c*e*x + c*d)*sqrt(log(
c*e*x + c*d)))/(c*e)
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \sqrt{\log(c(d+ex))}\,dx = -\frac{i\sqrt{\pi}\,\mathrm{erf}\left(-i\sqrt{\log(cex+cd)}\right)}{2\,ce} + \frac{(cex+cd)\sqrt{\log(cex+cd)}}{ce}$$

```
[In] integrate(log(c*(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*I*sqrt(pi)*erf(-I*sqrt(log(c*e*x + c*d)))/(c*e) + (c*e*x + c*d)*sqrt(l
og(c*e*x + c*d))/(c*e)
```

## Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \sqrt{\log(c(d+ex))}\,dx = \frac{\sqrt{\ln\left(c\,\left(d+e\,x\right)\right)}\,\left(d+e\,x\right)}{e} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\ln\left(c\,\left(d+e\,x\right)\right)}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,c\,e}$$

```
[In] int(log(c*(d + e*x))^(1/2),x)
```

```
[Out] (log(c*(d + e*x))^(1/2)*(d + e*x))/e + (pi^(1/2)*erf(log(c*(d + e*x))^(1/2)
*1i)*1i)/(2*c*e)
```

# 3.12 $\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx = \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{ce}$$

```
[Out] erfi(ln(c*(e*x+d))^(1/2))*Pi^(1/2)/c/e
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2436, 2336, 2211, 2235}

$$\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx = \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{ce}$$

```
[In] Int[1/Sqrt[Log[c*(d + e*x)]],x]
[Out] (Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/(c*e)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\log(cx)}}\,dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{e^x}{\sqrt{x}}\,dx, x, \log(c(d+ex))\right)}{ce} \\
&= \frac{2\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\log(c(d+ex))}\right)}{ce} \\
&= \frac{\sqrt{\pi}\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{ce}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx = \frac{\sqrt{\pi}\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{ce}$$

```
[In] Integrate[1/Sqrt[Log[c*(d + e*x)]],x]
[Out] (Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/(c*e)
```

## Maple [F]

$$\int \frac{1}{\sqrt{\ln \left(c \left(e x+d \right)\right)}}d x$$

```
[In] int(1/ln(c*(e*x+d))^(1/2),x)
```

```
[Out] int(1/ln(c*(e*x+d))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/log(c*(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(20) = 40$.

Time = 0.96 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx = \begin{cases} 0 & \text{for}\: c = 0 \\ \frac{x}{\sqrt{\log{\left(c d \right)}}} & \text{for}\: e = 0 \\ \frac{\sqrt{\pi} \sqrt{- \log{\left(c d + c e x \right)}} \operatorname{erfc}{\left(\sqrt{- \log{\left(c d + c e x \right)}} \right)}}{c e \sqrt{\log{\left(c d + c e x \right)}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/ln(c*(e*x+d))**(1/2),x)
```

```
[Out] Piecewise((0, Eq(c, 0)), (x/sqrt(log(c*d)), Eq(e, 0)), (sqrt(pi)*sqrt(-log(
c*d + c*e*x))*erfc(sqrt(-log(c*d + c*e*x)))/(c*e*sqrt(log(c*d + c*e*x))), T
rue))
```

## Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx = -\frac{i\sqrt{\pi}\,\mathrm{erf}\left(i\sqrt{\log\left(cex+cd\right)}\right)}{ce}$$

```
[In] integrate(1/log(c*(e*x+d))^(1/2),x, algorithm="maxima")
[Out] -I*sqrt(pi)*erf(I*sqrt(log(c*e*x + c*d)))/(c*e)
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx = \frac{i\sqrt{\pi}\,\mathrm{erf}\left(-i\sqrt{\log\left(cex+cd\right)}\right)}{ce}$$

```
[In] integrate(1/log(c*(e*x+d))^(1/2),x, algorithm="giac")
[Out] I*sqrt(pi)*erf(-I*sqrt(log(c*e*x + c*d)))/(c*e)
```

## Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sqrt{\log(c(d+ex))}}\,dx = \frac{\sqrt{\pi}\,\sqrt{-\ln\left(c\,\left(d+e\,x\right)\right)}\,\mathrm{erfc}\left(\sqrt{-\ln\left(c\,\left(d+e\,x\right)\right)}\right)}{c\,e\,\sqrt{\ln\left(c\,\left(d+e\,x\right)\right)}}$$

```
[In] int(1/log(c*(d + e*x))^(1/2),x)
[Out] (pi^(1/2)*(-log(c*(d + e*x)))^(1/2)*erfc((-log(c*(d + e*x)))^(1/2)))/(c*e*l
og(c*(d + e*x))^(1/2))
```

# 3.13 $\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 49

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx = \frac{2\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{ce} - \frac{2(d+ex)}{e\sqrt{\log(c(d+ex))}}$$

```
[Out] 2*erfi(ln(c*(e*x+d))^(1/2))*Pi^(1/2)/c/e-2*(e*x+d)/e/ln(c*(e*x+d))^(1/2)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2436, 2334, 2336, 2211, 2235}

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx = \frac{2\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{ce} - \frac{2(d+ex)}{e\sqrt{\log(c(d+ex))}}$$

```
[In] Int[Log[c*(d + e*x)]^(-3/2),x]
```

```
[Out] (2*Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/(c*e) - (2*(d + e*x))/(e*Sqrt[Log
[c*(d + e*x)]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\log^{\frac{3}{2}}(cx)}\,dx, x, d+ex\right)}{e}\\
&= -\frac{2(d+ex)}{e\sqrt{\log(c(d+ex))}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{\log(cx)}}\,dx, x, d+ex\right)}{e}\\
&= -\frac{2(d+ex)}{e\sqrt{\log(c(d+ex))}} + \frac{2\text{Subst}\left(\int \frac{e^x}{\sqrt{x}}\,dx, x, \log(c(d+ex))\right)}{ce}\\
&= -\frac{2(d+ex)}{e\sqrt{\log(c(d+ex))}} + \frac{4\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\log(c(d+ex))}\right)}{ce}\\
&= \frac{2\sqrt{\pi}\,\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{ce} - \frac{2(d+ex)}{e\sqrt{\log(c(d+ex))}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx=\frac{-2c(d+ex)+2\Gamma\left(\frac{1}{2},-\log(c(d+ex))\right)\sqrt{-\log(c(d+ex))}}{ce\sqrt{\log(c(d+ex))}}$$

```
[In] Integrate[Log[c*(d + e*x)]^(-3/2),x]
[Out] (-2*c*(d + e*x) + 2*Gamma[1/2, -Log[c*(d + e*x)]]*Sqrt[-Log[c*(d + e*x)]])/
(c*e*Sqrt[Log[c*(d + e*x)]])
```

## Maple [F]

$$\int \frac{1}{\ln\left(c\left(ex+d\right)\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/ln(c*(e*x+d))^(3/2),x)
[Out] int(1/ln(c*(e*x+d))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/log(c*(e*x+d))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(44) = 88.

Time = 13.49 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.88

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx=\begin{cases} 0 & \text{for}\ c=0 \\ \frac{x}{\log\left(cd\right)^{\frac{3}{2}}} & \text{for}\ e=0 \\ \frac{\left(-\log\left(cd+cex\right)\right)^{\frac{3}{2}}\left(-2\sqrt{\pi}\operatorname{erfc}\left(\sqrt{-\log\left(cd+cex\right)}\right)+\frac{2(cd+cex)}{\sqrt{-\log\left(cd+cex\right)}}\right)}{ce\log\left(cd+cex\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/ln(c*(e*x+d))**(3/2),x)
```

```
[Out] Piecewise((0, Eq(c, 0)), (x/log(c*d)**(3/2), Eq(e, 0)), ((-log(c*d + c*e*x)
)**(3/2)*(-2*sqrt(pi)*erfc(sqrt(-log(c*d + c*e*x))) + 2*(c*d + c*e*x)/sqrt(
-log(c*d + c*e*x)))/(c*e*log(c*d + c*e*x)**(3/2)), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx = -\frac{\sqrt{-\log(cex+cd)}\Gamma\left(-\frac{1}{2}, -\log(cex+cd)\right)}{ce\sqrt{\log(cex+cd)}}$$

```
[In] integrate(1/log(c*(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-log(c*e*x + c*d))*gamma(-1/2, -log(c*e*x + c*d))/(c*e*sqrt(log(c*e*x
 + c*d)))
```

## Giac [F]

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx = \int \frac{1}{\log\left((ex+d)c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/log(c*(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(log((e*x + d)*c)^(-3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \frac{1}{\log^{\frac{3}{2}}(c(d+ex))}\,dx = -\frac{2\,(d+e\,x)}{e\,\sqrt{\ln\left(c\,\left(d+e\,x\right)\right)}} - \frac{2\,\sqrt{\pi}\,{\left(-\ln\left(c\,\left(d+e\,x\right)\right)\right)}^{3/2}\,\mathrm{erfc}\left(\sqrt{-\ln\left(c\,\left(d+e\,x\right)\right)}\right)}{c\,e\,{\ln\left(c\,\left(d+e\,x\right)\right)}^{3/2}}$$

```
[In] int(1/log(c*(d + e*x))^(3/2),x)
```

```
[Out] - (2*(d + e*x))/(e*log(c*(d + e*x))^(1/2)) - (2*pi^(1/2)*(-log(c*(d + e*x))
)^(3/2)*erfc((-log(c*(d + e*x)))^(1/2)))/(c*e*log(c*(d + e*x))^(3/2))
```

# 3.14 $\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))} dx$



## Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))} dx = \frac{4\sqrt{\pi}\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{3ce} - \frac{2(d+ex)}{3e\log^{\frac{3}{2}}(c(d+ex))} - \frac{4(d+ex)}{3e\sqrt{\log(c(d+ex))}}$$

```
[Out] -2/3*(e*x+d)/e/ln(c*(e*x+d))^(3/2)+4/3*erfi(ln(c*(e*x+d))^(1/2))*Pi^(1/2)/c
/e-4/3*(e*x+d)/e/ln(c*(e*x+d))^(1/2)
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2436, 2334, 2336, 2211, 2235}

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))} dx = \frac{4\sqrt{\pi}\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{3ce} - \frac{2(d+ex)}{3e\log^{\frac{3}{2}}(c(d+ex))} - \frac{4(d+ex)}{3e\sqrt{\log(c(d+ex))}}$$

```
[In] Int[Log[c*(d + e*x)]^(-5/2),x]
```

```
[Out] (4*Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/(3*c*e) - (2*(d + e*x))/(3*e*Log[
c*(d + e*x)]^(3/2)) - (4*(d + e*x))/(3*e*Sqrt[Log[c*(d + e*x)]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\log^{\frac{5}{2}}(cx)}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{2(d+ex)}{3e\log^{\frac{3}{2}}(c(d+ex))} + \frac{2\text{Subst}\left(\int \frac{1}{\log^{\frac{3}{2}}(cx)}\,dx, x, d+ex\right)}{3e}$$

$$= -\frac{2(d+ex)}{3e\log^{\frac{3}{2}}(c(d+ex))} - \frac{4(d+ex)}{3e\sqrt{\log(c(d+ex))}} + \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{\log(cx)}}\,dx, x, d+ex\right)}{3e}$$

$$= -\frac{2(d+ex)}{3e\log^{\frac{3}{2}}(c(d+ex))} - \frac{4(d+ex)}{3e\sqrt{\log(c(d+ex))}} + \frac{4\text{Subst}\left(\int \frac{e^x}{\sqrt{x}}\,dx, x, \log(c(d+ex))\right)}{3ce}$$

$$= -\frac{2(d+ex)}{3e\log^{\frac{3}{2}}(c(d+ex))} - \frac{4(d+ex)}{3e\sqrt{\log(c(d+ex))}} + \frac{8\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\log(c(d+ex))}\right)}{3ce}$$

$$= \frac{4\sqrt{\pi}\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{3ce} - \frac{2(d+ex)}{3e\log^{\frac{3}{2}}(c(d+ex))} - \frac{4(d+ex)}{3e\sqrt{\log(c(d+ex))}}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}\,dx = -\frac{2\left(2\Gamma\left(\frac{1}{2}, -\log(c(d+ex))\right)(-\log(c(d+ex)))^{3/2} + c(d+ex)(1+2\log(c(d+ex)))\right)}{3ce\log^{\frac{3}{2}}(c(d+ex))}$$

```
[In] Integrate[Log[c*(d + e*x)]^(-5/2),x]
[Out] (-2*(2*Gamma[1/2, -Log[c*(d + e*x)]]*(-Log[c*(d + e*x)])^(3/2) + c*(d + e*x
)*(1 + 2*Log[c*(d + e*x)])))/(3*c*e*Log[c*(d + e*x)]^(3/2))
```

## Maple [F]

$$\int \frac{1}{\ln\left(c\left(ex+d\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/ln(c*(e*x+d))^(5/2),x)
[Out] int(1/ln(c*(e*x+d))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/log(c*(e*x+d))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}\,dx = \text{Timed out}$$

```
[In] integrate(1/ln(c*(e*x+d))**(5/2),x)
[Out] Timed out
```

## Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}\,dx = -\frac{(-\log{(cex+cd)})^{\frac{3}{2}}\,\Gamma\left(-\frac{3}{2},-\log{(cex+cd)}\right)}{ce\log{(cex+cd)}^{\frac{3}{2}}}$$

```
[In] integrate(1/log(c*(e*x+d))^(5/2),x, algorithm="maxima")
[Out] -(-log(c*e*x + c*d))^(3/2)*gamma(-3/2, -log(c*e*x + c*d))/(c*e*log(c*e*x +
c*d)^(3/2))
```

## Giac [F]

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}\,dx = \int \frac{1}{\log{((ex+d)c)}^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/log(c*(e*x+d))^(5/2),x, algorithm="giac")
[Out] integrate(log((e*x + d)*c)^(-5/2), x)
```

## Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int \frac{1}{\log^{\frac{5}{2}}(c(d+ex))}\,dx = \frac{4\sqrt{\pi}\,(-\ln{(c\,(d+e\,x))})^{5/2}\,\mathrm{erfc}\left(\sqrt{-\ln{(c\,(d+e\,x))}}\right)}{3\,c\,e\ln{(c\,(d+e\,x))}^{5/2}} - \frac{4\,d\ln{(c\,(d+e\,x))}^2+2\,d\,\ln{(c\,(d+e\,x))}+2\,e\,x\,\ln{(c\,(d+e\,x))}+4\,e\,x\ln{(c\,(d+e\,x))}^2}{3\,e\ln{(c\,(d+e\,x))}^{5/2}}$$

```
[In] int(1/log(c*(d + e*x))^(5/2),x)
```

```
[Out] (4*pi^(1/2)*(-log(c*(d + e*x)))^(5/2)*erfc((-log(c*(d + e*x)))^(1/2)))/(3*c
*e*log(c*(d + e*x))^(5/2)) - (4*d*log(c*(d + e*x))^2 + 2*d*log(c*(d + e*x))
 + 2*e*x*log(c*(d + e*x)) + 4*e*x*log(c*(d + e*x))^2)/(3*e*log(c*(d + e*x))
^(5/2))
```

# 3.15 $\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx=\frac{8\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{15ce}-\frac{2(d+ex)}{5e\log^{\frac{5}{2}}(c(d+ex))}-\frac{4(d+ex)}{15e\log^{\frac{3}{2}}(c(d+ex))}-\frac{8(d+ex)}{15e\sqrt{\log(c(d+ex))}}$$

```
[Out] -2/5*(e*x+d)/e/ln(c*(e*x+d))^(5/2)-4/15*(e*x+d)/e/ln(c*(e*x+d))^(3/2)+8/15*
erfi(ln(c*(e*x+d))^(1/2))*Pi^(1/2)/c/e-8/15*(e*x+d)/e/ln(c*(e*x+d))^(1/2)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2436, 2334, 2336, 2211, 2235}

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx=\frac{8\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{15ce}-\frac{4(d+ex)}{15e\log^{\frac{3}{2}}(c(d+ex))}-\frac{2(d+ex)}{5e\log^{\frac{5}{2}}(c(d+ex))}-\frac{8(d+ex)}{15e\sqrt{\log(c(d+ex))}}$$

```
[In] Int[Log[c*(d + e*x)]^(-7/2),x]
```

```
[Out] (8*Sqrt[Pi]*Erfi[Sqrt[Log[c*(d + e*x)]]])/(15*c*e) - (2*(d + e*x))/(5*e*Log
[c*(d + e*x)]^(5/2)) - (4*(d + e*x))/(15*e*Log[c*(d + e*x)]^(3/2)) - (8*(d
+ e*x))/(15*e*Sqrt[Log[c*(d + e*x)]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\log^{\frac{7}{2}}(cx)}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{2(d+ex)}{5e\log^{\frac{5}{2}}(c(d+ex))} + \frac{2\text{Subst}\left(\int \frac{1}{\log^{\frac{5}{2}}(cx)}\,dx, x, d+ex\right)}{5e}$$

$$= -\frac{2(d+ex)}{5e\log^{\frac{5}{2}}(c(d+ex))} - \frac{4(d+ex)}{15e\log^{\frac{3}{2}}(c(d+ex))} + \frac{4\text{Subst}\left(\int \frac{1}{\log^{\frac{3}{2}}(cx)}\,dx, x, d+ex\right)}{15e}$$

$$= -\frac{2(d+ex)}{5e\log^{\frac{5}{2}}(c(d+ex))} - \frac{4(d+ex)}{15e\log^{\frac{3}{2}}(c(d+ex))} - \frac{8(d+ex)}{15e\sqrt{\log(c(d+ex))}} + \frac{8\text{Subst}\left(\int \frac{1}{\sqrt{\log(cx)}}\,dx, x, d+ex\right)}{15e}$$

$$= -\frac{2(d+ex)}{5e\log^{\frac{5}{2}}(c(d+ex))} - \frac{4(d+ex)}{15e\log^{\frac{3}{2}}(c(d+ex))} - \frac{8(d+ex)}{15e\sqrt{\log(c(d+ex))}} + \frac{8\text{Subst}\left(\int \frac{e^x}{\sqrt{x}}\,dx, x, \log(c(d+ex))\right)}{15ce}$$

$$= -\frac{2(d+ex)}{5e\log^{\frac{5}{2}}(c(d+ex))} - \frac{4(d+ex)}{15e\log^{\frac{3}{2}}(c(d+ex))} - \frac{8(d+ex)}{15e\sqrt{\log(c(d+ex))}} + \frac{16\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\log(c(d+ex))}\right)}{15ce}$$

$$= \frac{8\sqrt{\pi}\text{erfi}\left(\sqrt{\log(c(d+ex))}\right)}{15ce} - \frac{2(d+ex)}{5e\log^{\frac{5}{2}}(c(d+ex))} - \frac{4(d+ex)}{15e\log^{\frac{3}{2}}(c(d+ex))} - \frac{8(d+ex)}{15e\sqrt{\log(c(d+ex))}}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.84

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx = \frac{8\Gamma\left(\frac{1}{2}, -\log(c(d+ex))\right)(-\log(c(d+ex)))^{5/2} - 2c(d+ex)\left(3+2\log(c(d+ex))+4\log^2(c(d+ex))\right)}{15ce\log^{\frac{5}{2}}(c(d+ex))}$$

```
[In] Integrate[Log[c*(d + e*x)]^(-7/2),x]
```

```
[Out] (8*Gamma[1/2, -Log[c*(d + e*x)]]*(-Log[c*(d + e*x)])^(5/2) - 2*c*(d + e*x)*
(3 + 2*Log[c*(d + e*x)] + 4*Log[c*(d + e*x)]^2))/(15*c*e*Log[c*(d + e*x)]^(
5/2))
```

## Maple [F]

$$\int \frac{1}{\ln \left(c \left(e x+d\right)\right)^{\frac{7}{2}}}d x$$

```
[In] int(1/ln(c*(e*x+d))^(7/2),x)
```

```
[Out] int(1/ln(c*(e*x+d))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/log(c*(e*x+d))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx = \text{Timed out}$$

```
[In] integrate(1/ln(c*(e*x+d))**(7/2),x)
```

```
[Out] Timed out
```

## Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx = -\frac{\left(-\log\left(cex+cd\right)\right)^{\frac{5}{2}}\Gamma\left(-\frac{5}{2},-\log\left(cex+cd\right)\right)}{ce\log\left(cex+cd\right)^{\frac{5}{2}}}$$

```
[In] integrate(1/log(c*(e*x+d))^(7/2),x, algorithm="maxima")
```

```
[Out] -(-log(c*e*x + c*d))^(5/2)*gamma(-5/2, -log(c*e*x + c*d))/(c*e*log(c*e*x +
c*d)^(5/2))
```

## Giac [F]

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx = \int \frac{1}{\log\left((ex+d)c\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/log(c*(e*x+d))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(log((e*x + d)*c)^(-7/2), x)
```

## Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.39

$$\int \frac{1}{\log^{\frac{7}{2}}(c(d+ex))}\,dx =$$

$$-\frac{4\,d\ln\left(c\left(d+e\,x\right)\right)^2+8\,d\ln\left(c\left(d+e\,x\right)\right)^3+6\,d\,\ln\left(c\left(d+e\,x\right)\right)+6\,e\,x\,\ln\left(c\left(d+e\,x\right)\right)+4\,e\,x\ln\left(c\left(d+e\,x\right.\right.}{15\,e\ln\left(c\,\left(d+e\,x\right)\right)^{7/2}}$$

$$-\frac{8\,\sqrt{\pi}\,\left(-\ln\left(c\left(d+e\,x\right)\right)\right)^{7/2}\,\mathrm{erfc}\left(\sqrt{-\ln\left(c\,\left(d+e\,x\right)\right)}\right)}{15\,c\,e\ln\left(c\,\left(d+e\,x\right)\right)^{7/2}}$$

```
[In] int(1/log(c*(d + e*x))^(7/2),x)
```

```
[Out] - (4*d*log(c*(d + e*x))^2 + 8*d*log(c*(d + e*x))^3 + 6*d*log(c*(d + e*x)) +
 6*e*x*log(c*(d + e*x)) + 4*e*x*log(c*(d + e*x))^2 + 8*e*x*log(c*(d + e*x))
^3)/(15*e*log(c*(d + e*x))^(7/2)) - (8*pi^(1/2)*(-log(c*(d + e*x)))^(7/2)*e
rfc((-log(c*(d + e*x)))^(1/2)))/(15*c*e*log(c*(d + e*x))^(7/2))
```

# 3.16 $\int \log^p(c(d+ex))\,dx$



## Optimal result

Integrand size = 10, antiderivative size = 45

$$\int \log^p(c(d+ex))\,dx = \frac{\Gamma(1+p,-\log(c(d+ex)))(-\log(c(d+ex)))^{-p}\log^p(c(d+ex))}{ce}$$

```
[Out] GAMMA(p+1,-ln(c*(e*x+d)))*ln(c*(e*x+d))^p/c/e/((-ln(c*(e*x+d)))^p)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2436, 2336, 2212}

$$\int \log^p(c(d+ex))\,dx = \frac{(-\log(c(d+ex)))^{-p}\log^p(c(d+ex))\Gamma(p+1,-\log(c(d+ex)))}{ce}$$

```
[In] Int[Log[c*(d + e*x)]^p,x]
```

```
[Out] (Gamma[1 + p, -Log[c*(d + e*x)]]*Log[c*(d + e*x)]^p)/(c*e*(-Log[c*(d + e*x)
])^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \log^p(cx)\,dx, x, d+ex\right)}{e}\\ &= \frac{\text{Subst}\left(\int e^x x^p\,dx, x, \log(c(d+ex))\right)}{ce}\\ &= \frac{\Gamma(1+p,-\log(c(d+ex)))(-\log(c(d+ex)))^{-p}\log^p(c(d+ex))}{ce}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \log^p(c(d+ex))\,dx = \frac{\Gamma(1+p,-\log(c(d+ex)))(-\log(c(d+ex)))^{-p}\log^p(c(d+ex))}{ce}$$

```
[In] Integrate[Log[c*(d + e*x)]^p,x]
[Out] (Gamma[1 + p, -Log[c*(d + e*x)]]*Log[c*(d + e*x)]^p)/(c*e*(-Log[c*(d + e*x)
])^p)
```

## Maple [F]

$$\int \ln\left(c(ex+d)\right)^p\,dx$$

```
[In] int(ln(c*(e*x+d))^p,x)
[Out] int(ln(c*(e*x+d))^p,x)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.60

$$\int \log^p(c(d+ex))\,dx = \frac{e^{(-i\,\pi p)}\Gamma(p+1,-\log(cex+cd))}{ce}$$

```
[In] integrate(log(c*(e*x+d))^p,x, algorithm="fricas")
[Out] e^(-I*pi*p)*gamma(p + 1, -log(c*e*x + c*d))/(c*e)
```

## Sympy [A] (verification not implemented)

Time = 3.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \log^p(c(d+ex))\,dx = \begin{cases} \tilde{\infty}^p x & \text{for } c=0 \\ x\log(cd)^p & \text{for } e=0 \\ \frac{(-\log(cd+cex))^{-p}\log(cd+cex)^p\Gamma(p+1,-\log(cd+cex))}{ce} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(e*x+d))**p,x)
[Out] Piecewise((zoo**p*x, Eq(c, 0)), (x*log(c*d)**p, Eq(e, 0)), (log(c*d + c*e*x
)**p*uppergamma(p + 1, -log(c*d + c*e*x))/(c*e*(-log(c*d + c*e*x))**p), Tru
e))
```

## Maxima [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \log^p(c(d+ex))\,dx = -\frac{(-\log(cex+cd))^{-p-1}\log(cex+cd)^{p+1}\,\Gamma(p+1,-\log(cex+cd))}{ce}$$

```
[In] integrate(log(c*(e*x+d))^p,x, algorithm="maxima")
[Out] -(-log(c*e*x + c*d))^(-p - 1)*log(c*e*x + c*d)^(p + 1)*gamma(p + 1, -log(c*
e*x + c*d))/(c*e)
```

## Giac [F]

$$\int \log^p(c(d+ex))\,dx = \int \log\left((ex+d)c\right)^p\,dx$$

```
[In] integrate(log(c*(e*x+d))^p,x, algorithm="giac")
```

```
[Out] integrate(log((e*x + d)*c)^p, x)
```

## Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \log^p(c(d+ex))\,dx = \frac{\ln\left(c\,(d+e\,x)\right)^p\,\Gamma(p+1,-\ln\left(c\,(d+e\,x)\right))}{c\,e\,\left(-\ln\left(c\,\left(d+e\,x\right)\right)\right)^p}$$

```
[In] int(log(c*(d + e*x))^p,x)
```

```
[Out] (log(c*(d + e*x))^p*igamma(p + 1, -log(c*(d + e*x))))/(c*e*(-log(c*(d + e*x
)))^p)
```

# 3.17 $\int (a+b\log (c(d+ex)^n))^4 \, dx$



## Optimal result

Integrand size = 16, antiderivative size = 131

$$\int (a+b\log (c(d+ex)^n))^4 \, dx = -24ab^3n^3x+24b^4n^4x-\frac{24b^4n^3(d+ex)\log (c(d+ex)^n)}{e}+\frac{12b^2n^2(d+ex)\,(a+b\log (c(d+ex)^n))^2}{e}-\frac{4bn(d+ex)\,(a+b\log (c(d+ex)^n))^3}{e}+\frac{(d+ex)\,(a+b\log (c(d+ex)^n))^4}{e}$$

```
[Out] -24*a*b^3*n^3*x+24*b^4*n^4*x-24*b^4*n^3*(e*x+d)*ln(c*(e*x+d)^n)/e+12*b^2*n^
2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e-4*b*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e
+(e*x+d)*(a+b*ln(c*(e*x+d)^n))^4/e
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2436, 2333, 2332}

$$\int (a+b\log (c(d+ex)^n))^4 \, dx = -24ab^3n^3x+\frac{12b^2n^2(d+ex)\,(a+b\log (c(d+ex)^n))^2}{e}-\frac{4bn(d+ex)\,(a+b\log (c(d+ex)^n))^3}{e}+\frac{(d+ex)\,(a+b\log (c(d+ex)^n))^4}{e}-\frac{24b^4n^3(d+ex)\log (c(d+ex)^n)}{e}+24b^4n^4x$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^4,x]
```

```
[Out] -24*a*b^3*n^3*x + 24*b^4*n^4*x - (24*b^4*n^3*(d + e*x)*Log[c*(d + e*x)^n])/
e + (12*b^2*n^2*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e - (4*b*n*(d + e*x
)*(a + b*Log[c*(d + e*x)^n])^3)/e + ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^4
)/e
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+b\log(cx^n))^4\,dx, x, d+ex\right)}{e}\\
&= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e} - \frac{(4bn)\text{Subst}\left(\int (a+b\log(cx^n))^3\,dx, x, d+ex\right)}{e}\\
&= -\frac{4bn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e} + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e}\\
&\quad + \frac{(12b^2n^2)\,\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx, x, d+ex\right)}{e}\\
&= \frac{12b^2n^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} - \frac{4bn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e}\\
&\quad + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e} - \frac{(24b^3n^3)\,\text{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+ex\right)}{e}\\
&= -24ab^3n^3x + \frac{12b^2n^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}\\
&\quad - \frac{4bn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e} + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e}\\
&\quad - \frac{(24b^4n^3)\,\text{Subst}\left(\int \log(cx^n)\,dx, x, d+ex\right)}{e}
\end{aligned}$$

$$= -24ab^3n^3x + 24b^4n^4x - \frac{24b^4n^3(d+ex)\log\left(c(d+ex)^n\right)}{e} + \frac{12b^2n^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} - \frac{4bn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e} + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \left(a+b\log\left(c(d+ex)^n\right)\right)^4\,dx = \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4 - 4bn\big((d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3 - 3bn\big((d+ex)\left(a+b\log\left(c(d\right.\right.}{e}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^4,x]
```

```
[Out] ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^4 - 4*b*n*((d + e*x)*(a + b*Log[c*(d
+ e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 - 2*b*n*(e*(a
- b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n]))))/e
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. 2(131) = 262.

Time = 1.36 (sec) , antiderivative size = 525, normalized size of antiderivative = 4.01

| method | result |
|---|---|
| parallelrisch | $\frac{-12x\ln(c(ex+d)^n)a^2b^2e\,n^2+4x\ln(c(ex+d)^n)a^3ben+4x\ln(c(ex+d)^n)^3a\,b^3en-12x\ln(c(ex+d)^n)^2a\,b^3e\,n^2+24x\ln(c(ex+d)^n)}{}$ |
| risch | Expression too large to display |

```
[In] int((a+b*ln(c*(e*x+d)^n))^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-12*x*ln(c*(e*x+d)^n)*a^2*b^2*e*n^2+4*x*ln(c*(e*x+d)^n)*a^3*b*e*n+4*x*ln(c
*(e*x+d)^n)^3*a*b^3*e*n-12*x*ln(c*(e*x+d)^n)^2*a*b^3*e*n^2+24*x*ln(c*(e*x+d
)^n)*a*b^3*e*n^3+6*x*ln(c*(e*x+d)^n)^2*a^2*b^2*e*n+24*a*b^3*d*n^4-12*a^2*b^
2*d*n^3+4*a^3*b*d*n^2+ln(c*(e*x+d)^n)^4*b^4*d*n-4*ln(c*(e*x+d)^n)^3*b^4*d*n
^2+12*ln(c*(e*x+d)^n)^2*b^4*d*n^3-24*ln(c*(e*x+d)^n)*b^4*d*n^4+x*a^4*e*n+24
*x*b^4*e*n^5+12*x*a^2*b^2*e*n^3+4*ln(c*(e*x+d)^n)^3*a*b^3*d*n-12*ln(c*(e*x+
d)^n)^2*a*b^3*d*n^2+24*ln(c*(e*x+d)^n)*a*b^3*d*n^3-4*x*a^3*b*e*n^2+6*ln(c*(
e*x+d)^n)^2*a^2*b^2*d*n-12*ln(c*(e*x+d)^n)*a^2*b^2*d*n^2+4*ln(c*(e*x+d)^n)*
a^3*b*d*n+x*ln(c*(e*x+d)^n)^4*b^4*e*n-4*x*ln(c*(e*x+d)^n)^3*b^4*e*n^2+12*x*
ln(c*(e*x+d)^n)^2*b^4*e*n^3-24*x*ln(c*(e*x+d)^n)*b^4*e*n^4-24*x*a*b^3*e*n^4
-24*b^4*d*n^5-a^4*d*n)/e/n
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(131) = 262.

Time = 0.30 (sec) , antiderivative size = 614, normalized size of antiderivative = 4.69

$$\int (a + b\log(c(d + ex)^n))^4 \, dx$$
$$= \frac{b^4ex\log(c)^4 + (b^4en^4x + b^4dn^4)\log(ex + d)^4 - 4(b^4en - ab^3e)x\log(c)^3 - 4(b^4dn^4 - ab^3dn^3 + (b^4en^4 - a}{}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^4,x, algorithm="fricas")
```

```
[Out] (b^4*e*x*log(c)^4 + (b^4*e*n^4*x + b^4*d*n^4)*log(e*x + d)^4 - 4*(b^4*e*n -
 a*b^3*e)*x*log(c)^3 - 4*(b^4*d*n^4 - a*b^3*d*n^3 + (b^4*e*n^4 - a*b^3*e*n^
3)*x - (b^4*e*n^3*x + b^4*d*n^3)*log(c))*log(e*x + d)^3 + 6*(2*b^4*e*n^2 -
2*a*b^3*e*n + a^2*b^2*e)*x*log(c)^2 + 6*(2*b^4*d*n^4 - 2*a*b^3*d*n^3 + a^2*
b^2*d*n^2 + (b^4*e*n^2*x + b^4*d*n^2)*log(c)^2 + (2*b^4*e*n^4 - 2*a*b^3*e*n
^3 + a^2*b^2*e*n^2)*x - 2*(b^4*d*n^3 - a*b^3*d*n^2 + (b^4*e*n^3 - a*b^3*e*n
^2)*x)*log(c))*log(e*x + d)^2 - 4*(6*b^4*e*n^3 - 6*a*b^3*e*n^2 + 3*a^2*b^2*
e*n - a^3*b*e)*x*log(c) + (24*b^4*e*n^4 - 24*a*b^3*e*n^3 + 12*a^2*b^2*e*n^2
 - 4*a^3*b*e*n + a^4*e)*x - 4*(6*b^4*d*n^4 - 6*a*b^3*d*n^3 + 3*a^2*b^2*d*n^
2 - a^3*b*d*n - (b^4*e*n*x + b^4*d*n)*log(c)^3 + 3*(b^4*d*n^2 - a*b^3*d*n +
 (b^4*e*n^2 - a*b^3*e*n)*x)*log(c)^2 + (6*b^4*e*n^4 - 6*a*b^3*e*n^3 + 3*a^2
*b^2*e*n^2 - a^3*b*e*n)*x - 3*(2*b^4*d*n^3 - 2*a*b^3*d*n^2 + a^2*b^2*d*n +
(2*b^4*e*n^3 - 2*a*b^3*e*n^2 + a^2*b^2*e*n)*x)*log(c))*log(e*x + d))/e
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(126) = 252.

Time = 0.90 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.78

$$\int (a + b\log(c(d + ex)^n))^4 \, dx$$
$$= \begin{cases} a^4x + \frac{4a^3bd\log(c(d+ex)^n)}{e} - 4a^3bnx + 4a^3bx\log(c(d + ex)^n) - \frac{12a^2b^2dn\log(c(d+ex)^n)}{e} + \frac{6a^2b^2d\log(c(d+ex)^n)^2}{e} + 12 \\ x(a + b\log(cd^n))^4 \end{cases}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*d*log(c*(d + e*x)**n)/e - 4*a**3*b*n*x + 4*a**
3*b*x*log(c*(d + e*x)**n) - 12*a**2*b**2*d*n*log(c*(d + e*x)**n)/e + 6*a**2
*b**2*d*log(c*(d + e*x)**n)**2/e + 12*a**2*b**2*n**2*x - 12*a**2*b**2*n*x*l
og(c*(d + e*x)**n) + 6*a**2*b**2*x*log(c*(d + e*x)**n)**2 + 24*a*b**3*d*n**
2*log(c*(d + e*x)**n)/e - 12*a*b**3*d*n*log(c*(d + e*x)**n)**2/e + 4*a*b**3
```

```
*d*log(c*(d + e*x)**n)**3/e - 24*a*b**3*n**3*x + 24*a*b**3*n**2*x*log(c*(d
+ e*x)**n) - 12*a*b**3*n*x*log(c*(d + e*x)**n)**2 + 4*a*b**3*x*log(c*(d + e
*x)**n)**3 - 24*b**4*d*n**3*log(c*(d + e*x)**n)/e + 12*b**4*d*n**2*log(c*(d
 + e*x)**n)**2/e - 4*b**4*d*n*log(c*(d + e*x)**n)**3/e + b**4*d*log(c*(d +
e*x)**n)**4/e + 24*b**4*n**4*x - 24*b**4*n**3*x*log(c*(d + e*x)**n) + 12*b*
*4*n**2*x*log(c*(d + e*x)**n)**2 - 4*b**4*n*x*log(c*(d + e*x)**n)**3 + b**4
*x*log(c*(d + e*x)**n)**4, Ne(e, 0)), (x*(a + b*log(c*d**n))**4, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(131) = 262$.

Time = 0.21 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.82

$$\int (a + b\log(c(d+ex)^n))^4 \, dx = b^4x\log((ex+d)^nc)^4 + 4\,ab^3x\log((ex+d)^nc)^3$$
$$- 4\,a^3ben\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) + 6\,a^2b^2x\log((ex+d)^nc)^2 + 4\,a^3bx\log((ex+d)^nc)$$
$$-6\left(2\,en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc) + \frac{(d\log(ex+d)^2 - 2\,ex + 2\,d\log(ex+d))n^2}{e}\right)a^2b^2$$
$$-4\left(3\,en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc)^2 - en\left(\frac{(d\log(ex+d)^3 + 3\,d\log(ex+d)^2 - 6\,ex + 6\,d\,\mathrm{l}}{e^2}\right.\right.$$
$$-\left(4\,en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc)^3 + \left(en\left(\frac{(d\log(ex+d)^4 + 4\,d\log(ex+d)^3 + 12\,d\log(e}{e^3}\right.\right.\right.$$
$$+ a^4x$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log((e*x + d)^n*c)^4 + 4*a*b^3*x*log((e*x + d)^n*c)^3 - 4*a^3*b*e*n*(
x/e - d*log(e*x + d)/e^2) + 6*a^2*b^2*x*log((e*x + d)^n*c)^2 + 4*a^3*b*x*lo
g((e*x + d)^n*c) - 6*(2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c) +
 (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*a^2*b^2 - 4*(3*e*n*(x
/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^2 - e*n*((d*log(e*x + d)^3 + 3*
d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*n^2/e^2 - 3*(d*log(e*x + d)^2
- 2*e*x + 2*d*log(e*x + d))*n*log((e*x + d)^n*c)/e^2))*a*b^3 - (4*e*n*(x/e
- d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^3 + (e*n*((d*log(e*x + d)^4 + 4*d*
log(e*x + d)^3 + 12*d*log(e*x + d)^2 - 24*e*x + 24*d*log(e*x + d))*n^2/e^3
- 4*(d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*n*lo
g((e*x + d)^n*c)/e^3) + 6*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n*l
og((e*x + d)^n*c)^2/e^2)*e*n)*b^4 + a^4*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(131) = 262$.

Time = 0.34 (sec) , antiderivative size = 758, normalized size of antiderivative = 5.79

$$
\begin{aligned}
\int (a+b\log(c(d+ex)^n))^4\,dx = {} & \frac{(ex+d)b^4n^4\log(ex+d)^4}{e} - \frac{4(ex+d)b^4n^4\log(ex+d)^3}{e} \\
& + \frac{4(ex+d)b^4n^3\log(ex+d)^3\log(c)}{e} \\
& + \frac{12(ex+d)b^4n^4\log(ex+d)^2}{e} \\
& + \frac{4(ex+d)ab^3n^3\log(ex+d)^3}{e} \\
& - \frac{12(ex+d)b^4n^3\log(ex+d)^2\log(c)}{e} \\
& + \frac{6(ex+d)b^4n^2\log(ex+d)^2\log(c)^2}{e} \\
& - \frac{24(ex+d)b^4n^4\log(ex+d)}{e} \\
& - \frac{12(ex+d)ab^3n^3\log(ex+d)^2}{e} \\
& + \frac{24(ex+d)b^4n^3\log(ex+d)\log(c)}{e} \\
& + \frac{12(ex+d)ab^3n^2\log(ex+d)^2\log(c)}{e} \\
& - \frac{12(ex+d)b^4n^2\log(ex+d)\log(c)^2}{e} \\
& + \frac{4(ex+d)b^4n\log(ex+d)\log(c)^3}{e} \\
& + \frac{24(ex+d)b^4n^4}{e} + \frac{24(ex+d)ab^3n^3\log(ex+d)}{e} \\
& + \frac{6(ex+d)a^2b^2n^2\log(ex+d)^2}{e} - \frac{24(ex+d)b^4n^3\log(c)}{e} \\
& - \frac{24(ex+d)ab^3n^2\log(ex+d)\log(c)}{e} \\
& + \frac{12(ex+d)b^4n^2\log(c)^2}{e} \\
& + \frac{12(ex+d)ab^3n\log(ex+d)\log(c)^2}{e} \\
& - \frac{4(ex+d)b^4n\log(c)^3}{e} + \frac{(ex+d)b^4\log(c)^4}{e} \\
& - \frac{24(ex+d)ab^3n^3}{e} - \frac{12(ex+d)a^2b^2n^2\log(ex+d)}{e} \\
& + \frac{24(ex+d)ab^3n^2\log(c)}{e} \\
& + \frac{12(ex+d)a^2b^2n\log(ex+d)\log(c)}{e} \\
& - \frac{12(ex+d)ab^3n\log(c)^2}{e} + \frac{4(ex+d)ab^3\log(c)^3}{e} \\
& + \frac{12(ex+d)a^2b^2n^2}{e} + \frac{4(ex+d)a^3bn\log(ex+d)}{e} \\
& - \frac{12(ex+d)a^2b^2n\log(c)}{e} + \frac{6(ex+d)a^2b^2\log(c)^2}{e}
\end{aligned}
$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^4,x, algorithm="giac")
```

```
[Out] (e*x + d)*b^4*n^4*log(e*x + d)^4/e - 4*(e*x + d)*b^4*n^4*log(e*x + d)^3/e +
 4*(e*x + d)*b^4*n^3*log(e*x + d)^3*log(c)/e + 12*(e*x + d)*b^4*n^4*log(e*x
 + d)^2/e + 4*(e*x + d)*a*b^3*n^3*log(e*x + d)^3/e - 12*(e*x + d)*b^4*n^3*l
og(e*x + d)^2*log(c)/e + 6*(e*x + d)*b^4*n^2*log(e*x + d)^2*log(c)^2/e - 24
*(e*x + d)*b^4*n^4*log(e*x + d)/e - 12*(e*x + d)*a*b^3*n^3*log(e*x + d)^2/e
 + 24*(e*x + d)*b^4*n^3*log(e*x + d)*log(c)/e + 12*(e*x + d)*a*b^3*n^2*log(
e*x + d)^2*log(c)/e - 12*(e*x + d)*b^4*n^2*log(e*x + d)*log(c)^2/e + 4*(e*x
 + d)*b^4*n*log(e*x + d)*log(c)^3/e + 24*(e*x + d)*b^4*n^4/e + 24*(e*x + d)
*a*b^3*n^3*log(e*x + d)/e + 6*(e*x + d)*a^2*b^2*n^2*log(e*x + d)^2/e - 24*(
e*x + d)*b^4*n^3*log(c)/e - 24*(e*x + d)*a*b^3*n^2*log(e*x + d)*log(c)/e +
12*(e*x + d)*b^4*n^2*log(c)^2/e + 12*(e*x + d)*a*b^3*n*log(e*x + d)*log(c)^
2/e - 4*(e*x + d)*b^4*n*log(c)^3/e + (e*x + d)*b^4*log(c)^4/e - 24*(e*x + d
)*a*b^3*n^3/e - 12*(e*x + d)*a^2*b^2*n^2*log(e*x + d)/e + 24*(e*x + d)*a*b^
3*n^2*log(c)/e + 12*(e*x + d)*a^2*b^2*n*log(e*x + d)*log(c)/e - 12*(e*x + d
)*a*b^3*n*log(c)^2/e + 4*(e*x + d)*a*b^3*log(c)^3/e + 12*(e*x + d)*a^2*b^2*
n^2/e + 4*(e*x + d)*a^3*b*n*log(e*x + d)/e - 12*(e*x + d)*a^2*b^2*n*log(c)/
e + 6*(e*x + d)*a^2*b^2*log(c)^2/e - 4*(e*x + d)*a^3*b*n/e + 4*(e*x + d)*a^
3*b*log(c)/e + (e*x + d)*a^4/e
```

## Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.10

$$
\begin{aligned}
\int (a+b\log(c(d+ex)^n))^4\,dx
&= \ln\left(c\,(d+e\,x)^n\right)^2\left(\frac{6\,(d\,a^2\,b^2-2\,d\,a\,b^3\,n+2\,d\,b^4\,n^2)}{e}+6\,b^2\,x\,\left(a^2-2\,a\,b\,n+2\,b^2\,n^2\right)\right)\\
&\quad+x\,\left(a^4-4\,a^3\,b\,n+12\,a^2\,b^2\,n^2-24\,a\,b^3\,n^3+24\,b^4\,n^4\right)+\ln\left(c\,(d+e\,x)^n\right)^4\left(b^4\,x+\frac{b^4\,d}{e}\right)\\
&\quad+\ln\left(c\,(d+e\,x)^n\right)^3\left(\frac{4\,(a\,b^3\,d-b^4\,d\,n)}{e}+4\,b^3\,x\,(a-b\,n)\right)\\
&\quad-\frac{\ln(d+e\,x)\,\left(-4\,d\,a^3\,b\,n+12\,d\,a^2\,b^2\,n^2-24\,d\,a\,b^3\,n^3+24\,d\,b^4\,n^4\right)}{e}\\
&\quad+4\,b\,x\,\ln\left(c\,(d+e\,x)^n\right)\,\left(a^3-3\,a^2\,b\,n+6\,a\,b^2\,n^2-6\,b^3\,n^3\right)
\end{aligned}
$$

```
[In] int((a + b*log(c*(d + e*x)^n))^4,x)
```

```
[Out] log(c*(d + e*x)^n)^2*((6*(a^2*b^2*d + 2*b^4*d*n^2 - 2*a*b^3*d*n))/e + 6*b^2
*x*(a^2 + 2*b^2*n^2 - 2*a*b*n)) + x*(a^4 + 24*b^4*n^4 - 24*a*b^3*n^3 + 12*a
^2*b^2*n^2 - 4*a^3*b*n) + log(c*(d + e*x)^n)^4*(b^4*x + (b^4*d)/e) + log(c*
(d + e*x)^n)^3*((4*(a*b^3*d - b^4*d*n))/e + 4*b^3*x*(a - b*n)) - (log(d + e
*x)*(24*b^4*d*n^4 + 12*a^2*b^2*d*n^2 - 4*a^3*b*d*n - 24*a*b^3*d*n^3))/e + 4
*b*x*log(c*(d + e*x)^n)*(a^3 - 6*b^3*n^3 + 6*a*b^2*n^2 - 3*a^2*b*n)
```

# 3.18 $\int (a+b\log(c(d+ex)^n))^3\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 99

$$\int (a+b\log(c(d+ex)^n))^3\,dx = 6ab^2n^2x-6b^3n^3x+\frac{6b^3n^2(d+ex)\log(c(d+ex)^n)}{e}-\frac{3bn(d+ex)(a+b\log(c(d+ex)^n))^2}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{e}$$

```
[Out] 6*a*b^2*n^2*x-6*b^3*n^3*x+6*b^3*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e-3*b*n*(e*x+d)
*(a+b*ln(c*(e*x+d)^n))^2/e+(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2436, 2333, 2332}

$$\int (a+b\log(c(d+ex)^n))^3\,dx = 6ab^2n^2x-\frac{3bn(d+ex)(a+b\log(c(d+ex)^n))^2}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{e}+\frac{6b^3n^2(d+ex)\log(c(d+ex)^n)}{e}-6b^3n^3x$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] 6*a*b^2*n^2*x - 6*b^3*n^3*x + (6*b^3*n^2*(d + e*x)*Log[c*(d + e*x)^n])/e -
(3*b*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e + ((d + e*x)*(a + b*Log[c*
(d + e*x)^n])^3)/e
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+b\log(cx^n))^3 \, dx, x, d+ex\right)}{e} \\
&= \frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{e} - \frac{(3bn)\text{Subst}\left(\int (a+b\log(cx^n))^2 \, dx, x, d+ex\right)}{e} \\
&= -\frac{3bn(d+ex)(a+b\log(c(d+ex)^n))^2}{e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{e} \\
&\quad + \frac{(6b^2n^2)\,\text{Subst}\left(\int (a+b\log(cx^n)) \, dx, x, d+ex\right)}{e} \\
&= 6ab^2n^2x - \frac{3bn(d+ex)(a+b\log(c(d+ex)^n))^2}{e} \\
&\quad + \frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{e} + \frac{(6b^3n^2)\,\text{Subst}\left(\int \log(cx^n) \, dx, x, d+ex\right)}{e} \\
&= 6ab^2n^2x - 6b^3n^3x + \frac{6b^3n^2(d+ex)\log(c(d+ex)^n)}{e} \\
&\quad - \frac{3bn(d+ex)(a+b\log(c(d+ex)^n))^2}{e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{e}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int (a+b\log (c(d+ex)^n))^3\,dx$$
$$=\frac{(d+ex)\,(a+b\log (c(d+ex)^n))^3-3bn\big((d+ex)\,(a+b\log (c(d+ex)^n))^2-2bn(e(a-bn)x+b(d+ex)\log$$
$$e$$

[In] Integrate[(a + b*Log[c*(d + e*x)^n])^3,x]

[Out] ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 - 2*b*n*(e*(a - b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n])))/e

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(99) = 198.

Time = 0.62 (sec) , antiderivative size = 322, normalized size of antiderivative = 3.25

| method | result |
|---|---|
| parallelrisch | $\frac{x\ln(c(ex+d)^n)^3b^3en-3x\ln(c(ex+d)^n)^2b^3e\,n^2+6x\ln(c(ex+d)^n)b^3e\,n^3-6x\,b^3e\,n^4+3x\ln(c(ex+d)^n)^2a\,b^2en-6x\ln(c(ex+d)^n)}{}$ |
| risch | Expression too large to display |

[In] int((a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)

[Out] (x*ln(c*(e*x+d)^n)^3*b^3*e*n-3*x*ln(c*(e*x+d)^n)^2*b^3*e*n^2+6*x*ln(c*(e*x+d)^n)*b^3*e*n^3-6*x*b^3*e*n^4+3*x*ln(c*(e*x+d)^n)^2*a*b^2*e*n-6*x*ln(c*(e*x+d)^n)*a*b^2*e*n^2+6*x*a*b^2*e*n^3+ln(c*(e*x+d)^n)^3*b^3*d*n-3*ln(c*(e*x+d)^n)^2*b^3*d*n^2+6*ln(c*(e*x+d)^n)*b^3*d*n^3+6*b^3*d*n^4+3*x*ln(c*(e*x+d)^n)*a^2*b*e*n-3*x*a^2*b*e*n^2+3*ln(c*(e*x+d)^n)^2*a*b^2*d*n-6*ln(c*(e*x+d)^n)*a*b^2*d*n^2-6*a*b^2*d*n^3+x*a^3*e*n+3*ln(c*(e*x+d)^n)*a^2*b*d*n+3*a^2*b*d*n^2-a^3*d*n)/e/n

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(99) = 198.

Time = 0.29 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.27

$$\int (a+b\log (c(d+ex)^n))^3\,dx$$
$$=\frac{b^3ex\log (c)^3+(b^3en^3x+b^3dn^3)\log (ex+d)^3-3\,(b^3en-ab^2e)x\log (c)^2-3\,(b^3dn^3-ab^2dn^2+(b^3en^3-}{}$$

[In] integrate((a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")

```
[Out] (b^3*e*x*log(c)^3 + (b^3*e*n^3*x + b^3*d*n^3)*log(e*x + d)^3 - 3*(b^3*e*n -
 a*b^2*e)*x*log(c)^2 - 3*(b^3*d*n^3 - a*b^2*d*n^2 + (b^3*e*n^3 - a*b^2*e*n^
2)*x - (b^3*e*n^2*x + b^3*d*n^2)*log(c))*log(e*x + d)^2 + 3*(2*b^3*e*n^2 -
2*a*b^2*e*n + a^2*b*e)*x*log(c) - (6*b^3*e*n^3 - 6*a*b^2*e*n^2 + 3*a^2*b*e*
n - a^3*e)*x + 3*(2*b^3*d*n^3 - 2*a*b^2*d*n^2 + a^2*b*d*n + (b^3*e*n*x + b^
3*d*n)*log(c)^2 + (2*b^3*e*n^3 - 2*a*b^2*e*n^2 + a^2*b*e*n)*x - 2*(b^3*d*n^
2 - a*b^2*d*n + (b^3*e*n^2 - a*b^2*e*n)*x)*log(c))*log(e*x + d))/e
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(95) = 190.

Time = 0.50 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.97

$$\int (a+b\log (c(d+ex)^n))^3\,dx$$
$$= \begin{cases} a^3x + \frac{3a^2bd\log(c(d+ex)^n)}{e} - 3a^2bnx + 3a^2bx\log(c(d+ex)^n) - \frac{6ab^2dn\log(c(d+ex)^n)}{e} + \frac{3ab^2d\log(c(d+ex)^n)^2}{e} + 6ab^2 \\ x(a+b\log(cd^n))^3 \end{cases}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*d*log(c*(d + e*x)**n)/e - 3*a**2*b*n*x + 3*a**
2*b*x*log(c*(d + e*x)**n) - 6*a*b**2*d*n*log(c*(d + e*x)**n)/e + 3*a*b**2*d
*log(c*(d + e*x)**n)**2/e + 6*a*b**2*n**2*x - 6*a*b**2*n*x*log(c*(d + e*x)*
*n) + 3*a*b**2*x*log(c*(d + e*x)**n)**2 + 6*b**3*d*n**2*log(c*(d + e*x)**n)
/e - 3*b**3*d*n*log(c*(d + e*x)**n)**2/e + b**3*d*log(c*(d + e*x)**n)**3/e
- 6*b**3*n**3*x + 6*b**3*n**2*x*log(c*(d + e*x)**n) - 3*b**3*n*x*log(c*(d +
 e*x)**n)**2 + b**3*x*log(c*(d + e*x)**n)**3, Ne(e, 0)), (x*(a + b*log(c*d*
*n))**3, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(99) = 198.

Time = 0.22 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.85

$$\int (a+b\log (c(d+ex)^n))^3\,dx = b^3x\log((ex+d)^nc)^3 - 3\,a^2ben\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)$$
$$+ 3\,ab^2x\log((ex+d)^nc)^2 + 3\,a^2bx\log((ex+d)^nc)$$
$$-3\left(2\,en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc) + \frac{(d\log(ex+d)^2 - 2\,ex + 2\,d\log(ex+d))n^2}{e}\right)ab^2$$
$$-\left(3\,en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc)^2 - en\left(\frac{(d\log(ex+d)^3 + 3\,d\log(ex+d)^2 - 6\,ex + 6\,d\log(\right.\right.$$
$$+ a^3x$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log((e*x + d)^n*c)^3 - 3*a^2*b*e*n*(x/e - d*log(e*x + d)/e^2) + 3*a*b
^2*x*log((e*x + d)^n*c)^2 + 3*a^2*b*x*log((e*x + d)^n*c) - 3*(2*e*n*(x/e -
d*log(e*x + d)/e^2)*log((e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*lo
g(e*x + d))*n^2/e)*a*b^2 - (3*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^
n*c)^2 - e*n*((d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x
+ d))*n^2/e^2 - 3*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n*log((e*x
+ d)^n*c)/e^2))*b^3 + a^3*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(99) = 198$.

Time = 0.31 (sec) , antiderivative size = 399, normalized size of antiderivative = 4.03

$$\begin{aligned}
\int (a+b\log(c(d+ex)^n))^3\,dx &= \frac{(ex+d)b^3n^3\log(ex+d)^3}{e} - \frac{3\,(ex+d)b^3n^3\log(ex+d)^2}{e} \\
&+ \frac{3\,(ex+d)b^3n^2\log(ex+d)^2\log(c)}{e} \\
&+ \frac{6\,(ex+d)b^3n^3\log(ex+d)}{e} \\
&+ \frac{3\,(ex+d)ab^2n^2\log(ex+d)^2}{e} \\
&- \frac{6\,(ex+d)b^3n^2\log(ex+d)\log(c)}{e} \\
&+ \frac{3\,(ex+d)b^3n\log(ex+d)\log(c)^2}{e} - \frac{6\,(ex+d)b^3n^3}{e} \\
&- \frac{6\,(ex+d)ab^2n^2\log(ex+d)}{e} + \frac{6\,(ex+d)b^3n^2\log(c)}{e} \\
&+ \frac{6\,(ex+d)ab^2n\log(ex+d)\log(c)}{e} \\
&- \frac{3\,(ex+d)b^3n\log(c)^2}{e} + \frac{(ex+d)b^3\log(c)^3}{e} \\
&+ \frac{6\,(ex+d)ab^2n^2}{e} + \frac{3\,(ex+d)a^2bn\log(ex+d)}{e} \\
&- \frac{6\,(ex+d)ab^2n\log(c)}{e} + \frac{3\,(ex+d)ab^2\log(c)^2}{e} \\
&- \frac{3\,(ex+d)a^2bn}{e} + \frac{3\,(ex+d)a^2b\log(c)}{e} + \frac{(ex+d)a^3}{e}
\end{aligned}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

```
[Out] (e*x + d)*b^3*n^3*log(e*x + d)^3/e - 3*(e*x + d)*b^3*n^3*log(e*x + d)^2/e +
 3*(e*x + d)*b^3*n^2*log(e*x + d)^2*log(c)/e + 6*(e*x + d)*b^3*n^3*log(e*x
+ d)/e + 3*(e*x + d)*a*b^2*n^2*log(e*x + d)^2/e - 6*(e*x + d)*b^3*n^2*log(e
```

```
*x + d)*log(c)/e + 3*(e*x + d)*b^3*n*log(e*x + d)*log(c)^2/e - 6*(e*x + d)*
b^3*n^3/e - 6*(e*x + d)*a*b^2*n^2*log(e*x + d)/e + 6*(e*x + d)*b^3*n^2*log(
c)/e + 6*(e*x + d)*a*b^2*n*log(e*x + d)*log(c)/e - 3*(e*x + d)*b^3*n*log(c)
^2/e + (e*x + d)*b^3*log(c)^3/e + 6*(e*x + d)*a*b^2*n^2/e + 3*(e*x + d)*a^2
*b*n*log(e*x + d)/e - 6*(e*x + d)*a*b^2*n*log(c)/e + 3*(e*x + d)*a*b^2*log(
c)^2/e - 3*(e*x + d)*a^2*b*n/e + 3*(e*x + d)*a^2*b*log(c)/e + (e*x + d)*a^3
/e
```

## Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.74

$$\int (a+b\log(c(d+ex)^n))^3\,dx = x\left(a^3-3\,a^2\,b\,n+6\,a\,b^2\,n^2-6\,b^3\,n^3\right) + \ln\left(c\,(d+e\,x)^n\right)^3\left(b^3\,x+\frac{b^3\,d}{e}\right) + \ln\left(c\,(d+e\,x)^n\right)^2\left(\frac{3\,(a\,b^2\,d-b^3\,d\,n)}{e}+3\,b^2\,x\,(a-b\,n)\right) + \frac{\ln(d+e\,x)\,\left(3\,d\,a^2\,b\,n-6\,d\,a\,b^2\,n^2+6\,d\,b^3\,n^3\right)}{e} + 3\,b\,x\,\ln\left(c\,(d+e\,x)^n\right)\,\left(a^2-2\,a\,b\,n+2\,b^2\,n^2\right)$$

```
[In] int((a + b*log(c*(d + e*x)^n))^3,x)

[Out] x*(a^3 - 6*b^3*n^3 + 6*a*b^2*n^2 - 3*a^2*b*n) + log(c*(d + e*x)^n)^3*(b^3*x
 + (b^3*d)/e) + log(c*(d + e*x)^n)^2*((3*(a*b^2*d - b^3*d*n))/e + 3*b^2*x*(
a - b*n)) + (log(d + e*x)*(6*b^3*d*n^3 + 3*a^2*b*d*n - 6*a*b^2*d*n^2))/e +
3*b*x*log(c*(d + e*x)^n)*(a^2 + 2*b^2*n^2 - 2*a*b*n)
```

# 3.19 $\int (a+b\log(c(d+ex)^n))^2\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 65

$$\int (a+b\log(c(d+ex)^n))^2\,dx = -2abnx+2b^2n^2x-\frac{2b^2n(d+ex)\log(c(d+ex)^n)}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{e}$$

```
[Out] -2*a*b*n*x+2*b^2*n^2*x-2*b^2*n*(e*x+d)*ln(c*(e*x+d)^n)/e+(e*x+d)*(a+b*ln(c*
(e*x+d)^n))^2/e
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2436, 2333, 2332}

$$\int (a+b\log(c(d+ex)^n))^2\,dx = \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{e}-2abnx-\frac{2b^2n(d+ex)\log(c(d+ex)^n)}{e}+2b^2n^2x$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] -2*a*b*n*x + 2*b^2*n^2*x - (2*b^2*n*(d + e*x)*Log[c*(d + e*x)^n])/e + ((d +
 e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx, x, d+ex\right)}{e}\\
&= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} - \frac{(2bn)\text{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+ex\right)}{e}\\
&= -2abnx + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} - \frac{(2b^2n)\,\text{Subst}\left(\int \log(cx^n)\,dx, x, d+ex\right)}{e}\\
&= -2abnx + 2b^2n^2x - \frac{2b^2n(d+ex)\log\left(c(d+ex)^n\right)}{e} + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx = \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} - 2bn\left(ax-bnx+\frac{b(d+ex)\log\left(c(d+ex)^n\right)}{e}\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e - 2*b*n*(a*x - b*n*x + (b*(d + e
*x)*Log[c*(d + e*x)^n])/e)
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

| method | result |
|---|---|
| norman | $\left(2b^{2}n^{2}-2abn+a^{2}\right)x+b^{2}x\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)^{2}+\left(-2b^{2}n+2ab\right)x\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)+\frac{b^{2}d\ln\left(c\,\mathrm{e}^{n\ln\dots}\right.}{e}$ |
| default | $a^{2}x+b^{2}x\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)^{2}+\frac{b^{2}d\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)^{2}}{e}+2b^{2}n^{2}x-2b^{2}nx\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)-\frac{2n^{2}b^{2}d\ln(e\dots}{e}$ |
| parts | $a^{2}x+b^{2}x\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)^{2}+\frac{b^{2}d\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)^{2}}{e}+2b^{2}n^{2}x-2b^{2}nx\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)-\frac{2n^{2}b^{2}d\ln(e\dots}{e}$ |
| parallelrisch | $\frac{x\ln(c(ex+d)^{n})^{2}b^{2}den-2x\ln(c(ex+d)^{n})b^{2}de\,n^{2}+2x\,b^{2}de\,n^{3}+2x\ln(c(ex+d)^{n})abden-2xabde\,n^{2}+\ln(c(ex+d)^{n})^{2}b^{2}d^{2}n-2\ln(\dots}{end}$ |
| risch | Expression too large to display |

```
[In] int((a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (2*b^2*n^2-2*a*b*n+a^2)*x+b^2*x*ln(c*exp(n*ln(e*x+d)))^2+(-2*b^2*n+2*a*b)*x
*ln(c*exp(n*ln(e*x+d)))+b^2*d/e*ln(c*exp(n*ln(e*x+d)))^2+n*(-2*b^2*d*n+2*a*
b*d)/e*ln(e*x+d)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(65) = 130.

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.15

$$\int (a+b\log(c(d+ex)^{n}))^{2}\,dx$$
$$=\frac{b^{2}ex\log(c)^{2}+\left(b^{2}en^{2}x+b^{2}dn^{2}\right)\log(ex+d)^{2}-2\left(b^{2}en-abe\right)x\log(c)+\left(2\,b^{2}en^{2}-2\,aben+a^{2}e\right)x-2\,(b\dots}{e}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

```
[Out] (b^2*e*x*log(c)^2 + (b^2*e*n^2*x + b^2*d*n^2)*log(e*x + d)^2 - 2*(b^2*e*n -
 a*b*e)*x*log(c) + (2*b^2*e*n^2 - 2*a*b*e*n + a^2*e)*x - 2*(b^2*d*n^2 - a*b
*d*n + (b^2*e*n^2 - a*b*e*n)*x - (b^2*e*n*x + b^2*d*n)*log(c))*log(e*x + d)
)/e
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(63) = 126.

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.25

$$\int (a+b\log (c(d+ex)^n))^2 \, dx$$
$$= \begin{cases} a^2x + \frac{2abd\log(c(d+ex)^n)}{e} - 2abnx + 2abx\log(c(d+ex)^n) - \frac{2b^2dn\log(c(d+ex)^n)}{e} + \frac{b^2d\log(c(d+ex)^n)^2}{e} + 2b^2n^2x - 2 \\ x(a+b\log(cd^n))^2 \end{cases}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*d*log(c*(d + e*x)**n)/e - 2*a*b*n*x + 2*a*b*x*log
(c*(d + e*x)**n) - 2*b**2*d*n*log(c*(d + e*x)**n)/e + b**2*d*log(c*(d + e*x
)**n)**2/e + 2*b**2*n**2*x - 2*b**2*n*x*log(c*(d + e*x)**n) + b**2*x*log(c*
(d + e*x)**n)**2, Ne(e, 0)), (x*(a + b*log(c*d**n))**2, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(65) = 130.

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.02

$$\int (a+b\log (c(d+ex)^n))^2 \, dx$$
$$= -2\,aben\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) + b^2x\log\left((ex+d)^n c\right)^2 + 2\,abx\log\left((ex+d)^n c\right)$$
$$- \left(2\,en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log\left((ex+d)^n c\right) + \frac{\left(d\log(ex+d)^2 - 2\,ex + 2\,d\log(ex+d)\right)n^2}{e}\right)b^2$$
$$+ a^2x$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*e*n*(x/e - d*log(e*x + d)/e^2) + b^2*x*log((e*x + d)^n*c)^2 + 2*a*b*
x*log((e*x + d)^n*c) - (2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)
 + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*b^2 + a^2*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(65) = 130.

Time = 0.30 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.68

$$\int (a+b\log(c(d+ex)^n))^2\,dx = \frac{(ex+d)b^2n^2\log(ex+d)^2}{e} - \frac{2(ex+d)b^2n^2\log(ex+d)}{e} + \frac{2(ex+d)b^2n\log(ex+d)\log(c)}{e} + \frac{2(ex+d)b^2n^2}{e} + \frac{2(ex+d)abn\log(ex+d)}{e} - \frac{2(ex+d)b^2n\log(c)}{e} + \frac{(ex+d)b^2\log(c)^2}{e} - \frac{2(ex+d)abn}{e} + \frac{2(ex+d)ab\log(c)}{e} + \frac{(ex+d)a^2}{e}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] (e*x + d)*b^2*n^2*log(e*x + d)^2/e - 2*(e*x + d)*b^2*n^2*log(e*x + d)/e + 2
*(e*x + d)*b^2*n*log(e*x + d)*log(c)/e + 2*(e*x + d)*b^2*n^2/e + 2*(e*x + d
)*a*b*n*log(e*x + d)/e - 2*(e*x + d)*b^2*n*log(c)/e + (e*x + d)*b^2*log(c)^
2/e - 2*(e*x + d)*a*b*n/e + 2*(e*x + d)*a*b*log(c)/e + (e*x + d)*a^2/e
```

## Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

$$\int (a+b\log(c(d+ex)^n))^2\,dx = x\left(a^2-2\,a\,b\,n+2\,b^2\,n^2\right) + \ln\left(c\,(d+e\,x)^n\right)^2\left(b^2\,x+\frac{b^2\,d}{e}\right) - \frac{\ln(d+e\,x)\,\left(2\,b^2\,d\,n^2-2\,a\,b\,d\,n\right)}{e} + 2\,b\,x\,\ln\left(c\,(d+e\,x)^n\right)\,(a-b\,n)$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2,x)
```

```
[Out] x*(a^2 + 2*b^2*n^2 - 2*a*b*n) + log(c*(d + e*x)^n)^2*(b^2*x + (b^2*d)/e) -
(log(d + e*x)*(2*b^2*d*n^2 - 2*a*b*d*n))/e + 2*b*x*log(c*(d + e*x)^n)*(a -
b*n)
```

# 3.20 $\int (a+b\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 14, antiderivative size = 29

$$\int (a+b\log(c(d+ex)^n))\,dx = ax-bnx+\frac{b(d+ex)\log(c(d+ex)^n)}{e}$$

```
[Out] a*x-b*n*x+b*(e*x+d)*ln(c*(e*x+d)^n)/e
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2436, 2332}

$$\int (a+b\log(c(d+ex)^n))\,dx = ax+\frac{b(d+ex)\log(c(d+ex)^n)}{e}-bnx$$

```
[In] Int[a + b*Log[c*(d + e*x)^n],x]
```

```
[Out] a*x - b*n*x + (b*(d + e*x)*Log[c*(d + e*x)^n])/e
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax + b\int \log\left(c(d+ex)^n\right)\,dx\\ &= ax + \frac{b\text{Subst}\left(\int \log\left(cx^n\right)\,dx, x, d+ex\right)}{e}\\ &= ax - bnx + \frac{b(d+ex)\log\left(c(d+ex)^n\right)}{e}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = ax - bnx + \frac{b(d+ex)\log\left(c(d+ex)^n\right)}{e}$$

```
[In] Integrate[a + b*Log[c*(d + e*x)^n],x]
[Out] a*x - b*n*x + (b*(d + e*x)*Log[c*(d + e*x)^n])/e
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

| method | result |
|---|---|
| default | $ax+b\ln\left(c(ex+d)^n\right)x-bnx+\frac{bnd\ln(ex+d)}{e}$ |
| parts | $ax+b\ln\left(c(ex+d)^n\right)x-bnx+\frac{bnd\ln(ex+d)}{e}$ |
| norman | $(-bn+a)\,x+bx\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)+\frac{bnd\ln(ex+d)}{e}$ |
| parallelrisch | $\frac{b\left(x\ln(c(ex+d)^n)den-xde\,n^2+\ln(c(ex+d)^n)d^2n\right)}{den}+ax$ |
| risch | $ax+bx\ln\left((ex+d)^n\right)-\frac{ib\pi x\ \mathrm{csgn}(ic)\,\mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)}{2}+\frac{ib\pi x\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic(ex+d)^n)^2}{2}+$ ib$\pi$ |

```
[In] int(a+b*ln(c*(e*x+d)^n),x,method=_RETURNVERBOSE)
[Out] a*x+b*ln(c*(e*x+d)^n)*x-b*n*x+b/e*n*d*ln(e*x+d)
```

## Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int (a+b\log(c(d+ex)^n))\,dx = \frac{bex\log(c)-(ben-ae)x+(benx+bdn)\log(ex+d)}{e}$$

```
[In] integrate(a+b*log(c*(e*x+d)^n),x, algorithm="fricas")
```

```
[Out] (b*e*x*log(c) - (b*e*n - a*e)*x + (b*e*n*x + b*d*n)*log(e*x + d))/e
```

## Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int (a+b\log(c(d+ex)^n))\,dx = ax+b\left(\begin{cases}\frac{d\log(c(d+ex)^n)}{e}-nx+x\log(c(d+ex)^n) & \text{for } e\neq 0\\ x\log(cd^n) & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(a+b*ln(c*(e*x+d)**n),x)
```

```
[Out] a*x + b*Piecewise((d*log(c*(d + e*x)**n)/e - n*x + x*log(c*(d + e*x)**n), N
e(e, 0)), (x*log(c*d**n), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int (a+b\log(c(d+ex)^n))\,dx = -ben\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)+bx\log((ex+d)^nc)+ax$$

```
[In] integrate(a+b*log(c*(e*x+d)^n),x, algorithm="maxima")
```

```
[Out] -b*e*n*(x/e - d*log(e*x + d)/e^2) + b*x*log((e*x + d)^n*c) + a*x
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int (a+b\log(c(d+ex)^n))\,dx = \left(\frac{(ex+d)n\log(ex+d)}{e} - \frac{(ex+d)n}{e} + \frac{(ex+d)\log(c)}{e}\right)b + ax$$

```
[In] integrate(a+b*log(c*(e*x+d)^n),x, algorithm="giac")
[Out] ((e*x + d)*n*log(e*x + d)/e - (e*x + d)*n/e + (e*x + d)*log(c)/e)*b + a*x
```

## Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int (a+b\log(c(d+ex)^n))\,dx = x\,(a-b\,n) + b\,x\,\ln\left(c\,(d+e\,x)^n\right) + \frac{b\,d\,n\,\ln(d+e\,x)}{e}$$

```
[In] int(a + b*log(c*(d + e*x)^n),x)
[Out] x*(a - b*n) + b*x*log(c*(d + e*x)^n) + (b*d*n*log(d + e*x))/e
```

# 3.21 $\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 63

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben}$$

```
[Out] (e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b/e/exp(a/b/n)/n/((c*(e*x+d)^n)^(1/n)
)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2436, 2337, 2209}

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(-1),x]
```

```
[Out] ((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e*E^(a/(b*n)
)*n*(c*(d + e*x)^n)^n^(-1))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx, x, d+ex\right)}{e}\\ &= \frac{\left((d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{en}\\ &= \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\,\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(-1),x]
[Out] ((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e*E^(a/(b*n)
)*n*(c*(d + e*x)^n)^n^(-1))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.87 (sec) , antiderivative size = 309, normalized size of antiderivative = 4.90

| method | result |
|---|---|
| risch | $-\frac{(ex+d)((ex+d)^n)^{-\frac{1}{n}}c^{-\frac{1}{n}}\mathrm{e}^{-\frac{-ib\pi\,\text{csgn}(ic(ex+d)^n)\,\text{csgn}(ic)\,\text{csgn}(i(ex+d)^n)+i\pi\,\text{csgn}(ic)\,\text{csgn}(ic(ex+d)^n)^2b+i\pi\,\text{csgn}(i(ex+d)^n)\,\text{csgn}(ic(ex-}{2bn}}}{}$ |

```
[In] int(1/(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e/b/n*(e*x+d)*((e*x+d)^n)^(-1/n)*c^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*
x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+
I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b
+2*a)/b/n)*Ei(1,-ln(e*x+d)+1/2*I*(b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c
*(e*x+d)^n)-b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2-b*Pi*csgn(I*(e*x+d)^n)*csg
n(I*c*(e*x+d)^n)^2+b*Pi*csgn(I*c*(e*x+d)^n)^3+2*I*b*ln(c)+2*I*b*(ln((e*x+d)
^n)-n*ln(e*x+d))+2*I*a)/b/n)
```

## Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{e^{\left(-\frac{b\log(c)+a}{bn}\right)}\,\mathrm{log\_integral}\left((ex+d)e^{\left(\frac{b\log(c)+a}{bn}\right)}\right)}{ben}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

```
[Out] e^(-(b*log(c) + a)/(b*n))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n)))/
(b*e*n)
```

## Sympy [F]

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{1}{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n)),x)
```

```
[Out] Integral(1/(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [F]

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{1}{b\log\left((ex+d)^n c\right)+a}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*log((e*x + d)^n*c) + a), x)
```

## Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{\text{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}en}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e*n)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{1}{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e*x)^n)),x)
[Out] int(1/(a + b*log(c*(d + e*x)^n)), x)
```

# 3.22 $\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2en^2} - \frac{d+ex}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

```
[Out] (e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e/exp(a/b/n)/n^2/((c*(e*x+d)^n)^(
1/n))+(-e*x-d)/b/e/n/(a+b*ln(c*(e*x+d)^n))
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2436, 2334, 2337, 2209}

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2en^2} - \frac{d+ex}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(-2),x]
```

```
[Out] ((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b^2*e*E^(a/(b*
n))*n^2*(c*(d + e*x)^n)^n^(-1)) - (d + e*x)/(b*e*n*(a + b*Log[c*(d + e*x)^n
]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^2}\,dx, x, d+ex\right)}{e}\\
&= -\frac{d+ex}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx, x, d+ex\right)}{ben}\\
&= -\frac{d+ex}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}\\
&\quad + \frac{\left((d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{ben^2}\\
&= \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2en^2} - \frac{d+ex}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = -\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(be^{\frac{a}{bn}}n(c(d+ex)^n)^{\frac{1}{n}}-\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+\right.}{b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(-2),x]
```

```
[Out] -(((d + e*x)*(b*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1) - ExpIntegralEi[(a + b
*Log[c*(d + e*x)^n])/(b*n)]*(a + b*Log[c*(d + e*x)^n])))/(b^2*e*E^(a/(b*n))
*n^2*(c*(d + e*x)^n)^n^(-1)*(a + b*Log[c*(d + e*x)^n])))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.98 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.75

| method | result |
|---|---|
| risch | $-\frac{2(ex+d)}{\left(-ib\pi\,\text{csgn}(ic(ex+d)^n)\,\text{csgn}(ic)\,\text{csgn}(i(ex+d)^n)+i\pi\,\text{csgn}(ic)\,\text{csgn}(ic(ex+d)^n)^2b+i\pi\,\text{csgn}(i(ex+d)^n)\,\text{csgn}(ic(ex+d)^n)^2b-i\pi\,\text{csgn}(\right.}$ |

```
[In] int(1/(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*
csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi
*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln((e*x+d)^n)+2*b*ln(c)+2*a)/b/n/e*(e*x+d)-1/b
^2/n^2/e*(e*x+d)*((e*x+d)^n)^(-1/n)*c^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*
x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+
I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b
+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(
I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*
csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x
+d)^n)-n*ln(e*x+d))+2*a)/b/n)
```

## Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = -\frac{\left((benx+bdn)e^{\left(\frac{b\log(c)+a}{bn}\right)}-(bn\log(ex+d)+b\log(c)+a)\,\mathrm{log\_integral}\left((ex+d)e^{\left(\frac{b\log(c)+a}{bn}\right)}\right)\right)e^{\left(-\frac{b\log(c)+a}{bn}\right)}}{b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

```
[Out] -((b*e*n*x + b*d*n)*e^((b*log(c) + a)/(b*n)) - (b*n*log(e*x + d) + b*log(c)
 + a)*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n))))*e^(-(b*log(c) + a)/
(b*n))/(b^3*e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n^2)
```

## Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**(-2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{(b\log((ex+d)^nc)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] -(e*x + d)/(b^2*e*n*log((e*x + d)^n) + b^2*e*n*log(c) + a*b*e*n) + integrat
e(1/(b^2*n*log((e*x + d)^n) + b^2*n*log(c) + a*b*n), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(95) = 190$.

Time = 0.39 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.98

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{bn\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}\log(ex+d)}{(b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2)c^{\left(\frac{1}{n}\right)}} - \frac{(ex+d)bn}{b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2} + \frac{b\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}\log(c)}{(b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2)c^{\left(\frac{1}{n}\right)}} + \frac{a\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}}{(b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2)c^{\left(\frac{1}{n}\right)}}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] b*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)/((b^3*e
*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n^2)*c^(1/n)) - (e*x + d)*b*
n/(b^3*e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n^2) + b*Ei(log(c)/n
 + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)/((b^3*e*n^3*log(e*x + d) + b
^3*e*n^2*log(c) + a*b^2*e*n^2)*c^(1/n)) + a*Ei(log(c)/n + a/(b*n) + log(e*x
 + d))*e^(-a/(b*n))/((b^3*e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n
^2)*c^(1/n))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{(a+b\,\ln(c\,(d+e\,x)^n))^2}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e*x)^n))^2,x)
```

```
[Out] int(1/(a + b*log(c*(d + e*x)^n))^2, x)
```

# 3.23 $\int \frac{1}{(a+b\log(c(d+ex)^n))^3} \, dx$



## Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3} \, dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3en^3} - \frac{d+ex}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} - \frac{d+ex}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

```
[Out] 1/2*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^3/e/exp(a/b/n)/n^3/((c*(e*x+d)^
n)^(1/n))+1/2*(-e*x-d)/b/e/n/(a+b*ln(c*(e*x+d)^n))^2+1/2*(-e*x-d)/b^2/e/n^2
/(a+b*ln(c*(e*x+d)^n))
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2436, 2334, 2337, 2209}

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3} \, dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3en^3} - \frac{d+ex}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{d+ex}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(-3),x]
```

```
[Out] ((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(2*b^3*e*E^(a/(
b*n))*n^3*(c*(d + e*x)^n)^n^(-1)) - (d + e*x)/(2*b*e*n*(a + b*Log[c*(d + e*
x)^n])^2) - (d + e*x)/(2*b^2*e*n^2*(a + b*Log[c*(d + e*x)^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^3}\,dx, x, d+ex\right)}{e}\\
&= -\frac{d+ex}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}+\frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^2}\,dx, x, d+ex\right)}{2ben}\\
&= -\frac{d+ex}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}-\frac{d+ex}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx, x, d+ex\right)}{2b^2en^2}\\
&= -\frac{d+ex}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}-\frac{d+ex}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\\
&\quad+\frac{\left((d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{2b^2en^3}
\end{aligned}$$

$$= \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3en^3} - \frac{d+ex}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} - \frac{d+ex}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx = -\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(-\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2+be^{\frac{a}{bn}}n(c(\dots\right.}{2b^3en^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(-3),x]
```

```
[Out] -1/2*((d + e*x)*(-(ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)]*(a + b*L
og[c*(d + e*x)^n])^2) + b*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)*(a + b*n + b
*Log[c*(d + e*x)^n])))/(b^3*e*E^(a/(b*n))*n^3*(c*(d + e*x)^n)^n^(-1)*(a + b
*Log[c*(d + e*x)^n])^2)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.14 (sec) , antiderivative size = 734, normalized size of antiderivative = 5.44

| method | result |
|---|---|
| risch | $-\frac{2benx+2bdn+i\pi bd\ \operatorname{csgn}(i(ex+d)^n)\operatorname{csgn}(ic(ex+d)^n)^2+i\pi bd\ \operatorname{csgn}(ic)\operatorname{csgn}(ic(ex+d)^n)^2-i\pi bd\ \operatorname{csgn}(ic)\operatorname{csgn}(i(ex+d)^n)\operatorname{csgn}(ic(ex\dots}{\left(-ib\pi\ \operatorname{csgn}(ic(ex+d)^n)\operatorname{csgn}\dots\right.}$ |

```
[In] int(1/(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(2*b*e*n*x+2*b*d*n+I*Pi*b*d*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b
*d*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2-I*Pi*b*d*csgn(I*c)*csgn(I*(e*x+d)^n)*csg
n(I*c*(e*x+d)^n)-I*Pi*b*d*csgn(I*c*(e*x+d)^n)^3-I*Pi*b*e*x*csgn(I*c*(e*x+d)
^n)^3+I*Pi*b*e*x*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I*Pi*b*e*x*csgn(I*
c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*Pi*b*e*x*csgn(I*c)*csgn(I*c*(e*x
+d)^n)^2+2*ln(c)*b*e*x+2*b*e*x*ln((e*x+d)^n)+2*d*b*ln(c)+2*a*e*x+2*b*d*ln((
e*x+d)^n)+2*a*d)/(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I
*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+
d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln((e*x+d)^n)+2*b*ln(c)+2*a)^2/b
^2/n^2/e-1/2/b^3/n^3/e*(e*x+d)*c^(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1/2*(-I*b*P
```

```
i*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(
e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*
(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*c
sgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c
)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(128) = 256$.

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.95

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = -\frac{\left((b^2dn^2+abdn+(b^2en^2+aben)x+(b^2en^2x+b^2dn^2)\log(ex+d)+(b^2enx+b^2dn)\log(c))e^{\left(\frac{b\log(c)+a}{bn}\right)}\right.}{2\left(b^5en^5\log(ex+d)^2+b^5en^3\log(c)^2\right.}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

```
[Out] -1/2*((b^2*d*n^2 + a*b*d*n + (b^2*e*n^2 + a*b*e*n)*x + (b^2*e*n^2*x + b^2*d
*n^2)*log(e*x + d) + (b^2*e*n*x + b^2*d*n)*log(c))*e^((b*log(c) + a)/(b*n))
 - (b^2*n^2*log(e*x + d)^2 + b^2*log(c)^2 + 2*a*b*log(c) + a^2 + 2*(b^2*n*l
og(c) + a*b*n)*log(e*x + d))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n)
)))*e^(-(b*log(c) + a)/(b*n))/(b^5*e*n^5*log(e*x + d)^2 + b^5*e*n^3*log(c)^
2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3 + 2*(b^5*e*n^4*log(c) + a*b^4*e*n^
4)*log(e*x + d))
```

## Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**(-3), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^3}\,dx$$

[In] integrate(1/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")

[Out] -1/2*((d*n + d*log(c))*b + a*d + ((e*n + e*log(c))*b + a*e)*x + (b*e*x + b*d)*log((e*x + d)^n))/(b^4*e*n^2*log((e*x + d)^n)^2 + b^4*e*n^2*log(c)^2 + 2*a*b^3*e*n^2*log(c) + a^2*b^2*e*n^2 + 2*(b^4*e*n^2*log(c) + a*b^3*e*n^2)*log((e*x + d)^n)) + integrate(1/2/(b^3*n^2*log((e*x + d)^n) + b^3*n^2*log(c) + a*b^2*n^2), x)

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1218 vs. $2(128) = 256$.

Time = 0.41 (sec) , antiderivative size = 1218, normalized size of antiderivative = 9.02

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \text{Too large to display}$$

[In] integrate(1/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")

[Out] 1/2*b^2*n^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)^2/((b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n)) - 1/2*(e*x + d)*b^2*n^2*log(e*x + d)/(b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3) + b^2*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)*log(c)/((b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n)) - 1/2*(e*x + d)*b^2*n^2/(b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3) + a*b*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)/((b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n)) - 1/2*(e*x + d)*b^2*n*log(c)/(b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3) + 1/2*b^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)^2/((b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n)) - 1/2*(e*x + d)*a*b*n/(b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^

```
4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)
 + a*b*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)/((b^5*e*n^
5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x
+ d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n))
+ 1/2*a^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/((b^5*e*n^5*lo
g(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d)
 + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log{(c(d+ex)^n)})^3}\,dx = \int \frac{1}{\left(a+b\,\ln{\left(c\,{\left(d+e\,x\right)}^n\right)}\right)^3}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e*x)^n))^3,x)
[Out] int(1/(a + b*log(c*(d + e*x)^n))^3, x)
```

# 3.24 $\int (a+b\log(c(d+ex)^n))^{5/2}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 179

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx = -\frac{15b^{5/2}e^{-\frac{a}{bn}}n^{5/2}\sqrt{\pi}(d+ex)\,(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e} + \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} - \frac{5bn(d+ex)\,(a+b\log(c(d+ex)^n))^{3/2}}{2e} + \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^{5/2}}{e}$$

```
[Out] -5/2*b*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e+(e*x+d)*(a+b*ln(c*(e*x+d)^n)
)^(5/2)/e-15/8*b^(5/2)*n^(5/2)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(
1/2)/n^(1/2))*Pi^(1/2)/e/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+15/4*b^2*n^2*(e*x
+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e
```

## Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used

= {2436, 2333, 2337, 2211, 2235}

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx = -\frac{15\sqrt{\pi}b^{5/2}n^{5/2}e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e} + \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e} - \frac{5bn(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] (-15*b^(5/2)*n^(5/2)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]
/(Sqrt[b]*Sqrt[n])])/(8*e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (15*b^2*n^2
*(d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]])/(4*e) - (5*b*n*(d + e*x)*(a + b*
Log[c*(d + e*x)^n])^(3/2))/(2*e) + ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^(5
/2))/e
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+b\log(cx^n))^{5/2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e} - \frac{(5bn)\text{Subst}\left(\int (a+b\log(cx^n))^{3/2}\,dx, x, d+ex\right)}{2e}$$

$$= -\frac{5bn(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e} + \frac{(15b^2n^2)\,\text{Subst}\left(\int \sqrt{a+b\log(cx^n)}\,dx, x, d+ex\right)}{4e}$$

$$= \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} - \frac{5bn(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e} - \frac{(15b^3n^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{8e}$$

$$= \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} - \frac{5bn(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e} - \frac{\left(15b^3n^2(d+ex)(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log(c(d+ex)^n)\right)}{8e}$$

$$= \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} - \frac{5bn(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e} - \frac{\left(15b^2n^2(d+ex)(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log(c(d+ex)^n)}\right)}{4e}$$

$$= -\frac{15b^{5/2}e^{-\frac{a}{bn}}n^{5/2}\sqrt{\pi}(d+ex)(c(d+ex)^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e} + \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} - \frac{5bn(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e}$$

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.85

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx = \frac{(d+ex)\left(8(a+b\log(c(d+ex)^n))^{5/2}-5bn\left(3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}(c(d+ex)^n)^{-1/n}\,\mathrm{er}\right.\right.}{8}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] ((d + e*x)*(8*(a + b*Log[c*(d + e*x)^n])^(5/2) - 5*b*n*((3*b^(3/2)*n^(3/2)*
Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n
))*(c*(d + e*x)^n)^n^(-1)) + 2*Sqrt[a + b*Log[c*(d + e*x)^n]]*(2*a - 3*b*n
+ 2*b*Log[c*(d + e*x)^n]))))/(8*e)
```

## Maple [F]

$$\int (a+b\ln(c(ex+d)^n))^{\frac{5}{2}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (a+b\log (c(d+ex)^n))^{5/2}\,dx=\int (a+b\log (c(d+ex)^n))^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(5/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**(5/2), x)
```

## Maxima [F]

$$\int (a+b\log (c(d+ex)^n))^{5/2}\,dx=\int (b\log ((ex+d)^nc)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Giac [F]

$$\int (a+b\log (c(d+ex)^n))^{5/2}\,dx=\int (b\log ((ex+d)^nc)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\log (c(d+ex)^n))^{5/2}\,dx=\int {\left(a+b\,\ln \left(c\,{\left(d+e\,x\right)}^n\right)\right)}^{5/2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(5/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^(5/2), x)
```

# 3.25 $\int (a+b\log (c(d+ex)^n))^{3/2}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 143

$$\int (a+b\log (c(d+ex)^n))^{3/2}\,dx=\frac{3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}(d+ex)\,(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e}-\frac{3bn(d+ex)\sqrt{a+b\log (c(d+ex)^n)}}{2e}+\frac{(d+ex)\,(a+b\log (c(d+ex)^n))^{3/2}}{e}$$

```
[Out] (e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e+3/4*b^(3/2)*n^(3/2)*(e*x+d)*erfi((a+b
*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e/exp(a/b/n)/((c*(e*x+d)^
n)^(1/n))-3/2*b*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2436, 2333, 2337, 2211, 2235}

$$\int (a+b\log (c(d+ex)^n))^{3/2}\,dx=\frac{3\sqrt{\pi}b^{3/2}n^{3/2}e^{-\frac{a}{bn}}(d+ex)\,(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e}+\frac{(d+ex)\,(a+b\log (c(d+ex)^n))^{3/2}}{e}-\frac{3bn(d+ex)\sqrt{a+b\log (c(d+ex)^n)}}{2e}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] (3*b^(3/2)*n^(3/2)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(
Sqrt[b]*Sqrt[n])])/(4*e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) - (3*b*n*(d + e
*x)*Sqrt[a + b*Log[c*(d + e*x)^n]])/(2*e) + ((d + e*x)*(a + b*Log[c*(d + e*
x)^n])^(3/2))/e
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+b\log(cx^n))^{3/2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^{3/2}}{e} - \frac{(3bn)\text{Subst}\left(\int \sqrt{a+b\log(cx^n)}\,dx, x, d+ex\right)}{2e}$$

$$= -\frac{3bn(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{2e} + \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^{3/2}}{e} + \frac{(3b^2n^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{4e}$$

$$= -\frac{3bn(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{e} + \frac{\left(3b^2n(d+ex)(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log(c(d+ex)^n)\right)}{4e}$$

$$= -\frac{3bn(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{e} + \frac{\left(3bn(d+ex)(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log(c(d+ex)^n)}\right)}{2e}$$

$$= \frac{3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}(d+ex)(c(d+ex)^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e} - \frac{3bn(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{e}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int (a+b\log(c(d+ex)^n))^{3/2}\,dx = \frac{(d+ex)\left(3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}(c(d+ex)^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right) + 2\sqrt{a+b\log}\right.}{4e}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] ((d + e*x)*((3*b^(3/2)*n^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]
/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + 2*Sqrt[a + b*Lo
g[c*(d + e*x)^n]]*(2*a - 3*b*n + 2*b*Log[c*(d + e*x)^n])))/(4*e)
```

## Maple [F]

$$\int (a+b\ln(c(ex+d)^n))^{\frac{3}{2}}\,dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (a+b\log(c(d+ex)^n))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (a+b\log(c(d+ex)^n))^{3/2}\,dx = \int (a+b\log(c(d+ex)^n))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**(3/2), x)
```

## Maxima [F]

$$\int (a+b\log(c(d+ex)^n))^{3/2}\,dx = \int (b\log((ex+d)^n c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Giac [F]

$$\int (a+b\log(c(d+ex)^n))^{3/2}\,dx = \int (b\log((ex+d)^n c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a + b\log(c(d + ex)^n))^{3/2} \, dx = \int (a + b\,\ln(c\,(d + e\,x)^n))^{3/2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(3/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^(3/2), x)
```

# 3.26 $\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 111

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = -\frac{\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e} + \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e}$$

```
[Out] -1/2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*b^(1/2)*n^(1
/2)*Pi^(1/2)/e/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+(e*x+d)*(a+b*ln(c*(e*x+d)^n
))^(1/2)/e
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2436, 2333, 2337, 2211, 2235}

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e} - \frac{\sqrt{\pi}\sqrt{b}\sqrt{n}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e}$$

```
[In] Int[Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] -1/2*(Sqrt[b]*Sqrt[n]*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]
]/(Sqrt[b]*Sqrt[n])])/(e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + ((d + e*x)*S
qrt[a + b*Log[c*(d + e*x)^n]])/e
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{a+b\log\left(cx^n\right)}\,dx, x, d+ex\right)}{e}\\
&= \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e} - \frac{(bn)\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{2e}\\
&= \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e}\\
&\quad - \frac{\left(b(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{2e}
\end{aligned}
$$

$$= \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e} - \frac{\left((d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{e}$$

$$= -\frac{\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e} + \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{(d+ex)\left(-\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+2\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{2e}$$

```
[In] Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]],x]
[Out] ((d + e*x)*(-((Sqrt[b]*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]
/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1))) + 2*Sqrt[a + b*L
og[c*(d + e*x)^n]]))/(2*e)
```

## Maple [F]

$$\int \sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(1/2),x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral(sqrt(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [F]

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{b\log\left((ex+d)^n c\right)+a}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Giac [F]

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{b\log\left((ex+d)^n c\right)+a}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(1/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^(1/2), x)
```

# 3.27 $\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \frac{e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}$$

```
[Out] (e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e/exp(a/
b/n)/((c*(e*x+d)^n)^(1/n))/b^(1/2)/n^(1/2)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2436, 2337, 2211, 2235}

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}$$

```
[In] Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] (Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])
/(Sqrt[b]*e*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{e}\\
&= \frac{\left((d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{en}\\
&= \frac{\left(2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{ben}\\
&= \frac{e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \frac{e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}$$

```
[In] Integrate[1/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
[Out] (Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])
/(Sqrt[b]*e*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1))
```

## Maple [F]

$$\int \frac{1}{\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}dx$$

```
[In] int(1/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
[Out] int(1/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral(1/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left(\left(ex+d\right)^{n}c\right)+a}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{a + b\log\left(c(d + ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left((ex + d)^n c\right) + a}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b\log\left(c(d + ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{a + b\,\ln\left(c\,{\left(d + e\,x\right)}^n\right)}}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e*x)^n))^(1/2),x)
[Out] int(1/(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

# 3.28 $\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 116

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}en^{3/2}} - \frac{2(d+ex)}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

```
[Out] 2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(3/2
)/e/exp(a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(1/n))-2*(e*x+d)/b/e/n/(a+b*ln(c*(e*x
+d)^n))^(1/2)
```

## Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2436, 2334, 2337, 2211, 2235}

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}en^{3/2}} - \frac{2(d+ex)}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(-3/2),x]
```

```
[Out] (2*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])
])/(b^(3/2)*e*E^(a/(b*n))*n^(3/2)*(c*(d + e*x)^n)^n^(-1)) - (2*(d + e*x))/(
b*e*n*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^{3/2}}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{2(d+ex)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{ben}$$

$$= -\frac{2(d+ex)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{ben^2}$$

$$= -\frac{2(d+ex)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(4(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^2en^2}$$

$$= \frac{2e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}en^{3/2}} - \frac{2(d+ex)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx =$$

$$-\frac{2e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(e^{\frac{a}{bn}}\left(c(d+ex)^n\right)^{\frac{1}{n}} - \Gamma\left(\frac{1}{2}, -\frac{a+b\log(c(d+ex)^n)}{bn}\right)\sqrt{-\frac{a+b\log(c(d+ex)^n)}{bn}}\right)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(-3/2),x]
[Out] (-2*(d + e*x)*(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1) - Gamma[1/2, -((a + b*Log
[c*(d + e*x)^n])/(b*n))]*Sqrt[-((a + b*Log[c*(d + e*x)^n])/(b*n))]))/(b*e*E
^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

## Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
[Out] int(1/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(a + b\log(c(d + ex)^n))^{3/2}}\,dx = \int \frac{1}{(a + b\log(c(d + ex)^n))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**(-3/2), x)
```

## Maxima [F]

$$\int \frac{1}{(a + b\log(c(d + ex)^n))^{3/2}}\,dx = \int \frac{1}{(b\log((ex + d)^n c) + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(-3/2), x)
```

## Giac [F]

$$\int \frac{1}{(a + b\log(c(d + ex)^n))^{3/2}}\,dx = \int \frac{1}{(b\log((ex + d)^n c) + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(-3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\log(c(d + ex)^n))^{3/2}}\,dx = \int \frac{1}{(a + b\,\ln(c\,(d + e\,x)^n))^{3/2}}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e*x)^n))^(3/2),x)
[Out] int(1/(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

# 3.29 $\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 156

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \frac{4e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}en^{5/2}} - \frac{2(d+ex)}{3ben(a+b\log(c(d+ex)^n))^{3/2}} - \frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log(c(d+ex)^n)}}$$

```
[Out] -2/3*(e*x+d)/b/e/n/(a+b*ln(c*(e*x+d)^n))^(3/2)+4/3*(e*x+d)*erfi((a+b*ln(c*(
e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)/e/exp(a/b/n)/n^(5/2)/((c
*(e*x+d)^n)^(1/n))-4/3*(e*x+d)/b^2/e/n^2/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2436, 2334, 2337, 2211, 2235}

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \frac{4\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}en^{5/2}} - \frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log(c(d+ex)^n)}} - \frac{2(d+ex)}{3ben(a+b\log(c(d+ex)^n))^{3/2}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(-5/2),x]
```

```
[Out] (4*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])
])/(3*b^(5/2)*e*E^(a/(b*n))*n^(5/2)*(c*(d + e*x)^n)^n^(-1)) - (2*(d + e*x))
```

```
/(3*b*e*n*(a + b*Log[c*(d + e*x)^n])^(3/2)) - (4*(d + e*x))/(3*b^2*e*n^2*Sq
rt[a + b*Log[c*(d + e*x)^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^{5/2}}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{2(d+ex)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^{3/2}}\,dx, x, d+ex\right)}{3ben}$$

$$= -\frac{2(d+ex)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{3b^2en^2}$$

$$= -\frac{2(d+ex)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(4(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{3b^2en^3}$$

$$= -\frac{2(d+ex)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(8(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{3b^3en^3}$$

$$= \frac{4e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}en^{5/2}} - \frac{2(d+ex)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx = -\frac{2e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(2bn\Gamma\left(\frac{1}{2}, -\frac{a+b\log(c(d+ex)^n)}{bn}\right)\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{3/2} + e^{\frac{a}{bn}}\left(c(d+ex)^n\right)^{\frac{1}{n}}(2a\right.}{3b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(-5/2),x]
```

```
[Out] (-2*(d + e*x)*(2*b*n*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(-((a
+ b*Log[c*(d + e*x)^n])/(b*n)))^(3/2) + E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*
(2*a + b*n + 2*b*Log[c*(d + e*x)^n])))/(3*b^2*e*E^(a/(b*n))*n^2*(c*(d + e*x
)^n)^n^(-1)*(a + b*Log[c*(d + e*x)^n])^(3/2))
```

## Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

```
[Out] int(1/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{1}{(a+b\log(c\,(d+ex)^n))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n))**(5/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**(-5/2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(-5/2), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(-5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{1}{\left(a+b\,\ln\left(c\left(d+e\,x\right)^n\right)\right)^{5/2}}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e*x)^n))^(5/2),x)
[Out] int(1/(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

# 3.30 $\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 192

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}\,dx = \frac{8e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{15b^{7/2}en^{7/2}} - \frac{2(d+ex)}{5ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}} - \frac{4(d+ex)}{15b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{8(d+ex)}{15b^3en^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

```
[Out] -2/5*(e*x+d)/b/e/n/(a+b*ln(c*(e*x+d)^n))^(5/2)-4/15*(e*x+d)/b^2/e/n^2/(a+b*
ln(c*(e*x+d)^n))^(3/2)+8/15*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2
)/n^(1/2))*Pi^(1/2)/b^(7/2)/e/exp(a/b/n)/n^(7/2)/((c*(e*x+d)^n)^(1/n))-8/15
*(e*x+d)/b^3/e/n^3/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

## Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2436, 2334, 2337, 2211, 2235}

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}\,dx = \frac{8\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{15b^{7/2}en^{7/2}} - \frac{8(d+ex)}{15b^3en^3\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{4(d+ex)}{15b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{2(d+ex)}{5ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(-7/2),x]
```

```
[Out] (8*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])
])/(15*b^(7/2)*e*E^(a/(b*n))*n^(7/2)*(c*(d + e*x)^n)^n^(-1)) - (2*(d + e*x)
)/(5*b*e*n*(a + b*Log[c*(d + e*x)^n])^(5/2)) - (4*(d + e*x))/(15*b^2*e*n^2*
(a + b*Log[c*(d + e*x)^n])^(3/2)) - (8*(d + e*x))/(15*b^3*e*n^3*Sqrt[a + b*
Log[c*(d + e*x)^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^{7/2}}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{2(d+ex)}{5ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^{5/2}}\,dx, x, d+ex\right)}{5ben}$$

$$= -\frac{2(d+ex)}{5ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}} - \frac{4(d+ex)}{15b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{4\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^{3/2}}\,dx, x, d+ex\right)}{15b^2en^2}$$

$$= -\frac{2(d+ex)}{5ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}} - \frac{4(d+ex)}{15b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{8(d+ex)}{15b^3en^3\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{8\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{15b^3en^3}$$

$$= -\frac{2(d+ex)}{5ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}} - \frac{4(d+ex)}{15b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{8(d+ex)}{15b^3en^3\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(8(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{15b^3en^4}$$

$$= -\frac{2(d+ex)}{5ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}} - \frac{4(d+ex)}{15b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{8(d+ex)}{15b^3en^3\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(16(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{15b^4en^4}$$

$$= \frac{8e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{15b^{7/2}en^{7/2}} - \frac{2(d+ex)}{5ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}} - \frac{4(d+ex)}{15b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{8(d+ex)}{15b^3en^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{7/2}}\,dx = -\frac{2e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(-4b^2n^2\Gamma\left(\frac{1}{2},-\frac{a+b\log(c(d+ex)^n)}{bn}\right)\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{5/2} + e^{\frac{a}{bn}}\left(c(d+ex)^n\right)^{\frac{1}{n}}\left(\right.\right.}{15b^3en^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{5}}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(-7/2),x]
```

```
[Out] (-2*(d + e*x)*(-4*b^2*n^2*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(
-((a + b*Log[c*(d + e*x)^n])/(b*n)))^(5/2) + E^(a/(b*n))*(c*(d + e*x)^n)^n^
(-1)*(4*a^2 + 2*a*b*n + 3*b^2*n^2 + 2*b*(4*a + b*n)*Log[c*(d + e*x)^n] + 4*
b^2*Log[c*(d + e*x)^n]^2)))/(15*b^3*e*E^(a/(b*n))*n^3*(c*(d + e*x)^n)^n^(-1
)*(a + b*Log[c*(d + e*x)^n])^(5/2))
```

## Maple [F]

$$\int \frac{1}{(a+b\ln(c(ex+d)^n))^{\frac{7}{2}}}dx$$

```
[In] int(1/(a+b*ln(c*(e*x+d)^n))^(7/2),x)
```

```
[Out] int(1/(a+b*ln(c*(e*x+d)^n))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{7/2}}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n))**(7/2),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**(-7/2), x)
```

## Maxima [F]

$$\int \frac{1}{(a + b\log(c(d + ex)^n))^{7/2}}\,dx = \int \frac{1}{(b\log((ex + d)^n c) + a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(7/2),x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(-7/2), x)
```

## Giac [F]

$$\int \frac{1}{(a + b\log(c(d + ex)^n))^{7/2}}\,dx = \int \frac{1}{(b\log((ex + d)^n c) + a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(7/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(-7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\log(c(d + ex)^n))^{7/2}}\,dx = \int \frac{1}{(a + b\,\ln(c\,(d + e\,x)^n))^{7/2}}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e*x)^n))^(7/2),x)
[Out] int(1/(a + b*log(c*(d + e*x)^n))^(7/2), x)
```

# 3.31 $\int (a+b\log(c(d+ex)^n))^p\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 103

$$\int (a+b\log(c(d+ex)^n))^p\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\Gamma\left(1+p,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^p\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-p}}{e}$$

```
[Out] (e*x+d)*GAMMA(p+1,(-a-b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(e*x+d)^n))^p/e/exp
(a/b/n)/((c*(e*x+d)^n)^(1/n))/(((-a-b*ln(c*(e*x+d)^n))/b/n)^p)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2436, 2337, 2212}

$$\int (a+b\log(c(d+ex)^n))^p\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}(a+b\log(c(d+ex)^n))^p\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{e}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^p,x]
```

```
[Out] ((d + e*x)*Gamma[1 + p, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(a + b*Log[c*(
d + e*x)^n])^p)/(e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(-((a + b*Log[c*(d +
e*x)^n])/(b*n)))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+b\log(cx^n))^p\,dx, x, d+ex\right)}{e}$$

$$= \frac{\left((d+ex)\,(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int e^{\frac{x}{n}}(a+bx)^p\,dx, x, \log(c(d+ex)^n)\right)}{en}$$

$$= \frac{e^{-\frac{a}{bn}}(d+ex)\,(c(d+ex)^n)^{-1/n}\,\Gamma\left(1+p,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^p\left(-\frac{a+b\log(c(d+ex)^n}{bn}\right.}{e}$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

$$\int (a+b\log(c(d+ex)^n))^p\,dx$$
$$= \frac{e^{-\frac{a}{bn}}(d+ex)\,(c(d+ex)^n)^{-1/n}\,\Gamma\left(1+p,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^p\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-p}}{e}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^p,x]
```

```
[Out] ((d + e*x)*Gamma[1 + p, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(a + b*Log[c*(
d + e*x)^n])^p)/(e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(-((a + b*Log[c*(d +
e*x)^n])/(b*n)))^p)
```

## Maple [F]

$$\int \left(a+b\ln \left(c(ex+d)^{n}\right)\right)^{p}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^p,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^p,x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int (a+b\log (c(d+ex)^n))^p\,dx=\frac{e^{\left(-\frac{bnp\log\left(-\frac{1}{bn}\right)+b\log(c)+a}{bn}\right)}\Gamma\left(p+1,-\frac{bn\log(ex+d)+b\log(c)+a}{bn}\right)}{e}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^p,x, algorithm="fricas")
[Out] e^(-(b*n*p*log(-1/(b*n)) + b*log(c) + a)/(b*n))*gamma(p + 1, -(b*n*log(e*x
+ d) + b*log(c) + a)/(b*n))/e
```

## Sympy [F]

$$\int (a+b\log (c(d+ex)^n))^p\,dx=\int (a+b\log (c(d+ex)^n))^p\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**p,x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**p, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (a+b\log (c(d+ex)^n))^p\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^p,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

## Giac [F]

$$\int (a+b\log(c(d+ex)^n))^p \, dx = \int (b\log((ex+d)^n c)+a)^p \, dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^p, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d+ex)^n))^p \, dx = \int (a+b\,\ln(c\,(d+e\,x)^n))^p \, dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^p,x)
[Out] int((a + b*log(c*(d + e*x)^n))^p, x)
```

# 3.32 $\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 88

$$\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\,dx = \frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\sqrt{d+ex}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{d+ex}\right)}{b}\right)^{-p}}{c^2e}$$

```
[Out] GAMMA(p+1,-2*(a+b*ln(c*(e*x+d)^(1/2)))/b)*(a+b*ln(c*(e*x+d)^(1/2)))^p/(2^p)
/c^2/e/exp(2*a/b)/(((-a-b*ln(c*(e*x+d)^(1/2)))/b)^p)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2436, 2336, 2212}

$$\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\,dx = \frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{d+ex}\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\sqrt{d+ex}\right)\right)}{b}\right)}{c^2e}$$

```
[In] Int[(a + b*Log[c*Sqrt[d + e*x]])^p,x]
```

```
[Out] (Gamma[1 + p, (-2*(a + b*Log[c*Sqrt[d + e*x]]))/b]*(a + b*Log[c*Sqrt[d + e*
x]])^p)/(2^p*c^2*e*E^((2*a)/b)*(-((a + b*Log[c*Sqrt[d + e*x]])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx,x,d+ex\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\sqrt{d+ex}\right)\right)}{c^2e}$$

$$= \frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\sqrt{d+ex}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{d+ex}\right)}{b}\right)^{-p}}{c^2e}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\,dx = \frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\sqrt{d+ex}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{d+ex}\right)}{b}\right)^{-p}}{c^2e}$$

```
[In] Integrate[(a + b*Log[c*Sqrt[d + e*x]])^p,x]
[Out] (Gamma[1 + p, (-2*(a + b*Log[c*Sqrt[d + e*x]]))/b]*(a + b*Log[c*Sqrt[d + e*
x]])^p)/(2^p*c^2*e*E^((2*a)/b)*(-((a + b*Log[c*Sqrt[d + e*x]])/b))^p)
```

## Maple [F]

$$\int \left(a + b\ln\left(c\sqrt{ex+d}\right)\right)^p dx$$

[In] int((a+b*ln(c*(e*x+d)^(1/2)))^p,x)

[Out] int((a+b*ln(c*(e*x+d)^(1/2)))^p,x)

## Fricas [F]

$$\int \left(a + b\log\left(c\sqrt{d+ex}\right)\right)^p dx = \int \left(b\log\left(\sqrt{ex+d}c\right) + a\right)^p dx$$

[In] integrate((a+b*log(c*(e*x+d)^(1/2)))^p,x, algorithm="fricas")

[Out] integral((b*log(sqrt(e*x + d)*c) + a)^p, x)

## Sympy [F]

$$\int \left(a + b\log\left(c\sqrt{d+ex}\right)\right)^p dx = \int \left(a + b\log\left(c\sqrt{d+ex}\right)\right)^p dx$$

[In] integrate((a+b*ln(c*(e*x+d)**(1/2)))**p,x)

[Out] Integral((a + b*log(c*sqrt(d + e*x)))**p, x)

## Maxima [A] (verification not implemented)

none

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int \left(a + b\log\left(c\sqrt{d+ex}\right)\right)^p dx = -\frac{2\left(b\log\left(\sqrt{ex+d}c\right) + a\right)^{p+1} e^{\left(-\frac{2a}{b}\right)} E_{-p}\left(-\frac{2\left(b\log\left(\sqrt{ex+d}c\right)+a\right)}{b}\right)}{bc^2e}$$

[In] integrate((a+b*log(c*(e*x+d)^(1/2)))^p,x, algorithm="maxima")

[Out] -2*(b*log(sqrt(e*x + d)*c) + a)^(p + 1)*e^(-2*a/b)*exp_integral_e(-p, -2*(b
*log(sqrt(e*x + d)*c) + a)/b)/(b*c^2*e)

## Giac [F]

$$\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\,dx=\int \left(b\log\left(\sqrt{ex+d}c\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^(1/2)))^p,x, algorithm="giac")
[Out] integrate((b*log(sqrt(e*x + d)*c) + a)^p, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\sqrt{d+ex}\right)\right)^p\,dx=\int \left(a+b\,\ln\left(c\,\sqrt{d+e\,x}\right)\right)^p\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^(1/2)))^p,x)
[Out] int((a + b*log(c*(d + e*x)^(1/2)))^p, x)
```

# 3.33 $\int \frac{(e+fx)^{-1+p}}{\log(d(e+fx)^p)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 20

$$\int \frac{(e+fx)^{-1+p}}{\log\left(d(e+fx)^p\right)}\,dx=\frac{\text{LogIntegral}\left(d(e+fx)^p\right)}{dfp}$$

```
[Out] Li(d*(f*x+e)^p)/d/f/p
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2437, 2344, 2335}

$$\int \frac{(e+fx)^{-1+p}}{\log\left(d(e+fx)^p\right)}\,dx=\frac{\text{LogIntegral}\left(d(e+fx)^p\right)}{dfp}$$

```
[In] Int[(e + f*x)^(-1 + p)/Log[d*(e + f*x)^p],x]
[Out] LogIntegral[d*(e + f*x)^p]/(d*f*p)
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2344

```
Int[(x_)^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :> Dist[1/n, Subst[Int[1/Log
[c*x], x], x, x^n], x] /; FreeQ[{c, m, n}, x] && EqQ[m, n - 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^{-1+p}}{\log(dx^p)}\,dx, x, e+fx\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{\log(dx)}\,dx, x, (e+fx)^p\right)}{fp}\\
&= \frac{\text{li}(d(e+fx)^p)}{dfp}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(e+fx)^{-1+p}}{\log(d(e+fx)^p)}\,dx = \frac{\text{LogIntegral}\left(d(e+fx)^p\right)}{dfp}$$

```
[In] Integrate[(e + f*x)^(-1 + p)/Log[d*(e + f*x)^p],x]
[Out] LogIntegral[d*(e + f*x)^p]/(d*f*p)
```

## Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

| method | result |
|---|---|
| default | $-\frac{\text{Ei}_1(-\ln(d(fx+e)^p))}{pfd}$ |
| risch | $-\frac{e^{\frac{i\pi\ \text{csgn}(id(fx+e)^p)(-\text{csgn}(id(fx+e)^p)+\text{csgn}(id))(-\text{csgn}(id(fx+e)^p)+\text{csgn}(i(fx+e)^p))}{2}}\ \text{Ei}_1\left(-\ln(d)-\ln((fx+e)^p)-\frac{i\pi\ \text{csgn}(i(fx+e)^p)\ \text{csg}}{2}\right.}{pfd}$ |

```
[In] int((f*x+e)^(-1+p)/ln(d*(f*x+e)^p),x,method=_RETURNVERBOSE)
[Out] -1/p/f/d*Ei(1,-ln(d*(f*x+e)^p))
```

## Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^{-1+p}}{\log (d(e+fx)^p)}\,dx = \frac{\text{Ei}(p\log (fx+e)+\log (d))}{dfp}$$

```
[In] integrate((f*x+e)^(-1+p)/log(d*(f*x+e)^p),x, algorithm="fricas")
[Out] Ei(p*log(f*x + e) + log(d))/(d*f*p)
```

## Sympy [F]

$$\int \frac{(e+fx)^{-1+p}}{\log (d(e+fx)^p)}\,dx = \int \frac{(e+fx)^{p-1}}{\log (d\,(e+fx)^p)}\,dx$$

```
[In] integrate((f*x+e)**(-1+p)/ln(d*(f*x+e)**p),x)
[Out] Integral((e + f*x)**(p - 1)/log(d*(e + f*x)**p), x)
```

## Maxima [F]

$$\int \frac{(e+fx)^{-1+p}}{\log (d(e+fx)^p)}\,dx = \int \frac{(fx+e)^{p-1}}{\log ((fx+e)^p d)}\,dx$$

```
[In] integrate((f*x+e)^(-1+p)/log(d*(f*x+e)^p),x, algorithm="maxima")
[Out] integrate((f*x + e)^(p - 1)/log((f*x + e)^p*d), x)
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^{-1+p}}{\log (d(e+fx)^p)}\,dx = \frac{\text{Ei}(p\log (fx+e)+\log (d))}{dfp}$$

```
[In] integrate((f*x+e)^(-1+p)/log(d*(f*x+e)^p),x, algorithm="giac")
[Out] Ei(p*log(f*x + e) + log(d))/(d*f*p)
```

## Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(e+fx)^{-1+p}}{\log\left(d(e+fx)^p\right)}\,dx = \frac{\operatorname{logint}(d\,(e+f\,x)^p)}{d\,f\,p}$$

```
[In] int((e + f*x)^(p - 1)/log(d*(e + f*x)^p),x)
[Out] logint(d*(e + f*x)^p)/(d*f*p)
```

# 3.34 $\int \frac{(eg+fgx)^{-1+p}}{\log(d(e+fx)^p)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 42

$$\int \frac{(eg+fgx)^{-1+p}}{\log(d(e+fx)^p)}\,dx = \frac{(e+fx)^{1-p}(g(e+fx))^{-1+p}\,\text{LogIntegral}\left(d(e+fx)^p\right)}{dfp}$$

```
[Out] (f*x+e)^(1-p)*(g*(f*x+e))^(-1+p)*Li(d*(f*x+e)^p)/d/f/p
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2437, 2345, 2344, 2335}

$$\int \frac{(eg+fgx)^{-1+p}}{\log\left(d(e+fx)^p\right)}\,dx = \frac{(e+fx)^{1-p}(g(e+fx))^{p-1}\,\text{LogIntegral}\left(d(e+fx)^p\right)}{dfp}$$

```
[In] Int[(e*g + f*g*x)^(-1 + p)/Log[d*(e + f*x)^p],x]
```

```
[Out] ((e + f*x)^(1 - p)*(g*(e + f*x))^(-1 + p)*LogIntegral[d*(e + f*x)^p])/(d*f*
p)
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2344

```
Int[(x_)^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :> Dist[1/n, Subst[Int[1/Log
[c*x], x], x, x^n], x] /; FreeQ[{c, m, n}, x] && EqQ[m, n - 1]
```

Rule 2345

```
Int[((d_)*(x_))^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :> Dist[(d*x)^m/x^m,
Int[x^m/Log[c*x^n], x], x] /; FreeQ[{c, d, m, n}, x] && EqQ[m, n - 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(gx)^{-1+p}}{\log(dx^p)}\,dx, x, e+fx\right)}{f} \\
&= \frac{\left((e+fx)^{1-p}(g(e+fx))^{-1+p}\right)\text{Subst}\left(\int \frac{x^{-1+p}}{\log(dx^p)}\,dx, x, e+fx\right)}{f} \\
&= \frac{\left((e+fx)^{1-p}(g(e+fx))^{-1+p}\right)\text{Subst}\left(\int \frac{1}{\log(dx)}\,dx, x, (e+fx)^p\right)}{fp} \\
&= \frac{(e+fx)^{1-p}(g(e+fx))^{-1+p}\text{li}(d(e+fx)^p)}{dfp}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$
\int \frac{(eg+fgx)^{-1+p}}{\log\left(d(e+fx)^p\right)}\,dx = \frac{(e+fx)^{1-p}(g(e+fx))^{-1+p}\,\text{LogIntegral}\left(d(e+fx)^p\right)}{dfp}
$$

```
[In] Integrate[(e*g + f*g*x)^(-1 + p)/Log[d*(e + f*x)^p],x]
[Out] ((e + f*x)^(1 - p)*(g*(e + f*x))^(-1 + p)*LogIntegral[d*(e + f*x)^p])/(d*f*
p)
```

## Maple [F]

$$\int \frac{(fgx+eg)^{-1+p}}{\ln\left(d\left(fx+e\right)^{p}\right)}dx$$

```
[In] int((f*g*x+e*g)^(-1+p)/ln(d*(f*x+e)^p),x)
[Out] int((f*g*x+e*g)^(-1+p)/ln(d*(f*x+e)^p),x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

$$\int \frac{(eg+fgx)^{-1+p}}{\log\left(d(e+fx)^p\right)}\,dx = \frac{g^{p-1}\mathrm{Ei}(p\log\left(fx+e\right)+\log\left(d\right))}{dfp}$$

```
[In] integrate((f*g*x+e*g)^(-1+p)/log(d*(f*x+e)^p),x, algorithm="fricas")
[Out] g^(p - 1)*Ei(p*log(f*x + e) + log(d))/(d*f*p)
```

## Sympy [F]

$$\int \frac{(eg+fgx)^{-1+p}}{\log\left(d(e+fx)^p\right)}\,dx = \int \frac{(g(e+fx))^{p-1}}{\log\left(d\left(e+fx\right)^{p}\right)}\,dx$$

```
[In] integrate((f*g*x+e*g)**(-1+p)/ln(d*(f*x+e)**p),x)
[Out] Integral((g*(e + f*x))**(p - 1)/log(d*(e + f*x)**p), x)
```

## Maxima [F]

$$\int \frac{(eg+fgx)^{-1+p}}{\log\left(d(e+fx)^p\right)}\,dx = \int \frac{(fgx+eg)^{p-1}}{\log\left((fx+e)^{p}d\right)}\,dx$$

```
[In] integrate((f*g*x+e*g)^(-1+p)/log(d*(f*x+e)^p),x, algorithm="maxima")
[Out] integrate((f*g*x + e*g)^(p - 1)/log((f*x + e)^p*d), x)
```

## Giac [F]

$$\int \frac{(eg + fgx)^{-1+p}}{\log(d(e + fx)^p)}\,dx = \int \frac{(fgx + eg)^{p-1}}{\log((fx + e)^p d)}\,dx$$

```
[In] integrate((f*g*x+e*g)^(-1+p)/log(d*(f*x+e)^p),x, algorithm="giac")
[Out] integrate((f*g*x + e*g)^(p - 1)/log((f*x + e)^p*d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(eg + fgx)^{-1+p}}{\log(d(e + fx)^p)}\,dx = \int \frac{(e\,g + f\,g\,x)^{p-1}}{\ln(d\,(e + f\,x)^p)}\,dx$$

```
[In] int((e*g + f*g*x)^(p - 1)/log(d*(e + f*x)^p),x)
[Out] int((e*g + f*g*x)^(p - 1)/log(d*(e + f*x)^p), x)
```

# 3.35 $\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 178

$$\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx = -\frac{b(ef-dg)^4nx}{5e^4} - \frac{b(ef-dg)^3n(f+gx)^2}{10e^3g} - \frac{b(ef-dg)^2n(f+gx)^3}{15e^2g} - \frac{b(ef-dg)n(f+gx)^4}{20eg} - \frac{bn(f+gx)^5}{25g} - \frac{b(ef-dg)^5n\log(d+ex)}{5e^5g} + \frac{(f+gx)^5(a+b\log(c(d+ex)^n))}{5g}$$

```
[Out] -1/5*b*(-d*g+e*f)^4*n*x/e^4-1/10*b*(-d*g+e*f)^3*n*(g*x+f)^2/e^3/g-1/15*b*(-
d*g+e*f)^2*n*(g*x+f)^3/e^2/g-1/20*b*(-d*g+e*f)*n*(g*x+f)^4/e/g-1/25*b*n*(g*
x+f)^5/g-1/5*b*(-d*g+e*f)^5*n*ln(e*x+d)/e^5/g+1/5*(g*x+f)^5*(a+b*ln(c*(e*x+
d)^n))/g
```

## Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2442, 45}

$$\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx = \frac{(f+gx)^5(a+b\log(c(d+ex)^n))}{5g} - \frac{bn(ef-dg)^5\log(d+ex)}{5e^5g} - \frac{bnx(ef-dg)^4}{5e^4} - \frac{bn(f+gx)^2(ef-dg)^3}{10e^3g} - \frac{bn(f+gx)^3(ef-dg)^2}{15e^2g} - \frac{bn(f+gx)^4(ef-dg)}{20eg} - \frac{bn(f+gx)^5}{25g}$$

```
[In] Int[(f + g*x)^4*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] -1/5*(b*(e*f - d*g)^4*n*x)/e^4 - (b*(e*f - d*g)^3*n*(f + g*x)^2)/(10*e^3*g)
 - (b*(e*f - d*g)^2*n*(f + g*x)^3)/(15*e^2*g) - (b*(e*f - d*g)*n*(f + g*x)^
4)/(20*e*g) - (b*n*(f + g*x)^5)/(25*g) - (b*(e*f - d*g)^5*n*Log[d + e*x])/(
5*e^5*g) + ((f + g*x)^5*(a + b*Log[c*(d + e*x)^n]))/(5*g)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^5(a+b\log(c(d+ex)^n))}{5g} - \frac{(ben)\int\frac{(f+gx)^5}{d+ex}\,dx}{5g}$$

$$= \frac{(f+gx)^5 (a+b\log(c(d+ex)^n))}{5g} - \frac{(ben)\int \left(\frac{g(ef-dg)^4}{e^5} + \frac{(ef-dg)^5}{e^5(d+ex)} + \frac{g(ef-dg)^3(f+gx)}{e^4} + \frac{g(ef-dg)^2(f+gx)^2}{e^3} + \frac{g(ef-dg)(f+gx)^3}{e^2} + \frac{g(f+gx)^4}{e}\right) dx}{5g}$$

$$= -\frac{b(ef-dg)^4nx}{5e^4} - \frac{b(ef-dg)^3n(f+gx)^2}{10e^3g} - \frac{b(ef-dg)^2n(f+gx)^3}{15e^2g} - \frac{b(ef-dg)n(f+gx)^4}{20eg} - \frac{bn(f+gx)^5}{25g} - \frac{b(ef-dg)^5n\log(d+ex)}{5e^5g} + \frac{(f+gx)^5 (a+b\log(c(d+ex)^n))}{5g}$$

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.77

$$\int (f+gx)^4 (a+b\log(c(d+ex)^n)) \, dx = \frac{ex(60ae^4(5f^4+10f^3gx+10f^2g^2x^2+5fg^3x^3+g^4x^4) - bn(60d^4g^4 - 30d^3eg^3(10f+gx) + 10d^2e^2g^2(60f}{}$$

```
[In] Integrate[(f + g*x)^4*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (e*x*(60*a*e^4*(5*f^4 + 10*f^3*g*x + 10*f^2*g^2*x^2 + 5*f*g^3*x^3 + g^4*x^4
) - b*n*(60*d^4*g^4 - 30*d^3*e*g^3*(10*f + g*x) + 10*d^2*e^2*g^2*(60*f^2 +
15*f*g*x + 2*g^2*x^2) - 5*d*e^3*g*(120*f^3 + 60*f^2*g*x + 20*f*g^2*x^2 + 3*
g^3*x^3) + e^4*(300*f^4 + 300*f^3*g*x + 200*f^2*g^2*x^2 + 75*f*g^3*x^3 + 12
*g^4*x^4))) + 60*b*d^2*g*(-10*e^3*f^3 + 10*d*e^2*f^2*g - 5*d^2*e*f*g^2 + d^
3*g^3)*n*Log[d + e*x] + 60*b*e^4*(5*d*f^4 + e*x*(5*f^4 + 10*f^3*g*x + 10*f^
2*g^2*x^2 + 5*f*g^3*x^3 + g^4*x^4))*Log[c*(d + e*x)^n])/(300*e^5)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(164) = 328$.

Time = 1.62 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.31

| method | result |
|---|---|
| parallelrisch | $\frac{300bd\,e^4f^4n+300x\ln(c(ex+d)^n)b\,e^5f^4-300xb\,e^5f^4n-300\ln(c(ex+d)^n)bd\,e^4f^4+60\ln(ex+d)b\,d^5g^4n+60x^5\ln(c(ex+d)^n)b\,e^5}{}$ |
| risch | Expression too large to display |

```
[In] int((g*x+f)^4*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/300*(300*b*d*e^4*f^4*n+300*x*ln(c*(e*x+d)^n)*b*e^5*f^4-300*x*b*e^5*f^4*n-
300*ln(c*(e*x+d)^n)*b*d*e^4*f^4+60*ln(e*x+d)*b*d^5*g^4*n+60*x^5*ln(c*(e*x+d
```

```
)^n)*b*e^5*g^4-12*x^5*b*e^5*g^4*n+300*x^4*a*e^5*f*g^3+600*x^3*a*e^5*f^2*g^2
+600*x^2*a*e^5*f^3*g+60*x^5*a*e^5*g^4+600*ln(e*x+d)*b*d*e^4*f^4*n+15*x^4*b*
d*e^4*g^4*n-75*x^4*b*e^5*f*g^3*n+600*x^3*ln(c*(e*x+d)^n)*b*e^5*f^2*g^2-20*x
^3*b*d^2*e^3*g^4*n-200*x^3*b*e^5*f^2*g^2*n+600*x^2*ln(c*(e*x+d)^n)*b*e^5*f^
3*g+30*x^2*b*d^3*e^2*g^4*n-300*x^2*b*e^5*f^3*g*n-60*x*b*d^4*e*g^4*n+300*x^4
*ln(c*(e*x+d)^n)*b*e^5*f*g^3-300*b*d^4*e*f*g^3*n+600*b*d^3*e^2*f^2*g^2*n-30
0*a*d*e^4*f^4-600*b*d^2*e^3*f^3*g*n+300*x*a*e^5*f^4-600*x*b*d^2*e^3*f^2*g^2
*n+600*x*b*d*e^4*f^3*g*n-300*ln(e*x+d)*b*d^4*e*f*g^3*n+600*ln(e*x+d)*b*d^3*
e^2*f^2*g^2*n+100*x^3*b*d*e^4*f*g^3*n-150*x^2*b*d^2*e^3*f*g^3*n+300*x^2*b*d
*e^4*f^2*g^2*n+300*x*b*d^3*e^2*f*g^3*n-600*ln(e*x+d)*b*d^2*e^3*f^3*g*n+60*b
*d^5*g^4*n)/e^5
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(164) = 328$.

Time = 0.32 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.65

$$\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx =$$
$$-\frac{12\,(be^5g^4n-5\,ae^5g^4)x^5-15\,(20\,ae^5fg^3-(5\,be^5fg^3-bde^4g^4)n)x^4-20\,(30\,ae^5f^2g^2-(10\,be^5f^2g^2-5\,b$$

```
[In] integrate((g*x+f)^4*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

```
[Out] -1/300*(12*(b*e^5*g^4*n - 5*a*e^5*g^4)*x^5 - 15*(20*a*e^5*f*g^3 - (5*b*e^5*
f*g^3 - b*d*e^4*g^4)*n)*x^4 - 20*(30*a*e^5*f^2*g^2 - (10*b*e^5*f^2*g^2 - 5*
b*d*e^4*f*g^3 + b*d^2*e^3*g^4)*n)*x^3 - 30*(20*a*e^5*f^3*g - (10*b*e^5*f^3*
g - 10*b*d*e^4*f^2*g^2 + 5*b*d^2*e^3*f*g^3 - b*d^3*e^2*g^4)*n)*x^2 - 60*(5*
a*e^5*f^4 - (5*b*e^5*f^4 - 10*b*d*e^4*f^3*g + 10*b*d^2*e^3*f^2*g^2 - 5*b*d^
3*e^2*f*g^3 + b*d^4*e*g^4)*n)*x - 60*(b*e^5*g^4*n*x^5 + 5*b*e^5*f*g^3*n*x^4
 + 10*b*e^5*f^2*g^2*n*x^3 + 10*b*e^5*f^3*g*n*x^2 + 5*b*e^5*f^4*n*x + (5*b*d
*e^4*f^4 - 10*b*d^2*e^3*f^3*g + 10*b*d^3*e^2*f^2*g^2 - 5*b*d^4*e*f*g^3 + b*
d^5*g^4)*n)*log(e*x + d) - 60*(b*e^5*g^4*x^5 + 5*b*e^5*f*g^3*x^4 + 10*b*e^5
*f^2*g^2*x^3 + 10*b*e^5*f^3*g*x^2 + 5*b*e^5*f^4*x)*log(c))/e^5
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(153) = 306$.

Time = 2.17 (sec) , antiderivative size = 568, normalized size of antiderivative = 3.19

$$\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx$$
$$=\begin{cases} af^4x+2af^3gx^2+2af^2g^2x^3+afg^3x^4+\frac{ag^4x^5}{5}+\frac{bd^5g^4\log(c(d+ex)^n)}{5e^5}-\frac{bd^4fg^3\log(c(d+ex)^n)}{e^4}-\frac{bd^4g^4nx}{5e^4}+\frac{2bd^3f^2g^2}{} \\ (a+b\log(cd^n))\left(f^4x+2f^3gx^2+2f^2g^2x^3+fg^3x^4+\frac{g^4x^5}{5}\right) \end{cases}$$

```
[In] integrate((g*x+f)**4*(a+b*ln(c*(e*x+d)**n)),x)
```

```
[Out] Piecewise((a*f**4*x + 2*a*f**3*g*x**2 + 2*a*f**2*g**2*x**3 + a*f*g**3*x**4
+ a*g**4*x**5/5 + b*d**5*g**4*log(c*(d + e*x)**n)/(5*e**5) - b*d**4*f*g**3*
log(c*(d + e*x)**n)/e**4 - b*d**4*g**4*n*x/(5*e**4) + 2*b*d**3*f**2*g**2*lo
g(c*(d + e*x)**n)/e**3 + b*d**3*f*g**3*n*x/e**3 + b*d**3*g**4*n*x**2/(10*e*
*3) - 2*b*d**2*f**3*g*log(c*(d + e*x)**n)/e**2 - 2*b*d**2*f**2*g**2*n*x/e**
2 - b*d**2*f*g**3*n*x**2/(2*e**2) - b*d**2*g**4*n*x**3/(15*e**2) + b*d*f**4
*log(c*(d + e*x)**n)/e + 2*b*d*f**3*g*n*x/e + b*d*f**2*g**2*n*x**2/e + b*d*
f*g**3*n*x**3/(3*e) + b*d*g**4*n*x**4/(20*e) - b*f**4*n*x + b*f**4*x*log(c*
(d + e*x)**n) - b*f**3*g*n*x**2 + 2*b*f**3*g*x**2*log(c*(d + e*x)**n) - 2*b
*f**2*g**2*n*x**3/3 + 2*b*f**2*g**2*x**3*log(c*(d + e*x)**n) - b*f*g**3*n*x
**4/4 + b*f*g**3*x**4*log(c*(d + e*x)**n) - b*g**4*n*x**5/25 + b*g**4*x**5*
log(c*(d + e*x)**n)/5, Ne(e, 0)), ((a + b*log(c*d**n))*(f**4*x + 2*f**3*g*x
**2 + 2*f**2*g**2*x**3 + f*g**3*x**4 + g**4*x**5/5), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(164) = 328$.

Time = 0.21 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.21

$$\begin{aligned}
&\int (f+gx)^4 (a+b\log(c(d+ex)^n))\,dx \\
&= \frac{1}{5}\,bg^4x^5\log\left((ex+d)^n c\right) + \frac{1}{5}\,ag^4x^5 + bfg^3x^4\log\left((ex+d)^n c\right) + afg^3x^4 \\
&\quad + 2\,bf^2g^2x^3\log\left((ex+d)^n c\right) + 2\,af^2g^2x^3 - bef^4n\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) \\
&\quad + \frac{1}{300}\,beg^4n\left(\frac{60\,d^5\log(ex+d)}{e^6} - \frac{12\,e^4x^5 - 15\,de^3x^4 + 20\,d^2e^2x^3 - 30\,d^3ex^2 + 60\,d^4x}{e^5}\right) \\
&\quad - \frac{1}{12}\,befg^3n\left(\frac{12\,d^4\log(ex+d)}{e^5} + \frac{3\,e^3x^4 - 4\,de^2x^3 + 6\,d^2ex^2 - 12\,d^3x}{e^4}\right) \\
&\quad + \frac{1}{3}\,bef^2g^2n\left(\frac{6\,d^3\log(ex+d)}{e^4} - \frac{2\,e^2x^3 - 3\,dex^2 + 6\,d^2x}{e^3}\right) \\
&\quad - bef^3gn\left(\frac{2\,d^2\log(ex+d)}{e^3} + \frac{ex^2 - 2\,dx}{e^2}\right) \\
&\quad + 2\,bf^3gx^2\log\left((ex+d)^n c\right) + 2\,af^3gx^2 + bf^4x\log\left((ex+d)^n c\right) + af^4x
\end{aligned}$$

```
[In] integrate((g*x+f)^4*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] 1/5*b*g^4*x^5*log((e*x + d)^n*c) + 1/5*a*g^4*x^5 + b*f*g^3*x^4*log((e*x + d
)^n*c) + a*f*g^3*x^4 + 2*b*f^2*g^2*x^3*log((e*x + d)^n*c) + 2*a*f^2*g^2*x^3
 - b*e*f^4*n*(x/e - d*log(e*x + d)/e^2) + 1/300*b*e*g^4*n*(60*d^5*log(e*x +
 d)/e^6 - (12*e^4*x^5 - 15*d*e^3*x^4 + 20*d^2*e^2*x^3 - 30*d^3*e*x^2 + 60*d
^4*x)/e^5) - 1/12*b*e*f*g^3*n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e
```

```
^2*x^3 + 6*d^2*e*x^2 - 12*d^3*x)/e^4) + 1/3*b*e*f^2*g^2*n*(6*d^3*log(e*x +
d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) - b*e*f^3*g*n*(2*d^2*log(e*
x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 2*b*f^3*g*x^2*log((e*x + d)^n*c) + 2*a*
f^3*g*x^2 + b*f^4*x*log((e*x + d)^n*c) + a*f^4*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1209 vs. $2(164) = 328$.

Time = 0.32 (sec) , antiderivative size = 1209, normalized size of antiderivative = 6.79

$$\int (f+gx)^4 (a+b\log (c(d+ex)^n))\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^4*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] (e*x + d)*b*f^4*n*log(e*x + d)/e + 2*(e*x + d)^2*b*f^3*g*n*log(e*x + d)/e^2
 - 4*(e*x + d)*b*d*f^3*g*n*log(e*x + d)/e^2 + 2*(e*x + d)^3*b*f^2*g^2*n*log
(e*x + d)/e^3 - 6*(e*x + d)^2*b*d*f^2*g^2*n*log(e*x + d)/e^3 + 6*(e*x + d)*
b*d^2*f^2*g^2*n*log(e*x + d)/e^3 + (e*x + d)^4*b*f*g^3*n*log(e*x + d)/e^4 -
 4*(e*x + d)^3*b*d*f*g^3*n*log(e*x + d)/e^4 + 6*(e*x + d)^2*b*d^2*f*g^3*n*l
og(e*x + d)/e^4 - 4*(e*x + d)*b*d^3*f*g^3*n*log(e*x + d)/e^4 + 1/5*(e*x + d
)^5*b*g^4*n*log(e*x + d)/e^5 - (e*x + d)^4*b*d*g^4*n*log(e*x + d)/e^5 + 2*(
e*x + d)^3*b*d^2*g^4*n*log(e*x + d)/e^5 - 2*(e*x + d)^2*b*d^3*g^4*n*log(e*x
 + d)/e^5 + (e*x + d)*b*d^4*g^4*n*log(e*x + d)/e^5 - (e*x + d)*b*f^4*n/e -
(e*x + d)^2*b*f^3*g*n/e^2 + 4*(e*x + d)*b*d*f^3*g*n/e^2 - 2/3*(e*x + d)^3*b
*f^2*g^2*n/e^3 + 3*(e*x + d)^2*b*d*f^2*g^2*n/e^3 - 6*(e*x + d)*b*d^2*f^2*g^
2*n/e^3 - 1/4*(e*x + d)^4*b*f*g^3*n/e^4 + 4/3*(e*x + d)^3*b*d*f*g^3*n/e^4 -
 3*(e*x + d)^2*b*d^2*f*g^3*n/e^4 + 4*(e*x + d)*b*d^3*f*g^3*n/e^4 - 1/25*(e*
x + d)^5*b*g^4*n/e^5 + 1/4*(e*x + d)^4*b*d*g^4*n/e^5 - 2/3*(e*x + d)^3*b*d^
2*g^4*n/e^5 + (e*x + d)^2*b*d^3*g^4*n/e^5 - (e*x + d)*b*d^4*g^4*n/e^5 + (e*
x + d)*b*f^4*log(c)/e + 2*(e*x + d)^2*b*f^3*g*log(c)/e^2 - 4*(e*x + d)*b*d*
f^3*g*log(c)/e^2 + 2*(e*x + d)^3*b*f^2*g^2*log(c)/e^3 - 6*(e*x + d)^2*b*d*f
^2*g^2*log(c)/e^3 + 6*(e*x + d)*b*d^2*f^2*g^2*log(c)/e^3 + (e*x + d)^4*b*f*
g^3*log(c)/e^4 - 4*(e*x + d)^3*b*d*f*g^3*log(c)/e^4 + 6*(e*x + d)^2*b*d^2*f
*g^3*log(c)/e^4 - 4*(e*x + d)*b*d^3*f*g^3*log(c)/e^4 + 1/5*(e*x + d)^5*b*g^
4*log(c)/e^5 - (e*x + d)^4*b*d*g^4*log(c)/e^5 + 2*(e*x + d)^3*b*d^2*g^4*log
(c)/e^5 - 2*(e*x + d)^2*b*d^3*g^4*log(c)/e^5 + (e*x + d)*b*d^4*g^4*log(c)/e
^5 + (e*x + d)*a*f^4/e + 2*(e*x + d)^2*a*f^3*g/e^2 - 4*(e*x + d)*a*d*f^3*g/
e^2 + 2*(e*x + d)^3*a*f^2*g^2/e^3 - 6*(e*x + d)^2*a*d*f^2*g^2/e^3 + 6*(e*x
+ d)*a*d^2*f^2*g^2/e^3 + (e*x + d)^4*a*f*g^3/e^4 - 4*(e*x + d)^3*a*d*f*g^3/
e^4 + 6*(e*x + d)^2*a*d^2*f*g^3/e^4 - 4*(e*x + d)*a*d^3*f*g^3/e^4 + 1/5*(e*
x + d)^5*a*g^4/e^5 - (e*x + d)^4*a*d*g^4/e^5 + 2*(e*x + d)^3*a*d^2*g^4/e^5
- 2*(e*x + d)^2*a*d^3*g^4/e^5 + (e*x + d)*a*d^4*g^4/e^5
```

## Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.96

$$\int (f+gx)^4 \left(a+b\log \left(c(d+ex)^n\right)\right)\,dx = x\left(\frac{5\,a\,e\,f^4+20\,a\,d\,f^3\,g-5\,b\,e\,f^4\,n}{5\,e} - \frac{d\left(\frac{d\left(\frac{d\left(\frac{g^3\,(a\,d\,g+4\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{d\,g^4\,(5\,a-b\,n)}{5\,e}\right)}{e}-\frac{2\,f\,g^2\,(2\,a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{e}\right)}{e}+\frac{2\,f^2\,g\,(3\,a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{e}\right)}{e}\right) - x^3\left(\frac{d\left(\frac{g^3\,(a\,d\,g+4\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{d\,g^4\,(5\,a-b\,n)}{5\,e}\right)}{3\,e}-\frac{2\,f\,g^2\,(2\,a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{3\,e}\right) + x^4\left(\frac{g^3\,(a\,d\,g+4\,a\,e\,f-b\,e\,f\,n)}{4\,e}-\frac{d\,g^4\,(5\,a-b\,n)}{20\,e}\right) + \ln\left(c\,{(d+e\,x)}^n\right)\,\left(b\,f^4\,x+2\,b\,f^3\,g\,x^2+2\,b\,f^2\,g^2\,x^3+b\,f\,g^3\,x^4+\frac{b\,g^4\,x^5}{5}\right) + x^2\left(\frac{d\left(\frac{d\left(\frac{g^3\,(a\,d\,g+4\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{d\,g^4\,(5\,a-b\,n)}{5\,e}\right)}{e}-\frac{2\,f\,g^2\,(2\,a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{e}\right)}{2\,e}+\frac{f^2\,g\,(3\,a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{e}\right) + \frac{g^4\,x^5\,(5\,a-b\,n)}{25} + \frac{\ln\left(d+e\,x\right)\,\left(b\,n\,d^5\,g^4-5\,b\,n\,d^4\,e\,f\,g^3+10\,b\,n\,d^3\,e^2\,f^2\,g^2-10\,b\,n\,d^2\,e^3\,f^3\,g+5\,b\,n\,d\,e^4\,f^4\right)}{5\,e^5}$$

```
[In] int((f + g*x)^4*(a + b*log(c*(d + e*x)^n)),x)
```

```
[Out] x*((5*a*e*f^4 + 20*a*d*f^3*g - 5*b*e*f^4*n)/(5*e) - (d*((d*((d*((g^3*(a*d*g
 + 4*a*e*f - b*e*f*n))/e - (d*g^4*(5*a - b*n))/(5*e)))/e - (2*f*g^2*(2*a*d*
g + 3*a*e*f - b*e*f*n))/e))/e + (2*f^2*g*(3*a*d*g + 2*a*e*f - b*e*f*n))/e))
/e) - x^3*((d*((g^3*(a*d*g + 4*a*e*f - b*e*f*n))/e - (d*g^4*(5*a - b*n))/(5
*e)))/(3*e) - (2*f*g^2*(2*a*d*g + 3*a*e*f - b*e*f*n))/(3*e)) + x^4*((g^3*(a
*d*g + 4*a*e*f - b*e*f*n))/(4*e) - (d*g^4*(5*a - b*n))/(20*e)) + log(c*(d +
 e*x)^n)*((b*g^4*x^5)/5 + b*f^4*x + 2*b*f^2*g^2*x^3 + 2*b*f^3*g*x^2 + b*f*g
^3*x^4) + x^2*((d*((d*((g^3*(a*d*g + 4*a*e*f - b*e*f*n))/e - (d*g^4*(5*a -
b*n))/(5*e)))/e - (2*f*g^2*(2*a*d*g + 3*a*e*f - b*e*f*n))/e))/(2*e) + (f^2*
g*(3*a*d*g + 2*a*e*f - b*e*f*n))/e) + (g^4*x^5*(5*a - b*n))/25 + (log(d + e
*x)*(b*d^5*g^4*n + 5*b*d*e^4*f^4*n + 10*b*d^3*e^2*f^2*g^2*n - 5*b*d^4*e*f*g
^3*n - 10*b*d^2*e^3*f^3*g*n))/(5*e^5)
```

# 3.36 $\int (f+gx)^3 (a+b\log (c(d+ex)^n)) \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 149

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n)) \, dx = -\frac{b(ef-dg)^3nx}{4e^3} - \frac{b(ef-dg)^2n(f+gx)^2}{8e^2g} - \frac{b(ef-dg)n(f+gx)^3}{12eg} - \frac{bn(f+gx)^4}{16g} - \frac{b(ef-dg)^4n\log(d+ex)}{4e^4g} + \frac{(f+gx)^4 (a+b\log (c(d+ex)^n))}{4g}$$

```
[Out] -1/4*b*(-d*g+e*f)^3*n*x/e^3-1/8*b*(-d*g+e*f)^2*n*(g*x+f)^2/e^2/g-1/12*b*(-d
*g+e*f)*n*(g*x+f)^3/e/g-1/16*b*n*(g*x+f)^4/g-1/4*b*(-d*g+e*f)^4*n*ln(e*x+d)
/e^4/g+1/4*(g*x+f)^4*(a+b*ln(c*(e*x+d)^n))/g
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2442, 45}

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n)) \, dx = \frac{(f+gx)^4 (a+b\log (c(d+ex)^n))}{4g} - \frac{bn(ef-dg)^4\log(d+ex)}{4e^4g} - \frac{bnx(ef-dg)^3}{4e^3} - \frac{bn(f+gx)^2(ef-dg)^2}{8e^2g} - \frac{bn(f+gx)^3(ef-dg)}{12eg} - \frac{bn(f+gx)^4}{16g}$$

```
[In] Int[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] -1/4*(b*(e*f - d*g)^3*n*x)/e^3 - (b*(e*f - d*g)^2*n*(f + g*x)^2)/(8*e^2*g)
- (b*(e*f - d*g)*n*(f + g*x)^3)/(12*e*g) - (b*n*(f + g*x)^4)/(16*g) - (b*(e
*f - d*g)^4*n*Log[d + e*x])/(4*e^4*g) + ((f + g*x)^4*(a + b*Log[c*(d + e*x)
^n]))/(4*g)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(f+gx)^4\,(a+b\log\left(c(d+ex)^n\right))}{4g}-\frac{(ben)\int\frac{(f+gx)^4}{d+ex}\,dx}{4g}\\
&= \frac{(f+gx)^4\,(a+b\log\left(c(d+ex)^n\right))}{4g}\\
&\quad-\frac{(ben)\int\left(\frac{g(ef-dg)^3}{e^4}+\frac{(ef-dg)^4}{e^4(d+ex)}+\frac{g(ef-dg)^2(f+gx)}{e^3}+\frac{g(ef-dg)(f+gx)^2}{e^2}+\frac{g(f+gx)^3}{e}\right)dx}{4g}\\
&= -\frac{b(ef-dg)^3nx}{4e^3}-\frac{b(ef-dg)^2n(f+gx)^2}{8e^2g}-\frac{b(ef-dg)n(f+gx)^3}{12eg}\\
&\quad-\frac{bn(f+gx)^4}{16g}-\frac{b(ef-dg)^4n\log(d+ex)}{4e^4g}+\frac{(f+gx)^4\,(a+b\log\left(c(d+ex)^n\right))}{4g}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.52

$$\int(f+gx)^3\,(a+b\log\left(c(d+ex)^n\right))\,dx$$
$$=\frac{ex(12ae^3(4f^3+6f^2gx+4fg^2x^2+g^3x^3)-bn(-12d^3g^3+6d^2eg^2(8f+gx)-4de^2g(18f^2+6fgx+g^2x^2)\cdots}{}$$

```
[In] Integrate[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (e*x*(12*a*e^3*(4*f^3 + 6*f^2*g*x + 4*f*g^2*x^2 + g^3*x^3) - b*n*(-12*d^3*g
^3 + 6*d^2*e*g^2*(8*f + g*x) - 4*d*e^2*g*(18*f^2 + 6*f*g*x + g^2*x^2) + e^3
*(48*f^3 + 36*f^2*g*x + 16*f*g^2*x^2 + 3*g^3*x^3))) - 12*b*d^2*g*(6*e^2*f^2
 - 4*d*e*f*g + d^2*g^2)*n*Log[d + e*x] + 12*b*e^3*(4*d*f^3 + e*x*(4*f^3 + 6
*f^2*g*x + 4*f*g^2*x^2 + g^3*x^3))*Log[c*(d + e*x)^n])/(48*e^4)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(137) = 274$.

Time = 1.07 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.87

| method | result |
|---|---|
| parallelrisch | $-\frac{-48b\,d^3 e f\,g^2 n+72b\,d^2 e^2 f^2 g n-72x^2 a\,e^4 f^2 g-48x\ln(c(ex+d)^n)b\,e^4 f^3+48xb\,e^4 f^3 n+48\ln(c(ex+d)^n)bd\,e^3 f^3+12\ln(ex+d)b}{}$ |
| risch | Expression too large to display |

```
[In] int((g*x+f)^3*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(-48*b*d^3*e*f*g^2*n+72*b*d^2*e^2*f^2*g*n-72*x^2*a*e^4*f^2*g-48*x*ln(
c*(e*x+d)^n)*b*e^4*f^3+48*x*b*e^4*f^3*n+48*ln(c*(e*x+d)^n)*b*d*e^3*f^3+12*l
n(e*x+d)*b*d^4*g^3*n-12*x^4*ln(c*(e*x+d)^n)*b*e^4*g^3+3*x^4*b*e^4*g^3*n-48*
x^3*a*e^4*f*g^2+48*a*d*f^3*e^3-48*x^3*ln(c*(e*x+d)^n)*b*e^4*f*g^2-4*x^3*b*d
*e^3*g^3*n+16*x^3*b*e^4*f*g^2*n-72*x^2*ln(c*(e*x+d)^n)*b*e^4*f^2*g+6*x^2*b*
d^2*e^2*g^3*n+36*x^2*b*e^4*f^2*g*n-12*x*b*d^3*e*g^3*n-96*ln(e*x+d)*b*d*e^3*
f^3*n-48*b*d*e^3*f^3*n+12*b*d^4*g^3*n-48*ln(e*x+d)*b*d^3*e*f*g^2*n+72*ln(e*
x+d)*b*d^2*e^2*f^2*g*n+48*x*b*d^2*e^2*f*g^2*n-72*x*b*d*e^3*f^2*g*n-24*x^2*b
*d*e^3*f*g^2*n-12*x^4*a*e^4*g^3-48*x*a*e^4*f^3)/e^4
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(137) = 274$.

Time = 0.29 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.28

$$\int (f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = -\frac{3\,(be^4g^3n-4\,ae^4g^3)x^4-4\,(12\,ae^4fg^2-(4\,be^4fg^2-bde^3g^3)n)x^3-6\,(12\,ae^4f^2g-(6\,be^4f^2g-4\,bde^3fg}{}$$

```
[In] integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

```
[Out] -1/48*(3*(b*e^4*g^3*n - 4*a*e^4*g^3)*x^4 - 4*(12*a*e^4*f*g^2 - (4*b*e^4*f*g
^2 - b*d*e^3*g^3)*n)*x^3 - 6*(12*a*e^4*f^2*g - (6*b*e^4*f^2*g - 4*b*d*e^3*f
*g^2 + b*d^2*e^2*g^3)*n)*x^2 - 12*(4*a*e^4*f^3 - (4*b*e^4*f^3 - 6*b*d*e^3*f
^2*g + 4*b*d^2*e^2*f*g^2 - b*d^3*e*g^3)*n)*x - 12*(b*e^4*g^3*n*x^4 + 4*b*e^
```

```
4*f*g^2*n*x^3 + 6*b*e^4*f^2*g*n*x^2 + 4*b*e^4*f^3*n*x + (4*b*d*e^3*f^3 - 6*
b*d^2*e^2*f^2*g + 4*b*d^3*e*f*g^2 - b*d^4*g^3)*n)*log(e*x + d) - 12*(b*e^4*
g^3*x^4 + 4*b*e^4*f*g^2*x^3 + 6*b*e^4*f^2*g*x^2 + 4*b*e^4*f^3*x)*log(c))/e^
4
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(128) = 256$.

Time = 1.16 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.75

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))\,dx = \begin{cases} af^3x + \frac{3af^2gx^2}{2} + afg^2x^3 + \frac{ag^3x^4}{4} - \frac{bd^4g^3\log(c(d+ex)^n)}{4e^4} + \frac{bd^3fg^2\log(c(d+ex)^n)}{e^3} + \frac{bd^3g^3nx}{4e^3} - \frac{3bd^2f^2g\log(c(d+ex)^n)}{2e^2} - \\ (a+b\log(cd^n))\left(f^3x + \frac{3f^2gx^2}{2} + fg^2x^3 + \frac{g^3x^4}{4}\right) \end{cases}$$

```
[In] integrate((g*x+f)**3*(a+b*ln(c*(e*x+d)**n)),x)
```

```
[Out] Piecewise((a*f**3*x + 3*a*f**2*g*x**2/2 + a*f*g**2*x**3 + a*g**3*x**4/4 - b
*d**4*g**3*log(c*(d + e*x)**n)/(4*e**4) + b*d**3*f*g**2*log(c*(d + e*x)**n)
/e**3 + b*d**3*g**3*n*x/(4*e**3) - 3*b*d**2*f**2*g*log(c*(d + e*x)**n)/(2*e
**2) - b*d**2*f*g**2*n*x/e**2 - b*d**2*g**3*n*x**2/(8*e**2) + b*d*f**3*log(
c*(d + e*x)**n)/e + 3*b*d*f**2*g*n*x/(2*e) + b*d*f*g**2*n*x**2/(2*e) + b*d*
g**3*n*x**3/(12*e) - b*f**3*n*x + b*f**3*x*log(c*(d + e*x)**n) - 3*b*f**2*g
*n*x**2/4 + 3*b*f**2*g*x**2*log(c*(d + e*x)**n)/2 - b*f*g**2*n*x**3/3 + b*f
*g**2*x**3*log(c*(d + e*x)**n) - b*g**3*n*x**4/16 + b*g**3*x**4*log(c*(d +
e*x)**n)/4, Ne(e, 0)), ((a + b*log(c*d**n))*(f**3*x + 3*f**2*g*x**2/2 + f*g
**2*x**3 + g**3*x**4/4), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(137) = 274$.

Time = 0.20 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.91

$$\begin{aligned}
&\int (f+gx)^3 (a+b\log(c(d+ex)^n))\,dx \\
&= \frac{1}{4}\,bg^3x^4\log\left((ex+d)^nc\right) + \frac{1}{4}\,ag^3x^4 + bfg^2x^3\log\left((ex+d)^nc\right) \\
&\quad + afg^2x^3 - bef^3n\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) \\
&\quad - \frac{1}{48}\,beg^3n\left(\frac{12\,d^4\log(ex+d)}{e^5} + \frac{3\,e^3x^4 - 4\,de^2x^3 + 6\,d^2ex^2 - 12\,d^3x}{e^4}\right) \\
&\quad + \frac{1}{6}\,befg^2n\left(\frac{6\,d^3\log(ex+d)}{e^4} - \frac{2\,e^2x^3 - 3\,dex^2 + 6\,d^2x}{e^3}\right) \\
&\quad - \frac{3}{4}\,bef^2gn\left(\frac{2\,d^2\log(ex+d)}{e^3} + \frac{ex^2 - 2\,dx}{e^2}\right) \\
&\quad + \frac{3}{2}\,bf^2gx^2\log\left((ex+d)^nc\right) + \frac{3}{2}\,af^2gx^2 + bf^3x\log\left((ex+d)^nc\right) + af^3x
\end{aligned}$$

```
[In] integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*g^3*x^4*log((e*x + d)^n*c) + 1/4*a*g^3*x^4 + b*f*g^2*x^3*log((e*x + d
)^n*c) + a*f*g^2*x^3 - b*e*f^3*n*(x/e - d*log(e*x + d)/e^2) - 1/48*b*e*g^3*
n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e^2*x^3 + 6*d^2*e*x^2 - 12*d^
3*x)/e^4) + 1/6*b*e*f*g^2*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^
2 + 6*d^2*x)/e^3) - 3/4*b*e*f^2*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*
x)/e^2) + 3/2*b*f^2*g*x^2*log((e*x + d)^n*c) + 3/2*a*f^2*g*x^2 + b*f^3*x*lo
g((e*x + d)^n*c) + a*f^3*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(137) = 274$.

Time = 0.32 (sec) , antiderivative size = 770, normalized size of antiderivative = 5.17

$$\begin{aligned}
\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)\,dx &= \frac{(ex+d)bf^3n\log(ex+d)}{e} \\
&+ \frac{3\,(ex+d)^2bf^2gn\log(ex+d)}{2\,e^2} \\
&- \frac{3\,(ex+d)bdf^2gn\log(ex+d)}{e^2} \\
&+ \frac{(ex+d)^3bfg^2n\log(ex+d)}{e^3} \\
&- \frac{3\,(ex+d)^2bdfg^2n\log(ex+d)}{e^3} \\
&+ \frac{3\,(ex+d)bd^2fg^2n\log(ex+d)}{e^3} \\
&+ \frac{(ex+d)^4bg^3n\log(ex+d)}{4\,e^4} \\
&- \frac{(ex+d)^3bdg^3n\log(ex+d)}{e^4} \\
&+ \frac{3\,(ex+d)^2bd^2g^3n\log(ex+d)}{2\,e^4} \\
&- \frac{(ex+d)bd^3g^3n\log(ex+d)}{e^4} - \frac{(ex+d)bf^3n}{e} \\
&- \frac{3\,(ex+d)^2bf^2gn}{4\,e^2} + \frac{3\,(ex+d)bdf^2gn}{e^2} \\
&- \frac{(ex+d)^3bfg^2n}{3\,e^3} + \frac{3\,(ex+d)^2bdfg^2n}{2\,e^3} \\
&- \frac{3\,(ex+d)bd^2fg^2n}{e^3} - \frac{(ex+d)^4bg^3n}{16\,e^4} \\
&+ \frac{(ex+d)^3bdg^3n}{3\,e^4} - \frac{3\,(ex+d)^2bd^2g^3n}{4\,e^4} \\
&+ \frac{(ex+d)bd^3g^3n}{e^4} + \frac{(ex+d)bf^3\log(c)}{e} \\
&+ \frac{3\,(ex+d)^2bf^2g\log(c)}{2\,e^2} - \frac{3\,(ex+d)bdf^2g\log(c)}{e^2} \\
&+ \frac{(ex+d)^3bfg^2\log(c)}{e^3} - \frac{3\,(ex+d)^2bdfg^2\log(c)}{e^3} \\
&+ \frac{3\,(ex+d)bd^2fg^2\log(c)}{e^3} \\
&+ \frac{(ex+d)^4bg^3\log(c)}{4\,e^4} - \frac{(ex+d)^3bdg^3\log(c)}{e^4} \\
&+ \frac{3\,(ex+d)^2bd^2g^3\log(c)}{2\,e^4} - \frac{(ex+d)bd^3g^3\log(c)}{e^4} \\
&+ \frac{(ex+d)af^3}{e} + \frac{3\,(ex+d)^2af^2g}{2\,e^2} \\
&- \frac{3\,(ex+d)adf^2g}{e^2} + \frac{(ex+d)^3afg^2}{e^3} \\
&- \frac{3\,(ex+d)^2adfg^2}{e^3} + \frac{3\,(ex+d)ad^2fg^2}{e^3} \\
&(ex+d)^4ag^3 \quad (ex+d)^3adg^3
\end{aligned}$$

```
[In] integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] (e*x + d)*b*f^3*n*log(e*x + d)/e + 3/2*(e*x + d)^2*b*f^2*g*n*log(e*x + d)/e
^2 - 3*(e*x + d)*b*d*f^2*g*n*log(e*x + d)/e^2 + (e*x + d)^3*b*f*g^2*n*log(e
*x + d)/e^3 - 3*(e*x + d)^2*b*d*f*g^2*n*log(e*x + d)/e^3 + 3*(e*x + d)*b*d^
2*f*g^2*n*log(e*x + d)/e^3 + 1/4*(e*x + d)^4*b*g^3*n*log(e*x + d)/e^4 - (e*
x + d)^3*b*d*g^3*n*log(e*x + d)/e^4 + 3/2*(e*x + d)^2*b*d^2*g^3*n*log(e*x +
 d)/e^4 - (e*x + d)*b*d^3*g^3*n*log(e*x + d)/e^4 - (e*x + d)*b*f^3*n/e - 3/
4*(e*x + d)^2*b*f^2*g*n/e^2 + 3*(e*x + d)*b*d*f^2*g*n/e^2 - 1/3*(e*x + d)^3
*b*f*g^2*n/e^3 + 3/2*(e*x + d)^2*b*d*f*g^2*n/e^3 - 3*(e*x + d)*b*d^2*f*g^2*
n/e^3 - 1/16*(e*x + d)^4*b*g^3*n/e^4 + 1/3*(e*x + d)^3*b*d*g^3*n/e^4 - 3/4*
(e*x + d)^2*b*d^2*g^3*n/e^4 + (e*x + d)*b*d^3*g^3*n/e^4 + (e*x + d)*b*f^3*l
og(c)/e + 3/2*(e*x + d)^2*b*f^2*g*log(c)/e^2 - 3*(e*x + d)*b*d*f^2*g*log(c)
/e^2 + (e*x + d)^3*b*f*g^2*log(c)/e^3 - 3*(e*x + d)^2*b*d*f*g^2*log(c)/e^3
+ 3*(e*x + d)*b*d^2*f*g^2*log(c)/e^3 + 1/4*(e*x + d)^4*b*g^3*log(c)/e^4 - (
e*x + d)^3*b*d*g^3*log(c)/e^4 + 3/2*(e*x + d)^2*b*d^2*g^3*log(c)/e^4 - (e*x
 + d)*b*d^3*g^3*log(c)/e^4 + (e*x + d)*a*f^3/e + 3/2*(e*x + d)^2*a*f^2*g/e^
2 - 3*(e*x + d)*a*d*f^2*g/e^2 + (e*x + d)^3*a*f*g^2/e^3 - 3*(e*x + d)^2*a*d
*f*g^2/e^3 + 3*(e*x + d)*a*d^2*f*g^2/e^3 + 1/4*(e*x + d)^4*a*g^3/e^4 - (e*x
 + d)^3*a*d*g^3/e^4 + 3/2*(e*x + d)^2*a*d^2*g^3/e^4 - (e*x + d)*a*d^3*g^3/e
^4
```

## Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.36

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))\,dx$$

$$= x\left(\frac{4\,a\,e\,f^3+12\,a\,d\,f^2\,g-4\,b\,e\,f^3\,n}{4\,e} + \frac{d\left(\frac{d\left(\frac{g^2\,(a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{d\,g^3\,(4\,a-b\,n)}{4\,e}\right)}{e}-\frac{3\,f\,g\,(2\,a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{2\,e}\right)}{e}\right)$$
$$+ x^3\left(\frac{g^2\,(a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{3\,e}-\frac{d\,g^3\,(4\,a-b\,n)}{12\,e}\right)$$
$$+ \ln\left(c\,(d+e\,x)^n\right)\,\left(b\,f^3\,x+\frac{3\,b\,f^2\,g\,x^2}{2}+b\,f\,g^2\,x^3+\frac{b\,g^3\,x^4}{4}\right)$$
$$- x^2\left(\frac{d\left(\frac{g^2\,(a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{d\,g^3\,(4\,a-b\,n)}{4\,e}\right)}{2\,e}-\frac{3\,f\,g\,(2\,a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{4\,e}\right)$$
$$- \frac{\ln(d+e\,x)\,\left(b\,n\,d^4\,g^3-4\,b\,n\,d^3\,e\,f\,g^2+6\,b\,n\,d^2\,e^2\,f^2\,g-4\,b\,n\,d\,e^3\,f^3\right)}{4\,e^4}$$
$$+ \frac{g^3\,x^4\,(4\,a-b\,n)}{16}$$

```
[In] int((f + g*x)^3*(a + b*log(c*(d + e*x)^n)),x)
```

```
[Out] x*((4*a*e*f^3 + 12*a*d*f^2*g - 4*b*e*f^3*n)/(4*e) + (d*((d*((g^2*(a*d*g + 3
*a*e*f - b*e*f*n))/e - (d*g^3*(4*a - b*n))/(4*e)))/e - (3*f*g*(2*a*d*g + 2*
a*e*f - b*e*f*n))/(2*e)))/e) + x^3*((g^2*(a*d*g + 3*a*e*f - b*e*f*n))/(3*e)
 - (d*g^3*(4*a - b*n))/(12*e)) + log(c*(d + e*x)^n)*((b*g^3*x^4)/4 + b*f^3*
x + (3*b*f^2*g*x^2)/2 + b*f*g^2*x^3) - x^2*((d*((g^2*(a*d*g + 3*a*e*f - b*e
*f*n))/e - (d*g^3*(4*a - b*n))/(4*e)))/(2*e) - (3*f*g*(2*a*d*g + 2*a*e*f -
b*e*f*n))/(4*e)) - (log(d + e*x)*(b*d^4*g^3*n - 4*b*d*e^3*f^3*n - 4*b*d^3*e
*f*g^2*n + 6*b*d^2*e^2*f^2*g*n))/(4*e^4) + (g^3*x^4*(4*a - b*n))/16
```

# 3.37 $\int (f + gx)^2 (a + b \log (c(d + ex)^n)) \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 120

$$\int (f + gx)^2 (a + b \log (c(d + ex)^n)) \, dx = -\frac{b(ef - dg)^2 nx}{3e^2} - \frac{b(ef - dg)n(f + gx)^2}{6eg} - \frac{bn(f + gx)^3}{9g} - \frac{b(ef - dg)^3 n \log(d + ex)}{3e^3 g} + \frac{(f + gx)^3 (a + b \log (c(d + ex)^n))}{3g}$$

```
[Out] -1/3*b*(-d*g+e*f)^2*n*x/e^2-1/6*b*(-d*g+e*f)*n*(g*x+f)^2/e/g-1/9*b*n*(g*x+f
)^3/g-1/3*b*(-d*g+e*f)^3*n*ln(e*x+d)/e^3/g+1/3*(g*x+f)^3*(a+b*ln(c*(e*x+d)^
n))/g
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2442, 45}

$$\int (f + gx)^2 (a + b \log (c(d + ex)^n)) \, dx = \frac{(f + gx)^3 (a + b \log (c(d + ex)^n))}{3g} - \frac{bn(ef - dg)^3 \log(d + ex)}{3e^3 g} - \frac{bnx(ef - dg)^2}{3e^2} - \frac{bn(f + gx)^2 (ef - dg)}{6eg} - \frac{bn(f + gx)^3}{9g}$$

```
[In] Int[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] -1/3*(b*(e*f - d*g)^2*n*x)/e^2 - (b*(e*f - d*g)*n*(f + g*x)^2)/(6*e*g) - (b
*n*(f + g*x)^3)/(9*g) - (b*(e*f - d*g)^3*n*Log[d + e*x])/(3*e^3*g) + ((f +
g*x)^3*(a + b*Log[c*(d + e*x)^n]))/(3*g)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}-\frac{(ben)\int\frac{(f+gx)^3}{d+ex}\,dx}{3g}\\
&= \frac{(f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}\\
&\quad-\frac{(ben)\int\left(\frac{g(ef-dg)^2}{e^3}+\frac{(ef-dg)^3}{e^3(d+ex)}+\frac{g(ef-dg)(f+gx)}{e^2}+\frac{g(f+gx)^2}{e}\right)dx}{3g}\\
&= -\frac{b(ef-dg)^2nx}{3e^2}-\frac{b(ef-dg)n(f+gx)^2}{6eg}-\frac{bn(f+gx)^3}{9g}\\
&\quad-\frac{b(ef-dg)^3n\log(d+ex)}{3e^3g}+\frac{(f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25

$$
\int(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx
$$
$$
=\frac{6bd^2g(-3ef+dg)n\log(d+ex)+e(x(6ae^2(3f^2+3fgx+g^2x^2)-bn(6d^2g^2-3deg(6f+gx)+e^2(18f^2+\cdots}{18e^3}
$$

```
[In] Integrate[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (6*b*d^2*g*(-3*e*f + d*g)*n*Log[d + e*x] + e*(x*(6*a*e^2*(3*f^2 + 3*f*g*x +
 g^2*x^2) - b*n*(6*d^2*g^2 - 3*d*e*g*(6*f + g*x) + e^2*(18*f^2 + 9*f*g*x +
2*g^2*x^2))) + 6*b*e*(3*d*f^2 + e*x*(3*f^2 + 3*f*g*x + g^2*x^2))*Log[c*(d +
 e*x)^n]))/(18*e^3)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(110) = 220$.

Time = 0.85 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.21

| method | result |
|---|---|
| parallelrisch | $\frac{6x^3\ln(c(ex+d)^n)bde^3g^2-2x^3bde^3g^2n+6x^3ade^3g^2+18x^2\ln(c(ex+d)^n)bde^3fg+3x^2bd^2e^2g^2n-9x^2bde^3fgn+6\ln(ex+d)bd^4g^2}{}$ |
| risch | $\frac{a g^2x^3}{3}+a f^2x-\frac{g^2bn x^3}{9}+gaf x^2+g\ln(c) bf x^2-\frac{\ln(ex+d)bf^3n}{3g}-\frac{ig^2\pi b x^3 \operatorname{csgn}(ic(ex+d)^n)^3}{6}+\frac{(gx+f)}{}$ |

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(6*x^3*ln(c*(e*x+d)^n)*b*d*e^3*g^2-2*x^3*b*d*e^3*g^2*n+6*x^3*a*d*e^3*g
^2+18*x^2*ln(c*(e*x+d)^n)*b*d*e^3*f*g+3*x^2*b*d^2*e^2*g^2*n-9*x^2*b*d*e^3*f
*g*n+6*ln(e*x+d)*b*d^4*g^2*n-18*ln(e*x+d)*b*d^3*e*f*g*n+36*ln(e*x+d)*b*d^2*
e^2*f^2*n+18*x^2*a*d*e^3*f*g+18*x*ln(c*(e*x+d)^n)*b*d*e^3*f^2-6*x*b*d^3*e*g
^2*n+18*x*b*d^2*e^2*f*g*n-18*x*b*d*e^3*f^2*n+18*x*a*d*e^3*f^2-18*ln(c*(e*x+
d)^n)*b*d^2*e^2*f^2)/d/e^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(110) = 220$.

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.84

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))\,dx = -\frac{2(be^3g^2n-3ae^3g^2)x^3-3(6ae^3fg-(3be^3fg-bde^2g^2)n)x^2-6(3ae^3f^2-(3be^3f^2-3bde^2fg+bd^2eg}{}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

```
[Out] -1/18*(2*(b*e^3*g^2*n - 3*a*e^3*g^2)*x^3 - 3*(6*a*e^3*f*g - (3*b*e^3*f*g -
b*d*e^2*g^2)*n)*x^2 - 6*(3*a*e^3*f^2 - (3*b*e^3*f^2 - 3*b*d*e^2*f*g + b*d^2
*e*g^2)*n)*x - 6*(b*e^3*g^2*n*x^3 + 3*b*e^3*f*g*n*x^2 + 3*b*e^3*f^2*n*x + (
3*b*d*e^2*f^2 - 3*b*d^2*e*f*g + b*d^3*g^2)*n)*log(e*x + d) - 6*(b*e^3*g^2*x
^3 + 3*b*e^3*f*g*x^2 + 3*b*e^3*f^2*x)*log(c))/e^3
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(102) = 204$.

Time = 0.66 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.10

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))\, dx = \begin{cases} af^2x + afgx^2 + \frac{ag^2x^3}{3} + \frac{bd^3g^2\log (c(d+ex)^n)}{3e^3} - \frac{bd^2fg\log (c(d+ex)^n)}{e^2} - \frac{bd^2g^2nx}{3e^2} + \frac{bdf^2\log (c(d+ex)^n)}{e} + \frac{bdfgnx}{e} + \frac{bdg^2n}{6e} \\ (a+b\log (cd^n)) \left(f^2x + fgx^2 + \frac{g^2x^3}{3}\right) \end{cases}$$

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n)),x)
```

```
[Out] Piecewise((a*f**2*x + a*f*g*x**2 + a*g**2*x**3/3 + b*d**3*g**2*log(c*(d + e
*x)**n)/(3*e**3) - b*d**2*f*g*log(c*(d + e*x)**n)/e**2 - b*d**2*g**2*n*x/(3
*e**2) + b*d*f**2*log(c*(d + e*x)**n)/e + b*d*f*g*n*x/e + b*d*g**2*n*x**2/(
6*e) - b*f**2*n*x + b*f**2*x*log(c*(d + e*x)**n) - b*f*g*n*x**2/2 + b*f*g*x
**2*log(c*(d + e*x)**n) - b*g**2*n*x**3/9 + b*g**2*x**3*log(c*(d + e*x)**n)
/3, Ne(e, 0)), ((a + b*log(c*d**n))*(f**2*x + f*g*x**2 + g**2*x**3/3), True
))
```

## Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.56

$$\begin{aligned} \int (f+gx)^2 (a+b\log (c(d+ex)^n))\, dx &= \frac{1}{3}\, bg^2x^3 \log \left((ex+d)^n c\right) + \frac{1}{3}\, ag^2x^3 - bef^2n\left(\frac{x}{e} - \frac{d\log (ex+d)}{e^2}\right) \\ &+ \frac{1}{18}\, beg^2n\left(\frac{6\, d^3 \log (ex+d)}{e^4} - \frac{2\, e^2x^3 - 3\, dex^2 + 6\, d^2x}{e^3}\right) \\ &- \frac{1}{2}\, befgn\left(\frac{2\, d^2 \log (ex+d)}{e^3} + \frac{ex^2 - 2\, dx}{e^2}\right) \\ &+ bfgx^2 \log \left((ex+d)^n c\right) + afgx^2 + bf^2x \log \left((ex+d)^n c\right) + af^2x \end{aligned}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*g^2*x^3*log((e*x + d)^n*c) + 1/3*a*g^2*x^3 - b*e*f^2*n*(x/e - d*log(e
*x + d)/e^2) + 1/18*b*e*g^2*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*
x^2 + 6*d^2*x)/e^3) - 1/2*b*e*f*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*
x)/e^2) + b*f*g*x^2*log((e*x + d)^n*c) + a*f*g*x^2 + b*f^2*x*log((e*x + d)^
n*c) + a*f^2*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(110) = 220.

Time = 0.31 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.53

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))\,dx = \frac{(ex+d)bf^2n\log(ex+d)}{e} + \frac{(ex+d)^2bfgn\log(ex+d)}{e^2} - \frac{2(ex+d)bdfgn\log(ex+d)}{e^2} + \frac{(ex+d)^3bg^2n\log(ex+d)}{3e^3} - \frac{(ex+d)^2bdg^2n\log(ex+d)}{e^3} + \frac{(ex+d)bd^2g^2n\log(ex+d)}{e^3} - \frac{(ex+d)bf^2n}{e} - \frac{(ex+d)^2bfgn}{2e^2} + \frac{2(ex+d)bdfgn}{e^2} - \frac{(ex+d)^3bg^2n}{9e^3} + \frac{(ex+d)^2bdg^2n}{2e^3} - \frac{(ex+d)bd^2g^2n}{e^3} + \frac{(ex+d)bf^2\log(c)}{e} + \frac{(ex+d)^2bfg\log(c)}{e^2} - \frac{2(ex+d)bdfg\log(c)}{e^2} + \frac{(ex+d)^3bg^2\log(c)}{3e^3} - \frac{(ex+d)^2bdg^2\log(c)}{e^3} + \frac{(ex+d)bd^2g^2\log(c)}{e^3} + \frac{(ex+d)af^2}{e} + \frac{(ex+d)^2afg}{e^2} - \frac{2(ex+d)adfg}{e^2} + \frac{(ex+d)^3ag^2}{3e^3} - \frac{(ex+d)^2adg^2}{e^3} + \frac{(ex+d)ad^2g^2}{e^3}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] (e*x + d)*b*f^2*n*log(e*x + d)/e + (e*x + d)^2*b*f*g*n*log(e*x + d)/e^2 - 2
*(e*x + d)*b*d*f*g*n*log(e*x + d)/e^2 + 1/3*(e*x + d)^3*b*g^2*n*log(e*x + d
)/e^3 - (e*x + d)^2*b*d*g^2*n*log(e*x + d)/e^3 + (e*x + d)*b*d^2*g^2*n*log(
e*x + d)/e^3 - (e*x + d)*b*f^2*n/e - 1/2*(e*x + d)^2*b*f*g*n/e^2 + 2*(e*x +
 d)*b*d*f*g*n/e^2 - 1/9*(e*x + d)^3*b*g^2*n/e^3 + 1/2*(e*x + d)^2*b*d*g^2*n
/e^3 - (e*x + d)*b*d^2*g^2*n/e^3 + (e*x + d)*b*f^2*log(c)/e + (e*x + d)^2*b
*f*g*log(c)/e^2 - 2*(e*x + d)*b*d*f*g*log(c)/e^2 + 1/3*(e*x + d)^3*b*g^2*lo
g(c)/e^3 - (e*x + d)^2*b*d*g^2*log(c)/e^3 + (e*x + d)*b*d^2*g^2*log(c)/e^3
```

```
+ (e*x + d)*a*f^2/e + (e*x + d)^2*a*f*g/e^2 - 2*(e*x + d)*a*d*f*g/e^2 + 1/3
*(e*x + d)^3*a*g^2/e^3 - (e*x + d)^2*a*d*g^2/e^3 + (e*x + d)*a*d^2*g^2/e^3
```

## Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.77

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))\, dx = x^2 \left(\frac{g\,(a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{2\,e}-\frac{d\,g^2\,(3\,a-b\,n)}{6\,e}\right) + x\left(\frac{3\,a\,e\,f^2-3\,b\,e\,f^2\,n+6\,a\,d\,f\,g}{3\,e}-\frac{d\left(\frac{g\,(a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{d\,g^2\,(3\,a-b\,n)}{3\,e}\right)}{e}\right) + \ln\left(c\,(d+e\,x)^n\right)\left(b\,f^2\,x+b\,f\,g\,x^2+\frac{b\,g^2\,x^3}{3}\right)+\frac{g^2\,x^3\,(3\,a-b\,n)}{9} + \frac{\ln\left(d+e\,x\right)\,\left(b\,n\,d^3\,g^2-3\,b\,n\,d^2\,e\,f\,g+3\,b\,n\,d\,e^2\,f^2\right)}{3\,e^3}$$

```
[In] int((f + g*x)^2*(a + b*log(c*(d + e*x)^n)),x)
```

```
[Out] x^2*((g*(a*d*g + 2*a*e*f - b*e*f*n))/(2*e) - (d*g^2*(3*a - b*n))/(6*e)) + x
*((3*a*e*f^2 - 3*b*e*f^2*n + 6*a*d*f*g)/(3*e) - (d*((g*(a*d*g + 2*a*e*f - b
*e*f*n))/e - (d*g^2*(3*a - b*n))/(3*e)))/e) + log(c*(d + e*x)^n)*((b*g^2*x^
3)/3 + b*f^2*x + b*f*g*x^2) + (g^2*x^3*(3*a - b*n))/9 + (log(d + e*x)*(b*d^
3*g^2*n + 3*b*d*e^2*f^2*n - 3*b*d^2*e*f*g*n))/(3*e^3)
```

# 3.38 $\int (f+gx) (a+b\log (c(d+ex)^n)) \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 91

$$\int (f+gx) (a+b\log (c(d+ex)^n)) \, dx = -\frac{b(ef-dg)nx}{2e} - \frac{bn(f+gx)^2}{4g} - \frac{b(ef-dg)^2 n\log(d+ex)}{2e^2g} + \frac{(f+gx)^2 (a+b\log (c(d+ex)^n))}{2g}$$

```
[Out] -1/2*b*(-d*g+e*f)*n*x/e-1/4*b*n*(g*x+f)^2/g-1/2*b*(-d*g+e*f)^2*n*ln(e*x+d)/
e^2/g+1/2*(g*x+f)^2*(a+b*ln(c*(e*x+d)^n))/g
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2442, 45}

$$\int (f+gx) (a+b\log (c(d+ex)^n)) \, dx = \frac{(f+gx)^2 (a+b\log (c(d+ex)^n))}{2g} - \frac{bn(ef-dg)^2 \log(d+ex)}{2e^2g} - \frac{bnx(ef-dg)}{2e} - \frac{bn(f+gx)^2}{4g}$$

```
[In] Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] -1/2*(b*(e*f - d*g)*n*x)/e - (b*n*(f + g*x)^2)/(4*g) - (b*(e*f - d*g)^2*n*L
og[d + e*x])/(2*e^2*g) + ((f + g*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*g)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2g}-\frac{(ben)\int\frac{(f+gx)^2}{d+ex}\,dx}{2g}\\
&= \frac{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2g}-\frac{(ben)\int\left(\frac{g(ef-dg)}{e^2}+\frac{(ef-dg)^2}{e^2(d+ex)}+\frac{g(f+gx)}{e}\right)dx}{2g}\\
&= -\frac{b(ef-dg)nx}{2e}-\frac{bn(f+gx)^2}{4g}-\frac{b(ef-dg)^2n\log(d+ex)}{2e^2g}\\
&\quad+\frac{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2g}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04

$$\begin{aligned}
\int(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)dx &= afx-bfnx+\frac{1}{2}agx^2\\
&\quad-\frac{1}{2}bgn\left(-\frac{dx}{e}+\frac{x^2}{2}+\frac{d^2\log(d+ex)}{e^2}\right)\\
&\quad+\frac{1}{2}bgx^2\log\left(c(d+ex)^n\right)\\
&\quad+\frac{bf(d+ex)\log\left(c(d+ex)^n\right)}{e}
\end{aligned}$$

```
[In] Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] a*f*x - b*f*n*x + (a*g*x^2)/2 - (b*g*n*(-((d*x)/e) + x^2/2 + (d^2*Log[d + e
*x])/e^2))/2 + (b*g*x^2*Log[c*(d + e*x)^n])/2 + (b*f*(d + e*x)*Log[c*(d + e
*x)^n])/e
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

| method | result |
|---|---|
| parts | $a\left(\frac{1}{2}g\,x^{2}+fx\right)+b\Big(f\ln\left(c(ex+d)^{n}\right)x-fnx+\frac{fnd\ln(ex+d)}{e}+\frac{x^{2}g\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)}{2}-\frac{gn\,x^{2}}{4}-\frac{n\,d^{2}g\ln}{2e}$ |
| norman | $\left(-\frac{1}{4}bgn+\frac{1}{2}ag\right)x^{2}+bfx\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)+\frac{(dgbn-2befn+2aef)x}{2e}+\frac{bg\,x^{2}\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)}{2}-\frac{n(b\,d^{2}g-2bde}{2e}$ |
| default | $afx+\frac{ag\,x^{2}}{2}+bf\ln\left(c(ex+d)^{n}\right)x-bfnx+\frac{bfnd\ln(ex+d)}{e}+\frac{bg\,x^{2}\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)}{2}-\frac{bgn\,x^{2}}{4}-\frac{nb\,d^{2}g\ln}{2e^{2}}$ |
| parallelrisch | $-\frac{-2x^{2}\ln(c(ex+d)^{n})b\,e^{2}g+b\,e^{2}gn\,x^{2}+2\ln(ex+d)b\,d^{2}gn-8\ln(ex+d)bdefn-2a\,e^{2}g\,x^{2}-4x\ln(c(ex+d)^{n})b\,e^{2}f-2bdegnx+4b\,e^{2}f}{4e^{2}}$ |
| risch | $\frac{bx(gx+2f)\ln((ex+d)^{n})}{2}-\frac{i\pi bfx\ \mathrm{csgn}(ic)\,\mathrm{csgn}(i(ex+d)^{n})\,\mathrm{csgn}(ic(ex+d)^{n})}{2}-\frac{i\pi bfx\,\mathrm{csgn}(ic(ex+d)^{n})^{3}}{2}-\frac{i\pi bg\,x^{2}\,\mathrm{csgn}(ic)\,c}{}$ |

```
[In] int((g*x+f)*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*g*x^2+f*x)+b*(f*ln(c*(e*x+d)^n)*x-f*n*x+f/e*n*d*ln(e*x+d)+1/2*x^2*g*
ln(c*exp(n*ln(e*x+d)))-1/4*g*n*x^2-1/2*n*d^2*g/e^2*ln(e*x+d)+1/2*d*g*n/e*x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.31

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^{n}\right)\right)\,dx =$$
$$-\frac{(be^{2}gn-2\,ae^{2}g)x^{2}-2\,(2\,ae^{2}f-(2\,be^{2}f-bdeg)n)x-2\,(be^{2}gnx^{2}+2\,be^{2}fnx+(2\,bdef-bd^{2}g)n)\log(e}{4\,e^{2}}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

```
[Out] -1/4*((b*e^2*g*n - 2*a*e^2*g)*x^2 - 2*(2*a*e^2*f - (2*b*e^2*f - b*d*e*g)*n)
*x - 2*(b*e^2*g*n*x^2 + 2*b*e^2*f*n*x + (2*b*d*e*f - b*d^2*g)*n)*log(e*x +
d) - 2*(b*e^2*g*x^2 + 2*b*e^2*f*x)*log(c))/e^2
```

## Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^{n}\right)\right)\,dx$$
$$=\begin{cases} afx+\frac{agx^{2}}{2}-\frac{bd^{2}g\log\left(c(d+ex)^{n}\right)}{2e^{2}}+\frac{bdf\log\left(c(d+ex)^{n}\right)}{e}+\frac{bdgnx}{2e}-bfnx+bfx\log\left(c(d+ex)^{n}\right)-\frac{bgnx^{2}}{4}+\frac{bgx^{2}\log(c(d}{2} \\ (a+b\log\left(cd^{n}\right))\left(fx+\frac{gx^{2}}{2}\right) \end{cases}$$

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n)),x)
```

```
[Out] Piecewise((a*f*x + a*g*x**2/2 - b*d**2*g*log(c*(d + e*x)**n)/(2*e**2) + b*d
*f*log(c*(d + e*x)**n)/e + b*d*g*n*x/(2*e) - b*f*n*x + b*f*x*log(c*(d + e*x
)**n) - b*g*n*x**2/4 + b*g*x**2*log(c*(d + e*x)**n)/2, Ne(e, 0)), ((a + b*l
og(c*d**n))*(f*x + g*x**2/2), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = -befn\left(\frac{x}{e}-\frac{d\log\left(ex+d\right)}{e^2}\right) - \frac{1}{4}\,begn\left(\frac{2\,d^2\log\left(ex+d\right)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right) + \frac{1}{2}\,bgx^2\log\left((ex+d)^nc\right)+\frac{1}{2}\,agx^2 + bfx\log\left((ex+d)^nc\right)+afx$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] -b*e*f*n*(x/e - d*log(e*x + d)/e^2) - 1/4*b*e*g*n*(2*d^2*log(e*x + d)/e^3 +
 (e*x^2 - 2*d*x)/e^2) + 1/2*b*g*x^2*log((e*x + d)^n*c) + 1/2*a*g*x^2 + b*f*
x*log((e*x + d)^n*c) + a*f*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(83) = 166$.

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.01

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = \frac{(ex+d)bfn\log\left(ex+d\right)}{e}+\frac{(ex+d)^2bgn\log\left(ex+d\right)}{2\,e^2} - \frac{(ex+d)bdgn\log\left(ex+d\right)}{e^2}-\frac{(ex+d)bfn}{e} - \frac{(ex+d)^2bgn}{4\,e^2}+\frac{(ex+d)bdgn}{e^2}+\frac{(ex+d)bf\log\left(c\right)}{e} + \frac{(ex+d)^2bg\log\left(c\right)}{2\,e^2}-\frac{(ex+d)bdg\log\left(c\right)}{e^2} + \frac{(ex+d)af}{e}+\frac{(ex+d)^2ag}{2\,e^2}-\frac{(ex+d)adg}{e^2}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] (e*x + d)*b*f*n*log(e*x + d)/e + 1/2*(e*x + d)^2*b*g*n*log(e*x + d)/e^2 - (
e*x + d)*b*d*g*n*log(e*x + d)/e^2 - (e*x + d)*b*f*n/e - 1/4*(e*x + d)^2*b*g
*n/e^2 + (e*x + d)*b*d*g*n/e^2 + (e*x + d)*b*f*log(c)/e + 1/2*(e*x + d)^2*b
*g*log(c)/e^2 - (e*x + d)*b*d*g*log(c)/e^2 + (e*x + d)*a*f/e + 1/2*(e*x + d
)^2*a*g/e^2 - (e*x + d)*a*d*g/e^2
```

## Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = x\left(\frac{2\,a\,d\,g+2\,a\,e\,f-2\,b\,e\,f\,n}{2\,e}-\frac{d\,g\,(2\,a-b\,n)}{2\,e}\right) + \ln\left(c\,(d+e\,x)^n\right)\,\left(\frac{b\,g\,x^2}{2}+b\,f\,x\right) - \frac{\ln\left(d+e\,x\right)\,\left(b\,d^2\,g\,n-2\,b\,d\,e\,f\,n\right)}{2\,e^2} + \frac{g\,x^2\,(2\,a-b\,n)}{4}$$

```
[In] int((f + g*x)*(a + b*log(c*(d + e*x)^n)),x)
```

```
[Out] x*((2*a*d*g + 2*a*e*f - 2*b*e*f*n)/(2*e) - (d*g*(2*a - b*n))/(2*e)) + log(c
*(d + e*x)^n)*(b*f*x + (b*g*x^2)/2) - (log(d + e*x)*(b*d^2*g*n - 2*b*d*e*f*
n))/(2*e^2) + (g*x^2*(2*a - b*n))/4
```

# 3.39 $\int (a+b\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 14, antiderivative size = 29

$$\int (a+b\log(c(d+ex)^n))\,dx = ax-bnx+\frac{b(d+ex)\log(c(d+ex)^n)}{e}$$

```
[Out] a*x-b*n*x+b*(e*x+d)*ln(c*(e*x+d)^n)/e
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2436, 2332}

$$\int (a+b\log(c(d+ex)^n))\,dx = ax+\frac{b(d+ex)\log(c(d+ex)^n)}{e}-bnx$$

```
[In] Int[a + b*Log[c*(d + e*x)^n],x]
```

```
[Out] a*x - b*n*x + (b*(d + e*x)*Log[c*(d + e*x)^n])/e
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = ax + b\int \log\left(c(d+ex)^n\right)\,dx$$

$$= ax + \frac{b\text{Subst}\left(\int \log\left(cx^n\right)\,dx, x, d+ex\right)}{e}$$

$$= ax - bnx + \frac{b(d+ex)\log\left(c(d+ex)^n\right)}{e}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = ax - bnx + \frac{b(d+ex)\log\left(c(d+ex)^n\right)}{e}$$

```
[In] Integrate[a + b*Log[c*(d + e*x)^n],x]
[Out] a*x - b*n*x + (b*(d + e*x)*Log[c*(d + e*x)^n])/e
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

| method | result |
|---|---|
| default | $ax+b\ln\left(c(ex+d)^n\right)x-bnx+\frac{bnd\ln(ex+d)}{e}$ |
| parts | $ax+b\ln\left(c(ex+d)^n\right)x-bnx+\frac{bnd\ln(ex+d)}{e}$ |
| norman | $(-bn+a)\,x+bx\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)+\frac{bnd\ln(ex+d)}{e}$ |
| parallelrisch | $\frac{b\left(x\ln\left(c(ex+d)^n\right)den-xde\,n^2+\ln\left(c(ex+d)^n\right)d^2n\right)}{den}+ax$ |
| risch | $ax+bx\ln\left((ex+d)^n\right)-\frac{ib\pi x\ \mathrm{csgn}(ic)\,\mathrm{csgn}\left(i(ex+d)^n\right)\mathrm{csgn}\left(ic(ex+d)^n\right)}{2}+\frac{ib\pi x\ \mathrm{csgn}(ic)\,\mathrm{csgn}\left(ic(ex+d)^n\right)^2}{2}+\frac{ib\pi x}{}$ |

```
[In] int(a+b*ln(c*(e*x+d)^n),x,method=_RETURNVERBOSE)
[Out] a*x+b*ln(c*(e*x+d)^n)*x-b*n*x+b/e*n*d*ln(e*x+d)
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int (a+b\log(c(d+ex)^n))\,dx = \frac{bex\log(c) - (ben-ae)x + (benx+bdn)\log(ex+d)}{e}$$

```
[In] integrate(a+b*log(c*(e*x+d)^n),x, algorithm="fricas")
[Out] (b*e*x*log(c) - (b*e*n - a*e)*x + (b*e*n*x + b*d*n)*log(e*x + d))/e
```

## Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int (a+b\log(c(d+ex)^n))\,dx = ax+b\left(\begin{cases} \frac{d\log(c(d+ex)^n)}{e} - nx + x\log(c(d+ex)^n) & \text{for } e\neq 0 \\ x\log(cd^n) & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(a+b*ln(c*(e*x+d)**n),x)
[Out] a*x + b*Piecewise((d*log(c*(d + e*x)**n)/e - n*x + x*log(c*(d + e*x)**n), N
e(e, 0)), (x*log(c*d**n), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int (a+b\log(c(d+ex)^n))\,dx = -ben\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) + bx\log((ex+d)^n c) + ax$$

```
[In] integrate(a+b*log(c*(e*x+d)^n),x, algorithm="maxima")
[Out] -b*e*n*(x/e - d*log(e*x + d)/e^2) + b*x*log((e*x + d)^n*c) + a*x
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int (a+b\log (c(d+ex)^n))\ dx=\left(\frac{(ex+d)n\log (ex+d)}{e}-\frac{(ex+d)n}{e}+\frac{(ex+d)\log (c)}{e}\right)b+ax$$

```
[In] integrate(a+b*log(c*(e*x+d)^n),x, algorithm="giac")
[Out] ((e*x + d)*n*log(e*x + d)/e - (e*x + d)*n/e + (e*x + d)*log(c)/e)*b + a*x
```

## Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int (a+b\log (c(d+ex)^n))\ dx=x\,(a-b\,n)+b\,x\,\ln \left(c\,{(d+e\,x)}^n\right)+\frac{b\,d\,n\,\ln (d+e\,x)}{e}$$

```
[In] int(a + b*log(c*(d + e*x)^n),x)
[Out] x*(a - b*n) + b*x*log(c*(d + e*x)^n) + (b*d*n*log(d + e*x))/e
```

# 3.40 $\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[Out] (a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g+b*n*polylog(2,-g*(e*x+d)/(
-d*g+e*f))/g
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2441, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g + (b*n*PolyLo
g[2, -((g*(d + e*x))/(e*f - d*g))])/g
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}-\frac{(ben)\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}-\frac{(bn)\text{Subst}\left(\int\frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{bn\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int\frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx=\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,\frac{g(d+ex)}{-ef+dg}\right)}{g}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x),x]
[Out] ((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g + (b*n*PolyLo
g[2, (g*(d + e*x))/(-(e*f) + d*g)])/g
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.70 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.44

| method | result |
|---|---|
| risch | $\frac{b\ln\left((ex+d)^{n}\right)\ln(gx+f)}{g}-\frac{bn\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g}-\frac{bn\ln(gx+f)\ln\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g}+\frac{\left(-\frac{ib\pi\ \mathrm{csgn}\left(ic(ex+d)^{n}\right)\mathrm{csgn}(ic)\,\mathrm{csgn}}{2}\right.}{}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] b*ln((e*x+d)^n)*ln(g*x+f)/g-b/g*n*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b/g*
n*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I
*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+
1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x
+d)^n)^3+b*ln(c)+a)*ln(g*x+f)/g
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{f+gx}\,dx=\int \frac{b\log\left((ex+d)^{n}c\right)+a}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{f+gx}\,dx=\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{f+gx}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))/(f + g*x), x)
```

## Maxima [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{f + gx}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{gx + f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
[Out] b*integrate((log((e*x + d)^n) + log(c))/(g*x + f), x) + a*log(g*x + f)/g
```

## Giac [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{f + gx}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{gx + f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{f + gx}\,dx = \int \frac{a + b\,\ln\left(c\,{\left(d + e\,x\right)}^n\right)}{f + g\,x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(f + g*x), x)
```

# 3.41 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = \frac{ben\log(d+ex)}{g(ef-dg)} - \frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)} - \frac{ben\log(f+gx)}{g(ef-dg)}$$

```
[Out] b*e*n*ln(e*x+d)/g/(-d*g+e*f)+(-a-b*ln(c*(e*x+d)^n))/g/(g*x+f)-b*e*n*ln(g*x+
f)/g/(-d*g+e*f)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2442, 36, 31}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = -\frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)} + \frac{ben\log(d+ex)}{g(ef-dg)} - \frac{ben\log(f+gx)}{g(ef-dg)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^2,x]
```

```
[Out] (b*e*n*Log[d + e*x])/(g*(e*f - d*g)) - (a + b*Log[c*(d + e*x)^n])/(g*(f + g
*x)) - (b*e*n*Log[f + g*x])/(g*(e*f - d*g))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)}+\frac{(ben)\int\frac{1}{(d+ex)(f+gx)}\,dx}{g}\\ &= -\frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)}-\frac{(ben)\int\frac{1}{f+gx}\,dx}{ef-dg}+\frac{(be^2n)\int\frac{1}{d+ex}\,dx}{g(ef-dg)}\\ &= \frac{ben\log(d+ex)}{g(ef-dg)}-\frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)}-\frac{ben\log(f+gx)}{g(ef-dg)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int\frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx=\frac{-\frac{a+b\log(c(d+ex)^n)}{f+gx}+\frac{ben(\log(d+ex)-\log(f+gx))}{ef-dg}}{g}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^2,x]
```

```
[Out] (-((a + b*Log[c*(d + e*x)^n])/(f + g*x)) + (b*e*n*(Log[d + e*x] - Log[f + g
*x]))/(e*f - d*g))/g
```

## Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

| method | result |
|---|---|
| parallelrisch | $-\frac{\ln(ex+d)xb\,e^2gn-\ln(gx+f)xb\,e^2gn+\ln(ex+d)b\,e^2fn-\ln(gx+f)b\,e^2fn+\ln(c(ex+d)^n)bdeg-\ln(c(ex+d)^n)b\,e^2f+adeg-a\,e^2f}{(dg-ef)(gx+f)eg}$ |
| risch | $-\frac{b\ln((ex+d)^n)}{g(gx+f)}-\frac{i\pi\ \mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)^2bdg+i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic(ex+d)^n)^2bdg+i\pi bef\,\mathrm{csgn}(ic(ex+d)^n)^3-i\pi\ c}{}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(ln(e*x+d)*x*b*e^2*g*n-ln(g*x+f)*x*b*e^2*g*n+ln(e*x+d)*b*e^2*f*n-ln(g*x+f)
*b*e^2*f*n+ln(c*(e*x+d)^n)*b*d*e*g-ln(c*(e*x+d)^n)*b*e^2*f+a*d*e*g-a*e^2*f)
/(d*g-e*f)/(g*x+f)/e/g
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = -\frac{aef-adg-(begnx+bdgn)\log(ex+d)+(begnx+befn)\log(gx+f)+(bef-bdg)\log(c)}{ef^2g-dfg^2+(efg^2-dg^3)x}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] -(a*e*f - a*d*g - (b*e*g*n*x + b*d*g*n)*log(e*x + d) + (b*e*g*n*x + b*e*f*n
)*log(g*x + f) + (b*e*f - b*d*g)*log(c))/(e*f^2*g - d*f*g^2 + (e*f*g^2 - d*
g^3)*x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(61) = 122.

Time = 3.12 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.50

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = \begin{cases} \frac{ax+\frac{bd\log\left(c(d+ex)^n\right)}{e}-bnx+bx\log\left(c(d+ex)^n\right)}{f^2} \\ -\frac{a}{fg+g^2x}-\frac{bn}{fg+g^2x}-\frac{b\log\left(c\left(\frac{ef}{g}+ex\right)^n\right)}{fg+g^2x} \\ -\frac{adg}{dfg^2+dg^3x-ef^2g-efg^2x}+\frac{aef}{dfg^2+dg^3x-ef^2g-efg^2x}-\frac{bdg\log\left(c(d+ex)^n\right)}{dfg^2+dg^3x-ef^2g-efg^2x}+\frac{befn\log\left(\frac{f}{g}+x\right)}{dfg^2+dg^3x-ef^2g-efg^2x}+\frac{begnx\log\left(\frac{f}{g}\right.}{dfg^2+dg^3x-ef^2} \end{cases}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**2,x)
```

```
[Out] Piecewise(((a*x + b*d*log(c*(d + e*x)**n)/e - b*n*x + b*x*log(c*(d + e*x)**
n))/f**2, Eq(g, 0)), (-a/(f*g + g**2*x) - b*n/(f*g + g**2*x) - b*log(c*(e*f
/g + e*x)**n)/(f*g + g**2*x), Eq(d, e*f/g)), (-a*d*g/(d*f*g**2 + d*g**3*x -
 e*f**2*g - e*f*g**2*x) + a*e*f/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*
x) - b*d*g*log(c*(d + e*x)**n)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x
) + b*e*f*n*log(f/g + x)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x) + b*
e*g*n*x*log(f/g + x)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x) - b*e*g*
x*log(c*(d + e*x)**n)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = ben\left(\frac{\log\left(ex+d\right)}{efg-dg^2}-\frac{\log\left(gx+f\right)}{efg-dg^2}\right) - \frac{b\log\left((ex+d)^nc\right)}{g^2x+fg}-\frac{a}{g^2x+fg}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] b*e*n*(log(e*x + d)/(e*f*g - d*g^2) - log(g*x + f)/(e*f*g - d*g^2)) - b*log
((e*x + d)^n*c)/(g^2*x + f*g) - a/(g^2*x + f*g)
```

## Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = \frac{ben\log\left(ex+d\right)}{efg-dg^2}-\frac{ben\log\left(gx+f\right)}{efg-dg^2} - \frac{bn\log\left(ex+d\right)}{g^2x+fg}-\frac{b\log\left(c\right)+a}{g^2x+fg}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] b*e*n*log(e*x + d)/(e*f*g - d*g^2) - b*e*n*log(g*x + f)/(e*f*g - d*g^2) - b
*n*log(e*x + d)/(g^2*x + f*g) - (b*log(c) + a)/(g^2*x + f*g)
```

## Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = -\frac{a}{x\,g^2+f\,g}-\frac{b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{g\,\left(f+g\,x\right)}+\frac{b\,e\,n\,\mathrm{atan}\left(\frac{e\,f\,2{}\mathrm{i}+e\,g\,x\,2{}\mathrm{i}}{d\,g-e\,f}+1{}\mathrm{i}\right)\,2{}\mathrm{i}}{g\,\left(d\,g-e\,f\right)}$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^2,x)
```

```
[Out] (b*e*n*atan((e*f*2i + e*g*x*2i)/(d*g - e*f) + 1i)*2i)/(g*(d*g - e*f)) - (b*
log(c*(d + e*x)^n))/(g*(f + g*x)) - a/(f*g + g^2*x)
```

# 3.42 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^3}\,dx=\frac{ben}{2g(ef-dg)(f+gx)}+\frac{be^2n\log(d+ex)}{2g(ef-dg)^2}-\frac{a+b\log\left(c(d+ex)^n\right)}{2g(f+gx)^2}-\frac{be^2n\log(f+gx)}{2g(ef-dg)^2}$$

```
[Out] 1/2*b*e*n/g/(-d*g+e*f)/(g*x+f)+1/2*b*e^2*n*ln(e*x+d)/g/(-d*g+e*f)^2+1/2*(-a
-b*ln(c*(e*x+d)^n))/g/(g*x+f)^2-1/2*b*e^2*n*ln(g*x+f)/g/(-d*g+e*f)^2
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2442, 46}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^3}\,dx=-\frac{a+b\log\left(c(d+ex)^n\right)}{2g(f+gx)^2}+\frac{be^2n\log(d+ex)}{2g(ef-dg)^2}-\frac{be^2n\log(f+gx)}{2g(ef-dg)^2}+\frac{ben}{2g(f+gx)(ef-dg)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^3,x]
```

```
[Out] (b*e*n)/(2*g*(e*f - d*g)*(f + g*x)) + (b*e^2*n*Log[d + e*x])/(2*g*(e*f - d*
g)^2) - (a + b*Log[c*(d + e*x)^n])/(2*g*(f + g*x)^2) - (b*e^2*n*Log[f + g*x
])/(2*g*(e*f - d*g)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\log\left(c(d+ex)^n\right)}{2g(f+gx)^2}+\frac{(ben)\int\frac{1}{(d+ex)(f+gx)^2}\,dx}{2g}\\ &= -\frac{a+b\log\left(c(d+ex)^n\right)}{2g(f+gx)^2}+\frac{(ben)\int\left(\frac{e^2}{(ef-dg)^2(d+ex)}-\frac{g}{(ef-dg)(f+gx)^2}-\frac{eg}{(ef-dg)^2(f+gx)}\right)dx}{2g}\\ &= \frac{ben}{2g(ef-dg)(f+gx)}+\frac{be^2n\log(d+ex)}{2g(ef-dg)^2}-\frac{a+b\log\left(c(d+ex)^n\right)}{2g(f+gx)^2}-\frac{be^2n\log(f+gx)}{2g(ef-dg)^2}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

$$\int\frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^3}\,dx = -\frac{a+b\log\left(c(d+ex)^n\right)-\frac{ben(f+gx)(ef-dg+e(f+gx)\log(d+ex)-e(f+gx)\log(f+gx))}{(ef-dg)^2}}{2g(f+gx)^2}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^3,x]
```

```
[Out] -1/2*(a + b*Log[c*(d + e*x)^n] - (b*e*n*(f + g*x)*(e*f - d*g + e*(f + g*x)*
Log[d + e*x] - e*(f + g*x)*Log[f + g*x]))/(e*f - d*g)^2)/(g*(f + g*x)^2)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(107) = 214$.

Time = 1.08 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.53

| method | result |
|---|---|
| parallelrisch | $\frac{-xbd\,e^{2}g^{3}n+xb\,e^{3}f\,g^{2}n+b\,e^{3}f^{2}gn+2ad\,e^{2}f\,g^{2}-\ln(c(ex+d)^{n})b\,d^{2}e\,g^{3}-\ln(c(ex+d)^{n})b\,e^{3}f^{2}g-bd\,e^{2}f\,g^{2}n+\ln(ex+d)b\,e^{3}f^{2}gn-}{2(g}$ |
| risch | $-\frac{b\ln((ex+d)^{n})}{2g(gx+f)^{2}}-\frac{2\ln(gx+f)b\,e^{2}f^{2}n-2\ln(-ex-d)b\,e^{2}f^{2}n+2a\,e^{2}f^{2}+2\ln(gx+f)b\,e^{2}g^{2}n\,x^{2}-2\ln(-ex-d)b\,e^{2}g^{2}n\,x^{2}-4\ln(}{}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-x*b*d*e^2*g^3*n+x*b*e^3*f*g^2*n+b*e^3*f^2*g*n+2*a*d*e^2*f*g^2-ln(c*(e
*x+d)^n)*b*d^2*e*g^3-ln(c*(e*x+d)^n)*b*e^3*f^2*g-b*d*e^2*f*g^2*n+ln(e*x+d)*
b*e^3*f^2*g*n-ln(g*x+f)*b*e^3*f^2*g*n+2*ln(c*(e*x+d)^n)*b*d*e^2*f*g^2+ln(e*
x+d)*x^2*b*e^3*g^3*n-ln(g*x+f)*x^2*b*e^3*g^3*n+2*ln(e*x+d)*x*b*e^3*f*g^2*n-
2*ln(g*x+f)*x*b*e^3*f*g^2*n-a*d^2*e*g^3-a*e^3*f^2*g)/(d^2*g^2-2*d*e*f*g+e^2
*f^2)/(g*x+f)^2/e/g^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(104) = 208$.

Time = 0.30 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.45

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{(f+gx)^{3}}\,dx = -\frac{ae^{2}f^{2}-2\,adefg+ad^{2}g^{2}-(be^{2}fg-bdeg^{2})nx-(be^{2}f^{2}-bdefg)n-(be^{2}g^{2}nx^{2}+2\,be^{2}fgnx+(2\,bdef}{2\left(e^{2}f^{4}g-2\,def^{3}g^{2}+d^{2}f^{2}g^{3}+(e^{2}f^{2}g^{3}-2\right.}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*e^2*f^2 - 2*a*d*e*f*g + a*d^2*g^2 - (b*e^2*f*g - b*d*e*g^2)*n*x - (
b*e^2*f^2 - b*d*e*f*g)*n - (b*e^2*g^2*n*x^2 + 2*b*e^2*f*g*n*x + (2*b*d*e*f*
g - b*d^2*g^2)*n)*log(e*x + d) + (b*e^2*g^2*n*x^2 + 2*b*e^2*f*g*n*x + b*e^2
*f^2*n)*log(g*x + f) + (b*e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*log(c))/(e^2*f
^4*g - 2*d*e*f^3*g^2 + d^2*f^2*g^3 + (e^2*f^2*g^3 - 2*d*e*f*g^4 + d^2*g^5)*
x^2 + 2*(e^2*f^3*g^2 - 2*d*e*f^2*g^3 + d^2*f*g^4)*x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1945 vs. 2(97) = 194.

Time = 13.16 (sec) , antiderivative size = 1945, normalized size of antiderivative = 17.37

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{(f + gx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**3,x)
```

```
[Out] Piecewise(((a*x + b*d*log(c*(d + e*x)**n)/e - b*n*x + b*x*log(c*(d + e*x)**
n))/f**3, Eq(g, 0)), (-2*a/(4*f**2*g + 8*f*g**2*x + 4*g**3*x**2) - b*n/(4*f
**2*g + 8*f*g**2*x + 4*g**3*x**2) - 2*b*log(c*(e*f/g + e*x)**n)/(4*f**2*g +
 8*f*g**2*x + 4*g**3*x**2), Eq(d, e*f/g)), (-a*d**2*g**2/(2*d**2*f**2*g**3
+ 4*d**2*f*g**4*x + 2*d**2*g**5*x**2 - 4*d*e*f**3*g**2 - 8*d*e*f**2*g**3*x
- 4*d*e*f*g**4*x**2 + 2*e**2*f**4*g + 4*e**2*f**3*g**2*x + 2*e**2*f**2*g**3
*x**2) + 2*a*d*e*f*g/(2*d**2*f**2*g**3 + 4*d**2*f*g**4*x + 2*d**2*g**5*x**2
 - 4*d*e*f**3*g**2 - 8*d*e*f**2*g**3*x - 4*d*e*f*g**4*x**2 + 2*e**2*f**4*g
+ 4*e**2*f**3*g**2*x + 2*e**2*f**2*g**3*x**2) - a*e**2*f**2/(2*d**2*f**2*g*
*3 + 4*d**2*f*g**4*x + 2*d**2*g**5*x**2 - 4*d*e*f**3*g**2 - 8*d*e*f**2*g**3
*x - 4*d*e*f*g**4*x**2 + 2*e**2*f**4*g + 4*e**2*f**3*g**2*x + 2*e**2*f**2*g
**3*x**2) - b*d**2*g**2*log(c*(d + e*x)**n)/(2*d**2*f**2*g**3 + 4*d**2*f*g*
*4*x + 2*d**2*g**5*x**2 - 4*d*e*f**3*g**2 - 8*d*e*f**2*g**3*x - 4*d*e*f*g**
4*x**2 + 2*e**2*f**4*g + 4*e**2*f**3*g**2*x + 2*e**2*f**2*g**3*x**2) - b*d*
e*f*g*n/(2*d**2*f**2*g**3 + 4*d**2*f*g**4*x + 2*d**2*g**5*x**2 - 4*d*e*f**3
*g**2 - 8*d*e*f**2*g**3*x - 4*d*e*f*g**4*x**2 + 2*e**2*f**4*g + 4*e**2*f**3
*g**2*x + 2*e**2*f**2*g**3*x**2) + 2*b*d*e*f*g*log(c*(d + e*x)**n)/(2*d**2*
f**2*g**3 + 4*d**2*f*g**4*x + 2*d**2*g**5*x**2 - 4*d*e*f**3*g**2 - 8*d*e*f*
*2*g**3*x - 4*d*e*f*g**4*x**2 + 2*e**2*f**4*g + 4*e**2*f**3*g**2*x + 2*e**2
*f**2*g**3*x**2) - b*d*e*g**2*n*x/(2*d**2*f**2*g**3 + 4*d**2*f*g**4*x + 2*d
**2*g**5*x**2 - 4*d*e*f**3*g**2 - 8*d*e*f**2*g**3*x - 4*d*e*f*g**4*x**2 + 2
*e**2*f**4*g + 4*e**2*f**3*g**2*x + 2*e**2*f**2*g**3*x**2) - b*e**2*f**2*n*
log(f/g + x)/(2*d**2*f**2*g**3 + 4*d**2*f*g**4*x + 2*d**2*g**5*x**2 - 4*d*e
*f**3*g**2 - 8*d*e*f**2*g**3*x - 4*d*e*f*g**4*x**2 + 2*e**2*f**4*g + 4*e**2
*f**3*g**2*x + 2*e**2*f**2*g**3*x**2) + b*e**2*f**2*n/(2*d**2*f**2*g**3 + 4
*d**2*f*g**4*x + 2*d**2*g**5*x**2 - 4*d*e*f**3*g**2 - 8*d*e*f**2*g**3*x - 4
*d*e*f*g**4*x**2 + 2*e**2*f**4*g + 4*e**2*f**3*g**2*x + 2*e**2*f**2*g**3*x*
*2) - 2*b*e**2*f*g*n*x*log(f/g + x)/(2*d**2*f**2*g**3 + 4*d**2*f*g**4*x + 2
*d**2*g**5*x**2 - 4*d*e*f**3*g**2 - 8*d*e*f**2*g**3*x - 4*d*e*f*g**4*x**2 +
 2*e**2*f**4*g + 4*e**2*f**3*g**2*x + 2*e**2*f**2*g**3*x**2) + b*e**2*f*g*n
*x/(2*d**2*f**2*g**3 + 4*d**2*f*g**4*x + 2*d**2*g**5*x**2 - 4*d*e*f**3*g**2
 - 8*d*e*f**2*g**3*x - 4*d*e*f*g**4*x**2 + 2*e**2*f**4*g + 4*e**2*f**3*g**2
*x + 2*e**2*f**2*g**3*x**2) + 2*b*e**2*f*g*x*log(c*(d + e*x)**n)/(2*d**2*f*
*2*g**3 + 4*d**2*f*g**4*x + 2*d**2*g**5*x**2 - 4*d*e*f**3*g**2 - 8*d*e*f**2
*g**3*x - 4*d*e*f*g**4*x**2 + 2*e**2*f**4*g + 4*e**2*f**3*g**2*x + 2*e**2*f
```

```
**2*g**3*x**2) - b*e**2*g**2*n*x**2*log(f/g + x)/(2*d**2*f**2*g**3 + 4*d**2
*f*g**4*x + 2*d**2*g**5*x**2 - 4*d*e*f**3*g**2 - 8*d*e*f**2*g**3*x - 4*d*e*
f*g**4*x**2 + 2*e**2*f**4*g + 4*e**2*f**3*g**2*x + 2*e**2*f**2*g**3*x**2) +
 b*e**2*g**2*x**2*log(c*(d + e*x)**n)/(2*d**2*f**2*g**3 + 4*d**2*f*g**4*x +
 2*d**2*g**5*x**2 - 4*d*e*f**3*g**2 - 8*d*e*f**2*g**3*x - 4*d*e*f*g**4*x**2
 + 2*e**2*f**4*g + 4*e**2*f**3*g**2*x + 2*e**2*f**2*g**3*x**2), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.49

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^3}\,dx = \frac{1}{2}\,ben\left(\frac{e\log\left(ex+d\right)}{e^2f^2g-2\,defg^2+d^2g^3}-\frac{e\log\left(gx+f\right)}{e^2f^2g-2\,defg^2+d^2g^3}+\frac{1}{ef^2g-dfg^2+(efg^2-dg^3)x}\right) - \frac{b\log\left((ex+d)^nc\right)}{2\left(g^3x^2+2\,fg^2x+f^2g\right)}-\frac{a}{2\left(g^3x^2+2\,fg^2x+f^2g\right)}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*e*n*(e*log(e*x + d)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) - e*log(g*x +
 f)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) + 1/(e*f^2*g - d*f*g^2 + (e*f*g^2 -
 d*g^3)*x)) - 1/2*b*log((e*x + d)^n*c)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1/2*
a/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

## Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.79

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^3}\,dx = \frac{be^2n\log\left(ex+d\right)}{2\left(e^2f^2g-2\,defg^2+d^2g^3\right)} - \frac{be^2n\log\left(gx+f\right)}{2\left(e^2f^2g-2\,defg^2+d^2g^3\right)}-\frac{bn\log\left(ex+d\right)}{2\left(g^3x^2+2\,fg^2x+f^2g\right)} + \frac{begnx+befn-bef\log\left(c\right)+bdg\log\left(c\right)-aef+adg}{2\left(efg^3x^2-dg^4x^2+2\,ef^2g^2x-2\,dfg^3x+ef^3g-df^2g^2\right)}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] 1/2*b*e^2*n*log(e*x + d)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) - 1/2*b*e^2*n*
log(g*x + f)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) - 1/2*b*n*log(e*x + d)/(g^
3*x^2 + 2*f*g^2*x + f^2*g) + 1/2*(b*e*g*n*x + b*e*f*n - b*e*f*log(c) + b*d*
g*log(c) - a*e*f + a*d*g)/(e*f*g^3*x^2 - d*g^4*x^2 + 2*e*f^2*g^2*x - 2*d*f*
g^3*x + e*f^3*g - d*f^2*g^2)
```

## Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.54

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^3}\,dx = \frac{b\,e^2\,n\,\mathrm{atanh}\left(\frac{2\,d^2\,g^3-2\,e^2\,f^2\,g}{2\,g\,{\left(d\,g-e\,f\right)}^2}+\frac{2\,e\,g\,x}{d\,g-e\,f}\right)}{g\,{\left(d\,g-e\,f\right)}^2} - \frac{b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{2\,g\,\left(f^2+2\,f\,g\,x+g^2\,x^2\right)} - \frac{\frac{a\,d\,g-a\,e\,f+b\,e\,f\,n}{d\,g-e\,f}+\frac{b\,e\,g\,n\,x}{d\,g-e\,f}}{2\,f^2\,g+4\,f\,g^2\,x+2\,g^3\,x^2}$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^3,x)
```

```
[Out] (b*e^2*n*atanh((2*d^2*g^3 - 2*e^2*f^2*g)/(2*g*(d*g - e*f)^2) + (2*e*g*x)/(d
*g - e*f)))/(g*(d*g - e*f)^2) - (b*log(c*(d + e*x)^n))/(2*g*(f^2 + g^2*x^2
+ 2*f*g*x)) - ((a*d*g - a*e*f + b*e*f*n)/(d*g - e*f) + (b*e*g*n*x)/(d*g - e
*f))/(2*f^2*g + 2*g^3*x^2 + 4*f*g^2*x)
```

# 3.43 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^4} dx$



## Optimal result

Integrand size = 22, antiderivative size = 141

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^4} dx = \frac{ben}{6g(ef-dg)(f+gx)^2} + \frac{be^2n}{3g(ef-dg)^2(f+gx)} + \frac{be^3n\log(d+ex)}{3g(ef-dg)^3} - \frac{a+b\log(c(d+ex)^n)}{3g(f+gx)^3} - \frac{be^3n\log(f+gx)}{3g(ef-dg)^3}$$

```
[Out] 1/6*b*e*n/g/(-d*g+e*f)/(g*x+f)^2+1/3*b*e^2*n/g/(-d*g+e*f)^2/(g*x+f)+1/3*b*e
^3*n*ln(e*x+d)/g/(-d*g+e*f)^3+1/3*(-a-b*ln(c*(e*x+d)^n))/g/(g*x+f)^3-1/3*b*
e^3*n*ln(g*x+f)/g/(-d*g+e*f)^3
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2442, 46}

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^4} dx = -\frac{a+b\log(c(d+ex)^n)}{3g(f+gx)^3} + \frac{be^3n\log(d+ex)}{3g(ef-dg)^3} - \frac{be^3n\log(f+gx)}{3g(ef-dg)^3} + \frac{be^2n}{3g(f+gx)(ef-dg)^2} + \frac{ben}{6g(f+gx)^2(ef-dg)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^4,x]
```

```
[Out] (b*e*n)/(6*g*(e*f - d*g)*(f + g*x)^2) + (b*e^2*n)/(3*g*(e*f - d*g)^2*(f + g
*x)) + (b*e^3*n*Log[d + e*x])/(3*g*(e*f - d*g)^3) - (a + b*Log[c*(d + e*x)^
n])/(3*g*(f + g*x)^3) - (b*e^3*n*Log[f + g*x])/(3*g*(e*f - d*g)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\log\left(c(d+ex)^n\right)}{3g(f+gx)^3}+\frac{(ben)\int\frac{1}{(d+ex)(f+gx)^3}\,dx}{3g}\\
&= -\frac{a+b\log\left(c(d+ex)^n\right)}{3g(f+gx)^3}\\
&\quad+\frac{(ben)\int\left(\frac{e^3}{(ef-dg)^3(d+ex)}-\frac{g}{(ef-dg)(f+gx)^3}-\frac{eg}{(ef-dg)^2(f+gx)^2}-\frac{e^2g}{(ef-dg)^3(f+gx)}\right)dx}{3g}\\
&= \frac{ben}{6g(ef-dg)(f+gx)^2}+\frac{be^2n}{3g(ef-dg)^2(f+gx)}\\
&\quad+\frac{be^3n\log(d+ex)}{3g(ef-dg)^3}-\frac{a+b\log\left(c(d+ex)^n\right)}{3g(f+gx)^3}-\frac{be^3n\log(f+gx)}{3g(ef-dg)^3}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.78

$$\int\frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^4}\,dx=\frac{-2(a+b\log\left(c(d+ex)^n\right))+\frac{ben(f+gx)\left((ef-dg)(3ef-dg+2egx)+2e^2(f+gx)^2\log(d+ex)-2e^2(f+gx)^2\log(f+gx)\right)}{(ef-dg)^3}}{6g(f+gx)^3}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^4,x]
```

```
[Out] (-2*(a + b*Log[c*(d + e*x)^n]) + (b*e*n*(f + g*x)*((e*f - d*g)*(3*e*f - d*g
 + 2*e*g*x) + 2*e^2*(f + g*x)^2*Log[d + e*x] - 2*e^2*(f + g*x)^2*Log[f + g*
x]))/(e*f - d*g)^3)/(6*g*(f + g*x)^3)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(134) = 268$.

Time = 1.61 (sec) , antiderivative size = 455, normalized size of antiderivative = 3.23

| method | result |
|---|---|
| parallelrisch | $-\frac{2\ln(c(ex+d)^n)b\,d^3e\,g^5-2\ln(c(ex+d)^n)b\,e^4f^3g^2+3b\,e^4f^3g^2n-6a\,d^2e^2f\,g^4+6ad\,e^3f^2g^3+6\ln(ex+d)x^2b\,e^4f\,g^4n-6\ln(gx+}{}$ |
| risch | Expression too large to display |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(2*ln(c*(e*x+d)^n)*b*d^3*e*g^5-2*ln(c*(e*x+d)^n)*b*e^4*f^3*g^2+3*b*e^4
*f^3*g^2*n-6*a*d^2*e^2*f*g^4+6*a*d*e^3*f^2*g^3+6*ln(e*x+d)*x^2*b*e^4*f*g^4*
n-6*ln(g*x+f)*x^2*b*e^4*f*g^4*n+6*ln(e*x+d)*x*b*e^4*f^2*g^3*n+2*x^2*b*e^4*f
*g^4*n+x*b*d^2*e^2*g^5*n+5*x*b*e^4*f^2*g^3*n-6*ln(c*(e*x+d)^n)*b*d^2*e^2*f*
g^4+6*ln(c*(e*x+d)^n)*b*d*e^3*f^2*g^3+2*ln(e*x+d)*x^3*b*e^4*g^5*n-2*ln(g*x+
f)*x^3*b*e^4*g^5*n+2*ln(e*x+d)*b*e^4*f^3*g^2*n-2*ln(g*x+f)*b*e^4*f^3*g^2*n+
b*d^2*e^2*f*g^4*n-4*b*d*e^3*f^2*g^3*n+2*a*d^3*e*g^5-2*a*e^4*f^3*g^2-2*x^2*b
*d*e^3*g^5*n-6*ln(g*x+f)*x*b*e^4*f^2*g^3*n-6*x*b*d*e^3*f*g^4*n)/(d^3*g^3-3*
d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)/(g*x+f)^3/g^3/e
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(131) = 262$.

Time = 0.32 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.60

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^4}\,dx = -\frac{2\,ae^3f^3-6\,ade^2f^2g+6\,ad^2efg^2-2\,ad^3g^3-2\,(be^3fg^2-bde^2g^3)nx^2-(5\,be^3f^2g-6\,bde^2fg^2+bd^2eg^3}{6\,(e^3f^6g-3\,de^2f^5g}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*e^3*f^3 - 6*a*d*e^2*f^2*g + 6*a*d^2*e*f*g^2 - 2*a*d^3*g^3 - 2*(b*
e^3*f*g^2 - b*d*e^2*g^3)*n*x^2 - (5*b*e^3*f^2*g - 6*b*d*e^2*f*g^2 + b*d^2*e
*g^3)*n*x - (3*b*e^3*f^3 - 4*b*d*e^2*f^2*g + b*d^2*e*f*g^2)*n - 2*(b*e^3*g^
3*n*x^3 + 3*b*e^3*f*g^2*n*x^2 + 3*b*e^3*f^2*g*n*x + (3*b*d*e^2*f^2*g - 3*b*
d^2*e*f*g^2 + b*d^3*g^3)*n)*log(e*x + d) + 2*(b*e^3*g^3*n*x^3 + 3*b*e^3*f*g
^2*n*x^2 + 3*b*e^3*f^2*g*n*x + b*e^3*f^3*n)*log(g*x + f) + 2*(b*e^3*f^3 - 3
*b*d*e^2*f^2*g + 3*b*d^2*e*f*g^2 - b*d^3*g^3)*log(c))/(e^3*f^6*g - 3*d*e^2*
f^5*g^2 + 3*d^2*e*f^4*g^3 - d^3*f^3*g^4 + (e^3*f^3*g^4 - 3*d*e^2*f^2*g^5 +
3*d^2*e*f*g^6 - d^3*g^7)*x^3 + 3*(e^3*f^4*g^3 - 3*d*e^2*f^3*g^4 + 3*d^2*e*f
^2*g^5 - d^3*f*g^6)*x^2 + 3*(e^3*f^5*g^2 - 3*d*e^2*f^4*g^3 + 3*d^2*e*f^3*g^
4 - d^3*f^2*g^5)*x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^4}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**4,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(131) = 262$.

Time = 0.21 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.13

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^4}\,dx = \frac{1}{6}\left(\frac{2\,e^2\log\left(ex+d\right)}{e^3f^3g-3\,de^2f^2g^2+3\,d^2efg^3-d^3g^4}-\frac{2\,e^2\log\left(gx+f\right)}{e^3f^3g-3\,de^2f^2g^2+3\,d^2efg^3-d^3g^4}+\frac{}{e^2f^4g-2\,def^3g^2+d^2\ldots}\right. - \frac{b\log\left((ex+d)^nc\right)}{3\,(g^4x^3+3\,fg^3x^2+3\,f^2g^2x+f^3g)}-\frac{a}{3\,(g^4x^3+3\,fg^3x^2+3\,f^2g^2x+f^3g)}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^4,x, algorithm="maxima")
[Out] 1/6*(2*e^2*log(e*x + d)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 - d^3*
g^4) - 2*e^2*log(g*x + f)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 - d^
3*g^4) + (2*e*g*x + 3*e*f - d*g)/(e^2*f^4*g - 2*d*e*f^3*g^2 + d^2*f^2*g^3 +
 (e^2*f^2*g^3 - 2*d*e*f*g^4 + d^2*g^5)*x^2 + 2*(e^2*f^3*g^2 - 2*d*e*f^2*g^3
 + d^2*f*g^4)*x))*b*e*n - 1/3*b*log((e*x + d)^n*c)/(g^4*x^3 + 3*f*g^3*x^2 +
 3*f^2*g^2*x + f^3*g) - 1/3*a/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(131) = 262$.

Time = 0.31 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.84

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^4}\,dx = \frac{be^3n\log\left(ex+d\right)}{3\,(e^3f^3g-3\,de^2f^2g^2+3\,d^2efg^3-d^3g^4)} - \frac{be^3n\log\left(gx+f\right)}{3\,(e^3f^3g-3\,de^2f^2g^2+3\,d^2efg^3-d^3g^4)}-\frac{bn\log\left(ex+d\right)}{3\,(g^4x^3+3\,fg^3x^2+3\,f^2g^2x+f^3g)} + \frac{2\,be^2g^2nx^2+5\,be^2fgnx-bdeg^2nx+3\,be^2f^2n-bdefgn-2\,be^2f^2\log\left(c\right)+4\,bdefg\log\left(c\right)-2\,bd^2g^2\log\ldots}{6\,(e^2f^2g^4x^3-2\,defg^5x^3+d^2g^6x^3+3\,e^2f^3g^3x^2-6\,def^2g^4x^2+3\,d^2fg^5x^2+3\,e^2f^4g^2x-6\,def^3g^3x+3\,d\ldots}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] 1/3*b*e^3*n*log(e*x + d)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 - d^3
*g^4) - 1/3*b*e^3*n*log(g*x + f)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g
^3 - d^3*g^4) - 1/3*b*n*log(e*x + d)/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x +
 f^3*g) + 1/6*(2*b*e^2*g^2*n*x^2 + 5*b*e^2*f*g*n*x - b*d*e*g^2*n*x + 3*b*e^
2*f^2*n - b*d*e*f*g*n - 2*b*e^2*f^2*log(c) + 4*b*d*e*f*g*log(c) - 2*b*d^2*g
^2*log(c) - 2*a*e^2*f^2 + 4*a*d*e*f*g - 2*a*d^2*g^2)/(e^2*f^2*g^4*x^3 - 2*d
*e*f*g^5*x^3 + d^2*g^6*x^3 + 3*e^2*f^3*g^3*x^2 - 6*d*e*f^2*g^4*x^2 + 3*d^2*
f*g^5*x^2 + 3*e^2*f^4*g^2*x - 6*d*e*f^3*g^3*x + 3*d^2*f^2*g^4*x + e^2*f^5*g
 - 2*d*e*f^4*g^2 + d^2*f^3*g^3)
```

## Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.01

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{(f+gx)^4}\,dx=\frac{2\,a\,d\,e\,f}{3\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}-\frac{a\,d^2\,g}{3\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}-\frac{b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{3\,g\,{\left(f+g\,x\right)}^3}-\frac{a\,e^2\,f^2}{3\,g\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}+\frac{5\,b\,e^2\,f\,n\,x}{6\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}+\frac{b\,e^2\,g\,n\,x^2}{3\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}-\frac{b\,d\,e\,f\,n}{6\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}+\frac{b\,e^2\,f^2\,n}{2\,g\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}-\frac{b\,d\,e\,g\,n\,x}{6\,{\left(f+g\,x\right)}^3\,{\left(d\,g-e\,f\right)}^2}+\frac{b\,e^3\,n\,\mathrm{atan}\left(\frac{d\,g\,1{}\mathrm{i}+e\,f\,1{}\mathrm{i}+e\,g\,x\,2{}\mathrm{i}}{d\,g-e\,f}\right)\,2{}\mathrm{i}}{3\,g\,{\left(d\,g-e\,f\right)}^3}$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^4,x)
```

```
[Out] (2*a*d*e*f)/(3*(f + g*x)^3*(d*g - e*f)^2) - (a*d^2*g)/(3*(f + g*x)^3*(d*g -
 e*f)^2) - (b*log(c*(d + e*x)^n))/(3*g*(f + g*x)^3) - (a*e^2*f^2)/(3*g*(f +
 g*x)^3*(d*g - e*f)^2) + (b*e^3*n*atan((d*g*1i + e*f*1i + e*g*x*2i)/(d*g -
e*f))*2i)/(3*g*(d*g - e*f)^3) + (5*b*e^2*f*n*x)/(6*(f + g*x)^3*(d*g - e*f)^
2) + (b*e^2*g*n*x^2)/(3*(f + g*x)^3*(d*g - e*f)^2) - (b*d*e*f*n)/(6*(f + g*
x)^3*(d*g - e*f)^2) + (b*e^2*f^2*n)/(2*g*(f + g*x)^3*(d*g - e*f)^2) - (b*d*
e*g*n*x)/(6*(f + g*x)^3*(d*g - e*f)^2)
```

# 3.44 $\int (f+gx)^3 (a+b\log (c(d+ex)^n))^2 \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 365

$$\begin{aligned}
\int (f+gx)^3 (a+b\log (c(d+ex)^n))^2 \, dx &= \frac{2b^2(ef-dg)^3n^2x}{e^3}+\frac{3b^2g(ef-dg)^2n^2(d+ex)^2}{4e^4}+\frac{2b^2g^2(ef-dg)n^2(d+ex)^3}{9e^4} \\
&\quad +\frac{b^2g^3n^2(d+ex)^4}{32e^4}+\frac{b^2(ef-dg)^4n^2\log^2(d+ex)}{4e^4g} \\
&\quad -\frac{2b(ef-dg)^3n(d+ex)(a+b\log (c(d+ex)^n))}{e^4} \\
&\quad -\frac{3bg(ef-dg)^2n(d+ex)^2(a+b\log (c(d+ex)^n))}{2e^4} \\
&\quad -\frac{2bg^2(ef-dg)n(d+ex)^3(a+b\log (c(d+ex)^n))}{3e^4} \\
&\quad -\frac{bg^3n(d+ex)^4(a+b\log (c(d+ex)^n))}{8e^4} \\
&\quad -\frac{b(ef-dg)^4n\log(d+ex)(a+b\log (c(d+ex)^n))}{2e^4g}+\frac{(f+gx)^4(a+b\log (c(d+ex)^n))^2}{4g}
\end{aligned}$$

```
[Out] 2*b^2*(-d*g+e*f)^3*n^2*x/e^3+3/4*b^2*g*(-d*g+e*f)^2*n^2*(e*x+d)^2/e^4+2/9*b
^2*g^2*(-d*g+e*f)*n^2*(e*x+d)^3/e^4+1/32*b^2*g^3*n^2*(e*x+d)^4/e^4+1/4*b^2*
(-d*g+e*f)^4*n^2*ln(e*x+d)^2/e^4/g-2*b*(-d*g+e*f)^3*n*(e*x+d)*(a+b*ln(c*(e*
x+d)^n))/e^4-3/2*b*g*(-d*g+e*f)^2*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^4-2/3
*b*g^2*(-d*g+e*f)*n*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))/e^4-1/8*b*g^3*n*(e*x+d)
^4*(a+b*ln(c*(e*x+d)^n))/e^4-1/2*b*(-d*g+e*f)^4*n*ln(e*x+d)*(a+b*ln(c*(e*x+
d)^n))/e^4/g+1/4*(g*x+f)^4*(a+b*ln(c*(e*x+d)^n))^2/g
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2445, 2458, 45, 2372, 12, 2338}

$$\int (f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx$$
$$=-\frac{2bg^2n(d+ex)^3(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^4}-\frac{bn(ef-dg)^4\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4g}-\frac{2bn(d+ex)(ef-dg)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^4}-\frac{3bgn(d+ex)^2(ef-dg)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4}-\frac{bg^3n(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{8e^4}+\frac{(f+gx)^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4g}+\frac{2b^2g^2n^2(d+ex)^3(ef-dg)}{9e^4}+\frac{3b^2gn^2(d+ex)^2(ef-dg)^2}{4e^4}+\frac{b^2n^2(ef-dg)^4\log^2(d+ex)}{4e^4g}+\frac{b^2g^3n^2(d+ex)^4}{32e^4}+\frac{2b^2n^2x(ef-dg)^3}{e^3}$$

```
[In] Int[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (2*b^2*(e*f - d*g)^3*n^2*x)/e^3 + (3*b^2*g*(e*f - d*g)^2*n^2*(d + e*x)^2)/(
4*e^4) + (2*b^2*g^2*(e*f - d*g)*n^2*(d + e*x)^3)/(9*e^4) + (b^2*g^3*n^2*(d
+ e*x)^4)/(32*e^4) + (b^2*(e*f - d*g)^4*n^2*Log[d + e*x]^2)/(4*e^4*g) - (2*
b*(e*f - d*g)^3*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n]))/e^4 - (3*b*g*(e*f -
 d*g)^2*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^4) - (2*b*g^2*(e*f -
 d*g)*n*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))/(3*e^4) - (b*g^3*n*(d + e*x
)^4*(a + b*Log[c*(d + e*x)^n]))/(8*e^4) - (b*(e*f - d*g)^4*n*Log[d + e*x]*(
a + b*Log[c*(d + e*x)^n]))/(2*e^4*g) + ((f + g*x)^4*(a + b*Log[c*(d + e*x)^
n])^2)/(4*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4g} - \frac{(ben)\int \frac{(f+gx)^4(a+b\log(c(d+ex)^n))}{d+ex}\,dx}{2g}$$

$$= \frac{(f+gx)^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4g} - \frac{(bn)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^4(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{2g}$$

$$= -\frac{2b(ef-dg)^3n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^4} - \frac{3bg(ef-dg)^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4} - \frac{2bg^2(ef-dg)n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^4} - \frac{bg^3n(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{8e^4} - \frac{b(ef-dg)^4n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4g} + \frac{(f+gx)^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4g} + \frac{(b^2n^2)\,\text{Subst}\left(\int \frac{48g(ef-dg)^3+36g^2(ef-dg)^2x+16g^3(ef-dg)x^2+3g^4x^3+\frac{12(ef-dg)^4\log(x)}{x}}{12e^4}\,dx,x,d+ex\right)}{2g}$$

$$= -\frac{2b(ef-dg)^3n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^4} - \frac{3bg(ef-dg)^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4} - \frac{2bg^2(ef-dg)n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^4} - \frac{bg^3n(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{8e^4} - \frac{b(ef-dg)^4n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4g} + \frac{(f+gx)^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4g} + \frac{(b^2n^2)\,\text{Subst}\left(\int \left(48g(ef-dg)^3+36g^2(ef-dg)^2x+16g^3(ef-dg)x^2+3g^4x^3+\frac{12(ef-dg)^4\log(x}{x}\right.\right.}{24e^4g}$$

$$
\begin{aligned}
&= \frac{2b^2(ef-dg)^3n^2x}{e^3} + \frac{3b^2g(ef-dg)^2n^2(d+ex)^2}{4e^4} + \frac{2b^2g^2(ef-dg)n^2(d+ex)^3}{9e^4} \\
&\quad + \frac{b^2g^3n^2(d+ex)^4}{32e^4} - \frac{2b(ef-dg)^3n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^4} \\
&\quad - \frac{3bg(ef-dg)^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4} \\
&\quad - \frac{2bg^2(ef-dg)n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^4} \\
&\quad - \frac{bg^3n(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{8e^4} \\
&\quad - \frac{b(ef-dg)^4n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4g} \\
&\quad + \frac{(f+gx)^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4g} \\
&\quad + \frac{\left(b^2(ef-dg)^4n^2\right)\text{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, d+ex\right)}{2e^4g} \\
&= \frac{2b^2(ef-dg)^3n^2x}{e^3} + \frac{3b^2g(ef-dg)^2n^2(d+ex)^2}{4e^4} + \frac{2b^2g^2(ef-dg)n^2(d+ex)^3}{9e^4} \\
&\quad + \frac{b^2g^3n^2(d+ex)^4}{32e^4} + \frac{b^2(ef-dg)^4n^2\log^2(d+ex)}{4e^4g} \\
&\quad - \frac{2b(ef-dg)^3n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^4} \\
&\quad - \frac{3bg(ef-dg)^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4} \\
&\quad - \frac{2bg^2(ef-dg)n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^4} \\
&\quad - \frac{bg^3n(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{8e^4} \\
&\quad - \frac{b(ef-dg)^4n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4g} \\
&\quad + \frac{(f+gx)^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4g}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.99

$$\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^2 \, dx = \frac{288(ef-dg)^3(d+ex) \left(a+b\log \left(c(d+ex)^n\right)\right)^2+432g(ef-dg)^2(d+ex)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^2+28}{}$$

```
[In] Integrate[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (288*(e*f - d*g)^3*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 + 432*g*(e*f - d*
g)^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2 + 288*g^2*(e*f - d*g)*(d + e*
x)^3*(a + b*Log[c*(d + e*x)^n])^2 + 72*g^3*(d + e*x)^4*(a + b*Log[c*(d + e*
x)^n])^2 - 576*b*(e*f - d*g)^3*n*(e*(a - b*n)*x + b*(d + e*x)*Log[c*(d + e*
x)^n]) + 216*b*g*(e*f - d*g)^2*n*(b*e*n*x*(2*d + e*x) - 2*(d + e*x)^2*(a +
b*Log[c*(d + e*x)^n])) + 64*b*g^2*(e*f - d*g)*n*(b*e*n*x*(3*d^2 + 3*d*e*x +
 e^2*x^2) - 3*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])) + 9*b*g^3*n*(b*e*n*x*
(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - 4*(d + e*x)^4*(a + b*Log[c*(d
 + e*x)^n])))/(288*e^4)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1342 vs. 2(347) = 694.

Time = 2.20 (sec) , antiderivative size = 1343, normalized size of antiderivative = 3.68

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1343 |
| risch | Expression too large to display | 6770 |

```
[In] int((g*x+f)^3*(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/288*(192*a*b*e^4*f*g^2*n*x^3+300*b^2*d^3*e*g^3*n^2*x-288*a^2*e^4*f^3*x+2
88*a^2*d*e^3*f^3-288*a*b*d*e^3*f*g^2*n*x^2-288*a^2*e^4*f*g^2*x^3+576*b*n*a*
e^4*f^3*x+36*a*b*e^4*g^3*n*x^4+28*b^2*d*e^3*g^3*n^2*x^3-64*b^2*e^4*f*g^2*n^
2*x^3-9*b^2*e^4*g^3*n^2*x^4+72*ln(c*(e*x+d)^n)^2*b^2*d^4*g^3-864*a*b*d*e^3*
f^2*g*n*x-576*b^2*n^2*e^4*f^3*x-300*b^2*d^4*g^3*n^2-288*ln(c*(e*x+d)^n)^2*b
^2*d*e^3*f^3+144*ln(c*(e*x+d)^n)*b^2*d^4*g^3*n-444*ln(e*x+d)*b^2*d^4*g^3*n^
2-72*a^2*e^4*g^3*x^4-78*b^2*d^2*e^2*g^3*n^2*x^2-216*b^2*e^4*f^2*g*n^2*x^2-4
32*a^2*e^4*f^2*g*x^2+240*b^2*d*e^3*f*g^2*n^2*x^2+432*a*b*e^4*f^2*g*n*x^2-10
56*b^2*d^2*e^2*f*g^2*n^2*x+1296*b^2*d*e^3*f^2*g*n^2*x+576*b^2*d*e^3*f^3*n^2
+144*a*b*d^4*g^3*n-72*x^4*ln(c*(e*x+d)^n)^2*b^2*e^4*g^3-288*x*ln(c*(e*x+d)^
n)^2*b^2*e^4*f^3+576*x*ln(c*(e*x+d)^n)*b^2*d^2*e^2*f*g^2*n-864*x*ln(c*(e*x+
d)^n)*b^2*d*e^3*f^2*g*n+1056*b^2*d^3*e*f*g^2*n^2-1296*b^2*d^2*e^2*f^2*g*n^2
-576*a*b*d*e^3*f^3*n-576*ln(e*x+d)*a*b*d^3*e*f*g^2*n+864*ln(e*x+d)*a*b*d^2*
```

```
e^2*f^2*g*n+72*a*b*d^2*e^2*g^3*n*x^2-48*a*b*d*e^3*g^3*n*x^3-144*a*b*d^3*e*g
^3*n*x-576*ln(c*(e*x+d)^n)*b^2*d^3*e*f*g^2*n+864*ln(c*(e*x+d)^n)*b^2*d^2*e^
2*f^2*g*n+36*x^4*ln(c*(e*x+d)^n)*b^2*e^4*g^3*n-144*x^4*ln(c*(e*x+d)^n)*a*b*
e^4*g^3-288*x^3*ln(c*(e*x+d)^n)^2*b^2*e^4*f*g^2-432*x^2*ln(c*(e*x+d)^n)^2*b
^2*e^4*f^2*g+576*x*ln(c*(e*x+d)^n)*b^2*e^4*f^3*n-576*x*ln(c*(e*x+d)^n)*a*b*
e^4*f^3-288*ln(c*(e*x+d)^n)^2*b^2*d^3*e*f*g^2+432*ln(c*(e*x+d)^n)^2*b^2*d^2
*e^2*f^2*g-576*ln(c*(e*x+d)^n)*b^2*d*e^3*f^3*n+576*ln(c*(e*x+d)^n)*a*b*d*e^
3*f^3-288*x^2*ln(c*(e*x+d)^n)*b^2*d*e^3*f*g^2*n-576*a*b*d^3*e*f*g^2*n+864*a
*b*d^2*e^2*f^2*g*n+576*a*b*d^2*e^2*f*g^2*n*x-48*x^3*ln(c*(e*x+d)^n)*b^2*d*e
^3*g^3*n+192*x^3*ln(c*(e*x+d)^n)*b^2*e^4*f*g^2*n-576*x^3*ln(c*(e*x+d)^n)*a*
b*e^4*f*g^2+72*x^2*ln(c*(e*x+d)^n)*b^2*d^2*e^2*g^3*n+432*x^2*ln(c*(e*x+d)^n
)*b^2*e^4*f^2*g*n-864*x^2*ln(c*(e*x+d)^n)*a*b*e^4*f^2*g-144*x*ln(c*(e*x+d)^
n)*b^2*d^3*e*g^3*n+1152*ln(e*x+d)*b^2*d*e^3*f^3*n^2+144*ln(e*x+d)*a*b*d^4*g
^3*n-1152*ln(e*x+d)*a*b*d*e^3*f^3*n+1632*ln(e*x+d)*b^2*d^3*e*f*g^2*n^2-2160
*ln(e*x+d)*b^2*d^2*e^2*f^2*g*n^2)/e^4
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(347) = 694$.

Time = 0.30 (sec) , antiderivative size = 1190, normalized size of antiderivative = 3.26

$$\int (f+gx)^3 \left(a+b\log\left(c(d+ex)^n\right)\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

```
[Out] 1/288*(9*(b^2*e^4*g^3*n^2 - 4*a*b*e^4*g^3*n + 8*a^2*e^4*g^3)*x^4 + 4*(72*a^
2*e^4*f*g^2 + (16*b^2*e^4*f*g^2 - 7*b^2*d*e^3*g^3)*n^2 - 12*(4*a*b*e^4*f*g^
2 - a*b*d*e^3*g^3)*n)*x^3 + 6*(72*a^2*e^4*f^2*g + (36*b^2*e^4*f^2*g - 40*b^
2*d*e^3*f*g^2 + 13*b^2*d^2*e^2*g^3)*n^2 - 12*(6*a*b*e^4*f^2*g - 4*a*b*d*e^3
*f*g^2 + a*b*d^2*e^2*g^3)*n)*x^2 + 72*(b^2*e^4*g^3*n^2*x^4 + 4*b^2*e^4*f*g^
2*n^2*x^3 + 6*b^2*e^4*f^2*g*n^2*x^2 + 4*b^2*e^4*f^3*n^2*x + (4*b^2*d*e^3*f^
3 - 6*b^2*d^2*e^2*f^2*g + 4*b^2*d^3*e*f*g^2 - b^2*d^4*g^3)*n^2)*log(e*x + d
)^2 + 72*(b^2*e^4*g^3*x^4 + 4*b^2*e^4*f*g^2*x^3 + 6*b^2*e^4*f^2*g*x^2 + 4*b
^2*e^4*f^3*x)*log(c)^2 + 12*(24*a^2*e^4*f^3 + (48*b^2*e^4*f^3 - 108*b^2*d*e
^3*f^2*g + 88*b^2*d^2*e^2*f*g^2 - 25*b^2*d^3*e*g^3)*n^2 - 12*(4*a*b*e^4*f^3
 - 6*a*b*d*e^3*f^2*g + 4*a*b*d^2*e^2*f*g^2 - a*b*d^3*e*g^3)*n)*x - 12*(3*(b
^2*e^4*g^3*n^2 - 4*a*b*e^4*g^3*n)*x^4 - 4*(12*a*b*e^4*f*g^2*n - (4*b^2*e^4*
f*g^2 - b^2*d*e^3*g^3)*n^2)*x^3 + (48*b^2*d*e^3*f^3 - 108*b^2*d^2*e^2*f^2*g
 + 88*b^2*d^3*e*f*g^2 - 25*b^2*d^4*g^3)*n^2 - 6*(12*a*b*e^4*f^2*g*n - (6*b^
2*e^4*f^2*g - 4*b^2*d*e^3*f*g^2 + b^2*d^2*e^2*g^3)*n^2)*x^2 - 12*(4*a*b*d*e
^3*f^3 - 6*a*b*d^2*e^2*f^2*g + 4*a*b*d^3*e*f*g^2 - a*b*d^4*g^3)*n - 12*(4*a
*b*e^4*f^3*n - (4*b^2*e^4*f^3 - 6*b^2*d*e^3*f^2*g + 4*b^2*d^2*e^2*f*g^2 - b
^2*d^3*e*g^3)*n^2)*x - 12*(b^2*e^4*g^3*n*x^4 + 4*b^2*e^4*f*g^2*n*x^3 + 6*b^
2*e^4*f^2*g*n*x^2 + 4*b^2*e^4*f^3*n*x + (4*b^2*d*e^3*f^3 - 6*b^2*d^2*e^2*f^
```

```
2*g + 4*b^2*d^3*e*f*g^2 - b^2*d^4*g^3)*n)*log(c))*log(e*x + d) - 12*(3*(b^2
*e^4*g^3*n - 4*a*b*e^4*g^3)*x^4 - 4*(12*a*b*e^4*f*g^2 - (4*b^2*e^4*f*g^2 -
b^2*d*e^3*g^3)*n)*x^3 - 6*(12*a*b*e^4*f^2*g - (6*b^2*e^4*f^2*g - 4*b^2*d*e^
3*f*g^2 + b^2*d^2*e^2*g^3)*n)*x^2 - 12*(4*a*b*e^4*f^3 - (4*b^2*e^4*f^3 - 6*
b^2*d*e^3*f^2*g + 4*b^2*d^2*e^2*f*g^2 - b^2*d^3*e*g^3)*n)*x)*log(c))/e^4
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1241 vs. 2(348) = 696.

Time = 2.35 (sec) , antiderivative size = 1241, normalized size of antiderivative = 3.40

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))^2 \, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)**3*(a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Piecewise((a**2*f**3*x + 3*a**2*f**2*g*x**2/2 + a**2*f*g**2*x**3 + a**2*g**
3*x**4/4 - a*b*d**4*g**3*log(c*(d + e*x)**n)/(2*e**4) + 2*a*b*d**3*f*g**2*l
og(c*(d + e*x)**n)/e**3 + a*b*d**3*g**3*n*x/(2*e**3) - 3*a*b*d**2*f**2*g*lo
g(c*(d + e*x)**n)/e**2 - 2*a*b*d**2*f*g**2*n*x/e**2 - a*b*d**2*g**3*n*x**2/
(4*e**2) + 2*a*b*d*f**3*log(c*(d + e*x)**n)/e + 3*a*b*d*f**2*g*n*x/e + a*b*
d*f*g**2*n*x**2/e + a*b*d*g**3*n*x**3/(6*e) - 2*a*b*f**3*n*x + 2*a*b*f**3*x
*log(c*(d + e*x)**n) - 3*a*b*f**2*g*n*x**2/2 + 3*a*b*f**2*g*x**2*log(c*(d +
 e*x)**n) - 2*a*b*f*g**2*n*x**3/3 + 2*a*b*f*g**2*x**3*log(c*(d + e*x)**n) -
 a*b*g**3*n*x**4/8 + a*b*g**3*x**4*log(c*(d + e*x)**n)/2 + 25*b**2*d**4*g**
3*n*log(c*(d + e*x)**n)/(24*e**4) - b**2*d**4*g**3*log(c*(d + e*x)**n)**2/(
4*e**4) - 11*b**2*d**3*f*g**2*n*log(c*(d + e*x)**n)/(3*e**3) + b**2*d**3*f*
g**2*log(c*(d + e*x)**n)**2/e**3 - 25*b**2*d**3*g**3*n**2*x/(24*e**3) + b**
2*d**3*g**3*n*x*log(c*(d + e*x)**n)/(2*e**3) + 9*b**2*d**2*f**2*g*n*log(c*(
d + e*x)**n)/(2*e**2) - 3*b**2*d**2*f**2*g*log(c*(d + e*x)**n)**2/(2*e**2)
+ 11*b**2*d**2*f*g**2*n**2*x/(3*e**2) - 2*b**2*d**2*f*g**2*n*x*log(c*(d + e
*x)**n)/e**2 + 13*b**2*d**2*g**3*n**2*x**2/(48*e**2) - b**2*d**2*g**3*n*x**
2*log(c*(d + e*x)**n)/(4*e**2) - 2*b**2*d*f**3*n*log(c*(d + e*x)**n)/e + b*
*2*d*f**3*log(c*(d + e*x)**n)**2/e - 9*b**2*d*f**2*g*n**2*x/(2*e) + 3*b**2*
d*f**2*g*n*x*log(c*(d + e*x)**n)/e - 5*b**2*d*f*g**2*n**2*x**2/(6*e) + b**2
*d*f*g**2*n*x**2*log(c*(d + e*x)**n)/e - 7*b**2*d*g**3*n**2*x**3/(72*e) + b
**2*d*g**3*n*x**3*log(c*(d + e*x)**n)/(6*e) + 2*b**2*f**3*n**2*x - 2*b**2*f
**3*n*x*log(c*(d + e*x)**n) + b**2*f**3*x*log(c*(d + e*x)**n)**2 + 3*b**2*f
**2*g*n**2*x**2/4 - 3*b**2*f**2*g*n*x**2*log(c*(d + e*x)**n)/2 + 3*b**2*f**
2*g*x**2*log(c*(d + e*x)**n)**2/2 + 2*b**2*f*g**2*n**2*x**3/9 - 2*b**2*f*g*
*2*n*x**3*log(c*(d + e*x)**n)/3 + b**2*f*g**2*x**3*log(c*(d + e*x)**n)**2 +
 b**2*g**3*n**2*x**4/32 - b**2*g**3*n*x**4*log(c*(d + e*x)**n)/8 + b**2*g**
3*x**4*log(c*(d + e*x)**n)**2/4, Ne(e, 0)), ((a + b*log(c*d**n))**2*(f**3*x
 + 3*f**2*g*x**2/2 + f*g**2*x**3 + g**3*x**4/4), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 827 vs. 2(347) = 694.

Time = 0.21 (sec) , antiderivative size = 827, normalized size of antiderivative = 2.27

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^2\,dx$$
$$= \frac{1}{4}b^2g^3x^4\log((ex+d)^nc)^2 + \frac{1}{2}abg^3x^4\log((ex+d)^nc) + b^2fg^2x^3\log((ex+d)^nc)^2 + \frac{1}{4}a^2g^3x^4 + 2abfg^2x^3\log((ex+d)^nc) + \frac{3}{2}b^2f^2gx^2\log((ex+d)^nc)^2 + a^2fg^2x^3 - 2abef^3n\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) - \frac{1}{24}abeg^3n\left(\frac{12d^4\log(ex+d)}{e^5} + \frac{3e^3x^4 - 4de^2x^3 + 6d^2ex^2 - 12d^3x}{e^4}\right) + \frac{1}{3}abefg^2n\left(\frac{6d^3\log(ex+d)}{e^4} - \frac{2e^2x^3 - 3dex^2 + 6d^2x}{e^3}\right) - \frac{3}{2}abef^2gn\left(\frac{2d^2\log(ex+d)}{e^3} + \frac{ex^2 - 2dx}{e^2}\right) + 3abf^2gx^2\log((ex+d)^nc) + b^2f^3x\log((ex+d)^nc)^2 + \frac{3}{2}a^2f^2gx^2 + 2abf^3x\log((ex+d)^nc) - \left(2en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc) + \frac{(d\log(ex+d)^2 - 2ex + 2d\log(ex+d))n^2}{e}\right)b^2f^3 - \frac{3}{4}\left(2en\left(\frac{2d^2\log(ex+d)}{e^3} + \frac{ex^2-2dx}{e^2}\right)\log((ex+d)^nc) - \frac{(e^2x^2 + 2d^2\log(ex+d)^2 - 6dex + 6d^2\log\right.}{e^2} + \frac{1}{18}\left(6en\left(\frac{6d^3\log(ex+d)}{e^4} - \frac{2e^2x^3 - 3dex^2 + 6d^2x}{e^3}\right)\log((ex+d)^nc) + \frac{(4e^3x^3 - 15de^2x^2 - 18d^3\log\right.}{} - \frac{1}{288}\left(12en\left(\frac{12d^4\log(ex+d)}{e^5} + \frac{3e^3x^4 - 4de^2x^3 + 6d^2ex^2 - 12d^3x}{e^4}\right)\log((ex+d)^nc) - \frac{(9e^4x^4 - 28}{}\right. + a^2f^3x$$

```
[In] integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*g^3*x^4*log((e*x + d)^n*c)^2 + 1/2*a*b*g^3*x^4*log((e*x + d)^n*c) +
 b^2*f*g^2*x^3*log((e*x + d)^n*c)^2 + 1/4*a^2*g^3*x^4 + 2*a*b*f*g^2*x^3*log
((e*x + d)^n*c) + 3/2*b^2*f^2*g*x^2*log((e*x + d)^n*c)^2 + a^2*f*g^2*x^3 -
2*a*b*e*f^3*n*(x/e - d*log(e*x + d)/e^2) - 1/24*a*b*e*g^3*n*(12*d^4*log(e*x
 + d)/e^5 + (3*e^3*x^4 - 4*d*e^2*x^3 + 6*d^2*e*x^2 - 12*d^3*x)/e^4) + 1/3*a
*b*e*f*g^2*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^
3) - 3/2*a*b*e*f^2*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 3*a
*b*f^2*g*x^2*log((e*x + d)^n*c) + b^2*f^3*x*log((e*x + d)^n*c)^2 + 3/2*a^2*
```

```
f^2*g*x^2 + 2*a*b*f^3*x*log((e*x + d)^n*c) - (2*e*n*(x/e - d*log(e*x + d)/e
^2)*log((e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/
e)*b^2*f^3 - 3/4*(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log(
(e*x + d)^n*c) - (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x
+ d))*n^2/e^2)*b^2*f^2*g + 1/18*(6*e*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3
 - 3*d*e*x^2 + 6*d^2*x)/e^3)*log((e*x + d)^n*c) + (4*e^3*x^3 - 15*d*e^2*x^2
 - 18*d^3*log(e*x + d)^2 + 66*d^2*e*x - 66*d^3*log(e*x + d))*n^2/e^3)*b^2*f
*g^2 - 1/288*(12*e*n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e^2*x^3 +
6*d^2*e*x^2 - 12*d^3*x)/e^4)*log((e*x + d)^n*c) - (9*e^4*x^4 - 28*d*e^3*x^3
 + 78*d^2*e^2*x^2 + 72*d^4*log(e*x + d)^2 - 300*d^3*e*x + 300*d^4*log(e*x +
 d))*n^2/e^4)*b^2*g^3 + a^2*f^3*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2345 vs. 2(347) = 694.

Time = 0.34 (sec) , antiderivative size = 2345, normalized size of antiderivative = 6.42

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^2 \, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] (e*x + d)*b^2*f^3*n^2*log(e*x + d)^2/e + 3/2*(e*x + d)^2*b^2*f^2*g*n^2*log(
e*x + d)^2/e^2 - 3*(e*x + d)*b^2*d*f^2*g*n^2*log(e*x + d)^2/e^2 + (e*x + d)
^3*b^2*f*g^2*n^2*log(e*x + d)^2/e^3 - 3*(e*x + d)^2*b^2*d*f*g^2*n^2*log(e*x
 + d)^2/e^3 + 3*(e*x + d)*b^2*d^2*f*g^2*n^2*log(e*x + d)^2/e^3 + 1/4*(e*x +
 d)^4*b^2*g^3*n^2*log(e*x + d)^2/e^4 - (e*x + d)^3*b^2*d*g^3*n^2*log(e*x +
d)^2/e^4 + 3/2*(e*x + d)^2*b^2*d^2*g^3*n^2*log(e*x + d)^2/e^4 - (e*x + d)*b
^2*d^3*g^3*n^2*log(e*x + d)^2/e^4 - 2*(e*x + d)*b^2*f^3*n^2*log(e*x + d)/e
- 3/2*(e*x + d)^2*b^2*f^2*g*n^2*log(e*x + d)/e^2 + 6*(e*x + d)*b^2*d*f^2*g*
n^2*log(e*x + d)/e^2 - 2/3*(e*x + d)^3*b^2*f*g^2*n^2*log(e*x + d)/e^3 + 3*(
e*x + d)^2*b^2*d*f*g^2*n^2*log(e*x + d)/e^3 - 6*(e*x + d)*b^2*d^2*f*g^2*n^2
*log(e*x + d)/e^3 - 1/8*(e*x + d)^4*b^2*g^3*n^2*log(e*x + d)/e^4 + 2/3*(e*x
 + d)^3*b^2*d*g^3*n^2*log(e*x + d)/e^4 - 3/2*(e*x + d)^2*b^2*d^2*g^3*n^2*lo
g(e*x + d)/e^4 + 2*(e*x + d)*b^2*d^3*g^3*n^2*log(e*x + d)/e^4 + 2*(e*x + d)
*b^2*f^3*n*log(e*x + d)*log(c)/e + 3*(e*x + d)^2*b^2*f^2*g*n*log(e*x + d)*l
og(c)/e^2 - 6*(e*x + d)*b^2*d*f^2*g*n*log(e*x + d)*log(c)/e^2 + 2*(e*x + d)
^3*b^2*f*g^2*n*log(e*x + d)*log(c)/e^3 - 6*(e*x + d)^2*b^2*d*f*g^2*n*log(e*
x + d)*log(c)/e^3 + 6*(e*x + d)*b^2*d^2*f*g^2*n*log(e*x + d)*log(c)/e^3 + 1
/2*(e*x + d)^4*b^2*g^3*n*log(e*x + d)*log(c)/e^4 - 2*(e*x + d)^3*b^2*d*g^3*
n*log(e*x + d)*log(c)/e^4 + 3*(e*x + d)^2*b^2*d^2*g^3*n*log(e*x + d)*log(c)
/e^4 - 2*(e*x + d)*b^2*d^3*g^3*n*log(e*x + d)*log(c)/e^4 + 2*(e*x + d)*b^2*
f^3*n^2/e + 3/4*(e*x + d)^2*b^2*f^2*g*n^2/e^2 - 6*(e*x + d)*b^2*d*f^2*g*n^2
/e^2 + 2/9*(e*x + d)^3*b^2*f*g^2*n^2/e^3 - 3/2*(e*x + d)^2*b^2*d*f*g^2*n^2/
e^3 + 6*(e*x + d)*b^2*d^2*f*g^2*n^2/e^3 + 1/32*(e*x + d)^4*b^2*g^3*n^2/e^4
```

```
- 2/9*(e*x + d)^3*b^2*d*g^3*n^2/e^4 + 3/4*(e*x + d)^2*b^2*d^2*g^3*n^2/e^4 -
 2*(e*x + d)*b^2*d^3*g^3*n^2/e^4 + 2*(e*x + d)*a*b*f^3*n*log(e*x + d)/e + 3
*(e*x + d)^2*a*b*f^2*g*n*log(e*x + d)/e^2 - 6*(e*x + d)*a*b*d*f^2*g*n*log(e
*x + d)/e^2 + 2*(e*x + d)^3*a*b*f*g^2*n*log(e*x + d)/e^3 - 6*(e*x + d)^2*a*
b*d*f*g^2*n*log(e*x + d)/e^3 + 6*(e*x + d)*a*b*d^2*f*g^2*n*log(e*x + d)/e^3
 + 1/2*(e*x + d)^4*a*b*g^3*n*log(e*x + d)/e^4 - 2*(e*x + d)^3*a*b*d*g^3*n*l
og(e*x + d)/e^4 + 3*(e*x + d)^2*a*b*d^2*g^3*n*log(e*x + d)/e^4 - 2*(e*x + d
)*a*b*d^3*g^3*n*log(e*x + d)/e^4 - 2*(e*x + d)*b^2*f^3*n*log(c)/e - 3/2*(e*
x + d)^2*b^2*f^2*g*n*log(c)/e^2 + 6*(e*x + d)*b^2*d*f^2*g*n*log(c)/e^2 - 2/
3*(e*x + d)^3*b^2*f*g^2*n*log(c)/e^3 + 3*(e*x + d)^2*b^2*d*f*g^2*n*log(c)/e
^3 - 6*(e*x + d)*b^2*d^2*f*g^2*n*log(c)/e^3 - 1/8*(e*x + d)^4*b^2*g^3*n*log
(c)/e^4 + 2/3*(e*x + d)^3*b^2*d*g^3*n*log(c)/e^4 - 3/2*(e*x + d)^2*b^2*d^2*
g^3*n*log(c)/e^4 + 2*(e*x + d)*b^2*d^3*g^3*n*log(c)/e^4 + (e*x + d)*b^2*f^3
*log(c)^2/e + 3/2*(e*x + d)^2*b^2*f^2*g*log(c)^2/e^2 - 3*(e*x + d)*b^2*d*f^
2*g*log(c)^2/e^2 + (e*x + d)^3*b^2*f*g^2*log(c)^2/e^3 - 3*(e*x + d)^2*b^2*d
*f*g^2*log(c)^2/e^3 + 3*(e*x + d)*b^2*d^2*f*g^2*log(c)^2/e^3 + 1/4*(e*x + d
)^4*b^2*g^3*log(c)^2/e^4 - (e*x + d)^3*b^2*d*g^3*log(c)^2/e^4 + 3/2*(e*x +
d)^2*b^2*d^2*g^3*log(c)^2/e^4 - (e*x + d)*b^2*d^3*g^3*log(c)^2/e^4 - 2*(e*x
 + d)*a*b*f^3*n/e - 3/2*(e*x + d)^2*a*b*f^2*g*n/e^2 + 6*(e*x + d)*a*b*d*f^2
*g*n/e^2 - 2/3*(e*x + d)^3*a*b*f*g^2*n/e^3 + 3*(e*x + d)^2*a*b*d*f*g^2*n/e^
3 - 6*(e*x + d)*a*b*d^2*f*g^2*n/e^3 - 1/8*(e*x + d)^4*a*b*g^3*n/e^4 + 2/3*(
e*x + d)^3*a*b*d*g^3*n/e^4 - 3/2*(e*x + d)^2*a*b*d^2*g^3*n/e^4 + 2*(e*x + d
)*a*b*d^3*g^3*n/e^4 + 2*(e*x + d)*a*b*f^3*log(c)/e + 3*(e*x + d)^2*a*b*f^2*
g*log(c)/e^2 - 6*(e*x + d)*a*b*d*f^2*g*log(c)/e^2 + 2*(e*x + d)^3*a*b*f*g^2
*log(c)/e^3 - 6*(e*x + d)^2*a*b*d*f*g^2*log(c)/e^3 + 6*(e*x + d)*a*b*d^2*f*
g^2*log(c)/e^3 + 1/2*(e*x + d)^4*a*b*g^3*log(c)/e^4 - 2*(e*x + d)^3*a*b*d*g
^3*log(c)/e^4 + 3*(e*x + d)^2*a*b*d^2*g^3*log(c)/e^4 - 2*(e*x + d)*a*b*d^3*
g^3*log(c)/e^4 + (e*x + d)*a^2*f^3/e + 3/2*(e*x + d)^2*a^2*f^2*g/e^2 - 3*(e
*x + d)*a^2*d*f^2*g/e^2 + (e*x + d)^3*a^2*f*g^2/e^3 - 3*(e*x + d)^2*a^2*d*f
*g^2/e^3 + 3*(e*x + d)*a^2*d^2*f*g^2/e^3 + 1/4*(e*x + d)^4*a^2*g^3/e^4 - (e
*x + d)^3*a^2*d*g^3/e^4 + 3/2*(e*x + d)^2*a^2*d^2*g^3/e^4 - (e*x + d)*a^2*d
^3*g^3/e^4
```

## Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 1051, normalized size of antiderivative = 2.88

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^2 \, dx$$

$$= x\left(\frac{72\,a^2\,d\,e^2\,f^2\,g+24\,a^2\,e^3\,f^3-48\,a\,b\,e^3\,f^3\,n-12\,b^2\,d^3\,g^3\,n^2+48\,b^2\,d^2\,e\,f\,g^2\,n^2-72\,b^2\,d\,e^2\,f^2\,g\,n^2+48}{24\,e^3}\right.$$

$$+\frac{d\left(\frac{d\left(\frac{g^2\,\left(6\,a^2\,d\,g+18\,a^2\,e\,f-b^2\,d\,g\,n^2+4\,b^2\,e\,f\,n^2-12\,a\,b\,e\,f\,n\right)}{6\,e}-\frac{d\,g^3\,\left(8\,a^2-4\,a\,b\,n+b^2\,n^2\right)}{8\,e}\right)}{e}-\frac{g\,(12\,a^2\,d\,e\,f\,g+12\,a^2\,e^2\,f^2-12\,a\,b\,e^2\,f^2\,n+b^2\,d^2}{4\,e^2}\right.}{e}$$

$$-x^2\left(\frac{d\left(\frac{g^2\,(6\,a^2\,d\,g+18\,a^2\,e\,f-b^2\,d\,g\,n^2+4\,b^2\,e\,f\,n^2-12\,a\,b\,e\,f\,n)}{6\,e}-\frac{d\,g^3\,(8\,a^2-4\,a\,b\,n+b^2\,n^2)}{8\,e}\right)}{2\,e}\right.$$

$$\left.-\frac{g\,(12\,a^2\,d\,e\,f\,g+12\,a^2\,e^2\,f^2-12\,a\,b\,e^2\,f^2\,n+b^2\,d^2\,g^2\,n^2-4\,b^2\,d\,e\,f\,g\,n^2+6\,b^2\,e^2\,f^2\,n^2)}{8\,e^2}\right)$$

$$+\ln(c\,(d+e\,x)^n)^2\left(b^2\,f^3\,x-\frac{d\,(b^2\,d^3\,g^3-4\,b^2\,d^2\,e\,f\,g^2+6\,b^2\,d\,e^2\,f^2\,g-4\,b^2\,e^3\,f^3)}{4\,e^4}\right.$$

$$\left.+\frac{b^2\,g^3\,x^4}{4}+\frac{3\,b^2\,f^2\,g\,x^2}{2}+b^2\,f\,g^2\,x^3\right)$$

$$+x^3\left(\frac{g^2\,(6\,a^2\,d\,g+18\,a^2\,e\,f-b^2\,d\,g\,n^2+4\,b^2\,e\,f\,n^2-12\,a\,b\,e\,f\,n)}{18\,e}\right.$$

$$\left.-\frac{d\,g^3\,(8\,a^2-4\,a\,b\,n+b^2\,n^2)}{24\,e}\right)$$

$$+\ln(c\,(d+e\,x)^n)\left(\frac{x\left(\frac{d\left(\frac{d\left(\frac{8\,b\,g^2\,(a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{2\,b\,d\,g^3\,(4\,a-b\,n)}{e}\right)}{e}-\frac{12\,b\,f\,g\,(2\,a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{e}\right)}{2\,e}+\frac{4\,b\,f^2\,(3\,a\,d\,g+a\,e\,f-b}{e}\right.}{2}\right.$$

$$+\frac{x^3\left(\frac{4\,b\,g^2\,(a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{3\,e}-\frac{b\,d\,g^3\,(4\,a-b\,n)}{3\,e}\right)}{2}$$

$$x^2\left(\frac{d\left(\frac{8\,b\,g^2\,(a\,d\,g+3\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{2\,b\,d\,g^3\,(4\,a-b\,n)}{e}\right)}{4\,e}-\frac{3\,b\,f\,g\,(2\,a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{e}\right)$$

```
[In] int((f + g*x)^3*(a + b*log(c*(d + e*x)^n))^2,x)
```

```
[Out] x*((24*a^2*e^3*f^3 - 12*b^2*d^3*g^3*n^2 + 48*b^2*e^3*f^3*n^2 - 48*a*b*e^3*f
^3*n + 72*a^2*d*e^2*f^2*g - 72*b^2*d*e^2*f^2*g*n^2 + 48*b^2*d^2*e*f*g^2*n^2
)/(24*e^3) + (d*((d*((g^2*(6*a^2*d*g + 18*a^2*e*f - b^2*d*g*n^2 + 4*b^2*e*f
*n^2 - 12*a*b*e*f*n))/(6*e) - (d*g^3*(8*a^2 + b^2*n^2 - 4*a*b*n))/(8*e)))/e
 - (g*(12*a^2*e^2*f^2 + b^2*d^2*g^2*n^2 + 6*b^2*e^2*f^2*n^2 - 12*a*b*e^2*f^
2*n + 12*a^2*d*e*f*g - 4*b^2*d*e*f*g*n^2))/(4*e^2)))/e) - x^2*((d*((g^2*(6*
a^2*d*g + 18*a^2*e*f - b^2*d*g*n^2 + 4*b^2*e*f*n^2 - 12*a*b*e*f*n))/(6*e) -
 (d*g^3*(8*a^2 + b^2*n^2 - 4*a*b*n))/(8*e)))/(2*e) - (g*(12*a^2*e^2*f^2 + b
^2*d^2*g^2*n^2 + 6*b^2*e^2*f^2*n^2 - 12*a*b*e^2*f^2*n + 12*a^2*d*e*f*g - 4*
b^2*d*e*f*g*n^2))/(8*e^2)) + log(c*(d + e*x)^n)^2*(b^2*f^3*x - (d*(b^2*d^3*
g^3 - 4*b^2*e^3*f^3 + 6*b^2*d*e^2*f^2*g - 4*b^2*d^2*e*f*g^2))/(4*e^4) + (b^
2*g^3*x^4)/4 + (3*b^2*f^2*g*x^2)/2 + b^2*f*g^2*x^3) + x^3*((g^2*(6*a^2*d*g
+ 18*a^2*e*f - b^2*d*g*n^2 + 4*b^2*e*f*n^2 - 12*a*b*e*f*n))/(18*e) - (d*g^3
*(8*a^2 + b^2*n^2 - 4*a*b*n))/(24*e)) + log(c*(d + e*x)^n)*((x*((d*((d*((8*
b*g^2*(a*d*g + 3*a*e*f - b*e*f*n))/e - (2*b*d*g^3*(4*a - b*n))/e))/e - (12*
b*f*g*(2*a*d*g + 2*a*e*f - b*e*f*n))/e))/(2*e) + (4*b*f^2*(3*a*d*g + a*e*f
- b*e*f*n))/e))/2 + (x^3*((4*b*g^2*(a*d*g + 3*a*e*f - b*e*f*n))/(3*e) - (b*
d*g^3*(4*a - b*n))/(3*e)))/2 - (x^2*((d*((8*b*g^2*(a*d*g + 3*a*e*f - b*e*f*
n))/e - (2*b*d*g^3*(4*a - b*n))/e))/(4*e) - (3*b*f*g*(2*a*d*g + 2*a*e*f - b
*e*f*n))/e))/2 + (b*g^3*x^4*(4*a - b*n))/8) + (log(d + e*x)*(25*b^2*d^4*g^3
*n^2 - 12*a*b*d^4*g^3*n - 48*b^2*d*e^3*f^3*n^2 - 88*b^2*d^3*e*f*g^2*n^2 + 4
8*a*b*d*e^3*f^3*n + 108*b^2*d^2*e^2*f^2*g*n^2 - 72*a*b*d^2*e^2*f^2*g*n + 48
*a*b*d^3*e*f*g^2*n))/(24*e^4) + (g^3*x^4*(8*a^2 + b^2*n^2 - 4*a*b*n))/32
```

# 3.45 $\int (f+gx)^2 (a+b\log(c(d+ex)^n))^2 \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 287

$$\begin{aligned}
&\int (f+gx)^2 (a+b\log(c(d+ex)^n))^2 \, dx \\
&= \frac{2b^2(ef-dg)^2n^2x}{e^2} + \frac{b^2g(ef-dg)n^2(d+ex)^2}{2e^3} + \frac{2b^2g^2n^2(d+ex)^3}{27e^3} \\
&\quad + \frac{b^2(ef-dg)^3n^2\log^2(d+ex)}{3e^3g} - \frac{2b(ef-dg)^2n(d+ex)(a+b\log(c(d+ex)^n))}{e^3} \\
&\quad - \frac{bg(ef-dg)n(d+ex)^2(a+b\log(c(d+ex)^n))}{e^3} \\
&\quad - \frac{2bg^2n(d+ex)^3(a+b\log(c(d+ex)^n))}{9e^3} \\
&\quad - \frac{2b(ef-dg)^3n\log(d+ex)(a+b\log(c(d+ex)^n))}{3e^3g} \\
&\quad + \frac{(f+gx)^3(a+b\log(c(d+ex)^n))^2}{3g}
\end{aligned}$$

```
[Out] 2*b^2*(-d*g+e*f)^2*n^2*x/e^2+1/2*b^2*g*(-d*g+e*f)*n^2*(e*x+d)^2/e^3+2/27*b^
2*g^2*n^2*(e*x+d)^3/e^3+1/3*b^2*(-d*g+e*f)^3*n^2*ln(e*x+d)^2/e^3/g-2*b*(-d*
g+e*f)^2*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))/e^3-b*g*(-d*g+e*f)*n*(e*x+d)^2*(a+
b*ln(c*(e*x+d)^n))/e^3-2/9*b*g^2*n*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))/e^3-2/3*
b*(-d*g+e*f)^3*n*ln(e*x+d)*(a+b*ln(c*(e*x+d)^n))/e^3/g+1/3*(g*x+f)^3*(a+b*l
n(c*(e*x+d)^n))^2/g
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2445, 2458, 45, 2372, 12, 14, 2338}

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^2\,dx = -\frac{2bn(ef-dg)^3\log(d+ex)(a+b\log(c(d+ex)^n))}{3e^3g} - \frac{2bn(d+ex)(ef-dg)^2(a+b\log(c(d+ex)^n))}{e^3} - \frac{bgn(d+ex)^2(ef-dg)(a+b\log(c(d+ex)^n))}{e^3} - \frac{2bg^2n(d+ex)^3(a+b\log(c(d+ex)^n))}{9e^3} + \frac{(f+gx)^3(a+b\log(c(d+ex)^n))^2}{3g} + \frac{b^2gn^2(d+ex)^2(ef-dg)}{2e^3} + \frac{b^2n^2(ef-dg)^3\log^2(d+ex)}{3e^3g} + \frac{2b^2g^2n^2(d+ex)^3}{27e^3} + \frac{2b^2n^2x(ef-dg)^2}{e^2}$$

```
[In] Int[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (2*b^2*(e*f - d*g)^2*n^2*x)/e^2 + (b^2*g*(e*f - d*g)*n^2*(d + e*x)^2)/(2*e^
3) + (2*b^2*g^2*n^2*(d + e*x)^3)/(27*e^3) + (b^2*(e*f - d*g)^3*n^2*Log[d +
e*x]^2)/(3*e^3*g) - (2*b*(e*f - d*g)^2*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n
]))/e^3 - (b*g*(e*f - d*g)*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/e^3 -
(2*b*g^2*n*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))/(9*e^3) - (2*b*(e*f - d*
g)^3*n*Log[d + e*x]*(a + b*Log[c*(d + e*x)^n]))/(3*e^3*g) + ((f + g*x)^3*(a
 + b*Log[c*(d + e*x)^n])^2)/(3*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^3\,(a+b\log\left(c(d+ex)^n\right))^2}{3g} - \frac{(2ben)\int \frac{(f+gx)^3(a+b\log(c(d+ex)^n))}{d+ex}\,dx}{3g}$$

$$= \frac{(f+gx)^3\,(a+b\log\left(c(d+ex)^n\right))^2}{3g} - \frac{(2bn)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^3(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{3g}$$

$$= -\frac{2b(ef-dg)^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} - \frac{bg(ef-dg)n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} - \frac{2bg^2n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3} - \frac{2b(ef-dg)^3n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^3g} + \frac{(f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g} + \frac{(2b^2n^2)\operatorname{Subst}\left(\int \frac{gx\left(18e^2f^2+9efg(-4d+x)+g^2\left(18d^2-9dx+2x^2\right)\right)+6(ef-dg)^3\log(x)}{6e^3x}\,dx, x, d+ex\right)}{3g}$$

$$= -\frac{2b(ef-dg)^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} - \frac{bg(ef-dg)n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} - \frac{2bg^2n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3} - \frac{2b(ef-dg)^3n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^3g} + \frac{(f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g} + \frac{(b^2n^2)\operatorname{Subst}\left(\int \frac{gx\left(18e^2f^2+9efg(-4d+x)+g^2\left(18d^2-9dx+2x^2\right)\right)+6(ef-dg)^3\log(x)}{x}\,dx, x, d+ex\right)}{9e^3g}$$

$$= -\frac{2b(ef-dg)^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} - \frac{bg(ef-dg)n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} - \frac{2bg^2n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3} - \frac{2b(ef-dg)^3n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^3g} + \frac{(f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g} + \frac{(b^2n^2)\operatorname{Subst}\left(\int \left(g(18(ef-dg)^2+9g(ef-dg)x+2g^2x^2)+\frac{6(ef-dg)^3\log(x)}{x}\right)dx, x, d+ex\right)}{9e^3g}$$

$$
\begin{aligned}
&= -\frac{2b(ef-dg)^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} \\
&\quad - \frac{bg(ef-dg)n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} \\
&\quad - \frac{2bg^2n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3} \\
&\quad - \frac{2b(ef-dg)^3n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^3g} \\
&\quad + \frac{(f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g} \\
&\quad + \frac{(b^2n^2)\,\text{Subst}\left(\int\left(18(ef-dg)^2+9g(ef-dg)x+2g^2x^2\right)\,dx,x,d+ex\right)}{9e^3} \\
&\quad + \frac{(2b^2(ef-dg)^3n^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,d+ex\right)}{3e^3g} \\
&= \frac{2b^2(ef-dg)^2n^2x}{e^2}+\frac{b^2g(ef-dg)n^2(d+ex)^2}{2e^3}+\frac{2b^2g^2n^2(d+ex)^3}{27e^3} \\
&\quad + \frac{b^2(ef-dg)^3n^2\log^2(d+ex)}{3e^3g}-\frac{2b(ef-dg)^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} \\
&\quad - \frac{bg(ef-dg)n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} \\
&\quad - \frac{2bg^2n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3} \\
&\quad - \frac{2b(ef-dg)^3n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^3g} \\
&\quad + \frac{(f+gx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.86

$$
\int(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx
= \frac{54(ef-dg)^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2+54g(ef-dg)(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2+18g^2(\cdots}{\cdots}
$$

```
[In] Integrate[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (54*(e*f - d*g)^2*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 + 54*g*(e*f - d*g)
*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2 + 18*g^2*(d + e*x)^3*(a + b*Log[c
*(d + e*x)^n])^2 - 108*b*(e*f - d*g)^2*n*(e*(a - b*n)*x + b*(d + e*x)*Log[c
*(d + e*x)^n]) + 27*b*g*(e*f - d*g)*n*(b*e*n*x*(2*d + e*x) - 2*(d + e*x)^2*
(a + b*Log[c*(d + e*x)^n])) + 4*b*g^2*n*(b*e*n*x*(3*d^2 + 3*d*e*x + e^2*x^2
) - 3*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])))/(54*e^3)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 849 vs. 2(275) = 550.

Time = 1.48 (sec) , antiderivative size = 850, normalized size of antiderivative = 2.96

| method | result |
|---|---|
| parallelrisch | $\frac{108x^2\ln(c(ex+d)^n)ab\,e^3fgn-108b^2d\,e^2f^2n^3+36ab\,d^3g^2n^2-54a^2d\,e^2f^2n+4x^3b^2e^3g^2n^3+18x^3a^2e^3g^2n+108x\,b^2e^3f^2n^3+18\ln(}{}$ |
| risch | Expression too large to display |

[In] int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)

[Out] 1/54*(108*x^2*ln(c*(e*x+d)^n)*a*b*e^3*f*g*n-108*b^2*d*e^2*f^2*n^3+36*a*b*d^3*g^2*n^2-54*a^2*d*e^2*f^2*n+4*x^3*b^2*e^3*g^2*n^3+18*x^3*a^2*e^3*g^2*n+108*x*b^2*e^3*f^2*n^3+18*ln(c*(e*x+d)^n)^2*b^2*d^3*g^2*n-66*ln(c*(e*x+d)^n)*b^2*d^3*g^2*n^2+54*x*a^2*e^3*f^2*n+162*b^2*d^2*e*f*g*n^3+108*a*b*d*e^2*f^2*n^2+18*x^3*ln(c*(e*x+d)^n)^2*b^2*e^3*g^2*n-12*x^3*ln(c*(e*x+d)^n)*b^2*e^3*g^2*n^2-12*x^3*a*b*e^3*g^2*n^2-15*x^2*b^2*d*e^2*g^2*n^3+27*x^2*b^2*e^3*f*g*n^3+54*x*ln(c*(e*x+d)^n)^2*b^2*e^3*f^2*n-108*x*ln(c*(e*x+d)^n)*b^2*e^3*f^2*n^2+66*x*b^2*d^2*e*g^2*n^3+108*x*ln(c*(e*x+d)^n)*b^2*d*e^2*f*g*n^2+108*x*a*b*d*e^2*f*g*n^2-108*ln(c*(e*x+d)^n)*a*b*d^2*e*f*g*n+54*x^2*a^2*e^3*f*g*n-108*x*a*b*e^3*f^2*n^2+54*ln(c*(e*x+d)^n)^2*b^2*d*e^2*f^2*n-108*ln(c*(e*x+d)^n)*b^2*d*e^2*f^2*n^2+36*ln(c*(e*x+d)^n)*a*b*d^3*g^2*n-66*b^2*d^3*g^2*n^3-108*a*b*d^2*e*f*g*n^2+36*x^3*ln(c*(e*x+d)^n)*a*b*e^3*g^2*n+54*x^2*ln(c*(e*x+d)^n)^2*b^2*e^3*f*g*n+18*x^2*ln(c*(e*x+d)^n)*b^2*d*e^2*g^2*n^2-54*x^2*ln(c*(e*x+d)^n)*b^2*e^3*f*g*n^2+18*x^2*a*b*d*e^2*g^2*n^2-54*x^2*a*b*e^3*f*g*n^2-36*x*ln(c*(e*x+d)^n)*b^2*d^2*e*g^2*n^2-162*x*b^2*d*e^2*f*g*n^3+108*x*ln(c*(e*x+d)^n)*a*b*e^3*f^2*n-36*x*a*b*d^2*e*g^2*n^2-54*ln(c*(e*x+d)^n)^2*b^2*d^2*e*f*g*n+162*ln(c*(e*x+d)^n)*b^2*d^2*e*f*g*n^2+108*ln(c*(e*x+d)^n)*a*b*d*e^2*f^2*n)/n/e^3

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. 2(275) = 550.

Time = 0.30 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.65

$$\int (f+gx)^2\,(a+b\log{(c(d+ex)^n)})^2\,dx$$
$$= \frac{2\,(2\,b^2e^3g^2n^2-6\,abe^3g^2n+9\,a^2e^3g^2)x^3+3\,(18\,a^2e^3fg+(9\,b^2e^3fg-5\,b^2de^2g^2)n^2-6\,(3\,abe^3fg-abde^2g^2)}{}$$

[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")

[Out] 1/54*(2*(2*b^2*e^3*g^2*n^2 - 6*a*b*e^3*g^2*n + 9*a^2*e^3*g^2)*x^3 + 3*(18*a^2*e^3*f*g + (9*b^2*e^3*f*g - 5*b^2*d*e^2*g^2)*n^2 - 6*(3*a*b*e^3*f*g - a*b

```
*d*e^2*g^2)*n)*x^2 + 18*(b^2*e^3*g^2*n^2*x^3 + 3*b^2*e^3*f*g*n^2*x^2 + 3*b^
2*e^3*f^2*n^2*x + (3*b^2*d*e^2*f^2 - 3*b^2*d^2*e*f*g + b^2*d^3*g^2)*n^2)*lo
g(e*x + d)^2 + 18*(b^2*e^3*g^2*x^3 + 3*b^2*e^3*f*g*x^2 + 3*b^2*e^3*f^2*x)*l
og(c)^2 + 6*(9*a^2*e^3*f^2 + (18*b^2*e^3*f^2 - 27*b^2*d*e^2*f*g + 11*b^2*d^
2*e*g^2)*n^2 - 6*(3*a*b*e^3*f^2 - 3*a*b*d*e^2*f*g + a*b*d^2*e*g^2)*n)*x - 6
*(2*(b^2*e^3*g^2*n^2 - 3*a*b*e^3*g^2*n)*x^3 + (18*b^2*d*e^2*f^2 - 27*b^2*d^
2*e*f*g + 11*b^2*d^3*g^2)*n^2 - 3*(6*a*b*e^3*f*g*n - (3*b^2*e^3*f*g - b^2*d
*e^2*g^2)*n^2)*x^2 - 6*(3*a*b*d*e^2*f^2 - 3*a*b*d^2*e*f*g + a*b*d^3*g^2)*n
- 6*(3*a*b*e^3*f^2*n - (3*b^2*e^3*f^2 - 3*b^2*d*e^2*f*g + b^2*d^2*e*g^2)*n^
2)*x - 6*(b^2*e^3*g^2*n*x^3 + 3*b^2*e^3*f*g*n*x^2 + 3*b^2*e^3*f^2*n*x + (3*
b^2*d*e^2*f^2 - 3*b^2*d^2*e*f*g + b^2*d^3*g^2)*n)*log(c))*log(e*x + d) - 6*
(2*(b^2*e^3*g^2*n - 3*a*b*e^3*g^2)*x^3 - 3*(6*a*b*e^3*f*g - (3*b^2*e^3*f*g
- b^2*d*e^2*g^2)*n)*x^2 - 6*(3*a*b*e^3*f^2 - (3*b^2*e^3*f^2 - 3*b^2*d*e^2*f
*g + b^2*d^2*e*g^2)*n)*x)*log(c))/e^3
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(274) = 548$.

Time = 1.23 (sec) , antiderivative size = 774, normalized size of antiderivative = 2.70

$$\int (f+gx)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^2 \, dx$$
$$= \begin{cases} a^2f^2x + a^2fgx^2 + \frac{a^2g^2x^3}{3} + \frac{2abd^3g^2\log\left(c(d+ex)^n\right)}{3e^3} - \frac{2abd^2fg\log\left(c(d+ex)^n\right)}{e^2} - \frac{2abd^2g^2nx}{3e^2} + \frac{2abdf^2\log\left(c(d+ex)^n\right)}{e} + 2abd \\ \left(a+b\log\left(cd^n\right)\right)^2\left(f^2x + fgx^2 + \frac{g^2x^3}{3}\right) \end{cases}$$

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Piecewise((a**2*f**2*x + a**2*f*g*x**2 + a**2*g**2*x**3/3 + 2*a*b*d**3*g**2
*log(c*(d + e*x)**n)/(3*e**3) - 2*a*b*d**2*f*g*log(c*(d + e*x)**n)/e**2 - 2
*a*b*d**2*g**2*n*x/(3*e**2) + 2*a*b*d*f**2*log(c*(d + e*x)**n)/e + 2*a*b*d*
f*g*n*x/e + a*b*d*g**2*n*x**2/(3*e) - 2*a*b*f**2*n*x + 2*a*b*f**2*x*log(c*(
d + e*x)**n) - a*b*f*g*n*x**2 + 2*a*b*f*g*x**2*log(c*(d + e*x)**n) - 2*a*b*
g**2*n*x**3/9 + 2*a*b*g**2*x**3*log(c*(d + e*x)**n)/3 - 11*b**2*d**3*g**2*n
*log(c*(d + e*x)**n)/(9*e**3) + b**2*d**3*g**2*log(c*(d + e*x)**n)**2/(3*e*
*3) + 3*b**2*d**2*f*g*n*log(c*(d + e*x)**n)/e**2 - b**2*d**2*f*g*log(c*(d +
 e*x)**n)**2/e**2 + 11*b**2*d**2*g**2*n**2*x/(9*e**2) - 2*b**2*d**2*g**2*n*
x*log(c*(d + e*x)**n)/(3*e**2) - 2*b**2*d*f**2*n*log(c*(d + e*x)**n)/e + b*
*2*d*f**2*log(c*(d + e*x)**n)**2/e - 3*b**2*d*f*g*n**2*x/e + 2*b**2*d*f*g*n
*x*log(c*(d + e*x)**n)/e - 5*b**2*d*g**2*n**2*x**2/(18*e) + b**2*d*g**2*n*x
**2*log(c*(d + e*x)**n)/(3*e) + 2*b**2*f**2*n**2*x - 2*b**2*f**2*n*x*log(c*
(d + e*x)**n) + b**2*f**2*x*log(c*(d + e*x)**n)**2 + b**2*f*g*n**2*x**2/2 -
 b**2*f*g*n*x**2*log(c*(d + e*x)**n) + b**2*f*g*x**2*log(c*(d + e*x)**n)**2
 + 2*b**2*g**2*n**2*x**3/27 - 2*b**2*g**2*n*x**3*log(c*(d + e*x)**n)/9 + b*
```

```
*2*g**2*x**3*log(c*(d + e*x)**n)**2/3, Ne(e, 0)), ((a + b*log(c*d**n))**2*(
f**2*x + f*g*x**2 + g**2*x**3/3), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(275) = 550.

Time = 0.21 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.93

$$\int (f+gx)^2(a+b\log(c(d+ex)^n))^2\,dx$$
$$= \frac{1}{3}b^2g^2x^3\log\left((ex+d)^nc\right)^2 + \frac{2}{3}abg^2x^3\log\left((ex+d)^nc\right) + b^2fgx^2\log\left((ex+d)^nc\right)^2 + \frac{1}{3}a^2g^2x^3 - 2abef^2n\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) + \frac{1}{9}abeg^2n\left(\frac{6d^3\log(ex+d)}{e^4} - \frac{2e^2x^3 - 3dex^2 + 6d^2x}{e^3}\right) - abefgn\left(\frac{2d^2\log(ex+d)}{e^3} + \frac{ex^2 - 2dx}{e^2}\right) + 2abfgx^2\log\left((ex+d)^nc\right) + b^2f^2x\log\left((ex+d)^nc\right)^2 + a^2fgx^2 + 2abf^2x\log\left((ex+d)^nc\right) - \left(2en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log\left((ex+d)^nc\right) + \frac{\left(d\log(ex+d)^2 - 2ex + 2d\log(ex+d)\right)n^2}{e}\right)b^2f^2 - \frac{1}{2}\left(2en\left(\frac{2d^2\log(ex+d)}{e^3} + \frac{ex^2 - 2dx}{e^2}\right)\log\left((ex+d)^nc\right) - \frac{\left(e^2x^2 + 2d^2\log(ex+d)^2 - 6dex + 6d^2\log\right.}{e^2}\right. + \frac{1}{54}\left(6en\left(\frac{6d^3\log(ex+d)}{e^4} - \frac{2e^2x^3 - 3dex^2 + 6d^2x}{e^3}\right)\log\left((ex+d)^nc\right) + \frac{\left(4e^3x^3 - 15de^2x^2 - 18d^3\log\right.}{}\right. + a^2f^2x$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*g^2*x^3*log((e*x + d)^n*c)^2 + 2/3*a*b*g^2*x^3*log((e*x + d)^n*c) +
 b^2*f*g*x^2*log((e*x + d)^n*c)^2 + 1/3*a^2*g^2*x^3 - 2*a*b*e*f^2*n*(x/e -
d*log(e*x + d)/e^2) + 1/9*a*b*e*g^2*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3
- 3*d*e*x^2 + 6*d^2*x)/e^3) - a*b*e*f*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2
- 2*d*x)/e^2) + 2*a*b*f*g*x^2*log((e*x + d)^n*c) + b^2*f^2*x*log((e*x + d)^
n*c)^2 + a^2*f*g*x^2 + 2*a*b*f^2*x*log((e*x + d)^n*c) - (2*e*n*(x/e - d*log
(e*x + d)/e^2)*log((e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x
 + d))*n^2/e)*b^2*f^2 - 1/2*(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x
)/e^2)*log((e*x + d)^n*c) - (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d
^2*log(e*x + d))*n^2/e^2)*b^2*f*g + 1/54*(6*e*n*(6*d^3*log(e*x + d)/e^4 - (
2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3)*log((e*x + d)^n*c) + (4*e^3*x^3 - 15*
d*e^2*x^2 - 18*d^3*log(e*x + d)^2 + 66*d^2*e*x - 66*d^3*log(e*x + d))*n^2/e
^3)*b^2*g^2 + a^2*f^2*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. 2(275) = 550.

Time = 0.41 (sec) , antiderivative size = 1315, normalized size of antiderivative = 4.58

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^2\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] (e*x + d)*b^2*f^2*n^2*log(e*x + d)^2/e + (e*x + d)^2*b^2*f*g*n^2*log(e*x +
d)^2/e^2 - 2*(e*x + d)*b^2*d*f*g*n^2*log(e*x + d)^2/e^2 + 1/3*(e*x + d)^3*b
^2*g^2*n^2*log(e*x + d)^2/e^3 - (e*x + d)^2*b^2*d*g^2*n^2*log(e*x + d)^2/e^
3 + (e*x + d)*b^2*d^2*g^2*n^2*log(e*x + d)^2/e^3 - 2*(e*x + d)*b^2*f^2*n^2*
log(e*x + d)/e - (e*x + d)^2*b^2*f*g*n^2*log(e*x + d)/e^2 + 4*(e*x + d)*b^2
*d*f*g*n^2*log(e*x + d)/e^2 - 2/9*(e*x + d)^3*b^2*g^2*n^2*log(e*x + d)/e^3
+ (e*x + d)^2*b^2*d*g^2*n^2*log(e*x + d)/e^3 - 2*(e*x + d)*b^2*d^2*g^2*n^2*
log(e*x + d)/e^3 + 2*(e*x + d)*b^2*f^2*n*log(e*x + d)*log(c)/e + 2*(e*x + d
)^2*b^2*f*g*n*log(e*x + d)*log(c)/e^2 - 4*(e*x + d)*b^2*d*f*g*n*log(e*x + d
)*log(c)/e^2 + 2/3*(e*x + d)^3*b^2*g^2*n*log(e*x + d)*log(c)/e^3 - 2*(e*x +
 d)^2*b^2*d*g^2*n*log(e*x + d)*log(c)/e^3 + 2*(e*x + d)*b^2*d^2*g^2*n*log(e
*x + d)*log(c)/e^3 + 2*(e*x + d)*b^2*f^2*n^2/e + 1/2*(e*x + d)^2*b^2*f*g*n^
2/e^2 - 4*(e*x + d)*b^2*d*f*g*n^2/e^2 + 2/27*(e*x + d)^3*b^2*g^2*n^2/e^3 -
1/2*(e*x + d)^2*b^2*d*g^2*n^2/e^3 + 2*(e*x + d)*b^2*d^2*g^2*n^2/e^3 + 2*(e*
x + d)*a*b*f^2*n*log(e*x + d)/e + 2*(e*x + d)^2*a*b*f*g*n*log(e*x + d)/e^2
- 4*(e*x + d)*a*b*d*f*g*n*log(e*x + d)/e^2 + 2/3*(e*x + d)^3*a*b*g^2*n*log(
e*x + d)/e^3 - 2*(e*x + d)^2*a*b*d*g^2*n*log(e*x + d)/e^3 + 2*(e*x + d)*a*b
*d^2*g^2*n*log(e*x + d)/e^3 - 2*(e*x + d)*b^2*f^2*n*log(c)/e - (e*x + d)^2*
b^2*f*g*n*log(c)/e^2 + 4*(e*x + d)*b^2*d*f*g*n*log(c)/e^2 - 2/9*(e*x + d)^3
*b^2*g^2*n*log(c)/e^3 + (e*x + d)^2*b^2*d*g^2*n*log(c)/e^3 - 2*(e*x + d)*b^
2*d^2*g^2*n*log(c)/e^3 + (e*x + d)*b^2*f^2*log(c)^2/e + (e*x + d)^2*b^2*f*g
*log(c)^2/e^2 - 2*(e*x + d)*b^2*d*f*g*log(c)^2/e^2 + 1/3*(e*x + d)^3*b^2*g^
2*log(c)^2/e^3 - (e*x + d)^2*b^2*d*g^2*log(c)^2/e^3 + (e*x + d)*b^2*d^2*g^2
*log(c)^2/e^3 - 2*(e*x + d)*a*b*f^2*n/e - (e*x + d)^2*a*b*f*g*n/e^2 + 4*(e*
x + d)*a*b*d*f*g*n/e^2 - 2/9*(e*x + d)^3*a*b*g^2*n/e^3 + (e*x + d)^2*a*b*d*
g^2*n/e^3 - 2*(e*x + d)*a*b*d^2*g^2*n/e^3 + 2*(e*x + d)*a*b*f^2*log(c)/e +
2*(e*x + d)^2*a*b*f*g*log(c)/e^2 - 4*(e*x + d)*a*b*d*f*g*log(c)/e^2 + 2/3*(
e*x + d)^3*a*b*g^2*log(c)/e^3 - 2*(e*x + d)^2*a*b*d*g^2*log(c)/e^3 + 2*(e*x
 + d)*a*b*d^2*g^2*log(c)/e^3 + (e*x + d)*a^2*f^2/e + (e*x + d)^2*a^2*f*g/e^
2 - 2*(e*x + d)*a^2*d*f*g/e^2 + 1/3*(e*x + d)^3*a^2*g^2/e^3 - (e*x + d)^2*a
^2*d*g^2/e^3 + (e*x + d)*a^2*d^2*g^2/e^3
```

## Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 591, normalized size of antiderivative = 2.06

$$\int (f+gx)^2\left(a+b\log \left(c(d+ex)^n\right)\right)^2\,dx$$

$$= \ln\left(c\,(d+e\,x)^n\right)\left(\frac{x^2\left(\frac{3\,b\,g\,(a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{b\,d\,g^2\,(3\,a-b\,n)}{e}\right)}{3}-\frac{x\left(\frac{d\left(\frac{18\,b\,g\,(a\,d\,g+2\,a\,e\,f-b\,e\,f\,n)}{e}-\frac{6\,b\,d\,g^2\,(3\,a-b\,n)}{e}\right)}{3\,e}-\frac{6\,b\,f\,(2\,a\,d\,g+a\,e\,f-b\,e\,f\,n)}{e}\right)}{3}+\frac{2\,b\,g^2\,x^3\,(3\,a-b\,n)}{9}\right)+x\left(\frac{18\,a^2\,d\,e\,f\,g+9\,a^2\,e^2\,f^2-18\,a\,b\,e^2\,f^2\,n+6\,b^2\,d^2\,g^2\,n^2-18\,b^2\,d\,e\,f\,g\,n^2+18\,b^2\,e^2\,f^2\,n^2}{9\,e^2}-\frac{d\left(\frac{g\,(3\,a^2\,d\,g+6\,a^2\,e\,f-b^2\,d\,g\,n^2+3\,b^2\,e\,f\,n^2-6\,a\,b\,e\,f\,n)}{3\,e}-\frac{d\,g^2\,(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2)}{9\,e}\right)}{e}\right)+x^2\left(\frac{g\,(3\,a^2\,d\,g+6\,a^2\,e\,f-b^2\,d\,g\,n^2+3\,b^2\,e\,f\,n^2-6\,a\,b\,e\,f\,n)}{6\,e}-\frac{d\,g^2\,(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2)}{18\,e}\right)+\ln\left(c\,(d+e\,x)^n\right)^2\left(b^2\,f^2\,x+\frac{b^2\,g^2\,x^3}{3}+\frac{d\,(b^2\,d^2\,g^2-3\,b^2\,d\,e\,f\,g+3\,b^2\,e^2\,f^2)}{3\,e^3}+b^2\,f\,g\,x^2\right)-\frac{\ln(d+e\,x)\,(11\,b^2\,d^3\,g^2\,n^2-27\,b^2\,d^2\,e\,f\,g\,n^2+18\,b^2\,d\,e^2\,f^2\,n^2-6\,a\,b\,d^3\,g^2\,n+18\,a\,b\,d^2\,e\,f\,g\,n-18\,a\,b\,d\,e}{9\,e^3}+\frac{g^2\,x^3\,(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2)}{27}$$

```
[In] int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^2,x)
```

```
[Out] log(c*(d + e*x)^n)*((x^2*((3*b*g*(a*d*g + 2*a*e*f - b*e*f*n))/e - (b*d*g^2*
(3*a - b*n))/e))/3 - (x*((d*((18*b*g*(a*d*g + 2*a*e*f - b*e*f*n))/e - (6*b*
d*g^2*(3*a - b*n))/e))/(3*e) - (6*b*f*(2*a*d*g + a*e*f - b*e*f*n))/e))/3 +
(2*b*g^2*x^3*(3*a - b*n))/9) + x*((9*a^2*e^2*f^2 + 6*b^2*d^2*g^2*n^2 + 18*b
^2*e^2*f^2*n^2 - 18*a*b*e^2*f^2*n + 18*a^2*d*e*f*g - 18*b^2*d*e*f*g*n^2)/(9
*e^2) - (d*((g*(3*a^2*d*g + 6*a^2*e*f - b^2*d*g*n^2 + 3*b^2*e*f*n^2 - 6*a*b
*e*f*n))/(3*e) - (d*g^2*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/(9*e)))/e) + x^2*((g
*(3*a^2*d*g + 6*a^2*e*f - b^2*d*g*n^2 + 3*b^2*e*f*n^2 - 6*a*b*e*f*n))/(6*e)
```

```
 - (d*g^2*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/(18*e)) + log(c*(d + e*x)^n)^2*(b^
2*f^2*x + (b^2*g^2*x^3)/3 + (d*(b^2*d^2*g^2 + 3*b^2*e^2*f^2 - 3*b^2*d*e*f*g
))/(3*e^3) + b^2*f*g*x^2) - (log(d + e*x)*(11*b^2*d^3*g^2*n^2 - 6*a*b*d^3*g
^2*n + 18*b^2*d*e^2*f^2*n^2 - 18*a*b*d*e^2*f^2*n - 27*b^2*d^2*e*f*g*n^2 + 1
8*a*b*d^2*e*f*g*n))/(9*e^3) + (g^2*x^3*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/27
```

# 3.46 $\int (f+gx) (a+b\log (c(d+ex)^n))^2 \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 186

$$
\begin{aligned}
\int (f+gx) (a+b\log (c(d+ex)^n))^2 \, dx = &-\frac{2ab(ef-dg)nx}{e} \\
&+\frac{2b^2(ef-dg)n^2x}{e}+\frac{b^2gn^2(d+ex)^2}{4e^2} \\
&-\frac{2b^2(ef-dg)n(d+ex)\log (c(d+ex)^n)}{e^2} \\
&-\frac{bgn(d+ex)^2(a+b\log (c(d+ex)^n))}{2e^2} \\
&+\frac{(ef-dg)(d+ex)(a+b\log (c(d+ex)^n))^2}{e^2} \\
&+\frac{g(d+ex)^2(a+b\log (c(d+ex)^n))^2}{2e^2}
\end{aligned}
$$

```
[Out] -2*a*b*(-d*g+e*f)*n*x/e+2*b^2*(-d*g+e*f)*n^2*x/e+1/4*b^2*g*n^2*(e*x+d)^2/e^
2-2*b^2*(-d*g+e*f)*n*(e*x+d)*ln(c*(e*x+d)^n)/e^2-1/2*b*g*n*(e*x+d)^2*(a+b*l
n(c*(e*x+d)^n))/e^2+(-d*g+e*f)*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^2+1/2*g*(e
*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2
```

## Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2 dx = \frac{(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2} - \frac{bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2} + \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2} - \frac{2abnx(ef-dg)}{e} - \frac{2b^2n(d+ex)(ef-dg)\log\left(c(d+ex)^n\right)}{e^2} + \frac{b^2gn^2(d+ex)^2}{4e^2} + \frac{2b^2n^2x(ef-dg)}{e}$$

```
[In] Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (-2*a*b*(e*f - d*g)*n*x)/e + (2*b^2*(e*f - d*g)*n^2*x)/e + (b^2*g*n^2*(d + e*x)^2)/(4*e^2) - (2*b^2*(e*f - d*g)*n*(d + e*x)*Log[c*(d + e*x)^n])/e^2 - (b*g*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^2) + ((e*f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e^2 + (g*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}+\frac{g(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}\right)dx\\
&= \frac{g\int (d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx}{e}+\frac{(ef-dg)\int \left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx}{e}\\
&= \frac{g\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+ex\right)}{e^2}\\
&\quad+\frac{(ef-dg)\text{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+ex\right)}{e^2}\\
&= \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}\\
&\quad-\frac{(bgn)\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)\,dx,x,d+ex\right)}{e^2}\\
&\quad-\frac{(2b(ef-dg)n)\text{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)\,dx,x,d+ex\right)}{e^2}
\end{aligned}$$

$$= -\frac{2ab(ef-dg)nx}{e} + \frac{b^2gn^2(d+ex)^2}{4e^2} - \frac{bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2} + \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2} + \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2} - \frac{(2b^2(ef-dg)n)\,\text{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{e^2}$$

$$= -\frac{2ab(ef-dg)nx}{e} + \frac{2b^2(ef-dg)n^2x}{e} + \frac{b^2gn^2(d+ex)^2}{4e^2} - \frac{2b^2(ef-dg)n(d+ex)\log\left(c(d+ex)^n\right)}{e^2} - \frac{bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2} + \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2} + \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.77

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx = \frac{4(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2 + 2g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2 - 8b(ef-dg)n(e(a-\cdots}{4e^2}$$

```
[In] Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (4*(e*f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 + 2*g*(d + e*x)^2*(a
+ b*Log[c*(d + e*x)^n])^2 - 8*b*(e*f - d*g)*n*(e*(a - b*n)*x + b*(d + e*x)*
Log[c*(d + e*x)^n]) + b*g*n*(b*e*n*x*(2*d + e*x) - 2*(d + e*x)^2*(a + b*Log
[c*(d + e*x)^n])))/(4*e^2)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(180) = 360.

Time = 0.61 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.47

| method | result |
|---|---|
| parallelrisch | $-\frac{2x^2\ln(c(ex+d)^n)b^2e^2gn-4x^2\ln(c(ex+d)^n)ab\,e^2g+8x\ln(c(ex+d)^n)b^2e^2fn-8x\ln(c(ex+d)^n)ab\,e^2f-8\ln(c(ex+d)^n)b^2def\cdots}{}$ |
| risch | Expression too large to display |

```
[In] int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(2*x^2*ln(c*(e*x+d)^n)*b^2*e^2*g*n-4*x^2*ln(c*(e*x+d)^n)*a*b*e^2*g+8*x
*ln(c*(e*x+d)^n)*b^2*e^2*f*n-8*x*ln(c*(e*x+d)^n)*a*b*e^2*f-8*ln(c*(e*x+d)^n
)*b^2*d*e*f*n+8*ln(c*(e*x+d)^n)*a*b*d*e*f-10*ln(e*x+d)*b^2*d^2*g*n^2-6*b^2*
d^2*g*n^2-b^2*e^2*g*n^2*x^2+8*b^2*d*e*f*n^2+16*ln(e*x+d)*b^2*d*e*f*n^2+4*ln
(e*x+d)*a*b*d^2*g*n-4*ln(c*(e*x+d)^n)^2*b^2*d*e*f+4*ln(c*(e*x+d)^n)*b^2*d^2
*g*n-2*x^2*ln(c*(e*x+d)^n)^2*b^2*e^2*g-4*x*ln(c*(e*x+d)^n)^2*b^2*e^2*f+4*a*
b*d^2*g*n+2*a*b*e^2*g*n*x^2+6*b^2*d*e*g*n^2*x+8*a*b*e^2*f*n*x-8*b^2*e^2*f*n
^2*x-8*a*b*d*e*f*n-4*a*b*d*e*g*n*x-16*ln(e*x+d)*a*b*d*e*f*n-2*a^2*e^2*g*x^2
-4*a^2*e^2*f*x+4*a^2*d*e*f-4*x*ln(c*(e*x+d)^n)*b^2*d*e*g*n+2*ln(c*(e*x+d)^n
)^2*b^2*d^2*g)/e^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(180) = 360$.

Time = 0.28 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.16

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx = \frac{(b^2e^2gn^2-2\,abe^2gn+2\,a^2e^2g)x^2+2\left(b^2e^2gn^2x^2+2\,b^2e^2fn^2x+(2\,b^2def-b^2d^2g)n^2\right)\log\left(ex+d\right)^2+2\,(b^2e}{}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*e^2*g*n^2 - 2*a*b*e^2*g*n + 2*a^2*e^2*g)*x^2 + 2*(b^2*e^2*g*n^2*x
^2 + 2*b^2*e^2*f*n^2*x + (2*b^2*d*e*f - b^2*d^2*g)*n^2)*log(e*x + d)^2 + 2*
(b^2*e^2*g*x^2 + 2*b^2*e^2*f*x)*log(c)^2 + 2*(2*a^2*e^2*f + (4*b^2*e^2*f -
3*b^2*d*e*g)*n^2 - 2*(2*a*b*e^2*f - a*b*d*e*g)*n)*x - 2*((4*b^2*d*e*f - 3*b
^2*d^2*g)*n^2 + (b^2*e^2*g*n^2 - 2*a*b*e^2*g*n)*x^2 - 2*(2*a*b*d*e*f - a*b*
d^2*g)*n - 2*(2*a*b*e^2*f*n - (2*b^2*e^2*f - b^2*d*e*g)*n^2)*x - 2*(b^2*e^2
*g*n*x^2 + 2*b^2*e^2*f*n*x + (2*b^2*d*e*f - b^2*d^2*g)*n)*log(c))*log(e*x +
 d) - 2*((b^2*e^2*g*n - 2*a*b*e^2*g)*x^2 - 2*(2*a*b*e^2*f - (2*b^2*e^2*f -
b^2*d*e*g)*n)*x)*log(c))/e^2
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(177) = 354$.

Time = 0.69 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.12

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx = \begin{cases} a^2fx+\frac{a^2gx^2}{2}-\frac{abd^2g\log\left(c(d+ex)^n\right)}{e^2}+\frac{2abdf\log\left(c(d+ex)^n\right)}{e}+\frac{abdgnx}{e}-2abfnx+2abfx\log\left(c(d+ex)^n\right)-\frac{abgnx^2}{2} \\ (a+b\log\left(cd^n\right))^2\left(fx+\frac{gx^2}{2}\right) \end{cases}$$

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Piecewise((a**2*f*x + a**2*g*x**2/2 - a*b*d**2*g*log(c*(d + e*x)**n)/e**2 +
 2*a*b*d*f*log(c*(d + e*x)**n)/e + a*b*d*g*n*x/e - 2*a*b*f*n*x + 2*a*b*f*x*
log(c*(d + e*x)**n) - a*b*g*n*x**2/2 + a*b*g*x**2*log(c*(d + e*x)**n) + 3*b
**2*d**2*g*n*log(c*(d + e*x)**n)/(2*e**2) - b**2*d**2*g*log(c*(d + e*x)**n)
**2/(2*e**2) - 2*b**2*d*f*n*log(c*(d + e*x)**n)/e + b**2*d*f*log(c*(d + e*x
)**n)**2/e - 3*b**2*d*g*n**2*x/(2*e) + b**2*d*g*n*x*log(c*(d + e*x)**n)/e +
 2*b**2*f*n**2*x - 2*b**2*f*n*x*log(c*(d + e*x)**n) + b**2*f*x*log(c*(d + e
*x)**n)**2 + b**2*g*n**2*x**2/4 - b**2*g*n*x**2*log(c*(d + e*x)**n)/2 + b**
2*g*x**2*log(c*(d + e*x)**n)**2/2, Ne(e, 0)), ((a + b*log(c*d**n))**2*(f*x
+ g*x**2/2), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.69

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx = \frac{1}{2}\,b^2gx^2\log\left((ex+d)^nc\right)^2 - 2\,abefn\left(\frac{x}{e}-\frac{d\log\left(ex+d\right)}{e^2}\right) - \frac{1}{2}\,abegn\left(\frac{2\,d^2\log\left(ex+d\right)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right) + abgx^2\log\left((ex+d)^nc\right) + b^2fx\log\left((ex+d)^nc\right)^2 + \frac{1}{2}\,a^2gx^2 + 2\,abfx\log\left((ex+d)^nc\right) - \left(2\,en\left(\frac{x}{e}-\frac{d\log\left(ex+d\right)}{e^2}\right)\log\left((ex+d)^nc\right) + \frac{\left(d\log\left(ex+d\right)^2-2\,ex+2\,d\log\left(ex+d\right)\right)n^2}{e}\right)b^2f - \frac{1}{4}\left(2\,en\left(\frac{2\,d^2\log\left(ex+d\right)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right)\log\left((ex+d)^nc\right) - \frac{\left(e^2x^2+2\,d^2\log\left(ex+d\right)^2-6\,dex+6\,d^2\log\left(ex+d\right)\right)n^2}{e^2}\right)b^2g + a^2fx$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*g*x^2*log((e*x + d)^n*c)^2 - 2*a*b*e*f*n*(x/e - d*log(e*x + d)/e^2)
 - 1/2*a*b*e*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + a*b*g*x^2
*log((e*x + d)^n*c) + b^2*f*x*log((e*x + d)^n*c)^2 + 1/2*a^2*g*x^2 + 2*a*b*
f*x*log((e*x + d)^n*c) - (2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*
c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*b^2*f - 1/4*(2*e*
n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)^n*c) - (e^2*
x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n^2/e^2)*b^2*g +
 a^2*f*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(180) = 360$.

Time = 0.38 (sec) , antiderivative size = 583, normalized size of antiderivative = 3.13

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx = \frac{(ex+d)b^2fn^2\log(ex+d)^2}{e} + \frac{(ex+d)^2b^2gn^2\log(ex+d)^2}{2\,e^2} - \frac{(ex+d)b^2dgn^2\log(ex+d)^2}{e^2} - \frac{2\,(ex+d)b^2fn^2\log(ex+d)}{e} - \frac{(ex+d)^2b^2gn^2\log(ex+d)}{2\,e^2} + \frac{2\,(ex+d)b^2dgn^2\log(ex+d)}{e^2} + \frac{2\,(ex+d)b^2fn\log(ex+d)\log(c)}{e} + \frac{(ex+d)^2b^2gn\log(ex+d)\log(c)}{e^2} - \frac{2\,(ex+d)b^2dgn\log(ex+d)\log(c)}{e^2} + \frac{2\,(ex+d)b^2fn^2}{e} + \frac{(ex+d)^2b^2gn^2}{4\,e^2} - \frac{2\,(ex+d)b^2dgn^2}{e^2} + \frac{2\,(ex+d)abfn\log(ex+d)}{e} + \frac{(ex+d)^2abgn\log(ex+d)}{e^2} - \frac{2\,(ex+d)abdgn\log(ex+d)}{e^2} - \frac{2\,(ex+d)b^2fn\log(c)}{e} - \frac{(ex+d)^2b^2gn\log(c)}{2\,e^2} + \frac{2\,(ex+d)b^2dgn\log(c)}{e^2} + \frac{(ex+d)b^2f\log(c)^2}{e} + \frac{(ex+d)^2b^2g\log(c)^2}{2\,e^2} - \frac{(ex+d)b^2dg\log(c)^2}{e^2} - \frac{2\,(ex+d)abfn}{e} - \frac{(ex+d)^2abgn}{2\,e^2} + \frac{2\,(ex+d)abdgn}{e^2} + \frac{2\,(ex+d)abf\log(c)}{e} + \frac{(ex+d)^2abg\log(c)}{e^2} - \frac{2\,(ex+d)abdg\log(c)}{e^2} + \frac{(ex+d)a^2f}{e} + \frac{(ex+d)^2a^2g}{2\,e^2} - \frac{(ex+d)a^2dg}{e^2}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] (e*x + d)*b^2*f*n^2*log(e*x + d)^2/e + 1/2*(e*x + d)^2*b^2*g*n^2*log(e*x +
d)^2/e^2 - (e*x + d)*b^2*d*g*n^2*log(e*x + d)^2/e^2 - 2*(e*x + d)*b^2*f*n^2
*log(e*x + d)/e - 1/2*(e*x + d)^2*b^2*g*n^2*log(e*x + d)/e^2 + 2*(e*x + d)*
b^2*d*g*n^2*log(e*x + d)/e^2 + 2*(e*x + d)*b^2*f*n*log(e*x + d)*log(c)/e +
(e*x + d)^2*b^2*g*n*log(e*x + d)*log(c)/e^2 - 2*(e*x + d)*b^2*d*g*n*log(e*x
 + d)*log(c)/e^2 + 2*(e*x + d)*b^2*f*n^2/e + 1/4*(e*x + d)^2*b^2*g*n^2/e^2
- 2*(e*x + d)*b^2*d*g*n^2/e^2 + 2*(e*x + d)*a*b*f*n*log(e*x + d)/e + (e*x +
 d)^2*a*b*g*n*log(e*x + d)/e^2 - 2*(e*x + d)*a*b*d*g*n*log(e*x + d)/e^2 - 2
*(e*x + d)*b^2*f*n*log(c)/e - 1/2*(e*x + d)^2*b^2*g*n*log(c)/e^2 + 2*(e*x +
 d)*b^2*d*g*n*log(c)/e^2 + (e*x + d)*b^2*f*log(c)^2/e + 1/2*(e*x + d)^2*b^2
*g*log(c)^2/e^2 - (e*x + d)*b^2*d*g*log(c)^2/e^2 - 2*(e*x + d)*a*b*f*n/e -
1/2*(e*x + d)^2*a*b*g*n/e^2 + 2*(e*x + d)*a*b*d*g*n/e^2 + 2*(e*x + d)*a*b*f
*log(c)/e + (e*x + d)^2*a*b*g*log(c)/e^2 - 2*(e*x + d)*a*b*d*g*log(c)/e^2 +
 (e*x + d)*a^2*f/e + 1/2*(e*x + d)^2*a^2*g/e^2 - (e*x + d)*a^2*d*g/e^2
```

## Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.44

$$\int (f+gx)\,(a+b\log (c(d+ex)^n))^2\,dx = \ln (c\,(d+e\,x)^n)^2\left(\frac{b^2\,g\,x^2}{2}-\frac{d\,(b^2\,d\,g-2\,b^2\,e\,f)}{2\,e^2}+b^2\,f\,x\right) + x\left(\frac{2\,a^2\,d\,g+2\,a^2\,e\,f-2\,b^2\,d\,g\,n^2+4\,b^2\,e\,f\,n^2-4\,a\,b\,e\,f\,n}{2\,e} - \frac{d\,g\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{2\,e}\right)+\ln (c\,(d+e\,x)^n)\,\left(\frac{b\,g\,(2\,a-b\,n)\,x^2}{2} + \left(\frac{2\,b\,(a\,d\,g+a\,e\,f-b\,e\,f\,n)}{e}-\frac{b\,d\,g\,(2\,a-b\,n)}{e}\right)\,x\right)+\frac{g\,x^2\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{4} + \frac{\ln (d+e\,x)\,\left(3\,g\,b^2\,d^2\,n^2-4\,e\,f\,b^2\,d\,n^2-2\,a\,g\,b\,d^2\,n+4\,a\,e\,f\,b\,d\,n\right)}{2\,e^2}$$

```
[In] int((f + g*x)*(a + b*log(c*(d + e*x)^n))^2,x)
```

```
[Out] log(c*(d + e*x)^n)^2*((b^2*g*x^2)/2 - (d*(b^2*d*g - 2*b^2*e*f))/(2*e^2) + b
^2*f*x) + x*((2*a^2*d*g + 2*a^2*e*f - 2*b^2*d*g*n^2 + 4*b^2*e*f*n^2 - 4*a*b
*e*f*n)/(2*e) - (d*g*(2*a^2 + b^2*n^2 - 2*a*b*n))/(2*e)) + log(c*(d + e*x)^
n)*(x*((2*b*(a*d*g + a*e*f - b*e*f*n))/e - (b*d*g*(2*a - b*n))/e) + (b*g*x^
2*(2*a - b*n))/2) + (g*x^2*(2*a^2 + b^2*n^2 - 2*a*b*n))/4 + (log(d + e*x)*(
3*b^2*d^2*g*n^2 - 4*b^2*d*e*f*n^2 - 2*a*b*d^2*g*n + 4*a*b*d*e*f*n))/(2*e^2)
```

# 3.47 $\int (a+b\log(c(d+ex)^n))^2\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 65

$$\int (a+b\log(c(d+ex)^n))^2\,dx = -2abnx+2b^2n^2x-\frac{2b^2n(d+ex)\log(c(d+ex)^n)}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{e}$$

```
[Out] -2*a*b*n*x+2*b^2*n^2*x-2*b^2*n*(e*x+d)*ln(c*(e*x+d)^n)/e+(e*x+d)*(a+b*ln(c*
(e*x+d)^n))^2/e
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2436, 2333, 2332}

$$\int (a+b\log(c(d+ex)^n))^2\,dx = \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{e}-2abnx-\frac{2b^2n(d+ex)\log(c(d+ex)^n)}{e}+2b^2n^2x$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] -2*a*b*n*x + 2*b^2*n^2*x - (2*b^2*n*(d + e*x)*Log[c*(d + e*x)^n])/e + ((d +
 e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx, x, d+ex\right)}{e} \\
&= \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^2}{e} - \frac{(2bn)\text{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+ex\right)}{e} \\
&= -2abnx + \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^2}{e} - \frac{(2b^2n)\,\text{Subst}\left(\int \log(cx^n)\,dx, x, d+ex\right)}{e} \\
&= -2abnx + 2b^2n^2x - \frac{2b^2n(d+ex)\log(c(d+ex)^n)}{e} + \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^2}{e}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int (a+b\log(c(d+ex)^n))^2\,dx = \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^2}{e} - 2bn\left(ax - bnx + \frac{b(d+ex)\log(c(d+ex)^n)}{e}\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e - 2*b*n*(a*x - b*n*x + (b*(d + e
*x)*Log[c*(d + e*x)^n])/e)
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

| method | result |
|---|---|
| norman | $(2b^2n^2-2abn+a^2)\,x+b^2x\ln\left(c\,e^{n\ln(ex+d)}\right)^2+(-2b^2n+2ab)\,x\ln\left(c\,e^{n\ln(ex+d)}\right)+\frac{b^2d\ln(c\,e^{n\ln}}{e}$ |
| default | $a^2x+b^2x\ln\left(c\,e^{n\ln(ex+d)}\right)^2+\frac{b^2d\ln\left(c\,e^{n\ln(ex+d)}\right)^2}{e}+2b^2n^2x-2b^2nx\ln\left(c\,e^{n\ln(ex+d)}\right)-\frac{2n^2b^2d\ln(ex}{e}$ |
| parts | $a^2x+b^2x\ln\left(c\,e^{n\ln(ex+d)}\right)^2+\frac{b^2d\ln\left(c\,e^{n\ln(ex+d)}\right)^2}{e}+2b^2n^2x-2b^2nx\ln\left(c\,e^{n\ln(ex+d)}\right)-\frac{2n^2b^2d\ln(ex}{e}$ |
| parallelrisch | $\frac{x\ln(c(ex+d)^n)^2b^2den-2x\ln(c(ex+d)^n)b^2den^2+2xb^2den^3+2x\ln(c(ex+d)^n)abden-2xabden^2+\ln(c(ex+d)^n)^2b^2d^2n-2\ln(c}{end}$ |
| risch | Expression too large to display |

```
[In] int((a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (2*b^2*n^2-2*a*b*n+a^2)*x+b^2*x*ln(c*exp(n*ln(e*x+d)))^2+(-2*b^2*n+2*a*b)*x
*ln(c*exp(n*ln(e*x+d)))+b^2*d/e*ln(c*exp(n*ln(e*x+d)))^2+n*(-2*b^2*d*n+2*a*
b*d)/e*ln(e*x+d)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(65) = 130.

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.15

$$\int (a+b\log(c(d+ex)^n))^2\,dx$$
$$=\frac{b^2ex\log(c)^2+(b^2en^2x+b^2dn^2)\log(ex+d)^2-2\,(b^2en-abe)x\log(c)+(2\,b^2en^2-2\,aben+a^2e)x-2\,(b}{e}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

```
[Out] (b^2*e*x*log(c)^2 + (b^2*e*n^2*x + b^2*d*n^2)*log(e*x + d)^2 - 2*(b^2*e*n -
 a*b*e)*x*log(c) + (2*b^2*e*n^2 - 2*a*b*e*n + a^2*e)*x - 2*(b^2*d*n^2 - a*b
*d*n + (b^2*e*n^2 - a*b*e*n)*x - (b^2*e*n*x + b^2*d*n)*log(c))*log(e*x + d)
)/e
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(63) = 126$.

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.25

$$\int (a + b\log (c(d+ex)^n))^2 \, dx$$
$$= \begin{cases} a^2x + \frac{2abd\log (c(d+ex)^n)}{e} - 2abnx + 2abx\log (c(d+ex)^n) - \frac{2b^2dn\log (c(d+ex)^n)}{e} + \frac{b^2d\log (c(d+ex)^n)^2}{e} + 2b^2n^2x - 2 \\ x(a + b\log (cd^n))^2 \end{cases}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*d*log(c*(d + e*x)**n)/e - 2*a*b*n*x + 2*a*b*x*log
(c*(d + e*x)**n) - 2*b**2*d*n*log(c*(d + e*x)**n)/e + b**2*d*log(c*(d + e*x
)**n)**2/e + 2*b**2*n**2*x - 2*b**2*n*x*log(c*(d + e*x)**n) + b**2*x*log(c*
(d + e*x)**n)**2, Ne(e, 0)), (x*(a + b*log(c*d**n))**2, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(65) = 130$.

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.02

$$\int (a + b\log (c(d+ex)^n))^2 \, dx$$
$$= -2\,aben\left(\frac{x}{e} - \frac{d\log (ex+d)}{e^2}\right) + b^2x\log ((ex+d)^n c)^2 + 2\,abx\log ((ex+d)^n c)$$
$$- \left(2\,en\left(\frac{x}{e} - \frac{d\log (ex+d)}{e^2}\right)\log ((ex+d)^n c) + \frac{(d\log (ex+d)^2 - 2\,ex + 2\,d\log (ex+d))n^2}{e}\right)b^2$$
$$+ a^2x$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*e*n*(x/e - d*log(e*x + d)/e^2) + b^2*x*log((e*x + d)^n*c)^2 + 2*a*b*
x*log((e*x + d)^n*c) - (2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)
 + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*b^2 + a^2*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(65) = 130.

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.68

$$\int (a+b\log(c(d+ex)^n))^2\,dx = \frac{(ex+d)b^2n^2\log(ex+d)^2}{e} - \frac{2\,(ex+d)b^2n^2\log(ex+d)}{e} + \frac{2\,(ex+d)b^2n\log(ex+d)\log(c)}{e} + \frac{2\,(ex+d)b^2n^2}{e} + \frac{2\,(ex+d)abn\log(ex+d)}{e} - \frac{2\,(ex+d)b^2n\log(c)}{e} + \frac{(ex+d)b^2\log(c)^2}{e} - \frac{2\,(ex+d)abn}{e} + \frac{2\,(ex+d)ab\log(c)}{e} + \frac{(ex+d)a^2}{e}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] (e*x + d)*b^2*n^2*log(e*x + d)^2/e - 2*(e*x + d)*b^2*n^2*log(e*x + d)/e + 2
*(e*x + d)*b^2*n*log(e*x + d)*log(c)/e + 2*(e*x + d)*b^2*n^2/e + 2*(e*x + d
)*a*b*n*log(e*x + d)/e - 2*(e*x + d)*b^2*n*log(c)/e + (e*x + d)*b^2*log(c)^
2/e - 2*(e*x + d)*a*b*n/e + 2*(e*x + d)*a*b*log(c)/e + (e*x + d)*a^2/e
```

## Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

$$\int (a+b\log(c(d+ex)^n))^2\,dx = x\left(a^2-2\,a\,b\,n+2\,b^2\,n^2\right)+\ln\left(c\,(d+e\,x)^n\right)^2\left(b^2\,x+\frac{b^2\,d}{e}\right) - \frac{\ln(d+e\,x)\,\left(2\,b^2\,d\,n^2-2\,a\,b\,d\,n\right)}{e} + 2\,b\,x\,\ln\left(c\,(d+e\,x)^n\right)\,(a-b\,n)$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2,x)
```

```
[Out] x*(a^2 + 2*b^2*n^2 - 2*a*b*n) + log(c*(d + e*x)^n)^2*(b^2*x + (b^2*d)/e) -
(log(d + e*x)*(2*b^2*d*n^2 - 2*a*b*d*n))/e + 2*b*x*log(c*(d + e*x)^n)*(a -
b*n)
```

# 3.48 $\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[Out] (a+b*ln(c*(e*x+d)^n))^2*ln(e*(g*x+f)/(-d*g+e*f))/g+2*b*n*(a+b*ln(c*(e*x+d)^
n))*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g-2*b^2*n^2*polylog(3,-g*(e*x+d)/(-d*g
+e*f))/g
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2443, 2481, 2421, 6724}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \frac{2bn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g} + \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g} - \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(f + g*x))/(e*f - d*g)])/g + (2*b*n*(a
 + b*Log[c*(d + e*x)^n])*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g - (2*b
^2*n^2*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))])/g
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} - \frac{(2ben)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} - \frac{(2bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}dx,x,d+ex\right)}{g}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{(2b^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}dx,x,d+ex\right)}{g}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{2b^2n^2\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.75

$$\int\frac{(a+b\log(c(d+ex)^n))^2}{f+gx}dx = \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f+gx)+2bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))\left(\log(\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x),x]
```

```
[Out] ((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x] + 2*b*n*(a -
b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*Log[(e*(f + g*x))/(e
*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + b^2*n^2*(Log[d + e
*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*
x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]))/g
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.27 (sec) , antiderivative size = 737, normalized size of antiderivative = 6.64

| method | result |
|---|---|
| risch | $\frac{b^2\ln(g(ex+d)-dg+ef)\ln(ex+d)^2n^2}{g} - \frac{2b^2\ln(g(ex+d)-dg+ef)\ln((ex+d)^n)\ln(ex+d)n}{g} + \frac{b^2\ln(g(ex+d)-dg+ef)\ln((ex+d)^n)^2}{g}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*ln(g*(e*x+d)-d*g+e*f)/g*ln(e*x+d)^2*n^2-2*b^2*ln(g*(e*x+d)-d*g+e*f)/g*l
n((e*x+d)^n)*ln(e*x+d)*n+b^2*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)^2+b^2*n^
2/g*ln(e*x+d)^2*ln(1-g*(e*x+d)/(d*g-e*f))+2*b^2*n^2/g*ln(e*x+d)*polylog(2,g
*(e*x+d)/(d*g-e*f))-2*b^2*n^2/g*polylog(3,g*(e*x+d)/(d*g-e*f))-2*b^2*n^2*di
log((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln(e*x+d)+2*b^2*n*dilog((g*(e*x+d)-d*
g+e*f)/(-d*g+e*f))/g*ln((e*x+d)^n)-2*b^2*n^2*ln(e*x+d)^2*ln((g*(e*x+d)-d*g+
e*f)/(-d*g+e*f))/g+2*b^2*n*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*l
n((e*x+d)^n)+(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*
csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n
)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b*(ln((e*x+d)^n)*ln(g*x+f
)/g-1/g*n*e*(dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e+ln(g*x+f)*ln(((g*x+f)*e
+d*g-e*f)/(d*g-e*f))/e))+1/4*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*
(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*cs
gn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^2*ln(g*x+
f)/g
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x +
 f), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**2/(f + g*x), x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="maxima")
```

```
[Out] a^2*log(g*x + f)/g + integrate((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a
*b*log(c) + 2*(b^2*log(c) + a*b)*log((e*x + d)^n))/(g*x + f), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(a+b\ln(c(d+e\,x)^n))^2}{f+g\,x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x), x)
```

# 3.49 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2} dx = \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{(ef-dg)(f+gx)} - \frac{2ben(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} - \frac{2b^2en^2\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)}$$

```
[Out] (e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/(-d*g+e*f)/(g*x+f)-2*b*e*n*(a+b*ln(c*(e*x+d
)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g/(-d*g+e*f)-2*b^2*e*n^2*polylog(2,-g*(e*x+d
)/(-d*g+e*f))/g/(-d*g+e*f)
```

## Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2444, 2441, 2440, 2438}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2} dx = -\frac{2ben\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g(ef-dg)} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{(f+gx)(ef-dg)} - \frac{2b^2en^2\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^2,x]
```

```
[Out] ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/((e*f - d*g)*(f + g*x)) - (2*b*e*n
*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/(g*(e*f - d*g))
 - (2*b^2*e*n^2*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/(g*(e*f - d*g))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(ef-dg)(f+gx)} - \frac{(2ben)\int\frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{ef-dg}$$

$$= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(ef-dg)(f+gx)} - \frac{2ben(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} + \frac{(2b^2e^2n^2)\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g(ef-dg)}$$

$$= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(ef-dg)(f+gx)} - \frac{2ben(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} + \frac{(2b^2en^2)\,\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{g(ef-dg)}$$

$$= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(ef-dg)(f+gx)} - \frac{2ben(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} - \frac{2b^2en^2\mathrm{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(f+gx)^2}\,dx = \frac{-\left((a+b\log\left(c(d+ex)^n\right))\left(ag(d+ex)+bg(d+ex)\log\left(c(d+ex)^n\right)-2ben(f+gx)\log\left(\frac{e(f+gx)}{ef-dg}\right)\right)\right)+2}{g(-ef+dg)(f+gx)}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^2,x]
```

```
[Out] (-((a + b*Log[c*(d + e*x)^n])*(a*g*(d + e*x) + b*g*(d + e*x)*Log[c*(d + e*x
)^n] - 2*b*e*n*(f + g*x)*Log[(e*(f + g*x))/(e*f - d*g)])) + 2*b^2*e*n^2*(f
+ g*x)*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)])/(g*(-(e*f) + d*g)*(f + g*x
))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.91 (sec) , antiderivative size = 537, normalized size of antiderivative = 4.07

| method | result |
|---|---|
| risch | $-\frac{b^2\ln\left((ex+d)^n\right)^2}{(gx+f)g} - \frac{2b^2ne\ln\left((ex+d)^n\right)\ln(ex+d)}{g(dg-ef)} + \frac{2b^2ne\ln\left((ex+d)^n\right)\ln(gx+f)}{g(dg-ef)} - \frac{2b^2n^2e\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g(dg-ef)} - \frac{2b^2n^2}{}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*ln((e*x+d)^n)^2/(g*x+f)/g-2*b^2/g*n*e*ln((e*x+d)^n)/(d*g-e*f)*ln(e*x+d
)+2*b^2/g*n*e*ln((e*x+d)^n)/(d*g-e*f)*ln(g*x+f)-2*b^2/g*n^2*e/(d*g-e*f)*dil
og(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-2*b^2/g*n^2*e/(d*g-e*f)*ln(g*x+f)*ln(((g*
x+f)*e+d*g-e*f)/(d*g-e*f))+b^2/g*n^2*e/(d*g-e*f)*ln(e*x+d)^2+(-I*b*Pi*csgn(
```

```
I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^
n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)
^n)^3*b+2*b*ln(c)+2*a)*b*(-ln((e*x+d)^n)/(g*x+f)/g+1/g*n*e*(-1/(d*g-e*f)*ln
(e*x+d)+1/(d*g-e*f)*ln(g*x+f)))-1/4*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*
csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d
)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^2/
(g*x+f)/g
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^2*x
^2 + 2*f*g*x + f^2), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**2/(f + g*x)**2, x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*e*n*(log(e*x + d)/(e*f*g - d*g^2) - log(g*x + f)/(e*f*g - d*g^2)) - b
^2*(log((e*x + d)^n)^2/(g^2*x + f*g) - integrate((e*g*x*log(c)^2 + d*g*log(
c)^2 + 2*(e*f*n + d*g*log(c) + (e*g*n + e*g*log(c))*x)*log((e*x + d)^n))/(e
*g^3*x^3 + d*f^2*g + (2*e*f*g^2 + d*g^3)*x^2 + (e*f^2*g + 2*d*f*g^2)*x), x)
) - 2*a*b*log((e*x + d)^n*c)/(g^2*x + f*g) - a^2/(g^2*x + f*g)
```

## Giac [F]

$$\int \frac{(a+b\log{(c(d+ex)^n)})^2}{(f+gx)^2}\,dx = \int \frac{(b\log{((ex+d)^n c)}+a)^2}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(c(d+ex)^n)})^2}{(f+gx)^2}\,dx = \int \frac{(a+b\ln{(c\,(d+e\,x)^n)})^2}{(f+g\,x)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^2,x)
[Out] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^2, x)
```

# 3.50 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3} dx$



## Optimal result

Integrand size = 24, antiderivative size = 202

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3} dx = -\frac{ben(d+ex)(a+b\log(c(d+ex)^n))}{(ef-dg)^2(f+gx)} - \frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2} + \frac{b^2e^2n^2\log(f+gx)}{g(ef-dg)^2} - \frac{be^2n(a+b\log(c(d+ex)^n))\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2} + \frac{b^2e^2n^2\,\mathrm{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2}$$

```
[Out] -b*e*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))/(-d*g+e*f)^2/(g*x+f)-1/2*(a+b*ln(c*(e*
x+d)^n))^2/g/(g*x+f)^2+b^2*e^2*n^2*ln(g*x+f)/g/(-d*g+e*f)^2-b*e^2*n*(a+b*ln
(c*(e*x+d)^n))*ln(1+(-d*g+e*f)/g/(e*x+d))/g/(-d*g+e*f)^2+b^2*e^2*n^2*polylo
g(2,(d*g-e*f)/g/(e*x+d))/g/(-d*g+e*f)^2
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{2445, 2458, 2389, 2379, 2438, 2351, 31\}$

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx = -\frac{be^2n\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))}{g(ef-dg)^2} - \frac{ben(d+ex)(a+b\log(c(d+ex)^n))}{(f+gx)(ef-dg)^2} - \frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2} + \frac{b^2e^2n^2\,\mathrm{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2} + \frac{b^2e^2n^2\log(f+gx)}{g(ef-dg)^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^3,x]
```

```
[Out] -((b*e*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n]))/((e*f - d*g)^2*(f + g*x))) -
 (a + b*Log[c*(d + e*x)^n])^2/(2*g*(f + g*x)^2) + (b^2*e^2*n^2*Log[f + g*x]
)/(g*(e*f - d*g)^2) - (b*e^2*n*(a + b*Log[c*(d + e*x)^n])*Log[1 + (e*f - d*
g)/(g*(d + e*x))])/(g*(e*f - d*g)^2) + (b^2*e^2*n^2*PolyLog[2, -((e*f - d*g
)/(g*(d + e*x)))])/(g*(e*f - d*g)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2}+\frac{(ben)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^2}\,dx}{g}\\
&= -\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2}+\frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^2}\,dx,x,d+ex\right)}{g}\\
&= -\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2}-\frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^2}\,dx,x,d+ex\right)}{ef-dg}\\
&\quad+\frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}\,dx,x,d+ex\right)}{g(ef-dg)}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{ben(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)^2(f+gx)} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g(f+gx)^2} \\
&\quad - \frac{be^2n(a+b\log\left(c(d+ex)^n\right))\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2} \\
&\quad + \frac{(b^2en^2)\,\mathrm{Subst}\left(\int \frac{1}{\frac{ef-dg}{e}+\frac{gx}{e}}\,dx, x, d+ex\right)}{(ef-dg)^2} \\
&\quad + \frac{(b^2e^2n^2)\,\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{ef-dg}{gx}\right)}{x}\,dx, x, d+ex\right)}{g(ef-dg)^2} \\
&= -\frac{ben(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)^2(f+gx)} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g(f+gx)^2} \\
&\quad + \frac{b^2e^2n^2\log(f+gx)}{g(ef-dg)^2} - \frac{be^2n(a+b\log\left(c(d+ex)^n\right))\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2} \\
&\quad + \frac{b^2e^2n^2\mathrm{Li}_2\left(-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$
\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(f+gx)^3}\,dx
= \frac{-\left(a+b\log\left(c(d+ex)^n\right)\right)^2 + \frac{e(f+gx)\left(2b(ef-dg)n(a+b\log(c(d+ex)^n))+e(f+gx)(a+b\log(c(d+ex)^n))^2-2b^2en^2(f+gx)(\log(d+ex)-\cdots\right.}{(ef-d\cdots}}{2g(f+gx)^2}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^3,x]
```

```
[Out] (-(a + b*Log[c*(d + e*x)^n])^2 + (e*(f + g*x)*(2*b*(e*f - d*g)*n*(a + b*Log
[c*(d + e*x)^n]) + e*(f + g*x)*(a + b*Log[c*(d + e*x)^n])^2 - 2*b^2*e*n^2*(
f + g*x)*(Log[d + e*x] - Log[f + g*x]) - 2*b*e*n*(f + g*x)*(a + b*Log[c*(d
+ e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] - 2*b^2*e*n^2*(f + g*x)*PolyLog[2
, (g*(d + e*x))/(-(e*f) + d*g)]))/(e*f - d*g)^2)/(2*g*(f + g*x)^2)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.35 (sec) , antiderivative size = 661, normalized size of antiderivative = 3.27

| method | result |
|---|---|
| risch | $-\frac{b^{2} \ln \left(\left(e x+d\right)^{n}\right)^{2}}{2 \left(g x+f\right)^{2} g}+\frac{b^{2} n \,e^{2} \ln \left(\left(e x+d\right)^{n}\right) \ln \left(e x+d\right)}{g \left(d g-e f\right)^{2}}-\frac{b^{2} n e \ln \left(\left(e x+d\right)^{n}\right)}{g \left(d g-e f\right) \left(g x+f\right)}-\frac{b^{2} n \,e^{2} \ln \left(\left(e x+d\right)^{n}\right) \ln \left(g x+f\right)}{g \left(d g-e f\right)^{2}}-\frac{b^{2} n^{2} e^{2} \ln \left(e x+d\right)^{2}}{2 g \left(d g-e f\right)^{2}}-$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2*ln((e*x+d)^n)^2/(g*x+f)^2/g+b^2/g*n*e^2*ln((e*x+d)^n)/(d*g-e*f)^2*
ln(e*x+d)-b^2/g*n*e*ln((e*x+d)^n)/(d*g-e*f)/(g*x+f)-b^2/g*n*e^2*ln((e*x+d)^
n)/(d*g-e*f)^2*ln(g*x+f)-1/2*b^2/g*n^2*e^2/(d*g-e*f)^2*ln(e*x+d)^2-b^2/g*n^
2*e^2/(d*g-e*f)^2*ln(e*x+d)+b^2/g*n^2*e^2/(d*g-e*f)^2*ln(g*x+f)+b^2/g*n^2*e
^2/(d*g-e*f)^2*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+b^2/g*n^2*e^2/(d*g-e*f)
^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-I*b*Pi*csgn(I*c*(e*x+d)^n)
*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*cs
gn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln
(c)+2*a)*b*(-1/2*ln((e*x+d)^n)/(g*x+f)^2/g+1/2/g*n*e*(e/(d*g-e*f)^2*ln(e*x+
d)-1/(d*g-e*f)/(g*x+f)-e/(d*g-e*f)^2*ln(g*x+f)))-1/8*(-I*b*Pi*csgn(I*c*(e*x
+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I
*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+
2*b*ln(c)+2*a)^2/(g*x+f)^2/g
```

## Fricas [F]

$$\int \frac{(a+b\log\left(c(d+ex)^n\right))^2}{(f+gx)^3}\,dx=\int \frac{\left(b\log\left(\left(ex+d\right)^n c\right)+a\right)^2}{\left(gx+f\right)^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^3*x
^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**2/(f + g*x)**3, x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{(gx+f)^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] a*b*e*n*(e*log(e*x + d)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) - e*log(g*x + f
)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) + 1/(e*f^2*g - d*f*g^2 + (e*f*g^2 - d
*g^3)*x)) - 1/2*b^2*(log((e*x + d)^n)^2/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 2*i
ntegrate((e*g*x*log(c)^2 + d*g*log(c)^2 + (e*f*n + 2*d*g*log(c) + (e*g*n +
2*e*g*log(c))*x)*log((e*x + d)^n))/(e*g^4*x^4 + d*f^3*g + (3*e*f*g^3 + d*g^
4)*x^3 + 3*(e*f^2*g^2 + d*f*g^3)*x^2 + (e*f^3*g + 3*d*f^2*g^2)*x), x)) - a*
b*log((e*x + d)^n*c)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1/2*a^2/(g^3*x^2 + 2*f
*g^2*x + f^2*g)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{(gx+f)^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^3}\,dx = \int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2}{{\left(f+g\,x\right)}^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^3,x)
[Out] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^3, x)
```

# 3.51 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 317

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx = &-\frac{b^2e^2n^2}{3g(ef-dg)^2(f+gx)}-\frac{b^2e^3n^2\log(d+ex)}{3g(ef-dg)^3}\\&+\frac{ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)(f+gx)^2}\\&-\frac{2be^2n(d+ex)(a+b\log(c(d+ex)^n))}{3(ef-dg)^3(f+gx)}\\&-\frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3}+\frac{b^2e^3n^2\log(f+gx)}{g(ef-dg)^3}\\&-\frac{2be^3n(a+b\log(c(d+ex)^n))\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{3g(ef-dg)^3}\\&+\frac{2b^2e^3n^2\,\mathrm{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{3g(ef-dg)^3}\end{aligned}$$

```
[Out] -1/3*b^2*e^2*n^2/g/(-d*g+e*f)^2/(g*x+f)-1/3*b^2*e^3*n^2*ln(e*x+d)/g/(-d*g+e
*f)^3+1/3*b*e*n*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f)/(g*x+f)^2-2/3*b*e^2*n*(e
*x+d)*(a+b*ln(c*(e*x+d)^n))/(-d*g+e*f)^3/(g*x+f)-1/3*(a+b*ln(c*(e*x+d)^n))^
2/g/(g*x+f)^3+b^2*e^3*n^2*ln(g*x+f)/g/(-d*g+e*f)^3-2/3*b*e^3*n*(a+b*ln(c*(e
*x+d)^n))*ln(1+(-d*g+e*f)/g/(e*x+d))/g/(-d*g+e*f)^3+2/3*b^2*e^3*n^2*polylog
(2,(d*g-e*f)/g/(e*x+d))/g/(-d*g+e*f)^3
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx = -\frac{2be^3n\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))}{3g(ef-dg)^3} - \frac{2be^2n(d+ex)(a+b\log(c(d+ex)^n))}{3(f+gx)(ef-dg)^3} + \frac{ben(a+b\log(c(d+ex)^n))}{3g(f+gx)^2(ef-dg)} - \frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3} + \frac{2b^2e^3n^2\operatorname{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{3g(ef-dg)^3} - \frac{b^2e^3n^2\log(d+ex)}{3g(ef-dg)^3} + \frac{b^2e^3n^2\log(f+gx)}{g(ef-dg)^3} - \frac{b^2e^2n^2}{3g(f+gx)(ef-dg)^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^4,x]
```

```
[Out] -1/3*(b^2*e^2*n^2)/(g*(e*f - d*g)^2*(f + g*x)) - (b^2*e^3*n^2*Log[d + e*x])
/(3*g*(e*f - d*g)^3) + (b*e*n*(a + b*Log[c*(d + e*x)^n]))/(3*g*(e*f - d*g)*
(f + g*x)^2) - (2*b*e^2*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n]))/(3*(e*f - d
*g)^3*(f + g*x)) - (a + b*Log[c*(d + e*x)^n])^2/(3*g*(f + g*x)^3) + (b^2*e^
3*n^2*Log[f + g*x])/(g*(e*f - d*g)^3) - (2*b*e^3*n*(a + b*Log[c*(d + e*x)^n
])*Log[1 + (e*f - d*g)/(g*(d + e*x))])/(3*g*(e*f - d*g)^3) + (2*b^2*e^3*n^2
*PolyLog[2, -((e*f - d*g)/(g*(d + e*x)))])/(3*g*(e*f - d*g)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
```

```
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = -\frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3} + \frac{(2ben)\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3} + \frac{(2bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^3}\,dx,x,d+ex\right)}{3g}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3} - \frac{(2bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^3}\,dx,x,d+ex\right)}{3(ef-dg)} + \frac{(2ben)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^2}\,dx,x,d+ex\right)}{3g(ef-dg)}$$

$$= \frac{ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)(f+gx)^2} - \frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3} - \frac{(2ben)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^2}\,dx,x,d+ex\right)}{3(ef-dg)^2} + \frac{(2be^2n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}\,dx,x,d+ex\right)}{3g(ef-dg)^2} - \frac{(b^2en^2)\,\text{Subst}\left(\int\frac{1}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^2}\,dx,x,d+ex\right)}{3g(ef-dg)}$$

$$= \frac{ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)(f+gx)^2} - \frac{2be^2n(d+ex)\,(a+b\log(c(d+ex)^n))}{3(ef-dg)^3(f+gx)} - \frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3} - \frac{2be^3n(a+b\log(c(d+ex)^n))\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{3g(ef-dg)^3} + \frac{(2b^2e^2n^2)\,\text{Subst}\left(\int\frac{1}{\frac{ef-dg}{e}+\frac{gx}{e}}\,dx,x,d+ex\right)}{3(ef-dg)^3} + \frac{(2b^2e^3n^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{ef-dg}{gx}\right)}{x}\,dx,x,d+ex\right)}{3g(ef-dg)^3} - \frac{(b^2en^2)\,\text{Subst}\left(\int\left(\frac{e^2}{(ef-dg)^2x}-\frac{e^2g}{(ef-dg)(ef-dg+gx)^2}-\frac{e^2g}{(ef-dg)^2(ef-dg+gx)}\right)\,dx,x,d+ex\right)}{3g(ef-dg)}$$

$$= -\frac{b^2e^2n^2}{3g(ef-dg)^2(f+gx)} - \frac{b^2e^3n^2\log(d+ex)}{3g(ef-dg)^3} + \frac{ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)(f+gx)^2} - \frac{2be^2n(d+ex)(a+b\log(c(d+ex)^n))}{3(ef-dg)^3(f+gx)} - \frac{(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^3} + \frac{b^2e^3n^2\log(f+gx)}{g(ef-dg)^3} - \frac{2be^3n(a+b\log(c(d+ex)^n))\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{3g(ef-dg)^3} + \frac{2b^2e^3n^2\text{Li}_2\left(-\frac{ef-dg}{g(d+ex)}\right)}{3g(ef-dg)^3}$$

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx$$

$$= \frac{-(a+b\log(c(d+ex)^n))^2 + \frac{e(f+gx)\left(b(ef-dg)^2n(a+b\log(c(d+ex)^n))+2be(ef-dg)n(f+gx)(a+b\log(c(d+ex)^n))+e^2(f+gx)^2(a+b}{}}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^4,x]
```

```
[Out] (-(a + b*Log[c*(d + e*x)^n])^2 + (e*(f + g*x)*(b*(e*f - d*g)^2*n*(a + b*Log
[c*(d + e*x)^n]) + 2*b*e*(e*f - d*g)*n*(f + g*x)*(a + b*Log[c*(d + e*x)^n])
 + e^2*(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^2 - 2*b^2*e^2*n^2*(f + g*x)^2
*(Log[d + e*x] - Log[f + g*x]) - b^2*e*n^2*(f + g*x)*(e*f - d*g + e*(f + g*
x)*Log[d + e*x] - e*(f + g*x)*Log[f + g*x]) - 2*b*e^2*n*(f + g*x)^2*(a + b*
Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] - 2*b^2*e^2*n^2*(f + g*x
)^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]))/(e*f - d*g)^3)/(3*g*(f + g*x
)^3)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.19 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.38

| method | result |
|---|---|
| risch | $-\frac{b^2\ln((ex+d)^n)^2}{3(gx+f)^3g} - \frac{2b^2ne^3\ln((ex+d)^n)\ln(ex+d)}{3g(dg-ef)^3} - \frac{b^2ne\ln((ex+d)^n)}{3g(dg-ef)(gx+f)^2} + \frac{2b^2ne^3\ln((ex+d)^n)\ln(gx+f)}{3g(dg-ef)^3} + \frac{2b^2ne^2\ln((ex-}{3g(dg-ef)^2(g}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^2*ln((e*x+d)^n)^2/(g*x+f)^3/g-2/3*b^2/g*n*e^3*ln((e*x+d)^n)/(d*g-e*f
)^3*ln(e*x+d)-1/3*b^2/g*n*e*ln((e*x+d)^n)/(d*g-e*f)/(g*x+f)^2+2/3*b^2/g*n*e
^3*ln((e*x+d)^n)/(d*g-e*f)^3*ln(g*x+f)+2/3*b^2/g*n*e^2*ln((e*x+d)^n)/(d*g-e
*f)^2/(g*x+f)+b^2/g*n^2*e^3/(d*g-e*f)^3*ln(e*x+d)-1/3*b^2/g*n^2*e^2/(d*g-e*
f)^2/(g*x+f)-b^2/g*n^2*e^3/(d*g-e*f)^3*ln(g*x+f)+1/3*b^2/g*n^2*e^3/(d*g-e*f
)^3*ln(e*x+d)^2-2/3*b^2/g*n^2*e^3/(d*g-e*f)^3*dilog(((g*x+f)*e+d*g-e*f)/(d*
g-e*f))-2/3*b^2/g*n^2*e^3/(d*g-e*f)^3*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g
-e*f))+(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I
*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-
I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b*(-1/3*ln((e*x+d)^n)/(g*x+f)^3
/g+1/3/g*n*e*(-e^2/(d*g-e*f)^3*ln(e*x+d)-1/2/(d*g-e*f)/(g*x+f)^2+e^2/(d*g-e
*f)^3*ln(g*x+f)+e/(d*g-e*f)^2/(g*x+f)))-1/12*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*c
sgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c
)+2*a)^2/(g*x+f)^3/g
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2}{(gx+f)^4}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^4*x
^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx=\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^4}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**4,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**2/(f + g*x)**4, x)
```

## Maxima [F]

$$\int \frac{(a + b\log{(c(d + ex)^n)})^2}{(f + gx)^4}\,dx = \int \frac{(b\log{((ex + d)^n c)} + a)^2}{(gx + f)^4}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(2*e^2*log(e*x + d)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 - d^3*
g^4) - 2*e^2*log(g*x + f)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 - d^
3*g^4) + (2*e*g*x + 3*e*f - d*g)/(e^2*f^4*g - 2*d*e*f^3*g^2 + d^2*f^2*g^3 +
 (e^2*f^2*g^3 - 2*d*e*f*g^4 + d^2*g^5)*x^2 + 2*(e^2*f^3*g^2 - 2*d*e*f^2*g^3
 + d^2*f*g^4)*x))*a*b*e*n - 1/3*b^2*(log((e*x + d)^n)^2/(g^4*x^3 + 3*f*g^3*
x^2 + 3*f^2*g^2*x + f^3*g) - 3*integrate(1/3*(3*e*g*x*log(c)^2 + 3*d*g*log(
c)^2 + 2*(e*f*n + 3*d*g*log(c) + (e*g*n + 3*e*g*log(c))*x)*log((e*x + d)^n)
)/(e*g^5*x^5 + d*f^4*g + (4*e*f*g^4 + d*g^5)*x^4 + 2*(3*e*f^2*g^3 + 2*d*f*g
^4)*x^3 + 2*(2*e*f^3*g^2 + 3*d*f^2*g^3)*x^2 + (e*f^4*g + 4*d*f^3*g^2)*x), x
)) - 2/3*a*b*log((e*x + d)^n*c)/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*
g) - 1/3*a^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

## Giac [F]

$$\int \frac{(a + b\log{(c(d + ex)^n)})^2}{(f + gx)^4}\,dx = \int \frac{(b\log{((ex + d)^n c)} + a)^2}{(gx + f)^4}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log{(c(d + ex)^n)})^2}{(f + gx)^4}\,dx = \int \frac{(a + b\,\ln{(c\,(d + e\,x)^n)})^2}{(f + g\,x)^4}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^4,x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^4, x)
```

# 3.52 $\int (f+gx)^3 (a+b\log (c(d+ex)^n))^3 \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 598

$$\begin{aligned}
&\int (f+gx)^3 (a+b\log (c(d+ex)^n))^3 \, dx \\
&= \frac{6ab^2(ef-dg)^3n^2x}{e^3} - \frac{6b^3(ef-dg)^3n^3x}{e^3} - \frac{9b^3g(ef-dg)^2n^3(d+ex)^2}{8e^4} \\
&\quad - \frac{2b^3g^2(ef-dg)n^3(d+ex)^3}{9e^4} - \frac{3b^3g^3n^3(d+ex)^4}{128e^4} \\
&\quad + \frac{6b^3(ef-dg)^3n^2(d+ex)\log (c(d+ex)^n)}{e^4} \\
&\quad + \frac{9b^2g(ef-dg)^2n^2(d+ex)^2 (a+b\log (c(d+ex)^n))}{4e^4} \\
&\quad + \frac{2b^2g^2(ef-dg)n^2(d+ex)^3 (a+b\log (c(d+ex)^n))}{3e^4} \\
&\quad + \frac{3b^2g^3n^2(d+ex)^4 (a+b\log (c(d+ex)^n))}{32e^4} \\
&\quad - \frac{3b(ef-dg)^3n(d+ex) (a+b\log (c(d+ex)^n))^2}{e^4} \\
&\quad - \frac{9bg(ef-dg)^2n(d+ex)^2 (a+b\log (c(d+ex)^n))^2}{4e^4} \\
&\quad - \frac{bg^2(ef-dg)n(d+ex)^3 (a+b\log (c(d+ex)^n))^2}{e^4} \\
&\quad - \frac{3bg^3n(d+ex)^4 (a+b\log (c(d+ex)^n))^2}{16e^4} + \frac{(ef-dg)^3(d+ex) (a+b\log (c(d+ex)^n))^3}{e^4} \\
&\quad + \frac{3g(ef-dg)^2(d+ex)^2 (a+b\log (c(d+ex)^n))^3}{2e^4} \\
&\quad + \frac{g^2(ef-dg)(d+ex)^3 (a+b\log (c(d+ex)^n))^3}{e^4} + \frac{g^3(d+ex)^4 (a+b\log (c(d+ex)^n))^3}{4e^4}
\end{aligned}$$

```
[Out] 6*a*b^2*(-d*g+e*f)^3*n^2*x/e^3-6*b^3*(-d*g+e*f)^3*n^3*x/e^3-9/8*b^3*g*(-d*g
+e*f)^2*n^3*(e*x+d)^2/e^4-2/9*b^3*g^2*(-d*g+e*f)*n^3*(e*x+d)^3/e^4-3/128*b^
3*g^3*n^3*(e*x+d)^4/e^4+6*b^3*(-d*g+e*f)^3*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e^4+
9/4*b^2*g*(-d*g+e*f)^2*n^2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^4+2/3*b^2*g^2*
(-d*g+e*f)*n^2*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))/e^4+3/32*b^2*g^3*n^2*(e*x+d)
^4*(a+b*ln(c*(e*x+d)^n))/e^4-3*b*(-d*g+e*f)^3*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n
))^2/e^4-9/4*b*g*(-d*g+e*f)^2*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^4-b*g^2
*(-d*g+e*f)*n*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))^2/e^4-3/16*b*g^3*n*(e*x+d)^4*
(a+b*ln(c*(e*x+d)^n))^2/e^4+(-d*g+e*f)^3*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e^
4+3/2*g*(-d*g+e*f)^2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^3/e^4+g^2*(-d*g+e*f)*(
e*x+d)^3*(a+b*ln(c*(e*x+d)^n))^3/e^4+1/4*g^3*(e*x+d)^4*(a+b*ln(c*(e*x+d)^n)
)^3/e^4
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{2448, 2436, 2333, 2332, 2437, 2342, 2341\}$

$$\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx$$
$$= \frac{2b^2g^2n^2(d+ex)^3(ef-dg)\left(a+b\log \left(c(d+ex)^n\right)\right)}{3e^4} + \frac{9b^2gn^2(d+ex)^2(ef-dg)^2\left(a+b\log \left(c(d+ex)^n\right)\right)}{4e^4} + \frac{3b^2g^3n^2(d+ex)^4\left(a+b\log \left(c(d+ex)^n\right)\right)}{32e^4} + \frac{6ab^2n^2x(ef-dg)^3}{e^3} + \frac{g^2(d+ex)^3(ef-dg)\left(a+b\log \left(c(d+ex)^n\right)\right)^3}{e^4} - \frac{bg^2n(d+ex)^3(ef-dg)\left(a+b\log \left(c(d+ex)^n\right)\right)^2}{e^4} + \frac{3g(d+ex)^2(ef-dg)^2\left(a+b\log \left(c(d+ex)^n\right)\right)^3}{2e^4} - \frac{9bgn(d+ex)^2(ef-dg)^2\left(a+b\log \left(c(d+ex)^n\right)\right)^2}{4e^4} + \frac{(d+ex)(ef-dg)^3\left(a+b\log \left(c(d+ex)^n\right)\right)^3}{e^4} - \frac{3bn(d+ex)(ef-dg)^3\left(a+b\log \left(c(d+ex)^n\right)\right)^2}{e^4} + \frac{g^3(d+ex)^4\left(a+b\log \left(c(d+ex)^n\right)\right)^3}{4e^4} - \frac{3bg^3n(d+ex)^4\left(a+b\log \left(c(d+ex)^n\right)\right)^2}{16e^4} + \frac{6b^3n^2(d+ex)(ef-dg)^3\log \left(c(d+ex)^n\right)}{e^4} - \frac{2b^3g^2n^3(d+ex)^3(ef-dg)}{9e^4} - \frac{9b^3gn^3(d+ex)^2(ef-dg)^2}{8e^4} - \frac{3b^3g^3n^3(d+ex)^4}{128e^4} - \frac{6b^3n^3x(ef-dg)^3}{e^3}$$

```
[In] Int[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] (6*a*b^2*(e*f - d*g)^3*n^2*x)/e^3 - (6*b^3*(e*f - d*g)^3*n^3*x)/e^3 - (9*b^
3*g*(e*f - d*g)^2*n^3*(d + e*x)^2)/(8*e^4) - (2*b^3*g^2*(e*f - d*g)*n^3*(d
+ e*x)^3)/(9*e^4) - (3*b^3*g^3*n^3*(d + e*x)^4)/(128*e^4) + (6*b^3*(e*f - d
*g)^3*n^2*(d + e*x)*Log[c*(d + e*x)^n])/e^4 + (9*b^2*g*(e*f - d*g)^2*n^2*(d
 + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(4*e^4) + (2*b^2*g^2*(e*f - d*g)*n^2*
(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))/(3*e^4) + (3*b^2*g^3*n^2*(d + e*x)^
4*(a + b*Log[c*(d + e*x)^n]))/(32*e^4) - (3*b*(e*f - d*g)^3*n*(d + e*x)*(a
+ b*Log[c*(d + e*x)^n])^2)/e^4 - (9*b*g*(e*f - d*g)^2*n*(d + e*x)^2*(a + b*
Log[c*(d + e*x)^n])^2)/(4*e^4) - (b*g^2*(e*f - d*g)*n*(d + e*x)^3*(a + b*Lo
g[c*(d + e*x)^n])^2)/e^4 - (3*b*g^3*n*(d + e*x)^4*(a + b*Log[c*(d + e*x)^n]
)^2)/(16*e^4) + ((e*f - d*g)^3*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/e^4
+ (3*g*(e*f - d*g)^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^3)/(2*e^4) + (g
^2*(e*f - d*g)*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])^3)/e^4 + (g^3*(d + e*
x)^4*(a + b*Log[c*(d + e*x)^n])^3)/(4*e^4)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(ef-dg)^3 (a+b\log(c(d+ex)^n))^3}{e^3} + \frac{3g(ef-dg)^2(d+ex)(a+b\log(c(d+ex)^n))^3}{e^3} + \frac{3g^2(ef-dg)(d+ex)^2(a+b\log(c(d+ex)^n))^3}{e^3} + \frac{g^3(d+ex)^3(a+b\log(c(d+ex)^n))^3}{e^3}\right) dx$$

$$= \frac{g^3 \int (d+ex)^3 (a+b\log(c(d+ex)^n))^3 \, dx}{e^3} + \frac{(3g^2(ef-dg)) \int (d+ex)^2 (a+b\log(c(d+ex)^n))^3 \, dx}{e^3} + \frac{(3g(ef-dg)^2) \int (d+ex)(a+b\log(c(d+ex)^n))^3 \, dx}{e^3} + \frac{(ef-dg)^3 \int (a+b\log(c(d+ex)^n))^3 \, dx}{e^3}$$

$$= \frac{g^3 \text{Subst}\left(\int x^3(a+b\log(cx^n))^3 \, dx, x, d+ex\right)}{e^4} + \frac{(3g^2(ef-dg)) \text{Subst}\left(\int x^2(a+b\log(cx^n))^3 \, dx, x, d+ex\right)}{e^4} + \frac{(3g(ef-dg)^2) \text{Subst}\left(\int x(a+b\log(cx^n))^3 \, dx, x, d+ex\right)}{e^4} + \frac{(ef-dg)^3 \text{Subst}\left(\int (a+b\log(cx^n))^3 \, dx, x, d+ex\right)}{e^4}$$

$$= \frac{(ef-dg)^3(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^4} + \frac{3g(ef-dg)^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2e^4} + \frac{g^2(ef-dg)(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^4} + \frac{g^3(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{4e^4} - \frac{(3bg^3n)\,\text{Subst}\left(\int x^3(a+b\log(cx^n))^2\,dx,x,d+ex\right)}{4e^4} - \frac{(3bg^2(ef-dg)n)\,\text{Subst}\left(\int x^2(a+b\log(cx^n))^2\,dx,x,d+ex\right)}{e^4} - \frac{(9bg(ef-dg)^2n)\,\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx,x,d+ex\right)}{2e^4} - \frac{(3b(ef-dg)^3n)\,\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx,x,d+ex\right)}{e^4}$$

$$= -\frac{3b(ef-dg)^3n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^4} - \frac{9bg(ef-dg)^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4e^4} - \frac{bg^2(ef-dg)n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^4} - \frac{3bg^3n(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{16e^4} + \frac{(ef-dg)^3(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^4} + \frac{3g(ef-dg)^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2e^4} + \frac{g^2(ef-dg)(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^4} + \frac{g^3(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{4e^4} + \frac{(3b^2g^3n^2)\,\text{Subst}\left(\int x^3(a+b\log(cx^n))\,dx,x,d+ex\right)}{8e^4} + \frac{(2b^2g^2(ef-dg)n^2)\,\text{Subst}\left(\int x^2(a+b\log(cx^n))\,dx,x,d+ex\right)}{e^4} + \frac{(9b^2g(ef-dg)^2n^2)\,\text{Subst}\left(\int x(a+b\log(cx^n))\,dx,x,d+ex\right)}{2e^4} + \frac{(6b^2(ef-dg)^3n^2)\,\text{Subst}\left(\int (a+b\log(cx^n))\,dx,x,d+ex\right)}{e^4}$$

$$
\begin{aligned}
&= \frac{6ab^2(ef-dg)^3n^2x}{e^3} - \frac{9b^3g(ef-dg)^2n^3(d+ex)^2}{8e^4} - \frac{2b^3g^2(ef-dg)n^3(d+ex)^3}{9e^4} \\
&\quad - \frac{3b^3g^3n^3(d+ex)^4}{128e^4} + \frac{9b^2g(ef-dg)^2n^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{4e^4} \\
&\quad + \frac{2b^2g^2(ef-dg)n^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^4} \\
&\quad + \frac{3b^2g^3n^2(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{32e^4} \\
&\quad - \frac{3b(ef-dg)^3n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^4} \\
&\quad - \frac{9bg(ef-dg)^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4e^4} \\
&\quad - \frac{bg^2(ef-dg)n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^4} \\
&\quad - \frac{3bg^3n(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{16e^4} \\
&\quad + \frac{(ef-dg)^3(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^4} \\
&\quad + \frac{3g(ef-dg)^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2e^4} \\
&\quad + \frac{g^2(ef-dg)(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^4} \\
&\quad + \frac{g^3(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{4e^4} \\
&\quad + \frac{\left(6b^3(ef-dg)^3n^2\right)\text{Subst}\left(\int \log\left(cx^n\right)\,dx, x, d+ex\right)}{e^4}
\end{aligned}
$$

$$= \frac{6ab^2(ef-dg)^3n^2x}{e^3} - \frac{6b^3(ef-dg)^3n^3x}{e^3} - \frac{9b^3g(ef-dg)^2n^3(d+ex)^2}{8e^4} - \frac{2b^3g^2(ef-dg)n^3(d+ex)^3}{9e^4} - \frac{3b^3g^3n^3(d+ex)^4}{128e^4} + \frac{6b^3(ef-dg)^3n^2(d+ex)\log(c(d+ex)^n)}{e^4} + \frac{9b^2g(ef-dg)^2n^2(d+ex)^2(a+b\log(c(d+ex)^n))}{4e^4} + \frac{2b^2g^2(ef-dg)n^2(d+ex)^3(a+b\log(c(d+ex)^n))}{3e^4} + \frac{3b^2g^3n^2(d+ex)^4(a+b\log(c(d+ex)^n))}{32e^4} - \frac{3b(ef-dg)^3n(d+ex)(a+b\log(c(d+ex)^n))^2}{e^4} - \frac{9bg(ef-dg)^2n(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^4} - \frac{bg^2(ef-dg)n(d+ex)^3(a+b\log(c(d+ex)^n))^2}{e^4} - \frac{3bg^3n(d+ex)^4(a+b\log(c(d+ex)^n))^2}{16e^4} + \frac{(ef-dg)^3(d+ex)(a+b\log(c(d+ex)^n))^3}{e^4} + \frac{3g(ef-dg)^2(d+ex)^2(a+b\log(c(d+ex)^n))^3}{2e^4} + \frac{g^2(ef-dg)(d+ex)^3(a+b\log(c(d+ex)^n))^3}{e^4} + \frac{g^3(d+ex)^4(a+b\log(c(d+ex)^n))^3}{4e^4}$$

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 475, normalized size of antiderivative = 0.79

$$\int (f+gx)^3(a+b\log(c(d+ex)^n))^3\,dx$$
$$= \frac{1152(ef-dg)^3(d+ex)(a+b\log(c(d+ex)^n))^3 + 1728g(ef-dg)^2(d+ex)^2(a+b\log(c(d+ex)^n))^3 +}{}$$

```
[In] Integrate[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] (1152*(e*f - d*g)^3*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3 + 1728*g*(e*f -
d*g)^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^3 + 1152*g^2*(e*f - d*g)*(d +
 e*x)^3*(a + b*Log[c*(d + e*x)^n])^3 + 288*g^3*(d + e*x)^4*(a + b*Log[c*(d
+ e*x)^n])^3 - 3456*b*(e*f - d*g)^3*n*((d + e*x)*(a + b*Log[c*(d + e*x)^n])
```

```
^2 - 2*b*n*(e*(a - b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n])) - 1296*b*g*(e*
f - d*g)^2*n*(2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2 + b*n*(b*e*n*x*(2*
d + e*x) - 2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))) - 128*b*g^2*(e*f - d*
g)*n*(9*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])^2 + 2*b*n*(b*e*n*x*(3*d^2 +
3*d*e*x + e^2*x^2) - 3*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))) - 27*b*g^3*
n*(8*(d + e*x)^4*(a + b*Log[c*(d + e*x)^n])^2 + b*n*(b*e*n*x*(4*d^3 + 6*d^2
*e*x + 4*d*e^2*x^2 + e^3*x^3) - 4*(d + e*x)^4*(a + b*Log[c*(d + e*x)^n]))))
/(1152*e^4)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2718 vs. 2(578) = 1156.

Time = 5.42 (sec) , antiderivative size = 2719, normalized size of antiderivative = 4.55

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 2719 |
| risch | Expression too large to display | 30495 |

```
[In] int((g*x+f)^3*(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1152*(-108*a*b^2*e^4*g^3*n^2*x^4-1152*a^3*e^4*f^3*x+1152*a^3*d*e^3*f^3-1
48*b^3*d*e^3*g^3*n^3*x^3+256*b^3*e^4*f*g^2*n^3*x^3+216*a^2*b*e^4*g^3*n*x^4+
690*b^3*d^2*e^2*g^3*n^3*x^2+1296*b^3*e^4*f^2*g*n^3*x^2-4980*b^3*d^3*e*g^3*n
^3*x-16320*b^3*d^3*e*f*g^2*n^3+18144*b^3*d^2*e^2*f^2*g*n^3+6912*a*b^2*d*e^3
*f^3*n^2-288*a^3*e^4*g^3*x^4+6912*b^3*n^3*e^4*f^3*x+3456*b*n*a^2*e^4*f^3*x-
6912*b^2*n^2*a*e^4*f^3*x-3456*a^2*b*d^3*e*f*g^2*n+5184*a^2*b*d^2*e^2*f^2*g*
n+2880*a*b^2*d*e^3*f*g^2*n^2*x^2-12672*a*b^2*d^2*e^2*f*g^2*n^2*x+15552*a*b^
2*d*e^3*f^2*g*n^2*x+8580*ln(e*x+d)*b^3*d^4*g^3*n^3+4980*b^3*d^4*g^3*n^3+432
*a^2*b*d^2*e^2*g^3*n*x^2+864*a^2*b*d^4*g^3*n+27*b^3*e^4*g^3*n^3*x^4-1152*a^
3*e^4*f*g^2*x^3-1728*a^3*e^4*f^2*g*x^2-6912*b^3*d*e^3*f^3*n^3-3600*a*b^2*d^
4*g^3*n^2-1728*a^2*b*d*e^3*f*g^2*n*x^2+2592*a^2*b*e^4*f^2*g*n*x^2+3600*a*b^
2*d^3*e*g^3*n^2*x+336*a*b^2*d*e^3*g^3*n^2*x^3-768*a*b^2*e^4*f*g^2*n^2*x^3-1
824*b^3*d*e^3*f*g^2*n^3*x^2+1152*a^2*b*e^4*f*g^2*n*x^3-936*a*b^2*d^2*e^2*g^
3*n^2*x^2-2592*a*b^2*e^4*f^2*g*n^2*x^2+16320*b^3*d^2*e^2*f*g^2*n^3*x-18144*
b^3*d*e^3*f^2*g*n^3*x+12672*a*b^2*d^3*e*f*g^2*n^2-15552*a*b^2*d^2*e^2*f^2*g
*n^2-288*x^4*ln(c*(e*x+d)^n)^3*b^3*e^4*g^3-1152*x*ln(c*(e*x+d)^n)^3*b^3*e^4
*f^3-1152*ln(c*(e*x+d)^n)^3*b^3*d*e^3*f^3-1800*ln(c*(e*x+d)^n)^2*b^3*d^4*g^
3*n-3600*ln(c*(e*x+d)^n)*b^3*d^4*g^3*n^2+864*ln(c*(e*x+d)^n)^2*a*b^2*d^4*g^
3-13824*ln(e*x+d)*b^3*d*e^3*f^3*n^3-5328*ln(e*x+d)*a*b^2*d^4*g^3*n^2+864*ln
(e*x+d)*a^2*b*d^4*g^3*n+288*ln(c*(e*x+d)^n)^3*b^3*d^4*g^3-3456*a^2*b*d*e^3*
f^3*n-3456*x^2*ln(c*(e*x+d)^n)*a*b^2*d*e^3*f*g^2*n+6912*x*ln(c*(e*x+d)^n)*a
*b^2*d^2*e^2*f*g^2*n-10368*x*ln(c*(e*x+d)^n)*a*b^2*d*e^3*f^2*g*n+2304*x^3*l
n(c*(e*x+d)^n)*a*b^2*e^4*f*g^2*n-1728*x^2*ln(c*(e*x+d)^n)^2*b^3*d*e^3*f*g^2
*n+2880*x^2*ln(c*(e*x+d)^n)*b^3*d*e^3*f*g^2*n^2+864*x^2*ln(c*(e*x+d)^n)*a*b
```

```
^2*d^2*e^2*g^3*n+5184*x^2*ln(c*(e*x+d)^n)*a*b^2*e^4*f^2*g*n+3456*x*ln(c*(e*
x+d)^n)^2*b^3*d^2*e^2*f*g^2*n-5184*x*ln(c*(e*x+d)^n)^2*b^3*d*e^3*f^2*g*n-12
672*x*ln(c*(e*x+d)^n)*b^3*d^2*e^2*f*g^2*n^2+15552*x*ln(c*(e*x+d)^n)*b^3*d*e
^3*f^2*g*n^2-1728*x*ln(c*(e*x+d)^n)*a*b^2*d^3*e*g^3*n-6912*ln(c*(e*x+d)^n)*
a*b^2*d^3*e*f*g^2*n+10368*ln(c*(e*x+d)^n)*a*b^2*d^2*e^2*f^2*g*n-1152*x^3*ln
(c*(e*x+d)^n)^3*b^3*e^4*f*g^2-864*x^4*ln(c*(e*x+d)^n)*a^2*b*e^4*g^3-1728*x^
2*ln(c*(e*x+d)^n)^3*b^3*e^4*f^2*g+3456*x*ln(c*(e*x+d)^n)^2*b^3*e^4*f^3*n-69
12*x*ln(c*(e*x+d)^n)*b^3*e^4*f^3*n^2-3456*x*ln(c*(e*x+d)^n)^2*a*b^2*e^4*f^3
-1152*ln(c*(e*x+d)^n)^3*b^3*d^3*e*f*g^2+1728*ln(c*(e*x+d)^n)^3*b^3*d^2*e^2*
f^2*g+3456*ln(c*(e*x+d)^n)^2*b^3*d*e^3*f^3*n+6912*ln(c*(e*x+d)^n)*b^3*d*e^3
*f^3*n^2-3456*x*ln(c*(e*x+d)^n)*a^2*b*e^4*f^3-3456*ln(c*(e*x+d)^n)^2*a*b^2*
d*e^3*f^3+1728*ln(c*(e*x+d)^n)*a*b^2*d^4*g^3*n+3456*ln(c*(e*x+d)^n)*a^2*b*d
*e^3*f^3-5184*x^2*ln(c*(e*x+d)^n)^2*a*b^2*e^4*f^2*g-864*x*ln(c*(e*x+d)^n)^2
*b^3*d^3*e*g^3*n+3600*x*ln(c*(e*x+d)^n)*b^3*d^3*e*g^3*n^2-5184*x^2*ln(c*(e*
x+d)^n)*a^2*b*e^4*f^2*g+6912*x*ln(c*(e*x+d)^n)*a*b^2*e^4*f^3*n+6336*ln(c*(e
*x+d)^n)^2*b^3*d^3*e*f*g^2*n-7776*ln(c*(e*x+d)^n)^2*b^3*d^2*e^2*f^2*g*n+126
72*ln(c*(e*x+d)^n)*b^3*d^3*e*f*g^2*n^2-15552*ln(c*(e*x+d)^n)*b^3*d^2*e^2*f^
2*g*n^2-3456*ln(c*(e*x+d)^n)^2*a*b^2*d^3*e*f*g^2+5184*ln(c*(e*x+d)^n)^2*a*b
^2*d^2*e^2*f^2*g-6912*ln(c*(e*x+d)^n)*a*b^2*d*e^3*f^3*n+19584*ln(e*x+d)*a*b
^2*d^3*e*f*g^2*n^2-25920*ln(e*x+d)*a*b^2*d^2*e^2*f^2*g*n^2+216*x^4*ln(c*(e*
x+d)^n)^2*b^3*e^4*g^3*n-108*x^4*ln(c*(e*x+d)^n)*b^3*e^4*g^3*n^2-864*x^4*ln(
c*(e*x+d)^n)^2*a*b^2*e^4*g^3-3456*ln(e*x+d)*a^2*b*d^3*e*f*g^2*n+5184*ln(e*x
+d)*a^2*b*d^2*e^2*f^2*g*n-288*a^2*b*d*e^3*g^3*n*x^3-864*a^2*b*d^3*e*g^3*n*x
-28992*ln(e*x+d)*b^3*d^3*e*f*g^2*n^3+33696*ln(e*x+d)*b^3*d^2*e^2*f^2*g*n^3+
13824*ln(e*x+d)*a*b^2*d*e^3*f^3*n^2-6912*ln(e*x+d)*a^2*b*d*e^3*f^3*n+432*x^
4*ln(c*(e*x+d)^n)*a*b^2*e^4*g^3*n-288*x^3*ln(c*(e*x+d)^n)^2*b^3*d*e^3*g^3*n
+1152*x^3*ln(c*(e*x+d)^n)^2*b^3*e^4*f*g^2*n+336*x^3*ln(c*(e*x+d)^n)*b^3*d*e
^3*g^3*n^2-768*x^3*ln(c*(e*x+d)^n)*b^3*e^4*f*g^2*n^2-3456*x^3*ln(c*(e*x+d)^
n)^2*a*b^2*e^4*f*g^2+432*x^2*ln(c*(e*x+d)^n)^2*b^3*d^2*e^2*g^3*n+2592*x^2*l
n(c*(e*x+d)^n)^2*b^3*e^4*f^2*g*n-936*x^2*ln(c*(e*x+d)^n)*b^3*d^2*e^2*g^3*n^
2-2592*x^2*ln(c*(e*x+d)^n)*b^3*e^4*f^2*g*n^2-3456*x^3*ln(c*(e*x+d)^n)*a^2*b
*e^4*f*g^2+3456*a^2*b*d^2*e^2*f*g^2*n*x-5184*a^2*b*d*e^3*f^2*g*n*x-576*x^3*
ln(c*(e*x+d)^n)*a*b^2*d*e^3*g^3*n)/e^4
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2802 vs. $2(578) = 1156$.

Time = 0.38 (sec) , antiderivative size = 2802, normalized size of antiderivative = 4.69

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^3 \, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

```
[Out] -1/1152*(9*(3*b^3*e^4*g^3*n^3 - 12*a*b^2*e^4*g^3*n^2 + 24*a^2*b*e^4*g^3*n -
 32*a^3*e^4*g^3)*x^4 - 4*(288*a^3*e^4*f*g^2 - (64*b^3*e^4*f*g^2 - 37*b^3*d*
e^3*g^3)*n^3 + 12*(16*a*b^2*e^4*f*g^2 - 7*a*b^2*d*e^3*g^3)*n^2 - 72*(4*a^2*
b*e^4*f*g^2 - a^2*b*d*e^3*g^3)*n)*x^3 - 288*(b^3*e^4*g^3*n^3*x^4 + 4*b^3*e^
4*f*g^2*n^3*x^3 + 6*b^3*e^4*f^2*g*n^3*x^2 + 4*b^3*e^4*f^3*n^3*x + (4*b^3*d*
e^3*f^3 - 6*b^3*d^2*e^2*f^2*g + 4*b^3*d^3*e*f*g^2 - b^3*d^4*g^3)*n^3)*log(e
*x + d)^3 - 288*(b^3*e^4*g^3*x^4 + 4*b^3*e^4*f*g^2*x^3 + 6*b^3*e^4*f^2*g*x^
2 + 4*b^3*e^4*f^3*x)*log(c)^3 - 6*(288*a^3*e^4*f^2*g - (216*b^3*e^4*f^2*g -
 304*b^3*d*e^3*f*g^2 + 115*b^3*d^2*e^2*g^3)*n^3 + 12*(36*a*b^2*e^4*f^2*g -
40*a*b^2*d*e^3*f*g^2 + 13*a*b^2*d^2*e^2*g^3)*n^2 - 72*(6*a^2*b*e^4*f^2*g -
4*a^2*b*d*e^3*f*g^2 + a^2*b*d^2*e^2*g^3)*n)*x^2 + 72*(3*(b^3*e^4*g^3*n^3 -
4*a*b^2*e^4*g^3*n^2)*x^4 + (48*b^3*d*e^3*f^3 - 108*b^3*d^2*e^2*f^2*g + 88*b
^3*d^3*e*f*g^2 - 25*b^3*d^4*g^3)*n^3 - 4*(12*a*b^2*e^4*f*g^2*n^2 - (4*b^3*e
^4*f*g^2 - b^3*d*e^3*g^3)*n^3)*x^3 - 12*(4*a*b^2*d*e^3*f^3 - 6*a*b^2*d^2*e^
2*f^2*g + 4*a*b^2*d^3*e*f*g^2 - a*b^2*d^4*g^3)*n^2 - 6*(12*a*b^2*e^4*f^2*g*
n^2 - (6*b^3*e^4*f^2*g - 4*b^3*d*e^3*f*g^2 + b^3*d^2*e^2*g^3)*n^3)*x^2 - 12
*(4*a*b^2*e^4*f^3*n^2 - (4*b^3*e^4*f^3 - 6*b^3*d*e^3*f^2*g + 4*b^3*d^2*e^2*
f*g^2 - b^3*d^3*e*g^3)*n^3)*x - 12*(b^3*e^4*g^3*n^2*x^4 + 4*b^3*e^4*f*g^2*n
^2*x^3 + 6*b^3*e^4*f^2*g*n^2*x^2 + 4*b^3*e^4*f^3*n^2*x + (4*b^3*d*e^3*f^3 -
 6*b^3*d^2*e^2*f^2*g + 4*b^3*d^3*e*f*g^2 - b^3*d^4*g^3)*n^2)*log(c))*log(e*
x + d)^2 + 72*(3*(b^3*e^4*g^3*n - 4*a*b^2*e^4*g^3)*x^4 - 4*(12*a*b^2*e^4*f*
g^2 - (4*b^3*e^4*f*g^2 - b^3*d*e^3*g^3)*n)*x^3 - 6*(12*a*b^2*e^4*f^2*g - (6
*b^3*e^4*f^2*g - 4*b^3*d*e^3*f*g^2 + b^3*d^2*e^2*g^3)*n)*x^2 - 12*(4*a*b^2*
e^4*f^3 - (4*b^3*e^4*f^3 - 6*b^3*d*e^3*f^2*g + 4*b^3*d^2*e^2*f*g^2 - b^3*d^
3*e*g^3)*n)*x)*log(c)^2 - 12*(96*a^3*e^4*f^3 - (576*b^3*e^4*f^3 - 1512*b^3*
d*e^3*f^2*g + 1360*b^3*d^2*e^2*f*g^2 - 415*b^3*d^3*e*g^3)*n^3 + 12*(48*a*b^
2*e^4*f^3 - 108*a*b^2*d*e^3*f^2*g + 88*a*b^2*d^2*e^2*f*g^2 - 25*a*b^2*d^3*e
*g^3)*n^2 - 72*(4*a^2*b*e^4*f^3 - 6*a^2*b*d*e^3*f^2*g + 4*a^2*b*d^2*e^2*f*g
^2 - a^2*b*d^3*e*g^3)*n)*x - 12*(9*(b^3*e^4*g^3*n^3 - 4*a*b^2*e^4*g^3*n^2 +
 8*a^2*b*e^4*g^3*n)*x^4 + (576*b^3*d*e^3*f^3 - 1512*b^3*d^2*e^2*f^2*g + 136
0*b^3*d^3*e*f*g^2 - 415*b^3*d^4*g^3)*n^3 + 4*(72*a^2*b*e^4*f*g^2*n + (16*b^
3*e^4*f*g^2 - 7*b^3*d*e^3*g^3)*n^3 - 12*(4*a*b^2*e^4*f*g^2 - a*b^2*d*e^3*g^
3)*n^2)*x^3 - 12*(48*a*b^2*d*e^3*f^3 - 108*a*b^2*d^2*e^2*f^2*g + 88*a*b^2*d
^3*e*f*g^2 - 25*a*b^2*d^4*g^3)*n^2 + 6*(72*a^2*b*e^4*f^2*g*n + (36*b^3*e^4*
f^2*g - 40*b^3*d*e^3*f*g^2 + 13*b^3*d^2*e^2*g^3)*n^3 - 12*(6*a*b^2*e^4*f^2*
g - 4*a*b^2*d*e^3*f*g^2 + a*b^2*d^2*e^2*g^3)*n^2)*x^2 + 72*(b^3*e^4*g^3*n*x
^4 + 4*b^3*e^4*f*g^2*n*x^3 + 6*b^3*e^4*f^2*g*n*x^2 + 4*b^3*e^4*f^3*n*x + (4
*b^3*d*e^3*f^3 - 6*b^3*d^2*e^2*f^2*g + 4*b^3*d^3*e*f*g^2 - b^3*d^4*g^3)*n)*
log(c)^2 + 72*(4*a^2*b*d*e^3*f^3 - 6*a^2*b*d^2*e^2*f^2*g + 4*a^2*b*d^3*e*f*
g^2 - a^2*b*d^4*g^3)*n + 12*(24*a^2*b*e^4*f^3*n + (48*b^3*e^4*f^3 - 108*b^3
*d*e^3*f^2*g + 88*b^3*d^2*e^2*f*g^2 - 25*b^3*d^3*e*g^3)*n^3 - 12*(4*a*b^2*e
^4*f^3 - 6*a*b^2*d*e^3*f^2*g + 4*a*b^2*d^2*e^2*f*g^2 - a*b^2*d^3*e*g^3)*n^2
)*x - 12*(3*(b^3*e^4*g^3*n^2 - 4*a*b^2*e^4*g^3*n)*x^4 - 4*(12*a*b^2*e^4*f*g
^2*n - (4*b^3*e^4*f*g^2 - b^3*d*e^3*g^3)*n^2)*x^3 + (48*b^3*d*e^3*f^3 - 108
*b^3*d^2*e^2*f^2*g + 88*b^3*d^3*e*f*g^2 - 25*b^3*d^4*g^3)*n^2 - 6*(12*a*b^2
```

```
*e^4*f^2*g*n - (6*b^3*e^4*f^2*g - 4*b^3*d*e^3*f*g^2 + b^3*d^2*e^2*g^3)*n^2)
*x^2 - 12*(4*a*b^2*d*e^3*f^3 - 6*a*b^2*d^2*e^2*f^2*g + 4*a*b^2*d^3*e*f*g^2
- a*b^2*d^4*g^3)*n - 12*(4*a*b^2*e^4*f^3*n - (4*b^3*e^4*f^3 - 6*b^3*d*e^3*f
^2*g + 4*b^3*d^2*e^2*f*g^2 - b^3*d^3*e*g^3)*n^2)*x)*log(c))*log(e*x + d) -
12*(9*(b^3*e^4*g^3*n^2 - 4*a*b^2*e^4*g^3*n + 8*a^2*b*e^4*g^3)*x^4 + 4*(72*a
^2*b*e^4*f*g^2 + (16*b^3*e^4*f*g^2 - 7*b^3*d*e^3*g^3)*n^2 - 12*(4*a*b^2*e^4
*f*g^2 - a*b^2*d*e^3*g^3)*n)*x^3 + 6*(72*a^2*b*e^4*f^2*g + (36*b^3*e^4*f^2*
g - 40*b^3*d*e^3*f*g^2 + 13*b^3*d^2*e^2*g^3)*n^2 - 12*(6*a*b^2*e^4*f^2*g -
4*a*b^2*d*e^3*f*g^2 + a*b^2*d^2*e^2*g^3)*n)*x^2 + 12*(24*a^2*b*e^4*f^3 + (4
8*b^3*e^4*f^3 - 108*b^3*d*e^3*f^2*g + 88*b^3*d^2*e^2*f*g^2 - 25*b^3*d^3*e*g
^3)*n^2 - 12*(4*a*b^2*e^4*f^3 - 6*a*b^2*d*e^3*f^2*g + 4*a*b^2*d^2*e^2*f*g^2
 - a*b^2*d^3*e*g^3)*n)*x)*log(c))/e^4
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2594 vs. 2(583) = 1166.

Time = 4.44 (sec) , antiderivative size = 2594, normalized size of antiderivative = 4.34

$$\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)**3*(a+b*ln(c*(e*x+d)**n))**3,x)
```

```
[Out] Piecewise((a**3*f**3*x + 3*a**3*f**2*g*x**2/2 + a**3*f*g**2*x**3 + a**3*g**
3*x**4/4 - 3*a**2*b*d**4*g**3*log(c*(d + e*x)**n)/(4*e**4) + 3*a**2*b*d**3*
f*g**2*log(c*(d + e*x)**n)/e**3 + 3*a**2*b*d**3*g**3*n*x/(4*e**3) - 9*a**2*
b*d**2*f**2*g*log(c*(d + e*x)**n)/(2*e**2) - 3*a**2*b*d**2*f*g**2*n*x/e**2
- 3*a**2*b*d**2*g**3*n*x**2/(8*e**2) + 3*a**2*b*d*f**3*log(c*(d + e*x)**n)/
e + 9*a**2*b*d*f**2*g*n*x/(2*e) + 3*a**2*b*d*f*g**2*n*x**2/(2*e) + a**2*b*d
*g**3*n*x**3/(4*e) - 3*a**2*b*f**3*n*x + 3*a**2*b*f**3*x*log(c*(d + e*x)**n
) - 9*a**2*b*f**2*g*n*x**2/4 + 9*a**2*b*f**2*g*x**2*log(c*(d + e*x)**n)/2 -
 a**2*b*f*g**2*n*x**3 + 3*a**2*b*f*g**2*x**3*log(c*(d + e*x)**n) - 3*a**2*b
*g**3*n*x**4/16 + 3*a**2*b*g**3*x**4*log(c*(d + e*x)**n)/4 + 25*a*b**2*d**4
*g**3*n*log(c*(d + e*x)**n)/(8*e**4) - 3*a*b**2*d**4*g**3*log(c*(d + e*x)**
n)**2/(4*e**4) - 11*a*b**2*d**3*f*g**2*n*log(c*(d + e*x)**n)/e**3 + 3*a*b**
2*d**3*f*g**2*log(c*(d + e*x)**n)**2/e**3 - 25*a*b**2*d**3*g**3*n**2*x/(8*e
**3) + 3*a*b**2*d**3*g**3*n*x*log(c*(d + e*x)**n)/(2*e**3) + 27*a*b**2*d**2
*f**2*g*n*log(c*(d + e*x)**n)/(2*e**2) - 9*a*b**2*d**2*f**2*g*log(c*(d + e*
x)**n)**2/(2*e**2) + 11*a*b**2*d**2*f*g**2*n**2*x/e**2 - 6*a*b**2*d**2*f*g*
*2*n*x*log(c*(d + e*x)**n)/e**2 + 13*a*b**2*d**2*g**3*n**2*x**2/(16*e**2) -
 3*a*b**2*d**2*g**3*n*x**2*log(c*(d + e*x)**n)/(4*e**2) - 6*a*b**2*d*f**3*n
*log(c*(d + e*x)**n)/e + 3*a*b**2*d*f**3*log(c*(d + e*x)**n)**2/e - 27*a*b*
*2*d*f**2*g*n**2*x/(2*e) + 9*a*b**2*d*f**2*g*n*x*log(c*(d + e*x)**n)/e - 5*
a*b**2*d*f*g**2*n**2*x**2/(2*e) + 3*a*b**2*d*f*g**2*n*x**2*log(c*(d + e*x)*
*n)/e - 7*a*b**2*d*g**3*n**2*x**3/(24*e) + a*b**2*d*g**3*n*x**3*log(c*(d +
```

```
e*x)**n)/(2*e) + 6*a*b**2*f**3*n**2*x - 6*a*b**2*f**3*n*x*log(c*(d + e*x)**
n) + 3*a*b**2*f**3*x*log(c*(d + e*x)**n)**2 + 9*a*b**2*f**2*g*n**2*x**2/4 -
 9*a*b**2*f**2*g*n*x**2*log(c*(d + e*x)**n)/2 + 9*a*b**2*f**2*g*x**2*log(c*
(d + e*x)**n)**2/2 + 2*a*b**2*f*g**2*n**2*x**3/3 - 2*a*b**2*f*g**2*n*x**3*l
og(c*(d + e*x)**n) + 3*a*b**2*f*g**2*x**3*log(c*(d + e*x)**n)**2 + 3*a*b**2
*g**3*n**2*x**4/32 - 3*a*b**2*g**3*n*x**4*log(c*(d + e*x)**n)/8 + 3*a*b**2*
g**3*x**4*log(c*(d + e*x)**n)**2/4 - 415*b**3*d**4*g**3*n**2*log(c*(d + e*x
)**n)/(96*e**4) + 25*b**3*d**4*g**3*n*log(c*(d + e*x)**n)**2/(16*e**4) - b*
*3*d**4*g**3*log(c*(d + e*x)**n)**3/(4*e**4) + 85*b**3*d**3*f*g**2*n**2*log
(c*(d + e*x)**n)/(6*e**3) - 11*b**3*d**3*f*g**2*n*log(c*(d + e*x)**n)**2/(2
*e**3) + b**3*d**3*f*g**2*log(c*(d + e*x)**n)**3/e**3 + 415*b**3*d**3*g**3*
n**3*x/(96*e**3) - 25*b**3*d**3*g**3*n**2*x*log(c*(d + e*x)**n)/(8*e**3) +
3*b**3*d**3*g**3*n*x*log(c*(d + e*x)**n)**2/(4*e**3) - 63*b**3*d**2*f**2*g*
n**2*log(c*(d + e*x)**n)/(4*e**2) + 27*b**3*d**2*f**2*g*n*log(c*(d + e*x)**
n)**2/(4*e**2) - 3*b**3*d**2*f**2*g*log(c*(d + e*x)**n)**3/(2*e**2) - 85*b*
*3*d**2*f*g**2*n**3*x/(6*e**2) + 11*b**3*d**2*f*g**2*n**2*x*log(c*(d + e*x)
**n)/e**2 - 3*b**3*d**2*f*g**2*n*x*log(c*(d + e*x)**n)**2/e**2 - 115*b**3*d
**2*g**3*n**3*x**2/(192*e**2) + 13*b**3*d**2*g**3*n**2*x**2*log(c*(d + e*x)
**n)/(16*e**2) - 3*b**3*d**2*g**3*n*x**2*log(c*(d + e*x)**n)**2/(8*e**2) +
6*b**3*d*f**3*n**2*log(c*(d + e*x)**n)/e - 3*b**3*d*f**3*n*log(c*(d + e*x)*
*n)**2/e + b**3*d*f**3*log(c*(d + e*x)**n)**3/e + 63*b**3*d*f**2*g*n**3*x/(
4*e) - 27*b**3*d*f**2*g*n**2*x*log(c*(d + e*x)**n)/(2*e) + 9*b**3*d*f**2*g*
n*x*log(c*(d + e*x)**n)**2/(2*e) + 19*b**3*d*f*g**2*n**3*x**2/(12*e) - 5*b*
*3*d*f*g**2*n**2*x**2*log(c*(d + e*x)**n)/(2*e) + 3*b**3*d*f*g**2*n*x**2*lo
g(c*(d + e*x)**n)**2/(2*e) + 37*b**3*d*g**3*n**3*x**3/(288*e) - 7*b**3*d*g*
*3*n**2*x**3*log(c*(d + e*x)**n)/(24*e) + b**3*d*g**3*n*x**3*log(c*(d + e*x
)**n)**2/(4*e) - 6*b**3*f**3*n**3*x + 6*b**3*f**3*n**2*x*log(c*(d + e*x)**n
) - 3*b**3*f**3*n*x*log(c*(d + e*x)**n)**2 + b**3*f**3*x*log(c*(d + e*x)**n
)**3 - 9*b**3*f**2*g*n**3*x**2/8 + 9*b**3*f**2*g*n**2*x**2*log(c*(d + e*x)*
*n)/4 - 9*b**3*f**2*g*n*x**2*log(c*(d + e*x)**n)**2/4 + 3*b**3*f**2*g*x**2*
log(c*(d + e*x)**n)**3/2 - 2*b**3*f*g**2*n**3*x**3/9 + 2*b**3*f*g**2*n**2*x
**3*log(c*(d + e*x)**n)/3 - b**3*f*g**2*n*x**3*log(c*(d + e*x)**n)**2 + b**
3*f*g**2*x**3*log(c*(d + e*x)**n)**3 - 3*b**3*g**3*n**3*x**4/128 + 3*b**3*g
**3*n**2*x**4*log(c*(d + e*x)**n)/32 - 3*b**3*g**3*n*x**4*log(c*(d + e*x)**
n)**2/16 + b**3*g**3*x**4*log(c*(d + e*x)**n)**3/4, Ne(e, 0)), ((a + b*log(
c*d**n))**3*(f**3*x + 3*f**2*g*x**2/2 + f*g**2*x**3 + g**3*x**4/4), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1687 vs. 2(578) = 1156.

Time = 0.26 (sec) , antiderivative size = 1687, normalized size of antiderivative = 2.82

$$\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*g^3*x^4*log((e*x + d)^n*c)^3 + 3/4*a*b^2*g^3*x^4*log((e*x + d)^n*c)
^2 + b^3*f*g^2*x^3*log((e*x + d)^n*c)^3 + 3/4*a^2*b*g^3*x^4*log((e*x + d)^n
*c) + 3*a*b^2*f*g^2*x^3*log((e*x + d)^n*c)^2 + 3/2*b^3*f^2*g*x^2*log((e*x +
 d)^n*c)^3 + 1/4*a^3*g^3*x^4 + 3*a^2*b*f*g^2*x^3*log((e*x + d)^n*c) + 9/2*a
*b^2*f^2*g*x^2*log((e*x + d)^n*c)^2 + b^3*f^3*x*log((e*x + d)^n*c)^3 + a^3*
f*g^2*x^3 - 3*a^2*b*e*f^3*n*(x/e - d*log(e*x + d)/e^2) - 1/16*a^2*b*e*g^3*n
*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e^2*x^3 + 6*d^2*e*x^2 - 12*d^3
*x)/e^4) + 1/2*a^2*b*e*f*g^2*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e
*x^2 + 6*d^2*x)/e^3) - 9/4*a^2*b*e*f^2*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2
 - 2*d*x)/e^2) + 9/2*a^2*b*f^2*g*x^2*log((e*x + d)^n*c) + 3*a*b^2*f^3*x*log
((e*x + d)^n*c)^2 + 3/2*a^3*f^2*g*x^2 + 3*a^2*b*f^3*x*log((e*x + d)^n*c) -
3*(2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c) + (d*log(e*x + d)^2
- 2*e*x + 2*d*log(e*x + d))*n^2/e)*a*b^2*f^3 - (3*e*n*(x/e - d*log(e*x + d)
/e^2)*log((e*x + d)^n*c)^2 - e*n*((d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 -
6*e*x + 6*d*log(e*x + d))*n^2/e^2 - 3*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e
*x + d))*n*log((e*x + d)^n*c)/e^2))*b^3*f^3 - 9/4*(2*e*n*(2*d^2*log(e*x + d
)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)^n*c) - (e^2*x^2 + 2*d^2*log(e*x
+ d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n^2/e^2)*a*b^2*f^2*g - 3/8*(6*e*n*(2
*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)^n*c)^2 + e*n*((4
*d^2*log(e*x + d)^3 + 3*e^2*x^2 + 18*d^2*log(e*x + d)^2 - 42*d*e*x + 42*d^2
*log(e*x + d))*n^2/e^3 - 6*(e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^
2*log(e*x + d))*n*log((e*x + d)^n*c)/e^3))*b^3*f^2*g + 1/6*(6*e*n*(6*d^3*lo
g(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3)*log((e*x + d)^n*c)
+ (4*e^3*x^3 - 15*d*e^2*x^2 - 18*d^3*log(e*x + d)^2 + 66*d^2*e*x - 66*d^3*l
og(e*x + d))*n^2/e^3)*a*b^2*f*g^2 + 1/36*(18*e*n*(6*d^3*log(e*x + d)/e^4 -
(2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3)*log((e*x + d)^n*c)^2 - e*n*((8*e^3*x
^3 - 36*d^3*log(e*x + d)^3 - 57*d*e^2*x^2 - 198*d^3*log(e*x + d)^2 + 510*d^
2*e*x - 510*d^3*log(e*x + d))*n^2/e^4 - 6*(4*e^3*x^3 - 15*d*e^2*x^2 - 18*d^
3*log(e*x + d)^2 + 66*d^2*e*x - 66*d^3*log(e*x + d))*n*log((e*x + d)^n*c)/e
^4))*b^3*f*g^2 - 1/96*(12*e*n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e
^2*x^3 + 6*d^2*e*x^2 - 12*d^3*x)/e^4)*log((e*x + d)^n*c) - (9*e^4*x^4 - 28*
d*e^3*x^3 + 78*d^2*e^2*x^2 + 72*d^4*log(e*x + d)^2 - 300*d^3*e*x + 300*d^4*
log(e*x + d))*n^2/e^4)*a*b^2*g^3 - 1/1152*(72*e*n*(12*d^4*log(e*x + d)/e^5
+ (3*e^3*x^4 - 4*d*e^2*x^3 + 6*d^2*e*x^2 - 12*d^3*x)/e^4)*log((e*x + d)^n*c
)^2 + e*n*((27*e^4*x^4 - 148*d*e^3*x^3 + 288*d^4*log(e*x + d)^3 + 690*d^2*e
```

```
^2*x^2 + 1800*d^4*log(e*x + d)^2 - 4980*d^3*e*x + 4980*d^4*log(e*x + d))*n^
2/e^5 - 12*(9*e^4*x^4 - 28*d*e^3*x^3 + 78*d^2*e^2*x^2 + 72*d^4*log(e*x + d)
^2 - 300*d^3*e*x + 300*d^4*log(e*x + d))*n*log((e*x + d)^n*c)/e^5))*b^3*g^3
 + a^3*f^3*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5182 vs. $2(578) = 1156$.

Time = 0.41 (sec) , antiderivative size = 5182, normalized size of antiderivative = 8.67

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))^3 \, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

```
[Out] (e*x + d)*b^3*f^3*n^3*log(e*x + d)^3/e + 3/2*(e*x + d)^2*b^3*f^2*g*n^3*log(
e*x + d)^3/e^2 - 3*(e*x + d)*b^3*d*f^2*g*n^3*log(e*x + d)^3/e^2 + (e*x + d)
^3*b^3*f*g^2*n^3*log(e*x + d)^3/e^3 - 3*(e*x + d)^2*b^3*d*f*g^2*n^3*log(e*x
 + d)^3/e^3 + 3*(e*x + d)*b^3*d^2*f*g^2*n^3*log(e*x + d)^3/e^3 + 1/4*(e*x +
 d)^4*b^3*g^3*n^3*log(e*x + d)^3/e^4 - (e*x + d)^3*b^3*d*g^3*n^3*log(e*x +
d)^3/e^4 + 3/2*(e*x + d)^2*b^3*d^2*g^3*n^3*log(e*x + d)^3/e^4 - (e*x + d)*b
^3*d^3*g^3*n^3*log(e*x + d)^3/e^4 - 3*(e*x + d)*b^3*f^3*n^3*log(e*x + d)^2/
e - 9/4*(e*x + d)^2*b^3*f^2*g*n^3*log(e*x + d)^2/e^2 + 9*(e*x + d)*b^3*d*f^
2*g*n^3*log(e*x + d)^2/e^2 - (e*x + d)^3*b^3*f*g^2*n^3*log(e*x + d)^2/e^3 +
 9/2*(e*x + d)^2*b^3*d*f*g^2*n^3*log(e*x + d)^2/e^3 - 9*(e*x + d)*b^3*d^2*f
*g^2*n^3*log(e*x + d)^2/e^3 - 3/16*(e*x + d)^4*b^3*g^3*n^3*log(e*x + d)^2/e
^4 + (e*x + d)^3*b^3*d*g^3*n^3*log(e*x + d)^2/e^4 - 9/4*(e*x + d)^2*b^3*d^2
*g^3*n^3*log(e*x + d)^2/e^4 + 3*(e*x + d)*b^3*d^3*g^3*n^3*log(e*x + d)^2/e^
4 + 3*(e*x + d)*b^3*f^3*n^2*log(e*x + d)^2*log(c)/e + 9/2*(e*x + d)^2*b^3*f
^2*g*n^2*log(e*x + d)^2*log(c)/e^2 - 9*(e*x + d)*b^3*d*f^2*g*n^2*log(e*x +
d)^2*log(c)/e^2 + 3*(e*x + d)^3*b^3*f*g^2*n^2*log(e*x + d)^2*log(c)/e^3 - 9
*(e*x + d)^2*b^3*d*f*g^2*n^2*log(e*x + d)^2*log(c)/e^3 + 9*(e*x + d)*b^3*d^
2*f*g^2*n^2*log(e*x + d)^2*log(c)/e^3 + 3/4*(e*x + d)^4*b^3*g^3*n^2*log(e*x
 + d)^2*log(c)/e^4 - 3*(e*x + d)^3*b^3*d*g^3*n^2*log(e*x + d)^2*log(c)/e^4
+ 9/2*(e*x + d)^2*b^3*d^2*g^3*n^2*log(e*x + d)^2*log(c)/e^4 - 3*(e*x + d)*b
^3*d^3*g^3*n^2*log(e*x + d)^2*log(c)/e^4 + 6*(e*x + d)*b^3*f^3*n^3*log(e*x
+ d)/e + 9/4*(e*x + d)^2*b^3*f^2*g*n^3*log(e*x + d)/e^2 - 18*(e*x + d)*b^3*
d*f^2*g*n^3*log(e*x + d)/e^2 + 2/3*(e*x + d)^3*b^3*f*g^2*n^3*log(e*x + d)/e
^3 - 9/2*(e*x + d)^2*b^3*d*f*g^2*n^3*log(e*x + d)/e^3 + 18*(e*x + d)*b^3*d^
2*f*g^2*n^3*log(e*x + d)/e^3 + 3/32*(e*x + d)^4*b^3*g^3*n^3*log(e*x + d)/e^
4 - 2/3*(e*x + d)^3*b^3*d*g^3*n^3*log(e*x + d)/e^4 + 9/4*(e*x + d)^2*b^3*d^
2*g^3*n^3*log(e*x + d)/e^4 - 6*(e*x + d)*b^3*d^3*g^3*n^3*log(e*x + d)/e^4 +
 3*(e*x + d)*a*b^2*f^3*n^2*log(e*x + d)^2/e + 9/2*(e*x + d)^2*a*b^2*f^2*g*n
^2*log(e*x + d)^2/e^2 - 9*(e*x + d)*a*b^2*d*f^2*g*n^2*log(e*x + d)^2/e^2 +
3*(e*x + d)^3*a*b^2*f*g^2*n^2*log(e*x + d)^2/e^3 - 9*(e*x + d)^2*a*b^2*d*f*
```

g^2*n^2*log(e*x + d)^2/e^3 + 9*(e*x + d)*a*b^2*d^2*f*g^2*n^2*log(e*x + d)^2
/e^3 + 3/4*(e*x + d)^4*a*b^2*g^3*n^2*log(e*x + d)^2/e^4 - 3*(e*x + d)^3*a*b
^2*d*g^3*n^2*log(e*x + d)^2/e^4 + 9/2*(e*x + d)^2*a*b^2*d^2*g^3*n^2*log(e*x
 + d)^2/e^4 - 3*(e*x + d)*a*b^2*d^3*g^3*n^2*log(e*x + d)^2/e^4 - 6*(e*x + d
)*b^3*f^3*n^2*log(e*x + d)*log(c)/e - 9/2*(e*x + d)^2*b^3*f^2*g*n^2*log(e*x
 + d)*log(c)/e^2 + 18*(e*x + d)*b^3*d*f^2*g*n^2*log(e*x + d)*log(c)/e^2 - 2
*(e*x + d)^3*b^3*f*g^2*n^2*log(e*x + d)*log(c)/e^3 + 9*(e*x + d)^2*b^3*d*f*
g^2*n^2*log(e*x + d)*log(c)/e^3 - 18*(e*x + d)*b^3*d^2*f*g^2*n^2*log(e*x +
d)*log(c)/e^3 - 3/8*(e*x + d)^4*b^3*g^3*n^2*log(e*x + d)*log(c)/e^4 + 2*(e*
x + d)^3*b^3*d*g^3*n^2*log(e*x + d)*log(c)/e^4 - 9/2*(e*x + d)^2*b^3*d^2*g^
3*n^2*log(e*x + d)*log(c)/e^4 + 6*(e*x + d)*b^3*d^3*g^3*n^2*log(e*x + d)*lo
g(c)/e^4 + 3*(e*x + d)*b^3*f^3*n*log(e*x + d)*log(c)^2/e + 9/2*(e*x + d)^2*
b^3*f^2*g*n*log(e*x + d)*log(c)^2/e^2 - 9*(e*x + d)*b^3*d*f^2*g*n*log(e*x +
 d)*log(c)^2/e^2 + 3*(e*x + d)^3*b^3*f*g^2*n*log(e*x + d)*log(c)^2/e^3 - 9*
(e*x + d)^2*b^3*d*f*g^2*n*log(e*x + d)*log(c)^2/e^3 + 9*(e*x + d)*b^3*d^2*f
*g^2*n*log(e*x + d)*log(c)^2/e^3 + 3/4*(e*x + d)^4*b^3*g^3*n*log(e*x + d)*l
og(c)^2/e^4 - 3*(e*x + d)^3*b^3*d*g^3*n*log(e*x + d)*log(c)^2/e^4 + 9/2*(e*
x + d)^2*b^3*d^2*g^3*n*log(e*x + d)*log(c)^2/e^4 - 3*(e*x + d)*b^3*d^3*g^3*
n*log(e*x + d)*log(c)^2/e^4 - 6*(e*x + d)*b^3*f^3*n^3/e - 9/8*(e*x + d)^2*b
^3*f^2*g*n^3/e^2 + 18*(e*x + d)*b^3*d*f^2*g*n^3/e^2 - 2/9*(e*x + d)^3*b^3*f
*g^2*n^3/e^3 + 9/4*(e*x + d)^2*b^3*d*f*g^2*n^3/e^3 - 18*(e*x + d)*b^3*d^2*f
*g^2*n^3/e^3 - 3/128*(e*x + d)^4*b^3*g^3*n^3/e^4 + 2/9*(e*x + d)^3*b^3*d*g^
3*n^3/e^4 - 9/8*(e*x + d)^2*b^3*d^2*g^3*n^3/e^4 + 6*(e*x + d)*b^3*d^3*g^3*n
^3/e^4 - 6*(e*x + d)*a*b^2*f^3*n^2*log(e*x + d)/e - 9/2*(e*x + d)^2*a*b^2*f
^2*g*n^2*log(e*x + d)/e^2 + 18*(e*x + d)*a*b^2*d*f^2*g*n^2*log(e*x + d)/e^2
 - 2*(e*x + d)^3*a*b^2*f*g^2*n^2*log(e*x + d)/e^3 + 9*(e*x + d)^2*a*b^2*d*f
*g^2*n^2*log(e*x + d)/e^3 - 18*(e*x + d)*a*b^2*d^2*f*g^2*n^2*log(e*x + d)/e
^3 - 3/8*(e*x + d)^4*a*b^2*g^3*n^2*log(e*x + d)/e^4 + 2*(e*x + d)^3*a*b^2*d
*g^3*n^2*log(e*x + d)/e^4 - 9/2*(e*x + d)^2*a*b^2*d^2*g^3*n^2*log(e*x + d)/
e^4 + 6*(e*x + d)*a*b^2*d^3*g^3*n^2*log(e*x + d)/e^4 + 6*(e*x + d)*b^3*f^3*
n^2*log(c)/e + 9/4*(e*x + d)^2*b^3*f^2*g*n^2*log(c)/e^2 - 18*(e*x + d)*b^3*
d*f^2*g*n^2*log(c)/e^2 + 2/3*(e*x + d)^3*b^3*f*g^2*n^2*log(c)/e^3 - 9/2*(e*
x + d)^2*b^3*d*f*g^2*n^2*log(c)/e^3 + 18*(e*x + d)*b^3*d^2*f*g^2*n^2*log(c)
/e^3 + 3/32*(e*x + d)^4*b^3*g^3*n^2*log(c)/e^4 - 2/3*(e*x + d)^3*b^3*d*g^3*
n^2*log(c)/e^4 + 9/4*(e*x + d)^2*b^3*d^2*g^3*n^2*log(c)/e^4 - 6*(e*x + d)*b
^3*d^3*g^3*n^2*log(c)/e^4 + 6*(e*x + d)*a*b^2*f^3*n*log(e*x + d)*log(c)/e +
 9*(e*x + d)^2*a*b^2*f^2*g*n*log(e*x + d)*log(c)/e^2 - 18*(e*x + d)*a*b^2*d
*f^2*g*n*log(e*x + d)*log(c)/e^2 + 6*(e*x + d)^3*a*b^2*f*g^2*n*log(e*x + d)
*log(c)/e^3 - 18*(e*x + d)^2*a*b^2*d*f*g^2*n*log(e*x + d)*log(c)/e^3 + 18*(
e*x + d)*a*b^2*d^2*f*g^2*n*log(e*x + d)*log(c)/e^3 + 3/2*(e*x + d)^4*a*b^2*
g^3*n*log(e*x + d)*log(c)/e^4 - 6*(e*x + d)^3*a*b^2*d*g^3*n*log(e*x + d)*lo
g(c)/e^4 + 9*(e*x + d)^2*a*b^2*d^2*g^3*n*log(e*x + d)*log(c)/e^4 - 6*(e*x +
 d)*a*b^2*d^3*g^3*n*log(e*x + d)*log(c)/e^4 - 3*(e*x + d)*b^3*f^3*n*log(c)^
2/e - 9/4*(e*x + d)^2*b^3*f^2*g*n*log(c)^2/e^2 + 9*(e*x + d)*b^3*d*f^2*g*n*
log(c)^2/e^2 - (e*x + d)^3*b^3*f*g^2*n*log(c)^2/e^3 + 9/2*(e*x + d)^2*b^3*d

*f*g^2*n*log(c)^2/e^3 - 9*(e*x + d)*b^3*d^2*f*g^2*n*log(c)^2/e^3 - 3/16*(e*x + d)^4*b^3*g^3*n*log(c)^2/e^4 + (e*x + d)^3*b^3*d*g^3*n*log(c)^2/e^4 - 9/4*(e*x + d)^2*b^3*d^2*g^3*n*log(c)^2/e^4 + 3*(e*x + d)*b^3*d^3*g^3*n*log(c)^2/e^4 + (e*x + d)*b^3*f^3*log(c)^3/e + 3/2*(e*x + d)^2*b^3*f^2*g*log(c)^3/e^2 - 3*(e*x + d)*b^3*d*f^2*g*log(c)^3/e^2 + (e*x + d)^3*b^3*f*g^2*log(c)^3/e^3 - 3*(e*x + d)^2*b^3*d*f*g^2*log(c)^3/e^3 + 3*(e*x + d)*b^3*d^2*f*g^2*log(c)^3/e^3 + 1/4*(e*x + d)^4*b^3*g^3*log(c)^3/e^4 - (e*x + d)^3*b^3*d*g^3*log(c)^3/e^4 + 3/2*(e*x + d)^2*b^3*d^2*g^3*log(c)^3/e^4 - (e*x + d)*b^3*d^3*g^3*log(c)^3/e^4 + 6*(e*x + d)*a*b^2*f^3*n^2/e + 9/4*(e*x + d)^2*a*b^2*f^2*g*n^2/e^2 - 18*(e*x + d)*a*b^2*d*f^2*g*n^2/e^2 + 2/3*(e*x + d)^3*a*b^2*f*g^2*n^2/e^3 - 9/2*(e*x + d)^2*a*b^2*d*f*g^2*n^2/e^3 + 18*(e*x + d)*a*b^2*d^2*f*g^2*n^2/e^3 + 3/32*(e*x + d)^4*a*b^2*g^3*n^2/e^4 - 2/3*(e*x + d)^3*a*b^2*d*g^3*n^2/e^4 + 9/4*(e*x + d)^2*a*b^2*d^2*g^3*n^2/e^4 - 6*(e*x + d)*a*b^2*d^3*g^3*n^2/e^4 + 3*(e*x + d)*a^2*b*f^3*n*log(e*x + d)/e + 9/2*(e*x + d)^2*a^2*b*f^2*g*n*log(e*x + d)/e^2 - 9*(e*x + d)*a^2*b*d*f^2*g*n*log(e*x + d)/e^2 + 3*(e*x + d)^3*a^2*b*f*g^2*n*log(e*x + d)/e^3 - 9*(e*x + d)^2*a^2*b*d*f*g^2*n*log(e*x + d)/e^3 + 9*(e*x + d)*a^2*b*d^2*f*g^2*n*log(e*x + d)/e^3 + 3/4*(e*x + d)^4*a^2*b*g^3*n*log(e*x + d)/e^4 - 3*(e*x + d)^3*a^2*b*d*g^3*n*log(e*x + d)/e^4 + 9/2*(e*x + d)^2*a^2*b*d^2*g^3*n*log(e*x + d)/e^4 - 3*(e*x + d)*a^2*b*d^3*g^3*n*log(e*x + d)/e^4 - 6*(e*x + d)*a*b^2*f^3*n*log(c)/e - 9/2*(e*x + d)^2*a*b^2*f^2*g*n*log(c)/e^2 + 18*(e*x + d)*a*b^2*d*f^2*g*n*log(c)/e^2 - 2*(e*x + d)^3*a*b^2*f*g^2*n*log(c)/e^3 + 9*(e*x + d)^2*a*b^2*d*f*g^2*n*log(c)/e^3 - 18*(e*x + d)*a*b^2*d^2*f*g^2*n*log(c)/e^3 - 3/8*(e*x + d)^4*a*b^2*g^3*n*log(c)/e^4 + 2*(e*x + d)^3*a*b^2*d*g^3*n*log(c)/e^4 - 9/2*(e*x + d)^2*a*b^2*d^2*g^3*n*log(c)/e^4 + 6*(e*x + d)*a*b^2*d^3*g^3*n*log(c)/e^4 + 3*(e*x + d)*a*b^2*f^3*log(c)^2/e + 9/2*(e*x + d)^2*a*b^2*f^2*g*log(c)^2/e^2 - 9*(e*x + d)*a*b^2*d*f^2*g*log(c)^2/e^2 + 3*(e*x + d)^3*a*b^2*f*g^2*log(c)^2/e^3 - 9*(e*x + d)^2*a*b^2*d*f*g^2*log(c)^2/e^3 + 9*(e*x + d)*a*b^2*d^2*f*g^2*log(c)^2/e^3 + 3/4*(e*x + d)^4*a*b^2*g^3*log(c)^2/e^4 - 3*(e*x + d)^3*a*b^2*d*g^3*log(c)^2/e^4 + 9/2*(e*x + d)^2*a*b^2*d^2*g^3*log(c)^2/e^4 - 3*(e*x + d)*a*b^2*d^3*g^3*log(c)^2/e^4 - 3*(e*x + d)*a^2*b*f^3*n/e - 9/4*(e*x + d)^2*a^2*b*f^2*g*n/e^2 + 9*(e*x + d)*a^2*b*d*f^2*g*n/e^2 - (e*x + d)^3*a^2*b*f*g^2*n/e^3 + 9/2*(e*x + d)^2*a^2*b*d*f*g^2*n/e^3 - 9*(e*x + d)*a^2*b*d^2*f*g^2*n/e^3 - 3/16*(e*x + d)^4*a^2*b*g^3*n/e^4 + (e*x + d)^3*a^2*b*d*g^3*n/e^4 - 9/4*(e*x + d)^2*a^2*b*d^2*g^3*n/e^4 + 3*(e*x + d)*a^2*b*d^3*g^3*n/e^4 + 3*(e*x + d)*a^2*b*f^3*log(c)/e + 9/2*(e*x + d)^2*a^2*b*f^2*g*log(c)/e^2 - 9*(e*x + d)*a^2*b*d*f^2*g*log(c)/e^2 + 3*(e*x + d)^3*a^2*b*f*g^2*log(c)/e^3 - 9*(e*x + d)^2*a^2*b*d*f*g^2*log(c)/e^3 + 9*(e*x + d)*a^2*b*d^2*f*g^2*log(c)/e^3 + 3/4*(e*x + d)^4*a^2*b*g^3*log(c)/e^4 - 3*(e*x + d)^3*a^2*b*d*g^3*log(c)/e^4 + 9/2*(e*x + d)^2*a^2*b*d^2*g^3*log(c)/e^4 - 3*(e*x + d)*a^2*b*d^3*g^3*log(c)/e^4 + (e*x + d)*a^3*f^3/e + 3/2*(e*x + d)^2*a^3*f^2*g/e^2 - 3*(e*x + d)*a^3*d*f^2*g/e^2 + (e*x + d)^3*a^3*f*g^2/e^3 - 3*(e*x + d)^2*a^3*d*f*g^2/e^3 + 3*(e*x + d)*a^3*d^2*f*g^2/e^3 + 1/4*(e*x + d)^4*a^3*g^3/e^4 - (e*x + d)^3*a^3*d*g^3/e^4 + 3/2*(e*x + d)^2*a^3*d^2*g^3/e^4 - (e*x + d)*a^3*d^3*g^3/e^4

## Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 2133, normalized size of antiderivative = 3.57

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^3 \, dx = \text{Too large to display}$$

```
[In] int((f + g*x)^3*(a + b*log(c*(d + e*x)^n))^3,x)
```

```
[Out] x^3*((g^2*(24*a^3*d*g + 72*a^3*e*f + 7*b^3*d*g*n^3 - 16*b^3*e*f*n^3 - 12*a*
b^2*d*g*n^2 + 48*a*b^2*e*f*n^2 - 72*a^2*b*e*f*n))/(72*e) - (d*g^3*(32*a^3 -
 3*b^3*n^3 + 12*a*b^2*n^2 - 24*a^2*b*n))/(96*e)) + log(c*(d + e*x)^n)^3*(b^
3*f^3*x - (d*(b^3*d^3*g^3 - 4*b^3*e^3*f^3 + 6*b^3*d*e^2*f^2*g - 4*b^3*d^2*e
*f*g^2))/(4*e^4) + (b^3*g^3*x^4)/4 + (3*b^3*f^2*g*x^2)/2 + b^3*f*g^2*x^3) -
 x^2*((d*((g^2*(24*a^3*d*g + 72*a^3*e*f + 7*b^3*d*g*n^3 - 16*b^3*e*f*n^3 -
12*a*b^2*d*g*n^2 + 48*a*b^2*e*f*n^2 - 72*a^2*b*e*f*n))/(24*e) - (d*g^3*(32*
a^3 - 3*b^3*n^3 + 12*a*b^2*n^2 - 24*a^2*b*n))/(32*e)))/(2*e) - (g*(48*a^3*e
^2*f^2 - 13*b^3*d^2*g^2*n^3 - 36*b^3*e^2*f^2*n^3 - 72*a^2*b*e^2*f^2*n + 48*
a^3*d*e*f*g + 12*a*b^2*d^2*g^2*n^2 + 72*a*b^2*e^2*f^2*n^2 + 40*b^3*d*e*f*g*
n^3 - 48*a*b^2*d*e*f*g*n^2))/(32*e^2)) + log(c*(d + e*x)^n)^2*((x^3*((4*b^2
*g^2*(a*d*g + 3*a*e*f - b*e*f*n))/e - (b^2*d*g^3*(4*a - b*n))/e))/4 - (x^2*
((d*((48*b^2*g^2*(a*d*g + 3*a*e*f - b*e*f*n))/e - (12*b^2*d*g^3*(4*a - b*n)
)/e))/(8*e) - (9*b^2*f*g*(2*a*d*g + 2*a*e*f - b*e*f*n))/e))/4 + (x*((d*((d*
((48*b^2*g^2*(a*d*g + 3*a*e*f - b*e*f*n))/e - (12*b^2*d*g^3*(4*a - b*n))/e)
)/e - (72*b^2*f*g*(2*a*d*g + 2*a*e*f - b*e*f*n))/e))/(4*e) + (12*b^2*f^2*(3
*a*d*g + a*e*f - b*e*f*n))/e))/4 - (12*a*b^2*d^4*g^3 - 25*b^3*d^4*g^3*n - 4
8*a*b^2*d*e^3*f^3 + 48*b^3*d*e^3*f^3*n + 72*a*b^2*d^2*e^2*f^2*g - 108*b^3*d
^2*e^2*f^2*g*n - 48*a*b^2*d^3*e*f*g^2 + 88*b^3*d^3*e*f*g^2*n)/(16*e^4) + (3
*b^2*g^3*x^4*(4*a - b*n))/16) + x*((96*a^3*e^3*f^3 + 300*b^3*d^3*g^3*n^3 -
576*b^3*e^3*f^3*n^3 + 288*a^3*d*e^2*f^2*g - 288*a^2*b*e^3*f^3*n - 144*a*b^2
*d^3*g^3*n^2 + 576*a*b^2*e^3*f^3*n^2 + 1296*b^3*d*e^2*f^2*g*n^3 - 1056*b^3*
d^2*e*f*g^2*n^3 - 864*a*b^2*d*e^2*f^2*g*n^2 + 576*a*b^2*d^2*e*f*g^2*n^2)/(9
6*e^3) + (d*((d*((g^2*(24*a^3*d*g + 72*a^3*e*f + 7*b^3*d*g*n^3 - 16*b^3*e*f
*n^3 - 12*a*b^2*d*g*n^2 + 48*a*b^2*e*f*n^2 - 72*a^2*b*e*f*n))/(24*e) - (d*g
^3*(32*a^3 - 3*b^3*n^3 + 12*a*b^2*n^2 - 24*a^2*b*n))/(32*e)))/e - (g*(48*a^
3*e^2*f^2 - 13*b^3*d^2*g^2*n^3 - 36*b^3*e^2*f^2*n^3 - 72*a^2*b*e^2*f^2*n +
48*a^3*d*e*f*g + 12*a*b^2*d^2*g^2*n^2 + 72*a*b^2*e^2*f^2*n^2 + 40*b^3*d*e*f
*g*n^3 - 48*a*b^2*d*e*f*g*n^2))/(16*e^2)))/e) - (log(d + e*x)*(415*b^3*d^4*
g^3*n^3 + 72*a^2*b*d^4*g^3*n - 300*a*b^2*d^4*g^3*n^2 - 576*b^3*d*e^3*f^3*n^
3 + 576*a*b^2*d*e^3*f^3*n^2 - 1360*b^3*d^3*e*f*g^2*n^3 + 1512*b^3*d^2*e^2*f
^2*g*n^3 - 288*a^2*b*d*e^3*f^3*n - 1296*a*b^2*d^2*e^2*f^2*g*n^2 - 288*a^2*b
*d^3*e*f*g^2*n + 432*a^2*b*d^2*e^2*f^2*g*n + 1056*a*b^2*d^3*e*f*g^2*n^2))/(
96*e^4) + (g^3*x^4*(32*a^3 - 3*b^3*n^3 + 12*a*b^2*n^2 - 24*a^2*b*n))/128 +
(log(c*(d + e*x)^n)*((x^3*(32*b*e^3*g^2*(6*a^2*d*g + 18*a^2*e*f - b^2*d*g*n
^2 + 4*b^2*e*f*n^2 - 12*a*b*e*f*n) - 24*b*d*e^3*g^3*(8*a^2 + b^2*n^2 - 4*a*
```

```
b*n)))/(24*e^2) - (x^2*((d*(32*b*e^3*g^2*(6*a^2*d*g + 18*a^2*e*f - b^2*d*g*
n^2 + 4*b^2*e*f*n^2 - 12*a*b*e*f*n) - 24*b*d*e^3*g^3*(8*a^2 + b^2*n^2 - 4*a
*b*n)))/e - 48*b*e^2*g*(12*a^2*e^2*f^2 + b^2*d^2*g^2*n^2 + 6*b^2*e^2*f^2*n^
2 - 12*a*b*e^2*f^2*n + 12*a^2*d*e*f*g - 4*b^2*d*e*f*g*n^2)))/(16*e^2) + (x*
((192*a^2*b*e^5*f^3 + 384*b^3*e^5*f^3*n^2 - 96*b^3*d^3*e^2*g^3*n^2 - 384*a*
b^2*e^5*f^3*n - 576*b^3*d*e^4*f^2*g*n^2 + 384*b^3*d^2*e^3*f*g^2*n^2 + 576*a
^2*b*d*e^4*f^2*g)/e + (d*((d*(32*b*e^3*g^2*(6*a^2*d*g + 18*a^2*e*f - b^2*d*
g*n^2 + 4*b^2*e*f*n^2 - 12*a*b*e*f*n) - 24*b*d*e^3*g^3*(8*a^2 + b^2*n^2 - 4
*a*b*n)))/e - 48*b*e^2*g*(12*a^2*e^2*f^2 + b^2*d^2*g^2*n^2 + 6*b^2*e^2*f^2*
n^2 - 12*a*b*e^2*f^2*n + 12*a^2*d*e*f*g - 4*b^2*d*e*f*g*n^2)))/e))/(8*e^2)
+ (3*b*e^2*g^3*x^4*(8*a^2 + b^2*n^2 - 4*a*b*n))/4))/(8*e^2)
```

# 3.53 $\int (f+gx)^2 (a+b\log(c(d+ex)^n))^3 \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 432

$$\begin{aligned}\int (f+gx)^2 (a+b\log(c(d+ex)^n))^3 \, dx &= \frac{6ab^2(ef-dg)^2n^2x}{e^2} - \frac{6b^3(ef-dg)^2n^3x}{e^2} - \frac{3b^3g(ef-dg)n^3(d+ex)^2}{4e^3} \\ &- \frac{2b^3g^2n^3(d+ex)^3}{27e^3} + \frac{6b^3(ef-dg)^2n^2(d+ex)\log(c(d+ex)^n)}{e^3} \\ &+ \frac{3b^2g(ef-dg)n^2(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^3} \\ &+ \frac{2b^2g^2n^2(d+ex)^3(a+b\log(c(d+ex)^n))}{9e^3} \\ &- \frac{3b(ef-dg)^2n(d+ex)(a+b\log(c(d+ex)^n))^2}{e^3} \\ &- \frac{3bg(ef-dg)n(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^3} \\ &- \frac{bg^2n(d+ex)^3(a+b\log(c(d+ex)^n))^2}{3e^3} + \frac{(ef-dg)^2(d+ex)(a+b\log(c(d+ex)^n))^3}{e^3} \\ &+ \frac{g(ef-dg)(d+ex)^2(a+b\log(c(d+ex)^n))^3}{e^3} + \frac{g^2(d+ex)^3(a+b\log(c(d+ex)^n))^3}{3e^3}\end{aligned}$$

```
[Out] 6*a*b^2*(-d*g+e*f)^2*n^2*x/e^2-6*b^3*(-d*g+e*f)^2*n^3*x/e^2-3/4*b^3*g*(-d*g
+e*f)*n^3*(e*x+d)^2/e^3-2/27*b^3*g^2*n^3*(e*x+d)^3/e^3+6*b^3*(-d*g+e*f)^2*n
^2*(e*x+d)*ln(c*(e*x+d)^n)/e^3+3/2*b^2*g*(-d*g+e*f)*n^2*(e*x+d)^2*(a+b*ln(c
*(e*x+d)^n))/e^3+2/9*b^2*g^2*n^2*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))/e^3-3*b*(-
d*g+e*f)^2*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^3-3/2*b*g*(-d*g+e*f)*n*(e*x+
d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^3-1/3*b*g^2*n*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n)
)^2/e^3+(-d*g+e*f)^2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e^3+g*(-d*g+e*f)*(e*x+
d)^2*(a+b*ln(c*(e*x+d)^n))^3/e^3+1/3*g^2*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))^3/
e^3
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^3\,dx$$
$$=\frac{3b^2gn^2(d+ex)^2(ef-dg)(a+b\log(c(d+ex)^n))}{2e^3}+\frac{2b^2g^2n^2(d+ex)^3(a+b\log(c(d+ex)^n))}{9e^3}+\frac{6ab^2n^2x(ef-dg)^2}{e^2}-\frac{3bgn(d+ex)^2(ef-dg)(a+b\log(c(d+ex)^n))^2}{2e^3}-\frac{3bn(d+ex)(ef-dg)^2(a+b\log(c(d+ex)^n))^2}{e^3}+\frac{g(d+ex)^2(ef-dg)(a+b\log(c(d+ex)^n))^3}{e^3}+\frac{(d+ex)(ef-dg)^2(a+b\log(c(d+ex)^n))^3}{e^3}-\frac{bg^2n(d+ex)^3(a+b\log(c(d+ex)^n))^2}{3e^3}+\frac{g^2(d+ex)^3(a+b\log(c(d+ex)^n))^3}{3e^3}+\frac{6b^3n^2(d+ex)(ef-dg)^2\log(c(d+ex)^n)}{e^3}-\frac{3b^3gn^3(d+ex)^2(ef-dg)}{4e^3}-\frac{2b^3g^2n^3(d+ex)^3}{27e^3}-\frac{6b^3n^3x(ef-dg)^2}{e^2}$$

```
[In] Int[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] (6*a*b^2*(e*f - d*g)^2*n^2*x)/e^2 - (6*b^3*(e*f - d*g)^2*n^3*x)/e^2 - (3*b^
3*g*(e*f - d*g)*n^3*(d + e*x)^2)/(4*e^3) - (2*b^3*g^2*n^3*(d + e*x)^3)/(27*
e^3) + (6*b^3*(e*f - d*g)^2*n^2*(d + e*x)*Log[c*(d + e*x)^n])/e^3 + (3*b^2*
g*(e*f - d*g)*n^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^3) + (2*b^2*
g^2*n^2*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))/(9*e^3) - (3*b*(e*f - d*g)^
2*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e^3 - (3*b*g*(e*f - d*g)*n*(d +
 e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^3) - (b*g^2*n*(d + e*x)^3*(a + b
*Log[c*(d + e*x)^n])^2)/(3*e^3) + ((e*f - d*g)^2*(d + e*x)*(a + b*Log[c*(d
+ e*x)^n])^3)/e^3 + (g*(e*f - d*g)*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^3
)/e^3 + (g^2*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])^3)/(3*e^3)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
```

```
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(ef-dg)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^3}{e^2} + \frac{2g(ef-dg)(d+ex) \left(a+b\log \left(c(d+ex)^n\right)\right)^3}{e^2} + \frac{g^2(d+ex)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^3}{e^2}\right) dx$$

$$
\begin{aligned}
&= \frac{g^2 \int (d+ex)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^3\, dx}{e^2} \\
&\quad + \frac{(2g(ef-dg)) \int (d+ex) \left(a+b\log\left(c(d+ex)^n\right)\right)^3\, dx}{e^2} \\
&\quad + \frac{(ef-dg)^2 \int \left(a+b\log\left(c(d+ex)^n\right)\right)^3\, dx}{e^2} \\
&= \frac{g^2 \text{Subst}\left(\int x^2 (a+b\log(cx^n))^3\, dx, x, d+ex\right)}{e^3} \\
&\quad + \frac{(2g(ef-dg)) \text{Subst}\left(\int x (a+b\log(cx^n))^3\, dx, x, d+ex\right)}{e^3} \\
&\quad + \frac{(ef-dg)^2 \text{Subst}\left(\int (a+b\log(cx^n))^3\, dx, x, d+ex\right)}{e^3} \\
&= \frac{(ef-dg)^2 (d+ex) \left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3} \\
&\quad + \frac{g(ef-dg)(d+ex)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3} \\
&\quad + \frac{g^2 (d+ex)^3 \left(a+b\log\left(c(d+ex)^n\right)\right)^3}{3e^3} \\
&\quad - \frac{(bg^2 n) \text{Subst}\left(\int x^2 (a+b\log(cx^n))^2\, dx, x, d+ex\right)}{e^3} \\
&\quad - \frac{(3bg(ef-dg)n) \text{Subst}\left(\int x (a+b\log(cx^n))^2\, dx, x, d+ex\right)}{e^3} \\
&\quad - \frac{(3b(ef-dg)^2 n) \text{Subst}\left(\int (a+b\log(cx^n))^2\, dx, x, d+ex\right)}{e^3} \\
&= -\frac{3b(ef-dg)^2 n(d+ex) \left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^3} \\
&\quad - \frac{3bg(ef-dg)n(d+ex)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^3} \\
&\quad - \frac{bg^2 n(d+ex)^3 \left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3e^3} \\
&\quad + \frac{(ef-dg)^2 (d+ex) \left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3} \\
&\quad + \frac{g(ef-dg)(d+ex)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3} \\
&\quad + \frac{g^2 (d+ex)^3 \left(a+b\log\left(c(d+ex)^n\right)\right)^3}{3e^3} \\
&\quad + \frac{(2b^2 g^2 n^2) \text{Subst}\left(\int x^2 (a+b\log(cx^n))\, dx, x, d+ex\right)}{3e^3} \\
&\quad + \frac{(3b^2 g(ef-dg)n^2) \text{Subst}\left(\int x (a+b\log(cx^n))\, dx, x, d+ex\right)}{e^3} \\
&\quad + \frac{(6b^2 (ef-dg)^2 n^2) \text{Subst}\left(\int (a+b\log(cx^n))\, dx, x, d+ex\right)}{e^3}
\end{aligned}
$$

$$= \frac{6ab^2(ef-dg)^2n^2x}{e^2} - \frac{3b^3g(ef-dg)n^3(d+ex)^2}{4e^3} - \frac{2b^3g^2n^3(d+ex)^3}{27e^3} + \frac{3b^2g(ef-dg)n^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^3} + \frac{2b^2g^2n^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3} - \frac{3b(ef-dg)^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^3} - \frac{3bg(ef-dg)n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^3} - \frac{bg^2n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3e^3} + \frac{(ef-dg)^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3} + \frac{g(ef-dg)(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3} + \frac{g^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{3e^3} + \frac{\left(6b^3(ef-dg)^2n^2\right)\text{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{e^3}$$

$$= \frac{6ab^2(ef-dg)^2n^2x}{e^2} - \frac{6b^3(ef-dg)^2n^3x}{e^2} - \frac{3b^3g(ef-dg)n^3(d+ex)^2}{4e^3} - \frac{2b^3g^2n^3(d+ex)^3}{27e^3} + \frac{6b^3(ef-dg)^2n^2(d+ex)\log\left(c(d+ex)^n\right)}{e^3} + \frac{3b^2g(ef-dg)n^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^3} + \frac{2b^2g^2n^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3} - \frac{3b(ef-dg)^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^3} - \frac{3bg(ef-dg)n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^3} - \frac{bg^2n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3e^3} + \frac{(ef-dg)^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3} + \frac{g(ef-dg)(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3} + \frac{g^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{3e^3}$$

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.77

$$\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx$$
$$= \frac{108(ef-dg)^2(d+ex)\left(a+b\log \left(c(d+ex)^n\right)\right)^3+108g(ef-dg)(d+ex)^2\left(a+b\log \left(c(d+ex)^n\right)\right)^3+36g^2(}{}$$

```
[In] Integrate[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] (108*(e*f - d*g)^2*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3 + 108*g*(e*f - d*
g)*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^3 + 36*g^2*(d + e*x)^3*(a + b*Log
[c*(d + e*x)^n])^3 - 324*b*(e*f - d*g)^2*n*((d + e*x)*(a + b*Log[c*(d + e*x
)^n])^2 - 2*b*n*(e*(a - b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n])) - 81*b*g*
(e*f - d*g)*n*(2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2 + b*n*(b*e*n*x*(2
*d + e*x) - 2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))) - 4*b*g^2*n*(9*(d +
e*x)^3*(a + b*Log[c*(d + e*x)^n])^2 + 2*b*n*(b*e*n*x*(3*d^2 + 3*d*e*x + e^2
*x^2) - 3*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))))/(108*e^3)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1732 vs. 2(418) = 836.

Time = 3.55 (sec) , antiderivative size = 1733, normalized size of antiderivative = 4.01

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1733 |
| risch | Expression too large to display | 20417 |

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/108*(-108*a^2*b*d^2*e*g^2*n*x-81*b^3*e^3*f*g*n^3*x^2-36*a^2*b*e^3*g^2*n*x
^3-510*b^3*d^2*e*g^2*n^3*x+36*x^3*ln(c*(e*x+d)^n)^3*b^3*e^3*g^2+108*x*ln(c*
(e*x+d)^n)^3*b^3*e^3*f^2+108*ln(c*(e*x+d)^n)^3*b^3*d*e^2*f^2-198*ln(c*(e*x+
d)^n)^2*b^3*d^3*g^2*n-396*ln(c*(e*x+d)^n)*b^3*d^3*g^2*n^2+108*ln(c*(e*x+d)^
n)^2*a*b^2*d^3*g^2-90*a*b^2*d*e^2*g^2*n^2*x^2+162*a*b^2*e^3*f*g*n^2*x^2+113
4*b^3*d*e^2*f*g*n^3*x-162*a^2*b*e^3*f*g*n*x^2+396*a*b^2*d^2*e*g^2*n^2*x-648
*b^3*e^3*f^2*n^3*x+648*b^3*d*e^2*f^2*n^3-396*a*b^2*d^3*g^2*n^2+648*a*b^2*e^
3*f^2*n^2*x-324*a^2*b*e^3*f^2*n*x+108*a^2*b*d^3*g^2*n-1134*b^3*d^2*e*f*g*n^
3-648*a*b^2*d*e^2*f^2*n^2+36*a^3*e^3*g^2*x^3+324*a^2*b*d*e^2*f*g*n*x-8*b^3*
e^3*g^2*n^3*x^3+108*a^3*e^3*f*g*x^2+24*a*b^2*e^3*g^2*n^2*x^3+57*b^3*d*e^2*g
^2*n^3*x^2-108*a^3*d*e^2*f^2+108*a^3*e^3*f^2*x+510*b^3*d^3*g^2*n^3+906*ln(e
*x+d)*b^3*d^3*g^2*n^3-972*a*b^2*d*e^2*f*g*n^2*x+324*a^2*b*d*e^2*f^2*n+36*ln
(c*(e*x+d)^n)^3*b^3*d^3*g^2+972*a*b^2*d^2*e*f*g*n^2+108*x^2*ln(c*(e*x+d)^n)
*a*b^2*d*e^2*g^2*n+1296*ln(e*x+d)*b^3*d*e^2*f^2*n^3-612*ln(e*x+d)*a*b^2*d^3
```

```
*g^2*n^2+108*ln(e*x+d)*a^2*b*d^3*g^2*n+324*x*ln(c*(e*x+d)^n)^2*a*b^2*e^3*f^
2-108*ln(c*(e*x+d)^n)^3*b^3*d^2*e*f*g-324*ln(c*(e*x+d)^n)^2*b^3*d*e^2*f^2*n
-648*ln(c*(e*x+d)^n)*b^3*d*e^2*f^2*n^2+324*x*ln(c*(e*x+d)^n)*a^2*b*e^3*f^2+
324*ln(c*(e*x+d)^n)^2*a*b^2*d*e^2*f^2+216*ln(c*(e*x+d)^n)*a*b^2*d^3*g^2*n-3
24*ln(c*(e*x+d)^n)*a^2*b*d*e^2*f^2-324*a^2*b*d^2*e*f*g*n-324*x^2*ln(c*(e*x+
d)^n)*a*b^2*e^3*f*g*n+324*x*ln(c*(e*x+d)^n)^2*b^3*d*e^2*f*g*n-972*x*ln(c*(e
*x+d)^n)*b^3*d*e^2*f*g*n^2-216*x*ln(c*(e*x+d)^n)*a*b^2*d^2*e*g^2*n-648*ln(c
*(e*x+d)^n)*a*b^2*d^2*e*f*g*n-36*x^3*ln(c*(e*x+d)^n)^2*b^3*e^3*g^2*n+24*x^3
*ln(c*(e*x+d)^n)*b^3*e^3*g^2*n^2+108*x^3*ln(c*(e*x+d)^n)^2*a*b^2*e^3*g^2+10
8*x^2*ln(c*(e*x+d)^n)^3*b^3*e^3*f*g+108*x^3*ln(c*(e*x+d)^n)*a^2*b*e^3*g^2-3
24*x*ln(c*(e*x+d)^n)^2*b^3*e^3*f^2*n+648*x*ln(c*(e*x+d)^n)*b^3*e^3*f^2*n^2-
72*x^3*ln(c*(e*x+d)^n)*a*b^2*e^3*g^2*n+54*x^2*ln(c*(e*x+d)^n)^2*b^3*d*e^2*g
^2*n-162*x^2*ln(c*(e*x+d)^n)^2*b^3*e^3*f*g*n-90*x^2*ln(c*(e*x+d)^n)*b^3*d*e
^2*g^2*n^2+162*x^2*ln(c*(e*x+d)^n)*b^3*e^3*f*g*n^2+324*x^2*ln(c*(e*x+d)^n)^
2*a*b^2*e^3*f*g-108*x*ln(c*(e*x+d)^n)^2*b^3*d^2*e*g^2*n+396*x*ln(c*(e*x+d)^
n)*b^3*d^2*e*g^2*n^2+324*x^2*ln(c*(e*x+d)^n)*a^2*b*e^3*f*g-648*x*ln(c*(e*x+
d)^n)*a*b^2*e^3*f^2*n+486*ln(c*(e*x+d)^n)^2*b^3*d^2*e*f*g*n+972*ln(c*(e*x+d
)^n)*b^3*d^2*e*f*g*n^2-324*ln(c*(e*x+d)^n)^2*a*b^2*d^2*e*f*g+648*ln(c*(e*x+
d)^n)*a*b^2*d*e^2*f^2*n+648*x*ln(c*(e*x+d)^n)*a*b^2*d*e^2*f*g*n+1620*ln(e*x
+d)*a*b^2*d^2*e*f*g*n^2-324*ln(e*x+d)*a^2*b*d^2*e*f*g*n+54*a^2*b*d*e^2*g^2*
n*x^2-2106*ln(e*x+d)*b^3*d^2*e*f*g*n^3-1296*ln(e*x+d)*a*b^2*d*e^2*f^2*n^2+6
48*ln(e*x+d)*a^2*b*d*e^2*f^2*n)/e^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1771 vs. $2(418) = 836$.

Time = 0.34 (sec) , antiderivative size = 1771, normalized size of antiderivative = 4.10

$$\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

```
[Out] -1/108*(4*(2*b^3*e^3*g^2*n^3 - 6*a*b^2*e^3*g^2*n^2 + 9*a^2*b*e^3*g^2*n - 9*
a^3*e^3*g^2)*x^3 - 36*(b^3*e^3*g^2*n^3*x^3 + 3*b^3*e^3*f*g*n^3*x^2 + 3*b^3*
e^3*f^2*n^3*x + (3*b^3*d*e^2*f^2 - 3*b^3*d^2*e*f*g + b^3*d^3*g^2)*n^3)*log(
e*x + d)^3 - 36*(b^3*e^3*g^2*x^3 + 3*b^3*e^3*f*g*x^2 + 3*b^3*e^3*f^2*x)*log
(c)^3 - 3*(36*a^3*e^3*f*g - (27*b^3*e^3*f*g - 19*b^3*d*e^2*g^2)*n^3 + 6*(9*
a*b^2*e^3*f*g - 5*a*b^2*d*e^2*g^2)*n^2 - 18*(3*a^2*b*e^3*f*g - a^2*b*d*e^2*
g^2)*n)*x^2 + 18*((18*b^3*d*e^2*f^2 - 27*b^3*d^2*e*f*g + 11*b^3*d^3*g^2)*n^
3 + 2*(b^3*e^3*g^2*n^3 - 3*a*b^2*e^3*g^2*n^2)*x^3 - 6*(3*a*b^2*d*e^2*f^2 -
3*a*b^2*d^2*e*f*g + a*b^2*d^3*g^2)*n^2 - 3*(6*a*b^2*e^3*f*g*n^2 - (3*b^3*e^
3*f*g - b^3*d*e^2*g^2)*n^3)*x^2 - 6*(3*a*b^2*e^3*f^2*n^2 - (3*b^3*e^3*f^2 -
 3*b^3*d*e^2*f*g + b^3*d^2*e*g^2)*n^3)*x - 6*(b^3*e^3*g^2*n^2*x^3 + 3*b^3*e
^3*f*g*n^2*x^2 + 3*b^3*e^3*f^2*n^2*x + (3*b^3*d*e^2*f^2 - 3*b^3*d^2*e*f*g +
```

```
 b^3*d^3*g^2)*n^2)*log(c))*log(e*x + d)^2 + 18*(2*(b^3*e^3*g^2*n - 3*a*b^2*
e^3*g^2)*x^3 - 3*(6*a*b^2*e^3*f*g - (3*b^3*e^3*f*g - b^3*d*e^2*g^2)*n)*x^2
- 6*(3*a*b^2*e^3*f^2 - (3*b^3*e^3*f^2 - 3*b^3*d*e^2*f*g + b^3*d^2*e*g^2)*n)
*x)*log(c)^2 - 6*(18*a^3*e^3*f^2 - (108*b^3*e^3*f^2 - 189*b^3*d*e^2*f*g + 8
5*b^3*d^2*e*g^2)*n^3 + 6*(18*a*b^2*e^3*f^2 - 27*a*b^2*d*e^2*f*g + 11*a*b^2*
d^2*e*g^2)*n^2 - 18*(3*a^2*b*e^3*f^2 - 3*a^2*b*d*e^2*f*g + a^2*b*d^2*e*g^2)
*n)*x - 6*((108*b^3*d*e^2*f^2 - 189*b^3*d^2*e*f*g + 85*b^3*d^3*g^2)*n^3 + 2
*(2*b^3*e^3*g^2*n^3 - 6*a*b^2*e^3*g^2*n^2 + 9*a^2*b*e^3*g^2*n)*x^3 - 6*(18*
a*b^2*d*e^2*f^2 - 27*a*b^2*d^2*e*f*g + 11*a*b^2*d^3*g^2)*n^2 + 3*(18*a^2*b*
e^3*f*g*n + (9*b^3*e^3*f*g - 5*b^3*d*e^2*g^2)*n^3 - 6*(3*a*b^2*e^3*f*g - a*
b^2*d*e^2*g^2)*n^2)*x^2 + 18*(b^3*e^3*g^2*n*x^3 + 3*b^3*e^3*f*g*n*x^2 + 3*b
^3*e^3*f^2*n*x + (3*b^3*d*e^2*f^2 - 3*b^3*d^2*e*f*g + b^3*d^3*g^2)*n)*log(c
)^2 + 18*(3*a^2*b*d*e^2*f^2 - 3*a^2*b*d^2*e*f*g + a^2*b*d^3*g^2)*n + 6*(9*a
^2*b*e^3*f^2*n + (18*b^3*e^3*f^2 - 27*b^3*d*e^2*f*g + 11*b^3*d^2*e*g^2)*n^3
 - 6*(3*a*b^2*e^3*f^2 - 3*a*b^2*d*e^2*f*g + a*b^2*d^2*e*g^2)*n^2)*x - 6*(2*
(b^3*e^3*g^2*n^2 - 3*a*b^2*e^3*g^2*n)*x^3 + (18*b^3*d*e^2*f^2 - 27*b^3*d^2*
e*f*g + 11*b^3*d^3*g^2)*n^2 - 3*(6*a*b^2*e^3*f*g*n - (3*b^3*e^3*f*g - b^3*d
*e^2*g^2)*n^2)*x^2 - 6*(3*a*b^2*d*e^2*f^2 - 3*a*b^2*d^2*e*f*g + a*b^2*d^3*g
^2)*n - 6*(3*a*b^2*e^3*f^2*n - (3*b^3*e^3*f^2 - 3*b^3*d*e^2*f*g + b^3*d^2*e
*g^2)*n^2)*x)*log(c))*log(e*x + d) - 6*(2*(2*b^3*e^3*g^2*n^2 - 6*a*b^2*e^3*
g^2*n + 9*a^2*b*e^3*g^2)*x^3 + 3*(18*a^2*b*e^3*f*g + (9*b^3*e^3*f*g - 5*b^3
*d*e^2*g^2)*n^2 - 6*(3*a*b^2*e^3*f*g - a*b^2*d*e^2*g^2)*n)*x^2 + 6*(9*a^2*b
*e^3*f^2 + (18*b^3*e^3*f^2 - 27*b^3*d*e^2*f*g + 11*b^3*d^2*e*g^2)*n^2 - 6*(
3*a*b^2*e^3*f^2 - 3*a*b^2*d*e^2*f*g + a*b^2*d^2*e*g^2)*n)*x)*log(c))/e^3
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1578 vs. 2(422) = 844.

Time = 2.41 (sec) , antiderivative size = 1578, normalized size of antiderivative = 3.65

$$\int (f+gx)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n))**3,x)
```

```
[Out] Piecewise((a**3*f**2*x + a**3*f*g*x**2 + a**3*g**2*x**3/3 + a**2*b*d**3*g**
2*log(c*(d + e*x)**n)/e**3 - 3*a**2*b*d**2*f*g*log(c*(d + e*x)**n)/e**2 - a
**2*b*d**2*g**2*n*x/e**2 + 3*a**2*b*d*f**2*log(c*(d + e*x)**n)/e + 3*a**2*b
*d*f*g*n*x/e + a**2*b*d*g**2*n*x**2/(2*e) - 3*a**2*b*f**2*n*x + 3*a**2*b*f*
*2*x*log(c*(d + e*x)**n) - 3*a**2*b*f*g*n*x**2/2 + 3*a**2*b*f*g*x**2*log(c*
(d + e*x)**n) - a**2*b*g**2*n*x**3/3 + a**2*b*g**2*x**3*log(c*(d + e*x)**n)
 - 11*a*b**2*d**3*g**2*n*log(c*(d + e*x)**n)/(3*e**3) + a*b**2*d**3*g**2*lo
g(c*(d + e*x)**n)**2/e**3 + 9*a*b**2*d**2*f*g*n*log(c*(d + e*x)**n)/e**2 -
3*a*b**2*d**2*f*g*log(c*(d + e*x)**n)**2/e**2 + 11*a*b**2*d**2*g**2*n**2*x/
(3*e**2) - 2*a*b**2*d**2*g**2*n*x*log(c*(d + e*x)**n)/e**2 - 6*a*b**2*d*f**
```

```
2*n*log(c*(d + e*x)**n)/e + 3*a*b**2*d*f**2*log(c*(d + e*x)**n)**2/e - 9*a*
b**2*d*f*g*n**2*x/e + 6*a*b**2*d*f*g*n*x*log(c*(d + e*x)**n)/e - 5*a*b**2*d
*g**2*n**2*x**2/(6*e) + a*b**2*d*g**2*n*x**2*log(c*(d + e*x)**n)/e + 6*a*b*
*2*f**2*n**2*x - 6*a*b**2*f**2*n*x*log(c*(d + e*x)**n) + 3*a*b**2*f**2*x*lo
g(c*(d + e*x)**n)**2 + 3*a*b**2*f*g*n**2*x**2/2 - 3*a*b**2*f*g*n*x**2*log(c
*(d + e*x)**n) + 3*a*b**2*f*g*x**2*log(c*(d + e*x)**n)**2 + 2*a*b**2*g**2*n
**2*x**3/9 - 2*a*b**2*g**2*n*x**3*log(c*(d + e*x)**n)/3 + a*b**2*g**2*x**3*
log(c*(d + e*x)**n)**2 + 85*b**3*d**3*g**2*n**2*log(c*(d + e*x)**n)/(18*e**
3) - 11*b**3*d**3*g**2*n*log(c*(d + e*x)**n)**2/(6*e**3) + b**3*d**3*g**2*l
og(c*(d + e*x)**n)**3/(3*e**3) - 21*b**3*d**2*f*g*n**2*log(c*(d + e*x)**n)/
(2*e**2) + 9*b**3*d**2*f*g*n*log(c*(d + e*x)**n)**2/(2*e**2) - b**3*d**2*f*
g*log(c*(d + e*x)**n)**3/e**2 - 85*b**3*d**2*g**2*n**3*x/(18*e**2) + 11*b**
3*d**2*g**2*n**2*x*log(c*(d + e*x)**n)/(3*e**2) - b**3*d**2*g**2*n*x*log(c*
(d + e*x)**n)**2/e**2 + 6*b**3*d*f**2*n**2*log(c*(d + e*x)**n)/e - 3*b**3*d
*f**2*n*log(c*(d + e*x)**n)**2/e + b**3*d*f**2*log(c*(d + e*x)**n)**3/e + 2
1*b**3*d*f*g*n**3*x/(2*e) - 9*b**3*d*f*g*n**2*x*log(c*(d + e*x)**n)/e + 3*b
**3*d*f*g*n*x*log(c*(d + e*x)**n)**2/e + 19*b**3*d*g**2*n**3*x**2/(36*e) -
5*b**3*d*g**2*n**2*x**2*log(c*(d + e*x)**n)/(6*e) + b**3*d*g**2*n*x**2*log(
c*(d + e*x)**n)**2/(2*e) - 6*b**3*f**2*n**3*x + 6*b**3*f**2*n**2*x*log(c*(d
 + e*x)**n) - 3*b**3*f**2*n*x*log(c*(d + e*x)**n)**2 + b**3*f**2*x*log(c*(d
 + e*x)**n)**3 - 3*b**3*f*g*n**3*x**2/4 + 3*b**3*f*g*n**2*x**2*log(c*(d + e
*x)**n)/2 - 3*b**3*f*g*n*x**2*log(c*(d + e*x)**n)**2/2 + b**3*f*g*x**2*log(
c*(d + e*x)**n)**3 - 2*b**3*g**2*n**3*x**3/27 + 2*b**3*g**2*n**2*x**3*log(c
*(d + e*x)**n)/9 - b**3*g**2*n*x**3*log(c*(d + e*x)**n)**2/3 + b**3*g**2*x*
*3*log(c*(d + e*x)**n)**3/3, Ne(e, 0)), ((a + b*log(c*d**n))**3*(f**2*x + f
*g*x**2 + g**2*x**3/3), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1140 vs. $2(418) = 836$.

Time = 0.24 (sec) , antiderivative size = 1140, normalized size of antiderivative = 2.64

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^3 \, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*g^2*x^3*log((e*x + d)^n*c)^3 + a*b^2*g^2*x^3*log((e*x + d)^n*c)^2 +
 b^3*f*g*x^2*log((e*x + d)^n*c)^3 + a^2*b*g^2*x^3*log((e*x + d)^n*c) + 3*a*
b^2*f*g*x^2*log((e*x + d)^n*c)^2 + b^3*f^2*x*log((e*x + d)^n*c)^3 + 1/3*a^3
*g^2*x^3 - 3*a^2*b*e*f^2*n*(x/e - d*log(e*x + d)/e^2) + 1/6*a^2*b*e*g^2*n*(
6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) - 3/2*a^2*b
*e*f*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 3*a^2*b*f*g*x^2*l
og((e*x + d)^n*c) + 3*a*b^2*f^2*x*log((e*x + d)^n*c)^2 + a^3*f*g*x^2 + 3*a^
2*b*f^2*x*log((e*x + d)^n*c) - 3*(2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x
```

```
 + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*a*b^2*f^2
 - (3*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^2 - e*n*((d*log(e*x
 + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*n^2/e^2 - 3*(d*log
(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n*log((e*x + d)^n*c)/e^2))*b^3*f^2
- 3/2*(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)^n
*c) - (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n^2/e
^2)*a*b^2*f*g - 1/4*(6*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*l
og((e*x + d)^n*c)^2 + e*n*((4*d^2*log(e*x + d)^3 + 3*e^2*x^2 + 18*d^2*log(e
*x + d)^2 - 42*d*e*x + 42*d^2*log(e*x + d))*n^2/e^3 - 6*(e^2*x^2 + 2*d^2*lo
g(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n*log((e*x + d)^n*c)/e^3))*b^3
*f*g + 1/18*(6*e*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2
*x)/e^3)*log((e*x + d)^n*c) + (4*e^3*x^3 - 15*d*e^2*x^2 - 18*d^3*log(e*x +
d)^2 + 66*d^2*e*x - 66*d^3*log(e*x + d))*n^2/e^3)*a*b^2*g^2 + 1/108*(18*e*n
*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3)*log((e*x
+ d)^n*c)^2 - e*n*((8*e^3*x^3 - 36*d^3*log(e*x + d)^3 - 57*d*e^2*x^2 - 198*
d^3*log(e*x + d)^2 + 510*d^2*e*x - 510*d^3*log(e*x + d))*n^2/e^4 - 6*(4*e^3
*x^3 - 15*d*e^2*x^2 - 18*d^3*log(e*x + d)^2 + 66*d^2*e*x - 66*d^3*log(e*x +
 d))*n*log((e*x + d)^n*c)/e^4))*b^3*g^2 + a^3*f^2*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2932 vs. $2(418) = 836$.

Time = 0.38 (sec) , antiderivative size = 2932, normalized size of antiderivative = 6.79

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^3 \, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

```
[Out] (e*x + d)*b^3*f^2*n^3*log(e*x + d)^3/e + (e*x + d)^2*b^3*f*g*n^3*log(e*x +
d)^3/e^2 - 2*(e*x + d)*b^3*d*f*g*n^3*log(e*x + d)^3/e^2 + 1/3*(e*x + d)^3*b
^3*g^2*n^3*log(e*x + d)^3/e^3 - (e*x + d)^2*b^3*d*g^2*n^3*log(e*x + d)^3/e^
3 + (e*x + d)*b^3*d^2*g^2*n^3*log(e*x + d)^3/e^3 - 3*(e*x + d)*b^3*f^2*n^3*
log(e*x + d)^2/e - 3/2*(e*x + d)^2*b^3*f*g*n^3*log(e*x + d)^2/e^2 + 6*(e*x
+ d)*b^3*d*f*g*n^3*log(e*x + d)^2/e^2 - 1/3*(e*x + d)^3*b^3*g^2*n^3*log(e*x
 + d)^2/e^3 + 3/2*(e*x + d)^2*b^3*d*g^2*n^3*log(e*x + d)^2/e^3 - 3*(e*x + d
)*b^3*d^2*g^2*n^3*log(e*x + d)^2/e^3 + 3*(e*x + d)*b^3*f^2*n^2*log(e*x + d)
^2*log(c)/e + 3*(e*x + d)^2*b^3*f*g*n^2*log(e*x + d)^2*log(c)/e^2 - 6*(e*x
+ d)*b^3*d*f*g*n^2*log(e*x + d)^2*log(c)/e^2 + (e*x + d)^3*b^3*g^2*n^2*log(
e*x + d)^2*log(c)/e^3 - 3*(e*x + d)^2*b^3*d*g^2*n^2*log(e*x + d)^2*log(c)/e
^3 + 3*(e*x + d)*b^3*d^2*g^2*n^2*log(e*x + d)^2*log(c)/e^3 + 6*(e*x + d)*b^
3*f^2*n^3*log(e*x + d)/e + 3/2*(e*x + d)^2*b^3*f*g*n^3*log(e*x + d)/e^2 - 1
2*(e*x + d)*b^3*d*f*g*n^3*log(e*x + d)/e^2 + 2/9*(e*x + d)^3*b^3*g^2*n^3*lo
g(e*x + d)/e^3 - 3/2*(e*x + d)^2*b^3*d*g^2*n^3*log(e*x + d)/e^3 + 6*(e*x +
d)*b^3*d^2*g^2*n^3*log(e*x + d)/e^3 + 3*(e*x + d)*a*b^2*f^2*n^2*log(e*x + d
```

)^2/e + 3*(e*x + d)^2*a*b^2*f*g*n^2*log(e*x + d)^2/e^2 - 6*(e*x + d)*a*b^2*
d*f*g*n^2*log(e*x + d)^2/e^2 + (e*x + d)^3*a*b^2*g^2*n^2*log(e*x + d)^2/e^3
 - 3*(e*x + d)^2*a*b^2*d*g^2*n^2*log(e*x + d)^2/e^3 + 3*(e*x + d)*a*b^2*d^2
*g^2*n^2*log(e*x + d)^2/e^3 - 6*(e*x + d)*b^3*f^2*n^2*log(e*x + d)*log(c)/e
 - 3*(e*x + d)^2*b^3*f*g*n^2*log(e*x + d)*log(c)/e^2 + 12*(e*x + d)*b^3*d*f
*g*n^2*log(e*x + d)*log(c)/e^2 - 2/3*(e*x + d)^3*b^3*g^2*n^2*log(e*x + d)*l
og(c)/e^3 + 3*(e*x + d)^2*b^3*d*g^2*n^2*log(e*x + d)*log(c)/e^3 - 6*(e*x +
d)*b^3*d^2*g^2*n^2*log(e*x + d)*log(c)/e^3 + 3*(e*x + d)*b^3*f^2*n*log(e*x
+ d)*log(c)^2/e + 3*(e*x + d)^2*b^3*f*g*n*log(e*x + d)*log(c)^2/e^2 - 6*(e*
x + d)*b^3*d*f*g*n*log(e*x + d)*log(c)^2/e^2 + (e*x + d)^3*b^3*g^2*n*log(e*
x + d)*log(c)^2/e^3 - 3*(e*x + d)^2*b^3*d*g^2*n*log(e*x + d)*log(c)^2/e^3 +
 3*(e*x + d)*b^3*d^2*g^2*n*log(e*x + d)*log(c)^2/e^3 - 6*(e*x + d)*b^3*f^2*
n^3/e - 3/4*(e*x + d)^2*b^3*f*g*n^3/e^2 + 12*(e*x + d)*b^3*d*f*g*n^3/e^2 -
2/27*(e*x + d)^3*b^3*g^2*n^3/e^3 + 3/4*(e*x + d)^2*b^3*d*g^2*n^3/e^3 - 6*(e
*x + d)*b^3*d^2*g^2*n^3/e^3 - 6*(e*x + d)*a*b^2*f^2*n^2*log(e*x + d)/e - 3*
(e*x + d)^2*a*b^2*f*g*n^2*log(e*x + d)/e^2 + 12*(e*x + d)*a*b^2*d*f*g*n^2*l
og(e*x + d)/e^2 - 2/3*(e*x + d)^3*a*b^2*g^2*n^2*log(e*x + d)/e^3 + 3*(e*x +
 d)^2*a*b^2*d*g^2*n^2*log(e*x + d)/e^3 - 6*(e*x + d)*a*b^2*d^2*g^2*n^2*log(
e*x + d)/e^3 + 6*(e*x + d)*b^3*f^2*n^2*log(c)/e + 3/2*(e*x + d)^2*b^3*f*g*n
^2*log(c)/e^2 - 12*(e*x + d)*b^3*d*f*g*n^2*log(c)/e^2 + 2/9*(e*x + d)^3*b^3
*g^2*n^2*log(c)/e^3 - 3/2*(e*x + d)^2*b^3*d*g^2*n^2*log(c)/e^3 + 6*(e*x + d
)*b^3*d^2*g^2*n^2*log(c)/e^3 + 6*(e*x + d)*a*b^2*f^2*n*log(e*x + d)*log(c)/
e + 6*(e*x + d)^2*a*b^2*f*g*n*log(e*x + d)*log(c)/e^2 - 12*(e*x + d)*a*b^2*
d*f*g*n*log(e*x + d)*log(c)/e^2 + 2*(e*x + d)^3*a*b^2*g^2*n*log(e*x + d)*lo
g(c)/e^3 - 6*(e*x + d)^2*a*b^2*d*g^2*n*log(e*x + d)*log(c)/e^3 + 6*(e*x + d
)*a*b^2*d^2*g^2*n*log(e*x + d)*log(c)/e^3 - 3*(e*x + d)*b^3*f^2*n*log(c)^2/
e - 3/2*(e*x + d)^2*b^3*f*g*n*log(c)^2/e^2 + 6*(e*x + d)*b^3*d*f*g*n*log(c)
^2/e^2 - 1/3*(e*x + d)^3*b^3*g^2*n*log(c)^2/e^3 + 3/2*(e*x + d)^2*b^3*d*g^2
*n*log(c)^2/e^3 - 3*(e*x + d)*b^3*d^2*g^2*n*log(c)^2/e^3 + (e*x + d)*b^3*f^
2*log(c)^3/e + (e*x + d)^2*b^3*f*g*log(c)^3/e^2 - 2*(e*x + d)*b^3*d*f*g*log
(c)^3/e^2 + 1/3*(e*x + d)^3*b^3*g^2*log(c)^3/e^3 - (e*x + d)^2*b^3*d*g^2*lo
g(c)^3/e^3 + (e*x + d)*b^3*d^2*g^2*log(c)^3/e^3 + 6*(e*x + d)*a*b^2*f^2*n^2
/e + 3/2*(e*x + d)^2*a*b^2*f*g*n^2/e^2 - 12*(e*x + d)*a*b^2*d*f*g*n^2/e^2 +
 2/9*(e*x + d)^3*a*b^2*g^2*n^2/e^3 - 3/2*(e*x + d)^2*a*b^2*d*g^2*n^2/e^3 +
6*(e*x + d)*a*b^2*d^2*g^2*n^2/e^3 + 3*(e*x + d)*a^2*b*f^2*n*log(e*x + d)/e
+ 3*(e*x + d)^2*a^2*b*f*g*n*log(e*x + d)/e^2 - 6*(e*x + d)*a^2*b*d*f*g*n*lo
g(e*x + d)/e^2 + (e*x + d)^3*a^2*b*g^2*n*log(e*x + d)/e^3 - 3*(e*x + d)^2*a
^2*b*d*g^2*n*log(e*x + d)/e^3 + 3*(e*x + d)*a^2*b*d^2*g^2*n*log(e*x + d)/e^
3 - 6*(e*x + d)*a*b^2*f^2*n*log(c)/e - 3*(e*x + d)^2*a*b^2*f*g*n*log(c)/e^2
 + 12*(e*x + d)*a*b^2*d*f*g*n*log(c)/e^2 - 2/3*(e*x + d)^3*a*b^2*g^2*n*log(
c)/e^3 + 3*(e*x + d)^2*a*b^2*d*g^2*n*log(c)/e^3 - 6*(e*x + d)*a*b^2*d^2*g^2
*n*log(c)/e^3 + 3*(e*x + d)*a*b^2*f^2*log(c)^2/e + 3*(e*x + d)^2*a*b^2*f*g*
log(c)^2/e^2 - 6*(e*x + d)*a*b^2*d*f*g*log(c)^2/e^2 + (e*x + d)^3*a*b^2*g^2
*log(c)^2/e^3 - 3*(e*x + d)^2*a*b^2*d*g^2*log(c)^2/e^3 + 3*(e*x + d)*a*b^2*
d^2*g^2*log(c)^2/e^3 - 3*(e*x + d)*a^2*b*f^2*n/e - 3/2*(e*x + d)^2*a^2*b*f*

```
g*n/e^2 + 6*(e*x + d)*a^2*b*d*f*g*n/e^2 - 1/3*(e*x + d)^3*a^2*b*g^2*n/e^3 +
 3/2*(e*x + d)^2*a^2*b*d*g^2*n/e^3 - 3*(e*x + d)*a^2*b*d^2*g^2*n/e^3 + 3*(e
*x + d)*a^2*b*f^2*log(c)/e + 3*(e*x + d)^2*a^2*b*f*g*log(c)/e^2 - 6*(e*x +
d)*a^2*b*d*f*g*log(c)/e^2 + (e*x + d)^3*a^2*b*g^2*log(c)/e^3 - 3*(e*x + d)^
2*a^2*b*d*g^2*log(c)/e^3 + 3*(e*x + d)*a^2*b*d^2*g^2*log(c)/e^3 + (e*x + d)
*a^3*f^2/e + (e*x + d)^2*a^3*f*g/e^2 - 2*(e*x + d)*a^3*d*f*g/e^2 + 1/3*(e*x
 + d)^3*a^3*g^2/e^3 - (e*x + d)^2*a^3*d*g^2/e^3 + (e*x + d)*a^3*d^2*g^2/e^3
```

## Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 1157, normalized size of antiderivative = 2.68

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))^3 \, dx$$

$$= \ln \left(c\,{\left(d+e\,x\right)}^n\right)^2\,\left(x^2\,\left(\frac{3\,b^2\,g\,\left(a\,d\,g+2\,a\,e\,f-b\,e\,f\,n\right)}{2\,e}-\frac{b^2\,d\,g^2\,\left(3\,a-b\,n\right)}{2\,e}\right)-x\,\left(\frac{d\,\left(\frac{3\,b^2\,g\,\left(a\,d\,g+2\,a\,e\,f-b\,e\,f\,n\right)}{e}-\frac{b^2\,d\,g^2\,\left(3\,a-b\,n\right)}{e}\right)}{e}-\frac{3\,b^2\,f\,\left(2\,a\,d\,g+a\,e\,f-b\,e\,f\,n\right)}{e}\right)+\frac{d\,\left(-11\,n\,b^3\,d^2\,g^2+27\,n\,b^3\,d\,e\,f\,g-18\,n\,b^3\,e^2\,f^2+6\,a\,b^2\,d^2\,g^2-18\,a\,b^2\,d\,e\,f\,g+18\,a\,b^2\,e^2\,f^2\right)}{6\,e^3}+\frac{b^2\,g^2\,x^3\,\left(3\,a-b\,n\right)}{3}\right)$$

$$+x\left(\frac{36\,a^3\,d\,e\,f\,g+18\,a^3\,e^2\,f^2-54\,a^2\,b\,e^2\,f^2\,n+36\,a\,b^2\,d^2\,g^2\,n^2-108\,a\,b^2\,d\,e\,f\,g\,n^2+108\,a\,b^2\,e^2\,f^2\,n^2-\cdots}{18\,e^2}-\frac{d\,\left(\frac{g\,\left(6\,a^3\,d\,g+12\,a^3\,e\,f+5\,b^3\,d\,g\,n^3-9\,b^3\,e\,f\,n^3-6\,a\,b^2\,d\,g\,n^2+18\,a\,b^2\,e\,f\,n^2-18\,a^2\,b\,e\,f\,n\right)}{6\,e}-\frac{d\,g^2\,\left(9\,a^3-9\,a^2\,b\,n+6\,a\,b^2\,n^2-2\,b^3\,n^3\right)}{9\,e}\right)}{e}\right)$$

$$+x^2\left(\frac{g\,\left(6\,a^3\,d\,g+12\,a^3\,e\,f+5\,b^3\,d\,g\,n^3-9\,b^3\,e\,f\,n^3-6\,a\,b^2\,d\,g\,n^2+18\,a\,b^2\,e\,f\,n^2-18\,a^2\,b\,e\,f\,n\right)}{12\,e}-\frac{d\,g^2\,\left(9\,a^3-9\,a^2\,b\,n+6\,a\,b^2\,n^2-2\,b^3\,n^3\right)}{18\,e}\right)$$

$$+\ln \left(c\,{\left(d+e\,x\right)}^n\right)^3\,\left(b^3\,f^2\,x+\frac{b^3\,g^2\,x^3}{3}+\frac{d\,\left(b^3\,d^2\,g^2-3\,b^3\,d\,e\,f\,g+3\,b^3\,e^2\,f^2\right)}{3\,e^3}+b^3\,f\,g\,x^2\right)$$

$$+\frac{g^2\,x^3\,\left(9\,a^3-9\,a^2\,b\,n+6\,a\,b^2\,n^2-2\,b^3\,n^3\right)}{27}$$

$$+\frac{\ln \left(d+e\,x\right)\,\left(18\,a^2\,b\,d^3\,g^2\,n-54\,a^2\,b\,d^2\,e\,f\,g\,n+54\,a^2\,b\,d\,e^2\,f^2\,n-66\,a\,b^2\,d^3\,g^2\,n^2+162\,a\,b^2\,d^2\,e\,f\,g\,n^2-\cdots\right)}{18\,e^3}$$

$$+\frac{\ln \left(c\,{\left(d+e\,x\right)}^n\right)\,\left(\frac{x^2\,\left(9\,b\,e\,g\,\left(3\,a^2\,d\,g+6\,a^2\,e\,f-b^2\,d\,g\,n^2+3\,b^2\,e\,f\,n^2-6\,a\,b\,e\,f\,n\right)-3\,b\,d\,e\,g^2\,\left(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2\right)\right)}{6\,e}+\frac{x\,\left(54\,a^2\,b\,d\,e^2\,f\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^3,x)
```

```
[Out] log(c*(d + e*x)^n)^2*(x^2*((3*b^2*g*(a*d*g + 2*a*e*f - b*e*f*n))/(2*e) - (b
^2*d*g^2*(3*a - b*n))/(2*e)) - x*((d*((3*b^2*g*(a*d*g + 2*a*e*f - b*e*f*n))
/e - (b^2*d*g^2*(3*a - b*n))/e))/e - (3*b^2*f*(2*a*d*g + a*e*f - b*e*f*n))/
```

```
e) + (d*(6*a*b^2*d^2*g^2 + 18*a*b^2*e^2*f^2 - 11*b^3*d^2*g^2*n - 18*b^3*e^2
*f^2*n - 18*a*b^2*d*e*f*g + 27*b^3*d*e*f*g*n))/(6*e^3) + (b^2*g^2*x^3*(3*a
- b*n))/3) + x*((18*a^3*e^2*f^2 - 66*b^3*d^2*g^2*n^3 - 108*b^3*e^2*f^2*n^3
- 54*a^2*b*e^2*f^2*n + 36*a^3*d*e*f*g + 36*a*b^2*d^2*g^2*n^2 + 108*a*b^2*e^
2*f^2*n^2 + 162*b^3*d*e*f*g*n^3 - 108*a*b^2*d*e*f*g*n^2)/(18*e^2) - (d*((g*
(6*a^3*d*g + 12*a^3*e*f + 5*b^3*d*g*n^3 - 9*b^3*e*f*n^3 - 6*a*b^2*d*g*n^2 +
 18*a*b^2*e*f*n^2 - 18*a^2*b*e*f*n))/(6*e) - (d*g^2*(9*a^3 - 2*b^3*n^3 + 6*
a*b^2*n^2 - 9*a^2*b*n))/(9*e)))/e) + x^2*((g*(6*a^3*d*g + 12*a^3*e*f + 5*b^
3*d*g*n^3 - 9*b^3*e*f*n^3 - 6*a*b^2*d*g*n^2 + 18*a*b^2*e*f*n^2 - 18*a^2*b*e
*f*n))/(12*e) - (d*g^2*(9*a^3 - 2*b^3*n^3 + 6*a*b^2*n^2 - 9*a^2*b*n))/(18*e
)) + log(c*(d + e*x)^n)^3*(b^3*f^2*x + (b^3*g^2*x^3)/3 + (d*(b^3*d^2*g^2 +
3*b^3*e^2*f^2 - 3*b^3*d*e*f*g))/(3*e^3) + b^3*f*g*x^2) + (g^2*x^3*(9*a^3 -
2*b^3*n^3 + 6*a*b^2*n^2 - 9*a^2*b*n))/27 + (log(d + e*x)*(85*b^3*d^3*g^2*n^
3 + 18*a^2*b*d^3*g^2*n - 66*a*b^2*d^3*g^2*n^2 + 108*b^3*d*e^2*f^2*n^3 - 108
*a*b^2*d*e^2*f^2*n^2 + 54*a^2*b*d*e^2*f^2*n - 189*b^3*d^2*e*f*g*n^3 + 162*a
*b^2*d^2*e*f*g*n^2 - 54*a^2*b*d^2*e*f*g*n))/(18*e^3) + (log(c*(d + e*x)^n)*
((x^2*(9*b*e*g*(3*a^2*d*g + 6*a^2*e*f - b^2*d*g*n^2 + 3*b^2*e*f*n^2 - 6*a*b
*e*f*n) - 3*b*d*e*g^2*(9*a^2 + 2*b^2*n^2 - 6*a*b*n)))/(6*e) + (x*((27*a^2*b
*e^3*f^2 + 54*b^3*e^3*f^2*n^2 - 54*a*b^2*e^3*f^2*n + 18*b^3*d^2*e*g^2*n^2 +
 54*a^2*b*d*e^2*f*g - 54*b^3*d*e^2*f*g*n^2)/e - (d*(9*b*e*g*(3*a^2*d*g + 6*
a^2*e*f - b^2*d*g*n^2 + 3*b^2*e*f*n^2 - 6*a*b*e*f*n) - 3*b*d*e*g^2*(9*a^2 +
 2*b^2*n^2 - 6*a*b*n)))/e))/(3*e) + (b*e*g^2*x^3*(9*a^2 + 2*b^2*n^2 - 6*a*b
*n))/3))/(3*e)
```

# 3.54 $\int (f+gx)\,(a+b\log(c(d+ex)^n))^3\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 265

$$\begin{aligned}\int (f+gx)\,(a+b\log(c(d+ex)^n))^3\,dx =\ & \frac{6ab^2(ef-dg)n^2x}{e} - \frac{6b^3(ef-dg)n^3x}{e} \\ & - \frac{3b^3gn^3(d+ex)^2}{8e^2} \\ & + \frac{6b^3(ef-dg)n^2(d+ex)\log(c(d+ex)^n)}{e^2} \\ & + \frac{3b^2gn^2(d+ex)^2\,(a+b\log(c(d+ex)^n))}{4e^2} \\ & - \frac{3b(ef-dg)n(d+ex)\,(a+b\log(c(d+ex)^n))^2}{e^2} \\ & - \frac{3bgn(d+ex)^2\,(a+b\log(c(d+ex)^n))^2}{4e^2} \\ & + \frac{(ef-dg)(d+ex)\,(a+b\log(c(d+ex)^n))^3}{e^2} \\ & + \frac{g(d+ex)^2\,(a+b\log(c(d+ex)^n))^3}{2e^2}\end{aligned}$$

```
[Out] 6*a*b^2*(-d*g+e*f)*n^2*x/e-6*b^3*(-d*g+e*f)*n^3*x/e-3/8*b^3*g*n^3*(e*x+d)^2
/e^2+6*b^3*(-d*g+e*f)*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e^2+3/4*b^2*g*n^2*(e*x+d)
^2*(a+b*ln(c*(e*x+d)^n))/e^2-3*b*(-d*g+e*f)*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))
^2/e^2-3/4*b*g*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2+(-d*g+e*f)*(e*x+d)*(
a+b*ln(c*(e*x+d)^n))^3/e^2+1/2*g*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^3/e^2
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx=\frac{3b^2gn^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{4e^2}+\frac{6ab^2n^2x(ef-dg)}{e}-\frac{3bn(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2}+\frac{(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}-\frac{3bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2e^2}+\frac{6b^3n^2(d+ex)(ef-dg)\log\left(c(d+ex)^n\right)}{e^2}-\frac{3b^3gn^3(d+ex)^2}{8e^2}-\frac{6b^3n^3x(ef-dg)}{e}$$

```
[In] Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] (6*a*b^2*(e*f - d*g)*n^2*x)/e - (6*b^3*(e*f - d*g)*n^3*x)/e - (3*b^3*g*n^3*
(d + e*x)^2)/(8*e^2) + (6*b^3*(e*f - d*g)*n^2*(d + e*x)*Log[c*(d + e*x)^n])
/e^2 + (3*b^2*g*n^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(4*e^2) - (3*b*
(e*f - d*g)*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e^2 - (3*b*g*n*(d + e
*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(4*e^2) + ((e*f - d*g)*(d + e*x)*(a + b
*Log[c*(d + e*x)^n])^3)/e^2 + (g*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^3)/
(2*e^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e}+\frac{g(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e}\right)dx\\
&= \frac{g\int (d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx}{e}+\frac{(ef-dg)\int \left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx}{e}\\
&= \frac{g\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+ex\right)}{e^2}\\
&\quad+\frac{(ef-dg)\text{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+ex\right)}{e^2}
\end{aligned}$$

$$= \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2e^2} - \frac{(3bgn)\text{Subst}\left(\int x(a+b\log\left(cx^n\right))^2\,dx,x,d+ex\right)}{2e^2} - \frac{(3b(ef-dg)n)\text{Subst}\left(\int (a+b\log\left(cx^n\right))^2\,dx,x,d+ex\right)}{e^2}$$

$$= -\frac{3b(ef-dg)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2} - \frac{3bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4e^2} + \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2} + \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2e^2} + \frac{(3b^2gn^2)\,\text{Subst}\left(\int x(a+b\log\left(cx^n\right))\,dx,x,d+ex\right)}{2e^2} + \frac{(6b^2(ef-dg)n^2)\,\text{Subst}\left(\int (a+b\log\left(cx^n\right))\,dx,x,d+ex\right)}{e^2}$$

$$= \frac{6ab^2(ef-dg)n^2x}{e} - \frac{3b^3gn^3(d+ex)^2}{8e^2} + \frac{3b^2gn^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{4e^2} - \frac{3b(ef-dg)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2} - \frac{3bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4e^2} + \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2} + \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2e^2} + \frac{(6b^3(ef-dg)n^2)\,\text{Subst}\left(\int \log\left(cx^n\right)\,dx,x,d+ex\right)}{e^2}$$

$$= \frac{6ab^2(ef-dg)n^2x}{e} - \frac{6b^3(ef-dg)n^3x}{e} - \frac{3b^3gn^3(d+ex)^2}{8e^2} + \frac{6b^3(ef-dg)n^2(d+ex)\log(c(d+ex)^n)}{e^2} + \frac{3b^2gn^2(d+ex)^2(a+b\log(c(d+ex)^n))}{4e^2} - \frac{3b(ef-dg)n(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2} - \frac{3bgn(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2} + \frac{(ef-dg)(d+ex)(a+b\log(c(d+ex)^n))^3}{e^2} + \frac{g(d+ex)^2(a+b\log(c(d+ex)^n))^3}{2e^2}$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.76

$$\int (f+gx)(a+b\log(c(d+ex)^n))^3\,dx = \frac{8(ef-dg)(d+ex)(a+b\log(c(d+ex)^n))^3 + 4g(d+ex)^2(a+b\log(c(d+ex)^n))^3 - 24b(ef-dg)n((d+\cdots}{}$$

```
[In] Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] (8*(e*f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3 + 4*g*(d + e*x)^2*(a
+ b*Log[c*(d + e*x)^n])^3 - 24*b*(e*f - d*g)*n*((d + e*x)*(a + b*Log[c*(d +
 e*x)^n])^2 - 2*b*n*(e*(a - b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n])) - 3*b
*g*n*(2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2 + b*n*(b*e*n*x*(2*d + e*x)
 - 2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))))/(8*e^2)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. 2(257) = 514.

Time = 1.74 (sec) , antiderivative size = 906, normalized size of antiderivative = 3.42

| method | result |
|---|---|
| parallelrisch | $-\frac{-6ab^2e^2gn^2x^2-42b^3degn^3x+6a^2be^2gnx^2-48ab^2e^2fn^2x+24a^2be^2fnx-48b^3defn^3+3b^3e^2gn^3x^2+48b^3e^2fn^3x-36ab\cdots}{}$ |
| risch | Expression too large to display |

```
[In] int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-6*a*b^2*e^2*g*n^2*x^2-42*b^3*d*e*g*n^3*x+6*a^2*b*e^2*g*n*x^2-48*a*b^
2*e^2*f*n^2*x+24*a^2*b*e^2*f*n*x-48*b^3*d*e*f*n^3+3*b^3*e^2*g*n^3*x^2+48*b^
3*e^2*f*n^3*x-36*a*b^2*d^2*g*n^2+36*a*b^2*d*e*g*n^2*x+12*a^2*b*d^2*g*n-4*a^
3*e^2*g*x^2-8*a^3*e^2*f*x+8*a^3*d*e*f+42*b^3*d^2*g*n^3-96*ln(e*x+d)*b^3*d*e
*f*n^3-60*ln(e*x+d)*a*b^2*d^2*g*n^2+12*ln(e*x+d)*a^2*b*d^2*g*n-4*x^2*ln(c*(
e*x+d)^n)^3*b^3*e^2*g-8*x*ln(c*(e*x+d)^n)^3*b^3*e^2*f-8*ln(c*(e*x+d)^n)^3*b
^3*d*e*f-18*ln(c*(e*x+d)^n)^2*b^3*d^2*g*n-36*ln(c*(e*x+d)^n)*b^3*d^2*g*n^2+
12*ln(c*(e*x+d)^n)^2*a*b^2*d^2*g+6*x^2*ln(c*(e*x+d)^n)^2*b^3*e^2*g*n-6*x^2*
ln(c*(e*x+d)^n)*b^3*e^2*g*n^2-12*x^2*ln(c*(e*x+d)^n)^2*a*b^2*e^2*g+24*x*ln(
c*(e*x+d)^n)^2*b^3*e^2*f*n-48*x*ln(c*(e*x+d)^n)*b^3*e^2*f*n^2-12*x^2*ln(c*(
e*x+d)^n)*a^2*b*e^2*g-24*x*ln(c*(e*x+d)^n)^2*a*b^2*e^2*f+24*ln(c*(e*x+d)^n)
^2*b^3*d*e*f*n+48*ln(c*(e*x+d)^n)*b^3*d*e*f*n^2-24*x*ln(c*(e*x+d)^n)*a^2*b*
e^2*f-24*ln(c*(e*x+d)^n)^2*a*b^2*d*e*f+24*ln(c*(e*x+d)^n)*a*b^2*d^2*g*n+24*
ln(c*(e*x+d)^n)*a^2*b*d*e*f+48*a*b^2*d*e*f*n^2-24*a^2*b*d*e*f*n+78*ln(e*x+d
)*b^3*d^2*g*n^3-12*a^2*b*d*e*g*n*x+4*ln(c*(e*x+d)^n)^3*b^3*d^2*g+12*x^2*ln(
c*(e*x+d)^n)*a*b^2*e^2*g*n-12*x*ln(c*(e*x+d)^n)^2*b^3*d*e*g*n+36*x*ln(c*(e*
x+d)^n)*b^3*d*e*g*n^2+96*ln(e*x+d)*a*b^2*d*e*f*n^2-48*ln(e*x+d)*a^2*b*d*e*f
*n+48*x*ln(c*(e*x+d)^n)*a*b^2*e^2*f*n-48*ln(c*(e*x+d)^n)*a*b^2*d*e*f*n-24*x
*ln(c*(e*x+d)^n)*a*b^2*d*e*g*n)/e^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(257) = 514$.

Time = 0.31 (sec) , antiderivative size = 923, normalized size of antiderivative = 3.48

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx = \frac{4\left(b^3e^2gn^3x^2+2\,b^3e^2fn^3x+(2\,b^3def-b^3d^2g)n^3\right)\log\left(ex+d\right)^3+4\left(b^3e^2gx^2+2\,b^3e^2fx\right)\log\left(c\right)^3-\left(3\,b^3e^2g\right.}{}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^3*e^2*g*n^3*x^2 + 2*b^3*e^2*f*n^3*x + (2*b^3*d*e*f - b^3*d^2*g)*n
^3)*log(e*x + d)^3 + 4*(b^3*e^2*g*x^2 + 2*b^3*e^2*f*x)*log(c)^3 - (3*b^3*e^
2*g*n^3 - 6*a*b^2*e^2*g*n^2 + 6*a^2*b*e^2*g*n - 4*a^3*e^2*g)*x^2 - 6*((4*b^
3*d*e*f - 3*b^3*d^2*g)*n^3 - 2*(2*a*b^2*d*e*f - a*b^2*d^2*g)*n^2 + (b^3*e^2
*g*n^3 - 2*a*b^2*e^2*g*n^2)*x^2 - 2*(2*a*b^2*e^2*f*n^2 - (2*b^3*e^2*f - b^3
*d*e*g)*n^3)*x - 2*(b^3*e^2*g*n^2*x^2 + 2*b^3*e^2*f*n^2*x + (2*b^3*d*e*f -
b^3*d^2*g)*n^2)*log(c))*log(e*x + d)^2 - 6*((b^3*e^2*g*n - 2*a*b^2*e^2*g)*x
^2 - 2*(2*a*b^2*e^2*f - (2*b^3*e^2*f - b^3*d*e*g)*n)*x)*log(c)^2 + 2*(4*a^3
*e^2*f - 3*(8*b^3*e^2*f - 7*b^3*d*e*g)*n^3 + 6*(4*a*b^2*e^2*f - 3*a*b^2*d*e
*g)*n^2 - 6*(2*a^2*b*e^2*f - a^2*b*d*e*g)*n)*x + 6*((8*b^3*d*e*f - 7*b^3*d^
2*g)*n^3 - 2*(4*a*b^2*d*e*f - 3*a*b^2*d^2*g)*n^2 + (b^3*e^2*g*n^3 - 2*a*b^2
*e^2*g*n^2 + 2*a^2*b*e^2*g*n)*x^2 + 2*(b^3*e^2*g*n*x^2 + 2*b^3*e^2*f*n*x +
```

```
(2*b^3*d*e*f - b^3*d^2*g)*n)*log(c)^2 + 2*(2*a^2*b*d*e*f - a^2*b*d^2*g)*n +
 2*(2*a^2*b*e^2*f*n + (4*b^3*e^2*f - 3*b^3*d*e*g)*n^3 - 2*(2*a*b^2*e^2*f -
a*b^2*d*e*g)*n^2)*x - 2*((4*b^3*d*e*f - 3*b^3*d^2*g)*n^2 + (b^3*e^2*g*n^2 -
 2*a*b^2*e^2*g*n)*x^2 - 2*(2*a*b^2*d*e*f - a*b^2*d^2*g)*n - 2*(2*a*b^2*e^2*
f*n - (2*b^3*e^2*f - b^3*d*e*g)*n^2)*x)*log(c))*log(e*x + d) + 6*((b^3*e^2*
g*n^2 - 2*a*b^2*e^2*g*n + 2*a^2*b*e^2*g)*x^2 + 2*(2*a^2*b*e^2*f + (4*b^3*e^
2*f - 3*b^3*d*e*g)*n^2 - 2*(2*a*b^2*e^2*f - a*b^2*d*e*g)*n)*x)*log(c))/e^2
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 836 vs. 2(258) = 516.

Time = 1.27 (sec) , antiderivative size = 836, normalized size of antiderivative = 3.15

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx$$
$$=\begin{cases} a^3fx+\frac{a^3gx^2}{2}-\frac{3a^2bd^2g\log\left(c(d+ex)^n\right)}{2e^2}+\frac{3a^2bdf\log\left(c(d+ex)^n\right)}{e}+\frac{3a^2bdgnx}{2e}-3a^2bfnx+3a^2bfx\log\left(c(d+ex)^n\right)- \\ \left(a+b\log\left(cd^n\right)\right)^3\left(fx+\frac{gx^2}{2}\right) \end{cases}$$

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**3,x)
```

```
[Out] Piecewise((a**3*f*x + a**3*g*x**2/2 - 3*a**2*b*d**2*g*log(c*(d + e*x)**n)/(
2*e**2) + 3*a**2*b*d*f*log(c*(d + e*x)**n)/e + 3*a**2*b*d*g*n*x/(2*e) - 3*a
**2*b*f*n*x + 3*a**2*b*f*x*log(c*(d + e*x)**n) - 3*a**2*b*g*n*x**2/4 + 3*a*
*2*b*g*x**2*log(c*(d + e*x)**n)/2 + 9*a*b**2*d**2*g*n*log(c*(d + e*x)**n)/(
2*e**2) - 3*a*b**2*d**2*g*log(c*(d + e*x)**n)**2/(2*e**2) - 6*a*b**2*d*f*n*
log(c*(d + e*x)**n)/e + 3*a*b**2*d*f*log(c*(d + e*x)**n)**2/e - 9*a*b**2*d*
g*n**2*x/(2*e) + 3*a*b**2*d*g*n*x*log(c*(d + e*x)**n)/e + 6*a*b**2*f*n**2*x
 - 6*a*b**2*f*n*x*log(c*(d + e*x)**n) + 3*a*b**2*f*x*log(c*(d + e*x)**n)**2
 + 3*a*b**2*g*n**2*x**2/4 - 3*a*b**2*g*n*x**2*log(c*(d + e*x)**n)/2 + 3*a*b
**2*g*x**2*log(c*(d + e*x)**n)**2/2 - 21*b**3*d**2*g*n**2*log(c*(d + e*x)**
n)/(4*e**2) + 9*b**3*d**2*g*n*log(c*(d + e*x)**n)**2/(4*e**2) - b**3*d**2*g
*log(c*(d + e*x)**n)**3/(2*e**2) + 6*b**3*d*f*n**2*log(c*(d + e*x)**n)/e -
3*b**3*d*f*n*log(c*(d + e*x)**n)**2/e + b**3*d*f*log(c*(d + e*x)**n)**3/e +
 21*b**3*d*g*n**3*x/(4*e) - 9*b**3*d*g*n**2*x*log(c*(d + e*x)**n)/(2*e) + 3
*b**3*d*g*n*x*log(c*(d + e*x)**n)**2/(2*e) - 6*b**3*f*n**3*x + 6*b**3*f*n**
2*x*log(c*(d + e*x)**n) - 3*b**3*f*n*x*log(c*(d + e*x)**n)**2 + b**3*f*x*lo
g(c*(d + e*x)**n)**3 - 3*b**3*g*n**3*x**2/8 + 3*b**3*g*n**2*x**2*log(c*(d +
 e*x)**n)/4 - 3*b**3*g*n*x**2*log(c*(d + e*x)**n)**2/4 + b**3*g*x**2*log(c*
(d + e*x)**n)**3/2, Ne(e, 0)), ((a + b*log(c*d**n))**3*(f*x + g*x**2/2), Tr
ue))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(257) = 514.

Time = 0.22 (sec) , antiderivative size = 662, normalized size of antiderivative = 2.50

$$\int (f+gx)(a+b\log(c(d+ex)^n))^3\,dx$$
$$= \frac{1}{2}b^3gx^2\log((ex+d)^nc)^3 + \frac{3}{2}ab^2gx^2\log((ex+d)^nc)^2 + b^3fx\log((ex+d)^nc)^3 - 3a^2befn\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) - \frac{3}{4}a^2begn\left(\frac{2d^2\log(ex+d)}{e^3} + \frac{ex^2-2dx}{e^2}\right) + \frac{3}{2}a^2bgx^2\log((ex+d)^nc) + 3ab^2fx\log((ex+d)^nc)^2 + \frac{1}{2}a^3gx^2 + 3a^2bfx\log((ex+d)^nc) - 3\left(2en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc) + \frac{(d\log(ex+d)^2 - 2ex + 2d\log(ex+d))n^2}{e}\right)ab^2f - \left(3en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc)^2 - en\left(\frac{(d\log(ex+d)^3 + 3d\log(ex+d)^2 - 6ex + 6d\log(\right.\right.$$
$$-\frac{3}{4}\left(2en\left(\frac{2d^2\log(ex+d)}{e^3} + \frac{ex^2-2dx}{e^2}\right)\log((ex+d)^nc) - \frac{(e^2x^2 + 2d^2\log(ex+d)^2 - 6dex + 6d^2\log}{e^2}\right.$$
$$-\frac{1}{8}\left(6en\left(\frac{2d^2\log(ex+d)}{e^3} + \frac{ex^2-2dx}{e^2}\right)\log((ex+d)^nc)^2 + en\left(\frac{(4d^2\log(ex+d)^3 + 3e^2x^2 + 18d^2\log}{}\right.\right.$$
$$+ a^3fx$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*g*x^2*log((e*x + d)^n*c)^3 + 3/2*a*b^2*g*x^2*log((e*x + d)^n*c)^2 +
 b^3*f*x*log((e*x + d)^n*c)^3 - 3*a^2*b*e*f*n*(x/e - d*log(e*x + d)/e^2) -
3/4*a^2*b*e*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 3/2*a^2*b*
g*x^2*log((e*x + d)^n*c) + 3*a*b^2*f*x*log((e*x + d)^n*c)^2 + 1/2*a^3*g*x^2
 + 3*a^2*b*f*x*log((e*x + d)^n*c) - 3*(2*e*n*(x/e - d*log(e*x + d)/e^2)*log
((e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*a*b^
2*f - (3*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^2 - e*n*((d*log(
e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*n^2/e^2 - 3*(d*
log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n*log((e*x + d)^n*c)/e^2))*b^3*f
 - 3/4*(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)^
n*c) - (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n^2/
e^2)*a*b^2*g - 1/8*(6*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*lo
g((e*x + d)^n*c)^2 + e*n*((4*d^2*log(e*x + d)^3 + 3*e^2*x^2 + 18*d^2*log(e*
x + d)^2 - 42*d*e*x + 42*d^2*log(e*x + d))*n^2/e^3 - 6*(e^2*x^2 + 2*d^2*log
(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n*log((e*x + d)^n*c)/e^3))*b^3*
g + a^3*f*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1321 vs. 2(257) = 514.

Time = 0.33 (sec) , antiderivative size = 1321, normalized size of antiderivative = 4.98

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

```
[Out] (e*x + d)*b^3*f*n^3*log(e*x + d)^3/e + 1/2*(e*x + d)^2*b^3*g*n^3*log(e*x +
d)^3/e^2 - (e*x + d)*b^3*d*g*n^3*log(e*x + d)^3/e^2 - 3*(e*x + d)*b^3*f*n^3
*log(e*x + d)^2/e - 3/4*(e*x + d)^2*b^3*g*n^3*log(e*x + d)^2/e^2 + 3*(e*x +
 d)*b^3*d*g*n^3*log(e*x + d)^2/e^2 + 3*(e*x + d)*b^3*f*n^2*log(e*x + d)^2*l
og(c)/e + 3/2*(e*x + d)^2*b^3*g*n^2*log(e*x + d)^2*log(c)/e^2 - 3*(e*x + d)
*b^3*d*g*n^2*log(e*x + d)^2*log(c)/e^2 + 6*(e*x + d)*b^3*f*n^3*log(e*x + d)
/e + 3/4*(e*x + d)^2*b^3*g*n^3*log(e*x + d)/e^2 - 6*(e*x + d)*b^3*d*g*n^3*l
og(e*x + d)/e^2 + 3*(e*x + d)*a*b^2*f*n^2*log(e*x + d)^2/e + 3/2*(e*x + d)^
2*a*b^2*g*n^2*log(e*x + d)^2/e^2 - 3*(e*x + d)*a*b^2*d*g*n^2*log(e*x + d)^2
/e^2 - 6*(e*x + d)*b^3*f*n^2*log(e*x + d)*log(c)/e - 3/2*(e*x + d)^2*b^3*g*
n^2*log(e*x + d)*log(c)/e^2 + 6*(e*x + d)*b^3*d*g*n^2*log(e*x + d)*log(c)/e
^2 + 3*(e*x + d)*b^3*f*n*log(e*x + d)*log(c)^2/e + 3/2*(e*x + d)^2*b^3*g*n*
log(e*x + d)*log(c)^2/e^2 - 3*(e*x + d)*b^3*d*g*n*log(e*x + d)*log(c)^2/e^2
 - 6*(e*x + d)*b^3*f*n^3/e - 3/8*(e*x + d)^2*b^3*g*n^3/e^2 + 6*(e*x + d)*b^
3*d*g*n^3/e^2 - 6*(e*x + d)*a*b^2*f*n^2*log(e*x + d)/e - 3/2*(e*x + d)^2*a*
b^2*g*n^2*log(e*x + d)/e^2 + 6*(e*x + d)*a*b^2*d*g*n^2*log(e*x + d)/e^2 + 6
*(e*x + d)*b^3*f*n^2*log(c)/e + 3/4*(e*x + d)^2*b^3*g*n^2*log(c)/e^2 - 6*(e
*x + d)*b^3*d*g*n^2*log(c)/e^2 + 6*(e*x + d)*a*b^2*f*n*log(e*x + d)*log(c)/
e + 3*(e*x + d)^2*a*b^2*g*n*log(e*x + d)*log(c)/e^2 - 6*(e*x + d)*a*b^2*d*g
*n*log(e*x + d)*log(c)/e^2 - 3*(e*x + d)*b^3*f*n*log(c)^2/e - 3/4*(e*x + d)
^2*b^3*g*n*log(c)^2/e^2 + 3*(e*x + d)*b^3*d*g*n*log(c)^2/e^2 + (e*x + d)*b^
3*f*log(c)^3/e + 1/2*(e*x + d)^2*b^3*g*log(c)^3/e^2 - (e*x + d)*b^3*d*g*log
(c)^3/e^2 + 6*(e*x + d)*a*b^2*f*n^2/e + 3/4*(e*x + d)^2*a*b^2*g*n^2/e^2 - 6
*(e*x + d)*a*b^2*d*g*n^2/e^2 + 3*(e*x + d)*a^2*b*f*n*log(e*x + d)/e + 3/2*(
e*x + d)^2*a^2*b*g*n*log(e*x + d)/e^2 - 3*(e*x + d)*a^2*b*d*g*n*log(e*x + d
)/e^2 - 6*(e*x + d)*a*b^2*f*n*log(c)/e - 3/2*(e*x + d)^2*a*b^2*g*n*log(c)/e
^2 + 6*(e*x + d)*a*b^2*d*g*n*log(c)/e^2 + 3*(e*x + d)*a*b^2*f*log(c)^2/e +
3/2*(e*x + d)^2*a*b^2*g*log(c)^2/e^2 - 3*(e*x + d)*a*b^2*d*g*log(c)^2/e^2 -
 3*(e*x + d)*a^2*b*f*n/e - 3/4*(e*x + d)^2*a^2*b*g*n/e^2 + 3*(e*x + d)*a^2*
b*d*g*n/e^2 + 3*(e*x + d)*a^2*b*f*log(c)/e + 3/2*(e*x + d)^2*a^2*b*g*log(c)
/e^2 - 3*(e*x + d)*a^2*b*d*g*log(c)/e^2 + (e*x + d)*a^3*f/e + 1/2*(e*x + d)
^2*a^3*g/e^2 - (e*x + d)*a^3*d*g/e^2
```

## Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.93

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx$$
$$=\ln\left(c\,(d+e\,x)^n\right)^3\left(\frac{b^3\,g\,x^2}{2}-\frac{d\,(b^3\,d\,g-2\,b^3\,e\,f)}{2\,e^2}+b^3\,f\,x\right)$$
$$+\ln\left(c\,(d+e\,x)^n\right)\left(\frac{x\left(\frac{12\,a^2\,b\,d\,g+12\,a^2\,b\,e\,f-12\,b^3\,d\,g\,n^2+24\,b^3\,e\,f\,n^2-24\,a\,b^2\,e\,f\,n}{2\,e}-\frac{3\,b\,d\,g\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{e}\right)}{2}+\frac{3\,b\,g\,x^2\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{4}\right)$$
$$+\ln\left(c\,(d+e\,x)^n\right)^2\left(\frac{x\left(\frac{6\,b^2\,(a\,d\,g+a\,e\,f-b\,e\,f\,n)}{e}-\frac{3\,b^2\,d\,g\,(2\,a-b\,n)}{e}\right)}{2}-\frac{3\,d\,(2\,a\,b^2\,d\,g-4\,a\,b^2\,e\,f-3\,b^3\,d\,g\,n+4\,b^3\,e\,f\,n)}{4\,e^2}+\frac{3\,b^2\,g\,x^2\,(2\,a-b\,n)}{4}\right)$$
$$+x\left(\frac{4\,a^3\,d\,g+4\,a^3\,e\,f+18\,b^3\,d\,g\,n^3-24\,b^3\,e\,f\,n^3-12\,a\,b^2\,d\,g\,n^2+24\,a\,b^2\,e\,f\,n^2-12\,a^2\,b\,e\,f\,n}{4\,e}-\frac{d\,g\,(4\,a^3-6\,a^2\,b\,n+6\,a\,b^2\,n^2-3\,b^3\,n^3)}{4\,e}\right)+\frac{g\,x^2\,(4\,a^3-6\,a^2\,b\,n+6\,a\,b^2\,n^2-3\,b^3\,n^3)}{8}$$
$$-\frac{\ln(d+e\,x)\,\left(6\,g\,a^2\,b\,d^2\,n-12\,e\,f\,a^2\,b\,d\,n-18\,g\,a\,b^2\,d^2\,n^2+24\,e\,f\,a\,b^2\,d\,n^2+21\,g\,b^3\,d^2\,n^3-24\,e\,f\,b^3\,d\,n^3\right)}{4\,e^2}$$

```
[In] int((f + g*x)*(a + b*log(c*(d + e*x)^n))^3,x)
```

```
[Out] log(c*(d + e*x)^n)^3*((b^3*g*x^2)/2 - (d*(b^3*d*g - 2*b^3*e*f))/(2*e^2) + b
^3*f*x) + log(c*(d + e*x)^n)*((x*((12*a^2*b*d*g + 12*a^2*b*e*f - 12*b^3*d*g
*n^2 + 24*b^3*e*f*n^2 - 24*a*b^2*e*f*n)/(2*e) - (3*b*d*g*(2*a^2 + b^2*n^2 -
 2*a*b*n))/e))/2 + (3*b*g*x^2*(2*a^2 + b^2*n^2 - 2*a*b*n))/4) + log(c*(d +
e*x)^n)^2*((x*((6*b^2*(a*d*g + a*e*f - b*e*f*n))/e - (3*b^2*d*g*(2*a - b*n)
)/e))/2 - (3*d*(2*a*b^2*d*g - 4*a*b^2*e*f - 3*b^3*d*g*n + 4*b^3*e*f*n))/(4*
e^2) + (3*b^2*g*x^2*(2*a - b*n))/4) + x*((4*a^3*d*g + 4*a^3*e*f + 18*b^3*d*
g*n^3 - 24*b^3*e*f*n^3 - 12*a*b^2*d*g*n^2 + 24*a*b^2*e*f*n^2 - 12*a^2*b*e*f
*n)/(4*e) - (d*g*(4*a^3 - 3*b^3*n^3 + 6*a*b^2*n^2 - 6*a^2*b*n))/(4*e)) + (g
*x^2*(4*a^3 - 3*b^3*n^3 + 6*a*b^2*n^2 - 6*a^2*b*n))/8 - (log(d + e*x)*(21*b
^3*d^2*g*n^3 + 6*a^2*b*d^2*g*n - 24*b^3*d*e*f*n^3 - 18*a*b^2*d^2*g*n^2 - 12
*a^2*b*d*e*f*n + 24*a*b^2*d*e*f*n^2))/(4*e^2)
```

# 3.55 $\int (a+b\log (c(d+ex)^n))^3 \, dx$



## Optimal result

Integrand size = 16, antiderivative size = 99

$$\int (a+b\log (c(d+ex)^n))^3 \, dx = 6ab^2n^2x-6b^3n^3x+\frac{6b^3n^2(d+ex)\log (c(d+ex)^n)}{e} - \frac{3bn(d+ex)\left(a+b\log (c(d+ex)^n)\right)^2}{e} + \frac{(d+ex)\left(a+b\log (c(d+ex)^n)\right)^3}{e}$$

```
[Out] 6*a*b^2*n^2*x-6*b^3*n^3*x+6*b^3*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e-3*b*n*(e*x+d)
*(a+b*ln(c*(e*x+d)^n))^2/e+(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2436, 2333, 2332}

$$\int (a+b\log (c(d+ex)^n))^3 \, dx = 6ab^2n^2x-\frac{3bn(d+ex)\left(a+b\log (c(d+ex)^n)\right)^2}{e} + \frac{(d+ex)\left(a+b\log (c(d+ex)^n)\right)^3}{e} + \frac{6b^3n^2(d+ex)\log (c(d+ex)^n)}{e}-6b^3n^3x$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] 6*a*b^2*n^2*x - 6*b^3*n^3*x + (6*b^3*n^2*(d + e*x)*Log[c*(d + e*x)^n])/e -
(3*b*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e + ((d + e*x)*(a + b*Log[c*
(d + e*x)^n])^3)/e
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+b\log(cx^n))^3\,dx, x, d+ex\right)}{e}\\
&= \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^3}{e} - \frac{(3bn)\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx, x, d+ex\right)}{e}\\
&= -\frac{3bn(d+ex)\,(a+b\log(c(d+ex)^n))^2}{e} + \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^3}{e}\\
&\quad + \frac{(6b^2n^2)\,\text{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+ex\right)}{e}\\
&= 6ab^2n^2x - \frac{3bn(d+ex)\,(a+b\log(c(d+ex)^n))^2}{e}\\
&\quad + \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^3}{e} + \frac{(6b^3n^2)\,\text{Subst}\left(\int \log(cx^n)\,dx, x, d+ex\right)}{e}\\
&= 6ab^2n^2x - 6b^3n^3x + \frac{6b^3n^2(d+ex)\log(c(d+ex)^n)}{e}\\
&\quad - \frac{3bn(d+ex)\,(a+b\log(c(d+ex)^n))^2}{e} + \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^3}{e}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int (a+b\log (c(d+ex)^n))^3 \, dx$$
$$= \frac{(d+ex)(a+b\log (c(d+ex)^n))^3 - 3bn\big((d+ex)(a+b\log (c(d+ex)^n))^2 - 2bn(e(a-bn)x+b(d+ex)\mathrm{l}}{e}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d
+ e*x)^n])^2 - 2*b*n*(e*(a - b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n])))/e
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(99) = 198.

Time = 0.26 (sec) , antiderivative size = 322, normalized size of antiderivative = 3.25

| method | result |
|---|---|
| parallelrisch | $\frac{x\ln(c(ex+d)^n)^3b^3en-3x\ln(c(ex+d)^n)^2b^3e\,n^2+6x\ln(c(ex+d)^n)b^3e\,n^3-6x\,b^3e\,n^4+3x\ln(c(ex+d)^n)^2a\,b^2en-6x\ln(c(ex+d)^n)}{}$ |
| risch | Expression too large to display |

```
[In] int((a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (x*ln(c*(e*x+d)^n)^3*b^3*e*n-3*x*ln(c*(e*x+d)^n)^2*b^3*e*n^2+6*x*ln(c*(e*x+
d)^n)*b^3*e*n^3-6*x*b^3*e*n^4+3*x*ln(c*(e*x+d)^n)^2*a*b^2*e*n-6*x*ln(c*(e*x
+d)^n)*a*b^2*e*n^2+6*x*a*b^2*e*n^3+ln(c*(e*x+d)^n)^3*b^3*d*n-3*ln(c*(e*x+d)
^n)^2*b^3*d*n^2+6*ln(c*(e*x+d)^n)*b^3*d*n^3+6*b^3*d*n^4+3*x*ln(c*(e*x+d)^n)
*a^2*b*e*n-3*x*a^2*b*e*n^2+3*ln(c*(e*x+d)^n)^2*a*b^2*d*n-6*ln(c*(e*x+d)^n)*
a*b^2*d*n^2-6*a*b^2*d*n^3+x*a^3*e*n+3*ln(c*(e*x+d)^n)*a^2*b*d*n+3*a^2*b*d*n
^2-a^3*d*n)/e/n
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(99) = 198.

Time = 0.31 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.27

$$\int (a+b\log (c(d+ex)^n))^3 \, dx$$
$$= \frac{b^3ex\log (c)^3 + (b^3en^3x+b^3dn^3)\log (ex+d)^3 - 3(b^3en-ab^2e)x\log (c)^2 - 3(b^3dn^3-ab^2dn^2+(b^3en^3-}{}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

```
[Out] (b^3*e*x*log(c)^3 + (b^3*e*n^3*x + b^3*d*n^3)*log(e*x + d)^3 - 3*(b^3*e*n -
 a*b^2*e)*x*log(c)^2 - 3*(b^3*d*n^3 - a*b^2*d*n^2 + (b^3*e*n^3 - a*b^2*e*n^
2)*x - (b^3*e*n^2*x + b^3*d*n^2)*log(c))*log(e*x + d)^2 + 3*(2*b^3*e*n^2 -
2*a*b^2*e*n + a^2*b*e)*x*log(c) - (6*b^3*e*n^3 - 6*a*b^2*e*n^2 + 3*a^2*b*e*
n - a^3*e)*x + 3*(2*b^3*d*n^3 - 2*a*b^2*d*n^2 + a^2*b*d*n + (b^3*e*n*x + b^
3*d*n)*log(c)^2 + (2*b^3*e*n^3 - 2*a*b^2*e*n^2 + a^2*b*e*n)*x - 2*(b^3*d*n^
2 - a*b^2*d*n + (b^3*e*n^2 - a*b^2*e*n)*x)*log(c))*log(e*x + d))/e
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(95) = 190$.

Time = 0.50 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.97

$$\int (a + b\log(c(d+ex)^n))^3\,dx$$
$$= \begin{cases} a^3x + \frac{3a^2bd\log(c(d+ex)^n)}{e} - 3a^2bnx + 3a^2bx\log(c(d+ex)^n) - \frac{6ab^2dn\log(c(d+ex)^n)}{e} + \frac{3ab^2d\log(c(d+ex)^n)^2}{e} + 6ab^2n \\ x(a+b\log(cd^n))^3 \end{cases}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*d*log(c*(d + e*x)**n)/e - 3*a**2*b*n*x + 3*a**
2*b*x*log(c*(d + e*x)**n) - 6*a*b**2*d*n*log(c*(d + e*x)**n)/e + 3*a*b**2*d
*log(c*(d + e*x)**n)**2/e + 6*a*b**2*n**2*x - 6*a*b**2*n*x*log(c*(d + e*x)*
*n) + 3*a*b**2*x*log(c*(d + e*x)**n)**2 + 6*b**3*d*n**2*log(c*(d + e*x)**n)
/e - 3*b**3*d*n*log(c*(d + e*x)**n)**2/e + b**3*d*log(c*(d + e*x)**n)**3/e
- 6*b**3*n**3*x + 6*b**3*n**2*x*log(c*(d + e*x)**n) - 3*b**3*n*x*log(c*(d +
 e*x)**n)**2 + b**3*x*log(c*(d + e*x)**n)**3, Ne(e, 0)), (x*(a + b*log(c*d*
*n))**3, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(99) = 198$.

Time = 0.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.85

$$\int (a + b\log(c(d+ex)^n))^3\,dx = b^3x\log((ex+d)^nc)^3 - 3\,a^2ben\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)$$
$$+ 3\,ab^2x\log((ex+d)^nc)^2 + 3\,a^2bx\log((ex+d)^nc)$$
$$-3\left(2\,en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc) + \frac{(d\log(ex+d)^2 - 2\,ex + 2\,d\log(ex+d))n^2}{e}\right)ab^2$$
$$-\left(3\,en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc)^2 - en\left(\frac{(d\log(ex+d)^3 + 3\,d\log(ex+d)^2 - 6\,ex + 6\,d\log(\right.\right.$$
$$+ a^3x$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log((e*x + d)^n*c)^3 - 3*a^2*b*e*n*(x/e - d*log(e*x + d)/e^2) + 3*a*b
^2*x*log((e*x + d)^n*c)^2 + 3*a^2*b*x*log((e*x + d)^n*c) - 3*(2*e*n*(x/e -
d*log(e*x + d)/e^2)*log((e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*lo
g(e*x + d))*n^2/e)*a*b^2 - (3*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^
n*c)^2 - e*n*((d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x
+ d))*n^2/e^2 - 3*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n*log((e*x
+ d)^n*c)/e^2))*b^3 + a^3*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(99) = 198.

Time = 0.33 (sec) , antiderivative size = 399, normalized size of antiderivative = 4.03

$$
\begin{aligned}
\int (a+b\log(c(d+ex)^n))^3\,dx = {} & \frac{(ex+d)b^3n^3\log(ex+d)^3}{e} - \frac{3(ex+d)b^3n^3\log(ex+d)^2}{e} \\
& + \frac{3(ex+d)b^3n^2\log(ex+d)^2\log(c)}{e} \\
& + \frac{6(ex+d)b^3n^3\log(ex+d)}{e} \\
& + \frac{3(ex+d)ab^2n^2\log(ex+d)^2}{e} \\
& - \frac{6(ex+d)b^3n^2\log(ex+d)\log(c)}{e} \\
& + \frac{3(ex+d)b^3n\log(ex+d)\log(c)^2}{e} - \frac{6(ex+d)b^3n^3}{e} \\
& - \frac{6(ex+d)ab^2n^2\log(ex+d)}{e} + \frac{6(ex+d)b^3n^2\log(c)}{e} \\
& + \frac{6(ex+d)ab^2n\log(ex+d)\log(c)}{e} \\
& - \frac{3(ex+d)b^3n\log(c)^2}{e} + \frac{(ex+d)b^3\log(c)^3}{e} \\
& + \frac{6(ex+d)ab^2n^2}{e} + \frac{3(ex+d)a^2bn\log(ex+d)}{e} \\
& - \frac{6(ex+d)ab^2n\log(c)}{e} + \frac{3(ex+d)ab^2\log(c)^2}{e} \\
& - \frac{3(ex+d)a^2bn}{e} + \frac{3(ex+d)a^2b\log(c)}{e} + \frac{(ex+d)a^3}{e}
\end{aligned}
$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

```
[Out] (e*x + d)*b^3*n^3*log(e*x + d)^3/e - 3*(e*x + d)*b^3*n^3*log(e*x + d)^2/e +
 3*(e*x + d)*b^3*n^2*log(e*x + d)^2*log(c)/e + 6*(e*x + d)*b^3*n^3*log(e*x
+ d)/e + 3*(e*x + d)*a*b^2*n^2*log(e*x + d)^2/e - 6*(e*x + d)*b^3*n^2*log(e
```

```
*x + d)*log(c)/e + 3*(e*x + d)*b^3*n*log(e*x + d)*log(c)^2/e - 6*(e*x + d)*
b^3*n^3/e - 6*(e*x + d)*a*b^2*n^2*log(e*x + d)/e + 6*(e*x + d)*b^3*n^2*log(
c)/e + 6*(e*x + d)*a*b^2*n*log(e*x + d)*log(c)/e - 3*(e*x + d)*b^3*n*log(c)
^2/e + (e*x + d)*b^3*log(c)^3/e + 6*(e*x + d)*a*b^2*n^2/e + 3*(e*x + d)*a^2
*b*n*log(e*x + d)/e - 6*(e*x + d)*a*b^2*n*log(c)/e + 3*(e*x + d)*a*b^2*log(
c)^2/e - 3*(e*x + d)*a^2*b*n/e + 3*(e*x + d)*a^2*b*log(c)/e + (e*x + d)*a^3
/e
```

## Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.74

$$\begin{aligned}\int (a+b\log (c(d+ex)^n))^3\,dx &= x\left(a^3-3a^2bn+6ab^2n^2-6b^3n^3\right)\\ &\quad+\ln \left(c\left(d+ex\right)^n\right)^3\left(b^3x+\frac{b^3d}{e}\right)\\ &\quad+\ln \left(c\left(d+ex\right)^n\right)^2\left(\frac{3\left(ab^2d-b^3dn\right)}{e}+3b^2x\left(a-bn\right)\right)\\ &\quad+\frac{\ln \left(d+ex\right)\,\left(3da^2bn-6dab^2n^2+6db^3n^3\right)}{e}\\ &\quad+3bx\,\ln \left(c\left(d+ex\right)^n\right)\,\left(a^2-2abn+2b^2n^2\right)\end{aligned}$$

```
[In] int((a + b*log(c*(d + e*x)^n))^3,x)
```

```
[Out] x*(a^3 - 6*b^3*n^3 + 6*a*b^2*n^2 - 3*a^2*b*n) + log(c*(d + e*x)^n)^3*(b^3*x
 + (b^3*d)/e) + log(c*(d + e*x)^n)^2*((3*(a*b^2*d - b^3*d*n))/e + 3*b^2*x*(
a - b*n)) + (log(d + e*x)*(6*b^3*d*n^3 + 3*a^2*b*d*n - 6*a*b^2*d*n^2))/e +
3*b*x*log(c*(d + e*x)^n)*(a^2 + 2*b^2*n^2 - 2*a*b*n)
```

# 3.56 $\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{3bn(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{6b^2n^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g} + \frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[Out] (a+b*ln(c*(e*x+d)^n))^3*ln(e*(g*x+f)/(-d*g+e*f))/g+3*b*n*(a+b*ln(c*(e*x+d)^
n))^2*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g-6*b^2*n^2*(a+b*ln(c*(e*x+d)^n))*po
lylog(3,-g*(e*x+d)/(-d*g+e*f))/g+6*b^3*n^3*polylog(4,-g*(e*x+d)/(-d*g+e*f))
/g
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used

$= \{2443, 2481, 2421, 2430, 6724\}$

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = -\frac{6b^2n^2\,\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g} + \frac{3bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g} + \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g} + \frac{6b^3n^3\,\text{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])^3*Log[(e*(f + g*x))/(e*f - d*g)])/g + (3*b*n*(a
 + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g - (6
*b^2*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))
])/g + (6*b^3*n^3*PolyLog[4, -((g*(d + e*x))/(e*f - d*g))])/g
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
```

```
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} - \frac{(3ben)\int\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} - \frac{(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{3bn(a+b\log(c(d+ex)^n))^2\,\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{(6b^2n^2)\,\text{Subst}\left(\int\frac{(a+b\log(cx^n))\text{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{3bn(a+b\log(c(d+ex)^n))^2\,\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{6b^2n^2(a+b\log(c(d+ex)^n))\,\text{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} + \frac{(6b^3n^3)\,\text{Subst}\left(\int\frac{\text{Li}_3\left(-\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{3bn(a+b\log(c(d+ex)^n))^2\,\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{6b^2n^2(a+b\log(c(d+ex)^n))\,\text{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} + \frac{6b^3n^3\text{Li}_4\left(-\frac{g(d+ex)}{ef-dg}\right)}{g}
\end{aligned}
$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 335 vs. $2(158) = 316$.

Time = 0.18 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.12

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$$
$$= \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^3\log(f+gx)+3bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\left(\log\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x),x]
```

```
[Out] ((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*Log[f + g*x] + 3*b*n*(a -
b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(Log[d + e*x]*Log[(e*(f + g*x))/
(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + 6*b^2*n^2*(a - b
*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((Log[d + e*x]^2*Log[(e*(f + g*x))/
(e*f - d*g)])/2 + Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - P
olyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]) + b^3*n^3*(Log[d + e*x]^3*Log[(e*(
f + g*x))/(e*f - d*g)] + 3*Log[d + e*x]^2*PolyLog[2, (g*(d + e*x))/(-(e*f)
+ d*g)] - 6*Log[d + e*x]*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] + 6*PolyL
og[4, (g*(d + e*x))/(-(e*f) + d*g)]))/g
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.99 (sec) , antiderivative size = 1396, normalized size of antiderivative = 8.84

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1396 |

```
[In] int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] -b^3*ln(g*(e*x+d)-d*g+e*f)/g*ln(e*x+d)^3*n^3+3*b^3*ln(g*(e*x+d)-d*g+e*f)/g*
ln((e*x+d)^n)*ln(e*x+d)^2*n^2-3*b^3*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)^2
*ln(e*x+d)*n+b^3*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)^3-2*b^3*n^3/g*ln(e*x
+d)^3*ln(1-g*(e*x+d)/(d*g-e*f))-3*b^3*n^3/g*ln(e*x+d)^2*polylog(2,g*(e*x+d)
/(d*g-e*f))+6*b^3*n^3/g*polylog(4,g*(e*x+d)/(d*g-e*f))+3*b^3*n^3*dilog((g*(
e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln(e*x+d)^2-6*b^3*n^2*dilog((g*(e*x+d)-d*g+e*
f)/(-d*g+e*f))/g*ln((e*x+d)^n)*ln(e*x+d)+3*b^3*n*dilog((g*(e*x+d)-d*g+e*f)/
(-d*g+e*f))/g*ln((e*x+d)^n)^2+3*b^3*n^3*ln(e*x+d)^3*ln((g*(e*x+d)-d*g+e*f)/
(-d*g+e*f))/g-6*b^3*n^2*ln(e*x+d)^2*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln
((e*x+d)^n)+3*b^3*n*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln((e*x+
d)^n)^2+3*b^3*n^2/g*ln((e*x+d)^n)*ln(e*x+d)^2*ln(1-g*(e*x+d)/(d*g-e*f))+6*b
^3*n^2/g*ln((e*x+d)^n)*ln(e*x+d)*polylog(2,g*(e*x+d)/(d*g-e*f))-6*b^3*n^2/g
```

```
*ln((e*x+d)^n)*polylog(3,g*(e*x+d)/(d*g-e*f))+1/8*(-I*b*Pi*csgn(I*c*(e*x+d)
^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi
*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b
*ln(c)+2*a)^3*ln(g*x+f)/g+3/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I
*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*c
sgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b^2*((ln
((e*x+d)^n)-n*ln(e*x+d))^2*ln(g*(e*x+d)-d*g+e*f)/g+n^2/g*ln(e*x+d)^2*ln(1-g
*(e*x+d)/(d*g-e*f))+2*n^2/g*ln(e*x+d)*polylog(2,g*(e*x+d)/(d*g-e*f))-2*n^2/
g*polylog(3,g*(e*x+d)/(d*g-e*f))+2*n*(ln((e*x+d)^n)-n*ln(e*x+d))*dilog((g*(
e*x+d)-d*g+e*f)/(-d*g+e*f))/g+2*n*(ln((e*x+d)^n)-n*ln(e*x+d))*ln(e*x+d)*ln(
(g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g)+3/4*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*
c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*
x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)
^2*b*(ln((e*x+d)^n)*ln(g*x+f)/g-1/g*n*e*(dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f
))/e+ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e))
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*b
*log((e*x + d)^n*c) + a^3)/(g*x + f), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**3/(f + g*x), x)
```

## Maxima [F]

$$\int \frac{(a + b\log (c(d + ex)^n))^3}{f + gx}\,dx = \int \frac{(b\log ((ex + d)^n c) + a)^3}{gx + f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="maxima")
```

```
[Out] a^3*log(g*x + f)/g + integrate((b^3*log((e*x + d)^n)^3 + b^3*log(c)^3 + 3*a
*b^2*log(c)^2 + 3*a^2*b*log(c) + 3*(b^3*log(c) + a*b^2)*log((e*x + d)^n)^2
+ 3*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log((e*x + d)^n))/(g*x + f), x)
```

## Giac [F]

$$\int \frac{(a + b\log (c(d + ex)^n))^3}{f + gx}\,dx = \int \frac{(b\log ((ex + d)^n c) + a)^3}{gx + f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^3/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log (c(d + ex)^n))^3}{f + gx}\,dx = \int \frac{(a + b\,\ln (c\,(d + e\,x)^n))^3}{f + g\,x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^3/(f + g*x),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^3/(f + g*x), x)
```

# 3.57 $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 190

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx = \frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{(ef-dg)(f+gx)} - \frac{3ben(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} - \frac{6b^2en^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)} + \frac{6b^3en^3\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)}$$

```
[Out] (e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/(-d*g+e*f)/(g*x+f)-3*b*e*n*(a+b*ln(c*(e*x+d
)^n))^2*ln(e*(g*x+f)/(-d*g+e*f))/g/(-d*g+e*f)-6*b^2*e*n^2*(a+b*ln(c*(e*x+d)
^n))*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g/(-d*g+e*f)+6*b^3*e*n^3*polylog(3,-g
*(e*x+d)/(-d*g+e*f))/g/(-d*g+e*f)
```

## Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used

$= \{2444, 2443, 2481, 2421, 6724\}$

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx = -\frac{6b^2en^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g(ef-dg)} - \frac{3ben\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g(ef-dg)} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{(f+gx)(ef-dg)} + \frac{6b^3en^3\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x)^2,x]
```

```
[Out] ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/((e*f - d*g)*(f + g*x)) - (3*b*e*n
*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(f + g*x))/(e*f - d*g)])/(g*(e*f - d*g
)) - (6*b^2*e*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((g*(d + e*x))/(e*
f - d*g))])/(g*(e*f - d*g)) + (6*b^3*e*n^3*PolyLog[3, -((g*(d + e*x))/(e*f
- d*g))])/(g*(e*f - d*g))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(ef-dg)(f+gx)} - \frac{(3ben)\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx}{ef-dg}$$

$$= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(ef-dg)(f+gx)} - \frac{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} + \frac{(6b^2e^2n^2)\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g(ef-dg)}$$

$$= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(ef-dg)(f+gx)} - \frac{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} + \frac{(6b^2en^2)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{g(ef-dg)}$$

$$= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(ef-dg)(f+gx)} - \frac{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} - \frac{6b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)} + \frac{(6b^3en^3)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{g(ef-dg)}$$

$$= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(ef-dg)(f+gx)} - \frac{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} - \frac{6b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)} + \frac{6b^3en^3\text{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 410 vs. $2(190) = 380$.

Time = 0.25 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.16

$$\int \frac{(a + b\log(c(d + ex)^n))^3}{(f + gx)^2} dx$$
$$= \frac{-3b(ef - dg)n\log(d + ex)(a - bn\log(d + ex) + b\log(c(d + ex)^n))^2 + 3ben(f + gx)\log(d + ex)(a - bn\,\mathrm{lo}}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x)^2,x]
```

```
[Out] (-3*b*(e*f - d*g)*n*Log[d + e*x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^
n])^2 + 3*b*e*n*(f + g*x)*Log[d + e*x]*(a - b*n*Log[d + e*x] + b*Log[c*(d +
 e*x)^n])^2 - (e*f - d*g)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3 -
 3*b*e*n*(f + g*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f +
g*x] + 3*b^2*n^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x
]*(g*(d + e*x)*Log[d + e*x] - 2*e*(f + g*x)*Log[(e*(f + g*x))/(e*f - d*g)])
 - 2*e*(f + g*x)*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + b^3*n^3*(Log[d
 + e*x]^2*(g*(d + e*x)*Log[d + e*x] - 3*e*(f + g*x)*Log[(e*(f + g*x))/(e*f
- d*g)]) - 6*e*(f + g*x)*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*
g)] + 6*e*(f + g*x)*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]))/(g*(e*f - d*
g)*(f + g*x))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.02 (sec) , antiderivative size = 1268, normalized size of antiderivative = 6.67

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1268 |

```
[In] int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3*ln((e*x+d)^n)^3/(g*x+f)/g+3*b^3/g*n^3*e/(d*g-e*f)*ln(g*(e*x+d)-d*g+e*f
)*ln(e*x+d)^2-6*b^3/g*n^2*e/(d*g-e*f)*ln(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)*l
n(e*x+d)+3*b^3/g*n*e/(d*g-e*f)*ln(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)^2+3*b^3/
g*n^2*e/(d*g-e*f)*ln(e*x+d)^2*ln((e*x+d)^n)-3*b^3/g*n*e/(d*g-e*f)*ln(e*x+d)
*ln((e*x+d)^n)^2+3*b^3/g*n^3*e/(d*g-e*f)*ln(e*x+d)^2*ln(1+g*(e*x+d)/(-d*g+e
*f))+6*b^3/g*n^3*e/(d*g-e*f)*ln(e*x+d)*polylog(2,-g*(e*x+d)/(-d*g+e*f))-6*b
^3/g*n^3*e/(d*g-e*f)*polylog(3,-g*(e*x+d)/(-d*g+e*f))+b^3/g*n^3*e/(-d*g+e*f
)*ln(e*x+d)^3-6*b^3/g*n^3*e/(d*g-e*f)*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))
*ln(e*x+d)+6*b^3/g*n^2*e/(d*g-e*f)*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln
```

```
((e*x+d)^n)-6*b^3/g*n^3*e/(d*g-e*f)*ln(e*x+d)^2*ln((g*(e*x+d)-d*g+e*f)/(-d*
g+e*f))+6*b^3/g*n^2*e/(d*g-e*f)*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f)
)*ln((e*x+d)^n)-1/8*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n
)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e
*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^3/(g*x+f)/g+3/2*(-
I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(
I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn
(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b^2*(-ln((e*x+d)^n)^2/(g*x+f)/g+2/g*n*e*
(-ln((e*x+d)^n)/(d*g-e*f)*ln(e*x+d)+ln((e*x+d)^n)/(d*g-e*f)*ln(g*x+f)-e*n*(
1/(d*g-e*f)*(dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e+ln(g*x+f)*ln(((g*x+f)*e
+d*g-e*f)/(d*g-e*f))/e)-1/2/(d*g-e*f)/e*ln(e*x+d)^2)))+3/4*(-I*b*Pi*csgn(I*
c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)
^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n
)^3*b+2*b*ln(c)+2*a)^2*b*(-ln((e*x+d)^n)/(g*x+f)/g+1/g*n*e*(-1/(d*g-e*f)*ln
(e*x+d)+1/(d*g-e*f)*ln(g*x+f)))
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*b
*log((e*x + d)^n*c) + a^3)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f)**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**3/(f + g*x)**2, x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] 3*a^2*b*e*n*(log(e*x + d)/(e*f*g - d*g^2) - log(g*x + f)/(e*f*g - d*g^2)) -
 b^3*log((e*x + d)^n)^3/(g^2*x + f*g) - 3*a^2*b*log((e*x + d)^n*c)/(g^2*x +
 f*g) - a^3/(g^2*x + f*g) + integrate((b^3*d*g*log(c)^3 + 3*a*b^2*d*g*log(c
)^2 + 3*(a*b^2*d*g + (e*f*n + d*g*log(c))*b^3 + (a*b^2*e*g + (e*g*n + e*g*l
og(c))*b^3)*x)*log((e*x + d)^n)^2 + (b^3*e*g*log(c)^3 + 3*a*b^2*e*g*log(c)^
2)*x + 3*(b^3*d*g*log(c)^2 + 2*a*b^2*d*g*log(c) + (b^3*e*g*log(c)^2 + 2*a*b
^2*e*g*log(c))*x)*log((e*x + d)^n))/(e*g^3*x^3 + d*f^2*g + (2*e*f*g^2 + d*g
^3)*x^2 + (e*f^2*g + 2*d*f*g^2)*x), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^3/(g*x + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^3}{(f+g\,x)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^3/(f + g*x)^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^3/(f + g*x)^2, x)
```

# 3.58 $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 342

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx = -\frac{3ben(d+ex)(a+b\log(c(d+ex)^n))^2}{2(ef-dg)^2(f+gx)} - \frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2} + \frac{3b^2e^2n^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)^2} - \frac{3be^2n(a+b\log(c(d+ex)^n))^2\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{2g(ef-dg)^2} + \frac{3b^2e^2n^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2} + \frac{3b^3e^2n^3\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)^2} + \frac{3b^3e^2n^3\operatorname{PolyLog}\left(3,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2}$$

```
[Out] -3/2*b*e*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/(-d*g+e*f)^2/(g*x+f)-1/2*(a+b*ln
(c*(e*x+d)^n))^3/g/(g*x+f)^2+3*b^2*e^2*n^2*(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+
f)/(-d*g+e*f))/g/(-d*g+e*f)^2-3/2*b*e^2*n*(a+b*ln(c*(e*x+d)^n))^2*ln(1+(-d*
g+e*f)/g/(e*x+d))/g/(-d*g+e*f)^2+3*b^2*e^2*n^2*(a+b*ln(c*(e*x+d)^n))*polylo
g(2,(d*g-e*f)/g/(e*x+d))/g/(-d*g+e*f)^2+3*b^3*e^2*n^3*polylog(2,-g*(e*x+d)/
(-d*g+e*f))/g/(-d*g+e*f)^2+3*b^3*e^2*n^3*polylog(3,(d*g-e*f)/g/(e*x+d))/g/(
-d*g+e*f)^2
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438}

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^3}\,dx = \frac{3b^2e^2n^2\,\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)(a+b\log(c(d+ex)^n))}{g(ef-dg)^2} + \frac{3b^2e^2n^2\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g(ef-dg)^2} - \frac{3be^2n\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))^2}{2g(ef-dg)^2} - \frac{3ben(d+ex)(a+b\log(c(d+ex)^n))^2}{2(f+gx)(ef-dg)^2} - \frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2} + \frac{3b^3e^2n^3\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)^2} + \frac{3b^3e^2n^3\,\text{PolyLog}\left(3,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x)^3,x]
```

```
[Out] (-3*b*e*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(2*(e*f - d*g)^2*(f + g*x
)) - (a + b*Log[c*(d + e*x)^n])^3/(2*g*(f + g*x)^2) + (3*b^2*e^2*n^2*(a + b
*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/(g*(e*f - d*g)^2) - (3
*b*e^2*n*(a + b*Log[c*(d + e*x)^n])^2*Log[1 + (e*f - d*g)/(g*(d + e*x))])/(
2*g*(e*f - d*g)^2) + (3*b^2*e^2*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -
((e*f - d*g)/(g*(d + e*x)))])/(g*(e*f - d*g)^2) + (3*b^3*e^2*n^3*PolyLog[2,
 -((g*(d + e*x))/(e*f - d*g))])/(g*(e*f - d*g)^2) + (3*b^3*e^2*n^3*PolyLog[
3, -((e*f - d*g)/(g*(d + e*x)))])/(g*(e*f - d*g)^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
```

```
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2} + \frac{(3ben)\int \frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(f+gx)^2}\,dx}{2g}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2} + \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^2}\,dx, x, d+ex\right)}{2g}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2} - \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^2}\,dx, x, d+ex\right)}{2(ef-dg)} + \frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}\,dx, x, d+ex\right)}{2g(ef-dg)}$$

$$= -\frac{3ben(d+ex)\left(a+b\log(c(d+ex)^n)\right)^2}{2(ef-dg)^2(f+gx)} - \frac{(a+b\log(c(d+ex)^n))^3}{2g(f+gx)^2} - \frac{3be^2n(a+b\log(c(d+ex)^n))^2\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{2g(ef-dg)^2} + \frac{(3b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\frac{ef-dg}{e}+\frac{gx}{e}}\,dx, x, d+ex\right)}{(ef-dg)^2} + \frac{(3b^2e^2n^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{ef-dg}{gx}\right)(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{g(ef-dg)^2}$$

$$= -\frac{3ben(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2(ef-dg)^2(f+gx)} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2g(f+gx)^2} + \frac{3b^2e^2n^2(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)^2} - \frac{3be^2n(a+b\log\left(c(d+ex)^n\right))^2\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{2g(ef-dg)^2} + \frac{3b^2e^2n^2(a+b\log\left(c(d+ex)^n\right))\,\mathrm{Li}_2\left(-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2} - \frac{(3b^3e^2n^3)\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}dx, x, d+ex\right)}{g(ef-dg)^2} - \frac{(3b^3e^2n^3)\,\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2\left(-\frac{ef-dg}{gx}\right)}{x}dx, x, d+ex\right)}{g(ef-dg)^2}$$

$$= -\frac{3ben(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2(ef-dg)^2(f+gx)} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2g(f+gx)^2} + \frac{3b^2e^2n^2(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)^2} - \frac{3be^2n(a+b\log\left(c(d+ex)^n\right))^2\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{2g(ef-dg)^2} + \frac{3b^2e^2n^2(a+b\log\left(c(d+ex)^n\right))\,\mathrm{Li}_2\left(-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2} + \frac{3b^3e^2n^3\mathrm{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)^2} + \frac{3b^3e^2n^3\mathrm{Li}_3\left(-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^2}$$

## Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.81

$$\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(f+gx)^3}dx =$$

$$-\frac{-3be(ef-dg)n(f+gx)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2+3b(ef-dg)^2n\log(d+ex)\left(a-b\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x)^3,x]
```

```
[Out] -1/2*(-3*b*e*(e*f - d*g)*n*(f + g*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e
*x)^n])^2 + 3*b*(e*f - d*g)^2*n*Log[d + e*x]*(a - b*n*Log[d + e*x] + b*Log[
```

```
c*(d + e*x)^n])^2 - 3*b*e^2*n*(f + g*x)^2*Log[d + e*x]*(a - b*n*Log[d + e*x
] + b*Log[c*(d + e*x)^n])^2 + (e*f - d*g)^2*(a - b*n*Log[d + e*x] + b*Log[c
*(d + e*x)^n])^3 + 3*b*e^2*n*(f + g*x)^2*(a - b*n*Log[d + e*x] + b*Log[c*(d
 + e*x)^n])^2*Log[f + g*x] + 3*b^2*n^2*(a - b*n*Log[d + e*x] + b*Log[c*(d +
 e*x)^n])*(g*(d + e*x)*(d*g - e*(2*f + g*x))*Log[d + e*x]^2 - 2*e^2*(f + g*
x)^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*e*(f + g*x)*Log[d + e*x]*(g*(d + e*
x) + e*(f + g*x)*Log[(e*(f + g*x))/(e*f - d*g)]) + 2*e^2*(f + g*x)^2*PolyLo
g[2, (g*(d + e*x))/(-(e*f) + d*g)]) + b^3*n^3*(g*(d + e*x)*(d*g - e*(2*f +
g*x))*Log[d + e*x]^3 + 3*e*(f + g*x)*Log[d + e*x]^2*(g*(d + e*x) + e*(f + g
*x)*Log[(e*(f + g*x))/(e*f - d*g)]) - 6*e^2*(f + g*x)^2*Log[d + e*x]*(Log[(
e*(f + g*x))/(e*f - d*g)] - PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) - 6*e
^2*(f + g*x)^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 6*e^2*(f + g*x)^2
*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]))/(g*(e*f - d*g)^2*(f + g*x)^2)
```

## Maple [F]

$$\int \frac{(a+b\ln (c(ex+d)^n))^3}{(gx+f)^3}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)^3,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)^3,x)
```

## Fricas [F]

$$\int \frac{(a+b\log (c(d+ex)^n))^3}{(f+gx)^3}\,dx=\int \frac{(b\log ((ex+d)^nc)+a)^3}{(gx+f)^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^3,x, algorithm="fricas")
[Out] integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*b
*log((e*x + d)^n*c) + a^3)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

## Sympy [F]

$$\int \frac{(a+b\log (c(d+ex)^n))^3}{(f+gx)^3}\,dx=\int \frac{(a+b\log (c(d+ex)^n))^3}{(f+gx)^3}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f)**3,x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**3/(f + g*x)**3, x)
```

## Maxima [F]

$$\int \frac{(a+b\log (c(d+ex)^n))^3}{(f+gx)^3}\,dx=\int \frac{(b\log \left((ex+d)^nc\right)+a)^3}{(gx+f)^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] 3/2*a^2*b*e*n*(e*log(e*x + d)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) - e*log(g
*x + f)/(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3) + 1/(e*f^2*g - d*f*g^2 + (e*f*g
^2 - d*g^3)*x)) - 1/2*b^3*log((e*x + d)^n)^3/(g^3*x^2 + 2*f*g^2*x + f^2*g)
- 3/2*a^2*b*log((e*x + d)^n*c)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1/2*a^3/(g^3
*x^2 + 2*f*g^2*x + f^2*g) + integrate(1/2*(2*b^3*d*g*log(c)^3 + 6*a*b^2*d*g
*log(c)^2 + 3*(2*a*b^2*d*g + (e*f*n + 2*d*g*log(c))*b^3 + (2*a*b^2*e*g + (e
*g*n + 2*e*g*log(c))*b^3)*x)*log((e*x + d)^n)^2 + 2*(b^3*e*g*log(c)^3 + 3*a
*b^2*e*g*log(c)^2)*x + 6*(b^3*d*g*log(c)^2 + 2*a*b^2*d*g*log(c) + (b^3*e*g*
log(c)^2 + 2*a*b^2*e*g*log(c))*x)*log((e*x + d)^n))/(e*g^4*x^4 + d*f^3*g +
(3*e*f*g^3 + d*g^4)*x^3 + 3*(e*f^2*g^2 + d*f*g^3)*x^2 + (e*f^3*g + 3*d*f^2*
g^2)*x), x)
```

## Giac [F]

$$\int \frac{(a+b\log (c(d+ex)^n))^3}{(f+gx)^3}\,dx=\int \frac{(b\log \left((ex+d)^nc\right)+a)^3}{(gx+f)^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^3/(g*x + f)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log (c(d+ex)^n))^3}{(f+gx)^3}\,dx=\int \frac{\left(a+b\,\ln \left(c\,{\left(d+e\,x\right)}^n\right)\right)^3}{\left(f+g\,x\right)^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^3/(f + g*x)^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^3/(f + g*x)^3, x)
```

# 3.59 $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4} dx$

## Optimal result

Integrand size = 24, antiderivative size = 564

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx = \frac{b^2e^2n^2(d+ex)\,(a+b\log(c(d+ex)^n))}{(ef-dg)^3(f+gx)} + \frac{ben(a+b\log(c(d+ex)^n))^2}{2g(ef-dg)(f+gx)^2} - \frac{be^2n(d+ex)\,(a+b\log(c(d+ex)^n))^2}{(ef-dg)^3(f+gx)} - \frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3} - \frac{b^3e^3n^3\log(f+gx)}{g(ef-dg)^3} + \frac{2b^2e^3n^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)^3} + \frac{b^2e^3n^2(a+b\log(c(d+ex)^n))\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} - \frac{be^3n(a+b\log(c(d+ex)^n))^2\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} - \frac{b^3e^3n^3\,\mathrm{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} + \frac{2b^2e^3n^2(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} + \frac{2b^3e^3n^3\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)^3} + \frac{2b^3e^3n^3\,\mathrm{PolyLog}\left(3,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3}$$

```
[Out] b^2*e^2*n^2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))/(-d*g+e*f)^3/(g*x+f)+1/2*b*e*n*(a
+b*ln(c*(e*x+d)^n))^2/g/(-d*g+e*f)/(g*x+f)^2-b*e^2*n*(e*x+d)*(a+b*ln(c*(e*x
+d)^n))^2/(-d*g+e*f)^3/(g*x+f)-1/3*(a+b*ln(c*(e*x+d)^n))^3/g/(g*x+f)^3-b^3*
e^3*n^3*ln(g*x+f)/g/(-d*g+e*f)^3+2*b^2*e^3*n^2*(a+b*ln(c*(e*x+d)^n))*ln(e*(
g*x+f)/(-d*g+e*f))/g/(-d*g+e*f)^3+b^2*e^3*n^2*(a+b*ln(c*(e*x+d)^n))*ln(1+(-
d*g+e*f)/g/(e*x+d))/g/(-d*g+e*f)^3-b*e^3*n*(a+b*ln(c*(e*x+d)^n))^2*ln(1+(-d
*g+e*f)/g/(e*x+d))/g/(-d*g+e*f)^3-b^3*e^3*n^3*polylog(2,(d*g-e*f)/g/(e*x+d)
)/g/(-d*g+e*f)^3+2*b^2*e^3*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(2,(d*g-e*f)/g/
(e*x+d))/g/(-d*g+e*f)^3+2*b^3*e^3*n^3*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g/(-
d*g+e*f)^3+2*b^3*e^3*n^3*polylog(3,(d*g-e*f)/g/(e*x+d))/g/(-d*g+e*f)^3
```

## Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx = \frac{2b^2e^3n^2\,\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)(a+b\log(c(d+ex)^n))}{g(ef-dg)^3} + \frac{2b^2e^3n^2\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g(ef-dg)^3} + \frac{b^2e^3n^2\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))}{g(ef-dg)^3} + \frac{b^2e^2n^2(d+ex)(a+b\log(c(d+ex)^n))}{(f+gx)(ef-dg)^3} - \frac{be^3n\log\left(\frac{ef-dg}{g(d+ex)}+1\right)(a+b\log(c(d+ex)^n))^2}{g(ef-dg)^3} - \frac{be^2n(d+ex)(a+b\log(c(d+ex)^n))^2}{(f+gx)(ef-dg)^3} + \frac{ben(a+b\log(c(d+ex)^n))^2}{2g(f+gx)^2(ef-dg)} - \frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3} - \frac{b^3e^3n^3\,\text{PolyLog}\left(2,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} + \frac{2b^3e^3n^3\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)^3} + \frac{2b^3e^3n^3\,\text{PolyLog}\left(3,-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} - \frac{b^3e^3n^3\log(f+gx)}{g(ef-dg)^3}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x)^4,x]
```

```
[Out] (b^2*e^2*n^2*(d + e*x)*(a + b*Log[c*(d + e*x)^n]))/((e*f - d*g)^3*(f + g*x)
) + (b*e*n*(a + b*Log[c*(d + e*x)^n])^2)/(2*g*(e*f - d*g)*(f + g*x)^2) - (b
*e^2*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/((e*f - d*g)^3*(f + g*x)) -
(a + b*Log[c*(d + e*x)^n])^3/(3*g*(f + g*x)^3) - (b^3*e^3*n^3*Log[f + g*x])
/(g*(e*f - d*g)^3) + (2*b^2*e^3*n^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f +
g*x))/(e*f - d*g)])/(g*(e*f - d*g)^3) + (b^2*e^3*n^2*(a + b*Log[c*(d + e*x)
^n])*Log[1 + (e*f - d*g)/(g*(d + e*x))])/(g*(e*f - d*g)^3) - (b*e^3*n*(a +
b*Log[c*(d + e*x)^n])^2*Log[1 + (e*f - d*g)/(g*(d + e*x))])/(g*(e*f - d*g)^
3) - (b^3*e^3*n^3*PolyLog[2, -((e*f - d*g)/(g*(d + e*x)))])/(g*(e*f - d*g)^
3) + (2*b^2*e^3*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((e*f - d*g)/(g*
```

```
(d + e*x))))])/(g*(e*f - d*g)^3) + (2*b^3*e^3*n^3*PolyLog[2, -((g*(d + e*x))
/(e*f - d*g))])/(g*(e*f - d*g)^3) + (2*b^3*e^3*n^3*PolyLog[3, -((e*f - d*g)
/(g*(d + e*x)))])/(g*(e*f - d*g)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3}+\frac{(ben)\int\frac{(a+b\log(c(d+ex)^n))^2}{(d+ex)(f+gx)^3}\,dx}{g}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3} + \frac{(bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^3}\,dx, x, d+ex\right)}{g}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3} - \frac{(bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^3}\,dx, x, d+ex\right)}{ef-dg} + \frac{(ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^2}\,dx, x, d+ex\right)}{g(ef-dg)}$$

$$= \frac{ben(a+b\log(c(d+ex)^n))^2}{2g(ef-dg)(f+gx)^2} - \frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3} - \frac{(ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^2}\,dx, x, d+ex\right)}{(ef-dg)^2} + \frac{(be^2n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}\,dx, x, d+ex\right)}{g(ef-dg)^2} - \frac{(b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^2}\,dx, x, d+ex\right)}{g(ef-dg)}$$

$$= \frac{ben(a+b\log(c(d+ex)^n))^2}{2g(ef-dg)(f+gx)^2} - \frac{be^2n(d+ex)\left(a+b\log(c(d+ex)^n)\right)^2}{(ef-dg)^3(f+gx)} - \frac{(a+b\log(c(d+ex)^n))^3}{3g(f+gx)^3} - \frac{be^3n(a+b\log(c(d+ex)^n))^2\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} + \frac{(2b^2e^2n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\frac{ef-dg}{e}+\frac{gx}{e}}\,dx, x, d+ex\right)}{(ef-dg)^3} + \frac{(2b^2e^3n^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{ef-dg}{gx}\right)(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{g(ef-dg)^3} + \frac{(b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^2}\,dx, x, d+ex\right)}{(ef-dg)^2} - \frac{(b^2e^2n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}\,dx, x, d+ex\right)}{g(ef-dg)^2}$$

$$= \frac{b^2e^2n^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)^3(f+gx)} + \frac{ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g(ef-dg)(f+gx)^2}$$
$$- \frac{be^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(ef-dg)^3(f+gx)} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{3g(f+gx)^3}$$
$$+ \frac{2b^2e^3n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)^3}$$
$$+ \frac{b^2e^3n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3}$$
$$- \frac{be^3n\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3}$$
$$+ \frac{2b^2e^3n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3}$$
$$- \frac{(b^3e^2n^3)\operatorname{Subst}\left(\int\frac{1}{\frac{ef-dg}{e}+\frac{gx}{e}}\,dx, x, d+ex\right)}{(ef-dg)^3}$$
$$- \frac{(b^3e^3n^3)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{ef-dg}{gx}\right)}{x}\,dx, x, d+ex\right)}{g(ef-dg)^3}$$
$$- \frac{(2b^3e^3n^3)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{g(ef-dg)^3}$$
$$- \frac{(2b^3e^3n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{ef-dg}{gx}\right)}{x}\,dx, x, d+ex\right)}{g(ef-dg)^3}$$

$$= \frac{b^2e^2n^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)^3(f+gx)} + \frac{ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g(ef-dg)(f+gx)^2} - \frac{be^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(ef-dg)^3(f+gx)} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{3g(f+gx)^3} - \frac{b^3e^3n^3\log(f+gx)}{g(ef-dg)^3} + \frac{2b^2e^3n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)^3} + \frac{b^2e^3n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} - \frac{be^3n\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(1+\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} - \frac{b^3e^3n^3\operatorname{Li}_2\left(-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} + \frac{2b^2e^3n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3} + \frac{2b^3e^3n^3\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)^3} + \frac{2b^3e^3n^3\operatorname{Li}_3\left(-\frac{ef-dg}{g(d+ex)}\right)}{g(ef-dg)^3}$$

## Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.49

$$\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(f+gx)^4}\,dx$$

$$= \frac{3be(ef-dg)^2n(f+gx)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2 + 6be^2(ef-dg)n(f+gx)^2\left(a-bn\log\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x)^4,x]
```

```
[Out] (3*b*e*(e*f - d*g)^2*n*(f + g*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^
n])^2 + 6*b*e^2*(e*f - d*g)*n*(f + g*x)^2*(a - b*n*Log[d + e*x] + b*Log[c*(
d + e*x)^n])^2 - 6*b*(e*f - d*g)^3*n*Log[d + e*x]*(a - b*n*Log[d + e*x] + b
*Log[c*(d + e*x)^n])^2 + 6*b*e^3*n*(f + g*x)^3*Log[d + e*x]*(a - b*n*Log[d
+ e*x] + b*Log[c*(d + e*x)^n])^2 - 2*(e*f - d*g)^3*(a - b*n*Log[d + e*x] +
b*Log[c*(d + e*x)^n])^3 - 6*b*e^3*n*(f + g*x)^3*(a - b*n*Log[d + e*x] + b*L
og[c*(d + e*x)^n])^2*Log[f + g*x] + 6*b^2*n^2*(a - b*n*Log[d + e*x] + b*Log
[c*(d + e*x)^n])*(e^2*g*(d + e*x)*(f + g*x)^2 + g*(3*d*e^2*f^2 - 3*d^2*e*f*
g + d^3*g^2 + e^3*x*(3*f^2 + 3*f*g*x + g^2*x^2))*Log[d + e*x]^2 + 3*e^3*(f
+ g*x)^3*Log[(e*(f + g*x))/(e*f - d*g)] + e*(f + g*x)*Log[d + e*x]*(g^2*(d
+ e*x)^2 - 4*e*g*(d + e*x)*(f + g*x) - 2*e^2*(f + g*x)^2*Log[(e*(f + g*x))/
(e*f - d*g)]) - 2*e^3*(f + g*x)^3*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)])
 + b^3*n^3*(2*g*(3*d*e^2*f^2 - 3*d^2*e*f*g + d^3*g^2 + e^3*x*(3*f^2 + 3*f*g
*x + g^2*x^2))*Log[d + e*x]^3 - 6*e^3*(f + g*x)^3*Log[(e*(f + g*x))/(e*f -
```

```
d*g)] + 3*e*(f + g*x)*Log[d + e*x]^2*(g^2*(d + e*x)^2 - 4*e*g*(d + e*x)*(f
+ g*x) - 2*e^2*(f + g*x)^2*Log[(e*(f + g*x))/(e*f - d*g)]) + 18*e^3*(f + g*
x)^3*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] + 6*e^2*(f + g*x)^2*Log[d + e
*x]*(g*(d + e*x) + 3*e*(f + g*x)*Log[(e*(f + g*x))/(e*f - d*g)] - 2*e*(f +
g*x)*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + 12*e^3*(f + g*x)^3*PolyLog
[3, (g*(d + e*x))/(-(e*f) + d*g)]))/(6*g*(e*f - d*g)^3*(f + g*x)^3)
```

## Maple [F]

$$\int \frac{(a + b\ln(c(ex + d)^n))^3}{(gx + f)^4} dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)^4,x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)^4,x)
```

## Fricas [F]

$$\int \frac{(a + b\log(c(d + ex)^n))^3}{(f + gx)^4} dx = \int \frac{(b\log((ex + d)^n c) + a)^3}{(gx + f)^4} dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*b
*log((e*x + d)^n*c) + a^3)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g
*x + f^4), x)
```

## Sympy [F]

$$\int \frac{(a + b\log(c(d + ex)^n))^3}{(f + gx)^4} dx = \int \frac{(a + b\log(c(d + ex)^n))^3}{(f + gx)^4} dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f)**4,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**3/(f + g*x)**4, x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)^4}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^4,x, algorithm="maxima")

[Out] 1/2*(2*e^2*log(e*x + d)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 - d^3*g^4) - 2*e^2*log(g*x + f)/(e^3*f^3*g - 3*d*e^2*f^2*g^2 + 3*d^2*e*f*g^3 - d^3*g^4) + (2*e*g*x + 3*e*f - d*g)/(e^2*f^4*g - 2*d*e*f^3*g^2 + d^2*f^2*g^3 + (e^2*f^2*g^3 - 2*d*e*f*g^4 + d^2*g^5)*x^2 + 2*(e^2*f^3*g^2 - 2*d*e*f^2*g^3 + d^2*f*g^4)*x))*a^2*b*e*n - 1/3*b^3*log((e*x + d)^n)^3/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) - a^2*b*log((e*x + d)^n*c)/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) - 1/3*a^3/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + integrate((b^3*d*g*log(c)^3 + 3*a*b^2*d*g*log(c)^2 + (3*a*b^2*d*g + (e*f*n + 3*d*g*log(c))*b^3 + (3*a*b^2*e*g + (e*g*n + 3*e*g*log(c))*b^3)*x)*log((e*x + d)^n)^2 + (b^3*e*g*log(c)^3 + 3*a*b^2*e*g*log(c)^2)*x + 3*(b^3*d*g*log(c)^2 + 2*a*b^2*d*g*log(c) + (b^3*e*g*log(c)^2 + 2*a*b^2*e*g*log(c))*x)*log((e*x + d)^n))/(e*g^5*x^5 + d*f^4*g + (4*e*f*g^4 + d*g^5)*x^4 + 2*(3*e*f^2*g^3 + 2*d*f*g^4)*x^3 + 2*(2*e*f^3*g^2 + 3*d*f^2*g^3)*x^2 + (e*f^4*g + 4*d*f^3*g^2)*x), x)

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)^4}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)^4,x, algorithm="giac")

[Out] integrate((b*log((e*x + d)^n*c) + a)^3/(g*x + f)^4, x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)^4}\,dx = \int \frac{(a+b\ln(c\,(d+e\,x)^n))^3}{(f+g\,x)^4}\,dx$$

[In] int((a + b*log(c*(d + e*x)^n))^3/(f + g*x)^4,x)

[Out] int((a + b*log(c*(d + e*x)^n))^3/(f + g*x)^4, x)

# 3.60 $\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^4\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 340

$$\begin{aligned}
&\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^4\,dx\\
&= -\frac{24ab^3(ef-dg)n^3x}{e}+\frac{24b^4(ef-dg)n^4x}{e}+\frac{3b^4gn^4(d+ex)^2}{4e^2}\\
&\quad-\frac{24b^4(ef-dg)n^3(d+ex)\log\left(c(d+ex)^n\right)}{e^2}-\frac{3b^3gn^3(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2}\\
&\quad+\frac{12b^2(ef-dg)n^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2}\\
&\quad+\frac{3b^2gn^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}\\
&\quad-\frac{4b(ef-dg)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}\\
&\quad-\frac{bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}\\
&\quad+\frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{2e^2}
\end{aligned}$$

```
[Out] -24*a*b^3*(-d*g+e*f)*n^3*x/e+24*b^4*(-d*g+e*f)*n^4*x/e+3/4*b^4*g*n^4*(e*x+d
)^2/e^2-24*b^4*(-d*g+e*f)*n^3*(e*x+d)*ln(c*(e*x+d)^n)/e^2-3/2*b^3*g*n^3*(e*
x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2+12*b^2*(-d*g+e*f)*n^2*(e*x+d)*(a+b*ln(c*(e
*x+d)^n))^2/e^2+3/2*b^2*g*n^2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2-4*b*(-d
*g+e*f)*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e^2-b*g*n*(e*x+d)^2*(a+b*ln(c*(e*
x+d)^n))^3/e^2+(-d*g+e*f)*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^4/e^2+1/2*g*(e*x+d)
^2*(a+b*ln(c*(e*x+d)^n))^4/e^2
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\begin{aligned}
&\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^4\,dx\\
&=-\frac{3b^3gn^3(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2}-\frac{24ab^3n^3x(ef-dg)}{e}\\
&+\frac{12b^2n^2(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2}\\
&+\frac{3b^2gn^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}\\
&-\frac{4bn(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}\\
&+\frac{(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e^2}\\
&-\frac{bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{2e^2}\\
&-\frac{24b^4n^3(d+ex)(ef-dg)\log\left(c(d+ex)^n\right)}{e^2}+\frac{3b^4gn^4(d+ex)^2}{4e^2}+\frac{24b^4n^4x(ef-dg)}{e}
\end{aligned}$$

```
[In] Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^4,x]
```

```
[Out] (-24*a*b^3*(e*f - d*g)*n^3*x)/e + (24*b^4*(e*f - d*g)*n^4*x)/e + (3*b^4*g*n
^4*(d + e*x)^2)/(4*e^2) - (24*b^4*(e*f - d*g)*n^3*(d + e*x)*Log[c*(d + e*x)
^n])/e^2 - (3*b^3*g*n^3*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^2) + (
12*b^2*(e*f - d*g)*n^2*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e^2 + (3*b^2
*g*n^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^2) - (4*b*(e*f - d*g)
*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/e^2 - (b*g*n*(d + e*x)^2*(a + b*
Log[c*(d + e*x)^n])^3)/e^2 + ((e*f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^
n])^4)/e^2 + (g*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^4)/(2*e^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(ef-dg)\,(a+b\log\left(c(d+ex)^n\right))^4}{e}+\frac{g(d+ex)\,(a+b\log\left(c(d+ex)^n\right))^4}{e}\right)\,dx\\
&= \frac{g\int (d+ex)\,(a+b\log\left(c(d+ex)^n\right))^4\,dx}{e}+\frac{(ef-dg)\int (a+b\log\left(c(d+ex)^n\right))^4\,dx}{e}\\
&= \frac{g\text{Subst}\left(\int x(a+b\log\left(cx^n\right))^4\,dx,x,d+ex\right)}{e^2}\\
&\quad+\frac{(ef-dg)\text{Subst}\left(\int (a+b\log\left(cx^n\right))^4\,dx,x,d+ex\right)}{e^2}
\end{aligned}$$

$$= \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{2e^2}$$
$$-\frac{(2bgn)\mathrm{Subst}\left(\int x(a+b\log\left(cx^n\right))^3\,dx,x,d+ex\right)}{e^2}$$
$$-\frac{(4b(ef-dg)n)\mathrm{Subst}\left(\int (a+b\log\left(cx^n\right))^3\,dx,x,d+ex\right)}{e^2}$$

$$= -\frac{4b(ef-dg)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}$$
$$-\frac{bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}$$
$$+\frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e^2}$$
$$+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{2e^2}$$
$$+\frac{(3b^2gn^2)\,\mathrm{Subst}\left(\int x(a+b\log\left(cx^n\right))^2\,dx,x,d+ex\right)}{e^2}$$
$$+\frac{(12b^2(ef-dg)n^2)\,\mathrm{Subst}\left(\int (a+b\log\left(cx^n\right))^2\,dx,x,d+ex\right)}{e^2}$$

$$= \frac{12b^2(ef-dg)n^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2}$$
$$+\frac{3b^2gn^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}$$
$$-\frac{4b(ef-dg)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}$$
$$-\frac{bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}$$
$$+\frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e^2}$$
$$+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{2e^2}$$
$$-\frac{(3b^3gn^3)\,\mathrm{Subst}\left(\int x(a+b\log\left(cx^n\right))\,dx,x,d+ex\right)}{e^2}$$
$$-\frac{(24b^3(ef-dg)n^3)\,\mathrm{Subst}\left(\int (a+b\log\left(cx^n\right))\,dx,x,d+ex\right)}{e^2}$$

$$= -\frac{24ab^3(ef-dg)n^3x}{e} + \frac{3b^4gn^4(d+ex)^2}{4e^2} - \frac{3b^3gn^3(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2}$$
$$+ \frac{12b^2(ef-dg)n^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2}$$
$$+ \frac{3b^2gn^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}$$
$$- \frac{4b(ef-dg)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}$$
$$- \frac{bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}$$
$$+ \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e^2}$$
$$+ \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{2e^2}$$
$$- \frac{\left(24b^4(ef-dg)n^3\right)\text{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{e^2}$$

$$= -\frac{24ab^3(ef-dg)n^3x}{e} + \frac{24b^4(ef-dg)n^4x}{e} + \frac{3b^4gn^4(d+ex)^2}{4e^2}$$
$$- \frac{24b^4(ef-dg)n^3(d+ex)\log\left(c(d+ex)^n\right)}{e^2}$$
$$- \frac{3b^3gn^3(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2}$$
$$+ \frac{12b^2(ef-dg)n^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2}$$
$$+ \frac{3b^2gn^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2}$$
$$- \frac{4b(ef-dg)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}$$
$$- \frac{bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}$$
$$+ \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e^2}$$
$$+ \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{2e^2}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.76

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^4\,dx$$
$$=\frac{4(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4+2g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^4-16b(ef-dg)n\left((d\right.}{}$$

```
[In] Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^4,x]
```

```
[Out] (4*(e*f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^4 + 2*g*(d + e*x)^2*(a
+ b*Log[c*(d + e*x)^n])^4 - 16*b*(e*f - d*g)*n*((d + e*x)*(a + b*Log[c*(d +
 e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 - 2*b*n*(e*(a -
 b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n]))) - b*g*n*(4*(d + e*x)^2*(a + b*L
og[c*(d + e*x)^n])^3 - 3*b*n*(2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2 +
b*n*(b*e*n*x*(2*d + e*x) - 2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])))))/(4*
e^2)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1492 vs. 2(332) = 664.

Time = 3.52 (sec) , antiderivative size = 1493, normalized size of antiderivative = 4.39

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1493 |
| risch | Expression too large to display | 37938 |

```
[In] int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-16*a^3*b*d*e*f*n-90*b^4*n^4*d^2*g+4*a^4*d*e*f-2*a^4*e^2*g*x^2+8*a^3*
b*d^2*g*n-4*a^4*e^2*f*x-36*b^2*n^2*a^2*d^2*g+84*b^3*n^3*a*d^2*g-3*b^4*n^4*e
^2*g*x^2-96*b^4*n^4*e^2*f*x+6*b^3*n^3*a*e^2*g*x^2+90*b^4*n^4*d*e*g*x-6*b^2*
n^2*a^2*e^2*g*x^2+96*b^3*n^3*a*e^2*f*x-48*b^2*n^2*a^2*e^2*f*x+4*b*n*a^3*e^2
*g*x^2+16*b*n*a^3*e^2*f*x-174*ln(e*x+d)*b^4*d^2*g*n^4-96*b^3*n^3*a*d*e*f+48
*b^2*n^2*a^2*d*e*f+96*b^4*n^4*d*e*f*n^4+192*ln(e*x+d)*b^4*d*e*f*n^4+156*ln(e*x+
d)*a*b^3*d^2*g*n^3-60*ln(e*x+d)*a^2*b^2*d^2*g*n^2+8*ln(e*x+d)*a^3*b*d^2*g*n
-2*x^2*ln(c*(e*x+d)^n)^4*b^4*e^2*g-4*x*ln(c*(e*x+d)^n)^4*b^4*e^2*f-4*ln(c*(
e*x+d)^n)^4*b^4*d*e*f-12*ln(c*(e*x+d)^n)^3*b^4*d^2*g*n+42*ln(c*(e*x+d)^n)^2
*b^4*d^2*g*n^2+84*ln(c*(e*x+d)^n)*b^4*d^2*g*n^3+8*ln(c*(e*x+d)^n)^3*a*b^3*d
^2*g+12*ln(c*(e*x+d)^n)^2*a^2*b^2*d^2*g+2*ln(c*(e*x+d)^n)^4*b^4*d^2*g+4*x^2
*ln(c*(e*x+d)^n)^3*b^4*e^2*g*n-6*x^2*ln(c*(e*x+d)^n)^2*b^4*e^2*g*n^2+6*x^2*
ln(c*(e*x+d)^n)*b^4*e^2*g*n^3-8*x^2*ln(c*(e*x+d)^n)^3*a*b^3*e^2*g+16*x*ln(c
*(e*x+d)^n)^3*b^4*e^2*f*n-48*x*ln(c*(e*x+d)^n)^2*b^4*e^2*f*n^2+96*x*ln(c*(e
*x+d)^n)*b^4*e^2*f*n^3-12*x^2*ln(c*(e*x+d)^n)^2*a^2*b^2*e^2*g-16*x*ln(c*(e*
```

```
x+d)^n)^3*a*b^3*e^2*f+16*ln(c*(e*x+d)^n)^3*b^4*d*e*f*n-84*b^3*n^3*a*d*e*g*x
+36*b^2*n^2*a^2*d*e*g*x-192*ln(e*x+d)*a*b^3*d*e*f*n^3+96*ln(e*x+d)*a^2*b^2*
d*e*f*n^2-32*ln(e*x+d)*a^3*b*d*e*f*n-24*x*ln(c*(e*x+d)^n)^2*a*b^3*d*e*g*n-4
8*ln(c*(e*x+d)^n)^2*b^4*d*e*f*n^2-96*ln(c*(e*x+d)^n)*b^4*d*e*f*n^3-8*x^2*ln
(c*(e*x+d)^n)*a^3*b*e^2*g-24*x*ln(c*(e*x+d)^n)^2*a^2*b^2*e^2*f-16*ln(c*(e*x
+d)^n)^3*a*b^3*d*e*f-36*ln(c*(e*x+d)^n)^2*a*b^3*d^2*g*n-72*ln(c*(e*x+d)^n)*
a*b^3*d^2*g*n^2-16*x*ln(c*(e*x+d)^n)*a^3*b*e^2*f-24*ln(c*(e*x+d)^n)^2*a^2*b
^2*d*e*f+24*ln(c*(e*x+d)^n)*a^2*b^2*d^2*g*n+16*ln(c*(e*x+d)^n)*a^3*b*d*e*f+
12*x^2*ln(c*(e*x+d)^n)^2*a*b^3*e^2*g*n-12*x^2*ln(c*(e*x+d)^n)*a*b^3*e^2*g*n
^2-8*x*ln(c*(e*x+d)^n)^3*b^4*d*e*g*n+36*x*ln(c*(e*x+d)^n)^2*b^4*d*e*g*n^2-8
4*x*ln(c*(e*x+d)^n)*b^4*d*e*g*n^3+12*x^2*ln(c*(e*x+d)^n)*a^2*b^2*e^2*g*n+48
*x*ln(c*(e*x+d)^n)^2*a*b^3*e^2*f*n-96*x*ln(c*(e*x+d)^n)*a*b^3*e^2*f*n^2+48*
x*ln(c*(e*x+d)^n)*a^2*b^2*e^2*f*n+48*ln(c*(e*x+d)^n)^2*a*b^3*d*e*f*n+96*ln(
c*(e*x+d)^n)*a*b^3*d*e*f*n^2-48*ln(c*(e*x+d)^n)*a^2*b^2*d*e*f*n+72*x*ln(c*(
e*x+d)^n)*a*b^3*d*e*g*n^2-24*x*ln(c*(e*x+d)^n)*a^2*b^2*d*e*g*n-8*a^3*b*d*e*
g*n*x)/e^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1756 vs. $2(332) = 664$.

Time = 0.31 (sec) , antiderivative size = 1756, normalized size of antiderivative = 5.16

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^4\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^4*e^2*g*n^4*x^2 + 2*b^4*e^2*f*n^4*x + (2*b^4*d*e*f - b^4*d^2*g)*n
^4)*log(e*x + d)^4 + 2*(b^4*e^2*g*x^2 + 2*b^4*e^2*f*x)*log(c)^4 - 4*((4*b^4
*d*e*f - 3*b^4*d^2*g)*n^4 - 2*(2*a*b^3*d*e*f - a*b^3*d^2*g)*n^3 + (b^4*e^2*
g*n^4 - 2*a*b^3*e^2*g*n^3)*x^2 - 2*(2*a*b^3*e^2*f*n^3 - (2*b^4*e^2*f - b^4*
d*e*g)*n^4)*x - 2*(b^4*e^2*g*n^3*x^2 + 2*b^4*e^2*f*n^3*x + (2*b^4*d*e*f - b
^4*d^2*g)*n^3)*log(c))*log(e*x + d)^3 - 4*((b^4*e^2*g*n - 2*a*b^3*e^2*g)*x^
2 - 2*(2*a*b^3*e^2*f - (2*b^4*e^2*f - b^4*d*e*g)*n)*x)*log(c)^3 + (3*b^4*e^
2*g*n^4 - 6*a*b^3*e^2*g*n^3 + 6*a^2*b^2*e^2*g*n^2 - 4*a^3*b*e^2*g*n + 2*a^4
*e^2*g)*x^2 + 6*((8*b^4*d*e*f - 7*b^4*d^2*g)*n^4 - 2*(4*a*b^3*d*e*f - 3*a*b
^3*d^2*g)*n^3 + 2*(2*a^2*b^2*d*e*f - a^2*b^2*d^2*g)*n^2 + (b^4*e^2*g*n^4 -
2*a*b^3*e^2*g*n^3 + 2*a^2*b^2*e^2*g*n^2)*x^2 + 2*(b^4*e^2*g*n^2*x^2 + 2*b^4
*e^2*f*n^2*x + (2*b^4*d*e*f - b^4*d^2*g)*n^2)*log(c)^2 + 2*(2*a^2*b^2*e^2*f
*n^2 + (4*b^4*e^2*f - 3*b^4*d*e*g)*n^4 - 2*(2*a*b^3*e^2*f - a*b^3*d*e*g)*n^
3)*x - 2*((4*b^4*d*e*f - 3*b^4*d^2*g)*n^3 - 2*(2*a*b^3*d*e*f - a*b^3*d^2*g)
*n^2 + (b^4*e^2*g*n^3 - 2*a*b^3*e^2*g*n^2)*x^2 - 2*(2*a*b^3*e^2*f*n^2 - (2*
b^4*e^2*f - b^4*d*e*g)*n^3)*x)*log(c))*log(e*x + d)^2 + 6*((b^4*e^2*g*n^2 -
 2*a*b^3*e^2*g*n + 2*a^2*b^2*e^2*g)*x^2 + 2*(2*a^2*b^2*e^2*f + (4*b^4*e^2*f
 - 3*b^4*d*e*g)*n^2 - 2*(2*a*b^3*e^2*f - a*b^3*d*e*g)*n)*x)*log(c)^2 + 2*(2
```

```
*a^4*e^2*f + 3*(16*b^4*e^2*f - 15*b^4*d*e*g)*n^4 - 6*(8*a*b^3*e^2*f - 7*a*b
^3*d*e*g)*n^3 + 6*(4*a^2*b^2*e^2*f - 3*a^2*b^2*d*e*g)*n^2 - 4*(2*a^3*b*e^2*
f - a^3*b*d*e*g)*n)*x - 2*(3*(16*b^4*d*e*f - 15*b^4*d^2*g)*n^4 - 6*(8*a*b^3
*d*e*f - 7*a*b^3*d^2*g)*n^3 - 4*(b^4*e^2*g*n*x^2 + 2*b^4*e^2*f*n*x + (2*b^4
*d*e*f - b^4*d^2*g)*n)*log(c)^3 + 6*(4*a^2*b^2*d*e*f - 3*a^2*b^2*d^2*g)*n^2
 + (3*b^4*e^2*g*n^4 - 6*a*b^3*e^2*g*n^3 + 6*a^2*b^2*e^2*g*n^2 - 4*a^3*b*e^2
*g*n)*x^2 + 6*((4*b^4*d*e*f - 3*b^4*d^2*g)*n^2 + (b^4*e^2*g*n^2 - 2*a*b^3*e
^2*g*n)*x^2 - 2*(2*a*b^3*d*e*f - a*b^3*d^2*g)*n - 2*(2*a*b^3*e^2*f*n - (2*b
^4*e^2*f - b^4*d*e*g)*n^2)*x)*log(c)^2 - 4*(2*a^3*b*d*e*f - a^3*b*d^2*g)*n
- 2*(4*a^3*b*e^2*f*n - 3*(8*b^4*e^2*f - 7*b^4*d*e*g)*n^4 + 6*(4*a*b^3*e^2*f
 - 3*a*b^3*d*e*g)*n^3 - 6*(2*a^2*b^2*e^2*f - a^2*b^2*d*e*g)*n^2)*x - 6*((8*
b^4*d*e*f - 7*b^4*d^2*g)*n^3 - 2*(4*a*b^3*d*e*f - 3*a*b^3*d^2*g)*n^2 + (b^4
*e^2*g*n^3 - 2*a*b^3*e^2*g*n^2 + 2*a^2*b^2*e^2*g*n)*x^2 + 2*(2*a^2*b^2*d*e*
f - a^2*b^2*d^2*g)*n + 2*(2*a^2*b^2*e^2*f*n + (4*b^4*e^2*f - 3*b^4*d*e*g)*n
^3 - 2*(2*a*b^3*e^2*f - a*b^3*d*e*g)*n^2)*x)*log(c))*log(e*x + d) - 2*((3*b
^4*e^2*g*n^3 - 6*a*b^3*e^2*g*n^2 + 6*a^2*b^2*e^2*g*n - 4*a^3*b*e^2*g)*x^2 -
 2*(4*a^3*b*e^2*f - 3*(8*b^4*e^2*f - 7*b^4*d*e*g)*n^3 + 6*(4*a*b^3*e^2*f -
3*a*b^3*d*e*g)*n^2 - 6*(2*a^2*b^2*e^2*f - a^2*b^2*d*e*g)*n)*x)*log(c))/e^2
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1372 vs. $2(332) = 664$.

Time = 2.31 (sec) , antiderivative size = 1372, normalized size of antiderivative = 4.04

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^4\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**4,x)
```

```
[Out] Piecewise((a**4*f*x + a**4*g*x**2/2 - 2*a**3*b*d**2*g*log(c*(d + e*x)**n)/e
**2 + 4*a**3*b*d*f*log(c*(d + e*x)**n)/e + 2*a**3*b*d*g*n*x/e - 4*a**3*b*f*
n*x + 4*a**3*b*f*x*log(c*(d + e*x)**n) - a**3*b*g*n*x**2 + 2*a**3*b*g*x**2*
log(c*(d + e*x)**n) + 9*a**2*b**2*d**2*g*n*log(c*(d + e*x)**n)/e**2 - 3*a**
2*b**2*d**2*g*log(c*(d + e*x)**n)**2/e**2 - 12*a**2*b**2*d*f*n*log(c*(d + e
*x)**n)/e + 6*a**2*b**2*d*f*log(c*(d + e*x)**n)**2/e - 9*a**2*b**2*d*g*n**2
*x/e + 6*a**2*b**2*d*g*n*x*log(c*(d + e*x)**n)/e + 12*a**2*b**2*f*n**2*x -
12*a**2*b**2*f*n*x*log(c*(d + e*x)**n) + 6*a**2*b**2*f*x*log(c*(d + e*x)**n
)**2 + 3*a**2*b**2*g*n**2*x**2/2 - 3*a**2*b**2*g*n*x**2*log(c*(d + e*x)**n)
 + 3*a**2*b**2*g*x**2*log(c*(d + e*x)**n)**2 - 21*a*b**3*d**2*g*n**2*log(c*
(d + e*x)**n)/e**2 + 9*a*b**3*d**2*g*n*log(c*(d + e*x)**n)**2/e**2 - 2*a*b*
*3*d**2*g*log(c*(d + e*x)**n)**3/e**2 + 24*a*b**3*d*f*n**2*log(c*(d + e*x)*
*n)/e - 12*a*b**3*d*f*n*log(c*(d + e*x)**n)**2/e + 4*a*b**3*d*f*log(c*(d +
e*x)**n)**3/e + 21*a*b**3*d*g*n**3*x/e - 18*a*b**3*d*g*n**2*x*log(c*(d + e*
x)**n)/e + 6*a*b**3*d*g*n*x*log(c*(d + e*x)**n)**2/e - 24*a*b**3*f*n**3*x +
 24*a*b**3*f*n**2*x*log(c*(d + e*x)**n) - 12*a*b**3*f*n*x*log(c*(d + e*x)**
```

```
n)**2 + 4*a*b**3*f*x*log(c*(d + e*x)**n)**3 - 3*a*b**3*g*n**3*x**2/2 + 3*a*
b**3*g*n**2*x**2*log(c*(d + e*x)**n) - 3*a*b**3*g*n*x**2*log(c*(d + e*x)**n
)**2 + 2*a*b**3*g*x**2*log(c*(d + e*x)**n)**3 + 45*b**4*d**2*g*n**3*log(c*(
d + e*x)**n)/(2*e**2) - 21*b**4*d**2*g*n**2*log(c*(d + e*x)**n)**2/(2*e**2)
 + 3*b**4*d**2*g*n*log(c*(d + e*x)**n)**3/e**2 - b**4*d**2*g*log(c*(d + e*x
)**n)**4/(2*e**2) - 24*b**4*d*f*n**3*log(c*(d + e*x)**n)/e + 12*b**4*d*f*n*
*2*log(c*(d + e*x)**n)**2/e - 4*b**4*d*f*n*log(c*(d + e*x)**n)**3/e + b**4*
d*f*log(c*(d + e*x)**n)**4/e - 45*b**4*d*g*n**4*x/(2*e) + 21*b**4*d*g*n**3*
x*log(c*(d + e*x)**n)/e - 9*b**4*d*g*n**2*x*log(c*(d + e*x)**n)**2/e + 2*b*
*4*d*g*n*x*log(c*(d + e*x)**n)**3/e + 24*b**4*f*n**4*x - 24*b**4*f*n**3*x*l
og(c*(d + e*x)**n) + 12*b**4*f*n**2*x*log(c*(d + e*x)**n)**2 - 4*b**4*f*n*x
*log(c*(d + e*x)**n)**3 + b**4*f*x*log(c*(d + e*x)**n)**4 + 3*b**4*g*n**4*x
**2/4 - 3*b**4*g*n**3*x**2*log(c*(d + e*x)**n)/2 + 3*b**4*g*n**2*x**2*log(c
*(d + e*x)**n)**2/2 - b**4*g*n*x**2*log(c*(d + e*x)**n)**3 + b**4*g*x**2*lo
g(c*(d + e*x)**n)**4/2, Ne(e, 0)), ((a + b*log(c*d**n))**4*(f*x + g*x**2/2)
, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. $2(332) = 664$.

Time = 0.24 (sec) , antiderivative size = 1163, normalized size of antiderivative = 3.42

$$\int (f + gx) (a + b\log (c(d + ex)^n))^4 \, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^4,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*g*x^2*log((e*x + d)^n*c)^4 + 2*a*b^3*g*x^2*log((e*x + d)^n*c)^3 + b
^4*f*x*log((e*x + d)^n*c)^4 + 3*a^2*b^2*g*x^2*log((e*x + d)^n*c)^2 + 4*a*b^
3*f*x*log((e*x + d)^n*c)^3 - 4*a^3*b*e*f*n*(x/e - d*log(e*x + d)/e^2) - a^3
*b*e*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 2*a^3*b*g*x^2*log
((e*x + d)^n*c) + 6*a^2*b^2*f*x*log((e*x + d)^n*c)^2 + 1/2*a^4*g*x^2 + 4*a^
3*b*f*x*log((e*x + d)^n*c) - 6*(2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x +
 d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*a^2*b^2*f -
 4*(3*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^2 - e*n*((d*log(e*x
 + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*n^2/e^2 - 3*(d*log
(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n*log((e*x + d)^n*c)/e^2))*a*b^3*f
- (4*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^3 + (e*n*((d*log(e*x
 + d)^4 + 4*d*log(e*x + d)^3 + 12*d*log(e*x + d)^2 - 24*e*x + 24*d*log(e*x
+ d))*n^2/e^3 - 4*(d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(
e*x + d))*n*log((e*x + d)^n*c)/e^3) + 6*(d*log(e*x + d)^2 - 2*e*x + 2*d*log
(e*x + d))*n*log((e*x + d)^n*c)^2/e^2)*e*n)*b^4*f - 3/2*(2*e*n*(2*d^2*log(e
*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)^n*c) - (e^2*x^2 + 2*d^2*lo
g(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n^2/e^2)*a^2*b^2*g - 1/2*(6*e*
n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)^n*c)^2 + e*n
```

```
*((4*d^2*log(e*x + d)^3 + 3*e^2*x^2 + 18*d^2*log(e*x + d)^2 - 42*d*e*x + 42
*d^2*log(e*x + d))*n^2/e^3 - 6*(e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x +
6*d^2*log(e*x + d))*n*log((e*x + d)^n*c)/e^3))*a*b^3*g - 1/4*(4*e*n*(2*d^2*
log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)^n*c)^3 - (e*n*((2*d^2
*log(e*x + d)^4 + 12*d^2*log(e*x + d)^3 + 3*e^2*x^2 + 42*d^2*log(e*x + d)^2
 - 90*d*e*x + 90*d^2*log(e*x + d))*n^2/e^4 - 2*(4*d^2*log(e*x + d)^3 + 3*e^
2*x^2 + 18*d^2*log(e*x + d)^2 - 42*d*e*x + 42*d^2*log(e*x + d))*n*log((e*x
+ d)^n*c)/e^4) + 6*(e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*
x + d))*n*log((e*x + d)^n*c)^2/e^3)*e*n)*b^4*g + a^4*f*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2488 vs. $2(332) = 664$.

Time = 0.35 (sec) , antiderivative size = 2488, normalized size of antiderivative = 7.32

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^4\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^4,x, algorithm="giac")
```

```
[Out] (e*x + d)*b^4*f*n^4*log(e*x + d)^4/e + 1/2*(e*x + d)^2*b^4*g*n^4*log(e*x +
d)^4/e^2 - (e*x + d)*b^4*d*g*n^4*log(e*x + d)^4/e^2 - 4*(e*x + d)*b^4*f*n^4
*log(e*x + d)^3/e - (e*x + d)^2*b^4*g*n^4*log(e*x + d)^3/e^2 + 4*(e*x + d)*
b^4*d*g*n^4*log(e*x + d)^3/e^2 + 4*(e*x + d)*b^4*f*n^3*log(e*x + d)^3*log(c
)/e + 2*(e*x + d)^2*b^4*g*n^3*log(e*x + d)^3*log(c)/e^2 - 4*(e*x + d)*b^4*d
*g*n^3*log(e*x + d)^3*log(c)/e^2 + 12*(e*x + d)*b^4*f*n^4*log(e*x + d)^2/e
+ 3/2*(e*x + d)^2*b^4*g*n^4*log(e*x + d)^2/e^2 - 12*(e*x + d)*b^4*d*g*n^4*l
og(e*x + d)^2/e^2 + 4*(e*x + d)*a*b^3*f*n^3*log(e*x + d)^3/e + 2*(e*x + d)^
2*a*b^3*g*n^3*log(e*x + d)^3/e^2 - 4*(e*x + d)*a*b^3*d*g*n^3*log(e*x + d)^3
/e^2 - 12*(e*x + d)*b^4*f*n^3*log(e*x + d)^2*log(c)/e - 3*(e*x + d)^2*b^4*g
*n^3*log(e*x + d)^2*log(c)/e^2 + 12*(e*x + d)*b^4*d*g*n^3*log(e*x + d)^2*lo
g(c)/e^2 + 6*(e*x + d)*b^4*f*n^2*log(e*x + d)^2*log(c)^2/e + 3*(e*x + d)^2*
b^4*g*n^2*log(e*x + d)^2*log(c)^2/e^2 - 6*(e*x + d)*b^4*d*g*n^2*log(e*x + d
)^2*log(c)^2/e^2 - 24*(e*x + d)*b^4*f*n^4*log(e*x + d)/e - 3/2*(e*x + d)^2*
b^4*g*n^4*log(e*x + d)/e^2 + 24*(e*x + d)*b^4*d*g*n^4*log(e*x + d)/e^2 - 12
*(e*x + d)*a*b^3*f*n^3*log(e*x + d)^2/e - 3*(e*x + d)^2*a*b^3*g*n^3*log(e*x
 + d)^2/e^2 + 12*(e*x + d)*a*b^3*d*g*n^3*log(e*x + d)^2/e^2 + 24*(e*x + d)*
b^4*f*n^3*log(e*x + d)*log(c)/e + 3*(e*x + d)^2*b^4*g*n^3*log(e*x + d)*log(
c)/e^2 - 24*(e*x + d)*b^4*d*g*n^3*log(e*x + d)*log(c)/e^2 + 12*(e*x + d)*a*
b^3*f*n^2*log(e*x + d)^2*log(c)/e + 6*(e*x + d)^2*a*b^3*g*n^2*log(e*x + d)^
2*log(c)/e^2 - 12*(e*x + d)*a*b^3*d*g*n^2*log(e*x + d)^2*log(c)/e^2 - 12*(e
*x + d)*b^4*f*n^2*log(e*x + d)*log(c)^2/e - 3*(e*x + d)^2*b^4*g*n^2*log(e*x
 + d)*log(c)^2/e^2 + 12*(e*x + d)*b^4*d*g*n^2*log(e*x + d)*log(c)^2/e^2 + 4
*(e*x + d)*b^4*f*n*log(e*x + d)*log(c)^3/e + 2*(e*x + d)^2*b^4*g*n*log(e*x
+ d)*log(c)^3/e^2 - 4*(e*x + d)*b^4*d*g*n*log(e*x + d)*log(c)^3/e^2 + 24*(e
```

```
*x + d)*b^4*f*n^4/e + 3/4*(e*x + d)^2*b^4*g*n^4/e^2 - 24*(e*x + d)*b^4*d*g*
n^4/e^2 + 24*(e*x + d)*a*b^3*f*n^3*log(e*x + d)/e + 3*(e*x + d)^2*a*b^3*g*n
^3*log(e*x + d)/e^2 - 24*(e*x + d)*a*b^3*d*g*n^3*log(e*x + d)/e^2 + 6*(e*x
+ d)*a^2*b^2*f*n^2*log(e*x + d)^2/e + 3*(e*x + d)^2*a^2*b^2*g*n^2*log(e*x +
 d)^2/e^2 - 6*(e*x + d)*a^2*b^2*d*g*n^2*log(e*x + d)^2/e^2 - 24*(e*x + d)*b
^4*f*n^3*log(c)/e - 3/2*(e*x + d)^2*b^4*g*n^3*log(c)/e^2 + 24*(e*x + d)*b^4
*d*g*n^3*log(c)/e^2 - 24*(e*x + d)*a*b^3*f*n^2*log(e*x + d)*log(c)/e - 6*(e
*x + d)^2*a*b^3*g*n^2*log(e*x + d)*log(c)/e^2 + 24*(e*x + d)*a*b^3*d*g*n^2*
log(e*x + d)*log(c)/e^2 + 12*(e*x + d)*b^4*f*n^2*log(c)^2/e + 3/2*(e*x + d)
^2*b^4*g*n^2*log(c)^2/e^2 - 12*(e*x + d)*b^4*d*g*n^2*log(c)^2/e^2 + 12*(e*x
 + d)*a*b^3*f*n*log(e*x + d)*log(c)^2/e + 6*(e*x + d)^2*a*b^3*g*n*log(e*x +
 d)*log(c)^2/e^2 - 12*(e*x + d)*a*b^3*d*g*n*log(e*x + d)*log(c)^2/e^2 - 4*(
e*x + d)*b^4*f*n*log(c)^3/e - (e*x + d)^2*b^4*g*n*log(c)^3/e^2 + 4*(e*x + d
)*b^4*d*g*n*log(c)^3/e^2 + (e*x + d)*b^4*f*log(c)^4/e + 1/2*(e*x + d)^2*b^4
*g*log(c)^4/e^2 - (e*x + d)*b^4*d*g*log(c)^4/e^2 - 24*(e*x + d)*a*b^3*f*n^3
/e - 3/2*(e*x + d)^2*a*b^3*g*n^3/e^2 + 24*(e*x + d)*a*b^3*d*g*n^3/e^2 - 12*
(e*x + d)*a^2*b^2*f*n^2*log(e*x + d)/e - 3*(e*x + d)^2*a^2*b^2*g*n^2*log(e*
x + d)/e^2 + 12*(e*x + d)*a^2*b^2*d*g*n^2*log(e*x + d)/e^2 + 24*(e*x + d)*a
*b^3*f*n^2*log(c)/e + 3*(e*x + d)^2*a*b^3*g*n^2*log(c)/e^2 - 24*(e*x + d)*a
*b^3*d*g*n^2*log(c)/e^2 + 12*(e*x + d)*a^2*b^2*f*n*log(e*x + d)*log(c)/e +
6*(e*x + d)^2*a^2*b^2*g*n*log(e*x + d)*log(c)/e^2 - 12*(e*x + d)*a^2*b^2*d*
g*n*log(e*x + d)*log(c)/e^2 - 12*(e*x + d)*a*b^3*f*n*log(c)^2/e - 3*(e*x +
d)^2*a*b^3*g*n*log(c)^2/e^2 + 12*(e*x + d)*a*b^3*d*g*n*log(c)^2/e^2 + 4*(e*
x + d)*a*b^3*f*log(c)^3/e + 2*(e*x + d)^2*a*b^3*g*log(c)^3/e^2 - 4*(e*x + d
)*a*b^3*d*g*log(c)^3/e^2 + 12*(e*x + d)*a^2*b^2*f*n^2/e + 3/2*(e*x + d)^2*a
^2*b^2*g*n^2/e^2 - 12*(e*x + d)*a^2*b^2*d*g*n^2/e^2 + 4*(e*x + d)*a^3*b*f*n
*log(e*x + d)/e + 2*(e*x + d)^2*a^3*b*g*n*log(e*x + d)/e^2 - 4*(e*x + d)*a^
3*b*d*g*n*log(e*x + d)/e^2 - 12*(e*x + d)*a^2*b^2*f*n*log(c)/e - 3*(e*x + d
)^2*a^2*b^2*g*n*log(c)/e^2 + 12*(e*x + d)*a^2*b^2*d*g*n*log(c)/e^2 + 6*(e*x
 + d)*a^2*b^2*f*log(c)^2/e + 3*(e*x + d)^2*a^2*b^2*g*log(c)^2/e^2 - 6*(e*x
+ d)*a^2*b^2*d*g*log(c)^2/e^2 - 4*(e*x + d)*a^3*b*f*n/e - (e*x + d)^2*a^3*b
*g*n/e^2 + 4*(e*x + d)*a^3*b*d*g*n/e^2 + 4*(e*x + d)*a^3*b*f*log(c)/e + 2*(
e*x + d)^2*a^3*b*g*log(c)/e^2 - 4*(e*x + d)*a^3*b*d*g*log(c)/e^2 + (e*x + d
)*a^4*f/e + 1/2*(e*x + d)^2*a^4*g/e^2 - (e*x + d)*a^4*d*g/e^2
```

### Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 823, normalized size of antiderivative = 2.42

$$\int (f+gx) (a+b\log (c(d+ex)^n))^4 \, dx$$
$$= x \left( \frac{2 a^4 d g + 2 a^4 e f - 42 b^4 d g n^4 + 48 b^4 e f n^4 + 36 a b^3 d g n^3 - 48 a b^3 e f n^3 - 12 a^2 b^2 d g n^2 + 24 a^2 b^2}{2 e} - \frac{d g (2 a^4 - 4 a^3 b n + 6 a^2 b^2 n^2 - 6 a b^3 n^3 + 3 b^4 n^4)}{2 e} \right)$$
$$+ \ln (c (d + e x)^n)^4 \left( \frac{b^4 g x^2}{2} - \frac{d (b^4 d g - 2 b^4 e f)}{2 e^2} + b^4 f x \right)$$
$$+ \ln (c (d + e x)^n) \left( \frac{b g (4 a^3 - 6 a^2 b n + 6 a b^2 n^2 - 3 b^3 n^3) x^2}{2} + \left( \frac{4 a^3 b d g + 4 a^3 b e f + 18 b^4 d g n^3 - 24 b^4 e f n^3 - 12 a^2 b^2 e f n - 12 a b^3 d g n^2 + 24 a b^3 e f n^2}{e} - \frac{b d g (4 a^3 - 6 a^2 b n + 6 a b^2 n^2 - 3 b^3 n^3)}{e} \right) x \right)$$
$$+ \ln (c (d + e x)^n)^3 \left( x \left( \frac{4 b^3 (a d g + a e f - b e f n)}{e} - \frac{2 b^3 d g (2 a - b n)}{e} \right) - \frac{d (2 a b^3 d g - 4 a b^3 e f - 3 b^4 d g n + 4 b^4 e f n)}{e^2} + b^3 g x^2 (2 a - b n) \right)$$
$$+ \ln (c (d + e x)^n)^2 \left( x \left( \frac{6 a^2 b^2 d g + 6 a^2 b^2 e f - 6 b^4 d g n^2 + 12 b^4 e f n^2 - 12 a b^3 e f n}{e} - \frac{3 b^2 d g (2 a^2 - 2 a b n + b^2 n^2)}{e} \right) - \frac{3 d (2 a^2 b^2 d g - 4 a^2 b^2 e f + 7 b^4 d g n^2 - 8 b^4 e f n^2 - 6 a b^3 d g n + 8 a b^3 e f n)}{2 e^2} + \frac{3 b^2 g x^2 (2 a^2 - 2 a b n + b^2 n^2)}{2} \right)$$
$$+ \frac{\ln (d + e x) (-4 g a^3 b d^2 n + 8 e f a^3 b d n + 18 g a^2 b^2 d^2 n^2 - 24 e f a^2 b^2 d n^2 - 42 g a b^3 d^2 n^3 + 48 e f a}{2 e^2}$$
$$+ \frac{g x^2 (2 a^4 - 4 a^3 b n + 6 a^2 b^2 n^2 - 6 a b^3 n^3 + 3 b^4 n^4)}{4}$$

```
[In] int((f + g*x)*(a + b*log(c*(d + e*x)^n))^4,x)
```

```
[Out] x*((2*a^4*d*g + 2*a^4*e*f - 42*b^4*d*g*n^4 + 48*b^4*e*f*n^4 + 36*a*b^3*d*g*
n^3 - 48*a*b^3*e*f*n^3 - 12*a^2*b^2*d*g*n^2 + 24*a^2*b^2*e*f*n^2 - 8*a^3*b*
e*f*n)/(2*e) - (d*g*(2*a^4 + 3*b^4*n^4 - 6*a*b^3*n^3 + 6*a^2*b^2*n^2 - 4*a^
3*b*n))/(2*e)) + log(c*(d + e*x)^n)^4*((b^4*g*x^2)/2 - (d*(b^4*d*g - 2*b^4*
e*f))/(2*e^2) + b^4*f*x) + log(c*(d + e*x)^n)*(x*((4*a^3*b*d*g + 4*a^3*b*e*
f + 18*b^4*d*g*n^3 - 24*b^4*e*f*n^3 - 12*a^2*b^2*e*f*n - 12*a*b^3*d*g*n^2 +
```

```
 24*a*b^3*e*f*n^2)/e - (b*d*g*(4*a^3 - 3*b^3*n^3 + 6*a*b^2*n^2 - 6*a^2*b*n)
)/e) + (b*g*x^2*(4*a^3 - 3*b^3*n^3 + 6*a*b^2*n^2 - 6*a^2*b*n))/2) + log(c*(
d + e*x)^n)^3*(x*((4*b^3*(a*d*g + a*e*f - b*e*f*n))/e - (2*b^3*d*g*(2*a - b
*n))/e) - (d*(2*a*b^3*d*g - 4*a*b^3*e*f - 3*b^4*d*g*n + 4*b^4*e*f*n))/e^2 +
 b^3*g*x^2*(2*a - b*n)) + log(c*(d + e*x)^n)^2*(x*((6*a^2*b^2*d*g + 6*a^2*b
^2*e*f - 6*b^4*d*g*n^2 + 12*b^4*e*f*n^2 - 12*a*b^3*e*f*n)/e - (3*b^2*d*g*(2
*a^2 + b^2*n^2 - 2*a*b*n))/e) - (3*d*(2*a^2*b^2*d*g - 4*a^2*b^2*e*f + 7*b^4
*d*g*n^2 - 8*b^4*e*f*n^2 - 6*a*b^3*d*g*n + 8*a*b^3*e*f*n))/(2*e^2) + (3*b^2
*g*x^2*(2*a^2 + b^2*n^2 - 2*a*b*n))/2) + (log(d + e*x)*(45*b^4*d^2*g*n^4 -
4*a^3*b*d^2*g*n - 48*b^4*d*e*f*n^4 - 42*a*b^3*d^2*g*n^3 + 18*a^2*b^2*d^2*g*
n^2 + 8*a^3*b*d*e*f*n + 48*a*b^3*d*e*f*n^3 - 24*a^2*b^2*d*e*f*n^2))/(2*e^2)
 + (g*x^2*(2*a^4 + 3*b^4*n^4 - 6*a*b^3*n^3 + 6*a^2*b^2*n^2 - 4*a^3*b*n))/4
```

# 3.61 $\int (a+b\log(c(d+ex)^n))^4\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 131

$$\int (a+b\log(c(d+ex)^n))^4\,dx = -24ab^3n^3x+24b^4n^4x-\frac{24b^4n^3(d+ex)\log(c(d+ex)^n)}{e}+\frac{12b^2n^2(d+ex)\left(a+b\log(c(d+ex)^n)\right)^2}{e}-\frac{4bn(d+ex)\left(a+b\log(c(d+ex)^n)\right)^3}{e}+\frac{(d+ex)\left(a+b\log(c(d+ex)^n)\right)^4}{e}$$

```
[Out] -24*a*b^3*n^3*x+24*b^4*n^4*x-24*b^4*n^3*(e*x+d)*ln(c*(e*x+d)^n)/e+12*b^2*n^
2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e-4*b*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e
+(e*x+d)*(a+b*ln(c*(e*x+d)^n))^4/e
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2436, 2333, 2332}

$$\int (a+b\log(c(d+ex)^n))^4\,dx = -24ab^3n^3x+\frac{12b^2n^2(d+ex)\left(a+b\log(c(d+ex)^n)\right)^2}{e}-\frac{4bn(d+ex)\left(a+b\log(c(d+ex)^n)\right)^3}{e}+\frac{(d+ex)\left(a+b\log(c(d+ex)^n)\right)^4}{e}-\frac{24b^4n^3(d+ex)\log(c(d+ex)^n)}{e}+24b^4n^4x$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^4,x]
```

```
[Out] -24*a*b^3*n^3*x + 24*b^4*n^4*x - (24*b^4*n^3*(d + e*x)*Log[c*(d + e*x)^n])/
e + (12*b^2*n^2*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e - (4*b*n*(d + e*x
)*(a + b*Log[c*(d + e*x)^n])^3)/e + ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^4
)/e
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+b\log(cx^n))^4\,dx,x,d+ex\right)}{e}\\
&= \frac{(d+ex)(a+b\log(c(d+ex)^n))^4}{e}-\frac{(4bn)\text{Subst}\left(\int (a+b\log(cx^n))^3\,dx,x,d+ex\right)}{e}\\
&= -\frac{4bn(d+ex)(a+b\log(c(d+ex)^n))^3}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^4}{e}\\
&\quad+\frac{(12b^2n^2)\,\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx,x,d+ex\right)}{e}\\
&= \frac{12b^2n^2(d+ex)(a+b\log(c(d+ex)^n))^2}{e}-\frac{4bn(d+ex)(a+b\log(c(d+ex)^n))^3}{e}\\
&\quad+\frac{(d+ex)(a+b\log(c(d+ex)^n))^4}{e}-\frac{(24b^3n^3)\,\text{Subst}\left(\int (a+b\log(cx^n))\,dx,x,d+ex\right)}{e}\\
&= -24ab^3n^3x+\frac{12b^2n^2(d+ex)(a+b\log(c(d+ex)^n))^2}{e}\\
&\quad-\frac{4bn(d+ex)(a+b\log(c(d+ex)^n))^3}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^4}{e}\\
&\quad-\frac{(24b^4n^3)\,\text{Subst}\left(\int \log(cx^n)\,dx,x,d+ex\right)}{e}
\end{aligned}$$

$$= -24ab^3n^3x + 24b^4n^4x - \frac{24b^4n^3(d+ex)\log\left(c(d+ex)^n\right)}{e} + \frac{12b^2n^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} - \frac{4bn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e} + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{e}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \left(a+b\log\left(c(d+ex)^n\right)\right)^4 \, dx = \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4 - 4bn\Big((d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3 - 3bn\Big((d+ex)\left(a+b\log\left(c(d\right.\right.}{e}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^4,x]
```

```
[Out] ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^4 - 4*b*n*((d + e*x)*(a + b*Log[c*(d
+ e*x)^n])^3 - 3*b*n*((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2 - 2*b*n*(e*(a
- b*n)*x + b*(d + e*x)*Log[c*(d + e*x)^n]))))/e
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. 2(131) = 262.

Time = 0.59 (sec) , antiderivative size = 525, normalized size of antiderivative = 4.01

| method | result |
|---|---|
| parallelrisch | $\frac{-12x\ln(c(ex+d)^n)a^2b^2e\,n^2+4x\ln(c(ex+d)^n)a^3ben+4x\ln(c(ex+d)^n)^3a\,b^3en-12x\ln(c(ex+d)^n)^2a\,b^3e\,n^2+24x\ln(c(ex+d)^n)}{}$ |
| risch | Expression too large to display |

```
[In] int((a+b*ln(c*(e*x+d)^n))^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-12*x*ln(c*(e*x+d)^n)*a^2*b^2*e*n^2+4*x*ln(c*(e*x+d)^n)*a^3*b*e*n+4*x*ln(c
*(e*x+d)^n)^3*a*b^3*e*n-12*x*ln(c*(e*x+d)^n)^2*a*b^3*e*n^2+24*x*ln(c*(e*x+d
)^n)*a*b^3*e*n^3+6*x*ln(c*(e*x+d)^n)^2*a^2*b^2*e*n+24*a*b^3*d*n^4-12*a^2*b^
2*d*n^3+4*a^3*b*d*n^2+ln(c*(e*x+d)^n)^4*b^4*d*n-4*ln(c*(e*x+d)^n)^3*b^4*d*n
^2+12*ln(c*(e*x+d)^n)^2*b^4*d*n^3-24*ln(c*(e*x+d)^n)*b^4*d*n^4+x*a^4*e*n+24
*x*b^4*e*n^5+12*x*a^2*b^2*e*n^3+4*ln(c*(e*x+d)^n)^3*a*b^3*d*n-12*ln(c*(e*x+
d)^n)^2*a*b^3*d*n^2+24*ln(c*(e*x+d)^n)*a*b^3*d*n^3-4*x*a^3*b*e*n^2+6*ln(c*(
e*x+d)^n)^2*a^2*b^2*d*n-12*ln(c*(e*x+d)^n)*a^2*b^2*d*n^2+4*ln(c*(e*x+d)^n)*
a^3*b*d*n+x*ln(c*(e*x+d)^n)^4*b^4*e*n-4*x*ln(c*(e*x+d)^n)^3*b^4*e*n^2+12*x*
ln(c*(e*x+d)^n)^2*b^4*e*n^3-24*x*ln(c*(e*x+d)^n)*b^4*e*n^4-24*x*a*b^3*e*n^4
-24*b^4*d*n^5-a^4*d*n)/e/n
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(131) = 262$.

Time = 0.30 (sec) , antiderivative size = 614, normalized size of antiderivative = 4.69

$$\int (a + b\log(c(d+ex)^n))^4 \, dx$$
$$= \frac{b^4ex\log(c)^4 + (b^4en^4x + b^4dn^4)\log(ex+d)^4 - 4(b^4en - ab^3e)x\log(c)^3 - 4(b^4dn^4 - ab^3dn^3 + (b^4en^4 - a}{}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^4,x, algorithm="fricas")
```

```
[Out] (b^4*e*x*log(c)^4 + (b^4*e*n^4*x + b^4*d*n^4)*log(e*x + d)^4 - 4*(b^4*e*n -
 a*b^3*e)*x*log(c)^3 - 4*(b^4*d*n^4 - a*b^3*d*n^3 + (b^4*e*n^4 - a*b^3*e*n^
3)*x - (b^4*e*n^3*x + b^4*d*n^3)*log(c))*log(e*x + d)^3 + 6*(2*b^4*e*n^2 -
2*a*b^3*e*n + a^2*b^2*e)*x*log(c)^2 + 6*(2*b^4*d*n^4 - 2*a*b^3*d*n^3 + a^2*
b^2*d*n^2 + (b^4*e*n^2*x + b^4*d*n^2)*log(c)^2 + (2*b^4*e*n^4 - 2*a*b^3*e*n
^3 + a^2*b^2*e*n^2)*x - 2*(b^4*d*n^3 - a*b^3*d*n^2 + (b^4*e*n^3 - a*b^3*e*n
^2)*x)*log(c))*log(e*x + d)^2 - 4*(6*b^4*e*n^3 - 6*a*b^3*e*n^2 + 3*a^2*b^2*
e*n - a^3*b*e)*x*log(c) + (24*b^4*e*n^4 - 24*a*b^3*e*n^3 + 12*a^2*b^2*e*n^2
 - 4*a^3*b*e*n + a^4*e)*x - 4*(6*b^4*d*n^4 - 6*a*b^3*d*n^3 + 3*a^2*b^2*d*n^
2 - a^3*b*d*n - (b^4*e*n*x + b^4*d*n)*log(c)^3 + 3*(b^4*d*n^2 - a*b^3*d*n +
 (b^4*e*n^2 - a*b^3*e*n)*x)*log(c)^2 + (6*b^4*e*n^4 - 6*a*b^3*e*n^3 + 3*a^2
*b^2*e*n^2 - a^3*b*e*n)*x - 3*(2*b^4*d*n^3 - 2*a*b^3*d*n^2 + a^2*b^2*d*n +
(2*b^4*e*n^3 - 2*a*b^3*e*n^2 + a^2*b^2*e*n)*x)*log(c))*log(e*x + d))/e
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(126) = 252$.

Time = 0.90 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.78

$$\int (a + b\log(c(d+ex)^n))^4 \, dx$$
$$= \begin{cases} a^4x + \frac{4a^3bd\log(c(d+ex)^n)}{e} - 4a^3bnx + 4a^3bx\log(c(d+ex)^n) - \frac{12a^2b^2dn\log(c(d+ex)^n)}{e} + \frac{6a^2b^2d\log(c(d+ex)^n)^2}{e} + 12 \\ x(a+b\log(cd^n))^4 \end{cases}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*d*log(c*(d + e*x)**n)/e - 4*a**3*b*n*x + 4*a**
3*b*x*log(c*(d + e*x)**n) - 12*a**2*b**2*d*n*log(c*(d + e*x)**n)/e + 6*a**2
*b**2*d*log(c*(d + e*x)**n)**2/e + 12*a**2*b**2*n**2*x - 12*a**2*b**2*n*x*l
og(c*(d + e*x)**n) + 6*a**2*b**2*x*log(c*(d + e*x)**n)**2 + 24*a*b**3*d*n**
2*log(c*(d + e*x)**n)/e - 12*a*b**3*d*n*log(c*(d + e*x)**n)**2/e + 4*a*b**3
```

```
*d*log(c*(d + e*x)**n)**3/e - 24*a*b**3*n**3*x + 24*a*b**3*n**2*x*log(c*(d
+ e*x)**n) - 12*a*b**3*n*x*log(c*(d + e*x)**n)**2 + 4*a*b**3*x*log(c*(d + e
*x)**n)**3 - 24*b**4*d*n**3*log(c*(d + e*x)**n)/e + 12*b**4*d*n**2*log(c*(d
 + e*x)**n)**2/e - 4*b**4*d*n*log(c*(d + e*x)**n)**3/e + b**4*d*log(c*(d +
e*x)**n)**4/e + 24*b**4*n**4*x - 24*b**4*n**3*x*log(c*(d + e*x)**n) + 12*b*
*4*n**2*x*log(c*(d + e*x)**n)**2 - 4*b**4*n*x*log(c*(d + e*x)**n)**3 + b**4
*x*log(c*(d + e*x)**n)**4, Ne(e, 0)), (x*(a + b*log(c*d**n))**4, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(131) = 262$.

Time = 0.22 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.82

$$\int (a+b\log(c(d+ex)^n))^4\,dx = b^4x\log\left((ex+d)^nc\right)^4 + 4\,ab^3x\log\left((ex+d)^nc\right)^3$$
$$-4\,a^3ben\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right) + 6\,a^2b^2x\log\left((ex+d)^nc\right)^2 + 4\,a^3bx\log\left((ex+d)^nc\right)$$
$$-6\left(2\,en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log\left((ex+d)^nc\right) + \frac{(d\log(ex+d)^2-2\,ex+2\,d\log(ex+d))n^2}{e}\right)a^2b^2$$
$$-4\left(3\,en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log\left((ex+d)^nc\right)^2 - en\left(\frac{(d\log(ex+d)^3+3\,d\log(ex+d)^2-6\,ex+6\,d\mathrm{l}}{e^2}\right.\right.$$
$$-\left(4\,en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log\left((ex+d)^nc\right)^3 + \left(en\left(\frac{(d\log(ex+d)^4+4\,d\log(ex+d)^3+12\,d\log(e}{e^3}\right.\right.\right.$$
$$+a^4x$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log((e*x + d)^n*c)^4 + 4*a*b^3*x*log((e*x + d)^n*c)^3 - 4*a^3*b*e*n*(
x/e - d*log(e*x + d)/e^2) + 6*a^2*b^2*x*log((e*x + d)^n*c)^2 + 4*a^3*b*x*lo
g((e*x + d)^n*c) - 6*(2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c) +
 (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*a^2*b^2 - 4*(3*e*n*(x
/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^2 - e*n*((d*log(e*x + d)^3 + 3*
d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*n^2/e^2 - 3*(d*log(e*x + d)^2
- 2*e*x + 2*d*log(e*x + d))*n*log((e*x + d)^n*c)/e^2))*a*b^3 - (4*e*n*(x/e
- d*log(e*x + d)/e^2)*log((e*x + d)^n*c)^3 + (e*n*((d*log(e*x + d)^4 + 4*d*
log(e*x + d)^3 + 12*d*log(e*x + d)^2 - 24*e*x + 24*d*log(e*x + d))*n^2/e^3
- 4*(d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e*x + d))*n*lo
g((e*x + d)^n*c)/e^3) + 6*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n*l
og((e*x + d)^n*c)^2/e^2)*e*n)*b^4 + a^4*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(131) = 262$.

Time = 0.33 (sec) , antiderivative size = 758, normalized size of antiderivative = 5.79

$$
\begin{aligned}
\int (a+b\log(c(d+ex)^n))^4\,dx = {} & \frac{(ex+d)b^4n^4\log(ex+d)^4}{e} - \frac{4(ex+d)b^4n^4\log(ex+d)^3}{e} \\
& + \frac{4(ex+d)b^4n^3\log(ex+d)^3\log(c)}{e} \\
& + \frac{12(ex+d)b^4n^4\log(ex+d)^2}{e} \\
& + \frac{4(ex+d)ab^3n^3\log(ex+d)^3}{e} \\
& - \frac{12(ex+d)b^4n^3\log(ex+d)^2\log(c)}{e} \\
& + \frac{6(ex+d)b^4n^2\log(ex+d)^2\log(c)^2}{e} \\
& - \frac{24(ex+d)b^4n^4\log(ex+d)}{e} \\
& - \frac{12(ex+d)ab^3n^3\log(ex+d)^2}{e} \\
& + \frac{24(ex+d)b^4n^3\log(ex+d)\log(c)}{e} \\
& + \frac{12(ex+d)ab^3n^2\log(ex+d)^2\log(c)}{e} \\
& - \frac{12(ex+d)b^4n^2\log(ex+d)\log(c)^2}{e} \\
& + \frac{4(ex+d)b^4n\log(ex+d)\log(c)^3}{e} \\
& + \frac{24(ex+d)b^4n^4}{e} + \frac{24(ex+d)ab^3n^3\log(ex+d)}{e} \\
& + \frac{6(ex+d)a^2b^2n^2\log(ex+d)^2}{e} - \frac{24(ex+d)b^4n^3\log(c)}{e} \\
& - \frac{24(ex+d)ab^3n^2\log(ex+d)\log(c)}{e} \\
& + \frac{12(ex+d)b^4n^2\log(c)^2}{e} \\
& + \frac{12(ex+d)ab^3n\log(ex+d)\log(c)^2}{e} \\
& - \frac{4(ex+d)b^4n\log(c)^3}{e} + \frac{(ex+d)b^4\log(c)^4}{e} \\
& - \frac{24(ex+d)ab^3n^3}{e} - \frac{12(ex+d)a^2b^2n^2\log(ex+d)}{e} \\
& + \frac{24(ex+d)ab^3n^2\log(c)}{e} \\
& + \frac{12(ex+d)a^2b^2n\log(ex+d)\log(c)}{e} \\
& - \frac{12(ex+d)ab^3n\log(c)^2}{e} + \frac{4(ex+d)ab^3\log(c)^3}{e} \\
& + \frac{12(ex+d)a^2b^2n^2}{e} + \frac{4(ex+d)a^3bn\log(ex+d)}{e} \\
& 12(ex+d)a^2b^2n\log(c) \quad 6(ex+d)a^2b^2\log(c)^2
\end{aligned}
$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^4,x, algorithm="giac")
```

```
[Out] (e*x + d)*b^4*n^4*log(e*x + d)^4/e - 4*(e*x + d)*b^4*n^4*log(e*x + d)^3/e +
 4*(e*x + d)*b^4*n^3*log(e*x + d)^3*log(c)/e + 12*(e*x + d)*b^4*n^4*log(e*x
 + d)^2/e + 4*(e*x + d)*a*b^3*n^3*log(e*x + d)^3/e - 12*(e*x + d)*b^4*n^3*l
og(e*x + d)^2*log(c)/e + 6*(e*x + d)*b^4*n^2*log(e*x + d)^2*log(c)^2/e - 24
*(e*x + d)*b^4*n^4*log(e*x + d)/e - 12*(e*x + d)*a*b^3*n^3*log(e*x + d)^2/e
 + 24*(e*x + d)*b^4*n^3*log(e*x + d)*log(c)/e + 12*(e*x + d)*a*b^3*n^2*log(
e*x + d)^2*log(c)/e - 12*(e*x + d)*b^4*n^2*log(e*x + d)*log(c)^2/e + 4*(e*x
 + d)*b^4*n*log(e*x + d)*log(c)^3/e + 24*(e*x + d)*b^4*n^4/e + 24*(e*x + d)
*a*b^3*n^3*log(e*x + d)/e + 6*(e*x + d)*a^2*b^2*n^2*log(e*x + d)^2/e - 24*(
e*x + d)*b^4*n^3*log(c)/e - 24*(e*x + d)*a*b^3*n^2*log(e*x + d)*log(c)/e +
12*(e*x + d)*b^4*n^2*log(c)^2/e + 12*(e*x + d)*a*b^3*n*log(e*x + d)*log(c)^
2/e - 4*(e*x + d)*b^4*n*log(c)^3/e + (e*x + d)*b^4*log(c)^4/e - 24*(e*x + d
)*a*b^3*n^3/e - 12*(e*x + d)*a^2*b^2*n^2*log(e*x + d)/e + 24*(e*x + d)*a*b^
3*n^2*log(c)/e + 12*(e*x + d)*a^2*b^2*n*log(e*x + d)*log(c)/e - 12*(e*x + d
)*a*b^3*n*log(c)^2/e + 4*(e*x + d)*a*b^3*log(c)^3/e + 12*(e*x + d)*a^2*b^2*
n^2/e + 4*(e*x + d)*a^3*b*n*log(e*x + d)/e - 12*(e*x + d)*a^2*b^2*n*log(c)/
e + 6*(e*x + d)*a^2*b^2*log(c)^2/e - 4*(e*x + d)*a^3*b*n/e + 4*(e*x + d)*a^
3*b*log(c)/e + (e*x + d)*a^4/e
```

## Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.10

$$\int (a+b\log(c(d+ex)^n))^4\,dx$$
$$= \ln\left(c\,(d+e\,x)^n\right)^2\left(\frac{6\,(d\,a^2\,b^2-2\,d\,a\,b^3\,n+2\,d\,b^4\,n^2)}{e}+6\,b^2\,x\,\left(a^2-2\,a\,b\,n+2\,b^2\,n^2\right)\right)$$
$$+x\,\left(a^4-4\,a^3\,b\,n+12\,a^2\,b^2\,n^2-24\,a\,b^3\,n^3+24\,b^4\,n^4\right)+\ln\left(c\,(d+e\,x)^n\right)^4\left(b^4\,x+\frac{b^4\,d}{e}\right)$$
$$+\ln\left(c\,(d+e\,x)^n\right)^3\left(\frac{4\,(a\,b^3\,d-b^4\,d\,n)}{e}+4\,b^3\,x\,(a-b\,n)\right)$$
$$-\frac{\ln(d+e\,x)\,\left(-4\,d\,a^3\,b\,n+12\,d\,a^2\,b^2\,n^2-24\,d\,a\,b^3\,n^3+24\,d\,b^4\,n^4\right)}{e}$$
$$+4\,b\,x\,\ln\left(c\,(d+e\,x)^n\right)\,\left(a^3-3\,a^2\,b\,n+6\,a\,b^2\,n^2-6\,b^3\,n^3\right)$$

```
[In] int((a + b*log(c*(d + e*x)^n))^4,x)
```

```
[Out] log(c*(d + e*x)^n)^2*((6*(a^2*b^2*d + 2*b^4*d*n^2 - 2*a*b^3*d*n))/e + 6*b^2
*x*(a^2 + 2*b^2*n^2 - 2*a*b*n)) + x*(a^4 + 24*b^4*n^4 - 24*a*b^3*n^3 + 12*a
^2*b^2*n^2 - 4*a^3*b*n) + log(c*(d + e*x)^n)^4*(b^4*x + (b^4*d)/e) + log(c*
(d + e*x)^n)^3*((4*(a*b^3*d - b^4*d*n))/e + 4*b^3*x*(a - b*n)) - (log(d + e
*x)*(24*b^4*d*n^4 + 12*a^2*b^2*d*n^2 - 4*a^3*b*d*n - 24*a*b^3*d*n^3))/e + 4
*b*x*log(c*(d + e*x)^n)*(a^3 - 6*b^3*n^3 + 6*a*b^2*n^2 - 3*a^2*b*n)
```

# 3.62 $\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 205

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx &= \frac{(a+b\log(c(d+ex)^n))^4\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} \\ &\quad+\frac{4bn(a+b\log(c(d+ex)^n))^3\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g} \\ &\quad-\frac{12b^2n^2(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g} \\ &\quad+\frac{24b^3n^3(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g} \\ &\quad-\frac{24b^4n^4\operatorname{PolyLog}\left(5,-\frac{g(d+ex)}{ef-dg}\right)}{g}\end{aligned}$$

```
[Out] (a+b*ln(c*(e*x+d)^n))^4*ln(e*(g*x+f)/(-d*g+e*f))/g+4*b*n*(a+b*ln(c*(e*x+d)^
n))^3*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g-12*b^2*n^2*(a+b*ln(c*(e*x+d)^n))^2
*polylog(3,-g*(e*x+d)/(-d*g+e*f))/g+24*b^3*n^3*(a+b*ln(c*(e*x+d)^n))*polylo
g(4,-g*(e*x+d)/(-d*g+e*f))/g-24*b^4*n^4*polylog(5,-g*(e*x+d)/(-d*g+e*f))/g
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2443, 2481, 2421, 2430, 6724}

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx=\frac{24b^3n^3\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g}-\frac{12b^2n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g}+\frac{4bn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g}+\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^4}{g}-\frac{24b^4n^4\operatorname{PolyLog}\left(5,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^4/(f + g*x),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])^4*Log[(e*(f + g*x))/(e*f - d*g)])/g + (4*b*n*(a
 + b*Log[c*(d + e*x)^n])^3*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g - (1
2*b^2*n^2*(a + b*Log[c*(d + e*x)^n])^2*PolyLog[3, -((g*(d + e*x))/(e*f - d*
g))])/g + (24*b^3*n^3*(a + b*Log[c*(d + e*x)^n])*PolyLog[4, -((g*(d + e*x))
/(e*f - d*g))])/g - (24*b^4*n^4*PolyLog[5, -((g*(d + e*x))/(e*f - d*g))])/g
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
```

```
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+b\log(c(d+ex)^n))^4\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}-\frac{(4ben)\int\frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))^4\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}\\
&\quad-\frac{(4bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^3\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))^4\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{4bn(a+b\log(c(d+ex)^n))^3\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g}\\
&\quad-\frac{(12b^2n^2)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))^4\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{4bn(a+b\log(c(d+ex)^n))^3\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g}\\
&\quad-\frac{12b^2n^2(a+b\log(c(d+ex)^n))^2\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g}\\
&\quad+\frac{(24b^3n^3)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\operatorname{Li}_3\left(-\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g}
\end{aligned}$$

$$= \frac{(a+b\log(c(d+ex)^n))^4\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{4bn(a+b\log(c(d+ex)^n))^3\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{12b^2n^2(a+b\log(c(d+ex)^n))^2\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} + \frac{24b^3n^3(a+b\log(c(d+ex)^n))\operatorname{Li}_4\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{(24b^4n^4)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_4\left(-\frac{gx}{ef-dg}\right)}{x}dx, x, d+ex\right)}{g}$$

$$= \frac{(a+b\log(c(d+ex)^n))^4\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{4bn(a+b\log(c(d+ex)^n))^3\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{12b^2n^2(a+b\log(c(d+ex)^n))^2\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} + \frac{24b^3n^3(a+b\log(c(d+ex)^n))\operatorname{Li}_4\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{24b^4n^4\operatorname{Li}_5\left(-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 503 vs. $2(205) = 410$.

Time = 0.18 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.45

$$\int\frac{(a+b\log(c(d+ex)^n))^4}{f+gx}dx$$

$$= \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^4\log(f+gx)+4bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))^3\left(\log\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^4/(f + g*x),x]
```

```
[Out] ((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^4*Log[f + g*x] + 4*b*n*(a -
b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*(Log[d + e*x]*Log[(e*(f + g*x))/
(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + 6*b^2*n^2*(a - b
*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(Log[d + e*x]^2*Log[(e*(f + g*x))
/(e*f - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2
*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]) - 4*b^3*n^3*(-a + b*n*Log[d + e*
x] - b*Log[c*(d + e*x)^n])*(Log[d + e*x]^3*Log[(e*(f + g*x))/(e*f - d*g)] +
 3*Log[d + e*x]^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 6*Log[d + e*x]
*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] + 6*PolyLog[4, (g*(d + e*x))/(-(e
*f) + d*g)]) + b^4*n^4*(Log[d + e*x]^4*Log[(e*(f + g*x))/(e*f - d*g)] + 4*L
og[d + e*x]^3*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 12*Log[d + e*x]^2*
PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] + 24*Log[d + e*x]*PolyLog[4, (g*(d
 + e*x))/(-(e*f) + d*g)] - 24*PolyLog[5, (g*(d + e*x))/(-(e*f) + d*g)]))/g
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.02 (sec) , antiderivative size = 2172, normalized size of antiderivative = 10.60

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 2172 |

```
[In] int((a+b*ln(c*(e*x+d)^n))^4/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] -12*b^4*n^2/g*ln((e*x+d)^n)^2*polylog(3,g*(e*x+d)/(d*g-e*f))+24*b^4*n^3/g*l
n((e*x+d)^n)*polylog(4,g*(e*x+d)/(d*g-e*f))+3*b^4*n^4/g*ln(e*x+d)^4*ln(1-g*
(e*x+d)/(d*g-e*f))+4*b^4*n^4/g*ln(e*x+d)^3*polylog(2,g*(e*x+d)/(d*g-e*f))+b
^4*ln(g*(e*x+d)-d*g+e*f)/g*ln(e*x+d)^4*n^4-4*b^4*n^4*dilog((g*(e*x+d)-d*g+e
*f)/(-d*g+e*f))/g*ln(e*x+d)^3+4*b^4*n*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))
/g*ln((e*x+d)^n)^3-4*b^4*n^4*ln(e*x+d)^4*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))
/g+1/16*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(
I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b
-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^4*ln(g*x+f)/g+3/2*(-I*b*Pi*csg
n(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d
)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+
d)^n)^3*b+2*b*ln(c)+2*a)^2*b^2*((ln((e*x+d)^n)-n*ln(e*x+d))^2*ln(g*(e*x+d)-
d*g+e*f)/g+n^2/g*ln(e*x+d)^2*ln(1-g*(e*x+d)/(d*g-e*f))+2*n^2/g*ln(e*x+d)*po
lylog(2,g*(e*x+d)/(d*g-e*f))-2*n^2/g*polylog(3,g*(e*x+d)/(d*g-e*f))+2*n*(ln
((e*x+d)^n)-n*ln(e*x+d))*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g+2*n*(ln((e
*x+d)^n)-n*ln(e*x+d))*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g)+1/2*(
-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn
(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csg
n(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^3*b*(ln((e*x+d)^n)*ln(g*x+f)/g-1/g*n*e*
(dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e+ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d
*g-e*f))/e))+2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*P
i*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)
^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b^3/e*(e*(ln((e*x+d)^n)
-n*ln(e*x+d))^3*ln(g*(e*x+d)-d*g+e*f)/g+e*n^3/g*(ln(e*x+d)^3*ln(1-g*(e*x+d)
/(d*g-e*f))+3*ln(e*x+d)^2*polylog(2,g*(e*x+d)/(d*g-e*f))-6*ln(e*x+d)*polylo
g(3,g*(e*x+d)/(d*g-e*f))+6*polylog(4,g*(e*x+d)/(d*g-e*f)))+3*e*n*(ln((e*x+d
)^n)-n*ln(e*x+d))^2*(dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g+ln(e*x+d)*ln((
g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g)+3*e*n^2*(ln((e*x+d)^n)-n*ln(e*x+d))/g*(ln
(e*x+d)^2*ln(1-g*(e*x+d)/(d*g-e*f))+2*ln(e*x+d)*polylog(2,g*(e*x+d)/(d*g-e*
f))-2*polylog(3,g*(e*x+d)/(d*g-e*f))))-4*b^4*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*
x+d)^n)*ln(e*x+d)^3*n^3+6*b^4*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)^2*ln(e*
x+d)^2*n^2-4*b^4*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)^3*ln(e*x+d)*n+12*b^4
*n^3*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln((e*x+d)^n)*ln(e*x+d)^2-12*b
^4*n^2*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln((e*x+d)^n)^2*ln(e*x+d)+12
*b^4*n^3*ln(e*x+d)^3*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln((e*x+d)^n)-12*
```

```
b^4*n^2*ln(e*x+d)^2*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln((e*x+d)^n)^2+4*
b^4*n*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln((e*x+d)^n)^3-8*b^4*
n^3/g*ln((e*x+d)^n)*ln(e*x+d)^3*ln(1-g*(e*x+d)/(d*g-e*f))+6*b^4*n^2/g*ln((e
*x+d)^n)^2*ln(e*x+d)^2*ln(1-g*(e*x+d)/(d*g-e*f))-12*b^4*n^3/g*ln((e*x+d)^n)
*ln(e*x+d)^2*polylog(2,g*(e*x+d)/(d*g-e*f))+12*b^4*n^2/g*ln((e*x+d)^n)^2*ln
(e*x+d)*polylog(2,g*(e*x+d)/(d*g-e*f))-24*b^4*n^4/g*polylog(5,g*(e*x+d)/(d*
g-e*f))+b^4*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)^4
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx=\int \frac{(b\log((ex+d)^n c)+a)^4}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^4/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((b^4*log((e*x + d)^n*c)^4 + 4*a*b^3*log((e*x + d)^n*c)^3 + 6*a^2*b
^2*log((e*x + d)^n*c)^2 + 4*a^3*b*log((e*x + d)^n*c) + a^4)/(g*x + f), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx=\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**4/(g*x+f),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**4/(f + g*x), x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx=\int \frac{(b\log((ex+d)^n c)+a)^4}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^4/(g*x+f),x, algorithm="maxima")
```

```
[Out] a^4*log(g*x + f)/g + integrate((b^4*log((e*x + d)^n)^4 + b^4*log(c)^4 + 4*a
*b^3*log(c)^3 + 6*a^2*b^2*log(c)^2 + 4*a^3*b*log(c) + 4*(b^4*log(c) + a*b^3
)*log((e*x + d)^n)^3 + 6*(b^4*log(c)^2 + 2*a*b^3*log(c) + a^2*b^2)*log((e*x
 + d)^n)^2 + 4*(b^4*log(c)^3 + 3*a*b^3*log(c)^2 + 3*a^2*b^2*log(c) + a^3*b)
*log((e*x + d)^n))/(g*x + f), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^4}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^4/(g*x+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^4/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{f+gx}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^4}{f+g\,x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^4/(f + g*x),x)
[Out] int((a + b*log(c*(d + e*x)^n))^4/(f + g*x), x)
```

# 3.63 $\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 248

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2} dx = \frac{(d+ex)(a+b\log(c(d+ex)^n))^4}{(ef-dg)(f+gx)} - \frac{4ben(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} - \frac{12b^2en^2(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)} + \frac{24b^3en^3(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)} - \frac{24b^4en^4\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)}$$

```
[Out] (e*x+d)*(a+b*ln(c*(e*x+d)^n))^4/(-d*g+e*f)/(g*x+f)-4*b*e*n*(a+b*ln(c*(e*x+d
)^n))^3*ln(e*(g*x+f)/(-d*g+e*f))/g/(-d*g+e*f)-12*b^2*e*n^2*(a+b*ln(c*(e*x+d
)^n))^2*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g/(-d*g+e*f)+24*b^3*e*n^3*(a+b*ln(
c*(e*x+d)^n))*polylog(3,-g*(e*x+d)/(-d*g+e*f))/g/(-d*g+e*f)-24*b^4*e*n^4*po
lylog(4,-g*(e*x+d)/(-d*g+e*f))/g/(-d*g+e*f)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2444, 2443, 2481, 2421, 2430, 6724}

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx = \frac{24b^3en^3\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g(ef-dg)} - \frac{12b^2en^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g(ef-dg)} - \frac{4ben\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g(ef-dg)} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^4}{(f+gx)(ef-dg)} - \frac{24b^4en^4\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^4/(f + g*x)^2,x]
```

```
[Out] ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^4)/((e*f - d*g)*(f + g*x)) - (4*b*e*n
*(a + b*Log[c*(d + e*x)^n])^3*Log[(e*(f + g*x))/(e*f - d*g)])/(g*(e*f - d*g
)) - (12*b^2*e*n^2*(a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((g*(d + e*x))/
(e*f - d*g))])/(g*(e*f - d*g)) + (24*b^3*e*n^3*(a + b*Log[c*(d + e*x)^n])*P
olyLog[3, -((g*(d + e*x))/(e*f - d*g))])/(g*(e*f - d*g)) - (24*b^4*e*n^4*Po
lyLog[4, -((g*(d + e*x))/(e*f - d*g))])/(g*(e*f - d*g))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{(ef-dg)(f+gx)}-\frac{(4ben)\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx}{ef-dg}\\
&= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{(ef-dg)(f+gx)}-\frac{4ben\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)}\\
&\quad+\frac{\left(12b^2e^2n^2\right)\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g(ef-dg)}\\
&= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{(ef-dg)(f+gx)}-\frac{4ben\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)}\\
&\quad+\frac{\left(12b^2en^2\right)\operatorname{Subst}\left(\int\frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g(ef-dg)}
\end{aligned}
$$

$$= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{(ef-dg)(f+gx)} - \frac{4ben(a+b\log\left(c(d+ex)^n\right))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} - \frac{12b^2en^2(a+b\log\left(c(d+ex)^n\right))^2\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)} + \frac{(24b^3en^3)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}dx,x,d+ex\right)}{g(ef-dg)}$$

$$= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{(ef-dg)(f+gx)} - \frac{4ben(a+b\log\left(c(d+ex)^n\right))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} - \frac{12b^2en^2(a+b\log\left(c(d+ex)^n\right))^2\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)} + \frac{24b^3en^3(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)} - \frac{(24b^4en^4)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{gx}{ef-dg}\right)}{x}dx,x,d+ex\right)}{g(ef-dg)}$$

$$= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{(ef-dg)(f+gx)} - \frac{4ben(a+b\log\left(c(d+ex)^n\right))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)} - \frac{12b^2en^2(a+b\log\left(c(d+ex)^n\right))^2\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)} + \frac{24b^3en^3(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)} - \frac{24b^4en^4\operatorname{Li}_4\left(-\frac{g(d+ex)}{ef-dg}\right)}{g(ef-dg)}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 531 vs. 2(248) = 496.

Time = 0.46 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.14

$$\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^4}{(f+gx)^2}\,dx$$
$$= \frac{-\left((ef-dg)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^4\right)+4bn(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right))^3\left(\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^4/(f + g*x)^2,x]
```

```
[Out] (-((e*f - d*g)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^4) + 4*b*n*(a
- b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*(g*(d + e*x)*Log[d + e*x] - e*
```

```
(f + g*x)*Log[(e*(f + g*x))/(e*f - d*g)]) + 6*b^2*n^2*(a - b*n*Log[d + e*x]
 + b*Log[c*(d + e*x)^n])^2*(Log[d + e*x]*(g*(d + e*x)*Log[d + e*x] - 2*e*(f
 + g*x)*Log[(e*(f + g*x))/(e*f - d*g)]) - 2*e*(f + g*x)*PolyLog[2, (g*(d +
e*x))/(-(e*f) + d*g)]) + 4*b^3*n^3*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x
)^n])*(Log[d + e*x]^2*(g*(d + e*x)*Log[d + e*x] - 3*e*(f + g*x)*Log[(e*(f +
 g*x))/(e*f - d*g)]) - 6*e*(f + g*x)*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/
(-(e*f) + d*g)] + 6*e*(f + g*x)*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]) +
 b^4*n^4*(g*(d + e*x)*Log[d + e*x]^4 - 4*e*(f + g*x)*Log[d + e*x]^3*Log[(e*
(f + g*x))/(e*f - d*g)] - 12*e*(f + g*x)*Log[d + e*x]^2*PolyLog[2, (g*(d +
e*x))/(-(e*f) + d*g)] + 24*e*(f + g*x)*Log[d + e*x]*PolyLog[3, (g*(d + e*x)
)/(-(e*f) + d*g)] - 24*e*(f + g*x)*PolyLog[4, (g*(d + e*x))/(-(e*f) + d*g)]
))/(g*(e*f - d*g)*(f + g*x))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.22 (sec) , antiderivative size = 2156, normalized size of antiderivative = 8.69

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 2156 |

```
[In] int((a+b*ln(c*(e*x+d)^n))^4/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 24*b^4/g*n^4*e/(d*g-e*f)*polylog(4,-g*(e*x+d)/(-d*g+e*f))-3*b^4/g*n^4*e/(-d
*g+e*f)*ln(e*x+d)^4-2*b^4/g*n^4*e/(d*g-e*f)*ln(e*x+d)^4-12*b^4/g*n^4*e/(d*g
-e*f)*ln(e*x+d)^2*polylog(2,-g*(e*x+d)/(-d*g+e*f))-4*b^4/g*n^4*e/(d*g-e*f)*
ln(g*(e*x+d)-d*g+e*f)*ln(e*x+d)^3+4*b^4/g*n*e/(d*g-e*f)*ln(g*(e*x+d)-d*g+e*
f)*ln((e*x+d)^n)^3+4*b^4/g*n^3*e/(-d*g+e*f)*ln(e*x+d)^3*ln((e*x+d)^n)-8*b^4
/g*n^4*e/(d*g-e*f)*ln(e*x+d)^3*ln(1+g*(e*x+d)/(-d*g+e*f))+12*b^4/g*n^3*e/(d
*g-e*f)*ln(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)*ln(e*x+d)^2-12*b^4/g*n^2*e/(d*g
-e*f)*ln(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)^2*ln(e*x+d)-24*b^4/g*n^3*e/(d*g-e
*f)*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln((e*x+d)^n)*ln(e*x+d)-24*b^4/g*
n^3*e/(d*g-e*f)*ln(e*x+d)^2*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln((e*x+d)^n
)+12*b^4/g*n^2*e/(d*g-e*f)*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln(
(e*x+d)^n)^2+12*b^4/g*n^3*e/(d*g-e*f)*ln(e*x+d)^2*ln(1+g*(e*x+d)/(-d*g+e*f)
)*ln((e*x+d)^n)+24*b^4/g*n^3*e/(d*g-e*f)*ln(e*x+d)*polylog(2,-g*(e*x+d)/(-d
*g+e*f))*ln((e*x+d)^n)+6*b^4/g*n^2*e/(d*g-e*f)*ln(e*x+d)^2*ln((e*x+d)^n)^2-
4*b^4/g*n*e/(d*g-e*f)*ln(e*x+d)*ln((e*x+d)^n)^3+12*b^4/g*n^2*e/(d*g-e*f)*di
log((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln((e*x+d)^n)^2+12*b^4/g*n^4*e/(d*g-e*f
)*ln(e*x+d)^3*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))-24*b^4/g*n^3*e/(d*g-e*f)*p
olylog(3,-g*(e*x+d)/(-d*g+e*f))*ln((e*x+d)^n)+12*b^4/g*n^4*e/(d*g-e*f)*dilo
g((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln(e*x+d)^2+1/2*(-I*b*Pi*csgn(I*c*(e*x+d)
^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi
*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b
*ln(c)+2*a)^3*b*(-ln((e*x+d)^n)/(g*x+f)/g+1/g*n*e*(-1/(d*g-e*f)*ln(e*x+d)+1
```

```
/(d*g-e*f)*ln(g*x+f)))-1/16*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(
e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csg
n(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^4/(g*x+f)/
g-b^4*ln((e*x+d)^n)^4/(g*x+f)/g+2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*cs
gn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^
n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b^3*
(-ln((e*x+d)^n)^3/(g*x+f)/g+3/g*n*(e*(ln((e*x+d)^n)-n*ln(e*x+d))^2*(1/(d*g-
e*f)*ln(g*(e*x+d)-d*g+e*f)-1/(d*g-e*f)*ln(e*x+d))+e*n^2*(1/(d*g-e*f)*(ln(e*
x+d)^2*ln(1+g*(e*x+d)/(-d*g+e*f))+2*ln(e*x+d)*polylog(2,-g*(e*x+d)/(-d*g+e*
f))-2*polylog(3,-g*(e*x+d)/(-d*g+e*f)))+1/3/(-d*g+e*f)*ln(e*x+d)^3)+2*e*n*(
ln((e*x+d)^n)-n*ln(e*x+d))*(g/(d*g-e*f)*(dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*
f))/g+ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g)-1/2/(d*g-e*f)*ln(e*x+
d)^2)))+3/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*c
sgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)
^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^2*b^2*(-ln((e*x+d)^n)^2/(g
*x+f)/g+2/g*n*e*(-ln((e*x+d)^n)/(d*g-e*f)*ln(e*x+d)+ln((e*x+d)^n)/(d*g-e*f)
*ln(g*x+f)-e*n*(1/(d*g-e*f)*(dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e+ln(g*x+
f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e)-1/2/(d*g-e*f)/e*ln(e*x+d)^2)))
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^4}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^4/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((b^4*log((e*x + d)^n*c)^4 + 4*a*b^3*log((e*x + d)^n*c)^3 + 6*a^2*b
^2*log((e*x + d)^n*c)^2 + 4*a^3*b*log((e*x + d)^n*c) + a^4)/(g^2*x^2 + 2*f*
g*x + f^2), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^4}{(f+gx)^2}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**4/(g*x+f)**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**4/(f + g*x)**2, x)
```

## Maxima [F]

$$\int \frac{(a+b\log (c(d+ex)^n))^4}{(f+gx)^2}\,dx = \int \frac{(b\log \left((ex+d)^n c\right)+a)^4}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^4/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] 4*a^3*b*e*n*(log(e*x + d)/(e*f*g - d*g^2) - log(g*x + f)/(e*f*g - d*g^2)) -
 b^4*log((e*x + d)^n)^4/(g^2*x + f*g) - 4*a^3*b*log((e*x + d)^n*c)/(g^2*x +
 f*g) - a^4/(g^2*x + f*g) + integrate((b^4*d*g*log(c)^4 + 4*a*b^3*d*g*log(c
)^3 + 6*a^2*b^2*d*g*log(c)^2 + 4*(a*b^3*d*g + (e*f*n + d*g*log(c))*b^4 + (a
*b^3*e*g + (e*g*n + e*g*log(c))*b^4)*x)*log((e*x + d)^n)^3 + 6*(b^4*d*g*log
(c)^2 + 2*a*b^3*d*g*log(c) + a^2*b^2*d*g + (b^4*e*g*log(c)^2 + 2*a*b^3*e*g*
log(c) + a^2*b^2*e*g)*x)*log((e*x + d)^n)^2 + (b^4*e*g*log(c)^4 + 4*a*b^3*e
*g*log(c)^3 + 6*a^2*b^2*e*g*log(c)^2)*x + 4*(b^4*d*g*log(c)^3 + 3*a*b^3*d*g
*log(c)^2 + 3*a^2*b^2*d*g*log(c) + (b^4*e*g*log(c)^3 + 3*a*b^3*e*g*log(c)^2
 + 3*a^2*b^2*e*g*log(c))*x)*log((e*x + d)^n))/(e*g^3*x^3 + d*f^2*g + (2*e*f
*g^2 + d*g^3)*x^2 + (e*f^2*g + 2*d*f*g^2)*x), x)
```

## Giac [F]

$$\int \frac{(a+b\log (c(d+ex)^n))^4}{(f+gx)^2}\,dx = \int \frac{(b\log \left((ex+d)^n c\right)+a)^4}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^4/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^4/(g*x + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log (c(d+ex)^n))^4}{(f+gx)^2}\,dx = \int \frac{(a+b\,\ln \left(c\,(d+e\,x)^n\right))^4}{(f+g\,x)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^4/(f + g*x)^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^4/(f + g*x)^2, x)
```

# 3.64 $\int \log(a+bx)\,dx$



## Optimal result

Integrand size = 6, antiderivative size = 19

$$\int \log(a+bx)\,dx = -x + \frac{(a+bx)\log(a+bx)}{b}$$

```
[Out] -x+(b*x+a)*ln(b*x+a)/b
```

## Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2436, 2332}

$$\int \log(a+bx)\,dx = \frac{(a+bx)\log(a+bx)}{b} - x$$

```
[In] Int[Log[a + b*x],x]
```

```
[Out] -x + ((a + b*x)*Log[a + b*x])/b
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}(\int \log(x)\,dx, x, a+bx)}{b}$$
$$= -x + \frac{(a+bx)\log(a+bx)}{b}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \log(a+bx)\,dx = -x + \frac{(a+bx)\log(a+bx)}{b}$$

```
[In] Integrate[Log[a + b*x],x]
[Out] -x + ((a + b*x)*Log[a + b*x])/b
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

| method | result | size |
|---|---|---|
| norman | $x\ln(bx+a)+\frac{a\ln(bx+a)}{b}-x$ | 24 |
| risch | $x\ln(bx+a)+\frac{a\ln(bx+a)}{b}-x$ | 24 |
| derivativedivides | $\frac{(bx+a)\ln(bx+a)-bx-a}{b}$ | 25 |
| default | $\frac{(bx+a)\ln(bx+a)-bx-a}{b}$ | 25 |
| parallelrisch | $\frac{\ln(bx+a)xb-bx+a\ln(bx+a)+a}{b}$ | 28 |
| parts | $x\ln(bx+a)-b\left(\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}\right)$ | 31 |

```
[In] int(ln(b*x+a),x,method=_RETURNVERBOSE)
[Out] x*ln(b*x+a)+a/b*ln(b*x+a)-x
```

## Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \log(a+bx)\,dx = -\frac{bx-(bx+a)\log(bx+a)}{b}$$

```
[In] integrate(log(b*x+a),x, algorithm="fricas")
[Out] -(b*x - (b*x + a)*log(b*x + a))/b
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \log(a+bx)\,dx = -b\left(-\frac{a\log(a+bx)}{b^2}+\frac{x}{b}\right)+x\log(a+bx)$$

```
[In] integrate(ln(b*x+a),x)
[Out] -b*(-a*log(a + b*x)/b**2 + x/b) + x*log(a + b*x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \log(a+bx)\,dx = -\frac{bx-(bx+a)\log(bx+a)+a}{b}$$

```
[In] integrate(log(b*x+a),x, algorithm="maxima")
[Out] -(b*x - (b*x + a)*log(b*x + a) + a)/b
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \log(a+bx)\,dx = -\frac{bx-(bx+a)\log(bx+a)+a}{b}$$

```
[In] integrate(log(b*x+a),x, algorithm="giac")
[Out] -(b*x - (b*x + a)*log(b*x + a) + a)/b
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \log(a+bx)\,dx = x\ \ln(a+b\,x) - x + \frac{a\ \ln(a+b\,x)}{b}$$

```
[In] int(log(a + b*x),x)
[Out] x*log(a + b*x) - x + (a*log(a + b*x))/b
```

# 3.65 $\int \log^2(a+bx)\,dx$



## Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \log^2(a+bx)\,dx = 2x - \frac{2(a+bx)\log(a+bx)}{b} + \frac{(a+bx)\log^2(a+bx)}{b}$$

```
[Out] 2*x-2*(b*x+a)*ln(b*x+a)/b+(b*x+a)*ln(b*x+a)^2/b
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2436, 2333, 2332}

$$\int \log^2(a+bx)\,dx = \frac{(a+bx)\log^2(a+bx)}{b} - \frac{2(a+bx)\log(a+bx)}{b} + 2x$$

```
[In] Int[Log[a + b*x]^2,x]
```

```
[Out] 2*x - (2*(a + b*x)*Log[a + b*x])/b + ((a + b*x)*Log[a + b*x]^2)/b
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \log^2(x)\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)\log^2(a+bx)}{b} - \frac{2\text{Subst}\left(\int \log(x)\,dx, x, a+bx\right)}{b}\\ &= 2x - \frac{2(a+bx)\log(a+bx)}{b} + \frac{(a+bx)\log^2(a+bx)}{b}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \log^2(a+bx)\,dx = \frac{2bx - 2(a+bx)\log(a+bx) + (a+bx)\log^2(a+bx)}{b}$$

```
[In] Integrate[Log[a + b*x]^2,x]
[Out] (2*b*x - 2*(a + b*x)*Log[a + b*x] + (a + b*x)*Log[a + b*x]^2)/b
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(bx+a)^2(bx+a)-2(bx+a)\ln(bx+a)+2bx+2a}{b}$ | 40 |
| default | $\frac{\ln(bx+a)^2(bx+a)-2(bx+a)\ln(bx+a)+2bx+2a}{b}$ | 40 |
| risch | $\frac{(bx+a)\ln(bx+a)^2}{b} - 2x\ln(bx+a) + 2x - \frac{2a\ln(bx+a)}{b}$ | 43 |
| norman | $x\ln(bx+a)^2 + \frac{a\ln(bx+a)^2}{b} + 2x - 2x\ln(bx+a) - \frac{2a\ln(bx+a)}{b}$ | 49 |
| parallelrisch | $\frac{x\ln(bx+a)^2b-2\ln(bx+a)xb+\ln(bx+a)^2a+2bx-2a\ln(bx+a)-2a}{b}$ | 53 |

```
[In] int(ln(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(ln(b*x+a)^2*(b*x+a)-2*(b*x+a)*ln(b*x+a)+2*b*x+2*a)
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \log^2(a+bx)\,dx = \frac{(bx+a)\log{(bx+a)}^2 + 2\,bx - 2\,(bx+a)\log{(bx+a)}}{b}$$

```
[In] integrate(log(b*x+a)^2,x, algorithm="fricas")
[Out] ((b*x + a)*log(b*x + a)^2 + 2*b*x - 2*(b*x + a)*log(b*x + a))/b
```

## Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \log^2(a+bx)\,dx = 2b\left(-\frac{a\log{(a+bx)}}{b^2} + \frac{x}{b}\right) - 2x\log{(a+bx)} + \frac{(a+bx)\log{(a+bx)}^2}{b}$$

```
[In] integrate(ln(b*x+a)**2,x)
[Out] 2*b*(-a*log(a + b*x)/b**2 + x/b) - 2*x*log(a + b*x) + (a + b*x)*log(a + b*x
)**2/b
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \log^2(a+bx)\,dx = \frac{(bx+a)\left(\log{(bx+a)}^2 - 2\,\log{(bx+a)} + 2\right)}{b}$$

```
[In] integrate(log(b*x+a)^2,x, algorithm="maxima")
[Out] (b*x + a)*(log(b*x + a)^2 - 2*log(b*x + a) + 2)/b
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \log^2(a+bx)\,dx = \frac{(bx+a)\log(bx+a)^2}{b} - \frac{2\,(bx+a)\log(bx+a)}{b} + \frac{2\,(bx+a)}{b}$$

```
[In] integrate(log(b*x+a)^2,x, algorithm="giac")
[Out] (b*x + a)*log(b*x + a)^2/b - 2*(b*x + a)*log(b*x + a)/b + 2*(b*x + a)/b
```

## Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \log^2(a+bx)\,dx = 2\,x - 2\,x\,\ln(a+b\,x) + x\ln(a+b\,x)^2 + \frac{a\ln(a+b\,x)^2}{b} - \frac{2\,a\,\ln(a+b\,x)}{b}$$

```
[In] int(log(a + b*x)^2,x)
[Out] 2*x - 2*x*log(a + b*x) + x*log(a + b*x)^2 + (a*log(a + b*x)^2)/b - (2*a*log
(a + b*x))/b
```

# 3.66 $\int \log^3(a+bx)\,dx$



## Optimal result

Integrand size = 8, antiderivative size = 55

$$\int \log^3(a+bx)\,dx = -6x + \frac{6(a+bx)\log(a+bx)}{b} - \frac{3(a+bx)\log^2(a+bx)}{b} + \frac{(a+bx)\log^3(a+bx)}{b}$$

```
[Out] -6*x+6*(b*x+a)*ln(b*x+a)/b-3*(b*x+a)*ln(b*x+a)^2/b+(b*x+a)*ln(b*x+a)^3/b
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2436, 2333, 2332}

$$\int \log^3(a+bx)\,dx = \frac{(a+bx)\log^3(a+bx)}{b} - \frac{3(a+bx)\log^2(a+bx)}{b} + \frac{6(a+bx)\log(a+bx)}{b} - 6x$$

```
[In] Int[Log[a + b*x]^3,x]
```

```
[Out] -6*x + (6*(a + b*x)*Log[a + b*x])/b - (3*(a + b*x)*Log[a + b*x]^2)/b + ((a
+ b*x)*Log[a + b*x]^3)/b
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \log^3(x)\,dx, x, a+bx\right)}{b}\\
&= \frac{(a+bx)\log^3(a+bx)}{b} - \frac{3\text{Subst}\left(\int \log^2(x)\,dx, x, a+bx\right)}{b}\\
&= -\frac{3(a+bx)\log^2(a+bx)}{b} + \frac{(a+bx)\log^3(a+bx)}{b} + \frac{6\text{Subst}\left(\int \log(x)\,dx, x, a+bx\right)}{b}\\
&= -6x + \frac{6(a+bx)\log(a+bx)}{b} - \frac{3(a+bx)\log^2(a+bx)}{b} + \frac{(a+bx)\log^3(a+bx)}{b}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \log^3(a+bx)\,dx = \frac{-6bx + 6(a+bx)\log(a+bx) - 3(a+bx)\log^2(a+bx) + (a+bx)\log^3(a+bx)}{b}$$

```
[In] Integrate[Log[a + b*x]^3,x]
[Out] (-6*b*x + 6*(a + b*x)*Log[a + b*x] - 3*(a + b*x)*Log[a + b*x]^2 + (a + b*x)
*Log[a + b*x]^3)/b
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| derivativedivides | $\frac{\ln(bx+a)^3(bx+a)-3\ln(bx+a)^2(bx+a)+6(bx+a)\ln(bx+a)-6bx-6a}{b}$ |
| default | $\frac{\ln(bx+a)^3(bx+a)-3\ln(bx+a)^2(bx+a)+6(bx+a)\ln(bx+a)-6bx-6a}{b}$ |
| risch | $\frac{(bx+a)\ln(bx+a)^3}{b}-\frac{3(bx+a)\ln(bx+a)^2}{b}+6x\ln(bx+a)-6x+\frac{6a\ln(bx+a)}{b}$ |
| norman | $x\ln(bx+a)^3+\frac{a\ln(bx+a)^3}{b}-6x+6x\ln(bx+a)-3x\ln(bx+a)^2+\frac{6a\ln(bx+a)}{b}-\frac{3a\ln(bx}{b}$ |
| parallelrisch | $\frac{x\ln(bx+a)^3b-3x\ln(bx+a)^2b+\ln(bx+a)^3a+6\ln(bx+a)xb-3\ln(bx+a)^2a-6bx+6a\ln(bx+a)+6a}{b}$ |

```
[In] int(ln(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(ln(b*x+a)^3*(b*x+a)-3*ln(b*x+a)^2*(b*x+a)+6*(b*x+a)*ln(b*x+a)-6*b*x-6*
a)
```

## Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \log^3(a+bx)\,dx = \frac{(bx+a)\log(bx+a)^3-3(bx+a)\log(bx+a)^2-6bx+6(bx+a)\log(bx+a)}{b}$$

```
[In] integrate(log(b*x+a)^3,x, algorithm="fricas")
[Out] ((b*x + a)*log(b*x + a)^3 - 3*(b*x + a)*log(b*x + a)^2 - 6*b*x + 6*(b*x + a
)*log(b*x + a))/b
```

## Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \log^3(a+bx)\,dx = -6b\left(-\frac{a\log(a+bx)}{b^2}+\frac{x}{b}\right)+6x\log(a+bx) + \frac{(-3a-3bx)\log(a+bx)^2}{b}+\frac{(a+bx)\log(a+bx)^3}{b}$$

```
[In] integrate(ln(b*x+a)**3,x)
[Out] -6*b*(-a*log(a + b*x)/b**2 + x/b) + 6*x*log(a + b*x) + (-3*a - 3*b*x)*log(a
 + b*x)**2/b + (a + b*x)*log(a + b*x)**3/b
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int \log^3(a+bx)\,dx = \frac{\left(\log\left(bx+a\right)^3 - 3\,\log\left(bx+a\right)^2 + 6\,\log\left(bx+a\right) - 6\right)\left(bx+a\right)}{b}$$

```
[In] integrate(log(b*x+a)^3,x, algorithm="maxima")
[Out] (log(b*x + a)^3 - 3*log(b*x + a)^2 + 6*log(b*x + a) - 6)*(b*x + a)/b
```

## Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int \log^3(a+bx)\,dx = \frac{(bx+a)\log\left(bx+a\right)^3}{b} - \frac{3\,(bx+a)\log\left(bx+a\right)^2}{b} + \frac{6\,(bx+a)\log\left(bx+a\right)}{b} - \frac{6\,(bx+a)}{b}$$

```
[In] integrate(log(b*x+a)^3,x, algorithm="giac")
[Out] (b*x + a)*log(b*x + a)^3/b - 3*(b*x + a)*log(b*x + a)^2/b + 6*(b*x + a)*log
(b*x + a)/b - 6*(b*x + a)/b
```

## Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \log^3(a+bx)\,dx = 6\,x\,\ln\left(a+b\,x\right) - 6\,x - 3\,x\ln\left(a+b\,x\right)^2 + x\ln\left(a+b\,x\right)^3 - \frac{3\,a\ln\left(a+b\,x\right)^2}{b} + \frac{a\ln\left(a+b\,x\right)^3}{b} + \frac{6\,a\,\ln\left(a+b\,x\right)}{b}$$

```
[In] int(log(a + b*x)^3,x)
[Out] 6*x*log(a + b*x) - 6*x - 3*x*log(a + b*x)^2 + x*log(a + b*x)^3 - (3*a*log(a
 + b*x)^2)/b + (a*log(a + b*x)^3)/b + (6*a*log(a + b*x))/b
```

# 3.67 $\int \log(a+bx+cx)\,dx$



## Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \log(a+bx+cx)\,dx = -x+\frac{(a+(b+c)x)\log(a+(b+c)x)}{b+c}$$

```
[Out] -x+(a+(b+c)*x)*ln(a+(b+c)*x)/(b+c)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2494, 2436, 2332}

$$\int \log(a+bx+cx)\,dx = \frac{(a+x(b+c))\log(a+x(b+c))}{b+c}-x$$

```
[In] Int[Log[a + b*x + c*x],x]
```

```
[Out] -x + ((a + (b + c)*x)*Log[a + (b + c)*x])/(b + c)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \log(a+(b+c)x)\,dx\\&= \frac{\text{Subst}(\int \log(x)\,dx, x, a+(b+c)x)}{b+c}\\&= -x+\frac{(a+(b+c)x)\log(a+(b+c)x)}{b+c}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \log(a+bx+cx)\,dx = -x+\frac{(a+(b+c)x)\log(a+(b+c)x)}{b+c}$$

```
[In] Integrate[Log[a + b*x + c*x],x]
[Out] -x + ((a + (b + c)*x)*Log[a + (b + c)*x])/(b + c)
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

| method | result | size |
|---|---|---|
| norman | $x\ln(bx+xc+a)+\frac{a\ln(bx+xc+a)}{b+c}-x$ | 32 |
| derivativedivides | $\frac{(a+(b+c)x)\ln(a+(b+c)x)-a-(b+c)x}{b+c}$ | 33 |
| default | $\frac{(a+(b+c)x)\ln(a+(b+c)x)-a-(b+c)x}{b+c}$ | 33 |
| parts | $x\ln(bx+xc+a)-(b+c)\left(\frac{x}{b+c}-\frac{a\ln(bx+xc+a)}{(b+c)^2}\right)$ | 43 |
| risch | $x\ln(bx+xc+a)+\frac{a\ln(a+(b+c)x)}{b+c}-\frac{bx}{b+c}-\frac{xc}{b+c}$ | 46 |
| parallelrisch | $\frac{x\ln(bx+xc+a)ab+x\ln(bx+xc+a)ac-abx-xca+\ln(bx+xc+a)a^2}{a(b+c)}$ | 60 |

```
[In] int(ln(b*x+c*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(b*x+c*x+a)+a/(b+c)*ln(b*x+c*x+a)-x
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \log(a+bx+cx)\,dx = -\frac{(b+c)x-((b+c)x+a)\log((b+c)x+a)}{b+c}$$

```
[In] integrate(log(b*x+c*x+a),x, algorithm="fricas")
[Out] -((b + c)*x - ((b + c)*x + a)*log((b + c)*x + a))/(b + c)
```

## Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \log(a+bx+cx)\,dx = x\log(a+bx+cx) + (-b-c)\left(-\frac{a\log(a+x(b+c))}{(b+c)^2} + \frac{x}{b+c}\right)$$

```
[In] integrate(ln(b*x+c*x+a),x)
[Out] x*log(a + b*x + c*x) + (-b - c)*(-a*log(a + x*(b + c))/(b + c)**2 + x/(b +
c))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \log(a+bx+cx)\,dx = -\frac{bx+cx-(bx+cx+a)\log(bx+cx+a)+a}{b+c}$$

```
[In] integrate(log(b*x+c*x+a),x, algorithm="maxima")
[Out] -(b*x + c*x - (b*x + c*x + a)*log(b*x + c*x + a) + a)/(b + c)
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \log(a+bx+cx)\,dx=-\frac{bx+cx-(bx+cx+a)\log(bx+cx+a)+a}{b+c}$$

```
[In] integrate(log(b*x+c*x+a),x, algorithm="giac")
[Out] -(b*x + c*x - (b*x + c*x + a)*log(b*x + c*x + a) + a)/(b + c)
```

## Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \log(a+bx+cx)\,dx=x\,\ln(a+b\,x+c\,x)-x+\frac{a\,\ln(a+b\,x+c\,x)}{b+c}$$

```
[In] int(log(a + b*x + c*x),x)
[Out] x*log(a + b*x + c*x) - x + (a*log(a + b*x + c*x))/(b + c)
```

# 3.68 $\int \log^2(a+bx+cx)\,dx$



## Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \log^2(a+bx+cx)\,dx = 2x - \frac{2(a+(b+c)x)\log(a+(b+c)x)}{b+c} + \frac{(a+(b+c)x)\log^2(a+(b+c)x)}{b+c}$$

```
[Out] 2*x-2*(a+(b+c)*x)*ln(a+(b+c)*x)/(b+c)+(a+(b+c)*x)*ln(a+(b+c)*x)^2/(b+c)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2494, 2436, 2333, 2332}

$$\int \log^2(a+bx+cx)\,dx = \frac{(a+x(b+c))\log^2(a+x(b+c))}{b+c} - \frac{2(a+x(b+c))\log(a+x(b+c))}{b+c} + 2x$$

```
[In] Int[Log[a + b*x + c*x]^2,x]
```

```
[Out] 2*x - (2*(a + (b + c)*x)*Log[a + (b + c)*x])/(b + c) + ((a + (b + c)*x)*Log
[a + (b + c)*x]^2)/(b + c)
```

<u>Rule 2332</u>

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \log^2(a+(b+c)x)\,dx \\
&= \frac{\text{Subst}\left(\int \log^2(x)\,dx, x, a+(b+c)x\right)}{b+c} \\
&= \frac{(a+(b+c)x)\log^2(a+(b+c)x)}{b+c} - \frac{2\text{Subst}\left(\int \log(x)\,dx, x, a+(b+c)x\right)}{b+c} \\
&= 2x - \frac{2(a+(b+c)x)\log(a+(b+c)x)}{b+c} + \frac{(a+(b+c)x)\log^2(a+(b+c)x)}{b+c}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \log^2(a+bx+cx)\,dx = \frac{2(b+c)x - 2(a+(b+c)x)\log(a+(b+c)x) + (a+(b+c)x)\log^2(a+(b+c)x)}{b+c}$$

```
[In] Integrate[Log[a + b*x + c*x]^2,x]
[Out] (2*(b + c)*x - 2*(a + (b + c)*x)*Log[a + (b + c)*x] + (a + (b + c)*x)*Log[a
 + (b + c)*x]^2)/(b + c)
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

| method | result |
|---|---|
| derivativedivides | $\frac{\ln(a+(b+c)x)^2(a+(b+c)x)-2(a+(b+c)x)\ln(a+(b+c)x)+2a+2(b+c)x}{b+c}$ |
| default | $\frac{\ln(a+(b+c)x)^2(a+(b+c)x)-2(a+(b+c)x)\ln(a+(b+c)x)+2a+2(b+c)x}{b+c}$ |
| norman | $x\ln(bx+xc+a)^2+\frac{a\ln(bx+xc+a)^2}{b+c}+2x-2x\ln(bx+xc+a)-\frac{2a\ln(bx+xc+a)}{b+c}$ |
| risch | $\frac{\ln(bx+xc+a)^2(bx+xc+a)}{b+c}-2x\ln(bx+xc+a)-\frac{2a\ln(a+(b+c)x)}{b+c}+\frac{2bx}{b+c}+\frac{2xc}{b+c}$ |
| parallelrisch | $\frac{x\ln(bx+xc+a)^2ab+x\ln(bx+xc+a)^2ac-2x\ln(bx+xc+a)ab-2x\ln(bx+xc+a)ac+\ln(bx+xc+a)^2a^2+2abx+2xca-2\ln(b}{a(b+c)}$ |

```
[In] int(ln(b*x+c*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/(b+c)*(ln(a+(b+c)*x)^2*(a+(b+c)*x)-2*(a+(b+c)*x)*ln(a+(b+c)*x)+2*a+2*(b+c
)*x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \log^2(a+bx+cx)\,dx = \frac{((b+c)x+a)\log((b+c)x+a)^2+2(b+c)x-2((b+c)x+a)\log((b+c)x+a)}{b+c}$$

```
[In] integrate(log(b*x+c*x+a)^2,x, algorithm="fricas")
[Out] (((b + c)*x + a)*log((b + c)*x + a)^2 + 2*(b + c)*x - 2*((b + c)*x + a)*log
((b + c)*x + a))/(b + c)
```

## Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \log^2(a+bx+cx)\,dx = -2x\log(a+bx+cx)+(2b+2c)\left(-\frac{a\log(a+x(b+c))}{(b+c)^2}+\frac{x}{b+c}\right) + \frac{(a+bx+cx)\log(a+bx+cx)^2}{b+c}$$

```
[In] integrate(ln(b*x+c*x+a)**2,x)
[Out] -2*x*log(a + b*x + c*x) + (2*b + 2*c)*(-a*log(a + x*(b + c))/(b + c)**2 + x
/(b + c)) + (a + b*x + c*x)*log(a + b*x + c*x)**2/(b + c)
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \log^2(a+bx+cx)\,dx = \frac{(bx+cx+a)\left(\log\left(bx+cx+a\right)^2-2\,\log\left(bx+cx+a\right)+2\right)}{b+c}$$

```
[In] integrate(log(b*x+c*x+a)^2,x, algorithm="maxima")
[Out] (b*x + c*x + a)*(log(b*x + c*x + a)^2 - 2*log(b*x + c*x + a) + 2)/(b + c)
```

## Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

$$\int \log^2(a+bx+cx)\,dx = \frac{(bx+cx+a)\log\left(bx+cx+a\right)^2}{b+c} - \frac{2\,(bx+cx+a)\log\left(bx+cx+a\right)}{b+c} + \frac{2\,(bx+cx+a)}{b+c}$$

```
[In] integrate(log(b*x+c*x+a)^2,x, algorithm="giac")
[Out] (b*x + c*x + a)*log(b*x + c*x + a)^2/(b + c) - 2*(b*x + c*x + a)*log(b*x +
c*x + a)/(b + c) + 2*(b*x + c*x + a)/(b + c)
```

## Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.92

$$\int \log^2(a+bx+cx)\,dx = \frac{2\,b\,x+2\,c\,x-2\,a\,\ln\left(a+b\,x+c\,x\right)+a\ln\left(a+b\,x+c\,x\right)^2+b\,x\ln\left(a+b\,x+c\,x\right)^2+c\,x\ln\left(a+b\,x+c\,x\right)^2-}{b+c}$$

```
[In] int(log(a + b*x + c*x)^2,x)
[Out] (2*b*x + 2*c*x - 2*a*log(a + b*x + c*x) + a*log(a + b*x + c*x)^2 + b*x*log(
a + b*x + c*x)^2 + c*x*log(a + b*x + c*x)^2 - 2*b*x*log(a + b*x + c*x) - 2*
c*x*log(a + b*x + c*x))/(b + c)
```

# 3.69 $\int \log^3(a+bx+cx)\,dx$



## Optimal result

Integrand size = 11, antiderivative size = 73

$$\int \log^3(a+bx+cx)\,dx = -6x+\frac{6(a+(b+c)x)\log(a+(b+c)x)}{b+c} - \frac{3(a+(b+c)x)\log^2(a+(b+c)x)}{b+c} + \frac{(a+(b+c)x)\log^3(a+(b+c)x)}{b+c}$$

```
[Out] -6*x+6*(a+(b+c)*x)*ln(a+(b+c)*x)/(b+c)-3*(a+(b+c)*x)*ln(a+(b+c)*x)^2/(b+c)+
(a+(b+c)*x)*ln(a+(b+c)*x)^3/(b+c)
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2494, 2436, 2333, 2332}

$$\int \log^3(a+bx+cx)\,dx = \frac{(a+x(b+c))\log^3(a+x(b+c))}{b+c} - \frac{3(a+x(b+c))\log^2(a+x(b+c))}{b+c} + \frac{6(a+x(b+c))\log(a+x(b+c))}{b+c} - 6x$$

```
[In] Int[Log[a + b*x + c*x]^3,x]
```

```
[Out] -6*x + (6*(a + (b + c)*x)*Log[a + (b + c)*x])/(b + c) - (3*(a + (b + c)*x)*
Log[a + (b + c)*x]^2)/(b + c) + ((a + (b + c)*x)*Log[a + (b + c)*x]^3)/(b +
 c)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \log^3(a+(b+c)x)\,dx \\
&= \frac{\text{Subst}\left(\int \log^3(x)\,dx, x, a+(b+c)x\right)}{b+c} \\
&= \frac{(a+(b+c)x)\log^3(a+(b+c)x)}{b+c} - \frac{3\text{Subst}\left(\int \log^2(x)\,dx, x, a+(b+c)x\right)}{b+c} \\
&= -\frac{3(a+(b+c)x)\log^2(a+(b+c)x)}{b+c} + \frac{(a+(b+c)x)\log^3(a+(b+c)x)}{b+c} \\
&\quad + \frac{6\text{Subst}\left(\int \log(x)\,dx, x, a+(b+c)x\right)}{b+c} \\
&= -6x + \frac{6(a+(b+c)x)\log(a+(b+c)x)}{b+c} \\
&\quad - \frac{3(a+(b+c)x)\log^2(a+(b+c)x)}{b+c} + \frac{(a+(b+c)x)\log^3(a+(b+c)x)}{b+c}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \log^3(a+bx+cx)\,dx$$
$$= \frac{-6(b+c)x+6(a+(b+c)x)\log(a+(b+c)x)-3(a+(b+c)x)\log^2(a+(b+c)x)+(a+(b+c)x)\log^3}{b+c}$$

```
[In] Integrate[Log[a + b*x + c*x]^3,x]
```

```
[Out] (-6*(b + c)*x + 6*(a + (b + c)*x)*Log[a + (b + c)*x] - 3*(a + (b + c)*x)*Lo
g[a + (b + c)*x]^2 + (a + (b + c)*x)*Log[a + (b + c)*x]^3)/(b + c)
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $\frac{\ln(a+(b+c)x)^3(a+(b+c)x)-3\ln(a+(b+c)x)^2(a+(b+c)x)+6(a+(b+c)x)\ln(a+(b+c)x)-6a-6(b+c)x}{b+c}$ |
| default | $\frac{\ln(a+(b+c)x)^3(a+(b+c)x)-3\ln(a+(b+c)x)^2(a+(b+c)x)+6(a+(b+c)x)\ln(a+(b+c)x)-6a-6(b+c)x}{b+c}$ |
| norman | $x\ln\left(bx+xc+a\right)^3+\frac{a\ln(bx+xc+a)^3}{b+c}-6x+6x\ln\left(bx+xc+a\right)-3x\ln\left(bx+xc+a\right)^2+$ |
| risch | $\frac{\ln(bx+xc+a)^3(bx+xc+a)}{b+c}-\frac{3\ln(bx+xc+a)^2(bx+xc+a)}{b+c}+6x\ln\left(bx+xc+a\right)+\frac{6a\ln(a+(b+c)x)}{b+c}-\frac{6bx}{b+c}$ |
| parallelrisch | $\frac{x\ln(bx+xc+a)^3ab+x\ln(bx+xc+a)^3ac-3x\ln(bx+xc+a)^2ab-3x\ln(bx+xc+a)^2ac+\ln(bx+xc+a)^3a^2+6x\ln(bx+xc+a)}{(b+c)a}$ |

```
[In] int(ln(b*x+c*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b+c)*(ln(a+(b+c)*x)^3*(a+(b+c)*x)-3*ln(a+(b+c)*x)^2*(a+(b+c)*x)+6*(a+(b+
c)*x)*ln(a+(b+c)*x)-6*a-6*(b+c)*x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \log^3(a+bx+cx)\,dx$$
$$= \frac{((b+c)x+a)\log\left((b+c)x+a\right)^3-3\left((b+c)x+a\right)\log\left((b+c)x+a\right)^2-6\left(b+c\right)x+6\left((b+c)x+a\right)\log}{b+c}$$

```
[In] integrate(log(b*x+c*x+a)^3,x, algorithm="fricas")
```

```
[Out] (((b + c)*x + a)*log((b + c)*x + a)^3 - 3*((b + c)*x + a)*log((b + c)*x + a
)^2 - 6*(b + c)*x + 6*((b + c)*x + a)*log((b + c)*x + a))/(b + c)
```

## Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.30

$$\int \log^3(a+bx+cx)\,dx = 6x\log(a+bx+cx) + (-6b-6c)\left(-\frac{a\log(a+x(b+c))}{(b+c)^2} + \frac{x}{b+c}\right) + \frac{(-3a-3bx-3cx)\log(a+bx+cx)^2}{b+c} + \frac{(a+bx+cx)\log(a+bx+cx)^3}{b+c}$$

```
[In] integrate(ln(b*x+c*x+a)**3,x)
```

```
[Out] 6*x*log(a + b*x + c*x) + (-6*b - 6*c)*(-a*log(a + x*(b + c))/(b + c)**2 + x
/(b + c)) + (-3*a - 3*b*x - 3*c*x)*log(a + b*x + c*x)**2/(b + c) + (a + b*x
 + c*x)*log(a + b*x + c*x)**3/(b + c)
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \log^3(a+bx+cx)\,dx = \frac{\left(\log(bx+cx+a)^3 - 3\,\log(bx+cx+a)^2 + 6\,\log(bx+cx+a) - 6\right)(bx+cx+a)}{b+c}$$

```
[In] integrate(log(b*x+c*x+a)^3,x, algorithm="maxima")
```

```
[Out] (log(b*x + c*x + a)^3 - 3*log(b*x + c*x + a)^2 + 6*log(b*x + c*x + a) - 6)*
(b*x + c*x + a)/(b + c)
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \log^3(a+bx+cx)\,dx=\frac{(bx+cx+a)\log(bx+cx+a)^3}{b+c}-\frac{3\,(bx+cx+a)\log(bx+cx+a)^2}{b+c}+\frac{6\,(bx+cx+a)\log(bx+cx+a)}{b+c}-\frac{6\,(bx+cx+a)}{b+c}$$

```
[In] integrate(log(b*x+c*x+a)^3,x, algorithm="giac")
```

```
[Out] (b*x + c*x + a)*log(b*x + c*x + a)^3/(b + c) - 3*(b*x + c*x + a)*log(b*x +
c*x + a)^2/(b + c) + 6*(b*x + c*x + a)*log(b*x + c*x + a)/(b + c) - 6*(b*x
+ c*x + a)/(b + c)
```

## Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.89

$$\int \log^3(a+bx+cx)\,dx$$
$$=\frac{6\,a\,\ln(a+b\,x+c\,x)-6\,c\,x-6\,b\,x-3\,a\ln(a+b\,x+c\,x)^2+a\ln(a+b\,x+c\,x)^3-3\,b\,x\ln(a+b\,x+c\,x)}{}$$

```
[In] int(log(a + b*x + c*x)^3,x)
```

```
[Out] (6*a*log(a + b*x + c*x) - 6*c*x - 6*b*x - 3*a*log(a + b*x + c*x)^2 + a*log(
a + b*x + c*x)^3 - 3*b*x*log(a + b*x + c*x)^2 + b*x*log(a + b*x + c*x)^3 -
3*c*x*log(a + b*x + c*x)^2 + c*x*log(a + b*x + c*x)^3 + 6*b*x*log(a + b*x +
 c*x) + 6*c*x*log(a + b*x + c*x))/(b + c)
```

# 3.70 $\int \log\left(c(d+ex)^n\right)\,dx$



## Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \log\left(c(d+ex)^n\right)\,dx = -nx + \frac{(d+ex)\log\left(c(d+ex)^n\right)}{e}$$

```
[Out] -n*x+(e*x+d)*ln(c*(e*x+d)^n)/e
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2436, 2332}

$$\int \log\left(c(d+ex)^n\right)\,dx = \frac{(d+ex)\log\left(c(d+ex)^n\right)}{e} - nx$$

```
[In] Int[Log[c*(d + e*x)^n],x]
```

```
[Out] -(n*x) + ((d + e*x)*Log[c*(d + e*x)^n])/e
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \log(cx^n)\,dx, x, d+ex\right)}{e}$$
$$= -nx + \frac{(d+ex)\log(c(d+ex)^n)}{e}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \log(c(d+ex)^n)\,dx = -nx + \frac{(d+ex)\log(c(d+ex)^n)}{e}$$

```
[In] Integrate[Log[c*(d + e*x)^n],x]
[Out] -(n*x) + ((d + e*x)*Log[c*(d + e*x)^n])/e
```

## Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| norman | $x\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)+\frac{nd\ln(ex+d)}{e}-nx$ |
| default | $\ln\left(c(ex+d)^n\right)x-en\left(\frac{x}{e}-\frac{d\ln(ex+d)}{e^2}\right)$ |
| parts | $\ln\left(c(ex+d)^n\right)x-en\left(\frac{x}{e}-\frac{d\ln(ex+d)}{e^2}\right)$ |
| parallelrisch | $\frac{x\ln(c(ex+d)^n)den-xden^2+\ln(c(ex+d)^n)d^2n}{den}$ |
| risch | $x\ln\left((ex+d)^n\right)-\frac{i\pi x\,\mathrm{csgn}(ic)\,\mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)}{2}+\frac{i\pi x\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic(ex+d)^n)^2}{2}+\frac{i\pi x\,\mathrm{csgn}(i(ex$ |

```
[In] int(ln(c*(e*x+d)^n),x,method=_RETURNVERBOSE)
[Out] x*ln(c*exp(n*ln(e*x+d)))+n*d/e*ln(e*x+d)-n*x
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \log \left(c(d+ex)^n\right)\,dx = -\frac{enx - ex\log(c) - (enx + dn)\log(ex+d)}{e}$$

```
[In] integrate(log(c*(e*x+d)^n),x, algorithm="fricas")
[Out] -(e*n*x - e*x*log(c) - (e*n*x + d*n)*log(e*x + d))/e
```

## Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \log \left(c(d+ex)^n\right)\,dx = \begin{cases} \frac{d\log\left(c(d+ex)^n\right)}{e} - nx + x\log\left(c(d+ex)^n\right) & \text{for } e \neq 0 \\ x\log\left(cd^n\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(e*x+d)**n),x)
[Out] Piecewise((d*log(c*(d + e*x)**n)/e - n*x + x*log(c*(d + e*x)**n), Ne(e, 0))
, (x*log(c*d**n), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \log \left(c(d+ex)^n\right)\,dx = -en\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) + x\log\left((ex+d)^n c\right)$$

```
[In] integrate(log(c*(e*x+d)^n),x, algorithm="maxima")
[Out] -e*n*(x/e - d*log(e*x + d)/e^2) + x*log((e*x + d)^n*c)
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \log \left(c(d+ex)^n\right) \, dx = \frac{(ex+d)n \log (ex+d)}{e} - \frac{(ex+d)n}{e} + \frac{(ex+d) \log (c)}{e}$$

```
[In] integrate(log(c*(e*x+d)^n),x, algorithm="giac")
[Out] (e*x + d)*n*log(e*x + d)/e - (e*x + d)*n/e + (e*x + d)*log(c)/e
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \log \left(c(d+ex)^n\right) \, dx = x \ln \left(c\,(d+e\,x)^n\right) - n\,x + \frac{d\,n\,\ln (d+e\,x)}{e}$$

```
[In] int(log(c*(d + e*x)^n),x)
[Out] x*log(c*(d + e*x)^n) - n*x + (d*n*log(d + e*x))/e
```

# 3.71 $\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 24

$$\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx = -\frac{\operatorname{PolyLog}\left(2,\frac{e(f+gx)}{ef-dg}\right)}{g}$$

```
[Out] -polylog(2,e*(g*x+f)/(-d*g+e*f))/g
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2440, 2438}

$$\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx = -\frac{\operatorname{PolyLog}\left(2,\frac{e(f+gx)}{ef-dg}\right)}{g}$$

```
[In] Int[Log[-((g*(d + e*x))/(e*f - d*g))]/(f + g*x),x]
[Out] -(PolyLog[2, (e*(f + g*x))/(e*f - d*g)]/g)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{ex}{ef-dg}\right)}{x}\,dx, x, f+gx\right)}{g}$$
$$= -\frac{\text{Li}_2\left(\frac{e(f+gx)}{ef-dg}\right)}{g}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx = -\frac{\text{PolyLog}\left(2, \frac{e(f+gx)}{ef-dg}\right)}{g}$$

```
[In] Integrate[Log[-((g*(d + e*x))/(e*f - d*g))]/(f + g*x),x]
[Out] -(PolyLog[2, (e*(f + g*x))/(e*f - d*g)]/g)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.

Time = 1.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{(-dg+ef)\,\text{dilog}\left(-\frac{gex}{-dg+ef}-\frac{dg}{-dg+ef}\right)}{g(dg-ef)}$ | 54 |
| default | $\frac{(-dg+ef)\,\text{dilog}\left(-\frac{gex}{-dg+ef}-\frac{dg}{-dg+ef}\right)}{g(dg-ef)}$ | 54 |
| risch | $-\frac{\text{dilog}\left(-\frac{gex}{-dg+ef}-\frac{dg}{-dg+ef}\right)d}{dg-ef}+\frac{\text{dilog}\left(-\frac{gex}{-dg+ef}-\frac{dg}{-dg+ef}\right)ef}{g(dg-ef)}$ | 93 |
| parts | $\frac{\ln\left(-\frac{g(ex+d)}{-dg+ef}\right)\ln(gx+f)}{g}-\frac{e\left(\frac{\text{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{e}+\frac{\ln(gx+f)\ln\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{e}\right)}{g}$ | 106 |

```
[In] int(ln(-g*(e*x+d)/(-d*g+e*f))/(g*x+f),x,method=_RETURNVERBOSE)
[Out] 1/g*(-d*g+e*f)/(d*g-e*f)*dilog(-g*e/(-d*g+e*f)*x-d*g/(-d*g+e*f))
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{\log \left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx = -\frac{\mathrm{Li}_2\left(\frac{egx+dg}{ef-dg}+1\right)}{g}$$

```
[In] integrate(log(-g*(e*x+d)/(-d*g+e*f))/(g*x+f),x, algorithm="fricas")
[Out] -dilog((e*g*x + d*g)/(e*f - d*g) + 1)/g
```

## Sympy [F]

$$\int \frac{\log \left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx = \int \frac{\log \left(-\frac{dg}{-dg+ef}-\frac{egx}{-dg+ef}\right)}{f+gx}\,dx$$

```
[In] integrate(ln(-g*(e*x+d)/(-d*g+e*f))/(g*x+f),x)
[Out] Integral(log(-d*g/(-d*g + e*f) - e*g*x/(-d*g + e*f))/(f + g*x), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(23) = 46.

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.25

$$\int \frac{\log \left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx = -\frac{\log(ex+d)\log(gx+f)}{g}+\frac{\log(gx+f)\log\left(-\frac{(ex+d)g}{ef-dg}\right)}{g} + \frac{\log(ex+d)\log\left(\frac{egx+dg}{ef-dg}+1\right)+\mathrm{Li}_2\left(-\frac{egx+dg}{ef-dg}\right)}{g}$$

```
[In] integrate(log(-g*(e*x+d)/(-d*g+e*f))/(g*x+f),x, algorithm="maxima")
[Out] -log(e*x + d)*log(g*x + f)/g + log(g*x + f)*log(-(e*x + d)*g/(e*f - d*g))/g
 + (log(e*x + d)*log((e*g*x + d*g)/(e*f - d*g) + 1) + dilog(-(e*g*x + d*g)/
(e*f - d*g)))/g
```

## Giac [F]

$$\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx = \int \frac{\log\left(-\frac{(ex+d)g}{ef-dg}\right)}{gx+f}\,dx$$

```
[In] integrate(log(-g*(e*x+d)/(-d*g+e*f))/(g*x+f),x, algorithm="giac")
[Out] integrate(log(-(e*x + d)*g/(e*f - d*g))/(g*x + f), x)
```

## Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{\log\left(-\frac{g(d+ex)}{ef-dg}\right)}{f+gx}\,dx = -\frac{\mathrm{Li}_2\left(\frac{g\,(d+e\,x)}{d\,g-e\,f}\right)}{g}$$

```
[In] int(log((g*(d + e*x))/(d*g - e*f))/(f + g*x),x)
[Out] -dilog((g*(d + e*x))/(d*g - e*f))/g
```

# 3.72 $\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = a\log(x) - b\operatorname{PolyLog}(2,-cex)$$

```
[Out] a*ln(x)-b*polylog(2,-c*e*x)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2439, 2438}

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = a\log(x) - b\operatorname{PolyLog}(2,-cex)$$

```
[In] Int[(a + b*Log[c*(c^(-1) + e*x)])/x,x]
```

```
[Out] a*Log[x] - b*PolyLog[2, -(c*e*x)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + e*(x/d)]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= a\log(x) + b\int \frac{\log(1+cex)}{x}\,dx \\ &= a\log(x) - b\text{Li}_2(-cex)\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = a\log(x) - b\,\text{PolyLog}(2,-cex)$$

```
[In] Integrate[(a + b*Log[c*(c^(-1) + e*x)])/x,x]
[Out] a*Log[x] - b*PolyLog[2, -(c*e*x)]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| risch | $\ln(x)\,a - b\,\text{dilog}(cex+1)$ | 16 |
| parts | $\ln(x)\,a - b\,\text{dilog}(cex+1)$ | 16 |
| derivativedivides | $a\ln(cex) - b\,\text{dilog}(cex+1)$ | 19 |
| default | $a\ln(cex) - b\,\text{dilog}(cex+1)$ | 19 |

```
[In] int((a+b*ln(c*(1/c+e*x)))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*a-b*dilog(c*e*x+1)
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = -b\text{Li}_2(-cex) + a\log(x)$$

```
[In] integrate((a+b*log(c*(1/c+e*x)))/x,x, algorithm="fricas")
[Out] -b*dilog(-c*e*x) + a*log(x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = a\log\left(x\right) - b\operatorname{Li}_2\left(cexe^{i\pi}\right)$$

```
[In] integrate((a+b*ln(c*(1/c+e*x)))/x,x)
[Out] a*log(x) - b*polylog(2, c*e*x*exp_polar(I*pi))
```

## Maxima [F]

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = \int \frac{b\log\left(\left(ex+\frac{1}{c}\right)c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*log(c*(1/c+e*x)))/x,x, algorithm="maxima")
[Out] b*integrate(log(c*e*x + 1)/x, x) + a*log(x)
```

## Giac [F]

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = \int \frac{b\log\left(\left(ex+\frac{1}{c}\right)c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*log(c*(1/c+e*x)))/x,x, algorithm="giac")
[Out] integrate((b*log((e*x + 1/c)*c) + a)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c\left(\frac{1}{c}+ex\right)\right)}{x}\,dx = a\,\ln\left(x\right) - b\,\mathrm{polylog}(2,-c\,e\,x)$$

```
[In] int((a + b*log(c*(e*x + 1/c)))/x,x)
[Out] a*log(x) - b*polylog(2, -c*e*x)
```

# 3.73 $\int \frac{\log(3+ex)}{x} \, dx$



## Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{\log(3+ex)}{x} \, dx = \log(3)\log(x) - \text{PolyLog}\left(2, -\frac{ex}{3}\right)$$

```
[Out] ln(3)*ln(x)-polylog(2,-1/3*e*x)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2439, 2438}

$$\int \frac{\log(3+ex)}{x} \, dx = \log(3)\log(x) - \text{PolyLog}\left(2, -\frac{ex}{3}\right)$$

```
[In] Int[Log[3 + e*x]/x,x]
```

```
[Out] Log[3]*Log[x] - PolyLog[2, -1/3*(e*x)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + e*(x/d)]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \log(3)\log(x) + \int \frac{\log\left(1+\frac{ex}{3}\right)}{x}\,dx \\ &= \log(3)\log(x) - \text{Li}_2\left(-\frac{ex}{3}\right)\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\log(3+ex)}{x}\,dx = \log(3)\log(x) - \text{PolyLog}\left(2, -\frac{ex}{3}\right)$$

```
[In] Integrate[Log[3 + e*x]/x,x]
[Out] Log[3]*Log[x] - PolyLog[2, -1/3*(e*x)]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

| method | result | size |
|---|---|---|
| derivativedivides | $\left(\ln\left(ex+3\right)-\ln\left(\frac{ex}{3}+1\right)\right)\ln\left(-\frac{ex}{3}\right)-\text{dilog}\left(\frac{ex}{3}+1\right)$ | 33 |
| default | $\left(\ln\left(ex+3\right)-\ln\left(\frac{ex}{3}+1\right)\right)\ln\left(-\frac{ex}{3}\right)-\text{dilog}\left(\frac{ex}{3}+1\right)$ | 33 |
| risch | $\left(\ln\left(ex+3\right)-\ln\left(\frac{ex}{3}+1\right)\right)\ln\left(-\frac{ex}{3}\right)-\text{dilog}\left(\frac{ex}{3}+1\right)$ | 33 |
| parts | $\ln\left(ex+3\right)\ln\left(x\right)-e\left(\frac{\text{dilog}\left(\frac{ex}{3}+1\right)}{e}+\frac{\ln(x)\ln\left(\frac{ex}{3}+1\right)}{e}\right)$ | 39 |

```
[In] int(ln(e*x+3)/x,x,method=_RETURNVERBOSE)
[Out] (ln(e*x+3)-ln(1/3*e*x+1))*ln(-1/3*e*x)-dilog(1/3*e*x+1)
```

## Fricas [F]

$$\int \frac{\log(3+ex)}{x}\,dx = \int \frac{\log(ex+3)}{x}\,dx$$

```
[In] integrate(log(e*x+3)/x,x, algorithm="fricas")
[Out] integral(log(e*x + 3)/x, x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{\log(3+ex)}{x}\,dx = \begin{cases} -\operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \log(3)\log(x) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{for } |x| < 1 \\ -\log(3)\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(3) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(3) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(e*x+3)/x,x)
[Out] Piecewise((-polylog(2, e*x*exp_polar(I*pi)/3), (Abs(x) < 1) & (1/Abs(x) < 1
)), (log(3)*log(x) - polylog(2, e*x*exp_polar(I*pi)/3), Abs(x) < 1), (-log(
3)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/3), 1/Abs(x) < 1), (-meijerg((
(), (1, 1)), ((0, 0), ()), x)*log(3) + meijerg(((1, 1), ()), ((), (0, 0)),
x)*log(3) - polylog(2, e*x*exp_polar(I*pi)/3), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\log(3+ex)}{x}\,dx = \log(ex+3)\log\left(-\frac{1}{3}\,ex\right) + \operatorname{Li}_2\left(\frac{1}{3}\,ex+1\right)$$

```
[In] integrate(log(e*x+3)/x,x, algorithm="maxima")
[Out] log(e*x + 3)*log(-1/3*e*x) + dilog(1/3*e*x + 1)
```

## Giac [F]

$$\int \frac{\log(3+ex)}{x}\,dx = \int \frac{\log(ex+3)}{x}\,dx$$

```
[In] integrate(log(e*x+3)/x,x, algorithm="giac")
[Out] integrate(log(e*x + 3)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\log(3+ex)}{x}\,dx = \mathrm{Li}_2\left(-\frac{e\,x}{3}\right) + \ln(e\,x+3)\,\ln\left(-\frac{e\,x}{3}\right)$$

```
[In] int(log(e*x + 3)/x,x)
[Out] dilog(-(e*x)/3) + log(e*x + 3)*log(-(e*x)/3)
```

# 3.74 $\int \frac{\log(2+ex)}{x} \, dx$



## Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{\log(2+ex)}{x} \, dx = \log(2)\log(x) - \text{PolyLog}\left(2, -\frac{ex}{2}\right)$$

```
[Out] ln(2)*ln(x)-polylog(2,-1/2*e*x)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2439, 2438}

$$\int \frac{\log(2+ex)}{x} \, dx = \log(2)\log(x) - \text{PolyLog}\left(2, -\frac{ex}{2}\right)$$

```
[In] Int[Log[2 + e*x]/x,x]
```

```
[Out] Log[2]*Log[x] - PolyLog[2, -1/2*(e*x)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + e*(x/d)]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rubi steps

$$\text{integral} = \log(2)\log(x) + \int \frac{\log\left(1+\frac{ex}{2}\right)}{x}\,dx$$
$$= \log(2)\log(x) - \text{Li}_2\left(-\frac{ex}{2}\right)$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\log(2+ex)}{x}\,dx = \log(2)\log(x) - \text{PolyLog}\left(2,-\frac{ex}{2}\right)$$

```
[In] Integrate[Log[2 + e*x]/x,x]
[Out] Log[2]*Log[x] - PolyLog[2, -1/2*(e*x)]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

| method | result | size |
|---|---|---|
| derivativedivides | $\left(\ln\left(ex+2\right)-\ln\left(\frac{ex}{2}+1\right)\right)\ln\left(-\frac{ex}{2}\right)-\text{dilog}\left(\frac{ex}{2}+1\right)$ | 33 |
| default | $\left(\ln\left(ex+2\right)-\ln\left(\frac{ex}{2}+1\right)\right)\ln\left(-\frac{ex}{2}\right)-\text{dilog}\left(\frac{ex}{2}+1\right)$ | 33 |
| risch | $\left(\ln\left(ex+2\right)-\ln\left(\frac{ex}{2}+1\right)\right)\ln\left(-\frac{ex}{2}\right)-\text{dilog}\left(\frac{ex}{2}+1\right)$ | 33 |
| parts | $\ln\left(ex+2\right)\ln\left(x\right)-e\left(\frac{\text{dilog}\left(\frac{ex}{2}+1\right)}{e}+\frac{\ln(x)\ln\left(\frac{ex}{2}+1\right)}{e}\right)$ | 39 |

```
[In] int(ln(e*x+2)/x,x,method=_RETURNVERBOSE)
[Out] (ln(e*x+2)-ln(1/2*e*x+1))*ln(-1/2*e*x)-dilog(1/2*e*x+1)
```

## Fricas [F]

$$\int \frac{\log(2+ex)}{x}\,dx = \int \frac{\log(ex+2)}{x}\,dx$$

```
[In] integrate(log(e*x+2)/x,x, algorithm="fricas")
[Out] integral(log(e*x + 2)/x, x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{\log(2+ex)}{x}\,dx = \begin{cases} -\operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \log(2)\log(x) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{for } |x| < 1 \\ -\log(2)\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(2) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(2) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(e*x+2)/x,x)
[Out] Piecewise((-polylog(2, e*x*exp_polar(I*pi)/2), (Abs(x) < 1) & (1/Abs(x) < 1
)), (log(2)*log(x) - polylog(2, e*x*exp_polar(I*pi)/2), Abs(x) < 1), (-log(
2)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/2), 1/Abs(x) < 1), (-meijerg((
(), (1, 1)), ((0, 0), ()), x)*log(2) + meijerg(((1, 1), ()), ((), (0, 0)),
x)*log(2) - polylog(2, e*x*exp_polar(I*pi)/2), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\log(2+ex)}{x}\,dx = \log(ex+2)\log\left(-\frac{1}{2}\,ex\right) + \operatorname{Li}_2\left(\frac{1}{2}\,ex+1\right)$$

```
[In] integrate(log(e*x+2)/x,x, algorithm="maxima")
[Out] log(e*x + 2)*log(-1/2*e*x) + dilog(1/2*e*x + 1)
```

## Giac [F]

$$\int \frac{\log(2+ex)}{x}\,dx = \int \frac{\log(ex+2)}{x}\,dx$$

```
[In] integrate(log(e*x+2)/x,x, algorithm="giac")
[Out] integrate(log(e*x + 2)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\log(2+ex)}{x}\,dx = \mathrm{Li}_2\left(-\frac{e\,x}{2}\right) + \ln(e\,x+2)\,\ln\left(-\frac{e\,x}{2}\right)$$

```
[In] int(log(e*x + 2)/x,x)
[Out] dilog(-(e*x)/2) + log(e*x + 2)*log(-(e*x)/2)
```

# 3.75 $\int \frac{\log(1+ex)}{x} \, dx$



## Optimal result

Integrand size = 10, antiderivative size = 8

$$\int \frac{\log(1+ex)}{x} \, dx = -\operatorname{PolyLog}(2,-ex)$$

```
[Out] -polylog(2,-e*x)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{2438\}$

$$\int \frac{\log(1+ex)}{x} \, dx = -\operatorname{PolyLog}(2,-ex)$$

```
[In] Int[Log[1 + e*x]/x,x]
```

```
[Out] -PolyLog[2, -(e*x)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = -\operatorname{Li}_2(-ex)$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log(1+ex)}{x}\,dx = -\operatorname{PolyLog}(2,-ex)$$

```
[In] Integrate[Log[1 + e*x]/x,x]
[Out] -PolyLog[2, -(e*x)]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

| method | result | size |
|---|---|---|
| derivativedivides | $-\operatorname{dilog}(ex+1)$ | 9 |
| default | $-\operatorname{dilog}(ex+1)$ | 9 |
| meijerg | $-\operatorname{Li}_2(-ex)$ | 9 |
| risch | $-\operatorname{dilog}(ex+1)$ | 9 |
| parts | $\ln(ex+1)\ln(x)-e\left(\frac{\operatorname{dilog}(ex+1)}{e}+\frac{\ln(x)\ln(ex+1)}{e}\right)$ | 37 |

```
[In] int(1/x*ln(e*x+1),x,method=_RETURNVERBOSE)
[Out] -dilog(e*x+1)
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\log(1+ex)}{x}\,dx = -\operatorname{Li}_2(-ex)$$

```
[In] integrate(log(e*x+1)/x,x, algorithm="fricas")
[Out] -dilog(-e*x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{\log(1+ex)}{x}\,dx = -\operatorname{Li}_2\left(exe^{i\pi}\right)$$

```
[In] integrate(ln(e*x+1)/x,x)
[Out] -polylog(2, e*x*exp_polar(I*pi))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \frac{\log(1+ex)}{x}\,dx = \log\left(ex+1\right)\log\left(-ex\right) + \operatorname{Li}_2(ex+1)$$

```
[In] integrate(log(e*x+1)/x,x, algorithm="maxima")
[Out] log(e*x + 1)*log(-e*x) + dilog(e*x + 1)
```

## Giac [F]

$$\int \frac{\log(1+ex)}{x}\,dx = \int \frac{\log\left(ex+1\right)}{x}\,dx$$

```
[In] integrate(log(e*x+1)/x,x, algorithm="giac")
[Out] integrate(log(e*x + 1)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{\log(1+ex)}{x}\,dx = \operatorname{Li}_2(-e\,x) + \ln\left(e\,x+1\right)\,\ln\left(-e\,x\right)$$

```
[In] int(log(e*x + 1)/x,x)
[Out] dilog(-e*x) + log(e*x + 1)*log(-e*x)
```

# 3.76 $\int \frac{\log(ex)}{x} \, dx$



## Optimal result

Integrand size = 8, antiderivative size = 10

$$\int \frac{\log(ex)}{x} \, dx = \frac{1}{2} \log^2(ex)$$

```
[Out] 1/2*ln(e*x)^2
```

## Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2338}

$$\int \frac{\log(ex)}{x} \, dx = \frac{1}{2} \log^2(ex)$$

```
[In] Int[Log[e*x]/x,x]
```

```
[Out] Log[e*x]^2/2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2} \log^2(ex)$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\log(ex)}{x}\,dx = \frac{1}{2}\log^2(ex)$$

```
[In] Integrate[Log[e*x]/x,x]
[Out] Log[e*x]^2/2
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(ex)^2}{2}$ | 9 |
| default | $\frac{\ln(ex)^2}{2}$ | 9 |
| norman | $\frac{\ln(ex)^2}{2}$ | 9 |
| risch | $\frac{\ln(ex)^2}{2}$ | 9 |
| parts | $\ln(ex)\ln(x) - \frac{\ln(x)^2}{2}$ | 15 |

```
[In] int(ln(e*x)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(e*x)^2
```

## Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log(ex)}{x}\,dx = \frac{1}{2}\log(ex)^2$$

```
[In] integrate(log(e*x)/x,x, algorithm="fricas")
[Out] 1/2*log(e*x)^2
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{\log(ex)}{x}\,dx = \frac{\log{(ex)}^2}{2}$$

```
[In] integrate(ln(e*x)/x,x)
[Out] log(e*x)**2/2
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log(ex)}{x}\,dx = \frac{1}{2}\log{(ex)}^2$$

```
[In] integrate(log(e*x)/x,x, algorithm="maxima")
[Out] 1/2*log(e*x)^2
```

## Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log(ex)}{x}\,dx = \frac{1}{2}\log{(ex)}^2$$

```
[In] integrate(log(e*x)/x,x, algorithm="giac")
[Out] 1/2*log(e*x)^2
```

## Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log(ex)}{x}\,dx = \frac{\ln{(e\,x)}^2}{2}$$

```
[In] int(log(e*x)/x,x)
[Out] log(e*x)^2/2
```

# 3.77 $\int \frac{\log(-1+ex)}{x} dx$



## Optimal result

Integrand size = 10, antiderivative size = 20

$$\int \frac{\log(-1+ex)}{x} dx = \log(ex)\log(-1+ex) + \text{PolyLog}(2, 1-ex)$$

```
[Out] ln(e*x)*ln(e*x-1)+polylog(2,-e*x+1)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2441, 2352}

$$\int \frac{\log(-1+ex)}{x} dx = \text{PolyLog}(2, 1-ex) + \log(ex)\log(ex-1)$$

```
[In] Int[Log[-1 + e*x]/x,x]
```

```
[Out] Log[e*x]*Log[-1 + e*x] + PolyLog[2, 1 - e*x]
```

<u>Rule 2352</u>

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

<u>Rule 2441</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \log(ex)\log(-1+ex) - e\int \frac{\log(ex)}{-1+ex}\,dx\\ &= \log(ex)\log(-1+ex) + \text{Li}_2(1-ex)\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log(-1+ex)}{x}\,dx = \log(ex)\log(-1+ex) + \text{PolyLog}(2,1-ex)$$

```
[In] Integrate[Log[-1 + e*x]/x,x]
[Out] Log[e*x]*Log[-1 + e*x] + PolyLog[2, 1 - e*x]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| derivativedivides | $\text{dilog}(ex) + \ln(ex)\ln(ex-1)$ | 17 |
| default | $\text{dilog}(ex) + \ln(ex)\ln(ex-1)$ | 17 |
| risch | $\text{dilog}(ex) + \ln(ex)\ln(ex-1)$ | 17 |
| parts | $\ln(ex-1)\ln(x) - e\left(\frac{(\ln(x)-\ln(ex))\ln(-ex+1)}{e} - \frac{\text{dilog}(ex)}{e}\right)$ | 44 |

```
[In] int(ln(e*x-1)/x,x,method=_RETURNVERBOSE)
[Out] dilog(e*x)+ln(e*x)*ln(e*x-1)
```

## Fricas [F]

$$\int \frac{\log(-1+ex)}{x}\,dx = \int \frac{\log(ex-1)}{x}\,dx$$

```
[In] integrate(log(e*x-1)/x,x, algorithm="fricas")
[Out] integral(log(e*x - 1)/x, x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.00

$$\int \frac{\log(-1+ex)}{x}\,dx = \begin{cases} -\operatorname{Li}_2(ex) & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ i\pi\log(x) - \operatorname{Li}_2(ex) & \text{for } |x| < 1 \\ -i\pi\log\left(\frac{1}{x}\right) - \operatorname{Li}_2(ex) & \text{for } \frac{1}{|x|} < 1 \\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right) + i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right) - \operatorname{Li}_2(ex) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(e*x-1)/x,x)
```

```
[Out] Piecewise((-polylog(2, e*x), (Abs(x) < 1) & (1/Abs(x) < 1)), (I*pi*log(x) -
 polylog(2, e*x), Abs(x) < 1), (-I*pi*log(1/x) - polylog(2, e*x), 1/Abs(x)
< 1), (-I*pi*meijerg(((), (1, 1)), ((0, 0), ()), x) + I*pi*meijerg(((1, 1),
 ()), ((), (0, 0)), x) - polylog(2, e*x), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\log(-1+ex)}{x}\,dx = \log(ex-1)\log(ex) + \operatorname{Li}_2(-ex+1)$$

```
[In] integrate(log(e*x-1)/x,x, algorithm="maxima")
```

```
[Out] log(e*x - 1)*log(e*x) + dilog(-e*x + 1)
```

## Giac [F]

$$\int \frac{\log(-1+ex)}{x}\,dx = \int \frac{\log(ex-1)}{x}\,dx$$

```
[In] integrate(log(e*x-1)/x,x, algorithm="giac")
```

```
[Out] integrate(log(e*x - 1)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\log(-1+ex)}{x}\,dx = \mathrm{Li}_2(e\,x) + \ln\left(e\,x-1\right)\,\ln\left(e\,x\right)$$

```
[In] int(log(e*x - 1)/x,x)
[Out] dilog(e*x) + log(e*x - 1)*log(e*x)
```

# 3.78 $\int \frac{\log(-2+ex)}{x}\,dx$



## Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{\log(-2+ex)}{x}\,dx = \log\left(\frac{ex}{2}\right)\log(-2+ex)+\operatorname{PolyLog}\left(2,1-\frac{ex}{2}\right)$$

```
[Out] ln(1/2*e*x)*ln(e*x-2)+polylog(2,1-1/2*e*x)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2441, 2352}

$$\int \frac{\log(-2+ex)}{x}\,dx = \operatorname{PolyLog}\left(2,1-\frac{ex}{2}\right)+\log\left(\frac{ex}{2}\right)\log(ex-2)$$

```
[In] Int[Log[-2 + e*x]/x,x]
[Out] Log[(e*x)/2]*Log[-2 + e*x] + PolyLog[2, 1 - (e*x)/2]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\text{integral} = \log\left(\frac{ex}{2}\right)\log(-2+ex) - e\int \frac{\log\left(\frac{ex}{2}\right)}{-2+ex}\,dx$$
$$= \log\left(\frac{ex}{2}\right)\log(-2+ex) + \text{Li}_2\left(1-\frac{ex}{2}\right)$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\log(-2+ex)}{x}\,dx = \log\left(\frac{ex}{2}\right)\log(-2+ex) + \text{PolyLog}\left(2,\frac{1}{2}(2-ex)\right)$$

```
[In] Integrate[Log[-2 + e*x]/x,x]
[Out] Log[(e*x)/2]*Log[-2 + e*x] + PolyLog[2, (2 - e*x)/2]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

| method | result | size |
|---|---|---|
| derivativedivides | $\text{dilog}\left(\frac{ex}{2}\right)+\ln\left(\frac{ex}{2}\right)\ln\left(ex-2\right)$ | 19 |
| default | $\text{dilog}\left(\frac{ex}{2}\right)+\ln\left(\frac{ex}{2}\right)\ln\left(ex-2\right)$ | 19 |
| risch | $\text{dilog}\left(\frac{ex}{2}\right)+\ln\left(\frac{ex}{2}\right)\ln\left(ex-2\right)$ | 19 |
| parts | $\ln\left(ex-2\right)\ln\left(x\right)-e\left(\frac{\left(\ln(x)-\ln\left(\frac{ex}{2}\right)\right)\ln\left(1-\frac{ex}{2}\right)}{e}-\frac{\text{dilog}\left(\frac{ex}{2}\right)}{e}\right)$ | 46 |

```
[In] int(ln(e*x-2)/x,x,method=_RETURNVERBOSE)
[Out] dilog(1/2*e*x)+ln(1/2*e*x)*ln(e*x-2)
```

## Fricas [F]

$$\int \frac{\log(-2+ex)}{x}\,dx = \int \frac{\log\left(ex-2\right)}{x}\,dx$$

```
[In] integrate(log(e*x-2)/x,x, algorithm="fricas")
[Out] integral(log(e*x - 2)/x, x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.08

$$\int \frac{\log(-2+ex)}{x}\,dx = \begin{cases} -\operatorname{Li}_2\left(\frac{ex}{2}\right) \\ \log(2)\log(x) + 3i\pi\log(x) - \operatorname{Li}_2\left(\frac{ex}{2}\right) \\ -\log(2)\log\left(\frac{1}{x}\right) - 3i\pi\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{ex}{2}\right) \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(2) - 3i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(2) + 3i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\right. \end{cases}$$

```
[In] integrate(ln(e*x-2)/x,x)
```

```
[Out] Piecewise((-polylog(2, e*x/2), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(2)*log(
x) + 3*I*pi*log(x) - polylog(2, e*x/2), Abs(x) < 1), (-log(2)*log(1/x) - 3*
I*pi*log(1/x) - polylog(2, e*x/2), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), (
(0, 0), ()), x)*log(2) - 3*I*pi*meijerg(((), (1, 1)), ((0, 0), ()), x) + me
ijerg(((1, 1), ()), ((), (0, 0)), x)*log(2) + 3*I*pi*meijerg(((1, 1), ()),
((), (0, 0)), x) - polylog(2, e*x/2), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\log(-2+ex)}{x}\,dx = \log(ex-2)\log\left(\frac{1}{2}\,ex\right) + \operatorname{Li}_2\left(-\frac{1}{2}\,ex+1\right)$$

```
[In] integrate(log(e*x-2)/x,x, algorithm="maxima")
```

```
[Out] log(e*x - 2)*log(1/2*e*x) + dilog(-1/2*e*x + 1)
```

## Giac [F]

$$\int \frac{\log(-2+ex)}{x}\,dx = \int \frac{\log\left(ex-2\right)}{x}\,dx$$

```
[In] integrate(log(e*x-2)/x,x, algorithm="giac")
[Out] integrate(log(e*x - 2)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{\log(-2+ex)}{x}\,dx = \mathrm{Li}_2\left(\frac{e\,x}{2}\right) + \ln\left(e\,x-2\right)\,\ln\left(\frac{e\,x}{2}\right)$$

```
[In] int(log(e*x - 2)/x,x)
[Out] dilog((e*x)/2) + log(e*x - 2)*log((e*x)/2)
```

# 3.79 $\int \frac{a+b\log(3+ex)}{x}\,dx$



## Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{a+b\log(3+ex)}{x}\,dx=(a+b\log(3))\log(x)-b\,\text{PolyLog}\left(2,-\frac{ex}{3}\right)$$

```
[Out] (a+b*ln(3))*ln(x)-b*polylog(2,-1/3*e*x)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2439, 2438}

$$\int \frac{a+b\log(3+ex)}{x}\,dx=\log(x)(a+b\log(3))-b\,\text{PolyLog}\left(2,-\frac{ex}{3}\right)$$

```
[In] Int[(a + b*Log[3 + e*x])/x,x]
```

```
[Out] (a + b*Log[3])*Log[x] - b*PolyLog[2, -1/3*(e*x)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + e*(x/d)]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rubi steps

$$\text{integral} = (a + b\log(3))\log(x) + b\int \frac{\log\left(1 + \frac{ex}{3}\right)}{x}\,dx$$
$$= (a + b\log(3))\log(x) - b\text{Li}_2\left(-\frac{ex}{3}\right)$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{a + b\log(3 + ex)}{x}\,dx = a\log(x) + b\log(3)\log(x) - b\,\text{PolyLog}\left(2, -\frac{ex}{3}\right)$$

```
[In] Integrate[(a + b*Log[3 + e*x])/x,x]
[Out] a*Log[x] + b*Log[3]*Log[x] - b*PolyLog[2, -1/3*(e*x)]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

| method | result | size |
|---|---|---|
| parts | $\ln(x)\,a + b\left(\left(\ln(ex+3) - \ln\left(\frac{ex}{3}+1\right)\right)\ln\left(-\frac{ex}{3}\right) - \text{dilog}\left(\frac{ex}{3}+1\right)\right)$ | 40 |
| derivativedivides | $a\ln(ex) + b\left(\left(\ln(ex+3) - \ln\left(\frac{ex}{3}+1\right)\right)\ln\left(-\frac{ex}{3}\right) - \text{dilog}\left(\frac{ex}{3}+1\right)\right)$ | 42 |
| default | $a\ln(ex) + b\left(\left(\ln(ex+3) - \ln\left(\frac{ex}{3}+1\right)\right)\ln\left(-\frac{ex}{3}\right) - \text{dilog}\left(\frac{ex}{3}+1\right)\right)$ | 42 |
| risch | $\ln(x)\,a + \ln(ex+3)\ln\left(-\frac{ex}{3}\right)b - \ln\left(\frac{ex}{3}+1\right)\ln\left(-\frac{ex}{3}\right)b - \text{dilog}\left(\frac{ex}{3}+1\right)b$ | 44 |

```
[In] int((a+b*ln(e*x+3))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*a+b*((ln(e*x+3)-ln(1/3*e*x+1))*ln(-1/3*e*x)-dilog(1/3*e*x+1))
```

## Fricas [F]

$$\int \frac{a + b\log(3 + ex)}{x}\,dx = \int \frac{b\log(ex + 3) + a}{x}\,dx$$

```
[In] integrate((a+b*log(e*x+3))/x,x, algorithm="fricas")
[Out] integral((b*log(e*x + 3) + a)/x, x)
```

## Sympy [A] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.48

$$\int \frac{a + b\log(3 + ex)}{x}\,dx = a\log(x) + b\left(\begin{cases} -\operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \log(3)\log(x) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{for } |x| < 1 \\ -\log(3)\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x\right)\log(3) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x\right)\log(3) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{3}\right) & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((a+b*ln(e*x+3))/x,x)
[Out] a*log(x) + b*Piecewise((-polylog(2, e*x*exp_polar(I*pi)/3), (Abs(x) < 1) &
(1/Abs(x) < 1)), (log(3)*log(x) - polylog(2, e*x*exp_polar(I*pi)/3), Abs(x)
 < 1), (-log(3)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/3), 1/Abs(x) < 1)
, (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(3) + meijerg(((1, 1), ()), (
(), (0, 0)), x)*log(3) - polylog(2, e*x*exp_polar(I*pi)/3), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{a + b\log(3 + ex)}{x}\,dx = \left(\log(ex + 3)\log\left(-\frac{1}{3}\,ex\right) + \operatorname{Li}_2\left(\frac{1}{3}\,ex + 1\right)\right)b + a\log(x)$$

```
[In] integrate((a+b*log(e*x+3))/x,x, algorithm="maxima")
[Out] (log(e*x + 3)*log(-1/3*e*x) + dilog(1/3*e*x + 1))*b + a*log(x)
```

## Giac [F]

$$\int \frac{a + b\log(3 + ex)}{x}\,dx = \int \frac{b\log(ex + 3) + a}{x}\,dx$$

```
[In] integrate((a+b*log(e*x+3))/x,x, algorithm="giac")
[Out] integrate((b*log(e*x + 3) + a)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a + b\log(3 + ex)}{x}\,dx = b\,\mathrm{Li}_2\left(-\frac{e\,x}{3}\right) + a\,\ln(x) + b\,\ln(e\,x + 3)\,\ln\left(-\frac{e\,x}{3}\right)$$

```
[In] int((a + b*log(e*x + 3))/x,x)
[Out] b*dilog(-(e*x)/3) + a*log(x) + b*log(e*x + 3)*log(-(e*x)/3)
```

# 3.80 $\int \frac{a+b\log(2+ex)}{x}\,dx$

Contents



## Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{a+b\log(2+ex)}{x}\,dx = (a+b\log(2))\log(x) - b\,\mathrm{PolyLog}\left(2,-\frac{ex}{2}\right)$$

```
[Out] (a+b*ln(2))*ln(x)-b*polylog(2,-1/2*e*x)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2439, 2438}

$$\int \frac{a+b\log(2+ex)}{x}\,dx = \log(x)(a+b\log(2)) - b\,\mathrm{PolyLog}\left(2,-\frac{ex}{2}\right)$$

```
[In] Int[(a + b*Log[2 + e*x])/x,x]
```

```
[Out] (a + b*Log[2])*Log[x] - b*PolyLog[2, -1/2*(e*x)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + e*(x/d)]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rubi steps

$$\text{integral} = (a + b\log(2))\log(x) + b\int \frac{\log\left(1+\frac{ex}{2}\right)}{x}\,dx$$
$$= (a + b\log(2))\log(x) - b\text{Li}_2\left(-\frac{ex}{2}\right)$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log(2+ex)}{x}\,dx = a\log(x) + b\log(2)\log(x) - b\,\text{PolyLog}\left(2,-\frac{ex}{2}\right)$$

```
[In] Integrate[(a + b*Log[2 + e*x])/x,x]
[Out] a*Log[x] + b*Log[2]*Log[x] - b*PolyLog[2, -1/2*(e*x)]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

| method | result | size |
|---|---|---|
| parts | $\ln(x)\,a + b\left(\left(\ln(ex+2) - \ln\left(\frac{ex}{2}+1\right)\right)\ln\left(-\frac{ex}{2}\right) - \text{dilog}\left(\frac{ex}{2}+1\right)\right)$ | 40 |
| derivativedivides | $a\ln(ex) + b\left(\left(\ln(ex+2) - \ln\left(\frac{ex}{2}+1\right)\right)\ln\left(-\frac{ex}{2}\right) - \text{dilog}\left(\frac{ex}{2}+1\right)\right)$ | 42 |
| default | $a\ln(ex) + b\left(\left(\ln(ex+2) - \ln\left(\frac{ex}{2}+1\right)\right)\ln\left(-\frac{ex}{2}\right) - \text{dilog}\left(\frac{ex}{2}+1\right)\right)$ | 42 |
| risch | $\ln(x)\,a + \ln(ex+2)\ln\left(-\frac{ex}{2}\right)b - \ln\left(\frac{ex}{2}+1\right)\ln\left(-\frac{ex}{2}\right)b - \text{dilog}\left(\frac{ex}{2}+1\right)b$ | 44 |

```
[In] int((a+b*ln(e*x+2))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*a+b*((ln(e*x+2)-ln(1/2*e*x+1))*ln(-1/2*e*x)-dilog(1/2*e*x+1))
```

## Fricas [F]

$$\int \frac{a+b\log(2+ex)}{x}\,dx=\int \frac{b\log(ex+2)+a}{x}\,dx$$

```
[In] integrate((a+b*log(e*x+2))/x,x, algorithm="fricas")
[Out] integral((b*log(e*x + 2) + a)/x, x)
```

## Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.48

$$\int \frac{a+b\log(2+ex)}{x}\,dx=a\log(x)+b\left(\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{for } \frac{1}{|x|}<1\wedge|x|<1\\ \log(2)\log(x)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{for } |x|<1\\ -\log(2)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(2)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(2)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{2}\right) & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a+b*ln(e*x+2))/x,x)
[Out] a*log(x) + b*Piecewise((-polylog(2, e*x*exp_polar(I*pi)/2), (Abs(x) < 1) &
(1/Abs(x) < 1)), (log(2)*log(x) - polylog(2, e*x*exp_polar(I*pi)/2), Abs(x)
 < 1), (-log(2)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/2), 1/Abs(x) < 1)
, (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(2) + meijerg(((1, 1), ()), (
(), (0, 0)), x)*log(2) - polylog(2, e*x*exp_polar(I*pi)/2), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{a+b\log(2+ex)}{x}\,dx=\left(\log(ex+2)\log\left(-\frac{1}{2}\,ex\right)+\operatorname{Li}_2\left(\frac{1}{2}\,ex+1\right)\right)b+a\log(x)$$

```
[In] integrate((a+b*log(e*x+2))/x,x, algorithm="maxima")
[Out] (log(e*x + 2)*log(-1/2*e*x) + dilog(1/2*e*x + 1))*b + a*log(x)
```

## Giac [F]

$$\int \frac{a+b\log(2+ex)}{x}\,dx = \int \frac{b\log\left(ex+2\right)+a}{x}\,dx$$

```
[In] integrate((a+b*log(e*x+2))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(e*x + 2) + a)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log(2+ex)}{x}\,dx = b\,\mathrm{Li}_2\left(-\frac{e\,x}{2}\right) + a\,\ln\left(x\right) + b\,\ln\left(e\,x+2\right)\,\ln\left(-\frac{e\,x}{2}\right)$$

```
[In] int((a + b*log(e*x + 2))/x,x)
```

```
[Out] b*dilog(-(e*x)/2) + a*log(x) + b*log(e*x + 2)*log(-(e*x)/2)
```

# 3.81 $\int \frac{a+b\log(1+ex)}{x}\,dx$



## Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{a+b\log(1+ex)}{x}\,dx = a\log(x) - b\,\text{PolyLog}(2,-ex)$$

```
[Out] a*ln(x)-b*polylog(2,-e*x)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2439, 2438}

$$\int \frac{a+b\log(1+ex)}{x}\,dx = a\log(x) - b\,\text{PolyLog}(2,-ex)$$

```
[In] Int[(a + b*Log[1 + e*x])/x,x]
```

```
[Out] a*Log[x] - b*PolyLog[2, -(e*x)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + e*(x/d)]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= a\log(x) + b\int \frac{\log(1+ex)}{x}\,dx \\ &= a\log(x) - b\text{Li}_2(-ex) \end{aligned}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log(1+ex)}{x}\,dx = a\log(x) - b\,\text{PolyLog}(2,-ex)$$

```
[In] Integrate[(a + b*Log[1 + e*x])/x,x]
[Out] a*Log[x] - b*PolyLog[2, -(e*x)]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| risch | $\ln(x)\,a - b\,\text{dilog}(ex+1)$ | 15 |
| parts | $\ln(x)\,a - b\,\text{dilog}(ex+1)$ | 15 |
| derivativedivides | $a\ln(ex) - b\,\text{dilog}(ex+1)$ | 17 |
| default | $a\ln(ex) - b\,\text{dilog}(ex+1)$ | 17 |

```
[In] int((a+b*ln(e*x+1))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*a-b*dilog(e*x+1)
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{a+b\log(1+ex)}{x}\,dx = -b\text{Li}_2(-ex) + a\log(x)$$

```
[In] integrate((a+b*log(e*x+1))/x,x, algorithm="fricas")
[Out] -b*dilog(-e*x) + a*log(x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log(1+ex)}{x}\,dx = a\log(x) - b\,\mathrm{Li}_2\left(exe^{i\pi}\right)$$

```
[In] integrate((a+b*ln(e*x+1))/x,x)
[Out] a*log(x) - b*polylog(2, e*x*exp_polar(I*pi))
```

## Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{a+b\log(1+ex)}{x}\,dx = (\log(ex+1)\log(-ex) + \mathrm{Li}_2(ex+1))b + a\log(x)$$

```
[In] integrate((a+b*log(e*x+1))/x,x, algorithm="maxima")
[Out] (log(e*x + 1)*log(-e*x) + dilog(e*x + 1))*b + a*log(x)
```

## Giac [F]

$$\int \frac{a+b\log(1+ex)}{x}\,dx = \int \frac{b\log(ex+1)+a}{x}\,dx$$

```
[In] integrate((a+b*log(e*x+1))/x,x, algorithm="giac")
[Out] integrate((b*log(e*x + 1) + a)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log(1+ex)}{x}\,dx = a\,\ln(x) - b\,\mathrm{polylog}(2,-e\,x)$$

```
[In] int((a + b*log(e*x + 1))/x,x)
[Out] a*log(x) - b*polylog(2, -e*x)
```

# 3.82 $\int \frac{a+b\log(ex)}{x}\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \frac{a+b\log(ex)}{x}\,dx = \frac{(a+b\log(ex))^2}{2b}$$

```
[Out] 1/2*(a+b*ln(e*x))^2/b
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = $\{2338\}$

$$\int \frac{a+b\log(ex)}{x}\,dx = \frac{(a+b\log(ex))^2}{2b}$$

```
[In] Int[(a + b*Log[e*x])/x,x]
```

```
[Out] (a + b*Log[e*x])^2/(2*b)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = \frac{(a+b\log(ex))^2}{2b}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{a+b\log(ex)}{x}\,dx = a\log(x)+\frac{1}{2}b\log^2(ex)$$

```
[In] Integrate[(a + b*Log[e*x])/x,x]
[Out] a*Log[x] + (b*Log[e*x]^2)/2
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| risch | $\frac{b\ln(ex)^2}{2}+\ln(x)\,a$ | 15 |
| parts | $\frac{b\ln(ex)^2}{2}+\ln(x)\,a$ | 15 |
| derivativedivides | $\frac{b\ln(ex)^2}{2}+a\ln(ex)$ | 17 |
| default | $\frac{b\ln(ex)^2}{2}+a\ln(ex)$ | 17 |
| norman | $\frac{b\ln(ex)^2}{2}+a\ln(ex)$ | 17 |
| parallelrisch | $\frac{b\ln(ex)^2}{2}+a\ln(ex)$ | 17 |

```
[In] int((a+b*ln(e*x))/x,x,method=_RETURNVERBOSE)
[Out] 1/2*b*ln(e*x)^2+ln(x)*a
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{a+b\log(ex)}{x}\,dx = \frac{1}{2}\,b\log(ex)^2+a\log(ex)$$

```
[In] integrate((a+b*log(e*x))/x,x, algorithm="fricas")
[Out] 1/2*b*log(e*x)^2 + a*log(e*x)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+b\log(ex)}{x}\,dx = a\log(x) + \frac{b\log(ex)^2}{2}$$

```
[In] integrate((a+b*ln(e*x))/x,x)
[Out] a*log(x) + b*log(e*x)**2/2
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+b\log(ex)}{x}\,dx = \frac{(b\log(ex)+a)^2}{2b}$$

```
[In] integrate((a+b*log(e*x))/x,x, algorithm="maxima")
[Out] 1/2*(b*log(e*x) + a)^2/b
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+b\log(ex)}{x}\,dx = \frac{1}{2}b\log(ex)^2 + a\log(x)$$

```
[In] integrate((a+b*log(e*x))/x,x, algorithm="giac")
[Out] 1/2*b*log(e*x)^2 + a*log(x)
```

## Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+b\log(ex)}{x}\,dx = \frac{b\ln(e\,x)^2}{2} + a\,\ln(x)$$

```
[In] int((a + b*log(e*x))/x,x)
[Out] a*log(x) + (b*log(e*x)^2)/2
```

# 3.83 $\int \frac{a+b\log(-1+ex)}{x} \, dx$



## Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \frac{a+b\log(-1+ex)}{x} \, dx = \log(ex)(a+b\log(-1+ex)) + b\,\mathrm{PolyLog}(2,1-ex)$$

```
[Out] ln(e*x)*(a+b*ln(e*x-1))+b*polylog(2,-e*x+1)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2441, 2352}

$$\int \frac{a+b\log(-1+ex)}{x} \, dx = \log(ex)(a+b\log(ex-1)) + b\,\mathrm{PolyLog}(2,1-ex)$$

```
[In] Int[(a + b*Log[-1 + e*x])/x,x]
```

```
[Out] Log[e*x]*(a + b*Log[-1 + e*x]) + b*PolyLog[2, 1 - e*x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \log(ex)(a+b\log(-1+ex)) - (be)\int\frac{\log(ex)}{-1+ex}\,dx\\ &= \log(ex)(a+b\log(-1+ex)) + b\text{Li}_2(1-ex)\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int\frac{a+b\log(-1+ex)}{x}\,dx = a\log(x) + b\log(ex)\log(-1+ex) + b\,\text{PolyLog}(2,1-ex)$$

```
[In] Integrate[(a + b*Log[-1 + e*x])/x,x]
[Out] a*Log[x] + b*Log[e*x]*Log[-1 + e*x] + b*PolyLog[2, 1 - e*x]
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| risch | $\ln(x)\,a+\ln(ex-1)\ln(ex)\,b+\text{dilog}(ex)\,b$ | 24 |
| parts | $\ln(x)\,a+b(\text{dilog}(ex)+\ln(ex)\ln(ex-1))$ | 24 |
| derivativedivides | $a\ln(ex)+b(\text{dilog}(ex)+\ln(ex)\ln(ex-1))$ | 26 |
| default | $a\ln(ex)+b(\text{dilog}(ex)+\ln(ex)\ln(ex-1))$ | 26 |

```
[In] int((a+b*ln(e*x-1))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*a+ln(e*x-1)*ln(e*x)*b+dilog(e*x)*b
```

## Fricas [F]

$$\int \frac{a+b\log(-1+ex)}{x}\,dx = \int \frac{b\log\left(ex-1\right)+a}{x}\,dx$$

```
[In] integrate((a+b*log(e*x-1))/x,x, algorithm="fricas")
[Out] integral((b*log(e*x - 1) + a)/x, x)
```

## Sympy [A] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \frac{a+b\log(-1+ex)}{x}\,dx = a\log\left(x\right) + b\left(\begin{cases} -\operatorname{Li}_{2}\left(ex\right) & \text{for}\: \frac{1}{|x|} < 1 \wedge |x| < 1 \\ i\pi\log\left(x\right) - \operatorname{Li}_{2}\left(ex\right) & \text{for}\: |x| < 1 \\ -i\pi\log\left(\frac{1}{x}\right) - \operatorname{Li}_{2}\left(ex\right) & \text{for}\: \frac{1}{|x|} < 1 \\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1, 1 \\ 0, 0 & \end{matrix} \middle| x\right) + i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1, 1 & \\ & 0, 0 \end{matrix} \middle| x\right) - \operatorname{Li}_{2}\left(ex\right) & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((a+b*ln(e*x-1))/x,x)
[Out] a*log(x) + b*Piecewise((-polylog(2, e*x), (Abs(x) < 1) & (1/Abs(x) < 1)), (
I*pi*log(x) - polylog(2, e*x), Abs(x) < 1), (-I*pi*log(1/x) - polylog(2, e*
x), 1/Abs(x) < 1), (-I*pi*meijerg(((), (1, 1)), ((0, 0), ()), x) + I*pi*mei
jerg(((1, 1), ()), ((), (0, 0)), x) - polylog(2, e*x), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log(-1+ex)}{x}\,dx = \left(\log\left(ex-1\right)\log\left(ex\right) + \operatorname{Li}_2(-ex+1)\right)b + a\log\left(x\right)$$

```
[In] integrate((a+b*log(e*x-1))/x,x, algorithm="maxima")
[Out] (log(e*x - 1)*log(e*x) + dilog(-e*x + 1))*b + a*log(x)
```

## Giac [F]

$$\int \frac{a+b\log(-1+ex)}{x}\,dx=\int \frac{b\log\left(ex-1\right)+a}{x}\,dx$$

```
[In] integrate((a+b*log(e*x-1))/x,x, algorithm="giac")
[Out] integrate((b*log(e*x - 1) + a)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{a+b\log(-1+ex)}{x}\,dx=b\,\mathrm{Li}_2(e\,x)+a\,\ln\left(x\right)+b\,\ln\left(e\,x-1\right)\,\ln\left(e\,x\right)$$

```
[In] int((a + b*log(e*x - 1))/x,x)
[Out] b*dilog(e*x) + a*log(x) + b*log(e*x - 1)*log(e*x)
```

# 3.84 $\int \frac{a+b\log(-2+ex)}{x}\,dx$



## Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{a+b\log(-2+ex)}{x}\,dx=\log\left(\frac{ex}{2}\right)(a+b\log(-2+ex))+b\,\mathrm{PolyLog}\left(2,1-\frac{ex}{2}\right)$$

```
[Out] ln(1/2*e*x)*(a+b*ln(e*x-2))+b*polylog(2,1-1/2*e*x)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2441, 2352}

$$\int \frac{a+b\log(-2+ex)}{x}\,dx=\log\left(\frac{ex}{2}\right)(a+b\log(ex-2))+b\,\mathrm{PolyLog}\left(2,1-\frac{ex}{2}\right)$$

```
[In] Int[(a + b*Log[-2 + e*x])/x,x]
```

```
[Out] Log[(e*x)/2]*(a + b*Log[-2 + e*x]) + b*PolyLog[2, 1 - (e*x)/2]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \log\left(\frac{ex}{2}\right)(a+b\log(-2+ex)) - (be)\int\frac{\log\left(\frac{ex}{2}\right)}{-2+ex}\,dx \\
&= \log\left(\frac{ex}{2}\right)(a+b\log(-2+ex)) + b\text{Li}_2\left(1-\frac{ex}{2}\right)
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$
\int\frac{a+b\log(-2+ex)}{x}\,dx = a\log(x)+b\log\left(\frac{ex}{2}\right)\log(-2+ex)+b\,\text{PolyLog}\left(2,\frac{1}{2}(2-ex)\right)
$$

```
[In] Integrate[(a + b*Log[-2 + e*x])/x,x]
[Out] a*Log[x] + b*Log[(e*x)/2]*Log[-2 + e*x] + b*PolyLog[2, (2 - e*x)/2]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

| method | result | size |
|---|---|---|
| risch | $\ln(x)\,a+\ln(ex-2)\ln\left(\frac{ex}{2}\right)b+\text{dilog}\left(\frac{ex}{2}\right)b$ | 26 |
| parts | $\ln(x)\,a+b\left(\text{dilog}\left(\frac{ex}{2}\right)+\ln\left(\frac{ex}{2}\right)\ln(ex-2)\right)$ | 26 |
| derivativedivides | $a\ln(ex)+b\left(\text{dilog}\left(\frac{ex}{2}\right)+\ln\left(\frac{ex}{2}\right)\ln(ex-2)\right)$ | 28 |
| default | $a\ln(ex)+b\left(\text{dilog}\left(\frac{ex}{2}\right)+\ln\left(\frac{ex}{2}\right)\ln(ex-2)\right)$ | 28 |

```
[In] int((a+b*ln(e*x-2))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*a+ln(e*x-2)*ln(1/2*e*x)*b+dilog(1/2*e*x)*b
```

## Fricas [F]

$$\int \frac{a+b\log(-2+ex)}{x}\,dx = \int \frac{b\log(ex-2)+a}{x}\,dx$$

```
[In] integrate((a+b*log(e*x-2))/x,x, algorithm="fricas")
[Out] integral((b*log(e*x - 2) + a)/x, x)
```

## Sympy [A] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int \frac{a+b\log(-2+ex)}{x}\,dx = a\log(x) + b\left(\begin{cases} -\operatorname{Li}_2\left(\frac{ex}{2}\right) \\ \log(2)\log(x) + 3i\pi\log(x) - \operatorname{Li}_2\left(\frac{ex}{2}\right) \\ -\log(2)\log\left(\frac{1}{x}\right) - 3i\pi\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{ex}{2}\right) \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(2) - 3i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(2) + 3i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 \end{matrix}\right.\end{cases}\right)$$

```
[In] integrate((a+b*ln(e*x-2))/x,x)
[Out] a*log(x) + b*Piecewise((-polylog(2, e*x/2), (Abs(x) < 1) & (1/Abs(x) < 1)),
 (log(2)*log(x) + 3*I*pi*log(x) - polylog(2, e*x/2), Abs(x) < 1), (-log(2)*
log(1/x) - 3*I*pi*log(1/x) - polylog(2, e*x/2), 1/Abs(x) < 1), (-meijerg(((
), (1, 1)), ((0, 0), ()), x)*log(2) - 3*I*pi*meijerg(((), (1, 1)), ((0, 0),
 ()), x) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(2) + 3*I*pi*meijerg((
(1, 1), ()), ((), (0, 0)), x) - polylog(2, e*x/2), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{a+b\log(-2+ex)}{x}\,dx = \left(\log(ex-2)\log\left(\frac{1}{2}ex\right) + \operatorname{Li}_2\left(-\frac{1}{2}ex+1\right)\right)b + a\log(x)$$

```
[In] integrate((a+b*log(e*x-2))/x,x, algorithm="maxima")
[Out] (log(e*x - 2)*log(1/2*e*x) + dilog(-1/2*e*x + 1))*b + a*log(x)
```

## Giac [F]

$$\int \frac{a+b\log(-2+ex)}{x}\,dx = \int \frac{b\log\left(ex-2\right)+a}{x}\,dx$$

```
[In] integrate((a+b*log(e*x-2))/x,x, algorithm="giac")
[Out] integrate((b*log(e*x - 2) + a)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{a+b\log(-2+ex)}{x}\,dx = b\,\mathrm{Li}_2\left(\frac{e\,x}{2}\right)+a\,\ln\left(x\right)+b\,\ln\left(e\,x-2\right)\,\ln\left(\frac{e\,x}{2}\right)$$

```
[In] int((a + b*log(e*x - 2))/x,x)
[Out] b*dilog((e*x)/2) + a*log(x) + b*log(e*x - 2)*log((e*x)/2)
```

# 3.85 $\int x^2 \log^2 (c(a+bx)^n) \, dx$



## Optimal result

Integrand size = 16, antiderivative size = 187

$$\begin{aligned}\int x^2 \log^2 (c(a+bx)^n) \, dx = {} & \frac{2a^2n^2x}{b^2} - \frac{an^2(a+bx)^2}{2b^3} + \frac{2n^2(a+bx)^3}{27b^3} \\ & - \frac{a^3n^2\log^2(a+bx)}{3b^3} - \frac{2a^2n(a+bx)\log(c(a+bx)^n)}{b^3} \\ & + \frac{an(a+bx)^2\log(c(a+bx)^n)}{b^3} - \frac{2n(a+bx)^3\log(c(a+bx)^n)}{9b^3} \\ & + \frac{2a^3n\log(a+bx)\log(c(a+bx)^n)}{3b^3} + \frac{1}{3}x^3\log^2(c(a+bx)^n)\end{aligned}$$

```
[Out] 2*a^2*n^2*x/b^2-1/2*a*n^2*(b*x+a)^2/b^3+2/27*n^2*(b*x+a)^3/b^3-1/3*a^3*n^2*
ln(b*x+a)^2/b^3-2*a^2*n*(b*x+a)*ln(c*(b*x+a)^n)/b^3+a*n*(b*x+a)^2*ln(c*(b*x
+a)^n)/b^3-2/9*n*(b*x+a)^3*ln(c*(b*x+a)^n)/b^3+2/3*a^3*n*ln(b*x+a)*ln(c*(b*
x+a)^n)/b^3+1/3*x^3*ln(c*(b*x+a)^n)^2
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2445, 2458, 45, 2372, 12, 14, 2338}

$$\begin{aligned}\int x^2 \log^2 (c(a+bx)^n) \, dx = {} & \frac{2a^3n\log(a+bx)\log(c(a+bx)^n)}{3b^3} - \frac{a^3n^2\log^2(a+bx)}{3b^3} \\ & - \frac{2a^2n(a+bx)\log(c(a+bx)^n)}{b^3} + \frac{2a^2n^2x}{b^2} \\ & + \frac{an(a+bx)^2\log(c(a+bx)^n)}{b^3} - \frac{2n(a+bx)^3\log(c(a+bx)^n)}{9b^3} \\ & - \frac{an^2(a+bx)^2}{2b^3} + \frac{2n^2(a+bx)^3}{27b^3} + \frac{1}{3}x^3\log^2(c(a+bx)^n)\end{aligned}$$

```
[In] Int[x^2*Log[c*(a + b*x)^n]^2,x]
```

```
[Out] (2*a^2*n^2*x)/b^2 - (a*n^2*(a + b*x)^2)/(2*b^3) + (2*n^2*(a + b*x)^3)/(27*b
^3) - (a^3*n^2*Log[a + b*x]^2)/(3*b^3) - (2*a^2*n*(a + b*x)*Log[c*(a + b*x)
^n])/b^3 + (a*n*(a + b*x)^2*Log[c*(a + b*x)^n])/b^3 - (2*n*(a + b*x)^3*Log[
c*(a + b*x)^n])/(9*b^3) + (2*a^3*n*Log[a + b*x]*Log[c*(a + b*x)^n])/(3*b^3)
 + (x^3*Log[c*(a + b*x)^n]^2)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\log^2\left(c(a+bx)^n\right)-\frac{1}{3}(2bn)\int\frac{x^3\log\left(c(a+bx)^n\right)}{a+bx}\,dx\\
&= \frac{1}{3}x^3\log^2\left(c(a+bx)^n\right)-\frac{1}{3}(2n)\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\log\left(cx^n\right)}{x}\,dx,x,a+bx\right)\\
&= -\frac{2a^2n(a+bx)\log\left(c(a+bx)^n\right)}{b^3}+\frac{an(a+bx)^2\log\left(c(a+bx)^n\right)}{b^3}\\
&\quad-\frac{2n(a+bx)^3\log\left(c(a+bx)^n\right)}{9b^3}\\
&\quad+\frac{2a^3n\log(a+bx)\log\left(c(a+bx)^n\right)}{3b^3}+\frac{1}{3}x^3\log^2\left(c(a+bx)^n\right)\\
&\quad+\frac{1}{3}\left(2n^2\right)\text{Subst}\left(\int\frac{18a^2x-9ax^2+2x^3-6a^3\log(x)}{6b^3x}\,dx,x,a+bx\right)\\
&= -\frac{2a^2n(a+bx)\log\left(c(a+bx)^n\right)}{b^3}+\frac{an(a+bx)^2\log\left(c(a+bx)^n\right)}{b^3}\\
&\quad-\frac{2n(a+bx)^3\log\left(c(a+bx)^n\right)}{9b^3}+\frac{2a^3n\log(a+bx)\log\left(c(a+bx)^n\right)}{3b^3}\\
&\quad+\frac{1}{3}x^3\log^2\left(c(a+bx)^n\right)+\frac{n^2\text{Subst}\left(\int\frac{18a^2x-9ax^2+2x^3-6a^3\log(x)}{x}\,dx,x,a+bx\right)}{9b^3}\\
&= -\frac{2a^2n(a+bx)\log\left(c(a+bx)^n\right)}{b^3}+\frac{an(a+bx)^2\log\left(c(a+bx)^n\right)}{b^3}\\
&\quad-\frac{2n(a+bx)^3\log\left(c(a+bx)^n\right)}{9b^3}+\frac{2a^3n\log(a+bx)\log\left(c(a+bx)^n\right)}{3b^3}\\
&\quad+\frac{1}{3}x^3\log^2\left(c(a+bx)^n\right)+\frac{n^2\text{Subst}\left(\int\left(18a^2-9ax+2x^2-\frac{6a^3\log(x)}{x}\right)\,dx,x,a+bx\right)}{9b^3}\\
&= \frac{2a^2n^2x}{b^2}-\frac{an^2(a+bx)^2}{2b^3}+\frac{2n^2(a+bx)^3}{27b^3}\\
&\quad-\frac{2a^2n(a+bx)\log\left(c(a+bx)^n\right)}{b^3}+\frac{an(a+bx)^2\log\left(c(a+bx)^n\right)}{b^3}\\
&\quad-\frac{2n(a+bx)^3\log\left(c(a+bx)^n\right)}{9b^3}+\frac{2a^3n\log(a+bx)\log\left(c(a+bx)^n\right)}{3b^3}\\
&\quad+\frac{1}{3}x^3\log^2\left(c(a+bx)^n\right)-\frac{\left(2a^3n^2\right)\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,a+bx\right)}{3b^3}
\end{aligned}$$

$$= \frac{2a^2n^2x}{b^2} - \frac{an^2(a+bx)^2}{2b^3} + \frac{2n^2(a+bx)^3}{27b^3} - \frac{a^3n^2\log^2(a+bx)}{3b^3} - \frac{2a^2n(a+bx)\log\left(c(a+bx)^n\right)}{b^3} + \frac{an(a+bx)^2\log\left(c(a+bx)^n\right)}{b^3} - \frac{2n(a+bx)^3\log\left(c(a+bx)^n\right)}{9b^3} + \frac{2a^3n\log(a+bx)\log\left(c(a+bx)^n\right)}{3b^3} + \frac{1}{3}x^3\log^2\left(c(a+bx)^n\right)$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.87

$$\int x^2\log^2\left(c(a+bx)^n\right)\,dx = \frac{11a^2n^2x}{9b^2} - \frac{5an^2x^2}{18b} + \frac{2n^2x^3}{27} - \frac{11a^3n\log\left(c(a+bx)^n\right)}{9b^3} - \frac{2a^2nx\log\left(c(a+bx)^n\right)}{3b^2} + \frac{anx^2\log\left(c(a+bx)^n\right)}{3b} - \frac{2}{9}nx^3\log\left(c(a+bx)^n\right) + \frac{a^3\log^2\left(c(a+bx)^n\right)}{3b^3} + \frac{1}{3}x^3\log^2\left(c(a+bx)^n\right)$$

```
[In] Integrate[x^2*Log[c*(a + b*x)^n]^2,x]
```

```
[Out] (11*a^2*n^2*x)/(9*b^2) - (5*a*n^2*x^2)/(18*b) + (2*n^2*x^3)/27 - (11*a^3*n*
Log[c*(a + b*x)^n])/(9*b^3) - (2*a^2*n*x*Log[c*(a + b*x)^n])/(3*b^2) + (a*n
*x^2*Log[c*(a + b*x)^n])/(3*b) - (2*n*x^3*Log[c*(a + b*x)^n])/9 + (a^3*Log[
c*(a + b*x)^n]^2)/(3*b^3) + (x^3*Log[c*(a + b*x)^n]^2)/3
```

## Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| parallelrisch | $-\frac{-18x^3\ln(c(bx+a)^n)^2b^3+12x^3\ln(c(bx+a)^n)b^3n-4b^3n^2x^3-18x^2\ln(c(bx+a)^n)a\,b^2n+15a\,b^2n^2x^2+102\ln(bx+a)a^3n^2+36x\ln}{54b^3}$ |
| risch | Expression too large to display |

```
[In] int(x^2*ln(c*(b*x+a)^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/54*(-18*x^3*ln(c*(b*x+a)^n)^2*b^3+12*x^3*ln(c*(b*x+a)^n)*b^3*n-4*b^3*n^2
*x^3-18*x^2*ln(c*(b*x+a)^n)*a*b^2*n+15*a*b^2*n^2*x^2+102*ln(b*x+a)*a^3*n^2+
36*x*ln(c*(b*x+a)^n)*a^2*b*n-66*a^2*b*n^2*x-18*ln(c*(b*x+a)^n)^2*a^3-36*ln(
c*(b*x+a)^n)*a^3*n+66*a^3*n^2)/b^3
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.96

$$\int x^2 \log^2 \left(c(a+bx)^n\right) \, dx$$
$$= \frac{4\, b^3 n^2 x^3 + 18\, b^3 x^3 \log\left(c\right)^2 - 15\, a b^2 n^2 x^2 + 66\, a^2 b n^2 x + 18\, \left(b^3 n^2 x^3 + a^3 n^2\right) \log\left(bx + a\right)^2 - 6\, \left(2\, b^3 n^2 x^3 - 3\, \right.}{}$$

```
[In] integrate(x^2*log(c*(b*x+a)^n)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(4*b^3*n^2*x^3 + 18*b^3*x^3*log(c)^2 - 15*a*b^2*n^2*x^2 + 66*a^2*b*n^2
*x + 18*(b^3*n^2*x^3 + a^3*n^2)*log(b*x + a)^2 - 6*(2*b^3*n^2*x^3 - 3*a*b^2
*n^2*x^2 + 6*a^2*b*n^2*x + 11*a^3*n^2 - 6*(b^3*n*x^3 + a^3*n)*log(c))*log(b
*x + a) - 6*(2*b^3*n*x^3 - 3*a*b^2*n*x^2 + 6*a^2*b*n*x)*log(c))/b^3
```

## Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.93

$$\int x^2 \log^2 \left(c(a+bx)^n\right) \, dx$$
$$= \begin{cases} -\frac{11a^3 n \log\left(c(a+bx)^n\right)}{9b^3} + \frac{a^3 \log\left(c(a+bx)^n\right)^2}{3b^3} + \frac{11a^2n^2x}{9b^2} - \frac{2a^2nx \log\left(c(a+bx)^n\right)}{3b^2} - \frac{5an^2x^2}{18b} + \frac{anx^2 \log\left(c(a+bx)^n\right)}{3b} + \frac{2n^2x^3}{27} - \\ \frac{x^3 \log\left(a^n c\right)^2}{3} \end{cases}$$

```
[In] integrate(x**2*ln(c*(b*x+a)**n)**2,x)
```

```
[Out] Piecewise((-11*a**3*n*log(c*(a + b*x)**n)/(9*b**3) + a**3*log(c*(a + b*x)**
n)**2/(3*b**3) + 11*a**2*n**2*x/(9*b**2) - 2*a**2*n*x*log(c*(a + b*x)**n)/(
3*b**2) - 5*a*n**2*x**2/(18*b) + a*n*x**2*log(c*(a + b*x)**n)/(3*b) + 2*n**
2*x**3/27 - 2*n*x**3*log(c*(a + b*x)**n)/9 + x**3*log(c*(a + b*x)**n)**2/3,
 Ne(b, 0)), (x**3*log(a**n*c)**2/3, True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.70

$$\int x^2\log^2\left(c(a+bx)^n\right)\,dx = \frac{1}{3}x^3\log\left((bx+a)^nc\right)^2 + \frac{1}{9}bn\left(\frac{6a^3\log(bx+a)}{b^4}-\frac{2b^2x^3-3abx^2+6a^2x}{b^3}\right)\log\left((bx+a)^nc\right) + \frac{\left(4b^3x^3-15ab^2x^2-18a^3\log(bx+a)^2+66a^2bx-66a^3\log(bx+a)\right)n^2}{54b^3}$$

```
[In] integrate(x^2*log(c*(b*x+a)^n)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((b*x + a)^n*c)^2 + 1/9*b*n*(6*a^3*log(b*x + a)/b^4 - (2*b^2*x^3
 - 3*a*b*x^2 + 6*a^2*x)/b^3)*log((b*x + a)^n*c) + 1/54*(4*b^3*x^3 - 15*a*b^
2*x^2 - 18*a^3*log(b*x + a)^2 + 66*a^2*b*x - 66*a^3*log(b*x + a))*n^2/b^3
```

## Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.83

$$\int x^2\log^2\left(c(a+bx)^n\right)\,dx = \frac{(bx+a)^3n^2\log(bx+a)^2}{3b^3} - \frac{(bx+a)^2an^2\log(bx+a)^2}{b^3} + \frac{(bx+a)a^2n^2\log(bx+a)^2}{b^3} - \frac{2(bx+a)^3n^2\log(bx+a)}{9b^3} + \frac{(bx+a)^2an^2\log(bx+a)}{b^3} - \frac{2(bx+a)a^2n^2\log(bx+a)}{b^3} + \frac{2(bx+a)^3n\log(bx+a)\log(c)}{3b^3} - \frac{2(bx+a)^2an\log(bx+a)\log(c)}{b^3} + \frac{2(bx+a)a^2n\log(bx+a)\log(c)}{b^3} + \frac{2(bx+a)^3n^2}{27b^3} - \frac{(bx+a)^2an^2}{2b^3} + \frac{2(bx+a)a^2n^2}{b^3} - \frac{2(bx+a)^3n\log(c)}{9b^3} + \frac{(bx+a)^2an\log(c)}{b^3} - \frac{2(bx+a)a^2n\log(c)}{b^3} + \frac{(bx+a)^3\log(c)^2}{3b^3} - \frac{(bx+a)^2a\log(c)^2}{b^3} + \frac{(bx+a)a^2\log(c)^2}{b^3}$$

```
[In] integrate(x^2*log(c*(b*x+a)^n)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a)^3*n^2*log(b*x + a)^2/b^3 - (b*x + a)^2*a*n^2*log(b*x + a)^2/b
^3 + (b*x + a)*a^2*n^2*log(b*x + a)^2/b^3 - 2/9*(b*x + a)^3*n^2*log(b*x + a
)/b^3 + (b*x + a)^2*a*n^2*log(b*x + a)/b^3 - 2*(b*x + a)*a^2*n^2*log(b*x +
a)/b^3 + 2/3*(b*x + a)^3*n*log(b*x + a)*log(c)/b^3 - 2*(b*x + a)^2*a*n*log(
b*x + a)*log(c)/b^3 + 2*(b*x + a)*a^2*n*log(b*x + a)*log(c)/b^3 + 2/27*(b*x
 + a)^3*n^2/b^3 - 1/2*(b*x + a)^2*a*n^2/b^3 + 2*(b*x + a)*a^2*n^2/b^3 - 2/9
*(b*x + a)^3*n*log(c)/b^3 + (b*x + a)^2*a*n*log(c)/b^3 - 2*(b*x + a)*a^2*n*
log(c)/b^3 + 1/3*(b*x + a)^3*log(c)^2/b^3 - (b*x + a)^2*a*log(c)^2/b^3 + (b
*x + a)*a^2*log(c)^2/b^3
```

## Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.62

$$\int x^2 \log^2\left(c(a+bx)^n\right)\,dx = \frac{2\,n^2\,x^3}{27} + \ln\left(c\,(a+b\,x)^n\right)^2\left(\frac{x^3}{3}+\frac{a^3}{3\,b^3}\right) - \ln\left(c\,(a+b\,x)^n\right)\left(\frac{2\,n\,x^3}{9}-\frac{a\,n\,x^2}{3\,b}+\frac{2\,a^2\,n\,x}{3\,b^2}\right) - \frac{11\,a^3\,n^2\,\ln(a+b\,x)}{9\,b^3} - \frac{5\,a\,n^2\,x^2}{18\,b} + \frac{11\,a^2\,n^2\,x}{9\,b^2}$$

```
[In] int(x^2*log(c*(a + b*x)^n)^2,x)
```

```
[Out] (2*n^2*x^3)/27 + log(c*(a + b*x)^n)^2*(x^3/3 + a^3/(3*b^3)) - log(c*(a + b*
x)^n)*((2*n*x^3)/9 - (a*n*x^2)/(3*b) + (2*a^2*n*x)/(3*b^2)) - (11*a^3*n^2*l
og(a + b*x))/(9*b^3) - (5*a*n^2*x^2)/(18*b) + (11*a^2*n^2*x)/(9*b^2)
```

# 3.86 $\int \frac{\log^2(c(a+bx)^n)}{x^4} \, dx$



## Optimal result

Integrand size = 16, antiderivative size = 177

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{x^4} \, dx = -\frac{b^2n^2}{3a^2x}-\frac{b^3n^2\log(x)}{a^3}+\frac{b^3n^2\log(a+bx)}{3a^3}-\frac{bn\log\left(c(a+bx)^n\right)}{3ax^2}+\frac{2b^2n(a+bx)\log\left(c(a+bx)^n\right)}{3a^3x}-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}+\frac{2b^3n\log\left(c(a+bx)^n\right)\log\left(1-\frac{a}{a+bx}\right)}{3a^3}-\frac{2b^3n^2\operatorname{PolyLog}\left(2,\frac{a}{a+bx}\right)}{3a^3}$$

```
[Out] -1/3*b^2*n^2/a^2/x-b^3*n^2*ln(x)/a^3+1/3*b^3*n^2*ln(b*x+a)/a^3-1/3*b*n*ln(c
*(b*x+a)^n)/a/x^2+2/3*b^2*n*(b*x+a)*ln(c*(b*x+a)^n)/a^3/x-1/3*ln(c*(b*x+a)^
n)^2/x^3+2/3*b^3*n*ln(c*(b*x+a)^n)*ln(1-a/(b*x+a))/a^3-2/3*b^3*n^2*polylog(
2,a/(b*x+a))/a^3
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{x^4} \, dx = \frac{2b^3n\log\left(1-\frac{a}{a+bx}\right)\log\left(c(a+bx)^n\right)}{3a^3}-\frac{2b^3n^2\operatorname{PolyLog}\left(2,\frac{a}{a+bx}\right)}{3a^3}-\frac{b^3n^2\log(x)}{a^3}+\frac{b^3n^2\log(a+bx)}{3a^3}+\frac{2b^2n(a+bx)\log\left(c(a+bx)^n\right)}{3a^3x}-\frac{b^2n^2}{3a^2x}-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}-\frac{bn\log\left(c(a+bx)^n\right)}{3ax^2}$$

```
[In] Int[Log[c*(a + b*x)^n]^2/x^4,x]
```

```
[Out] -1/3*(b^2*n^2)/(a^2*x) - (b^3*n^2*Log[x])/a^3 + (b^3*n^2*Log[a + b*x])/(3*a
^3) - (b*n*Log[c*(a + b*x)^n])/(3*a*x^2) + (2*b^2*n*(a + b*x)*Log[c*(a + b*
x)^n])/(3*a^3*x) - Log[c*(a + b*x)^n]^2/(3*x^3) + (2*b^3*n*Log[c*(a + b*x)^
n]*Log[1 - a/(a + b*x)])/(3*a^3) - (2*b^3*n^2*PolyLog[2, a/(a + b*x)])/(3*a
^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}+\frac{1}{3}(2bn)\int \frac{\log\left(c(a+bx)^n\right)}{x^3(a+bx)}\,dx\\
&= -\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}+\frac{1}{3}(2n)\text{Subst}\left(\int \frac{\log\left(cx^n\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx,x,a+bx\right)\\
&= -\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}+\frac{(2n)\text{Subst}\left(\int \frac{\log(cx^n)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx,x,a+bx\right)}{3a}\\
&\quad-\frac{(2bn)\text{Subst}\left(\int \frac{\log(cx^n)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx,x,a+bx\right)}{3a}\\
&= -\frac{bn\log\left(c(a+bx)^n\right)}{3ax^2}-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}-\frac{(2bn)\text{Subst}\left(\int \frac{\log(cx^n)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx,x,a+bx\right)}{3a^2}\\
&\quad+\frac{(2b^2n)\,\text{Subst}\left(\int \frac{\log(cx^n)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx,x,a+bx\right)}{3a^2}+\frac{(bn^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx,x,a+bx\right)}{3a}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{bn\log\left(c(a+bx)^n\right)}{3ax^2}+\frac{2b^2n(a+bx)\log\left(c(a+bx)^n\right)}{3a^3x} \\
&\quad -\frac{\log^2\left(c(a+bx)^n\right)}{3x^3}+\frac{2b^3n\log\left(c(a+bx)^n\right)\log\left(1-\frac{a}{a+bx}\right)}{3a^3} \\
&\quad +\frac{(bn^2)\,\text{Subst}\left(\int\left(\frac{b^2}{a(a-x)^2}+\frac{b^2}{a^2(a-x)}+\frac{b^2}{a^2x}\right)dx,x,a+bx\right)}{3a} \\
&\quad -\frac{(2b^2n^2)\,\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x}{b}}\,dx,x,a+bx\right)}{3a^3}-\frac{(2b^3n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{a}{x}\right)}{x}\,dx,x,a+bx\right)}{3a^3} \\
&= -\frac{b^2n^2}{3a^2x}-\frac{b^3n^2\log(x)}{a^3}+\frac{b^3n^2\log(a+bx)}{3a^3}-\frac{bn\log\left(c(a+bx)^n\right)}{3ax^2} \\
&\quad +\frac{2b^2n(a+bx)\log\left(c(a+bx)^n\right)}{3a^3x}-\frac{\log^2\left(c(a+bx)^n\right)}{3x^3} \\
&\quad +\frac{2b^3n\log\left(c(a+bx)^n\right)\log\left(1-\frac{a}{a+bx}\right)}{3a^3}-\frac{2b^3n^2\text{Li}_2\left(\frac{a}{a+bx}\right)}{3a^3}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.97

$$
\int\frac{\log^2\left(c(a+bx)^n\right)}{x^4}\,dx = -\frac{ab^2n^2x^2+3b^3n^2x^3\log(x)-3b^3n^2x^3\log(a+bx)+a^2bnx\log\left(c(a+bx)^n\right)-2ab^2nx^2\log\left(c(a+bx)^n\right)-
$$

```
[In] Integrate[Log[c*(a + b*x)^n]^2/x^4,x]
```

```
[Out] -1/3*(a*b^2*n^2*x^2 + 3*b^3*n^2*x^3*Log[x] - 3*b^3*n^2*x^3*Log[a + b*x] + a
^2*b*n*x*Log[c*(a + b*x)^n] - 2*a*b^2*n*x^2*Log[c*(a + b*x)^n] - 2*b^3*n*x^
3*Log[-((b*x)/a)]*Log[c*(a + b*x)^n] + a^3*Log[c*(a + b*x)^n]^2 + b^3*x^3*L
og[c*(a + b*x)^n]^2 - 2*b^3*n^2*x^3*PolyLog[2, 1 + (b*x)/a])/(a^3*x^3)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.73

| method | result |
|---|---|
| risch | $-\frac{\ln((bx+a)^n)^2}{3x^3}-\frac{2b^3n\ln((bx+a)^n)\ln(bx+a)}{3a^3}-\frac{bn\ln((bx+a)^n)}{3a\,x^2}+\frac{2b^3n\ln((bx+a)^n)\ln(x)}{3a^3}+\frac{2b^2n\ln((bx+a)^n)}{3a^2x}+\frac{b^3n^2\ln(b}{a^3}$ |

```
[In] int(ln(c*(b*x+a)^n)^2/x^4,x,method=_RETURNVERBOSE)
```

[Out] -1/3*ln((b*x+a)^n)^2/x^3-2/3*b^3*n*ln((b*x+a)^n)/a^3*ln(b*x+a)-1/3*b*n*ln((b*x+a)^n)/a/x^2+2/3*b^3*n*ln((b*x+a)^n)/a^3*ln(x)+2/3*b^2*n*ln((b*x+a)^n)/a^2/x+b^3*n^2*ln(b*x+a)/a^3-1/3*b^2*n^2/a^2/x-b^3*n^2*ln(x)/a^3-2/3*b^3*n^2/a^3*dilog((b*x+a)/a)-2/3*b^3*n^2/a^3*ln(x)*ln((b*x+a)/a)+1/3*b^3*n^2/a^3*ln(b*x+a)^2+(-I*Pi*csgn(I*c*(b*x+a)^n)^3+I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*c)-I*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^n)*csgn(I*c)+2*ln(c))*(-1/3*ln((b*x+a)^n)/x^3+1/3*b*n*(-b^2/a^3*ln(b*x+a)-1/2/a/x^2+b^2/a^3*ln(x)+b/a^2/x))-1/12*(-I*Pi*csgn(I*c*(b*x+a)^n)^3+I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*c)-I*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^n)*csgn(I*c)+2*ln(c))^2/x^3

## Fricas [F]

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{x^4}\,dx = \int \frac{\log\left(\left(bx+a\right)^n c\right)^2}{x^4}\,dx$$

[In] integrate(log(c*(b*x+a)^n)^2/x^4,x, algorithm="fricas")

[Out] integral(log((b*x + a)^n*c)^2/x^4, x)

## Sympy [F]

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{x^4}\,dx = \int \frac{\log\left(c(a+bx)^n\right)^2}{x^4}\,dx$$

[In] integrate(ln(c*(b*x+a)**n)**2/x**4,x)

[Out] Integral(log(c*(a + b*x)**n)**2/x**4, x)

## Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{x^4}\,dx = -\frac{1}{3}\,b^2n^2\left(\frac{2\left(\log\left(\frac{bx}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)b}{a^3} - \frac{3\,b\log\left(bx+a\right)}{a^3} - \frac{bx\log\left(bx+a\right)^2 - 3\,bx\log\left(x\right) - a}{a^3x}\right) - \frac{1}{3}\,bn\left(\frac{2\,b^2\log\left(bx+a\right)}{a^3} - \frac{2\,b^2\log\left(x\right)}{a^3} - \frac{2\,bx-a}{a^2x^2}\right)\log\left(\left(bx+a\right)^n c\right) - \frac{\log\left(\left(bx+a\right)^n c\right)^2}{3\,x^3}$$

```
[In] integrate(log(c*(b*x+a)^n)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*n^2*(2*(log(b*x/a + 1)*log(x) + dilog(-b*x/a))*b/a^3 - 3*b*log(b*x
 + a)/a^3 - (b*x*log(b*x + a)^2 - 3*b*x*log(x) - a)/(a^3*x)) - 1/3*b*n*(2*b
^2*log(b*x + a)/a^3 - 2*b^2*log(x)/a^3 - (2*b*x - a)/(a^2*x^2))*log((b*x +
a)^n*c) - 1/3*log((b*x + a)^n*c)^2/x^3
```

## Giac [F]

$$\int \frac{\log^2(c(a+bx)^n)}{x^4}\,dx = \int \frac{\log\left((bx+a)^n c\right)^2}{x^4}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^n*c)^2/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(c(a+bx)^n)}{x^4}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)^2}{x^4}\,dx$$

```
[In] int(log(c*(a + b*x)^n)^2/x^4,x)
```

```
[Out] int(log(c*(a + b*x)^n)^2/x^4, x)
```

# 3.87 $\int x^2 \log^3\left(c(a+bx)^n\right)\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 285

$$\int x^2 \log^3\left(c(a+bx)^n\right)\,dx = -\frac{6a^2n^3x}{b^2}+\frac{3an^3(a+bx)^2}{4b^3}-\frac{2n^3(a+bx)^3}{27b^3}+\frac{6a^2n^2(a+bx)\log\left(c(a+bx)^n\right)}{b^3}-\frac{3an^2(a+bx)^2\log\left(c(a+bx)^n\right)}{2b^3}+\frac{2n^2(a+bx)^3\log\left(c(a+bx)^n\right)}{9b^3}-\frac{3a^2n(a+bx)\log^2\left(c(a+bx)^n\right)}{b^3}+\frac{3an(a+bx)^2\log^2\left(c(a+bx)^n\right)}{2b^3}-\frac{n(a+bx)^3\log^2\left(c(a+bx)^n\right)}{3b^3}+\frac{a^2(a+bx)\log^3\left(c(a+bx)^n\right)}{b^3}-\frac{a(a+bx)^2\log^3\left(c(a+bx)^n\right)}{b^3}+\frac{(a+bx)^3\log^3\left(c(a+bx)^n\right)}{3b^3}$$

```
[Out] -6*a^2*n^3*x/b^2+3/4*a*n^3*(b*x+a)^2/b^3-2/27*n^3*(b*x+a)^3/b^3+6*a^2*n^2*(
b*x+a)*ln(c*(b*x+a)^n)/b^3-3/2*a*n^2*(b*x+a)^2*ln(c*(b*x+a)^n)/b^3+2/9*n^2*
(b*x+a)^3*ln(c*(b*x+a)^n)/b^3-3*a^2*n*(b*x+a)*ln(c*(b*x+a)^n)^2/b^3+3/2*a*n
*(b*x+a)^2*ln(c*(b*x+a)^n)^2/b^3-1/3*n*(b*x+a)^3*ln(c*(b*x+a)^n)^2/b^3+a^2*
(b*x+a)*ln(c*(b*x+a)^n)^3/b^3-a*(b*x+a)^2*ln(c*(b*x+a)^n)^3/b^3+1/3*(b*x+a)
^3*ln(c*(b*x+a)^n)^3/b^3
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\int x^2\log^3\left(c(a+bx)^n\right)\,dx = \frac{6a^2n^2(a+bx)\log\left(c(a+bx)^n\right)}{b^3}+\frac{a^2(a+bx)\log^3\left(c(a+bx)^n\right)}{b^3} -\frac{3a^2n(a+bx)\log^2\left(c(a+bx)^n\right)}{b^3}-\frac{6a^2n^3x}{b^2} +\frac{2n^2(a+bx)^3\log\left(c(a+bx)^n\right)}{9b^3} -\frac{3an^2(a+bx)^2\log\left(c(a+bx)^n\right)}{2b^3} +\frac{(a+bx)^3\log^3\left(c(a+bx)^n\right)}{3b^3}-\frac{a(a+bx)^2\log^3\left(c(a+bx)^n\right)}{b^3} -\frac{n(a+bx)^3\log^2\left(c(a+bx)^n\right)}{3b^3}+\frac{3an(a+bx)^2\log^2\left(c(a+bx)^n\right)}{2b^3} -\frac{2n^3(a+bx)^3}{27b^3}+\frac{3an^3(a+bx)^2}{4b^3}$$

```
[In] Int[x^2*Log[c*(a + b*x)^n]^3,x]
```

```
[Out] (-6*a^2*n^3*x)/b^2 + (3*a*n^3*(a + b*x)^2)/(4*b^3) - (2*n^3*(a + b*x)^3)/(2
7*b^3) + (6*a^2*n^2*(a + b*x)*Log[c*(a + b*x)^n])/b^3 - (3*a*n^2*(a + b*x)^
2*Log[c*(a + b*x)^n])/(2*b^3) + (2*n^2*(a + b*x)^3*Log[c*(a + b*x)^n])/(9*b
^3) - (3*a^2*n*(a + b*x)*Log[c*(a + b*x)^n]^2)/b^3 + (3*a*n*(a + b*x)^2*Log
[c*(a + b*x)^n]^2)/(2*b^3) - (n*(a + b*x)^3*Log[c*(a + b*x)^n]^2)/(3*b^3) +
 (a^2*(a + b*x)*Log[c*(a + b*x)^n]^3)/b^3 - (a*(a + b*x)^2*Log[c*(a + b*x)^
n]^3)/b^3 + ((a + b*x)^3*Log[c*(a + b*x)^n]^3)/(3*b^3)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2 \log^3\left(c(a+bx)^n\right)}{b^2} - \frac{2a(a+bx)\log^3\left(c(a+bx)^n\right)}{b^2} + \frac{(a+bx)^2\log^3\left(c(a+bx)^n\right)}{b^2}\right) dx$$

$$= \frac{\int (a+bx)^2 \log^3\left(c(a+bx)^n\right)\,dx}{b^2} - \frac{(2a)\int (a+bx)\log^3\left(c(a+bx)^n\right)\,dx}{b^2} + \frac{a^2\int \log^3\left(c(a+bx)^n\right)\,dx}{b^2}$$

$$= \frac{\text{Subst}\left(\int x^2\log^3\left(cx^n\right)\,dx, x, a+bx\right)}{b^3} - \frac{(2a)\text{Subst}\left(\int x\log^3\left(cx^n\right)\,dx, x, a+bx\right)}{b^3} + \frac{a^2\text{Subst}\left(\int \log^3\left(cx^n\right)\,dx, x, a+bx\right)}{b^3}$$

$$
\begin{aligned}
&= \frac{a^2(a+bx)\log^3\left(c(a+bx)^n\right)}{b^3} - \frac{a(a+bx)^2\log^3\left(c(a+bx)^n\right)}{b^3} \\
&\quad + \frac{(a+bx)^3\log^3\left(c(a+bx)^n\right)}{3b^3} - \frac{n\text{Subst}\left(\int x^2\log^2\left(cx^n\right)\,dx, x, a+bx\right)}{b^3} \\
&\quad + \frac{(3an)\text{Subst}\left(\int x\log^2\left(cx^n\right)\,dx, x, a+bx\right)}{b^3} \\
&\quad - \frac{\left(3a^2n\right)\text{Subst}\left(\int \log^2\left(cx^n\right)\,dx, x, a+bx\right)}{b^3} \\
&= -\frac{3a^2n(a+bx)\log^2\left(c(a+bx)^n\right)}{b^3} + \frac{3an(a+bx)^2\log^2\left(c(a+bx)^n\right)}{2b^3} \\
&\quad - \frac{n(a+bx)^3\log^2\left(c(a+bx)^n\right)}{3b^3} + \frac{a^2(a+bx)\log^3\left(c(a+bx)^n\right)}{b^3} \\
&\quad - \frac{a(a+bx)^2\log^3\left(c(a+bx)^n\right)}{b^3} + \frac{(a+bx)^3\log^3\left(c(a+bx)^n\right)}{3b^3} \\
&\quad + \frac{\left(2n^2\right)\text{Subst}\left(\int x^2\log\left(cx^n\right)\,dx, x, a+bx\right)}{3b^3} \\
&\quad - \frac{\left(3an^2\right)\text{Subst}\left(\int x\log\left(cx^n\right)\,dx, x, a+bx\right)}{b^3} \\
&\quad + \frac{\left(6a^2n^2\right)\text{Subst}\left(\int \log\left(cx^n\right)\,dx, x, a+bx\right)}{b^3} \\
&= -\frac{6a^2n^3x}{b^2} + \frac{3an^3(a+bx)^2}{4b^3} - \frac{2n^3(a+bx)^3}{27b^3} + \frac{6a^2n^2(a+bx)\log\left(c(a+bx)^n\right)}{b^3} \\
&\quad - \frac{3an^2(a+bx)^2\log\left(c(a+bx)^n\right)}{2b^3} + \frac{2n^2(a+bx)^3\log\left(c(a+bx)^n\right)}{9b^3} \\
&\quad - \frac{3a^2n(a+bx)\log^2\left(c(a+bx)^n\right)}{b^3} + \frac{3an(a+bx)^2\log^2\left(c(a+bx)^n\right)}{2b^3} \\
&\quad - \frac{n(a+bx)^3\log^2\left(c(a+bx)^n\right)}{3b^3} + \frac{a^2(a+bx)\log^3\left(c(a+bx)^n\right)}{b^3} \\
&\quad - \frac{a(a+bx)^2\log^3\left(c(a+bx)^n\right)}{b^3} + \frac{(a+bx)^3\log^3\left(c(a+bx)^n\right)}{3b^3}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.91

$$\begin{aligned}\int x^2 \log^3\left(c(a+bx)^n\right) dx = & -\frac{85a^2n^3x}{18b^2}+\frac{19an^3x^2}{36b}-\frac{2n^3x^3}{27} \\ & +\frac{85a^3n^2\log\left(c(a+bx)^n\right)}{18b^3}+\frac{11a^2n^2x\log\left(c(a+bx)^n\right)}{3b^2} \\ & -\frac{5an^2x^2\log\left(c(a+bx)^n\right)}{6b}+\frac{2}{9}n^2x^3\log\left(c(a+bx)^n\right) \\ & -\frac{11a^3n\log^2\left(c(a+bx)^n\right)}{6b^3}-\frac{a^2nx\log^2\left(c(a+bx)^n\right)}{b^2} \\ & +\frac{anx^2\log^2\left(c(a+bx)^n\right)}{2b}-\frac{1}{3}nx^3\log^2\left(c(a+bx)^n\right) \\ & +\frac{a^3\log^3\left(c(a+bx)^n\right)}{3b^3}+\frac{1}{3}x^3\log^3\left(c(a+bx)^n\right)\end{aligned}$$

```
[In] Integrate[x^2*Log[c*(a + b*x)^n]^3,x]
```

```
[Out] (-85*a^2*n^3*x)/(18*b^2) + (19*a*n^3*x^2)/(36*b) - (2*n^3*x^3)/27 + (85*a^3
*n^2*Log[c*(a + b*x)^n])/(18*b^3) + (11*a^2*n^2*x*Log[c*(a + b*x)^n])/(3*b^
2) - (5*a*n^2*x^2*Log[c*(a + b*x)^n])/(6*b) + (2*n^2*x^3*Log[c*(a + b*x)^n]
)/9 - (11*a^3*n*Log[c*(a + b*x)^n]^2)/(6*b^3) - (a^2*n*x*Log[c*(a + b*x)^n]
^2)/b^2 + (a*n*x^2*Log[c*(a + b*x)^n]^2)/(2*b) - (n*x^3*Log[c*(a + b*x)^n]^
2)/3 + (a^3*Log[c*(a + b*x)^n]^3)/(3*b^3) + (x^3*Log[c*(a + b*x)^n]^3)/3
```

## Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| parallelrisch | $\frac{36x^3\ln\left(c(bx+a)^n\right)^3b^3-36x^3\ln\left(c(bx+a)^n\right)^2b^3n+24x^3\ln\left(c(bx+a)^n\right)b^3n^2-8b^3n^3x^3+54x^2\ln\left(c(bx+a)^n\right)^2a\,b^2n-90x^2\ln(c(bx+a)}{}$ |
| risch | Expression too large to display |

```
[In] int(x^2*ln(c*(b*x+a)^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/108*(36*x^3*ln(c*(b*x+a)^n)^3*b^3-36*x^3*ln(c*(b*x+a)^n)^2*b^3*n+24*x^3*l
n(c*(b*x+a)^n)*b^3*n^2-8*b^3*n^3*x^3+54*x^2*ln(c*(b*x+a)^n)^2*a*b^2*n-90*x^
2*ln(c*(b*x+a)^n)*a*b^2*n^2+57*a*b^2*n^3*x^2+906*ln(b*x+a)*a^3*n^3-108*x*ln
(c*(b*x+a)^n)^2*a^2*b*n+396*x*ln(c*(b*x+a)^n)*a^2*b*n^2-510*a^2*b*n^3*x+36*
ln(c*(b*x+a)^n)^3*a^3-198*ln(c*(b*x+a)^n)^2*a^3*n-396*ln(c*(b*x+a)^n)*a^3*n
^2+510*a^3*n^3)/b^3
```

## Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.20

$$\int x^2 \log^3\left(c(a+bx)^n\right)\,dx =$$
$$-\frac{8\,b^3n^3x^3 - 36\,b^3x^3\log\left(c\right)^3 - 57\,ab^2n^3x^2 + 510\,a^2bn^3x - 36\,(b^3n^3x^3 + a^3n^3)\log\left(bx+a\right)^3 + 18\,(2\,b^3n^3x^3}{}$$

```
[In] integrate(x^2*log(c*(b*x+a)^n)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(8*b^3*n^3*x^3 - 36*b^3*x^3*log(c)^3 - 57*a*b^2*n^3*x^2 + 510*a^2*b*
n^3*x - 36*(b^3*n^3*x^3 + a^3*n^3)*log(b*x + a)^3 + 18*(2*b^3*n^3*x^3 - 3*a
*b^2*n^3*x^2 + 6*a^2*b*n^3*x + 11*a^3*n^3 - 6*(b^3*n^2*x^3 + a^3*n^2)*log(c
))*log(b*x + a)^2 + 18*(2*b^3*n*x^3 - 3*a*b^2*n*x^2 + 6*a^2*b*n*x)*log(c)^2
 - 6*(4*b^3*n^3*x^3 - 15*a*b^2*n^3*x^2 + 66*a^2*b*n^3*x + 85*a^3*n^3 + 18*(
b^3*n*x^3 + a^3*n)*log(c)^2 - 6*(2*b^3*n^2*x^3 - 3*a*b^2*n^2*x^2 + 6*a^2*b*
n^2*x + 11*a^3*n^2)*log(c))*log(b*x + a) - 6*(4*b^3*n^2*x^3 - 15*a*b^2*n^2*
x^2 + 66*a^2*b*n^2*x)*log(c))/b^3
```

## Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.94

$$\int x^2 \log^3\left(c(a+bx)^n\right)\,dx$$
$$= \begin{cases} \frac{85a^3n^2\log\left(c(a+bx)^n\right)}{18b^3} - \frac{11a^3n\log\left(c(a+bx)^n\right)^2}{6b^3} + \frac{a^3\log\left(c(a+bx)^n\right)^3}{3b^3} - \frac{85a^2n^3x}{18b^2} + \frac{11a^2n^2x\log\left(c(a+bx)^n\right)}{3b^2} - \frac{a^2nx\log\left(c(a+bx)^n\right)^2}{b^2} \\ \frac{x^3\log\left(a^nc\right)^3}{3} \end{cases}$$

```
[In] integrate(x**2*ln(c*(b*x+a)**n)**3,x)
```

```
[Out] Piecewise((85*a**3*n**2*log(c*(a + b*x)**n)/(18*b**3) - 11*a**3*n*log(c*(a
+ b*x)**n)**2/(6*b**3) + a**3*log(c*(a + b*x)**n)**3/(3*b**3) - 85*a**2*n**
3*x/(18*b**2) + 11*a**2*n**2*x*log(c*(a + b*x)**n)/(3*b**2) - a**2*n*x*log(
c*(a + b*x)**n)**2/b**2 + 19*a*n**3*x**2/(36*b) - 5*a*n**2*x**2*log(c*(a +
b*x)**n)/(6*b) + a*n*x**2*log(c*(a + b*x)**n)**2/(2*b) - 2*n**3*x**3/27 + 2
*n**2*x**3*log(c*(a + b*x)**n)/9 - n*x**3*log(c*(a + b*x)**n)**2/3 + x**3*l
og(c*(a + b*x)**n)**3/3, Ne(b, 0)), (x**3*log(a**n*c)**3/3, True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.75

$$\int x^2 \log^3\left(c(a+bx)^n\right)\,dx = \frac{1}{3}\,x^3\log\left((bx+a)^n c\right)^3 + \frac{1}{6}\,bn\left(\frac{6\,a^3\log\left(bx+a\right)}{b^4} - \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{b^3}\right)\log\left((bx+a)^n c\right)^2 - \frac{1}{108}\,bn\left(\frac{\left(8\,b^3x^3 - 36\,a^3\log\left(bx+a\right)^3 - 57\,ab^2x^2 - 198\,a^3\log\left(bx+a\right)^2 + 510\,a^2bx - 510\,a^3\log\left(bx+a\right)\right)}{b^4}\right.$$

```
[In] integrate(x^2*log(c*(b*x+a)^n)^3,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((b*x + a)^n*c)^3 + 1/6*b*n*(6*a^3*log(b*x + a)/b^4 - (2*b^2*x^3
 - 3*a*b*x^2 + 6*a^2*x)/b^3)*log((b*x + a)^n*c)^2 - 1/108*b*n*((8*b^3*x^3 -
 36*a^3*log(b*x + a)^3 - 57*a*b^2*x^2 - 198*a^3*log(b*x + a)^2 + 510*a^2*b*
x - 510*a^3*log(b*x + a))*n^2/b^4 - 6*(4*b^3*x^3 - 15*a*b^2*x^2 - 18*a^3*lo
g(b*x + a)^2 + 66*a^2*b*x - 66*a^3*log(b*x + a))*n*log((b*x + a)^n*c)/b^4)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(271) = 542.

Time = 0.32 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.20

$$\int x^2 \log^3\left(c(a+bx)^n\right)\,dx = \frac{(bx+a)^3n^3\log(bx+a)^3}{3\,b^3} - \frac{(bx+a)^2an^3\log(bx+a)^3}{b^3} + \frac{(bx+a)a^2n^3\log(bx+a)^3}{b^3} - \frac{(bx+a)^3n^3\log(bx+a)^2}{3\,b^3} + \frac{3\,(bx+a)^2an^3\log(bx+a)^2}{2\,b^3} - \frac{3\,(bx+a)a^2n^3\log(bx+a)^2}{b^3} + \frac{(bx+a)^3n^2\log(bx+a)^2\log(c)}{b^3} - \frac{3\,(bx+a)^2an^2\log(bx+a)^2\log(c)}{b^3} + \frac{3\,(bx+a)a^2n^2\log(bx+a)^2\log(c)}{b^3} + \frac{2\,(bx+a)^3n^3\log(bx+a)}{9\,b^3} - \frac{3\,(bx+a)^2an^3\log(bx+a)}{2\,b^3} + \frac{6\,(bx+a)a^2n^3\log(bx+a)}{b^3} - \frac{2\,(bx+a)^3n^2\log(bx+a)\log(c)}{3\,b^3} + \frac{3\,(bx+a)^2an^2\log(bx+a)\log(c)}{b^3} - \frac{6\,(bx+a)a^2n^2\log(bx+a)\log(c)}{b^3} + \frac{(bx+a)^3n\log(bx+a)\log(c)^2}{b^3} - \frac{3\,(bx+a)^2an\log(bx+a)\log(c)^2}{b^3} + \frac{3\,(bx+a)a^2n\log(bx+a)\log(c)^2}{b^3} - \frac{2\,(bx+a)^3n^3}{27\,b^3} + \frac{3\,(bx+a)^2an^3}{4\,b^3} - \frac{6\,(bx+a)a^2n^3}{b^3} + \frac{2\,(bx+a)^3n^2\log(c)}{9\,b^3} - \frac{3\,(bx+a)^2an^2\log(c)}{2\,b^3} + \frac{6\,(bx+a)a^2n^2\log(c)}{b^3} - \frac{(bx+a)^3n\log(c)^2}{3\,b^3} + \frac{3\,(bx+a)^2an\log(c)^2}{2\,b^3} - \frac{3\,(bx+a)a^2n\log(c)^2}{b^3} + \frac{(bx+a)^3\log(c)^3}{3\,b^3} - \frac{(bx+a)^2a\log(c)^3}{b^3} + \frac{(bx+a)a^2\log(c)^3}{b^3}$$

```
[In] integrate(x^2*log(c*(b*x+a)^n)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a)^3*n^3*log(b*x + a)^3/b^3 - (b*x + a)^2*a*n^3*log(b*x + a)^3/b
^3 + (b*x + a)*a^2*n^3*log(b*x + a)^3/b^3 - 1/3*(b*x + a)^3*n^3*log(b*x + a
```

```
)^2/b^3 + 3/2*(b*x + a)^2*a*n^3*log(b*x + a)^2/b^3 - 3*(b*x + a)*a^2*n^3*lo
g(b*x + a)^2/b^3 + (b*x + a)^3*n^2*log(b*x + a)^2*log(c)/b^3 - 3*(b*x + a)^
2*a*n^2*log(b*x + a)^2*log(c)/b^3 + 3*(b*x + a)*a^2*n^2*log(b*x + a)^2*log(
c)/b^3 + 2/9*(b*x + a)^3*n^3*log(b*x + a)/b^3 - 3/2*(b*x + a)^2*a*n^3*log(b
*x + a)/b^3 + 6*(b*x + a)*a^2*n^3*log(b*x + a)/b^3 - 2/3*(b*x + a)^3*n^2*lo
g(b*x + a)*log(c)/b^3 + 3*(b*x + a)^2*a*n^2*log(b*x + a)*log(c)/b^3 - 6*(b*
x + a)*a^2*n^2*log(b*x + a)*log(c)/b^3 + (b*x + a)^3*n*log(b*x + a)*log(c)^
2/b^3 - 3*(b*x + a)^2*a*n*log(b*x + a)*log(c)^2/b^3 + 3*(b*x + a)*a^2*n*log
(b*x + a)*log(c)^2/b^3 - 2/27*(b*x + a)^3*n^3/b^3 + 3/4*(b*x + a)^2*a*n^3/b
^3 - 6*(b*x + a)*a^2*n^3/b^3 + 2/9*(b*x + a)^3*n^2*log(c)/b^3 - 3/2*(b*x +
a)^2*a*n^2*log(c)/b^3 + 6*(b*x + a)*a^2*n^2*log(c)/b^3 - 1/3*(b*x + a)^3*n*
log(c)^2/b^3 + 3/2*(b*x + a)^2*a*n*log(c)^2/b^3 - 3*(b*x + a)*a^2*n*log(c)^
2/b^3 + 1/3*(b*x + a)^3*log(c)^3/b^3 - (b*x + a)^2*a*log(c)^3/b^3 + (b*x +
a)*a^2*log(c)^3/b^3
```

## Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.60

$$\int x^2 \log^3 \left(c(a+bx)^n\right)\,dx = \ln\left(c\,(a+b\,x)^n\right)^3\left(\frac{x^3}{3}+\frac{a^3}{3\,b^3}\right)-\frac{2\,n^3\,x^3}{27} - \ln\left(c\,(a+b\,x)^n\right)^2\left(\frac{n\,x^3}{3}+\frac{11\,a^3\,n}{6\,b^3}-\frac{a\,n\,x^2}{2\,b}+\frac{a^2\,n\,x}{b^2}\right) + \frac{\ln\left(c\,(a+b\,x)^n\right)\left(\frac{2\,b\,n^2\,x^3}{3}-\frac{5\,a\,n^2\,x^2}{2}+\frac{11\,a^2\,n^2\,x}{b}\right)}{3\,b} + \frac{85\,a^3\,n^3\,\ln\left(a+b\,x\right)}{18\,b^3}+\frac{19\,a\,n^3\,x^2}{36\,b}-\frac{85\,a^2\,n^3\,x}{18\,b^2}$$

```
[In] int(x^2*log(c*(a + b*x)^n)^3,x)
```

```
[Out] log(c*(a + b*x)^n)^3*(x^3/3 + a^3/(3*b^3)) - (2*n^3*x^3)/27 - log(c*(a + b*
x)^n)^2*((n*x^3)/3 + (11*a^3*n)/(6*b^3) - (a*n*x^2)/(2*b) + (a^2*n*x)/b^2)
+ (log(c*(a + b*x)^n)*((2*b*n^2*x^3)/3 - (5*a*n^2*x^2)/2 + (11*a^2*n^2*x)/b
))/(3*b) + (85*a^3*n^3*log(a + b*x))/(18*b^3) + (19*a*n^3*x^2)/(36*b) - (85
*a^2*n^3*x)/(18*b^2)
```

# 3.88 $\int \frac{(f+gx)^3}{a+b\log(c(d+ex)^n)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 299

$$\begin{aligned}\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx &= \frac{e^{-\frac{a}{bn}}(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^4n}\\
&+\frac{3e^{-\frac{2a}{bn}}g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4n}\\
&+\frac{3e^{-\frac{3a}{bn}}g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4n}\\
&+\frac{e^{-\frac{4a}{bn}}g^3(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{ExpIntegralEi}\left(\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4n}\end{aligned}$$

```
[Out] (-d*g+e*f)^3*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b/e^4/exp(a/b/n)/n/((c*(
e*x+d)^n)^(1/n))+3*g*(-d*g+e*f)^2*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n))/b/n)
/b/e^4/exp(2*a/b/n)/n/((c*(e*x+d)^n)^(2/n))+3*g^2*(-d*g+e*f)*(e*x+d)^3*Ei(3
*(a+b*ln(c*(e*x+d)^n))/b/n)/b/e^4/exp(3*a/b/n)/n/((c*(e*x+d)^n)^(3/n))+g^3*
(e*x+d)^4*Ei(4*(a+b*ln(c*(e*x+d)^n))/b/n)/b/e^4/exp(4*a/b/n)/n/((c*(e*x+d)^
n)^(4/n))
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2446, 2436, 2337, 2209, 2437, 2347}

$$\int \frac{(f+gx)^3}{a+b\log(c(d+ex)^n)}\,dx = \frac{3g^2e^{-\frac{3a}{bn}}(d+ex)^3(ef-dg)\left(c(d+ex)^n\right)^{-3/n}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4n} + \frac{3ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4n} + \frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)^3\left(c(d+ex)^n\right)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^4n} + \frac{g^3e^{-\frac{4a}{bn}}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{ExpIntegralEi}\left(\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4n}$$

```
[In] Int[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] ((e*f - d*g)^3*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(
b*e^4*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)) + (3*g*(e*f - d*g)^2*(d + e*x)^
2*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^4*E^((2*a)/(b*n
))*n*(c*(d + e*x)^n)^(2/n)) + (3*g^2*(e*f - d*g)*(d + e*x)^3*ExpIntegralEi[
(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^4*E^((3*a)/(b*n))*n*(c*(d + e*x
)^n)^(3/n)) + (g^3*(d + e*x)^4*ExpIntegralEi[(4*(a + b*Log[c*(d + e*x)^n]))
/(b*n)])/(b*e^4*E^((4*a)/(b*n))*n*(c*(d + e*x)^n)^(4/n))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(ef-dg)^3}{e^3\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{3g(ef-dg)^2(d+ex)}{e^3\left(a+b\log\left(c(d+ex)^n\right)\right)}\right.\\
&\qquad\left.+\frac{3g^2(ef-dg)(d+ex)^2}{e^3\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{g^3(d+ex)^3}{e^3\left(a+b\log\left(c(d+ex)^n\right)\right)}\right)dx\\
&= \frac{g^3\int\frac{(d+ex)^3}{a+b\log(c(d+ex)^n)}\,dx}{e^3}+\frac{\left(3g^2(ef-dg)\right)\int\frac{(d+ex)^2}{a+b\log(c(d+ex)^n)}\,dx}{e^3}\\
&\quad+\frac{\left(3g(ef-dg)^2\right)\int\frac{d+ex}{a+b\log(c(d+ex)^n)}\,dx}{e^3}+\frac{(ef-dg)^3\int\frac{1}{a+b\log(c(d+ex)^n)}\,dx}{e^3}\\
&= \frac{g^3\text{Subst}\left(\int\frac{x^3}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{e^4}\\
&\quad+\frac{\left(3g^2(ef-dg)\right)\text{Subst}\left(\int\frac{x^2}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{e^4}\\
&\quad+\frac{\left(3g(ef-dg)^2\right)\text{Subst}\left(\int\frac{x}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{e^4}\\
&\quad+\frac{(ef-dg)^3\text{Subst}\left(\int\frac{1}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{e^4}
\end{aligned}$$

$$= \frac{\left(g^3(d+ex)^4 (c(d+ex)^n)^{-4/n}\right) \text{Subst}\left(\int \frac{e^{\frac{4x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{e^4 n} + \frac{\left(3g^2(ef-dg)(d+ex)^3 (c(d+ex)^n)^{-3/n}\right) \text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{e^4 n} + \frac{\left(3g(ef-dg)^2(d+ex)^2 (c(d+ex)^n)^{-2/n}\right) \text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{e^4 n} + \frac{\left((ef-dg)^3(d+ex) (c(d+ex)^n)^{-1/n}\right) \text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{e^4 n}$$

$$= \frac{e^{-\frac{a}{bn}}(ef-dg)^3(d+ex) (c(d+ex)^n)^{-1/n} \text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^4 n} + \frac{3e^{-\frac{2a}{bn}} g(ef-dg)^2(d+ex)^2 (c(d+ex)^n)^{-2/n} \text{Ei}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4 n} + \frac{3e^{-\frac{3a}{bn}} g^2(ef-dg)(d+ex)^3 (c(d+ex)^n)^{-3/n} \text{Ei}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4 n} + \frac{e^{-\frac{4a}{bn}} g^3(d+ex)^4 (c(d+ex)^n)^{-4/n} \text{Ei}\left(\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)}{be^4 n}$$

## Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{e^{-\frac{4a}{bn}}(d+ex) (c(d+ex)^n)^{-4/n} \left(e^{\frac{3a}{bn}}(ef-dg)^3 (c(d+ex)^n)^{3/n} \text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right) + g(d+ex)\right.}{}$$

```
[In] Integrate[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] ((d + e*x)*(E^((3*a)/(b*n))*(e*f - d*g)^3*(c*(d + e*x)^n)^(3/n)*ExpIntegral
Ei[(a + b*Log[c*(d + e*x)^n])/(b*n)] + g*(d + e*x)*(3*E^((2*a)/(b*n))*(e*f
- d*g)^2*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))
/(b*n)] - g*(d + e*x)*(-3*E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*Ex
pIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)] - g*(d + e*x)*ExpIntegral
Ei[(4*(a + b*Log[c*(d + e*x)^n]))/(b*n)]))))/(b*e^4*E^((4*a)/(b*n))*n*(c*(d
 + e*x)^n)^(4/n))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.54 (sec) , antiderivative size = 9346, normalized size of antiderivative = 31.26

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 9346 |

```
[In] int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

## Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.02

$$\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{\left(g^3\,\mathrm{log\_integral}\left((e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4)e^{\left(\frac{4\,(b\log(c)+a)}{bn}\right)}\right)+3\,(efg^2-dg^3)e^{\left(\frac{b\log(c)+a}{bn}\right)}\,\mathrm{lo}\right.}{}$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] (g^3*log_integral((e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)
*e^(4*(b*log(c) + a)/(b*n))) + 3*(e*f*g^2 - d*g^3)*e^((b*log(c) + a)/(b*n))
*log_integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*e^(3*(b*log(c) + a)
/(b*n))) + 3*(e^2*f^2*g - 2*d*e*f*g^2 + d^2*g^3)*e^(2*(b*log(c) + a)/(b*n))
*log_integral((e^2*x^2 + 2*d*e*x + d^2)*e^(2*(b*log(c) + a)/(b*n))) + (e^3*
f^3 - 3*d*e^2*f^2*g + 3*d^2*e*f*g^2 - d^3*g^3)*e^(3*(b*log(c) + a)/(b*n))*l
og_integral((e*x + d)*e^((b*log(c) + a)/(b*n))))*e^(-4*(b*log(c) + a)/(b*n)
)/(b*e^4*n)
```

## Sympy [F]

$$\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{(f+gx)^3}{a+b\log\left(c\,(d+ex)^n\right)}\,dx$$

```
[In] integrate((g*x+f)**3/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Integral((f + g*x)**3/(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [F]

$$\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{(gx+f)^3}{b\log\left((ex+d)^n c\right)+a}\,dx$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
[Out] integrate((g*x + f)^3/(b*log((e*x + d)^n*c) + a), x)
```

## Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.95

$$\begin{aligned}\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx = & \frac{f^3\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}en} \\ & - \frac{3\,df^2g\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}e^2n} \\ & + \frac{3\,d^2fg^2\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}e^3n} \\ & - \frac{d^3g^3\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}e^4n} \\ & + \frac{3\,f^2g\mathrm{Ei}\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+2\,\log\left(ex+d\right)\right)e^{\left(-\frac{2\,a}{bn}\right)}}{bc^{\frac{2}{n}}e^2n} \\ & - \frac{6\,dfg^2\mathrm{Ei}\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+2\,\log\left(ex+d\right)\right)e^{\left(-\frac{2\,a}{bn}\right)}}{bc^{\frac{2}{n}}e^3n} \\ & + \frac{3\,d^2g^3\mathrm{Ei}\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+2\,\log\left(ex+d\right)\right)e^{\left(-\frac{2\,a}{bn}\right)}}{bc^{\frac{2}{n}}e^4n} \\ & + \frac{3\,fg^2\mathrm{Ei}\left(\frac{3\,\log(c)}{n}+\frac{3\,a}{bn}+3\,\log\left(ex+d\right)\right)e^{\left(-\frac{3\,a}{bn}\right)}}{bc^{\frac{3}{n}}e^3n} \\ & - \frac{3\,dg^3\mathrm{Ei}\left(\frac{3\,\log(c)}{n}+\frac{3\,a}{bn}+3\,\log\left(ex+d\right)\right)e^{\left(-\frac{3\,a}{bn}\right)}}{bc^{\frac{3}{n}}e^4n} \\ & + \frac{g^3\mathrm{Ei}\left(\frac{4\,\log(c)}{n}+\frac{4\,a}{bn}+4\,\log\left(ex+d\right)\right)e^{\left(-\frac{4\,a}{bn}\right)}}{bc^{\frac{4}{n}}e^4n}\end{aligned}$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] f^3*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e*n) - 3*
d*f^2*g*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e^2*n
) + 3*d^2*f*g^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/
n)*e^3*n) - d^3*g^3*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c
^(1/n)*e^4*n) + 3*f^2*g*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a
/(b*n))/(b*c^(2/n)*e^2*n) - 6*d*f*g^2*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x
 + d))*e^(-2*a/(b*n))/(b*c^(2/n)*e^3*n) + 3*d^2*g^3*Ei(2*log(c)/n + 2*a/(b*
n) + 2*log(e*x + d))*e^(-2*a/(b*n))/(b*c^(2/n)*e^4*n) + 3*f*g^2*Ei(3*log(c)
/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))/(b*c^(3/n)*e^3*n) - 3*d*g^3
*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))/(b*c^(3/n)*e^4*
n) + g^3*Ei(4*log(c)/n + 4*a/(b*n) + 4*log(e*x + d))*e^(-4*a/(b*n))/(b*c^(4
/n)*e^4*n)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{{\left(f+g\,x\right)}^3}{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

```
[In] int((f + g*x)^3/(a + b*log(c*(d + e*x)^n)),x)
```

```
[Out] int((f + g*x)^3/(a + b*log(c*(d + e*x)^n)), x)
```

# 3.89 $\int \frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \frac{(f+gx)^2}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^3n} + \frac{2e^{-\frac{2a}{bn}}g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^3n} + \frac{e^{-\frac{3a}{bn}}g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{be^3n}$$

```
[Out] (-d*g+e*f)^2*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b/e^3/exp(a/b/n)/n/((c*(
e*x+d)^n)^(1/n))+2*g*(-d*g+e*f)*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n))/b/n)/b
/e^3/exp(2*a/b/n)/n/((c*(e*x+d)^n)^(2/n))+g^2*(e*x+d)^3*Ei(3*(a+b*ln(c*(e*x
+d)^n))/b/n)/b/e^3/exp(3*a/b/n)/n/((c*(e*x+d)^n)^(3/n))
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{2446, 2436, 2337, 2209, 2437, 2347\}$

$$\int \frac{(f+gx)^2}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{2ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^3n} + \frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^3n} + \frac{g^2e^{-\frac{3a}{bn}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{be^3n}$$

```
[In] Int[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] ((e*f - d*g)^2*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(
b*e^3*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)) + (2*g*(e*f - d*g)*(d + e*x)^2*
ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^3*E^((2*a)/(b*n))
*n*(c*(d + e*x)^n)^(2/n)) + (g^2*(d + e*x)^3*ExpIntegralEi[(3*(a + b*Log[c*
(d + e*x)^n]))/(b*n)])/(b*e^3*E^((3*a)/(b*n))*n*(c*(d + e*x)^n)^(3/n))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(ef-dg)^2}{e^2\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{2g(ef-dg)(d+ex)}{e^2\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{g^2(d+ex)^2}{e^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\right)dx$$

$$=\frac{g^2\int\frac{(d+ex)^2}{a+b\log(c(d+ex)^n)}\,dx}{e^2}+\frac{(2g(ef-dg))\int\frac{d+ex}{a+b\log(c(d+ex)^n)}\,dx}{e^2}+\frac{(ef-dg)^2\int\frac{1}{a+b\log(c(d+ex)^n)}\,dx}{e^2}$$

$$=\frac{g^2\text{Subst}\left(\int\frac{x^2}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{e^3}+\frac{(2g(ef-dg))\text{Subst}\left(\int\frac{x}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{e^3}+\frac{(ef-dg)^2\text{Subst}\left(\int\frac{1}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{e^3}$$

$$=\frac{\left(g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^3n}+\frac{\left(2g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^3n}+\frac{\left((ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^3n}$$

$$=\frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^3n}+\frac{2e^{-\frac{2a}{bn}}g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{Ei}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^3n}+\frac{e^{-\frac{3a}{bn}}g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\text{Ei}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{be^3n}$$

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx)^2}{a+b\log\left(c(d+ex)^n\right)}\,dx$$
$$= \frac{e^{-\frac{3a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-3/n}\left(e^{\frac{2a}{bn}}(ef-dg)^2\left(c(d+ex)^n\right)^{2/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)-g(d+ex\right.}{}$$

```
[In] Integrate[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] ((d + e*x)*(E^((2*a)/(b*n))*(e*f - d*g)^2*(c*(d + e*x)^n)^(2/n)*ExpIntegral
Ei[(a + b*Log[c*(d + e*x)^n])/(b*n)] - g*(d + e*x)*(-2*E^(a/(b*n))*(e*f - d
*g)*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*
n)] - g*(d + e*x)*ExpIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)])))/(b
*e^3*E^((3*a)/(b*n))*n*(c*(d + e*x)^n)^(3/n))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.34 (sec) , antiderivative size = 1889, normalized size of antiderivative = 8.63

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1889 |

```
[In] int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^3*g^2/b/n*(e*x+d)^3*((e*x+d)^n)^(-3/n)*c^(-3/n)*exp(-3/2*(-I*b*Pi*csgn
(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)
^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d
)^n)^3*b+2*a)/b/n)*Ei(1,-3*ln(e*x+d)-3/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(
I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(
e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*
b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)-1/e*f^2/b/n*(e*x+d)*((e*x+d)^n)^(-1
/n)*c^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)
^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*
(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(
-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn
(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csg
n(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)-1/
e^3*g^2*d^2/b/n*(e*x+d)*((e*x+d)^n)^(-1/n)*c^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(
I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^
n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)
^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c
```

```
)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x
+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(
ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)+2/e^3*d*g^2/b/n*(e*x+d)^2*((e*x+d)^n)^
(-2/n)*c^(-2/n)*exp(-(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^
n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(
e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-2*ln(e*x+d)-(-I*
b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*
c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I
*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)-2/e^2
*f*g/b/n*(e*x+d)^2*((e*x+d)^n)^(-2/n)*c^(-2/n)*exp(-(-I*b*Pi*csgn(I*c*(e*x+
d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*
Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2
*a)/b/n)*Ei(1,-2*ln(e*x+d)-(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e
*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn
(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^
n)-n*ln(e*x+d))+2*a)/b/n)+2/e^2*d*f*g/b/n*(e*x+d)*((e*x+d)^n)^(-1/n)*c^(-1/
n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*c
sgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)
^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*cs
gn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+
d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x
+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx)^2}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{\left(g^2\,\mathrm{log\_integral}\left((e^3x^3+3\,de^2x^2+3\,d^2ex+d^3)e^{\left(\frac{3\,(b\log(c)+a)}{bn}\right)}\right)+2\,(efg-dg^2)e^{\left(\frac{b\log(c)+a}{bn}\right)}\,\mathrm{log\_integral}\left(\left(\right.\right.}{}$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

```
[Out] (g^2*log_integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*e^(3*(b*log(c)
+ a)/(b*n))) + 2*(e*f*g - d*g^2)*e^((b*log(c) + a)/(b*n))*log_integral((e^2
*x^2 + 2*d*e*x + d^2)*e^(2*(b*log(c) + a)/(b*n))) + (e^2*f^2 - 2*d*e*f*g +
d^2*g^2)*e^(2*(b*log(c) + a)/(b*n))*log_integral((e*x + d)*e^((b*log(c) + a
)/(b*n))))*e^(-3*(b*log(c) + a)/(b*n))/(b*e^3*n)
```

## Sympy [F]

$$\int \frac{(f+gx)^2}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{(f+gx)^2}{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

```
[In] integrate((g*x+f)**2/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Integral((f + g*x)**2/(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [F]

$$\int \frac{(f+gx)^2}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{(gx+f)^2}{b\log\left((ex+d)^n c\right)+a}\,dx$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
[Out] integrate((g*x + f)^2/(b*log((e*x + d)^n*c) + a), x)
```

## Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.54

$$\int \frac{(f+gx)^2}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{f^2\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}en} - \frac{2\,dfg\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}e^2n} + \frac{d^2g^2\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}e^3n} + \frac{2\,fg\mathrm{Ei}\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+2\,\log(ex+d)\right)e^{\left(-\frac{2\,a}{bn}\right)}}{bc^{\frac{2}{n}}e^2n} - \frac{2\,dg^2\mathrm{Ei}\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+2\,\log(ex+d)\right)e^{\left(-\frac{2\,a}{bn}\right)}}{bc^{\frac{2}{n}}e^3n} + \frac{g^2\mathrm{Ei}\left(\frac{3\,\log(c)}{n}+\frac{3\,a}{bn}+3\,\log(ex+d)\right)e^{\left(-\frac{3\,a}{bn}\right)}}{bc^{\frac{3}{n}}e^3n}$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] f^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e*n) - 2*
d*f*g*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e^2*n)
+ d^2*g^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e^3
*n) + 2*f*g*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))/(b*c
^(2/n)*e^2*n) - 2*d*g^2*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a
/(b*n))/(b*c^(2/n)*e^3*n) + g^2*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x + d))
*e^(-3*a/(b*n))/(b*c^(3/n)*e^3*n)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{\left(f+g\,x\right)^2}{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

```
[In] int((f + g*x)^2/(a + b*log(c*(d + e*x)^n)),x)
[Out] int((f + g*x)^2/(a + b*log(c*(d + e*x)^n)), x)
```

# 3.90 $\int \frac{f+gx}{a+b\log(c(d+ex)^n)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{f+gx}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^2n} + \frac{e^{-\frac{2a}{bn}}g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^2n}$$

```
[Out] (-d*g+e*f)*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b/e^2/exp(a/b/n)/n/((c*(e*
x+d)^n)^(1/n))+g*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n))/b/n)/b/e^2/exp(2*a/b/
n)/n/((c*(e*x+d)^n)^(2/n))
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2446, 2436, 2337, 2209, 2437, 2347}

$$\int \frac{f+gx}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^2n} + \frac{ge^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^2n}$$

```
[In] Int[(f + g*x)/(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] ((e*f - d*g)*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*
e^2*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)) + (g*(d + e*x)^2*ExpIntegralEi[(2
*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b*e^2*E^((2*a)/(b*n))*n*(c*(d + e*x)^
n)^(2/n))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{ef-dg}{e\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{g(d+ex)}{e\left(a+b\log\left(c(d+ex)^n\right)\right)}\right)\,dx$$

$$= \frac{g\int \frac{d+ex}{a+b\log(c(d+ex)^n)}\,dx}{e} + \frac{(ef-dg)\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx}{e}$$

$$= \frac{g\text{Subst}\left(\int \frac{x}{a+b\log(cx^n)}\,dx, x, d+ex\right)}{e^2} + \frac{(ef-dg)\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx, x, d+ex\right)}{e^2}$$

$$= \frac{\left(g(d+ex)^2\,(c(d+ex)^n)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{a+bx}\,dx, x, \log(c(d+ex)^n)\right)}{e^2n} + \frac{\left((ef-dg)(d+ex)\,(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log(c(d+ex)^n)\right)}{e^2n}$$

$$= \frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex)\,(c(d+ex)^n)^{-1/n}\,\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{be^2n} + \frac{e^{-\frac{2a}{bn}}g(d+ex)^2\,(c(d+ex)^n)^{-2/n}\,\text{Ei}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{be^2n}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \frac{f+gx}{a+b\log(c(d+ex)^n)}\,dx = \frac{e^{-\frac{2a}{bn}}(d+ex)\,(c(d+ex)^n)^{-2/n}\left(e^{\frac{a}{bn}}(ef-dg)\,(c(d+ex)^n)^{\frac{1}{n}}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right) + g(d+ex)\,\text{I}\right.}{be^2n}$$

```
[In] Integrate[(f + g*x)/(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] ((d + e*x)*(E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(a
 + b*Log[c*(d + e*x)^n])/(b*n)] + g*(d + e*x)*ExpIntegralEi[(2*(a + b*Log[c
*(d + e*x)^n]))/(b*n)]))/(b*e^2*E^((2*a)/(b*n))*n*(c*(d + e*x)^n)^(2/n))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.96 (sec) , antiderivative size = 937, normalized size of antiderivative = 6.74

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 937 |

```
[In] int((g*x+f)/(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2*g/b/n*(e*x+d)^2*c^(-2/n)*((e*x+d)^n)^(-2/n)*exp(-(-I*b*Pi*csgn(I*c*(
e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*
```

```
b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3
*b+2*a)/b/n)*Ei(1,-2*ln(e*x+d)-(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(
I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*
csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x
+d)^n)-n*ln(e*x+d))+2*a)/b/n)-1/e*f/b/n*(e*x+d)*c^(-1/n)*((e*x+d)^n)^(-1/n)
*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csg
n(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2
*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn
(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)
^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d
)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)+1/e^2*d*g/b/n*
(e*x+d)*c^(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*c
sgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n
)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)
^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*
(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*
ln(e*x+d))+2*a)/b/n)
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

$$\int \frac{f+gx}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{\left((ef-dg)e^{\left(\frac{b\log(c)+a}{bn}\right)}\operatorname{log\_integral}\left((ex+d)e^{\left(\frac{b\log(c)+a}{bn}\right)}\right)+g\operatorname{log\_integral}\left((e^2x^2+2\,dex+d^2)e^{\left(\frac{2\,(b\log(c)+a)}{bn}\right)}\right)\right)e^{\left(-\frac{2\,(b\log(c)+a)}{bn}\right)}}{be^2n}$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

```
[Out] ((e*f - d*g)*e^((b*log(c) + a)/(b*n))*log_integral((e*x + d)*e^((b*log(c) +
 a)/(b*n))) + g*log_integral((e^2*x^2 + 2*d*e*x + d^2)*e^(2*(b*log(c) + a)/
(b*n))))*e^(-2*(b*log(c) + a)/(b*n))/(b*e^2*n)
```

## Sympy [F]

$$\int \frac{f+gx}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{f+gx}{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

```
[In] integrate((g*x+f)/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Integral((f + g*x)/(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [F]

$$\int \frac{f+gx}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{gx+f}{b\log\left((ex+d)^nc\right)+a}\,dx$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
[Out] integrate((g*x + f)/(b*log((e*x + d)^n*c) + a), x)
```

## Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.14

$$\int \frac{f+gx}{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{f\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}en}-\frac{dg\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}e^2n}+\frac{g\mathrm{Ei}\left(\frac{2\log(c)}{n}+\frac{2a}{bn}+2\log\left(ex+d\right)\right)e^{\left(-\frac{2a}{bn}\right)}}{bc^{\frac{2}{n}}e^2n}$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] f*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e*n) - d*g*
Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e^2*n) + g*Ei
(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))/(b*c^(2/n)*e^2*n)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{f + gx}{a + b\log\left(c(d + ex)^n\right)}\, dx = \int \frac{f + g\,x}{a + b\,\ln\left(c\,{\left(d + e\,x\right)}^n\right)}\, dx$$

```
[In] int((f + g*x)/(a + b*log(c*(d + e*x)^n)),x)
[Out] int((f + g*x)/(a + b*log(c*(d + e*x)^n)), x)
```

# 3.91 $\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 63

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben}$$

```
[Out] (e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b/e/exp(a/b/n)/n/((c*(e*x+d)^n)^(1/n)
)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2436, 2337, 2209}

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(-1),x]
```

```
[Out] ((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e*E^(a/(b*n)
)*n*(c*(d + e*x)^n)^n^(-1))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx, x, d+ex\right)}{e} \\ &= \frac{\left((d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{en} \\ &= \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben} \end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{ben}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(-1),x]
[Out] ((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e*E^(a/(b*n)
)*n*(c*(d + e*x)^n)^n^(-1))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 309, normalized size of antiderivative = 4.90

| method | result |
|---|---|
| risch | $-\frac{(ex+d)c^{-\frac{1}{n}}\left((ex+d)^n\right)^{-\frac{1}{n}}\mathrm{e}^{-\frac{-ib\pi\ \mathrm{csgn}\left(ic(ex+d)^n\right)\mathrm{csgn}(ic)\,\mathrm{csgn}\left(i(ex+d)^n\right)+i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}\left(ic(ex+d)^n\right)^2b+i\pi\ \mathrm{csgn}\left(i(ex+d)^n\right)\mathrm{csgn}\left(ic(ex+d\right.}{2bn}}}{}$ |

```
[In] int(1/(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e/b/n*(e*x+d)*c^(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*
x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+
I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b
+2*a)/b/n)*Ei(1,-ln(e*x+d)+1/2*I*(b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c
*(e*x+d)^n)-b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2-b*Pi*csgn(I*(e*x+d)^n)*csg
n(I*c*(e*x+d)^n)^2+b*Pi*csgn(I*c*(e*x+d)^n)^3+2*I*b*ln(c)+2*I*b*(ln((e*x+d)
^n)-n*ln(e*x+d))+2*I*a)/b/n)
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{e^{\left(-\frac{b\log(c)+a}{bn}\right)}\operatorname{log\_integral}\left((ex+d)e^{\left(\frac{b\log(c)+a}{bn}\right)}\right)}{ben}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

```
[Out] e^(-(b*log(c) + a)/(b*n))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n)))/
(b*e*n)
```

## Sympy [F]

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{1}{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n)),x)
```

```
[Out] Integral(1/(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [F]

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{1}{b\log\left((ex+d)^nc\right)+a}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*log((e*x + d)^n*c) + a), x)
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{\operatorname{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(ex+d\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}en}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/(b*c^(1/n)*e*n)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{1}{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e*x)^n)),x)
[Out] int(1/(a + b*log(c*(d + e*x)^n)), x)
```

# 3.92 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \text{Int}\left(\frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])),x]
```

```
[Out] Defer[Int][1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

```
[In] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])),x]
[Out] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])), x]
```

## Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)}dx$$

```
[In] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
[Out] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^nc\right)+a)}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral(1/(a*g*x + a*f + (b*g*x + b*f)*log((e*x + d)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^{n}\right)\right)(f+gx)}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Integral(1/((a + b*log(c*(d + e*x)**n))*(f + g*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{\left(f+g\,x\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))),x)
[Out] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))), x)
```

# 3.93 $\int \frac{1}{(f+gx)^2(a+b\log(c(d+ex)^n))}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])),x]
```

```
[Out] Defer[Int][1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2 (a+b\log(c(d+ex)^n))} dx = \int \frac{1}{(f+gx)^2 (a+b\log(c(d+ex)^n))} dx$$

```
[In] Integrate[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])),x]
[Out] Integrate[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])), x]
```

## Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2 (a+b\ln(c(ex+d)^n))} dx$$

```
[In] int(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n)),x)
[Out] int(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.29

$$\int \frac{1}{(f+gx)^2 (a+b\log(c(d+ex)^n))} dx = \int \frac{1}{(gx+f)^2(b\log((ex+d)^n c)+a)} dx$$

```
[In] integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral(1/(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 + 2*b*f*g*x + b*f^2)
*log((e*x + d)^n*c)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2\,(a+b\log{(c(d+ex)^n)})}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)**2/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2\,(a+b\log{(c(d+ex)^n)})}\,dx = \int \frac{1}{(gx+f)^2(b\log{((ex+d)^n c)}+a)}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)^2*(b*log((e*x + d)^n*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2\,(a+b\log{(c(d+ex)^n)})}\,dx = \int \frac{1}{(gx+f)^2(b\log{((ex+d)^n c)}+a)}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)^2*(b*log((e*x + d)^n*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2\,(a+b\log{(c(d+ex)^n)})}\,dx = \int \frac{1}{{\left(f+g\,x\right)}^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)^2*(a + b*log(c*(d + e*x)^n))),x)
[Out] int(1/((f + g*x)^2*(a + b*log(c*(d + e*x)^n))), x)
```

# 3.94 $\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 339

$$\begin{aligned}\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx &= \frac{e^{-\frac{a}{bn}}(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^4n^2}\\&\quad+\frac{6e^{-\frac{2a}{bn}}g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^4n^2}\\&\quad+\frac{9e^{-\frac{3a}{bn}}g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^4n^2}\\&\quad+\frac{4e^{-\frac{4a}{bn}}g^3(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{ExpIntegralEi}\left(\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^4n^2}\\&\quad-\frac{(d+ex)(f+gx)^3}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}\end{aligned}$$

```
[Out] (-d*g+e*f)^3*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e^4/exp(a/b/n)/n^2/(
(c*(e*x+d)^n)^(1/n))+6*g*(-d*g+e*f)^2*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n))/
b/n)/b^2/e^4/exp(2*a/b/n)/n^2/((c*(e*x+d)^n)^(2/n))+9*g^2*(-d*g+e*f)*(e*x+d
)^3*Ei(3*(a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e^4/exp(3*a/b/n)/n^2/((c*(e*x+d)^n)
^(3/n))+4*g^3*(e*x+d)^4*Ei(4*(a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e^4/exp(4*a/b/n
)/n^2/((c*(e*x+d)^n)^(4/n))-(e*x+d)*(g*x+f)^3/b/e/n/(a+b*ln(c*(e*x+d)^n))
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2447, 2446, 2436, 2337, 2209, 2437, 2347}

$$\begin{aligned}&\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx\\&=\frac{9g^2e^{-\frac{3a}{bn}}(d+ex)^3(ef-dg)\left(c(d+ex)^n\right)^{-3/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^4n^2}\\&\quad+\frac{6ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)^2\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^4n^2}\\&\quad+\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)^3\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^4n^2}\\&\quad+\frac{4g^3e^{-\frac{4a}{bn}}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\text{ExpIntegralEi}\left(\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^4n^2}\\&\quad-\frac{(d+ex)(f+gx)^3}{ben\,(a+b\log(c(d+ex)^n))}\end{aligned}$$

```
[In] Int[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] ((e*f - d*g)^3*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(
b^2*e^4*E^(a/(b*n))*n^2*(c*(d + e*x)^n)^n^(-1)) + (6*g*(e*f - d*g)^2*(d + e
*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b^2*e^4*E^((2*a
)/(b*n))*n^2*(c*(d + e*x)^n)^(2/n)) + (9*g^2*(e*f - d*g)*(d + e*x)^3*ExpInt
egralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b^2*e^4*E^((3*a)/(b*n))*n^2
*(c*(d + e*x)^n)^(3/n)) + (4*g^3*(d + e*x)^4*ExpIntegralEi[(4*(a + b*Log[c*
(d + e*x)^n]))/(b*n)])/(b^2*e^4*E^((4*a)/(b*n))*n^2*(c*(d + e*x)^n)^(4/n))
- ((d + e*x)*(f + g*x)^3)/(b*e*n*(a + b*Log[c*(d + e*x)^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{(d+ex)(f+gx)^3}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{4\int\frac{(f+gx)^3}{a+b\log(c(d+ex)^n)}\,dx}{bn}\\ &\quad-\frac{(3(ef-dg))\int\frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}\,dx}{ben} \end{aligned}$$

$$= -\frac{(d+ex)(f+gx)^3}{ben\,(a+b\log(c(d+ex)^n))} + \frac{4\int\left(\frac{(ef-dg)^3}{e^3(a+b\log(c(d+ex)^n))}+\frac{3g(ef-dg)^2(d+ex)}{e^3(a+b\log(c(d+ex)^n))}+\frac{3g^2(ef-dg)(d+ex)^2}{e^3(a+b\log(c(d+ex)^n))}+\frac{g^3(d+ex)^3}{e^3(a+b\log(c(d+ex)^n))}\right)dx}{bn} - \frac{(3(ef-dg))\int\left(\frac{(ef-dg)^2}{e^2(a+b\log(c(d+ex)^n))}+\frac{2g(ef-dg)(d+ex)}{e^2(a+b\log(c(d+ex)^n))}+\frac{g^2(d+ex)^2}{e^2(a+b\log(c(d+ex)^n))}\right)dx}{ben}$$

$$= -\frac{(d+ex)(f+gx)^3}{ben\,(a+b\log(c(d+ex)^n))} + \frac{(4g^3)\int\frac{(d+ex)^3}{a+b\log(c(d+ex)^n)}\,dx}{be^3n} - \frac{(3g^2(ef-dg))\int\frac{(d+ex)^2}{a+b\log(c(d+ex)^n)}\,dx}{be^3n} + \frac{(12g^2(ef-dg))\int\frac{(d+ex)^2}{a+b\log(c(d+ex)^n)}\,dx}{be^3n} - \frac{(6g(ef-dg)^2)\int\frac{d+ex}{a+b\log(c(d+ex)^n)}\,dx}{be^3n} + \frac{(12g(ef-dg)^2)\int\frac{d+ex}{a+b\log(c(d+ex)^n)}\,dx}{be^3n} - \frac{(3(ef-dg)^3)\int\frac{1}{a+b\log(c(d+ex)^n)}\,dx}{be^3n} + \frac{(4(ef-dg)^3)\int\frac{1}{a+b\log(c(d+ex)^n)}\,dx}{be^3n}$$

$$= -\frac{(d+ex)(f+gx)^3}{ben\,(a+b\log(c(d+ex)^n))} + \frac{(4g^3)\,\text{Subst}\left(\int\frac{x^3}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^4n} - \frac{(3g^2(ef-dg))\,\text{Subst}\left(\int\frac{x^2}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^4n} + \frac{(12g^2(ef-dg))\,\text{Subst}\left(\int\frac{x^2}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^4n} - \frac{(6g(ef-dg)^2)\,\text{Subst}\left(\int\frac{x}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^4n} + \frac{(12g(ef-dg)^2)\,\text{Subst}\left(\int\frac{x}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^4n} - \frac{(3(ef-dg)^3)\,\text{Subst}\left(\int\frac{1}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^4n} + \frac{(4(ef-dg)^3)\,\text{Subst}\left(\int\frac{1}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^4n}$$

$$= -\frac{(d+ex)(f+gx)^3}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{\left(4g^3(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\right)\text{Subst}\left(\int \frac{e^{\frac{4x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^4n^2} - \frac{\left(3g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^4n^2} + \frac{\left(12g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^4n^2} - \frac{\left(6g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^4n^2} + \frac{\left(12g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^4n^2} - \frac{\left(3(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^4n^2} + \frac{\left(4(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^4n^2}$$

$$= \frac{e^{-\frac{a}{bn}}(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^4n^2} + \frac{6e^{-\frac{2a}{bn}}g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{Ei}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^4n^2} + \frac{9e^{-\frac{3a}{bn}}g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\text{Ei}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^4n^2} + \frac{4e^{-\frac{4a}{bn}}g^3(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\text{Ei}\left(\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^4n^2} - \frac{(d+ex)(f+gx)^3}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1674 vs. 2(339) = 678.

Time = 0.46 (sec) , antiderivative size = 1674, normalized size of antiderivative = 4.94

$$\int \frac{(f+gx)^3}{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

$$= \frac{e^{-\frac{4a}{bn}}\left(c(d+ex)^n\right)^{-4/n}\left(-bde^3e^{\frac{4a}{bn}}f^3n\left(c(d+ex)^n\right)^{4/n} - be^4e^{\frac{4a}{bn}}f^3nx\left(c(d+ex)^n\right)^{4/n} - 3bde^3e^{\frac{4a}{bn}}f^2gnx(c(d+\right.}{}$$

```
[In] Integrate[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n])^2,x]

[Out] (-(b*d*e^3*E^((4*a)/(b*n))*f^3*n*(c*(d + e*x)^n)^(4/n)) - b*e^4*E^((4*a)/(b
*n))*f^3*n*x*(c*(d + e*x)^n)^(4/n) - 3*b*d*e^3*E^((4*a)/(b*n))*f^2*g*n*x*(c
*(d + e*x)^n)^(4/n) - 3*b*e^4*E^((4*a)/(b*n))*f^2*g*n*x^2*(c*(d + e*x)^n)^(
4/n) - 3*b*d*e^3*E^((4*a)/(b*n))*f*g^2*n*x^2*(c*(d + e*x)^n)^(4/n) - 3*b*e^
4*E^((4*a)/(b*n))*f*g^2*n*x^3*(c*(d + e*x)^n)^(4/n) - b*d*e^3*E^((4*a)/(b*n
))*g^3*n*x^3*(c*(d + e*x)^n)^(4/n) - b*e^4*E^((4*a)/(b*n))*g^3*n*x^4*(c*(d
+ e*x)^n)^(4/n) + a*e^3*E^((3*a)/(b*n))*f^3*(d + e*x)*(c*(d + e*x)^n)^(3/n)
*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)] - 3*a*d*e^2*E^((3*a)/(b*n)
)*f^2*g*(d + e*x)*(c*(d + e*x)^n)^(3/n)*ExpIntegralEi[(a + b*Log[c*(d + e*x
)^n])/(b*n)] + 3*a*d^2*e*E^((3*a)/(b*n))*f*g^2*(d + e*x)*(c*(d + e*x)^n)^(3
/n)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)] - a*d^3*E^((3*a)/(b*n))
*g^3*(d + e*x)*(c*(d + e*x)^n)^(3/n)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n
])/(b*n)] + 6*a*e^2*E^((2*a)/(b*n))*f^2*g*(d + e*x)^2*(c*(d + e*x)^n)^(2/n)
*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)] - 12*a*d*e*E^((2*a)/(b
*n))*f*g^2*(d + e*x)^2*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(2*(a + b*Log[c*
(d + e*x)^n]))/(b*n)] + 6*a*d^2*E^((2*a)/(b*n))*g^3*(d + e*x)^2*(c*(d + e*x
)^n)^(2/n)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)] + 9*a*e*E^(a
/(b*n))*f*g^2*(d + e*x)^3*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(3*(a + b*Lo
g[c*(d + e*x)^n]))/(b*n)] - 9*a*d*E^(a/(b*n))*g^3*(d + e*x)^3*(c*(d + e*x)^
n)^n^(-1)*ExpIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)] + 4*a*g^3*(d
+ e*x)^4*ExpIntegralEi[(4*(a + b*Log[c*(d + e*x)^n]))/(b*n)] + b*e^3*E^((3*
a)/(b*n))*f^3*(d + e*x)*(c*(d + e*x)^n)^(3/n)*ExpIntegralEi[(a + b*Log[c*(d
 + e*x)^n])/(b*n)]*Log[c*(d + e*x)^n] - 3*b*d*e^2*E^((3*a)/(b*n))*f^2*g*(d
+ e*x)*(c*(d + e*x)^n)^(3/n)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)
]*Log[c*(d + e*x)^n] + 3*b*d^2*e*E^((3*a)/(b*n))*f*g^2*(d + e*x)*(c*(d + e*
x)^n)^(3/n)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)]*Log[c*(d + e*x)
^n] - b*d^3*E^((3*a)/(b*n))*g^3*(d + e*x)*(c*(d + e*x)^n)^(3/n)*ExpIntegral
Ei[(a + b*Log[c*(d + e*x)^n])/(b*n)]*Log[c*(d + e*x)^n] + 6*b*e^2*E^((2*a)/
(b*n))*f^2*g*(d + e*x)^2*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(2*(a + b*Log[
c*(d + e*x)^n]))/(b*n)]*Log[c*(d + e*x)^n] - 12*b*d*e*E^((2*a)/(b*n))*f*g^2
*(d + e*x)^2*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^
n]))/(b*n)]*Log[c*(d + e*x)^n] + 6*b*d^2*E^((2*a)/(b*n))*g^3*(d + e*x)^2*(c
*(d + e*x)^n)^(2/n)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*Log
[c*(d + e*x)^n] + 9*b*e*E^(a/(b*n))*f*g^2*(d + e*x)^3*(c*(d + e*x)^n)^n^(-1
)*ExpIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*Log[c*(d + e*x)^n] -
9*b*d*E^(a/(b*n))*g^3*(d + e*x)^3*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(3*(
a + b*Log[c*(d + e*x)^n]))/(b*n)]*Log[c*(d + e*x)^n] + 4*b*g^3*(d + e*x)^4*
ExpIntegralEi[(4*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*Log[c*(d + e*x)^n])/(b^
2*e^4*E^((4*a)/(b*n))*n^2*(c*(d + e*x)^n)^(4/n)*(a + b*Log[c*(d + e*x)^n]))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.70 (sec) , antiderivative size = 9517, normalized size of antiderivative = 28.07

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 9517 |

```
[In] int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

## Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.01

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{\left(9\left(aefg^2-adg^3+(befg^2-bdg^3)n\log(ex+d)+(befg^2-bdg^3)\log(c)\right)e^{\left(\frac{b\log(c)+a}{bn}\right)}\log\_integral\left((e^3x^3\right.\right.}{}$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
[Out] (9*(a*e*f*g^2 - a*d*g^3 + (b*e*f*g^2 - b*d*g^3)*n*log(e*x + d) + (b*e*f*g^2
 - b*d*g^3)*log(c))*e^((b*log(c) + a)/(b*n))*log_integral((e^3*x^3 + 3*d*e^
2*x^2 + 3*d^2*e*x + d^3)*e^(3*(b*log(c) + a)/(b*n))) + 6*(a*e^2*f^2*g - 2*a
*d*e*f*g^2 + a*d^2*g^3 + (b*e^2*f^2*g - 2*b*d*e*f*g^2 + b*d^2*g^3)*n*log(e*
x + d) + (b*e^2*f^2*g - 2*b*d*e*f*g^2 + b*d^2*g^3)*log(c))*e^(2*(b*log(c) +
 a)/(b*n))*log_integral((e^2*x^2 + 2*d*e*x + d^2)*e^(2*(b*log(c) + a)/(b*n)
)) + (a*e^3*f^3 - 3*a*d*e^2*f^2*g + 3*a*d^2*e*f*g^2 - a*d^3*g^3 + (b*e^3*f^
3 - 3*b*d*e^2*f^2*g + 3*b*d^2*e*f*g^2 - b*d^3*g^3)*n*log(e*x + d) + (b*e^3*
f^3 - 3*b*d*e^2*f^2*g + 3*b*d^2*e*f*g^2 - b*d^3*g^3)*log(c))*e^(3*(b*log(c)
 + a)/(b*n))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n))) - (b*e^4*g^3*
n*x^4 + b*d*e^3*f^3*n + (3*b*e^4*f*g^2 + b*d*e^3*g^3)*n*x^3 + 3*(b*e^4*f^2*
g + b*d*e^3*f*g^2)*n*x^2 + (b*e^4*f^3 + 3*b*d*e^3*f^2*g)*n*x)*e^(4*(b*log(c
) + a)/(b*n)) + 4*(b*g^3*n*log(e*x + d) + b*g^3*log(c) + a*g^3)*log_integra
l((e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)*e^(4*(b*log(c)
+ a)/(b*n))))*e^(-4*(b*log(c) + a)/(b*n))/(b^3*e^4*n^3*log(e*x + d) + b^3*e
^4*n^2*log(c) + a*b^2*e^4*n^2)
```

## Sympy [F]

$$\int \frac{(f+gx)^3}{(a+b\log{(c(d+ex)^n)})^2}\,dx = \int \frac{(f+gx)^3}{(a+b\log{(c(d+ex)^n)})^2}\,dx$$

```
[In] integrate((g*x+f)**3/(a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Integral((f + g*x)**3/(a + b*log(c*(d + e*x)**n))**2, x)
```

## Maxima [F]

$$\int \frac{(f+gx)^3}{(a+b\log{(c(d+ex)^n)})^2}\,dx = \int \frac{(gx+f)^3}{(b\log{((ex+d)^n c)}+a)^2}\,dx$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] -(e*g^3*x^4 + d*f^3 + (3*e*f*g^2 + d*g^3)*x^3 + 3*(e*f^2*g + d*f*g^2)*x^2 +
 (e*f^3 + 3*d*f^2*g)*x)/(b^2*e*n*log((e*x + d)^n) + b^2*e*n*log(c) + a*b*e*
n) + integrate((4*e*g^3*x^3 + e*f^3 + 3*d*f^2*g + 3*(3*e*f*g^2 + d*g^3)*x^2
 + 6*(e*f^2*g + d*f*g^2)*x)/(b^2*e*n*log((e*x + d)^n) + b^2*e*n*log(c) + a*
b*e*n), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3473 vs. $2(341) = 682$.

Time = 0.42 (sec) , antiderivative size = 3473, normalized size of antiderivative = 10.24

$$\int \frac{(f+gx)^3}{(a+b\log{(c(d+ex)^n)})^2}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] b*e^3*f^3*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)
/((b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(1/n))
- 3*b*d*e^2*f^2*g*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(
e*x + d)/((b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c
^(1/n)) + 3*b*d^2*e*f*g^2*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*
n))*log(e*x + d)/((b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^
4*n^2)*c^(1/n)) - b*d^3*g^3*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(
b*n))*log(e*x + d)/((b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*
e^4*n^2)*c^(1/n)) - (e*x + d)*b*e^3*f^3*n/(b^3*e^4*n^3*log(e*x + d) + b^3*e
^4*n^2*log(c) + a*b^2*e^4*n^2) - 3*(e*x + d)^2*b*e^2*f^2*g*n/(b^3*e^4*n^3*l
```

og(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2) + 3*(e*x + d)*b*d*e^2*f^2
*g*n/(b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2) - 3*(e
*x + d)^3*b*e*f*g^2*n/(b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^
2*e^4*n^2) + 6*(e*x + d)^2*b*d*e*f*g^2*n/(b^3*e^4*n^3*log(e*x + d) + b^3*e^
4*n^2*log(c) + a*b^2*e^4*n^2) - 3*(e*x + d)*b*d^2*e*f*g^2*n/(b^3*e^4*n^3*lo
g(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2) - (e*x + d)^4*b*g^3*n/(b^3
*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2) + 3*(e*x + d)^3
*b*d*g^3*n/(b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)
- 3*(e*x + d)^2*b*d^2*g^3*n/(b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c)
+ a*b^2*e^4*n^2) + (e*x + d)*b*d^3*g^3*n/(b^3*e^4*n^3*log(e*x + d) + b^3*e^
4*n^2*log(c) + a*b^2*e^4*n^2) + 6*b*e^2*f^2*g*n*Ei(2*log(c)/n + 2*a/(b*n) +
2*log(e*x + d))*e^(-2*a/(b*n))*log(e*x + d)/((b^3*e^4*n^3*log(e*x + d) + b
^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(2/n)) - 12*b*d*e*f*g^2*n*Ei(2*log(c)/
n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(e*x + d)/((b^3*e^4*n^3*l
og(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(2/n)) + 6*b*d^2*g^3*n*
Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(e*x + d)/((b
^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(2/n)) + b*
e^3*f^3*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)/((b^3*e^4
*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(1/n)) - 3*b*d*e^
2*f^2*g*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)/((b^3*e^4
*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(1/n)) + 3*b*d^2*
e*f*g^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)/((b^3*e^4
*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(1/n)) - b*d^3*g^
3*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)/((b^3*e^4*n^3*l
og(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(1/n)) + a*e^3*f^3*Ei(l
og(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/((b^3*e^4*n^3*log(e*x + d) +
b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(1/n)) - 3*a*d*e^2*f^2*g*Ei(log(c)/n
+ a/(b*n) + log(e*x + d))*e^(-a/(b*n))/((b^3*e^4*n^3*log(e*x + d) + b^3*e^
4*n^2*log(c) + a*b^2*e^4*n^2)*c^(1/n)) + 3*a*d^2*e*f*g^2*Ei(log(c)/n + a/(b
*n) + log(e*x + d))*e^(-a/(b*n))/((b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*l
og(c) + a*b^2*e^4*n^2)*c^(1/n)) - a*d^3*g^3*Ei(log(c)/n + a/(b*n) + log(e*x
+ d))*e^(-a/(b*n))/((b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2
*e^4*n^2)*c^(1/n)) + 9*b*e*f*g^2*n*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x +
d))*e^(-3*a/(b*n))*log(e*x + d)/((b^3*e^4*n^3*log(e*x + d) + b^3*e^4*n^2*lo
g(c) + a*b^2*e^4*n^2)*c^(3/n)) - 9*b*d*g^3*n*Ei(3*log(c)/n + 3*a/(b*n) + 3*
log(e*x + d))*e^(-3*a/(b*n))*log(e*x + d)/((b^3*e^4*n^3*log(e*x + d) + b^3*
e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(3/n)) + 6*b*e^2*f^2*g*Ei(2*log(c)/n + 2*
a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(c)/((b^3*e^4*n^3*log(e*x + d)
+ b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(2/n)) - 12*b*d*e*f*g^2*Ei(2*log(c)
/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(c)/((b^3*e^4*n^3*log(e*
x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(2/n)) + 6*b*d^2*g^3*Ei(2*lo
g(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(c)/((b^3*e^4*n^3*lo
g(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(2/n)) + 6*a*e^2*f^2*g*E
i(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))/((b^3*e^4*n^3*log
(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(2/n)) - 12*a*d*e*f*g^2*E

```
i(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))/((b^3*e^4*n^3*log
(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(2/n)) + 6*a*d^2*g^3*Ei(2
*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))/((b^3*e^4*n^3*log(e*
x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(2/n)) + 4*b*g^3*n*Ei(4*log(
c)/n + 4*a/(b*n) + 4*log(e*x + d))*e^(-4*a/(b*n))*log(e*x + d)/((b^3*e^4*n^
3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(4/n)) + 9*b*e*f*g^2
*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))*log(c)/((b^3*e^
4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(3/n)) - 9*b*d*g
^3*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))*log(c)/((b^3*
e^4*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(3/n)) + 9*a*e
*f*g^2*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))/((b^3*e^4
*n^3*log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(3/n)) - 9*a*d*g^
3*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))/((b^3*e^4*n^3*
log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(3/n)) + 4*b*g^3*Ei(4*
log(c)/n + 4*a/(b*n) + 4*log(e*x + d))*e^(-4*a/(b*n))*log(c)/((b^3*e^4*n^3*
log(e*x + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(4/n)) + 4*a*g^3*Ei(4*
log(c)/n + 4*a/(b*n) + 4*log(e*x + d))*e^(-4*a/(b*n))/((b^3*e^4*n^3*log(e*x
 + d) + b^3*e^4*n^2*log(c) + a*b^2*e^4*n^2)*c^(4/n))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(f+g\,x)^3}{(a+b\,\ln(c\,(d+e\,x)^n))^2}\,dx$$

```
[In] int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^2,x)
[Out] int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^2, x)
```

# 3.95 $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 259

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^3n^2} + \frac{4e^{-\frac{2a}{bn}}g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^3n^2} + \frac{3e^{-\frac{3a}{bn}}g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^3n^2} - \frac{(d+ex)(f+gx)^2}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

```
[Out] (-d*g+e*f)^2*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e^3/exp(a/b/n)/n^2/(
(c*(e*x+d)^n)^(1/n))+4*g*(-d*g+e*f)*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n))/b/
n)/b^2/e^3/exp(2*a/b/n)/n^2/((c*(e*x+d)^n)^(2/n))+3*g^2*(e*x+d)^3*Ei(3*(a+b
*ln(c*(e*x+d)^n))/b/n)/b^2/e^3/exp(3*a/b/n)/n^2/((c*(e*x+d)^n)^(3/n))-(e*x+
d)*(g*x+f)^2/b/e/n/(a+b*ln(c*(e*x+d)^n))
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2447, 2446, 2436, 2337, 2209, 2437, 2347}

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{4ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^3n^2} + \frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^3n^2} + \frac{3g^2e^{-\frac{3a}{bn}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^3n^2} - \frac{(d+ex)(f+gx)^2}{ben\,(a+b\log(c(d+ex)^n))}$$

```
[In] Int[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] ((e*f - d*g)^2*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(
b^2*e^3*E^(a/(b*n))*n^2*(c*(d + e*x)^n)^n^(-1)) + (4*g*(e*f - d*g)*(d + e*x
)^2*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b^2*e^3*E^((2*a)/
(b*n))*n^2*(c*(d + e*x)^n)^(2/n)) + (3*g^2*(d + e*x)^3*ExpIntegralEi[(3*(a
+ b*Log[c*(d + e*x)^n]))/(b*n)])/(b^2*e^3*E^((3*a)/(b*n))*n^2*(c*(d + e*x)^
n)^(3/n)) - ((d + e*x)*(f + g*x)^2)/(b*e*n*(a + b*Log[c*(d + e*x)^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+ex)(f+gx)^2}{ben\,(a+b\log\left(c(d+ex)^n\right))}+\frac{3\int\frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}\,dx}{bn}\\
&\quad-\frac{(2(ef-dg))\int\frac{f+gx}{a+b\log(c(d+ex)^n)}\,dx}{ben}\\
&=-\frac{(d+ex)(f+gx)^2}{ben\,(a+b\log\left(c(d+ex)^n\right))}\\
&\quad+\frac{3\int\left(\frac{(ef-dg)^2}{e^2(a+b\log(c(d+ex)^n))}+\frac{2g(ef-dg)(d+ex)}{e^2(a+b\log(c(d+ex)^n))}+\frac{g^2(d+ex)^2}{e^2(a+b\log(c(d+ex)^n))}\right)dx}{bn}\\
&\quad-\frac{(2(ef-dg))\int\left(\frac{ef-dg}{e(a+b\log(c(d+ex)^n))}+\frac{g(d+ex)}{e(a+b\log(c(d+ex)^n))}\right)dx}{ben}
\end{aligned}$$

$$= -\frac{(d+ex)(f+gx)^2}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{(3g^2)\int \frac{(d+ex)^2}{a+b\log(c(d+ex)^n)}\,dx}{be^2n} - \frac{(2g(ef-dg))\int \frac{d+ex}{a+b\log(c(d+ex)^n)}\,dx}{be^2n} + \frac{(6g(ef-dg))\int \frac{d+ex}{a+b\log(c(d+ex)^n)}\,dx}{be^2n} - \frac{(2(ef-dg)^2)\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx}{be^2n} + \frac{(3(ef-dg)^2)\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx}{be^2n}$$

$$= -\frac{(d+ex)(f+gx)^2}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{(3g^2)\,\text{Subst}\left(\int \frac{x^2}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^3n} - \frac{(2g(ef-dg))\text{Subst}\left(\int \frac{x}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^3n} + \frac{(6g(ef-dg))\text{Subst}\left(\int \frac{x}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^3n} - \frac{(2(ef-dg)^2)\,\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^3n} + \frac{(3(ef-dg)^2)\,\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^3n}$$

$$= -\frac{(d+ex)(f+gx)^2}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{\left(3g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^3n^2} - \frac{\left(2g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^3n^2} + \frac{\left(6g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^3n^2} - \frac{\left(2(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^3n^2} + \frac{\left(3(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^3n^2}$$

$$= \frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^3n^2}$$
$$+ \frac{4e^{-\frac{2a}{bn}}g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{Ei}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^3n^2}$$
$$+ \frac{3e^{-\frac{3a}{bn}}g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{Ei}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^3n^2}$$
$$- \frac{(d+ex)(f+gx)^2}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1015 vs. $2(259) = 518$.

Time = 0.25 (sec) , antiderivative size = 1015, normalized size of antiderivative = 3.92

$$\int \frac{(f+gx)^2}{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$
$$= \frac{e^{-\frac{3a}{bn}}\left(c(d+ex)^n\right)^{-3/n}\left(-bde^2e^{\frac{3a}{bn}}f^2n\left(c(d+ex)^n\right)^{3/n} - be^3e^{\frac{3a}{bn}}f^2nx\left(c(d+ex)^n\right)^{3/n} - 2bde^2e^{\frac{3a}{bn}}fgnx(c(d+\right.}{}$$

```
[In] Integrate[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (-(b*d*e^2*E^((3*a)/(b*n))*f^2*n*(c*(d + e*x)^n)^(3/n)) - b*e^3*E^((3*a)/(b
*n))*f^2*n*x*(c*(d + e*x)^n)^(3/n) - 2*b*d*e^2*E^((3*a)/(b*n))*f*g*n*x*(c*(
d + e*x)^n)^(3/n) - 2*b*e^3*E^((3*a)/(b*n))*f*g*n*x^2*(c*(d + e*x)^n)^(3/n)
 - b*d*e^2*E^((3*a)/(b*n))*g^2*n*x^2*(c*(d + e*x)^n)^(3/n) - b*e^3*E^((3*a)
/(b*n))*g^2*n*x^3*(c*(d + e*x)^n)^(3/n) + a*e^2*E^((2*a)/(b*n))*f^2*(d + e*
x)*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)] -
2*a*d*e*E^((2*a)/(b*n))*f*g*(d + e*x)*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(
a + b*Log[c*(d + e*x)^n])/(b*n)] + a*d^2*E^((2*a)/(b*n))*g^2*(d + e*x)*(c*(
d + e*x)^n)^(2/n)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)] + 4*a*e*E
^(a/(b*n))*f*g*(d + e*x)^2*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(2*(a + b*L
og[c*(d + e*x)^n]))/(b*n)] - 4*a*d*E^(a/(b*n))*g^2*(d + e*x)^2*(c*(d + e*x)
^n)^n^(-1)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)] + 3*a*g^2*(d
 + e*x)^3*ExpIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)] + b*e^2*E^((2
*a)/(b*n))*f^2*(d + e*x)*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(a + b*Log[c*(
d + e*x)^n])/(b*n)]*Log[c*(d + e*x)^n] - 2*b*d*e*E^((2*a)/(b*n))*f*g*(d + e
*x)*(c*(d + e*x)^n)^(2/n)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)]*L
og[c*(d + e*x)^n] + b*d^2*E^((2*a)/(b*n))*g^2*(d + e*x)*(c*(d + e*x)^n)^(2/
n)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)]*Log[c*(d + e*x)^n] + 4*b
*e*E^(a/(b*n))*f*g*(d + e*x)^2*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(2*(a +
 b*Log[c*(d + e*x)^n]))/(b*n)]*Log[c*(d + e*x)^n] - 4*b*d*E^(a/(b*n))*g^2*(
d + e*x)^2*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n
```

```
]))/(b*n)]*Log[c*(d + e*x)^n] + 3*b*g^2*(d + e*x)^3*ExpIntegralEi[(3*(a + b
*Log[c*(d + e*x)^n]))/(b*n)]*Log[c*(d + e*x)^n])/(b^2*e^3*E^((3*a)/(b*n))*n
^2*(c*(d + e*x)^n)^(3/n)*(a + b*Log[c*(d + e*x)^n]))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.27 (sec) , antiderivative size = 5089, normalized size of antiderivative = 19.65

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 5089 |

```
[In] int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

## Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.67

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{\left(4\left(aefg-adg^2+(befg-bdg^2)n\log(ex+d)+(befg-bdg^2)\log(c)\right)e^{\left(\frac{b\log(c)+a}{bn}\right)}\operatorname{log\_integral}\left(\left(e^2x^2+2\right.\right.\right.}{}$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

```
[Out] (4*(a*e*f*g - a*d*g^2 + (b*e*f*g - b*d*g^2)*n*log(e*x + d) + (b*e*f*g - b*d
*g^2)*log(c))*e^((b*log(c) + a)/(b*n))*log_integral((e^2*x^2 + 2*d*e*x + d^
2)*e^(2*(b*log(c) + a)/(b*n))) + (a*e^2*f^2 - 2*a*d*e*f*g + a*d^2*g^2 + (b*
e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*n*log(e*x + d) + (b*e^2*f^2 - 2*b*d*e*f*
g + b*d^2*g^2)*log(c))*e^(2*(b*log(c) + a)/(b*n))*log_integral((e*x + d)*e^
((b*log(c) + a)/(b*n))) - (b*e^3*g^2*n*x^3 + b*d*e^2*f^2*n + (2*b*e^3*f*g +
 b*d*e^2*g^2)*n*x^2 + (b*e^3*f^2 + 2*b*d*e^2*f*g)*n*x)*e^(3*(b*log(c) + a)/
(b*n)) + 3*(b*g^2*n*log(e*x + d) + b*g^2*log(c) + a*g^2)*log_integral((e^3*
x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*e^(3*(b*log(c) + a)/(b*n))))*e^(-3*(b*
log(c) + a)/(b*n))/(b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e
^3*n^2)
```

## Sympy [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx$$

```
[In] integrate((g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Integral((f + g*x)**2/(a + b*log(c*(d + e*x)**n))**2, x)
```

## Maxima [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(gx+f)^2}{(b\log((ex+d)^n c)+a)^2}\,dx$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] -(e*g^2*x^3 + d*f^2 + (2*e*f*g + d*g^2)*x^2 + (e*f^2 + 2*d*f*g)*x)/(b^2*e*n
*log((e*x + d)^n) + b^2*e*n*log(c) + a*b*e*n) + integrate((3*e*g^2*x^2 + e*
f^2 + 2*d*f*g + 2*(2*e*f*g + d*g^2)*x)/(b^2*e*n*log((e*x + d)^n) + b^2*e*n*
log(c) + a*b*e*n), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2031 vs. 2(260) = 520.

Time = 0.39 (sec) , antiderivative size = 2031, normalized size of antiderivative = 7.84

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] b*e^2*f^2*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)
/((b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(1/n))
- 2*b*d*e*f*g*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x
+ d)/((b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(1/
n)) + b*d^2*g^2*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*
x + d)/((b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(
1/n)) - (e*x + d)*b*e^2*f^2*n/(b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c
) + a*b^2*e^3*n^2) - 2*(e*x + d)^2*b*e*f*g*n/(b^3*e^3*n^3*log(e*x + d) + b^
3*e^3*n^2*log(c) + a*b^2*e^3*n^2) + 2*(e*x + d)*b*d*e*f*g*n/(b^3*e^3*n^3*lo
g(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2) - (e*x + d)^3*b*g^2*n/(b^3
*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2) + 2*(e*x + d)^2
```

```
*b*d*g^2*n/(b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)
- (e*x + d)*b*d^2*g^2*n/(b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*
b^2*e^3*n^2) + 4*b*e*f*g*n*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-
2*a/(b*n))*log(e*x + d)/((b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a
*b^2*e^3*n^2)*c^(2/n)) - 4*b*d*g^2*n*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x
+ d))*e^(-2*a/(b*n))*log(e*x + d)/((b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*
log(c) + a*b^2*e^3*n^2)*c^(2/n)) + b*e^2*f^2*Ei(log(c)/n + a/(b*n) + log(e*
x + d))*e^(-a/(b*n))*log(c)/((b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c)
 + a*b^2*e^3*n^2)*c^(1/n)) - 2*b*d*e*f*g*Ei(log(c)/n + a/(b*n) + log(e*x +
d))*e^(-a/(b*n))*log(c)/((b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a
*b^2*e^3*n^2)*c^(1/n)) + b*d^2*g^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^
(-a/(b*n))*log(c)/((b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e
^3*n^2)*c^(1/n)) + a*e^2*f^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b
*n))/((b^3*e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(1/
n)) - 2*a*d*e*f*g*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/((b^3*
e^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(1/n)) + a*d^2
*g^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/((b^3*e^3*n^3*log(e
*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(1/n)) + 3*b*g^2*n*Ei(3*log
(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))*log(e*x + d)/((b^3*e^3*n
^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(3/n)) + 4*b*e*f*g*
Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(c)/((b^3*e^3
*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(2/n)) - 4*b*d*g^
2*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(c)/((b^3*e
^3*n^3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(2/n)) + 4*a*e*
f*g*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))/((b^3*e^3*n^
3*log(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(2/n)) - 4*a*d*g^2*E
i(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))/((b^3*e^3*n^3*log
(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(2/n)) + 3*b*g^2*Ei(3*log
(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))*log(c)/((b^3*e^3*n^3*log
(e*x + d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(3/n)) + 3*a*g^2*Ei(3*log
(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))/((b^3*e^3*n^3*log(e*x +
d) + b^3*e^3*n^2*log(c) + a*b^2*e^3*n^2)*c^(3/n))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx=\int \frac{{\left(f+g\,x\right)}^2}{{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}^2}\,dx$$

```
[In] int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^2,x)
[Out] int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^2, x)
```

# 3.96 $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 177

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex)\,(c(d+ex)^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2} + \frac{2e^{-\frac{2a}{bn}}g(d+ex)^2\,(c(d+ex)^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^2n^2} - \frac{(d+ex)(f+gx)}{ben\,(a+b\log(c(d+ex)^n))}$$

```
[Out] (-d*g+e*f)*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e^2/exp(a/b/n)/n^2/((c
*(e*x+d)^n)^(1/n))+2*g*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e^2/ex
p(2*a/b/n)/n^2/((c*(e*x+d)^n)^(2/n))-(e*x+d)*(g*x+f)/b/e/n/(a+b*ln(c*(e*x+d
)^n))
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used

$= \{2447, 2446, 2436, 2337, 2209, 2437, 2347\}$

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2} + \frac{2ge^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^2n^2} - \frac{(d+ex)(f+gx)}{ben\,(a+b\log(c(d+ex)^n))}$$

```
[In] Int[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] ((e*f - d*g)*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b^
2*e^2*E^(a/(b*n))*n^2*(c*(d + e*x)^n)^n^(-1)) + (2*g*(d + e*x)^2*ExpIntegra
lEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b^2*e^2*E^((2*a)/(b*n))*n^2*(c*
(d + e*x)^n)^(2/n)) - ((d + e*x)*(f + g*x))/(b*e*n*(a + b*Log[c*(d + e*x)^n
]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rubi steps

$$\text{integral} = -\frac{(d+ex)(f+gx)}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{2\int\frac{f+gx}{a+b\log(c(d+ex)^n)}\,dx}{bn}-\frac{(ef-dg)\int\frac{1}{a+b\log(c(d+ex)^n)}\,dx}{ben}$$

$$= -\frac{(d+ex)(f+gx)}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{2\int\left(\frac{ef-dg}{e(a+b\log(c(d+ex)^n))}+\frac{g(d+ex)}{e(a+b\log(c(d+ex)^n))}\right)dx}{bn}-\frac{(ef-dg)\text{Subst}\left(\int\frac{1}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^2n}$$

$$= -\frac{(d+ex)(f+gx)}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{(2g)\int\frac{d+ex}{a+b\log(c(d+ex)^n)}\,dx}{ben}+\frac{(2(ef-dg))\int\frac{1}{a+b\log(c(d+ex)^n)}\,dx}{ben}-\frac{\left((ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^2n^2}$$

$$= -\frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2}-\frac{(d+ex)(f+gx)}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{(2g)\text{Subst}\left(\int\frac{x}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^2n}+\frac{(2(ef-dg))\text{Subst}\left(\int\frac{1}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{be^2n}$$

$$= -\frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2} - \frac{(d+ex)(f+gx)}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{\left(2g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^2n^2} + \frac{\left(2(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^2n^2}$$

$$= \frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2e^2n^2} + \frac{2e^{-\frac{2a}{bn}}g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{Ei}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^2e^2n^2} - \frac{(d+ex)(f+gx)}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.18

$$\int\frac{f+gx}{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = -\frac{e^{-\frac{2a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(bee^{\frac{2a}{bn}}n\left(c(d+ex)^n\right)^{2/n}(f+gx) - e^{\frac{a}{bn}}(ef-dg)\left(c(d+ex)^n\right)^{\frac{1}{n}}\text{ExpIntegr}\right.}{b^2e^2n^2\left(\right.}$$

```
[In] Integrate[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] -(((d + e*x)*(b*e*E^((2*a)/(b*n))*n*(c*(d + e*x)^n)^(2/n)*(f + g*x) - E^(a/
(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(a + b*Log[c*(d + e
*x)^n])/(b*n)]*(a + b*Log[c*(d + e*x)^n]) - 2*g*(d + e*x)*ExpIntegralEi[(2*
(a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log[c*(d + e*x)^n])))/(b^2*e^2*E^
((2*a)/(b*n))*n^2*(c*(d + e*x)^n)^(2/n)*(a + b*Log[c*(d + e*x)^n])))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.01 (sec) , antiderivative size = 2300, normalized size of antiderivative = 12.99

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 2300 |

```
[In] int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(e*x+d)*(g*x+f)/b/e/n/(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x
+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I
*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln((e*x+d)^n)+2*b*ln(c)+
2*a)-1/b^2/n^2*f*((e*x+d)^n)^(-1/n)*c^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*
x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+
I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b
+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(
I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*
csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x
+d)^n)-n*ln(e*x+d))+2*a)/b/n)*x-1/b^2/e/n^2*f*((e*x+d)^n)^(-1/n)*c^(-1/n)*e
xp(-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(
I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b
-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I
*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n
)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^
n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)*d-2/b^2/n^2*g*((
e*x+d)^n)^(-2/n)*c^(-2/n)*exp(-(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(
I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*
csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-2*ln(e
*x+d)-(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*
c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I
*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/
b/n)*x^2-4/b^2/e/n^2*g*((e*x+d)^n)^(-2/n)*c^(-2/n)*exp(-(-I*b*Pi*csgn(I*c*(
e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*
b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3
*b+2*a)/b/n)*Ei(1,-2*ln(e*x+d)-(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(
I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*
csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x
+d)^n)-n*ln(e*x+d))+2*a)/b/n)*d*x-2/b^2/e^2/n^2*g*((e*x+d)^n)^(-2/n)*c^(-2/
n)*exp(-(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(
I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b
-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-2*ln(e*x+d)-(-I*b*Pi*csgn(I*c
*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^
2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)
^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)*d^2+1/b^2/e/n^2*d*
```

```
g*((e*x+d)^n)^(-1/n)*c^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*
c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*
x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1
,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*
Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d
)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x
+d))+2*a)/b/n)*x+1/b^2/e^2/n^2*d^2*g*((e*x+d)^n)^(-1/n)*c^(-1/n)*exp(-1/2*(
-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn
(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csg
n(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d
)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*P
i*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*
b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.35

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx$$
$$= \frac{\left((aef-adg+(bef-bdg)n\log(ex+d)+(bef-bdg)\log(c))e^{\left(\frac{b\log(c)+a}{bn}\right)}\operatorname{log\_integral}\left((ex+d)e^{\left(\frac{b\log(c)+a}{bn}\right.}\right.\right.}{}$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

```
[Out] ((a*e*f - a*d*g + (b*e*f - b*d*g)*n*log(e*x + d) + (b*e*f - b*d*g)*log(c))*
e^((b*log(c) + a)/(b*n))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n))) -
 (b*e^2*g*n*x^2 + b*d*e*f*n + (b*e^2*f + b*d*e*g)*n*x)*e^(2*(b*log(c) + a)/
(b*n)) + 2*(b*g*n*log(e*x + d) + b*g*log(c) + a*g)*log_integral((e^2*x^2 +
2*d*e*x + d^2)*e^(2*(b*log(c) + a)/(b*n))))*e^(-2*(b*log(c) + a)/(b*n))/(b^
3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2)
```

## Sympy [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx$$

```
[In] integrate((g*x+f)/(a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Integral((f + g*x)/(a + b*log(c*(d + e*x)**n))**2, x)
```

## Maxima [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{gx+f}{(b\log((ex+d)^n c)+a)^2}\,dx$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] -(e*g*x^2 + d*f + (e*f + d*g)*x)/(b^2*e*n*log((e*x + d)^n) + b^2*e*n*log(c)
 + a*b*e*n) + integrate((2*e*g*x + e*f + d*g)/(b^2*e*n*log((e*x + d)^n) + b
^2*e*n*log(c) + a*b*e*n), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 973 vs. $2(177) = 354$.

Time = 0.33 (sec) , antiderivative size = 973, normalized size of antiderivative = 5.50

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] b*e*f*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)/((b
^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2)*c^(1/n)) - b*
d*g*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)/((b^3
*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2)*c^(1/n)) - (e*x
 + d)*b*e*f*n/(b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^
2) - (e*x + d)^2*b*g*n/(b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b
^2*e^2*n^2) + (e*x + d)*b*d*g*n/(b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log
(c) + a*b^2*e^2*n^2) + 2*b*g*n*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*
e^(-2*a/(b*n))*log(e*x + d)/((b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c)
 + a*b^2*e^2*n^2)*c^(2/n)) + b*e*f*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^
(-a/(b*n))*log(c)/((b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e
^2*n^2)*c^(1/n)) - b*d*g*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))
*log(c)/((b^3*e^2*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2)*c^
(1/n)) + a*e*f*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/((b^3*e^2
*n^3*log(e*x + d) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2)*c^(1/n)) - a*d*g*Ei
(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/((b^3*e^2*n^3*log(e*x + d)
 + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2)*c^(1/n)) + 2*b*g*Ei(2*log(c)/n + 2*a
/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(c)/((b^3*e^2*n^3*log(e*x + d) +
 b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2)*c^(2/n)) + 2*a*g*Ei(2*log(c)/n + 2*a/(
b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))/((b^3*e^2*n^3*log(e*x + d) + b^3*e^2*
n^2*log(c) + a*b^2*e^2*n^2)*c^(2/n))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{f+g\,x}{\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^2}\,dx$$

```
[In] int((f + g*x)/(a + b*log(c*(d + e*x)^n))^2,x)
[Out] int((f + g*x)/(a + b*log(c*(d + e*x)^n))^2, x)
```

# 3.97 $\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2en^2} - \frac{d+ex}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

```
[Out] (e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e/exp(a/b/n)/n^2/((c*(e*x+d)^n)^(
1/n))+(-e*x-d)/b/e/n/(a+b*ln(c*(e*x+d)^n))
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2436, 2334, 2337, 2209}

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2en^2} - \frac{d+ex}{ben\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(-2),x]
```

```
[Out] ((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b^2*e*E^(a/(b*
n))*n^2*(c*(d + e*x)^n)^n^(-1)) - (d + e*x)/(b*e*n*(a + b*Log[c*(d + e*x)^n
]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^2}\,dx, x, d+ex\right)}{e}\\
&= -\frac{d+ex}{ben\,(a+b\log(c(d+ex)^n))}+\frac{\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx, x, d+ex\right)}{ben}\\
&= -\frac{d+ex}{ben\,(a+b\log(c(d+ex)^n))}\\
&\quad+\frac{\left((d+ex)\,(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log(c(d+ex)^n)\right)}{ben^2}\\
&= \frac{e^{-\frac{a}{bn}}(d+ex)\,(c(d+ex)^n)^{-1/n}\,\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2en^2}-\frac{d+ex}{ben\,(a+b\log(c(d+ex)^n))}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = -\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(be^{\frac{a}{bn}}n(c(d+ex)^n)^{\frac{1}{n}} - \text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+\ldots\right.}{b^2en^2(a+b\log(c(d+ex)^n))}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(-2),x]
```

```
[Out] -(((d + e*x)*(b*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1) - ExpIntegralEi[(a + b
*Log[c*(d + e*x)^n])/(b*n)]*(a + b*Log[c*(d + e*x)^n])))/(b^2*e*E^(a/(b*n))
*n^2*(c*(d + e*x)^n)^n^(-1)*(a + b*Log[c*(d + e*x)^n])))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.75

| method | result |
|---|---|
| risch | $-\frac{2(ex+d)}{\left(-ib\pi\ \text{csgn}(ic(ex+d)^n)\ \text{csgn}(ic)\ \text{csgn}(i(ex+d)^n)+i\pi\ \text{csgn}(ic)\ \text{csgn}(ic(ex+d)^n)^2b+i\pi\ \text{csgn}(i(ex+d)^n)\ \text{csgn}(ic(ex+d)^n)^2b-i\pi\ \text{csg}\ldots\right.}$ |

```
[In] int(1/(a+b*ln(c*(e*x+d)^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*
csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi
*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln((e*x+d)^n)+2*b*ln(c)+2*a)/b/n/e*(e*x+d)-1/b
^2/n^2/e*(e*x+d)*c^(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*
x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+
I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b
+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(
I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*
csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x
+d)^n)-n*ln(e*x+d))+2*a)/b/n)
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = -\frac{\left((benx+bdn)e^{\left(\frac{b\log(c)+a}{bn}\right)}-(bn\log(ex+d)+b\log(c)+a)\,\mathrm{log\_integral}\left((ex+d)e^{\left(\frac{b\log(c)+a}{bn}\right)}\right)\right)e^{\left(-\frac{b\log(}{b}\right.}}{b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

```
[Out] -((b*e*n*x + b*d*n)*e^((b*log(c) + a)/(b*n)) - (b*n*log(e*x + d) + b*log(c)
 + a)*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n))))*e^(-(b*log(c) + a)/
(b*n))/(b^3*e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n^2)
```

## Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**(-2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] -(e*x + d)/(b^2*e*n*log((e*x + d)^n) + b^2*e*n*log(c) + a*b*e*n) + integrat
e(1/(b^2*n*log((e*x + d)^n) + b^2*n*log(c) + a*b*n), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(95) = 190.

Time = 0.32 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.98

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \frac{bn\text{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}\log(ex+d)}{(b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2)c^{\left(\frac{1}{n}\right)}} - \frac{(ex+d)bn}{b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2} + \frac{b\text{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}\log(c)}{(b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2)c^{\left(\frac{1}{n}\right)}} + \frac{a\text{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(ex+d)\right)e^{\left(-\frac{a}{bn}\right)}}{(b^3en^3\log(ex+d)+b^3en^2\log(c)+ab^2en^2)c^{\left(\frac{1}{n}\right)}}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] b*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)/((b^3*e
*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n^2)*c^(1/n)) - (e*x + d)*b*
n/(b^3*e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n^2) + b*Ei(log(c)/n
 + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)/((b^3*e*n^3*log(e*x + d) + b
^3*e*n^2*log(c) + a*b^2*e*n^2)*c^(1/n)) + a*Ei(log(c)/n + a/(b*n) + log(e*x
 + d))*e^(-a/(b*n))/((b^3*e*n^3*log(e*x + d) + b^3*e*n^2*log(c) + a*b^2*e*n
^2)*c^(1/n))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e*x)^n))^2,x)
```

```
[Out] int(1/(a + b*log(c*(d + e*x)^n))^2, x)
```

# 3.98 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2} dx = \operatorname{Int}\left(\frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}, x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2} dx = \int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2} dx$$

```
[In] Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
```

```
[Out] Defer[Int][1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2} dx$$

## Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
[Out] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2), x]
```

## Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(a+b\ln\left(c\,(ex+d)^n\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
[Out] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*g*x + a^2*f + (b^2*g*x + b^2*f)*log((e*x + d)^n*c)^2 + 2*(a
*b*g*x + a*b*f)*log((e*x + d)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)^2(f+gx)}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**2,x)
[Out] Integral(1/((a + b*log(c*(d + e*x)**n))**2*(f + g*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 7.83

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^nc\right)+a)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
[Out] (e*f - d*g)*integrate(1/(b^2*e*f^2*n*log(c) + a*b*e*f^2*n + (b^2*e*g^2*n*lo
g(c) + a*b*e*g^2*n)*x^2 + 2*(b^2*e*f*g*n*log(c) + a*b*e*f*g*n)*x + (b^2*e*g
^2*n*x^2 + 2*b^2*e*f*g*n*x + b^2*e*f^2*n)*log((e*x + d)^n)), x) - (e*x + d)
/(b^2*e*f*n*log(c) + a*b*e*f*n + (b^2*e*g*n*log(c) + a*b*e*g*n)*x + (b^2*e*
g*n*x + b^2*e*f*n)*log((e*x + d)^n))
```

## Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^nc\right)+a)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\,(a+b\log{(c(d+ex)^n)})^2}\,dx = \int \frac{1}{(f+g\,x)\,\left(a+b\,\ln{\left(c\,(d+e\,x)^n\right)}\right)^2}\,dx$$

```
[In] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^2),x)
[Out] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^2), x)
```

# 3.99 $\int \frac{1}{(f+gx)^2(a+b\log(c(d+ex)^n))^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

```
[In] Int[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^2),x]
[Out] Defer[Int][1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^2),x]
[Out] Integrate[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^2), x]
```

## Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(gx+f\right)^2\left(a+b\ln\left(c\left(ex+d\right)^n\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^2,x)
[Out] int(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.33

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{\left(gx+f\right)^2\left(b\log\left(\left(ex+d\right)^nc\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*g^2*x^2 + 2*a^2*f*g*x + a^2*f^2 + (b^2*g^2*x^2 + 2*b^2*f*g*
x + b^2*f^2)*log((e*x + d)^n*c)^2 + 2*(a*b*g^2*x^2 + 2*a*b*f*g*x + a*b*f^2)
*log((e*x + d)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 11.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)^2 (a+b\log(c(d+ex)^n))^2} dx = \int \frac{1}{(a+b\log(c(d+ex)^n))^2 (f+gx)^2} dx$$

```
[In] integrate(1/(g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**2,x)
[Out] Integral(1/((a + b*log(c*(d + e*x)**n))**2*(f + g*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 12.04

$$\int \frac{1}{(f+gx)^2 (a+b\log(c(d+ex)^n))^2} dx = \int \frac{1}{(gx+f)^2 (b\log((ex+d)^n c)+a)^2} dx$$

```
[In] integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
[Out] -(e*x + d)/(b^2*e*f^2*n*log(c) + a*b*e*f^2*n + (b^2*e*g^2*n*log(c) + a*b*e*
g^2*n)*x^2 + 2*(b^2*e*f*g*n*log(c) + a*b*e*f*g*n)*x + (b^2*e*g^2*n*x^2 + 2*
b^2*e*f*g*n*x + b^2*e*f^2*n)*log((e*x + d)^n)) - integrate((e*g*x - e*f + 2
*d*g)/(b^2*e*f^3*n*log(c) + a*b*e*f^3*n + (b^2*e*g^3*n*log(c) + a*b*e*g^3*n
)*x^3 + 3*(b^2*e*f*g^2*n*log(c) + a*b*e*f*g^2*n)*x^2 + 3*(b^2*e*f^2*g*n*log
(c) + a*b*e*f^2*g*n)*x + (b^2*e*g^3*n*x^3 + 3*b^2*e*f*g^2*n*x^2 + 3*b^2*e*f
^2*g*n*x + b^2*e*f^3*n)*log((e*x + d)^n)), x)
```

## Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2 (a+b\log(c(d+ex)^n))^2} dx = \int \frac{1}{(gx+f)^2 (b\log((ex+d)^n c)+a)^2} dx$$

```
[In] integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
[Out] integrate(1/((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2 (a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{(f+g\,x)^2\,(a+b\,\ln(c\,(d+e\,x)^n))^2}\,dx$$

```
[In] int(1/((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^2),x)
[Out] int(1/((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^2), x)
```

# 3.100 $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 351

$$\begin{aligned}\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx &= \frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3e^3n^3}\\
&+\frac{4e^{-\frac{2a}{bn}}g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^3e^3n^3}\\
&+\frac{9e^{-\frac{3a}{bn}}g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{2b^3e^3n^3}\\
&-\frac{(d+ex)(f+gx)^2}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}+\frac{(ef-dg)(d+ex)(f+gx)}{b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\\
&-\frac{3(d+ex)(f+gx)^2}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\end{aligned}$$

```
[Out] 1/2*(-d*g+e*f)^2*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^3/e^3/exp(a/b/n)/n
^3/((c*(e*x+d)^n)^(1/n))+4*g*(-d*g+e*f)*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n)
)/b/n)/b^3/e^3/exp(2*a/b/n)/n^3/((c*(e*x+d)^n)^(2/n))+9/2*g^2*(e*x+d)^3*Ei(
3*(a+b*ln(c*(e*x+d)^n))/b/n)/b^3/e^3/exp(3*a/b/n)/n^3/((c*(e*x+d)^n)^(3/n))
-1/2*(e*x+d)*(g*x+f)^2/b/e/n/(a+b*ln(c*(e*x+d)^n))^2+(-d*g+e*f)*(e*x+d)*(g*
x+f)/b^2/e^2/n^2/(a+b*ln(c*(e*x+d)^n))-3/2*(e*x+d)*(g*x+f)^2/b^2/e/n^2/(a+b
*ln(c*(e*x+d)^n))
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2447, 2446, 2436, 2337, 2209, 2437, 2347}

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx = \frac{4ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\,(c(d+ex)^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^3e^3n^3} + \frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\,(c(d+ex)^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3e^3n^3} + \frac{9g^2e^{-\frac{3a}{bn}}(d+ex)^3\,(c(d+ex)^n)^{-3/n}\,\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{2b^3e^3n^3} + \frac{(d+ex)(f+gx)(ef-dg)}{b^2e^2n^2\,(a+b\log(c(d+ex)^n))} - \frac{3(d+ex)(f+gx)^2}{2b^2en^2\,(a+b\log(c(d+ex)^n))} - \frac{(d+ex)(f+gx)^2}{2ben\,(a+b\log(c(d+ex)^n))^2}$$

```
[In] Int[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] ((e*f - d*g)^2*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(
2*b^3*e^3*E^(a/(b*n))*n^3*(c*(d + e*x)^n)^n^(-1)) + (4*g*(e*f - d*g)*(d + e
*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b^3*e^3*E^((2*a
)/(b*n))*n^3*(c*(d + e*x)^n)^(2/n)) + (9*g^2*(d + e*x)^3*ExpIntegralEi[(3*(
a + b*Log[c*(d + e*x)^n]))/(b*n)])/(2*b^3*e^3*E^((3*a)/(b*n))*n^3*(c*(d + e
*x)^n)^(3/n)) - ((d + e*x)*(f + g*x)^2)/(2*b*e*n*(a + b*Log[c*(d + e*x)^n])
^2) + ((e*f - d*g)*(d + e*x)*(f + g*x))/(b^2*e^2*n^2*(a + b*Log[c*(d + e*x)
^n])) - (3*(d + e*x)*(f + g*x)^2)/(2*b^2*e*n^2*(a + b*Log[c*(d + e*x)^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rubi steps

$$\text{integral} = -\frac{(d+ex)(f+gx)^2}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}+\frac{3\int\frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^2}\,dx}{2bn}$$
$$-\frac{(ef-dg)\int\frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx}{ben}$$

$$= -\frac{(d+ex)(f+gx)^2}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} + \frac{(ef-dg)(d+ex)(f+gx)}{b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{3(d+ex)(f+gx)^2}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{9\int \frac{(f+gx)^2}{a+b\log(c(d+ex)^n)}\,dx}{2b^2n^2} - \frac{(2(ef-dg))\int \frac{f+gx}{a+b\log(c(d+ex)^n)}\,dx}{b^2en^2} - \frac{(3(ef-dg))\int \frac{f+gx}{a+b\log(c(d+ex)^n)}\,dx}{b^2en^2} + \frac{(ef-dg)^2\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx}{b^2e^2n^2}$$

$$= -\frac{(d+ex)(f+gx)^2}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} + \frac{(ef-dg)(d+ex)(f+gx)}{b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{3(d+ex)(f+gx)^2}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{9\int \left(\frac{(ef-dg)^2}{e^2(a+b\log(c(d+ex)^n))} + \frac{2g(ef-dg)(d+ex)}{e^2(a+b\log(c(d+ex)^n))} + \frac{g^2(d+ex)^2}{e^2(a+b\log(c(d+ex)^n))}\right)dx}{2b^2n^2} - \frac{(2(ef-dg))\int \left(\frac{ef-dg}{e(a+b\log(c(d+ex)^n))} + \frac{g(d+ex)}{e(a+b\log(c(d+ex)^n))}\right)dx}{b^2en^2} - \frac{(3(ef-dg))\int \left(\frac{ef-dg}{e(a+b\log(c(d+ex)^n))} + \frac{g(d+ex)}{e(a+b\log(c(d+ex)^n))}\right)dx}{b^2en^2} + \frac{(ef-dg)^2\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx, x, d+ex\right)}{b^2e^3n^2}$$

$$= -\frac{(d+ex)(f+gx)^2}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} + \frac{(ef-dg)(d+ex)(f+gx)}{b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{3(d+ex)(f+gx)^2}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{(9g^2)\int \frac{(d+ex)^2}{a+b\log(c(d+ex)^n)}\,dx}{2b^2e^2n^2} - \frac{(2g(ef-dg))\int \frac{d+ex}{a+b\log(c(d+ex)^n)}\,dx}{b^2e^2n^2} - \frac{(3g(ef-dg))\int \frac{d+ex}{a+b\log(c(d+ex)^n)}\,dx}{b^2e^2n^2} + \frac{(9g(ef-dg))\int \frac{d+ex}{a+b\log(c(d+ex)^n)}\,dx}{b^2e^2n^2} - \frac{(2(ef-dg)^2)\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx}{b^2e^2n^2} - \frac{(3(ef-dg)^2)\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx}{b^2e^2n^2} + \frac{(9(ef-dg)^2)\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx}{2b^2e^2n^2} + \frac{\left((ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{b^2e^3n^3}$$

$$= \frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^3e^3n^3} - \frac{(d+ex)(f+gx)^2}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} + \frac{(ef-dg)(d+ex)(f+gx)}{b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{3(d+ex)(f+gx)^2}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{(9g^2)\operatorname{Subst}\left(\int \frac{x^2}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{2b^2e^3n^2} - \frac{(2g(ef-dg))\operatorname{Subst}\left(\int \frac{x}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{b^2e^3n^2} - \frac{(3g(ef-dg))\operatorname{Subst}\left(\int \frac{x}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{b^2e^3n^2} + \frac{(9g(ef-dg))\operatorname{Subst}\left(\int \frac{x}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{b^2e^3n^2} - \frac{(2(ef-dg)^2)\operatorname{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{b^2e^3n^2} - \frac{(3(ef-dg)^2)\operatorname{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{b^2e^3n^2} + \frac{(9(ef-dg)^2)\operatorname{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{2b^2e^3n^2}$$

$$= \frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^3e^3n^3} - \frac{(d+ex)(f+gx)^2}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} + \frac{(ef-dg)(d+ex)(f+gx)}{b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{3(d+ex)(f+gx)^2}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{\left(9g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{a+bx}dx,x,\log\left(c(d+ex)^n\right)\right)}{2b^2e^3n^3} - \frac{\left(2g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{a+bx}dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^3n^3} - \frac{\left(3g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{a+bx}dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^3n^3} + \frac{\left(9g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{a+bx}dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^3n^3} - \frac{\left(2(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{a+bx}dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^3n^3} - \frac{\left(3(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{a+bx}dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^3n^3} + \frac{\left(9(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{a+bx}dx,x,\log\left(c(d+ex)^n\right)\right)}{2b^2e^3n^3}$$

$$= \frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3e^3n^3} + \frac{4e^{-\frac{2a}{bn}}g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{Ei}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^3e^3n^3} + \frac{9e^{-\frac{3a}{bn}}g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\text{Ei}\left(\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)}{2b^3e^3n^3} - \frac{(d+ex)(f+gx)^2}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} + \frac{(ef-dg)(d+ex)(f+gx)}{b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{3(d+ex)(f+gx)^2}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx$$
$$= \frac{e^{-\frac{3a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-3/n}\left(e^{\frac{2a}{bn}}(ef-dg)^2\left(c(d+ex)^n\right)^{2/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c\right.}{}$$

```
[In] Integrate[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] ((d + e*x)*(E^((2*a)/(b*n))*(e*f - d*g)^2*(c*(d + e*x)^n)^(2/n)*ExpIntegral
Ei[(a + b*Log[c*(d + e*x)^n])/(b*n)]*(a + b*Log[c*(d + e*x)^n])^2 - 8*E^(a/
(b*n))*g*(-(e*f) + d*g)*(d + e*x)*(c*(d + e*x)^n)^n^(-1)*ExpIntegralEi[(2*(
a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log[c*(d + e*x)^n])^2 + 9*g^2*(d +
 e*x)^2*ExpIntegralEi[(3*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log[c*(d
 + e*x)^n])^2 - b*e*E^((3*a)/(b*n))*n*(c*(d + e*x)^n)^(3/n)*(f + g*x)*(b*e*
n*(f + g*x) + a*(e*f + 2*d*g + 3*e*g*x) + b*(2*d*g + e*(f + 3*g*x))*Log[c*(
d + e*x)^n])))/(2*b^3*e^3*E^((3*a)/(b*n))*n^3*(c*(d + e*x)^n)^(3/n)*(a + b*
Log[c*(d + e*x)^n])^2)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.73 (sec) , antiderivative size = 6545, normalized size of antiderivative = 18.65

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 6545 |

```
[In] int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1090 vs. 2(344) = 688.

Time = 0.31 (sec) , antiderivative size = 1090, normalized size of antiderivative = 3.11

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(8*(a^2*e*f*g - a^2*d*g^2 + (b^2*e*f*g - b^2*d*g^2)*n^2*log(e*x + d)^2
+ (b^2*e*f*g - b^2*d*g^2)*log(c)^2 + 2*((b^2*e*f*g - b^2*d*g^2)*n*log(c) +
(a*b*e*f*g - a*b*d*g^2)*n)*log(e*x + d) + 2*(a*b*e*f*g - a*b*d*g^2)*log(c))
*e^((b*log(c) + a)/(b*n))*log_integral((e^2*x^2 + 2*d*e*x + d^2)*e^(2*(b*lo
g(c) + a)/(b*n))) + (a^2*e^2*f^2 - 2*a^2*d*e*f*g + a^2*d^2*g^2 + (b^2*e^2*f
^2 - 2*b^2*d*e*f*g + b^2*d^2*g^2)*n^2*log(e*x + d)^2 + (b^2*e^2*f^2 - 2*b^2
*d*e*f*g + b^2*d^2*g^2)*log(c)^2 + 2*((b^2*e^2*f^2 - 2*b^2*d*e*f*g + b^2*d^
2*g^2)*n*log(c) + (a*b*e^2*f^2 - 2*a*b*d*e*f*g + a*b*d^2*g^2)*n)*log(e*x +
d) + 2*(a*b*e^2*f^2 - 2*a*b*d*e*f*g + a*b*d^2*g^2)*log(c))*e^(2*(b*log(c) +
 a)/(b*n))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n))) - (b^2*d*e^2*f^
2*n^2 + (b^2*e^3*g^2*n^2 + 3*a*b*e^3*g^2*n)*x^3 + ((2*b^2*e^3*f*g + b^2*d*e
^2*g^2)*n^2 + (4*a*b*e^3*f*g + 5*a*b*d*e^2*g^2)*n)*x^2 + (a*b*d*e^2*f^2 + 2
*a*b*d^2*e*f*g)*n + ((b^2*e^3*f^2 + 2*b^2*d*e^2*f*g)*n^2 + (a*b*e^3*f^2 + 6
*a*b*d*e^2*f*g + 2*a*b*d^2*e*g^2)*n)*x + (3*b^2*e^3*g^2*n^2*x^3 + (4*b^2*e^
3*f*g + 5*b^2*d*e^2*g^2)*n^2*x^2 + (b^2*e^3*f^2 + 6*b^2*d*e^2*f*g + 2*b^2*d
^2*e*g^2)*n^2*x + (b^2*d*e^2*f^2 + 2*b^2*d^2*e*f*g)*n^2)*log(e*x + d) + (3*
b^2*e^3*g^2*n*x^3 + (4*b^2*e^3*f*g + 5*b^2*d*e^2*g^2)*n*x^2 + (b^2*e^3*f^2
+ 6*b^2*d*e^2*f*g + 2*b^2*d^2*e*g^2)*n*x + (b^2*d*e^2*f^2 + 2*b^2*d^2*e*f*g
)*n)*log(c))*e^(3*(b*log(c) + a)/(b*n)) + 9*(b^2*g^2*n^2*log(e*x + d)^2 + b
^2*g^2*log(c)^2 + 2*a*b*g^2*log(c) + a^2*g^2 + 2*(b^2*g^2*n*log(c) + a*b*g^
2*n)*log(e*x + d))*log_integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*e
^(3*(b*log(c) + a)/(b*n))))*e^(-3*(b*log(c) + a)/(b*n))/(b^5*e^3*n^5*log(e*
x + d)^2 + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3
+ 2*(b^5*e^3*n^4*log(c) + a*b^4*e^3*n^4)*log(e*x + d))
```

## Sympy [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx$$

```
[In] integrate((g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**3,x)
[Out] Integral((f + g*x)**2/(a + b*log(c*(d + e*x)**n))**3, x)
```

## Maxima [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{(gx+f)^2}{(b\log((ex+d)^n c)+a)^3}\,dx$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
[Out] -1/2*((3*a*e^2*g^2 + (e^2*g^2*n + 3*e^2*g^2*log(c))*b)*x^3 + ((4*e^2*f*g +
5*d*e*g^2)*a + (2*e^2*f*g*n + d*e*g^2*n + (4*e^2*f*g + 5*d*e*g^2)*log(c))*b
```

```
)*x^2 + (d*e*f^2 + 2*d^2*f*g)*a + (d*e*f^2*n + (d*e*f^2 + 2*d^2*f*g)*log(c)
)*b + ((e^2*f^2 + 6*d*e*f*g + 2*d^2*g^2)*a + (e^2*f^2*n + 2*d*e*f*g*n + (e^
2*f^2 + 6*d*e*f*g + 2*d^2*g^2)*log(c))*b)*x + (3*b*e^2*g^2*x^3 + (4*e^2*f*g
 + 5*d*e*g^2)*b*x^2 + (e^2*f^2 + 6*d*e*f*g + 2*d^2*g^2)*b*x + (d*e*f^2 + 2*
d^2*f*g)*b)*log((e*x + d)^n))/(b^4*e^2*n^2*log((e*x + d)^n)^2 + b^4*e^2*n^2
*log(c)^2 + 2*a*b^3*e^2*n^2*log(c) + a^2*b^2*e^2*n^2 + 2*(b^4*e^2*n^2*log(c
) + a*b^3*e^2*n^2)*log((e*x + d)^n)) + integrate(1/2*(9*e^2*g^2*x^2 + e^2*f
^2 + 6*d*e*f*g + 2*d^2*g^2 + 2*(4*e^2*f*g + 5*d*e*g^2)*x)/(b^3*e^2*n^2*log(
(e*x + d)^n) + b^3*e^2*n^2*log(c) + a*b^2*e^2*n^2), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8422 vs. $2(344) = 688$.

Time = 0.49 (sec) , antiderivative size = 8422, normalized size of antiderivative = 23.99

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*e^2*f^2*n^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(
e*x + d)^2/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c)
 + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*lo
g(c) + a^2*b^3*e^3*n^3)*c^(1/n)) - b^2*d*e*f*g*n^2*Ei(log(c)/n + a/(b*n) +
log(e*x + d))*e^(-a/(b*n))*log(e*x + d)^2/((b^5*e^3*n^5*log(e*x + d)^2 + 2*
b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^
3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) + 1/2*b^2*d
^2*g^2*n^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)^
2/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^
4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^
2*b^3*e^3*n^3)*c^(1/n)) - 1/2*(e*x + d)*b^2*e^2*f^2*n^2*log(e*x + d)/(b^5*e
^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4
*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3
*n^3) - 2*(e*x + d)^2*b^2*e*f*g*n^2*log(e*x + d)/(b^5*e^3*n^5*log(e*x + d)^
2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*
e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) + (e*x + d)*b^
2*d*e*f*g*n^2*log(e*x + d)/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(
e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a
*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) - 3/2*(e*x + d)^3*b^2*g^2*n^2*log(e*
x + d)/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*
a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c)
+ a^2*b^3*e^3*n^3) + 2*(e*x + d)^2*b^2*d*g^2*n^2*log(e*x + d)/(b^5*e^3*n^5*
log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*
x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) -
```

1/2*(e*x + d)*b^2*d^2*g^2*n^2*log(e*x + d)/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) + 4*b^2*e*f*g*n^2*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(e*x + d)^2/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(2/n)) - 4*b^2*d*g^2*n^2*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(e*x + d)^2/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(2/n)) + b^2*e^2*f^2*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)*log(c)/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) - 2*b^2*d*e*f*g*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)*log(c)/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) + b^2*d^2*g^2*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)*log(c)/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) - 1/2*(e*x + d)*b^2*e^2*f^2*n^2/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) - (e*x + d)^2*b^2*e*f*g*n^2/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) + (e*x + d)*b^2*d*e*f*g*n^2/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) - 1/2*(e*x + d)^3*b^2*g^2*n^2/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) + (e*x + d)^2*b^2*d*g^2*n^2/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) - 1/2*(e*x + d)*b^2*d^2*g^2*n^2/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) + a*b*e^2*f^2*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) - 2*a*b*d*e*f*g*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) + a*b*d^2*g^2*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b

^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) + 9/2*b^2*g^2*n^2*Ei(3*log(c)
/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))*log(e*x + d)^2/((b^5*e^3*n^
5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(
e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)
*c^(3/n)) - 1/2*(e*x + d)*b^2*e^2*f^2*n*log(c)/(b^5*e^3*n^5*log(e*x + d)^2
+ 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^
3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) - 2*(e*x + d)^2*
b^2*e*f*g*n*log(c)/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)
*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3
*n^3*log(c) + a^2*b^3*e^3*n^3) + (e*x + d)*b^2*d*e*f*g*n*log(c)/(b^5*e^3*n^
5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(
e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)
 - 3/2*(e*x + d)^3*b^2*g^2*n*log(c)/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3
*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c
)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) + 2*(e*x + d)^2*b^2*d*g^2*n
*log(c)/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2
*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c)
 + a^2*b^3*e^3*n^3) - 1/2*(e*x + d)*b^2*d^2*g^2*n*log(c)/(b^5*e^3*n^5*log(e
*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d
) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) + 8*b^
2*e*f*g*n*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(e*
x + d)*log(c)/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log
(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3
*log(c) + a^2*b^3*e^3*n^3)*c^(2/n)) - 8*b^2*d*g^2*n*Ei(2*log(c)/n + 2*a/(b*
n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(e*x + d)*log(c)/((b^5*e^3*n^5*log(e
*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d
) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(2/n
)) + 1/2*b^2*e^2*f^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log
(c)^2/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*
a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c)
+ a^2*b^3*e^3*n^3)*c^(1/n)) - b^2*d*e*f*g*Ei(log(c)/n + a/(b*n) + log(e*x +
 d))*e^(-a/(b*n))*log(c)^2/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log
(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*
a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) + 1/2*b^2*d^2*g^2*Ei(log(c
)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)^2/((b^5*e^3*n^5*log(e*x +
 d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) +
b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) -
 1/2*(e*x + d)*a*b*e^2*f^2*n/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*lo
g(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2
*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) - 2*(e*x + d)^2*a*b*e*f*g*n/(b^5*e
^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4
*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3
*n^3) + (e*x + d)*a*b*d*e*f*g*n/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4
*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2
+ 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) - 3/2*(e*x + d)^3*a*b*g^2*n/(b^

5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*
n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*
e^3*n^3) + 2*(e*x + d)^2*a*b*d*g^2*n/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^
3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(
c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3) - 1/2*(e*x + d)*a*b*d^2*g^
2*n/(b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b
^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a
^2*b^3*e^3*n^3) + 8*a*b*e*f*g*n*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))
*e^(-2*a/(b*n))*log(e*x + d)/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*l
og(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 +
2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(2/n)) - 8*a*b*d*g^2*n*Ei(2*log
(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(e*x + d)/((b^5*e^3*n
^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log
(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3
)*c^(2/n)) + a*b*e^2*f^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))
*log(c)/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) +
2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c
) + a^2*b^3*e^3*n^3)*c^(1/n)) - 2*a*b*d*e*f*g*Ei(log(c)/n + a/(b*n) + log(e
*x + d))*e^(-a/(b*n))*log(c)/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*l
og(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 +
2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) + a*b*d^2*g^2*Ei(log(c)/
n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)/((b^5*e^3*n^5*log(e*x + d)^
2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*
e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) + 9*b
^2*g^2*n*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))*log(e*x
 + d)*log(c)/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(
c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*
log(c) + a^2*b^3*e^3*n^3)*c^(3/n)) + 4*b^2*e*f*g*Ei(2*log(c)/n + 2*a/(b*n)
+ 2*log(e*x + d))*e^(-2*a/(b*n))*log(c)^2/((b^5*e^3*n^5*log(e*x + d)^2 + 2*
b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^
3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(2/n)) - 4*b^2*d*g
^2*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(c)^2/((b^
5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*
n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*
e^3*n^3)*c^(2/n)) + 1/2*a^2*e^2*f^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e
^(-a/(b*n))/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c
) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*l
og(c) + a^2*b^3*e^3*n^3)*c^(1/n)) - a^2*d*e*f*g*Ei(log(c)/n + a/(b*n) + log
(e*x + d))*e^(-a/(b*n))/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*
x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b
^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) + 1/2*a^2*d^2*g^2*Ei(log(c)/n
 + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^
5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*
log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(1/n)) + 9*a*b*g^2*n
*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))*log(e*x + d)/((

```
b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^
3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^
3*e^3*n^3)*c^(3/n)) + 8*a*b*e*f*g*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d
))*e^(-2*a/(b*n))*log(c)/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e
*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*
b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(2/n)) - 8*a*b*d*g^2*Ei(2*log(c)/n
+ 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(c)/((b^5*e^3*n^5*log(e*x +
 d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) +
b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(2/n)) +
 9/2*b^2*g^2*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))*log
(c)^2/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*
a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c)
+ a^2*b^3*e^3*n^3)*c^(3/n)) + 4*a^2*e*f*g*Ei(2*log(c)/n + 2*a/(b*n) + 2*log
(e*x + d))*e^(-2*a/(b*n))/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(
e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a
*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(2/n)) - 4*a^2*d*g^2*Ei(2*log(c)/n
 + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))/((b^5*e^3*n^5*log(e*x + d)^2
+ 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^
3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*e^3*n^3)*c^(2/n)) + 9*a*b
*g^2*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x + d))*e^(-3*a/(b*n))*log(c)/((b^
5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^3*
n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n^3*log(c) + a^2*b^3*
e^3*n^3)*c^(3/n)) + 9/2*a^2*g^2*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(e*x + d))
*e^(-3*a/(b*n))/((b^5*e^3*n^5*log(e*x + d)^2 + 2*b^5*e^3*n^4*log(e*x + d)*l
og(c) + 2*a*b^4*e^3*n^4*log(e*x + d) + b^5*e^3*n^3*log(c)^2 + 2*a*b^4*e^3*n
^3*log(c) + a^2*b^3*e^3*n^3)*c^(3/n))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{(f+g\,x)^2}{(a+b\,\ln(c\,(d+e\,x)^n))^3}\,dx$$

```
[In] int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^3,x)
[Out] int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^3, x)
```

# 3.101 $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 261

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx = \frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3e^2n^3} + \frac{2e^{-\frac{2a}{bn}}g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^3e^2n^3} - \frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} + \frac{(ef-dg)(d+ex)}{2b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{(d+ex)(f+gx)}{b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

```
[Out] 1/2*(-d*g+e*f)*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^3/e^2/exp(a/b/n)/n^3
/((c*(e*x+d)^n)^(1/n))+2*g*(e*x+d)^2*Ei(2*(a+b*ln(c*(e*x+d)^n))/b/n)/b^3/e^
2/exp(2*a/b/n)/n^3/((c*(e*x+d)^n)^(2/n))-1/2*(e*x+d)*(g*x+f)/b/e/n/(a+b*ln(
c*(e*x+d)^n))^2+1/2*(-d*g+e*f)*(e*x+d)/b^2/e^2/n^2/(a+b*ln(c*(e*x+d)^n))-(e
*x+d)*(g*x+f)/b^2/e/n^2/(a+b*ln(c*(e*x+d)^n))
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2447, 2446, 2436, 2337, 2209, 2437, 2347, 2334}

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3e^2n^3} + \frac{2ge^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^3e^2n^3} + \frac{(d+ex)(ef-dg)}{2b^2e^2n^2(a+b\log(c(d+ex)^n))} - \frac{(d+ex)(f+gx)}{b^2en^2(a+b\log(c(d+ex)^n))} - \frac{(d+ex)(f+gx)}{2ben(a+b\log(c(d+ex)^n))^2}$$

```
[In] Int[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] ((e*f - d*g)*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(2*
b^3*e^2*E^(a/(b*n))*n^3*(c*(d + e*x)^n)^n^(-1)) + (2*g*(d + e*x)^2*ExpInteg
ralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)])/(b^3*e^2*E^((2*a)/(b*n))*n^3*(
c*(d + e*x)^n)^(2/n)) - ((d + e*x)*(f + g*x))/(2*b*e*n*(a + b*Log[c*(d + e*
x)^n])^2) + ((e*f - d*g)*(d + e*x))/(2*b^2*e^2*n^2*(a + b*Log[c*(d + e*x)^n
])) - ((d + e*x)*(f + g*x))/(b^2*e*n^2*(a + b*Log[c*(d + e*x)^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rubi steps

$$\text{integral} = -\frac{(d+ex)(f+gx)}{2ben\,(a+b\log(c(d+ex)^n))^2} + \frac{\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^2}\,dx}{bn} - \frac{(ef-dg)\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx}{2ben}$$

$$= -\frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} - \frac{(d+ex)(f+gx)}{b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{2\int \frac{f+gx}{a+b\log(c(d+ex)^n)}\,dx}{b^2n^2} - \frac{(ef-dg)\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx}{b^2en^2} - \frac{(ef-dg)\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^2}\,dx, x, d+ex\right)}{2be^2n}$$

$$= -\frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} + \frac{(ef-dg)(d+ex)}{2b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{(d+ex)(f+gx)}{b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{2\int \left(\frac{ef-dg}{e(a+b\log(c(d+ex)^n))} + \frac{g(d+ex)}{e(a+b\log(c(d+ex)^n))}\right)dx}{b^2n^2} - \frac{(ef-dg)\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx, x, d+ex\right)}{2b^2e^2n^2} - \frac{(ef-dg)\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx, x, d+ex\right)}{b^2e^2n^2}$$

$$= -\frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} + \frac{(ef-dg)(d+ex)}{2b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{(d+ex)(f+gx)}{b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{(2g)\int \frac{d+ex}{a+b\log(c(d+ex)^n)}\,dx}{b^2en^2} + \frac{(2(ef-dg))\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx}{b^2en^2} - \frac{\left((ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{2b^2e^2n^3} - \frac{\left((ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{b^2e^2n^3}$$

$$= -\frac{3e^{-\frac{a}{bn}}(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3e^2n^3} - \frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} + \frac{(ef-dg)(d+ex)}{2b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{(d+ex)(f+gx)}{b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{(2g)\text{Subst}\left(\int \frac{x}{a+b\log(cx^n)}\,dx, x, d+ex\right)}{b^2e^2n^2} + \frac{(2(ef-dg))\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx, x, d+ex\right)}{b^2e^2n^2}$$

$$= -\frac{3e^{-\frac{a}{bn}}(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3e^2n^3} - \frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} + \frac{(ef-dg)(d+ex)}{2b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{(d+ex)(f+gx)}{b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{\left(2g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{b^2e^2n^3} + \frac{\left(2(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{b^2e^2n^3}$$

$$= \frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3e^2n^3} + \frac{2e^{-\frac{2a}{bn}}g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{Ei}\left(\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)}{b^3e^2n^3} - \frac{(d+ex)(f+gx)}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} + \frac{(ef-dg)(d+ex)}{2b^2e^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)} - \frac{(d+ex)(f+gx)}{b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.98

$$\int\frac{f+gx}{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx = -\frac{e^{-\frac{2a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(-e^{\frac{a}{bn}}(ef-dg)\left(c(d+ex)^n\right)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log$$

```
[In] Integrate[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] -1/2*((d + e*x)*(-(E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*ExpIntegr
alEi[(a + b*Log[c*(d + e*x)^n])/(b*n)]*(a + b*Log[c*(d + e*x)^n])^2) - 4*g*
(d + e*x)*ExpIntegralEi[(2*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log[c*
(d + e*x)^n])^2 + b*E^((2*a)/(b*n))*n*(c*(d + e*x)^n)^(2/n)*(b*e*n*(f + g*x
) + a*(e*f + d*g + 2*e*g*x) + b*(d*g + e*(f + 2*g*x))*Log[c*(d + e*x)^n]))
/(b^3*e^2*E^((2*a)/(b*n))*n^3*(c*(d + e*x)^n)^(2/n)*(a + b*Log[c*(d + e*x)^
n])^2)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.25 (sec) , antiderivative size = 3114, normalized size of antiderivative = 11.93

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 3114 |

```
[In] int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(2*ln(c)*b*d^2*g-I*Pi*b*d^2*g*csgn(I*c*(e*x+d)^n)^3+6*b*d*e*g*x*ln((e*x+d)
^n)-3*I*Pi*b*d*e*g*x*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+6*ln(c
)*b*d*e*g*x+2*b*e^2*g*n*x^2+2*b*e^2*f*n*x+2*b*e^2*f*x*ln((e*x+d)^n)+2*b*d*e
*f*ln((e*x+d)^n)+I*Pi*b*d^2*g*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b*d^2*g*
csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2+2*ln(c)*b*d*e*f-2*I*Pi*b*e^2*g*x^2*
csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+3*I*Pi*b*d*e*g*x*csgn(I*c)*
csgn(I*c*(e*x+d)^n)^2+2*b*d*e*g*n*x+4*b*e^2*g*x^2*ln((e*x+d)^n)+4*ln(c)*b*e
^2*g*x^2+2*ln(c)*b*e^2*f*x+6*a*d*e*g*x+3*I*Pi*b*d*e*g*x*csgn(I*(e*x+d)^n)*c
sgn(I*c*(e*x+d)^n)^2-I*Pi*b*e^2*f*x*csgn(I*c*(e*x+d)^n)^3-2*I*Pi*b*e^2*g*x^
2*csgn(I*c*(e*x+d)^n)^3+2*a*d^2*g+4*a*e^2*g*x^2+2*a*e^2*f*x+2*b*d^2*g*ln((e
*x+d)^n)-I*Pi*b*d*e*f*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+2*I*P
i*b*e^2*g*x^2*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2-I*Pi*b*d^2*g*csgn(I*c)*csgn(I
*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+2*I*Pi*b*e^2*g*x^2*csgn(I*(e*x+d)^n)*csgn(I
*c*(e*x+d)^n)^2-3*I*Pi*b*d*e*g*x*csgn(I*c*(e*x+d)^n)^3-I*Pi*b*d*e*f*csgn(I*
c*(e*x+d)^n)^3+2*b*d*e*f*n-I*Pi*b*e^2*f*x*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(
I*c*(e*x+d)^n)+2*a*d*e*f+I*Pi*b*d*e*f*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)
^2+I*Pi*b*e^2*f*x*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b*d*e*f*csgn(I*c)*cs
gn(I*c*(e*x+d)^n)^2+I*Pi*b*e^2*f*x*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2)
/b^2/e^2/n^2/(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*
csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n
)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln((e*x+d)^n)+2*b*ln(c)+2*a)^2-1/2/b
^3/n^3*f*c^(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*
csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csg
n(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/
n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d
)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c
*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n
*ln(e*x+d))+2*a)/b/n)*x-1/2/e/b^3/n^3*f*c^(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1/
2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*c
sgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*
csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*
x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+
I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b
+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)*d-2/b^3/n^3*g*c^(-2/n)
*((e*x+d)^n)^(-2/n)*exp(-(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x
```

```
+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I
*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-2*ln(e*x+d)-
(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csg
n(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*cs
gn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)*x
^2-4/e/b^3/n^3*g*c^(-2/n)*((e*x+d)^n)^(-2/n)*exp(-(-I*b*Pi*csgn(I*c*(e*x+d)
^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi
*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a
)/b/n)*Ei(1,-2*ln(e*x+d)-(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x
+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I
*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)
-n*ln(e*x+d))+2*a)/b/n)*d*x-2/e^2/b^3/n^3*g*c^(-2/n)*((e*x+d)^n)^(-2/n)*exp
(-(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*c
sgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*
csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-2*ln(e*x+d)-(-I*b*Pi*csgn(I*c*(e*x+
d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*
Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2
*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)*d^2+1/2/e/b^3/n^3*d*g*c^
(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*c
sgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)
^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln
(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*c
sgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)
^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))
+2*a)/b/n)*x+1/2/e^2/b^3/n^3*d^2*g*c^(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1/2*(-I
*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I
*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(
I*c*(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^
n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*
csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*
ln(c)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(255) = 510$.

Time = 0.32 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.25

$$\int \frac{f + gx}{(a + b\log{(c(d + ex)^n)})^3}\,dx$$
$$= \frac{\left(((b^2ef - b^2dg)n^2\log{(ex+d)}^2 + a^2ef - a^2dg + (b^2ef - b^2dg)\log{(c)}^2 + 2\,((b^2ef - b^2dg)n\log{(c)} + (ab\right.}{}$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(((b^2*e*f - b^2*d*g)*n^2*log(e*x + d)^2 + a^2*e*f - a^2*d*g + (b^2*e*f
 - b^2*d*g)*log(c)^2 + 2*((b^2*e*f - b^2*d*g)*n*log(c) + (a*b*e*f - a*b*d*g
)*n)*log(e*x + d) + 2*(a*b*e*f - a*b*d*g)*log(c))*e^((b*log(c) + a)/(b*n))*
log_integral((e*x + d)*e^((b*log(c) + a)/(b*n))) - (b^2*d*e*f*n^2 + (b^2*e^
2*g*n^2 + 2*a*b*e^2*g*n)*x^2 + (a*b*d*e*f + a*b*d^2*g)*n + ((b^2*e^2*f + b^
2*d*e*g)*n^2 + (a*b*e^2*f + 3*a*b*d*e*g)*n)*x + (2*b^2*e^2*g*n^2*x^2 + (b^2
*e^2*f + 3*b^2*d*e*g)*n^2*x + (b^2*d*e*f + b^2*d^2*g)*n^2)*log(e*x + d) + (
2*b^2*e^2*g*n*x^2 + (b^2*e^2*f + 3*b^2*d*e*g)*n*x + (b^2*d*e*f + b^2*d^2*g)
*n)*log(c))*e^(2*(b*log(c) + a)/(b*n)) + 4*(b^2*g*n^2*log(e*x + d)^2 + b^2*
g*log(c)^2 + 2*a*b*g*log(c) + a^2*g + 2*(b^2*g*n*log(c) + a*b*g*n)*log(e*x
+ d))*log_integral((e^2*x^2 + 2*d*e*x + d^2)*e^(2*(b*log(c) + a)/(b*n))))*e
^(-2*(b*log(c) + a)/(b*n))/(b^5*e^2*n^5*log(e*x + d)^2 + b^5*e^2*n^3*log(c)
^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3 + 2*(b^5*e^2*n^4*log(c) + a*b
^4*e^2*n^4)*log(e*x + d))
```

## Sympy [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx$$

```
[In] integrate((g*x+f)/(a+b*ln(c*(e*x+d)**n))**3,x)
```

```
[Out] Integral((f + g*x)/(a + b*log(c*(d + e*x)**n))**3, x)
```

## Maxima [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{gx+f}{(b\log((ex+d)^n c)+a)^3}\,dx$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*e^2*g + (e^2*g*n + 2*e^2*g*log(c))*b)*x^2 + (d*e*f + d^2*g)*a +
(d*e*f*n + (d*e*f + d^2*g)*log(c))*b + ((e^2*f + 3*d*e*g)*a + (e^2*f*n + d*
e*g*n + (e^2*f + 3*d*e*g)*log(c))*b)*x + (2*b*e^2*g*x^2 + (e^2*f + 3*d*e*g)
*b*x + (d*e*f + d^2*g)*b)*log((e*x + d)^n))/(b^4*e^2*n^2*log((e*x + d)^n)^2
 + b^4*e^2*n^2*log(c)^2 + 2*a*b^3*e^2*n^2*log(c) + a^2*b^2*e^2*n^2 + 2*(b^4
*e^2*n^2*log(c) + a*b^3*e^2*n^2)*log((e*x + d)^n)) + integrate(1/2*(4*e*g*x
 + e*f + 3*d*g)/(b^3*e*n^2*log((e*x + d)^n) + b^3*e*n^2*log(c) + a*b^2*e*n^
2), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4112 vs. 2(255) = 510.

Time = 0.43 (sec) , antiderivative size = 4112, normalized size of antiderivative = 15.75

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*e*f*n^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x
+ d)^2/((b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2
*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c)
 + a^2*b^3*e^2*n^3)*c^(1/n)) - 1/2*b^2*d*g*n^2*Ei(log(c)/n + a/(b*n) + log(
e*x + d))*e^(-a/(b*n))*log(e*x + d)^2/((b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*
e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*lo
g(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3)*c^(1/n)) - 1/2*(e*x + d)
*b^2*e*f*n^2*log(e*x + d)/(b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e
*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*
b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3) - (e*x + d)^2*b^2*g*n^2*log(e*x + d)/
(b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e
^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b
^3*e^2*n^3) + 1/2*(e*x + d)*b^2*d*g*n^2*log(e*x + d)/(b^5*e^2*n^5*log(e*x +
 d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) +
b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3) + 2*b^2*g*
n^2*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(e*x + d)
^2/((b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b
^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a
^2*b^3*e^2*n^3)*c^(2/n)) + b^2*e*f*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*
e^(-a/(b*n))*log(e*x + d)*log(c)/((b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n
^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^
2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3)*c^(1/n)) - b^2*d*g*n*Ei(log(c
)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)*log(c)/((b^5*e^2*n^
5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(
e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3)
*c^(1/n)) - 1/2*(e*x + d)*b^2*e*f*n^2/(b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e
^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log
(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3) - 1/2*(e*x + d)^2*b^2*g*n
^2/(b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^
4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^
2*b^3*e^2*n^3) + 1/2*(e*x + d)*b^2*d*g*n^2/(b^5*e^2*n^5*log(e*x + d)^2 + 2*
b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^
3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3) + a*b*e*f*n*Ei(log(c
)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)/((b^5*e^2*n^5*log(e
*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d
```

) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3)*c^(1/n
)) - a*b*d*g*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x +
d)/((b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*
b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) +
a^2*b^3*e^2*n^3)*c^(1/n)) - 1/2*(e*x + d)*b^2*e*f*n*log(c)/(b^5*e^2*n^5*log
(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x +
d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3) - (e
*x + d)^2*b^2*g*n*log(c)/(b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*
x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b
^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3) + 1/2*(e*x + d)*b^2*d*g*n*log(c)/(b^5*
e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^
4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^
2*n^3) + 4*b^2*g*n*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n
))*log(e*x + d)*log(c)/((b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x
+ d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^
4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3)*c^(2/n)) + 1/2*b^2*e*f*Ei(log(c)/n + a/
(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)^2/((b^5*e^2*n^5*log(e*x + d)^2 +
2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*
n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3)*c^(1/n)) - 1/2*b^2
*d*g*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)^2/((b^5*e^2*
n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*lo
g(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^
3)*c^(1/n)) - 1/2*(e*x + d)*a*b*e*f*n/(b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e
^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log
(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3) - (e*x + d)^2*a*b*g*n/(b^
5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*
n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*
e^2*n^3) + 1/2*(e*x + d)*a*b*d*g*n/(b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*
n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)
^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3) + 4*a*b*g*n*Ei(2*log(c)/n +
2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))*log(e*x + d)/((b^5*e^2*n^5*log(e
*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d
) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3)*c^(2/n
)) + a*b*e*f*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)/((b^
5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*
n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*
e^2*n^3)*c^(1/n)) - a*b*d*g*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*
n))*log(c)/((b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c)
+ 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*lo
g(c) + a^2*b^3*e^2*n^3)*c^(1/n)) + 2*b^2*g*Ei(2*log(c)/n + 2*a/(b*n) + 2*lo
g(e*x + d))*e^(-2*a/(b*n))*log(c)^2/((b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^
2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(
c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3)*c^(2/n)) + 1/2*a^2*e*f*Ei(
log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/((b^5*e^2*n^5*log(e*x + d)^
2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*

```
e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3)*c^(1/n)) - 1/2
*a^2*d*g*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/((b^5*e^2*n^5*l
og(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x
 + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3)*c^
(1/n)) + 4*a*b*g*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e*x + d))*e^(-2*a/(b*n))
*log(c)/((b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*x + d)*log(c) +
2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b^4*e^2*n^3*log(c
) + a^2*b^3*e^2*n^3)*c^(2/n)) + 2*a^2*g*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(e
*x + d))*e^(-2*a/(b*n))/((b^5*e^2*n^5*log(e*x + d)^2 + 2*b^5*e^2*n^4*log(e*
x + d)*log(c) + 2*a*b^4*e^2*n^4*log(e*x + d) + b^5*e^2*n^3*log(c)^2 + 2*a*b
^4*e^2*n^3*log(c) + a^2*b^3*e^2*n^3)*c^(2/n))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{f+g\,x}{\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^3}\,dx$$

```
[In] int((f + g*x)/(a + b*log(c*(d + e*x)^n))^3,x)
[Out] int((f + g*x)/(a + b*log(c*(d + e*x)^n))^3, x)
```

# 3.102 $\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3en^3} - \frac{d+ex}{2ben(a+b\log(c(d+ex)^n))^2} - \frac{d+ex}{2b^2en^2(a+b\log(c(d+ex)^n))}$$

```
[Out] 1/2*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^3/e/exp(a/b/n)/n^3/((c*(e*x+d)^
n)^(1/n))+1/2*(-e*x-d)/b/e/n/(a+b*ln(c*(e*x+d)^n))^2+1/2*(-e*x-d)/b^2/e/n^2
/(a+b*ln(c*(e*x+d)^n))
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2436, 2334, 2337, 2209}

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3en^3} - \frac{d+ex}{2b^2en^2(a+b\log(c(d+ex)^n))} - \frac{d+ex}{2ben(a+b\log(c(d+ex)^n))^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(-3),x]
```

```
[Out] ((d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(2*b^3*e*E^(a/(
b*n))*n^3*(c*(d + e*x)^n)^n^(-1)) - (d + e*x)/(2*b*e*n*(a + b*Log[c*(d + e*
x)^n])^2) - (d + e*x)/(2*b^2*e*n^2*(a + b*Log[c*(d + e*x)^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^3}\,dx, x, d+ex\right)}{e}\\
&= -\frac{d+ex}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}+\frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^2}\,dx, x, d+ex\right)}{2ben}\\
&= -\frac{d+ex}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}-\frac{d+ex}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx, x, d+ex\right)}{2b^2en^2}\\
&= -\frac{d+ex}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2}-\frac{d+ex}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\\
&\quad+\frac{\left((d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{2b^2en^3}
\end{aligned}$$

$$= \frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{2b^3en^3} - \frac{d+ex}{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2} - \frac{d+ex}{2b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx = -\frac{e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(-\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2+be^{\frac{a}{bn}}n(c(d}{2b^3en^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(-3),x]
```

```
[Out] -1/2*((d + e*x)*(-(ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)]*(a + b*L
og[c*(d + e*x)^n])^2) + b*E^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)*(a + b*n + b
*Log[c*(d + e*x)^n])))/(b^3*e*E^(a/(b*n))*n^3*(c*(d + e*x)^n)^n^(-1)*(a + b
*Log[c*(d + e*x)^n])^2)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 734, normalized size of antiderivative = 5.44

| method | result |
|---|---|
| risch | $-\frac{2benx+2bdn+i\pi bd\,\operatorname{csgn}(i(ex+d)^n)\operatorname{csgn}(ic(ex+d)^n)^2+i\pi bd\,\operatorname{csgn}(ic)\operatorname{csgn}(ic(ex+d)^n)^2-i\pi bd\,\operatorname{csgn}(ic)\operatorname{csgn}(i(ex+d)^n)\operatorname{csgn}(ic(ex+}{\left(-ib\pi\,\operatorname{csgn}(ic(ex+d)^n)\operatorname{csgn}(i}$ |

```
[In] int(1/(a+b*ln(c*(e*x+d)^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(2*b*e*n*x+2*b*d*n+I*Pi*b*d*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b
*d*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2-I*Pi*b*d*csgn(I*c)*csgn(I*(e*x+d)^n)*csg
n(I*c*(e*x+d)^n)-I*Pi*b*d*csgn(I*c*(e*x+d)^n)^3-I*Pi*b*e*x*csgn(I*c*(e*x+d)
^n)^3+I*Pi*b*e*x*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I*Pi*b*e*x*csgn(I*
c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*Pi*b*e*x*csgn(I*c)*csgn(I*c*(e*x
+d)^n)^2+2*ln(c)*b*e*x+2*b*e*x*ln((e*x+d)^n)+2*d*b*ln(c)+2*a*e*x+2*b*d*ln((
e*x+d)^n)+2*a*d)/(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I
*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+
d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln((e*x+d)^n)+2*b*ln(c)+2*a)^2/b
^2/n^2/e-1/2/b^3/n^3/e*(e*x+d)*c^(-1/n)*((e*x+d)^n)^(-1/n)*exp(-1/2*(-I*b*P
```

```
i*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(
e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*
(e*x+d)^n)^3*b+2*a)/b/n)*Ei(1,-ln(e*x+d)-1/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*c
sgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c
)+2*b*(ln((e*x+d)^n)-n*ln(e*x+d))+2*a)/b/n)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(128) = 256$.

Time = 0.31 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.95

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = -\frac{\left((b^2dn^2+abdn+(b^2en^2+aben)x+(b^2en^2x+b^2dn^2)\log(ex+d)+(b^2enx+b^2dn)\log(c))e^{\left(\frac{b\log(c)+a}{bn}\right)} \right.}{2\left(b^5en^5\log(ex+d)^2+b^5en^3\log(c\right.}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

```
[Out] -1/2*((b^2*d*n^2 + a*b*d*n + (b^2*e*n^2 + a*b*e*n)*x + (b^2*e*n^2*x + b^2*d
*n^2)*log(e*x + d) + (b^2*e*n*x + b^2*d*n)*log(c))*e^((b*log(c) + a)/(b*n))
 - (b^2*n^2*log(e*x + d)^2 + b^2*log(c)^2 + 2*a*b*log(c) + a^2 + 2*(b^2*n*l
og(c) + a*b*n)*log(e*x + d))*log_integral((e*x + d)*e^((b*log(c) + a)/(b*n)
)))*e^(-(b*log(c) + a)/(b*n))/(b^5*e*n^5*log(e*x + d)^2 + b^5*e*n^3*log(c)^
2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3 + 2*(b^5*e*n^4*log(c) + a*b^4*e*n^
4)*log(e*x + d))
```

## Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**(-3), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^3}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((d*n + d*log(c))*b + a*d + ((e*n + e*log(c))*b + a*e)*x + (b*e*x + b*
d)*log((e*x + d)^n))/(b^4*e*n^2*log((e*x + d)^n)^2 + b^4*e*n^2*log(c)^2 + 2
*a*b^3*e*n^2*log(c) + a^2*b^2*e*n^2 + 2*(b^4*e*n^2*log(c) + a*b^3*e*n^2)*lo
g((e*x + d)^n)) + integrate(1/2/(b^3*n^2*log((e*x + d)^n) + b^3*n^2*log(c)
+ a*b^2*n^2), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1218 vs. $2(128) = 256$.

Time = 0.33 (sec) , antiderivative size = 1218, normalized size of antiderivative = 9.02

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*n^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)
^2/((b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e
*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n
^3)*c^(1/n)) - 1/2*(e*x + d)*b^2*n^2*log(e*x + d)/(b^5*e*n^5*log(e*x + d)^2
 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3
*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3) + b^2*n*Ei(log(c)/n + a/(
b*n) + log(e*x + d))*e^(-a/(b*n))*log(e*x + d)*log(c)/((b^5*e*n^5*log(e*x +
 d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*
e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n)) - 1/2*(e*x
+ d)*b^2*n^2/(b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) +
2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^
2*b^3*e*n^3) + a*b*n*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log
(e*x + d)/((b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*
a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*
b^3*e*n^3)*c^(1/n)) - 1/2*(e*x + d)*b^2*n*log(c)/(b^5*e*n^5*log(e*x + d)^2
+ 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*
log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3) + 1/2*b^2*Ei(log(c)/n + a/
(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)^2/((b^5*e*n^5*log(e*x + d)^2 + 2*
b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d) + b^5*e*n^3*log(
c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n)) - 1/2*(e*x + d)*a*b*n
/(b^5*e*n^5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^
```

```
4*log(e*x + d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)
 + a*b*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))*log(c)/((b^5*e*n^
5*log(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x
+ d) + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n))
+ 1/2*a^2*Ei(log(c)/n + a/(b*n) + log(e*x + d))*e^(-a/(b*n))/((b^5*e*n^5*lo
g(e*x + d)^2 + 2*b^5*e*n^4*log(e*x + d)*log(c) + 2*a*b^4*e*n^4*log(e*x + d)
 + b^5*e*n^3*log(c)^2 + 2*a*b^4*e*n^3*log(c) + a^2*b^3*e*n^3)*c^(1/n))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^3}\,dx=\int \frac{1}{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^3}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e*x)^n))^3,x)
[Out] int(1/(a + b*log(c*(d + e*x)^n))^3, x)
```

# 3.103 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^3} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx = \operatorname{Int}\left(\frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^3,x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx = \int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx$$

```
[In] Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^3),x]
```

```
[Out] Defer[Int][1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx$$

```
[In] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^3),x]
```

```
[Out] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^3), x]
```

## Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{3}}dx$$

```
[In] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^3,x)
```

```
[Out] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.96

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^nc\right)+a)^3}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(a^3*g*x + a^3*f + (b^3*g*x + b^3*f)*log((e*x + d)^n*c)^3 + 3*(a
*b^2*g*x + a*b^2*f)*log((e*x + d)^n*c)^2 + 3*(a^2*b*g*x + a^2*b*f)*log((e*x
 + d)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 6.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)^3(f+gx)}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**3,x)
[Out] Integral(1/((a + b*log(c*(d + e*x)**n))**3*(f + g*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 703, normalized size of antiderivative = 29.29

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx=\int \frac{1}{(gx+f)\left(b\log\left((ex+d)^nc\right)+a\right)^3}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
[Out] -1/2*(b*e^2*g*n*x^2 + (d*e*f - d^2*g)*a + (d*e*f*n + (d*e*f - d^2*g)*log(c)
)*b + ((e^2*f - d*e*g)*a + (e^2*f*n + d*e*g*n + (e^2*f - d*e*g)*log(c))*b)*
x + ((e^2*f - d*e*g)*b*x + (d*e*f - d^2*g)*b)*log((e*x + d)^n))/(b^4*e^2*f^
2*n^2*log(c)^2 + 2*a*b^3*e^2*f^2*n^2*log(c) + a^2*b^2*e^2*f^2*n^2 + (b^4*e^
2*g^2*n^2*log(c)^2 + 2*a*b^3*e^2*g^2*n^2*log(c) + a^2*b^2*e^2*g^2*n^2)*x^2
+ (b^4*e^2*g^2*n^2*x^2 + 2*b^4*e^2*f*g*n^2*x + b^4*e^2*f^2*n^2)*log((e*x +
d)^n)^2 + 2*(b^4*e^2*f*g*n^2*log(c)^2 + 2*a*b^3*e^2*f*g*n^2*log(c) + a^2*b^
2*e^2*f*g*n^2)*x + 2*(b^4*e^2*f^2*n^2*log(c) + a*b^3*e^2*f^2*n^2 + (b^4*e^2
*g^2*n^2*log(c) + a*b^3*e^2*g^2*n^2)*x^2 + 2*(b^4*e^2*f*g*n^2*log(c) + a*b^
3*e^2*f*g*n^2)*x)*log((e*x + d)^n)) + integrate(1/2*(e^2*f^2 - 3*d*e*f*g +
2*d^2*g^2 - (e^2*f*g - d*e*g^2)*x)/(b^3*e^2*f^3*n^2*log(c) + a*b^2*e^2*f^3*
n^2 + (b^3*e^2*g^3*n^2*log(c) + a*b^2*e^2*g^3*n^2)*x^3 + 3*(b^3*e^2*f*g^2*n
^2*log(c) + a*b^2*e^2*f*g^2*n^2)*x^2 + 3*(b^3*e^2*f^2*g*n^2*log(c) + a*b^2*
e^2*f^2*g*n^2)*x + (b^3*e^2*g^3*n^2*x^3 + 3*b^3*e^2*f*g^2*n^2*x^2 + 3*b^3*e
^2*f^2*g*n^2*x + b^3*e^2*f^3*n^2)*log((e*x + d)^n)), x)
```

## Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^nc\right)+a)^3}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx = \int \frac{1}{(f+g\,x)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^3}\,dx$$

```
[In] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^3),x)
[Out] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^3), x)
```

# 3.104 $\int \frac{1}{(f+gx)^2(a+b\log(c(d+ex)^n))^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx=\operatorname{Int}\left(\frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^3,x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx=\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx$$

```
[In] Int[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^3),x]
```

```
[Out] Defer[Int][1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^3), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2 (a+b\log(c(d+ex)^n))^3} \, dx = \int \frac{1}{(f+gx)^2 (a+b\log(c(d+ex)^n))^3} \, dx$$

```
[In] Integrate[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^3),x]
[Out] Integrate[1/((f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^3), x]
```

## Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2 (a+b\ln(c(ex+d)^n))^3} dx$$

```
[In] int(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^3,x)
[Out] int(1/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 6.38

$$\int \frac{1}{(f+gx)^2 (a+b\log(c(d+ex)^n))^3} \, dx = \int \frac{1}{(gx+f)^2 (b\log((ex+d)^n c)+a)^3} \, dx$$

```
[In] integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
[Out] integral(1/(a^3*g^2*x^2 + 2*a^3*f*g*x + a^3*f^2 + (b^3*g^2*x^2 + 2*b^3*f*g*
x + b^3*f^2)*log((e*x + d)^n*c)^3 + 3*(a*b^2*g^2*x^2 + 2*a*b^2*f*g*x + a*b^
2*f^2)*log((e*x + d)^n*c)^2 + 3*(a^2*b*g^2*x^2 + 2*a^2*b*f*g*x + a^2*b*f^2)
*log((e*x + d)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 35.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)^2\,(a+b\log{(c(d+ex)^n)})^3}\,dx = \int \frac{1}{(a+b\log{(c\,(d+ex)^n)})^3\,(f+gx)^2}\,dx$$

```
[In] integrate(1/(g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**3,x)
[Out] Integral(1/((a + b*log(c*(d + e*x)**n))**3*(f + g*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 934, normalized size of antiderivative = 38.92

$$\int \frac{1}{(f+gx)^2\,(a+b\log{(c(d+ex)^n)})^3}\,dx = \int \frac{1}{(gx+f)^2(b\log{((ex+d)^n c)}+a)^3}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
[Out] 1/2*((a*e^2*g - (e^2*g*n - e^2*g*log(c))*b)*x^2 - (d*e*f - 2*d^2*g)*a - (d*
e*f*n + (d*e*f - 2*d^2*g)*log(c))*b - ((e^2*f - 3*d*e*g)*a + (e^2*f*n + d*e
*g*n + (e^2*f - 3*d*e*g)*log(c))*b)*x + (b*e^2*g*x^2 - (e^2*f - 3*d*e*g)*b*
x - (d*e*f - 2*d^2*g)*b)*log((e*x + d)^n))/(b^4*e^2*f^3*n^2*log(c)^2 + 2*a*
b^3*e^2*f^3*n^2*log(c) + a^2*b^2*e^2*f^3*n^2 + (b^4*e^2*g^3*n^2*log(c)^2 +
2*a*b^3*e^2*g^3*n^2*log(c) + a^2*b^2*e^2*g^3*n^2)*x^3 + 3*(b^4*e^2*f*g^2*n^
2*log(c)^2 + 2*a*b^3*e^2*f*g^2*n^2*log(c) + a^2*b^2*e^2*f*g^2*n^2)*x^2 + (b
^4*e^2*g^3*n^2*x^3 + 3*b^4*e^2*f*g^2*n^2*x^2 + 3*b^4*e^2*f^2*g*n^2*x + b^4*
e^2*f^3*n^2)*log((e*x + d)^n)^2 + 3*(b^4*e^2*f^2*g*n^2*log(c)^2 + 2*a*b^3*e
^2*f^2*g*n^2*log(c) + a^2*b^2*e^2*f^2*g*n^2)*x + 2*(b^4*e^2*f^3*n^2*log(c)
+ a*b^3*e^2*f^3*n^2 + (b^4*e^2*g^3*n^2*log(c) + a*b^3*e^2*g^3*n^2)*x^3 + 3*
(b^4*e^2*f*g^2*n^2*log(c) + a*b^3*e^2*f*g^2*n^2)*x^2 + 3*(b^4*e^2*f^2*g*n^2
*log(c) + a*b^3*e^2*f^2*g*n^2)*x)*log((e*x + d)^n)) + integrate(1/2*(e^2*g^
2*x^2 + e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2 - 2*(2*e^2*f*g - 3*d*e*g^2)*x)/(b^3
*e^2*f^4*n^2*log(c) + a*b^2*e^2*f^4*n^2 + (b^3*e^2*g^4*n^2*log(c) + a*b^2*e
^2*g^4*n^2)*x^4 + 4*(b^3*e^2*f*g^3*n^2*log(c) + a*b^2*e^2*f*g^3*n^2)*x^3 +
6*(b^3*e^2*f^2*g^2*n^2*log(c) + a*b^2*e^2*f^2*g^2*n^2)*x^2 + 4*(b^3*e^2*f^3
*g*n^2*log(c) + a*b^2*e^2*f^3*g*n^2)*x + (b^3*e^2*g^4*n^2*x^4 + 4*b^3*e^2*f
*g^3*n^2*x^3 + 6*b^3*e^2*f^2*g^2*n^2*x^2 + 4*b^3*e^2*f^3*g*n^2*x + b^3*e^2*
f^4*n^2)*log((e*x + d)^n)), x)
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx=\int \frac{1}{(gx+f)^2(b\log\left((ex+d)^nc\right)+a)^3}\,dx$$

```
[In] integrate(1/(g*x+f)^2/(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
[Out] integrate(1/((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}\,dx=\int \frac{1}{\left(f+g\,x\right)^2\,\left(a+b\,\ln\left(c\,\left(d+e\,x\right)^n\right)\right)^3}\,dx$$

```
[In] int(1/((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^3),x)
[Out] int(1/((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^3), x)
```

# 3.105 $\int (f+gx)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 404

$$
\begin{aligned}
&\int (f+gx)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx \\
&= -\frac{\sqrt{b}e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{n}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^3} \\
&\quad-\frac{\sqrt{b}e^{-\frac{2a}{bn}}g(ef-dg)\sqrt{n}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^3} \\
&\quad-\frac{\sqrt{b}e^{-\frac{3a}{bn}}g^2\sqrt{n}\sqrt{\frac{\pi}{3}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{6e^3} \\
&\quad+\frac{(ef-dg)^2(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3} \\
&\quad+\frac{g(ef-dg)(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3}+\frac{g^2(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{3e^3}
\end{aligned}
$$

```
[Out] -1/18*g^2*(e*x+d)^3*erfi(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2
))*b^(1/2)*n^(1/2)*3^(1/2)*Pi^(1/2)/e^3/exp(3*a/b/n)/((c*(e*x+d)^n)^(3/n))-
1/4*g*(-d*g+e*f)*(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)
/n^(1/2))*b^(1/2)*n^(1/2)*2^(1/2)*Pi^(1/2)/e^3/exp(2*a/b/n)/((c*(e*x+d)^n)^
(2/n))-1/2*(-d*g+e*f)^2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^
(1/2))*b^(1/2)*n^(1/2)*Pi^(1/2)/e^3/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+(-d*g+
e*f)^2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^3+g*(-d*g+e*f)*(e*x+d)^2*(a+b*
ln(c*(e*x+d)^n))^(1/2)/e^3+1/3*g^2*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^
3
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2448, 2436, 2333, 2337, 2211, 2235, 2437, 2342, 2347}

$$\begin{aligned}
&\int (f+gx)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx \\
&= -\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}g\sqrt{n}e^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^3} \\
&\quad -\frac{\sqrt{\pi}\sqrt{b}\sqrt{n}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^3} \\
&\quad -\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}g^2\sqrt{n}e^{-\frac{3a}{bn}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{6e^3} \\
&\quad +\frac{g(d+ex)^2(ef-dg)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3} \\
&\quad +\frac{(d+ex)(ef-dg)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3}+\frac{g^2(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{3e^3}
\end{aligned}$$

```
[In] Int[(f + g*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] -1/2*(Sqrt[b]*(e*f - d*g)^2*Sqrt[n]*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[
c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(e^3*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)
) - (Sqrt[b]*g*(e*f - d*g)*Sqrt[n]*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqr
t[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(2*e^3*E^((2*a)/(b*n))*(c*
(d + e*x)^n)^(2/n)) - (Sqrt[b]*g^2*Sqrt[n]*Sqrt[Pi/3]*(d + e*x)^3*Erfi[(Sqr
t[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(6*e^3*E^((3*a)/(b
*n))*(c*(d + e*x)^n)^(3/n)) + ((e*f - d*g)^2*(d + e*x)*Sqrt[a + b*Log[c*(d
+ e*x)^n]])/e^3 + (g*(e*f - d*g)*(d + e*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]]
)/e^3 + (g^2*(d + e*x)^3*Sqrt[a + b*Log[c*(d + e*x)^n]])/(3*e^3)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(ef-dg)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^2} + \frac{2g(ef-dg)(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^2} + \frac{g^2(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^2}\right) dx$$

$$= \frac{g^2\int (d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx}{e^2} + \frac{(2g(ef-dg))\int (d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx}{e^2} + \frac{(ef-dg)^2\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx}{e^2}$$

$$= \frac{g^2\text{Subst}\left(\int x^2\sqrt{a+b\log\left(cx^n\right)}\,dx,x,d+ex\right)}{e^3} + \frac{(2g(ef-dg))\text{Subst}\left(\int x\sqrt{a+b\log\left(cx^n\right)}\,dx,x,d+ex\right)}{e^3} + \frac{(ef-dg)^2\text{Subst}\left(\int \sqrt{a+b\log\left(cx^n\right)}\,dx,x,d+ex\right)}{e^3}$$

$$= \frac{(ef-dg)^2(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3} + \frac{g(ef-dg)(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3} + \frac{g^2(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{3e^3} - \frac{\left(bg^2n\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{6e^3} - \frac{(bg(ef-dg)n)\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{2e^3} - \frac{\left(b(ef-dg)^2n\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{2e^3}$$

$$= \frac{(ef-dg)^2(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3} + \frac{g(ef-dg)(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3} + \frac{g^2(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{3e^3} - \frac{\left(bg^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{6e^3} - \frac{\left(bg(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{2e^3} - \frac{\left(b(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{2e^3}$$

$$= \frac{(ef-dg)^2(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3} + \frac{g(ef-dg)(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3} + \frac{g^2(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{3e^3} - \frac{\left(g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{3e^3} - \frac{\left(g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{e^3} - \frac{\left((ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{e^3}$$

$$= -\frac{\sqrt{b}e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{n}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^3} - \frac{\sqrt{b}e^{-\frac{2a}{bn}}g(ef-dg)\sqrt{n}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^3} - \frac{\sqrt{b}e^{-\frac{3a}{bn}}g^2\sqrt{n}\sqrt{\frac{\pi}{3}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{6e^3} + \frac{(ef-dg)^2(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3} + \frac{g(ef-dg)(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^3} + \frac{g^2(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{3e^3}$$

## Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.93

$$\int (f+gx)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$$
$$=\frac{(d+ex)\left(-18\sqrt{b}e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{n}\sqrt{\pi}(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+9\sqrt{b}e^{-\frac{2a}{bn}}g(-ef+dg)\sqrt{}\right.}{}$$

```
[In] Integrate[(f + g*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] ((d + e*x)*((-18*Sqrt[b]*(e*f - d*g)^2*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Log
[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) +
 (9*Sqrt[b]*g*(-(e*f) + d*g)*Sqrt[n]*Sqrt[2*Pi]*(d + e*x)*Erfi[(Sqrt[2]*Sqr
t[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(E^((2*a)/(b*n))*(c*(d + e
*x)^n)^(2/n)) - (2*Sqrt[b]*g^2*Sqrt[n]*Sqrt[3*Pi]*(d + e*x)^2*Erfi[(Sqrt[3]
*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(E^((3*a)/(b*n))*(c*(d
 + e*x)^n)^(3/n)) + 36*(e*f - d*g)^2*Sqrt[a + b*Log[c*(d + e*x)^n]] + 36*g*
(e*f - d*g)*(d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]] + 12*g^2*(d + e*x)^2*S
qrt[a + b*Log[c*(d + e*x)^n]]))/(36*e^3)
```

## Maple [F]

$$\int (gx+f)^2\sqrt{a+b\ln\left(c\,(ex+d)^n\right)}dx$$

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (f+gx)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (f+gx)^2\sqrt{a+b\log{(c(d+ex)^n)}}\,dx = \int \sqrt{a+b\log{(c\,(d+ex)^n)}}(f+gx)^2\,dx$$

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral(sqrt(a + b*log(c*(d + e*x)**n))*(f + g*x)**2, x)
```

## Maxima [F]

$$\int (f+gx)^2\sqrt{a+b\log{(c(d+ex)^n)}}\,dx = \int (gx+f)^2\sqrt{b\log{((ex+d)^n c)}+a}\,dx$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^2*sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Giac [F]

$$\int (f+gx)^2\sqrt{a+b\log{(c(d+ex)^n)}}\,dx = \int (gx+f)^2\sqrt{b\log{((ex+d)^n c)}+a}\,dx$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^2*sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\sqrt{a+b\log{(c(d+ex)^n)}}\,dx = \int (f+g\,x)^2\,\sqrt{a+b\,\ln{(c\,(d+e\,x)^n)}}\,dx$$

```
[In] int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^(1/2),x)
[Out] int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

# 3.106 $\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 255

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = -\frac{\sqrt{b}e^{-\frac{a}{bn}}(ef-dg)\sqrt{n}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^2} - \frac{\sqrt{b}e^{-\frac{2a}{bn}}g\sqrt{n}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^2} + \frac{(ef-dg)(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^2} + \frac{g(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2}$$

```
[Out] -1/8*g*(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*
b^(1/2)*n^(1/2)*2^(1/2)*Pi^(1/2)/e^2/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))-1/2
*(-d*g+e*f)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*b^(1/
2)*n^(1/2)*Pi^(1/2)/e^2/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+(-d*g+e*f)*(e*x+d)
*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^2+1/2*g*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^(1/2
)/e^2
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

$= \{2448, 2436, 2333, 2337, 2211, 2235, 2437, 2342, 2347\}$

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$$
$$= -\frac{\sqrt{\pi}\sqrt{b}\sqrt{n}e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^2}$$
$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}g\sqrt{n}e^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^2}$$
$$+\frac{(d+ex)(ef-dg)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^2}+\frac{g(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2}$$

```
[In] Int[(f + g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] -1/2*(Sqrt[b]*(e*f - d*g)*Sqrt[n]*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*
(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(e^2*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1))
- (Sqrt[b]*g*Sqrt[n]*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*
(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(4*e^2*E^((2*a)/(b*n))*(c*(d + e*x)^n)^(
2/n)) + ((e*f - d*g)*(d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]])/e^2 + (g*(d
+ e*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(2*e^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(ef-dg)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e}+\frac{g(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e}\right)dx \\
&= \frac{g\int (d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx}{e}+\frac{(ef-dg)\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx}{e} \\
&= \frac{g\text{Subst}\left(\int x\sqrt{a+b\log\left(cx^n\right)}\,dx,x,d+ex\right)}{e^2} \\
&\quad+\frac{(ef-dg)\text{Subst}\left(\int \sqrt{a+b\log\left(cx^n\right)}\,dx,x,d+ex\right)}{e^2}
\end{aligned}$$

$$= \frac{(ef-dg)(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^2} + \frac{g(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2} - \frac{(bgn)\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{4e^2} - \frac{(b(ef-dg)n)\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{2e^2}$$

$$= \frac{(ef-dg)(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^2} + \frac{g(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2} - \frac{\left(bg(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{4e^2} - \frac{\left(b(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{2e^2}$$

$$= \frac{(ef-dg)(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^2} + \frac{g(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2} - \frac{\left(g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{2e^2} - \frac{\left((ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{e^2}$$

$$= -\frac{\sqrt{b}e^{-\frac{a}{bn}}(ef-dg)\sqrt{n}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e^2} - \frac{\sqrt{b}e^{-\frac{2a}{bn}}g\sqrt{n}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^2} + \frac{(ef-dg)(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e^2} + \frac{g(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2}$$

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.92

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = -\frac{e^{-\frac{2a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(4\sqrt{b}e^{\frac{a}{bn}}(ef-dg)\sqrt{n}\sqrt{\pi}\left(c(d+ex)^n\right)^{\frac{1}{n}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+\sqrt{b}g\sqrt{n}\sqrt{\phantom{x}}\right.}{8e^{\phantom{2}}}$$

```
[In] Integrate[(f + g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] -1/8*((d + e*x)*(4*Sqrt[b]*E^(a/(b*n))*(e*f - d*g)*Sqrt[n]*Sqrt[Pi]*(c*(d +
 e*x)^n)^n^(-1)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])] + Sq
```

```
rt[b]*g*Sqrt[n]*Sqrt[2*Pi]*(d + e*x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*
x)^n]])/(Sqrt[b]*Sqrt[n])] - 4*E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)*(2*e*f
 - d*g + e*g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]))/(e^2*E^((2*a)/(b*n))*(c*(d
 + e*x)^n)^(2/n))
```

## Maple [F]

$$\int (gx+f)\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}dx$$

```
[In] int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^{n}\right)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^{n}\right)}\,dx=\int\sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}(f+gx)\,dx$$

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*log(c*(d + e*x)**n))*(f + g*x), x)
```

## Maxima [F]

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^{n}\right)}\,dx=\int (gx+f)\sqrt{b\log\left(\left(ex+d\right)^{n}c\right)+a}\,dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)*sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Giac [F]

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int (gx+f)\sqrt{b\log\left((ex+d)^n c\right)+a}\,dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)*sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int (f+g\,x)\,\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

```
[In] int((f + g*x)*(a + b*log(c*(d + e*x)^n))^(1/2),x)
[Out] int((f + g*x)*(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

# 3.107 $\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 111

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = -\frac{\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e} + \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e}$$

```
[Out] -1/2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*b^(1/2)*n^(1
/2)*Pi^(1/2)/e/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+(e*x+d)*(a+b*ln(c*(e*x+d)^n
))^(1/2)/e
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2436, 2333, 2337, 2211, 2235}

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e} - \frac{\sqrt{\pi}\sqrt{b}\sqrt{n}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e}$$

```
[In] Int[Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] -1/2*(Sqrt[b]*Sqrt[n]*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]
]/(Sqrt[b]*Sqrt[n])])/(e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + ((d + e*x)*S
qrt[a + b*Log[c*(d + e*x)^n]])/e
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{a+b\log\left(cx^n\right)}\,dx, x, d+ex\right)}{e}\\
&= \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e} - \frac{(bn)\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{2e}\\
&= \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{e}\\
&\quad - \frac{\left(b(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{2e}
\end{aligned}
$$

$$= \frac{(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{e} - \frac{\left((d+ex)\,(c(d+ex)^n)^{-1/n}\right)\mathrm{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log(c(d+ex)^n)}\right)}{e}$$

$$= -\frac{\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}(d+ex)\,(c(d+ex)^n)^{-1/n}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2e} + \frac{(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{e}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \sqrt{a+b\log(c(d+ex)^n)}\,dx = \frac{(d+ex)\left(-\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}(c(d+ex)^n)^{-1/n}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+2\sqrt{a+b\log(c(d+ex)^n)}\right)}{2e}$$

```
[In] Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]],x]
[Out] ((d + e*x)*(-((Sqrt[b]*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]
/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1))) + 2*Sqrt[a + b*L
og[c*(d + e*x)^n]]))/(2*e)
```

## Maple [F]

$$\int \sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(1/2),x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\log(c(d+ex)^n)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral(sqrt(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [F]

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{b\log\left((ex+d)^n c\right)+a}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Giac [F]

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{b\log\left((ex+d)^n c\right)+a}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \sqrt{a+b\,\ln\left(c\left(d+e\,x\right)^n\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(1/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^(1/2), x)
```

# 3.108 $\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx}\,dx = \text{Int}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f),x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx}\,dx = \int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx}\,dx$$

```
[In] Int[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x),x]
[Out] Defer[Int][Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx=\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx$$

```
[In] Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x),x]
[Out] Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x), x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\ln\left(c\,(ex+d)^{n}\right)}}{gx+f}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f),x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx=\int \frac{\sqrt{a+b\log\left(c\,(d+ex)^{n}\right)}}{f+gx}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(1/2)/(g*x+f),x)
[Out] Integral(sqrt(a + b*log(c*(d + e*x)**n))/(f + g*x), x)
```

## Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx=\int \frac{\sqrt{b\log\left((ex+d)^n c\right)+a}}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f),x, algorithm="maxima")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx=\int \frac{\sqrt{b\log\left((ex+d)^n c\right)+a}}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f),x, algorithm="giac")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

## Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{f+gx}\,dx=\int \frac{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}{f+g\,x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x),x)
[Out] int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x), x)
```

# 3.109 $\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{(f+gx)^2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx = \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(ef-dg)(f+gx)} - \frac{ben\text{Int}\left(\frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}},x\right)}{2(ef-dg)}$$

```
[Out] (e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/(-d*g+e*f)/(g*x+f)-1/2*b*e*n*Unintegrab
le(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)/(-d*g+e*f)
```

## Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx = \int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx$$

```
[In] Int[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^2,x]
```

```
[Out] ((d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]])/((e*f - d*g)*(f + g*x)) - (b*e*n
*Defer[Int][1/((f + g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]), x])/(2*(e*f - d*g
))
```

Rubi steps

$$\text{integral} = \frac{(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(ef-dg)(f+gx)} - \frac{(ben)\int \frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{2(ef-dg)}$$

## Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx=\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx$$

```
[In] Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^2,x]
[Out] Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^2, x]
```

## Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}{\left(gx+f\right)^{2}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^2,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx=\int \frac{\sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}}{\left(f+gx\right)^{2}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(1/2)/(g*x+f)**2,x)
[Out] Integral(sqrt(a + b*log(c*(d + e*x)**n))/(f + g*x)**2, x)
```

## Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx = \int \frac{\sqrt{b\log\left((ex+d)^n c\right)+a}}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx = \int \frac{\sqrt{b\log\left((ex+d)^n c\right)+a}}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^2,x, algorithm="giac")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^2}\,dx = \int \frac{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}{{\left(f+g\,x\right)}^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^2,x)
[Out] int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^2, x)
```

# 3.110 $\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{(f+gx)^3}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}\,dx = -\frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2g(f+gx)^2} + \frac{ben\text{Int}\left(\frac{1}{(d+ex)(f+gx)^2\sqrt{a+b\log(c(d+ex)^n)}},x\right)}{4g}$$

```
[Out] -1/2*(a+b*ln(c*(e*x+d)^n))^(1/2)/g/(g*x+f)^2+1/4*b*e*n*Unintegrable(1/(e*x+
d)/(g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(1/2),x)/g
```

## Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}\,dx = \int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}\,dx$$

```
[In] Int[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^3,x]
```

```
[Out] -1/2*Sqrt[a + b*Log[c*(d + e*x)^n]]/(g*(f + g*x)^2) + (b*e*n*Defer[Int][1/(
(d + e*x)*(f + g*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]]), x])/(4*g)
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2g(f+gx)^2} + \frac{(ben)\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{4g}$$

## Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}\,dx=\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}\,dx$$

```
[In] Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^3,x]
[Out] Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^3, x]
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}{\left(gx+f\right)^{3}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^3,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^3,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^3,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 8.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}\,dx=\int \frac{\sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}}{\left(f+gx\right)^{3}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(1/2)/(g*x+f)**3,x)
[Out] Integral(sqrt(a + b*log(c*(d + e*x)**n))/(f + g*x)**3, x)
```

## Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}\,dx=\int \frac{\sqrt{b\log\left((ex+d)^n c\right)+a}}{(gx+f)^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^3,x, algorithm="maxima")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f)^3, x)
```

## Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}\,dx=\int \frac{\sqrt{b\log\left((ex+d)^n c\right)+a}}{(gx+f)^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^3,x, algorithm="giac")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^3}\,dx=\int \frac{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}{{\left(f+g\,x\right)}^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^3,x)
[Out] int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^3, x)
```

# 3.111 $\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 526

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2}\,dx = \frac{3b^{3/2}e^{-\frac{a}{bn}}(ef-dg)^2n^{3/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^3}$$
$$+\frac{3b^{3/2}e^{-\frac{2a}{bn}}g(ef-dg)n^{3/2}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^3}$$
$$+\frac{b^{3/2}e^{-\frac{3a}{bn}}g^2n^{3/2}\sqrt{\frac{\pi}{3}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{12e^3}$$
$$-\frac{3b(ef-dg)^2n(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{2e^3}$$
$$-\frac{3bg(ef-dg)n(d+ex)^2\sqrt{a+b\log(c(d+ex)^n)}}{4e^3}$$
$$-\frac{bg^2n(d+ex)^3\sqrt{a+b\log(c(d+ex)^n)}}{6e^3}+\frac{(ef-dg)^2(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{e^3}$$
$$+\frac{g(ef-dg)(d+ex)^2(a+b\log(c(d+ex)^n))^{3/2}}{e^3}+\frac{g^2(d+ex)^3(a+b\log(c(d+ex)^n))^{3/2}}{3e^3}$$

```
[Out] (-d*g+e*f)^2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^3+g*(-d*g+e*f)*(e*x+d)^2
*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^3+1/3*g^2*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))^(3
/2)/e^3+1/36*b^(3/2)*g^2*n^(3/2)*(e*x+d)^3*erfi(3^(1/2)*(a+b*ln(c*(e*x+d)^n
))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2)*Pi^(1/2)/e^3/exp(3*a/b/n)/((c*(e*x+d)^n)^
(3/n))+3/16*b^(3/2)*g*(-d*g+e*f)*n^(3/2)*(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(
e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/e^3/exp(2*a/b/n)/((c*(e*
x+d)^n)^(2/n))+3/4*b^(3/2)*(-d*g+e*f)^2*n^(3/2)*(e*x+d)*erfi((a+b*ln(c*(e*x
+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e^3/exp(a/b/n)/((c*(e*x+d)^n)^(1/n)
```

```
)-3/2*b*(-d*g+e*f)^2*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^3-3/4*b*g*(-d*
g+e*f)*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^3-1/6*b*g^2*n*(e*x+d)^3*(a
+b*ln(c*(e*x+d)^n))^(1/2)/e^3
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2448, 2436, 2333, 2337, 2211, 2235, 2437, 2342, 2347}

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2}\,dx = \frac{3\sqrt{\frac{\pi}{2}}b^{3/2}gn^{3/2}e^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^3}$$
$$+\frac{3\sqrt{\pi}b^{3/2}n^{3/2}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^3}$$
$$+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}g^2n^{3/2}e^{-\frac{3a}{bn}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{12e^3}$$
$$+\frac{g(d+ex)^2(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^3}$$
$$+\frac{(d+ex)(ef-dg)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^3}$$
$$-\frac{3bgn(d+ex)^2(ef-dg)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^3}$$
$$-\frac{3bn(d+ex)(ef-dg)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^3}$$
$$+\frac{g^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{3e^3}-\frac{bg^2n(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{6e^3}$$

```
[In] Int[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] (3*b^(3/2)*(e*f - d*g)^2*n^(3/2)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(
d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(4*e^3*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1))
 + (3*b^(3/2)*g*(e*f - d*g)*n^(3/2)*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sq
rt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(8*e^3*E^((2*a)/(b*n))*(c
*(d + e*x)^n)^(2/n)) + (b^(3/2)*g^2*n^(3/2)*Sqrt[Pi/3]*(d + e*x)^3*Erfi[(Sq
rt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(12*e^3*E^((3*a)/
(b*n))*(c*(d + e*x)^n)^(3/n)) - (3*b*(e*f - d*g)^2*n*(d + e*x)*Sqrt[a + b*L
og[c*(d + e*x)^n]])/(2*e^3) - (3*b*g*(e*f - d*g)*n*(d + e*x)^2*Sqrt[a + b*L
og[c*(d + e*x)^n]])/(4*e^3) - (b*g^2*n*(d + e*x)^3*Sqrt[a + b*Log[c*(d + e*
x)^n]])/(6*e^3) + ((e*f - d*g)^2*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^(3/2)
)/e^3 + (g*(e*f - d*g)*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^(3/2))/e^3 +
(g^2*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])^(3/2))/(3*e^3)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(ef-dg)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^2} + \frac{2g(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^2} + \frac{g^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^2}\right) dx$$

$$= \frac{g^2\int (d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx}{e^2} + \frac{(2g(ef-dg))\int (d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx}{e^2} + \frac{(ef-dg)^2\int \left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx}{e^2}$$

$$= \frac{g^2\text{Subst}\left(\int x^2(a+b\log(cx^n))^{3/2}\,dx, x, d+ex\right)}{e^3} + \frac{(2g(ef-dg))\text{Subst}\left(\int x(a+b\log(cx^n))^{3/2}\,dx, x, d+ex\right)}{e^3} + \frac{(ef-dg)^2\text{Subst}\left(\int (a+b\log(cx^n))^{3/2}\,dx, x, d+ex\right)}{e^3}$$

$$= \frac{(ef-dg)^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^3} + \frac{g(ef-dg)(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^3} + \frac{g^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{3e^3} - \frac{(bg^2n)\,\text{Subst}\left(\int x^2\sqrt{a+b\log\left(cx^n\right)}\,dx,x,d+ex\right)}{2e^3} - \frac{(3bg(ef-dg)n)\text{Subst}\left(\int x\sqrt{a+b\log\left(cx^n\right)}\,dx,x,d+ex\right)}{2e^3} - \frac{(3b(ef-dg)^2n)\,\text{Subst}\left(\int\sqrt{a+b\log\left(cx^n\right)}\,dx,x,d+ex\right)}{2e^3}$$

$$= -\frac{3b(ef-dg)^2n(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^3} - \frac{3bg(ef-dg)n(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^3} - \frac{bg^2n(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{6e^3} + \frac{(ef-dg)^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^3} + \frac{g(ef-dg)(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^3} + \frac{g^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{3e^3} + \frac{(b^2g^2n^2)\,\text{Subst}\left(\int\frac{x^2}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{12e^3} + \frac{(3b^2g(ef-dg)n^2)\,\text{Subst}\left(\int\frac{x}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{8e^3} + \frac{(3b^2(ef-dg)^2n^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{4e^3}$$

$$= -\frac{3b(ef-dg)^2 n(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^3} - \frac{3bg(ef-dg)n(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^3} - \frac{bg^2 n(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{6e^3} + \frac{(ef-dg)^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^3} + \frac{g(ef-dg)(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^3} + \frac{g^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{3e^3} + \frac{\left(b^2g^2n(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{12e^3} + \frac{\left(3b^2g(ef-dg)n(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{8e^3} + \frac{\left(3b^2(ef-dg)^2n(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{4e^3}$$

$$= -\frac{3b(ef-dg)^2 n(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^3} - \frac{3bg(ef-dg)n(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^3} - \frac{bg^2 n(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{6e^3} + \frac{(ef-dg)^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^3} + \frac{g(ef-dg)(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^3} + \frac{g^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{3e^3} + \frac{\left(bg^2n(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{6e^3} + \frac{\left(3bg(ef-dg)n(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{4e^3} + \frac{\left(3b(ef-dg)^2n(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{2e^3}$$

$$= \frac{3b^{3/2}e^{-\frac{a}{bn}}(ef-dg)^2n^{3/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^3}$$
$$+ \frac{3b^{3/2}e^{-\frac{2a}{bn}}g(ef-dg)n^{3/2}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^3}$$
$$+ \frac{b^{3/2}e^{-\frac{3a}{bn}}g^2n^{3/2}\sqrt{\frac{\pi}{3}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{12e^3}$$
$$- \frac{3b(ef-dg)^2n(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^3}$$
$$- \frac{3bg(ef-dg)n(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^3}$$
$$- \frac{bg^2n(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{6e^3}$$
$$+ \frac{(ef-dg)^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^3}$$
$$+ \frac{g(ef-dg)(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^3}$$
$$+ \frac{g^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{3e^3}$$

## Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.85

$$\int (f+gx)^2 (a+b\log\left(c(d+ex)^n\right))^{3/2}\,dx = \frac{(d+ex)\left(144(ef-dg)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}+144g(ef-dg)(d+ex)(a+\right.}{}$$

```
[In] Integrate[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] ((d + e*x)*(144*(e*f - d*g)^2*(a + b*Log[c*(d + e*x)^n])^(3/2) + 144*g*(e*f
 - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^(3/2) + 48*g^2*(d + e*x)^2*(a
+ b*Log[c*(d + e*x)^n])^(3/2) + 4*b*g^2*n*(d + e*x)^2*((Sqrt[b]*Sqrt[n]*Sqr
t[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(
E^((3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)) - 6*Sqrt[a + b*Log[c*(d + e*x)^n]])
+ 27*b*g*(e*f - d*g)*n*(d + e*x)*((Sqrt[b]*Sqrt[n]*Sqrt[2*Pi]*Erfi[(Sqrt[2]
*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(E^((2*a)/(b*n))*(c*(d
 + e*x)^n)^(2/n)) - 4*Sqrt[a + b*Log[c*(d + e*x)^n]]) + 108*b*(e*f - d*g)^2
*n*((Sqrt[b]*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*
Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) - 2*Sqrt[a + b*Log[c*(d + e
*x)^n]])))/(144*e^3)
```

## Maple [F]

$$\int (gx+f)^2 (a+b\ln(c(ex+d)^n))^{\frac{3}{2}} dx$$

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2} \, dx = \int (a+b\log(c(d+ex)^n))^{\frac{3}{2}} (f+gx)^2 \, dx$$

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n))**(3/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**(3/2)*(f + g*x)**2, x)
```

## Maxima [F]

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2} \, dx = \int (gx+f)^2 (b\log((ex+d)^n c)+a)^{\frac{3}{2}} \, dx$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Giac [F]

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2}\,dx = \int (gx+f)^2 (b\log((ex+d)^n c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
[Out] integrate((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{3/2}\,dx = \int (f+g\,x)^2 \left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^{3/2}\,dx$$

```
[In] int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^(3/2),x)
[Out] int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

# 3.112 $\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 330

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx=\frac{3b^{3/2}e^{-\frac{a}{bn}}(ef-dg)n^{3/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^2}+\frac{3b^{3/2}e^{-\frac{2a}{bn}}gn^{3/2}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{16e^2}-\frac{3b(ef-dg)n(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2}-\frac{3bgn(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{8e^2}+\frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2}$$

```
[Out] (-d*g+e*f)*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^2+1/2*g*(e*x+d)^2*(a+b*ln(
c*(e*x+d)^n))^(3/2)/e^2+3/32*b^(3/2)*g*n^(3/2)*(e*x+d)^2*erfi(2^(1/2)*(a+b*
ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/e^2/exp(2*a/b/n)/(
(c*(e*x+d)^n)^(2/n))+3/4*b^(3/2)*(-d*g+e*f)*n^(3/2)*(e*x+d)*erfi((a+b*ln(c*
(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e^2/exp(a/b/n)/((c*(e*x+d)^n)^(
1/n))-3/2*b*(-d*g+e*f)*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^2-3/8*b*g*n*
(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^2
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2448, 2436, 2333, 2337, 2211, 2235, 2437, 2342, 2347}

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx=\frac{3\sqrt{\pi}b^{3/2}n^{3/2}e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^2}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}gn^{3/2}e^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{16e^2}+\frac{(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^2}-\frac{3bn(d+ex)(ef-dg)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2}-\frac{3bgn(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{8e^2}$$

```
[In] Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] (3*b^(3/2)*(e*f - d*g)*n^(3/2)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d
+ e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(4*e^2*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) +
 (3*b^(3/2)*g*n^(3/2)*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c
*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(16*e^2*E^((2*a)/(b*n))*(c*(d + e*x)^n)
^(2/n)) - (3*b*(e*f - d*g)*n*(d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]])/(2*e
^2) - (3*b*g*n*(d + e*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(8*e^2) + ((e*f
- d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^(3/2))/e^2 + (g*(d + e*x)^2*(a
+ b*Log[c*(d + e*x)^n])^(3/2))/(2*e^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
```

```
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

integral

$$= \int \left( \frac{(ef - dg)\left(a + b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e} + \frac{g(d+ex)\left(a + b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e} \right) dx$$

$$=\frac{g\int (d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx}{e}+\frac{(ef-dg)\int \left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx}{e}$$

$$=\frac{g\text{Subst}\left(\int x(a+b\log(cx^n))^{3/2}\,dx,x,d+ex\right)}{e^2}+\frac{(ef-dg)\text{Subst}\left(\int (a+b\log(cx^n))^{3/2}\,dx,x,d+ex\right)}{e^2}$$

$$=\frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2}-\frac{(3bgn)\text{Subst}\left(\int x\sqrt{a+b\log(cx^n)}\,dx,x,d+ex\right)}{4e^2}-\frac{(3b(ef-dg)n)\text{Subst}\left(\int \sqrt{a+b\log(cx^n)}\,dx,x,d+ex\right)}{2e^2}$$

$$=-\frac{3b(ef-dg)n(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2}-\frac{3bgn(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{8e^2}+\frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2}+\frac{(3b^2gn^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{16e^2}+\frac{(3b^2(ef-dg)n^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{4e^2}$$

$$=-\frac{3b(ef-dg)n(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2}-\frac{3bgn(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{8e^2}+\frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2}+\frac{\left(3b^2gn(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{16e^2}+\frac{\left(3b^2(ef-dg)n(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{4e^2}$$

$$=-\frac{3b(ef-dg)n(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2}-\frac{3bgn(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{8e^2}+\frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2}+\frac{\left(3bgn(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{8e^2}+\frac{\left(3b(ef-dg)n(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{2e^2}$$

$$= \frac{3b^{3/2}e^{-\frac{a}{bn}}(ef-dg)n^{3/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e^2} + \frac{3b^{3/2}e^{-\frac{2a}{bn}}gn^{3/2}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{16e^2} - \frac{3b(ef-dg)n(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e^2} - \frac{3bgn(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{8e^2} + \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e^2} + \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2}$$

## Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.85

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx = \frac{(d+ex)\left(32(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}+16g(d+ex)\left(a+b\log\left(c(d+\right.\right.}{}$$

```
[In] Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] ((d + e*x)*(32*(e*f - d*g)*(a + b*Log[c*(d + e*x)^n])^(3/2) + 16*g*(d + e*x
)*(a + b*Log[c*(d + e*x)^n])^(3/2) + 3*b*g*n*(d + e*x)*((Sqrt[b]*Sqrt[n]*Sq
rt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/
(E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) - 4*Sqrt[a + b*Log[c*(d + e*x)^n]])
 + 24*b*(e*f - d*g)*n*((Sqrt[b]*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d +
 e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) - 2*Sqrt
[a + b*Log[c*(d + e*x)^n]])))/(32*e^2)
```

## Maple [F]

$$\int (gx+f)\left(a+b\ln\left(c(ex+d)^n\right)\right)^{\frac{3}{2}}dx$$

```
[In] int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (f + gx) (a + b \log (c(d + ex)^n))^{3/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (f + gx) (a + b \log (c(d + ex)^n))^{3/2} \, dx = \int (a + b \log (c(d + ex)^n))^{\frac{3}{2}} (f + gx) \, dx$$

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**(3/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**(3/2)*(f + g*x), x)
```

## Maxima [F]

$$\int (f + gx) (a + b \log (c(d + ex)^n))^{3/2} \, dx = \int (gx + f)(b \log ((ex + d)^n c) + a)^{\frac{3}{2}} \, dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
[Out] integrate((g*x + f)*(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Giac [F]

$$\int (f + gx) (a + b \log (c(d + ex)^n))^{3/2} \, dx = \int (gx + f)(b \log ((ex + d)^n c) + a)^{\frac{3}{2}} \, dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
[Out] integrate((g*x + f)*(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx=\int \left(f+g\,x\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^{3/2}\,dx$$

```
[In] int((f + g*x)*(a + b*log(c*(d + e*x)^n))^(3/2),x)
[Out] int((f + g*x)*(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

# 3.113 $\int (a+b\log (c(d+ex)^n))^{3/2}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 143

$$\int (a+b\log (c(d+ex)^n))^{3/2}\,dx=\frac{3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}(d+ex)\,(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e}-\frac{3bn(d+ex)\sqrt{a+b\log (c(d+ex)^n)}}{2e}+\frac{(d+ex)\,(a+b\log (c(d+ex)^n))^{3/2}}{e}$$

```
[Out] (e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e+3/4*b^(3/2)*n^(3/2)*(e*x+d)*erfi((a+b
*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e/exp(a/b/n)/((c*(e*x+d)^
n)^(1/n))-3/2*b*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e
```

## Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2436, 2333, 2337, 2211, 2235}

$$\int (a+b\log (c(d+ex)^n))^{3/2}\,dx=\frac{3\sqrt{\pi}b^{3/2}n^{3/2}e^{-\frac{a}{bn}}(d+ex)\,(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e}+\frac{(d+ex)\,(a+b\log (c(d+ex)^n))^{3/2}}{e}-\frac{3bn(d+ex)\sqrt{a+b\log (c(d+ex)^n)}}{2e}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] (3*b^(3/2)*n^(3/2)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(
Sqrt[b]*Sqrt[n])])/(4*e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) - (3*b*n*(d + e
*x)*Sqrt[a + b*Log[c*(d + e*x)^n]])/(2*e) + ((d + e*x)*(a + b*Log[c*(d + e*
x)^n])^(3/2))/e
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+b\log(cx^n))^{3/2}\, dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{e} - \frac{(3bn)\text{Subst}\left(\int \sqrt{a+b\log(cx^n)}\, dx, x, d+ex\right)}{2e}$$

$$= -\frac{3bn(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{e} + \frac{(3b^2n^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\, dx, x, d+ex\right)}{4e}$$

$$= -\frac{3bn(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e} + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e} + \frac{\left(3b^2n(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{4e}$$

$$= -\frac{3bn(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e} + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e} + \frac{\left(3bn(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{2e}$$

$$= \frac{3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{4e} - \frac{3bn(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{2e} + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{e}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int (a+b\log\left(c(d+ex)^n\right))^{3/2}\,dx = \frac{(d+ex)\left(3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right) + 2\sqrt{a+b\log\ldots}\right.}{4e}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2),x]
[Out] ((d + e*x)*((3*b^(3/2)*n^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]
/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + 2*Sqrt[a + b*Lo
g[c*(d + e*x)^n]]*(2*a - 3*b*n + 2*b*Log[c*(d + e*x)^n])))/(4*e)
```

## Maple [F]

$$\int (a+b\ln\left(c(ex+d)^n\right))^{\frac{3}{2}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(3/2),x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (a + b \log (c(d + ex)^n))^{3/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (a + b \log (c(d + ex)^n))^{3/2} \, dx = \int (a + b \log (c(d + ex)^n))^{\frac{3}{2}} \, dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(3/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**(3/2), x)
```

## Maxima [F]

$$\int (a + b \log (c(d + ex)^n))^{3/2} \, dx = \int (b \log ((ex + d)^n c) + a)^{\frac{3}{2}} \, dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Giac [F]

$$\int (a + b \log (c(d + ex)^n))^{3/2} \, dx = \int (b \log ((ex + d)^n c) + a)^{\frac{3}{2}} \, dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d+ex)^n))^{3/2}\,dx = \int (a+b\,\ln(c\,(d+e\,x)^n))^{3/2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(3/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^(3/2), x)
```

# 3.114 $\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx = \operatorname{Int}\left(\frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x),x]
```

```
[Out] Defer[Int][(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x),x]
[Out] Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x), x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\ln(c(ex+d)^n))^{\frac{3}{2}}}{gx+f}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f),x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(3/2)/(g*x+f),x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f),x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(3/2)/(g*x + f), x)
```

## Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(3/2)/(g*x + f), x)
```

## Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}{f+g\,x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(3/2)/(f + g*x),x)
[Out] int((a + b*log(c*(d + e*x)^n))^(3/2)/(f + g*x), x)
```

# 3.115 $\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx = \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^{3/2}}{(ef-dg)(f+gx)} - \frac{3ben\text{Int}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx},x\right)}{2(ef-dg)}$$

```
[Out] (e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/(-d*g+e*f)/(g*x+f)-3/2*b*e*n*Unintegrab
le((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f),x)/(-d*g+e*f)
```

## Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x)^2,x]
```

```
[Out] ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^(3/2))/((e*f - d*g)*(f + g*x)) - (3*b
*e*n*Defer[Int][Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x), x])/(2*(e*f - d*g
))
```

Rubi steps

$$\text{integral} = \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^{3/2}}{(ef-dg)(f+gx)} - \frac{(3ben)\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{f+gx}\,dx}{2(ef-dg)}$$

## Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx=\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x)^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x)^2, x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\ln(c(ex+d)^n))^{\frac{3}{2}}}{(gx+f)^2}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f)^2,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f)^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f)^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(3/2)/(g*x+f)**2,x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^{\frac{3}{2}}}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f)^2,x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(3/2)/(g*x + f)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^{\frac{3}{2}}}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(3/2)/(g*x + f)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}{(f+g\,x)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(3/2)/(f + g*x)^2,x)
[Out] int((a + b*log(c*(d + e*x)^n))^(3/2)/(f + g*x)^2, x)
```

# 3.116 $\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3} dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3} dx = -\frac{(a+b\log(c(d+ex)^n))^{3/2}}{2g(f+gx)^2} + \frac{3ben\text{Int}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{(d+ex)(f+gx)^2}, x\right)}{4g}$$

```
[Out] -1/2*(a+b*ln(c*(e*x+d)^n))^(3/2)/g/(g*x+f)^2+3/4*b*e*n*Unintegrable((a+b*ln
(c*(e*x+d)^n))^(1/2)/(e*x+d)/(g*x+f)^2,x)/g
```

## Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3} dx = \int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3} dx$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x)^3,x]
```

```
[Out] -1/2*(a + b*Log[c*(d + e*x)^n])^(3/2)/(g*(f + g*x)^2) + (3*b*e*n*Defer[Int]
[Sqrt[a + b*Log[c*(d + e*x)^n]]/((d + e*x)*(f + g*x)^2), x])/(4*g)
```

Rubi steps

$$\text{integral} = -\frac{(a+b\log(c(d+ex)^n))^{3/2}}{2g(f+gx)^2} + \frac{(3ben)\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{(d+ex)(f+gx)^2} dx}{4g}$$

## Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx=\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x)^3,x]
[Out] Integrate[(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x)^3, x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\ln(c(ex+d)^n))^{\frac{3}{2}}}{(gx+f)^3}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f)^3,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f)^3,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f)^3,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(3/2)/(g*x+f)**3,x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}{(gx+f)^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f)^3,x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(3/2)/(g*x + f)^3, x)
```

## Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}{(gx+f)^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(3/2)/(g*x+f)^3,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(3/2)/(g*x + f)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(f+gx)^3}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}{(f+g\,x)^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(3/2)/(f + g*x)^3,x)
[Out] int((a + b*log(c*(d + e*x)^n))^(3/2)/(f + g*x)^3, x)
```

# 3.117 $\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{5/2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 660

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{5/2}\,dx = -\frac{15b^{5/2}e^{-\frac{a}{bn}}(ef-dg)^2n^{5/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^3} - \frac{15b^{5/2}e^{-\frac{2a}{bn}}g(ef-dg)n^{5/2}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{32e^3} - \frac{5b^{5/2}e^{-\frac{3a}{bn}}g^2n^{5/2}\sqrt{\frac{\pi}{3}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{72e^3} + \frac{15b^2(ef-dg)^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e^3} + \frac{15b^2g(ef-dg)n^2(d+ex)^2\sqrt{a+b\log(c(d+ex)^n)}}{16e^3} + \frac{5b^2g^2n^2(d+ex)^3\sqrt{a+b\log(c(d+ex)^n)}}{36e^3} - \frac{5b(ef-dg)^2n(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e^3} - \frac{5bg(ef-dg)n(d+ex)^2(a+b\log(c(d+ex)^n))^{3/2}}{4e^3} - \frac{5bg^2n(d+ex)^3(a+b\log(c(d+ex)^n))^{3/2}}{18e^3} + \frac{(ef-dg)^2(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e^3} + \frac{g(ef-dg)(d+ex)^2(a+b\log(c(d+ex)^n))^{5/2}}{e^3} + \frac{g^2(d+ex)^3(a+b\log(c(d+ex)^n))^{5/2}}{3e^3}$$

```
[Out] -5/2*b*(-d*g+e*f)^2*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^3-5/4*b*g*(-d*g
+e*f)*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^3-5/18*b*g^2*n*(e*x+d)^3*(a
```

```
+b*ln(c*(e*x+d)^n))^(3/2)/e^3+(-d*g+e*f)^2*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(5
/2)/e^3+g*(-d*g+e*f)*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^(5/2)/e^3+1/3*g^2*(e*x
+d)^3*(a+b*ln(c*(e*x+d)^n))^(5/2)/e^3-5/216*b^(5/2)*g^2*n^(5/2)*(e*x+d)^3*e
rfi(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2)*Pi^(1/2)/e
^3/exp(3*a/b/n)/((c*(e*x+d)^n)^(3/n))-15/64*b^(5/2)*g*(-d*g+e*f)*n^(5/2)*(e
*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*P
i^(1/2)/e^3/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))-15/8*b^(5/2)*(-d*g+e*f)^2*n^
(5/2)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e^
3/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+15/4*b^2*(-d*g+e*f)^2*n^2*(e*x+d)*(a+b*l
n(c*(e*x+d)^n))^(1/2)/e^3+15/16*b^2*g*(-d*g+e*f)*n^2*(e*x+d)^2*(a+b*ln(c*(e
*x+d)^n))^(1/2)/e^3+5/36*b^2*g^2*n^2*(e*x+d)^3*(a+b*ln(c*(e*x+d)^n))^(1/2)/
e^3
```

## Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {2448, 2436, 2333, 2337, 2211, 2235, 2437, 2342, 2347}

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{5/2}\,dx =$$

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}gn^{5/2}e^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\,(c(d+ex)^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{32e^3}$$
$$-\frac{15\sqrt{\pi}b^{5/2}n^{5/2}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\,(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^3}$$
$$-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}g^2n^{5/2}e^{-\frac{3a}{bn}}(d+ex)^3\,(c(d+ex)^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{72e^3}$$
$$+\frac{15b^2gn^2(d+ex)^2(ef-dg)\sqrt{a+b\log(c(d+ex)^n)}}{16e^3}$$
$$+\frac{15b^2n^2(d+ex)(ef-dg)^2\sqrt{a+b\log(c(d+ex)^n)}}{4e^3}$$
$$+\frac{5b^2g^2n^2(d+ex)^3\sqrt{a+b\log(c(d+ex)^n)}}{36e^3}+\frac{g(d+ex)^2(ef-dg)\,(a+b\log(c(d+ex)^n))^{5/2}}{e^3}$$
$$+\frac{(d+ex)(ef-dg)^2\,(a+b\log(c(d+ex)^n))^{5/2}}{e^3}$$
$$-\frac{5bgn(d+ex)^2(ef-dg)\,(a+b\log(c(d+ex)^n))^{3/2}}{4e^3}$$
$$-\frac{5bn(d+ex)(ef-dg)^2\,(a+b\log(c(d+ex)^n))^{3/2}}{2e^3}$$
$$+\frac{g^2(d+ex)^3\,(a+b\log(c(d+ex)^n))^{5/2}}{3e^3}-\frac{5bg^2n(d+ex)^3\,(a+b\log(c(d+ex)^n))^{3/2}}{18e^3}$$

```
[In] Int[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] (-15*b^(5/2)*(e*f - d*g)^2*n^(5/2)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c
*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(8*e^3*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1
)) - (15*b^(5/2)*g*(e*f - d*g)*n^(5/2)*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]
*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(32*e^3*E^((2*a)/(b*n)
)*(c*(d + e*x)^n)^(2/n)) - (5*b^(5/2)*g^2*n^(5/2)*Sqrt[Pi/3]*(d + e*x)^3*Er
fi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(72*e^3*E^(
(3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)) + (15*b^2*(e*f - d*g)^2*n^2*(d + e*x)*S
qrt[a + b*Log[c*(d + e*x)^n]])/(4*e^3) + (15*b^2*g*(e*f - d*g)*n^2*(d + e*x
)^2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(16*e^3) + (5*b^2*g^2*n^2*(d + e*x)^3*S
qrt[a + b*Log[c*(d + e*x)^n]])/(36*e^3) - (5*b*(e*f - d*g)^2*n*(d + e*x)*(a
 + b*Log[c*(d + e*x)^n])^(3/2))/(2*e^3) - (5*b*g*(e*f - d*g)*n*(d + e*x)^2*
(a + b*Log[c*(d + e*x)^n])^(3/2))/(4*e^3) - (5*b*g^2*n*(d + e*x)^3*(a + b*L
og[c*(d + e*x)^n])^(3/2))/(18*e^3) + ((e*f - d*g)^2*(d + e*x)*(a + b*Log[c*
(d + e*x)^n])^(5/2))/e^3 + (g*(e*f - d*g)*(d + e*x)^2*(a + b*Log[c*(d + e*x
)^n])^(5/2))/e^3 + (g^2*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n])^(5/2))/(3*e^
3)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(ef-dg)^2 (a+b\log(c(d+ex)^n))^{5/2}}{e^2} + \frac{2g(ef-dg)(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e^2} + \frac{g^2(d+ex)^2 (a+b\log(c(d+ex)^n))^{5/2}}{e^2}\right) dx$$

$$= \frac{g^2 \int (d+ex)^2 (a+b\log(c(d+ex)^n))^{5/2}\, dx}{e^2} + \frac{(2g(ef-dg)) \int (d+ex)(a+b\log(c(d+ex)^n))^{5/2}\, dx}{e^2} + \frac{(ef-dg)^2 \int (a+b\log(c(d+ex)^n))^{5/2}\, dx}{e^2}$$

$$= \frac{g^2 \text{Subst}\left(\int x^2 (a+b\log(cx^n))^{5/2}\,dx, x, d+ex\right)}{e^3} + \frac{(2g(ef-dg))\text{Subst}\left(\int x (a+b\log(cx^n))^{5/2}\,dx, x, d+ex\right)}{e^3} + \frac{(ef-dg)^2 \text{Subst}\left(\int (a+b\log(cx^n))^{5/2}\,dx, x, d+ex\right)}{e^3}$$

$$= \frac{(ef-dg)^2 (d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^3} + \frac{g(ef-dg)(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^3} + \frac{g^2 (d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{3e^3} - \frac{(5bg^2 n)\,\text{Subst}\left(\int x^2 (a+b\log(cx^n))^{3/2}\,dx, x, d+ex\right)}{6e^3} - \frac{(5bg(ef-dg)n)\text{Subst}\left(\int x (a+b\log(cx^n))^{3/2}\,dx, x, d+ex\right)}{2e^3} - \frac{(5b(ef-dg)^2 n)\,\text{Subst}\left(\int (a+b\log(cx^n))^{3/2}\,dx, x, d+ex\right)}{2e^3}$$

$$= -\frac{5b(ef-dg)^2 n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^3} - \frac{5bg(ef-dg)n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{4e^3} - \frac{5bg^2 n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{18e^3} + \frac{(ef-dg)^2 (d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^3} + \frac{g(ef-dg)(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^3} + \frac{g^2 (d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{3e^3} + \frac{(5b^2 g^2 n^2)\,\text{Subst}\left(\int x^2 \sqrt{a+b\log(cx^n)}\,dx, x, d+ex\right)}{12e^3} + \frac{(15b^2 g(ef-dg)n^2)\,\text{Subst}\left(\int x \sqrt{a+b\log(cx^n)}\,dx, x, d+ex\right)}{8e^3} + \frac{(15b^2 (ef-dg)^2 n^2)\,\text{Subst}\left(\int \sqrt{a+b\log(cx^n)}\,dx, x, d+ex\right)}{4e^3}$$

$$
\begin{aligned}
&= \frac{15b^2(ef-dg)^2n^2(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^3} \\
&\quad+ \frac{15b^2g(ef-dg)n^2(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{16e^3} \\
&\quad+ \frac{5b^2g^2n^2(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{36e^3} \\
&\quad- \frac{5b(ef-dg)^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^3} \\
&\quad- \frac{5bg(ef-dg)n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{4e^3} \\
&\quad- \frac{5bg^2n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{18e^3} \\
&\quad+ \frac{(ef-dg)^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^3} \\
&\quad+ \frac{g(ef-dg)(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^3} \\
&\quad+ \frac{g^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{3e^3} \\
&\quad- \frac{(5b^3g^2n^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{72e^3} \\
&\quad- \frac{(15b^3g(ef-dg)n^3)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{32e^3} \\
&\quad- \frac{(15b^3(ef-dg)^2n^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{8e^3}
\end{aligned}
$$

$$= \frac{15b^2(ef-dg)^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e^3} + \frac{15b^2g(ef-dg)n^2(d+ex)^2\sqrt{a+b\log(c(d+ex)^n)}}{16e^3} + \frac{5b^2g^2n^2(d+ex)^3\sqrt{a+b\log(c(d+ex)^n)}}{36e^3} - \frac{5b(ef-dg)^2n(d+ex)\left(a+b\log(c(d+ex)^n)\right)^{3/2}}{2e^3} - \frac{5bg(ef-dg)n(d+ex)^2\left(a+b\log(c(d+ex)^n)\right)^{3/2}}{4e^3} - \frac{5bg^2n(d+ex)^3\left(a+b\log(c(d+ex)^n)\right)^{3/2}}{18e^3} + \frac{(ef-dg)^2(d+ex)\left(a+b\log(c(d+ex)^n)\right)^{5/2}}{e^3} + \frac{g(ef-dg)(d+ex)^2\left(a+b\log(c(d+ex)^n)\right)^{5/2}}{e^3} + \frac{g^2(d+ex)^3\left(a+b\log(c(d+ex)^n)\right)^{5/2}}{3e^3} - \frac{\left(5b^3g^2n^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}dx,x,\log(c(d+ex)^n)\right)}{72e^3} - \frac{\left(15b^3g(ef-dg)n^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}dx,x,\log(c(d+ex)^n)\right)}{32e^3} - \frac{\left(15b^3(ef-dg)^2n^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}dx,x,\log(c(d+ex)^n)\right)}{8e^3}$$

$$= \frac{15b^2(ef-dg)^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e^3} + \frac{15b^2g(ef-dg)n^2(d+ex)^2\sqrt{a+b\log(c(d+ex)^n)}}{16e^3} + \frac{5b^2g^2n^2(d+ex)^3\sqrt{a+b\log(c(d+ex)^n)}}{36e^3} - \frac{5b(ef-dg)^2n(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e^3} - \frac{5bg(ef-dg)n(d+ex)^2(a+b\log(c(d+ex)^n))^{3/2}}{4e^3} - \frac{5bg^2n(d+ex)^3(a+b\log(c(d+ex)^n))^{3/2}}{18e^3} + \frac{(ef-dg)^2(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e^3} + \frac{g(ef-dg)(d+ex)^2(a+b\log(c(d+ex)^n))^{5/2}}{e^3} + \frac{g^2(d+ex)^3(a+b\log(c(d+ex)^n))^{5/2}}{3e^3} - \frac{\left(5b^2g^2n^2(d+ex)^3(c(d+ex)^n)^{-3/n}\right)\text{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx, x, \sqrt{a+b\log(c(d+ex)^n)}\right)}{36e^3} - \frac{\left(15b^2g(ef-dg)n^2(d+ex)^2(c(d+ex)^n)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log(c(d+ex)^n)}\right)}{16e^3} - \frac{\left(15b^2(ef-dg)^2n^2(d+ex)(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log(c(d+ex)^n)}\right)}{4e^3}$$

$$= -\frac{15b^{5/2}e^{-\frac{a}{bn}}(ef-dg)^2n^{5/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^3}$$
$$-\frac{15b^{5/2}e^{-\frac{2a}{bn}}g(ef-dg)n^{5/2}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{32e^3}$$
$$-\frac{5b^{5/2}e^{-\frac{3a}{bn}}g^2n^{5/2}\sqrt{\frac{\pi}{3}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{72e^3}$$
$$+\frac{15b^2(ef-dg)^2n^2(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^3}$$
$$+\frac{15b^2g(ef-dg)n^2(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{16e^3}$$
$$+\frac{5b^2g^2n^2(d+ex)^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}{36e^3}$$
$$-\frac{5b(ef-dg)^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^3}$$
$$-\frac{5bg(ef-dg)n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{4e^3}$$
$$-\frac{5bg^2n(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{18e^3}$$
$$+\frac{(ef-dg)^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^3}$$
$$+\frac{g(ef-dg)(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^3}$$
$$+\frac{g^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{3e^3}$$

## Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 511, normalized size of antiderivative = 0.77

$$\int (f+gx)^2 (a+b\log\left(c(d+ex)^n\right))^{5/2}\,dx = \frac{(d+ex)\left(1728(ef-dg)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}+1728g(ef-dg)(d+ex)\left(a\right.\right.}{}$$

```
[In] Integrate[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] ((d + e*x)*(1728*(e*f - d*g)^2*(a + b*Log[c*(d + e*x)^n])^(5/2) + 1728*g*(e
*f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^(5/2) + 576*g^2*(d + e*x)^2*
(a + b*Log[c*(d + e*x)^n])^(5/2) - 1080*b*(e*f - d*g)^2*n*((3*b^(3/2)*n^(3/
2)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(
b*n))*(c*(d + e*x)^n)^n^(-1)) + 2*Sqrt[a + b*Log[c*(d + e*x)^n]]*(2*a - 3*b
*n + 2*b*Log[c*(d + e*x)^n])) - 40*b*g^2*n*(d + e*x)^2*((b^(3/2)*n^(3/2)*Sq
```

```
rt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/
(E^((3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)) + 6*Sqrt[a + b*Log[c*(d + e*x)^n]]*
(2*a - b*n + 2*b*Log[c*(d + e*x)^n])) - 135*b*g*(e*f - d*g)*n*(d + e*x)*((3
*b^(3/2)*n^(3/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(
Sqrt[b]*Sqrt[n])])/(E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) + 4*Sqrt[a + b*L
og[c*(d + e*x)^n]]*(4*a - 3*b*n + 4*b*Log[c*(d + e*x)^n]))))/(1728*e^3)
```

## Maple [F]

$$\int (gx+f)^2 (a+b\ln(c(ex+d)^n))^{\frac{5}{2}} dx$$

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{5/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^{5/2} \, dx = \int (a+b\log(c(d+ex)^n))^{\frac{5}{2}} (f+gx)^2 \, dx$$

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n))**(5/2),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**(5/2)*(f + g*x)**2, x)
```

## Maxima [F]

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))^{5/2}\,dx = \int (gx+f)^2(b\log ((ex+d)^n c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Giac [F]

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))^{5/2}\,dx = \int (gx+f)^2(b\log ((ex+d)^n c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
[Out] integrate((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))^{5/2}\,dx = \int (f+g\,x)^2\,(a+b\,\ln (c\,(d+e\,x)^n))^{5/2}\,dx$$

```
[In] int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^(5/2),x)
[Out] int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

# 3.118 $\int (f+gx)(a+b\log(c(d+ex)^n))^{5/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 413

$$\begin{aligned}\int (f+gx)(a+b\log(c(d+ex)^n))^{5/2}\,dx &= -\frac{15b^{5/2}e^{-\frac{a}{bn}}(ef-dg)n^{5/2}\sqrt{\pi}(d+ex)(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^2}\\
&\quad-\frac{15b^{5/2}e^{-\frac{2a}{bn}}gn^{5/2}\sqrt{\frac{\pi}{2}}(d+ex)^2(c(d+ex)^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{64e^2}\\
&\quad+\frac{15b^2(ef-dg)n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e^2}\\
&\quad+\frac{15b^2gn^2(d+ex)^2\sqrt{a+b\log(c(d+ex)^n)}}{32e^2}\\
&\quad-\frac{5b(ef-dg)n(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e^2}\\
&\quad-\frac{5bgn(d+ex)^2(a+b\log(c(d+ex)^n))^{3/2}}{8e^2}\\
&\quad+\frac{(ef-dg)(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e^2}\\
&\quad+\frac{g(d+ex)^2(a+b\log(c(d+ex)^n))^{5/2}}{2e^2}\end{aligned}$$

```
[Out] -5/2*b*(-d*g+e*f)*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^2-5/8*b*g*n*(e*x+
d)^2*(a+b*ln(c*(e*x+d)^n))^(3/2)/e^2+(-d*g+e*f)*(e*x+d)*(a+b*ln(c*(e*x+d)^n
))^(5/2)/e^2+1/2*g*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^(5/2)/e^2-15/128*b^(5/2)
*g*n^(5/2)*(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/
2))*2^(1/2)*Pi^(1/2)/e^2/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))-15/8*b^(5/2)*(-
d*g+e*f)*n^(5/2)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*
```

```
Pi^(1/2)/e^2/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+15/4*b^2*(-d*g+e*f)*n^2*(e*x+
d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e^2+15/32*b^2*g*n^2*(e*x+d)^2*(a+b*ln(c*(e*x
+d)^n))^(1/2)/e^2
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2448, 2436, 2333, 2337, 2211, 2235, 2437, 2342, 2347}

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}\,dx = -\frac{15\sqrt{\pi}b^{5/2}n^{5/2}e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^2} - \frac{15\sqrt{\frac{\pi}{2}}b^{5/2}gn^{5/2}e^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{64e^2} + \frac{15b^2n^2(d+ex)(ef-dg)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^2} + \frac{15b^2gn^2(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{32e^2} + \frac{(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^2} - \frac{5bn(d+ex)(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2} + \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{2e^2} - \frac{5bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{8e^2}$$

```
[In] Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] (-15*b^(5/2)*(e*f - d*g)*n^(5/2)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(
d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(8*e^2*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1))
 - (15*b^(5/2)*g*n^(5/2)*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Lo
g[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(64*e^2*E^((2*a)/(b*n))*(c*(d + e*x)
^n)^(2/n)) + (15*b^2*(e*f - d*g)*n^2*(d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n
]])/(4*e^2) + (15*b^2*g*n^2*(d + e*x)^2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(32
*e^2) - (5*b*(e*f - d*g)*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^(3/2))/(2*e
^2) - (5*b*g*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^(3/2))/(8*e^2) + ((e*
f - d*g)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^(5/2))/e^2 + (g*(d + e*x)^2*(
a + b*Log[c*(d + e*x)^n])^(5/2))/(2*e^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

integral

$$= \int \left(\frac{(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e}+\frac{g(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e}\right)dx$$

$$= \frac{g\int (d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}\,dx}{e}+\frac{(ef-dg)\int \left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}\,dx}{e}$$

$$= \frac{g\mathrm{Subst}\left(\int x(a+b\log\left(cx^n\right))^{5/2}\,dx,x,d+ex\right)}{e^2}+\frac{(ef-dg)\mathrm{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)^{5/2}\,dx,x,d+ex\right)}{e^2}$$

$$= \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{2e^2}-\frac{(5bgn)\mathrm{Subst}\left(\int x(a+b\log\left(cx^n\right))^{3/2}\,dx,x,d+ex\right)}{4e^2}-\frac{(5b(ef-dg)n)\mathrm{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)^{3/2}\,dx,x,d+ex\right)}{2e^2}$$

$$= -\frac{5b(ef-dg)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2}-\frac{5bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{8e^2}+\frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^2}+\frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{2e^2}+\frac{(15b^2gn^2)\,\mathrm{Subst}\left(\int x\sqrt{a+b\log\left(cx^n\right)}\,dx,x,d+ex\right)}{16e^2}+\frac{(15b^2(ef-dg)n^2)\,\mathrm{Subst}\left(\int \sqrt{a+b\log\left(cx^n\right)}\,dx,x,d+ex\right)}{4e^2}$$

$$= \frac{15b^2(ef-dg)n^2(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^2} + \frac{15b^2gn^2(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{32e^2} - \frac{5b(ef-dg)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2} - \frac{5bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{8e^2} + \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^2} + \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{2e^2} - \frac{\left(15b^3gn^3\right)\operatorname{Subst}\left(\int\frac{x}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{64e^2} - \frac{\left(15b^3(ef-dg)n^3\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{8e^2}$$

$$= \frac{15b^2(ef-dg)n^2(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^2} + \frac{15b^2gn^2(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{32e^2} - \frac{5b(ef-dg)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2} - \frac{5bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{8e^2} + \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^2} + \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{2e^2} - \frac{\left(15b^3gn^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{64e^2} - \frac{\left(15b^3(ef-dg)n^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{8e^2}$$

$$= \frac{15b^2(ef-dg)n^2(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^2} + \frac{15b^2gn^2(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{32e^2} - \frac{5b(ef-dg)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2} - \frac{5bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{8e^2} + \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^2} + \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{2e^2} - \frac{\left(15b^2gn^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{32e^2} - \frac{\left(15b^2(ef-dg)n^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{4e^2}$$

$$= -\frac{15b^{5/2}e^{-\frac{a}{bn}}(ef-dg)n^{5/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e^2} - \frac{15b^{5/2}e^{-\frac{2a}{bn}}gn^{5/2}\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{64e^2} + \frac{15b^2(ef-dg)n^2(d+ex)\sqrt{a+b\log\left(c(d+ex)^n\right)}}{4e^2} + \frac{15b^2gn^2(d+ex)^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{32e^2} - \frac{5b(ef-dg)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{2e^2} - \frac{5bgn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}{8e^2} + \frac{(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{e^2} + \frac{g(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}{2e^2}$$

## Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.79

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}\,dx = \frac{(d+ex)\left(128(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}+64g(d+ex)\left(a+b\log\left(c(d+\right.\right.}{}$$

```
[In] Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] ((d + e*x)*(128*(e*f - d*g)*(a + b*Log[c*(d + e*x)^n])^(5/2) + 64*g*(d + e*
x)*(a + b*Log[c*(d + e*x)^n])^(5/2) - 80*b*(e*f - d*g)*n*((3*b^(3/2)*n^(3/2
)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b
*n))*(c*(d + e*x)^n)^n^(-1)) + 2*Sqrt[a + b*Log[c*(d + e*x)^n]]*(2*a - 3*b*
n + 2*b*Log[c*(d + e*x)^n])) - 5*b*g*n*(d + e*x)*((3*b^(3/2)*n^(3/2)*Sqrt[2
*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(E^(
(2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) + 4*Sqrt[a + b*Log[c*(d + e*x)^n]]*(4*a
 - 3*b*n + 4*b*Log[c*(d + e*x)^n]))))/(128*e^2)
```

## Maple [F]

$$\int (gx+f)\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{\frac{5}{2}}dx$$

```
[In] int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}\,dx=\int\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{\frac{5}{2}}(f+gx)\,dx$$

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**(5/2),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**(5/2)*(f + g*x), x)
```

## Maxima [F]

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}\,dx=\int (gx+f)(b\log\left((ex+d)^n c\right)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
[Out] integrate((g*x + f)*(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Giac [F]

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}\,dx=\int (gx+f)(b\log\left((ex+d)^n c\right)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
[Out] integrate((g*x + f)*(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}\,dx=\int (f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^{5/2}\,dx$$

```
[In] int((f + g*x)*(a + b*log(c*(d + e*x)^n))^(5/2),x)
[Out] int((f + g*x)*(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

# 3.119 $\int (a+b\log (c(d+ex)^n))^{5/2}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 179

$$\int (a+b\log (c(d+ex)^n))^{5/2}\,dx = -\frac{15b^{5/2}e^{-\frac{a}{bn}}n^{5/2}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e} + \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} - \frac{5bn(d+ex)\left(a+b\log(c(d+ex)^n)\right)^{3/2}}{2e} + \frac{(d+ex)\left(a+b\log(c(d+ex)^n)\right)^{5/2}}{e}$$

```
[Out] -5/2*b*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^(3/2)/e+(e*x+d)*(a+b*ln(c*(e*x+d)^n)
)^(5/2)/e-15/8*b^(5/2)*n^(5/2)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(
1/2)/n^(1/2))*Pi^(1/2)/e/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))+15/4*b^2*n^2*(e*x
+d)*(a+b*ln(c*(e*x+d)^n))^(1/2)/e
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used

$= \{2436, 2333, 2337, 2211, 2235\}$

$$\int (a+b\log (c(d+ex)^n))^{5/2}\,dx = -\frac{15\sqrt{\pi}b^{5/2}n^{5/2}e^{-\frac{a}{bn}}(d+ex)\,(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e} + \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} + \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^{5/2}}{e} - \frac{5bn(d+ex)\,(a+b\log(c(d+ex)^n))^{3/2}}{2e}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] (-15*b^(5/2)*n^(5/2)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]
/(Sqrt[b]*Sqrt[n])])/(8*e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (15*b^2*n^2
*(d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]])/(4*e) - (5*b*n*(d + e*x)*(a + b*
Log[c*(d + e*x)^n])^(3/2))/(2*e) + ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^(5
/2))/e
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+b\log(cx^n))^{5/2}\,dx,x,d+ex\right)}{e}$$

$$= \frac{(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e} - \frac{(5bn)\text{Subst}\left(\int (a+b\log(cx^n))^{3/2}\,dx,x,d+ex\right)}{2e}$$

$$= -\frac{5bn(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e} + \frac{(15b^2n^2)\,\text{Subst}\left(\int \sqrt{a+b\log(cx^n)}\,dx,x,d+ex\right)}{4e}$$

$$= \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} - \frac{5bn(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e} - \frac{(15b^3n^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{8e}$$

$$= \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} - \frac{5bn(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e} - \frac{\left(15b^3n^2(d+ex)(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log(c(d+ex)^n)\right)}{8e}$$

$$= \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} - \frac{5bn(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e} - \frac{\left(15b^2n^2(d+ex)(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx,x,\sqrt{a+b\log(c(d+ex)^n)}\right)}{4e}$$

$$= -\frac{15b^{5/2}e^{-\frac{a}{bn}}n^{5/2}\sqrt{\pi}(d+ex)(c(d+ex)^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{8e} + \frac{15b^2n^2(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}{4e} - \frac{5bn(d+ex)(a+b\log(c(d+ex)^n))^{3/2}}{2e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^{5/2}}{e}$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.85

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx = \frac{(d+ex)\left(8(a+b\log(c(d+ex)^n))^{5/2}-5bn\left(3b^{3/2}e^{-\frac{a}{bn}}n^{3/2}\sqrt{\pi}(c(d+ex)^n)^{-1/n}\,\mathrm{er}\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] ((d + e*x)*(8*(a + b*Log[c*(d + e*x)^n])^(5/2) - 5*b*n*((3*b^(3/2)*n^(3/2)*
Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n
))*(c*(d + e*x)^n)^n^(-1)) + 2*Sqrt[a + b*Log[c*(d + e*x)^n]]*(2*a - 3*b*n
+ 2*b*Log[c*(d + e*x)^n]))))/(8*e)
```

## Maple [F]

$$\int (a+b\ln(c(ex+d)^n))^{\frac{5}{2}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx=\int (a+b\log(c(d+ex)^n))^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(5/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**(5/2), x)
```

## Maxima [F]

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx=\int (b\log((ex+d)^n c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Giac [F]

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx=\int (b\log((ex+d)^n c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d+ex)^n))^{5/2}\,dx=\int (a+b\,\ln(c\,(d+e\,x)^n))^{5/2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(5/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^(5/2), x)
```

# 3.120 $\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx} dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx} dx = \operatorname{Int}\left(\frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}, x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx} dx = \int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx} dx$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x),x]
```

```
[Out] Defer[Int][(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx} dx$$

## Mathematica [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx=\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x),x]
[Out] Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x), x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\ln(c(ex+d)^n))^{\frac{5}{2}}}{gx+f}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f),x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(5/2)/(g*x+f),x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^{\frac{5}{2}}}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f),x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(5/2)/(g*x + f), x)
```

## Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^{\frac{5}{2}}}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(5/2)/(g*x + f), x)
```

## Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{f+gx}\,dx=\int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^{5/2}}{f+g\,x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(5/2)/(f + g*x),x)
[Out] int((a + b*log(c*(d + e*x)^n))^(5/2)/(f + g*x), x)
```

# 3.121 $\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx = \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^{5/2}}{(ef-dg)(f+gx)} - \frac{5ben\text{Int}\left(\frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx},x\right)}{2(ef-dg)}$$

```
[Out] (e*x+d)*(a+b*ln(c*(e*x+d)^n))^(5/2)/(-d*g+e*f)/(g*x+f)-5/2*b*e*n*Unintegrab
le((a+b*ln(c*(e*x+d)^n))^(3/2)/(g*x+f),x)/(-d*g+e*f)
```

## Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x)^2,x]
```

```
[Out] ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^(5/2))/((e*f - d*g)*(f + g*x)) - (5*b
*e*n*Defer[Int][(a + b*Log[c*(d + e*x)^n])^(3/2)/(f + g*x), x])/(2*(e*f - d
*g))
```

Rubi steps

$$\text{integral} = \frac{(d+ex)\,(a+b\log(c(d+ex)^n))^{5/2}}{(ef-dg)(f+gx)} - \frac{(5ben)\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{f+gx}\,dx}{2(ef-dg)}$$

## Mathematica [N/A]

Not integrable

Time = 7.92 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x)^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x)^2, x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\ln(c(ex+d)^n))^{\frac{5}{2}}}{(gx+f)^2}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f)^2,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f)^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f)^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(5/2)/(g*x+f)**2,x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^{\frac{5}{2}}}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f)^2,x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(5/2)/(g*x + f)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^{\frac{5}{2}}}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(5/2)/(g*x + f)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^2}\,dx=\int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^{5/2}}{(f+g\,x)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(5/2)/(f + g*x)^2,x)
[Out] int((a + b*log(c*(d + e*x)^n))^(5/2)/(f + g*x)^2, x)
```

# 3.122 $\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3} dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3} dx = -\frac{(a+b\log(c(d+ex)^n))^{5/2}}{2g(f+gx)^2} + \frac{5ben\text{Int}\left(\frac{(a+b\log(c(d+ex)^n))^{3/2}}{(d+ex)(f+gx)^2},x\right)}{4g}$$

```
[Out] -1/2*(a+b*ln(c*(e*x+d)^n))^(5/2)/g/(g*x+f)^2+5/4*b*e*n*Unintegrable((a+b*ln
(c*(e*x+d)^n))^(3/2)/(e*x+d)/(g*x+f)^2,x)/g
```

## Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3} dx = \int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3} dx$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x)^3,x]
```

```
[Out] -1/2*(a + b*Log[c*(d + e*x)^n])^(5/2)/(g*(f + g*x)^2) + (5*b*e*n*Defer[Int]
[(a + b*Log[c*(d + e*x)^n])^(3/2)/((d + e*x)*(f + g*x)^2), x])/(4*g)
```

Rubi steps

$$\text{integral} = -\frac{(a+b\log(c(d+ex)^n))^{5/2}}{2g(f+gx)^2} + \frac{(5ben)\int \frac{(a+b\log(c(d+ex)^n))^{3/2}}{(d+ex)(f+gx)^2} dx}{4g}$$

## Mathematica [N/A]

Not integrable

Time = 5.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx=\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x)^3,x]
[Out] Integrate[(a + b*Log[c*(d + e*x)^n])^(5/2)/(f + g*x)^3, x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\ln(c(ex+d)^n))^{\frac{5}{2}}}{(gx+f)^3}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f)^3,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(5/2)/(g*x+f)^3,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f)^3,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(5/2)/(g*x+f)**3,x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^{\frac{5}{2}}}{(gx+f)^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f)^3,x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(5/2)/(g*x + f)^3, x)
```

## Giac [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^{\frac{5}{2}}}{(gx+f)^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(5/2)/(g*x+f)^3,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(5/2)/(g*x + f)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex)^n))^{5/2}}{(f+gx)^3}\,dx=\int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^{5/2}}{(f+g\,x)^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(5/2)/(f + g*x)^3,x)
[Out] int((a + b*log(c*(d + e*x)^n))^(5/2)/(f + g*x)^3, x)
```

# 3.123 $\int \frac{(f+gx)^3}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 383

$$\begin{aligned}
\int \frac{(f+gx)^3}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx
&= \frac{e^{-\frac{a}{bn}}(ef-dg)^3\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}} \\
&\quad+\frac{e^{-\frac{4a}{bn}}g^3\sqrt{\pi}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2\sqrt{b}e^4\sqrt{n}} \\
&\quad+\frac{3e^{-\frac{2a}{bn}}g(ef-dg)^2\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}} \\
&\quad+\frac{e^{-\frac{3a}{bn}}g^2(ef-dg)\sqrt{3\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}}
\end{aligned}$$

```
[Out] 3/2*g*(-d*g+e*f)^2*(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/
2)/n^(1/2))*2^(1/2)*Pi^(1/2)/e^4/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))/b^(1/2)
/n^(1/2)+(-d*g+e*f)^3*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1
/2))*Pi^(1/2)/e^4/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))/b^(1/2)/n^(1/2)+1/2*g^3*
(e*x+d)^4*erfi(2*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e^4/
exp(4*a/b/n)/((c*(e*x+d)^n)^(4/n))/b^(1/2)/n^(1/2)+g^2*(-d*g+e*f)*(e*x+d)^3
*erfi(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2)*Pi^(1/2)
/e^4/exp(3*a/b/n)/((c*(e*x+d)^n)^(3/n))/b^(1/2)/n^(1/2)
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2448, 2436, 2337, 2211, 2235, 2437, 2347}

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \frac{\sqrt{3\pi}g^2e^{-\frac{3a}{bn}}(d+ex)^3(ef-dg)\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}} + \frac{3\sqrt{\frac{\pi}{2}}ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}} + \frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^3\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}} + \frac{\sqrt{\pi}g^3e^{-\frac{4a}{bn}}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2\sqrt{b}e^4\sqrt{n}}$$

```
[In] Int[(f + g*x)^3/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] ((e*f - d*g)^3*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt
[b]*Sqrt[n])])/(Sqrt[b]*e^4*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)) + (
g^3*Sqrt[Pi]*(d + e*x)^4*Erfi[(2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*S
qrt[n])])/(2*Sqrt[b]*e^4*E^((4*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(4/n)) + (
3*g*(e*f - d*g)^2*Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d
+ e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^4*E^((2*a)/(b*n))*Sqrt[n]*(c*(d
+ e*x)^n)^(2/n)) + (g^2*(e*f - d*g)*Sqrt[3*Pi]*(d + e*x)^3*Erfi[(Sqrt[3]*Sq
rt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^4*E^((3*a)/(b*
n))*Sqrt[n]*(c*(d + e*x)^n)^(3/n))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(ef-dg)^3}{e^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}+\frac{3g(ef-dg)^2(d+ex)}{e^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}\right.\\
&\qquad \left.+\frac{3g^2(ef-dg)(d+ex)^2}{e^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}+\frac{g^3(d+ex)^3}{e^3\sqrt{a+b\log\left(c(d+ex)^n\right)}}\right)dx\\
&= \frac{g^3\int\frac{(d+ex)^3}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{e^3}+\frac{\left(3g^2(ef-dg)\right)\int\frac{(d+ex)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{e^3}\\
&\quad+\frac{\left(3g(ef-dg)^2\right)\int\frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{e^3}+\frac{(ef-dg)^3\int\frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{e^3}
\end{aligned}$$

$$=\frac{g^3\text{Subst}\left(\int \frac{x^3}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{e^4}+\frac{(3g^2(ef-dg))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{e^4}+\frac{(3g(ef-dg)^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{e^4}+\frac{(ef-dg)^3\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{e^4}$$

$$=\frac{\left(g^3(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\right)\text{Subst}\left(\int \frac{e^{\frac{4x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^4n}+\frac{\left(3g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^4n}+\frac{\left(3g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^4n}+\frac{\left((ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^4n}$$

$$=\frac{\left(2g^3(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\right)\text{Subst}\left(\int e^{-\frac{4a}{bn}+\frac{4x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{be^4n}+\frac{\left(6g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{be^4n}+\frac{\left(6g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{be^4n}+\frac{\left(2(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{be^4n}$$

$$=\frac{e^{-\frac{a}{bn}}(ef-dg)^3\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}}+\frac{e^{-\frac{4a}{bn}}g^3\sqrt{\pi}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\text{erfi}\left(\frac{2\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{2\sqrt{b}e^4\sqrt{n}}+\frac{3e^{-\frac{2a}{bn}}g(ef-dg)^2\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}}+\frac{e^{-\frac{3a}{bn}}g^2(ef-dg)\sqrt{3\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^4\sqrt{n}}$$

## Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.86

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \frac{e^{-\frac{4a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-4/n}\left(2e^{\frac{3a}{bn}}(ef-dg)^3\left(c(d+ex)^n\right)^{3/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+g^3(d+ex)^3\text{er}\right.}{}$$

```
[In] Integrate[(f + g*x)^3/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] (Sqrt[Pi]*(d + e*x)*(2*E^((3*a)/(b*n))*(e*f - d*g)^3*(c*(d + e*x)^n)^(3/n)*
Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])] + g^3*(d + e*x)^3*Er
fi[(2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])] + 3*Sqrt[2]*E^((2*
a)/(b*n))*g*(e*f - d*g)^2*(d + e*x)*(c*(d + e*x)^n)^(2/n)*Erfi[(Sqrt[2]*Sqr
t[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])] + 2*Sqrt[3]*E^(a/(b*n))*g^2
*(e*f - d*g)*(d + e*x)^2*(c*(d + e*x)^n)^n^(-1)*Erfi[(Sqrt[3]*Sqrt[a + b*Lo
g[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]))/(2*Sqrt[b]*e^4*E^((4*a)/(b*n))*Sqrt
[n]*(c*(d + e*x)^n)^(4/n))
```

## Maple [F]

$$\int \frac{(gx+f)^3}{\sqrt{a+b\ln\left(c\left(ex+d\right)^n\right)}}dx$$

```
[In] int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

```
[Out] int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{(f+gx)^3}{\sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}}\,dx$$

```
[In] integrate((g*x+f)**3/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral((f + g*x)**3/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [F]

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{(gx+f)^3}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^3/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Giac [F]

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{(gx+f)^3}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^3/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{(f+g\,x)^3}{\sqrt{a+b\,\ln\left(c\,(d+e\,x)^n\right)}}\,dx$$

```
[In] int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^(1/2),x)
[Out] int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

# 3.124 $\int \frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 283

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \frac{e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}} + \frac{e^{-\frac{2a}{bn}}g(ef-dg)\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}} + \frac{e^{-\frac{3a}{bn}}g^2\sqrt{\frac{\pi}{3}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}}$$

```
[Out] 1/3*g^2*(e*x+d)^3*erfi(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))
*3^(1/2)*Pi^(1/2)/e^3/exp(3*a/b/n)/((c*(e*x+d)^n)^(3/n))/b^(1/2)/n^(1/2)+(-
d*g+e*f)^2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/
2)/e^3/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))/b^(1/2)/n^(1/2)+g*(-d*g+e*f)*(e*x+d
)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1
/2)/e^3/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))/b^(1/2)/n^(1/2)
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2448, 2436, 2337, 2211, 2235, 2437, 2347}

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$
$$=\frac{\sqrt{2\pi}ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}}$$
$$+\frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}}$$
$$+\frac{\sqrt{\frac{\pi}{3}}g^2e^{-\frac{3a}{bn}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}}$$

```
[In] Int[(f + g*x)^2/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] ((e*f - d*g)^2*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt
[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)) + (
g*(e*f - d*g)*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*
x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*
x)^n)^(2/n)) + (g^2*Sqrt[Pi/3]*(d + e*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(
d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(Sqrt[b]*e^3*E^((3*a)/(b*n))*Sqrt[n]*(c*(
d + e*x)^n)^(3/n))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(ef-dg)^2}{e^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{2g(ef-dg)(d+ex)}{e^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{g^2(d+ex)^2}{e^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}\right) dx$$

$$= \frac{g^2\int \frac{(d+ex)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{e^2} + \frac{(2g(ef-dg))\int \frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{e^2} + \frac{(ef-dg)^2\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{e^2}$$

$$= \frac{g^2\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{e^3} + \frac{(2g(ef-dg))\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{e^3} + \frac{(ef-dg)^2\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{e^3}$$

$$= \frac{\left(g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{e^3n} + \frac{\left(2g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{e^3n} + \frac{\left((ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{e^3n}$$

$$= \frac{\left(2g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\operatorname{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{be^3n} + \frac{\left(4g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{be^3n} + \frac{\left(2(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{be^3n}$$

$$= \frac{e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}} + \frac{e^{-\frac{2a}{bn}}g(ef-dg)\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}} + \frac{e^{-\frac{3a}{bn}}g^2\sqrt{\frac{\pi}{3}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^3\sqrt{n}}$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \frac{e^{-\frac{3a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-3/n}\left(3e^{\frac{2a}{bn}}(ef-dg)^2\left(c(d+ex)^n\right)^{2/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)+3\sqrt{2}e^{\frac{a}{bn}}g(ef-\right.}{3\sqrt{b}e^3\sqrt{n}}$$

```
[In] Integrate[(f + g*x)^2/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] (Sqrt[Pi]*(d + e*x)*(3*E^((2*a)/(b*n))*(e*f - d*g)^2*(c*(d + e*x)^n)^(2/n)*
Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])] + 3*Sqrt[2]*E^(a/(b*
n))*g*(e*f - d*g)*(d + e*x)*(c*(d + e*x)^n)^n^(-1)*Erfi[(Sqrt[2]*Sqrt[a + b
*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])] + Sqrt[3]*g^2*(d + e*x)^2*Erfi[(Sq
rt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]))/(3*Sqrt[b]*e^3*E
^((3*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(3/n))
```

## Maple [F]

$$\int \frac{(gx+f)^2}{\sqrt{a+b\ln\left(c\left(ex+d\right)^n\right)}}dx$$

```
[In] int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
[Out] int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}}\,dx$$

```
[In] integrate((g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral((f + g*x)**2/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [F]

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{(gx+f)^2}{\sqrt{b\log\left(\left(ex+d\right)^n c\right)+a}}\,dx$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^2/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Giac [F]

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{(gx+f)^2}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^2/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{(f+g\,x)^2}{\sqrt{a+b\,\ln\left(c\,(d+e\,x)^n\right)}}\,dx$$

```
[In] int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^(1/2),x)
[Out] int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

# 3.125 $\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 181

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \frac{e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}} + \frac{e^{-\frac{2a}{bn}}g\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}}$$

```
[Out] 1/2*g*(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2
^(1/2)*Pi^(1/2)/e^2/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))/b^(1/2)/n^(1/2)+(-d*
g+e*f)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e
^2/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))/b^(1/2)/n^(1/2)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2448, 2436, 2337, 2211, 2235, 2437, 2347}

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}} + \frac{\sqrt{\frac{\pi}{2}}ge^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}}$$

```
[In] Int[(f + g*x)/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] ((e*f - d*g)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b
]*Sqrt[n])])/(Sqrt[b]*e^2*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)) + (g*
Sqrt[Pi/2]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[
b]*Sqrt[n])])/(Sqrt[b]*e^2*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(2/n))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
```

```
+ e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{ef-dg}{e\sqrt{a+b\log\left(c(d+ex)^n\right)}}+\frac{g(d+ex)}{e\sqrt{a+b\log\left(c(d+ex)^n\right)}}\right)dx$$

$$=\frac{g\int \frac{d+ex}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx}{e}+\frac{(ef-dg)\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx}{e}$$

$$=\frac{g\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log\left(cx^n\right)}}\,dx,x,d+ex\right)}{e^2}+\frac{(ef-dg)\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log\left(cx^n\right)}}\,dx,x,d+ex\right)}{e^2}$$

$$=\frac{\left(g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^2n}$$
$$+\frac{\left((ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^2n}$$

$$=\frac{\left(2g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{be^2n}$$
$$+\frac{\left(2(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{be^2n}$$

$$=\frac{e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}}$$
$$+\frac{e^{-\frac{2a}{bn}}g\sqrt{\frac{\pi}{2}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e^2\sqrt{n}}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$
$$=\frac{e^{-\frac{2a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(2e^{\frac{a}{bn}}(ef-dg)\left(c(d+ex)^n\right)^{\frac{1}{n}}\text{erfi}\left(\frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{\sqrt{b}\sqrt{n}}\right)+\sqrt{2}g(d+ex)\text{erfi}\right.}{2\sqrt{b}e^2\sqrt{n}}$$

```
[In] Integrate[(f + g*x)/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] (Sqrt[Pi]*(d + e*x)*(2*E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*Erfi[
Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])] + Sqrt[2]*g*(d + e*x)*Erf
i[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]))/(2*Sqrt[b]*
e^2*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(2/n))
```

## Maple [F]

$$\int \frac{gx+f}{\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}dx$$

```
[In] int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
[Out] int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}\,dx = \int \frac{f+gx}{\sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}}\,dx$$

```
[In] integrate((g*x+f)/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral((f + g*x)/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [F]

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}\,dx = \int \frac{gx+f}{\sqrt{b\log\left(\left(ex+d\right)^{n}c\right)+a}}\,dx$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Giac [F]

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{gx+f}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{f+g\,x}{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}\,dx$$

```
[In] int((f + g*x)/(a + b*log(c*(d + e*x)^n))^(1/2),x)
[Out] int((f + g*x)/(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

# 3.126 $\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\frac{e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}$$

```
[Out] (e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/e/exp(a/
b/n)/((c*(e*x+d)^n)^(1/n))/b^(1/2)/n^(1/2)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2436, 2337, 2211, 2235}

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\frac{\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}$$

```
[In] Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] (Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])
/(Sqrt[b]*e*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{e} \\
&= \frac{\left((d+ex)\,(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{en} \\
&= \frac{\left(2(d+ex)\,(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{ben} \\
&= \frac{e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\,(c(d+ex)^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \frac{e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\,(c(d+ex)^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{\sqrt{b}e\sqrt{n}}$$

```
[In] Integrate[1/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
[Out] (Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])
/(Sqrt[b]*e*E^(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1))
```

## Maple [F]

$$\int \frac{1}{\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}dx$$

```
[In] int(1/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
[Out] int(1/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral(1/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^{n}\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left(\left(ex+d\right)^{n}c\right)+a}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b*log((e*x + d)^n*c) + a), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}\,dx$$

[In] int(1/(a + b*log(c*(d + e*x)^n))^(1/2),x)

[Out] int(1/(a + b*log(c*(d + e*x)^n))^(1/2), x)

# 3.127 $\int \frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \mathrm{Int}\left(\frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}}\,dx$$

```
[In] Int[1/((f + g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]),x]
[Out] Defer[Int][1/((f + g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx)\sqrt{a+b\log(c(d+ex)^n)}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

```
[In] Integrate[1/((f + g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]),x]
[Out] Integrate[1/((f + g*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]), x]
```

## Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(gx+f)\,\sqrt{a+b\ln\left(c\,(ex+d)^{n}\right)}}dx$$

```
[In] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
[Out] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{1}{\sqrt{a+b\log\left(c\,(d+ex)^{n}\right)}\,(f+gx)}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*log(c*(d + e*x)**n))*(f + g*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{(gx+f)\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)*sqrt(b*log((e*x + d)^n*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{(gx+f)\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate(1/((g*x + f)*sqrt(b*log((e*x + d)^n*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{(f+g\,x)\ \sqrt{a+b\,\ln\left(c\,(d+e\,x)^n\right)}}\,dx$$

```
[In] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^(1/2)),x)
[Out] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^(1/2)), x)
```

# 3.128 $\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 422

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bn}}(ef-dg)^3\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}}$$
$$+\frac{4e^{-\frac{4a}{bn}}g^3\sqrt{\pi}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}}$$
$$+\frac{6e^{-\frac{2a}{bn}}g(ef-dg)^2\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}}$$
$$+\frac{6e^{-\frac{3a}{bn}}g^2(ef-dg)\sqrt{3\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}}$$
$$-\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

```
[Out] 2*(-d*g+e*f)^3*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi
^(1/2)/b^(3/2)/e^4/exp(a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(1/n))+4*g^3*(e*x+d)^4
*erfi(2*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(3/2)/e^4/e
xp(4*a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(4/n))+6*g*(-d*g+e*f)^2*(e*x+d)^2*erfi(2
^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2
)/e^4/exp(2*a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(2/n))+6*g^2*(-d*g+e*f)*(e*x+d)^3
*erfi(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2)*Pi^(1/2)
/b^(3/2)/e^4/exp(3*a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(3/n))-2*(e*x+d)*(g*x+f)^3
/b/e/n/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

## Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2447, 2448, 2436, 2337, 2211, 2235, 2437, 2347}

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{6\sqrt{3\pi}g^2e^{-\frac{3a}{bn}}(d+ex)^3(ef-dg)\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}}$$
$$+\frac{6\sqrt{2\pi}ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}}$$
$$+\frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^3\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}}$$
$$+\frac{4\sqrt{\pi}g^3e^{-\frac{4a}{bn}}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}}$$
$$-\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

```
[In] Int[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] (2*(e*f - d*g)^3*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sq
rt[b]*Sqrt[n])])/(b^(3/2)*e^4*E^(a/(b*n))*n^(3/2)*(c*(d + e*x)^n)^n^(-1)) +
 (4*g^3*Sqrt[Pi]*(d + e*x)^4*Erfi[(2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[
b]*Sqrt[n])])/(b^(3/2)*e^4*E^((4*a)/(b*n))*n^(3/2)*(c*(d + e*x)^n)^(4/n)) +
 (6*g*(e*f - d*g)^2*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(
d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(b^(3/2)*e^4*E^((2*a)/(b*n))*n^(3/2)*(c*(
d + e*x)^n)^(2/n)) + (6*g^2*(e*f - d*g)*Sqrt[3*Pi]*(d + e*x)^3*Erfi[(Sqrt[3
]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(b^(3/2)*e^4*E^((3*a)
/(b*n))*n^(3/2)*(c*(d + e*x)^n)^(3/n)) - (2*(d + e*x)*(f + g*x)^3)/(b*e*n*S
qrt[a + b*Log[c*(d + e*x)^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = -\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{8\int \frac{(f+gx)^3}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{bn} - \frac{(6(ef-dg))\int \frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{ben}$$

$$= -\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{8\int \left(\frac{(ef-dg)^3}{e^3\sqrt{a+b\log(c(d+ex)^n)}} + \frac{3g(ef-dg)^2(d+ex)}{e^3\sqrt{a+b\log(c(d+ex)^n)}} + \frac{3g^2(ef-dg)(d+ex)^2}{e^3\sqrt{a+b\log(c(d+ex)^n)}} + \frac{g^3(d+ex)^3}{e^3\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{bn} - \frac{(6(ef-dg))\int \left(\frac{(ef-dg)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}} + \frac{2g(ef-dg)(d+ex)}{e^2\sqrt{a+b\log(c(d+ex)^n)}} + \frac{g^2(d+ex)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{ben}$$

$$= -\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{(8g^3)\int \frac{(d+ex)^3}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{be^3n} - \frac{(6g^2(ef-dg))\int \frac{(d+ex)^2}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{be^3n} + \frac{(24g^2(ef-dg))\int \frac{(d+ex)^2}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{be^3n} - \frac{(12g(ef-dg)^2)\int \frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{be^3n} + \frac{(24g(ef-dg)^2)\int \frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{be^3n} - \frac{(6(ef-dg)^3)\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{be^3n} + \frac{(8(ef-dg)^3)\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}dx}{be^3n}$$

$$= -\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{(8g^3)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+b\log(cx^n)}}dx, x, d+ex\right)}{be^4n} - \frac{(6g^2(ef-dg))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cx^n)}}dx, x, d+ex\right)}{be^4n} + \frac{(24g^2(ef-dg))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cx^n)}}dx, x, d+ex\right)}{be^4n} - \frac{(12g(ef-dg)^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}dx, x, d+ex\right)}{be^4n} + \frac{(24g(ef-dg)^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}dx, x, d+ex\right)}{be^4n} - \frac{(6(ef-dg)^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}dx, x, d+ex\right)}{be^4n} + \frac{(8(ef-dg)^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}dx, x, d+ex\right)}{be^4n}$$

$$= -\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(8g^3(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\right)\text{Subst}\left(\int \frac{e^{\frac{4x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^4n^2} - \frac{\left(6g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^4n^2} + \frac{\left(24g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^4n^2} - \frac{\left(12g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^4n^2} + \frac{\left(24g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^4n^2} - \frac{\left(6(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^4n^2} + \frac{\left(8(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^4n^2}$$

$$= -\frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(16g^3(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\right)\text{Subst}\left(\int e^{-\frac{4a}{bn}+\frac{4x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^2e^4n^2} - \frac{\left(12g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^2e^4n^2} + \frac{\left(48g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^2e^4n^2} - \frac{\left(24g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^2e^4n^2} + \frac{\left(48g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^2e^4n^2} - \frac{\left(12(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^2e^4n^2} + \frac{\left(16(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^2e^4n^2}$$

$$= \frac{2e^{-\frac{a}{bn}}(ef-dg)^3\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}} + \frac{4e^{-\frac{4a}{bn}}g^3\sqrt{\pi}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}} + \frac{6e^{-\frac{2a}{bn}}g(ef-dg)^2\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}} + \frac{6e^{-\frac{3a}{bn}}g^2(ef-dg)\sqrt{3\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^4n^{3/2}} - \frac{2(d+ex)(f+gx)^3}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1281 vs. 2(422) = 844.

Time = 1.70 (sec) , antiderivative size = 1281, normalized size of antiderivative = 3.04

$$\int \frac{(f+gx)^3}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \frac{2\left(-\sqrt{b}de^3f^3\sqrt{n}-\sqrt{b}e^4f^3\sqrt{n}x-3\sqrt{b}de^3f^2g\sqrt{n}x-3\sqrt{b}e^4f^2g\sqrt{n}x^2-3\right.}{}$$

```
[In] Integrate[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] (2*(-(Sqrt[b]*d*e^3*f^3*Sqrt[n]) - Sqrt[b]*e^4*f^3*Sqrt[n]*x - 3*Sqrt[b]*d*
e^3*f^2*g*Sqrt[n]*x - 3*Sqrt[b]*e^4*f^2*g*Sqrt[n]*x^2 - 3*Sqrt[b]*d*e^3*f*g
^2*Sqrt[n]*x^2 - 3*Sqrt[b]*e^4*f*g^2*Sqrt[n]*x^3 - Sqrt[b]*d*e^3*g^3*Sqrt[n
]*x^3 - Sqrt[b]*e^4*g^3*Sqrt[n]*x^4 - (6*d*e^2*f^2*g*Sqrt[Pi]*(d + e*x)*Erf
i[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e
*x)^n]])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (3*d^2*e*f*g^2*Sqrt[Pi]*(d
+ e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Lo
g[c*(d + e*x)^n]])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) - (d^3*g^3*Sqrt[Pi]
*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*Sqrt[a +
b*Log[c*(d + e*x)^n]])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (2*g^3*Sqrt[P
i]*(d + e*x)^4*Erfi[(2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]*S
qrt[a + b*Log[c*(d + e*x)^n]])/(E^((4*a)/(b*n))*(c*(d + e*x)^n)^(4/n)) + (3
*e^2*f^2*g*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^
n]])/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(E^((2*a)/(b*n))*(c
*(d + e*x)^n)^(2/n)) - (6*d*e*f*g^2*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sq
rt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)
^n]])/(E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) + (3*d^2*g^3*Sqrt[2*Pi]*(d +
e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]*Sqr
t[a + b*Log[c*(d + e*x)^n]])/(E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) + (3*e
*f*g^2*Sqrt[3*Pi]*(d + e*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

```
/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(E^((3*a)/(b*n))*(c*(d
+ e*x)^n)^(3/n)) - (3*d*g^3*Sqrt[3*Pi]*(d + e*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b
*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(E
^((3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)) + (Sqrt[b]*e^3*f^3*Sqrt[n]*(d + e*x)*
Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*Sqrt[-((a + b*Log[c*(d + e*
x)^n])/(b*n))])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (3*Sqrt[b]*d*e^2*f^2
*g*Sqrt[n]*(d + e*x)*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*Sqrt[-
((a + b*Log[c*(d + e*x)^n])/(b*n))])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1))))
/(b^(3/2)*e^4*n^(3/2)*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

## Maple [F]

$$\int \frac{(gx+f)^3}{(a+b\ln(c(ex+d)^n))^{\frac{3}{2}}}dx$$

```
[In] int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

```
[Out] int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)**3/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
```

```
[Out] Integral((f + g*x)**3/(a + b*log(c*(d + e*x)**n))**(3/2), x)
```

## Maxima [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(gx+f)^3}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^3/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Giac [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(gx+f)^3}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
[Out] integrate((g*x + f)^3/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(f+g\,x)^3}{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}\,dx$$

```
[In] int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^(3/2),x)
[Out] int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

# 3.129 $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 325

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{\pi}(d+ex)\,(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^3n^{3/2}} + \frac{4e^{-\frac{2a}{bn}}g(ef-dg)\sqrt{2\pi}(d+ex)^2\,(c(d+ex)^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^3n^{3/2}} + \frac{2e^{-\frac{3a}{bn}}g^2\sqrt{3\pi}(d+ex)^3\,(c(d+ex)^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^3n^{3/2}} - \frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

```
[Out] 2*(-d*g+e*f)^2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi
^(1/2)/b^(3/2)/e^3/exp(a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(1/n))+4*g*(-d*g+e*f)*
(e*x+d)^2*erfi(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)
*Pi^(1/2)/b^(3/2)/e^3/exp(2*a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(2/n))+2*g^2*(e*x
+d)^3*erfi(3^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2)*Pi^
(1/2)/b^(3/2)/e^3/exp(3*a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(3/n))-2*(e*x+d)*(g*x
+f)^2/b/e/n/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2447, 2448, 2436, 2337, 2211, 2235, 2437, 2347}

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{4\sqrt{2\pi}ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^3n^{3/2}} + \frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^3n^{3/2}} + \frac{2\sqrt{3\pi}g^2e^{-\frac{3a}{bn}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^3n^{3/2}} - \frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

```
[In] Int[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] (2*(e*f - d*g)^2*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sq
rt[b]*Sqrt[n])])/(b^(3/2)*e^3*E^(a/(b*n))*n^(3/2)*(c*(d + e*x)^n)^n^(-1)) +
 (4*g*(e*f - d*g)*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d
+ e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(b^(3/2)*e^3*E^((2*a)/(b*n))*n^(3/2)*(c*(d
+ e*x)^n)^(2/n)) + (2*g^2*Sqrt[3*Pi]*(d + e*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*L
og[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(b^(3/2)*e^3*E^((3*a)/(b*n))*n^(3/2
)*(c*(d + e*x)^n)^(3/n)) - (2*(d + e*x)*(f + g*x)^2)/(b*e*n*Sqrt[a + b*Log[
c*(d + e*x)^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = -\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}+\frac{6\int\frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{bn}$$
$$-\frac{(4(ef-dg))\int\frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{ben}$$

$$= -\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{6\int\left(\frac{(ef-dg)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}} + \frac{2g(ef-dg)(d+ex)}{e^2\sqrt{a+b\log(c(d+ex)^n)}} + \frac{g^2(d+ex)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{bn} - \frac{(4(ef-dg))\int\left(\frac{ef-dg}{e\sqrt{a+b\log(c(d+ex)^n)}} + \frac{g(d+ex)}{e\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{ben}$$

$$= -\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{(6g^2)\int\frac{(d+ex)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{be^2n} - \frac{(4g(ef-dg))\int\frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{be^2n} + \frac{(12g(ef-dg))\int\frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{be^2n} - \frac{(4(ef-dg)^2)\int\frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{be^2n} + \frac{(6(ef-dg)^2)\int\frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{be^2n}$$

$$= -\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{(6g^2)\,\text{Subst}\left(\int\frac{x^2}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{be^3n} - \frac{(4g(ef-dg))\text{Subst}\left(\int\frac{x}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{be^3n} + \frac{(12g(ef-dg))\text{Subst}\left(\int\frac{x}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{be^3n} - \frac{(4(ef-dg)^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{be^3n} + \frac{(6(ef-dg)^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{be^3n}$$

$$= -\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(6g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^3n^2} - \frac{\left(4g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^3n^2} + \frac{\left(12g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^3n^2} - \frac{\left(4(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^3n^2} + \frac{\left(6(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{be^3n^2}$$

$$= -\frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}} + \frac{\left(12g^2(d+ex)^3(c(d+ex)^n)^{-3/n}\right)\text{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx, x, \sqrt{a+b\log(c(d+ex)^n)}\right)}{b^2e^3n^2} - \frac{\left(8g(ef-dg)(d+ex)^2(c(d+ex)^n)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log(c(d+ex)^n)}\right)}{b^2e^3n^2} + \frac{\left(24g(ef-dg)(d+ex)^2(c(d+ex)^n)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log(c(d+ex)^n)}\right)}{b^2e^3n^2} - \frac{\left(8(ef-dg)^2(d+ex)(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log(c(d+ex)^n)}\right)}{b^2e^3n^2} + \frac{\left(12(ef-dg)^2(d+ex)(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log(c(d+ex)^n)}\right)}{b^2e^3n^2}$$

$$= \frac{2e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{\pi}(d+ex)(c(d+ex)^n)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^3n^{3/2}} + \frac{4e^{-\frac{2a}{bn}}g(ef-dg)\sqrt{2\pi}(d+ex)^2(c(d+ex)^n)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^3n^{3/2}} + \frac{2e^{-\frac{3a}{bn}}g^2\sqrt{3\pi}(d+ex)^3(c(d+ex)^n)^{-3/n}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^3n^{3/2}} - \frac{2(d+ex)(f+gx)^2}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 828 vs. $2(325) = 650$.

Time = 0.70 (sec) , antiderivative size = 828, normalized size of antiderivative = 2.55

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2\left(-\sqrt{b}de^2f^2\sqrt{n} - \sqrt{b}e^3f^2\sqrt{n}x - 2\sqrt{b}de^2fg\sqrt{n}x - 2\sqrt{b}e^3fg\sqrt{n}x^2 - \sqrt{}\right.}{}$$

```
[In] Integrate[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] (2*(-(Sqrt[b]*d*e^2*f^2*Sqrt[n]) - Sqrt[b]*e^3*f^2*Sqrt[n]*x - 2*Sqrt[b]*d*
e^2*f*g*Sqrt[n]*x - 2*Sqrt[b]*e^3*f*g*Sqrt[n]*x^2 - Sqrt[b]*d*e^2*g^2*Sqrt[
n]*x^2 - Sqrt[b]*e^3*g^2*Sqrt[n]*x^3 - (4*d*e*f*g*Sqrt[Pi]*(d + e*x)*Erfi[S
qrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)
^n]])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (d^2*g^2*Sqrt[Pi]*(d + e*x)*Er
fi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d +
e*x)^n]])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (2*e*f*g*Sqrt[2*Pi]*(d + e
```

```
*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]*Sqrt
[a + b*Log[c*(d + e*x)^n]])/(E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) - (2*d*
g^2*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(S
qrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(E^((2*a)/(b*n))*(c*(d + e
*x)^n)^(2/n)) + (g^2*Sqrt[3*Pi]*(d + e*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*
(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(E^((3*a)
/(b*n))*(c*(d + e*x)^n)^(3/n)) + (Sqrt[b]*e^2*f^2*Sqrt[n]*(d + e*x)*Gamma[1
/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*Sqrt[-((a + b*Log[c*(d + e*x)^n])/
(b*n))])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (2*Sqrt[b]*d*e*f*g*Sqrt[n]*
(d + e*x)*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*Sqrt[-((a + b*Log
[c*(d + e*x)^n])/(b*n))])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1))))/(b^(3/2)*e
^3*n^(3/2)*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

## Maple [F]

$$\int \frac{(gx+f)^2}{(a+b\ln(c(ex+d)^n))^{\frac{3}{2}}}dx$$

```
[In] int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
[Out] int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
[Out] Integral((f + g*x)**2/(a + b*log(c*(d + e*x)**n))**(3/2), x)
```

## Maxima [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(gx+f)^2}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^2/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Giac [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(gx+f)^2}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
[Out] integrate((g*x + f)^2/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(f+g\,x)^2}{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}\,dx$$

```
[In] int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^(3/2),x)
[Out] int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

# 3.130 $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 220

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}} + \frac{2e^{-\frac{2a}{bn}}g\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}} - \frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

```
[Out] 2*(-d*g+e*f)*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(
1/2)/b^(3/2)/e^2/exp(a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(1/n))+2*g*(e*x+d)^2*erf
i(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/b^(
3/2)/e^2/exp(2*a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(2/n))-2*(e*x+d)*(g*x+f)/b/e/n
/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2447, 2448, 2436, 2337, 2211, 2235, 2437, 2347}

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}} + \frac{2\sqrt{2\pi}ge^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}} - \frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}$$

[In] Int[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^(3/2),x]

[Out] (2*(e*f - d*g)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(b^(3/2)*e^2*E^(a/(b*n))*n^(3/2)*(c*(d + e*x)^n)^n^(-1)) + (2*g*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(b^(3/2)*e^2*E^((2*a)/(b*n))*n^(3/2)*(c*(d + e*x)^n)^(2/n)) - (2*(d + e*x)*(f + g*x))/(b*e*n*Sqrt[a + b*Log[c*(d + e*x)^n]])

Rule 2211

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2337

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rule 2347

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

Rule 2436

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

Rule 2437

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]

Rule 2447

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e

```
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}+\frac{4\int\frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{bn}\\
&\quad-\frac{(2(ef-dg))\int\frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{ben}\\
&= -\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}+\frac{4\int\left(\frac{ef-dg}{e\sqrt{a+b\log(c(d+ex)^n)}}+\frac{g(d+ex)}{e\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{bn}\\
&\quad-\frac{(2(ef-dg))\text{Subst}\left(\int\frac{1}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{be^2n}\\
&= -\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}+\frac{(4g)\int\frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{ben}\\
&\quad+\frac{(4(ef-dg))\int\frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{ben}\\
&\quad-\frac{\left(2(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^2n^2}\\
&= -\frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log(c(d+ex)^n)}}+\frac{(4g)\text{Subst}\left(\int\frac{x}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{be^2n}\\
&\quad+\frac{(4(ef-dg))\text{Subst}\left(\int\frac{1}{\sqrt{a+b\log(cx^n)}}\,dx,x,d+ex\right)}{be^2n}\\
&\quad-\frac{\left(4(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^2e^2n^2}
\end{aligned}
$$

$$= -\frac{2e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}} - \frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(4g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^2n^2} + \frac{\left(4(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{be^2n^2}$$

$$= -\frac{2e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}} - \frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(8g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^2e^2n^2} + \frac{\left(8(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^2e^2n^2}$$

$$= \frac{2e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}} + \frac{2e^{-\frac{2a}{bn}}g\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}e^2n^{3/2}} - \frac{2(d+ex)(f+gx)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

## Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.54

$$\int\frac{f+gx}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \frac{2e^{-\frac{2a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(-2de^{\frac{a}{bn}}g\sqrt{\pi}\left(c(d+ex)^n\right)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(}}{\sqrt{b}}\right.\right.}{}$$

```
[In] Integrate[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] (2*(d + e*x)*(-2*d*E^(a/(b*n))*g*Sqrt[Pi]*(c*(d + e*x)^n)^n^(-1)*Erfi[Sqrt[
a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n]]
 + g*Sqrt[2*Pi]*(d + e*x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sq
rt[b]*Sqrt[n])]*Sqrt[a + b*Log[c*(d + e*x)^n]] + Sqrt[b]*E^(a/(b*n))*Sqrt[n
]*(c*(d + e*x)^n)^n^(-1)*(-(e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(f + g*x))
```

```
 + (e*f + d*g)*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*Sqrt[-((a +
b*Log[c*(d + e*x)^n])/(b*n))])))/(b^(3/2)*e^2*E^((2*a)/(b*n))*n^(3/2)*(c*(d
 + e*x)^n)^(2/n)*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

## Maple [F]

$$\int \frac{gx+f}{\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{\frac{3}{2}}}dx$$

```
[In] int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
[Out] int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{f+gx}{\left(a+b\log\left(c\left(d+ex\right)^{n}\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
[Out] Integral((f + g*x)/(a + b*log(c*(d + e*x)**n))**(3/2), x)
```

## Maxima [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{gx+f}{\left(b\log\left(\left(ex+d\right)^{n}c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
[Out] integrate((g*x + f)/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Giac [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{gx+f}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
[Out] integrate((g*x + f)/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{f+g\,x}{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}\,dx$$

```
[In] int((f + g*x)/(a + b*log(c*(d + e*x)^n))^(3/2),x)
[Out] int((f + g*x)/(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

# 3.131 $\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 116

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}en^{3/2}} - \frac{2(d+ex)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

```
[Out] 2*(e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(3/2
)/e/exp(a/b/n)/n^(3/2)/((c*(e*x+d)^n)^(1/n))-2*(e*x+d)/b/e/n/(a+b*ln(c*(e*x
+d)^n))^(1/2)
```

## Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2436, 2334, 2337, 2211, 2235}

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \frac{2\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}en^{3/2}} - \frac{2(d+ex)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(-3/2),x]
```

```
[Out] (2*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])
])/(b^(3/2)*e*E^(a/(b*n))*n^(3/2)*(c*(d + e*x)^n)^n^(-1)) - (2*(d + e*x))/(
b*e*n*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^{3/2}}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{2(d+ex)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{ben}$$

$$= -\frac{2(d+ex)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{ben^2}$$

$$= -\frac{2(d+ex)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(4(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^2en^2}$$

$$= \frac{2e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{3/2}en^{3/2}} - \frac{2(d+ex)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = -\frac{2e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(e^{\frac{a}{bn}}\left(c(d+ex)^n\right)^{\frac{1}{n}} - \Gamma\left(\frac{1}{2}, -\frac{a+b\log(c(d+ex)^n)}{bn}\right)\sqrt{-\frac{a+b\log(c(d+ex)^n)}{bn}}\right)}{ben\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(-3/2),x]
[Out] (-2*(d + e*x)*(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1) - Gamma[1/2, -((a + b*Log
[c*(d + e*x)^n])/(b*n))]*Sqrt[-((a + b*Log[c*(d + e*x)^n])/(b*n))]))/(b*e*E
^(a/(b*n))*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

## Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
[Out] int(1/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**(-3/2), x)
```

## Maxima [F]

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\log\left((ex+d)^n c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(-3/2), x)
```

## Giac [F]

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\log\left((ex+d)^n c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(-3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \int \frac{1}{\left(a+b\,\ln\left(c\left(d+e\,x\right)^n\right)\right)^{3/2}}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e*x)^n))^(3/2),x)
[Out] int(1/(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

# 3.132 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \text{Int}\left(\frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx$$

```
[In] Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(3/2)),x]
[Out] Defer[Int][1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx$$

```
[In] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(3/2)),x]
[Out] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(3/2)), x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(gx+f)\left(a+b\ln\left(c\,(ex+d)^{n}\right)\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
[Out] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 5.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx=\int \frac{1}{\left(a+b\log\left(c\,(d+ex)^{n}\right)\right)^{\frac{3}{2}}(f+gx)}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
[Out] Integral(1/((a + b*log(c*(d + e*x)**n))**(3/2)*(f + g*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^(3/2)), x)
```

## Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx = \int \frac{1}{(f+g\,x)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^{3/2}}\,dx$$

```
[In] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^(3/2)),x)
[Out] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^(3/2)), x)
```

# 3.133 $\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 520

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \frac{4e^{-\frac{a}{bn}}(ef-dg)^3\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^4n^{5/2}}$$
$$+\frac{32e^{-\frac{4a}{bn}}g^3\sqrt{\pi}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^4n^{5/2}}$$
$$+\frac{8e^{-\frac{2a}{bn}}g(ef-dg)^2\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^4n^{5/2}}$$
$$+\frac{12e^{-\frac{3a}{bn}}g^2(ef-dg)\sqrt{3\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^4n^{5/2}}$$
$$-\frac{2(d+ex)(f+gx)^3}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}+\frac{4(ef-dg)(d+ex)(f+gx)^2}{b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$
$$-\frac{16(d+ex)(f+gx)^3}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

```
[Out] -2/3*(e*x+d)*(g*x+f)^3/b/e/n/(a+b*ln(c*(e*x+d)^n))^(3/2)+4/3*(-d*g+e*f)^3*(
e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)/e
^4/exp(a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(1/n))+32/3*g^3*(e*x+d)^4*erfi(2*(a+b*
ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)/e^4/exp(4*a/b/n)/n
^(5/2)/((c*(e*x+d)^n)^(4/n))+8*g*(-d*g+e*f)^2*(e*x+d)^2*erfi(2^(1/2)*(a+b*l
n(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/e^4/exp(2*a
/b/n)/n^(5/2)/((c*(e*x+d)^n)^(2/n))+12*g^2*(-d*g+e*f)*(e*x+d)^3*erfi(3^(1/2
)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)/e^4
/exp(3*a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(3/n))+4*(-d*g+e*f)*(e*x+d)*(g*x+f)^2/
b^2/e^2/n^2/(a+b*ln(c*(e*x+d)^n))^(1/2)-16/3*(e*x+d)*(g*x+f)^3/b^2/e/n^2/(a
+b*ln(c*(e*x+d)^n))^(1/2)
```

## Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 59, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2447, 2448, 2436, 2337, 2211, 2235, 2437, 2347}

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \frac{12\sqrt{3\pi}g^2e^{-\frac{3a}{bn}}(d+ex)^3(ef-dg)\,(c(d+ex)^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^4n^{5/2}}$$
$$+\frac{8\sqrt{2\pi}ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)^2\,(c(d+ex)^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^4n^{5/2}}$$
$$+\frac{4\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^3\,(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^4n^{5/2}}$$
$$+\frac{32\sqrt{\pi}g^3e^{-\frac{4a}{bn}}(d+ex)^4\,(c(d+ex)^n)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^4n^{5/2}}$$
$$+\frac{4(d+ex)(f+gx)^2(ef-dg)}{b^2e^2n^2\sqrt{a+b\log(c(d+ex)^n)}}-\frac{16(d+ex)(f+gx)^3}{3b^2en^2\sqrt{a+b\log(c(d+ex)^n)}}$$
$$-\frac{2(d+ex)(f+gx)^3}{3ben\,(a+b\log(c(d+ex)^n))^{3/2}}$$

```
[In] Int[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] (4*(e*f - d*g)^3*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sq
rt[b]*Sqrt[n])])/(3*b^(5/2)*e^4*E^(a/(b*n))*n^(5/2)*(c*(d + e*x)^n)^n^(-1))
 + (32*g^3*Sqrt[Pi]*(d + e*x)^4*Erfi[(2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sq
rt[b]*Sqrt[n])])/(3*b^(5/2)*e^4*E^((4*a)/(b*n))*n^(5/2)*(c*(d + e*x)^n)^(4/
n)) + (8*g*(e*f - d*g)^2*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Lo
g[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(b^(5/2)*e^4*E^((2*a)/(b*n))*n^(5/2)
*(c*(d + e*x)^n)^(2/n)) + (12*g^2*(e*f - d*g)*Sqrt[3*Pi]*(d + e*x)^3*Erfi[(
Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(b^(5/2)*e^4*E^
((3*a)/(b*n))*n^(5/2)*(c*(d + e*x)^n)^(3/n)) - (2*(d + e*x)*(f + g*x)^3)/(3
*b*e*n*(a + b*Log[c*(d + e*x)^n])^(3/2)) + (4*(e*f - d*g)*(d + e*x)*(f + g*
x)^2)/(b^2*e^2*n^2*Sqrt[a + b*Log[c*(d + e*x)^n]]) - (16*(d + e*x)*(f + g*x
)^3)/(3*b^2*e*n^2*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = -\frac{2(d+ex)(f+gx)^3}{3ben(a+b\log(c(d+ex)^n))^{3/2}} + \frac{8\int\frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx}{3bn} - \frac{(2(ef-dg))\int\frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx}{ben}$$

$$= -\frac{2(d+ex)(f+gx)^3}{3ben(a+b\log(c(d+ex)^n))^{3/2}} + \frac{4(ef-dg)(d+ex)(f+gx)^2}{b^2e^2n^2\sqrt{a+b\log(c(d+ex)^n)}} - \frac{16(d+ex)(f+gx)^3}{3b^2en^2\sqrt{a+b\log(c(d+ex)^n)}} + \frac{64\int\frac{(f+gx)^3}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{3b^2n^2} - \frac{(12(ef-dg))\int\frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2en^2} - \frac{(16(ef-dg))\int\frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2en^2} + \frac{(8(ef-dg)^2)\int\frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^2n^2}$$

$$= -\frac{2(d+ex)(f+gx)^3}{3ben(a+b\log(c(d+ex)^n))^{3/2}} + \frac{4(ef-dg)(d+ex)(f+gx)^2}{b^2e^2n^2\sqrt{a+b\log(c(d+ex)^n)}} - \frac{16(d+ex)(f+gx)^3}{3b^2en^2\sqrt{a+b\log(c(d+ex)^n)}} + \frac{64\int\left(\frac{(ef-dg)^3}{e^3\sqrt{a+b\log(c(d+ex)^n)}} + \frac{3g(ef-dg)^2(d+ex)}{e^3\sqrt{a+b\log(c(d+ex)^n)}} + \frac{3g^2(ef-dg)(d+ex)^2}{e^3\sqrt{a+b\log(c(d+ex)^n)}} + \frac{g^3(d+ex)^3}{e^3\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{3b^2n^2} - \frac{(12(ef-dg))\int\left(\frac{(ef-dg)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}} + \frac{2g(ef-dg)(d+ex)}{e^2\sqrt{a+b\log(c(d+ex)^n)}} + \frac{g^2(d+ex)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{b^2en^2} - \frac{(16(ef-dg))\int\left(\frac{(ef-dg)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}} + \frac{2g(ef-dg)(d+ex)}{e^2\sqrt{a+b\log(c(d+ex)^n)}} + \frac{g^2(d+ex)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{b^2en^2} + \frac{(8(ef-dg)^2)\int\left(\frac{ef-dg}{e\sqrt{a+b\log(c(d+ex)^n)}} + \frac{g(d+ex)}{e\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{b^2e^2n^2}$$

$$= -\frac{2(d+ex)(f+gx)^3}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{4(ef-dg)(d+ex)(f+gx)^2}{b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$
$$- \frac{16(d+ex)(f+gx)^3}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{(64g^3)\int \frac{(d+ex)^3}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{3b^2e^3n^2}$$
$$- \frac{(12g^2(ef-dg))\int \frac{(d+ex)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^3n^2} - \frac{(16g^2(ef-dg))\int \frac{(d+ex)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^3n^2}$$
$$+ \frac{(64g^2(ef-dg))\int \frac{(d+ex)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^3n^2} + \frac{(8g(ef-dg)^2)\int \frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^3n^2}$$
$$- \frac{(24g(ef-dg)^2)\int \frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^3n^2} - \frac{(32g(ef-dg)^2)\int \frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^3n^2}$$
$$+ \frac{(64g(ef-dg)^2)\int \frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^3n^2} + \frac{(8(ef-dg)^3)\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^3n^2}$$
$$- \frac{(12(ef-dg)^3)\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^3n^2} - \frac{(16(ef-dg)^3)\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^3n^2}$$
$$+ \frac{(64(ef-dg)^3)\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{3b^2e^3n^2}$$

$$= -\frac{2(d+ex)(f+gx)^3}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{4(ef-dg)(d+ex)(f+gx)^2}{b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$
$$- \frac{16(d+ex)(f+gx)^3}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{(64g^3)\,\mathrm{Subst}\left(\int \frac{x^3}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{3b^2e^4n^2}$$
$$- \frac{(12g^2(ef-dg))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^4n^2}$$
$$- \frac{(16g^2(ef-dg))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^4n^2}$$
$$+ \frac{(64g^2(ef-dg))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^4n^2}$$
$$+ \frac{(8g(ef-dg)^2)\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^4n^2}$$
$$- \frac{(24g(ef-dg)^2)\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^4n^2}$$
$$- \frac{(32g(ef-dg)^2)\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^4n^2}$$
$$+ \frac{(64g(ef-dg)^2)\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^4n^2}$$
$$+ \frac{(8(ef-dg)^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^4n^2}$$
$$- \frac{(12(ef-dg)^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^4n^2}$$
$$- \frac{(16(ef-dg)^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^4n^2}$$
$$+ \frac{(64(ef-dg)^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{3b^2e^4n^2}$$

$$= -\frac{2(d+ex)(f+gx)^3}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{4(ef-dg)(d+ex)(f+gx)^2}{b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$
$$- \frac{16(d+ex)(f+gx)^3}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$
$$+ \frac{\left(64g^3(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\right)\text{Subst}\left(\int\frac{e^{\frac{4x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{3b^2e^4n^3}$$
$$- \frac{\left(12g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^4n^3}$$
$$- \frac{\left(16g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^4n^3}$$
$$+ \frac{\left(64g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^4n^3}$$
$$+ \frac{\left(8g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^4n^3}$$
$$- \frac{\left(24g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^4n^3}$$
$$- \frac{\left(32g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^4n^3}$$
$$+ \frac{\left(64g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^4n^3}$$
$$+ \frac{\left(8(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^4n^3}$$
$$- \frac{\left(12(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^4n^3}$$
$$- \frac{\left(16(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{b^2e^4n^3}$$
$$+ \frac{\left(64(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{3b^2e^4n^3}$$

$$
\begin{aligned}
&= -\frac{2(d+ex)(f+gx)^3}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} \\
&+ \frac{4(ef-dg)(d+ex)(f+gx)^2}{b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{16(d+ex)(f+gx)^3}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} \\
&+ \frac{\left(128g^3(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\right)\operatorname{Subst}\left(\int e^{-\frac{4a}{bn}+\frac{4x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{3b^3e^4n^3} \\
&- \frac{\left(24g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\operatorname{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^4n^3} \\
&- \frac{\left(32g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\operatorname{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^4n^3} \\
&+ \frac{\left(128g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\operatorname{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^4n^3} \\
&+ \frac{\left(16g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^4n^3} \\
&- \frac{\left(48g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^4n^3} \\
&- \frac{\left(64g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^4n^3} \\
&+ \frac{\left(128g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^4n^3} \\
&+ \frac{\left(16(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^4n^3} \\
&- \frac{\left(24(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^4n^3} \\
&- \frac{\left(32(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^4n^3} \\
&+ \frac{\left(128(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{3b^3e^4n^3}
\end{aligned}
$$

$$= \frac{4e^{-\frac{a}{bn}}(ef-dg)^3\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^4n^{5/2}} + \frac{32e^{-\frac{4a}{bn}}g^3\sqrt{\pi}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^4n^{5/2}} + \frac{8e^{-\frac{2a}{bn}}g(ef-dg)^2\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^4n^{5/2}} + \frac{12e^{-\frac{3a}{bn}}g^2(ef-dg)\sqrt{3\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^4n^{5/2}} - \frac{2(d+ex)(f+gx)^3}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{4(ef-dg)(d+ex)(f+gx)^2}{b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{16(d+ex)(f+gx)^3}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1523 vs. 2(520) = 1040.

Time = 6.12 (sec) , antiderivative size = 1523, normalized size of antiderivative = 2.93

$$\int \frac{(f+gx)^3}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx = \frac{2(d+ex)\left(-24de^2e^{-\frac{a}{bn}}f^2g\sqrt{\pi}(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right) - \right.}{}$$

```
[In] Integrate[(f + g*x)^3/(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] (2*(d + e*x)*((-24*d*e^2*f^2*g*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]
/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) - (6*d^2*e*f*g^2*
Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n
))*(c*(d + e*x)^n)^n^(-1)) - (2*d^3*g^3*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d +
 e*x)^n]]/(Sqrt[b]*Sqrt[n])])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)) + (16*g^
3*Sqrt[Pi]*(d + e*x)^3*Erfi[(2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqr
t[n])])/(E^((4*a)/(b*n))*(c*(d + e*x)^n)^(4/n)) + (12*e^2*f^2*g*Sqrt[2*Pi]*
(d + e*x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])
/(E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) + (30*d*e*f*g^2*Sqrt[2*Pi]*(d + e*
x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(E^((2
*a)/(b*n))*(c*(d + e*x)^n)^(2/n)) + (6*d^2*g^3*Sqrt[2*Pi]*(d + e*x)*Erfi[(S
qrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(E^((2*a)/(b*n))
*(c*(d + e*x)^n)^(2/n)) + (16*d*g^3*Sqrt[Pi]*(d + e*x)*(3*Sqrt[2]*d*E^(a/(b
*n))*(c*(d + e*x)^n)^n^(-1)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(
Sqrt[b]*Sqrt[n])] - 2*Sqrt[3]*(d + e*x)*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d +
 e*x)^n]])/(Sqrt[b]*Sqrt[n])]))/(E^((3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)) - (
14*d*g^3*Sqrt[Pi]*(d + e*x)*(3*Sqrt[2]*d*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)
```

```
*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])] - Sqrt[3]
*(d + e*x)*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]
))/(E^((3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)) + (18*e*f*g^2*Sqrt[Pi]*(d + e*x)
*(-3*Sqrt[2]*d*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*Erfi[(Sqrt[2]*Sqrt[a + b*
Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])] + Sqrt[3]*(d + e*x)*Erfi[(Sqrt[3]*S
qrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]))/(E^((3*a)/(b*n))*(c*(d
+ e*x)^n)^(3/n)) - (b^(3/2)*e^3*n^(3/2)*(f + g*x)^3)/(a + b*Log[c*(d + e*x)
^n])^(3/2) - (2*a*Sqrt[b]*e^2*Sqrt[n]*(f + g*x)^2*(3*d*g + e*(f + 4*g*x)))/
(a + b*Log[c*(d + e*x)^n])^(3/2) - (2*b^(3/2)*e^2*Sqrt[n]*(f + g*x)^2*(3*d*
g + e*(f + 4*g*x))*Log[c*(d + e*x)^n])/(a + b*Log[c*(d + e*x)^n])^(3/2) + (
2*Sqrt[b]*e^3*f^3*Sqrt[n]*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*S
qrt[-((a + b*Log[c*(d + e*x)^n])/(b*n))])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-
1)*Sqrt[a + b*Log[c*(d + e*x)^n]]) + (18*Sqrt[b]*d*e^2*f^2*g*Sqrt[n]*Gamma[
1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*Sqrt[-((a + b*Log[c*(d + e*x)^n])
/(b*n))])/(E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*Sqrt[a + b*Log[c*(d + e*x)^n]
]) + (12*Sqrt[b]*d^2*e*f*g^2*Sqrt[n]*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n]
)/(b*n))]*Sqrt[-((a + b*Log[c*(d + e*x)^n])/(b*n))])/(E^(a/(b*n))*(c*(d + e
*x)^n)^n^(-1)*Sqrt[a + b*Log[c*(d + e*x)^n]])))/(3*b^(5/2)*e^4*n^(5/2))
```

## Maple [F]

$$\int \frac{(gx+f)^3}{(a+b\ln(c(ex+d)^n))^{\frac{5}{2}}}dx$$

```
[In] int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

```
[Out] int((g*x+f)^3/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)**3/(a+b*ln(c*(e*x+d)**n))**(5/2),x)
[Out] Integral((f + g*x)**3/(a + b*log(c*(d + e*x)**n))**(5/2), x)
```

## Maxima [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(gx+f)^3}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^3/(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Giac [F]

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(gx+f)^3}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)^3/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
[Out] integrate((g*x + f)^3/(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(f+g\,x)^3}{(a+b\,\ln(c\,(d+e\,x)^n))^{5/2}}\,dx$$

```
[In] int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^(5/2),x)
[Out] int((f + g*x)^3/(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

# 3.134 $\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 421

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \frac{4e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{\pi}(d+ex)\,(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^3n^{5/2}}$$
$$+\frac{16e^{-\frac{2a}{bn}}g(ef-dg)\sqrt{2\pi}(d+ex)^2\,(c(d+ex)^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^3n^{5/2}}$$
$$+\frac{4e^{-\frac{3a}{bn}}g^2\sqrt{3\pi}(d+ex)^3\,(c(d+ex)^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^3n^{5/2}}$$
$$-\frac{2(d+ex)(f+gx)^2}{3ben\,(a+b\log(c(d+ex)^n))^{3/2}}+\frac{8(ef-dg)(d+ex)(f+gx)}{3b^2e^2n^2\sqrt{a+b\log(c(d+ex)^n)}}$$
$$-\frac{4(d+ex)(f+gx)^2}{b^2en^2\sqrt{a+b\log(c(d+ex)^n)}}$$

```
[Out] -2/3*(e*x+d)*(g*x+f)^2/b/e/n/(a+b*ln(c*(e*x+d)^n))^(3/2)+4/3*(-d*g+e*f)^2*(
e*x+d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)/e
^3/exp(a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(1/n))+16/3*g*(-d*g+e*f)*(e*x+d)^2*erf
i(2^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/b^(
5/2)/e^3/exp(2*a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(2/n))+4*g^2*(e*x+d)^3*erfi(3^
(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)
/e^3/exp(3*a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(3/n))+8/3*(-d*g+e*f)*(e*x+d)*(g*x
+f)/b^2/e^2/n^2/(a+b*ln(c*(e*x+d)^n))^(1/2)-4*(e*x+d)*(g*x+f)^2/b^2/e/n^2/(
a+b*ln(c*(e*x+d)^n))^(1/2)
```

## Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2447, 2448, 2436, 2337, 2211, 2235, 2437, 2347}

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \frac{16\sqrt{2\pi}ge^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^3n^{5/2}}$$
$$+\frac{4\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^3n^{5/2}}$$
$$+\frac{4\sqrt{3\pi}g^2e^{-\frac{3a}{bn}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^3n^{5/2}}$$
$$+\frac{8(d+ex)(f+gx)(ef-dg)}{3b^2e^2n^2\sqrt{a+b\log(c(d+ex)^n)}}-\frac{4(d+ex)(f+gx)^2}{b^2en^2\sqrt{a+b\log(c(d+ex)^n)}}$$
$$-\frac{2(d+ex)(f+gx)^2}{3ben\,(a+b\log(c(d+ex)^n))^{3/2}}$$

```
[In] Int[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] (4*(e*f - d*g)^2*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sq
rt[b]*Sqrt[n])])/(3*b^(5/2)*e^3*E^(a/(b*n))*n^(5/2)*(c*(d + e*x)^n)^n^(-1))
 + (16*g*(e*f - d*g)*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*
(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(3*b^(5/2)*e^3*E^((2*a)/(b*n))*n^(5/2)*(
c*(d + e*x)^n)^(2/n)) + (4*g^2*Sqrt[3*Pi]*(d + e*x)^3*Erfi[(Sqrt[3]*Sqrt[a
+ b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])])/(b^(5/2)*e^3*E^((3*a)/(b*n))*n
^(5/2)*(c*(d + e*x)^n)^(3/n)) - (2*(d + e*x)*(f + g*x)^2)/(3*b*e*n*(a + b*L
og[c*(d + e*x)^n])^(3/2)) + (8*(e*f - d*g)*(d + e*x)*(f + g*x))/(3*b^2*e^2*
n^2*Sqrt[a + b*Log[c*(d + e*x)^n]]) - (4*(d + e*x)*(f + g*x)^2)/(b^2*e*n^2*
Sqrt[a + b*Log[c*(d + e*x)^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = -\frac{2(d+ex)(f+gx)^2}{3ben\,(a+b\log\left(c(d+ex)^n\right))^{3/2}} + \frac{2\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx}{bn} - \frac{(4(ef-dg))\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx}{3ben}$$

$$= -\frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{8(ef-dg)(d+ex)(f+gx)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{4(d+ex)(f+gx)^2}{b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{12\int\frac{(f+gx)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2n^2} - \frac{(16(ef-dg))\int\frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{3b^2en^2} - \frac{(8(ef-dg))\int\frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2en^2} + \frac{(8(ef-dg)^2)\int\frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{3b^2e^2n^2}$$

$$= -\frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{8(ef-dg)(d+ex)(f+gx)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{4(d+ex)(f+gx)^2}{b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{12\int\left(\frac{(ef-dg)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}} + \frac{2g(ef-dg)(d+ex)}{e^2\sqrt{a+b\log(c(d+ex)^n)}} + \frac{g^2(d+ex)^2}{e^2\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{b^2n^2} - \frac{(16(ef-dg))\int\left(\frac{ef-dg}{e\sqrt{a+b\log(c(d+ex)^n)}} + \frac{g(d+ex)}{e\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{3b^2en^2} - \frac{(8(ef-dg))\int\left(\frac{ef-dg}{e\sqrt{a+b\log(c(d+ex)^n)}} + \frac{g(d+ex)}{e\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{b^2en^2} + \frac{(8(ef-dg)^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{3b^2e^3n^2}$$

$$= -\frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{8(ef-dg)(d+ex)(f+gx)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{4(d+ex)(f+gx)^2}{b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{(12g^2)\int\frac{(d+ex)^2}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^2n^2} - \frac{(16g(ef-dg))\int\frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{3b^2e^2n^2} - \frac{(8g(ef-dg))\int\frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^2n^2} + \frac{(24g(ef-dg))\int\frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^2n^2} - \frac{(16(ef-dg)^2)\int\frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{3b^2e^2n^2} - \frac{(8(ef-dg)^2)\int\frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^2n^2} + \frac{(12(ef-dg)^2)\int\frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{b^2e^2n^2} + \frac{\left(8(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{3b^2e^3n^3}$$

$$= -\frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{8(ef-dg)(d+ex)(f+gx)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$
$$- \frac{4(d+ex)(f+gx)^2}{b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{(12g^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^3n^2}$$
$$- \frac{(16g(ef-dg))\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{3b^2e^3n^2}$$
$$- \frac{(8g(ef-dg))\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^3n^2}$$
$$+ \frac{(24g(ef-dg))\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^3n^2}$$
$$- \frac{(16(ef-dg)^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{3b^2e^3n^2}$$
$$- \frac{(8(ef-dg)^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^3n^2}$$
$$+ \frac{(12(ef-dg)^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{b^2e^3n^2}$$
$$+ \frac{\left(16(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{3b^3e^3n^3}$$

$$= \frac{8e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^3n^{5/2}}$$
$$- \frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$
$$+ \frac{8(ef-dg)(d+ex)(f+gx)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{4(d+ex)(f+gx)^2}{b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$
$$+ \frac{\left(12g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{b^2e^3n^3}$$
$$- \frac{\left(16g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{3b^2e^3n^3}$$
$$- \frac{\left(8g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{b^2e^3n^3}$$
$$+ \frac{\left(24g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{b^2e^3n^3}$$
$$- \frac{\left(16(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{3b^2e^3n^3}$$
$$- \frac{\left(8(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{b^2e^3n^3}$$
$$+ \frac{\left(12(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{b^2e^3n^3}$$

$$= \frac{8e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^3n^{5/2}}$$
$$- \frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$
$$+ \frac{8(ef-dg)(d+ex)(f+gx)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{4(d+ex)(f+gx)^2}{b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$
$$+ \frac{\left(24g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\operatorname{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^3n^3}$$
$$- \frac{\left(32g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{3b^3e^3n^3}$$
$$- \frac{\left(16g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^3n^3}$$
$$+ \frac{\left(48g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\operatorname{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^3n^3}$$
$$- \frac{\left(32(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{3b^3e^3n^3}$$
$$- \frac{\left(16(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^3n^3}$$
$$+ \frac{\left(24(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{b^3e^3n^3}$$

$$= \frac{4e^{-\frac{a}{bn}}(ef-dg)^2\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^3n^{5/2}}$$
$$+ \frac{16e^{-\frac{2a}{bn}}g(ef-dg)\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^3n^{5/2}}$$
$$+ \frac{4e^{-\frac{3a}{bn}}g^2\sqrt{3\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^3n^{5/2}}$$
$$- \frac{2(d+ex)(f+gx)^2}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{8(ef-dg)(d+ex)(f+gx)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$
$$- \frac{4(d+ex)(f+gx)^2}{b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

## Mathematica [A] (verified)

Time = 2.64 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.25

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx =$$

$$-\frac{2e^{-\frac{3a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-3/n}\left(2de^{\frac{2a}{bn}}g(8ef+dg)\sqrt{\pi}(c(d+ex)^n)^{2/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)(a+b\log(c(\right.}{}$$

```
[In] Integrate[(f + g*x)^2/(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] (-2*(d + e*x)*(2*d*E^((2*a)/(b*n))*g*(8*e*f + d*g)*Sqrt[Pi]*(c*(d + e*x)^n)
^(2/n)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])]*(a + b*Log[c*
(d + e*x)^n])^(3/2) + 8*E^(a/(b*n))*g*(-(e*f) + d*g)*Sqrt[2*Pi]*(d + e*x)*(
c*(d + e*x)^n)^n^(-1)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b
]*Sqrt[n])]*(a + b*Log[c*(d + e*x)^n])^(3/2) - 6*g^2*Sqrt[3*Pi]*(d + e*x)^2
*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])]*(a + b*Lo
g[c*(d + e*x)^n])^(3/2) + Sqrt[b]*E^((2*a)/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^(
2/n)*(2*b*(e^2*f^2 + 6*d*e*f*g + 2*d^2*g^2)*n*Gamma[1/2, -((a + b*Log[c*(d
+ e*x)^n])/(b*n))]*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^(3/2) + e*E^(a/(b*
n))*(c*(d + e*x)^n)^n^(-1)*(f + g*x)*(b*e*n*(f + g*x) + 2*a*(e*f + 2*d*g +
3*e*g*x) + 2*b*(2*d*g + e*(f + 3*g*x))*Log[c*(d + e*x)^n]))))/(3*b^(5/2)*e^
3*E^((3*a)/(b*n))*n^(5/2)*(c*(d + e*x)^n)^(3/n)*(a + b*Log[c*(d + e*x)^n])^
(3/2))
```

## Maple [F]

$$\int \frac{(gx+f)^2}{(a+b\ln(c(ex+d)^n))^{\frac{5}{2}}}\,dx$$

```
[In] int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

```
[Out] int((g*x+f)^2/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)**2/(a+b*ln(c*(e*x+d)**n))**(5/2),x)
[Out] Integral((f + g*x)**2/(a + b*log(c*(d + e*x)**n))**(5/2), x)
```

## Maxima [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(gx+f)^2}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^2/(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Giac [F]

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(gx+f)^2}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)^2/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
[Out] integrate((g*x + f)^2/(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{(f+g\,x)^2}{(a+b\,\ln(c\,(d+e\,x)^n))^{5/2}}\,dx$$

```
[In] int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^(5/2),x)
[Out] int((f + g*x)^2/(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

# 3.135 $\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 311

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \frac{4e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^2n^{5/2}} + \frac{8e^{-\frac{2a}{bn}}g\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^2n^{5/2}} - \frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{4(ef-dg)(d+ex)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{8(d+ex)(f+gx)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

```
[Out] -2/3*(e*x+d)*(g*x+f)/b/e/n/(a+b*ln(c*(e*x+d)^n))^(3/2)+4/3*(-d*g+e*f)*(e*x+
d)*erfi((a+b*ln(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)/e^2/e
xp(a/b/n)/n^(5/2)/((c*(e*x+d)^n)^(1/n))+8/3*g*(e*x+d)^2*erfi(2^(1/2)*(a+b*l
n(c*(e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/e^2/exp(2*a
/b/n)/n^(5/2)/((c*(e*x+d)^n)^(2/n))+4/3*(-d*g+e*f)*(e*x+d)/b^2/e^2/n^2/(a+b
*ln(c*(e*x+d)^n))^(1/2)-8/3*(e*x+d)*(g*x+f)/b^2/e/n^2/(a+b*ln(c*(e*x+d)^n))
^(1/2)
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2447, 2448, 2436, 2337, 2211, 2235, 2437, 2347, 2334}

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx=\frac{4\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)(ef-dg)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^2n^{5/2}}+\frac{8\sqrt{2\pi}ge^{-\frac{2a}{bn}}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^2n^{5/2}}+\frac{4(d+ex)(ef-dg)}{3b^2e^2n^2\sqrt{a+b\log(c(d+ex)^n)}}-\frac{8(d+ex)(f+gx)}{3b^2en^2\sqrt{a+b\log(c(d+ex)^n)}}-\frac{2(d+ex)(f+gx)}{3ben(a+b\log(c(d+ex)^n))^{3/2}}$$

```
[In] Int[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] (4*(e*f - d*g)*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt
[b]*Sqrt[n])])/(3*b^(5/2)*e^2*E^(a/(b*n))*n^(5/2)*(c*(d + e*x)^n)^n^(-1)) +
 (8*g*Sqrt[2*Pi]*(d + e*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/
(Sqrt[b]*Sqrt[n])])/(3*b^(5/2)*e^2*E^((2*a)/(b*n))*n^(5/2)*(c*(d + e*x)^n)^
(2/n)) - (2*(d + e*x)*(f + g*x))/(3*b*e*n*(a + b*Log[c*(d + e*x)^n])^(3/2))
 + (4*(e*f - d*g)*(d + e*x))/(3*b^2*e^2*n^2*Sqrt[a + b*Log[c*(d + e*x)^n]])
 - (8*(d + e*x)*(f + g*x))/(3*b^2*e*n^2*Sqrt[a + b*Log[c*(d + e*x)^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}+\frac{4\int\frac{f+gx}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx}{3bn}\\ &\quad-\frac{(2(ef-dg))\int\frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}\,dx}{3ben}\end{aligned}$$

$$= -\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{8(d+ex)(f+gx)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{16\int \frac{f+gx}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{3b^2n^2} - \frac{(8(ef-dg))\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{3b^2en^2} - \frac{(2(ef-dg))\mathrm{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^{3/2}}\,dx, x, d+ex\right)}{3be^2n}$$

$$= -\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{4(ef-dg)(d+ex)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{8(d+ex)(f+gx)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{16\int \left(\frac{ef-dg}{e\sqrt{a+b\log(c(d+ex)^n)}} + \frac{g(d+ex)}{e\sqrt{a+b\log(c(d+ex)^n)}}\right)dx}{3b^2n^2} - \frac{(4(ef-dg))\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{3b^2e^2n^2} - \frac{(8(ef-dg))\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{3b^2e^2n^2}$$

$$= -\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{4(ef-dg)(d+ex)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{8(d+ex)(f+gx)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{(16g)\int \frac{d+ex}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{3b^2en^2} + \frac{(16(ef-dg))\int \frac{1}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{3b^2en^2} - \frac{\left(4(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\mathrm{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{3b^2e^2n^3} - \frac{\left(8(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\mathrm{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{3b^2e^2n^3}$$

$$= -\frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{4(ef-dg)(d+ex)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{8(d+ex)(f+gx)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{(16g)\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{3b^2e^2n^2} + \frac{(16(ef-dg))\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{3b^2e^2n^2} - \frac{\left(8(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{3b^3e^2n^3} - \frac{\left(16(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{3b^3e^2n^3}$$

$$= -\frac{4e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^2n^{5/2}} - \frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{4(ef-dg)(d+ex)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{8(d+ex)(f+gx)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(16g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{3b^2e^2n^3} + \frac{\left(16(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx, x, \log\left(c(d+ex)^n\right)\right)}{3b^2e^2n^3}$$

$$= -\frac{4e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{b^{5/2}e^2n^{5/2}} - \frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{4(ef-dg)(d+ex)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{8(d+ex)(f+gx)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(32g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{3b^3e^2n^3} + \frac{\left(32(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{3b^3e^2n^3}$$

$$= \frac{4e^{-\frac{a}{bn}}(ef-dg)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^2n^{5/2}} + \frac{8e^{-\frac{2a}{bn}}g\sqrt{2\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}e^2n^{5/2}} - \frac{2(d+ex)(f+gx)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{4(ef-dg)(d+ex)}{3b^2e^2n^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{8(d+ex)(f+gx)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

## Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.14

$$\int \frac{f+gx}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx = \frac{2e^{-\frac{2a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(-8de^{\frac{a}{bn}}g\sqrt{\pi}\left(c(d+ex)^n\right)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c}}{\sqrt{b}\sqrt{}}\right.\right.}{}$$

```
[In] Integrate[(f + g*x)/(a + b*Log[c*(d + e*x)^n])^(5/2),x]
```

```
[Out] (2*(d + e*x)*(-8*d*E^(a/(b*n))*g*Sqrt[Pi]*(c*(d + e*x)^n)^n^(-1)*Erfi[Sqrt[
a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])] + 4*g*Sqrt[2*Pi]*(d + e*x)*Erf
i[(Sqrt[2]*Sqrt[a + b*Log[c*(d + e*x)^n]])/(Sqrt[b]*Sqrt[n])] - (Sqrt[b]*E^
(a/(b*n))*Sqrt[n]*(c*(d + e*x)^n)^n^(-1)*(2*b*(e*f + 3*d*g)*n*Gamma[1/2, -(
(a + b*Log[c*(d + e*x)^n])/(b*n))]*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^(3
/2) + E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(b*e*n*(f + g*x) + 2*a*(e*f + d*g
+ 2*e*g*x) + 2*b*(d*g + e*(f + 2*g*x))*Log[c*(d + e*x)^n])))/(a + b*Log[c*(
d + e*x)^n])^(3/2)))/(3*b^(5/2)*e^2*E^((2*a)/(b*n))*n^(5/2)*(c*(d + e*x)^n)
^(2/n))
```

## Maple [F]

$$\int \frac{gx+f}{\left(a+b\ln\left(c\left(ex+d\right)^n\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

```
[Out] int((g*x+f)/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)/(a+b*ln(c*(e*x+d)**n))**(5/2),x)
[Out] Integral((f + g*x)/(a + b*log(c*(d + e*x)**n))**(5/2), x)
```

## Maxima [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{gx+f}{(b\log((ex+d)^nc)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
[Out] integrate((g*x + f)/(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Giac [F]

$$\int \frac{f+gx}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{gx+f}{(b\log((ex+d)^nc)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
[Out] integrate((g*x + f)/(b*log((e*x + d)^n*c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(a+b\log{(c(d+ex)^n)})^{5/2}}\,dx=\int \frac{f+g\,x}{{\left(a+b\,\ln{\left(c\,{\left(d+e\,x\right)}^n\right)}\right)}^{5/2}}\,dx$$

```
[In] int((f + g*x)/(a + b*log(c*(d + e*x)^n))^(5/2),x)
[Out] int((f + g*x)/(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

# 3.136 $\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 156

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \frac{4e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}en^{5/2}} - \frac{2(d+ex)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

```
[Out] -2/3*(e*x+d)/b/e/n/(a+b*ln(c*(e*x+d)^n))^(3/2)+4/3*(e*x+d)*erfi((a+b*ln(c*(
e*x+d)^n))^(1/2)/b^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)/e/exp(a/b/n)/n^(5/2)/((c
*(e*x+d)^n)^(1/n))-4/3*(e*x+d)/b^2/e/n^2/(a+b*ln(c*(e*x+d)^n))^(1/2)
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2436, 2334, 2337, 2211, 2235}

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \frac{4\sqrt{\pi}e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}en^{5/2}} - \frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} - \frac{2(d+ex)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^(-5/2),x]
```

```
[Out] (4*Sqrt[Pi]*(d + e*x)*Erfi[Sqrt[a + b*Log[c*(d + e*x)^n]]/(Sqrt[b]*Sqrt[n])
])/(3*b^(5/2)*e*E^(a/(b*n))*n^(5/2)*(c*(d + e*x)^n)^n^(-1)) - (2*(d + e*x))
```

```
/(3*b*e*n*(a + b*Log[c*(d + e*x)^n])^(3/2)) - (4*(d + e*x))/(3*b^2*e*n^2*Sq
rt[a + b*Log[c*(d + e*x)^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^{5/2}}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{2(d+ex)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^{3/2}}\,dx, x, d+ex\right)}{3ben}$$

$$= -\frac{2(d+ex)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx, x, d+ex\right)}{3b^2en^2}$$

$$= -\frac{2(d+ex)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(4(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}dx, x, \log\left(c(d+ex)^n\right)\right)}{3b^2en^3}$$

$$= -\frac{2(d+ex)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}} + \frac{\left(8(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}dx, x, \sqrt{a+b\log\left(c(d+ex)^n\right)}\right)}{3b^3en^3}$$

$$= \frac{4e^{-\frac{a}{bn}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{b}\sqrt{n}}\right)}{3b^{5/2}en^{5/2}} - \frac{2(d+ex)}{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}} - \frac{4(d+ex)}{3b^2en^2\sqrt{a+b\log\left(c(d+ex)^n\right)}}$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}dx = -\frac{2e^{-\frac{a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\left(2bn\Gamma\left(\frac{1}{2},-\frac{a+b\log(c(d+ex)^n)}{bn}\right)\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{3/2}+e^{\frac{a}{bn}}\left(c(d+ex)^n\right)^{\frac{1}{n}}(2a\right.}{3b^2en^2\left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^(-5/2),x]
```

```
[Out] (-2*(d + e*x)*(2*b*n*Gamma[1/2, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(-((a
+ b*Log[c*(d + e*x)^n])/(b*n)))^(3/2) + E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*
(2*a + b*n + 2*b*Log[c*(d + e*x)^n])))/(3*b^2*e*E^(a/(b*n))*n^2*(c*(d + e*x
)^n)^n^(-1)*(a + b*Log[c*(d + e*x)^n])^(3/2))
```

## Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(ex+d\right)^n\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

```
[Out] int(1/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{1}{(a+b\log(c(d+ex)^n))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n))**(5/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**(-5/2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(-5/2), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{1}{(b\log((ex+d)^n c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(-5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d+ex)^n))^{5/2}}\,dx = \int \frac{1}{(a+b\,\ln(c\,(d+e\,x)^n))^{5/2}}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e*x)^n))^(5/2),x)
[Out] int(1/(a + b*log(c*(d + e*x)^n))^(5/2), x)
```

# 3.137 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx=\text{Int}\left(\frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx$$

```
[In] Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(5/2)),x]
```

```
[Out] Defer[Int][1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(5/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx$$

```
[In] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(5/2)),x]
[Out] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^(5/2)), x]
```

## Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(gx+f)\left(a+b\ln\left(c\,(ex+d)^{n}\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
[Out] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**(5/2),x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^(5/2)), x)
```

## Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^(5/2),x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^{5/2}}\,dx=\int \frac{1}{(f+g\,x)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^{5/2}}\,dx$$

```
[In] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^(5/2)),x)
[Out] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^(5/2)), x)
```

# 3.138 $\int (f+gx)^{3/2} (a+b\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 163

$$\int (f+gx)^{3/2} (a+b\log(c(d+ex)^n))\,dx = -\frac{4b(ef-dg)^2n\sqrt{f+gx}}{5e^2g} - \frac{4b(ef-dg)n(f+gx)^{3/2}}{15eg} - \frac{4bn(f+gx)^{5/2}}{25g} + \frac{4b(ef-dg)^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{5e^{5/2}g} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5g}$$

```
[Out] -4/15*b*(-d*g+e*f)*n*(g*x+f)^(3/2)/e/g-4/25*b*n*(g*x+f)^(5/2)/g+4/5*b*(-d*g
+e*f)^(5/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(5/2)/g+2/5
*(g*x+f)^(5/2)*(a+b*ln(c*(e*x+d)^n))/g-4/5*b*(-d*g+e*f)^2*n*(g*x+f)^(1/2)/e
^2/g
```

## Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2442, 52, 65, 214}

$$\int (f+gx)^{3/2} (a+b\log(c(d+ex)^n))\,dx = \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5g} + \frac{4bn(ef-dg)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{5e^{5/2}g} - \frac{4bn\sqrt{f+gx}(ef-dg)^2}{5e^2g} - \frac{4bn(f+gx)^{3/2}(ef-dg)}{15eg} - \frac{4bn(f+gx)^{5/2}}{25g}$$

```
[In] Int[(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (-4*b*(e*f - d*g)^2*n*Sqrt[f + g*x])/(5*e^2*g) - (4*b*(e*f - d*g)*n*(f + g*
x)^(3/2))/(15*e*g) - (4*b*n*(f + g*x)^(5/2))/(25*g) + (4*b*(e*f - d*g)^(5/2
)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(5*e^(5/2)*g) + (2*(f
 + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n]))/(5*g)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g} - \frac{(2ben)\int \frac{(f+gx)^{5/2}}{d+ex}\,dx}{5g}$$

$$= -\frac{4bn(f+gx)^{5/2}}{25g} + \frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g} - \frac{(2b(ef-dg)n)\int \frac{(f+gx)^{3/2}}{d+ex}\,dx}{5g}$$

$$= -\frac{4b(ef-dg)n(f+gx)^{3/2}}{15eg} - \frac{4bn(f+gx)^{5/2}}{25g} + \frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g} - \frac{\left(2b(ef-dg)^2n\right)\int \frac{\sqrt{f+gx}}{d+ex}\,dx}{5eg}$$

$$= -\frac{4b(ef-dg)^2n\sqrt{f+gx}}{5e^2g} - \frac{4b(ef-dg)n(f+gx)^{3/2}}{15eg} - \frac{4bn(f+gx)^{5/2}}{25g} + \frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g} - \frac{\left(2b(ef-dg)^3n\right)\int \frac{1}{(d+ex)\sqrt{f+gx}}\,dx}{5e^2g}$$

$$= -\frac{4b(ef-dg)^2n\sqrt{f+gx}}{5e^2g} - \frac{4b(ef-dg)n(f+gx)^{3/2}}{15eg} - \frac{4bn(f+gx)^{5/2}}{25g} + \frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g} - \frac{\left(4b(ef-dg)^3n\right)\text{Subst}\left(\int \frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{5e^2g^2}$$

$$= -\frac{4b(ef-dg)^2n\sqrt{f+gx}}{5e^2g} - \frac{4b(ef-dg)n(f+gx)^{3/2}}{15eg} - \frac{4bn(f+gx)^{5/2}}{25g} + \frac{4b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{5e^{5/2}g} + \frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.84

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = \frac{2\left(-\frac{2}{5}bn(f+gx)^{5/2} - \frac{2b(ef-dg)n\left(\sqrt{e}\sqrt{f+gx}(4ef-3dg+egx)-3(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\right)}{3e^{5/2}}\right.}{5g}$$

```
[In] Integrate[(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (2*((-2*b*n*(f + g*x)^(5/2))/5 - (2*b*(e*f - d*g)*n*(Sqrt[e]*Sqrt[f + g*x]*
(4*e*f - 3*d*g + e*g*x) - 3*(e*f - d*g)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x
])/Sqrt[e*f - d*g]]))/(3*e^(5/2)) + (f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^
n])))/(5*g)
```

## Maple [F]

$$\int (gx+f)^{\frac{3}{2}} (a+b\ln (c(ex+d)^n))\, dx$$

[In] int((g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n)),x)

[Out] int((g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n)),x)

## Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 538, normalized size of antiderivative = 3.30

$$\int (f+gx)^{3/2} (a+b\log (c(d+ex)^n))\, dx = \left[\frac{2\left(15\,(be^2f^2-2\,bdefg+bd^2g^2)n\sqrt{\frac{ef-dg}{e}}\log\left(\frac{egx+2\,ef-dg+2\,\sqrt{gx+f}e\sqrt{\frac{ef-dg}{e}}}{ex+d}\right)+(15\,a\right.}{}\right.$$

[In] integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")

[Out] [2/75*(15*(b*e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*n*sqrt((e*f - d*g)/e)*log((e*g*x + 2*e*f - d*g + 2*sqrt(g*x + f)*e*sqrt((e*f - d*g)/e))/(e*x + d)) + (15*a*e^2*f^2 - 3*(2*b*e^2*g^2*n - 5*a*e^2*g^2)*x^2 - 2*(23*b*e^2*f^2 - 35*b*d*e*f*g + 15*b*d^2*g^2)*n + 2*(15*a*e^2*f*g - (11*b*e^2*f*g - 5*b*d*e*g^2)*n)*x + 15*(b*e^2*g^2*n*x^2 + 2*b*e^2*f*g*n*x + b*e^2*f^2*n)*log(e*x + d) + 15*(b*e^2*g^2*x^2 + 2*b*e^2*f*g*x + b*e^2*f^2)*log(c))*sqrt(g*x + f))/(e^2*g), 2/75*(30*(b*e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*n*sqrt(-(e*f - d*g)/e)*arctan(-sqrt(g*x + f)*e*sqrt(-(e*f - d*g)/e)/(e*f - d*g)) + (15*a*e^2*f^2 - 3*(2*b*e^2*g^2*n - 5*a*e^2*g^2)*x^2 - 2*(23*b*e^2*f^2 - 35*b*d*e*f*g + 15*b*d^2*g^2)*n + 2*(15*a*e^2*f*g - (11*b*e^2*f*g - 5*b*d*e*g^2)*n)*x + 15*(b*e^2*g^2*n*x^2 + 2*b*e^2*f*g*n*x + b*e^2*f^2*n)*log(e*x + d) + 15*(b*e^2*g^2*x^2 + 2*b*e^2*f*g*x + b*e^2*f^2)*log(c))*sqrt(g*x + f))/(e^2*g)]

## Sympy [F]

$$\int (f+gx)^{3/2} (a+b\log (c(d+ex)^n))\, dx = \int (a+b\log (c(d+ex)^n)) (f+gx)^{\frac{3}{2}}\, dx$$

[In] integrate((g*x+f)**(3/2)*(a+b*ln(c*(e*x+d)**n)),x)

[Out] Integral((a + b*log(c*(d + e*x)**n))*(f + g*x)**(3/2), x)

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = \int (gx+f)^{\frac{3}{2}}\left(b\log\left((ex+d)^n c\right)+a\right)dx$$

```
[In] integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^(3/2)*(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx = \int (f+g\,x)^{3/2}\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)\,dx$$

```
[In] int((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n)),x)
```

```
[Out] int((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n)), x)
```

# 3.139 $\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx = -\frac{4b(ef-dg)n\sqrt{f+gx}}{3eg}-\frac{4bn(f+gx)^{3/2}}{9g}+\frac{4b(ef-dg)^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3e^{3/2}g}+\frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3g}$$

```
[Out] -4/9*b*n*(g*x+f)^(3/2)/g+4/3*b*(-d*g+e*f)^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^(
1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/g+2/3*(g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))/g-
4/3*b*(-d*g+e*f)*n*(g*x+f)^(1/2)/e/g
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2442, 52, 65, 214}

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx = \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3g}+\frac{4bn(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3e^{3/2}g}-\frac{4bn\sqrt{f+gx}(ef-dg)}{3eg}-\frac{4bn(f+gx)^{3/2}}{9g}$$

```
[In] Int[Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (-4*b*(e*f - d*g)*n*Sqrt[f + g*x])/(3*e*g) - (4*b*n*(f + g*x)^(3/2))/(9*g)
+ (4*b*(e*f - d*g)^(3/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
)/(3*e^(3/2)*g) + (2*(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n]))/(3*g)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}-\frac{(2ben)\int\frac{(f+gx)^{3/2}}{d+ex}\,dx}{3g}\\
&= -\frac{4bn(f+gx)^{3/2}}{9g}+\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}-\frac{(2b(ef-dg)n)\int\frac{\sqrt{f+gx}}{d+ex}\,dx}{3g}\\
&= -\frac{4b(ef-dg)n\sqrt{f+gx}}{3eg}-\frac{4bn(f+gx)^{3/2}}{9g}\\
&\quad+\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}-\frac{\left(2b(ef-dg)^2n\right)\int\frac{1}{(d+ex)\sqrt{f+gx}}\,dx}{3eg}
\end{aligned}$$

$$= -\frac{4b(ef-dg)n\sqrt{f+gx}}{3eg} - \frac{4bn(f+gx)^{3/2}}{9g} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3g} - \frac{(4b(ef-dg)^2n)\,\text{Subst}\left(\int \frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{3eg^2}$$

$$= -\frac{4b(ef-dg)n\sqrt{f+gx}}{3eg} - \frac{4bn(f+gx)^{3/2}}{9g} + \frac{4b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3e^{3/2}g} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3g}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx = \frac{2\left(6b(ef-dg)^{3/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right) + \sqrt{e}\sqrt{f+gx}(3ae(f+gx) - 2bn(4ef-3dg+egx) + 3be(f+gx)\right.}{9e^{3/2}g}$$

```
[In] Integrate[Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n]),x]
[Out] (2*(6*b*(e*f - d*g)^(3/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]
] + Sqrt[e]*Sqrt[f + g*x]*(3*a*e*(f + g*x) - 2*b*n*(4*e*f - 3*d*g + e*g*x)
+ 3*b*e*(f + g*x)*Log[c*(d + e*x)^n])))/(9*e^(3/2)*g)
```

## Maple [F]

$$\int \sqrt{gx+f}\,(a+b\ln(c(ex+d)^n))\,dx$$

```
[In] int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n)),x)
[Out] int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n)),x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.36

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx = \left[-\frac{2\left(3(bef-bdg)n\sqrt{\frac{ef-dg}{e}}\log\left(\frac{egx+2ef-dg-2\sqrt{gx+f}e\sqrt{\frac{ef-dg}{e}}}{ex+d}\right) - (3aef-2(4bef-3bdg)n-(2begn-3\right.}{9eg}\right.$$

```
[In] integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

```
[Out] [-2/9*(3*(b*e*f - b*d*g)*n*sqrt((e*f - d*g)/e)*log((e*g*x + 2*e*f - d*g - 2
*sqrt(g*x + f)*e*sqrt((e*f - d*g)/e))/(e*x + d)) - (3*a*e*f - 2*(4*b*e*f -
3*b*d*g)*n - (2*b*e*g*n - 3*a*e*g)*x + 3*(b*e*g*n*x + b*e*f*n)*log(e*x + d)
 + 3*(b*e*g*x + b*e*f)*log(c))*sqrt(g*x + f))/(e*g), 2/9*(6*(b*e*f - b*d*g)
*n*sqrt(-(e*f - d*g)/e)*arctan(-sqrt(g*x + f)*e*sqrt(-(e*f - d*g)/e)/(e*f -
 d*g)) + (3*a*e*f - 2*(4*b*e*f - 3*b*d*g)*n - (2*b*e*g*n - 3*a*e*g)*x + 3*(
b*e*g*n*x + b*e*f*n)*log(e*x + d) + 3*(b*e*g*x + b*e*f)*log(c))*sqrt(g*x +
f))/(e*g)]
```

## Sympy [F]

$$\int \sqrt{f+gx}(a+b\log{(c(d+ex)^n)})\,dx = \int (a+b\log{(c(d+ex)^n)})\sqrt{f+gx}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(a+b*ln(c*(e*x+d)**n)),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))*sqrt(f + g*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \sqrt{f+gx}(a+b\log{(c(d+ex)^n)})\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \sqrt{f+gx}(a+b\log{(c(d+ex)^n)})\,dx = \int \sqrt{gx+f}(b\log{((ex+d)^n c)}+a)\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)*(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))\,dx=\int \sqrt{f+g\,x}\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)\,dx$$

```
[In] int((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n)),x)
[Out] int((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n)), x)
```

# 3.140 $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f+gx}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx = -\frac{4bn\sqrt{f+gx}}{g} + \frac{4b\sqrt{ef-dg}\,n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}g} + \frac{2\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}$$

```
[Out] 4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(-d*g+e*f)^(1/2)/g/e^
(1/2)-4*b*n*(g*x+f)^(1/2)/g+2*(a+b*ln(c*(e*x+d)^n))*(g*x+f)^(1/2)/g
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2442, 52, 65, 214}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g} + \frac{4bn\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}g} - \frac{4bn\sqrt{f+gx}}{g}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/Sqrt[f + g*x],x]
```

```
[Out] (-4*b*n*Sqrt[f + g*x])/g + (4*b*Sqrt[e*f - d*g]*n*ArcTanh[(Sqrt[e]*Sqrt[f +
 g*x])/Sqrt[e*f - d*g]])/(Sqrt[e]*g) + (2*Sqrt[f + g*x]*(a + b*Log[c*(d + e
*x)^n]))/g
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g} - \frac{(2ben)\int\frac{\sqrt{f+gx}}{d+ex}\,dx}{g}\\
&= -\frac{4bn\sqrt{f+gx}}{g} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g} - \frac{(2b(ef-dg)n)\int\frac{1}{(d+ex)\sqrt{f+gx}}\,dx}{g}\\
&= -\frac{4bn\sqrt{f+gx}}{g} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g}\\
&\quad - \frac{(4b(ef-dg)n)\text{Subst}\left(\int\frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{g^2}\\
&= -\frac{4bn\sqrt{f+gx}}{g} + \frac{4b\sqrt{ef-dg}\,n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}g} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx = \frac{2\left(\frac{2b\sqrt{ef-dg}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}}+\sqrt{f+gx}(a-2bn+b\log\left(c(d+ex)^n\right))\right)}{g}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/Sqrt[f + g*x],x]
[Out] (2*((2*b*Sqrt[e*f - d*g]*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
)/Sqrt[e] + Sqrt[f + g*x]*(a - 2*b*n + b*Log[c*(d + e*x)^n])))/g
```

## Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2\sqrt{gx+f}\,a+2b\left(\ln\left(c\left(\frac{(gx+f)e+dg-ef}{g}\right)^{n}\right)\sqrt{gx+f}-2en\left(\frac{\sqrt{gx+f}}{e}+\frac{(-dg+ef)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e\sqrt{(dg-ef)e}}\right)\right)}{g}$ | 113 |
| default | $\frac{2\sqrt{gx+f}\,a+2b\left(\ln\left(c\left(\frac{(gx+f)e+dg-ef}{g}\right)^{n}\right)\sqrt{gx+f}-2en\left(\frac{\sqrt{gx+f}}{e}+\frac{(-dg+ef)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e\sqrt{(dg-ef)e}}\right)\right)}{g}$ | 113 |
| parts | $\frac{2a\sqrt{gx+f}}{g}+\frac{2b\left(\ln\left(c\left(\frac{(gx+f)e+dg-ef}{g}\right)^{n}\right)\sqrt{gx+f}-2en\left(\frac{\sqrt{gx+f}}{e}+\frac{(-dg+ef)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e\sqrt{(dg-ef)e}}\right)\right)}{g}$ | 116 |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/g*((g*x+f)^(1/2)*a+b*(ln(c*(((g*x+f)*e+d*g-e*f)/g)^n)*(g*x+f)^(1/2)-2*e*n
*((g*x+f)^(1/2)/e+(-d*g+e*f)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/(
(d*g-e*f)*e)^(1/2))))
```

## Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.91

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx = \left[\frac{2\left(bn\sqrt{\frac{ef-dg}{e}}\log\left(\frac{egx+2\,ef-dg+2\,\sqrt{gx+f}e\sqrt{\frac{ef-dg}{e}}}{ex+d}\right)+\left(bn\log\left(ex+d\right)-2\,bn+b\log\left(c\right)+a\right)\sqrt{gx+f}\right)}{g}, \frac{2\left(2\right.}{}\right.$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*(b*n*sqrt((e*f - d*g)/e)*log((e*g*x + 2*e*f - d*g + 2*sqrt(g*x + f)*e*sq
rt((e*f - d*g)/e))/(e*x + d)) + (b*n*log(e*x + d) - 2*b*n + b*log(c) + a)*s
qrt(g*x + f))/g, 2*(2*b*n*sqrt(-(e*f - d*g)/e)*arctan(-sqrt(g*x + f)*e*sqrt
(-(e*f - d*g)/e)/(e*f - d*g)) + (b*n*log(e*x + d) - 2*b*n + b*log(c) + a)*s
qrt(g*x + f))/g]
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx = \int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))/sqrt(f + g*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx = -\frac{2\left(\left(2\,e\left(\frac{(ef-dg)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+deg}e}+\frac{\sqrt{gx+f}}{e}\right)-\sqrt{gx+f}\log\left(ex+d\right)\right)bn-\sqrt{gx+f}b\log\left(c\right)-\sqrt{gx+f}a\right)}{g}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(1/2),x, algorithm="giac")
[Out] -2*((2*e*((e*f - d*g)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e
^2*f + d*e*g)*e) + sqrt(g*x + f)/e) - sqrt(g*x + f)*log(e*x + d))*b*n - sqr
t(g*x + f)*b*log(c) - sqrt(g*x + f)*a)/g
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx}}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{\sqrt{f+g\,x}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(1/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(1/2), x)
```

# 3.141 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 81

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{3/2}}\,dx=-\frac{4b\sqrt{e}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{g\sqrt{ef-dg}}-\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{g\sqrt{f+gx}}$$

```
[Out] -4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*e^(1/2)/g/(-d*g+e*f)
^(1/2)-2*(a+b*ln(c*(e*x+d)^n))/g/(g*x+f)^(1/2)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2442, 65, 214}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{3/2}}\,dx=-\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{g\sqrt{f+gx}}-\frac{4b\sqrt{e}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{g\sqrt{ef-dg}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(3/2),x]
```

```
[Out] (-4*b*Sqrt[e]*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(g*Sqrt[e
*f - d*g]) - (2*(a + b*Log[c*(d + e*x)^n]))/(g*Sqrt[f + g*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(a+b\log(c(d+ex)^n))}{g\sqrt{f+gx}}+\frac{(2ben)\int\frac{1}{(d+ex)\sqrt{f+gx}}\,dx}{g}\\
&= -\frac{2(a+b\log(c(d+ex)^n))}{g\sqrt{f+gx}}+\frac{(4ben)\text{Subst}\left(\int\frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}\,dx,x,\sqrt{f+gx}\right)}{g^2}\\
&= -\frac{4b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{g\sqrt{ef-dg}}-\frac{2(a+b\log(c(d+ex)^n))}{g\sqrt{f+gx}}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int\frac{a+b\log(c(d+ex)^n)}{(f+gx)^{3/2}}\,dx=\frac{2\left(-\frac{2b\sqrt{e}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\frac{a+b\log(c(d+ex)^n)}{\sqrt{f+gx}}\right)}{g}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(3/2),x]
[Out] (2*((-2*b*Sqrt[e]*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[
e*f - d*g] - (a + b*Log[c*(d + e*x)^n])/Sqrt[f + g*x]))/g
```

## Maple [F]

$$\int \frac{a+b\ln\left(c(ex+d)^{n}\right)}{(gx+f)^{\frac{3}{2}}}dx$$

[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(3/2),x)

[Out] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(3/2),x)

## Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.77

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{(f+gx)^{3/2}}\,dx = \left[\frac{2\left((bgnx+bfn)\sqrt{\frac{e}{ef-dg}}\log\left(\frac{egx+2\,ef-dg-2\,(ef-dg)\sqrt{gx+f}\sqrt{\frac{e}{ef-dg}}}{ex+d}\right)-(bn\log\left(ex\right.\right.}{g^{2}x+fg}\right.$$
$$\left.-\frac{2\left(2\,(bgnx+bfn)\sqrt{-\frac{e}{ef-dg}}\arctan\left(-\frac{(ef-dg)\sqrt{gx+f}\sqrt{-\frac{e}{ef-dg}}}{egx+ef}\right)+(bn\log\left(ex+d\right)+b\log\left(c\right)+a)\sqrt{gx+f}\right)}{g^{2}x+fg}\right]$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(3/2),x, algorithm="fricas")

[Out] [2*((b*g*n*x + b*f*n)*sqrt(e/(e*f - d*g))*log((e*g*x + 2*e*f - d*g - 2*(e*f - d*g)*sqrt(g*x + f)*sqrt(e/(e*f - d*g)))/(e*x + d)) - (b*n*log(e*x + d) + b*log(c) + a)*sqrt(g*x + f))/(g^2*x + f*g), -2*(2*(b*g*n*x + b*f*n)*sqrt(-e/(e*f - d*g))*arctan(-(e*f - d*g)*sqrt(g*x + f)*sqrt(-e/(e*f - d*g))/(e*g*x + e*f)) + (b*n*log(e*x + d) + b*log(c) + a)*sqrt(g*x + f))/(g^2*x + f*g)]

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{(f+gx)^{3/2}}\,dx = \int \frac{a+b\log\left(c(d+ex)^{n}\right)}{(f+gx)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**(3/2),x)

[Out] Integral((a + b*log(c*(d + e*x)**n))/(f + g*x)**(3/2), x)

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{(f+gx)^{3/2}}\,dx = \frac{4\,ben\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+deg}g} - \frac{2\,bn\log\left((gx+f)e-ef+dg\right)}{\sqrt{gx+f}g} + \frac{2\,(bn\log\left(g\right)-b\log\left(c\right)-a)}{\sqrt{gx+f}g}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 4*b*e*n*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*f + d*e*g)*
g) - 2*b*n*log((g*x + f)*e - e*f + d*g)/(sqrt(g*x + f)*g) + 2*(b*n*log(g) -
 b*log(c) - a)/(sqrt(g*x + f)*g)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{(f+gx)^{3/2}}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{{\left(f+g\,x\right)}^{3/2}} \,d x$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(3/2),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(3/2), x)
```

# 3.142 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx = \frac{4ben}{3g(ef-dg)\sqrt{f+gx}} - \frac{4be^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3g(ef-dg)^{3/2}} - \frac{2(a+b\log\left(c(d+ex)^n\right))}{3g(f+gx)^{3/2}}$$

```
[Out] -4/3*b*e^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/g/(-d*g+e*
f)^(3/2)-2/3*(a+b*ln(c*(e*x+d)^n))/g/(g*x+f)^(3/2)+4/3*b*e*n/g/(-d*g+e*f)/(
g*x+f)^(1/2)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2442, 53, 65, 214}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx = -\frac{2(a+b\log\left(c(d+ex)^n\right))}{3g(f+gx)^{3/2}} - \frac{4be^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3g(ef-dg)^{3/2}} + \frac{4ben}{3g\sqrt{f+gx}(ef-dg)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(5/2),x]
```

```
[Out] (4*b*e*n)/(3*g*(e*f - d*g)*Sqrt[f + g*x]) - (4*b*e^(3/2)*n*ArcTanh[(Sqrt[e]
*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(3*g*(e*f - d*g)^(3/2)) - (2*(a + b*Log[c
*(d + e*x)^n]))/(3*g*(f + g*x)^(3/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(a+b\log(c(d+ex)^n))}{3g(f+gx)^{3/2}}+\frac{(2ben)\int\frac{1}{(d+ex)(f+gx)^{3/2}}\,dx}{3g}\\
&= \frac{4ben}{3g(ef-dg)\sqrt{f+gx}}-\frac{2(a+b\log(c(d+ex)^n))}{3g(f+gx)^{3/2}}+\frac{(2be^2n)\int\frac{1}{(d+ex)\sqrt{f+gx}}\,dx}{3g(ef-dg)}\\
&= \frac{4ben}{3g(ef-dg)\sqrt{f+gx}}-\frac{2(a+b\log(c(d+ex)^n))}{3g(f+gx)^{3/2}}\\
&\quad+\frac{(4be^2n)\,\text{Subst}\left(\int\frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}\,dx,x,\sqrt{f+gx}\right)}{3g^2(ef-dg)}\\
&= \frac{4ben}{3g(ef-dg)\sqrt{f+gx}}-\frac{4be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3g(ef-dg)^{3/2}}-\frac{2(a+b\log(c(d+ex)^n))}{3g(f+gx)^{3/2}}
\end{aligned}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx = -\frac{4ben\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{e(f+gx)}{ef-dg}\right)}{3g(-ef+dg)\sqrt{f+gx}}-\frac{2(a+b\log \left(c(d+ex)^n\right))}{3g(f+gx)^{3/2}}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(5/2),x]
```

```
[Out] (-4*b*e*n*Hypergeometric2F1[-1/2, 1, 1/2, (e*(f + g*x))/(e*f - d*g)])/(3*g*
(-(e*f) + d*g)*Sqrt[f + g*x]) - (2*(a + b*Log[c*(d + e*x)^n]))/(3*g*(f + g*
x)^(3/2))
```

## Maple [F]

$$\int \frac{a+b\ln \left(c(ex+d)^n\right)}{(gx+f)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(5/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(5/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(94) = 188.

Time = 0.36 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.73

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx = \left[-\frac{2\left(\left(beg^2nx^2+2\,befgnx+bef^2n\right)\sqrt{\frac{e}{ef-dg}}\log\left(\frac{egx+2\,ef-dg+2\,(ef-dg)\sqrt{gx+f}\sqrt{\frac{e}{ef}}}{ex+d}\right.\right.}{3\,(ef^3g-df^2g}\right.$$

$$-\frac{2\left(2\left(beg^2nx^2+2\,befgnx+bef^2n\right)\sqrt{-\frac{e}{ef-dg}}\arctan\left(-\frac{(ef-dg)\sqrt{gx+f}\sqrt{-\frac{e}{ef-dg}}}{egx+ef}\right)-\left(2\,begnx+2\,befn-aef\right.\right.}{3\,(ef^3g-df^2g^2+(efg^3-dg^4)x^2+2\,(ef^2g^2-}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(5/2),x, algorithm="fricas")
```

```
[Out] [-2/3*((b*e*g^2*n*x^2 + 2*b*e*f*g*n*x + b*e*f^2*n)*sqrt(e/(e*f - d*g))*log(
(e*g*x + 2*e*f - d*g + 2*(e*f - d*g)*sqrt(g*x + f)*sqrt(e/(e*f - d*g)))/(e*
```

```
x + d)) - (2*b*e*g*n*x + 2*b*e*f*n - a*e*f + a*d*g - (b*e*f - b*d*g)*n*log(
e*x + d) - (b*e*f - b*d*g)*log(c))*sqrt(g*x + f))/(e*f^3*g - d*f^2*g^2 + (e
*f*g^3 - d*g^4)*x^2 + 2*(e*f^2*g^2 - d*f*g^3)*x), -2/3*(2*(b*e*g^2*n*x^2 +
2*b*e*f*g*n*x + b*e*f^2*n)*sqrt(-e/(e*f - d*g))*arctan(-(e*f - d*g)*sqrt(g*
x + f)*sqrt(-e/(e*f - d*g))/(e*g*x + e*f)) - (2*b*e*g*n*x + 2*b*e*f*n - a*e
*f + a*d*g - (b*e*f - b*d*g)*n*log(e*x + d) - (b*e*f - b*d*g)*log(c))*sqrt(
g*x + f))/(e*f^3*g - d*f^2*g^2 + (e*f*g^3 - d*g^4)*x^2 + 2*(e*f^2*g^2 - d*f
*g^3)*x)]
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**(5/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))/(f + g*x)**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.44

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx=\frac{4\,be^2n\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{3\,\sqrt{-e^2f+deg}(efg-dg^2)}-\frac{2\,bn\log\left((gx+f)e-ef+dg\right)}{3\,(gx+f)^{\frac{3}{2}}g}+\frac{2\,(befn\log\left(g\right)-bdgn\log\left(g\right)+2\,(gx+f)ben-bef\log\left(c\right)+bdg\log\left(c\right)-aef+adg)}{3\,\left((gx+f)^{\frac{3}{2}}efg-(gx+f)^{\frac{3}{2}}dg^2\right)}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*b*e^2*n*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*f + d*e
*g)*(e*f*g - d*g^2)) - 2/3*b*n*log((g*x + f)*e - e*f + d*g)/((g*x + f)^(3/2
)*g) + 2/3*(b*e*f*n*log(g) - b*d*g*n*log(g) + 2*(g*x + f)*b*e*n - b*e*f*log
(c) + b*d*g*log(c) - a*e*f + a*d*g)/((g*x + f)^(3/2)*e*f*g - (g*x + f)^(3/2
)*d*g^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{5/2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{{\left(f+g\,x\right)}^{5/2}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(5/2),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(5/2), x)
```

# 3.143 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^{7/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{7/2}}\,dx=\frac{4ben}{15g(ef-dg)(f+gx)^{3/2}}+\frac{4be^2n}{5g(ef-dg)^2\sqrt{f+gx}}-\frac{4be^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{5g(ef-dg)^{5/2}}-\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g(f+gx)^{5/2}}$$

```
[Out] 4/15*b*e*n/g/(-d*g+e*f)/(g*x+f)^(3/2)-4/5*b*e^(5/2)*n*arctanh(e^(1/2)*(g*x+
f)^(1/2)/(-d*g+e*f)^(1/2))/g/(-d*g+e*f)^(5/2)-2/5*(a+b*ln(c*(e*x+d)^n))/g/(
g*x+f)^(5/2)+4/5*b*e^2*n/g/(-d*g+e*f)^2/(g*x+f)^(1/2)
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2442, 53, 65, 214}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{7/2}}\,dx=-\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{5g(f+gx)^{5/2}}-\frac{4be^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{5g(ef-dg)^{5/2}}+\frac{4be^2n}{5g\sqrt{f+gx}(ef-dg)^2}+\frac{4ben}{15g(f+gx)^{3/2}(ef-dg)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(7/2),x]
```

```
[Out] (4*b*e*n)/(15*g*(e*f - d*g)*(f + g*x)^(3/2)) + (4*b*e^2*n)/(5*g*(e*f - d*g)
^2*Sqrt[f + g*x]) - (4*b*e^(5/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f
 - d*g]])/(5*g*(e*f - d*g)^(5/2)) - (2*(a + b*Log[c*(d + e*x)^n]))/(5*g*(f
+ g*x)^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(a+b\log(c(d+ex)^n))}{5g(f+gx)^{5/2}}+\frac{(2ben)\int\frac{1}{(d+ex)(f+gx)^{5/2}}\,dx}{5g}\\
&= \frac{4ben}{15g(ef-dg)(f+gx)^{3/2}}-\frac{2(a+b\log(c(d+ex)^n))}{5g(f+gx)^{5/2}}+\frac{(2be^2n)\int\frac{1}{(d+ex)(f+gx)^{3/2}}\,dx}{5g(ef-dg)}\\
&= \frac{4ben}{15g(ef-dg)(f+gx)^{3/2}}+\frac{4be^2n}{5g(ef-dg)^2\sqrt{f+gx}}\\
&\quad-\frac{2(a+b\log(c(d+ex)^n))}{5g(f+gx)^{5/2}}+\frac{(2be^3n)\int\frac{1}{(d+ex)\sqrt{f+gx}}\,dx}{5g(ef-dg)^2}
\end{aligned}$$

$$= \frac{4ben}{15g(ef-dg)(f+gx)^{3/2}} + \frac{4be^2n}{5g(ef-dg)^2\sqrt{f+gx}} - \frac{2(a+b\log(c(d+ex)^n))}{5g(f+gx)^{5/2}} + \frac{(4be^3n)\operatorname{Subst}\left(\int \frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{5g^2(ef-dg)^2}$$

$$= \frac{4ben}{15g(ef-dg)(f+gx)^{3/2}} + \frac{4be^2n}{5g(ef-dg)^2\sqrt{f+gx}} - \frac{4be^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{5g(ef-dg)^{5/2}} - \frac{2(a+b\log(c(d+ex)^n))}{5g(f+gx)^{5/2}}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.54

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^{7/2}}\,dx = \frac{2\left(\frac{2ben(f+gx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{e(f+gx)}{ef-dg}\right)}{ef-dg} - 3(a+b\log(c(d+ex)^n))\right)}{15g(f+gx)^{5/2}}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(7/2),x]
[Out] (2*((2*b*e*n*(f + g*x)*Hypergeometric2F1[-3/2, 1, -1/2, (e*(f + g*x))/(e*f
- d*g)])/(e*f - d*g) - 3*(a + b*Log[c*(d + e*x)^n])))/(15*g*(f + g*x)^(5/2)
)
```

## Maple [F]

$$\int \frac{a+b\ln(c(ex+d)^n)}{(gx+f)^{\frac{7}{2}}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(7/2),x)
[Out] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(7/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(121) = 242.

Time = 0.36 (sec) , antiderivative size = 789, normalized size of antiderivative = 5.44

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{7/2}}\,dx = \left[\frac{2\left(3\left(be^2g^3nx^3+3\,be^2fg^2nx^2+3\,be^2f^2gnx+be^2f^3n\right)\sqrt{\frac{e}{ef-dg}}\log\left(\frac{egx+2\,ef-d\cdots}{\cdots}\right.\right.}{\cdots}\right.$$

$$-\frac{2\left(6\left(be^2g^3nx^3+3\,be^2fg^2nx^2+3\,be^2f^2gnx+be^2f^3n\right)\sqrt{-\frac{e}{ef-dg}}\arctan\left(-\frac{(ef-dg)\sqrt{gx+f}\sqrt{-\frac{e}{ef-dg}}}{egx+ef}\right)-\left(6\,be^2g^{\cdots}\right.\right.}{15\left(e^2f^5g-2\,def^4g^2+d^2f^3g^3\cdots\right.}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(7/2),x, algorithm="fricas")
```

```
[Out] [2/15*(3*(b*e^2*g^3*n*x^3 + 3*b*e^2*f*g^2*n*x^2 + 3*b*e^2*f^2*g*n*x + b*e^2
*f^3*n)*sqrt(e/(e*f - d*g))*log((e*g*x + 2*e*f - d*g - 2*(e*f - d*g)*sqrt(g
*x + f)*sqrt(e/(e*f - d*g)))/(e*x + d)) + (6*b*e^2*g^2*n*x^2 - 3*a*e^2*f^2
+ 6*a*d*e*f*g - 3*a*d^2*g^2 + 2*(7*b*e^2*f*g - b*d*e*g^2)*n*x - 3*(b*e^2*f^
2 - 2*b*d*e*f*g + b*d^2*g^2)*n*log(e*x + d) + 2*(4*b*e^2*f^2 - b*d*e*f*g)*n
 - 3*(b*e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*log(c))*sqrt(g*x + f))/(e^2*f^5*
g - 2*d*e*f^4*g^2 + d^2*f^3*g^3 + (e^2*f^2*g^4 - 2*d*e*f*g^5 + d^2*g^6)*x^3
 + 3*(e^2*f^3*g^3 - 2*d*e*f^2*g^4 + d^2*f*g^5)*x^2 + 3*(e^2*f^4*g^2 - 2*d*e
*f^3*g^3 + d^2*f^2*g^4)*x), -2/15*(6*(b*e^2*g^3*n*x^3 + 3*b*e^2*f*g^2*n*x^2
 + 3*b*e^2*f^2*g*n*x + b*e^2*f^3*n)*sqrt(-e/(e*f - d*g))*arctan(-(e*f - d*g
)*sqrt(g*x + f)*sqrt(-e/(e*f - d*g))/(e*g*x + e*f)) - (6*b*e^2*g^2*n*x^2 -
3*a*e^2*f^2 + 6*a*d*e*f*g - 3*a*d^2*g^2 + 2*(7*b*e^2*f*g - b*d*e*g^2)*n*x -
 3*(b*e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*n*log(e*x + d) + 2*(4*b*e^2*f^2 -
b*d*e*f*g)*n - 3*(b*e^2*f^2 - 2*b*d*e*f*g + b*d^2*g^2)*log(c))*sqrt(g*x + f
))/(e^2*f^5*g - 2*d*e*f^4*g^2 + d^2*f^3*g^3 + (e^2*f^2*g^4 - 2*d*e*f*g^5 +
d^2*g^6)*x^3 + 3*(e^2*f^3*g^3 - 2*d*e*f^2*g^4 + d^2*f*g^5)*x^2 + 3*(e^2*f^4
*g^2 - 2*d*e*f^3*g^3 + d^2*f^2*g^4)*x)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**(7/2),x)
```

```
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{(f + gx)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(7/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{(f + gx)^{7/2}}\,dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{(gx + f)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(7/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/(g*x + f)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{(f + gx)^{7/2}}\,dx = \int \frac{a + b\,\ln\left(c\,{\left(d + e\,x\right)}^n\right)}{{\left(f + g\,x\right)}^{7/2}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(7/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(7/2), x)
```

# 3.144 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^{9/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 176

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx = \frac{4ben}{35g(ef-dg)(f+gx)^{5/2}}+\frac{4be^2n}{21g(ef-dg)^2(f+gx)^{3/2}} + \frac{4be^3n}{7g(ef-dg)^3\sqrt{f+gx}}-\frac{4be^{7/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{7g(ef-dg)^{7/2}}-\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{7g(f+gx)^{7/2}}$$

```
[Out] 4/35*b*e*n/g/(-d*g+e*f)/(g*x+f)^(5/2)+4/21*b*e^2*n/g/(-d*g+e*f)^2/(g*x+f)^(
3/2)-4/7*b*e^(7/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/g/(-d*
g+e*f)^(7/2)-2/7*(a+b*ln(c*(e*x+d)^n))/g/(g*x+f)^(7/2)+4/7*b*e^3*n/g/(-d*g+
e*f)^3/(g*x+f)^(1/2)
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2442, 53, 65, 214}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx = -\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{7g(f+gx)^{7/2}}-\frac{4be^{7/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{7g(ef-dg)^{7/2}} + \frac{4be^3n}{7g\sqrt{f+gx}(ef-dg)^3}+\frac{4be^2n}{21g(f+gx)^{3/2}(ef-dg)^2}+\frac{4ben}{35g(f+gx)^{5/2}(ef-dg)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(9/2),x]
```

```
[Out] (4*b*e*n)/(35*g*(e*f - d*g)*(f + g*x)^(5/2)) + (4*b*e^2*n)/(21*g*(e*f - d*g
)^2*(f + g*x)^(3/2)) + (4*b*e^3*n)/(7*g*(e*f - d*g)^3*Sqrt[f + g*x]) - (4*b
```

```
*e^(7/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(7*g*(e*f - d*
g)^(7/2)) - (2*(a + b*Log[c*(d + e*x)^n]))/(7*g*(f + g*x)^(7/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(a+b\log(c(d+ex)^n))}{7g(f+gx)^{7/2}}+\frac{(2ben)\int\frac{1}{(d+ex)(f+gx)^{7/2}}\,dx}{7g}\\
&= \frac{4ben}{35g(ef-dg)(f+gx)^{5/2}}-\frac{2(a+b\log(c(d+ex)^n))}{7g(f+gx)^{7/2}}+\frac{(2be^2n)\int\frac{1}{(d+ex)(f+gx)^{5/2}}\,dx}{7g(ef-dg)}\\
&= \frac{4ben}{35g(ef-dg)(f+gx)^{5/2}}+\frac{4be^2n}{21g(ef-dg)^2(f+gx)^{3/2}}\\
&\quad-\frac{2(a+b\log(c(d+ex)^n))}{7g(f+gx)^{7/2}}+\frac{(2be^3n)\int\frac{1}{(d+ex)(f+gx)^{3/2}}\,dx}{7g(ef-dg)^2}
\end{aligned}$$

$$= \frac{4ben}{35g(ef-dg)(f+gx)^{5/2}} + \frac{4be^2n}{21g(ef-dg)^2(f+gx)^{3/2}} + \frac{4be^3n}{7g(ef-dg)^3\sqrt{f+gx}} - \frac{2(a+b\log(c(d+ex)^n))}{7g(f+gx)^{7/2}} + \frac{(2be^4n)\int\frac{1}{(d+ex)\sqrt{f+gx}}dx}{7g(ef-dg)^3}$$

$$= \frac{4ben}{35g(ef-dg)(f+gx)^{5/2}} + \frac{4be^2n}{21g(ef-dg)^2(f+gx)^{3/2}} + \frac{4be^3n}{7g(ef-dg)^3\sqrt{f+gx}} - \frac{2(a+b\log(c(d+ex)^n))}{7g(f+gx)^{7/2}} + \frac{(4be^4n)\,\text{Subst}\left(\int\frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}dx, x, \sqrt{f+gx}\right)}{7g^2(ef-dg)^3}$$

$$= \frac{4ben}{35g(ef-dg)(f+gx)^{5/2}} + \frac{4be^2n}{21g(ef-dg)^2(f+gx)^{3/2}} + \frac{4be^3n}{7g(ef-dg)^3\sqrt{f+gx}} - \frac{4be^{7/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{7g(ef-dg)^{7/2}} - \frac{2(a+b\log(c(d+ex)^n))}{7g(f+gx)^{7/2}}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.44

$$\int\frac{a+b\log(c(d+ex)^n)}{(f+gx)^{9/2}}dx = \frac{2\left(\frac{2ben(f+gx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{e(f+gx)}{ef-dg}\right)}{ef-dg} - 5(a+b\log(c(d+ex)^n))\right)}{35g(f+gx)^{7/2}}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^(9/2),x]
[Out] (2*((2*b*e*n*(f + g*x)*Hypergeometric2F1[-5/2, 1, -3/2, (e*(f + g*x))/(e*f
- d*g)])/(e*f - d*g) - 5*(a + b*Log[c*(d + e*x)^n])))/(35*g*(f + g*x)^(7/2)
)
```

## Maple [F]

$$\int\frac{a+b\ln(c(ex+d)^n)}{(gx+f)^{\frac{9}{2}}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(9/2),x)
[Out] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^(9/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(148) = 296$.

Time = 0.43 (sec) , antiderivative size = 1252, normalized size of antiderivative = 7.11

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{(f + gx)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(9/2),x, algorithm="fricas")
```

```
[Out] [-2/105*(15*(b*e^3*g^4*n*x^4 + 4*b*e^3*f*g^3*n*x^3 + 6*b*e^3*f^2*g^2*n*x^2
+ 4*b*e^3*f^3*g*n*x + b*e^3*f^4*n)*sqrt(e/(e*f - d*g))*log((e*g*x + 2*e*f -
 d*g + 2*(e*f - d*g)*sqrt(g*x + f)*sqrt(e/(e*f - d*g)))/(e*x + d)) - (30*b*
e^3*g^3*n*x^3 - 15*a*e^3*f^3 + 45*a*d*e^2*f^2*g - 45*a*d^2*e*f*g^2 + 15*a*d
^3*g^3 + 10*(10*b*e^3*f*g^2 - b*d*e^2*g^3)*n*x^2 + 2*(58*b*e^3*f^2*g - 16*b
*d*e^2*f*g^2 + 3*b*d^2*e*g^3)*n*x - 15*(b*e^3*f^3 - 3*b*d*e^2*f^2*g + 3*b*d
^2*e*f*g^2 - b*d^3*g^3)*n*log(e*x + d) + 2*(23*b*e^3*f^3 - 11*b*d*e^2*f^2*g
 + 3*b*d^2*e*f*g^2)*n - 15*(b*e^3*f^3 - 3*b*d*e^2*f^2*g + 3*b*d^2*e*f*g^2 -
 b*d^3*g^3)*log(c))*sqrt(g*x + f))/(e^3*f^7*g - 3*d*e^2*f^6*g^2 + 3*d^2*e*f
^5*g^3 - d^3*f^4*g^4 + (e^3*f^3*g^5 - 3*d*e^2*f^2*g^6 + 3*d^2*e*f*g^7 - d^3
*g^8)*x^4 + 4*(e^3*f^4*g^4 - 3*d*e^2*f^3*g^5 + 3*d^2*e*f^2*g^6 - d^3*f*g^7)
*x^3 + 6*(e^3*f^5*g^3 - 3*d*e^2*f^4*g^4 + 3*d^2*e*f^3*g^5 - d^3*f^2*g^6)*x^
2 + 4*(e^3*f^6*g^2 - 3*d*e^2*f^5*g^3 + 3*d^2*e*f^4*g^4 - d^3*f^3*g^5)*x), -
2/105*(30*(b*e^3*g^4*n*x^4 + 4*b*e^3*f*g^3*n*x^3 + 6*b*e^3*f^2*g^2*n*x^2 +
4*b*e^3*f^3*g*n*x + b*e^3*f^4*n)*sqrt(-e/(e*f - d*g))*arctan(-(e*f - d*g)*s
qrt(g*x + f)*sqrt(-e/(e*f - d*g))/(e*g*x + e*f)) - (30*b*e^3*g^3*n*x^3 - 15
*a*e^3*f^3 + 45*a*d*e^2*f^2*g - 45*a*d^2*e*f*g^2 + 15*a*d^3*g^3 + 10*(10*b*
e^3*f*g^2 - b*d*e^2*g^3)*n*x^2 + 2*(58*b*e^3*f^2*g - 16*b*d*e^2*f*g^2 + 3*b
*d^2*e*g^3)*n*x - 15*(b*e^3*f^3 - 3*b*d*e^2*f^2*g + 3*b*d^2*e*f*g^2 - b*d^3
*g^3)*n*log(e*x + d) + 2*(23*b*e^3*f^3 - 11*b*d*e^2*f^2*g + 3*b*d^2*e*f*g^2
)*n - 15*(b*e^3*f^3 - 3*b*d*e^2*f^2*g + 3*b*d^2*e*f*g^2 - b*d^3*g^3)*log(c)
)*sqrt(g*x + f))/(e^3*f^7*g - 3*d*e^2*f^6*g^2 + 3*d^2*e*f^5*g^3 - d^3*f^4*g
^4 + (e^3*f^3*g^5 - 3*d*e^2*f^2*g^6 + 3*d^2*e*f*g^7 - d^3*g^8)*x^4 + 4*(e^3
*f^4*g^4 - 3*d*e^2*f^3*g^5 + 3*d^2*e*f^2*g^6 - d^3*f*g^7)*x^3 + 6*(e^3*f^5*
g^3 - 3*d*e^2*f^4*g^4 + 3*d^2*e*f^3*g^5 - d^3*f^2*g^6)*x^2 + 4*(e^3*f^6*g^2
 - 3*d*e^2*f^5*g^3 + 3*d^2*e*f^4*g^4 - d^3*f^3*g^5)*x)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**(9/2),x)
```

```
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)/(g*x + f)^(9/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^{9/2}}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{{\left(f+g\,x\right)}^{9/2}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(9/2),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^(9/2), x)
```

# 3.145 $\int (f+gx)^{3/2} (a+b\log(c(d+ex)^n))^2\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 590

$$\int (f+gx)^{3/2} (a+b\log(c(d+ex)^n))^2\,dx = \frac{368b^2(ef-dg)^2n^2\sqrt{f+gx}}{75e^2g} + \frac{128b^2(ef-dg)n^2(f+gx)^{3/2}}{225eg} + \frac{16b^2n^2(f+gx)^{5/2}}{125g} - \frac{368b^2(ef-dg)^{5/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{75e^{5/2}g} - \frac{8b^2(ef-dg)^{5/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5e^{5/2}g} - \frac{8b(ef-dg)^2n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{5e^2g} - \frac{8b(ef-dg)n(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{15eg} - \frac{8bn(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{25g} + \frac{8b(ef-dg)^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5e^{5/2}g} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))^2}{5g} + \frac{16b^2(ef-dg)^{5/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5e^{5/2}g} + \frac{8b^2(ef-dg)^{5/2}n^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5e^{5/2}g}$$

```
[Out] 128/225*b^2*(-d*g+e*f)*n^2*(g*x+f)^(3/2)/e/g+16/125*b^2*n^2*(g*x+f)^(5/2)/g
-368/75*b^2*(-d*g+e*f)^(5/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(
1/2))/e^(5/2)/g-8/5*b^2*(-d*g+e*f)^(5/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/
(-d*g+e*f)^(1/2))^2/e^(5/2)/g-8/15*b*(-d*g+e*f)*n*(g*x+f)^(3/2)*(a+b*ln(c*(
e*x+d)^n))/e/g-8/25*b*n*(g*x+f)^(5/2)*(a+b*ln(c*(e*x+d)^n))/g+8/5*b*(-d*g+e
*f)^(5/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+
d)^n))/e^(5/2)/g+2/5*(g*x+f)^(5/2)*(a+b*ln(c*(e*x+d)^n))^2/g+16/5*b^2*(-d*g
+e*f)^(5/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(
1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/e^(5/2)/g+8/5*b^2*(-d*g+e*f)^(5/2)*n^
2*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/e^(5/2)/g+368/7
5*b^2*(-d*g+e*f)^2*n^2*(g*x+f)^(1/2)/e^2/g-8/5*b*(-d*g+e*f)^2*n*(a+b*ln(c*(
e*x+d)^n))*(g*x+f)^(1/2)/e^2/g
```

## Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {2445, 2458, 2388, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 52}

$$\int (f+gx)^{3/2} (a+b\log(c(d+ex)^n))^2\,dx = \frac{8bn(ef-dg)^{5/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5e^{5/2}g} - \frac{8bn\sqrt{f+gx}(ef-dg)^2(a+b\log(c(d+ex)^n))}{5e^2g} - \frac{8bn(f+gx)^{3/2}(ef-dg)(a+b\log(c(d+ex)^n))}{15eg} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))^2}{5g} - \frac{8bn(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{25g} - \frac{8b^2n^2(ef-dg)^{5/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5e^{5/2}g} - \frac{368b^2n^2(ef-dg)^{5/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{75e^{5/2}g} + \frac{16b^2n^2(ef-dg)^{5/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5e^{5/2}g} + \frac{8b^2n^2(ef-dg)^{5/2}\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5e^{5/2}g} + \frac{368b^2n^2\sqrt{f+gx}(ef-dg)^2}{75e^2g} + \frac{128b^2n^2(f+gx)^{3/2}(ef-dg)}{225eg} + \frac{16b^2n^2(f+gx)^{5/2}}{125g}$$

```
[In] Int[(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (368*b^2*(e*f - d*g)^2*n^2*Sqrt[f + g*x])/(75*e^2*g) + (128*b^2*(e*f - d*g)
*n^2*(f + g*x)^(3/2))/(225*e*g) + (16*b^2*n^2*(f + g*x)^(5/2))/(125*g) - (3
68*b^2*(e*f - d*g)^(5/2)*n^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g
]])/(75*e^(5/2)*g) - (8*b^2*(e*f - d*g)^(5/2)*n^2*ArcTanh[(Sqrt[e]*Sqrt[f +
 g*x])/Sqrt[e*f - d*g]]^2)/(5*e^(5/2)*g) - (8*b*(e*f - d*g)^2*n*Sqrt[f + g*
x]*(a + b*Log[c*(d + e*x)^n]))/(5*e^2*g) - (8*b*(e*f - d*g)*n*(f + g*x)^(3/
2)*(a + b*Log[c*(d + e*x)^n]))/(15*e*g) - (8*b*n*(f + g*x)^(5/2)*(a + b*Log
[c*(d + e*x)^n]))/(25*g) + (8*b*(e*f - d*g)^(5/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f
 + g*x])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/(5*e^(5/2)*g) + (2*(f
 + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n])^2)/(5*g) + (16*b^2*(e*f - d*g)^(5/
2)*n^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]
*Sqrt[f + g*x])/Sqrt[e*f - d*g])])/(5*e^(5/2)*g) + (8*b^2*(e*f - d*g)^(5/2)
*n^2*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])])/(5*e^
(5/2)*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
```

```
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g} - \frac{(4ben)\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx}{5g}$$

$$= \frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g} - \frac{(4bn)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{5/2}(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{5g}$$

$$= \frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g} - \frac{(4bn)\text{Subst}\left(\int \left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx, x, d+ex\right)}{5e} - \frac{(4b(ef-dg)n)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{5eg}$$

$$
\begin{aligned}
&= -\frac{8bn(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{25g} + \frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g} \\
&\quad - \frac{(4b(ef-dg)n)\text{Subst}\left(\int \sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}(a+b\log\left(cx^n\right))\,dx,x,d+ex\right)}{5e^2} \\
&\quad - \frac{\left(4b(ef-dg)^2n\right)\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}(a+b\log(cx^n))}{x}\,dx,x,d+ex\right)}{5e^2g} \\
&\quad + \frac{\left(8b^2n^2\right)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{5/2}}{x}\,dx,x,d+ex\right)}{25g} \\
&= \frac{16b^2n^2(f+gx)^{5/2}}{125g} - \frac{8b(ef-dg)n(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{15eg} \\
&\quad - \frac{8bn(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{25g} + \frac{2(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{5g} \\
&\quad - \frac{\left(4b(ef-dg)^2n\right)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx,x,d+ex\right)}{5e^3} \\
&\quad - \frac{\left(4b(ef-dg)^3n\right)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx,x,d+ex\right)}{5e^3g} \\
&\quad + \frac{\left(8b^2(ef-dg)n^2\right)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}{x}\,dx,x,d+ex\right)}{25eg} \\
&\quad + \frac{\left(8b^2(ef-dg)n^2\right)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}{x}\,dx,x,d+ex\right)}{15eg}
\end{aligned}
$$

$$= \frac{128b^2(ef-dg)n^2(f+gx)^{3/2}}{225eg} + \frac{16b^2n^2(f+gx)^{5/2}}{125g}$$
$$- \frac{8b(ef-dg)^2n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{5e^2g}$$
$$- \frac{8b(ef-dg)n(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{15eg}$$
$$- \frac{8bn(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{25g}$$
$$+ \frac{8b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5e^{5/2}g}$$
$$+ \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))^2}{5g}$$
$$+ \frac{(8b^2(ef-dg)^2n^2)\,\text{Subst}\left(\int\frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}{x}\,dx,x,d+ex\right)}{25e^2g}$$
$$+ \frac{(8b^2(ef-dg)^2n^2)\,\text{Subst}\left(\int\frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}{x}\,dx,x,d+ex\right)}{15e^2g}$$
$$+ \frac{(8b^2(ef-dg)^2n^2)\,\text{Subst}\left(\int\frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}{x}\,dx,x,d+ex\right)}{5e^2g}$$
$$+ \frac{(4b^2(ef-dg)^3n^2)\,\text{Subst}\left(\int-\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}\,x}\,dx,x,d+ex\right)}{5e^3g}$$

$$= \frac{368b^2(ef-dg)^2n^2\sqrt{f+gx}}{75e^2g} + \frac{128b^2(ef-dg)n^2(f+gx)^{3/2}}{225eg} + \frac{16b^2n^2(f+gx)^{5/2}}{125g} - \frac{8b(ef-dg)^2n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{5e^2g} - \frac{8b(ef-dg)n(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{15eg} - \frac{8bn(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{25g} + \frac{8b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5e^{5/2}g} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))^2}{5g} - \frac{\left(8b^2(ef-dg)^{5/2}n^2\right)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{x}dx, x, d+ex\right)}{5e^{5/2}g} + \frac{(8b^2(ef-dg)^3n^2)\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}dx, x, d+ex\right)}{25e^3g} + \frac{(8b^2(ef-dg)^3n^2)\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}dx, x, d+ex\right)}{15e^3g} + \frac{(8b^2(ef-dg)^3n^2)\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}dx, x, d+ex\right)}{5e^3g}$$

$$
\begin{aligned}
&= \frac{368b^2(ef-dg)^2n^2\sqrt{f+gx}}{75e^2g} + \frac{128b^2(ef-dg)n^2(f+gx)^{3/2}}{225eg} \\
&\quad + \frac{16b^2n^2(f+gx)^{5/2}}{125g} - \frac{8b(ef-dg)^2n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{5e^2g} \\
&\quad - \frac{8b(ef-dg)n(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{15eg} \\
&\quad - \frac{8bn(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{25g} \\
&\quad + \frac{8b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5e^{5/2}g} \\
&\quad + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))^2}{5g} \\
&\quad - \frac{\left(16b^2(ef-dg)^{5/2}n^2\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{dg+e(-f+x^2)}\,dx, x, \sqrt{f+gx}\right)}{5e^{3/2}g} \\
&\quad + \frac{\left(16b^2(ef-dg)^3n^2\right)\text{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{25e^2g^2} \\
&\quad + \frac{\left(16b^2(ef-dg)^3n^2\right)\text{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{15e^2g^2} \\
&\quad + \frac{\left(16b^2(ef-dg)^3n^2\right)\text{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{5e^2g^2}
\end{aligned}
$$

$$=\frac{368b^2(ef-dg)^2n^2\sqrt{f+gx}}{75e^2g}+\frac{128b^2(ef-dg)n^2(f+gx)^{3/2}}{225eg}+\frac{16b^2n^2(f+gx)^{5/2}}{125g}-\frac{368b^2(ef-dg)^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{75e^{5/2}g}-\frac{8b(ef-dg)^2n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{5e^2g}-\frac{8b(ef-dg)n(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{15eg}-\frac{8bn(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{25g}+\frac{8b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5e^{5/2}g}+\frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))^2}{5g}-\frac{(16b^2(ef-dg)^{5/2}n^2)\,\mathrm{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{-ef+dg+ex^2}\,dx,x,\sqrt{f+gx}\right)}{5e^{3/2}g}$$

$$=\frac{368b^2(ef-dg)^2n^2\sqrt{f+gx}}{75e^2g}+\frac{128b^2(ef-dg)n^2(f+gx)^{3/2}}{225eg}+\frac{16b^2n^2(f+gx)^{5/2}}{125g}-\frac{368b^2(ef-dg)^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{75e^{5/2}g}-\frac{8b^2(ef-dg)^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5e^{5/2}g}-\frac{8b(ef-dg)^2n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{5e^2g}-\frac{8b(ef-dg)n(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{15eg}-\frac{8bn(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{25g}+\frac{8b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5e^{5/2}g}+\frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))^2}{5g}+\frac{(16b^2(ef-dg)^2n^2)\,\mathrm{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\,dx,x,\sqrt{f+gx}\right)}{5e^2g}$$

$$
\begin{aligned}
&= \frac{368b^2(ef-dg)^2n^2\sqrt{f+gx}}{75e^2g} + \frac{128b^2(ef-dg)n^2(f+gx)^{3/2}}{225eg} + \frac{16b^2n^2(f+gx)^{5/2}}{125g} \\
&\quad - \frac{368b^2(ef-dg)^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{75e^{5/2}g} - \frac{8b^2(ef-dg)^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5e^{5/2}g} \\
&\quad - \frac{8b(ef-dg)^2n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{5e^2g} \\
&\quad - \frac{8b(ef-dg)n(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{15eg} \\
&\quad - \frac{8bn(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{25g} \\
&\quad + \frac{8b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5e^{5/2}g} \\
&\quad + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))^2}{5g} \\
&\quad + \frac{16b^2(ef-dg)^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5e^{5/2}g} \\
&\quad - \frac{(16b^2(ef-dg)^2n^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\right)}{1-\frac{ex^2}{ef-dg}}\,dx, x, \sqrt{f+gx}\right)}{5e^2g}
\end{aligned}
$$

$$= \frac{368b^2(ef-dg)^2n^2\sqrt{f+gx}}{75e^2g} + \frac{128b^2(ef-dg)n^2(f+gx)^{3/2}}{225eg} + \frac{16b^2n^2(f+gx)^{5/2}}{125g}$$
$$- \frac{368b^2(ef-dg)^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{75e^{5/2}g} - \frac{8b^2(ef-dg)^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5e^{5/2}g}$$
$$- \frac{8b(ef-dg)^2n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{5e^2g}$$
$$- \frac{8b(ef-dg)n(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{15eg}$$
$$- \frac{8bn(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{25g}$$
$$+ \frac{8b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5e^{5/2}g}$$
$$+ \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))^2}{5g}$$
$$+ \frac{16b^2(ef-dg)^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5e^{5/2}g}$$
$$+ \frac{\left(16b^2(ef-dg)^{5/2}n^2\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5e^{5/2}g}$$

$$= \frac{368b^2(ef-dg)^2n^2\sqrt{f+gx}}{75e^2g} + \frac{128b^2(ef-dg)n^2(f+gx)^{3/2}}{225eg} + \frac{16b^2n^2(f+gx)^{5/2}}{125g}$$
$$- \frac{368b^2(ef-dg)^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{75e^{5/2}g} - \frac{8b^2(ef-dg)^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5e^{5/2}g}$$
$$- \frac{8b(ef-dg)^2n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{5e^2g}$$
$$- \frac{8b(ef-dg)n(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{15eg}$$
$$- \frac{8bn(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{25g}$$
$$+ \frac{8b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5e^{5/2}g}$$
$$+ \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))^2}{5g}$$
$$+ \frac{16b^2(ef-dg)^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5e^{5/2}g}$$
$$+ \frac{8b^2(ef-dg)^{5/2}n^2\mathrm{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5e^{5/2}g}$$

## Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 854, normalized size of antiderivative = 1.45

$$\int (f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2\,dx = \frac{2\left((f+gx)^{5/2}(a+b\log(c(d+ex)^n))^2 - \frac{bn\left(900a\sqrt{e}(ef-dg)^2\sqrt{f+gx}-1800b\sqrt{e}(ef-dg)^2n\sqrt{f\right.}{}\right.}{}$$

```
[In] Integrate[(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (2*((f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n])^2 - (b*n*(900*a*Sqrt[e]*(e*f
 - d*g)^2*Sqrt[f + g*x] - 1800*b*Sqrt[e]*(e*f - d*g)^2*n*Sqrt[f + g*x] + 18
00*b*(e*f - d*g)^(5/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] -
 200*b*(e*f - d*g)*n*(Sqrt[e]*Sqrt[f + g*x]*(4*e*f - 3*d*g + e*g*x) - 3*(e*
f - d*g)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]) - 24*b*n*(
3*e^(5/2)*(f + g*x)^(5/2) + 5*(e*f - d*g)*(Sqrt[e]*Sqrt[f + g*x]*(4*e*f - 3
*d*g + e*g*x) - 3*(e*f - d*g)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*
f - d*g]])) + 900*b*Sqrt[e]*(e*f - d*g)^2*Sqrt[f + g*x]*Log[c*(d + e*x)^n]
+ 300*e^(3/2)*(e*f - d*g)*(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n]) + 180*
```

```
e^(5/2)*(f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n]) + 450*(e*f - d*g)^(5/2)*
(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] - 4
50*(e*f - d*g)^(5/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] + Sqrt[
e]*Sqrt[f + g*x]] - 225*b*(e*f - d*g)^(5/2)*n*(Log[Sqrt[e*f - d*g] - Sqrt[e
]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] + 2*Log[(1 +
 (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) + 2*PolyLog[2, 1/2 - (Sqrt[e]
*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 225*b*(e*f - d*g)^(5/2)*n*(Log[Sqrt
[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f
+ g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 2*Pol
yLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2])))/(225*e^(5/2)))
/(5*g)
```

## Maple [F]

$$\int (gx+f)^{\frac{3}{2}} (a+b\ln (c(ex+d)^n))^2 dx$$

[In] `int((g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))^2,x)`

[Out] `int((g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))^2,x)`

## Fricas [F]

$$\int (f+gx)^{3/2} (a+b\log (c(d+ex)^n))^2 \, dx = \int (gx+f)^{\frac{3}{2}}(b\log ((ex+d)^n c)+a)^2 \, dx$$

[In] `integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")`

[Out]
```
integral((b^2*g*x + b^2*f)*sqrt(g*x + f)*log((e*x + d)^n*c)^2 + 2*(a*b*g*x
+ a*b*f)*sqrt(g*x + f)*log((e*x + d)^n*c) + (a^2*g*x + a^2*f)*sqrt(g*x + f)
, x)
```

## Sympy [F]

$$\int (f+gx)^{3/2} (a+b\log (c(d+ex)^n))^2 \, dx = \int (a+b\log (c(d+ex)^n))^2 (f+gx)^{\frac{3}{2}} \, dx$$

[In] `integrate((g*x+f)**(3/2)*(a+b*ln(c*(e*x+d)**n))**2,x)`

[Out] `Integral((a + b*log(c*(d + e*x)**n))**2*(f + g*x)**(3/2), x)`

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^{3/2} \left(a+b\log \left(c(d+ex)^n\right)\right)^2 \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int (f+gx)^{3/2} \left(a+b\log \left(c(d+ex)^n\right)\right)^2 \, dx = \int (gx+f)^{\frac{3}{2}} (b\log \left((ex+d)^n c\right)+a)^2 \, dx$$

```
[In] integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^(3/2)*(b*log((e*x + d)^n*c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^{3/2} \left(a+b\log \left(c(d+ex)^n\right)\right)^2 \, dx = \int (f+g\,x)^{3/2}\,\left(a+b\,\ln \left(c\,(d+e\,x)^n\right)\right)^2 \,dx$$

```
[In] int((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n))^2,x)
```

```
[Out] int((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n))^2, x)
```

# 3.146 $\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 510

$$
\begin{aligned}
&\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx \\
&= \frac{64b^2(ef-dg)n^2\sqrt{f+gx}}{9eg}+\frac{16b^2n^2(f+gx)^{3/2}}{27g}-\frac{64b^2(ef-dg)^{3/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{9e^{3/2}g} \\
&-\frac{8b^2(ef-dg)^{3/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3e^{3/2}g}-\frac{8b(ef-dg)n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{3eg} \\
&-\frac{8bn(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{9g} \\
&+\frac{8b(ef-dg)^{3/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3e^{3/2}g} \\
&+\frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g} \\
&+\frac{16b^2(ef-dg)^{3/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3e^{3/2}g} \\
&+\frac{8b^2(ef-dg)^{3/2}n^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3e^{3/2}g}
\end{aligned}
$$

```
[Out] 16/27*b^2*n^2*(g*x+f)^(3/2)/g-64/9*b^2*(-d*g+e*f)^(3/2)*n^2*arctanh(e^(1/2)
*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/g-8/3*b^2*(-d*g+e*f)^(3/2)*n^2*arc
tanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2/e^(3/2)/g-8/9*b*n*(g*x+f)^(3
/2)*(a+b*ln(c*(e*x+d)^n))/g+8/3*b*(-d*g+e*f)^(3/2)*n*arctanh(e^(1/2)*(g*x+f
)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+d)^n))/e^(3/2)/g+2/3*(g*x+f)^(3/2)
```

```
*(a+b*ln(c*(e*x+d)^n))^2/g+16/3*b^2*(-d*g+e*f)^(3/2)*n^2*arctanh(e^(1/2)*(g
*x+f)^(1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2
)))/e^(3/2)/g+8/3*b^2*(-d*g+e*f)^(3/2)*n^2*polylog(2,1-2/(1-e^(1/2)*(g*x+f)
^(1/2)/(-d*g+e*f)^(1/2)))/e^(3/2)/g+64/9*b^2*(-d*g+e*f)*n^2*(g*x+f)^(1/2)/e
/g-8/3*b*(-d*g+e*f)*n*(a+b*ln(c*(e*x+d)^n))*(g*x+f)^(1/2)/e/g
```

## Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {2445, 2458, 2388, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 52}

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx$$
$$= \frac{8bn(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3e^{3/2}g} - \frac{8bn(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{9g} - \frac{8bn\sqrt{f+gx}(ef-dg)(a+b\log(c(d+ex)^n))}{3eg} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g} - \frac{8b^2n^2(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3e^{3/2}g} - \frac{64b^2n^2(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{9e^{3/2}g} + \frac{16b^2n^2(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3e^{3/2}g} + \frac{8b^2n^2(ef-dg)^{3/2}\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3e^{3/2}g} + \frac{64b^2n^2\sqrt{f+gx}(ef-dg)}{9eg} + \frac{16b^2n^2(f+gx)^{3/2}}{27g}$$

```
[In] Int[Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (64*b^2*(e*f - d*g)*n^2*Sqrt[f + g*x])/(9*e*g) + (16*b^2*n^2*(f + g*x)^(3/2
))/(27*g) - (64*b^2*(e*f - d*g)^(3/2)*n^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/S
qrt[e*f - d*g]])/(9*e^(3/2)*g) - (8*b^2*(e*f - d*g)^(3/2)*n^2*ArcTanh[(Sqrt
[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]^2)/(3*e^(3/2)*g) - (8*b*(e*f - d*g)*n*S
qrt[f + g*x]*(a + b*Log[c*(d + e*x)^n]))/(3*e*g) - (8*b*n*(f + g*x)^(3/2)*(
a + b*Log[c*(d + e*x)^n]))/(9*g) + (8*b*(e*f - d*g)^(3/2)*n*ArcTanh[(Sqrt[e
]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/(3*e^(3/2)*g)
 + (2*(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])^2)/(3*g) + (16*b^2*(e*f -
d*g)^(3/2)*n^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*Log[2/(1 -
```

```
(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])])/(3*e^(3/2)*g) + (8*b^2*(e*f - d*
g)^(3/2)*n^2*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])
])/(3*e^(3/2)*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
```

```
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
```

```
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g} - \frac{(4ben)\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx}{3g}$$

$$= \frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g} - \frac{(4bn)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{3g}$$

$$= \frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g} - \frac{(4bn)\text{Subst}\left(\int \sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}\left(a+b\log\left(cx^n\right)\right)\,dx, x, d+ex\right)}{3e} - \frac{(4b(ef-dg)n)\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{3eg}$$

$$= -\frac{8bn(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{9g} + \frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g} - \frac{(4b(ef-dg)n)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{3e^2} - \frac{\left(4b(ef-dg)^2n\right)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{3e^2g} + \frac{\left(8b^2n^2\right)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}{x}\,dx, x, d+ex\right)}{9g}$$

$$= \frac{16b^2n^2(f+gx)^{3/2}}{27g} - \frac{8b(ef-dg)n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{3eg}$$
$$- \frac{8bn(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{9g}$$
$$+ \frac{8b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3e^{3/2}g}$$
$$+ \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g}$$
$$+ \frac{(8b^2(ef-dg)n^2)\,\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}{x}\,dx, x, d+ex\right)}{9eg}$$
$$+ \frac{(8b^2(ef-dg)n^2)\,\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}{x}\,dx, x, d+ex\right)}{3eg}$$
$$+ \frac{(4b^2(ef-dg)^2n^2)\,\text{Subst}\left(\int -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}x}\,dx, x, d+ex\right)}{3e^2g}$$

$$= \frac{64b^2(ef-dg)n^2\sqrt{f+gx}}{9eg} + \frac{16b^2n^2(f+gx)^{3/2}}{27g}$$
$$- \frac{8b(ef-dg)n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{3eg}$$
$$- \frac{8bn(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{9g}$$
$$+ \frac{8b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3e^{3/2}g}$$
$$+ \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g}$$
$$- \frac{(8b^2(ef-dg)^{3/2}n^2)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{x}\,dx, x, d+ex\right)}{3e^{3/2}g}$$
$$+ \frac{(8b^2(ef-dg)^2n^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{9e^2g}$$
$$+ \frac{(8b^2(ef-dg)^2n^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{3e^2g}$$

$$
\begin{aligned}
&= \frac{64b^2(ef-dg)n^2\sqrt{f+gx}}{9eg} + \frac{16b^2n^2(f+gx)^{3/2}}{27g} \\
&\quad - \frac{8b(ef-dg)n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{3eg} \\
&\quad - \frac{8bn(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{9g} \\
&\quad + \frac{8b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3e^{3/2}g} \\
&\quad + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g} \\
&\quad - \frac{\left(16b^2(ef-dg)^{3/2}n^2\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{dg+e(-f+x^2)}\,dx, x, \sqrt{f+gx}\right)}{3\sqrt{e}g} \\
&\quad + \frac{(16b^2(ef-dg)^2n^2)\text{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{9eg^2} \\
&\quad + \frac{(16b^2(ef-dg)^2n^2)\text{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{3eg^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{64b^2(ef-dg)n^2\sqrt{f+gx}}{9eg} + \frac{16b^2n^2(f+gx)^{3/2}}{27g} \\
&\quad - \frac{64b^2(ef-dg)^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{9e^{3/2}g} \\
&\quad - \frac{8b(ef-dg)n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{3eg} \\
&\quad - \frac{8bn(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{9g} \\
&\quad + \frac{8b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3e^{3/2}g} \\
&\quad + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g} \\
&\quad - \frac{\left(16b^2(ef-dg)^{3/2}n^2\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{-ef+dg+ex^2}\,dx, x, \sqrt{f+gx}\right)}{3\sqrt{e}g}
\end{aligned}
$$

$$= \frac{64b^2(ef-dg)n^2\sqrt{f+gx}}{9eg} + \frac{16b^2n^2(f+gx)^{3/2}}{27g} - \frac{64b^2(ef-dg)^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{9e^{3/2}g} - \frac{8b^2(ef-dg)^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3e^{3/2}g} - \frac{8b(ef-dg)n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{3eg} - \frac{8bn(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{9g} + \frac{8b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3e^{3/2}g} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g} + \frac{(16b^2(ef-dg)n^2)\operatorname{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}dx, x, \sqrt{f+gx}\right)}{3eg}$$

$$= \frac{64b^2(ef-dg)n^2\sqrt{f+gx}}{9eg} + \frac{16b^2n^2(f+gx)^{3/2}}{27g} - \frac{64b^2(ef-dg)^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{9e^{3/2}g} - \frac{8b^2(ef-dg)^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3e^{3/2}g} - \frac{8b(ef-dg)n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{3eg} - \frac{8bn(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{9g} + \frac{8b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3e^{3/2}g} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g} + \frac{16b^2(ef-dg)^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3e^{3/2}g} - \frac{(16b^2(ef-dg)n^2)\operatorname{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\right)}{1-\frac{ex^2}{ef-dg}}dx, x, \sqrt{f+gx}\right)}{3eg}$$

$$= \frac{64b^2(ef-dg)n^2\sqrt{f+gx}}{9eg} + \frac{16b^2n^2(f+gx)^{3/2}}{27g} - \frac{64b^2(ef-dg)^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{9e^{3/2}g} - \frac{8b^2(ef-dg)^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3e^{3/2}g} - \frac{8b(ef-dg)n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{3eg} - \frac{8bn(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{9g} + \frac{8b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3e^{3/2}g} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g} + \frac{16b^2(ef-dg)^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3e^{3/2}g} + \frac{(16b^2(ef-dg)^{3/2}n^2)\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3e^{3/2}g}$$

$$= \frac{64b^2(ef-dg)n^2\sqrt{f+gx}}{9eg} + \frac{16b^2n^2(f+gx)^{3/2}}{27g} - \frac{64b^2(ef-dg)^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{9e^{3/2}g} - \frac{8b^2(ef-dg)^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3e^{3/2}g} - \frac{8b(ef-dg)n\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{3eg} - \frac{8bn(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{9g} + \frac{8b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3e^{3/2}g} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2}{3g} + \frac{16b^2(ef-dg)^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3e^{3/2}g} + \frac{8b^2(ef-dg)^{3/2}n^2\operatorname{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3e^{3/2}g}$$

## Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.33

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx$$
$$= \frac{2\left((f+gx)^{3/2}(a+b\log(c(d+ex)^n))^2 - \frac{bn\left(36a\sqrt{e}(ef-dg)\sqrt{f+gx}-72b\sqrt{e}(ef-dg)n\sqrt{f+gx}-8b\sqrt{e}n\sqrt{f+gx}(4ef-3dg+egx)+\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (2*((f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])^2 - (b*n*(36*a*Sqrt[e]*(e*f
- d*g)*Sqrt[f + g*x] - 72*b*Sqrt[e]*(e*f - d*g)*n*Sqrt[f + g*x] - 8*b*Sqrt[
e]*n*Sqrt[f + g*x]*(4*e*f - 3*d*g + e*g*x) + 96*b*(e*f - d*g)^(3/2)*n*ArcTa
nh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] + 36*b*Sqrt[e]*(e*f - d*g)*Sqrt
[f + g*x]*Log[c*(d + e*x)^n] + 12*e^(3/2)*(f + g*x)^(3/2)*(a + b*Log[c*(d +
 e*x)^n]) + 18*(e*f - d*g)^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f -
d*g] - Sqrt[e]*Sqrt[f + g*x]] - 18*(e*f - d*g)^(3/2)*(a + b*Log[c*(d + e*x)
^n])*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] + 9*b*(e*f - d*g)^(3/2)*n
*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] + Sqrt[e
]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]
) - 9*b*(e*f - d*g)^(3/2)*n*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(L
og[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f +
g*x])/Sqrt[e*f - d*g])/2]) - 18*b*(e*f - d*g)^(3/2)*n*PolyLog[2, 1/2 - (Sqr
t[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])] + 18*b*(e*f - d*g)^(3/2)*n*PolyLog
[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]))/(9*e^(3/2))))/(3*g)
```

## Maple [F]

$$\int \sqrt{gx+f}\,(a+b\ln(c(ex+d)^n))^2dx$$

```
[In] int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

```
[Out] int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Fricas [F]

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx=\int \sqrt{gx+f}(b\log((ex+d)^nc)+a)^2\,dx$$

[In] integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(g*x + f)*b^2*log((e*x + d)^n*c)^2 + 2*sqrt(g*x + f)*a*b*log((e*x + d)^n*c) + sqrt(g*x + f)*a^2, x)

## Sympy [F]

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx=\int (a+b\log(c(d+ex)^n))^2\sqrt{f+gx}\,dx$$

[In] integrate((g*x+f)**(1/2)*(a+b*ln(c*(e*x+d)**n))**2,x)

[Out] Integral((a + b*log(c*(d + e*x)**n))**2*sqrt(f + g*x), x)

## Maxima [F(-2)]

Exception generated.

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx=\text{Exception raised: ValueError}$$

[In] integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for more de

## Giac [F]

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx=\int \sqrt{gx+f}(b\log((ex+d)^nc)+a)^2\,dx$$

[In] integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")

[Out] integrate(sqrt(g*x + f)*(b*log((e*x + d)^n*c) + a)^2, x)

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{f+gx}(a+b\log(c(d+ex)^n))^2\,dx=\int \sqrt{f+g\,x}\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2\,dx$$

```
[In] int((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))^2,x)
[Out] int((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))^2, x)
```

# 3.147 $\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 418

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx &= \frac{16b^2n^2\sqrt{f+gx}}{g}-\frac{16b^2\sqrt{ef-dg}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}g}\\&-\frac{8b^2\sqrt{ef-dg}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}g}\\&-\frac{8bn\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g}\\&+\frac{8b\sqrt{ef-dg}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}g}\\&+\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g}\\&+\frac{16b^2\sqrt{ef-dg}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}g}\\&+\frac{8b^2\sqrt{ef-dg}n^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}g}\end{aligned}$$

```
[Out] -16*b^2*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(-d*g+e*f)^(1/2
)/g/e^(1/2)-8*b^2*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2*(-d
*g+e*f)^(1/2)/g/e^(1/2)+8*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2
))*(a+b*ln(c*(e*x+d)^n))*(-d*g+e*f)^(1/2)/g/e^(1/2)+16*b^2*n^2*arctanh(e^(1
/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f
)^(1/2)))*(-d*g+e*f)^(1/2)/g/e^(1/2)+8*b^2*n^2*polylog(2,1-2/(1-e^(1/2)*(g*
```

```
x+f)^(1/2)/(-d*g+e*f)^(1/2)))*(-d*g+e*f)^(1/2)/g/e^(1/2)+16*b^2*n^2*(g*x+f)
^(1/2)/g-8*b*n*(a+b*ln(c*(e*x+d)^n))*(g*x+f)^(1/2)/g+2*(a+b*ln(c*(e*x+d)^n)
)^2*(g*x+f)^(1/2)/g
```

## Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {2445, 2458, 2388, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 52}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx = \frac{8bn\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}g} - \frac{8bn\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} - \frac{8b^2n^2\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}g} - \frac{16b^2n^2\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}g} + \frac{16b^2n^2\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}g} + \frac{8b^2n^2\sqrt{ef-dg}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}g} + \frac{16b^2n^2\sqrt{f+gx}}{g}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/Sqrt[f + g*x],x]
```

```
[Out] (16*b^2*n^2*Sqrt[f + g*x])/g - (16*b^2*Sqrt[e*f - d*g]*n^2*ArcTanh[(Sqrt[e]
*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(Sqrt[e]*g) - (8*b^2*Sqrt[e*f - d*g]*n^2*
ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]^2)/(Sqrt[e]*g) - (8*b*n*Sq
rt[f + g*x]*(a + b*Log[c*(d + e*x)^n]))/g + (8*b*Sqrt[e*f - d*g]*n*ArcTanh[
(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/(Sqrt[
e]*g) + (2*Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])^2)/g + (16*b^2*Sqrt[e*f
 - d*g]*n^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sq
rt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])])/(Sqrt[e]*g) + (8*b^2*Sqrt[e*f - d*g
]*n^2*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])])/(Sqr
t[e]*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
```

```
NeQ[q, 1]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} - \frac{(4ben)\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx}{g}$$

$$= \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} - \frac{(4bn)\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{g}$$

$$= \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} - \frac{(4bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{e} - \frac{(4b(ef-dg)n)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{eg}$$

$$= -\frac{8bn\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g} + \frac{8b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}g} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} + \frac{(8b^2n^2)\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}{x}\,dx, x, d+ex\right)}{g} + \frac{(4b^2(ef-dg)n^2)\text{Subst}\left(\int -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}x}\,dx, x, d+ex\right)}{eg}$$

$$= \frac{16b^2n^2\sqrt{f+gx}}{g} - \frac{8bn\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g} + \frac{8b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}g} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} - \frac{(8b^2\sqrt{ef-dg}n^2)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{x}dx,x,d+ex\right)}{\sqrt{e}g} + \frac{(8b^2(ef-dg)n^2)\,\text{Subst}\left(\int\frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}dx,x,d+ex\right)}{eg}$$

$$= \frac{16b^2n^2\sqrt{f+gx}}{g} - \frac{8bn\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g} + \frac{8b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}g} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} - \frac{(16b^2\sqrt{e}\sqrt{ef-dg}n^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{dg+e(-f+x^2)}dx,x,\sqrt{f+gx}\right)}{g} + \frac{(16b^2(ef-dg)n^2)\,\text{Subst}\left(\int\frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}dx,x,\sqrt{f+gx}\right)}{g^2}$$

$$= \frac{16b^2n^2\sqrt{f+gx}}{g} - \frac{16b^2\sqrt{ef-dg}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}g} - \frac{8bn\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g} + \frac{8b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}g} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} - \frac{(16b^2\sqrt{e}\sqrt{ef-dg}n^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{-ef+dg+ex^2}dx,x,\sqrt{f+gx}\right)}{g}$$

$$= \frac{16b^2n^2\sqrt{f+gx}}{g} - \frac{16b^2\sqrt{ef-dg}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}g} - \frac{8b^2\sqrt{ef-dg}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}g} - \frac{8bn\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g} + \frac{8b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}g} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} + \frac{(16b^2n^2)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{16b^2n^2\sqrt{f+gx}}{g} - \frac{16b^2\sqrt{ef-dg}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}g} - \frac{8b^2\sqrt{ef-dg}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}g} - \frac{8bn\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g} + \frac{8b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}g} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} + \frac{16b^2\sqrt{ef-dg}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}g} - \frac{(16b^2n^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\right)}{1-\frac{ex^2}{ef-dg}}dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{16b^2n^2\sqrt{f+gx}}{g} - \frac{16b^2\sqrt{ef-dg}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}g} - \frac{8b^2\sqrt{ef-dg}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}g} - \frac{8bn\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g} + \frac{8b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}g} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} + \frac{16b^2\sqrt{ef-dg}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}g} + \frac{\left(16b^2\sqrt{ef-dg}n^2\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}g}$$

$$= \frac{16b^2n^2\sqrt{f+gx}}{g} - \frac{16b^2\sqrt{ef-dg}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}g} - \frac{8b^2\sqrt{ef-dg}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}g} - \frac{8bn\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{g} + \frac{8b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}g} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2}{g} + \frac{16b^2\sqrt{ef-dg}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}g} + \frac{8b^2\sqrt{ef-dg}n^2\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}g}$$

## Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(\sqrt{f+gx}(a+b\log(c(d+ex)^n))^2 - \frac{bn\left(4a\sqrt{e}\sqrt{f+gx}-8b\sqrt{e}n\sqrt{f+gx}+8b\sqrt{ef-dg}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)+4b\sqrt{e}\sqrt{f+gx}\log(c(\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/Sqrt[f + g*x],x]
```

```
[Out] (2*(Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])^2 - (b*n*(4*a*Sqrt[e]*Sqrt[f +
 g*x] - 8*b*Sqrt[e]*n*Sqrt[f + g*x] + 8*b*Sqrt[e*f - d*g]*n*ArcTanh[(Sqrt[e
]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] + 4*b*Sqrt[e]*Sqrt[f + g*x]*Log[c*(d + e*
x)^n] + 2*Sqrt[e*f - d*g]*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] -
Sqrt[e]*Sqrt[f + g*x]] - 2*Sqrt[e*f - d*g]*(a + b*Log[c*(d + e*x)^n])*Log[S
qrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] - b*Sqrt[e*f - d*g]*n*(Log[Sqrt[e*f
 - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*
x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) + 2*PolyLog[2
, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + b*Sqrt[e*f - d*g]*n
*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] + Sqrt[
e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])
]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2])))/Sqrt[
e]))/g
```

## Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))^2}{\sqrt{gx+f}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(1/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(1/2),x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(g*x + f)*b^2*log((e*x + d)^n*c)^2 + 2*sqrt(g*x + f)*a*b*log(
(e*x + d)^n*c) + sqrt(g*x + f)*a^2)/(g*x + f), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**(1/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**2/sqrt(f + g*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/sqrt(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}}\,dx = \int \frac{(a+b\ln(c\,(d+e\,x)^n))^2}{\sqrt{f+g\,x}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(1/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(1/2), x)
```

# 3.148 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 312

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx = \frac{8b^2\sqrt{e}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{g\sqrt{ef-dg}} - \frac{8b\sqrt{e}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{ef-dg}} - \frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}} - \frac{16b^2\sqrt{e}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{g\sqrt{ef-dg}} - \frac{8b^2\sqrt{e}n^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{g\sqrt{ef-dg}}$$

```
[Out] 8*b^2*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2*e^(1/2)/g/(-d*g
+e*f)^(1/2)-8*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c
*(e*x+d)^n))*e^(1/2)/g/(-d*g+e*f)^(1/2)-16*b^2*n^2*arctanh(e^(1/2)*(g*x+f)^
(1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))*e^
(1/2)/g/(-d*g+e*f)^(1/2)-8*b^2*n^2*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(
-d*g+e*f)^(1/2)))*e^(1/2)/g/(-d*g+e*f)^(1/2)-2*(a+b*ln(c*(e*x+d)^n))^2/g/(g
*x+f)^(1/2)
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2445, 2458, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx = -\frac{8b\sqrt{e}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{ef-dg}} - \frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}} + \frac{8b^2\sqrt{e}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{g\sqrt{ef-dg}} - \frac{16b^2\sqrt{e}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{g\sqrt{ef-dg}} - \frac{8b^2\sqrt{e}n^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{g\sqrt{ef-dg}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(3/2),x]

[Out] (8*b^2*Sqrt[e]*n^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]^2)/(g*S
qrt[e*f - d*g]) - (8*b*Sqrt[e]*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f -
 d*g]]*(a + b*Log[c*(d + e*x)^n]))/(g*Sqrt[e*f - d*g]) - (2*(a + b*Log[c*(d
 + e*x)^n])^2)/(g*Sqrt[f + g*x]) - (16*b^2*Sqrt[e]*n^2*ArcTanh[(Sqrt[e]*Sqr
t[f + g*x])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f -
d*g])])/(g*Sqrt[e*f - d*g]) - (8*b^2*Sqrt[e]*n^2*PolyLog[2, 1 - 2/(1 - (Sqr
t[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])])/(g*Sqrt[e*f - d*g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
```

```
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = -\frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}} + \frac{(4ben)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f+gx}}\,dx}{g}$$

$$= -\frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}} + \frac{(4bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{g}$$

$$= -\frac{8b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{ef-dg}} - \frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}} - \frac{(4b^2n^2)\,\text{Subst}\left(\int -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}x}\,dx, x, d+ex\right)}{g}$$

$$= -\frac{8b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{ef-dg}} - \frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}} + \frac{(8b^2\sqrt{e}n^2)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{x}\,dx, x, d+ex\right)}{g\sqrt{ef-dg}}$$

$$= -\frac{8b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{ef-dg}} - \frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}} + \frac{(16b^2e^{3/2}n^2)\,\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{dg+e(-f+x^2)}\,dx, x, \sqrt{f+gx}\right)}{g\sqrt{ef-dg}}$$

$$= -\frac{8b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{ef-dg}} - \frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}} + \frac{(16b^2e^{3/2}n^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{-ef+dg+ex^2}\,dx, x, \sqrt{f+gx}\right)}{g\sqrt{ef-dg}}$$

$$= \frac{8b^2\sqrt{e}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{g\sqrt{ef-dg}} - \frac{8b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{ef-dg}} - \frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}} - \frac{(16b^2en^2)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\,dx, x, \sqrt{f+gx}\right)}{g(ef-dg)}$$

$$= \frac{8b^2\sqrt{e}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{g\sqrt{ef-dg}} - \frac{8b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{ef-dg}} - \frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}} - \frac{16b^2\sqrt{e}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{g\sqrt{ef-dg}} + \frac{(16b^2en^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\right)}{1-\frac{ex^2}{ef-dg}}\,dx, x, \sqrt{f+gx}\right)}{g(ef-dg)}$$

$$= \frac{8b^2\sqrt{e}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{g\sqrt{ef-dg}} - \frac{8b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{ef-dg}} - \frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}} - \frac{16b^2\sqrt{e}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{g\sqrt{ef-dg}} - \frac{(16b^2\sqrt{e}n^2)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{g\sqrt{ef-dg}}$$

$$= \frac{8b^2\sqrt{e}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{g\sqrt{ef-dg}} - \frac{8b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{ef-dg}} - \frac{2(a+b\log(c(d+ex)^n))^2}{g\sqrt{f+gx}} - \frac{16b^2\sqrt{e}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{g\sqrt{ef-dg}} - \frac{8b^2\sqrt{e}n^2\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{g\sqrt{ef-dg}}$$

## Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx = \frac{2\left(-\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{f+gx}} + \frac{b\sqrt{e}n\left(2(a+b\log(c(d+ex)^n))\log\left(\sqrt{ef-dg}-\sqrt{e}\sqrt{f+gx}\right)-2(a+b\log(\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-((a + b*Log[c*(d + e*x)^n])^2/Sqrt[f + g*x]) + (b*Sqrt[e]*n*(2*(a + b*
Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] - 2*(a + b
*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] - b*n*(Lo
g[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*S
qrt[f + g*x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) + 2
*PolyLog[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + b*n*(Log[
Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqr
t[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 2
*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2])))/Sqrt[e*f -
d*g]))/g
```

## Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))^2}{(gx+f)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(3/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(3/2),x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{3/2}}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(g*x + f)*b^2*log((e*x + d)^n*c)^2 + 2*sqrt(g*x + f)*a*b*log(
(e*x + d)^n*c) + sqrt(g*x + f)*a^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

## Sympy [F]

$$\int \frac{(a+b\log{(c(d+ex)^n)})^2}{(f+gx)^{3/2}}\,dx = \int \frac{(a+b\log{(c(d+ex)^n)})^2}{(f+gx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**(3/2),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**2/(f + g*x)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log{(c(d+ex)^n)})^2}{(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \frac{(a+b\log{(c(d+ex)^n)})^2}{(f+gx)^{3/2}}\,dx = \int \frac{(b\log{((ex+d)^nc)}+a)^2}{(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(c(d+ex)^n)})^2}{(f+gx)^{3/2}}\,dx = \int \frac{{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}^2}{{\left(f+g\,x\right)}^{3/2}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(3/2),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(3/2), x)
```

# 3.149 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 423

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx = \frac{16b^2e^{3/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3g(ef-dg)^{3/2}} + \frac{8b^2e^{3/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3g(ef-dg)^{3/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)\sqrt{f+gx}} - \frac{8be^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3g(ef-dg)^{3/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}} - \frac{16b^2e^{3/2}n^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3g(ef-dg)^{3/2}} - \frac{8b^2e^{3/2}n^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3g(ef-dg)^{3/2}}$$

```
[Out] 16/3*b^2*e^(3/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/g/(-d*
g+e*f)^(3/2)+8/3*b^2*e^(3/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(
1/2))^2/g/(-d*g+e*f)^(3/2)-8/3*b*e^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-
d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f)^(3/2)-2/3*(a+b*ln(c*(e*x
+d)^n))^2/g/(g*x+f)^(3/2)-16/3*b^2*e^(3/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2
)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/g/(-d*
g+e*f)^(3/2)-8/3*b^2*e^(3/2)*n^2*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d
*g+e*f)^(1/2)))/g/(-d*g+e*f)^(3/2)+8/3*b*e*n*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+
e*f)/(g*x+f)^(1/2)
```

## Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2445, 2458, 2389, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx = -\frac{8be^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3g(ef-dg)^{3/2}}+\frac{8ben(a+b\log(c(d+ex)^n))}{3g\sqrt{f+gx}(ef-dg)}-\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}}+\frac{8b^2e^{3/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3g(ef-dg)^{3/2}}+\frac{16b^2e^{3/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3g(ef-dg)^{3/2}}-\frac{16b^2e^{3/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3g(ef-dg)^{3/2}}-\frac{8b^2e^{3/2}n^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3g(ef-dg)^{3/2}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(5/2),x]
```

```
[Out] (16*b^2*e^(3/2)*n^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(3*g*
(e*f - d*g)^(3/2)) + (8*b^2*e^(3/2)*n^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqr
t[e*f - d*g]]^2)/(3*g*(e*f - d*g)^(3/2)) + (8*b*e*n*(a + b*Log[c*(d + e*x)^
n]))/(3*g*(e*f - d*g)*Sqrt[f + g*x]) - (8*b*e^(3/2)*n*ArcTanh[(Sqrt[e]*Sqrt
[f + g*x])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/(3*g*(e*f - d*g)^(3
/2)) - (2*(a + b*Log[c*(d + e*x)^n])^2)/(3*g*(f + g*x)^(3/2)) - (16*b^2*e^(
3/2)*n^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[
e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])])/(3*g*(e*f - d*g)^(3/2)) - (8*b^2*e^(3/
2)*n^2*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])])/(3*
g*(e*f - d*g)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
```

```
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = -\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}} + \frac{(4ben)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{3/2}}\,dx}{3g}$$

$$= -\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}} + \frac{(4bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{3g}$$

$$= -\frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}} - \frac{(4bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{3(ef-dg)} + \frac{(4ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{3g(ef-dg)}$$

$$= \frac{8ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)\sqrt{f+gx}} - \frac{8be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3g(ef-dg)^{3/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}} - \frac{(4b^2en^2)\,\text{Subst}\left(\int -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}x}\,dx, x, d+ex\right)}{3g(ef-dg)} - \frac{(8b^2en^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{3g(ef-dg)}$$

$$= \frac{8ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)\sqrt{f+gx}} - \frac{8be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3g(ef-dg)^{3/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}} + \frac{\left(8b^2e^{3/2}n^2\right)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{x}\,dx, x, d+ex\right)}{3g(ef-dg)^{3/2}} - \frac{(16b^2e^2n^2)\,\text{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{3g^2(ef-dg)}$$

$$= \frac{16b^2e^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3g(ef-dg)^{3/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)\sqrt{f+gx}} - \frac{8be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3g(ef-dg)^{3/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}} + \frac{\left(16b^2e^{5/2}n^2\right)\,\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{dg+e(-f+x^2)}\,dx, x, \sqrt{f+gx}\right)}{3g(ef-dg)^{3/2}}$$

$$= \frac{16b^2e^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3g(ef-dg)^{3/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)\sqrt{f+gx}} - \frac{8be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3g(ef-dg)^{3/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}} + \frac{(16b^2e^{5/2}n^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{-ef+dg+ex^2}\,dx,x,\sqrt{f+gx}\right)}{3g(ef-dg)^{3/2}}$$

$$= \frac{16b^2e^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3g(ef-dg)^{3/2}} + \frac{8b^2e^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3g(ef-dg)^{3/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)\sqrt{f+gx}} - \frac{8be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3g(ef-dg)^{3/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}} - \frac{(16b^2e^2n^2)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\,dx,x,\sqrt{f+gx}\right)}{3g(ef-dg)^2}$$

$$= \frac{16b^2e^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3g(ef-dg)^{3/2}} + \frac{8b^2e^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3g(ef-dg)^{3/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)\sqrt{f+gx}} - \frac{8be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3g(ef-dg)^{3/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}} - \frac{16b^2e^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3g(ef-dg)^{3/2}} + \frac{(16b^2e^2n^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\right)}{1-\frac{ex^2}{ef-dg}}\,dx,x,\sqrt{f+gx}\right)}{3g(ef-dg)^2}$$

$$= \frac{16b^2e^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3g(ef-dg)^{3/2}} + \frac{8b^2e^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3g(ef-dg)^{3/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)\sqrt{f+gx}} - \frac{8be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3g(ef-dg)^{3/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}} - \frac{16b^2e^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3g(ef-dg)^{3/2}} - \frac{(16b^2e^{3/2}n^2)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3g(ef-dg)^{3/2}}$$

$$= \frac{16b^2e^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3g(ef-dg)^{3/2}} + \frac{8b^2e^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{3g(ef-dg)^{3/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{3g(ef-dg)\sqrt{f+gx}} - \frac{8be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{3g(ef-dg)^{3/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{3g(f+gx)^{3/2}} - \frac{16b^2e^{3/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3g(ef-dg)^{3/2}} - \frac{8b^2e^{3/2}n^2\operatorname{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{3g(ef-dg)^{3/2}}$$

## Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx = \frac{2\left(-(a+b\log(c(d+ex)^n))^2 + \frac{ben(f+gx)\left(8b\sqrt{e}n\sqrt{f+gx}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)+4\sqrt{ef}\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(5/2),x]
```

```
[Out] (2*(-(a + b*Log[c*(d + e*x)^n])^2 + (b*e*n*(f + g*x)*(8*b*Sqrt[e]*n*Sqrt[f
+ g*x]*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] + 4*Sqrt[e*f - d*g]
*(a + b*Log[c*(d + e*x)^n]) + 2*Sqrt[e]*Sqrt[f + g*x]*(a + b*Log[c*(d + e*x
)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] - 2*Sqrt[e]*Sqrt[f + g*x
]*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] -
 b*Sqrt[e]*n*Sqrt[f + g*x]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(L
og[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f +
g*x])/Sqrt[e*f - d*g])/2]) + 2*PolyLog[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*
```

```
Sqrt[e*f - d*g])]) + b*Sqrt[e]*n*Sqrt[f + g*x]*(Log[Sqrt[e*f - d*g] + Sqrt[
e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2
 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 2*PolyLog[2, (1 + (Sqrt[
e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2])))/(e*f - d*g)^(3/2)))/(3*g*(f + g*x)
^(3/2))
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}{(gx+f)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(5/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(5/2),x)
```

## Fricas [F]

$$\int \frac{\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{(f+gx)^{5/2}}\,dx=\int \frac{\left(b\log\left((ex+d)^{n}c\right)+a\right)^{2}}{(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(g*x + f)*b^2*log((e*x + d)^n*c)^2 + 2*sqrt(g*x + f)*a*b*log(
(e*x + d)^n*c) + sqrt(g*x + f)*a^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^
3), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{(f+gx)^{5/2}}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**(5/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{5/2}}\,dx = \int \frac{(a+b\ln(c\,(d+e\,x)^n))^2}{(f+g\,x)^{5/2}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(5/2),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(5/2), x)
```

# 3.150 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 503

$$\begin{aligned}
\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx &= -\frac{16b^2e^2n^2}{15g(ef-dg)^2\sqrt{f+gx}} \\
&+\frac{64b^2e^{5/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15g(ef-dg)^{5/2}}+\frac{8b^2e^{5/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5g(ef-dg)^{5/2}} \\
&+\frac{8ben(a+b\log(c(d+ex)^n))}{15g(ef-dg)(f+gx)^{3/2}}+\frac{8be^2n(a+b\log(c(d+ex)^n))}{5g(ef-dg)^2\sqrt{f+gx}} \\
&-\frac{8be^{5/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5g(ef-dg)^{5/2}} \\
&-\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}-\frac{16b^2e^{5/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5g(ef-dg)^{5/2}} \\
&-\frac{8b^2e^{5/2}n^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5g(ef-dg)^{5/2}}
\end{aligned}$$

```
[Out] 64/15*b^2*e^(5/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/g/(-d
*g+e*f)^(5/2)+8/5*b^2*e^(5/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^
(1/2))^2/g/(-d*g+e*f)^(5/2)+8/15*b*e*n*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f)/(
g*x+f)^(3/2)-8/5*b*e^(5/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)
)*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f)^(5/2)-2/5*(a+b*ln(c*(e*x+d)^n))^2/g/(g
*x+f)^(5/2)-16/5*b^2*e^(5/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(
1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/g/(-d*g+e*f)^(5/2)-8
/5*b^2*e^(5/2)*n^2*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))
)/g/(-d*g+e*f)^(5/2)-16/15*b^2*e^2*n^2/g/(-d*g+e*f)^2/(g*x+f)^(1/2)+8/5*b*e
^2*n*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f)^2/(g*x+f)^(1/2)
```

## Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {2445, 2458, 2389, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 53}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx = -\frac{8be^{5/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5g(ef-dg)^{5/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{5g\sqrt{f+gx}(ef-dg)^2} + \frac{8ben(a+b\log(c(d+ex)^n))}{15g(f+gx)^{3/2}(ef-dg)} - \frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}} + \frac{8b^2e^{5/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5g(ef-dg)^{5/2}} + \frac{64b^2e^{5/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15g(ef-dg)^{5/2}} - \frac{16b^2e^{5/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5g(ef-dg)^{5/2}} - \frac{8b^2e^{5/2}n^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5g(ef-dg)^{5/2}} - \frac{16b^2e^2n^2}{15g\sqrt{f+gx}(ef-dg)^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(7/2),x]
```

```
[Out] (-16*b^2*e^2*n^2)/(15*g*(e*f - d*g)^2*Sqrt[f + g*x]) + (64*b^2*e^(5/2)*n^2*
ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(15*g*(e*f - d*g)^(5/2))
+ (8*b^2*e^(5/2)*n^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]^2)/(5
*g*(e*f - d*g)^(5/2)) + (8*b*e*n*(a + b*Log[c*(d + e*x)^n]))/(15*g*(e*f - d
*g)*(f + g*x)^(3/2)) + (8*b*e^2*n*(a + b*Log[c*(d + e*x)^n]))/(5*g*(e*f - d
*g)^2*Sqrt[f + g*x]) - (8*b*e^(5/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[
e*f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/(5*g*(e*f - d*g)^(5/2)) - (2*(a + b
*Log[c*(d + e*x)^n])^2)/(5*g*(f + g*x)^(5/2)) - (16*b^2*e^(5/2)*n^2*ArcTanh
[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f + g*x]
)/Sqrt[e*f - d*g])])/(5*g*(e*f - d*g)^(5/2)) - (8*b^2*e^(5/2)*n^2*PolyLog[2
, 1 - 2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])])/(5*g*(e*f - d*g)^(5
/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = -\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}} + \frac{(4ben)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}\,dx}{5g}$$

$$= -\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}} + \frac{(4bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{5/2}}\,dx, x, d+ex\right)}{5g}$$

$$= -\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}} - \frac{(4bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{5/2}}\,dx, x, d+ex\right)}{5(ef-dg)} + \frac{(4ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{5g(ef-dg)}$$

$$= \frac{8ben(a+b\log(c(d+ex)^n))}{15g(ef-dg)(f+gx)^{3/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}} - \frac{(4ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{5(ef-dg)^2} + \frac{(4be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{5g(ef-dg)^2} - \frac{(8b^2en^2)\,\text{Subst}\left(\int \frac{1}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{15g(ef-dg)}$$

$$= -\frac{16b^2e^2n^2}{15g(ef-dg)^2\sqrt{f+gx}} + \frac{8ben(a+b\log(c(d+ex)^n))}{15g(ef-dg)(f+gx)^{3/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{5g(ef-dg)^2\sqrt{f+gx}} - \frac{8be^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5g(ef-dg)^{5/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}} - \frac{(8b^2e^2n^2)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{15g(ef-dg)^2} - \frac{(4b^2e^2n^2)\operatorname{Subst}\left(\int -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}x}\,dx, x, d+ex\right)}{5g(ef-dg)^2} - \frac{(8b^2e^2n^2)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{5g(ef-dg)^2}$$

$$= -\frac{16b^2e^2n^2}{15g(ef-dg)^2\sqrt{f+gx}} + \frac{8ben(a+b\log(c(d+ex)^n))}{15g(ef-dg)(f+gx)^{3/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{5g(ef-dg)^2\sqrt{f+gx}} - \frac{8be^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5g(ef-dg)^{5/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}} + \frac{(8b^2e^{5/2}n^2)\operatorname{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{x}\,dx, x, d+ex\right)}{5g(ef-dg)^{5/2}} - \frac{(16b^2e^3n^2)\operatorname{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{15g^2(ef-dg)^2} - \frac{(16b^2e^3n^2)\operatorname{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{5g^2(ef-dg)^2}$$

$$=-\frac{16b^2e^2n^2}{15g(ef-dg)^2\sqrt{f+gx}}+\frac{64b^2e^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15g(ef-dg)^{5/2}}+\frac{8ben(a+b\log(c(d+ex)^n))}{15g(ef-dg)(f+gx)^{3/2}}+\frac{8be^2n(a+b\log(c(d+ex)^n))}{5g(ef-dg)^2\sqrt{f+gx}}-\frac{8be^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5g(ef-dg)^{5/2}}-\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}+\frac{(16b^2e^{7/2}n^2)\operatorname{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{dg+e(-f+x^2)}\,dx,x,\sqrt{f+gx}\right)}{5g(ef-dg)^{5/2}}$$

$$=-\frac{16b^2e^2n^2}{15g(ef-dg)^2\sqrt{f+gx}}+\frac{64b^2e^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15g(ef-dg)^{5/2}}+\frac{8ben(a+b\log(c(d+ex)^n))}{15g(ef-dg)(f+gx)^{3/2}}+\frac{8be^2n(a+b\log(c(d+ex)^n))}{5g(ef-dg)^2\sqrt{f+gx}}-\frac{8be^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5g(ef-dg)^{5/2}}-\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}+\frac{(16b^2e^{7/2}n^2)\operatorname{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{-ef+dg+ex^2}\,dx,x,\sqrt{f+gx}\right)}{5g(ef-dg)^{5/2}}$$

$$=-\frac{16b^2e^2n^2}{15g(ef-dg)^2\sqrt{f+gx}}+\frac{64b^2e^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15g(ef-dg)^{5/2}}+\frac{8b^2e^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5g(ef-dg)^{5/2}}+\frac{8ben(a+b\log(c(d+ex)^n))}{15g(ef-dg)(f+gx)^{3/2}}+\frac{8be^2n(a+b\log(c(d+ex)^n))}{5g(ef-dg)^2\sqrt{f+gx}}-\frac{8be^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5g(ef-dg)^{5/2}}-\frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}-\frac{(16b^2e^3n^2)\operatorname{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\,dx,x,\sqrt{f+gx}\right)}{5g(ef-dg)^3}$$

$$= -\frac{16b^2e^2n^2}{15g(ef-dg)^2\sqrt{f+gx}} + \frac{64b^2e^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15g(ef-dg)^{5/2}} + \frac{8b^2e^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5g(ef-dg)^{5/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{15g(ef-dg)(f+gx)^{3/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{5g(ef-dg)^2\sqrt{f+gx}} - \frac{8be^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5g(ef-dg)^{5/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}} - \frac{16b^2e^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5g(ef-dg)^{5/2}} + \frac{(16b^2e^3n^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\right)}{1-\frac{ex^2}{ef-dg}}\,dx, x, \sqrt{f+gx}\right)}{5g(ef-dg)^3}$$

$$= -\frac{16b^2e^2n^2}{15g(ef-dg)^2\sqrt{f+gx}} + \frac{64b^2e^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15g(ef-dg)^{5/2}} + \frac{8b^2e^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5g(ef-dg)^{5/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{15g(ef-dg)(f+gx)^{3/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{5g(ef-dg)^2\sqrt{f+gx}} - \frac{8be^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5g(ef-dg)^{5/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}} - \frac{16b^2e^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5g(ef-dg)^{5/2}} - \frac{(16b^2e^{5/2}n^2)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5g(ef-dg)^{5/2}}$$

$$= -\frac{16b^2e^2n^2}{15g(ef-dg)^2\sqrt{f+gx}} + \frac{64b^2e^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15g(ef-dg)^{5/2}}$$
$$+ \frac{8b^2e^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{5g(ef-dg)^{5/2}}$$
$$+ \frac{8ben(a+b\log(c(d+ex)^n))}{15g(ef-dg)(f+gx)^{3/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{5g(ef-dg)^2\sqrt{f+gx}}$$
$$- \frac{8be^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{5g(ef-dg)^{5/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{5g(f+gx)^{5/2}}$$
$$- \frac{16b^2e^{5/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5g(ef-dg)^{5/2}} - \frac{8b^2e^{5/2}n^2\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{5g(ef-dg)^{5/2}}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.22 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.27

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx = \frac{2\left(-3(a+b\log(c(d+ex)^n))^2 + \frac{ben(f+gx)\left(24be^{3/2}n(f+gx)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)-8\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(7/2),x]
```

```
[Out] (2*(-3*(a + b*Log[c*(d + e*x)^n])^2 + (b*e*n*(f + g*x)*(24*b*e^(3/2)*n*(f +
 g*x)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] - 8*b*e*Sqrt[e
*f - d*g]*n*(f + g*x)*Hypergeometric2F1[-1/2, 1, 1/2, (e*(f + g*x))/(e*f -
d*g)] + 4*(e*f - d*g)^(3/2)*(a + b*Log[c*(d + e*x)^n]) + 12*e*Sqrt[e*f - d*
g]*(f + g*x)*(a + b*Log[c*(d + e*x)^n]) + 6*e^(3/2)*(f + g*x)^(3/2)*(a + b*
Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] - 6*e^(3/2
)*(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] + Sqrt[e]*
Sqrt[f + g*x]] - 3*b*e^(3/2)*n*(f + g*x)^(3/2)*(Log[Sqrt[e*f - d*g] - Sqrt[
e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] + 2*Log[(1
+ (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) + 2*PolyLog[2, 1/2 - (Sqrt[e
]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 3*b*e^(3/2)*n*(f + g*x)^(3/2)*(Log
[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sq
rt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) +
2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2])))/(e*f - d*g
)^(5/2)))/(15*g*(f + g*x)^(5/2))
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c(ex+d)^n\right)\right)^2}{(gx+f)^{\frac{7}{2}}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(7/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(7/2),x)
```

## Fricas [F]

$$\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(f+gx)^{7/2}}\,dx = \int \frac{\left(b\log\left((ex+d)^n c\right)+a\right)^2}{(gx+f)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(g*x + f)*b^2*log((e*x + d)^n*c)^2 + 2*sqrt(g*x + f)*a*b*log(
(e*x + d)^n*c) + sqrt(g*x + f)*a^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2
+ 4*f^3*g*x + f^4), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(f+gx)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**(7/2),x)
```

```
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(f+gx)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(7/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{7/2}}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2}{(f+g\,x)^{7/2}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(7/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(7/2), x)
```

# 3.151 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 583

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx = & -\frac{16b^2e^2n^2}{105g(ef-dg)^2(f+gx)^{3/2}} \\ & -\frac{128b^2e^3n^2}{105g(ef-dg)^3\sqrt{f+gx}}+\frac{368b^2e^{7/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{105g(ef-dg)^{7/2}} \\ & +\frac{8b^2e^{7/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{7g(ef-dg)^{7/2}}+\frac{8ben(a+b\log(c(d+ex)^n))}{35g(ef-dg)(f+gx)^{5/2}} \\ & +\frac{8be^2n(a+b\log(c(d+ex)^n))}{21g(ef-dg)^2(f+gx)^{3/2}}+\frac{8be^3n(a+b\log(c(d+ex)^n))}{7g(ef-dg)^3\sqrt{f+gx}} \\ & -\frac{8be^{7/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{7g(ef-dg)^{7/2}} \\ & -\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}}-\frac{16b^2e^{7/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{7g(ef-dg)^{7/2}} \\ & -\frac{8b^2e^{7/2}n^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{7g(ef-dg)^{7/2}}\end{aligned}$$

```
[Out] -16/105*b^2*e^2*n^2/g/(-d*g+e*f)^2/(g*x+f)^(3/2)+368/105*b^2*e^(7/2)*n^2*ar
ctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/g/(-d*g+e*f)^(7/2)+8/7*b^2*e^
(7/2)*n^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2/g/(-d*g+e*f)^(7
/2)+8/35*b*e*n*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f)/(g*x+f)^(5/2)+8/21*b*e^2*
n*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f)^2/(g*x+f)^(3/2)-8/7*b*e^(7/2)*n*arctan
h(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+d)^n))/g/(-d*g+e*f
```

```
)^(7/2)-2/7*(a+b*ln(c*(e*x+d)^n))^2/g/(g*x+f)^(7/2)-16/7*b^2*e^(7/2)*n^2*ar
ctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)
/(-d*g+e*f)^(1/2)))/g/(-d*g+e*f)^(7/2)-8/7*b^2*e^(7/2)*n^2*polylog(2,1-2/(1
-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/g/(-d*g+e*f)^(7/2)-128/105*b^2*e^
3*n^2/g/(-d*g+e*f)^3/(g*x+f)^(1/2)+8/7*b*e^3*n*(a+b*ln(c*(e*x+d)^n))/g/(-d*
g+e*f)^3/(g*x+f)^(1/2)
```

## Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {2445, 2458, 2389, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 53}

$$\begin{aligned}
&\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx = \\
&-\frac{8be^{7/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{7g(ef-dg)^{7/2}} \\
&+\frac{8be^3n(a+b\log(c(d+ex)^n))}{7g\sqrt{f+gx}(ef-dg)^3}+\frac{8be^2n(a+b\log(c(d+ex)^n))}{21g(f+gx)^{3/2}(ef-dg)^2} \\
&+\frac{8ben(a+b\log(c(d+ex)^n))}{35g(f+gx)^{5/2}(ef-dg)}-\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} \\
&+\frac{8b^2e^{7/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{7g(ef-dg)^{7/2}}+\frac{368b^2e^{7/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{105g(ef-dg)^{7/2}} \\
&-\frac{16b^2e^{7/2}n^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{7g(ef-dg)^{7/2}} \\
&-\frac{8b^2e^{7/2}n^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{7g(ef-dg)^{7/2}} \\
&-\frac{128b^2e^3n^2}{105g\sqrt{f+gx}(ef-dg)^3}-\frac{16b^2e^2n^2}{105g(f+gx)^{3/2}(ef-dg)^2}
\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(9/2),x]
```

```
[Out] (-16*b^2*e^2*n^2)/(105*g*(e*f - d*g)^2*(f + g*x)^(3/2)) - (128*b^2*e^3*n^2)
/(105*g*(e*f - d*g)^3*Sqrt[f + g*x]) + (368*b^2*e^(7/2)*n^2*ArcTanh[(Sqrt[e
]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(105*g*(e*f - d*g)^(7/2)) + (8*b^2*e^(7/
2)*n^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]^2)/(7*g*(e*f - d*g)
^(7/2)) + (8*b*e*n*(a + b*Log[c*(d + e*x)^n]))/(35*g*(e*f - d*g)*(f + g*x)^
(5/2)) + (8*b*e^2*n*(a + b*Log[c*(d + e*x)^n]))/(21*g*(e*f - d*g)^2*(f + g*
x)^(3/2)) + (8*b*e^3*n*(a + b*Log[c*(d + e*x)^n]))/(7*g*(e*f - d*g)^3*Sqrt[
f + g*x]) - (8*b*e^(7/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
```

```
*(a + b*Log[c*(d + e*x)^n]))/(7*g*(e*f - d*g)^(7/2)) - (2*(a + b*Log[c*(d +
 e*x)^n])^2)/(7*g*(f + g*x)^(7/2)) - (16*b^2*e^(7/2)*n^2*ArcTanh[(Sqrt[e]*S
qrt[f + g*x])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f
- d*g])])/(7*g*(e*f - d*g)^(7/2)) - (8*b^2*e^(7/2)*n^2*PolyLog[2, 1 - 2/(1
- (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])])/(7*g*(e*f - d*g)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} + \frac{(4ben)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{7/2}}\,dx}{7g} \\
&= -\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} + \frac{(4bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{7/2}}\,dx, x, d+ex\right)}{7g} \\
&= -\frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} - \frac{(4bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{7/2}}\,dx, x, d+ex\right)}{7(ef-dg)} \\
&\quad + \frac{(4ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{5/2}}\,dx, x, d+ex\right)}{7g(ef-dg)}
\end{aligned}
$$

$$= \frac{8ben(a+b\log(c(d+ex)^n))}{35g(ef-dg)(f+gx)^{5/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} - \frac{(4ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{5/2}}\,dx, x, d+ex\right)}{7(ef-dg)^2} + \frac{(4be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{7g(ef-dg)^2} - \frac{(8b^2en^2)\,\text{Subst}\left(\int \frac{1}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{5/2}}\,dx, x, d+ex\right)}{35g(ef-dg)}$$

$$= -\frac{16b^2e^2n^2}{105g(ef-dg)^2(f+gx)^{3/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{35g(ef-dg)(f+gx)^{5/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{21g(ef-dg)^2(f+gx)^{3/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} - \frac{(4be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{7(ef-dg)^3} + \frac{(4be^3n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{7g(ef-dg)^3} - \frac{(8b^2e^2n^2)\,\text{Subst}\left(\int \frac{1}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{35g(ef-dg)^2} - \frac{(8b^2e^2n^2)\,\text{Subst}\left(\int \frac{1}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{21g(ef-dg)^2}$$

$$= -\frac{16b^2e^2n^2}{105g(ef-dg)^2(f+gx)^{3/2}} - \frac{128b^2e^3n^2}{105g(ef-dg)^3\sqrt{f+gx}} + \frac{8ben(a+b\log(c(d+ex)^n))}{35g(ef-dg)(f+gx)^{5/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{21g(ef-dg)^2(f+gx)^{3/2}} + \frac{8be^3n(a+b\log(c(d+ex)^n))}{7g(ef-dg)^3\sqrt{f+gx}} - \frac{8be^{7/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{7g(ef-dg)^{7/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} - \frac{(8b^2e^3n^2)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{35g(ef-dg)^3} - \frac{(8b^2e^3n^2)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{21g(ef-dg)^3} - \frac{(4b^2e^3n^2)\,\mathrm{Subst}\left(\int -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}x}\,dx, x, d+ex\right)}{7g(ef-dg)^3} - \frac{(8b^2e^3n^2)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{7g(ef-dg)^3}$$

$$= -\frac{16b^2e^2n^2}{105g(ef-dg)^2(f+gx)^{3/2}} - \frac{128b^2e^3n^2}{105g(ef-dg)^3\sqrt{f+gx}} + \frac{8ben(a+b\log(c(d+ex)^n))}{35g(ef-dg)(f+gx)^{5/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{21g(ef-dg)^2(f+gx)^{3/2}} + \frac{8be^3n(a+b\log(c(d+ex)^n))}{7g(ef-dg)^3\sqrt{f+gx}} - \frac{8be^{7/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{7g(ef-dg)^{7/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} + \frac{(8b^2e^{7/2}n^2)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{x}\,dx, x, d+ex\right)}{7g(ef-dg)^{7/2}} - \frac{(16b^2e^4n^2)\,\text{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{35g^2(ef-dg)^3} - \frac{(16b^2e^4n^2)\,\text{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{21g^2(ef-dg)^3} - \frac{(16b^2e^4n^2)\,\text{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{7g^2(ef-dg)^3}$$

$$= -\frac{16b^2e^2n^2}{105g(ef-dg)^2(f+gx)^{3/2}} - \frac{128b^2e^3n^2}{105g(ef-dg)^3\sqrt{f+gx}} + \frac{368b^2e^{7/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{105g(ef-dg)^{7/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{35g(ef-dg)(f+gx)^{5/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{21g(ef-dg)^2(f+gx)^{3/2}} + \frac{8be^3n(a+b\log(c(d+ex)^n))}{7g(ef-dg)^3\sqrt{f+gx}} - \frac{8be^{7/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{7g(ef-dg)^{7/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} + \frac{(16b^2e^{9/2}n^2)\,\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{dg+e(-f+x^2)}\,dx, x, \sqrt{f+gx}\right)}{7g(ef-dg)^{7/2}}$$

$$= -\frac{16b^2e^2n^2}{105g(ef-dg)^2(f+gx)^{3/2}} - \frac{128b^2e^3n^2}{105g(ef-dg)^3\sqrt{f+gx}} + \frac{368b^2e^{7/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{105g(ef-dg)^{7/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{35g(ef-dg)(f+gx)^{5/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{21g(ef-dg)^2(f+gx)^{3/2}} + \frac{8be^3n(a+b\log(c(d+ex)^n))}{7g(ef-dg)^3\sqrt{f+gx}} - \frac{8be^{7/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{7g(ef-dg)^{7/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} + \frac{(16b^2e^{9/2}n^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{-ef+dg+ex^2}\,dx, x, \sqrt{f+gx}\right)}{7g(ef-dg)^{7/2}}$$

$$= -\frac{16b^2e^2n^2}{105g(ef-dg)^2(f+gx)^{3/2}} - \frac{128b^2e^3n^2}{105g(ef-dg)^3\sqrt{f+gx}} + \frac{368b^2e^{7/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{105g(ef-dg)^{7/2}} + \frac{8b^2e^{7/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{7g(ef-dg)^{7/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{35g(ef-dg)(f+gx)^{5/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{21g(ef-dg)^2(f+gx)^{3/2}} + \frac{8be^3n(a+b\log(c(d+ex)^n))}{7g(ef-dg)^3\sqrt{f+gx}} - \frac{8be^{7/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{7g(ef-dg)^{7/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} - \frac{(16b^2e^4n^2)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\,dx, x, \sqrt{f+gx}\right)}{7g(ef-dg)^4}$$

$$= -\frac{16b^2e^2n^2}{105g(ef-dg)^2(f+gx)^{3/2}} - \frac{128b^2e^3n^2}{105g(ef-dg)^3\sqrt{f+gx}} + \frac{368b^2e^{7/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{105g(ef-dg)^{7/2}} + \frac{8b^2e^{7/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{7g(ef-dg)^{7/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{35g(ef-dg)(f+gx)^{5/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{21g(ef-dg)^2(f+gx)^{3/2}} + \frac{8be^3n(a+b\log(c(d+ex)^n))}{7g(ef-dg)^3\sqrt{f+gx}} - \frac{8be^{7/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{7g(ef-dg)^{7/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} - \frac{16b^2e^{7/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{7g(ef-dg)^{7/2}} + \frac{(16b^2e^4n^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\right)}{1-\frac{ex^2}{ef-dg}}\,dx, x, \sqrt{f+gx}\right)}{7g(ef-dg)^4}$$

$$= -\frac{16b^2e^2n^2}{105g(ef-dg)^2(f+gx)^{3/2}} - \frac{128b^2e^3n^2}{105g(ef-dg)^3\sqrt{f+gx}} + \frac{368b^2e^{7/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{105g(ef-dg)^{7/2}} + \frac{8b^2e^{7/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{7g(ef-dg)^{7/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{35g(ef-dg)(f+gx)^{5/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{21g(ef-dg)^2(f+gx)^{3/2}} + \frac{8be^3n(a+b\log(c(d+ex)^n))}{7g(ef-dg)^3\sqrt{f+gx}} - \frac{8be^{7/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{7g(ef-dg)^{7/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} - \frac{16b^2e^{7/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{7g(ef-dg)^{7/2}} - \frac{(16b^2e^{7/2}n^2)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{7g(ef-dg)^{7/2}}$$

$$= -\frac{16b^2e^2n^2}{105g(ef-dg)^2(f+gx)^{3/2}} - \frac{128b^2e^3n^2}{105g(ef-dg)^3\sqrt{f+gx}} + \frac{368b^2e^{7/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{105g(ef-dg)^{7/2}} + \frac{8b^2e^{7/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{7g(ef-dg)^{7/2}} + \frac{8ben(a+b\log(c(d+ex)^n))}{35g(ef-dg)(f+gx)^{5/2}} + \frac{8be^2n(a+b\log(c(d+ex)^n))}{21g(ef-dg)^2(f+gx)^{3/2}} + \frac{8be^3n(a+b\log(c(d+ex)^n))}{7g(ef-dg)^3\sqrt{f+gx}} - \frac{8be^{7/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{7g(ef-dg)^{7/2}} - \frac{2(a+b\log(c(d+ex)^n))^2}{7g(f+gx)^{7/2}} - \frac{16b^2e^{7/2}n^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{7g(ef-dg)^{7/2}} - \frac{8b^2e^{7/2}n^2\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{7g(ef-dg)^{7/2}}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.21 (sec) , antiderivative size = 728, normalized size of antiderivative = 1.25

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx = \frac{2\left(-15(a+b\log(c(d+ex)^n))^2 + \frac{ben(f+gx)\left(120be^{5/2}n(f+gx)^{5/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x)^(9/2),x]
```

```
[Out] (2*(-15*(a + b*Log[c*(d + e*x)^n])^2 + (b*e*n*(f + g*x)*(120*b*e^(5/2)*n*(f
 + g*x)^(5/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] - 8*b*e*(e*f
 - d*g)^(3/2)*n*(f + g*x)*Hypergeometric2F1[-3/2, 1, -1/2, (e*(f + g*x))/(e
*f - d*g)] - 40*b*e^2*Sqrt[e*f - d*g]*n*(f + g*x)^2*Hypergeometric2F1[-1/2,
 1, 1/2, (e*(f + g*x))/(e*f - d*g)] + 12*(e*f - d*g)^(5/2)*(a + b*Log[c*(d
+ e*x)^n]) + 20*e*(e*f - d*g)^(3/2)*(f + g*x)*(a + b*Log[c*(d + e*x)^n]) +
60*e^2*Sqrt[e*f - d*g]*(f + g*x)^2*(a + b*Log[c*(d + e*x)^n]) + 30*e^(5/2)*
(f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sq
rt[f + g*x]] - 30*e^(5/2)*(f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sq
rt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] - 15*b*e^(5/2)*n*(f + g*x)^(5/2)*(Lo
g[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*S
qrt[f + g*x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) + 2
*PolyLog[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 15*b*e^(5
/2)*n*(f + g*x)^(5/2)*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sq
rt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x]
)/(2*Sqrt[e*f - d*g])]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*
f - d*g])/2]))/(e*f - d*g)^(7/2)))/(105*g*(f + g*x)^(7/2))
```

## Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))^2}{(gx+f)^{\frac{9}{2}}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(9/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)^(9/2),x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(g*x + f)*b^2*log((e*x + d)^n*c)^2 + 2*sqrt(g*x + f)*a*b*log(
(e*x + d)^n*c) + sqrt(g*x + f)*a^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3
 + 10*f^3*g^2*x^2 + 5*f^4*g*x + f^5), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)**(9/2),x)
```

```
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)^{\frac{9}{2}}}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)^(9/2),x, algorithm="giac")

[Out] integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f)^(9/2), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)^{9/2}}\,dx = \int \frac{(a+b\ln(c\,(d+e\,x)^n))^2}{(f+g\,x)^{9/2}}\,dx$$

[In] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(9/2),x)

[Out] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x)^(9/2), x)

# 3.152 $\int \frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)}\,dx = \mathrm{Int}\left(\frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)},x\right)$$

```
[Out] Unintegrable((g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)}\,dx$$

```
[In] Int[(f + g*x)^(3/2)/(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] Defer[Int][(f + g*x)^(3/2)/(a + b*Log[c*(d + e*x)^n]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(f+gx)^{3/2}}{a+b\log(c(d+ex)^n)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^{3/2}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{(f+gx)^{3/2}}{a+b\log\left(c(d+ex)^n\right)}\,dx$$

```
[In] Integrate[(f + g*x)^(3/2)/(a + b*Log[c*(d + e*x)^n]),x]
[Out] Integrate[(f + g*x)^(3/2)/(a + b*Log[c*(d + e*x)^n]), x]
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(gx+f)^{\frac{3}{2}}}{a+b\ln\left(c\left(ex+d\right)^{n}\right)}dx$$

```
[In] int((g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n)),x)
[Out] int((g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^{3/2}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{(gx+f)^{\frac{3}{2}}}{b\log\left((ex+d)^{n}c\right)+a}\,dx$$

```
[In] integrate((g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral((g*x + f)^(3/2)/(b*log((e*x + d)^n*c) + a), x)
```

## Sympy [N/A]

Not integrable

Time = 21.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(f+gx)^{3/2}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{(f+gx)^{\frac{3}{2}}}{a+b\log\left(c\left(d+ex\right)^{n}\right)}\,dx$$

```
[In] integrate((g*x+f)**(3/2)/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Integral((f + g*x)**(3/2)/(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 7.85

$$\int \frac{(f+gx)^{3/2}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{(gx+f)^{\frac{3}{2}}}{b\log\left((ex+d)^n c\right)+a}\,dx$$

```
[In] integrate((g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] 2/5*(g^2*x^2 + 2*f*g*x + f^2)*sqrt(g*x + f)/(b*g*log((e*x + d)^n) + b*g*log
(c) + a*g) + integrate(2/5*(b*e*g^2*n*x^2 + 2*b*e*f*g*n*x + b*e*f^2*n)*sqrt
(g*x + f)/(b^2*d*g*log(c)^2 + 2*a*b*d*g*log(c) + a^2*d*g + (b^2*e*g*x + b^2
*d*g)*log((e*x + d)^n)^2 + (b^2*e*g*log(c)^2 + 2*a*b*e*g*log(c) + a^2*e*g)*
x + 2*(b^2*d*g*log(c) + a*b*d*g + (b^2*e*g*log(c) + a*b*e*g)*x)*log((e*x +
d)^n)), x)
```

## Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^{3/2}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{(gx+f)^{\frac{3}{2}}}{b\log\left((ex+d)^n c\right)+a}\,dx$$

```
[In] integrate((g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^(3/2)/(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^{3/2}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{(f+g\,x)^{3/2}}{a+b\,\ln\left(c\,(d+e\,x)^n\right)}\,dx$$

```
[In] int((f + g*x)^(3/2)/(a + b*log(c*(d + e*x)^n)),x)
```

```
[Out] int((f + g*x)^(3/2)/(a + b*log(c*(d + e*x)^n)), x)
```

# 3.153 $\int \frac{\sqrt{f+gx}}{a+b\log(c(d+ex)^n)}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\text{Int}\left(\frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)},x\right)$$

```
[Out] Unintegrable((g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx$$

```
[In] Int[Sqrt[f + g*x]/(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] Defer[Int][Sqrt[f + g*x]/(a + b*Log[c*(d + e*x)^n]), x]
```

Rubi steps

$$\text{integral}=\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx$$

```
[In] Integrate[Sqrt[f + g*x]/(a + b*Log[c*(d + e*x)^n]),x]
[Out] Integrate[Sqrt[f + g*x]/(a + b*Log[c*(d + e*x)^n]), x]
```

## Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{gx+f}}{a+b\ln\left(c\left(ex+d\right)^{n}\right)}dx$$

```
[In] int((g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n)),x)
[Out] int((g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{\sqrt{gx+f}}{b\log\left((ex+d)^{n}c\right)+a}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral(sqrt(g*x + f)/(b*log((e*x + d)^n*c) + a), x)
```

## Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{\sqrt{f+gx}}{a+b\log\left(c\left(d+ex\right)^{n}\right)}\,dx$$

```
[In] integrate((g*x+f)**(1/2)/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Integral(sqrt(f + g*x)/(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 6.69

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{\sqrt{gx+f}}{b\log\left((ex+d)^n c\right)+a}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] 2/3*(g*x + f)^(3/2)/(b*g*log((e*x + d)^n) + b*g*log(c) + a*g) + integrate(2
/3*(b*e*g*n*x + b*e*f*n)*sqrt(g*x + f)/(b^2*d*g*log(c)^2 + 2*a*b*d*g*log(c)
 + a^2*d*g + (b^2*e*g*x + b^2*d*g)*log((e*x + d)^n)^2 + (b^2*e*g*log(c)^2 +
 2*a*b*e*g*log(c) + a^2*e*g)*x + 2*(b^2*d*g*log(c) + a*b*d*g + (b^2*e*g*log
(c) + a*b*e*g)*x)*log((e*x + d)^n)), x)
```

## Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{\sqrt{gx+f}}{b\log\left((ex+d)^n c\right)+a}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{f+gx}}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{\sqrt{f+g\,x}}{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

```
[In] int((f + g*x)^(1/2)/(a + b*log(c*(d + e*x)^n)),x)
```

```
[Out] int((f + g*x)^(1/2)/(a + b*log(c*(d + e*x)^n)), x)
```

# 3.154 $\int \frac{1}{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx = \operatorname{Int}\left(\frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))},x\right)$$

```
[Out] Unintegrable(1/(a+b*ln(c*(e*x+d)^n))/(g*x+f)^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx$$

```
[In] Int[1/(Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])),x]
```

```
[Out] Defer[Int][1/(Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx=\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx$$

```
[In] Integrate[1/(Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])),x]
[Out] Integrate[1/(Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])), x]
```

## Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+b\ln(c(ex+d)^n))\sqrt{gx+f}}dx$$

```
[In] int(1/(a+b*ln(c*(e*x+d)^n))/(g*x+f)^(1/2),x)
[Out] int(1/(a+b*ln(c*(e*x+d)^n))/(g*x+f)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx=\int \frac{1}{\sqrt{gx+f}(b\log((ex+d)^n c)+a)}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(g*x + f)/(a*g*x + a*f + (b*g*x + b*f)*log((e*x + d)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx=\int \frac{1}{(a+b\log(c(d+ex)^n))\sqrt{f+gx}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(e*x+d)**n))/(g*x+f)**(1/2),x)
[Out] Integral(1/((a + b*log(c*(d + e*x)**n))*sqrt(f + g*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 6.69

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{\sqrt{gx+f}(b\log((ex+d)^n c)+a)}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(g*x + f)/(b*g*log((e*x + d)^n) + b*g*log(c) + a*g) + integrate(2*(b*
e*g*n*x + b*e*f*n)/((b^2*d*g*log(c)^2 + 2*a*b*d*g*log(c) + a^2*d*g + (b^2*e
*g*x + b^2*d*g)*log((e*x + d)^n)^2 + (b^2*e*g*log(c)^2 + 2*a*b*e*g*log(c) +
 a^2*e*g)*x + 2*(b^2*d*g*log(c) + a*b*d*g + (b^2*e*g*log(c) + a*b*e*g)*x)*l
og((e*x + d)^n))*sqrt(g*x + f)), x)
```

## Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{\sqrt{gx+f}(b\log((ex+d)^n c)+a)}\,dx$$

```
[In] integrate(1/(a+b*log(c*(e*x+d)^n))/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{f+gx}\,(a+b\log(c(d+ex)^n))}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))), x)
```

# 3.155 $\int \frac{1}{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \text{Int}\left(\frac{1}{(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{1}{(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])),x]
[Out] Defer[Int][1/((f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)^{3/2}\,(a+b\log{(c(d+ex)^n)})}\,dx = \int \frac{1}{(f+gx)^{3/2}\,(a+b\log{(c(d+ex)^n)})}\,dx$$

```
[In] Integrate[1/((f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])),x]
[Out] Integrate[1/((f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])), x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(gx+f)^{\frac{3}{2}}\,(a+b\ln{(c\,(ex+d)^n)})}dx$$

```
[In] int(1/(g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n)),x)
[Out] int(1/(g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \frac{1}{(f+gx)^{3/2}\,(a+b\log{(c(d+ex)^n)})}\,dx = \int \frac{1}{(gx+f)^{\frac{3}{2}}(b\log{((ex+d)^n c)}+a)}\,dx$$

```
[In] integrate(1/(g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral(sqrt(g*x + f)/(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 + 2*b*f*
g*x + b*f^2)*log((e*x + d)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 7.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx)^{3/2}\,(a+b\log\left(c(d+ex)^n\right))}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(g*x+f)**(3/2)/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Integral(1/((a + b*log(c*(d + e*x)**n))*(f + g*x)**(3/2)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 6.38

$$\int \frac{1}{(f+gx)^{3/2}\,(a+b\log\left(c(d+ex)^n\right))}\,dx = \int \frac{1}{(gx+f)^{\frac{3}{2}}(b\log\left((ex+d)^n c\right)+a)}\,dx$$

```
[In] integrate(1/(g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
[Out] -2*b*e*n*integrate(1/((b^2*d*g*log(c)^2 + 2*a*b*d*g*log(c) + a^2*d*g + (b^2
*e*g*x + b^2*d*g)*log((e*x + d)^n)^2 + (b^2*e*g*log(c)^2 + 2*a*b*e*g*log(c)
 + a^2*e*g)*x + 2*(b^2*d*g*log(c) + a*b*d*g + (b^2*e*g*log(c) + a*b*e*g)*x)
*log((e*x + d)^n))*sqrt(g*x + f)), x) - 2/((b*g*log((e*x + d)^n) + b*g*log(
c) + a*g)*sqrt(g*x + f))
```

## Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)^{3/2}\,(a+b\log\left(c(d+ex)^n\right))}\,dx = \int \frac{1}{(gx+f)^{\frac{3}{2}}(b\log\left((ex+d)^n c\right)+a)}\,dx$$

```
[In] integrate(1/(g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)^(3/2)*(b*log((e*x + d)^n*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^{3/2}\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n))),x)
[Out] int(1/((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n))), x)
```

# 3.156 $\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\frac{2(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}{3g}-\frac{ben\text{Int}\left(\frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+b\log(c(d+ex)^n)}},x\right)}{3g}$$

```
[Out] 2/3*(g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/g-1/3*b*e*n*Unintegrable((g*x
+f)^(3/2)/(e*x+d)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)/g
```

## Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$$

```
[In] Int[Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
```

```
[Out] (2*(f + g*x)^(3/2)*Sqrt[a + b*Log[c*(d + e*x)^n]])/(3*g) - (b*e*n*Defer[Int
][(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]), x])/(3*g)
```

Rubi steps

$$\text{integral}=\frac{2(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}{3g}-\frac{(ben)\int\frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{3g}$$

## Mathematica [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$$

```
[In] Integrate[Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
[Out] Integrate[Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]], x]
```

## Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \sqrt{gx+f}\,\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}dx$$

```
[In] int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
[Out] int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}\sqrt{f+gx}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral(sqrt(a + b*log(c*(d + e*x)**n))*sqrt(f + g*x), x)
```

## Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{gx+f}\sqrt{b\log\left((ex+d)^nc\right)+a}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)*sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Giac [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{gx+f}\sqrt{b\log\left((ex+d)^nc\right)+a}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)*sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{f+g\,x}\,\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

```
[In] int((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))^(1/2),x)
[Out] int((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

# 3.157 $\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{f+gx}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}{g} - \frac{ben\text{Int}\left(\frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+b\log(c(d+ex)^n)}},x\right)}{g}$$

```
[Out] 2*(g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))^(1/2)/g-b*e*n*Unintegrable((g*x+f)^(1
/2)/(e*x+d)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)/g
```

## Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{\sqrt{f+gx}}\,dx$$

```
[In] Int[Sqrt[a + b*Log[c*(d + e*x)^n]]/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]])/g - (b*e*n*Defer[Int][Sqrt
[f + g*x]/((d + e*x)*Sqrt[a + b*Log[c*(d + e*x)^n]]), x])/g
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}{g} - \frac{(ben)\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{g}$$

## Mathematica [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{f+gx}}\,dx=\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{f+gx}}\,dx$$

```
[In] Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/Sqrt[f + g*x],x]
[Out] Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/Sqrt[f + g*x], x]
```

## Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+b\ln(c(ex+d)^n)}}{\sqrt{gx+f}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^(1/2),x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{f+gx}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{f+gx}}\,dx=\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{\sqrt{f+gx}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(1/2)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + b*log(c*(d + e*x)**n))/sqrt(f + g*x), x)
```

## Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{b\log\left((ex+d)^n c\right)+a}}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a)/sqrt(g*x + f), x)
```

## Giac [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{b\log\left((ex+d)^n c\right)+a}}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a)/sqrt(g*x + f), x)
```

## Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}{\sqrt{f+g\,x}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^(1/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^(1/2), x)
```

# 3.158 $\int \frac{\sqrt{a+b\log(c(d+ex)^n)}}{(f+gx)^{3/2}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx = -\frac{2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{g\sqrt{f+gx}} + \frac{ben\text{Int}\left(\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+b\log(c(d+ex)^n)}},x\right)}{g}$$

```
[Out] -2*(a+b*ln(c*(e*x+d)^n))^(1/2)/g/(g*x+f)^(1/2)+b*e*n*Unintegrable(1/(e*x+d)
/(g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)/g
```

## Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx = \int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx$$

```
[In] Int[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*Log[c*(d + e*x)^n]])/(g*Sqrt[f + g*x]) + (b*e*n*Defer[Int][1
/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]]), x])/g
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{a+b\log\left(c(d+ex)^n\right)}}{g\sqrt{f+gx}} + \frac{(ben)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+b\log(c(d+ex)^n)}}\,dx}{g}$$

## Mathematica [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx = \int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx$$

```
[In] Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^(3/2),x]
[Out] Integrate[Sqrt[a + b*Log[c*(d + e*x)^n]]/(f + g*x)^(3/2), x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+b\ln\left(c\,(ex+d)^n\right)}}{(gx+f)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^(3/2),x)
[Out] int((a+b*ln(c*(e*x+d)^n))^(1/2)/(g*x+f)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 9.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx = \int \frac{\sqrt{a+b\log\left(c\,(d+ex)^n\right)}}{(f+gx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**(1/2)/(g*x+f)**(3/2),x)
[Out] Integral(sqrt(a + b*log(c*(d + e*x)**n))/(f + g*x)**(3/2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx=\int \frac{\sqrt{b\log\left((ex+d)^nc\right)+a}}{(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f)^(3/2), x)
```

## Giac [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx=\int \frac{\sqrt{b\log\left((ex+d)^nc\right)+a}}{(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^(1/2)/(g*x+f)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a)/(g*x + f)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\log\left(c(d+ex)^n\right)}}{(f+gx)^{3/2}}\,dx=\int \frac{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}{{\left(f+g\,x\right)}^{3/2}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^(3/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^(1/2)/(f + g*x)^(3/2), x)
```

# 3.159 $\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\text{Int}\left(\frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}},x\right)$$

```
[Out] Unintegrable((g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

```
[In] Int[Sqrt[f + g*x]/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
[Out] Defer[Int][Sqrt[f + g*x]/Sqrt[a + b*Log[c*(d + e*x)^n]], x]
```

Rubi steps

$$\text{integral}=\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

```
[In] Integrate[Sqrt[f + g*x]/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
[Out] Integrate[Sqrt[f + g*x]/Sqrt[a + b*Log[c*(d + e*x)^n]], x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{gx+f}}{\sqrt{a+b\ln\left(c\,(ex+d)^n\right)}}dx$$

```
[In] int((g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
[Out] int((g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c\,(d+ex)^n\right)}}\,dx$$

```
[In] integrate((g*x+f)**(1/2)/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral(sqrt(f + g*x)/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{\sqrt{gx+f}}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{\sqrt{gx+f}}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{\sqrt{f+g\,x}}{\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}\,dx$$

```
[In] int((f + g*x)^(1/2)/(a + b*log(c*(d + e*x)^n))^(1/2),x)
[Out] int((f + g*x)^(1/2)/(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

# 3.160 $\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log(c(d+ex)^n)}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \operatorname{Int}\left(\frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

```
[In] Int[1/(Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]]),x]
[Out] Defer[Int][1/(Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

```
[In] Integrate[1/(Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]]),x]
[Out] Integrate[1/(Sqrt[f + g*x]*Sqrt[a + b*Log[c*(d + e*x)^n]]), x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{gx+f}\,\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}dx$$

```
[In] int(1/(g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
[Out] int(1/(g*x+f)^(1/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas
")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{1}{\sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}\sqrt{f+gx}}\,dx$$

```
[In] integrate(1/(g*x+f)**(1/2)/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*log(c*(d + e*x)**n))*sqrt(f + g*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{1}{\sqrt{gx+f}\sqrt{b\log\left((ex+d)^nc\right)+a}}\,dx$$

```
[In] integrate(1/(g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima
")
[Out] integrate(1/(sqrt(g*x + f)*sqrt(b*log((e*x + d)^n*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{1}{\sqrt{gx+f}\sqrt{b\log\left((ex+d)^nc\right)+a}}\,dx$$

```
[In] integrate(1/(g*x+f)^(1/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(g*x + f)*sqrt(b*log((e*x + d)^n*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))^(1/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n))^(1/2)), x)
```

# 3.161 $\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log(c(d+ex)^n)}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\text{Int}\left(\frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

```
[In] Int[1/((f + g*x)^(3/2)*Sqrt[a + b*Log[c*(d + e*x)^n]]),x]
```

```
[Out] Defer[Int][1/((f + g*x)^(3/2)*Sqrt[a + b*Log[c*(d + e*x)^n]]), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

```
[In] Integrate[1/((f + g*x)^(3/2)*Sqrt[a + b*Log[c*(d + e*x)^n]]),x]
[Out] Integrate[1/((f + g*x)^(3/2)*Sqrt[a + b*Log[c*(d + e*x)^n]]), x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{(gx+f)^{\frac{3}{2}}\sqrt{a+b\ln\left(c\left(ex+d\right)^{n}\right)}}dx$$

```
[In] int(1/(g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
[Out] int(1/(g*x+f)^(3/2)/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas
")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 26.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d+ex\right)^{n}\right)}\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(g*x+f)**(3/2)/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*log(c*(d + e*x)**n))*(f + g*x)**(3/2)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{1}{(gx+f)^{\frac{3}{2}}\sqrt{b\log\left((ex+d)^{n}c\right)+a}}\,dx$$

```
[In] integrate(1/(g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima
")
[Out] integrate(1/((g*x + f)^(3/2)*sqrt(b*log((e*x + d)^n*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{1}{(gx+f)^{\frac{3}{2}}\sqrt{b\log\left((ex+d)^{n}c\right)+a}}\,dx$$

```
[In] integrate(1/(g*x+f)^(3/2)/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate(1/((g*x + f)^(3/2)*sqrt(b*log((e*x + d)^n*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(f+gx)^{3/2}\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{1}{{\left(f+g\,x\right)}^{3/2}\,\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}}\,dx$$

```
[In] int(1/((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n))^(1/2)),x)
[Out] int(1/((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n))^(1/2)), x)
```

# 3.162 $\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 94

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx = \frac{ben(f+gx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)(1+m)(2+m)} + \frac{(f+gx)^{1+m}(a+b\log(c(d+ex)^n))}{g(1+m)}$$

```
[Out] b*e*n*(g*x+f)^(2+m)*hypergeom([1, 2+m],[3+m],e*(g*x+f)/(-d*g+e*f))/g/(-d*g+
e*f)/(1+m)/(2+m)+(g*x+f)^(1+m)*(a+b*ln(c*(e*x+d)^n))/g/(1+m)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2442, 70}

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx = \frac{(f+gx)^{m+1}(a+b\log(c(d+ex)^n))}{g(m+1)} + \frac{ben(f+gx)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{e(f+gx)}{ef-dg}\right)}{g(m+1)(m+2)(ef-dg)}$$

```
[In] Int[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (b*e*n*(f + g*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (e*(f + g*x))/(
e*f - d*g)])/(g*(e*f - d*g)*(1 + m)*(2 + m)) + ((f + g*x)^(1 + m)*(a + b*Lo
g[c*(d + e*x)^n]))/(g*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^{1+m}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g(1+m)} - \frac{(ben)\int\frac{(f+gx)^{1+m}}{d+ex}\,dx}{g(1+m)}$$

$$= \frac{ben(f+gx)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{e(f+gx)}{ef-dg}\right)}{g(ef-dg)(1+m)(2+m)} + \frac{(f+gx)^{1+m}\left(a+b\log\left(c(d+ex)^n\right)\right)}{g(1+m)}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86

$$\int (f+gx)^m\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx$$
$$= \frac{(f+gx)^{1+m}\left(a+\frac{ben(f+gx)\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{e(f+gx)}{ef-dg}\right)}{(ef-dg)(2+m)}+b\log\left(c(d+ex)^n\right)\right)}{g(1+m)}$$

```
[In] Integrate[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] ((f + g*x)^(1 + m)*(a + (b*e*n*(f + g*x)*Hypergeometric2F1[1, 2 + m, 3 + m,
 (e*(f + g*x))/(e*f - d*g)])/((e*f - d*g)*(2 + m)) + b*Log[c*(d + e*x)^n]))
/(g*(1 + m))
```

## Maple [F]

$$\int (gx+f)^m (a+b\ln(c(ex+d)^n))\,dx$$

```
[In] int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n)),x)
[Out] int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n)),x)
```

## Fricas [F]

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx = \int (b\log((ex+d)^n c)+a)(gx+f)^m\,dx$$

```
[In] integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral((g*x + f)^m*b*log((e*x + d)^n*c) + (g*x + f)^m*a, x)
```

## Sympy [F(-2)]

Exception generated.

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((g*x+f)**m*(a+b*ln(c*(e*x+d)**n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [F]

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx = \int (b\log((ex+d)^n c)+a)(gx+f)^m\,dx$$

```
[In] integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
[Out] b*((g*x + f)*(g*x + f)^m*log((e*x + d)^n)/(g*(m + 1)) + integrate((d*g*(m +
 1)*log(c) - e*f*n + (e*g*(m + 1)*log(c) - e*g*n)*x)*(g*x + f)^m/(e*g*(m +
1)*x + d*g*(m + 1)), x)) + (g*x + f)^(m + 1)*a/(g*(m + 1))
```

## Giac [F]

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx = \int (b\log((ex+d)^n c)+a)(gx+f)^m\,dx$$

```
[In] integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*(g*x + f)^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))\,dx = \int (f+g\,x)^m\,(a+b\,\ln(c\,(d+e\,x)^n))\,dx$$

```
[In] int((f + g*x)^m*(a + b*log(c*(d + e*x)^n)),x)
[Out] int((f + g*x)^m*(a + b*log(c*(d + e*x)^n)), x)
```

# 3.163 $\int \frac{(f+gx)^m}{a+b\log(c(d+ex)^n)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx = \text{Int}\left(\frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)},x\right)$$

```
[Out] Unintegrable((g*x+f)^m/(a+b*ln(c*(e*x+d)^n)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx = \int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx$$

```
[In] Int[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n]),x]
[Out] Defer[Int][(f + g*x)^m/(a + b*Log[c*(d + e*x)^n]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx$$

```
[In] Integrate[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n]),x]
[Out] Integrate[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n]), x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^m}{a+b\ln\left(c\,(ex+d)^n\right)}dx$$

```
[In] int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n)),x)
[Out] int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{(gx+f)^m}{b\log\left((ex+d)^n c\right)+a}\,dx$$

```
[In] integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral((g*x + f)^m/(b*log((e*x + d)^n*c) + a), x)
```

## Sympy [N/A]

Not integrable

Time = 31.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx)^m}{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \frac{(f+gx)^m}{a+b\log\left(c\,(d+ex)^n\right)}\,dx$$

```
[In] integrate((g*x+f)**m/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Integral((f + g*x)**m/(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{(gx+f)^m}{b\log((ex+d)^n c)+a}\,dx$$

```
[In] integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
[Out] integrate((g*x + f)^m/(b*log((e*x + d)^n*c) + a), x)
```

## Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{(gx+f)^m}{b\log((ex+d)^n c)+a}\,dx$$

```
[In] integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] integrate((g*x + f)^m/(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{(f+g\,x)^m}{a+b\,\ln\left(c\,(d+e\,x)^n\right)}\,dx$$

```
[In] int((f + g*x)^m/(a + b*log(c*(d + e*x)^n)),x)
[Out] int((f + g*x)^m/(a + b*log(c*(d + e*x)^n)), x)
```

# 3.164 $\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx=\operatorname{Int}\left(\frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2},x\right)$$

```
[Out] Unintegrable((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx=\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx$$

```
[In] Int[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] Defer[Int][(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 5.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx=\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx$$

```
[In] Integrate[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^2,x]
[Out] Integrate[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^2, x]
```

## Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^m}{(a+b\ln(c(ex+d)^n))^2}dx$$

```
[In] int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^2,x)
[Out] int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx=\int \frac{(gx+f)^m}{(b\log((ex+d)^nc)+a)^2}\,dx$$

```
[In] integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
[Out] integral((g*x + f)^m/(b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) +
 a^2), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((g*x+f)**m/(a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 5.46

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(gx+f)^m}{(b\log((ex+d)^n c)+a)^2}\,dx$$

```
[In] integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] -(e*x + d)*(g*x + f)^m/(b^2*e*n*log((e*x + d)^n) + b^2*e*n*log(c) + a*b*e*n
) + integrate((e*g*(m + 1)*x + d*g*m + e*f)*(g*x + f)^m/(b^2*e*f*n*log(c) +
 a*b*e*f*n + (b^2*e*g*n*log(c) + a*b*e*g*n)*x + (b^2*e*g*n*x + b^2*e*f*n)*l
og((e*x + d)^n)), x)
```

## Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{(gx+f)^m}{(b\log((ex+d)^n c)+a)^2}\,dx$$

```
[In] integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^m/(b*log((e*x + d)^n*c) + a)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^2} \, dx = \int \frac{(f+g\,x)^m}{\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^2} \, dx$$

```
[In] int((f + g*x)^m/(a + b*log(c*(d + e*x)^n))^2,x)
[Out] int((f + g*x)^m/(a + b*log(c*(d + e*x)^n))^2, x)
```

# 3.165 $\int (f+gx)^m \left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int (f+gx)^m \left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx=\text{Int}\left((f+gx)^m \left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2},x\right)$$

```
[Out] Unintegrable((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (f+gx)^m \left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx=\int (f+gx)^m \left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx$$

```
[In] Int[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] Defer[Int][(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^(3/2), x]
```

Rubi steps

$$\text{integral}=\int (f+gx)^m \left(a+b\log\left(c(d+ex)^n\right)\right)^{3/2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 9.50 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^{3/2}\,dx=\int (f+gx)^m (a+b\log(c(d+ex)^n))^{3/2}\,dx$$

```
[In] Integrate[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^(3/2),x]
[Out] Integrate[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^(3/2), x]
```

## Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (gx+f)^m (a+b\ln(c(ex+d)^n))^{\frac{3}{2}}dx$$

```
[In] int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
[Out] int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^{3/2}\,dx=\int (b\log((ex+d)^n c)+a)^{\frac{3}{2}}(gx+f)^m\,dx$$

```
[In] integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
[Out] integral(((g*x + f)^m*b*log((e*x + d)^n*c) + (g*x + f)^m*a)*sqrt(b*log((e*x
 + d)^n*c) + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)**m*(a+b*ln(c*(e*x+d)**n))**(3/2),x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m \left(a+b\log \left(c(d+ex)^n\right)\right)^{3/2} \, dx = \int (b\log \left((ex+d)^n c\right)+a)^{\frac{3}{2}}(gx+f)^m \, dx$$

```
[In] integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(3/2)*(g*x + f)^m, x)
```

## Giac [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m \left(a+b\log \left(c(d+ex)^n\right)\right)^{3/2} \, dx = \int (b\log \left((ex+d)^n c\right)+a)^{\frac{3}{2}}(gx+f)^m \, dx$$

```
[In] integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^(3/2)*(g*x + f)^m, x)
```

## Mupad [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m \left(a+b\log \left(c(d+ex)^n\right)\right)^{3/2} \, dx = \int {\left(f+g\,x\right)}^m\,{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}^{3/2} \,d x$$

```
[In] int((f + g*x)^m*(a + b*log(c*(d + e*x)^n))^(3/2),x)
[Out] int((f + g*x)^m*(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

# 3.166 $\int (f+gx)^m \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int (f+gx)^m \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \operatorname{Int}\left((f+gx)^m \sqrt{a+b\log\left(c(d+ex)^n\right)},x\right)$$

```
[Out] Unintegrable((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (f+gx)^m \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx = \int (f+gx)^m \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$$

```
[In] Int[(f + g*x)^m*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
[Out] Defer[Int][(f + g*x)^m*Sqrt[a + b*Log[c*(d + e*x)^n]], x]
```

Rubi steps

$$\text{integral} = \int (f+gx)^m \sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (f+gx)^m\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int (f+gx)^m\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx$$

```
[In] Integrate[(f + g*x)^m*Sqrt[a + b*Log[c*(d + e*x)^n]],x]
[Out] Integrate[(f + g*x)^m*Sqrt[a + b*Log[c*(d + e*x)^n]], x]
```

## Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (gx+f)^m\sqrt{a+b\ln\left(c\left(ex+d\right)^n\right)}dx$$

```
[In] int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
[Out] int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{b\log\left((ex+d)^nc\right)+a}(gx+f)^m\,dx$$

```
[In] integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*log((e*x + d)^n*c) + a)*(g*x + f)^m, x)
```

## Sympy [F(-2)]

Exception generated.

$$\int (f+gx)^m\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((g*x+f)**m*(a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{b\log\left((ex+d)^nc\right)+a}(gx+f)^m\,dx$$

```
[In] integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a)*(g*x + f)^m, x)
```

## Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int \sqrt{b\log\left((ex+d)^nc\right)+a}(gx+f)^m\,dx$$

```
[In] integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*log((e*x + d)^n*c) + a)*(g*x + f)^m, x)
```

## Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (f+gx)^m\sqrt{a+b\log\left(c(d+ex)^n\right)}\,dx=\int {\left(f+g\,x\right)}^m\,\sqrt{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}\,dx$$

```
[In] int((f + g*x)^m*(a + b*log(c*(d + e*x)^n))^(1/2),x)
[Out] int((f + g*x)^m*(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

# 3.167 $\int \frac{(f+gx)^m}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \operatorname{Int}\left(\frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}},x\right)$$

```
[Out] Unintegrable((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx = \int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

```
[In] Int[(f + g*x)^m/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
[Out] Defer[Int][(f + g*x)^m/Sqrt[a + b*Log[c*(d + e*x)^n]], x]
```

Rubi steps

$$\text{integral} = \int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 7.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx$$

```
[In] Integrate[(f + g*x)^m/Sqrt[a + b*Log[c*(d + e*x)^n]],x]
[Out] Integrate[(f + g*x)^m/Sqrt[a + b*Log[c*(d + e*x)^n]], x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(gx+f)^m}{\sqrt{a+b\ln\left(c\left(ex+d\right)^n\right)}}dx$$

```
[In] int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
[Out] int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{(gx+f)^m}{\sqrt{b\log\left((ex+d)^nc\right)+a}}\,dx$$

```
[In] integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="fricas")
[Out] integral((g*x + f)^m/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Sympy [N/A]

Not integrable

Time = 4.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c(d+ex)^n\right)}}\,dx=\int \frac{(f+gx)^m}{\sqrt{a+b\log\left(c\left(d+ex\right)^n\right)}}\,dx$$

```
[In] integrate((g*x+f)**m/(a+b*ln(c*(e*x+d)**n))**(1/2),x)
[Out] Integral((f + g*x)**m/sqrt(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{(gx+f)^m}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

```
[In] integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^m/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{(gx+f)^m}{\sqrt{b\log\left((ex+d)^n c\right)+a}}\,dx$$

```
[In] integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^m/sqrt(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{\sqrt{a+b\log(c(d+ex)^n)}}\,dx = \int \frac{(f+g\,x)^m}{\sqrt{a+b\,\ln\left(c\,(d+e\,x)^n\right)}}\,dx$$

```
[In] int((f + g*x)^m/(a + b*log(c*(d + e*x)^n))^(1/2),x)
[Out] int((f + g*x)^m/(a + b*log(c*(d + e*x)^n))^(1/2), x)
```

# 3.168 $\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx=\text{Int}\left(\frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}},x\right)$$

```
[Out] Unintegrable((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx=\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$$

```
[In] Int[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
```

```
[Out] Defer[Int][(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^(3/2), x]
```

Rubi steps

$$\text{integral}=\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 8.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx=\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx$$

```
[In] Integrate[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^(3/2),x]
[Out] Integrate[(f + g*x)^m/(a + b*Log[c*(d + e*x)^n])^(3/2), x]
```

## Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(gx+f)^m}{(a+b\ln(c(ex+d)^n))^{\frac{3}{2}}}dx$$

```
[In] int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
[Out] int((g*x+f)^m/(a+b*ln(c*(e*x+d)^n))^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx=\int \frac{(gx+f)^m}{(b\log((ex+d)^nc)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*log((e*x + d)^n*c) + a)*(g*x + f)^m/(b^2*log((e*x + d)^n*c)
^2 + 2*a*b*log((e*x + d)^n*c) + a^2), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((g*x+f)**m/(a+b*ln(c*(e*x+d)**n))**(3/2),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(gx+f)^m}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^m/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(gx+f)^m}{(b\log((ex+d)^n c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)^m/(a+b*log(c*(e*x+d)^n))^(3/2),x, algorithm="giac")
[Out] integrate((g*x + f)^m/(b*log((e*x + d)^n*c) + a)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^m}{(a+b\log(c(d+ex)^n))^{3/2}}\,dx = \int \frac{(f+g\,x)^m}{(a+b\,\ln(c\,(d+e\,x)^n))^{3/2}}\,dx$$

```
[In] int((f + g*x)^m/(a + b*log(c*(d + e*x)^n))^(3/2),x)
[Out] int((f + g*x)^m/(a + b*log(c*(d + e*x)^n))^(3/2), x)
```

# 3.169 $\int (f+gx)^m (a+b\log (c(d+ex)^n))^n \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (f+gx)^m (a+b\log (c(d+ex)^n))^n \, dx = \text{Int}\left((f+gx)^m (a+b\log (c(d+ex)^n))^n, x\right)$$

```
[Out] Unintegrable((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^n,x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (f+gx)^m (a+b\log (c(d+ex)^n))^n \, dx = \int (f+gx)^m (a+b\log (c(d+ex)^n))^n \, dx$$

```
[In] Int[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^n,x]
```

```
[Out] Defer[Int][(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^n, x]
```

Rubi steps

$$\text{integral} = \int (f+gx)^m (a+b\log (c(d+ex)^n))^n \, dx$$

## Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^n \, dx = \int (f+gx)^m (a+b\log(c(d+ex)^n))^n \, dx$$

```
[In] Integrate[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^n,x]
[Out] Integrate[(f + g*x)^m*(a + b*Log[c*(d + e*x)^n])^n, x]
```

## Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (gx+f)^m (a+b\ln(c(ex+d)^n))^n dx$$

```
[In] int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^n,x)
[Out] int((g*x+f)^m*(a+b*ln(c*(e*x+d)^n))^n,x)
```

## Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^n \, dx = \int (gx+f)^m (b\log((ex+d)^n c)+a)^n \, dx$$

```
[In] integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="fricas")
[Out] integral((g*x + f)^m*(b*log((e*x + d)^n*c) + a)^n, x)
```

## Sympy [F(-2)]

Exception generated.

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^n \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((g*x+f)**m*(a+b*ln(c*(e*x+d)**n))**n,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^n \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

## Giac [F(-2)]

Exception generated.

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^n \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^m*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,0
,7,4,0,5,0,3,5,0,0,0]%%%}+%%%{5,[0,0,6,4,0,4,1,3,5,0,0,0]%%%}+%%%{2,[0,0,6,
3,1,5,0,3
```

## Mupad [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (f+gx)^m (a+b\log(c(d+ex)^n))^n \, dx = \int (f+g\,x)^m \,(a+b\,\ln(c\,(d+e\,x)^n))^n \, dx$$

```
[In] int((f + g*x)^m*(a + b*log(c*(d + e*x)^n))^n,x)
```

```
[Out] int((f + g*x)^m*(a + b*log(c*(d + e*x)^n))^n, x)
```

# 3.170 $\int (f+gx)^3 (a+b\log(c(d+ex)^n))^n\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 474

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^n\,dx$$
$$= \frac{4^{-1-n}e^{-\frac{4a}{bn}}g^3(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\Gamma\left(1+n,-\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(c(\cdots}{bn}\right.}{e^4}$$
$$+\frac{3^{-n}e^{-\frac{3a}{bn}}g^2(ef-dg)(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\Gamma\left(1+n,-\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)(a+b\log(c(d+ex)^n))^n\left(\cdots\right.}{e^4}$$
$$+\frac{3\,2^{-1-n}e^{-\frac{2a}{bn}}g(ef-dg)^2(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\Gamma\left(1+n,-\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)(a+b\log(c(d+ex)^n)\cdots}{e^4}$$
$$+\frac{e^{-\frac{a}{bn}}(ef-dg)^3(d+ex)\left(c(d+ex)^n\right)^{-1/n}\Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(\cdots}{}\right.}{e^4}$$

```
[Out] 4^(-1-n)*g^3*(e*x+d)^4*GAMMA(1+n,-4*(a+b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(e
*x+d)^n))^n/e^4/exp(4*a/b/n)/((c*(e*x+d)^n)^(4/n))/(((-a-b*ln(c*(e*x+d)^n))
/b/n)^n)+g^2*(-d*g+e*f)*(e*x+d)^3*GAMMA(1+n,-3*(a+b*ln(c*(e*x+d)^n))/b/n)*(
a+b*ln(c*(e*x+d)^n))^n/(3^n)/e^4/exp(3*a/b/n)/((c*(e*x+d)^n)^(3/n))/(((-a-b
*ln(c*(e*x+d)^n))/b/n)^n)+3*2^(-1-n)*g*(-d*g+e*f)^2*(e*x+d)^2*GAMMA(1+n,-2*
(a+b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(e*x+d)^n))^n/e^4/exp(2*a/b/n)/((c*(e*
x+d)^n)^(2/n))/(((-a-b*ln(c*(e*x+d)^n))/b/n)^n)+(-d*g+e*f)^3*(e*x+d)*GAMMA(
1+n,(-a-b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(e*x+d)^n))^n/e^4/exp(a/b/n)/((c*
(e*x+d)^n)^(1/n))/(((-a-b*ln(c*(e*x+d)^n))/b/n)^n)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2448, 2436, 2337, 2212, 2437, 2347}

$$\int (f+gx)^3 (a+b\log (c(d+ex)^n))^n \, dx$$
$$= \frac{g^2 3^{-n} e^{-\frac{3a}{bn}} (d+ex)^3 (ef-dg) \left(c(d+ex)^n\right)^{-3/n} (a+b\log (c(d+ex)^n))^n \left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n} \Gamma\Big(n+1,-\frac{3(}{}}{e^4}$$
$$+\frac{3g2^{-n-1} e^{-\frac{2a}{bn}} (d+ex)^2 (ef-dg)^2 \left(c(d+ex)^n\right)^{-2/n} (a+b\log (c(d+ex)^n))^n \left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n} \Gamma\Big(n+}{e^4}$$
$$+\frac{e^{-\frac{a}{bn}} (d+ex)(ef-dg)^3 \left(c(d+ex)^n\right)^{-1/n} (a+b\log (c(d+ex)^n))^n \left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n} \Gamma\Big(n+1,-\frac{a+b\log}{}}{e^4}$$
$$+\frac{g^3 4^{-n-1} e^{-\frac{4a}{bn}} (d+ex)^4 \left(c(d+ex)^n\right)^{-4/n} (a+b\log (c(d+ex)^n))^n \left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n} \Gamma\Big(n+1,-\frac{4(a+b\log}{}}{e^4}$$

```
[In] Int[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n])^n,x]
```

```
[Out] (4^(-1 - n)*g^3*(d + e*x)^4*Gamma[1 + n, (-4*(a + b*Log[c*(d + e*x)^n]))/(b
*n)]*(a + b*Log[c*(d + e*x)^n])^n)/(e^4*E^((4*a)/(b*n))*(c*(d + e*x)^n)^(4/
n)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n) + (g^2*(e*f - d*g)*(d + e*x)^3*
Gamma[1 + n, (-3*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log[c*(d + e*x)^
n])^n)/(3^n*e^4*E^((3*a)/(b*n))*(c*(d + e*x)^n)^(3/n)*(-((a + b*Log[c*(d +
e*x)^n])/(b*n)))^n) + (3*2^(-1 - n)*g*(e*f - d*g)^2*(d + e*x)^2*Gamma[1 + n
, (-2*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log[c*(d + e*x)^n])^n)/(e^4
*E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)*(-((a + b*Log[c*(d + e*x)^n])/(b*n))
)^n) + ((e*f - d*g)^3*(d + e*x)*Gamma[1 + n, -((a + b*Log[c*(d + e*x)^n])/(
b*n))]*(a + b*Log[c*(d + e*x)^n])^n)/(e^4*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1
)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(ef-dg)^3 (a+b\log (c(d+ex)^n))^n}{e^3} + \frac{3g(ef-dg)^2(d+ex) (a+b\log (c(d+ex)^n))^n}{e^3} + \frac{3g^2(ef-dg)(d+ex)^2 (a+b\log (c(d+ex)^n))^n}{e^3} + \frac{g^3(d+ex)^3 (a+b\log (c(d+ex)^n))^n}{e^3}\right) dx \\
&= \frac{g^3 \int (d+ex)^3 (a+b\log (c(d+ex)^n))^n \, dx}{e^3} + \frac{(3g^2(ef-dg)) \int (d+ex)^2 (a+b\log (c(d+ex)^n))^n \, dx}{e^3} + \frac{(3g(ef-dg)^2) \int (d+ex) (a+b\log (c(d+ex)^n))^n \, dx}{e^3} + \frac{(ef-dg)^3 \int (a+b\log (c(d+ex)^n))^n \, dx}{e^3}
\end{aligned}$$

$$= \frac{g^3\text{Subst}\left(\int x^3(a+b\log(cx^n))^n\,dx,x,d+ex\right)}{e^4} + \frac{(3g^2(ef-dg))\,\text{Subst}\left(\int x^2(a+b\log(cx^n))^n\,dx,x,d+ex\right)}{e^4} + \frac{(3g(ef-dg)^2)\,\text{Subst}\left(\int x(a+b\log(cx^n))^n\,dx,x,d+ex\right)}{e^4} + \frac{(ef-dg)^3\text{Subst}\left(\int (a+b\log(cx^n))^n\,dx,x,d+ex\right)}{e^4}$$

$$= \frac{\left(g^3(d+ex)^4\,(c(d+ex)^n)^{-4/n}\right)\text{Subst}\left(\int e^{\frac{4x}{n}}(a+bx)^n\,dx,x,\log(c(d+ex)^n)\right)}{e^4n} + \frac{\left(3g^2(ef-dg)(d+ex)^3\,(c(d+ex)^n)^{-3/n}\right)\text{Subst}\left(\int e^{\frac{3x}{n}}(a+bx)^n\,dx,x,\log(c(d+ex)^n)\right)}{e^4n} + \frac{\left(3g(ef-dg)^2(d+ex)^2\,(c(d+ex)^n)^{-2/n}\right)\text{Subst}\left(\int e^{\frac{2x}{n}}(a+bx)^n\,dx,x,\log(c(d+ex)^n)\right)}{e^4n} + \frac{\left((ef-dg)^3(d+ex)\,(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int e^{\frac{x}{n}}(a+bx)^n\,dx,x,\log(c(d+ex)^n)\right)}{e^4n}$$

$$= \frac{4^{-1-n}e^{-\frac{4a}{bn}}g^3(d+ex)^4\,(c(d+ex)^n)^{-4/n}\,\Gamma\left(1+n,-\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)(a+b\log(c(d+ex)^n))^n\left(-\frac{a\cdots}{}\right.}{e^4} + \frac{3^{-n}e^{-\frac{3a}{bn}}g^2(ef-dg)(d+ex)^3\,(c(d+ex)^n)^{-3/n}\,\Gamma\left(1+n,-\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)(a+b\log(c(d+ex\cdots}{e^4} + \frac{3\,2^{-1-n}e^{-\frac{2a}{bn}}g(ef-dg)^2(d+ex)^2\,(c(d+ex)^n)^{-2/n}\,\Gamma\left(1+n,-\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)(a+b\log(c(d+\cdots}{e^4} + \frac{e^{-\frac{a}{bn}}(ef-dg)^3(d+ex)\,(c(d+ex)^n)^{-1/n}\,\Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^n\left(-\cdots\right.}{e^4}$$

## Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.72

$$\int (f+gx)^3\,(a+b\log(c(d+ex)^n))^n\,dx$$

$$= \frac{3^{-n}4^{-1-n}e^{-\frac{4a}{bn}}(d+ex)\,(c(d+ex)^n)^{-4/n}\left(3^ng^3(d+ex)^3\Gamma\left(1+n,-\frac{4(a+b\log(c(d+ex)^n))}{bn}\right)+2^{1+n}e^{\frac{a}{bn}}(ef-dg)\,(\cdots\right.}{}$$

```
[In] Integrate[(f + g*x)^3*(a + b*Log[c*(d + e*x)^n])^n,x]
```

```
[Out] (4^(-1 - n)*(d + e*x)*(3^n*g^3*(d + e*x)^3*Gamma[1 + n, (-4*(a + b*Log[c*(d
 + e*x)^n]))/(b*n)] + 2^(1 + n)*E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(
```

```
-1)*(2^(1 + n)*g^2*(d + e*x)^2*Gamma[1 + n, (-3*(a + b*Log[c*(d + e*x)^n]))
/(b*n)] + 3^n*E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*(3*g*(d + e*x)
*Gamma[1 + n, (-2*(a + b*Log[c*(d + e*x)^n]))/(b*n)] + 2^(1 + n)*E^(a/(b*n)
)*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*Gamma[1 + n, -((a + b*Log[c*(d + e*x)^
n])/(b*n))])))*(a + b*Log[c*(d + e*x)^n])^n)/(3^n*e^4*E^((4*a)/(b*n))*(c*(d
 + e*x)^n)^(4/n)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n)
```

## Maple [F]

$$\int \left(gx+f\right)^{3} \left(a+b\ln \left(c(ex+d)^{n}\right)\right)^{n}dx$$

[In] `int((g*x+f)^3*(a+b*ln(c*(e*x+d)^n))^n,x)`

[Out] `int((g*x+f)^3*(a+b*ln(c*(e*x+d)^n))^n,x)`

## Fricas [F]

$$\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^n \, dx = \int \left(gx+f\right)^{3}{\left(b\log \left({\left(ex+d\right)}^{n}c\right)+a\right)}^{n} \, dx$$

[In] `integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="fricas")`

[Out] `integral((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*(b*log((e*x + d)^n*c) + a)^n, x)`

## Sympy [F]

$$\int (f+gx)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^n \, dx = \int \left(a+b\log \left(c(d+ex)^{n}\right)\right)^{n} \left(f+gx\right)^{3}\, dx$$

[In] `integrate((g*x+f)**3*(a+b*ln(c*(e*x+d)**n))**n,x)`

[Out] `Integral((a + b*log(c*(d + e*x)**n))**n*(f + g*x)**3, x)`

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^n\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

## Giac [F]

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^n\,dx = \int (gx+f)^3 (b\log((ex+d)^n c)+a)^n\,dx$$

```
[In] integrate((g*x+f)^3*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^3*(b*log((e*x + d)^n*c) + a)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 (a+b\log(c(d+ex)^n))^n\,dx = \int (f+g\,x)^3\,(a+b\,\ln(c\,(d+e\,x)^n))^n\,dx$$

```
[In] int((f + g*x)^3*(a + b*log(c*(d + e*x)^n))^n,x)
```

```
[Out] int((f + g*x)^3*(a + b*log(c*(d + e*x)^n))^n, x)
```

# 3.171 $\int (f+gx)^2 (a+b\log (c(d+ex)^n))^n \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 348

$$\int (f+gx)^2 (a+b\log (c(d+ex)^n))^n \, dx$$

$$= \frac{3^{-1-n}e^{-\frac{3a}{bn}}g^2(d+ex)^3 \left(c(d+ex)^n\right)^{-3/n} \Gamma\left(1+n,-\frac{3(a+b\log(c(d+ex)^n))}{bn}\right) (a+b\log (c(d+ex)^n))^n \left(-\frac{a+b\log(c(\cdots}{bn}\cdots\right.}{e^3}$$

$$+ \frac{2^{-n}e^{-\frac{2a}{bn}}g(ef-dg)(d+ex)^2 \left(c(d+ex)^n\right)^{-2/n} \Gamma\left(1+n,-\frac{2(a+b\log(c(d+ex)^n))}{bn}\right) (a+b\log (c(d+ex)^n))^n \left(-\cdots\right.}{e^3}$$

$$+ \frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex) \left(c(d+ex)^n\right)^{-1/n} \Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right) (a+b\log (c(d+ex)^n))^n \left(-\frac{a+b\log(\cdots}{\cdots}\right.}{e^3}$$

```
[Out] 3^(-1-n)*g^2*(e*x+d)^3*GAMMA(1+n,-3*(a+b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(e
*x+d)^n))^n/e^3/exp(3*a/b/n)/((c*(e*x+d)^n)^(3/n))/(((-a-b*ln(c*(e*x+d)^n))
/b/n)^n)+g*(-d*g+e*f)*(e*x+d)^2*GAMMA(1+n,-2*(a+b*ln(c*(e*x+d)^n))/b/n)*(a+
b*ln(c*(e*x+d)^n))^n/(2^n)/e^3/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))/(((-a-b*l
n(c*(e*x+d)^n))/b/n)^n)+(-d*g+e*f)^2*(e*x+d)*GAMMA(1+n,(-a-b*ln(c*(e*x+d)^n
))/b/n)*(a+b*ln(c*(e*x+d)^n))^n/e^3/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))/(((-a-
b*ln(c*(e*x+d)^n))/b/n)^n)
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2448, 2436, 2337, 2212, 2437, 2347}

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^n \, dx$$
$$= \frac{g2^{-n}e^{-\frac{2a}{bn}}(d+ex)^2(ef-dg)\left(c(d+ex)^n\right)^{-2/n}(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}\Gamma\Big(n+1,-\frac{2(a}{}}{e^3}$$
$$+\frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg)^2\left(c(d+ex)^n\right)^{-1/n}(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}\Gamma\Big(n+1,-\frac{a+b\log}{}}{e^3}$$
$$+\frac{g^2 3^{-n-1}e^{-\frac{3a}{bn}}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}\Gamma\Big(n+1,-\frac{3(a+b\log}{}}{e^3}$$

```
[In] Int[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^n,x]
```

```
[Out] (3^(-1 - n)*g^2*(d + e*x)^3*Gamma[1 + n, (-3*(a + b*Log[c*(d + e*x)^n]))/(b
*n)]*(a + b*Log[c*(d + e*x)^n])^n)/(e^3*E^((3*a)/(b*n))*(c*(d + e*x)^n)^(3/
n)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n) + (g*(e*f - d*g)*(d + e*x)^2*Ga
mma[1 + n, (-2*(a + b*Log[c*(d + e*x)^n]))/(b*n)]*(a + b*Log[c*(d + e*x)^n]
)^n)/(2^n*e^3*E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)*(-((a + b*Log[c*(d + e*
x)^n])/(b*n)))^n) + ((e*f - d*g)^2*(d + e*x)*Gamma[1 + n, -((a + b*Log[c*(d
 + e*x)^n])/(b*n))]*(a + b*Log[c*(d + e*x)^n])^n)/(e^3*E^(a/(b*n))*(c*(d +
e*x)^n)^n^(-1)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
```

```
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left( \frac{(ef-dg)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^n}{e^2} + \frac{2g(ef-dg)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^n}{e^2} + \frac{g^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^n}{e^2} \right) dx$$

$$= \frac{g^2 \int (d+ex)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^n \, dx}{e^2} + \frac{(2g(ef-dg)) \int (d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^n \, dx}{e^2} + \frac{(ef-dg)^2 \int \left(a+b\log\left(c(d+ex)^n\right)\right)^n \, dx}{e^2}$$

$$= \frac{g^2 \text{Subst}\left(\int x^2 \left(a+b\log\left(cx^n\right)\right)^n \, dx, x, d+ex\right)}{e^3} + \frac{(2g(ef-dg))\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^n \, dx, x, d+ex\right)}{e^3} + \frac{(ef-dg)^2\text{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)^n \, dx, x, d+ex\right)}{e^3}$$

$$= \frac{\left(g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\right)\mathrm{Subst}\left(\int e^{\frac{3x}{n}}(a+bx)^n\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^3n} + \frac{\left(2g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\mathrm{Subst}\left(\int e^{\frac{2x}{n}}(a+bx)^n\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^3n} + \frac{\left((ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\mathrm{Subst}\left(\int e^{\frac{x}{n}}(a+bx)^n\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^3n}$$

$$= \frac{3^{-1-n}e^{-\frac{3a}{bn}}g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\Gamma\left(1+n,-\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^n\left(-\frac{a-}{}\right.}{e^3} + \frac{2^{-n}e^{-\frac{2a}{bn}}g(ef-dg)(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\Gamma\left(1+n,-\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)\left(a+b\log\left(c(d+ex)\right.\right.}{e^3} + \frac{e^{-\frac{a}{bn}}(ef-dg)^2(d+ex)\left(c(d+ex)^n\right)^{-1/n}\Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^n\left(-\right.}{e^3}$$

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.75

$$\int (f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx = \frac{2^{-n}3^{-1-n}e^{-\frac{3a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-3/n}\left(2^ng^2(d+ex)^2\Gamma\left(1+n,-\frac{3(a+b\log(c(d+ex)^n))}{bn}\right)+3^{1+n}e^{\frac{a}{bn}}(ef-dg)\left(\right.\right.}{}$$

```
[In] Integrate[(f + g*x)^2*(a + b*Log[c*(d + e*x)^n])^n,x]
```

```
[Out] (3^(-1 - n)*(d + e*x)*(2^n*g^2*(d + e*x)^2*Gamma[1 + n, (-3*(a + b*Log[c*(d
 + e*x)^n]))/(b*n)] + 3^(1 + n)*E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(
-1)*(g*(d + e*x)*Gamma[1 + n, (-2*(a + b*Log[c*(d + e*x)^n]))/(b*n)] + 2^n*
E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*Gamma[1 + n, -((a + b*Log[c*
(d + e*x)^n])/(b*n))]))*(a + b*Log[c*(d + e*x)^n])^n)/(2^n*e^3*E^((3*a)/(b*
n))*(c*(d + e*x)^n)^(3/n)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n)
```

## Maple [F]

$$\int (gx+f)^2 (a+b\ln(c(ex+d)^n))^n dx$$

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^n,x)
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x+d)^n))^n,x)
```

## Fricas [F]

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^n \, dx = \int (gx+f)^2 (b\log((ex+d)^n c)+a)^n \, dx$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*(b*log((e*x + d)^n*c) + a)^n, x)
```

## Sympy [F]

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^n \, dx = \int (a+b\log(c(d+ex)^n))^n (f+gx)^2 \, dx$$

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x+d)**n))**n,x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**n*(f + g*x)**2, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^2 (a+b\log(c(d+ex)^n))^n \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

## Giac [F]

$$\int (f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx=\int (gx+f)^2(b\log\left((ex+d)^nc\right)+a)^n\,dx$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="giac")
[Out] integrate((g*x + f)^2*(b*log((e*x + d)^n*c) + a)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx=\int (f+g\,x)^2\,\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^n\,dx$$

```
[In] int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^n,x)
[Out] int((f + g*x)^2*(a + b*log(c*(d + e*x)^n))^n, x)
```

# 3.172 $\int (f+gx) (a+b\log (c(d+ex)^n))^n \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 225

$$\int (f+gx) (a+b\log (c(d+ex)^n))^n \, dx$$
$$= \frac{2^{-1-n}e^{-\frac{2a}{bn}}g(d+ex)^2 (c(d+ex)^n)^{-2/n}\Gamma\left(1+n,-\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)(a+b\log (c(d+ex)^n))^n \left(-\frac{a+b\log(c(d\cdots}{bn}\right.}{e^2}$$
$$+\frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex) (c(d+ex)^n)^{-1/n}\Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log (c(d+ex)^n))^n \left(-\frac{a+b\log(c\cdots}{b\cdots}\right.}{e^2}$$

```
[Out] 2^(-1-n)*g*(e*x+d)^2*GAMMA(1+n,-2*(a+b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(e*x
+d)^n))^n/e^2/exp(2*a/b/n)/((c*(e*x+d)^n)^(2/n))/(((-a-b*ln(c*(e*x+d)^n))/b
/n)^n)+(-d*g+e*f)*(e*x+d)*GAMMA(1+n,(-a-b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(
e*x+d)^n))^n/e^2/exp(a/b/n)/((c*(e*x+d)^n)^(1/n))/(((-a-b*ln(c*(e*x+d)^n))/
b/n)^n)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2448, 2436, 2337, 2212, 2437, 2347}

$$\int (f+gx) (a+b\log (c(d+ex)^n))^n \, dx$$
$$= \frac{e^{-\frac{a}{bn}}(d+ex)(ef-dg) (c(d+ex)^n)^{-1/n}(a+b\log (c(d+ex)^n))^n \left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\log(c\cdots}{b\cdots}\right.}{e^2}$$
$$+\frac{g2^{-n-1}e^{-\frac{2a}{bn}}(d+ex)^2 (c(d+ex)^n)^{-2/n}(a+b\log (c(d+ex)^n))^n \left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\log\cdots}{\cdots}\right.}{e^2}$$

```
[In] Int[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^n,x]
```

```
[Out] (2^(-1 - n)*g*(d + e*x)^2*Gamma[1 + n, (-2*(a + b*Log[c*(d + e*x)^n]))/(b*n
)]*(a + b*Log[c*(d + e*x)^n])^n)/(e^2*E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)
*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^n) + ((e*f - d*g)*(d + e*x)*Gamma[1
+ n, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(a + b*Log[c*(d + e*x)^n])^n)/(e^
2*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(-((a + b*Log[c*(d + e*x)^n])/(b*n)))^
n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
```

```
d*g, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(ef-dg)\left(a+b\log\left(c(d+ex)^n\right)\right)^n}{e}+\frac{g(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^n}{e}\right)dx$$

$$=\frac{g\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx}{e}+\frac{(ef-dg)\int\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx}{e}$$

$$=\frac{g\text{Subst}\left(\int x(a+b\log(cx^n))^n\,dx,x,d+ex\right)}{e^2}+\frac{(ef-dg)\text{Subst}\left(\int(a+b\log(cx^n))^n\,dx,x,d+ex\right)}{e^2}$$

$$=\frac{\left(g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\right)\text{Subst}\left(\int e^{\frac{2x}{n}}(a+bx)^n\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^2n}+\frac{\left((ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{\frac{x}{n}}(a+bx)^n\,dx,x,\log\left(c(d+ex)^n\right)\right)}{e^2n}$$

$$=\frac{2^{-1-n}e^{-\frac{2a}{bn}}g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\Gamma\left(1+n,-\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^n\left(-\frac{a}{\cdots}\right.}{e^2}+\frac{e^{-\frac{a}{bn}}(ef-dg)(d+ex)\left(c(d+ex)^n\right)^{-1/n}\Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^n\left(-\cdots\right.}{e^2}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.80

$$\int(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx$$

$$=\frac{2^{-1-n}e^{-\frac{2a}{bn}}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(g(d+ex)\Gamma\left(1+n,-\frac{2(a+b\log(c(d+ex)^n))}{bn}\right)+2^{1+n}e^{\frac{a}{bn}}(ef-dg)\left(c(d+e\cdots\right.\right.}{e^2}$$

```
[In] Integrate[(f + g*x)*(a + b*Log[c*(d + e*x)^n])^n,x]

[Out] (2^(-1 - n)*(d + e*x)*(g*(d + e*x)*Gamma[1 + n, (-2*(a + b*Log[c*(d + e*x)^
n]))/(b*n)] + 2^(1 + n)*E^(a/(b*n))*(e*f - d*g)*(c*(d + e*x)^n)^n^(-1)*Gamm
a[1 + n, -((a + b*Log[c*(d + e*x)^n])/(b*n))])*(a + b*Log[c*(d + e*x)^n])^n
)/(e^2*E^((2*a)/(b*n))*(c*(d + e*x)^n)^(2/n)*(-((a + b*Log[c*(d + e*x)^n])/
(b*n)))^n)
```

## Maple [F]

$$\int (gx + f)(a + b\ln (c(ex + d)^n))^n dx$$

```
[In] int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^n,x)
[Out] int((g*x+f)*(a+b*ln(c*(e*x+d)^n))^n,x)
```

## Fricas [F]

$$\int (f + gx)(a + b\log (c(d + ex)^n))^n\, dx = \int (gx + f)(b\log ((ex + d)^n c) + a)^n\, dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="fricas")
[Out] integral((g*x + f)*(b*log((e*x + d)^n*c) + a)^n, x)
```

## Sympy [F]

$$\int (f + gx)(a + b\log (c(d + ex)^n))^n\, dx = \int (a + b\log (c(d + ex)^n))^n (f + gx)\, dx$$

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x+d)**n))**n,x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**n*(f + g*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (f + gx)(a + b\log (c(d + ex)^n))^n\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

## Giac [F]

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx=\int (gx+f)(b\log\left((ex+d)^n c\right)+a)^n\,dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x+d)^n))^n,x, algorithm="giac")
[Out] integrate((g*x + f)*(b*log((e*x + d)^n*c) + a)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^n\,dx=\int (f+g\,x)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^n\,dx$$

```
[In] int((f + g*x)*(a + b*log(c*(d + e*x)^n))^n,x)
[Out] int((f + g*x)*(a + b*log(c*(d + e*x)^n))^n, x)
```

# 3.173 $\int (a+b\log(c(d+ex)^n))^n\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 103

$$\int (a+b\log(c(d+ex)^n))^n\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}\Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}}{e}$$

```
[Out] (e*x+d)*GAMMA(1+n,(-a-b*ln(c*(e*x+d)^n))/b/n)*(a+b*ln(c*(e*x+d)^n))^n/e/exp
(a/b/n)/((c*(e*x+d)^n)^(1/n))/(((-a-b*ln(c*(e*x+d)^n))/b/n)^n)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2436, 2337, 2212}

$$\int (a+b\log(c(d+ex)^n))^n\,dx = \frac{e^{-\frac{a}{bn}}(d+ex)(c(d+ex)^n)^{-1/n}(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{e}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^n,x]
```

```
[Out] ((d + e*x)*Gamma[1 + n, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(a + b*Log[c*(
d + e*x)^n])^n)/(e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(-((a + b*Log[c*(d +
e*x)^n])/(b*n)))^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+b\log(cx^n))^n\,dx, x, d+ex\right)}{e}$$

$$= \frac{\left((d+ex)\,(c(d+ex)^n)^{-1/n}\right)\text{Subst}\left(\int e^{\frac{x}{n}}(a+bx)^n\,dx, x, \log(c(d+ex)^n)\right)}{en}$$

$$= \frac{e^{-\frac{a}{bn}}(d+ex)\,(c(d+ex)^n)^{-1/n}\,\Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(c(d+e}{bn}\right.}{e}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

$$\int (a+b\log(c(d+ex)^n))^n\,dx$$
$$= \frac{e^{-\frac{a}{bn}}(d+ex)\,(c(d+ex)^n)^{-1/n}\,\Gamma\left(1+n,-\frac{a+b\log(c(d+ex)^n)}{bn}\right)(a+b\log(c(d+ex)^n))^n\left(-\frac{a+b\log(c(d+ex)^n)}{bn}\right)^{-n}}{e}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^n,x]
```

```
[Out] ((d + e*x)*Gamma[1 + n, -((a + b*Log[c*(d + e*x)^n])/(b*n))]*(a + b*Log[c*(
d + e*x)^n])^n)/(e*E^(a/(b*n))*(c*(d + e*x)^n)^n^(-1)*(-((a + b*Log[c*(d +
e*x)^n])/(b*n)))^n)
```

## Maple [F]

$$\int (a+b\ln(c(ex+d)^n))^n dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^n,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^n,x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int (a+b\log(c(d+ex)^n))^n\,dx = \frac{e^{\left(-\frac{bn^2\log\left(-\frac{1}{bn}\right)+b\log(c)+a}{bn}\right)}\Gamma\left(n+1,-\frac{bn\log(ex+d)+b\log(c)+a}{bn}\right)}{e}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^n,x, algorithm="fricas")
[Out] e^(-(b*n^2*log(-1/(b*n)) + b*log(c) + a)/(b*n))*gamma(n + 1, -(b*n*log(e*x
+ d) + b*log(c) + a)/(b*n))/e
```

## Sympy [F]

$$\int (a+b\log(c(d+ex)^n))^n\,dx = \int (a+b\log(c(d+ex)^n))^n\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**n,x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**n, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (a+b\log(c(d+ex)^n))^n\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^n,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

## Giac [F]

$$\int (a+b\log(c(d+ex)^n))^n\,dx=\int (b\log((ex+d)^nc)+a)^n\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^n,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d+ex)^n))^n\,dx=\int \left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^n\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^n,x)
[Out] int((a + b*log(c*(d + e*x)^n))^n, x)
```

# 3.174 $\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx = \operatorname{Int}\left(\frac{(a+b\log(c(d+ex)^n))^n}{f+gx},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(e*x+d)^n))^n/(g*x+f),x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^n/(f + g*x),x]
[Out] Defer[Int][(a + b*Log[c*(d + e*x)^n])^n/(f + g*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx=\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^n/(f + g*x),x]
[Out] Integrate[(a + b*Log[c*(d + e*x)^n])^n/(f + g*x), x]
```

## Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(ex+d)^n))^n}{gx+f}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^n/(g*x+f),x)
[Out] int((a+b*ln(c*(e*x+d)^n))^n/(g*x+f),x)
```

## Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx=\int \frac{(b\log((ex+d)^n c)+a)^n}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^n/(g*x+f),x, algorithm="fricas")
[Out] integral((b*log((e*x + d)^n*c) + a)^n/(g*x + f), x)
```

## Sympy [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx=\int \frac{(a+b\log(c(d+ex)^n))^n}{f+gx}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**n/(g*x+f),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**n/(f + g*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\log(c(d + ex)^n))^n}{f + gx} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^n/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

## Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a + b\log(c(d + ex)^n))^n}{f + gx} \, dx = \int \frac{(b\log((ex + d)^n c) + a)^n}{gx + f} \, dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^n/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^n/(g*x + f), x)
```

## Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a + b\log(c(d + ex)^n))^n}{f + gx} \, dx = \int \frac{(a + b\,\ln(c\,(d + e\,x)^n))^n}{f + g\,x} \, dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^n/(f + g*x),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^n/(f + g*x), x)
```

# 3.175 $\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 315

$$\begin{aligned}\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx &= -\frac{4bi(fh-ei)^3x}{df^4}-\frac{3bi^2(fh-ei)^2(e+fx)^2}{2df^5}\\ &\quad-\frac{4bi^3(fh-ei)(e+fx)^3}{9df^5}-\frac{bi^4(e+fx)^4}{16df^5}\\ &\quad-\frac{b(fh-ei)^4\log^2(e+fx)}{2df^5}\\ &\quad+\frac{4i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5}\\ &\quad+\frac{3i^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{df^5}\\ &\quad+\frac{4i^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{3df^5}\\ &\quad+\frac{i^4(e+fx)^4(a+b\log(c(e+fx)))}{4df^5}\\ &\quad+\frac{(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{df^5}\end{aligned}$$

```
[Out] -4*b*i*(-e*i+f*h)^3*x/d/f^4-3/2*b*i^2*(-e*i+f*h)^2*(f*x+e)^2/d/f^5-4/9*b*i^
3*(-e*i+f*h)*(f*x+e)^3/d/f^5-1/16*b*i^4*(f*x+e)^4/d/f^5-1/2*b*(-e*i+f*h)^4*
ln(f*x+e)^2/d/f^5+4*i*(-e*i+f*h)^3*(f*x+e)*(a+b*ln(c*(f*x+e)))/d/f^5+3*i^2*
(-e*i+f*h)^2*(f*x+e)^2*(a+b*ln(c*(f*x+e)))/d/f^5+4/3*i^3*(-e*i+f*h)*(f*x+e)
^3*(a+b*ln(c*(f*x+e)))/d/f^5+1/4*i^4*(f*x+e)^4*(a+b*ln(c*(f*x+e)))/d/f^5+(-
e*i+f*h)^4*ln(f*x+e)*(a+b*ln(c*(f*x+e)))/d/f^5
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2458, 12, 45, 2372, 2338}

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{4i^3(e+fx)^3(fh-ei)(a+b\log(c(e+fx)))}{3df^5} + \frac{3i^2(e+fx)^2(fh-ei)^2(a+b\log(c(e+fx)))}{df^5} + \frac{(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{df^5} + \frac{4i(e+fx)(fh-ei)^3(a+b\log(c(e+fx)))}{df^5} + \frac{i^4(e+fx)^4(a+b\log(c(e+fx)))}{4df^5} - \frac{4bi^3(e+fx)^3(fh-ei)}{9df^5} - \frac{3bi^2(e+fx)^2(fh-ei)^2}{2df^5} - \frac{b(fh-ei)^4\log^2(e+fx)}{2df^5} - \frac{bi^4(e+fx)^4}{16df^5} - \frac{4bix(fh-ei)^3}{df^4}$$

```
[In] Int[((h + i*x)^4*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

```
[Out] (-4*b*i*(f*h - e*i)^3*x)/(d*f^4) - (3*b*i^2*(f*h - e*i)^2*(e + f*x)^2)/(2*d
*f^5) - (4*b*i^3*(f*h - e*i)*(e + f*x)^3)/(9*d*f^5) - (b*i^4*(e + f*x)^4)/(
16*d*f^5) - (b*(f*h - e*i)^4*Log[e + f*x]^2)/(2*d*f^5) + (4*i*(f*h - e*i)^3
*(e + f*x)*(a + b*Log[c*(e + f*x)]))/(d*f^5) + (3*i^2*(f*h - e*i)^2*(e + f*
x)^2*(a + b*Log[c*(e + f*x)]))/(d*f^5) + (4*i^3*(f*h - e*i)*(e + f*x)^3*(a
+ b*Log[c*(e + f*x)]))/(3*d*f^5) + (i^4*(e + f*x)^4*(a + b*Log[c*(e + f*x)]
))/(4*d*f^5) + ((f*h - e*i)^4*Log[e + f*x]*(a + b*Log[c*(e + f*x)]))/(d*f^5
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^4(a+b\log(cx))}{dx}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^4(a+b\log(cx))}{x}\,dx, x, e+fx\right)}{df}$$

$$= \frac{4i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5} + \frac{3i^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{df^5} + \frac{4i^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{3df^5} + \frac{i^4(e+fx)^4(a+b\log(c(e+fx)))}{4df^5} + \frac{(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{df^5} - \frac{b\text{Subst}\left(\int \frac{48i(fh-ei)^3+36i^2(fh-ei)^2x+16i^3(fh-ei)x^2+3i^4x^3+\frac{12(fh-ei)^4\log(x)}{x}}{12f^4}\,dx, x, e+fx\right)}{df}$$

$$= \frac{4i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5} + \frac{3i^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{df^5} + \frac{4i^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{3df^5} + \frac{i^4(e+fx)^4(a+b\log(c(e+fx)))}{4df^5} + \frac{(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{df^5} - \frac{b\text{Subst}\left(\int \left(48i(fh-ei)^3+36i^2(fh-ei)^2x+16i^3(fh-ei)x^2+3i^4x^3+\frac{12(fh-ei)^4\log(x)}{x}\right)dx, x, e\right.}{12df^5}$$

$$= -\frac{4bi(fh-ei)^3x}{df^4} - \frac{3bi^2(fh-ei)^2(e+fx)^2}{2df^5} - \frac{4bi^3(fh-ei)(e+fx)^3}{9df^5} - \frac{bi^4(e+fx)^4}{16df^5} + \frac{4i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5} + \frac{3i^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{df^5} + \frac{4i^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{3df^5} + \frac{i^4(e+fx)^4(a+b\log(c(e+fx)))}{4df^5} + \frac{(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{df^5} - \frac{(b(fh-ei)^4)\,\text{Subst}\left(\int \frac{\log(x)}{x}dx, x, e+fx\right)}{df^5}$$

$$= -\frac{4bi(fh-ei)^3x}{df^4} - \frac{3bi^2(fh-ei)^2(e+fx)^2}{2df^5} - \frac{4bi^3(fh-ei)(e+fx)^3}{9df^5} - \frac{bi^4(e+fx)^4}{16df^5} - \frac{b(fh-ei)^4\log^2(e+fx)}{2df^5} + \frac{4i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5} + \frac{3i^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{df^5} + \frac{4i^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{3df^5} + \frac{i^4(e+fx)^4(a+b\log(c(e+fx)))}{4df^5} + \frac{(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{df^5}$$

## Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.87

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$= \frac{72a^2f^4h^4-288a^2ef^3h^3i+432a^2e^2f^2h^2i^2-288a^2e^3fhi^3+72a^2e^4i^4+576abf^4h^3ix-576b^2f^4h^3ix-864}{}$$

```
[In] Integrate[((h + i*x)^4*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

```
[Out] (72*a^2*f^4*h^4 - 288*a^2*e*f^3*h^3*i + 432*a^2*e^2*f^2*h^2*i^2 - 288*a^2*e
^3*f*h*i^3 + 72*a^2*e^4*i^4 + 576*a*b*f^4*h^3*i*x - 576*b^2*f^4*h^3*i*x - 8
64*a*b*e*f^3*h^2*i^2*x + 1296*b^2*e*f^3*h^2*i^2*x + 576*a*b*e^2*f^2*h*i^3*x
 - 1056*b^2*e^2*f^2*h*i^3*x - 144*a*b*e^3*f*i^4*x + 300*b^2*e^3*f*i^4*x + 4
32*a*b*f^4*h^2*i^2*x^2 - 216*b^2*f^4*h^2*i^2*x^2 - 288*a*b*e*f^3*h*i^3*x^2
+ 240*b^2*e*f^3*h*i^3*x^2 + 72*a*b*e^2*f^2*i^4*x^2 - 78*b^2*e^2*f^2*i^4*x^2
 + 192*a*b*f^4*h*i^3*x^3 - 64*b^2*f^4*h*i^3*x^3 - 48*a*b*e*f^3*i^4*x^3 + 28
*b^2*e*f^3*i^4*x^3 + 36*a*b*f^4*i^4*x^4 - 9*b^2*f^4*i^4*x^4 - 12*b^2*e^2*i^
2*(36*f^2*h^2 - 40*e*f*h*i + 13*e^2*i^2)*Log[e + f*x] + 12*b*(12*a*(f*h - e
*i)^4 + b*i*(-12*e^4*i^3 - 12*e^3*f*i^2*(-4*h + i*x) + 6*e^2*f^2*i*(-12*h^2
 + 8*h*i*x + i^2*x^2) + 4*e*f^3*(12*h^3 - 18*h^2*i*x - 6*h*i^2*x^2 - i^3*x^
3) + f^4*x*(48*h^3 + 36*h^2*i*x + 16*h*i^2*x^2 + 3*i^3*x^3)))*Log[c*(e + f*
x)] + 72*b^2*(f*h - e*i)^4*Log[c*(e + f*x)]^2)/(144*b*d*f^5)
```

## Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.76

| method | result |
|---|---|
| norman | $\frac{(12ae^4i^4-48ae^3fhi^3+72ae^2f^2h^2i^2-48aef^3h^3i+12ah^4f^4-25be^4i^4+88be^3fhi^3-108be^2f^2h^2i^2+48bef^3h^3i)\ln(c(fx}{12df^5}$ |
| parts | $\frac{a\left(\frac{i\left(\frac{f^3i^3x^4}{4}+\frac{((-ei+2fh)f^2i^2+2f^3i^2h)x^3}{3}+\frac{(2(-ei+2fh)f^2hi+fi(e^2i^2-2efhi+2f^2h^2))x^2}{2}+x(-ei+2fh)(e^2i^2-2efhi+2f^2h^2)\right.}{f^4}\right.}{d}$ |
| risch | $-\frac{9\ln(fx+e)be^2h^2i^2}{df^3}+\frac{4\ln(fx+e)beh^3i}{df^2}-\frac{2b\ln(c(fx+e))^2e^3hi^3}{df^4}+\frac{3b\ln(c(fx+e))^2e^2h^2i^2}{df^3}-\frac{2b\ln(c(fx+e))^2eh}{df^2}$ |
| parallelrisch | $144\ln(c(fx+e))ae^4i^4+144\ln(c(fx+e))af^4h^4+1872be^3fhi^3-2376be^2f^2h^2i^2+1152bef^3h^3i-288x^2\ln(c(fx+e))bef^3h$ |
| derivativedivides | $-\frac{4cae^3hi^3\ln(cfx+ce)}{f^3d}-\frac{4caeh^3i\ln(cfx+ce)}{fd}+\frac{6cae^2h^2i^2\ln(cfx+ce)}{f^2d}-\frac{6aehi^3(cfx+ce)^2}{cf^3d}-\frac{12behi^3\left(\frac{(cfx+ce)^2\ln(cfx+ce)}{2}-\frac{(cfx}{}\right.}{cf^3d}$ |
| default | $-\frac{4cae^3hi^3\ln(cfx+ce)}{f^3d}-\frac{4caeh^3i\ln(cfx+ce)}{fd}+\frac{6cae^2h^2i^2\ln(cfx+ce)}{f^2d}-\frac{6aehi^3(cfx+ce)^2}{cf^3d}-\frac{12behi^3\left(\frac{(cfx+ce)^2\ln(cfx+ce)}{2}-\frac{(cfx}{}\right.}{cf^3d}$ |

```
[In] int((i*x+h)^4*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(12*a*e^4*i^4-48*a*e^3*f*h*i^3+72*a*e^2*f^2*h^2*i^2-48*a*e*f^3*h^3*i+1
2*a*f^4*h^4-25*b*e^4*i^4+88*b*e^3*f*h*i^3-108*b*e^2*f^2*h^2*i^2+48*b*e*f^3*
h^3*i)/d/f^5*ln(c*(f*x+e))+1/2*b*(e^4*i^4-4*e^3*f*h*i^3+6*e^2*f^2*h^2*i^2-4
*e*f^3*h^3*i+f^4*h^4)/d/f^5*ln(c*(f*x+e))^2-1/12*i*(12*a*e^3*i^3-48*a*e^2*f
*h*i^2+72*a*e*f^2*h^2*i-48*a*f^3*h^3-25*b*e^3*i^3+88*b*e^2*f*h*i^2-108*b*e*
f^2*h^2*i+48*b*f^3*h^3)/d/f^4*x+1/24*i^2*(12*a*e^2*i^2-48*a*e*f*h*i+72*a*f^
2*h^2-13*b*e^2*i^2+40*b*e*f*h*i-36*b*f^2*h^2)/d/f^3*x^2-1/36*i^3*(12*a*e*i-
48*a*f*h-7*b*e*i+16*b*f*h)/d/f^2*x^3+1/16*i^4*(4*a-b)/d/f*x^4+1/4*b*i^4/d/f
*x^4*ln(c*(f*x+e))-b*i*(e^3*i^3-4*e^2*f*h*i^2+6*e*f^2*h^2*i-4*f^3*h^3)/f^4/
d*x*ln(c*(f*x+e))+1/2*b*i^2*(e^2*i^2-4*e*f*h*i+6*f^2*h^2)/d/f^3*x^2*ln(c*(f
*x+e))-1/3*b*i^3*(e*i-4*f*h)/d/f^2*x^3*ln(c*(f*x+e))
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.52

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$=\frac{9\,(4\,a-b)f^4i^4x^4+4\,(16\,(3\,a-b)f^4hi^3-(12\,a-7\,b)ef^3i^4)x^3+6\,(36\,(2\,a-b)f^4h^2i^2-8\,(6\,a-5\,b)ef^3hi^3}{}$$

```
[In] integrate((i*x+h)^4*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="fricas")
```

```
[Out] 1/144*(9*(4*a - b)*f^4*i^4*x^4 + 4*(16*(3*a - b)*f^4*h*i^3 - (12*a - 7*b)*e
*f^3*i^4)*x^3 + 6*(36*(2*a - b)*f^4*h^2*i^2 - 8*(6*a - 5*b)*e*f^3*h*i^3 + (
```

```
12*a - 13*b)*e^2*f^2*i^4)*x^2 + 72*(b*f^4*h^4 - 4*b*e*f^3*h^3*i + 6*b*e^2*f
^2*h^2*i^2 - 4*b*e^3*f*h*i^3 + b*e^4*i^4)*log(c*f*x + c*e)^2 + 12*(48*(a -
b)*f^4*h^3*i - 36*(2*a - 3*b)*e*f^3*h^2*i^2 + 8*(6*a - 11*b)*e^2*f^2*h*i^3
- (12*a - 25*b)*e^3*f*i^4)*x + 12*(3*b*f^4*i^4*x^4 + 12*a*f^4*h^4 - 48*(a -
 b)*e*f^3*h^3*i + 36*(2*a - 3*b)*e^2*f^2*h^2*i^2 - 8*(6*a - 11*b)*e^3*f*h*i
^3 + (12*a - 25*b)*e^4*i^4 + 4*(4*b*f^4*h*i^3 - b*e*f^3*i^4)*x^3 + 6*(6*b*f
^4*h^2*i^2 - 4*b*e*f^3*h*i^3 + b*e^2*f^2*i^4)*x^2 + 12*(4*b*f^4*h^3*i - 6*b
*e*f^3*h^2*i^2 + 4*b*e^2*f^2*h*i^3 - b*e^3*f*i^4)*x)*log(c*f*x + c*e))/(d*f
^5)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(289) = 578$.

Time = 0.84 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.17

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$= x^4\left(\frac{ai^4}{4df}-\frac{bi^4}{16df}\right)+x^3\left(-\frac{aei^4}{3df^2}+\frac{4ahi^3}{3df}+\frac{7bei^4}{36df^2}-\frac{4bhi^3}{9df}\right)$$
$$+x^2\left(\frac{ae^2i^4}{2df^3}-\frac{2aehi^3}{df^2}+\frac{3ah^2i^2}{df}-\frac{13be^2i^4}{24df^3}+\frac{5behi^3}{3df^2}-\frac{3bh^2i^2}{2df}\right)$$
$$+x\left(-\frac{ae^3i^4}{df^4}+\frac{4ae^2hi^3}{df^3}-\frac{6aeh^2i^2}{df^2}+\frac{4ah^3i}{df}+\frac{25be^3i^4}{12df^4}-\frac{22be^2hi^3}{3df^3}+\frac{9beh^2i^2}{df^2}-\frac{4bh^3i}{df}\right)$$
$$+\frac{(-12be^3i^4x+48be^2fhi^3x+6be^2fi^4x^2-72bef^2h^2i^2x-24bef^2hi^3x^2-4bef^2i^4x^3+48bf^3h^3ix+36bf^3}{12df^4}$$
$$+\frac{(be^4i^4-4be^3fhi^3+6be^2f^2h^2i^2-4bef^3h^3i+bf^4h^4)\log{(c(e+fx))}^2}{2df^5}$$
$$+\frac{(12ae^4i^4-48ae^3fhi^3+72ae^2f^2h^2i^2-48aef^3h^3i+12af^4h^4-25be^4i^4+88be^3fhi^3-108be^2f^2h^2i^2+}{12df^5}$$

```
[In] integrate((i*x+h)**4*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x)
```

```
[Out] x**4*(a*i**4/(4*d*f) - b*i**4/(16*d*f)) + x**3*(-a*e*i**4/(3*d*f**2) + 4*a*
h*i**3/(3*d*f) + 7*b*e*i**4/(36*d*f**2) - 4*b*h*i**3/(9*d*f)) + x**2*(a*e**
2*i**4/(2*d*f**3) - 2*a*e*h*i**3/(d*f**2) + 3*a*h**2*i**2/(d*f) - 13*b*e**2
*i**4/(24*d*f**3) + 5*b*e*h*i**3/(3*d*f**2) - 3*b*h**2*i**2/(2*d*f)) + x*(-
a*e**3*i**4/(d*f**4) + 4*a*e**2*h*i**3/(d*f**3) - 6*a*e*h**2*i**2/(d*f**2)
+ 4*a*h**3*i/(d*f) + 25*b*e**3*i**4/(12*d*f**4) - 22*b*e**2*h*i**3/(3*d*f**
3) + 9*b*e*h**2*i**2/(d*f**2) - 4*b*h**3*i/(d*f)) + (-12*b*e**3*i**4*x + 48
*b*e**2*f*h*i**3*x + 6*b*e**2*f*i**4*x**2 - 72*b*e*f**2*h**2*i**2*x - 24*b*
e*f**2*h*i**3*x**2 - 4*b*e*f**2*i**4*x**3 + 48*b*f**3*h**3*i*x + 36*b*f**3*
h**2*i**2*x**2 + 16*b*f**3*h*i**3*x**3 + 3*b*f**3*i**4*x**4)*log(c*(e + f*x
))/(12*d*f**4) + (b*e**4*i**4 - 4*b*e**3*f*h*i**3 + 6*b*e**2*f**2*h**2*i**2
```

```
 - 4*b*e*f**3*h**3*i + b*f**4*h**4)*log(c*(e + f*x))**2/(2*d*f**5) + (12*a*
e**4*i**4 - 48*a*e**3*f*h*i**3 + 72*a*e**2*f**2*h**2*i**2 - 48*a*e*f**3*h**
3*i + 12*a*f**4*h**4 - 25*b*e**4*i**4 + 88*b*e**3*f*h*i**3 - 108*b*e**2*f**
2*h**2*i**2 + 48*b*e*f**3*h**3*i)*log(e + f*x)/(12*d*f**5)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(303) = 606$.

Time = 0.23 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.40

$$\begin{aligned}
&\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx = 4\,bh^3i\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)\log(cfx+ce) \\
&+\frac{1}{12}\,bi^4\left(\frac{12\,e^4\log(fx+e)}{df^5}+\frac{3\,f^3x^4-4\,ef^2x^3+6\,e^2fx^2-12\,e^3x}{df^4}\right)\log(cfx+ce) \\
&-\frac{2}{3}\,bhi^3\left(\frac{6\,e^3\log(fx+e)}{df^4}-\frac{2\,f^2x^3-3\,efx^2+6\,e^2x}{df^3}\right)\log(cfx+ce) \\
&+3\,bh^2i^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)\log(cfx+ce) \\
&-\frac{1}{2}\,bh^4\left(\frac{2\,\log(cfx+ce)\log(dfx+de)}{df}-\frac{\log(fx+e)^2+2\,\log(fx+e)\log(c)}{df}\right) \\
&+4\,ah^3i\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right) \\
&+\frac{1}{12}\,ai^4\left(\frac{12\,e^4\log(fx+e)}{df^5}+\frac{3\,f^3x^4-4\,ef^2x^3+6\,e^2fx^2-12\,e^3x}{df^4}\right) \\
&-\frac{2}{3}\,ahi^3\left(\frac{6\,e^3\log(fx+e)}{df^4}-\frac{2\,f^2x^3-3\,efx^2+6\,e^2x}{df^3}\right) \\
&+3\,ah^2i^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)+\frac{bh^4\log(cfx+ce)\log(dfx+de)}{df} \\
&+\frac{ah^4\log(dfx+de)}{df}+\frac{2\left(e\log(fx+e)^2-2\,fx+2\,e\log(fx+e)\right)bh^3i}{df^2} \\
&-\frac{3\left(f^2x^2+2\,e^2\log(fx+e)^2-6\,efx+6\,e^2\log(fx+e)\right)bh^2i^2}{2\,df^3} \\
&-\frac{\left(4\,f^3x^3-15\,ef^2x^2-18\,e^3\log(fx+e)^2+66\,e^2fx-66\,e^3\log(fx+e)\right)bhi^3}{9\,df^4} \\
&-\frac{\left(9\,f^4x^4-28\,ef^3x^3+78\,e^2f^2x^2+72\,e^4\log(fx+e)^2-300\,e^3fx+300\,e^4\log(fx+e)\right)bi^4}{144\,df^5}
\end{aligned}$$

```
[In] integrate((i*x+h)^4*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="maxima")
```

```
[Out] 4*b*h^3*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) + 1/12*b*i^4*
(12*e^4*log(f*x + e)/(d*f^5) + (3*f^3*x^4 - 4*e*f^2*x^3 + 6*e^2*f*x^2 - 12*
```

```
e^3*x)/(d*f^4))*log(c*f*x + c*e) - 2/3*b*h*i^3*(6*e^3*log(f*x + e)/(d*f^4)
- (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3))*log(c*f*x + c*e) + 3*b*h^2*i^2
*(2*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2))*log(c*f*x + c*e) -
1/2*b*h^4*(2*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2 + 2*
log(f*x + e)*log(c))/(d*f)) + 4*a*h^3*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))
+ 1/12*a*i^4*(12*e^4*log(f*x + e)/(d*f^5) + (3*f^3*x^4 - 4*e*f^2*x^3 + 6*e^
2*f*x^2 - 12*e^3*x)/(d*f^4)) - 2/3*a*h*i^3*(6*e^3*log(f*x + e)/(d*f^4) - (2
*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3)) + 3*a*h^2*i^2*(2*e^2*log(f*x + e)/
(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2)) + b*h^4*log(c*f*x + c*e)*log(d*f*x + d*e
)/(d*f) + a*h^4*log(d*f*x + d*e)/(d*f) + 2*(e*log(f*x + e)^2 - 2*f*x + 2*e*
log(f*x + e))*b*h^3*i/(d*f^2) - 3/2*(f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f
*x + 6*e^2*log(f*x + e))*b*h^2*i^2/(d*f^3) - 1/9*(4*f^3*x^3 - 15*e*f^2*x^2
- 18*e^3*log(f*x + e)^2 + 66*e^2*f*x - 66*e^3*log(f*x + e))*b*h*i^3/(d*f^4)
 - 1/144*(9*f^4*x^4 - 28*e*f^3*x^3 + 78*e^2*f^2*x^2 + 72*e^4*log(f*x + e)^2
 - 300*e^3*f*x + 300*e^4*log(f*x + e))*b*i^4/(d*f^5)
```

## Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.82

$$
\begin{aligned}
\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx &= \frac{(4\,ai^4-bi^4)x^4}{16\,df} \\
&+ \frac{1}{12}\left(\frac{3\,bi^4x^4}{df} + \frac{4\,(4\,bfhi^3-bei^4)x^3}{df^2} + \frac{6\,(6\,bf^2h^2i^2-4\,befhi^3+be^2i^4)x^2}{df^3} + \frac{12\,(4\,bf^3h^3i-6\,bef^2h^2i^2+}{df^4}\right. \\
&\qquad + ce) + \frac{(48\,afhi^3-16\,bfhi^3-12\,aei^4+7\,bei^4)x^3}{36\,df^2} \\
&+ \frac{(72\,af^2h^2i^2-36\,bf^2h^2i^2-48\,aefhi^3+40\,befhi^3+12\,ae^2i^4-13\,be^2i^4)x^2}{24\,df^3} \\
&+ \frac{(48\,af^3h^3i-48\,bf^3h^3i-72\,aef^2h^2i^2+108\,bef^2h^2i^2+48\,ae^2fhi^3-88\,be^2fhi^3-12\,ae^3i^4+25\,be^3i^4)x}{12\,df^4} \\
&+ \frac{(bf^4h^4-4\,bef^3h^3i+6\,be^2f^2h^2i^2-4\,be^3fhi^3+be^4i^4)\log(cfx+ce)^2}{2\,df^5} \\
&+ \frac{(12\,af^4h^4-48\,aef^3h^3i+48\,bef^3h^3i+72\,ae^2f^2h^2i^2-108\,be^2f^2h^2i^2-48\,ae^3fhi^3+88\,be^3fhi^3+12\,ae}{12\,df^5}
\end{aligned}
$$

```
[In] integrate((i*x+h)^4*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="giac")
```

```
[Out] 1/16*(4*a*i^4 - b*i^4)*x^4/(d*f) + 1/12*(3*b*i^4*x^4/(d*f) + 4*(4*b*f*h*i^3
 - b*e*i^4)*x^3/(d*f^2) + 6*(6*b*f^2*h^2*i^2 - 4*b*e*f*h*i^3 + b*e^2*i^4)*x
^2/(d*f^3) + 12*(4*b*f^3*h^3*i - 6*b*e*f^2*h^2*i^2 + 4*b*e^2*f*h*i^3 - b*e^
3*i^4)*x/(d*f^4))*log(c*f*x + c*e) + 1/36*(48*a*f*h*i^3 - 16*b*f*h*i^3 - 12
*a*e*i^4 + 7*b*e*i^4)*x^3/(d*f^2) + 1/24*(72*a*f^2*h^2*i^2 - 36*b*f^2*h^2*i
```

```
^2 - 48*a*e*f*h*i^3 + 40*b*e*f*h*i^3 + 12*a*e^2*i^4 - 13*b*e^2*i^4)*x^2/(d*
f^3) + 1/12*(48*a*f^3*h^3*i - 48*b*f^3*h^3*i - 72*a*e*f^2*h^2*i^2 + 108*b*e
*f^2*h^2*i^2 + 48*a*e^2*f*h*i^3 - 88*b*e^2*f*h*i^3 - 12*a*e^3*i^4 + 25*b*e^
3*i^4)*x/(d*f^4) + 1/2*(b*f^4*h^4 - 4*b*e*f^3*h^3*i + 6*b*e^2*f^2*h^2*i^2 -
 4*b*e^3*f*h*i^3 + b*e^4*i^4)*log(c*f*x + c*e)^2/(d*f^5) + 1/12*(12*a*f^4*h
^4 - 48*a*e*f^3*h^3*i + 48*b*e*f^3*h^3*i + 72*a*e^2*f^2*h^2*i^2 - 108*b*e^2
*f^2*h^2*i^2 - 48*a*e^3*f*h*i^3 + 88*b*e^3*f*h*i^3 + 12*a*e^4*i^4 - 25*b*e^
4*i^4)*log(f*x + e)/(d*f^5)
```

## Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.10

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))}{de+dfx}\,dx = x^3\left(\frac{i^3\,(12\,a\,f\,h+b\,e\,i-4\,b\,f\,h)}{9\,d\,f^2}-\frac{e\,i^4\,(4\,a-b)}{12\,d\,f^2}\right) - x^2\left(\frac{e\left(\frac{i^3\,(12\,a\,f\,h+b\,e\,i-4\,b\,f\,h)}{3\,d\,f^2}-\frac{e\,i^4\,(4\,a-b)}{4\,d\,f^2}\right)}{2\,f} - \frac{i^2\,(12\,a\,f^2\,h^2-b\,e^2\,i^2-6\,b\,f^2\,h^2+4\,b\,e\,f\,h\,i)}{4\,d\,f^3}\right) + x\left(\frac{12\,b\,e^3\,i^4+48\,a\,f^3\,h^3\,i-48\,b\,f^3\,h^3\,i-48\,b\,e^2\,f\,h\,i^3+72\,b\,e\,f^2\,h^2\,i^2}{12\,d\,f^4} + \frac{e\left(\frac{e\left(\frac{i^3\,(12\,a\,f\,h+b\,e\,i-4\,b\,f\,h)}{3\,d\,f^2}-\frac{e\,i^4\,(4\,a-b)}{4\,d\,f^2}\right)}{f}-\frac{i^2\,(12\,a\,f^2\,h^2-b\,e^2\,i^2-6\,b\,f^2\,h^2+4\,b\,e\,f\,h\,i)}{2\,d\,f^3}\right)}{f}\right) + f\,\ln\left(c\,(e+f\,x)\right)\left(\frac{b\,i^4\,x^4}{4\,d\,f^2}+\frac{b\,i^2\,x^2\,(e^2\,i^2-4\,e\,f\,h\,i+6\,f^2\,h^2)}{2\,d\,f^4}-\frac{b\,i^3\,x^3\,(e\,i-4\,f\,h)}{3\,d\,f^3}-\frac{b\,i\,x\,(e^3\,i^3-4\,e^2\,f\,h\,i^2+6\,e\,f^2\,h^2\,i-4\,f^3\,h^3)}{d\,f^5}\right) + \frac{\ln\left(e+f\,x\right)\,(12\,a\,e^4\,i^4+12\,a\,f^4\,h^4-25\,b\,e^4\,i^4-48\,a\,e\,f^3\,h^3\,i-48\,a\,e^3\,f\,h\,i^3+48\,b\,e\,f^3\,h^3\,i+88\,b\,e^3\,f\,h\,i}{12\,d\,f^5} + \frac{b\ln\left(c\,(e+f\,x)\right)^2\,(e^4\,i^4-4\,e^3\,f\,h\,i^3+6\,e^2\,f^2\,h^2\,i^2-4\,e\,f^3\,h^3\,i+f^4\,h^4)}{2\,d\,f^5}+\frac{i^4\,x^4\,(4\,a-b)}{16\,d\,f}$$

```
[In] int(((h + i*x)^4*(a + b*log(c*(e + f*x))))/(d*e + d*f*x),x)
```

```
[Out] x^3*((i^3*(12*a*f*h + b*e*i - 4*b*f*h))/(9*d*f^2) - (e*i^4*(4*a - b))/(12*d
*f^2)) - x^2*((e*((i^3*(12*a*f*h + b*e*i - 4*b*f*h))/(3*d*f^2) - (e*i^4*(4*
a - b))/(4*d*f^2)))/(2*f) - (i^2*(12*a*f^2*h^2 - b*e^2*i^2 - 6*b*f^2*h^2 +
4*b*e*f*h*i))/(4*d*f^3)) + x*((12*b*e^3*i^4 + 48*a*f^3*h^3*i - 48*b*f^3*h^3
*i - 48*b*e^2*f*h*i^3 + 72*b*e*f^2*h^2*i^2)/(12*d*f^4) + (e*((e*((i^3*(12*a
*f*h + b*e*i - 4*b*f*h))/(3*d*f^2) - (e*i^4*(4*a - b))/(4*d*f^2)))/f - (i^2
*(12*a*f^2*h^2 - b*e^2*i^2 - 6*b*f^2*h^2 + 4*b*e*f*h*i))/(2*d*f^3)))/f) + f
*log(c*(e + f*x))*((b*i^4*x^4)/(4*d*f^2) + (b*i^2*x^2*(e^2*i^2 + 6*f^2*h^2
- 4*e*f*h*i))/(2*d*f^4) - (b*i^3*x^3*(e*i - 4*f*h))/(3*d*f^3) - (b*i*x*(e^3
*i^3 - 4*f^3*h^3 + 6*e*f^2*h^2*i - 4*e^2*f*h*i^2))/(d*f^5)) + (log(e + f*x)
*(12*a*e^4*i^4 + 12*a*f^4*h^4 - 25*b*e^4*i^4 - 48*a*e*f^3*h^3*i - 48*a*e^3*
f*h*i^3 + 48*b*e*f^3*h^3*i + 88*b*e^3*f*h*i^3 + 72*a*e^2*f^2*h^2*i^2 - 108*
b*e^2*f^2*h^2*i^2))/(12*d*f^5) + (b*log(c*(e + f*x))^2*(e^4*i^4 + f^4*h^4 +
 6*e^2*f^2*h^2*i^2 - 4*e*f^3*h^3*i - 4*e^3*f*h*i^3))/(2*d*f^5) + (i^4*x^4*(
4*a - b))/(16*d*f)
```

# 3.176 $\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 244

$$\begin{aligned}\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx = &-\frac{3bi(fh-ei)^2x}{df^3}-\frac{3bi^2(fh-ei)(e+fx)^2}{4df^4}\\ &-\frac{bi^3(e+fx)^3}{9df^4}-\frac{b(fh-ei)^3\log^2(e+fx)}{2df^4}\\ &+\frac{3i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4}\\ &+\frac{3i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{2df^4}\\ &+\frac{i^3(e+fx)^3(a+b\log(c(e+fx)))}{3df^4}\\ &+\frac{(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{df^4}\end{aligned}$$

```
[Out] -3*b*i*(-e*i+f*h)^2*x/d/f^3-3/4*b*i^2*(-e*i+f*h)*(f*x+e)^2/d/f^4-1/9*b*i^3*
(f*x+e)^3/d/f^4-1/2*b*(-e*i+f*h)^3*ln(f*x+e)^2/d/f^4+3*i*(-e*i+f*h)^2*(f*x+
e)*(a+b*ln(c*(f*x+e)))/d/f^4+3/2*i^2*(-e*i+f*h)*(f*x+e)^2*(a+b*ln(c*(f*x+e)
))/d/f^4+1/3*i^3*(f*x+e)^3*(a+b*ln(c*(f*x+e)))/d/f^4+(-e*i+f*h)^3*ln(f*x+e)
*(a+b*ln(c*(f*x+e)))/d/f^4
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2458, 12, 45, 2372, 14, 2338}

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{3i^2(e+fx)^2(fh-ei)(a+b\log(c(e+fx)))}{2df^4} + \frac{(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{df^4} + \frac{3i(e+fx)(fh-ei)^2(a+b\log(c(e+fx)))}{df^4} + \frac{i^3(e+fx)^3(a+b\log(c(e+fx)))}{3df^4} - \frac{3bi^2(e+fx)^2(fh-ei)}{4df^4} - \frac{b(fh-ei)^3\log^2(e+fx)}{2df^4} - \frac{bi^3(e+fx)^3}{9df^4} - \frac{3bix(fh-ei)^2}{df^3}$$

```
[In] Int[((h + i*x)^3*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

```
[Out] (-3*b*i*(f*h - e*i)^2*x)/(d*f^3) - (3*b*i^2*(f*h - e*i)*(e + f*x)^2)/(4*d*f
^4) - (b*i^3*(e + f*x)^3)/(9*d*f^4) - (b*(f*h - e*i)^3*Log[e + f*x]^2)/(2*d
*f^4) + (3*i*(f*h - e*i)^2*(e + f*x)*(a + b*Log[c*(e + f*x)]))/(d*f^4) + (3
*i^2*(f*h - e*i)*(e + f*x)^2*(a + b*Log[c*(e + f*x)]))/(2*d*f^4) + (i^3*(e
+ f*x)^3*(a + b*Log[c*(e + f*x)]))/(3*d*f^4) + ((f*h - e*i)^3*Log[e + f*x]*
(a + b*Log[c*(e + f*x)]))/(d*f^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3(a+b\log(cx))}{dx}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3(a+b\log(cx))}{x}\,dx, x, e+fx\right)}{df}$$

$$= \frac{3i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4} + \frac{3i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{2df^4} + \frac{i^3(e+fx)^3(a+b\log(c(e+fx)))}{3df^4} + \frac{(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{df^4} - \frac{b\text{Subst}\left(\int \frac{ix(18f^2h^2+9fhi(-4e+x)+i^2(18e^2-9ex+2x^2))+6(fh-ei)^3\log(x)}{6f^3x}\,dx, x, e+fx\right)}{df}$$

$$=\frac{3i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4}+\frac{3i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{2df^4}+\frac{i^3(e+fx)^3(a+b\log(c(e+fx)))}{3df^4}+\frac{(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{df^4}-\frac{b\text{Subst}\left(\int\frac{ix(18f^2h^2+9fhi(-4e+x)+i^2(18e^2-9ex+2x^2))+6(fh-ei)^3\log(x)}{x}\,dx,x,e+fx\right)}{6df^4}$$

$$=\frac{3i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4}+\frac{3i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{2df^4}+\frac{i^3(e+fx)^3(a+b\log(c(e+fx)))}{3df^4}+\frac{(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{df^4}-\frac{b\text{Subst}\left(\int\left(i(18(fh-ei)^2+9i(fh-ei)x+2i^2x^2)+\frac{6(fh-ei)^3\log(x)}{x}\right)\,dx,x,e+fx\right)}{6df^4}$$

$$=\frac{3i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4}+\frac{3i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{2df^4}+\frac{i^3(e+fx)^3(a+b\log(c(e+fx)))}{3df^4}+\frac{(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{df^4}-\frac{(bi)\text{Subst}\left(\int(18(fh-ei)^2+9i(fh-ei)x+2i^2x^2)\,dx,x,e+fx\right)}{6df^4}-\frac{(b(fh-ei)^3)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,e+fx\right)}{df^4}$$

$$=-\frac{3bi(fh-ei)^2x}{df^3}-\frac{3bi^2(fh-ei)(e+fx)^2}{4df^4}-\frac{bi^3(e+fx)^3}{9df^4}-\frac{b(fh-ei)^3\log^2(e+fx)}{2df^4}+\frac{3i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4}+\frac{3i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{2df^4}+\frac{i^3(e+fx)^3(a+b\log(c(e+fx)))}{3df^4}+\frac{(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{df^4}$$

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.54

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$= \frac{18a^2f^3h^3-54a^2ef^2h^2i+54a^2e^2fhi^2-18a^2e^3i^3+108abf^3h^2ix-108b^2f^3h^2ix-108abef^2hi^2x+162b^2ef}{}$$

```
[In] Integrate[((h + i*x)^3*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

```
[Out] (18*a^2*f^3*h^3 - 54*a^2*e*f^2*h^2*i + 54*a^2*e^2*f*h*i^2 - 18*a^2*e^3*i^3
+ 108*a*b*f^3*h^2*i*x - 108*b^2*f^3*h^2*i*x - 108*a*b*e*f^2*h*i^2*x + 162*b
^2*e*f^2*h*i^2*x + 36*a*b*e^2*f*i^3*x - 66*b^2*e^2*f*i^3*x + 54*a*b*f^3*h*i
^2*x^2 - 27*b^2*f^3*h*i^2*x^2 - 18*a*b*e*f^2*i^3*x^2 + 15*b^2*e*f^2*i^3*x^2
 + 12*a*b*f^3*i^3*x^3 - 4*b^2*f^3*i^3*x^3 + 6*b^2*e^2*i^2*(-9*f*h + 5*e*i)*
Log[e + f*x] + 6*b*(6*a*(f*h - e*i)^3 + b*i*(6*e^3*i^2 + 6*e^2*f*i*(-3*h +
i*x) + 3*e*f^2*(6*h^2 - 6*h*i*x - i^2*x^2) + f^3*x*(18*h^2 + 9*h*i*x + 2*i^
2*x^2)))*Log[c*(e + f*x)] + 18*b^2*(f*h - e*i)^3*Log[c*(e + f*x)]^2)/(36*b*
d*f^4)
```

## Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.46

| method | result |
|---|---|
| norman | $\frac{bi(e^2i^2-3efhi+3f^2h^2)x\ln(c(fx+e))}{d\,f^3}-\frac{(6a\,e^3i^3-18a\,e^2fh\,i^2+18ae\,f^2h^2i-6a\,f^3h^3-11b\,e^3i^3+27b\,e^2fh\,i^2-18be\,f^2h^2}{6d\,f^4}$ |
| parts | $\frac{a\left(\frac{i\left(\frac{1}{3}f^2i^2x^3-\frac{1}{2}ef\,i^2x^2+\frac{3}{2}f^2hi\,x^2+x\,e^2i^2-3xefhi+3x\,f^2h^2\right)}{f^3}+\frac{\left(-e^3i^3+3e^2fh\,i^2-3e\,f^2h^2i+f^3h^3\right)\ln(fx+e)}{f^4}\right)}{d}+\frac{b\left(-\frac{c\,e^3i^3}{}\right.}{}$ |
| risch | $-\frac{b\ln(c(fx+e))^2e^3i^3}{2d\,f^4}+\frac{3b\ln(c(fx+e))^2e^2h\,i^2}{2d\,f^3}-\frac{3b\ln(c(fx+e))^2e\,h^2i}{2d\,f^2}+\frac{b\ln(c(fx+e))^2h^3}{2df}+\frac{bix(2f^2i^2x^2-3ef\,i^2}{}$ |
| parallelrisch | $\frac{-108x\ln(c(fx+e))be\,f^2h\,i^2+12a\,f^3i^3x^3-4b\,f^3i^3x^3+108\ln(c(fx+e))a\,e^2fh\,i^2-108\ln(c(fx+e))ae\,f^2h^2i-162\ln(c(fx+}{}$ |
| derivativedivides | $\frac{-\frac{ca\,e^3i^3\ln(cfx+ce)}{f^3d}+\frac{3ca\,e^2h\,i^2\ln(cfx+ce)}{f^2d}-\frac{3cae\,h^2i\ln(cfx+ce)}{fd}+\frac{ca\,h^3\ln(cfx+ce)}{d}+\frac{3a\,e^2i^3(cfx+ce)}{f^3d}-\frac{6aeh\,i^2(cfx+ce)}{f^2d}+\frac{3a\,h^2}{}}{}$ |
| default | $\frac{-\frac{ca\,e^3i^3\ln(cfx+ce)}{f^3d}+\frac{3ca\,e^2h\,i^2\ln(cfx+ce)}{f^2d}-\frac{3cae\,h^2i\ln(cfx+ce)}{fd}+\frac{ca\,h^3\ln(cfx+ce)}{d}+\frac{3a\,e^2i^3(cfx+ce)}{f^3d}-\frac{6aeh\,i^2(cfx+ce)}{f^2d}+\frac{3a\,h^2}{}}{}$ |

```
[In] int((i*x+h)^3*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

```
[Out] b*i*(e^2*i^2-3*e*f*h*i+3*f^2*h^2)/d/f^3*x*ln(c*(f*x+e))-1/6*(6*a*e^3*i^3-18
*a*e^2*f*h*i^2+18*a*e*f^2*h^2*i-6*a*f^3*h^3-11*b*e^3*i^3+27*b*e^2*f*h*i^2-1
```

```
8*b*e*f^2*h^2*i)/d/f^4*ln(c*(f*x+e))-1/2*b*(e^3*i^3-3*e^2*f*h*i^2+3*e*f^2*h
^2*i-f^3*h^3)/d/f^4*ln(c*(f*x+e))^2+1/6*i*(6*a*e^2*i^2-18*a*e*f*h*i+18*a*f^
2*h^2-11*b*e^2*i^2+27*b*e*f*h*i-18*b*f^2*h^2)/d/f^3*x-1/12*i^2*(6*a*e*i-18*
a*f*h-5*b*e*i+9*b*f*h)/d/f^2*x^2+1/9*i^3*(3*a-b)/d/f*x^3+1/3*b*i^3/d/f*x^3*
ln(c*(f*x+e))-1/2*b*i^2*(e*i-3*f*h)/d/f^2*x^2*ln(c*(f*x+e))
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.26

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$=\frac{4\,(3\,a-b)f^3i^3x^3+3\,(9\,(2\,a-b)f^3hi^2-(6\,a-5\,b)ef^2i^3)x^2+18\,(bf^3h^3-3\,bef^2h^2i+3\,be^2fhi^2-be^3i^3)}{}$$

```
[In] integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="fricas")
```

```
[Out] 1/36*(4*(3*a - b)*f^3*i^3*x^3 + 3*(9*(2*a - b)*f^3*h*i^2 - (6*a - 5*b)*e*f^
2*i^3)*x^2 + 18*(b*f^3*h^3 - 3*b*e*f^2*h^2*i + 3*b*e^2*f*h*i^2 - b*e^3*i^3)
*log(c*f*x + c*e)^2 + 6*(18*(a - b)*f^3*h^2*i - 9*(2*a - 3*b)*e*f^2*h*i^2 +
 (6*a - 11*b)*e^2*f*i^3)*x + 6*(2*b*f^3*i^3*x^3 + 6*a*f^3*h^3 - 18*(a - b)*
e*f^2*h^2*i + 9*(2*a - 3*b)*e^2*f*h*i^2 - (6*a - 11*b)*e^3*i^3 + 3*(3*b*f^3
*h*i^2 - b*e*f^2*i^3)*x^2 + 6*(3*b*f^3*h^2*i - 3*b*e*f^2*h*i^2 + b*e^2*f*i^
3)*x)*log(c*f*x + c*e))/(d*f^4)
```

## Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.75

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$=x^3\left(\frac{ai^3}{3df}-\frac{bi^3}{9df}\right)+x^2\left(-\frac{aei^3}{2df^2}+\frac{3ahi^2}{2df}+\frac{5bei^3}{12df^2}-\frac{3bhi^2}{4df}\right)$$
$$+x\left(\frac{ae^2i^3}{df^3}-\frac{3aehi^2}{df^2}+\frac{3ah^2i}{df}-\frac{11be^2i^3}{6df^3}+\frac{9behi^2}{2df^2}-\frac{3bh^2i}{df}\right)$$
$$+\frac{(6be^2i^3x-18befhi^2x-3befi^3x^2+18bf^2h^2ix+9bf^2hi^2x^2+2bf^2i^3x^3)\log(c(e+fx))}{6df^3}$$
$$+\frac{(-be^3i^3+3be^2fhi^2-3bef^2h^2i+bf^3h^3)\log(c(e+fx))^2}{2df^4}$$
$$-\frac{(6ae^3i^3-18ae^2fhi^2+18aef^2h^2i-6af^3h^3-11be^3i^3+27be^2fhi^2-18bef^2h^2i)\log(e+fx)}{6df^4}$$

```
[In] integrate((i*x+h)**3*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x)
```

```
[Out] x**3*(a*i**3/(3*d*f) - b*i**3/(9*d*f)) + x**2*(-a*e*i**3/(2*d*f**2) + 3*a*h
*i**2/(2*d*f) + 5*b*e*i**3/(12*d*f**2) - 3*b*h*i**2/(4*d*f)) + x*(a*e**2*i*
*3/(d*f**3) - 3*a*e*h*i**2/(d*f**2) + 3*a*h**2*i/(d*f) - 11*b*e**2*i**3/(6*
d*f**3) + 9*b*e*h*i**2/(2*d*f**2) - 3*b*h**2*i/(d*f)) + (6*b*e**2*i**3*x -
18*b*e*f*h*i**2*x - 3*b*e*f*i**3*x**2 + 18*b*f**2*h**2*i*x + 9*b*f**2*h*i**
2*x**2 + 2*b*f**2*i**3*x**3)*log(c*(e + f*x))/(6*d*f**3) + (-b*e**3*i**3 +
3*b*e**2*f*h*i**2 - 3*b*e*f**2*h**2*i + b*f**3*h**3)*log(c*(e + f*x))**2/(2
*d*f**4) - (6*a*e**3*i**3 - 18*a*e**2*f*h*i**2 + 18*a*e*f**2*h**2*i - 6*a*f
**3*h**3 - 11*b*e**3*i**3 + 27*b*e**2*f*h*i**2 - 18*b*e*f**2*h**2*i)*log(e
+ f*x)/(6*d*f**4)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(234) = 468.

Time = 0.23 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.21

$$\begin{aligned}
&\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx \\
&= 3\,bh^2i\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)\log(cfx+ce) \\
&\quad -\frac{1}{6}\,bi^3\left(\frac{6\,e^3\log(fx+e)}{df^4}-\frac{2\,f^2x^3-3\,efx^2+6\,e^2x}{df^3}\right)\log(cfx+ce) \\
&\quad +\frac{3}{2}\,bhi^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)\log(cfx+ce) \\
&\quad -\frac{1}{2}\,bh^3\left(\frac{2\,\log(cfx+ce)\log(dfx+de)}{df}-\frac{\log(fx+e)^2+2\,\log(fx+e)\log(c)}{df}\right) \\
&\quad +3\,ah^2i\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)-\frac{1}{6}\,ai^3\left(\frac{6\,e^3\log(fx+e)}{df^4}-\frac{2\,f^2x^3-3\,efx^2+6\,e^2x}{df^3}\right) \\
&\quad +\frac{3}{2}\,ahi^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)+\frac{bh^3\log(cfx+ce)\log(dfx+de)}{df} \\
&\quad +\frac{ah^3\log(dfx+de)}{df}+\frac{3\left(e\log(fx+e)^2-2\,fx+2\,e\log(fx+e)\right)bh^2i}{2\,df^2} \\
&\quad -\frac{3\left(f^2x^2+2\,e^2\log(fx+e)^2-6\,efx+6\,e^2\log(fx+e)\right)bhi^2}{4\,df^3} \\
&\quad -\frac{\left(4\,f^3x^3-15\,ef^2x^2-18\,e^3\log(fx+e)^2+66\,e^2fx-66\,e^3\log(fx+e)\right)bi^3}{36\,df^4}
\end{aligned}$$

```
[In] integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="maxima")
```

```
[Out] 3*b*h^2*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) - 1/6*b*i^3*(
6*e^3*log(f*x + e)/(d*f^4) - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3))*log
```

```
(c*f*x + c*e) + 3/2*b*h*i^2*(2*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(
d*f^2))*log(c*f*x + c*e) - 1/2*b*h^3*(2*log(c*f*x + c*e)*log(d*f*x + d*e)/(
d*f) - (log(f*x + e)^2 + 2*log(f*x + e)*log(c))/(d*f)) + 3*a*h^2*i*(x/(d*f)
 - e*log(f*x + e)/(d*f^2)) - 1/6*a*i^3*(6*e^3*log(f*x + e)/(d*f^4) - (2*f^2
*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3)) + 3/2*a*h*i^2*(2*e^2*log(f*x + e)/(d*f
^3) + (f*x^2 - 2*e*x)/(d*f^2)) + b*h^3*log(c*f*x + c*e)*log(d*f*x + d*e)/(d
*f) + a*h^3*log(d*f*x + d*e)/(d*f) + 3/2*(e*log(f*x + e)^2 - 2*f*x + 2*e*lo
g(f*x + e))*b*h^2*i/(d*f^2) - 3/4*(f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x
 + 6*e^2*log(f*x + e))*b*h*i^2/(d*f^3) - 1/36*(4*f^3*x^3 - 15*e*f^2*x^2 - 1
8*e^3*log(f*x + e)^2 + 66*e^2*f*x - 66*e^3*log(f*x + e))*b*i^3/(d*f^4)
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.51

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{(3\,ai^3-bi^3)x^3}{9\,df} + \frac{1}{6}\left(\frac{2\,bi^3x^3}{df}+\frac{3\,(3\,bfhi^2-bei^3)x^2}{df^2}+\frac{6\,(3\,bf^2h^2i-3\,befhi^2+be^2i^3)x}{df^3}\right)\log(cfx+ce) + \frac{(18\,afhi^2-9\,bfhi^2-6\,aei^3+5\,bei^3)x^2}{12\,df^2} + \frac{(18\,af^2h^2i-18\,bf^2h^2i-18\,aefhi^2+27\,befhi^2+6\,ae^2i^3-11\,be^2i^3)x}{6\,df^3} + \frac{(bf^3h^3-3\,bef^2h^2i+3\,be^2fhi^2-be^3i^3)\log(cfx+ce)^2}{2\,df^4} + \frac{(6\,af^3h^3-18\,aef^2h^2i+18\,bef^2h^2i+18\,ae^2fhi^2-27\,be^2fhi^2-6\,ae^3i^3+11\,be^3i^3)\log(fx+e)}{6\,df^4}$$

```
[In] integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="giac")
```

```
[Out] 1/9*(3*a*i^3 - b*i^3)*x^3/(d*f) + 1/6*(2*b*i^3*x^3/(d*f) + 3*(3*b*f*h*i^2 -
 b*e*i^3)*x^2/(d*f^2) + 6*(3*b*f^2*h^2*i - 3*b*e*f*h*i^2 + b*e^2*i^3)*x/(d*
f^3))*log(c*f*x + c*e) + 1/12*(18*a*f*h*i^2 - 9*b*f*h*i^2 - 6*a*e*i^3 + 5*b
*e*i^3)*x^2/(d*f^2) + 1/6*(18*a*f^2*h^2*i - 18*b*f^2*h^2*i - 18*a*e*f*h*i^2
 + 27*b*e*f*h*i^2 + 6*a*e^2*i^3 - 11*b*e^2*i^3)*x/(d*f^3) + 1/2*(b*f^3*h^3
- 3*b*e*f^2*h^2*i + 3*b*e^2*f*h*i^2 - b*e^3*i^3)*log(c*f*x + c*e)^2/(d*f^4)
 + 1/6*(6*a*f^3*h^3 - 18*a*e*f^2*h^2*i + 18*b*e*f^2*h^2*i + 18*a*e^2*f*h*i^
2 - 27*b*e^2*f*h*i^2 - 6*a*e^3*i^3 + 11*b*e^3*i^3)*log(f*x + e)/(d*f^4)
```

## Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.61

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))}{de+dfx}\,dx = x^2\left(\frac{i^2\,(6\,a\,f\,h+b\,e\,i-3\,b\,f\,h)}{4\,d\,f^2}-\frac{e\,i^3\,(3\,a-b)}{6\,d\,f^2}\right) - x\left(\frac{e\left(\frac{i^2\,(6\,a\,f\,h+b\,e\,i-3\,b\,f\,h)}{2\,d\,f^2}-\frac{e\,i^3\,(3\,a-b)}{3\,d\,f^2}\right)}{f}-\frac{i\,(3\,a\,f^2\,h^2-b\,e^2\,i^2-3\,b\,f^2\,h^2+3\,b\,e\,f\,h\,i)}{d\,f^3}\right) + f\,\ln\left(c\,(e+f\,x)\right)\,\left(\frac{b\,i^3\,x^3}{3\,d\,f^2}+\frac{b\,i\,x\,(e^2\,i^2-3\,e\,f\,h\,i+3\,f^2\,h^2)}{d\,f^4}-\frac{b\,i^2\,x^2\,(e\,i-3\,f\,h)}{2\,d\,f^3}\right) + \frac{\ln\left(e+f\,x\right)\,\left(6\,a\,f^3\,h^3-6\,a\,e^3\,i^3+11\,b\,e^3\,i^3-18\,a\,e\,f^2\,h^2\,i+18\,a\,e^2\,f\,h\,i^2+18\,b\,e\,f^2\,h^2\,i-27\,b\,e^2\,f\,h\,i^2\right)}{6\,d\,f^4} + \frac{i^3\,x^3\,(3\,a-b)}{9\,d\,f}-\frac{b\ln\left(c\,(e+f\,x)\right)^2\,(e^3\,i^3-3\,e^2\,f\,h\,i^2+3\,e\,f^2\,h^2\,i-f^3\,h^3)}{2\,d\,f^4}$$

```
[In] int(((h + i*x)^3*(a + b*log(c*(e + f*x))))/(d*e + d*f*x),x)
```

```
[Out] x^2*((i^2*(6*a*f*h + b*e*i - 3*b*f*h))/(4*d*f^2) - (e*i^3*(3*a - b))/(6*d*f
^2)) - x*((e*((i^2*(6*a*f*h + b*e*i - 3*b*f*h))/(2*d*f^2) - (e*i^3*(3*a - b
))/(3*d*f^2)))/f - (i*(3*a*f^2*h^2 - b*e^2*i^2 - 3*b*f^2*h^2 + 3*b*e*f*h*i)
)/(d*f^3)) + f*log(c*(e + f*x))*((b*i^3*x^3)/(3*d*f^2) + (b*i*x*(e^2*i^2 +
3*f^2*h^2 - 3*e*f*h*i))/(d*f^4) - (b*i^2*x^2*(e*i - 3*f*h))/(2*d*f^3)) + (l
og(e + f*x)*(6*a*f^3*h^3 - 6*a*e^3*i^3 + 11*b*e^3*i^3 - 18*a*e*f^2*h^2*i +
18*a*e^2*f*h*i^2 + 18*b*e*f^2*h^2*i - 27*b*e^2*f*h*i^2))/(6*d*f^4) + (i^3*x
^3*(3*a - b))/(9*d*f) - (b*log(c*(e + f*x))^2*(e^3*i^3 - f^3*h^3 + 3*e*f^2*
h^2*i - 3*e^2*f*h*i^2))/(2*d*f^4)
```

# 3.177 $\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 157

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx = -\frac{b(4fh-3ei+fix)^2}{4df^3} - \frac{b(fh-ei)^2\log^2(e+fx)}{2df^3} + \frac{2i(fh-ei)(e+fx)(a+b\log(c(e+fx)))}{df^3} + \frac{i^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^3} + \frac{(fh-ei)^2\log(e+fx)(a+b\log(c(e+fx)))}{df^3}$$

```
[Out] -1/4*b*(f*i*x-3*e*i+4*f*h)^2/d/f^3-1/2*b*(-e*i+f*h)^2*ln(f*x+e)^2/d/f^3+2*i
*(-e*i+f*h)*(f*x+e)*(a+b*ln(c*(f*x+e)))/d/f^3+1/2*i^2*(f*x+e)^2*(a+b*ln(c*(
f*x+e)))/d/f^3+(-e*i+f*h)^2*ln(f*x+e)*(a+b*ln(c*(f*x+e)))/d/f^3
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2458, 12, 45, 2372, 14, 2338}

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{(fh-ei)^2\log(e+fx)(a+b\log(c(e+fx)))}{df^3} + \frac{2i(e+fx)(fh-ei)(a+b\log(c(e+fx)))}{df^3} + \frac{i^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^3} - \frac{b(-3ei+4fh+fix)^2}{4df^3} - \frac{b(fh-ei)^2\log^2(e+fx)}{2df^3}$$

```
[In] Int[((h + i*x)^2*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

```
[Out] -1/4*(b*(4*f*h - 3*e*i + f*i*x)^2)/(d*f^3) - (b*(f*h - e*i)^2*Log[e + f*x]^
2)/(2*d*f^3) + (2*i*(f*h - e*i)*(e + f*x)*(a + b*Log[c*(e + f*x)]))/(d*f^3)
 + (i^2*(e + f*x)^2*(a + b*Log[c*(e + f*x)]))/(2*d*f^3) + ((f*h - e*i)^2*Lo
g[e + f*x]*(a + b*Log[c*(e + f*x)]))/(d*f^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2(a+b\log(cx))}{dx}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2(a+b\log(cx))}{x}\,dx, x, e+fx\right)}{df}$$

$$= \frac{2i(fh-ei)(e+fx)(a+b\log(c(e+fx)))}{df^3} + \frac{i^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^3} + \frac{(fh-ei)^2\log(e+fx)(a+b\log(c(e+fx)))}{df^3} - \frac{b\text{Subst}\left(\int \frac{ix(4fh+i(-4e+x))+2(fh-ei)^2\log(x)}{2f^2x}\,dx, x, e+fx\right)}{df}$$

$$= \frac{2i(fh-ei)(e+fx)(a+b\log(c(e+fx)))}{df^3} + \frac{i^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^3} + \frac{(fh-ei)^2\log(e+fx)(a+b\log(c(e+fx)))}{df^3} - \frac{b\text{Subst}\left(\int \frac{ix(4fh+i(-4e+x))+2(fh-ei)^2\log(x)}{x}\,dx, x, e+fx\right)}{2df^3}$$

$$= \frac{2i(fh-ei)(e+fx)(a+b\log(c(e+fx)))}{df^3} + \frac{i^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^3} + \frac{(fh-ei)^2\log(e+fx)(a+b\log(c(e+fx)))}{df^3} - \frac{b\text{Subst}\left(\int \left(-i(-4fh+4ei-ix)+\frac{2(fh-ei)^2\log(x)}{x}\right)dx, x, e+fx\right)}{2df^3}$$

$$= -\frac{b(4fh-3ei+fix)^2}{4df^3} + \frac{2i(fh-ei)(e+fx)(a+b\log(c(e+fx)))}{df^3} + \frac{i^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^3} + \frac{(fh-ei)^2\log(e+fx)(a+b\log(c(e+fx)))}{df^3} - \frac{(b(fh-ei)^2)\,\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, e+fx\right)}{df^3}$$

$$= -\frac{b(4fh-3ei+fix)^2}{4df^3} - \frac{b(fh-ei)^2\log^2(e+fx)}{2df^3} + \frac{2i(fh-ei)(e+fx)(a+b\log(c(e+fx)))}{df^3} + \frac{i^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^3} + \frac{(fh-ei)^2\log(e+fx)(a+b\log(c(e+fx)))}{df^3}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.36

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{2a^2f^2h^2-4a^2efhi+2a^2e^2i^2+8abf^2hix-8b^2f^2hix-4abefi^2x+6b^2efi^2x+2abf^2i^2x^2-b^2f^2i^2x^2-2b^2}{}$$

```
[In] Integrate[((h + i*x)^2*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

```
[Out] (2*a^2*f^2*h^2 - 4*a^2*e*f*h*i + 2*a^2*e^2*i^2 + 8*a*b*f^2*h*i*x - 8*b^2*f^
2*h*i*x - 4*a*b*e*f*i^2*x + 6*b^2*e*f*i^2*x + 2*a*b*f^2*i^2*x^2 - b^2*f^2*i
^2*x^2 - 2*b^2*e^2*i^2*Log[e + f*x] + 2*b*(2*a*(f*h - e*i)^2 + b*i*(-2*e^2*
i + e*f*(4*h - 2*i*x) + f^2*x*(4*h + i*x)))*Log[c*(e + f*x)] + 2*b^2*(f*h -
 e*i)^2*Log[c*(e + f*x)]^2)/(4*b*d*f^3)
```

## Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| norman | $\frac{(2ae^2i^2-4aefhi+2af^2h^2-3be^2i^2+4befhi)\ln(c(fx+e))}{2df^3}+\frac{b(e^2i^2-2efhi+f^2h^2)\ln(c(fx+e))^2}{2df^3}-\frac{i(2aei-4afh-3be}{2df^2}$ |
| parts | $\frac{a\left(\frac{i\left(\frac{1}{2}fix^2-xei+2xfh\right)}{f^2}+\frac{(e^2i^2-2efhi+f^2h^2)\ln(fx+e)}{f^3}\right)}{d}+\frac{b\left(\frac{ce^2i^2\ln(cfx+ce)^2}{2f^2}-\frac{cehi\ln(cfx+ce)^2}{f}+\frac{ch^2\ln(cfx+ce)^2}{2}-\frac{2e}{}\right.}{}$ |
| risch | $\frac{b\ln(c(fx+e))^2e^2i^2}{2df^3}-\frac{b\ln(c(fx+e))^2ehi}{df^2}+\frac{b\ln(c(fx+e))^2h^2}{2df}-\frac{bix(-fix+2ei-4fh)\ln(c(fx+e))}{2df^2}+\frac{ai^2x^2}{2df}-\frac{bi^2}{4d}$ |
| parallelrisch | $\frac{6ae^2i^2-11be^2i^2-16aefhi+16befhi-4aefi^2x+8af^2hix+6befi^2x-8bf^2hix+2af^2i^2x^2-bf^2i^2x^2-4x\ln(c(fx+e))bef}{}$ |
| derivativedivides | $\frac{\frac{cae^2i^2\ln(cfx+ce)}{f^2d}-\frac{2caehi\ln(cfx+ce)}{fd}+\frac{cah^2\ln(cfx+ce)}{d}-\frac{2aei^2(cfx+ce)}{f^2d}+\frac{2ahi(cfx+ce)}{fd}+\frac{ai^2(cfx+ce)^2}{2cf^2d}+\frac{cbe^2i^2\ln(cfx+ce)^2}{2f^2d}}{}$ |
| default | $\frac{\frac{cae^2i^2\ln(cfx+ce)}{f^2d}-\frac{2caehi\ln(cfx+ce)}{fd}+\frac{cah^2\ln(cfx+ce)}{d}-\frac{2aei^2(cfx+ce)}{f^2d}+\frac{2ahi(cfx+ce)}{fd}+\frac{ai^2(cfx+ce)^2}{2cf^2d}+\frac{cbe^2i^2\ln(cfx+ce)^2}{2f^2d}}{}$ |

```
[In] int((i*x+h)^2*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*a*e^2*i^2-4*a*e*f*h*i+2*a*f^2*h^2-3*b*e^2*i^2+4*b*e*f*h*i)/d/f^3*ln(
c*(f*x+e))+1/2*b*(e^2*i^2-2*e*f*h*i+f^2*h^2)/d/f^3*ln(c*(f*x+e))^2-1/2*i*(2
*a*e*i-4*a*f*h-3*b*e*i+4*b*f*h)/d/f^2*x+1/4*i^2*(2*a-b)/d/f*x^2+1/2*b*i^2/d
/f*x^2*ln(c*(f*x+e))-b*i*(e*i-2*f*h)/d/f^2*x*ln(c*(f*x+e))
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{(2\,a-b)f^2i^2x^2+2\,(bf^2h^2-2\,befhi+be^2i^2)\log\left(cfx+ce\right)^2+2\,(4\,(a-b)f^2hi-(2\,a-3\,b)efi^2)x+2\,(b}{4\,df^3}$$

```
[In] integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="fricas")
```

```
[Out] 1/4*((2*a - b)*f^2*i^2*x^2 + 2*(b*f^2*h^2 - 2*b*e*f*h*i + b*e^2*i^2)*log(c*
f*x + c*e)^2 + 2*(4*(a - b)*f^2*h*i - (2*a - 3*b)*e*f*i^2)*x + 2*(b*f^2*i^2
*x^2 + 2*a*f^2*h^2 - 4*(a - b)*e*f*h*i + (2*a - 3*b)*e^2*i^2 + 2*(2*b*f^2*h
*i - b*e*f*i^2)*x)*log(c*f*x + c*e))/(d*f^3)
```

## Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.44

$$\begin{aligned}\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx &= x^2\left(\frac{ai^2}{2df}-\frac{bi^2}{4df}\right)+x\left(-\frac{aei^2}{df^2}+\frac{2ahi}{df}+\frac{3bei^2}{2df^2}-\frac{2bhi}{df}\right)\\ &+\frac{(-2bei^2x+4bfhix+bfi^2x^2)\log\left(c(e+fx)\right)}{2df^2}\\ &+\frac{(be^2i^2-2befhi+bf^2h^2)\log\left(c(e+fx)\right)^2}{2df^3}\\ &+\frac{(2ae^2i^2-4aefhi+2af^2h^2-3be^2i^2+4befhi)\log\left(e+fx\right)}{2df^3}\end{aligned}$$

```
[In] integrate((i*x+h)**2*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x)
```

```
[Out] x**2*(a*i**2/(2*d*f) - b*i**2/(4*d*f)) + x*(-a*e*i**2/(d*f**2) + 2*a*h*i/(d
*f) + 3*b*e*i**2/(2*d*f**2) - 2*b*h*i/(d*f)) + (-2*b*e*i**2*x + 4*b*f*h*i*x
 + b*f*i**2*x**2)*log(c*(e + f*x))/(2*d*f**2) + (b*e**2*i**2 - 2*b*e*f*h*i
+ b*f**2*h**2)*log(c*(e + f*x))**2/(2*d*f**3) + (2*a*e**2*i**2 - 4*a*e*f*h*
i + 2*a*f**2*h**2 - 3*b*e**2*i**2 + 4*b*e*f*h*i)*log(e + f*x)/(2*d*f**3)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(151) = 302.

Time = 0.23 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.24

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$= 2\,bhi\left(\frac{x}{df}-\frac{e\log\left(fx+e\right)}{df^2}\right)\log\left(cfx+ce\right)$$
$$+\frac{1}{2}\,bi^2\left(\frac{2\,e^2\log\left(fx+e\right)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)\log\left(cfx+ce\right)$$
$$-\frac{1}{2}\,bh^2\left(\frac{2\,\log\left(cfx+ce\right)\log\left(dfx+de\right)}{df}-\frac{\log\left(fx+e\right)^2+2\,\log\left(fx+e\right)\log\left(c\right)}{df}\right)$$
$$+2\,ahi\left(\frac{x}{df}-\frac{e\log\left(fx+e\right)}{df^2}\right)+\frac{1}{2}\,ai^2\left(\frac{2\,e^2\log\left(fx+e\right)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)$$
$$+\frac{bh^2\log\left(cfx+ce\right)\log\left(dfx+de\right)}{df}+\frac{ah^2\log\left(dfx+de\right)}{df}$$
$$+\frac{\left(e\log\left(fx+e\right)^2-2\,fx+2\,e\log\left(fx+e\right)\right)bhi}{df^2}$$
$$-\frac{\left(f^2x^2+2\,e^2\log\left(fx+e\right)^2-6\,efx+6\,e^2\log\left(fx+e\right)\right)bi^2}{4\,df^3}$$

```
[In] integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="maxima")
```

```
[Out] 2*b*h*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) + 1/2*b*i^2*(2*
e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2))*log(c*f*x + c*e) - 1/2*
b*h^2*(2*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2 + 2*log(
f*x + e)*log(c))/(d*f)) + 2*a*h*i*(x/(d*f) - e*log(f*x + e)/(d*f^2)) + 1/2*
a*i^2*(2*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2)) + b*h^2*log(c*
f*x + c*e)*log(d*f*x + d*e)/(d*f) + a*h^2*log(d*f*x + d*e)/(d*f) + (e*log(f
*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*b*h*i/(d*f^2) - 1/4*(f^2*x^2 + 2*e^2*
log(f*x + e)^2 - 6*e*f*x + 6*e^2*log(f*x + e))*b*i^2/(d*f^3)
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.33

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$=\frac{1}{2}\left(\frac{bi^2x^2}{df}+\frac{2\,(2\,bfhi-bei^2)x}{df^2}\right)\log\left(cfx+ce\right)+\frac{(2\,ai^2-bi^2)x^2}{4\,df}$$
$$+\frac{(4\,afhi-4\,bfhi-2\,aei^2+3\,bei^2)x}{2\,df^2}+\frac{(bf^2h^2-2\,befhi+be^2i^2)\log\left(cfx+ce\right)^2}{2\,df^3}$$
$$+\frac{(2\,af^2h^2-4\,aefhi+4\,befhi+2\,ae^2i^2-3\,be^2i^2)\log\left(fx+e\right)}{2\,df^3}$$

```
[In] integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="giac")
```

```
[Out] 1/2*(b*i^2*x^2/(d*f) + 2*(2*b*f*h*i - b*e*i^2)*x/(d*f^2))*log(c*f*x + c*e)
+ 1/4*(2*a*i^2 - b*i^2)*x^2/(d*f) + 1/2*(4*a*f*h*i - 4*b*f*h*i - 2*a*e*i^2
+ 3*b*e*i^2)*x/(d*f^2) + 1/2*(b*f^2*h^2 - 2*b*e*f*h*i + b*e^2*i^2)*log(c*f*
x + c*e)^2/(d*f^3) + 1/2*(2*a*f^2*h^2 - 4*a*e*f*h*i + 4*b*e*f*h*i + 2*a*e^2
*i^2 - 3*b*e^2*i^2)*log(f*x + e)/(d*f^3)
```

## Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.32

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$=x\left(\frac{i\,(2\,a\,f\,h+b\,e\,i-2\,b\,f\,h)}{d\,f^2}-\frac{e\,i^2\,(2\,a-b)}{2\,d\,f^2}\right)$$
$$+f\,\ln\left(c\,(e+f\,x)\right)\,\left(\frac{b\,i^2\,x^2}{2\,d\,f^2}-\frac{b\,i\,x\,(e\,i-2\,f\,h)}{d\,f^3}\right)$$
$$+\frac{\ln\left(e+f\,x\right)\,\left(2\,a\,e^2\,i^2+2\,a\,f^2\,h^2-3\,b\,e^2\,i^2-4\,a\,e\,f\,h\,i+4\,b\,e\,f\,h\,i\right)}{2\,d\,f^3}$$
$$+\frac{b\ln\left(c\,(e+f\,x)\right)^2\,\left(e^2\,i^2-2\,e\,f\,h\,i+f^2\,h^2\right)}{2\,d\,f^3}+\frac{i^2\,x^2\,(2\,a-b)}{4\,d\,f}$$

```
[In] int(((h + i*x)^2*(a + b*log(c*(e + f*x))))/(d*e + d*f*x),x)
```

```
[Out] x*((i*(2*a*f*h + b*e*i - 2*b*f*h))/(d*f^2) - (e*i^2*(2*a - b))/(2*d*f^2)) +
 f*log(c*(e + f*x))*((b*i^2*x^2)/(2*d*f^2) - (b*i*x*(e*i - 2*f*h))/(d*f^3))
 + (log(e + f*x)*(2*a*e^2*i^2 + 2*a*f^2*h^2 - 3*b*e^2*i^2 - 4*a*e*f*h*i + 4
*b*e*f*h*i))/(2*d*f^3) + (b*log(c*(e + f*x))^2*(e^2*i^2 + f^2*h^2 - 2*e*f*h
*i))/(2*d*f^3) + (i^2*x^2*(2*a - b))/(4*d*f)
```

# 3.178 $\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 79

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{aix}{df} - \frac{bix}{df} + \frac{bi(e+fx)\log(c(e+fx))}{df^2} + \frac{(fh-ei)(a+b\log(c(e+fx)))^2}{2bdf^2}$$

```
[Out] a*i*x/d/f-b*i*x/d/f+b*i*(f*x+e)*ln(c*(f*x+e))/d/f^2+1/2*(-e*i+f*h)*(a+b*ln(
c*(f*x+e)))^2/b/d/f^2
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2458, 12, 2388, 2338, 2332}

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{(fh-ei)(a+b\log(c(e+fx)))^2}{2bdf^2} + \frac{aix}{df} + \frac{bi(e+fx)\log(c(e+fx))}{df^2} - \frac{bix}{df}$$

```
[In] Int[((h + i*x)*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

```
[Out] (a*i*x)/(d*f) - (b*i*x)/(d*f) + (b*i*(e + f*x)*Log[c*(e + f*x)])/(d*f^2) +
((f*h - e*i)*(a + b*Log[c*(e + f*x)])^2)/(2*b*d*f^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)(a+b\log(cx))}{dx}\,dx, x, e+fx\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)(a+b\log(cx))}{x}\,dx, x, e+fx\right)}{df}\\
&= \frac{i\text{Subst}(\int (a+b\log(cx))\,dx, x, e+fx)}{df^2}+\frac{(fh-ei)\text{Subst}\left(\int \frac{a+b\log(cx)}{x}\,dx, x, e+fx\right)}{df^2}\\
&= \frac{aix}{df}+\frac{(fh-ei)(a+b\log(c(e+fx)))^2}{2bdf^2}+\frac{(bi)\text{Subst}(\int \log(cx)\,dx, x, e+fx)}{df^2}\\
&= \frac{aix}{df}-\frac{bix}{df}+\frac{bi(e+fx)\log(c(e+fx))}{df^2}+\frac{(fh-ei)(a+b\log(c(e+fx)))^2}{2bdf^2}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{2afix-2bfix+2bi(e+fx)\log(c(e+fx))+\frac{(fh-ei)(a+b\log(c(e+fx)))^2}{b}}{2df^2}$$

```
[In] Integrate[((h + i*x)*(a + b*Log[c*(e + f*x)]))/(d*e + d*f*x),x]
```

```
[Out] (2*a*f*i*x - 2*b*f*i*x + 2*b*i*(e + f*x)*Log[c*(e + f*x)] + ((f*h - e*i)*(a
 + b*Log[c*(e + f*x)])^2)/b)/(2*d*f^2)
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| norman | $\frac{i(a-b)x}{df}+\frac{bix\ln(c(fx+e))}{df}-\frac{(aei-afh-bei)\ln(c(fx+e))}{d\,f^2}-\frac{b(ei-fh)\ln(c(fx+e))^2}{2d\,f^2}$ |
| parts | $\frac{a\left(\frac{xi}{f}+\frac{(-ei+fh)\ln(fx+e)}{f^2}\right)}{d}+\frac{b\left(-\frac{cei\ln(cfx+ce)^2}{2f}+\frac{ch\ln(cfx+ce)^2}{2}+\frac{i((cfx+ce)\ln(cfx+ce)-cfx-ce)}{f}\right)}{dcf}$ |
| parallelrisch | $\frac{2x\ln(c(fx+e))b\,e^2fi-\ln(c(fx+e))^2b\,e^3i+\ln(c(fx+e))^2b\,e^2fh+2xa\,e^2fi-2xb\,e^2fi-2\ln(c(fx+e))a\,e^3i+2\ln(c(fx+e))a\,e}{2d\,e^2f^2}$ |
| risch | $-\frac{b\ln(c(fx+e))^2ei}{2d\,f^2}+\frac{b\ln(c(fx+e))^2h}{2df}+\frac{bix\ln(c(fx+e))}{df}-\frac{\ln(fx+e)aei}{d\,f^2}+\frac{\ln(fx+e)ah}{df}+\frac{\ln(fx+e)bei}{d\,f^2}+\frac{aix}{df}$ |
| derivativedivides | $\frac{-\frac{acei\ln(cfx+ce)}{fd}+\frac{ahc\ln(cfx+ce)}{d}+\frac{ai(cfx+ce)}{fd}-\frac{bcei\ln(cfx+ce)^2}{2fd}+\frac{bhc\ln(cfx+ce)^2}{2d}+\frac{bi((cfx+ce)\ln(cfx+ce)-cfx-ce)}{fd}}{cf}$ |
| default | $\frac{-\frac{acei\ln(cfx+ce)}{fd}+\frac{ahc\ln(cfx+ce)}{d}+\frac{ai(cfx+ce)}{fd}-\frac{bcei\ln(cfx+ce)^2}{2fd}+\frac{bhc\ln(cfx+ce)^2}{2d}+\frac{bi((cfx+ce)\ln(cfx+ce)-cfx-ce)}{fd}}{cf}$ |

```
[In] int((i*x+h)*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

```
[Out] i*(a-b)/d/f*x+b*i*x/d/f*ln(c*(f*x+e))-(a*e*i-a*f*h-b*e*i)/d/f^2*ln(c*(f*x+e
))-1/2*b*(e*i-f*h)/d/f^2*ln(c*(f*x+e))^2
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{2\,(a-b)fix+(bfh-bei)\log\left(cfx+ce\right)^2+2\,(bfix+afh-(a-b)ei)\log\left(cfx+ce\right)}{2\,df^2}$$

```
[In] integrate((i*x+h)*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a - b)*f*i*x + (b*f*h - b*e*i)*log(c*f*x + c*e)^2 + 2*(b*f*i*x + a*
f*h - (a - b)*e*i)*log(c*f*x + c*e))/(d*f^2)
```

## Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{bix\log\left(c(e+fx)\right)}{df}+x\left(\frac{ai}{df}-\frac{bi}{df}\right) + \frac{(-bei+bfh)\log\left(c(e+fx)\right)^2}{2df^2} - \frac{(aei-afh-bei)\log\left(e+fx\right)}{df^2}$$

```
[In] integrate((i*x+h)*(a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x)
```

```
[Out] b*i*x*log(c*(e + f*x))/(d*f) + x*(a*i/(d*f) - b*i/(d*f)) + (-b*e*i + b*f*h)
*log(c*(e + f*x))**2/(2*d*f**2) - (a*e*i - a*f*h - b*e*i)*log(e + f*x)/(d*f
**2)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(77) = 154$.

Time = 0.23 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.54

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$= bi\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)\log(cfx+ce)$$
$$-\frac{1}{2}bh\left(\frac{2\log(cfx+ce)\log(dfx+de)}{df}-\frac{\log(fx+e)^2+2\log(fx+e)\log(c)}{df}\right)$$
$$+ai\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)+\frac{bh\log(cfx+ce)\log(dfx+de)}{df}$$
$$+\frac{ah\log(dfx+de)}{df}+\frac{\left(e\log(fx+e)^2-2fx+2e\log(fx+e)\right)bi}{2df^2}$$

```
[In] integrate((i*x+h)*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="maxima")
```

```
[Out] b*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) - 1/2*b*h*(2*log(c*
f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2 + 2*log(f*x + e)*log(c)
)/(d*f)) + a*i*(x/(d*f) - e*log(f*x + e)/(d*f^2)) + b*h*log(c*f*x + c*e)*lo
g(d*f*x + d*e)/(d*f) + a*h*log(d*f*x + d*e)/(d*f) + 1/2*(e*log(f*x + e)^2 -
 2*f*x + 2*e*log(f*x + e))*b*i/(d*f^2)
```

## Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx = \frac{bix\log(cfx+ce)}{df}+\frac{(ai-bi)x}{df}+\frac{(bfh-bei)\log(cfx+ce)^2}{2df^2}+\frac{(afh-aei+bei)\log(fx+e)}{df^2}$$

```
[In] integrate((i*x+h)*(a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="giac")
```

```
[Out] b*i*x*log(c*f*x + c*e)/(d*f) + (a*i - b*i)*x/(d*f) + 1/2*(b*f*h - b*e*i)*lo
g(c*f*x + c*e)^2/(d*f^2) + (a*f*h - a*e*i + b*e*i)*log(f*x + e)/(d*f^2)
```

## Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.27

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))}{de+dfx}\,dx$$
$$=\frac{2\,a\,f\,i\,x-2\,b\,f\,i\,x-b\,e\,i\ln\left(c\,e+c\,f\,x\right)^2+b\,f\,h\ln\left(c\,e+c\,f\,x\right)^2-2\,a\,e\,i\,\ln\left(e+f\,x\right)+2\,a\,f\,h\,\ln\left(e+f\,x\right)}{2\,d\,f^2}$$

```
[In] int(((h + i*x)*(a + b*log(c*(e + f*x))))/(d*e + d*f*x),x)
```

```
[Out] (2*a*f*i*x - 2*b*f*i*x - b*e*i*log(c*e + c*f*x)^2 + b*f*h*log(c*e + c*f*x)^
2 - 2*a*e*i*log(e + f*x) + 2*a*f*h*log(e + f*x) + 2*b*e*i*log(e + f*x) + 2*
b*f*i*x*log(c*e + c*f*x))/(2*d*f^2)
```

# 3.179 $\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx = \frac{(a+b\log(c(e+fx)))^2}{2bdf}$$

```
[Out] 1/2*(a+b*ln(c*(f*x+e)))^2/b/d/f
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2437, 12, 2338}

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx = \frac{(a+b\log(c(e+fx)))^2}{2bdf}$$

```
[In] Int[(a + b*Log[c*(e + f*x)])/(d*e + d*f*x),x]
```

```
[Out] (a + b*Log[c*(e + f*x)])^2/(2*b*d*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\log(cx)}{dx}\,dx, x, e+fx\right)}{f} \\ &= \frac{\text{Subst}\left(\int \frac{a+b\log(cx)}{x}\,dx, x, e+fx\right)}{df} \\ &= \frac{(a+b\log(c(e+fx)))^2}{2bdf} \end{aligned}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx = \frac{(a+b\log(c(e+fx)))^2}{2bdf}$$

```
[In] Integrate[(a + b*Log[c*(e + f*x)])/(d*e + d*f*x),x]
[Out] (a + b*Log[c*(e + f*x)])^2/(2*b*d*f)
```

## Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

| method | result | size |
|---|---|---|
| risch | $\frac{b\ln(c(fx+e))^2}{2df}+\frac{a\ln(fx+e)}{df}$ | 35 |
| parallelrisch | $\frac{\ln(c(fx+e))^2 bf+2\ln(c(fx+e))af}{2d\,f^2}$ | 35 |
| parts | $\frac{b\ln(c(fx+e))^2}{2df}+\frac{a\ln(fx+e)}{df}$ | 35 |
| norman | $\frac{a\ln(c(fx+e))}{df}+\frac{b\ln(c(fx+e))^2}{2df}$ | 37 |
| derivativedivides | $\frac{\frac{ca\ln(cfx+ce)}{d}+\frac{cb\ln(cfx+ce)^2}{2d}}{cf}$ | 42 |
| default | $\frac{\frac{ca\ln(cfx+ce)}{d}+\frac{cb\ln(cfx+ce)^2}{2d}}{cf}$ | 42 |

```
[In] int((a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x,method=_RETURNVERBOSE)
[Out] 1/2*b/d/f*ln(c*(f*x+e))^2+a/d/f*ln(f*x+e)
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx=\frac{b\log\left(cfx+ce\right)^2+2\,a\log\left(cfx+ce\right)}{2\,df}$$

```
[In] integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="fricas")
[Out] 1/2*(b*log(c*f*x + c*e)^2 + 2*a*log(c*f*x + c*e))/(d*f)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx=\frac{a\log\left(de+dfx\right)}{df}+\frac{b\log\left(c(e+fx)\right)^2}{2df}$$

```
[In] integrate((a+b*ln(c*(f*x+e)))/(d*f*x+d*e),x)
[Out] a*log(d*e + d*f*x)/(d*f) + b*log(c*(e + f*x))**2/(2*d*f)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(25) = 50.

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.74

$$\begin{aligned}\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx&=-\frac{1}{2}\,b\left(\frac{2\,\log\left(cfx+ce\right)\log\left(dfx+de\right)}{df}-\frac{\log\left(fx+e\right)^2+2\,\log\left(fx+e\right)\log\left(c\right)}{df}\right)\\&\quad+\frac{b\log\left(cfx+ce\right)\log\left(dfx+de\right)}{df}+\frac{a\log\left(dfx+de\right)}{df}\end{aligned}$$

```
[In] integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="maxima")
[Out] -1/2*b*(2*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2 + 2*log
(f*x + e)*log(c))/(d*f)) + b*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) + a*lo
g(d*f*x + d*e)/(d*f)
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx = \frac{b\log\left(cfx+ce\right)^2}{2\,df} + \frac{a\log\left(fx+e\right)}{df}$$

```
[In] integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e),x, algorithm="giac")
[Out] 1/2*b*log(c*f*x + c*e)^2/(d*f) + a*log(f*x + e)/(d*f)
```

## Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{a+b\log(c(e+fx))}{de+dfx}\,dx = \frac{b\ln\left(c\,e+c\,f\,x\right)^2+2\,a\,\ln\left(e+f\,x\right)}{2\,d\,f}$$

```
[In] int((a + b*log(c*(e + f*x)))/(d*e + d*f*x),x)
[Out] (2*a*log(e + f*x) + b*log(c*e + c*f*x)^2)/(2*d*f)
```

# 3.180 $\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 87

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx = -\frac{(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} + \frac{b\,\mathrm{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)}$$

```
[Out] -(a+b*ln(c*(f*x+e)))*ln(1+(-e*i+f*h)/i/(f*x+e))/d/(-e*i+f*h)+b*polylog(2,(e
*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2458, 12, 2379, 2438}

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx = \frac{b\,\mathrm{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} - \frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))}{d(fh-ei)}$$

```
[In] Int[(a + b*Log[c*(e + f*x)])/((d*e + d*f*x)*(h + i*x)),x]
```

```
[Out] -(((a + b*Log[c*(e + f*x)])*Log[1 + (f*h - e*i)/(i*(e + f*x))])/(d*(f*h - e
*i))) + (b*PolyLog[2, -((f*h - e*i)/(i*(e + f*x)))])/(d*(f*h - e*i))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{a+b\log(cx)}{dx\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{a+b\log(cx)}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)}\,dx, x, e+fx\right)}{df}$$

$$= -\frac{(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} + \frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{fh-ei}{ix}\right)}{x}\,dx, x, e+fx\right)}{d(fh-ei)}$$

$$= -\frac{(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} + \frac{b\text{Li}_2\left(-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx$$
$$=\frac{(a+b\log(c(e+fx)))\left(a+b\log(c(e+fx))-2b\log\left(\frac{f(h+ix)}{fh-ei}\right)\right)-2b^2\operatorname{PolyLog}\left(2,\frac{i(e+fx)}{-fh+ei}\right)}{2bd(fh-ei)}$$

```
[In] Integrate[(a + b*Log[c*(e + f*x)])/((d*e + d*f*x)*(h + i*x)),x]
```

```
[Out] ((a + b*Log[c*(e + f*x)])*(a + b*Log[c*(e + f*x)] - 2*b*Log[(f*(h + i*x))/(
f*h - e*i)]) - 2*b^2*PolyLog[2, (i*(e + f*x))/(-(f*h) + e*i)])/(2*b*d*(f*h
- e*i))
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(86) = 172.

Time = 0.97 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.03

| method | result |
|---|---|
| parts | $\frac{a\left(\frac{\ln(ix+h)}{ei-fh}-\frac{\ln(fx+e)}{ei-fh}\right)}{d}-\frac{b\ln(cfx+ce)^2}{2d(ei-fh)}+\frac{b\operatorname{dilog}\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right)}{d(ei-fh)}+\frac{b\ln(cfx+ce)\ln\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right)}{d(ei-fh)}$ |
| risch | $\frac{a\ln(ix+h)}{d(ei-fh)}-\frac{a\ln(fx+e)}{d(ei-fh)}-\frac{b\ln(cfx+ce)^2}{2d(ei-fh)}+\frac{b\operatorname{dilog}\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right)}{d(ei-fh)}+\frac{b\ln(cfx+ce)\ln\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right.}{d(ei-fh)}$ |
| derivativedivides | $\frac{-\frac{cfa\ln(cfx+ce)}{d(ei-fh)}+\frac{cfa\ln(cei-hcf-i(cfx+ce))}{d(ei-fh)}-\frac{c^2fb\left(\frac{\ln(cfx+ce)^2}{2c(ei-fh)}-\frac{i\left(\frac{\operatorname{dilog}\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right)}{i}+\frac{\ln(cfx+ce)\ln\left(\frac{-cei+hcf}{-ce}\right.}{i}\right.}{c(ei-fh)}\right.}{d}}{cf}$ |
| default | $\frac{-\frac{cfa\ln(cfx+ce)}{d(ei-fh)}+\frac{cfa\ln(cei-hcf-i(cfx+ce))}{d(ei-fh)}-\frac{c^2fb\left(\frac{\ln(cfx+ce)^2}{2c(ei-fh)}-\frac{i\left(\frac{\operatorname{dilog}\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right)}{i}+\frac{\ln(cfx+ce)\ln\left(\frac{-cei+hcf}{-ce}\right.}{i}\right.}{c(ei-fh)}\right.}{d}}{cf}$ |

```
[In] int((a+b*ln(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h),x,method=_RETURNVERBOSE)
```

```
[Out] a/d*(1/(e*i-f*h)*ln(i*x+h)-1/(e*i-f*h)*ln(f*x+e))-1/2*b/d/(e*i-f*h)*ln(c*f*
x+c*e)^2+b/d/(e*i-f*h)*dilog((-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*e*i+c*f*h))+b
/d/(e*i-f*h)*ln(c*f*x+c*e)*ln((-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*e*i+c*f*h))
```

## Fricas [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx=\int \frac{b\log\left((fx+e)c\right)+a}{(dfx+de)(ix+h)}\,dx$$

[In] integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h),x, algorithm="fricas")

[Out] integral((b*log(c*f*x + c*e) + a)/(d*f*i*x^2 + d*e*h + (d*f*h + d*e*i)*x), x)

## Sympy [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx=\frac{\int \frac{a}{eh+eix+fhx+fix^2}\,dx+\int \frac{b\log\left(ce+cfx\right)}{eh+eix+fhx+fix^2}\,dx}{d}$$

[In] integrate((a+b*ln(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h),x)

[Out] (Integral(a/(e*h + e*i*x + f*h*x + f*i*x**2), x) + Integral(b*log(c*e + c*f*x)/(e*h + e*i*x + f*h*x + f*i*x**2), x))/d

## Maxima [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx=\int \frac{b\log\left((fx+e)c\right)+a}{(dfx+de)(ix+h)}\,dx$$

[In] integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h),x, algorithm="maxima")

[Out] a*(log(f*x + e)/(d*f*h - d*e*i) - log(i*x + h)/(d*f*h - d*e*i)) + b*integrate((log(f*x + e) + log(c))/(d*f*i*x^2 + d*e*h + (f*h + e*i)*d*x), x)

## Giac [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx=\int \frac{b\log\left((fx+e)c\right)+a}{(dfx+de)(ix+h)}\,dx$$

[In] integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h),x, algorithm="giac")

[Out] integrate((b*log((f*x + e)*c) + a)/((d*f*x + d*e)*(i*x + h)), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)}\,dx=\int \frac{a+b\,\ln\left(c\,\left(e+f\,x\right)\right)}{\left(h+i\,x\right)\,\left(d\,e+d\,f\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(e + f*x)))/((h + i*x)*(d*e + d*f*x)),x)
[Out] int((a + b*log(c*(e + f*x)))/((h + i*x)*(d*e + d*f*x)), x)
```

# 3.181 $\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 151

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx = -\frac{i(e+fx)(a+b\log(c(e+fx)))}{d(fh-ei)^2(h+ix)}+\frac{bf\log(h+ix)}{d(fh-ei)^2} - \frac{f(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2} + \frac{bf\,\mathrm{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2}$$

```
[Out] -i*(f*x+e)*(a+b*ln(c*(f*x+e)))/d/(-e*i+f*h)^2/(i*x+h)+b*f*ln(i*x+h)/d/(-e*i
+f*h)^2-f*(a+b*ln(c*(f*x+e)))*ln(1+(-e*i+f*h)/i/(f*x+e))/d/(-e*i+f*h)^2+b*f
*polylog(2,(e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)^2
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2458, 12, 2389, 2379, 2438, 2351, 31}

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx = -\frac{f\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))}{d(fh-ei)^2} - \frac{i(e+fx)(a+b\log(c(e+fx)))}{d(h+ix)(fh-ei)^2} + \frac{bf\,\mathrm{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2}+\frac{bf\log(h+ix)}{d(fh-ei)^2}$$

```
[In] Int[(a + b*Log[c*(e + f*x)])/((d*e + d*f*x)*(h + i*x)^2),x]
```

```
[Out] -((i*(e + f*x)*(a + b*Log[c*(e + f*x)]))/(d*(f*h - e*i)^2*(h + i*x))) + (b*
f*Log[h + i*x])/(d*(f*h - e*i)^2) - (f*(a + b*Log[c*(e + f*x)])*Log[1 + (f*
h - e*i)/(i*(e + f*x))])/(d*(f*h - e*i)^2) + (b*f*PolyLog[2, -((f*h - e*i)/
(i*(e + f*x)))])/(d*(f*h - e*i)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
```

```
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{a+b\log(cx)}{dx\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{a+b\log(cx)}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}\,dx, x, e+fx\right)}{df}$$

$$= \frac{\text{Subst}\left(\int \frac{a+b\log(cx)}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)}\,dx, x, e+fx\right)}{d(fh-ei)} - \frac{i\text{Subst}\left(\int \frac{a+b\log(cx)}{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}\,dx, x, e+fx\right)}{df(fh-ei)}$$

$$= -\frac{i(e+fx)(a+b\log(c(e+fx)))}{d(fh-ei)^2(h+ix)} - \frac{f(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2} + \frac{(bf)\text{Subst}\left(\int \frac{\log\left(1+\frac{fh-ei}{ix}\right)}{x}\,dx, x, e+fx\right)}{d(fh-ei)^2} + \frac{(bi)\text{Subst}\left(\int \frac{1}{\frac{fh-ei}{f}+\frac{ix}{f}}\,dx, x, e+fx\right)}{d(fh-ei)^2}$$

$$= -\frac{i(e+fx)(a+b\log(c(e+fx)))}{d(fh-ei)^2(h+ix)} + \frac{bf\log(h+ix)}{d(fh-ei)^2} - \frac{f(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2} + \frac{bf\text{Li}_2\left(-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.93

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx = \frac{\frac{2(fh-ei)(a+b\log(c(e+fx)))}{h+ix} + \frac{f(a+b\log(c(e+fx)))^2}{b} + 2bf(-\log(e+fx)+\log(h+ix)) - 2f(a+b\log(c(e+fx)))}{2d(fh-ei)^2}$$

```
[In] Integrate[(a + b*Log[c*(e + f*x)])/((d*e + d*f*x)*(h + i*x)^2),x]
[Out] ((2*(f*h - e*i)*(a + b*Log[c*(e + f*x)]))/(h + i*x) + (f*(a + b*Log[c*(e +
f*x)])^2)/b + 2*b*f*(-Log[e + f*x] + Log[h + i*x]) - 2*f*(a + b*Log[c*(e +
f*x)])*Log[(f*(h + i*x))/(f*h - e*i)] - 2*b*f*PolyLog[2, (i*(e + f*x))/(-(f
*h) + e*i)])/(2*d*(f*h - e*i)^2)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(150) = 300$.

Time = 1.07 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.10

| method | result |
|---|---|
| parts | $\frac{a\left(-\frac{1}{(ei-fh)(ix+h)}-\frac{f\ln(ix+h)}{(ei-fh)^2}+\frac{f\ln(fx+e)}{(ei-fh)^2}\right)}{d}+\frac{b\left(\frac{c f^2\ln(cfx+ce)^2}{2(ei-fh)^2}-\frac{c f^2 i\left(\frac{\operatorname{dilog}\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right)}{i}+\frac{\ln(cfx+ce)\ln\left(\cdots\right.}{\cdots}\right.}{(ei-fh)^2}\cdots\right.}{\cdots}$ |
| risch | $-\frac{a}{d(ei-fh)(ix+h)}-\frac{af\ln(ix+h)}{d(ei-fh)^2}+\frac{af\ln(fx+e)}{d(ei-fh)^2}+\frac{bf\ln(cfx+ce)^2}{2d(ei-fh)^2}-\frac{bf\operatorname{dilog}\left(\frac{-cei+hcf+i(cfx+ce)}{-cei+hcf}\right)}{d(ei-fh)^2}-\frac{bf\ln(c\cdots}{\cdots}$ |
| derivativedivides | $\frac{c^3f^2a\left(\frac{1}{c(ei-fh)(cei-hcf-i(cfx+ce))}-\frac{\ln(cei-hcf-i(cfx+ce))}{c^2(ei-fh)^2}+\frac{\ln(cfx+ce)}{c^2(ei-fh)^2}\right)}{d}+\frac{c^3f^2b\left(\frac{i\left(\frac{\ln(cei-hcf-i(cfx+ce))}{c(ei-fh)i}+\frac{\ln(cfx\cdots}{c(ei-fh)(ce\cdots}\right.}{c(ei-fh)}\cdots\right.}{cf}$ |
| default | $\frac{c^3f^2a\left(\frac{1}{c(ei-fh)(cei-hcf-i(cfx+ce))}-\frac{\ln(cei-hcf-i(cfx+ce))}{c^2(ei-fh)^2}+\frac{\ln(cfx+ce)}{c^2(ei-fh)^2}\right)}{d}+\frac{c^3f^2b\left(\frac{i\left(\frac{\ln(cei-hcf-i(cfx+ce))}{c(ei-fh)i}+\frac{\ln(cfx\cdots}{c(ei-fh)(ce\cdots}\right.}{c(ei-fh)}\cdots\right.}{cf}$ |

```
[In] int((a+b*ln(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a/d*(-1/(e*i-f*h)/(i*x+h)-f/(e*i-f*h)^2*ln(i*x+h)+f/(e*i-f*h)^2*ln(f*x+e))+
b/d/c/f*(1/2*c*f^2/(e*i-f*h)^2*ln(c*f*x+c*e)^2-c*f^2/(e*i-f*h)^2*i*(dilog((
-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*e*i+c*f*h))/i+ln(c*f*x+c*e)*ln((-c*e*i+h*c*
f+i*(c*f*x+c*e))/(-c*e*i+c*f*h))/i)+c^2*f^2/(e*i-f*h)*i*(1/c/(e*i-f*h)*ln(-
c*e*i+h*c*f+i*(c*f*x+c*e))/i-ln(c*f*x+c*e)*(c*f*x+c*e)/c/(e*i-f*h)/(-c*e*i+
h*c*f+i*(c*f*x+c*e))))
```

## Fricas [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx=\int \frac{b\log\left((fx+e)c\right)+a}{(dfx+de)(ix+h)^2}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*f*x + c*e) + a)/(d*f*i^2*x^3 + d*e*h^2 + (2*d*f*h*i + d*e
*i^2)*x^2 + (d*f*h^2 + 2*d*e*h*i)*x), x)
```

## Sympy [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx = \frac{\int \frac{a}{eh^2+2ehix+ei^2x^2+fh^2x+2fhix^2+fi^2x^3}\,dx + \int \frac{b\log(ce+cfx)}{eh^2+2ehix+ei^2x^2+fh^2x+2fhix^2+fi^2x^3}\,dx}{d}$$

```
[In] integrate((a+b*ln(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)**2,x)
```

```
[Out] (Integral(a/(e*h**2 + 2*e*h*i*x + e*i**2*x**2 + f*h**2*x + 2*f*h*i*x**2 + f
*i**2*x**3), x) + Integral(b*log(c*e + c*f*x)/(e*h**2 + 2*e*h*i*x + e*i**2*
x**2 + f*h**2*x + 2*f*h*i*x**2 + f*i**2*x**3), x))/d
```

## Maxima [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx = \int \frac{b\log((fx+e)c)+a}{(dfx+de)(ix+h)^2}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="maxima")
```

```
[Out] a*(f*log(f*x + e)/(d*f^2*h^2 - 2*d*e*f*h*i + d*e^2*i^2) - f*log(i*x + h)/(d
*f^2*h^2 - 2*d*e*f*h*i + d*e^2*i^2) + 1/(d*f*h^2 - d*e*h*i + (d*f*h*i - d*e
*i^2)*x)) + b*integrate((log(f*x + e) + log(c))/(d*f*i^2*x^3 + d*e*h^2 + (2
*f*h*i + e*i^2)*d*x^2 + (f*h^2 + 2*e*h*i)*d*x), x)
```

## Giac [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx = \int \frac{b\log((fx+e)c)+a}{(dfx+de)(ix+h)^2}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((f*x + e)*c) + a)/((d*f*x + d*e)*(i*x + h)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^2}\,dx=\int \frac{a+b\,\ln\left(c\,\left(e+f\,x\right)\right)}{{\left(h+i\,x\right)}^2\,\left(d\,e+d\,f\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(e + f*x)))/((h + i*x)^2*(d*e + d*f*x)),x)
[Out] int((a + b*log(c*(e + f*x)))/((h + i*x)^2*(d*e + d*f*x)), x)
```

# 3.182 $\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 250

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx = -\frac{bf}{2d(fh-ei)^2(h+ix)} - \frac{bf^2\log(e+fx)}{2d(fh-ei)^3} + \frac{a+b\log(c(e+fx))}{2d(fh-ei)(h+ix)^2} - \frac{fi(e+fx)(a+b\log(c(e+fx)))}{d(fh-ei)^3(h+ix)} + \frac{3bf^2\log(h+ix)}{2d(fh-ei)^3} - \frac{f^2(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} + \frac{bf^2\operatorname{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3}$$

```
[Out] -1/2*b*f/d/(-e*i+f*h)^2/(i*x+h)-1/2*b*f^2*ln(f*x+e)/d/(-e*i+f*h)^3+1/2*(a+b
*ln(c*(f*x+e)))/d/(-e*i+f*h)/(i*x+h)^2-f*i*(f*x+e)*(a+b*ln(c*(f*x+e)))/d/(-
e*i+f*h)^3/(i*x+h)+3/2*b*f^2*ln(i*x+h)/d/(-e*i+f*h)^3-f^2*(a+b*ln(c*(f*x+e)
))*ln(1+(-e*i+f*h)/i/(f*x+e))/d/(-e*i+f*h)^3+b*f^2*polylog(2,(e*i-f*h)/i/(f
*x+e))/d/(-e*i+f*h)^3
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

= {2458, 12, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx = -\frac{f^2\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))}{d(fh-ei)^3} - \frac{fi(e+fx)(a+b\log(c(e+fx)))}{d(h+ix)(fh-ei)^3} + \frac{a+b\log(c(e+fx))}{2d(h+ix)^2(fh-ei)} + \frac{bf^2\,\mathrm{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} - \frac{bf^2\log(e+fx)}{2d(fh-ei)^3} + \frac{3bf^2\log(h+ix)}{2d(fh-ei)^3} - \frac{bf}{2d(h+ix)(fh-ei)^2}$$

```
[In] Int[(a + b*Log[c*(e + f*x)])/((d*e + d*f*x)*(h + i*x)^3),x]
```

```
[Out] -1/2*(b*f)/(d*(f*h - e*i)^2*(h + i*x)) - (b*f^2*Log[e + f*x])/(2*d*(f*h - e
*i)^3) + (a + b*Log[c*(e + f*x)])/(2*d*(f*h - e*i)*(h + i*x)^2) - (f*i*(e +
 f*x)*(a + b*Log[c*(e + f*x)]))/(d*(f*h - e*i)^3*(h + i*x)) + (3*b*f^2*Log[
h + i*x])/(2*d*(f*h - e*i)^3) - (f^2*(a + b*Log[c*(e + f*x)])*Log[1 + (f*h
- e*i)/(i*(e + f*x))])/(d*(f*h - e*i)^3) + (b*f^2*PolyLog[2, -((f*h - e*i)/
(i*(e + f*x)))])/(d*(f*h - e*i)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{a+b\log(cx)}{dx\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{a+b\log(cx)}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3}\,dx, x, e+fx\right)}{df}$$

$$= \frac{\text{Subst}\left(\int \frac{a+b\log(cx)}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}\,dx, x, e+fx\right)}{d(fh-ei)} - \frac{i\text{Subst}\left(\int \frac{a+b\log(cx)}{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3}\,dx, x, e+fx\right)}{df(fh-ei)}$$

$$= \frac{a+b\log(c(e+fx))}{2d(fh-ei)(h+ix)^2} + \frac{f\text{Subst}\left(\int \frac{a+b\log(cx)}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)}\,dx, x, e+fx\right)}{d(fh-ei)^2} - \frac{i\text{Subst}\left(\int \frac{a+b\log(cx)}{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}\,dx, x, e+fx\right)}{d(fh-ei)^2} - \frac{b\text{Subst}\left(\int \frac{1}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}\,dx, x, e+fx\right)}{2d(fh-ei)}$$

$$= \frac{a+b\log(c(e+fx))}{2d(fh-ei)(h+ix)^2} - \frac{fi(e+fx)(a+b\log(c(e+fx)))}{d(fh-ei)^3(h+ix)} - \frac{f^2(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} + \frac{(bf^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{fh-ei}{ix}\right)}{x}\,dx, x, e+fx\right)}{d(fh-ei)^3} + \frac{(bfi)\text{Subst}\left(\int \frac{1}{\frac{fh-ei}{f}+\frac{ix}{f}}\,dx, x, e+fx\right)}{d(fh-ei)^3} - \frac{b\text{Subst}\left(\int \left(\frac{f^2}{(fh-ei)^2x} - \frac{f^2i}{(fh-ei)(fh-ei+ix)^2} - \frac{f^2i}{(fh-ei)^2(fh-ei+ix)}\right)dx, x, e+fx\right)}{2d(fh-ei)}$$

$$= -\frac{bf}{2d(fh-ei)^2(h+ix)} - \frac{bf^2\log(e+fx)}{2d(fh-ei)^3} + \frac{a+b\log(c(e+fx))}{2d(fh-ei)(h+ix)^2} - \frac{fi(e+fx)(a+b\log(c(e+fx)))}{d(fh-ei)^3(h+ix)} + \frac{3bf^2\log(h+ix)}{2d(fh-ei)^3} - \frac{f^2(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} + \frac{bf^2\text{Li}_2\left(-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3}$$

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.90

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx$$

$$= \frac{\frac{(fh-ei)^2(a+b\log(c(e+fx)))}{(h+ix)^2} + \frac{2f(fh-ei)(a+b\log(c(e+fx)))}{h+ix} + \frac{f^2(a+b\log(c(e+fx)))^2}{b} + 2bf^2(-\log(e+fx)+\log(h+ix)) -}{2d(f}$$

```
[In] Integrate[(a + b*Log[c*(e + f*x)])/((d*e + d*f*x)*(h + i*x)^3),x]
```

```
[Out] (((f*h - e*i)^2*(a + b*Log[c*(e + f*x)]))/(h + i*x)^2 + (2*f*(f*h - e*i)*(a
 + b*Log[c*(e + f*x)]))/(h + i*x) + (f^2*(a + b*Log[c*(e + f*x)])^2)/b + 2*
b*f^2*(-Log[e + f*x] + Log[h + i*x]) - (b*f*(f*h - e*i + f*(h + i*x)*Log[e
+ f*x] - f*(h + i*x)*Log[h + i*x]))/(h + i*x) - 2*f^2*(a + b*Log[c*(e + f*x
)])*Log[(f*(h + i*x))/(f*h - e*i)] - 2*b*f^2*PolyLog[2, (i*(e + f*x))/(-(f*
h) + e*i)])/(2*d*(f*h - e*i)^3)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(241) = 482.

Time = 1.20 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.04

| method | result |
|---|---|
| parts | $\frac{a\left(-\frac{1}{2(ei-fh)(ix+h)^{2}}+\frac{f^{2}\ln(ix+h)}{(ei-fh)^{3}}+\frac{f}{(ei-fh)^{2}(ix+h)}-\frac{f^{2}\ln(fx+e)}{(ei-fh)^{3}}\right)}{d}+\frac{b\left(-\frac{c f^{3}\ln(cfx+ce)^{2}}{2(ei-fh)^{3}}+\frac{c^{3}f^{3}i\left(-\frac{\ln(-cei+hcf+i(c}{i}\right.}{}\right.}{}$ |
| derivativedivides | $-\frac{c^{4}f^{3}a\left(\frac{\ln(cfx+ce)}{c^{3}(ei-fh)^{3}}+\frac{1}{c^{2}(ei-fh)^{2}(cei-hcf-i(cfx+ce))}-\frac{\ln(cei-hcf-i(cfx+ce))}{c^{3}(ei-fh)^{3}}+\frac{1}{2c(ei-fh)(cei-hcf-i(cfx+ce))^{2}}\right)}{d}-\frac{c^{4}f^{3}b}{}$ |
| default | $-\frac{c^{4}f^{3}a\left(\frac{\ln(cfx+ce)}{c^{3}(ei-fh)^{3}}+\frac{1}{c^{2}(ei-fh)^{2}(cei-hcf-i(cfx+ce))}-\frac{\ln(cei-hcf-i(cfx+ce))}{c^{3}(ei-fh)^{3}}+\frac{1}{2c(ei-fh)(cei-hcf-i(cfx+ce))^{2}}\right)}{d}-\frac{c^{4}f^{3}b}{}$ |
| risch | $-\frac{a}{2d(ei-fh)(ix+h)^{2}}+\frac{a f^{2}\ln(ix+h)}{d(ei-fh)^{3}}+\frac{af}{d(ei-fh)^{2}(ix+h)}-\frac{a f^{2}\ln(fx+e)}{d(ei-fh)^{3}}-\frac{b f^{2}\ln(cfx+ce)^{2}}{2d(ei-fh)^{3}}-\frac{3b f^{2}\ln(-c}{2c}$ |

```
[In] int((a+b*ln(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a/d*(-1/2/(e*i-f*h)/(i*x+h)^2+f^2/(e*i-f*h)^3*ln(i*x+h)+f/(e*i-f*h)^2/(i*x+
h)-f^2/(e*i-f*h)^3*ln(f*x+e))+b/d/c/f*(-1/2*c*f^3/(e*i-f*h)^3*ln(c*f*x+c*e)
^2+c^3*f^3/(e*i-f*h)*i*(-1/2/c^2/(e*i-f*h)^2*(ln(-c*e*i+h*c*f+i*(c*f*x+c*e)
)/i+c*(e*i-f*h)/i/(-c*e*i+h*c*f+i*(c*f*x+c*e)))+1/2*ln(c*f*x+c*e)*(-2*c*e*i
+2*h*c*f+i*(c*f*x+c*e))*(c*f*x+c*e)/(-c*e*i+h*c*f+i*(c*f*x+c*e))^2/c^2/(e*i
-f*h)^2)+c*f^3/(e*i-f*h)^3*i*(dilog((-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*e*i+c*
f*h))/i+ln(c*f*x+c*e)*ln((-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*e*i+c*f*h))/i)-c^
2*f^3/(e*i-f*h)^2*i*(1/c/(e*i-f*h)*ln(-c*e*i+h*c*f+i*(c*f*x+c*e))/i-ln(c*f*
x+c*e)*(c*f*x+c*e)/c/(e*i-f*h)/(-c*e*i+h*c*f+i*(c*f*x+c*e))))
```

## Fricas [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx = \int \frac{b\log\left((fx+e)c\right)+a}{(dfx+de)(ix+h)^3}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*f*x + c*e) + a)/(d*f*i^3*x^4 + d*e*h^3 + (3*d*f*h*i^2 + d
*e*i^3)*x^3 + 3*(d*f*h^2*i + d*e*h*i^2)*x^2 + (d*f*h^3 + 3*d*e*h^2*i)*x), x
)
```

## Sympy [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx = \frac{\int \frac{a}{eh^3+3eh^2ix+3ehi^2x^2+ei^3x^3+fh^3x+3fh^2ix^2+3fhi^2x^3+fi^3x^4}\,dx + \int \frac{b\log\left(ce+cfx\right)}{eh^3+3eh^2ix+3ehi^2x^2+ei^3x^3+fh^3x+3fh^2ix^2+3fhi^2x^3+fi^3x^4}\,dx}{d}$$

```
[In] integrate((a+b*ln(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)**3,x)
```

```
[Out] (Integral(a/(e*h**3 + 3*e*h**2*i*x + 3*e*h*i**2*x**2 + e*i**3*x**3 + f*h**3
*x + 3*f*h**2*i*x**2 + 3*f*h*i**2*x**3 + f*i**3*x**4), x) + Integral(b*log(
c*e + c*f*x)/(e*h**3 + 3*e*h**2*i*x + 3*e*h*i**2*x**2 + e*i**3*x**3 + f*h**
3*x + 3*f*h**2*i*x**2 + 3*f*h*i**2*x**3 + f*i**3*x**4), x))/d
```

## Maxima [F]

$$\int \frac{a+b\log(c(e+fx))}{(de+dfx)(h+ix)^3}\,dx = \int \frac{b\log\left((fx+e)c\right)+a}{(dfx+de)(ix+h)^3}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*f^2*log(f*x + e)/(d*f^3*h^3 - 3*d*e*f^2*h^2*i + 3*d*e^2*f*h*i^2 - d*
e^3*i^3) - 2*f^2*log(i*x + h)/(d*f^3*h^3 - 3*d*e*f^2*h^2*i + 3*d*e^2*f*h*i^
2 - d*e^3*i^3) + (2*f*i*x + 3*f*h - e*i)/(d*f^2*h^4 - 2*d*e*f*h^3*i + d*e^2
*h^2*i^2 + (d*f^2*h^2*i^2 - 2*d*e*f*h*i^3 + d*e^2*i^4)*x^2 + 2*(d*f^2*h^3*i
 - 2*d*e*f*h^2*i^2 + d*e^2*h*i^3)*x))*a + b*integrate((log(f*x + e) + log(c
))/(d*f*i^3*x^4 + d*e*h^3 + (3*f*h*i^2 + e*i^3)*d*x^3 + 3*(f*h^2*i + e*h*i^
2)*d*x^2 + (f*h^3 + 3*e*h^2*i)*d*x), x)
```

## Giac [F]

$$\int \frac{a + b\log(c(e + fx))}{(de + dfx)(h + ix)^3}\,dx = \int \frac{b\log\left((fx + e)c\right) + a}{(dfx + de)(ix + h)^3}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="giac")
[Out] integrate((b*log((f*x + e)*c) + a)/((d*f*x + d*e)*(i*x + h)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(c(e + fx))}{(de + dfx)(h + ix)^3}\,dx = \int \frac{a + b\,\ln\left(c\,(e + f\,x)\right)}{(h + i\,x)^3\,(d\,e + d\,f\,x)}\,dx$$

```
[In] int((a + b*log(c*(e + f*x)))/((h + i*x)^3*(d*e + d*f*x)),x)
[Out] int((a + b*log(c*(e + f*x)))/((h + i*x)^3*(d*e + d*f*x)), x)
```

# 3.183 $\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$

## Optimal result

Integrand size = 32, antiderivative size = 579

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = -\frac{4abi(fh-ei)^3x}{df^4}+\frac{8b^2i(fh-ei)^3x}{df^4}+\frac{3b^2i^2(fh-ei)^2(e+fx)^2}{2df^5}+\frac{8b^2i^3(fh-ei)(e+fx)^3}{27df^5}+\frac{b^2i^4(e+fx)^4}{32df^5}+\frac{7b^2(fh-ei)^4\log^2(e+fx)}{12df^5}-\frac{4b^2i(fh-ei)^3(e+fx)\log(c(e+fx))}{df^5}-\frac{4bi(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5}-\frac{3bi^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{df^5}-\frac{8bi^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{9df^5}-\frac{bi^4(e+fx)^4(a+b\log(c(e+fx)))}{8df^5}-\frac{7b(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{6df^5}+\frac{2i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))^2}{df^5}+\frac{i^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^5}+\frac{(fh-ei)(h+ix)^3(a+b\log(c(e+fx)))^2}{3df^2}+\frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{4df}+\frac{(fh-ei)^4(a+b\log(c(e+fx)))^3}{3bdf^5}$$

```
[Out] -4*a*b*i*(-e*i+f*h)^3*x/d/f^4+8*b^2*i*(-e*i+f*h)^3*x/d/f^4+3/2*b^2*i^2*(-e*
i+f*h)^2*(f*x+e)^2/d/f^5+8/27*b^2*i^3*(-e*i+f*h)*(f*x+e)^3/d/f^5+1/32*b^2*i
^4*(f*x+e)^4/d/f^5+7/12*b^2*(-e*i+f*h)^4*ln(f*x+e)^2/d/f^5-4*b^2*i*(-e*i+f*
h)^3*(f*x+e)*ln(c*(f*x+e))/d/f^5-4*b*i*(-e*i+f*h)^3*(f*x+e)*(a+b*ln(c*(f*x+
e)))/d/f^5-3*b*i^2*(-e*i+f*h)^2*(f*x+e)^2*(a+b*ln(c*(f*x+e)))/d/f^5-8/9*b*i
^3*(-e*i+f*h)*(f*x+e)^3*(a+b*ln(c*(f*x+e)))/d/f^5-1/8*b*i^4*(f*x+e)^4*(a+b*
ln(c*(f*x+e)))/d/f^5-7/6*b*(-e*i+f*h)^4*ln(f*x+e)*(a+b*ln(c*(f*x+e)))/d/f^5
+2*i*(-e*i+f*h)^3*(f*x+e)*(a+b*ln(c*(f*x+e)))^2/d/f^5+1/2*i^2*(-e*i+f*h)^2*
```

```
(f*x+e)^2*(a+b*ln(c*(f*x+e)))^2/d/f^5+1/3*(-e*i+f*h)*(i*x+h)^3*(a+b*ln(c*(f
*x+e)))^2/d/f^2+1/4*(i*x+h)^4*(a+b*ln(c*(f*x+e)))^2/d/f+1/3*(-e*i+f*h)^4*(a
+b*ln(c*(f*x+e)))^3/b/d/f^5
```

## Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.469, Rules used = {2458, 12, 2388, 2339, 30, 2333, 2332, 2367, 2342, 2341, 2356, 45, 2372, 14, 2338}

$$\begin{aligned}\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = &-\frac{8bi^3(e+fx)^3(fh-ei)(a+b\log(c(e+fx)))}{9df^5}\\ &+\frac{i^2(e+fx)^2(fh-ei)^2(a+b\log(c(e+fx)))^2}{2df^5}\\ &-\frac{3bi^2(e+fx)^2(fh-ei)^2(a+b\log(c(e+fx)))}{df^5}\\ &+\frac{(fh-ei)^4(a+b\log(c(e+fx)))^3}{3bdf^5}\\ &-\frac{7b(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{6df^5}\\ &+\frac{2i(e+fx)(fh-ei)^3(a+b\log(c(e+fx)))^2}{df^5}\\ &-\frac{4bi(e+fx)(fh-ei)^3(a+b\log(c(e+fx)))}{df^5}\\ &-\frac{bi^4(e+fx)^4(a+b\log(c(e+fx)))}{8df^5}\\ &+\frac{(h+ix)^3(fh-ei)(a+b\log(c(e+fx)))^2}{3df^2}\\ &+\frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{4df}\\ &-\frac{4abix(fh-ei)^3}{df^4}\\ &-\frac{4b^2i(e+fx)(fh-ei)^3\log(c(e+fx))}{df^5}\\ &+\frac{8b^2i^3(e+fx)^3(fh-ei)}{27df^5}\\ &+\frac{3b^2i^2(e+fx)^2(fh-ei)^2}{2df^5}\\ &+\frac{7b^2(fh-ei)^4\log^2(e+fx)}{12df^5}\\ &+\frac{b^2i^4(e+fx)^4}{32df^5}+\frac{8b^2ix(fh-ei)^3}{df^4}\end{aligned}$$

```
[In] Int[((h + i*x)^4*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

```
[Out] (-4*a*b*i*(f*h - e*i)^3*x)/(d*f^4) + (8*b^2*i*(f*h - e*i)^3*x)/(d*f^4) + (3
*b^2*i^2*(f*h - e*i)^2*(e + f*x)^2)/(2*d*f^5) + (8*b^2*i^3*(f*h - e*i)*(e +
 f*x)^3)/(27*d*f^5) + (b^2*i^4*(e + f*x)^4)/(32*d*f^5) + (7*b^2*(f*h - e*i)
^4*Log[e + f*x]^2)/(12*d*f^5) - (4*b^2*i*(f*h - e*i)^3*(e + f*x)*Log[c*(e +
 f*x)])/(d*f^5) - (4*b*i*(f*h - e*i)^3*(e + f*x)*(a + b*Log[c*(e + f*x)]))/
(d*f^5) - (3*b*i^2*(f*h - e*i)^2*(e + f*x)^2*(a + b*Log[c*(e + f*x)]))/(d*f
^5) - (8*b*i^3*(f*h - e*i)*(e + f*x)^3*(a + b*Log[c*(e + f*x)]))/(9*d*f^5)
- (b*i^4*(e + f*x)^4*(a + b*Log[c*(e + f*x)]))/(8*d*f^5) - (7*b*(f*h - e*i)
^4*Log[e + f*x]*(a + b*Log[c*(e + f*x)]))/(6*d*f^5) + (2*i*(f*h - e*i)^3*(e
 + f*x)*(a + b*Log[c*(e + f*x)])^2)/(d*f^5) + (i^2*(f*h - e*i)^2*(e + f*x)^
2*(a + b*Log[c*(e + f*x)])^2)/(2*d*f^5) + ((f*h - e*i)*(h + i*x)^3*(a + b*L
og[c*(e + f*x)])^2)/(3*d*f^2) + ((h + i*x)^4*(a + b*Log[c*(e + f*x)])^2)/(4
*d*f) + ((f*h - e*i)^4*(a + b*Log[c*(e + f*x)])^3)/(3*b*d*f^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
```

```
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
```

```
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^4 (a+b\log(cx))^2}{dx}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^4 (a+b\log(cx))^2}{x}\,dx, x, e+fx\right)}{df}$$

$$= \frac{i\text{Subst}\left(\int \left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3 (a+b\log(cx))^2\,dx, x, e+fx\right)}{df^2} + \frac{(fh-ei)\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3 (a+b\log(cx))^2}{x}\,dx, x, e+fx\right)}{df^2}$$

$$=\frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{4df}-\frac{b\text{Subst}\left(\int\frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^4(a+b\log(cx))}{x}dx,x,e+fx\right)}{2df}$$
$$+\frac{(i(fh-ei))\text{Subst}\left(\int\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2(a+b\log(cx))^2dx,x,e+fx\right)}{df^3}$$
$$+\frac{(fh-ei)^2\text{Subst}\left(\int\frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2(a+b\log(cx))^2}{x}dx,x,e+fx\right)}{df^3}$$

$$=-\frac{2bi(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5}$$
$$-\frac{3bi^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^5}$$
$$-\frac{2bi^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{3df^5}$$
$$-\frac{bi^4(e+fx)^4(a+b\log(c(e+fx)))}{8df^5}$$
$$-\frac{b(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{2df^5}$$
$$+\frac{(fh-ei)(h+ix)^3(a+b\log(c(e+fx)))^2}{3df^2}+\frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{4df}$$
$$+\frac{b^2\text{Subst}\left(\int\frac{48i(fh-ei)^3+36i^2(fh-ei)^2x+16i^3(fh-ei)x^2+3i^4x^3+\frac{12(fh-ei)^4\log(x)}{x}}{12f^4}dx,x,e+fx\right)}{2df}$$
$$-\frac{(2b(fh-ei))\text{Subst}\left(\int\frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3(a+b\log(cx))}{x}dx,x,e+fx\right)}{3df^2}$$
$$+\frac{(i(fh-ei)^2)\,\text{Subst}\left(\int\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)(a+b\log(cx))^2dx,x,e+fx\right)}{df^4}$$
$$+\frac{(fh-ei)^3\text{Subst}\left(\int\frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)(a+b\log(cx))^2}{x}dx,x,e+fx\right)}{df^4}$$

$$
\begin{aligned}
&= -\frac{4bi(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5} \\
&\quad -\frac{5bi^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^5} \\
&\quad -\frac{8bi^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{9df^5} \\
&\quad -\frac{bi^4(e+fx)^4(a+b\log(c(e+fx)))}{8df^5} \\
&\quad -\frac{7b(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{6df^5} \\
&\quad +\frac{(fh-ei)(h+ix)^3(a+b\log(c(e+fx)))^2}{3df^2}+\frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{4df} \\
&\quad +\frac{b^2\text{Subst}\left(\int\left(48i(fh-ei)^3+36i^2(fh-ei)^2x+16i^3(fh-ei)x^2+3i^4x^3+\frac{12(fh-ei)^4\log(x)}{x}\right)dx,x\right.}{24df^5} \\
&\quad +\frac{(2b^2(fh-ei))\,\text{Subst}\left(\int\frac{ix(18f^2h^2+9fhi(-4e+x)+i^2(18e^2-9ex+2x^2))+6(fh-ei)^3\log(x)}{6f^3x}dx,x,e+fx\right)}{3df^2} \\
&\quad +\frac{(i(fh-ei)^2)\,\text{Subst}\left(\int\left(\frac{(fh-ei)(a+b\log(cx))^2}{f}+\frac{ix(a+b\log(cx))^2}{f}\right)dx,x,e+fx\right)}{df^4} \\
&\quad +\frac{(i(fh-ei)^3)\,\text{Subst}\left(\int(a+b\log(cx))^2\,dx,x,e+fx\right)}{df^5} \\
&\quad +\frac{(fh-ei)^4\text{Subst}\left(\int\frac{(a+b\log(cx))^2}{x}dx,x,e+fx\right)}{df^5}
\end{aligned}
$$

$$= \frac{2b^2i(fh-ei)^3x}{df^4} + \frac{3b^2i^2(fh-ei)^2(e+fx)^2}{4df^5} + \frac{2b^2i^3(fh-ei)(e+fx)^3}{9df^5} + \frac{b^2i^4(e+fx)^4}{32df^5} - \frac{4bi(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5} - \frac{5bi^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^5} - \frac{8bi^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{9df^5} - \frac{bi^4(e+fx)^4(a+b\log(c(e+fx)))}{8df^5} - \frac{7b(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{6df^5} + \frac{i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))^2}{df^5} + \frac{(fh-ei)(h+ix)^3(a+b\log(c(e+fx)))^2}{3df^2} + \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{4df} + \frac{(b^2(fh-ei))\,\text{Subst}\left(\int \frac{ix(18f^2h^2+9fhi(-4e+x)+i^2(18e^2-9ex+2x^2))+6(fh-ei)^3\log(x)}{x}\,dx, x, e+fx\right)}{9df^5} + \frac{(i^2(fh-ei)^2)\,\text{Subst}\left(\int x(a+b\log(cx))^2\,dx, x, e+fx\right)}{df^5} + \frac{(i(fh-ei)^3)\,\text{Subst}\left(\int (a+b\log(cx))^2\,dx, x, e+fx\right)}{df^5} - \frac{(2bi(fh-ei)^3)\,\text{Subst}(\int (a+b\log(cx))\,dx, x, e+fx)}{df^5} + \frac{(fh-ei)^4\text{Subst}\left(\int x^2\,dx, x, a+b\log(c(e+fx))\right)}{bdf^5} + \frac{(b^2(fh-ei)^4)\,\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, e+fx\right)}{2df^5}$$

$$
\begin{aligned}
&= -\frac{2abi(fh-ei)^3x}{df^4} + \frac{2b^2i(fh-ei)^3x}{df^4} + \frac{3b^2i^2(fh-ei)^2(e+fx)^2}{4df^5} \\
&+ \frac{2b^2i^3(fh-ei)(e+fx)^3}{9df^5} + \frac{b^2i^4(e+fx)^4}{32df^5} + \frac{b^2(fh-ei)^4\log^2(e+fx)}{4df^5} \\
&- \frac{4bi(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5} \\
&- \frac{5bi^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^5} \\
&- \frac{8bi^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{9df^5} \\
&- \frac{bi^4(e+fx)^4(a+b\log(c(e+fx)))}{8df^5} \\
&- \frac{7b(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{6df^5} \\
&+ \frac{2i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))^2}{df^5} \\
&+ \frac{i^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^5} \\
&+ \frac{(fh-ei)(h+ix)^3(a+b\log(c(e+fx)))^2}{3df^2} \\
&+ \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{4df} + \frac{(fh-ei)^4(a+b\log(c(e+fx)))^3}{3bdf^5} \\
&+ \frac{(b^2(fh-ei))\operatorname{Subst}\left(\int \left(i(18(fh-ei)^2+9i(fh-ei)x+2i^2x^2)+\frac{6(fh-ei)^3\log(x)}{x}\right)dx, x, e+fx\right)}{9df^5} \\
&- \frac{(bi^2(fh-ei)^2)\operatorname{Subst}(\int x(a+b\log(cx))\,dx, x, e+fx)}{df^5} \\
&- \frac{(2bi(fh-ei)^3)\operatorname{Subst}(\int (a+b\log(cx))\,dx, x, e+fx)}{df^5} \\
&- \frac{(2b^2i(fh-ei)^3)\operatorname{Subst}(\int \log(cx)\,dx, x, e+fx)}{df^5}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4abi(fh-ei)^3x}{df^4} + \frac{4b^2i(fh-ei)^3x}{df^4} + \frac{b^2i^2(fh-ei)^2(e+fx)^2}{df^5} \\
&+ \frac{2b^2i^3(fh-ei)(e+fx)^3}{9df^5} + \frac{b^2i^4(e+fx)^4}{32df^5} \\
&+ \frac{b^2(fh-ei)^4\log^2(e+fx)}{4df^5} - \frac{2b^2i(fh-ei)^3(e+fx)\log(c(e+fx))}{df^5} \\
&- \frac{4bi(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5} \\
&- \frac{3bi^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{df^5} \\
&- \frac{8bi^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{9df^5} \\
&- \frac{bi^4(e+fx)^4(a+b\log(c(e+fx)))}{8df^5} \\
&- \frac{7b(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{6df^5} \\
&+ \frac{2i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))^2}{df^5} \\
&+ \frac{i^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^5} \\
&+ \frac{(fh-ei)(h+ix)^3(a+b\log(c(e+fx)))^2}{3df^2} \\
&+ \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{4df} + \frac{(fh-ei)^4(a+b\log(c(e+fx)))^3}{3bdf^5} \\
&+ \frac{(b^2i(fh-ei))\,\mathrm{Subst}\left(\int\left(18(fh-ei)^2+9i(fh-ei)x+2i^2x^2\right)dx,x,e+fx\right)}{9df^5} \\
&- \frac{(2b^2i(fh-ei)^3)\,\mathrm{Subst}(\int\log(cx)\,dx,x,e+fx)}{df^5} \\
&+ \frac{(2b^2(fh-ei)^4)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,e+fx\right)}{3df^5}
\end{aligned}
$$

$$= -\frac{4abi(fh-ei)^3x}{df^4} + \frac{8b^2i(fh-ei)^3x}{df^4} + \frac{3b^2i^2(fh-ei)^2(e+fx)^2}{2df^5} + \frac{8b^2i^3(fh-ei)(e+fx)^3}{27df^5} + \frac{b^2i^4(e+fx)^4}{32df^5} + \frac{7b^2(fh-ei)^4\log^2(e+fx)}{12df^5} - \frac{4b^2i(fh-ei)^3(e+fx)\log(c(e+fx))}{df^5} - \frac{4bi(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))}{df^5} - \frac{3bi^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))}{df^5} - \frac{8bi^3(fh-ei)(e+fx)^3(a+b\log(c(e+fx)))}{9df^5} - \frac{bi^4(e+fx)^4(a+b\log(c(e+fx)))}{8df^5} - \frac{7b(fh-ei)^4\log(e+fx)(a+b\log(c(e+fx)))}{6df^5} + \frac{2i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))^2}{df^5} + \frac{i^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^5} + \frac{(fh-ei)(h+ix)^3(a+b\log(c(e+fx)))^2}{3df^2} + \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{4df} + \frac{(fh-ei)^4(a+b\log(c(e+fx)))^3}{3bdf^5}$$

## Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.65

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= \frac{3456i(fh-ei)^3(e+fx)(a+b\log(c(e+fx)))^2 + 2592i^2(fh-ei)^2(e+fx)^2(a+b\log(c(e+fx)))^2 + 11\ldots}{}$$

```
[In] Integrate[((h + i*x)^4*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

```
[Out] (3456*i*(f*h - e*i)^3*(e + f*x)*(a + b*Log[c*(e + f*x)])^2 + 2592*i^2*(f*h
- e*i)^2*(e + f*x)^2*(a + b*Log[c*(e + f*x)])^2 + 1152*i^3*(f*h - e*i)*(e +
 f*x)^3*(a + b*Log[c*(e + f*x)])^2 + 216*i^4*(e + f*x)^4*(a + b*Log[c*(e +
f*x)])^2 + (288*(f*h - e*i)^4*(a + b*Log[c*(e + f*x)])^3)/b - 6912*b*i*(f*h
 - e*i)^3*((a - b)*f*x + b*(e + f*x)*Log[c*(e + f*x)]) + 1296*b*i^2*(f*h -
e*i)^2*(b*f*x*(2*e + f*x) - 2*(e + f*x)^2*(a + b*Log[c*(e + f*x)])) + 256*b
*i^3*(f*h - e*i)*(b*f*x*(3*e^2 + 3*e*f*x + f^2*x^2) - 3*(e + f*x)^3*(a + b*
```

```
Log[c*(e + f*x)])) + 27*b*i^4*(b*f*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3
*x^3) - 4*(e + f*x)^4*(a + b*Log[c*(e + f*x)])))/(864*d*f^5)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. 2(557) = 1114.

Time = 0.92 (sec) , antiderivative size = 1135, normalized size of antiderivative = 1.96

| method | result | size |
|---|---|---|
| norman | Expression too large to display | 1135 |
| risch | Expression too large to display | 1475 |
| parts | Expression too large to display | 1541 |
| derivativedivides | Expression too large to display | 1891 |
| default | Expression too large to display | 1891 |
| parallelrisch | Expression too large to display | 1912 |

```
[In] int((i*x+h)^4*(a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/72*(72*a^2*e^4*i^4-288*a^2*e^3*f*h*i^3+432*a^2*e^2*f^2*h^2*i^2-288*a^2*e*
f^3*h^3*i+72*a^2*f^4*h^4-300*a*b*e^4*i^4+1056*a*b*e^3*f*h*i^3-1296*a*b*e^2*
f^2*h^2*i^2+576*a*b*e*f^3*h^3*i+415*b^2*e^4*i^4-1360*b^2*e^3*f*h*i^3+1512*b
^2*e^2*f^2*h^2*i^2-576*b^2*e*f^3*h^3*i)/d/f^5*ln(c*(f*x+e))+1/12*b*(12*a*e^
4*i^4-48*a*e^3*f*h*i^3+72*a*e^2*f^2*h^2*i^2-48*a*e*f^3*h^3*i+12*a*f^4*h^4-2
5*b*e^4*i^4+88*b*e^3*f*h*i^3-108*b*e^2*f^2*h^2*i^2+48*b*e*f^3*h^3*i)/d/f^5*
ln(c*(f*x+e))^2+1/3*b^2*(e^4*i^4-4*e^3*f*h*i^3+6*e^2*f^2*h^2*i^2-4*e*f^3*h^
3*i+f^4*h^4)/d/f^5*ln(c*(f*x+e))^3-1/72*i*(72*a^2*e^3*i^3-288*a^2*e^2*f*h*i
^2+432*a^2*e*f^2*h^2*i-288*a^2*f^3*h^3-300*a*b*e^3*i^3+1056*a*b*e^2*f*h*i^2
-1296*a*b*e*f^2*h^2*i+576*a*b*f^3*h^3+415*b^2*e^3*i^3-1360*b^2*e^2*f*h*i^2+
1512*b^2*e*f^2*h^2*i-576*b^2*f^3*h^3)/d/f^4*x+1/144*i^2*(72*a^2*e^2*i^2-288
*a^2*e*f*h*i+432*a^2*f^2*h^2-156*a*b*e^2*i^2+480*a*b*e*f*h*i-432*a*b*f^2*h^
2+115*b^2*e^2*i^2-304*b^2*e*f*h*i+216*b^2*f^2*h^2)/d/f^3*x^2-1/216*i^3*(72*
a^2*e*i-288*a^2*f*h-84*a*b*e*i+192*a*b*f*h+37*b^2*e*i-64*b^2*f*h)/f^2/d*x^3
+1/32*i^4*(8*a^2-4*a*b+b^2)/d/f*x^4+1/4*b^2*i^4/d/f*x^4*ln(c*(f*x+e))^2-1/6
*b*i*(12*a*e^3*i^3-48*a*e^2*f*h*i^2+72*a*e*f^2*h^2*i-48*a*f^3*h^3-25*b*e^3*
i^3+88*b*e^2*f*h*i^2-108*b*e*f^2*h^2*i+48*b*f^3*h^3)/d/f^4*x*ln(c*(f*x+e))+
1/12*b*i^2*(12*a*e^2*i^2-48*a*e*f*h*i+72*a*f^2*h^2-13*b*e^2*i^2+40*b*e*f*h*
i-36*b*f^2*h^2)/d/f^3*x^2*ln(c*(f*x+e))-1/18*b*i^3*(12*a*e*i-48*a*f*h-7*b*e
*i+16*b*f*h)/d/f^2*x^3*ln(c*(f*x+e))+1/8*b*i^4*(4*a-b)/d/f*x^4*ln(c*(f*x+e)
)-b^2*i*(e^3*i^3-4*e^2*f*h*i^2+6*e*f^2*h^2*i-4*f^3*h^3)/d/f^4*x*ln(c*(f*x+e
))^2+1/2*b^2*i^2*(e^2*i^2-4*e*f*h*i+6*f^2*h^2)/d/f^3*x^2*ln(c*(f*x+e))^2-1/
3*b^2*i^3*(e*i-4*f*h)/d/f^2*x^3*ln(c*(f*x+e))^2
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 939, normalized size of antiderivative = 1.62

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= \frac{27\,(8\,a^2-4\,ab+b^2)f^4i^4x^4+4\,(32\,(9\,a^2-6\,ab+2\,b^2)f^4hi^3-(72\,a^2-84\,ab+37\,b^2)ef^3i^4)x^3+288\,(b^2f^4}{}$$

```
[In] integrate((i*x+h)^4*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="fricas
")
```

```
[Out] 1/864*(27*(8*a^2 - 4*a*b + b^2)*f^4*i^4*x^4 + 4*(32*(9*a^2 - 6*a*b + 2*b^2)
*f^4*h*i^3 - (72*a^2 - 84*a*b + 37*b^2)*e*f^3*i^4)*x^3 + 288*(b^2*f^4*h^4 -
 4*b^2*e*f^3*h^3*i + 6*b^2*e^2*f^2*h^2*i^2 - 4*b^2*e^3*f*h*i^3 + b^2*e^4*i^
4)*log(c*f*x + c*e)^3 + 6*(216*(2*a^2 - 2*a*b + b^2)*f^4*h^2*i^2 - 16*(18*a
^2 - 30*a*b + 19*b^2)*e*f^3*h*i^3 + (72*a^2 - 156*a*b + 115*b^2)*e^2*f^2*i^
4)*x^2 + 72*(3*b^2*f^4*i^4*x^4 + 12*a*b*f^4*h^4 - 48*(a*b - b^2)*e*f^3*h^3*
i + 36*(2*a*b - 3*b^2)*e^2*f^2*h^2*i^2 - 8*(6*a*b - 11*b^2)*e^3*f*h*i^3 + (
12*a*b - 25*b^2)*e^4*i^4 + 4*(4*b^2*f^4*h*i^3 - b^2*e*f^3*i^4)*x^3 + 6*(6*b
^2*f^4*h^2*i^2 - 4*b^2*e*f^3*h*i^3 + b^2*e^2*f^2*i^4)*x^2 + 12*(4*b^2*f^4*h
^3*i - 6*b^2*e*f^3*h^2*i^2 + 4*b^2*e^2*f^2*h*i^3 - b^2*e^3*f*i^4)*x)*log(c*
f*x + c*e)^2 + 12*(288*(a^2 - 2*a*b + 2*b^2)*f^4*h^3*i - 216*(2*a^2 - 6*a*b
 + 7*b^2)*e*f^3*h^2*i^2 + 16*(18*a^2 - 66*a*b + 85*b^2)*e^2*f^2*h*i^3 - (72
*a^2 - 300*a*b + 415*b^2)*e^3*f*i^4)*x + 12*(9*(4*a*b - b^2)*f^4*i^4*x^4 +
72*a^2*f^4*h^4 - 288*(a^2 - 2*a*b + 2*b^2)*e*f^3*h^3*i + 216*(2*a^2 - 6*a*b
 + 7*b^2)*e^2*f^2*h^2*i^2 - 16*(18*a^2 - 66*a*b + 85*b^2)*e^3*f*h*i^3 + (72
*a^2 - 300*a*b + 415*b^2)*e^4*i^4 + 4*(16*(3*a*b - b^2)*f^4*h*i^3 - (12*a*b
 - 7*b^2)*e*f^3*i^4)*x^3 + 6*(36*(2*a*b - b^2)*f^4*h^2*i^2 - 8*(6*a*b - 5*b
^2)*e*f^3*h*i^3 + (12*a*b - 13*b^2)*e^2*f^2*i^4)*x^2 + 12*(48*(a*b - b^2)*f
^4*h^3*i - 36*(2*a*b - 3*b^2)*e*f^3*h^2*i^2 + 8*(6*a*b - 11*b^2)*e^2*f^2*h*
i^3 - (12*a*b - 25*b^2)*e^3*f*i^4)*x)*log(c*f*x + c*e))/(d*f^5)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1479 vs. 2(534) = 1068.

Time = 1.88 (sec) , antiderivative size = 1479, normalized size of antiderivative = 2.55

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \text{Too large to display}$$

```
[In] integrate((i*x+h)**4*(a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e),x)
```

```
[Out] x**4*(a**2*i**4/(4*d*f) - a*b*i**4/(8*d*f) + b**2*i**4/(32*d*f)) + x**3*(-a
**2*e*i**4/(3*d*f**2) + 4*a**2*h*i**3/(3*d*f) + 7*a*b*e*i**4/(18*d*f**2) -
8*a*b*h*i**3/(9*d*f) - 37*b**2*e*i**4/(216*d*f**2) + 8*b**2*h*i**3/(27*d*f)
) + x**2*(a**2*e**2*i**4/(2*d*f**3) - 2*a**2*e*h*i**3/(d*f**2) + 3*a**2*h**
2*i**2/(d*f) - 13*a*b*e**2*i**4/(12*d*f**3) + 10*a*b*e*h*i**3/(3*d*f**2) -
3*a*b*h**2*i**2/(d*f) + 115*b**2*e**2*i**4/(144*d*f**3) - 19*b**2*e*h*i**3/
(9*d*f**2) + 3*b**2*h**2*i**2/(2*d*f)) + x*(-a**2*e**3*i**4/(d*f**4) + 4*a*
*2*e**2*h*i**3/(d*f**3) - 6*a**2*e*h**2*i**2/(d*f**2) + 4*a**2*h**3*i/(d*f)
 + 25*a*b*e**3*i**4/(6*d*f**4) - 44*a*b*e**2*h*i**3/(3*d*f**3) + 18*a*b*e*h
**2*i**2/(d*f**2) - 8*a*b*h**3*i/(d*f) - 415*b**2*e**3*i**4/(72*d*f**4) + 1
70*b**2*e**2*h*i**3/(9*d*f**3) - 21*b**2*e*h**2*i**2/(d*f**2) + 8*b**2*h**3
*i/(d*f)) + (-144*a*b*e**3*i**4*x + 576*a*b*e**2*f*h*i**3*x + 72*a*b*e**2*f
*i**4*x**2 - 864*a*b*e*f**2*h**2*i**2*x - 288*a*b*e*f**2*h*i**3*x**2 - 48*a
*b*e*f**2*i**4*x**3 + 576*a*b*f**3*h**3*i*x + 432*a*b*f**3*h**2*i**2*x**2 +
 192*a*b*f**3*h*i**3*x**3 + 36*a*b*f**3*i**4*x**4 + 300*b**2*e**3*i**4*x -
1056*b**2*e**2*f*h*i**3*x - 78*b**2*e**2*f*i**4*x**2 + 1296*b**2*e*f**2*h**
2*i**2*x + 240*b**2*e*f**2*h*i**3*x**2 + 28*b**2*e*f**2*i**4*x**3 - 576*b**
2*f**3*h**3*i*x - 216*b**2*f**3*h**2*i**2*x**2 - 64*b**2*f**3*h*i**3*x**3 -
 9*b**2*f**3*i**4*x**4)*log(c*(e + f*x))/(72*d*f**4) + (b**2*e**4*i**4 - 4*
b**2*e**3*f*h*i**3 + 6*b**2*e**2*f**2*h**2*i**2 - 4*b**2*e*f**3*h**3*i + b*
*2*f**4*h**4)*log(c*(e + f*x))**3/(3*d*f**5) + (72*a**2*e**4*i**4 - 288*a**
2*e**3*f*h*i**3 + 432*a**2*e**2*f**2*h**2*i**2 - 288*a**2*e*f**3*h**3*i + 7
2*a**2*f**4*h**4 - 300*a*b*e**4*i**4 + 1056*a*b*e**3*f*h*i**3 - 1296*a*b*e*
*2*f**2*h**2*i**2 + 576*a*b*e*f**3*h**3*i + 415*b**2*e**4*i**4 - 1360*b**2*
e**3*f*h*i**3 + 1512*b**2*e**2*f**2*h**2*i**2 - 576*b**2*e*f**3*h**3*i)*log
(e + f*x)/(72*d*f**5) + (12*a*b*e**4*i**4 - 48*a*b*e**3*f*h*i**3 + 72*a*b*e
**2*f**2*h**2*i**2 - 48*a*b*e*f**3*h**3*i + 12*a*b*f**4*h**4 - 25*b**2*e**4
*i**4 + 88*b**2*e**3*f*h*i**3 - 12*b**2*e**3*f*i**4*x - 108*b**2*e**2*f**2*
h**2*i**2 + 48*b**2*e**2*f**2*h*i**3*x + 6*b**2*e**2*f**2*i**4*x**2 + 48*b*
*2*e*f**3*h**3*i - 72*b**2*e*f**3*h**2*i**2*x - 24*b**2*e*f**3*h*i**3*x**2
- 4*b**2*e*f**3*i**4*x**3 + 48*b**2*f**4*h**3*i*x + 36*b**2*f**4*h**2*i**2*
x**2 + 16*b**2*f**4*h*i**3*x**3 + 3*b**2*f**4*i**4*x**4)*log(c*(e + f*x))**
2/(12*d*f**5)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1427 vs. $2(557) = 1114$.

Time = 0.27 (sec) , antiderivative size = 1427, normalized size of antiderivative = 2.46

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \text{Too large to display}$$

```
[In] integrate((i*x+h)^4*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="maxima
")
```

```
[Out] 8*a*b*h^3*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) + 1/6*a*b*i
^4*(12*e^4*log(f*x + e)/(d*f^5) + (3*f^3*x^4 - 4*e*f^2*x^3 + 6*e^2*f*x^2 -
12*e^3*x)/(d*f^4))*log(c*f*x + c*e) - 4/3*a*b*h*i^3*(6*e^3*log(f*x + e)/(d*
f^4) - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3))*log(c*f*x + c*e) + 6*a*b*
h^2*i^2*(2*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2))*log(c*f*x +
c*e) - a*b*h^4*(2*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2
 + 2*log(f*x + e)*log(c))/(d*f)) + 4*a^2*h^3*i*(x/(d*f) - e*log(f*x + e)/(d
*f^2)) + 1/12*a^2*i^4*(12*e^4*log(f*x + e)/(d*f^5) + (3*f^3*x^4 - 4*e*f^2*x
^3 + 6*e^2*f*x^2 - 12*e^3*x)/(d*f^4)) - 2/3*a^2*h*i^3*(6*e^3*log(f*x + e)/(
d*f^4) - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3)) + 3*a^2*h^2*i^2*(2*e^2*
log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2)) + 1/3*b^2*h^4*log(c*f*x + c
*e)^3/(d*f) + 2*a*b*h^4*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) + a^2*h^4*l
og(d*f*x + d*e)/(d*f) + 4*(e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*a*b
*h^3*i/(d*f^2) - 3*(f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x + 6*e^2*log(f*
x + e))*a*b*h^2*i^2/(d*f^3) - 4/3*(c^2*e*log(c*f*x + c*e)^3 - 3*(c*f*x + c*
e)*(c*log(c*f*x + c*e)^2 - 2*c*log(c*f*x + c*e) + 2*c))*b^2*h^3*i/(c^2*d*f^
2) - 2/9*(4*f^3*x^3 - 15*e*f^2*x^2 - 18*e^3*log(f*x + e)^2 + 66*e^2*f*x - 6
6*e^3*log(f*x + e))*a*b*h*i^3/(d*f^4) - 1/72*(9*f^4*x^4 - 28*e*f^3*x^3 + 78
*e^2*f^2*x^2 + 72*e^4*log(f*x + e)^2 - 300*e^3*f*x + 300*e^4*log(f*x + e))*
a*b*i^4/(d*f^5) + 1/2*(4*c^3*e^2*log(c*f*x + c*e)^3 + 3*(c*f*x + c*e)^2*(2*
c*log(c*f*x + c*e)^2 - 2*c*log(c*f*x + c*e) + c) - 24*(c^2*e*log(c*f*x + c*
e)^2 - 2*c^2*e*log(c*f*x + c*e) + 2*c^2*e)*(c*f*x + c*e))*b^2*h^2*i^2/(c^3*
d*f^3) - 1/27*(36*c^4*e^3*log(c*f*x + c*e)^3 - 4*(c*f*x + c*e)^3*(9*c*log(c
*f*x + c*e)^2 - 6*c*log(c*f*x + c*e) + 2*c) + 81*(2*c^2*e*log(c*f*x + c*e)^
2 - 2*c^2*e*log(c*f*x + c*e) + c^2*e)*(c*f*x + c*e)^2 - 324*(c^3*e^2*log(c*
f*x + c*e)^2 - 2*c^3*e^2*log(c*f*x + c*e) + 2*c^3*e^2)*(c*f*x + c*e))*b^2*h
*i^3/(c^4*d*f^4) + 1/864*(288*c^5*e^4*log(c*f*x + c*e)^3 + 27*(c*f*x + c*e)
^4*(8*c*log(c*f*x + c*e)^2 - 4*c*log(c*f*x + c*e) + c) - 128*(9*c^2*e*log(c
*f*x + c*e)^2 - 6*c^2*e*log(c*f*x + c*e) + 2*c^2*e)*(c*f*x + c*e)^3 + 1296*
(2*c^3*e^2*log(c*f*x + c*e)^2 - 2*c^3*e^2*log(c*f*x + c*e) + c^3*e^2)*(c*f*
x + c*e)^2 - 3456*(c^4*e^3*log(c*f*x + c*e)^2 - 2*c^4*e^3*log(c*f*x + c*e)
+ 2*c^4*e^3)*(c*f*x + c*e))*b^2*i^4/(c^5*d*f^5)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1213 vs. $2(557) = 1114$.

Time = 0.33 (sec) , antiderivative size = 1213, normalized size of antiderivative = 2.09

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \text{Too large to display}$$

```
[In] integrate((i*x+h)^4*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="giac")
```

```
[Out] 1/32*(8*a^2*i^4 - 4*a*b*i^4 + b^2*i^4)*x^4/(d*f) + 1/12*(3*b^2*i^4*x^4/(d*f
) + 4*(4*b^2*f*h*i^3 - b^2*e*i^4)*x^3/(d*f^2) + 6*(6*b^2*f^2*h^2*i^2 - 4*b^
```

```
2*e*f*h*i^3 + b^2*e^2*i^4)*x^2/(d*f^3) + 12*(4*b^2*f^3*h^3*i - 6*b^2*e*f^2*
h^2*i^2 + 4*b^2*e^2*f*h*i^3 - b^2*e^3*i^4)*x/(d*f^4) + (12*a*b*f^4*h^4 - 48
*a*b*e*f^3*h^3*i + 48*b^2*e*f^3*h^3*i + 72*a*b*e^2*f^2*h^2*i^2 - 108*b^2*e^
2*f^2*h^2*i^2 - 48*a*b*e^3*f*h*i^3 + 88*b^2*e^3*f*h*i^3 + 12*a*b*e^4*i^4 -
25*b^2*e^4*i^4)/(d*f^5))*log(c*f*x + c*e)^2 + 1/72*(9*(4*a*b*i^4 - b^2*i^4)
*x^4/(d*f) + 4*(48*a*b*f*h*i^3 - 16*b^2*f*h*i^3 - 12*a*b*e*i^4 + 7*b^2*e*i^
4)*x^3/(d*f^2) + 6*(72*a*b*f^2*h^2*i^2 - 36*b^2*f^2*h^2*i^2 - 48*a*b*e*f*h*
i^3 + 40*b^2*e*f*h*i^3 + 12*a*b*e^2*i^4 - 13*b^2*e^2*i^4)*x^2/(d*f^3) + 12*
(48*a*b*f^3*h^3*i - 48*b^2*f^3*h^3*i - 72*a*b*e*f^2*h^2*i^2 + 108*b^2*e*f^2
*h^2*i^2 + 48*a*b*e^2*f*h*i^3 - 88*b^2*e^2*f*h*i^3 - 12*a*b*e^3*i^4 + 25*b^
2*e^3*i^4)*x/(d*f^4))*log(c*f*x + c*e) + 1/216*(288*a^2*f*h*i^3 - 192*a*b*f
*h*i^3 + 64*b^2*f*h*i^3 - 72*a^2*e*i^4 + 84*a*b*e*i^4 - 37*b^2*e*i^4)*x^3/(
d*f^2) + 1/144*(432*a^2*f^2*h^2*i^2 - 432*a*b*f^2*h^2*i^2 + 216*b^2*f^2*h^2
*i^2 - 288*a^2*e*f*h*i^3 + 480*a*b*e*f*h*i^3 - 304*b^2*e*f*h*i^3 + 72*a^2*e
^2*i^4 - 156*a*b*e^2*i^4 + 115*b^2*e^2*i^4)*x^2/(d*f^3) + 1/3*(b^2*f^4*h^4
- 4*b^2*e*f^3*h^3*i + 6*b^2*e^2*f^2*h^2*i^2 - 4*b^2*e^3*f*h*i^3 + b^2*e^4*i
^4)*log(c*f*x + c*e)^3/(d*f^5) + 1/72*(288*a^2*f^3*h^3*i - 576*a*b*f^3*h^3*
i + 576*b^2*f^3*h^3*i - 432*a^2*e*f^2*h^2*i^2 + 1296*a*b*e*f^2*h^2*i^2 - 15
12*b^2*e*f^2*h^2*i^2 + 288*a^2*e^2*f*h*i^3 - 1056*a*b*e^2*f*h*i^3 + 1360*b^
2*e^2*f*h*i^3 - 72*a^2*e^3*i^4 + 300*a*b*e^3*i^4 - 415*b^2*e^3*i^4)*x/(d*f^
4) + 1/72*(72*a^2*f^4*h^4 - 288*a^2*e*f^3*h^3*i + 576*a*b*e*f^3*h^3*i - 576
*b^2*e*f^3*h^3*i + 432*a^2*e^2*f^2*h^2*i^2 - 1296*a*b*e^2*f^2*h^2*i^2 + 151
2*b^2*e^2*f^2*h^2*i^2 - 288*a^2*e^3*f*h*i^3 + 1056*a*b*e^3*f*h*i^3 - 1360*b
^2*e^3*f*h*i^3 + 72*a^2*e^4*i^4 - 300*a*b*e^4*i^4 + 415*b^2*e^4*i^4)*log(f*
x + e)/(d*f^5)
```

## Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 1346, normalized size of antiderivative = 2.32

$$\int \frac{(h+ix)^4(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \ln\left(c\left(e+f\,x\right)\right)^2\left(f\left(\frac{b^2\,i^4\,x^4}{4\,d\,f^2}-\frac{b^2\,i^3\,x^3\,\left(e\,i-4\,f\,h\right)}{3\,d\,f^3}-\frac{b^2\,i\,x\,\left(e^3\,i^3-4\,e^2\,f\,h\,i^2+6\,e\,f^2\,h^2\,i-4\,f^3\,h^3\right)}{d\,f^5}+\frac{b^2\,i^2\,x^2\,\left(e^2\,i^2-4\,e\,f\,h\,i+6\,f^2\,h^2\right)}{2\,d\,f^4}\right)+\frac{-25\,b^2\,e^4\,i^4+88\,b^2\,e^3\,f\,h\,i^3-108\,b^2\,e^2\,f^2\,h^2\,i^2+48\,b^2\,e\,f^3\,h^3\,i+12\,a\,b\,e^4\,i^4-48\,a\,b\,e^3\,f\,h\,i^3+72\,a\,b\,e^2}{12\,d\,f^5}\right.$$

$$-x^2\left(\frac{e\left(\frac{i^3\,\left(72\,a^2\,f\,h-7\,b^2\,e\,i+16\,b^2\,f\,h+12\,a\,b\,e\,i-48\,a\,b\,f\,h\right)}{18\,d\,f^2}-\frac{e\,i^4\,\left(8\,a^2-4\,a\,b+b^2\right)}{8\,d\,f^2}\right)}{2\,f}-\frac{i^2\,\left(72\,a^2\,f^2\,h^2-12\,a\,b\,e^2\,i^2+48\,a\,b\,e\,f\,h\,i-72\,a\,b\,f^2\,h^2+13\,b^2\,e^2\,i^2-40\,b^2\,e\,f\,h\,i+36\,b^2\,f^2\,h^2\right)}{24\,d\,f^3}\right)$$

$$+x^3\left(\frac{i^3\,\left(72\,a^2\,f\,h-7\,b^2\,e\,i+16\,b^2\,f\,h+12\,a\,b\,e\,i-48\,a\,b\,f\,h\right)}{54\,d\,f^2}-\frac{e\,i^4\,\left(8\,a^2-4\,a\,b+b^2\right)}{24\,d\,f^2}\right)$$

$$+x\left(\frac{288\,a^2\,f^3\,h^3\,i+144\,a\,b\,e^3\,i^4-576\,a\,b\,e^2\,f\,h\,i^3+864\,a\,b\,e\,f^2\,h^2\,i^2-576\,a\,b\,f^3\,h^3\,i-300\,b^2\,e^3\,i^4+105}{72\,d\,f^4}\right.$$

$$+\frac{e\left(\frac{e\left(\frac{i^3\,\left(72\,a^2\,f\,h-7\,b^2\,e\,i+16\,b^2\,f\,h+12\,a\,b\,e\,i-48\,a\,b\,f\,h\right)}{18\,d\,f^2}-\frac{e\,i^4\,\left(8\,a^2-4\,a\,b+b^2\right)}{8\,d\,f^2}\right)}{f}-\frac{i^2\,\left(72\,a^2\,f^2\,h^2-12\,a\,b\,e^2\,i^2+48\,a\,b\,e\,f\,h\,i-72\,a\,b\,f^2\,h^2+13\right.}{12\,d\,f^3}\right.}{f}$$

$$+f\,\ln\left(c\left(e+f\,x\right)\right)\left(\frac{x^3\,\left(7\,e\,b^2\,i^4-16\,f\,h\,b^2\,i^3-12\,a\,e\,b\,i^4+48\,a\,f\,h\,b\,i^3\right)}{18\,d\,f^3}-\frac{x^2\,\left(13\,b^2\,e^2\,i^4-40\,b^2\,e\,f\,h\,i^3+36\,b^2\,f^2\,h^2\,i^2-12\,a\,b\,e^2\,i^4+48\,a\,b\,e\,f\,h\,i^3-72\,a\,b\,f^2\,h^2\,i^2\right)}{12\,d\,f^4}+\frac{x\,\left(25\,b^2\,e^3\,i^4-88\,b^2\,e^2\,f\,h\,i^3+108\,b^2\,e\,f^2\,h^2\,i^2-48\,b^2\,f^3\,h^3\,i-12\,a\,b\,e^3\,i^4+48\,a\,b\,e^2\,f\,h\,i^3-72\,a\,b\,e\,f^2}{6\,d\,f^5}+\frac{b\,i^4\,x^4\,\left(4\,a-b\right)}{8\,d\,f^2}\right)$$

$$+\frac{\ln\left(e+f\,x\right)\,\left(72\,a^2\,e^4\,i^4-288\,a^2\,e^3\,f\,h\,i^3+432\,a^2\,e^2\,f^2\,h^2\,i^2-288\,a^2\,e\,f^3\,h^3\,i+72\,a^2\,f^4\,h^4-300\,a\,b\,e^4\,i}{}$$

$$+\frac{b^2\ln\left(c\left(e+f\,x\right)\right)^3\,\left(e^4\,i^4-4\,e^3\,f\,h\,i^3+6\,e^2\,f^2\,h^2\,i^2-4\,e\,f^3\,h^3\,i+f^4\,h^4\right)}{3\,d\,f^5}+\frac{i^4\,x^4\,\left(8\,a^2-4\,a\,b+b^2\right)}{32\,d\,f}$$

```
[In] int(((h + i*x)^4*(a + b*log(c*(e + f*x)))^2)/(d*e + d*f*x),x)

[Out] log(c*(e + f*x))^2*(f*((b^2*i^4*x^4)/(4*d*f^2) - (b^2*i^3*x^3*(e*i - 4*f*h)
)/(3*d*f^3) - (b^2*i*x*(e^3*i^3 - 4*f^3*h^3 + 6*e*f^2*h^2*i - 4*e^2*f*h*i^2
))/(d*f^5) + (b^2*i^2*x^2*(e^2*i^2 + 6*f^2*h^2 - 4*e*f*h*i))/(2*d*f^4)) + (
12*a*b*e^4*i^4 - 25*b^2*e^4*i^4 + 12*a*b*f^4*h^4 - 108*b^2*e^2*f^2*h^2*i^2
+ 48*b^2*e*f^3*h^3*i + 88*b^2*e^3*f*h*i^3 + 72*a*b*e^2*f^2*h^2*i^2 - 48*a*b
*e*f^3*h^3*i - 48*a*b*e^3*f*h*i^3)/(12*d*f^5)) - x^2*((e*((i^3*(72*a^2*f*h
- 7*b^2*e*i + 16*b^2*f*h + 12*a*b*e*i - 48*a*b*f*h))/(18*d*f^2) - (e*i^4*(8
*a^2 - 4*a*b + b^2))/(8*d*f^2)))/(2*f) - (i^2*(72*a^2*f^2*h^2 + 13*b^2*e^2*
i^2 + 36*b^2*f^2*h^2 - 12*a*b*e^2*i^2 - 72*a*b*f^2*h^2 - 40*b^2*e*f*h*i + 4
8*a*b*e*f*h*i))/(24*d*f^3)) + x^3*((i^3*(72*a^2*f*h - 7*b^2*e*i + 16*b^2*f*
h + 12*a*b*e*i - 48*a*b*f*h))/(54*d*f^2) - (e*i^4*(8*a^2 - 4*a*b + b^2))/(2
4*d*f^2)) + x*((288*a^2*f^3*h^3*i - 300*b^2*e^3*i^4 + 576*b^2*f^3*h^3*i + 1
44*a*b*e^3*i^4 - 576*a*b*f^3*h^3*i + 1056*b^2*e^2*f*h*i^3 - 1296*b^2*e*f^2*
h^2*i^2 - 576*a*b*e^2*f*h*i^3 + 864*a*b*e*f^2*h^2*i^2)/(72*d*f^4) + (e*((e*
((i^3*(72*a^2*f*h - 7*b^2*e*i + 16*b^2*f*h + 12*a*b*e*i - 48*a*b*f*h))/(18*
d*f^2) - (e*i^4*(8*a^2 - 4*a*b + b^2))/(8*d*f^2)))/f - (i^2*(72*a^2*f^2*h^2
 + 13*b^2*e^2*i^2 + 36*b^2*f^2*h^2 - 12*a*b*e^2*i^2 - 72*a*b*f^2*h^2 - 40*b
^2*e*f*h*i + 48*a*b*e*f*h*i))/(12*d*f^3)))/f) + f*log(c*(e + f*x))*((x^3*(7
*b^2*e*i^4 - 12*a*b*e*i^4 - 16*b^2*f*h*i^3 + 48*a*b*f*h*i^3))/(18*d*f^3) -
(x^2*(13*b^2*e^2*i^4 + 36*b^2*f^2*h^2*i^2 - 12*a*b*e^2*i^4 - 40*b^2*e*f*h*i
^3 - 72*a*b*f^2*h^2*i^2 + 48*a*b*e*f*h*i^3))/(12*d*f^4) + (x*(25*b^2*e^3*i^
4 - 48*b^2*f^3*h^3*i - 12*a*b*e^3*i^4 + 48*a*b*f^3*h^3*i - 88*b^2*e^2*f*h*i
^3 + 108*b^2*e*f^2*h^2*i^2 + 48*a*b*e^2*f*h*i^3 - 72*a*b*e*f^2*h^2*i^2))/(6
*d*f^5) + (b*i^4*x^4*(4*a - b))/(8*d*f^2)) + (log(e + f*x)*(72*a^2*e^4*i^4
+ 72*a^2*f^4*h^4 + 415*b^2*e^4*i^4 - 300*a*b*e^4*i^4 + 432*a^2*e^2*f^2*h^2*
i^2 + 1512*b^2*e^2*f^2*h^2*i^2 - 288*a^2*e*f^3*h^3*i - 288*a^2*e^3*f*h*i^3
- 576*b^2*e*f^3*h^3*i - 1360*b^2*e^3*f*h*i^3 - 1296*a*b*e^2*f^2*h^2*i^2 + 5
76*a*b*e*f^3*h^3*i + 1056*a*b*e^3*f*h*i^3))/(72*d*f^5) + (b^2*log(c*(e + f*
x))^3*(e^4*i^4 + f^4*h^4 + 6*e^2*f^2*h^2*i^2 - 4*e*f^3*h^3*i - 4*e^3*f*h*i^
3))/(3*d*f^5) + (i^4*x^4*(8*a^2 - 4*a*b + b^2))/(32*d*f)
```

# 3.184 $\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 464

$$
\begin{aligned}
\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = &-\frac{4abi(fh-ei)^2x}{df^3}+\frac{6b^2i(fh-ei)^2x}{df^3}\\
&+\frac{3b^2i^2(fh-ei)(e+fx)^2}{4df^4}+\frac{2b^2i^3(e+fx)^3}{27df^4}\\
&+\frac{b^2(fh-ei)^3\log^2(e+fx)}{3df^4}\\
&-\frac{4b^2i(fh-ei)^2(e+fx)\log(c(e+fx))}{df^4}\\
&-\frac{2bi(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4}\\
&-\frac{3bi^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{2df^4}\\
&-\frac{2bi^3(e+fx)^3(a+b\log(c(e+fx)))}{9df^4}\\
&-\frac{2b(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{3df^4}\\
&+\frac{2i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))^2}{df^4}\\
&+\frac{i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^4}\\
&+\frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{3df}\\
&+\frac{(fh-ei)^3(a+b\log(c(e+fx)))^3}{3bdf^4}
\end{aligned}
$$

```
[Out] -4*a*b*i*(-e*i+f*h)^2*x/d/f^3+6*b^2*i*(-e*i+f*h)^2*x/d/f^3+3/4*b^2*i^2*(-e*
i+f*h)*(f*x+e)^2/d/f^4+2/27*b^2*i^3*(f*x+e)^3/d/f^4+1/3*b^2*(-e*i+f*h)^3*ln
(f*x+e)^2/d/f^4-4*b^2*i*(-e*i+f*h)^2*(f*x+e)*ln(c*(f*x+e))/d/f^4-2*b*i*(-e*
i+f*h)^2*(f*x+e)*(a+b*ln(c*(f*x+e)))/d/f^4-3/2*b*i^2*(-e*i+f*h)*(f*x+e)^2*(
a+b*ln(c*(f*x+e)))/d/f^4-2/9*b*i^3*(f*x+e)^3*(a+b*ln(c*(f*x+e)))/d/f^4-2/3*
b*(-e*i+f*h)^3*ln(f*x+e)*(a+b*ln(c*(f*x+e)))/d/f^4+2*i*(-e*i+f*h)^2*(f*x+e)
*(a+b*ln(c*(f*x+e)))^2/d/f^4+1/2*i^2*(-e*i+f*h)*(f*x+e)^2*(a+b*ln(c*(f*x+e)
))^2/d/f^4+1/3*(i*x+h)^3*(a+b*ln(c*(f*x+e)))^2/d/f+1/3*(-e*i+f*h)^3*(a+b*ln
(c*(f*x+e)))^3/b/d/f^4
```

## Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {2458, 12, 2388, 2339, 30, 2333, 2332, 2367, 2342, 2341, 2356, 45, 2372, 14, 2338}

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{i^2(e+fx)^2(fh-ei)(a+b\log(c(e+fx)))^2}{2df^4} - \frac{3bi^2(e+fx)^2(fh-ei)(a+b\log(c(e+fx)))}{2df^4} + \frac{(fh-ei)^3(a+b\log(c(e+fx)))^3}{3bdf^4} - \frac{2b(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{3df^4} + \frac{2i(e+fx)(fh-ei)^2(a+b\log(c(e+fx)))^2}{df^4} - \frac{2bi(e+fx)(fh-ei)^2(a+b\log(c(e+fx)))}{df^4} - \frac{2bi^3(e+fx)^3(a+b\log(c(e+fx)))}{9df^4} + \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{3df} - \frac{4abix(fh-ei)^2}{df^3} - \frac{4b^2i(e+fx)(fh-ei)^2\log(c(e+fx))}{df^4} + \frac{3b^2i^2(e+fx)^2(fh-ei)}{4df^4} + \frac{b^2(fh-ei)^3\log^2(e+fx)}{3df^4} + \frac{2b^2i^3(e+fx)^3}{27df^4} + \frac{6b^2ix(fh-ei)^2}{df^3}$$

```
[In] Int[((h + i*x)^3*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

```
[Out] (-4*a*b*i*(f*h - e*i)^2*x)/(d*f^3) + (6*b^2*i*(f*h - e*i)^2*x)/(d*f^3) + (3
*b^2*i^2*(f*h - e*i)*(e + f*x)^2)/(4*d*f^4) + (2*b^2*i^3*(e + f*x)^3)/(27*d
*f^4) + (b^2*(f*h - e*i)^3*Log[e + f*x]^2)/(3*d*f^4) - (4*b^2*i*(f*h - e*i)
^2*(e + f*x)*Log[c*(e + f*x)])/(d*f^4) - (2*b*i*(f*h - e*i)^2*(e + f*x)*(a
+ b*Log[c*(e + f*x)]))/(d*f^4) - (3*b*i^2*(f*h - e*i)*(e + f*x)^2*(a + b*Lo
g[c*(e + f*x)]))/(2*d*f^4) - (2*b*i^3*(e + f*x)^3*(a + b*Log[c*(e + f*x)]))
/(9*d*f^4) - (2*b*(f*h - e*i)^3*Log[e + f*x]*(a + b*Log[c*(e + f*x)]))/(3*d
*f^4) + (2*i*(f*h - e*i)^2*(e + f*x)*(a + b*Log[c*(e + f*x)])^2)/(d*f^4) +
(i^2*(f*h - e*i)*(e + f*x)^2*(a + b*Log[c*(e + f*x)])^2)/(2*d*f^4) + ((h +
i*x)^3*(a + b*Log[c*(e + f*x)])^2)/(3*d*f) + ((f*h - e*i)^3*(a + b*Log[c*(e
 + f*x)])^3)/(3*b*d*f^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
```

```
 && EqQ[m, -1])
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3(a+b\log(cx))^2}{dx}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3(a+b\log(cx))^2}{x}\,dx, x, e+fx\right)}{df}$$

$$= \frac{i\text{Subst}\left(\int \left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2(a+b\log(cx))^2\,dx, x, e+fx\right)}{df^2} + \frac{(fh-ei)\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2(a+b\log(cx))^2}{x}\,dx, x, e+fx\right)}{df^2}$$

$$= \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{3df} - \frac{(2b)\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3(a+b\log(cx))}{x}\,dx, x, e+fx\right)}{3df} + \frac{(i(fh-ei))\text{Subst}\left(\int \left(\frac{fh-ei}{f}+\frac{ix}{f}\right)(a+b\log(cx))^2\,dx, x, e+fx\right)}{df^3} + \frac{(fh-ei)^2\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)(a+b\log(cx))^2}{x}\,dx, x, e+fx\right)}{df^3}$$

$$= -\frac{2bi(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4} - \frac{bi^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{df^4} - \frac{2bi^3(e+fx)^3(a+b\log(c(e+fx)))}{9df^4} - \frac{2b(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{3df^4} + \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{3df} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{ix(18f^2h^2+9fhi(-4e+x)+i^2(18e^2-9ex+2x^2))+6(fh-ei)^3\log(x)}{6f^3x}\,dx, x, e+fx\right)}{3df} + \frac{(i(fh-ei))\text{Subst}\left(\int \left(\frac{(fh-ei)(a+b\log(cx))^2}{f} + \frac{ix(a+b\log(cx))^2}{f}\right)\,dx, x, e+fx\right)}{df^3} + \frac{(i(fh-ei)^2)\,\text{Subst}\left(\int (a+b\log(cx))^2\,dx, x, e+fx\right)}{df^4} + \frac{(fh-ei)^3\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{x}\,dx, x, e+fx\right)}{df^4}$$

$$= -\frac{2bi(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4} - \frac{bi^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{df^4} - \frac{2bi^3(e+fx)^3(a+b\log(c(e+fx)))}{9df^4} - \frac{2b(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{3df^4} + \frac{i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))^2}{df^4} + \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{3df} + \frac{b^2\text{Subst}\left(\int \frac{ix(18f^2h^2+9fhi(-4e+x)+i^2(18e^2-9ex+2x^2))+6(fh-ei)^3\log(x)}{x}\,dx, x, e+fx\right)}{9df^4} + \frac{(i^2(fh-ei))\,\text{Subst}\left(\int x(a+b\log(cx))^2\,dx, x, e+fx\right)}{df^4} + \frac{(i(fh-ei)^2)\,\text{Subst}\left(\int (a+b\log(cx))^2\,dx, x, e+fx\right)}{df^4} - \frac{(2bi(fh-ei)^2)\,\text{Subst}(\int (a+b\log(cx))\,dx, x, e+fx)}{df^4} + \frac{(fh-ei)^3\text{Subst}\left(\int x^2\,dx, x, a+b\log(c(e+fx))\right)}{bdf^4}$$

$$
\begin{aligned}
&= -\frac{2abi(fh-ei)^2x}{df^3} - \frac{2bi(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4} \\
&- \frac{bi^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{df^4} \\
&- \frac{2bi^3(e+fx)^3(a+b\log(c(e+fx)))}{9df^4} \\
&- \frac{2b(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{3df^4} \\
&+ \frac{2i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))^2}{df^4} \\
&+ \frac{i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^4} \\
&+ \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{3df} + \frac{(fh-ei)^3(a+b\log(c(e+fx)))^3}{3bdf^4} \\
&+ \frac{b^2\text{Subst}\left(\int \left(i(18(fh-ei)^2+9i(fh-ei)x+2i^2x^2)+\frac{6(fh-ei)^3\log(x)}{x}\right)dx, x, e+fx\right)}{9df^4} \\
&- \frac{(bi^2(fh-ei))\,\text{Subst}(\int x(a+b\log(cx))\,dx, x, e+fx)}{df^4} \\
&- \frac{(2bi(fh-ei)^2)\,\text{Subst}(\int (a+b\log(cx))\,dx, x, e+fx)}{df^4} \\
&- \frac{(2b^2i(fh-ei)^2)\,\text{Subst}(\int \log(cx)\,dx, x, e+fx)}{df^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4abi(fh-ei)^2x}{df^3} + \frac{2b^2i(fh-ei)^2x}{df^3} + \frac{b^2i^2(fh-ei)(e+fx)^2}{4df^4} \\
&- \frac{2b^2i(fh-ei)^2(e+fx)\log(c(e+fx))}{df^4} \\
&- \frac{2bi(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4} \\
&- \frac{3bi^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{2df^4} \\
&- \frac{2bi^3(e+fx)^3(a+b\log(c(e+fx)))}{9df^4} \\
&- \frac{2b(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{3df^4} \\
&+ \frac{2i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))^2}{df^4} \\
&+ \frac{i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^4} \\
&+ \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{3df} + \frac{(fh-ei)^3(a+b\log(c(e+fx)))^3}{3bdf^4} \\
&+ \frac{(b^2i)\operatorname{Subst}\left(\int \left(18(fh-ei)^2+9i(fh-ei)x+2i^2x^2\right)\,dx, x, e+fx\right)}{9df^4} \\
&- \frac{(2b^2i(fh-ei)^2)\operatorname{Subst}(\int \log(cx)\,dx, x, e+fx)}{df^4} \\
&+ \frac{(2b^2(fh-ei)^3)\operatorname{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, e+fx\right)}{3df^4}
\end{aligned}
$$

$$= -\frac{4abi(fh-ei)^2x}{df^3} + \frac{6b^2i(fh-ei)^2x}{df^3} + \frac{3b^2i^2(fh-ei)(e+fx)^2}{4df^4} + \frac{2b^2i^3(e+fx)^3}{27df^4} + \frac{b^2(fh-ei)^3\log^2(e+fx)}{3df^4} - \frac{4b^2i(fh-ei)^2(e+fx)\log(c(e+fx))}{df^4} - \frac{2bi(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))}{df^4} - \frac{3bi^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))}{2df^4} - \frac{2bi^3(e+fx)^3(a+b\log(c(e+fx)))}{9df^4} - \frac{2b(fh-ei)^3\log(e+fx)(a+b\log(c(e+fx)))}{3df^4} + \frac{2i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))^2}{df^4} + \frac{i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^4} + \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{3df} + \frac{(fh-ei)^3(a+b\log(c(e+fx)))^3}{3bdf^4}$$

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.58

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= \frac{324i(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))^2 + 162i^2(fh-ei)(e+fx)^2(a+b\log(c(e+fx)))^2 + 36i^3(e}{}$$

```
[In] Integrate[((h + i*x)^3*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

```
[Out] (324*i*(f*h - e*i)^2*(e + f*x)*(a + b*Log[c*(e + f*x)])^2 + 162*i^2*(f*h -
e*i)*(e + f*x)^2*(a + b*Log[c*(e + f*x)])^2 + 36*i^3*(e + f*x)^3*(a + b*Log
[c*(e + f*x)])^2 + (36*(f*h - e*i)^3*(a + b*Log[c*(e + f*x)])^3)/b - 648*b*
i*(f*h - e*i)^2*((a - b)*f*x + b*(e + f*x)*Log[c*(e + f*x)]) + 81*b*i^2*(f*
h - e*i)*(b*f*x*(2*e + f*x) - 2*(e + f*x)^2*(a + b*Log[c*(e + f*x)])) + 8*b
*i^3*(b*f*x*(3*e^2 + 3*e*f*x + f^2*x^2) - 3*(e + f*x)^3*(a + b*Log[c*(e + f
*x)])))/(108*d*f^4)
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.55

| method | result |
|---|---|
| norman | $\frac{b^2 i(e^2 i^2-3efhi+3f^2h^2)x\ln(c(fx+e))^2}{d f^3}-\frac{(18a^2e^3i^3-54a^2e^2fh\,i^2+54a^2e\,f^2h^2i-18a^2f^3h^3-66ab\,e^3i^3+162ab\,e^2fh\,i^2}{18d\,f^4}$ |
| risch | $-\frac{b(-2b\,f^3i^3x^3+3be\,f^2i^3x^2-9b\,f^3h\,i^2x^2-6b\,e^2f\,i^3x+18be\,f^2h\,i^2x-18b\,f^3h^2ix+6a\,e^3i^3-18a\,e^2fh\,i^2+18ae\,f^2h^2i-6a\,f}{6d\,f^4}$ |
| parts | $\frac{a^2\left(\frac{i\left(\frac{1}{3}f^2i^2x^3-\frac{1}{2}ef\,i^2x^2+\frac{3}{2}f^2hi\,x^2+x\,e^2i^2-3xefhi+3x\,f^2h^2\right)}{f^3}+\frac{\left(-e^3i^3+3e^2fh\,i^2-3e\,f^2h^2i+f^3h^3\right)\ln(fx+e)}{f^4}\right)}{d}+\frac{b^2\left(-\frac{c\,e^3}{}\right.}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |
| parallelrisch | Expression too large to display |

```
[In] int((i*x+h)^3*(a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*i*(e^2*i^2-3*e*f*h*i+3*f^2*h^2)/d/f^3*x*ln(c*(f*x+e))^2-1/18*(18*a^2*e^
3*i^3-54*a^2*e^2*f*h*i^2+54*a^2*e*f^2*h^2*i-18*a^2*f^3*h^3-66*a*b*e^3*i^3+1
62*a*b*e^2*f*h*i^2-108*a*b*e*f^2*h^2*i+85*b^2*e^3*i^3-189*b^2*e^2*f*h*i^2+1
08*b^2*e*f^2*h^2*i)/d/f^4*ln(c*(f*x+e))-1/6*b*(6*a*e^3*i^3-18*a*e^2*f*h*i^2
+18*a*e*f^2*h^2*i-6*a*f^3*h^3-11*b*e^3*i^3+27*b*e^2*f*h*i^2-18*b*e*f^2*h^2*
i)/d/f^4*ln(c*(f*x+e))^2-1/3*b^2*(e^3*i^3-3*e^2*f*h*i^2+3*e*f^2*h^2*i-f^3*h
^3)/d/f^4*ln(c*(f*x+e))^3+1/18*i*(18*a^2*e^2*i^2-54*a^2*e*f*h*i+54*a^2*f^2*
h^2-66*a*b*e^2*i^2+162*a*b*e*f*h*i-108*a*b*f^2*h^2+85*b^2*e^2*i^2-189*b^2*e
*f*h*i+108*b^2*f^2*h^2)/d/f^3*x-1/36*i^2*(18*a^2*e*i-54*a^2*f*h-30*a*b*e*i+
54*a*b*f*h+19*b^2*e*i-27*b^2*f*h)/d/f^2*x^2+1/27*i^3*(9*a^2-6*a*b+2*b^2)/d/
f*x^3+1/3*b^2*i^3/d/f*x^3*ln(c*(f*x+e))^2+1/3*b*i*(6*a*e^2*i^2-18*a*e*f*h*i
+18*a*f^2*h^2-11*b*e^2*i^2+27*b*e*f*h*i-18*b*f^2*h^2)/d/f^3*x*ln(c*(f*x+e))
-1/6*b*i^2*(6*a*e*i-18*a*f*h-5*b*e*i+9*b*f*h)/d/f^2*x^2*ln(c*(f*x+e))+2/9*b
*i^3*(3*a-b)/d/f*x^3*ln(c*(f*x+e))-1/2*b^2*i^2*(e*i-3*f*h)/d/f^2*x^2*ln(c*(
f*x+e))^2
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 606, normalized size of antiderivative = 1.31

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$=\frac{4\left(9a^2-6ab+2b^2\right)f^3i^3x^3+36\left(b^2f^3h^3-3b^2ef^2h^2i+3b^2e^2fhi^2-b^2e^3i^3\right)\log\left(cfx+ce\right)^3+3\left(27\left(2a^2-\right.\right.}{}$$

```
[In] integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="fricas
")
```

```
[Out] 1/108*(4*(9*a^2 - 6*a*b + 2*b^2)*f^3*i^3*x^3 + 36*(b^2*f^3*h^3 - 3*b^2*e*f^
2*h^2*i + 3*b^2*e^2*f*h*i^2 - b^2*e^3*i^3)*log(c*f*x + c*e)^3 + 3*(27*(2*a^
2 - 2*a*b + b^2)*f^3*h*i^2 - (18*a^2 - 30*a*b + 19*b^2)*e*f^2*i^3)*x^2 + 18
*(2*b^2*f^3*i^3*x^3 + 6*a*b*f^3*h^3 - 18*(a*b - b^2)*e*f^2*h^2*i + 9*(2*a*b
 - 3*b^2)*e^2*f*h*i^2 - (6*a*b - 11*b^2)*e^3*i^3 + 3*(3*b^2*f^3*h*i^2 - b^2
*e*f^2*i^3)*x^2 + 6*(3*b^2*f^3*h^2*i - 3*b^2*e*f^2*h*i^2 + b^2*e^2*f*i^3)*x
)*log(c*f*x + c*e)^2 + 6*(54*(a^2 - 2*a*b + 2*b^2)*f^3*h^2*i - 27*(2*a^2 -
6*a*b + 7*b^2)*e*f^2*h*i^2 + (18*a^2 - 66*a*b + 85*b^2)*e^2*f*i^3)*x + 6*(4
*(3*a*b - b^2)*f^3*i^3*x^3 + 18*a^2*f^3*h^3 - 54*(a^2 - 2*a*b + 2*b^2)*e*f^
2*h^2*i + 27*(2*a^2 - 6*a*b + 7*b^2)*e^2*f*h*i^2 - (18*a^2 - 66*a*b + 85*b^
2)*e^3*i^3 + 3*(9*(2*a*b - b^2)*f^3*h*i^2 - (6*a*b - 5*b^2)*e*f^2*i^3)*x^2
+ 6*(18*(a*b - b^2)*f^3*h^2*i - 9*(2*a*b - 3*b^2)*e*f^2*h*i^2 + (6*a*b - 11
*b^2)*e^2*f*i^3)*x)*log(c*f*x + c*e))/(d*f^4)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(428) = 856$.

Time = 1.10 (sec) , antiderivative size = 918, normalized size of antiderivative = 1.98

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = x^3\left(\frac{a^2i^3}{3df}-\frac{2abi^3}{9df}+\frac{2b^2i^3}{27df}\right) + x^2\left(-\frac{a^2ei^3}{2df^2}+\frac{3a^2hi^2}{2df}+\frac{5abei^3}{6df^2}-\frac{3abhi^2}{2df}-\frac{19b^2ei^3}{36df^2}+\frac{3b^2hi^2}{4df}\right)+x\left(\frac{a^2e^2i^3}{df^3}-\frac{3a^2ehi^2}{df^2} + \frac{3a^2h^2i}{df}-\frac{11abe^2i^3}{3df^3}+\frac{9abehi^2}{df^2}-\frac{6abh^2i}{df}+\frac{85b^2e^2i^3}{18df^3}-\frac{21b^2ehi^2}{2df^2}+\frac{6b^2h^2i}{df}\right) + \frac{(36abe^2i^3x-108abefhi^2x-18abefi^3x^2+108abf^2h^2ix+54abf^2hi^2x^2+12abf^2i^3x^3-66b^2e^2i^3x+16}{18df^3} + \frac{(-b^2e^3i^3+3b^2e^2fhi^2-3b^2ef^2h^2i+b^2f^3h^3)\log{(c(e+fx))}^3}{3df^4} - \frac{(18a^2e^3i^3-54a^2e^2fhi^2+54a^2ef^2h^2i-18a^2f^3h^3-66abe^3i^3+162abe^2fhi^2-108abef^2h^2i+85b^2e^3i^3}{18df^4} + \frac{(-6abe^3i^3+18abe^2fhi^2-18abef^2h^2i+6abf^3h^3+11b^2e^3i^3-27b^2e^2fhi^2+6b^2e^2fi^3x+18b^2ef^2h^2i-}{6df^4}$$

```
[In] integrate((i*x+h)**3*(a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e),x)
```

```
[Out] x**3*(a**2*i**3/(3*d*f) - 2*a*b*i**3/(9*d*f) + 2*b**2*i**3/(27*d*f)) + x**2
*(-a**2*e*i**3/(2*d*f**2) + 3*a**2*h*i**2/(2*d*f) + 5*a*b*e*i**3/(6*d*f**2)
 - 3*a*b*h*i**2/(2*d*f) - 19*b**2*e*i**3/(36*d*f**2) + 3*b**2*h*i**2/(4*d*f
)) + x*(a**2*e**2*i**3/(d*f**3) - 3*a**2*e*h*i**2/(d*f**2) + 3*a**2*h**2*i/
(d*f) - 11*a*b*e**2*i**3/(3*d*f**3) + 9*a*b*e*h*i**2/(d*f**2) - 6*a*b*h**2*
```

```
i/(d*f) + 85*b**2*e**2*i**3/(18*d*f**3) - 21*b**2*e*h*i**2/(2*d*f**2) + 6*b
**2*h**2*i/(d*f)) + (36*a*b*e**2*i**3*x - 108*a*b*e*f*h*i**2*x - 18*a*b*e*f
*i**3*x**2 + 108*a*b*f**2*h**2*i*x + 54*a*b*f**2*h*i**2*x**2 + 12*a*b*f**2*
i**3*x**3 - 66*b**2*e**2*i**3*x + 162*b**2*e*f*h*i**2*x + 15*b**2*e*f*i**3*
x**2 - 108*b**2*f**2*h**2*i*x - 27*b**2*f**2*h*i**2*x**2 - 4*b**2*f**2*i**3
*x**3)*log(c*(e + f*x))/(18*d*f**3) + (-b**2*e**3*i**3 + 3*b**2*e**2*f*h*i*
*2 - 3*b**2*e*f**2*h**2*i + b**2*f**3*h**3)*log(c*(e + f*x))**3/(3*d*f**4)
- (18*a**2*e**3*i**3 - 54*a**2*e**2*f*h*i**2 + 54*a**2*e*f**2*h**2*i - 18*a
**2*f**3*h**3 - 66*a*b*e**3*i**3 + 162*a*b*e**2*f*h*i**2 - 108*a*b*e*f**2*h
**2*i + 85*b**2*e**3*i**3 - 189*b**2*e**2*f*h*i**2 + 108*b**2*e*f**2*h**2*i
)*log(e + f*x)/(18*d*f**4) + (-6*a*b*e**3*i**3 + 18*a*b*e**2*f*h*i**2 - 18*
a*b*e*f**2*h**2*i + 6*a*b*f**3*h**3 + 11*b**2*e**3*i**3 - 27*b**2*e**2*f*h*
i**2 + 6*b**2*e**2*f*i**3*x + 18*b**2*e*f**2*h**2*i - 18*b**2*e*f**2*h*i**2
*x - 3*b**2*e*f**2*i**3*x**2 + 18*b**2*f**3*h**2*i*x + 9*b**2*f**3*h*i**2*x
**2 + 2*b**2*f**3*i**3*x**3)*log(c*(e + f*x))**2/(6*d*f**4)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(446) = 892$.

Time = 0.28 (sec) , antiderivative size = 964, normalized size of antiderivative = 2.08

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = 6\,abh^2i\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)\log(cfx+ce)$$
$$-\frac{1}{3}\,abi^3\left(\frac{6\,e^3\log(fx+e)}{df^4}-\frac{2\,f^2x^3-3\,efx^2+6\,e^2x}{df^3}\right)\log(cfx+ce)$$
$$+3\,abhi^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)\log(cfx+ce)$$
$$-abh^3\left(\frac{2\,\log(cfx+ce)\log(dfx+de)}{df}-\frac{\log(fx+e)^2+2\,\log(fx+e)\log(c)}{df}\right)$$
$$+3\,a^2h^2i\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)-\frac{1}{6}\,a^2i^3\left(\frac{6\,e^3\log(fx+e)}{df^4}-\frac{2\,f^2x^3-3\,efx^2+6\,e^2x}{df^3}\right)$$
$$+\frac{3}{2}\,a^2hi^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)+\frac{b^2h^3\log(cfx+ce)^3}{3\,df}$$
$$+\frac{2\,abh^3\log(cfx+ce)\log(dfx+de)}{df}+\frac{a^2h^3\log(dfx+de)}{df}$$
$$+\frac{3\left(e\log(fx+e)^2-2\,fx+2\,e\log(fx+e)\right)abh^2i}{df^2}$$
$$-\frac{3\left(f^2x^2+2\,e^2\log(fx+e)^2-6\,efx+6\,e^2\log(fx+e)\right)abhi^2}{2\,df^3}$$
$$-\frac{\left(c^2e\log(cfx+ce)^3-3\,(cfx+ce)\left(c\log(cfx+ce)^2-2\,c\log(cfx+ce)+2\,c\right)\right)b^2h^2i}{c^2df^2}$$
$$-\frac{\left(4\,f^3x^3-15\,ef^2x^2-18\,e^3\log(fx+e)^2+66\,e^2fx-66\,e^3\log(fx+e)\right)abi^3}{18\,df^4}$$
$$+\frac{\left(4\,c^3e^2\log(cfx+ce)^3+3\,(cfx+ce)^2\left(2\,c\log(cfx+ce)^2-2\,c\log(cfx+ce)+c\right)-24\,\left(c^2e\log(cfx+\right.\right.}{4\,c^3df^3}$$
$$-\frac{\left(36\,c^4e^3\log(cfx+ce)^3-4\,(cfx+ce)^3\left(9\,c\log(cfx+ce)^2-6\,c\log(cfx+ce)+2\,c\right)+81\,\left(2\,c^2e\log(c\right.\right.}{}$$

```
[In] integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="maxima
")
```

```
[Out] 6*a*b*h^2*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) - 1/3*a*b*i
^3*(6*e^3*log(f*x + e)/(d*f^4) - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3))
*log(c*f*x + c*e) + 3*a*b*h*i^2*(2*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*
x)/(d*f^2))*log(c*f*x + c*e) - a*b*h^3*(2*log(c*f*x + c*e)*log(d*f*x + d*e)
/(d*f) - (log(f*x + e)^2 + 2*log(f*x + e)*log(c))/(d*f)) + 3*a^2*h^2*i*(x/(
d*f) - e*log(f*x + e)/(d*f^2)) - 1/6*a^2*i^3*(6*e^3*log(f*x + e)/(d*f^4) -
(2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/(d*f^3)) + 3/2*a^2*h*i^2*(2*e^2*log(f*x +
 e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2)) + 1/3*b^2*h^3*log(c*f*x + c*e)^3/(d*
f) + 2*a*b*h^3*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) + a^2*h^3*log(d*f*x
```

```
+ d*e)/(d*f) + 3*(e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*a*b*h^2*i/(d
*f^2) - 3/2*(f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x + 6*e^2*log(f*x + e))
*a*b*h*i^2/(d*f^3) - (c^2*e*log(c*f*x + c*e)^3 - 3*(c*f*x + c*e)*(c*log(c*f
*x + c*e)^2 - 2*c*log(c*f*x + c*e) + 2*c))*b^2*h^2*i/(c^2*d*f^2) - 1/18*(4*
f^3*x^3 - 15*e*f^2*x^2 - 18*e^3*log(f*x + e)^2 + 66*e^2*f*x - 66*e^3*log(f*
x + e))*a*b*i^3/(d*f^4) + 1/4*(4*c^3*e^2*log(c*f*x + c*e)^3 + 3*(c*f*x + c*
e)^2*(2*c*log(c*f*x + c*e)^2 - 2*c*log(c*f*x + c*e) + c) - 24*(c^2*e*log(c*
f*x + c*e)^2 - 2*c^2*e*log(c*f*x + c*e) + 2*c^2*e)*(c*f*x + c*e))*b^2*h*i^2
/(c^3*d*f^3) - 1/108*(36*c^4*e^3*log(c*f*x + c*e)^3 - 4*(c*f*x + c*e)^3*(9*
c*log(c*f*x + c*e)^2 - 6*c*log(c*f*x + c*e) + 2*c) + 81*(2*c^2*e*log(c*f*x
+ c*e)^2 - 2*c^2*e*log(c*f*x + c*e) + c^2*e)*(c*f*x + c*e)^2 - 324*(c^3*e^2
*log(c*f*x + c*e)^2 - 2*c^3*e^2*log(c*f*x + c*e) + 2*c^3*e^2)*(c*f*x + c*e)
)*b^2*i^3/(c^4*d*f^4)
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.66

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= \frac{1}{6}\left(\frac{2\,b^2i^3x^3}{df} + \frac{3\,(3\,b^2fhi^2 - b^2ei^3)x^2}{df^2} + \frac{6\,(3\,b^2f^2h^2i - 3\,b^2efhi^2 + b^2e^2i^3)x}{df^3} + \frac{6\,abf^3h^3 - 18\,abef^2h^2i + 18\right.$$
$$+ ce)^2 + \frac{(9\,a^2i^3 - 6\,abi^3 + 2\,b^2i^3)x^3}{27\,df}$$
$$+ \frac{1}{18}\left(\frac{4\,(3\,abi^3 - b^2i^3)x^3}{df} + \frac{3\,(18\,abfhi^2 - 9\,b^2fhi^2 - 6\,abei^3 + 5\,b^2ei^3)x^2}{df^2} + \frac{6\,(18\,abf^2h^2i - 18\,b^2f^2h^2i -\right.$$
$$+ ce) + \frac{(54\,a^2fhi^2 - 54\,abfhi^2 + 27\,b^2fhi^2 - 18\,a^2ei^3 + 30\,abei^3 - 19\,b^2ei^3)x^2}{36\,df^2}$$
$$+ \frac{(b^2f^3h^3 - 3\,b^2ef^2h^2i + 3\,b^2e^2fhi^2 - b^2e^3i^3)\log(cfx + ce)^3}{3\,df^4}$$
$$+ \frac{(54\,a^2f^2h^2i - 108\,abf^2h^2i + 108\,b^2f^2h^2i - 54\,a^2efhi^2 + 162\,abefhi^2 - 189\,b^2efhi^2 + 18\,a^2e^2i^3 - 66\,abe}{18\,df^3}$$
$$+ \frac{(18\,a^2f^3h^3 - 54\,a^2ef^2h^2i + 108\,abef^2h^2i - 108\,b^2ef^2h^2i + 54\,a^2e^2fhi^2 - 162\,abe^2fhi^2 + 189\,b^2e^2fhi^2 -}{18\,df^4}$$

```
[In] integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^2*i^3*x^3/(d*f) + 3*(3*b^2*f*h*i^2 - b^2*e*i^3)*x^2/(d*f^2) + 6*(3
*b^2*f^2*h^2*i - 3*b^2*e*f*h*i^2 + b^2*e^2*i^3)*x/(d*f^3) + (6*a*b*f^3*h^3
- 18*a*b*e*f^2*h^2*i + 18*b^2*e*f^2*h^2*i + 18*a*b*e^2*f*h*i^2 - 27*b^2*e^2
*f*h*i^2 - 6*a*b*e^3*i^3 + 11*b^2*e^3*i^3)/(d*f^4))*log(c*f*x + c*e)^2 + 1/
27*(9*a^2*i^3 - 6*a*b*i^3 + 2*b^2*i^3)*x^3/(d*f) + 1/18*(4*(3*a*b*i^3 - b^2
```

```
*i^3)*x^3/(d*f) + 3*(18*a*b*f*h*i^2 - 9*b^2*f*h*i^2 - 6*a*b*e*i^3 + 5*b^2*e
*i^3)*x^2/(d*f^2) + 6*(18*a*b*f^2*h^2*i - 18*b^2*f^2*h^2*i - 18*a*b*e*f*h*i
^2 + 27*b^2*e*f*h*i^2 + 6*a*b*e^2*i^3 - 11*b^2*e^2*i^3)*x/(d*f^3))*log(c*f*
x + c*e) + 1/36*(54*a^2*f*h*i^2 - 54*a*b*f*h*i^2 + 27*b^2*f*h*i^2 - 18*a^2*
e*i^3 + 30*a*b*e*i^3 - 19*b^2*e*i^3)*x^2/(d*f^2) + 1/3*(b^2*f^3*h^3 - 3*b^2
*e*f^2*h^2*i + 3*b^2*e^2*f*h*i^2 - b^2*e^3*i^3)*log(c*f*x + c*e)^3/(d*f^4)
+ 1/18*(54*a^2*f^2*h^2*i - 108*a*b*f^2*h^2*i + 108*b^2*f^2*h^2*i - 54*a^2*e
*f*h*i^2 + 162*a*b*e*f*h*i^2 - 189*b^2*e*f*h*i^2 + 18*a^2*e^2*i^3 - 66*a*b*
e^2*i^3 + 85*b^2*e^2*i^3)*x/(d*f^3) + 1/18*(18*a^2*f^3*h^3 - 54*a^2*e*f^2*h
^2*i + 108*a*b*e*f^2*h^2*i - 108*b^2*e*f^2*h^2*i + 54*a^2*e^2*f*h*i^2 - 162
*a*b*e^2*f*h*i^2 + 189*b^2*e^2*f*h*i^2 - 18*a^2*e^3*i^3 + 66*a*b*e^3*i^3 -
85*b^2*e^3*i^3)*log(f*x + e)/(d*f^4)
```

## Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.73

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= x^2\left(\frac{i^2\,(18\,a^2\,f\,h-5\,b^2\,e\,i+9\,b^2\,f\,h+6\,a\,b\,e\,i-18\,a\,b\,f\,h)}{12\,d\,f^2}-\frac{e\,i^3\,(9\,a^2-6\,a\,b+2\,b^2)}{18\,d\,f^2}\right)$$
$$+\ln(c\,(e+f\,x))^2\left(f\left(\frac{b^2\,i^3\,x^3}{3\,d\,f^2}-\frac{b^2\,i^2\,x^2\,(e\,i-3\,f\,h)}{2\,d\,f^3}+\frac{b^2\,i\,x\,(e^2\,i^2-3\,e\,f\,h\,i+3\,f^2\,h^2)}{d\,f^4}\right)\right.$$
$$\left.+\frac{11\,b^2\,e^3\,i^3-27\,b^2\,e^2\,f\,h\,i^2+18\,b^2\,e\,f^2\,h^2\,i-6\,a\,b\,e^3\,i^3+18\,a\,b\,e^2\,f\,h\,i^2-18\,a\,b\,e\,f^2\,h^2\,i+6\,a\,b\,f^3\,h^3}{6\,d\,f^4}\right)$$
$$+x\left(\frac{54\,a^2\,f^2\,h^2\,i-36\,a\,b\,e^2\,i^3+108\,a\,b\,e\,f\,h\,i^2-108\,a\,b\,f^2\,h^2\,i+66\,b^2\,e^2\,i^3-162\,b^2\,e\,f\,h\,i^2+108\,b^2\,f^2\,h}{18\,d\,f^3}\right.$$
$$\left.-\frac{e\left(\frac{i^2\,(18\,a^2\,f\,h-5\,b^2\,e\,i+9\,b^2\,f\,h+6\,a\,b\,e\,i-18\,a\,b\,f\,h)}{6\,d\,f^2}-\frac{e\,i^3\,(9\,a^2-6\,a\,b+2\,b^2)}{9\,d\,f^2}\right)}{f}\right)$$
$$+f\,\ln(c\,(e+f\,x))\left(\frac{x^2\,(5\,e\,b^2\,i^3-9\,f\,h\,b^2\,i^2-6\,a\,e\,b\,i^3+18\,a\,f\,h\,b\,i^2)}{6\,d\,f^3}\right.$$
$$-\frac{x\,(11\,b^2\,e^2\,i^3-27\,b^2\,e\,f\,h\,i^2+18\,b^2\,f^2\,h^2\,i-6\,a\,b\,e^2\,i^3+18\,a\,b\,e\,f\,h\,i^2-18\,a\,b\,f^2\,h^2\,i)}{3\,d\,f^4}$$
$$\left.+\frac{2\,b\,i^3\,x^3\,(3\,a-b)}{9\,d\,f^2}\right)$$
$$-\frac{\ln(e+f\,x)\,(18\,a^2\,e^3\,i^3-54\,a^2\,e^2\,f\,h\,i^2+54\,a^2\,e\,f^2\,h^2\,i-18\,a^2\,f^3\,h^3-66\,a\,b\,e^3\,i^3+162\,a\,b\,e^2\,f\,h\,i^2-1}{18\,d\,f^4}$$
$$+\frac{i^3\,x^3\,(9\,a^2-6\,a\,b+2\,b^2)}{27\,d\,f}-\frac{b^2\ln(c\,(e+f\,x))^3\,(e^3\,i^3-3\,e^2\,f\,h\,i^2+3\,e\,f^2\,h^2\,i-f^3\,h^3)}{3\,d\,f^4}$$

```
[In] int(((h + i*x)^3*(a + b*log(c*(e + f*x)))^2)/(d*e + d*f*x),x)
```

[Out] x^2*((i^2*(18*a^2*f*h - 5*b^2*e*i + 9*b^2*f*h + 6*a*b*e*i - 18*a*b*f*h))/(12*d*f^2) - (e*i^3*(9*a^2 - 6*a*b + 2*b^2))/(18*d*f^2)) + log(c*(e + f*x))^2*(f*((b^2*i^3*x^3)/(3*d*f^2) - (b^2*i^2*x^2*(e*i - 3*f*h))/(2*d*f^3) + (b^2*i*x*(e^2*i^2 + 3*f^2*h^2 - 3*e*f*h*i))/(d*f^4)) + (11*b^2*e^3*i^3 - 6*a*b*e^3*i^3 + 6*a*b*f^3*h^3 + 18*b^2*e*f^2*h^2*i - 27*b^2*e^2*f*h*i^2 - 18*a*b*e*f^2*h^2*i + 18*a*b*e^2*f*h*i^2)/(6*d*f^4)) + x*((66*b^2*e^2*i^3 + 54*a^2*f^2*h^2*i + 108*b^2*f^2*h^2*i - 36*a*b*e^2*i^3 - 108*a*b*f^2*h^2*i - 162*b^2*e*f*h*i^2 + 108*a*b*e*f*h*i^2)/(18*d*f^3) - (e*((i^2*(18*a^2*f*h - 5*b^2*e*i + 9*b^2*f*h + 6*a*b*e*i - 18*a*b*f*h))/(6*d*f^2) - (e*i^3*(9*a^2 - 6*a*b + 2*b^2))/(9*d*f^2)))/f) + f*log(c*(e + f*x))*((x^2*(5*b^2*e*i^3 - 6*a*b*e*i^3 - 9*b^2*f*h*i^2 + 18*a*b*f*h*i^2))/(6*d*f^3) - (x*(11*b^2*e^2*i^3 + 18*b^2*f^2*h^2*i - 6*a*b*e^2*i^3 - 18*a*b*f^2*h^2*i - 27*b^2*e*f*h*i^2 + 18*a*b*e*f*h*i^2))/(3*d*f^4) + (2*b*i^3*x^3*(3*a - b))/(9*d*f^2)) - (log(e + f*x)*(18*a^2*e^3*i^3 - 18*a^2*f^3*h^3 + 85*b^2*e^3*i^3 - 66*a*b*e^3*i^3 + 54*a^2*e*f^2*h^2*i - 54*a^2*e^2*f*h*i^2 + 108*b^2*e*f^2*h^2*i - 189*b^2*e^2*f*h*i^2 - 108*a*b*e*f^2*h^2*i + 162*a*b*e^2*f*h*i^2))/(18*d*f^4) + (i^3*x^3*(9*a^2 - 6*a*b + 2*b^2))/(27*d*f) - (b^2*log(c*(e + f*x))^3*(e^3*i^3 - f^3*h^3 + 3*e*f^2*h^2*i - 3*e^2*f*h*i^2))/(3*d*f^4)

# 3.185 $\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 238

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = -\frac{4abi(fh-ei)x}{df^2}+\frac{4b^2i(fh-ei)x}{df^2}+\frac{b^2i^2(e+fx)^2}{4df^3} - \frac{4b^2i(fh-ei)(e+fx)\log(c(e+fx))}{df^3} - \frac{bi^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^3} + \frac{2i(fh-ei)(e+fx)(a+b\log(c(e+fx)))^2}{df^3} + \frac{i^2(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^3} + \frac{(fh-ei)^2(a+b\log(c(e+fx)))^3}{3bdf^3}$$

```
[Out] -4*a*b*i*(-e*i+f*h)*x/d/f^2+4*b^2*i*(-e*i+f*h)*x/d/f^2+1/4*b^2*i^2*(f*x+e)^
2/d/f^3-4*b^2*i*(-e*i+f*h)*(f*x+e)*ln(c*(f*x+e))/d/f^3-1/2*b*i^2*(f*x+e)^2*
(a+b*ln(c*(f*x+e)))/d/f^3+2*i*(-e*i+f*h)*(f*x+e)*(a+b*ln(c*(f*x+e)))^2/d/f^
3+1/2*i^2*(f*x+e)^2*(a+b*ln(c*(f*x+e)))^2/d/f^3+1/3*(-e*i+f*h)^2*(a+b*ln(c*
(f*x+e)))^3/b/d/f^3
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2458, 12, 2388, 2339, 30, 2333, 2332, 2367, 2342, 2341}

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{(fh-ei)^2(a+b\log(c(e+fx)))^3}{3bdf^3} + \frac{2i(e+fx)(fh-ei)(a+b\log(c(e+fx)))^2}{df^3} + \frac{i^2(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^3} - \frac{bi^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^3} - \frac{4abix(fh-ei)}{df^2} - \frac{4b^2i(e+fx)(fh-ei)\log(c(e+fx))}{df^3} + \frac{b^2i^2(e+fx)^2}{4df^3} + \frac{4b^2ix(fh-ei)}{df^2}$$

```
[In] Int[((h + i*x)^2*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

```
[Out] (-4*a*b*i*(f*h - e*i)*x)/(d*f^2) + (4*b^2*i*(f*h - e*i)*x)/(d*f^2) + (b^2*i
^2*(e + f*x)^2)/(4*d*f^3) - (4*b^2*i*(f*h - e*i)*(e + f*x)*Log[c*(e + f*x)]
)/(d*f^3) - (b*i^2*(e + f*x)^2*(a + b*Log[c*(e + f*x)]))/(2*d*f^3) + (2*i*(
f*h - e*i)*(e + f*x)*(a + b*Log[c*(e + f*x)])^2)/(d*f^3) + (i^2*(e + f*x)^2
*(a + b*Log[c*(e + f*x)])^2)/(2*d*f^3) + ((f*h - e*i)^2*(a + b*Log[c*(e + f
*x)])^3)/(3*b*d*f^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2(a+b\log(cx))^2}{dx}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2(a+b\log(cx))^2}{x}\,dx, x, e+fx\right)}{df}$$

$$= \frac{i\text{Subst}\left(\int \left(\frac{fh-ei}{f}+\frac{ix}{f}\right)(a+b\log(cx))^2\,dx, x, e+fx\right)}{df^2} + \frac{(fh-ei)\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)(a+b\log(cx))^2}{x}\,dx, x, e+fx\right)}{df^2}$$

$$= \frac{i\text{Subst}\left(\int \left(\frac{(fh-ei)(a+b\log(cx))^2}{f}+\frac{ix(a+b\log(cx))^2}{f}\right)\,dx, x, e+fx\right)}{df^2} + \frac{(i(fh-ei))\text{Subst}\left(\int (a+b\log(cx))^2\,dx, x, e+fx\right)}{df^3} + \frac{(fh-ei)^2\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{x}\,dx, x, e+fx\right)}{df^3}$$

$$= \frac{i(fh-ei)(e+fx)(a+b\log(c(e+fx)))^2}{df^3} + \frac{i^2\text{Subst}\left(\int x(a+b\log(cx))^2\,dx, x, e+fx\right)}{df^3} + \frac{(i(fh-ei))\text{Subst}\left(\int (a+b\log(cx))^2\,dx, x, e+fx\right)}{df^3} - \frac{(2bi(fh-ei))\text{Subst}\left(\int (a+b\log(cx))\,dx, x, e+fx\right)}{df^3} + \frac{(fh-ei)^2\text{Subst}\left(\int x^2\,dx, x, a+b\log(c(e+fx))\right)}{bdf^3}$$

$$= -\frac{2abi(fh-ei)x}{df^2} + \frac{2i(fh-ei)(e+fx)(a+b\log(c(e+fx)))^2}{df^3} + \frac{i^2(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^3} + \frac{(fh-ei)^2(a+b\log(c(e+fx)))^3}{3bdf^3} - \frac{(bi^2)\operatorname{Subst}(\int x(a+b\log(cx))\,dx,x,e+fx)}{df^3} - \frac{(2bi(fh-ei))\operatorname{Subst}(\int (a+b\log(cx))\,dx,x,e+fx)}{df^3} - \frac{(2b^2i(fh-ei))\operatorname{Subst}(\int \log(cx)\,dx,x,e+fx)}{df^3}$$

$$= -\frac{4abi(fh-ei)x}{df^2} + \frac{2b^2i(fh-ei)x}{df^2} + \frac{b^2i^2(e+fx)^2}{4df^3} - \frac{2b^2i(fh-ei)(e+fx)\log(c(e+fx))}{df^3} - \frac{bi^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^3} + \frac{2i(fh-ei)(e+fx)(a+b\log(c(e+fx)))^2}{df^3} + \frac{i^2(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^3} + \frac{(fh-ei)^2(a+b\log(c(e+fx)))^3}{3bdf^3} - \frac{(2b^2i(fh-ei))\operatorname{Subst}(\int \log(cx)\,dx,x,e+fx)}{df^3}$$

$$= -\frac{4abi(fh-ei)x}{df^2} + \frac{4b^2i(fh-ei)x}{df^2} + \frac{b^2i^2(e+fx)^2}{4df^3} - \frac{4b^2i(fh-ei)(e+fx)\log(c(e+fx))}{df^3} - \frac{bi^2(e+fx)^2(a+b\log(c(e+fx)))}{2df^3} + \frac{2i(fh-ei)(e+fx)(a+b\log(c(e+fx)))^2}{df^3} + \frac{i^2(e+fx)^2(a+b\log(c(e+fx)))^2}{2df^3} + \frac{(fh-ei)^2(a+b\log(c(e+fx)))^3}{3bdf^3}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.72

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{24i(fh-ei)(e+fx)(a+b\log(c(e+fx)))^2 + 6i^2(e+fx)^2(a+b\log(c(e+fx)))^2 + \frac{4(fh-ei)^2(a+b\log(c(e+fx)))^3}{b}}{}$$

```
[In] Integrate[((h + i*x)^2*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
[Out] (24*i*(f*h - e*i)*(e + f*x)*(a + b*Log[c*(e + f*x)])^2 + 6*i^2*(e + f*x)^2*
(a + b*Log[c*(e + f*x)])^2 + (4*(f*h - e*i)^2*(a + b*Log[c*(e + f*x)])^3)/b
 - 48*b*i*(f*h - e*i)*((a - b)*f*x + b*(e + f*x)*Log[c*(e + f*x)]) + 3*b*i^
2*(b*f*x*(2*e + f*x) - 2*(e + f*x)^2*(a + b*Log[c*(e + f*x)])))/(12*d*f^3)
```

## Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.67

| method | result |
|---|---|
| norman | $\frac{(2a^2e^2i^2-4a^2efhi+2a^2f^2h^2-6ab\,e^2i^2+8abefhi+7b^2e^2i^2-8b^2efhi)\ln(c(fx+e))}{2d\,f^3}+\frac{b(2a\,e^2i^2-4aefhi+2a\,f^2h^2-3b\,e^2}{2d\,f^3}$ |
| risch | $\frac{b^2\ln(c(fx+e))^3e^2i^2}{3d\,f^3}-\frac{2b^2\ln(c(fx+e))^3ehi}{3d\,f^2}+\frac{b^2\ln(c(fx+e))^3h^2}{3df}+\frac{b(b\,f^2i^2x^2-2bef\,i^2x+4b\,f^2hix+2a\,e^2i^2-4aefh}{2d\,f^3}$ |
| parts | $\frac{a^2\left(\frac{i\left(\frac{1}{2}fi\,x^2-xei+2xfh\right)}{f^2}+\frac{\left(e^2i^2-2efhi+f^2h^2\right)\ln(fx+e)}{f^3}\right)}{d}+\frac{b^2\left(\frac{c\,e^2i^2\ln(cfx+ce)^3}{3f^2}-\frac{2cehi\ln(cfx+ce)^3}{3f}+\frac{c\,h^2\ln(cfx+ce)^3}{3}\right.}{}$ |
| derivativedivides | $\frac{\frac{c\,a^2e^2i^2\ln(cfx+ce)}{f^2d}-\frac{2c\,a^2ehi\ln(cfx+ce)}{fd}+\frac{c\,a^2h^2\ln(cfx+ce)}{d}-\frac{2a^2e\,i^2(cfx+ce)}{f^2d}+\frac{2a^2hi(cfx+ce)}{fd}+\frac{a^2i^2(cfx+ce)^2}{2c\,f^2d}+\frac{cab\,e^2i^2\ln(c}{f^2d}}{}$ |
| default | $\frac{\frac{c\,a^2e^2i^2\ln(cfx+ce)}{f^2d}-\frac{2c\,a^2ehi\ln(cfx+ce)}{fd}+\frac{c\,a^2h^2\ln(cfx+ce)}{d}-\frac{2a^2e\,i^2(cfx+ce)}{f^2d}+\frac{2a^2hi(cfx+ce)}{fd}+\frac{a^2i^2(cfx+ce)^2}{2c\,f^2d}+\frac{cab\,e^2i^2\ln(c}{f^2d}}{}$ |
| parallelrisch | $\frac{-66ab\,e^2i^2-12a^2ef\,i^2x+24a^2f^2hix-42b^2ef\,i^2x+48b^2f^2hix-48a^2efhi+36abef\,i^2x+4\ln(c(fx+e))^3b^2e^2i^2+4\ln(c(fx+}{}$ |

```
[In] int((i*x+h)^2*(a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*a^2*e^2*i^2-4*a^2*e*f*h*i+2*a^2*f^2*h^2-6*a*b*e^2*i^2+8*a*b*e*f*h*i+
7*b^2*e^2*i^2-8*b^2*e*f*h*i)/d/f^3*ln(c*(f*x+e))+1/2*b*(2*a*e^2*i^2-4*a*e*f
*h*i+2*a*f^2*h^2-3*b*e^2*i^2+4*b*e*f*h*i)/d/f^3*ln(c*(f*x+e))^2+1/3*b^2*(e^
2*i^2-2*e*f*h*i+f^2*h^2)/d/f^3*ln(c*(f*x+e))^3-1/2*i*(2*a^2*e*i-4*a^2*f*h-6
*a*b*e*i+8*a*b*f*h+7*b^2*e*i-8*b^2*f*h)/d/f^2*x+1/4*i^2*(2*a^2-2*a*b+b^2)/d
/f*x^2+1/2*b^2*i^2/d/f*x^2*ln(c*(f*x+e))^2-b*i*(2*a*e*i-4*a*f*h-3*b*e*i+4*b
*f*h)/d/f^2*x*ln(c*(f*x+e))+1/2*b*i^2*(2*a-b)/d/f*x^2*ln(c*(f*x+e))-b^2*i*(
e*i-2*f*h)/d/f^2*x*ln(c*(f*x+e))^2
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.41

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$=\frac{3\left(2a^2-2ab+b^2\right)f^2i^2x^2+4\left(b^2f^2h^2-2b^2efhi+b^2e^2i^2\right)\log\left(cfx+ce\right)^3+6\left(b^2f^2i^2x^2+2abf^2h^2-4\left(ab\right.\right.}{}$$

```
[In] integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="fricas
")
```

```
[Out] 1/12*(3*(2*a^2 - 2*a*b + b^2)*f^2*i^2*x^2 + 4*(b^2*f^2*h^2 - 2*b^2*e*f*h*i
+ b^2*e^2*i^2)*log(c*f*x + c*e)^3 + 6*(b^2*f^2*i^2*x^2 + 2*a*b*f^2*h^2 - 4*
(a*b - b^2)*e*f*h*i + (2*a*b - 3*b^2)*e^2*i^2 + 2*(2*b^2*f^2*h*i - b^2*e*f*
i^2)*x)*log(c*f*x + c*e)^2 + 6*(4*(a^2 - 2*a*b + 2*b^2)*f^2*h*i - (2*a^2 -
6*a*b + 7*b^2)*e*f*i^2)*x + 6*((2*a*b - b^2)*f^2*i^2*x^2 + 2*a^2*f^2*h^2 -
4*(a^2 - 2*a*b + 2*b^2)*e*f*h*i + (2*a^2 - 6*a*b + 7*b^2)*e^2*i^2 + 2*(4*(a
*b - b^2)*f^2*h*i - (2*a*b - 3*b^2)*e*f*i^2)*x)*log(c*f*x + c*e))/(d*f^3)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(218) = 436$.

Time = 0.60 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.99

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = x^2\left(\frac{a^2i^2}{2df}-\frac{abi^2}{2df}+\frac{b^2i^2}{4df}\right)+x\left(-\frac{a^2ei^2}{df^2}+\frac{2a^2hi}{df}+\frac{3abei^2}{df^2}-\frac{4abhi}{df}-\frac{7b^2ei^2}{2df^2}+\frac{4b^2hi}{df}\right) + \frac{(-4abei^2x+8abfhix+2abfi^2x^2+6b^2ei^2x-8b^2fhix-b^2fi^2x^2)\log(c(e+fx))}{2df^2} + \frac{(b^2e^2i^2-2b^2efhi+b^2f^2h^2)\log(c(e+fx))^3}{3df^3} + \frac{(2a^2e^2i^2-4a^2efhi+2a^2f^2h^2-6abe^2i^2+8abefhi+7b^2e^2i^2-8b^2efhi)\log(e+fx)}{2df^3} + \frac{(2abe^2i^2-4abefhi+2abf^2h^2-3b^2e^2i^2+4b^2efhi-2b^2efi^2x+4b^2f^2hix+b^2f^2i^2x^2)\log(c(e+fx))^2}{2df^3}$$

```
[In] integrate((i*x+h)**2*(a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e),x)
```

```
[Out] x**2*(a**2*i**2/(2*d*f) - a*b*i**2/(2*d*f) + b**2*i**2/(4*d*f)) + x*(-a**2*
e*i**2/(d*f**2) + 2*a**2*h*i/(d*f) + 3*a*b*e*i**2/(d*f**2) - 4*a*b*h*i/(d*f
) - 7*b**2*e*i**2/(2*d*f**2) + 4*b**2*h*i/(d*f)) + (-4*a*b*e*i**2*x + 8*a*b
*f*h*i*x + 2*a*b*f*i**2*x**2 + 6*b**2*e*i**2*x - 8*b**2*f*h*i*x - b**2*f*i*
*2*x**2)*log(c*(e + f*x))/(2*d*f**2) + (b**2*e**2*i**2 - 2*b**2*e*f*h*i + b
**2*f**2*h**2)*log(c*(e + f*x))**3/(3*d*f**3) + (2*a**2*e**2*i**2 - 4*a**2*
e*f*h*i + 2*a**2*f**2*h**2 - 6*a*b*e**2*i**2 + 8*a*b*e*f*h*i + 7*b**2*e**2*
i**2 - 8*b**2*e*f*h*i)*log(e + f*x)/(2*d*f**3) + (2*a*b*e**2*i**2 - 4*a*b*e
*f*h*i + 2*a*b*f**2*h**2 - 3*b**2*e**2*i**2 + 4*b**2*e*f*h*i - 2*b**2*e*f*i
**2*x + 4*b**2*f**2*h*i*x + b**2*f**2*i**2*x**2)*log(c*(e + f*x))**2/(2*d*f
**3)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(230) = 460.

Time = 0.24 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.46

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = 4\,abhi\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)\log(cfx+ce) + abi^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right)\log(cfx+ce) - abh^2\left(\frac{2\,\log(cfx+ce)\log(dfx+de)}{df}-\frac{\log(fx+e)^2+2\,\log(fx+e)\log(c)}{df}\right) + 2\,a^2hi\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)+\frac{1}{2}\,a^2i^2\left(\frac{2\,e^2\log(fx+e)}{df^3}+\frac{fx^2-2\,ex}{df^2}\right) + \frac{b^2h^2\log(cfx+ce)^3}{3\,df}+\frac{2\,abh^2\log(cfx+ce)\log(dfx+de)}{df} + \frac{a^2h^2\log(dfx+de)}{df}+\frac{2\left(e\log(fx+e)^2-2\,fx+2\,e\log(fx+e)\right)abhi}{df^2} - \frac{\left(f^2x^2+2\,e^2\log(fx+e)^2-6\,efx+6\,e^2\log(fx+e)\right)abi^2}{2\,df^3} - \frac{2\left(c^2e\log(cfx+ce)^3-3\,(cfx+ce)\left(c\log(cfx+ce)^2-2\,c\log(cfx+ce)+2\,c\right)\right)b^2hi}{3\,c^2df^2} + \frac{\left(4\,c^3e^2\log(cfx+ce)^3+3\,(cfx+ce)^2\left(2\,c\log(cfx+ce)^2-2\,c\log(cfx+ce)+c\right)-24\,\left(c^2e\log(cfx+\right.\right.}{12\,c^3df^3}$$

```
[In] integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="maxima
")
```

```
[Out] 4*a*b*h*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) + a*b*i^2*(2*
e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2))*log(c*f*x + c*e) - a*b*
h^2*(2*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2 + 2*log(f*
x + e)*log(c))/(d*f)) + 2*a^2*h*i*(x/(d*f) - e*log(f*x + e)/(d*f^2)) + 1/2*
a^2*i^2*(2*e^2*log(f*x + e)/(d*f^3) + (f*x^2 - 2*e*x)/(d*f^2)) + 1/3*b^2*h^
2*log(c*f*x + c*e)^3/(d*f) + 2*a*b*h^2*log(c*f*x + c*e)*log(d*f*x + d*e)/(d
*f) + a^2*h^2*log(d*f*x + d*e)/(d*f) + 2*(e*log(f*x + e)^2 - 2*f*x + 2*e*lo
g(f*x + e))*a*b*h*i/(d*f^2) - 1/2*(f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x
 + 6*e^2*log(f*x + e))*a*b*i^2/(d*f^3) - 2/3*(c^2*e*log(c*f*x + c*e)^3 - 3*
(c*f*x + c*e)*(c*log(c*f*x + c*e)^2 - 2*c*log(c*f*x + c*e) + 2*c))*b^2*h*i/
(c^2*d*f^2) + 1/12*(4*c^3*e^2*log(c*f*x + c*e)^3 + 3*(c*f*x + c*e)^2*(2*c*l
og(c*f*x + c*e)^2 - 2*c*log(c*f*x + c*e) + c) - 24*(c^2*e*log(c*f*x + c*e)^
2 - 2*c^2*e*log(c*f*x + c*e) + 2*c^2*e)*(c*f*x + c*e))*b^2*i^2/(c^3*d*f^3)
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.76

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$=\frac{1}{2}\left(\frac{b^2i^2x^2}{df}+\frac{2\,(2\,b^2fhi-b^2ei^2)x}{df^2}+\frac{2\,abf^2h^2-4\,abefhi+4\,b^2efhi+2\,abe^2i^2-3\,b^2e^2i^2}{df^3}\right)\log(cfx+ce)^2+\frac{1}{2}\left(\frac{(2\,abi^2-b^2i^2)x^2}{df}+\frac{2\,(4\,abfhi-4\,b^2fhi-2\,abei^2+3\,b^2ei^2)x}{df^2}\right)\log(cfx+ce)$$
$$+\frac{(2\,a^2i^2-2\,abi^2+b^2i^2)x^2}{4\,df}+\frac{(b^2f^2h^2-2\,b^2efhi+b^2e^2i^2)\log(cfx+ce)^3}{3\,df^3}$$
$$+\frac{(4\,a^2fhi-8\,abfhi+8\,b^2fhi-2\,a^2ei^2+6\,abei^2-7\,b^2ei^2)x}{2\,df^2}$$
$$+\frac{(2\,a^2f^2h^2-4\,a^2efhi+8\,abefhi-8\,b^2efhi+2\,a^2e^2i^2-6\,abe^2i^2+7\,b^2e^2i^2)\log(fx+e)}{2\,df^3}$$

```
[In] integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*i^2*x^2/(d*f) + 2*(2*b^2*f*h*i - b^2*e*i^2)*x/(d*f^2) + (2*a*b*f^2
*h^2 - 4*a*b*e*f*h*i + 4*b^2*e*f*h*i + 2*a*b*e^2*i^2 - 3*b^2*e^2*i^2)/(d*f^
3))*log(c*f*x + c*e)^2 + 1/2*((2*a*b*i^2 - b^2*i^2)*x^2/(d*f) + 2*(4*a*b*f*
h*i - 4*b^2*f*h*i - 2*a*b*e*i^2 + 3*b^2*e*i^2)*x/(d*f^2))*log(c*f*x + c*e)
+ 1/4*(2*a^2*i^2 - 2*a*b*i^2 + b^2*i^2)*x^2/(d*f) + 1/3*(b^2*f^2*h^2 - 2*b^
2*e*f*h*i + b^2*e^2*i^2)*log(c*f*x + c*e)^3/(d*f^3) + 1/2*(4*a^2*f*h*i - 8*
a*b*f*h*i + 8*b^2*f*h*i - 2*a^2*e*i^2 + 6*a*b*e*i^2 - 7*b^2*e*i^2)*x/(d*f^2
) + 1/2*(2*a^2*f^2*h^2 - 4*a^2*e*f*h*i + 8*a*b*e*f*h*i - 8*b^2*e*f*h*i + 2*
a^2*e^2*i^2 - 6*a*b*e^2*i^2 + 7*b^2*e^2*i^2)*log(f*x + e)/(d*f^3)
```

## Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.71

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$= x\left(\frac{i\,(2\,a^2\,f\,h-3\,b^2\,e\,i+4\,b^2\,f\,h+2\,a\,b\,e\,i-4\,a\,b\,f\,h)}{d\,f^2}-\frac{e\,i^2\,(2\,a^2-2\,a\,b+b^2)}{2\,d\,f^2}\right)$$
$$+\ln\left(c\,(e+f\,x)\right)^2\left(f\left(\frac{b^2\,i^2\,x^2}{2\,d\,f^2}-\frac{b^2\,i\,x\,(e\,i-2\,f\,h)}{d\,f^3}\right)+\frac{-3\,b^2\,e^2\,i^2+4\,b^2\,e\,f\,h\,i+2\,a\,b\,e^2\,i^2-4\,a\,b\,e\,f\,h\,i+2\,a\,b\,f^2\,h^2}{2\,d\,f^3}\right)$$
$$+f\,\ln\left(c\,(e+f\,x)\right)\left(\frac{x\,(3\,e\,b^2\,i^2-4\,f\,h\,b^2\,i-2\,a\,e\,b\,i^2+4\,a\,f\,h\,b\,i)}{d\,f^3}+\frac{b\,i^2\,x^2\,(2\,a-b)}{2\,d\,f^2}\right)$$
$$+\frac{\ln\left(e+f\,x\right)\,\left(2\,a^2\,e^2\,i^2-4\,a^2\,e\,f\,h\,i+2\,a^2\,f^2\,h^2-6\,a\,b\,e^2\,i^2+8\,a\,b\,e\,f\,h\,i+7\,b^2\,e^2\,i^2-8\,b^2\,e\,f\,h\,i\right)}{2\,d\,f^3}$$
$$+\frac{b^2\ln\left(c\,(e+f\,x)\right)^3\,(e^2\,i^2-2\,e\,f\,h\,i+f^2\,h^2)}{3\,d\,f^3}+\frac{i^2\,x^2\,(2\,a^2-2\,a\,b+b^2)}{4\,d\,f}$$

```
[In] int(((h + i*x)^2*(a + b*log(c*(e + f*x)))^2)/(d*e + d*f*x),x)
```

```
[Out] x*((i*(2*a^2*f*h - 3*b^2*e*i + 4*b^2*f*h + 2*a*b*e*i - 4*a*b*f*h))/(d*f^2)
- (e*i^2*(2*a^2 - 2*a*b + b^2))/(2*d*f^2)) + log(c*(e + f*x))^2*(f*((b^2*i^
2*x^2)/(2*d*f^2) - (b^2*i*x*(e*i - 2*f*h))/(d*f^3)) + (2*a*b*e^2*i^2 - 3*b^
2*e^2*i^2 + 2*a*b*f^2*h^2 + 4*b^2*e*f*h*i - 4*a*b*e*f*h*i)/(2*d*f^3)) + f*l
og(c*(e + f*x))*((x*(3*b^2*e*i^2 - 2*a*b*e*i^2 - 4*b^2*f*h*i + 4*a*b*f*h*i)
)/(d*f^3) + (b*i^2*x^2*(2*a - b))/(2*d*f^2)) + (log(e + f*x)*(2*a^2*e^2*i^2
 + 2*a^2*f^2*h^2 + 7*b^2*e^2*i^2 - 6*a*b*e^2*i^2 - 4*a^2*e*f*h*i - 8*b^2*e*
f*h*i + 8*a*b*e*f*h*i))/(2*d*f^3) + (b^2*log(c*(e + f*x))^3*(e^2*i^2 + f^2*
h^2 - 2*e*f*h*i))/(3*d*f^3) + (i^2*x^2*(2*a^2 - 2*a*b + b^2))/(4*d*f)
```

# 3.186 $\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 113

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = -\frac{2abix}{df}+\frac{2b^2ix}{df}-\frac{2b^2i(e+fx)\log(c(e+fx))}{df^2}+\frac{i(e+fx)(a+b\log(c(e+fx)))^2}{df^2}+\frac{(fh-ei)(a+b\log(c(e+fx)))^3}{3bdf^2}$$

```
[Out] -2*a*b*i*x/d/f+2*b^2*i*x/d/f-2*b^2*i*(f*x+e)*ln(c*(f*x+e))/d/f^2+i*(f*x+e)*
(a+b*ln(c*(f*x+e)))^2/d/f^2+1/3*(-e*i+f*h)*(a+b*ln(c*(f*x+e)))^3/b/d/f^2
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2458, 12, 2388, 2339, 30, 2333, 2332}

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{(fh-ei)(a+b\log(c(e+fx)))^3}{3bdf^2}+\frac{i(e+fx)(a+b\log(c(e+fx)))^2}{df^2}-\frac{2abix}{df}-\frac{2b^2i(e+fx)\log(c(e+fx))}{df^2}+\frac{2b^2ix}{df}$$

```
[In] Int[((h + i*x)*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

```
[Out] (-2*a*b*i*x)/(d*f) + (2*b^2*i*x)/(d*f) - (2*b^2*i*(e + f*x)*Log[c*(e + f*x)
])/(d*f^2) + (i*(e + f*x)*(a + b*Log[c*(e + f*x)])^2)/(d*f^2) + ((f*h - e*i
)*(a + b*Log[c*(e + f*x)])^3)/(3*b*d*f^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)(a+b\log(cx))^2}{dx}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)(a+b\log(cx))^2}{x}\,dx, x, e+fx\right)}{df}$$

$$= \frac{i\text{Subst}\left(\int (a+b\log(cx))^2\,dx, x, e+fx\right)}{df^2} + \frac{(fh-ei)\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{x}\,dx, x, e+fx\right)}{df^2}$$

$$= \frac{i(e+fx)(a+b\log(c(e+fx)))^2}{df^2} - \frac{(2bi)\text{Subst}\left(\int (a+b\log(cx))\,dx, x, e+fx\right)}{df^2} + \frac{(fh-ei)\text{Subst}\left(\int x^2\,dx, x, a+b\log(c(e+fx))\right)}{bdf^2}$$

$$= -\frac{2abix}{df} + \frac{i(e+fx)(a+b\log(c(e+fx)))^2}{df^2} + \frac{(fh-ei)(a+b\log(c(e+fx)))^3}{3bdf^2} - \frac{(2b^2i)\,\text{Subst}\left(\int \log(cx)\,dx, x, e+fx\right)}{df^2}$$

$$= -\frac{2abix}{df} + \frac{2b^2ix}{df} - \frac{2b^2i(e+fx)\log(c(e+fx))}{df^2} + \frac{i(e+fx)(a+b\log(c(e+fx)))^2}{df^2} + \frac{(fh-ei)(a+b\log(c(e+fx)))^3}{3bdf^2}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{-6(a-b)bfix - 6b^2i(e+fx)\log(c(e+fx)) + 3i(e+fx)(a+b\log(c(e+fx)))^2 + \frac{(fh-ei)(a+b\log(c(e+fx)))^3}{b}}{3df^2}$$

```
[In] Integrate[((h + i*x)*(a + b*Log[c*(e + f*x)])^2)/(d*e + d*f*x),x]
```

```
[Out] (-6*(a - b)*b*f*i*x - 6*b^2*i*(e + f*x)*Log[c*(e + f*x)] + 3*i*(e + f*x)*(a
 + b*Log[c*(e + f*x)])^2 + ((f*h - e*i)*(a + b*Log[c*(e + f*x)])^3)/b)/(3*d
*f^2)
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| norman | $\frac{i(a^2-2ab+2b^2)x}{df}+\frac{b^2 ix\ln(c(fx+e))^2}{df}-\frac{(a^2ei-a^2fh-2abei+2b^2ei)\ln(c(fx+e))}{d f^2}-\frac{b(aei-afh-bei)\ln(c(fx+e))^2}{d f^2}$ |
| parts | $\frac{a^2\left(\frac{xi}{f}+\frac{(-ei+fh)\ln(fx+e)}{f^2}\right)}{d}+\frac{b^2\left(-\frac{cei\ln(cfx+ce)^3}{3f}+\frac{ch\ln(cfx+ce)^3}{3}+\frac{i\left((cfx+ce)\ln(cfx+ce)^2-2(cfx+ce)\ln(cfx+ce)+2cfx\right.}{f}\right.}{dcf}$ |
| risch | $-\frac{b^2\ln(c(fx+e))^3 ei}{3d f^2}+\frac{b^2\ln(c(fx+e))^3 h}{3df}-\frac{b(-bfix+aei-afh-bei)\ln(c(fx+e))^2}{d f^2}+\frac{2bi(a-b)x\ln(c(fx+e))}{df}-\ln($ |
| parallelrisch | $3x\ln(c(fx+e))^2b^2fi-\ln(c(fx+e))^3b^2ei+\ln(c(fx+e))^3b^2fh+6x\ln(c(fx+e))abfi-6x\ln(c(fx+e))b^2fi-3\ln(c(fx+e))^2a$ |
| derivativedivides | $\frac{-\frac{a^2cei\ln(cfx+ce)}{fd}+\frac{a^2hc\ln(cfx+ce)}{d}+\frac{a^2i(cfx+ce)}{fd}-\frac{abcei\ln(cfx+ce)^2}{fd}+\frac{abhc\ln(cfx+ce)^2}{d}+\frac{2abi((cfx+ce)\ln(cfx+ce)-cfx-ce)}{fd}}{cf}$ |
| default | $\frac{-\frac{a^2cei\ln(cfx+ce)}{fd}+\frac{a^2hc\ln(cfx+ce)}{d}+\frac{a^2i(cfx+ce)}{fd}-\frac{abcei\ln(cfx+ce)^2}{fd}+\frac{abhc\ln(cfx+ce)^2}{d}+\frac{2abi((cfx+ce)\ln(cfx+ce)-cfx-ce)}{fd}}{cf}$ |

```
[In] int((i*x+h)*(a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e),x,method=_RETURNVERBOSE)
```

```
[Out] i*(a^2-2*a*b+2*b^2)/d/f*x+b^2*i*x/d/f*ln(c*(f*x+e))^2-(a^2*e*i-a^2*f*h-2*a*
b*e*i+2*b^2*e*i)/d/f^2*ln(c*(f*x+e))-b*(a*e*i-a*f*h-b*e*i)/d/f^2*ln(c*(f*x+
e))^2-1/3*b^2*(e*i-f*h)/d/f^2*ln(c*(f*x+e))^3+2*b*i*(a-b)/d/f*x*ln(c*(f*x+e
))
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.25

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$$
$$=\frac{3\left(a^2-2\,ab+2\,b^2\right)fix+\left(b^2fh-b^2ei\right)\log\left(cfx+ce\right)^3+3\left(b^2fix+abfh-\left(ab-b^2\right)ei\right)\log\left(cfx+ce\right)^2+3}{3\,df^2}$$

```
[In] integrate((i*x+h)*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2 - 2*a*b + 2*b^2)*f*i*x + (b^2*f*h - b^2*e*i)*log(c*f*x + c*e)^3
 + 3*(b^2*f*i*x + a*b*f*h - (a*b - b^2)*e*i)*log(c*f*x + c*e)^2 + 3*(a^2*f*
h + 2*(a*b - b^2)*f*i*x - (a^2 - 2*a*b + 2*b^2)*e*i)*log(c*f*x + c*e))/(d*f
^2)
```

## Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.55

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = x\left(\frac{a^2i}{df}-\frac{2abi}{df}+\frac{2b^2i}{df}\right) + \frac{(2abix-2b^2ix)\log(c(e+fx))}{df} + \frac{(-b^2ei+b^2fh)\log(c(e+fx))^3}{3df^2} - \frac{(a^2ei-a^2fh-2abei+2b^2ei)\log(e+fx)}{df^2} + \frac{(-abei+abfh+b^2ei+b^2fix)\log(c(e+fx))^2}{df^2}$$

```
[In] integrate((i*x+h)*(a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e),x)
```

```
[Out] x*(a**2*i/(d*f) - 2*a*b*i/(d*f) + 2*b**2*i/(d*f)) + (2*a*b*i*x - 2*b**2*i*x
)*log(c*(e + f*x))/(d*f) + (-b**2*e*i + b**2*f*h)*log(c*(e + f*x))**3/(3*d*
f**2) - (a**2*e*i - a**2*f*h - 2*a*b*e*i + 2*b**2*e*i)*log(e + f*x)/(d*f**2
) + (-a*b*e*i + a*b*f*h + b**2*e*i + b**2*f*i*x)*log(c*(e + f*x))**2/(d*f**
2)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(111) = 222.

Time = 0.23 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.69

$$\begin{aligned}\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx &= 2\,abi\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)\log(cfx+ce) \\ &\quad - abh\left(\frac{2\,\log(cfx+ce)\log(dfx+de)}{df}-\frac{\log(fx+e)^2+2\,\log(fx+e)\log(c)}{df}\right) \\ &\quad + a^2i\left(\frac{x}{df}-\frac{e\log(fx+e)}{df^2}\right)+\frac{b^2h\log(cfx+ce)^3}{3\,df}+\frac{2\,abh\log(cfx+ce)\log(dfx+de)}{df} \\ &\quad + \frac{a^2h\log(dfx+de)}{df}+\frac{\left(e\log(fx+e)^2-2\,fx+2\,e\log(fx+e)\right)abi}{df^2} \\ &\quad - \frac{\left(c^2e\log(cfx+ce)^3-3\,(cfx+ce)\left(c\log(cfx+ce)^2-2\,c\log(cfx+ce)+2\,c\right)\right)b^2i}{3\,c^2df^2}\end{aligned}$$

```
[In] integrate((i*x+h)*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="maxima")
```

```
[Out] 2*a*b*i*(x/(d*f) - e*log(f*x + e)/(d*f^2))*log(c*f*x + c*e) - a*b*h*(2*log(
c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2 + 2*log(f*x + e)*log(
c))/(d*f)) + a^2*i*(x/(d*f) - e*log(f*x + e)/(d*f^2)) + 1/3*b^2*h*log(c*f*x
 + c*e)^3/(d*f) + 2*a*b*h*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) + a^2*h*l
og(d*f*x + d*e)/(d*f) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*a*b*i
/(d*f^2) - 1/3*(c^2*e*log(c*f*x + c*e)^3 - 3*(c*f*x + c*e)*(c*log(c*f*x + c
*e)^2 - 2*c*log(c*f*x + c*e) + 2*c))*b^2*i/(c^2*d*f^2)
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.58

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \left(\frac{b^2ix}{df}+\frac{abfh-abei+b^2ei}{df^2}\right)\log\left(cfx+ce\right)^2 + \frac{2\left(abi-b^2i\right)x\log\left(cfx+ce\right)}{df} + \frac{\left(b^2fh-b^2ei\right)\log\left(cfx+ce\right)^3}{3\,df^2} + \frac{\left(a^2i-2\,abi+2\,b^2i\right)x}{df} + \frac{\left(a^2fh-a^2ei+2\,abei-2\,b^2ei\right)\log\left(fx+e\right)}{df^2}$$

```
[In] integrate((i*x+h)*(a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="giac")
```

```
[Out] (b^2*i*x/(d*f) + (a*b*f*h - a*b*e*i + b^2*e*i)/(d*f^2))*log(c*f*x + c*e)^2
+ 2*(a*b*i - b^2*i)*x*log(c*f*x + c*e)/(d*f) + 1/3*(b^2*f*h - b^2*e*i)*log(
c*f*x + c*e)^3/(d*f^2) + (a^2*i - 2*a*b*i + 2*b^2*i)*x/(d*f) + (a^2*f*h - a
^2*e*i + 2*a*b*e*i - 2*b^2*e*i)*log(f*x + e)/(d*f^2)
```

## Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.44

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \ln\left(c\left(e+f\,x\right)\right)^2\left(\frac{b\left(a\,f\,h-a\,e\,i+b\,e\,i\right)}{d\,f^2}+\frac{b^2\,i\,x}{d\,f}\right) - \frac{\ln\left(e+f\,x\right)\,\left(a^2\,e\,i-a^2\,f\,h+2\,b^2\,e\,i-2\,a\,b\,e\,i\right)}{d\,f^2} + \frac{i\,x\left(a^2-2\,a\,b+2\,b^2\right)}{d\,f} - \frac{b^2\ln\left(c\left(e+f\,x\right)\right)^3\left(e\,i-f\,h\right)}{3\,d\,f^2} + \frac{2\,b\,i\,x\,\ln\left(c\left(e+f\,x\right)\right)\,\left(a-b\right)}{d\,f}$$

```
[In] int(((h + i*x)*(a + b*log(c*(e + f*x)))^2)/(d*e + d*f*x),x)
```

```
[Out] log(c*(e + f*x))^2*((b*(a*f*h - a*e*i + b*e*i))/(d*f^2) + (b^2*i*x)/(d*f))
- (log(e + f*x)*(a^2*e*i - a^2*f*h + 2*b^2*e*i - 2*a*b*e*i))/(d*f^2) + (i*x
*(a^2 - 2*a*b + 2*b^2))/(d*f) - (b^2*log(c*(e + f*x))^3*(e*i - f*h))/(3*d*f
^2) + (2*b*i*x*log(c*(e + f*x))*(a - b))/(d*f)
```

# 3.187 $\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 27

$$\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{(a+b\log(c(e+fx)))^3}{3bdf}$$

```
[Out] 1/3*(a+b*ln(c*(f*x+e)))^3/b/d/f
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2437, 12, 2339, 30}

$$\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{(a+b\log(c(e+fx)))^3}{3bdf}$$

```
[In] Int[(a + b*Log[c*(e + f*x)])^2/(d*e + d*f*x),x]
```

```
[Out] (a + b*Log[c*(e + f*x)])^3/(3*b*d*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{dx}\,dx, x, e+fx\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{x}\,dx, x, e+fx\right)}{df}\\ &= \frac{\text{Subst}\left(\int x^2\,dx, x, a+b\log(c(e+fx))\right)}{bdf}\\ &= \frac{(a+b\log(c(e+fx)))^3}{3bdf}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{(a+b\log(c(e+fx)))^3}{3bdf}$$

```
[In] Integrate[(a + b*Log[c*(e + f*x)])^2/(d*e + d*f*x),x]
[Out] (a + b*Log[c*(e + f*x)])^3/(3*b*d*f)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(25) = 50.

Time = 0.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{\ln(c(fx+e))^3 b^2 f+3\ln(c(fx+e))^2 abf+3\ln(c(fx+e))a^2 f}{3d\,f^2}$ | 54 |
| risch | $\frac{b^2\ln(c(fx+e))^3}{3df}+\frac{ab\ln(c(fx+e))^2}{df}+\frac{a^2\ln(fx+e)}{df}$ | 58 |
| parts | $\frac{b^2\ln(c(fx+e))^3}{3df}+\frac{ab\ln(c(fx+e))^2}{df}+\frac{a^2\ln(fx+e)}{df}$ | 58 |
| norman | $\frac{a^2\ln(c(fx+e))}{df}+\frac{ab\ln(c(fx+e))^2}{df}+\frac{b^2\ln(c(fx+e))^3}{3df}$ | 60 |
| derivativedivides | $\frac{\frac{c\,a^2\ln(cfx+ce)}{d}+\frac{cab\ln(cfx+ce)^2}{d}+\frac{c\,b^2\ln(cfx+ce)^3}{3d}}{cf}$ | 64 |
| default | $\frac{\frac{c\,a^2\ln(cfx+ce)}{d}+\frac{cab\ln(cfx+ce)^2}{d}+\frac{c\,b^2\ln(cfx+ce)^3}{3d}}{cf}$ | 64 |

```
[In] int((a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e),x,method=_RETURNVERBOSE)
[Out] 1/3*(ln(c*(f*x+e))^3*b^2*f+3*ln(c*(f*x+e))^2*a*b*f+3*ln(c*(f*x+e))*a^2*f)/d
/f^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{b^2\log(cfx+ce)^3+3\,ab\log(cfx+ce)^2+3\,a^2\log(cfx+ce)}{3\,df}$$

```
[In] integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="fricas")
[Out] 1/3*(b^2*log(c*f*x + c*e)^3 + 3*a*b*log(c*f*x + c*e)^2 + 3*a^2*log(c*f*x +
c*e))/(d*f)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(19) = 38$.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{a^2\log(de+dfx)}{df}+\frac{ab\log(c(e+fx))^2}{df}+\frac{b^2\log(c(e+fx))^3}{3df}$$

```
[In] integrate((a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e),x)
[Out] a**2*log(d*e + d*f*x)/(d*f) + a*b*log(c*(e + f*x))**2/(d*f) + b**2*log(c*(e
 + f*x))**3/(3*d*f)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(25) = 50$.

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.74

$$\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = -ab\left(\frac{2\,\log(cfx+ce)\log(dfx+de)}{df} - \frac{\log(fx+e)^2 + 2\,\log(fx+e)\log(c)}{df}\right) + \frac{b^2\log(cfx+ce)^3}{3\,df} + \frac{2\,ab\log(cfx+ce)\log(dfx+de)}{df} + \frac{a^2\log(dfx+de)}{df}$$

```
[In] integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="maxima")
```

```
[Out] -a*b*(2*log(c*f*x + c*e)*log(d*f*x + d*e)/(d*f) - (log(f*x + e)^2 + 2*log(f
*x + e)*log(c))/(d*f)) + 1/3*b^2*log(c*f*x + c*e)^3/(d*f) + 2*a*b*log(c*f*x
 + c*e)*log(d*f*x + d*e)/(d*f) + a^2*log(d*f*x + d*e)/(d*f)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(25) = 50$.

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{b^2\log(cfx+ce)^3}{3\,df} + \frac{ab\log(cfx+ce)^2}{df} + \frac{a^2\log(fx+e)}{df}$$

```
[In] integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e),x, algorithm="giac")
```

```
[Out] 1/3*b^2*log(c*f*x + c*e)^3/(d*f) + a*b*log(c*f*x + c*e)^2/(d*f) + a^2*log(f
*x + e)/(d*f)
```

## Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \frac{(a+b\log(c(e+fx)))^2}{de+dfx}\,dx = \frac{3\,\ln(e+f\,x)\,a^2 + 3\,a\,b\ln(c\,e+c\,f\,x)^2 + b^2\ln(c\,e+c\,f\,x)^3}{3\,d\,f}$$

```
[In] int((a + b*log(c*(e + f*x)))^2/(d*e + d*f*x),x)
```

```
[Out] (b^2*log(c*e + c*f*x)^3 + 3*a^2*log(e + f*x) + 3*a*b*log(c*e + c*f*x)^2)/(3
*d*f)
```

# 3.188 $\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 142

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx = -\frac{(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} + \frac{2b(a+b\log(c(e+fx)))\operatorname{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} + \frac{2b^2\operatorname{PolyLog}\left(3,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)}$$

```
[Out] -(a+b*ln(c*(f*x+e)))^2*ln(1+(-e*i+f*h)/i/(f*x+e))/d/(-e*i+f*h)+2*b*(a+b*ln(
c*(f*x+e)))*polylog(2,(e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)+2*b^2*polylog(3,(e*
i-f*h)/i/(f*x+e))/d/(-e*i+f*h)
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2458, 12, 2379, 2421, 6724}

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx = \frac{2b\operatorname{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)(a+b\log(c(e+fx)))}{d(fh-ei)} - \frac{\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{d(fh-ei)} + \frac{2b^2\operatorname{PolyLog}\left(3,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)}$$

```
[In] Int[(a + b*Log[c*(e + f*x)])^2/((d*e + d*f*x)*(h + i*x)),x]
```

```
[Out] -(((a + b*Log[c*(e + f*x)])^2*Log[1 + (f*h - e*i)/(i*(e + f*x))])/(d*(f*h -
 e*i))) + (2*b*(a + b*Log[c*(e + f*x)])*PolyLog[2, -((f*h - e*i)/(i*(e + f*
x)))])/(d*(f*h - e*i)) + (2*b^2*PolyLog[3, -((f*h - e*i)/(i*(e + f*x)))])/(
d*(f*h - e*i))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{dx\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)}\,dx, x, e+fx\right)}{df}$$

$$= -\frac{(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} + \frac{(2b)\text{Subst}\left(\int \frac{\log\left(1+\frac{fh-ei}{ix}\right)(a+b\log(cx))}{x}\,dx, x, e+fx\right)}{d(fh-ei)}$$

$$= -\frac{(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} + \frac{2b(a+b\log(c(e+fx)))\text{Li}_2\left(-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} - \frac{(2b^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{fh-ei}{ix}\right)}{x}\,dx, x, e+fx\right)}{d(fh-ei)}$$

$$= -\frac{(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} + \frac{2b(a+b\log(c(e+fx)))\text{Li}_2\left(-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)} + \frac{2b^2\text{Li}_3\left(-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)}$$

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.33

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx$$
$$= \frac{3a^2\log(e+fx)+3ab\log^2(c(e+fx))+b^2\log^3(c(e+fx))-3a^2\log(h+ix)-6ab\log(c(e+fx))\log\left(\frac{f(h+}{fh-}\right.}{3a}$$

```
[In] Integrate[(a + b*Log[c*(e + f*x)])^2/((d*e + d*f*x)*(h + i*x)),x]
```

```
[Out] (3*a^2*Log[e + f*x] + 3*a*b*Log[c*(e + f*x)]^2 + b^2*Log[c*(e + f*x)]^3 - 3
*a^2*Log[h + i*x] - 6*a*b*Log[c*(e + f*x)]*Log[(f*(h + i*x))/(f*h - e*i)] -
 3*b^2*Log[c*(e + f*x)]^2*Log[(f*(h + i*x))/(f*h - e*i)] - 6*b*(a + b*Log[c
*(e + f*x)])*PolyLog[2, (i*(e + f*x))/(-(f*h) + e*i)] + 6*b^2*PolyLog[3, (i
*(e + f*x))/(-(f*h) + e*i)])/(3*d*(f*h - e*i))
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(140) = 280$.

Time = 0.94 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.35

| method | result |
|---|---|
| parts | $\frac{a^2\left(\frac{\ln(ix+h)}{ei-fh}-\frac{\ln(fx+e)}{ei-fh}\right)}{d}+\frac{b^2c\left(-\frac{\ln(cfx+ce)^3}{3c(ei-fh)}+\frac{\ln(cfx+ce)^2\ln\left(1+\frac{i(cfx+ce)}{-cei+hcf}\right)+2\ln(cfx+ce)\operatorname{Li}_2\left(-\frac{i(cfx+ce)}{-cei+hcf}\right)-2\operatorname{Li}_3\left(-\right.}{c(ei-fh)}}{d}$ |
| risch | $\frac{a^2\ln(ix+h)}{d(ei-fh)}-\frac{a^2\ln(fx+e)}{d(ei-fh)}-\frac{b^2\ln(cfx+ce)^3}{3d(ei-fh)}+\frac{b^2\ln(cfx+ce)^2\ln\left(1+\frac{i(cfx+ce)}{-cei+hcf}\right)}{d(ei-fh)}+\frac{2b^2\ln(cfx+ce)\operatorname{Li}_2\left(-\frac{i(cfx}{-cei+}\right.}{d(ei-fh)}$ |
| derivativedivides | $\frac{-\frac{cfa^2\ln(cfx+ce)}{d(ei-fh)}+\frac{cfa^2\ln(cei-hcf-i(cfx+ce))}{d(ei-fh)}-\frac{c^2fb^2\left(\frac{\ln(cfx+ce)^3}{3c(ei-fh)}-\frac{\ln(cfx+ce)^2\ln\left(1+\frac{i(cfx+ce)}{-cei+hcf}\right)+2\ln(cfx+ce)\operatorname{Li}_2\left(-\frac{i}{-}\right.}{c(ei-fh)}\right.}{d}}{}$ |
| default | $\frac{-\frac{cfa^2\ln(cfx+ce)}{d(ei-fh)}+\frac{cfa^2\ln(cei-hcf-i(cfx+ce))}{d(ei-fh)}-\frac{c^2fb^2\left(\frac{\ln(cfx+ce)^3}{3c(ei-fh)}-\frac{\ln(cfx+ce)^2\ln\left(1+\frac{i(cfx+ce)}{-cei+hcf}\right)+2\ln(cfx+ce)\operatorname{Li}_2\left(-\frac{i}{-}\right.}{c(ei-fh)}\right.}{d}}{}$ |

```
[In] int((a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h),x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(1/(e*i-f*h)*ln(i*x+h)-1/(e*i-f*h)*ln(f*x+e))+b^2/d*c*(-1/3/c/(e*i-f*
h)*ln(c*f*x+c*e)^3+1/c/(e*i-f*h)*(ln(c*f*x+c*e)^2*ln(1+i/(-c*e*i+c*f*h)*(c*
f*x+c*e))+2*ln(c*f*x+c*e)*polylog(2,-i/(-c*e*i+c*f*h)*(c*f*x+c*e))-2*polylo
g(3,-i/(-c*e*i+c*f*h)*(c*f*x+c*e))))-a*b/d/(e*i-f*h)*ln(c*f*x+c*e)^2+2*a*b/
d/(e*i-f*h)*dilog((-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*e*i+c*f*h))+2*a*b/d/(e*i
-f*h)*ln(c*f*x+c*e)*ln((-c*e*i+h*c*f+i*(c*f*x+c*e))/(-c*e*i+c*f*h))
```

## Fricas [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^2}{(dfx+de)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h),x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*f*x + c*e)^2 + 2*a*b*log(c*f*x + c*e) + a^2)/(d*f*i*x^2
 + d*e*h + (d*f*h + d*e*i)*x), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx = \frac{\int \frac{a^2}{eh+eix+fhx+fix^2}\,dx + \int \frac{b^2\log(ce+cfx)^2}{eh+eix+fhx+fix^2}\,dx + \int \frac{2ab\log(ce+cfx)}{eh+eix+fhx+fix^2}\,dx}{d}$$

```
[In] integrate((a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e)/(i*x+h),x)
[Out] (Integral(a**2/(e*h + e*i*x + f*h*x + f*i*x**2), x) + Integral(b**2*log(c*e
 + c*f*x)**2/(e*h + e*i*x + f*h*x + f*i*x**2), x) + Integral(2*a*b*log(c*e
+ c*f*x)/(e*h + e*i*x + f*h*x + f*i*x**2), x))/d
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(141) = 282.

Time = 0.27 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.33

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx = a^2\left(\frac{\log(fx+e)}{dfh-dei} - \frac{\log(ix+h)}{dfh-dei}\right) - \frac{\left(\log(fx+e)^2\log\left(\frac{fix+ei}{fh-ei}+1\right) + 2\,\mathrm{Li}_2\left(-\frac{fix+ei}{fh-ei}\right)\log(fx+e) - 2\,\mathrm{Li}_3\left(-\frac{fix+ei}{fh-ei}\right)\right)b^2}{(fh-ei)d} - \frac{2\left(b^2\log(c)+ab\right)\left(\log(fx+e)\log\left(\frac{fix+ei}{fh-ei}+1\right) + \mathrm{Li}_2\left(-\frac{fix+ei}{fh-ei}\right)\right)}{(fh-ei)d} - \frac{\left(b^2\log(c)^2+2\,ab\log(c)\right)\log(ix+h)}{(fh-ei)d} + \frac{b^2\log(fx+e)^3 + 3\left(b^2\log(c)+ab\right)\log(fx+e)^2 + 3\left(b^2\log(c)^2+2\,ab\log(c)\right)\log(fx+e)}{3\,(fh-ei)d}$$

```
[In] integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h),x, algorithm="maxima")
[Out] a^2*(log(f*x + e)/(d*f*h - d*e*i) - log(i*x + h)/(d*f*h - d*e*i)) - (log(f*
x + e)^2*log((f*i*x + e*i)/(f*h - e*i) + 1) + 2*dilog(-(f*i*x + e*i)/(f*h -
 e*i))*log(f*x + e) - 2*polylog(3, -(f*i*x + e*i)/(f*h - e*i)))*b^2/((f*h -
 e*i)*d) - 2*(b^2*log(c) + a*b)*(log(f*x + e)*log((f*i*x + e*i)/(f*h - e*i)
 + 1) + dilog(-(f*i*x + e*i)/(f*h - e*i)))/((f*h - e*i)*d) - (b^2*log(c)^2
+ 2*a*b*log(c))*log(i*x + h)/((f*h - e*i)*d) + 1/3*(b^2*log(f*x + e)^3 + 3*
(b^2*log(c) + a*b)*log(f*x + e)^2 + 3*(b^2*log(c)^2 + 2*a*b*log(c))*log(f*x
 + e))/((f*h - e*i)*d)
```

## Giac [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx=\int \frac{(b\log\left((fx+e)c\right)+a)^2}{(dfx+de)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h),x, algorithm="giac")
[Out] integrate((b*log((f*x + e)*c) + a)^2/((d*f*x + d*e)*(i*x + h)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)}\,dx=\int \frac{(a+b\,\ln\left(c\,(e+f\,x)\right))^2}{(h+i\,x)\,\left(d\,e+d\,f\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(e + f*x)))^2/((h + i*x)*(d*e + d*f*x)),x)
[Out] int((a + b*log(c*(e + f*x)))^2/((h + i*x)*(d*e + d*f*x)), x)
```

# 3.189 $\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 273

$$\begin{aligned}\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx = &-\frac{i(e+fx)(a+b\log(c(e+fx)))^2}{d(fh-ei)^2(h+ix)} \\ &+\frac{2bf(a+b\log(c(e+fx)))\log\left(\frac{f(h+ix)}{fh-ei}\right)}{d(fh-ei)^2} \\ &-\frac{f(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2} \\ &+\frac{2bf(a+b\log(c(e+fx)))\operatorname{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2} \\ &+\frac{2b^2f\operatorname{PolyLog}\left(2,-\frac{i(e+fx)}{fh-ei}\right)}{d(fh-ei)^2}+\frac{2b^2f\operatorname{PolyLog}\left(3,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2}\end{aligned}$$

```
[Out] -i*(f*x+e)*(a+b*ln(c*(f*x+e)))^2/d/(-e*i+f*h)^2/(i*x+h)+2*b*f*(a+b*ln(c*(f*
x+e)))*ln(f*(i*x+h)/(-e*i+f*h))/d/(-e*i+f*h)^2-f*(a+b*ln(c*(f*x+e)))^2*ln(1
+(-e*i+f*h)/i/(f*x+e))/d/(-e*i+f*h)^2+2*b*f*(a+b*ln(c*(f*x+e)))*polylog(2,(
e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)^2+2*b^2*f*polylog(2,-i*(f*x+e)/(-e*i+f*h))
/d/(-e*i+f*h)^2+2*b^2*f*polylog(3,(e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)^2
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2458, 12, 2389, 2379, 2421, 6724, 2355, 2354, 2438}

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx = \frac{2bf\,\text{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)(a+b\log(c(e+fx)))}{d(fh-ei)^2} + \frac{2bf\log\left(\frac{f(h+ix)}{fh-ei}\right)(a+b\log(c(e+fx)))}{d(fh-ei)^2} - \frac{i(e+fx)(a+b\log(c(e+fx)))^2}{d(h+ix)(fh-ei)^2} - \frac{f\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{d(fh-ei)^2} + \frac{2b^2f\,\text{PolyLog}\left(2,-\frac{i(e+fx)}{fh-ei}\right)}{d(fh-ei)^2} + \frac{2b^2f\,\text{PolyLog}\left(3,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2}$$

```
[In] Int[(a + b*Log[c*(e + f*x)])^2/((d*e + d*f*x)*(h + i*x)^2),x]
```

```
[Out] -((i*(e + f*x)*(a + b*Log[c*(e + f*x)])^2)/(d*(f*h - e*i)^2*(h + i*x))) + (
2*b*f*(a + b*Log[c*(e + f*x)])*Log[(f*(h + i*x))/(f*h - e*i)])/(d*(f*h - e*
i)^2) - (f*(a + b*Log[c*(e + f*x)])^2*Log[1 + (f*h - e*i)/(i*(e + f*x))])/(
d*(f*h - e*i)^2) + (2*b*f*(a + b*Log[c*(e + f*x)])*PolyLog[2, -((f*h - e*i)
/(i*(e + f*x)))])/(d*(f*h - e*i)^2) + (2*b^2*f*PolyLog[2, -((i*(e + f*x))/(
f*h - e*i))])/(d*(f*h - e*i)^2) + (2*b^2*f*PolyLog[3, -((f*h - e*i)/(i*(e +
 f*x)))])/(d*(f*h - e*i)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
```

```
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{dx\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}\,dx, x, e+fx\right)}{df}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)}\,dx, x, e+fx\right)}{d(fh-ei)} - \frac{i\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}\,dx, x, e+fx\right)}{df(fh-ei)}$$

$$= -\frac{i(e+fx)(a+b\log(c(e+fx)))^2}{d(fh-ei)^2(h+ix)} - \frac{f(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2} + \frac{(2bf)\text{Subst}\left(\int \frac{\log\left(1+\frac{fh-ei}{ix}\right)(a+b\log(cx))}{x}\,dx, x, e+fx\right)}{d(fh-ei)^2} + \frac{(2bi)\text{Subst}\left(\int \frac{a+b\log(cx)}{\frac{fh-ei}{f}+\frac{ix}{f}}\,dx, x, e+fx\right)}{d(fh-ei)^2}$$

$$= -\frac{i(e+fx)(a+b\log(c(e+fx)))^2}{d(fh-ei)^2(h+ix)} + \frac{2bf(a+b\log(c(e+fx)))\log\left(\frac{f(h+ix)}{fh-ei}\right)}{d(fh-ei)^2} - \frac{f(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2} + \frac{2bf(a+b\log(c(e+fx)))\text{Li}_2\left(-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2} - \frac{(2b^2f)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{ix}{fh-ei}\right)}{x}\,dx, x, e+fx\right)}{d(fh-ei)^2} - \frac{(2b^2f)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{fh-ei}{ix}\right)}{x}\,dx, x, e+fx\right)}{d(fh-ei)^2}$$

$$= -\frac{i(e+fx)(a+b\log(c(e+fx)))^2}{d(fh-ei)^2(h+ix)} + \frac{2bf(a+b\log(c(e+fx)))\log\left(\frac{f(h+ix)}{fh-ei}\right)}{d(fh-ei)^2} - \frac{f(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2} + \frac{2bf(a+b\log(c(e+fx)))\mathrm{Li}_2\left(-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2} + \frac{2b^2f\mathrm{Li}_2\left(-\frac{i(e+fx)}{fh-ei}\right)}{d(fh-ei)^2} + \frac{2b^2f\mathrm{Li}_3\left(-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^2}$$

## Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx$$
$$= \frac{3a^2(fh-ei)+3a^2f(h+ix)\log(e+fx)-3a^2f(h+ix)\log(h+ix)+3ab\Big(-2f(h+ix)\log(e+fx)+2(\ldots}{\ldots}$$

```
[In] Integrate[(a + b*Log[c*(e + f*x)])^2/((d*e + d*f*x)*(h + i*x)^2),x]
```

```
[Out] (3*a^2*(f*h - e*i) + 3*a^2*f*(h + i*x)*Log[e + f*x] - 3*a^2*f*(h + i*x)*Log
[h + i*x] + 3*a*b*(-2*f*(h + i*x)*Log[e + f*x] + 2*(f*h - e*i)*Log[c*(e + f
*x)] + f*(h + i*x)*Log[c*(e + f*x)]^2 + 2*f*(h + i*x)*Log[h + i*x] - 2*f*(h
 + i*x)*(Log[c*(e + f*x)]*Log[(f*(h + i*x))/(f*h - e*i)] + PolyLog[2, (i*(e
 + f*x))/(-(f*h) + e*i)])) + b^2*(Log[c*(e + f*x)]*(f*(h + i*x)*Log[c*(e +
f*x)]^2 + 6*f*(h + i*x)*Log[(f*(h + i*x))/(f*h - e*i)] - 3*Log[c*(e + f*x)]
*(i*(e + f*x) + f*(h + i*x)*Log[(f*(h + i*x))/(f*h - e*i)])) - 6*f*(h + i*x
)*(-1 + Log[c*(e + f*x)])*PolyLog[2, (i*(e + f*x))/(-(f*h) + e*i)] + 6*f*(h
 + i*x)*PolyLog[3, (i*(e + f*x))/(-(f*h) + e*i)]))/(3*d*(f*h - e*i)^2*(h +
i*x))
```

## Maple [F]

$$\int \frac{(a+b\ln(c(fx+e)))^2}{(dfx+de)(ix+h)^2}dx$$

```
[In] int((a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^2,x)
```

```
[Out] int((a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^2}{(dfx+de)(ix+h)^2}\,dx$$

[In] integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="fricas
")

[Out] integral((b^2*log(c*f*x + c*e)^2 + 2*a*b*log(c*f*x + c*e) + a^2)/(d*f*i^2*x
^3 + d*e*h^2 + (2*d*f*h*i + d*e*i^2)*x^2 + (d*f*h^2 + 2*d*e*h*i)*x), x)

## Sympy [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx$$
$$= \frac{\int \frac{a^2}{eh^2+2ehix+ei^2x^2+fh^2x+2fhix^2+fi^2x^3}\,dx + \int \frac{b^2\log\left(ce+cfx\right)^2}{eh^2+2ehix+ei^2x^2+fh^2x+2fhix^2+fi^2x^3}\,dx + \int \frac{2ab\log\left(ce+cfx\right)}{eh^2+2ehix+ei^2x^2+fh^2x+2fhix^2}}{d}$$

[In] integrate((a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e)/(i*x+h)**2,x)

[Out] (Integral(a**2/(e*h**2 + 2*e*h*i*x + e*i**2*x**2 + f*h**2*x + 2*f*h*i*x**2
+ f*i**2*x**3), x) + Integral(b**2*log(c*e + c*f*x)**2/(e*h**2 + 2*e*h*i*x
+ e*i**2*x**2 + f*h**2*x + 2*f*h*i*x**2 + f*i**2*x**3), x) + Integral(2*a*b
*log(c*e + c*f*x)/(e*h**2 + 2*e*h*i*x + e*i**2*x**2 + f*h**2*x + 2*f*h*i*x*
*2 + f*i**2*x**3), x))/d

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(271) = 542.

Time = 0.30 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.28

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx$$
$$= a^2\left(\frac{f\log(fx+e)}{df^2h^2-2\,defhi+de^2i^2}-\frac{f\log(ix+h)}{df^2h^2-2\,defhi+de^2i^2}+\frac{1}{dfh^2-dehi+(dfhi-dei^2)x}\right)$$
$$-\frac{\left(\log(fx+e)^2\log\left(\frac{fix+ei}{fh-ei}+1\right)+2\,\mathrm{Li}_2\left(-\frac{fix+ei}{fh-ei}\right)\log(fx+e)-2\,\mathrm{Li}_3(-\frac{fix+ei}{fh-ei})\right)b^2f}{(f^2h^2-2\,efhi+e^2i^2)d}$$
$$+\frac{3\,(fh-ei)b^2\log(c)^2+(b^2fix+b^2fh)\log(fx+e)^3+6\,(fh-ei)ab\log(c)+3\,(abfh+(fh\log(c)-ei)}{}$$
$$-\frac{2\,((f\log(c)-f)b^2+abf)\left(\log(fx+e)\log\left(\frac{fix+ei}{fh-ei}+1\right)+\mathrm{Li}_2\left(-\frac{fix+ei}{fh-ei}\right)\right)}{(f^2h^2-2\,efhi+e^2i^2)d}$$
$$-\frac{\left(2\,(f\log(c)-f)ab+\left(f\log(c)^2-2\,f\log(c)\right)b^2\right)\log(ix+h)}{(f^2h^2-2\,efhi+e^2i^2)d}$$

```
[In] integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="maxima
")
```

```
[Out] a^2*(f*log(f*x + e)/(d*f^2*h^2 - 2*d*e*f*h*i + d*e^2*i^2) - f*log(i*x + h)/
(d*f^2*h^2 - 2*d*e*f*h*i + d*e^2*i^2) + 1/(d*f*h^2 - d*e*h*i + (d*f*h*i - d
*e*i^2)*x)) - (log(f*x + e)^2*log((f*i*x + e*i)/(f*h - e*i) + 1) + 2*dilog(
-(f*i*x + e*i)/(f*h - e*i))*log(f*x + e) - 2*polylog(3, -(f*i*x + e*i)/(f*h
 - e*i)))*b^2*f/((f^2*h^2 - 2*e*f*h*i + e^2*i^2)*d) + 1/3*(3*(f*h - e*i)*b^
2*log(c)^2 + (b^2*f*i*x + b^2*f*h)*log(f*x + e)^3 + 6*(f*h - e*i)*a*b*log(c
) + 3*(a*b*f*h + (f*h*log(c) - e*i)*b^2 + (a*b*f*i + (f*i*log(c) - f*i)*b^2
)*x)*log(f*x + e)^2 + 3*(2*(f*h*log(c) - e*i)*a*b + (f*h*log(c)^2 - 2*e*i*l
og(c))*b^2 + (2*(f*i*log(c) - f*i)*a*b + (f*i*log(c)^2 - 2*f*i*log(c))*b^2)
*x)*log(f*x + e))/((f^2*h^2*i - 2*e*f*h*i^2 + e^2*i^3)*d*x + (f^2*h^3 - 2*e
*f*h^2*i + e^2*h*i^2)*d) - 2*((f*log(c) - f)*b^2 + a*b*f)*(log(f*x + e)*log
((f*i*x + e*i)/(f*h - e*i) + 1) + dilog(-(f*i*x + e*i)/(f*h - e*i)))/((f^2*
h^2 - 2*e*f*h*i + e^2*i^2)*d) - (2*(f*log(c) - f)*a*b + (f*log(c)^2 - 2*f*l
og(c))*b^2)*log(i*x + h)/((f^2*h^2 - 2*e*f*h*i + e^2*i^2)*d)
```

## Giac [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^2}{(dfx+de)(ix+h)^2}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="giac")
[Out] integrate((b*log((f*x + e)*c) + a)^2/((d*f*x + d*e)*(i*x + h)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(e+f\,x\right)\right)\right)^2}{\left(h+i\,x\right)^2\,\left(d\,e+d\,f\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(e + f*x)))^2/((h + i*x)^2*(d*e + d*f*x)),x)
[Out] int((a + b*log(c*(e + f*x)))^2/((h + i*x)^2*(d*e + d*f*x)), x)
```

# 3.190 $\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 485

$$\begin{aligned}\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx &= \frac{bfi(e+fx)(a+b\log(c(e+fx)))}{d(fh-ei)^3(h+ix)} + \frac{(a+b\log(c(e+fx)))^2}{2d(fh-ei)(h+ix)^2} \\ &\quad - \frac{fi(e+fx)(a+b\log(c(e+fx)))^2}{d(fh-ei)^3(h+ix)} - \frac{b^2f^2\log(h+ix)}{d(fh-ei)^3} \\ &\quad + \frac{2bf^2(a+b\log(c(e+fx)))\log\left(\frac{f(h+ix)}{fh-ei}\right)}{d(fh-ei)^3} \\ &\quad + \frac{bf^2(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} \\ &\quad - \frac{f^2(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} \\ &\quad - \frac{b^2f^2\operatorname{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} \\ &\quad + \frac{2bf^2(a+b\log(c(e+fx)))\operatorname{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} \\ &\quad + \frac{2b^2f^2\operatorname{PolyLog}\left(2,-\frac{i(e+fx)}{fh-ei}\right)}{d(fh-ei)^3} \\ &\quad + \frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3}\end{aligned}$$

```
[Out] b*f*i*(f*x+e)*(a+b*ln(c*(f*x+e)))/d/(-e*i+f*h)^3/(i*x+h)+1/2*(a+b*ln(c*(f*x
+e)))^2/d/(-e*i+f*h)/(i*x+h)^2-f*i*(f*x+e)*(a+b*ln(c*(f*x+e)))^2/d/(-e*i+f*
```

```
h)^3/(i*x+h)-b^2*f^2*ln(i*x+h)/d/(-e*i+f*h)^3+2*b*f^2*(a+b*ln(c*(f*x+e)))*l
n(f*(i*x+h)/(-e*i+f*h))/d/(-e*i+f*h)^3+b*f^2*(a+b*ln(c*(f*x+e)))*ln(1+(-e*i
+f*h)/i/(f*x+e))/d/(-e*i+f*h)^3-f^2*(a+b*ln(c*(f*x+e)))^2*ln(1+(-e*i+f*h)/i
/(f*x+e))/d/(-e*i+f*h)^3-b^2*f^2*polylog(2,(e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h
)^3+2*b*f^2*(a+b*ln(c*(f*x+e)))*polylog(2,(e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)
^3+2*b^2*f^2*polylog(2,-i*(f*x+e)/(-e*i+f*h))/d/(-e*i+f*h)^3+2*b^2*f^2*poly
log(3,(e*i-f*h)/i/(f*x+e))/d/(-e*i+f*h)^3
```

## Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2458, 12, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx = \frac{2bf^2\operatorname{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)(a+b\log(c(e+fx)))}{d(fh-ei)^3} + \frac{2bf^2\log\left(\frac{f(h+ix)}{fh-ei}\right)(a+b\log(c(e+fx)))}{d(fh-ei)^3} - \frac{f^2\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))^2}{d(fh-ei)^3} + \frac{bf^2\log\left(\frac{fh-ei}{i(e+fx)}+1\right)(a+b\log(c(e+fx)))}{d(fh-ei)^3} - \frac{fi(e+fx)(a+b\log(c(e+fx)))^2}{d(h+ix)(fh-ei)^3} + \frac{bfi(e+fx)(a+b\log(c(e+fx)))}{d(h+ix)(fh-ei)^3} + \frac{(a+b\log(c(e+fx)))^2}{2d(h+ix)^2(fh-ei)} - \frac{b^2f^2\operatorname{PolyLog}\left(2,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} + \frac{2b^2f^2\operatorname{PolyLog}\left(2,-\frac{i(e+fx)}{fh-ei}\right)}{d(fh-ei)^3} + \frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} - \frac{b^2f^2\log(h+ix)}{d(fh-ei)^3}$$

```
[In] Int[(a + b*Log[c*(e + f*x)])^2/((d*e + d*f*x)*(h + i*x)^3),x]
```

```
[Out] (b*f*i*(e + f*x)*(a + b*Log[c*(e + f*x)]))/(d*(f*h - e*i)^3*(h + i*x)) + (a
 + b*Log[c*(e + f*x)])^2/(2*d*(f*h - e*i)*(h + i*x)^2) - (f*i*(e + f*x)*(a
+ b*Log[c*(e + f*x)])^2)/(d*(f*h - e*i)^3*(h + i*x)) - (b^2*f^2*Log[h + i*x
])/(d*(f*h - e*i)^3) + (2*b*f^2*(a + b*Log[c*(e + f*x)])*Log[(f*(h + i*x))/
```

```
(f*h - e*i)])/(d*(f*h - e*i)^3) + (b*f^2*(a + b*Log[c*(e + f*x)])*Log[1 + (
f*h - e*i)/(i*(e + f*x))])/(d*(f*h - e*i)^3) - (f^2*(a + b*Log[c*(e + f*x)]
)^2*Log[1 + (f*h - e*i)/(i*(e + f*x))])/(d*(f*h - e*i)^3) - (b^2*f^2*PolyLo
g[2, -((f*h - e*i)/(i*(e + f*x)))])/(d*(f*h - e*i)^3) + (2*b*f^2*(a + b*Log
[c*(e + f*x)])*PolyLog[2, -((f*h - e*i)/(i*(e + f*x)))])/(d*(f*h - e*i)^3)
+ (2*b^2*f^2*PolyLog[2, -((i*(e + f*x))/(f*h - e*i))])/(d*(f*h - e*i)^3) +
(2*b^2*f^2*PolyLog[3, -((f*h - e*i)/(i*(e + f*x)))])/(d*(f*h - e*i)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{dx\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3}\,dx, x, e+fx\right)}{df}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}\,dx, x, e+fx\right)}{d(fh-ei)} - \frac{i\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3}\,dx, x, e+fx\right)}{df(fh-ei)}$$

$$= \frac{(a+b\log(c(e+fx)))^2}{2d(fh-ei)(h+ix)^2} + \frac{f\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)}\,dx, x, e+fx\right)}{d(fh-ei)^2} - \frac{i\text{Subst}\left(\int \frac{(a+b\log(cx))^2}{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}\,dx, x, e+fx\right)}{d(fh-ei)^2} - \frac{b\text{Subst}\left(\int \frac{a+b\log(cx)}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}\,dx, x, e+fx\right)}{d(fh-ei)}$$

$$= \frac{(a+b\log(c(e+fx)))^2}{2d(fh-ei)(h+ix)^2} - \frac{fi(e+fx)(a+b\log(c(e+fx)))^2}{d(fh-ei)^3(h+ix)} - \frac{f^2(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} + \frac{(2bf^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{fh-ei}{ix}\right)(a+b\log(cx))}{x}\,dx, x, e+fx\right)}{d(fh-ei)^3} + \frac{(2bfi)\text{Subst}\left(\int \frac{a+b\log(cx)}{\frac{fh-ei}{f}+\frac{ix}{f}}\,dx, x, e+fx\right)}{d(fh-ei)^3} - \frac{(bf)\text{Subst}\left(\int \frac{a+b\log(cx)}{x\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)}\,dx, x, e+fx\right)}{d(fh-ei)^2} + \frac{(bi)\text{Subst}\left(\int \frac{a+b\log(cx)}{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}\,dx, x, e+fx\right)}{d(fh-ei)^2}$$

$$= \frac{bfi(e+fx)(a+b\log(c(e+fx)))}{d(fh-ei)^3(h+ix)} + \frac{(a+b\log(c(e+fx)))^2}{2d(fh-ei)(h+ix)^2} - \frac{fi(e+fx)(a+b\log(c(e+fx)))^2}{d(fh-ei)^3(h+ix)} + \frac{2bf^2(a+b\log(c(e+fx)))\log\left(\frac{f(h+ix)}{fh-ei}\right)}{d(fh-ei)^3} + \frac{bf^2(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} - \frac{f^2(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} + \frac{2bf^2(a+b\log(c(e+fx)))\text{Li}_2\left(-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} - \frac{(b^2f^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{fh-ei}{ix}\right)}{x}\,dx, x, e+fx\right)}{d(fh-ei)^3} - \frac{(2b^2f^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{ix}{fh-ei}\right)}{x}\,dx, x, e+fx\right)}{d(fh-ei)^3} - \frac{(2b^2f^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{fh-ei}{ix}\right)}{x}\,dx, x, e+fx\right)}{d(fh-ei)^3} - \frac{(b^2fi)\,\text{Subst}\left(\int\frac{1}{\frac{fh-ei}{f}+\frac{ix}{f}}\,dx, x, e+fx\right)}{d(fh-ei)^3}$$

$$= \frac{bfi(e+fx)(a+b\log(c(e+fx)))}{d(fh-ei)^3(h+ix)} + \frac{(a+b\log(c(e+fx)))^2}{2d(fh-ei)(h+ix)^2} - \frac{fi(e+fx)(a+b\log(c(e+fx)))^2}{d(fh-ei)^3(h+ix)} - \frac{b^2f^2\log(h+ix)}{d(fh-ei)^3} + \frac{2bf^2(a+b\log(c(e+fx)))\log\left(\frac{f(h+ix)}{fh-ei}\right)}{d(fh-ei)^3} + \frac{bf^2(a+b\log(c(e+fx)))\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} - \frac{f^2(a+b\log(c(e+fx)))^2\log\left(1+\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} - \frac{b^2f^2\text{Li}_2\left(-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} + \frac{2bf^2(a+b\log(c(e+fx)))\text{Li}_2\left(-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3} + \frac{2b^2f^2\text{Li}_2\left(-\frac{i(e+fx)}{fh-ei}\right)}{d(fh-ei)^3} + \frac{2b^2f^2\text{Li}_3\left(-\frac{fh-ei}{i(e+fx)}\right)}{d(fh-ei)^3}$$

## Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.37

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx$$
$$= \frac{3a^2(fh-ei)^2+6a^2f(fh-ei)(h+ix)+6a^2f^2(h+ix)^2\log(e+fx)-6a^2f^2(h+ix)^2\log(h+ix)+6ab\big(}{}$$

```
[In] Integrate[(a + b*Log[c*(e + f*x)])^2/((d*e + d*f*x)*(h + i*x)^3),x]
```

```
[Out] (3*a^2*(f*h - e*i)^2 + 6*a^2*f*(f*h - e*i)*(h + i*x) + 6*a^2*f^2*(h + i*x)^
2*Log[e + f*x] - 6*a^2*f^2*(h + i*x)^2*Log[h + i*x] + 6*a*b*((f*h - e*i)^2*
Log[c*(e + f*x)] + f^2*(h + i*x)^2*Log[c*(e + f*x)]^2 - f*(h + i*x)*(f*h -
e*i + f*(h + i*x)*Log[e + f*x] - f*(h + i*x)*Log[h + i*x]) - 2*f*(h + i*x)*
(f*(h + i*x)*Log[e + f*x] + (-(f*h) + e*i)*Log[c*(e + f*x)] - f*(h + i*x)*L
og[h + i*x]) - 2*f^2*(h + i*x)^2*(Log[c*(e + f*x)]*Log[(f*(h + i*x))/(f*h -
 e*i)] + PolyLog[2, (i*(e + f*x))/(-(f*h) + e*i)])) + b^2*(2*f^2*(h + i*x)^
2*Log[c*(e + f*x)]^3 - 6*f*(h + i*x)*(Log[c*(e + f*x)]*(i*(e + f*x)*Log[c*(
e + f*x)] - 2*f*(h + i*x)*Log[(f*(h + i*x))/(f*h - e*i)]) - 2*f*(h + i*x)*P
olyLog[2, (i*(e + f*x))/(-(f*h) + e*i)]) + 3*((f*h - e*i)^2*Log[c*(e + f*x)
]^2 + f*(h + i*x)*(2*f*(h + i*x)*Log[e + f*x] - 2*(f*h - e*i)*Log[c*(e + f*
x)] - f*(h + i*x)*Log[c*(e + f*x)]^2 - 2*f*(h + i*x)*Log[h + i*x] + 2*f*(h
+ i*x)*Log[c*(e + f*x)]*Log[(f*(h + i*x))/(f*h - e*i)] + 2*f*(h + i*x)*Poly
Log[2, (i*(e + f*x))/(-(f*h) + e*i)])) - 6*f^2*(h + i*x)^2*(Log[c*(e + f*x)
]^2*Log[(f*(h + i*x))/(f*h - e*i)] + 2*Log[c*(e + f*x)]*PolyLog[2, (i*(e +
f*x))/(-(f*h) + e*i)] - 2*PolyLog[3, (i*(e + f*x))/(-(f*h) + e*i)])))/(6*d*
(f*h - e*i)^3*(h + i*x)^2)
```

## Maple [F]

$$\int \frac{(a+b\ln(c(fx+e)))^2}{(dfx+de)(ix+h)^3}dx$$

```
[In] int((a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^3,x)
```

```
[Out] int((a+b*ln(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^3,x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^2}{(dfx+de)(ix+h)^3}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="fricas
")
```

```
[Out] integral((b^2*log(c*f*x + c*e)^2 + 2*a*b*log(c*f*x + c*e) + a^2)/(d*f*i^3*x
^4 + d*e*h^3 + (3*d*f*h*i^2 + d*e*i^3)*x^3 + 3*(d*f*h^2*i + d*e*h*i^2)*x^2
+ (d*f*h^3 + 3*d*e*h^2*i)*x), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx$$
$$= \frac{\int \frac{a^2}{eh^3+3eh^2ix+3ehi^2x^2+ei^3x^3+fh^3x+3fh^2ix^2+3fhi^2x^3+fi^3x^4}\,dx + \int \frac{b^2\log(ce+cfx)^2}{eh^3+3eh^2ix+3ehi^2x^2+ei^3x^3+fh^3x+3fh^2ix^2+3fhi^2x^3+fi^3x^4}}{d}$$

```
[In] integrate((a+b*ln(c*(f*x+e)))**2/(d*f*x+d*e)/(i*x+h)**3,x)
```

```
[Out] (Integral(a**2/(e*h**3 + 3*e*h**2*i*x + 3*e*h*i**2*x**2 + e*i**3*x**3 + f*h
**3*x + 3*f*h**2*i*x**2 + 3*f*h*i**2*x**3 + f*i**3*x**4), x) + Integral(b**
2*log(c*e + c*f*x)**2/(e*h**3 + 3*e*h**2*i*x + 3*e*h*i**2*x**2 + e*i**3*x**
3 + f*h**3*x + 3*f*h**2*i*x**2 + 3*f*h*i**2*x**3 + f*i**3*x**4), x) + Integ
ral(2*a*b*log(c*e + c*f*x)/(e*h**3 + 3*e*h**2*i*x + 3*e*h*i**2*x**2 + e*i**
3*x**3 + f*h**3*x + 3*f*h**2*i*x**2 + 3*f*h*i**2*x**3 + f*i**3*x**4), x))/d
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1271 vs. $2(480) = 960$.

Time = 0.38 (sec) , antiderivative size = 1271, normalized size of antiderivative = 2.62

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="maxima
")
```

```
[Out] 1/2*(2*f^2*log(f*x + e)/(d*f^3*h^3 - 3*d*e*f^2*h^2*i + 3*d*e^2*f*h*i^2 - d*
e^3*i^3) - 2*f^2*log(i*x + h)/(d*f^3*h^3 - 3*d*e*f^2*h^2*i + 3*d*e^2*f*h*i^
2 - d*e^3*i^3) + (2*f*i*x + 3*f*h - e*i)/(d*f^2*h^4 - 2*d*e*f*h^3*i + d*e^2
```

```
*h^2*i^2 + (d*f^2*h^2*i^2 - 2*d*e*f*h*i^3 + d*e^2*i^4)*x^2 + 2*(d*f^2*h^3*i
 - 2*d*e*f*h^2*i^2 + d*e^2*h*i^3)*x))*a^2 - (log(f*x + e)^2*log((f*i*x + e*
i)/(f*h - e*i) + 1) + 2*dilog(-(f*i*x + e*i)/(f*h - e*i))*log(f*x + e) - 2*
polylog(3, -(f*i*x + e*i)/(f*h - e*i)))*b^2*f^2/((f^3*h^3 - 3*e*f^2*h^2*i +
 3*e^2*f*h*i^2 - e^3*i^3)*d) + 1/6*(2*(b^2*f^2*i^2*x^2 + 2*b^2*f^2*h*i*x +
b^2*f^2*h^2)*log(f*x + e)^3 - 6*(f^2*h^2 - e*f*h*i - (3*f^2*h^2 - 4*e*f*h*i
 + e^2*i^2)*log(c))*a*b + 3*((3*f^2*h^2 - 4*e*f*h*i + e^2*i^2)*log(c)^2 - 2
*(f^2*h^2 - e*f*h*i)*log(c))*b^2 + 3*(2*a*b*f^2*h^2 + (2*f^2*h^2*log(c) - 4
*e*f*h*i + e^2*i^2)*b^2 + (2*a*b*f^2*i^2 + (2*f^2*i^2*log(c) - 3*f^2*i^2)*b
^2)*x^2 + 2*(2*a*b*f^2*h*i + (2*f^2*h*i*log(c) - 2*f^2*h*i - e*f*i^2)*b^2)*
x)*log(f*x + e)^2 - 6*((f^2*h*i - e*f*i^2 - 2*(f^2*h*i - e*f*i^2)*log(c))*a
*b - ((f^2*h*i - e*f*i^2)*log(c)^2 - (f^2*h*i - e*f*i^2)*log(c))*b^2)*x + 6
*((2*f^2*h^2*log(c) - 4*e*f*h*i + e^2*i^2)*a*b + (f^2*h^2*log(c)^2 + e*f*h*
i - (4*e*f*h*i - e^2*i^2)*log(c))*b^2 + ((2*f^2*i^2*log(c) - 3*f^2*i^2)*a*b
 + (f^2*i^2*log(c)^2 - 3*f^2*i^2*log(c) + f^2*i^2)*b^2)*x^2 + (2*(2*f^2*h*i
*log(c) - 2*f^2*h*i - e*f*i^2)*a*b + (2*f^2*h*i*log(c)^2 + f^2*h*i + e*f*i^
2 - 2*(2*f^2*h*i + e*f*i^2)*log(c))*b^2)*x)*log(f*x + e))/((f^3*h^3*i^2 - 3
*e*f^2*h^2*i^3 + 3*e^2*f*h*i^4 - e^3*i^5)*d*x^2 + 2*(f^3*h^4*i - 3*e*f^2*h^
3*i^2 + 3*e^2*f*h^2*i^3 - e^3*h*i^4)*d*x + (f^3*h^5 - 3*e*f^2*h^4*i + 3*e^2
*f*h^3*i^2 - e^3*h^2*i^3)*d) - (2*a*b*f^2 + (2*f^2*log(c) - 3*f^2)*b^2)*(lo
g(f*x + e)*log((f*i*x + e*i)/(f*h - e*i) + 1) + dilog(-(f*i*x + e*i)/(f*h -
 e*i)))/((f^3*h^3 - 3*e*f^2*h^2*i + 3*e^2*f*h*i^2 - e^3*i^3)*d) - ((2*f^2*l
og(c) - 3*f^2)*a*b + (f^2*log(c)^2 - 3*f^2*log(c) + f^2)*b^2)*log(i*x + h)/
((f^3*h^3 - 3*e*f^2*h^2*i + 3*e^2*f*h*i^2 - e^3*i^3)*d)
```

## Giac [F]

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^2}{(dfx+de)(ix+h)^3}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))^2/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="giac")
[Out] integrate((b*log((f*x + e)*c) + a)^2/((d*f*x + d*e)*(i*x + h)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(e+fx)))^2}{(de+dfx)(h+ix)^3}\,dx = \int \frac{\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)^2}{\left(h+i\,x\right)^3\,\left(d\,e+d\,f\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(e + f*x)))^2/((h + i*x)^3*(d*e + d*f*x)),x)
[Out] int((a + b*log(c*(e + f*x)))^2/((h + i*x)^3*(d*e + d*f*x)), x)
```

# 3.191 $\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 230

$$\begin{aligned}\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx &= \frac{4e^{-\frac{a}{b}}i(fh-ei)^3\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^5}\\&\quad+\frac{6e^{-\frac{2a}{b}}i^2(fh-ei)^2\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2df^5}\\&\quad+\frac{4e^{-\frac{3a}{b}}i^3(fh-ei)\,\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(e+fx)))}{b}\right)}{bc^3df^5}\\&\quad+\frac{e^{-\frac{4a}{b}}i^4\,\text{ExpIntegralEi}\left(\frac{4(a+b\log(c(e+fx)))}{b}\right)}{bc^4df^5}+\frac{(fh-ei)^4\log(a+b\log(c(e+fx)))}{bdf^5}\end{aligned}$$

```
[Out] 4*i*(-e*i+f*h)^3*Ei((a+b*ln(c*(f*x+e)))/b)/b/c/d/exp(a/b)/f^5+6*i^2*(-e*i+f
*h)^2*Ei(2*(a+b*ln(c*(f*x+e)))/b)/b/c^2/d/exp(2*a/b)/f^5+4*i^3*(-e*i+f*h)*E
i(3*(a+b*ln(c*(f*x+e)))/b)/b/c^3/d/exp(3*a/b)/f^5+i^4*Ei(4*(a+b*ln(c*(f*x+e
)))/b)/b/c^4/d/exp(4*a/b)/f^5+(-e*i+f*h)^4*ln(a+b*ln(c*(f*x+e)))/b/d/f^5
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2458, 12, 2395, 2336, 2209, 2339, 29, 2346}

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{i^4e^{-\frac{4a}{b}}\operatorname{ExpIntegralEi}\left(\frac{4(a+b\log(c(e+fx)))}{b}\right)}{bc^4df^5} + \frac{4i^3e^{-\frac{3a}{b}}(fh-ei)\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(c(e+fx)))}{b}\right)}{bc^3df^5} + \frac{6i^2e^{-\frac{2a}{b}}(fh-ei)^2\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2df^5} + \frac{4ie^{-\frac{a}{b}}(fh-ei)^3\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^5} + \frac{(fh-ei)^4\log(a+b\log(c(e+fx)))}{bdf^5}$$

```
[In] Int[(h + i*x)^4/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

```
[Out] (4*i*(f*h - e*i)^3*ExpIntegralEi[(a + b*Log[c*(e + f*x)])/b])/(b*c*d*E^(a/b
)*f^5) + (6*i^2*(f*h - e*i)^2*ExpIntegralEi[(2*(a + b*Log[c*(e + f*x)]))/b]
)/(b*c^2*d*E^((2*a)/b)*f^5) + (4*i^3*(f*h - e*i)*ExpIntegralEi[(3*(a + b*Lo
g[c*(e + f*x)]))/b])/(b*c^3*d*E^((3*a)/b)*f^5) + (i^4*ExpIntegralEi[(4*(a +
 b*Log[c*(e + f*x)]))/b])/(b*c^4*d*E^((4*a)/b)*f^5) + ((f*h - e*i)^4*Log[a
+ b*Log[c*(e + f*x)]])/(b*d*f^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
```

```
c, p}, x] && IntegerQ[1/n]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^4}{dx(a+b\log(cx))}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^4}{x(a+b\log(cx))}\,dx, x, e+fx\right)}{df}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{4i(fh-ei)^3}{f^4(a+b\log(cx))}+\frac{(fh-ei)^4}{f^4x(a+b\log(cx))}+\frac{6i^2(fh-ei)^2x}{f^4(a+b\log(cx))}+\frac{4i^3(fh-ei)x^2}{f^4(a+b\log(cx))}+\frac{i^4x^3}{f^4(a+b\log(cx))}\right)\,dx, x, e+f\right.}{df}$$

$$= \frac{i^4 \text{Subst}\left(\int \frac{x^3}{a+b\log(cx)}\,dx, x, e+fx\right)}{df^5} + \frac{(4i^3(fh-ei))\,\text{Subst}\left(\int \frac{x^2}{a+b\log(cx)}\,dx, x, e+fx\right)}{df^5} + \frac{(6i^2(fh-ei)^2)\,\text{Subst}\left(\int \frac{x}{a+b\log(cx)}\,dx, x, e+fx\right)}{df^5} + \frac{(4i(fh-ei)^3)\,\text{Subst}\left(\int \frac{1}{a+b\log(cx)}\,dx, x, e+fx\right)}{df^5} + \frac{(fh-ei)^4 \text{Subst}\left(\int \frac{1}{x(a+b\log(cx))}\,dx, x, e+fx\right)}{df^5}$$

$$= \frac{i^4 \text{Subst}\left(\int \frac{e^{4x}}{a+bx}\,dx, x, \log(c(e+fx))\right)}{c^4 df^5} + \frac{(4i^3(fh-ei))\,\text{Subst}\left(\int \frac{e^{3x}}{a+bx}\,dx, x, \log(c(e+fx))\right)}{c^3 df^5} + \frac{(6i^2(fh-ei)^2)\,\text{Subst}\left(\int \frac{e^{2x}}{a+bx}\,dx, x, \log(c(e+fx))\right)}{c^2 df^5} + \frac{(4i(fh-ei)^3)\,\text{Subst}\left(\int \frac{e^x}{a+bx}\,dx, x, \log(c(e+fx))\right)}{cdf^5} + \frac{(fh-ei)^4 \text{Subst}\left(\int \frac{1}{x}\,dx, x, a+b\log(c(e+fx))\right)}{bdf^5}$$

$$= \frac{4e^{-\frac{a}{b}} i(fh-ei)^3 \text{Ei}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^5} + \frac{6e^{-\frac{2a}{b}} i^2(fh-ei)^2 \text{Ei}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2 df^5} + \frac{4e^{-\frac{3a}{b}} i^3(fh-ei) \text{Ei}\left(\frac{3(a+b\log(c(e+fx)))}{b}\right)}{bc^3 df^5} + \frac{e^{-\frac{4a}{b}} i^4 \text{Ei}\left(\frac{4(a+b\log(c(e+fx)))}{b}\right)}{bc^4 df^5} + \frac{(fh-ei)^4 \log(a+b\log(c(e+fx)))}{bdf^5}$$

## Mathematica [F]

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$$

```
[In] Integrate[(h + i*x)^4/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
[Out] Integrate[(h + i*x)^4/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])), x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(233) = 466$.

Time = 3.02 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.47

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{i^4 \mathrm{e}^{-\frac{4a}{b}} \mathrm{Ei}_1\left(-4\ln(cfx+ce)-\frac{4a}{b}\right)}{b}+\frac{c^4e^4i^4\ln(a+b\ln(cfx+ce))}{b}+\frac{c^4f^4h^4\ln(a+b\ln(cfx+ce))}{b}+\frac{4ce\,i^4 \mathrm{e}^{-\frac{3a}{b}} \mathrm{Ei}_1\left(-3\ln(cfx+ce)-\right.}{b}}{}$ |
| default | $\frac{-\frac{i^4 \mathrm{e}^{-\frac{4a}{b}} \mathrm{Ei}_1\left(-4\ln(cfx+ce)-\frac{4a}{b}\right)}{b}+\frac{c^4e^4i^4\ln(a+b\ln(cfx+ce))}{b}+\frac{c^4f^4h^4\ln(a+b\ln(cfx+ce))}{b}+\frac{4ce\,i^4 \mathrm{e}^{-\frac{3a}{b}} \mathrm{Ei}_1\left(-3\ln(cfx+ce)-\right.}{b}}{}$ |
| risch | $\frac{e^4i^4\ln(a+b\ln(cfx+ce))}{f^5db}-\frac{4e^3h\,i^3\ln(a+b\ln(cfx+ce))}{f^4db}+\frac{6e^2h^2i^2\ln(a+b\ln(cfx+ce))}{f^3db}-\frac{4e\,h^3i\ln(a+b\ln(cfx+ce)}{f^2db}$ |

```
[In] int((i*x+h)^4/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x,method=_RETURNVERBOSE)
[Out] 1/c^4/f^5/d*(-i^4/b*exp(-4*a/b)*Ei(1,-4*ln(c*f*x+c*e)-4*a/b)+c^4*e^4*i^4*ln
(a+b*ln(c*f*x+c*e))/b+c^4*f^4*h^4*ln(a+b*ln(c*f*x+c*e))/b+4*c*e*i^4/b*exp(-
3*a/b)*Ei(1,-3*ln(c*f*x+c*e)-3*a/b)-6*c^2*e^2*i^4/b*exp(-2*a/b)*Ei(1,-2*ln(
c*f*x+c*e)-2*a/b)+4*c^3*e^3*i^4/b*exp(-a/b)*Ei(1,-ln(c*f*x+c*e)-a/b)-4*c*f*
h*i^3/b*exp(-3*a/b)*Ei(1,-3*ln(c*f*x+c*e)-3*a/b)-6*c^2*f^2*h^2*i^2/b*exp(-2
*a/b)*Ei(1,-2*ln(c*f*x+c*e)-2*a/b)-4*c^3*f^3*h^3*i/b*exp(-a/b)*Ei(1,-ln(c*f
*x+c*e)-a/b)-4*c^4*e*f^3*h^3*i*ln(a+b*ln(c*f*x+c*e))/b+6*c^4*e^2*f^2*h^2*i^
2*ln(a+b*ln(c*f*x+c*e))/b-4*c^4*e^3*f*h*i^3*ln(a+b*ln(c*f*x+c*e))/b+12*c^2*
e*f*h*i^3/b*exp(-2*a/b)*Ei(1,-2*ln(c*f*x+c*e)-2*a/b)+12*c^3*e*f^2*h^2*i^2/b
*exp(-a/b)*Ei(1,-ln(c*f*x+c*e)-a/b)-12*c^3*e^2*f*h*i^3/b*exp(-a/b)*Ei(1,-ln
(c*f*x+c*e)-a/b))
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.74

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$$
$$= \frac{\left(i^4\,\mathrm{log\_integral}\left((c^4f^4x^4+4\,c^4ef^3x^3+6\,c^4e^2f^2x^2+4\,c^4e^3fx+c^4e^4)e^{\left(\frac{4\,a}{b}\right)}\right)+(c^4f^4h^4-4\,c^4ef^3h^3i+6\,c^4}{}$$

```
[In] integrate((i*x+h)^4/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="fricas")
```

```
[Out] (i^4*log_integral((c^4*f^4*x^4 + 4*c^4*e*f^3*x^3 + 6*c^4*e^2*f^2*x^2 + 4*c^
4*e^3*f*x + c^4*e^4)*e^(4*a/b)) + (c^4*f^4*h^4 - 4*c^4*e*f^3*h^3*i + 6*c^4*
e^2*f^2*h^2*i^2 - 4*c^4*e^3*f*h*i^3 + c^4*e^4*i^4)*e^(4*a/b)*log(b*log(c*f*
x + c*e) + a) + 4*(c*f*h*i^3 - c*e*i^4)*e^(a/b)*log_integral((c^3*f^3*x^3 +
 3*c^3*e*f^2*x^2 + 3*c^3*e^2*f*x + c^3*e^3)*e^(3*a/b)) + 6*(c^2*f^2*h^2*i^2
 - 2*c^2*e*f*h*i^3 + c^2*e^2*i^4)*e^(2*a/b)*log_integral((c^2*f^2*x^2 + 2*c
^2*e*f*x + c^2*e^2)*e^(2*a/b)) + 4*(c^3*f^3*h^3*i - 3*c^3*e*f^2*h^2*i^2 + 3
*c^3*e^2*f*h*i^3 - c^3*e^3*i^4)*e^(3*a/b)*log_integral((c*f*x + c*e)*e^(a/b
)))*e^(-4*a/b)/(b*c^4*d*f^5)
```

## Sympy [F]

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$$
$$= \frac{\int \frac{h^4}{ae+afx+be\log(ce+cfx)+bfx\log(ce+cfx)}\,dx+\int \frac{i^4x^4}{ae+afx+be\log(ce+cfx)+bfx\log(ce+cfx)}\,dx+\int \frac{4hi^3x^3}{ae+afx+be\log(ce+cfx)+bfx\,\mathrm{l}}}{d}$$

```
[In] integrate((i*x+h)**4/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x)
```

```
[Out] (Integral(h**4/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x)
), x) + Integral(i**4*x**4/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(
c*e + c*f*x)), x) + Integral(4*h*i**3*x**3/(a*e + a*f*x + b*e*log(c*e + c*f
*x) + b*f*x*log(c*e + c*f*x)), x) + Integral(6*h**2*i**2*x**2/(a*e + a*f*x
+ b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x)), x) + Integral(4*h**3*i*x/
(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x)), x))/d
```

## Maxima [F]

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(ix+h)^4}{(dfx+de)(b\log\left((fx+e)c\right)+a)}\,dx$$

```
[In] integrate((i*x+h)^4/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="maxima")
[Out] h^4*log((b*log(f*x + e) + b*log(c) + a)/b)/(b*d*f) + integrate((i^4*x^4 + 4
*h*i^3*x^3 + 6*h^2*i^2*x^2 + 4*h^3*i*x)/(b*d*e*log(c) + a*d*e + (b*d*f*log(
c) + a*d*f)*x + (b*d*f*x + b*d*e)*log(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(ix+h)^4}{(dfx+de)(b\log\left((fx+e)c\right)+a)}\,dx$$

```
[In] integrate((i*x+h)^4/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="giac")
[Out] integrate((i*x + h)^4/((d*f*x + d*e)*(b*log((f*x + e)*c) + a)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^4}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(h+i\,x)^4}{(d\,e+d\,f\,x)\,\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)}\,dx$$

```
[In] int((h + i*x)^4/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
[Out] int((h + i*x)^4/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))), x)
```

# 3.192 $\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 177

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{3e^{-\frac{a}{b}}i(fh-ei)^2\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^4} + \frac{3e^{-\frac{2a}{b}}i^2(fh-ei)\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2df^4} + \frac{e^{-\frac{3a}{b}}i^3\,\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(e+fx)))}{b}\right)}{bc^3df^4} + \frac{(fh-ei)^3\log(a+b\log(c(e+fx)))}{bdf^4}$$

```
[Out] 3*i*(-e*i+f*h)^2*Ei((a+b*ln(c*(f*x+e)))/b)/b/c/d/exp(a/b)/f^4+3*i^2*(-e*i+f
*h)*Ei(2*(a+b*ln(c*(f*x+e)))/b)/b/c^2/d/exp(2*a/b)/f^4+i^3*Ei(3*(a+b*ln(c*(
f*x+e)))/b)/b/c^3/d/exp(3*a/b)/f^4+(-e*i+f*h)^3*ln(a+b*ln(c*(f*x+e)))/b/d/f
^4
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{2458, 12, 2395, 2336, 2209, 2339, 29, 2346\}$

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$$
$$= \frac{i^3e^{-\frac{3a}{b}}\,\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(e+fx)))}{b}\right)}{bc^3df^4} + \frac{3i^2e^{-\frac{2a}{b}}(fh-ei)\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2df^4} + \frac{3ie^{-\frac{a}{b}}(fh-ei)^2\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^4} + \frac{(fh-ei)^3\log(a+b\log(c(e+fx)))}{bdf^4}$$

```
[In] Int[(h + i*x)^3/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

```
[Out] (3*i*(f*h - e*i)^2*ExpIntegralEi[(a + b*Log[c*(e + f*x)])/b])/(b*c*d*E^(a/b
)*f^4) + (3*i^2*(f*h - e*i)*ExpIntegralEi[(2*(a + b*Log[c*(e + f*x)]))/b])/
(b*c^2*d*E^((2*a)/b)*f^4) + (i^3*ExpIntegralEi[(3*(a + b*Log[c*(e + f*x)]))
/b])/(b*c^3*d*E^((3*a)/b)*f^4) + ((f*h - e*i)^3*Log[a + b*Log[c*(e + f*x)]]
)/(b*d*f^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3}{dx(a+b\log(cx))}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3}{x(a+b\log(cx))}\,dx, x, e+fx\right)}{df}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3i(fh-ei)^2}{f^3(a+b\log(cx))}+\frac{(fh-ei)^3}{f^3x(a+b\log(cx))}+\frac{3i^2(fh-ei)x}{f^3(a+b\log(cx))}+\frac{i^3x^2}{f^3(a+b\log(cx))}\right)dx, x, e+fx\right)}{df}$$

$$
\begin{aligned}
&= \frac{i^3 \text{Subst}\left(\int \frac{x^2}{a+b\log(cx)}\,dx, x, e+fx\right)}{df^4} \\
&+ \frac{(3i^2(fh-ei))\,\text{Subst}\left(\int \frac{x}{a+b\log(cx)}\,dx, x, e+fx\right)}{df^4} \\
&+ \frac{(3i(fh-ei)^2)\,\text{Subst}\left(\int \frac{1}{a+b\log(cx)}\,dx, x, e+fx\right)}{df^4} \\
&+ \frac{(fh-ei)^3 \text{Subst}\left(\int \frac{1}{x(a+b\log(cx))}\,dx, x, e+fx\right)}{df^4} \\
&= \frac{i^3 \text{Subst}\left(\int \frac{e^{3x}}{a+bx}\,dx, x, \log(c(e+fx))\right)}{c^3 df^4} \\
&+ \frac{(3i^2(fh-ei))\,\text{Subst}\left(\int \frac{e^{2x}}{a+bx}\,dx, x, \log(c(e+fx))\right)}{c^2 df^4} \\
&+ \frac{(3i(fh-ei)^2)\,\text{Subst}\left(\int \frac{e^{x}}{a+bx}\,dx, x, \log(c(e+fx))\right)}{cdf^4} \\
&+ \frac{(fh-ei)^3 \text{Subst}\left(\int \frac{1}{x}\,dx, x, a+b\log(c(e+fx))\right)}{bdf^4} \\
&= \frac{3e^{-\frac{a}{b}} i(fh-ei)^2 \text{Ei}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^4} + \frac{3e^{-\frac{2a}{b}} i^2(fh-ei)\text{Ei}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2df^4} \\
&+ \frac{e^{-\frac{3a}{b}} i^3 \text{Ei}\left(\frac{3(a+b\log(c(e+fx)))}{b}\right)}{bc^3df^4} + \frac{(fh-ei)^3 \log(a+b\log(c(e+fx)))}{bdf^4}
\end{aligned}
$$

## Mathematica [F]

$$
\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx
$$

```
[In] Integrate[(h + i*x)^3/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
[Out] Integrate[(h + i*x)^3/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])), x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(179) = 358.

Time = 2.46 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.04

| method | result |
|---|---|
| derivativedivides | $-\frac{\frac{i^3 e^{-\frac{3a}{b}} \operatorname{Ei}_1\left(-3\ln(cfx+ce)-\frac{3a}{b}\right)}{b}-\frac{c^3 f^3 h^3 \ln(a+b\ln(cfx+ce))}{b}+\frac{c^3 e^3 i^3 \ln(a+b\ln(cfx+ce))}{b}-\frac{3ce\, i^3 e^{-\frac{2a}{b}} \operatorname{Ei}_1\left(-2\ln(cfx+ce)-\frac{2a}{b}\right)}{b}}{}$ |
| default | $-\frac{\frac{i^3 e^{-\frac{3a}{b}} \operatorname{Ei}_1\left(-3\ln(cfx+ce)-\frac{3a}{b}\right)}{b}-\frac{c^3 f^3 h^3 \ln(a+b\ln(cfx+ce))}{b}+\frac{c^3 e^3 i^3 \ln(a+b\ln(cfx+ce))}{b}-\frac{3ce\, i^3 e^{-\frac{2a}{b}} \operatorname{Ei}_1\left(-2\ln(cfx+ce)-\frac{2a}{b}\right)}{b}}{}$ |
| risch | $-\frac{e^3 i^3 \ln(a+b\ln(cfx+ce))}{f^4 db}+\frac{3e^2 h\, i^2 \ln(a+b\ln(cfx+ce))}{f^3 db}-\frac{3e\, h^2 i \ln(a+b\ln(cfx+ce))}{f^2 db}+\frac{h^3 \ln(a+b\ln(cfx+ce))}{f db}-$ |

[In] int((i*x+h)^3/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x,method=_RETURNVERBOSE)

[Out] -1/c^3/f^4/d*(i^3/b*exp(-3*a/b)*Ei(1,-3*ln(c*f*x+c*e)-3*a/b)-c^3*f^3*h^3*ln(a+b*ln(c*f*x+c*e))/b+c^3*e^3*i^3*ln(a+b*ln(c*f*x+c*e))/b-3*c*e*i^3/b*exp(-2*a/b)*Ei(1,-2*ln(c*f*x+c*e)-2*a/b)+3*c^2*e^2*i^3/b*exp(-a/b)*Ei(1,-ln(c*f*x+c*e)-a/b)+3*c*f*h*i^2/b*exp(-2*a/b)*Ei(1,-2*ln(c*f*x+c*e)-2*a/b)+3*c^2*f^2*h^2*i/b*exp(-a/b)*Ei(1,-ln(c*f*x+c*e)-a/b)+3*c^3*e*f^2*h^2*i*ln(a+b*ln(c*f*x+c*e))/b-3*c^3*e^2*f*h*i^2*ln(a+b*ln(c*f*x+c*e))/b-6*c^2*e*f*h*i^2/b*exp(-a/b)*Ei(1,-ln(c*f*x+c*e)-a/b))

## Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.47

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\left(i^3 \operatorname{log\_integral}\left((c^3f^3x^3+3c^3ef^2x^2+3c^3e^2fx+c^3e^3)e^{\left(\frac{3a}{b}\right)}\right)+(c^3f^3h^3-3c^3ef^2h^2i+3c^3e^2fhi^2-c^3e^3\right.}{}$$

[In] integrate((i*x+h)^3/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="fricas")

[Out] (i^3*log_integral((c^3*f^3*x^3 + 3*c^3*e*f^2*x^2 + 3*c^3*e^2*f*x + c^3*e^3)*e^(3*a/b)) + (c^3*f^3*h^3 - 3*c^3*e*f^2*h^2*i + 3*c^3*e^2*f*h*i^2 - c^3*e^3*i^3)*e^(3*a/b)*log(b*log(c*f*x + c*e) + a) + 3*(c*f*h*i^2 - c*e*i^3)*e^(a/b)*log_integral((c^2*f^2*x^2 + 2*c^2*e*f*x + c^2*e^2)*e^(2*a/b)) + 3*(c^2*f^2*h^2*i - 2*c^2*e*f*h*i^2 + c^2*e^2*i^3)*e^(2*a/b)*log_integral((c*f*x + c*e)*e^(a/b)))*e^(-3*a/b)/(b*c^3*d*f^4)

## Sympy [F]

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\int \frac{h^3}{ae+afx+be\log(ce+cfx)+bfx\log(ce+cfx)}\,dx + \int \frac{i^3x^3}{ae+afx+be\log(ce+cfx)+bfx\log(ce+cfx)}\,dx + \int \frac{3hi^2x^2}{ae+afx+be\log(ce+cfx)+bf}}{d}$$

```
[In] integrate((i*x+h)**3/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x)
```

```
[Out] (Integral(h**3/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x)
), x) + Integral(i**3*x**3/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(
c*e + c*f*x)), x) + Integral(3*h*i**2*x**2/(a*e + a*f*x + b*e*log(c*e + c*f
*x) + b*f*x*log(c*e + c*f*x)), x) + Integral(3*h**2*i*x/(a*e + a*f*x + b*e*
log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x)), x))/d
```

## Maxima [F]

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(ix+h)^3}{(dfx+de)(b\log((fx+e)c)+a)}\,dx$$

```
[In] integrate((i*x+h)^3/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="maxima")
```

```
[Out] h^3*log((b*log(f*x + e) + b*log(c) + a)/b)/(b*d*f) + integrate((i^3*x^3 + 3
*h*i^2*x^2 + 3*h^2*i*x)/(b*d*e*log(c) + a*d*e + (b*d*f*log(c) + a*d*f)*x +
(b*d*f*x + b*d*e)*log(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(ix+h)^3}{(dfx+de)(b\log((fx+e)c)+a)}\,dx$$

```
[In] integrate((i*x+h)^3/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="giac")
```

```
[Out] integrate((i*x + h)^3/((d*f*x + d*e)*(b*log((f*x + e)*c) + a)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^3}{(de+dfx)(a+b\log(c(e+fx)))}\,dx=\int \frac{{\left(h+i\,x\right)}^3}{\left(d\,e+d\,f\,x\right)\,\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)}\,dx$$

```
[In] int((h + i*x)^3/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
[Out] int((h + i*x)^3/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))), x)
```

# 3.193 $\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 124

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{2e^{-\frac{a}{b}}i(fh-ei)\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^3} + \frac{e^{-\frac{2a}{b}}i^2\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2df^3} + \frac{(fh-ei)^2\log(a+b\log(c(e+fx)))}{bdf^3}$$

```
[Out] 2*i*(-e*i+f*h)*Ei((a+b*ln(c*(f*x+e)))/b)/b/c/d/exp(a/b)/f^3+i^2*Ei(2*(a+b*l
n(c*(f*x+e)))/b)/b/c^2/d/exp(2*a/b)/f^3+(-e*i+f*h)^2*ln(a+b*ln(c*(f*x+e)))/
b/d/f^3
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2458, 12, 2395, 2336, 2209, 2339, 29, 2346}

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{i^2e^{-\frac{2a}{b}}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2df^3} + \frac{2ie^{-\frac{a}{b}}(fh-ei)\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^3} + \frac{(fh-ei)^2\log(a+b\log(c(e+fx)))}{bdf^3}$$

```
[In] Int[(h + i*x)^2/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

```
[Out] (2*i*(f*h - e*i)*ExpIntegralEi[(a + b*Log[c*(e + f*x)])/b])/(b*c*d*E^(a/b)*
f^3) + (i^2*ExpIntegralEi[(2*(a + b*Log[c*(e + f*x)]))/b])/(b*c^2*d*E^((2*a
)/b)*f^3) + ((f*h - e*i)^2*Log[a + b*Log[c*(e + f*x)]])/(b*d*f^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}{dx(a+b\log(cx))}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2}{x(a+b\log(cx))}\,dx, x, e+fx\right)}{df}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{2i(fh-ei)}{f^2(a+b\log(cx))}+\frac{(fh-ei)^2}{f^2x(a+b\log(cx))}+\frac{i^2x}{f^2(a+b\log(cx))}\right)\,dx, x, e+fx\right)}{df}$$

$$= \frac{i^2\text{Subst}\left(\int \frac{x}{a+b\log(cx)}\,dx, x, e+fx\right)}{df^3}+\frac{(2i(fh-ei))\text{Subst}\left(\int \frac{1}{a+b\log(cx)}\,dx, x, e+fx\right)}{df^3}$$
$$+\frac{(fh-ei)^2\text{Subst}\left(\int \frac{1}{x(a+b\log(cx))}\,dx, x, e+fx\right)}{df^3}$$

$$= \frac{i^2\text{Subst}\left(\int \frac{e^{2x}}{a+bx}\,dx, x, \log(c(e+fx))\right)}{c^2df^3}$$
$$+\frac{(2i(fh-ei))\text{Subst}\left(\int \frac{e^x}{a+bx}\,dx, x, \log(c(e+fx))\right)}{cdf^3}$$
$$+\frac{(fh-ei)^2\text{Subst}\left(\int \frac{1}{x}\,dx, x, a+b\log(c(e+fx))\right)}{bdf^3}$$

$$= \frac{2e^{-\frac{a}{b}}i(fh-ei)\text{Ei}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^3}+\frac{e^{-\frac{2a}{b}}i^2\text{Ei}\left(\frac{2(a+b\log(c(e+fx)))}{b}\right)}{bc^2df^3}$$
$$+\frac{(fh-ei)^2\log(a+b\log(c(e+fx)))}{bdf^3}$$

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$$
$$= \frac{e^{-\frac{2a}{b}}\left(2ce^{a/b}i(fh-ei)\,\text{ExpIntegralEi}\left(\frac{a}{b}+\log(c(e+fx))\right)+i^2\,\text{ExpIntegralEi}\left(2\left(\frac{a}{b}+\log(c(e+fx))\right)\right)+c\right.}{bc^2df^3}$$

```
[In] Integrate[(h + i*x)^2/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

```
[Out] (2*c*E^(a/b)*i*(f*h - e*i)*ExpIntegralEi[a/b + Log[c*(e + f*x)]] + i^2*ExpI
ntegralEi[2*(a/b + Log[c*(e + f*x)])] + c^2*E^((2*a)/b)*(f*h - e*i)^2*Log[a
 + b*Log[c*(e + f*x)]])/(b*c^2*d*E^((2*a)/b)*f^3)
```

## Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.61

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{i^2 \mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}_1\left(-2\ln(cfx+ce)-\frac{2a}{b}\right)}{b}+\frac{c^2e^2i^2\ln(a+b\ln(cfx+ce))}{b}+\frac{c^2f^2h^2\ln(a+b\ln(cfx+ce))}{b}+\frac{2ce\,i^2\mathrm{e}^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\ln(cfx+ce)-\frac{a}{b}\right)}{b}-}{c^2f^3d}$ |
| default | $\frac{-\frac{i^2 \mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}_1\left(-2\ln(cfx+ce)-\frac{2a}{b}\right)}{b}+\frac{c^2e^2i^2\ln(a+b\ln(cfx+ce))}{b}+\frac{c^2f^2h^2\ln(a+b\ln(cfx+ce))}{b}+\frac{2ce\,i^2\mathrm{e}^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\ln(cfx+ce)-\frac{a}{b}\right)}{b}-}{c^2f^3d}$ |
| risch | $\frac{e^2i^2\ln(a+b\ln(cfx+ce))}{f^3db}-\frac{2ehi\ln(a+b\ln(cfx+ce))}{f^2db}+\frac{h^2\ln(a+b\ln(cfx+ce))}{fdb}+\frac{2e\,i^2\mathrm{e}^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\ln(cfx+ce)-\frac{a}{b}\right)}{c\,f^3db}-$ |

```
[In] int((i*x+h)^2/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2/f^3/d*(-i^2/b*exp(-2*a/b)*Ei(1,-2*ln(c*f*x+c*e)-2*a/b)+c^2*e^2*i^2*ln
(a+b*ln(c*f*x+c*e))/b+c^2*f^2*h^2*ln(a+b*ln(c*f*x+c*e))/b+2*c*e*i^2/b*exp(-
a/b)*Ei(1,-ln(c*f*x+c*e)-a/b)-2*c*f*h*i/b*exp(-a/b)*Ei(1,-ln(c*f*x+c*e)-a/b
)-2*c^2*e*f*h*i*ln(a+b*ln(c*f*x+c*e))/b)
```

## Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.20

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\left((c^2f^2h^2-2\,c^2efhi+c^2e^2i^2)e^{\left(\frac{2\,a}{b}\right)}\log\left(b\log\left(cfx+ce\right)+a\right)+i^2\operatorname{log\_integral}\left((c^2f^2x^2+2\,c^2efx+c^2e^2)\right.\right.}{bc^2df^3}$$

```
[In] integrate((i*x+h)^2/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="fricas")
```

```
[Out] ((c^2*f^2*h^2 - 2*c^2*e*f*h*i + c^2*e^2*i^2)*e^(2*a/b)*log(b*log(c*f*x + c*
e) + a) + i^2*log_integral((c^2*f^2*x^2 + 2*c^2*e*f*x + c^2*e^2)*e^(2*a/b))
 + 2*(c*f*h*i - c*e*i^2)*e^(a/b)*log_integral((c*f*x + c*e)*e^(a/b)))*e^(-2
*a/b)/(b*c^2*d*f^3)
```

## Sympy [F]

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\int \frac{h^2}{ae+afx+be\log\left(ce+cfx\right)+bfx\log\left(ce+cfx\right)}\,dx+\int \frac{i^2x^2}{ae+afx+be\log\left(ce+cfx\right)+bfx\log\left(ce+cfx\right)}\,dx+\int \frac{2hix}{ae+afx+be\log\left(ce+cfx\right)+bf}}{d}$$

```
[In] integrate((i*x+h)**2/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x)
```

```
[Out] (Integral(h**2/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x)
), x) + Integral(i**2*x**2/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(
c*e + c*f*x)), x) + Integral(2*h*i*x/(a*e + a*f*x + b*e*log(c*e + c*f*x) +
b*f*x*log(c*e + c*f*x)), x))/d
```

## Maxima [F]

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(ix+h)^2}{(dfx+de)(b\log\left((fx+e)c\right)+a)}\,dx$$

```
[In] integrate((i*x+h)^2/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="maxima")
```

```
[Out] h^2*log((b*log(f*x + e) + b*log(c) + a)/b)/(b*d*f) + integrate((i^2*x^2 + 2
*h*i*x)/(b*d*e*log(c) + a*d*e + (b*d*f*log(c) + a*d*f)*x + (b*d*f*x + b*d*e
)*log(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(ix+h)^2}{(dfx+de)(b\log((fx+e)c)+a)}\,dx$$

```
[In] integrate((i*x+h)^2/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="giac")
[Out] integrate((i*x + h)^2/((d*f*x + d*e)*(b*log((f*x + e)*c) + a)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^2}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{(h+i\,x)^2}{(d\,e+d\,f\,x)\,\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)}\,dx$$

```
[In] int((h + i*x)^2/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
[Out] int((h + i*x)^2/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))), x)
```

# 3.194 $\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 71

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{e^{-\frac{a}{b}}i\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^2} + \frac{(fh-ei)\log(a+b\log(c(e+fx)))}{bdf^2}$$

```
[Out] i*Ei((a+b*ln(c*(f*x+e)))/b)/b/c/d/exp(a/b)/f^2+(-e*i+f*h)*ln(a+b*ln(c*(f*x+
e)))/b/d/f^2
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2458, 12, 2395, 2336, 2209, 2339, 29}

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{ie^{-\frac{a}{b}}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^2} + \frac{(fh-ei)\log(a+b\log(c(e+fx)))}{bdf^2}$$

```
[In] Int[(h + i*x)/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

```
[Out] (i*ExpIntegralEi[(a + b*Log[c*(e + f*x)])/b])/(b*c*d*E^(a/b)*f^2) + ((f*h -
 e*i)*Log[a + b*Log[c*(e + f*x)]])/(b*d*f^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\frac{fh-ei}{f}+\frac{ix}{f}}{dx(a+b\log(cx))}\,dx, x, e+fx\right)}{f}$$

$$
= \frac{\text{Subst}\left(\int \frac{\frac{fh-ei}{f}+\frac{ix}{f}}{x(a+b\log(cx))}\,dx,x,e+fx\right)}{df}
$$

$$
= \frac{\text{Subst}\left(\int \left(\frac{i}{f(a+b\log(cx))}+\frac{fh-ei}{fx(a+b\log(cx))}\right)dx,x,e+fx\right)}{df}
$$

$$
= \frac{i\text{Subst}\left(\int \frac{1}{a+b\log(cx)}\,dx,x,e+fx\right)}{df^2}+\frac{(fh-ei)\text{Subst}\left(\int \frac{1}{x(a+b\log(cx))}\,dx,x,e+fx\right)}{df^2}
$$

$$
= \frac{i\text{Subst}\left(\int \frac{e^x}{a+bx}\,dx,x,\log(c(e+fx))\right)}{cdf^2}+\frac{(fh-ei)\text{Subst}\left(\int \frac{1}{x}\,dx,x,a+b\log(c(e+fx))\right)}{bdf^2}
$$

$$
= \frac{e^{-\frac{a}{b}}i\text{Ei}\left(\frac{a+b\log(c(e+fx))}{b}\right)}{bcdf^2}+\frac{(fh-ei)\log(a+b\log(c(e+fx)))}{bdf^2}
$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$
\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx
= \frac{e^{-\frac{a}{b}}\left(i\,\text{ExpIntegralEi}\left(\frac{a}{b}+\log(c(e+fx))\right)+ce^{a/b}(fh-ei)\log(a+b\log(c(e+fx)))\right)}{bcdf^2}
$$

```
[In] Integrate[(h + i*x)/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
[Out] (i*ExpIntegralEi[a/b + Log[c*(e + f*x)]] + c*E^(a/b)*(f*h - e*i)*Log[a + b*
Log[c*(e + f*x)]])/(b*c*d*E^(a/b)*f^2)
```

## Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\frac{i\,e^{-\frac{a}{b}}\,\text{Ei}_1\left(-\ln(cfx+ce)-\frac{a}{b}\right)}{b}-\frac{hcf\ln(a+b\ln(cfx+ce))}{b}+\frac{cei\ln(a+b\ln(cfx+ce))}{b}}{c\,f^2 d}$ | 88 |
| default | $-\frac{\frac{i\,e^{-\frac{a}{b}}\,\text{Ei}_1\left(-\ln(cfx+ce)-\frac{a}{b}\right)}{b}-\frac{hcf\ln(a+b\ln(cfx+ce))}{b}+\frac{cei\ln(a+b\ln(cfx+ce))}{b}}{c\,f^2 d}$ | 88 |
| risch | $-\frac{ei\ln(a+b\ln(cfx+ce))}{f^2db}+\frac{h\ln(a+b\ln(cfx+ce))}{fdb}-\frac{i\,e^{-\frac{a}{b}}\,\text{Ei}_1\left(-\ln(cfx+ce)-\frac{a}{b}\right)}{c\,f^2db}$ | 96 |

```
[In] int((i*x+h)/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x,method=_RETURNVERBOSE)
```

[Out] -1/c/f^2/d*(i/b*exp(-a/b)*Ei(1,-ln(c*f*x+c*e)-a/b)-h*c*f*ln(a+b*ln(c*f*x+c*e))/b+c*e*i*ln(a+b*ln(c*f*x+c*e))/b)

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\left((cfh-cei)e^{\frac{a}{b}}\log\left(b\log\left(cfx+ce\right)+a\right)+i\,\mathrm{log\_integral}\left((cfx+ce)e^{\frac{a}{b}}\right)\right)e^{\left(-\frac{a}{b}\right)}}{bcdf^2}$$

[In] integrate((i*x+h)/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="fricas")

[Out] ((c*f*h - c*e*i)*e^(a/b)*log(b*log(c*f*x + c*e) + a) + i*log_integral((c*f*x + c*e)*e^(a/b)))*e^(-a/b)/(b*c*d*f^2)

## Sympy [F]

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\int \frac{h}{ae+afx+be\log\left(ce+cfx\right)+bfx\log\left(ce+cfx\right)}\,dx+\int \frac{ix}{ae+afx+be\log\left(ce+cfx\right)+bfx\log\left(ce+cfx\right)}\,dx}{d}$$

[In] integrate((i*x+h)/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x)

[Out] (Integral(h/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x)), x) + Integral(i*x/(a*e + a*f*x + b*e*log(c*e + c*f*x) + b*f*x*log(c*e + c*f*x)), x))/d

## Maxima [F]

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \int \frac{ix+h}{(dfx+de)(b\log\left((fx+e)c\right)+a)}\,dx$$

[In] integrate((i*x+h)/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="maxima")

[Out] i*integrate(x/(b*d*e*log(c) + a*d*e + (b*d*f*log(c) + a*d*f)*x + (b*d*f*x + b*d*e)*log(f*x + e)), x) + h*log((b*log(f*x + e) + b*log(c) + a)/b)/(b*d*f)

## Giac [F]

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx=\int \frac{ix+h}{(dfx+de)(b\log\left((fx+e)c\right)+a)}\,dx$$

```
[In] integrate((i*x+h)/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="giac")
[Out] integrate((i*x + h)/((d*f*x + d*e)*(b*log((f*x + e)*c) + a)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{h+ix}{(de+dfx)(a+b\log(c(e+fx)))}\,dx=\int \frac{h+i\,x}{(d\,e+d\,f\,x)\,\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)}\,dx$$

```
[In] int((h + i*x)/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
[Out] int((h + i*x)/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))), x)
```

# 3.195 $\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 23

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\log(a+b\log(c(e+fx)))}{bdf}$$

```
[Out] ln(a+b*ln(c*(f*x+e)))/b/d/f
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2437, 12, 2339, 29}

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\log(a+b\log(c(e+fx)))}{bdf}$$

```
[In] Int[1/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
```

```
[Out] Log[a + b*Log[c*(e + f*x)]]/(b*d*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{dx(a+b\log(cx))}\,dx, x, e+fx\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x(a+b\log(cx))}\,dx, x, e+fx\right)}{df}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a+b\log(c(e+fx))\right)}{bdf}\\ &= \frac{\log(a+b\log(c(e+fx)))}{bdf}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\log(a+b\log(c(e+fx)))}{bdf}$$

```
[In] Integrate[1/((d*e + d*f*x)*(a + b*Log[c*(e + f*x)])),x]
[Out] Log[a + b*Log[c*(e + f*x)]]/(b*d*f)
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

| method | result | size |
|---|---|---|
| norman | $\frac{\ln(a+b\ln(c(fx+e)))}{bdf}$ | 24 |
| parallelrisch | $\frac{\ln(a+b\ln(c(fx+e)))}{bdf}$ | 24 |
| derivativedivides | $\frac{\ln(a+b\ln(cfx+ce))}{fdb}$ | 25 |
| default | $\frac{\ln(a+b\ln(cfx+ce))}{fdb}$ | 25 |
| risch | $\frac{\ln\left(\ln(c(fx+e))+\frac{a}{b}\right)}{bdf}$ | 26 |

```
[In] int(1/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x,method=_RETURNVERBOSE)
[Out] ln(a+b*ln(c*(f*x+e)))/b/d/f
```

## Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx=\frac{\log(b\log(cfx+ce)+a)}{bdf}$$

```
[In] integrate(1/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="fricas")
[Out] log(b*log(c*f*x + c*e) + a)/(b*d*f)
```

## Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx=\frac{\log\left(\frac{a}{b}+\log(c(e+fx))\right)}{bdf}$$

```
[In] integrate(1/(d*f*x+d*e)/(a+b*ln(c*(f*x+e))),x)
[Out] log(a/b + log(c*(e + f*x)))/(b*d*f)
```

## Maxima [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\log\left(\frac{b\log(fx+e)+b\log(c)+a}{b}\right)}{bdf}$$

```
[In] integrate(1/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="maxima")
[Out] log((b*log(f*x + e) + b*log(c) + a)/b)/(b*d*f)
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\log(b\log(cfx+ce)+a)}{bdf}$$

```
[In] integrate(1/(d*f*x+d*e)/(a+b*log(c*(f*x+e))),x, algorithm="giac")
[Out] log(b*log(c*f*x + c*e) + a)/(b*d*f)
```

## Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(de+dfx)(a+b\log(c(e+fx)))}\,dx = \frac{\ln(a+b\,\ln(c\,(e+f\,x)))}{b\,d\,f}$$

```
[In] int(1/((d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
[Out] log(a + b*log(c*(e + f*x)))/(b*d*f)
```

# 3.196 $\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))} dx$



## Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx = \frac{\log(a+b\log(c(e+fx)))}{bd(fh-ei)} - \frac{i\text{Int}\left(\frac{1}{(h+ix)(a+b\log(c(e+fx)))},x\right)}{d(fh-ei)}$$

```
[Out] ln(a+b*ln(c*(f*x+e)))/b/d/(-e*i+f*h)-i*Unintegrable(1/(i*x+h)/(a+b*ln(c*(f*
x+e))),x)/d/(-e*i+f*h)
```

## Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx$$

```
[In] Int[1/((d*e + d*f*x)*(h + i*x)*(a + b*Log[c*(e + f*x)])),x]
```

```
[Out] Log[a + b*Log[c*(e + f*x)]]/(b*d*(f*h - e*i)) - (i*Defer[Int][1/((h + i*x)*
(a + b*Log[c*(e + f*x)])), x])/(d*(f*h - e*i))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{f}{d(fh-ei)(e+fx)(a+b\log(c(e+fx)))} - \frac{i}{d(fh-ei)(h+ix)(a+b\log(c(e+fx)))}\right) dx \\
&= \frac{f\int \frac{1}{(e+fx)(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)} - \frac{i\int \frac{1}{(h+ix)(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+b\log(cx))}\,dx, x, e+fx\right)}{d(fh-ei)} - \frac{i\int \frac{1}{(h+ix)(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a+b\log(c(e+fx))\right)}{bd(fh-ei)} - \frac{i\int \frac{1}{(h+ix)(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)} \\
&= \frac{\log(a+b\log(c(e+fx)))}{bd(fh-ei)} - \frac{i\int \frac{1}{(h+ix)(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)}
\end{aligned}$$

## Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx$$

```
[In] Integrate[1/((d*e + d*f*x)*(h + i*x)*(a + b*Log[c*(e + f*x)])),x]
[Out] Integrate[1/((d*e + d*f*x)*(h + i*x)*(a + b*Log[c*(e + f*x)])), x]
```

## Maple [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dfx+de)\,(ix+h)\,(a+b\ln(c\,(fx+e)))}dx$$

```
[In] int(1/(d*f*x+d*e)/(i*x+h)/(a+b*ln(c*(f*x+e))),x)
[Out] int(1/(d*f*x+d*e)/(i*x+h)/(a+b*ln(c*(f*x+e))),x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.12

$$\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(dfx+de)(ix+h)(b\log\left((fx+e)c\right)+a)}\,dx$$

```
[In] integrate(1/(d*f*x+d*e)/(i*x+h)/(a+b*log(c*(f*x+e))),x, algorithm="fricas")
[Out] integral(1/(a*d*f*i*x^2 + a*d*e*h + (a*d*f*h + a*d*e*i)*x + (b*d*f*i*x^2 +
b*d*e*h + (b*d*f*h + b*d*e*i)*x)*log(c*f*x + c*e)), x)
```

## Sympy [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.09

$$\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx = \frac{\int \frac{1}{aeh+aeix+afhx+afix^2+beh\log\left(ce+cfx\right)+beix\log\left(ce+cfx\right)+bfhx\log\left(ce+cfx\right)+bfix^2\log\left(ce+cfx\right)}\,dx}{d}$$

```
[In] integrate(1/(d*f*x+d*e)/(i*x+h)/(a+b*ln(c*(f*x+e))),x)
[Out] Integral(1/(a*e*h + a*e*i*x + a*f*h*x + a*f*i*x**2 + b*e*h*log(c*e + c*f*x)
 + b*e*i*x*log(c*e + c*f*x) + b*f*h*x*log(c*e + c*f*x) + b*f*i*x**2*log(c*e
 + c*f*x)), x)/d
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(dfx+de)(ix+h)(b\log\left((fx+e)c\right)+a)}\,dx$$

```
[In] integrate(1/(d*f*x+d*e)/(i*x+h)/(a+b*log(c*(f*x+e))),x, algorithm="maxima")
[Out] integrate(1/((d*f*x + d*e)*(i*x + h)*(b*log((f*x + e)*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx$$
$$=\int \frac{1}{(dfx+de)(ix+h)(b\log\left((fx+e)c\right)+a)}\,dx$$

```
[In] integrate(1/(d*f*x+d*e)/(i*x+h)/(a+b*log(c*(f*x+e))),x, algorithm="giac")
[Out] integrate(1/((d*f*x + d*e)*(i*x + h)*(b*log((f*x + e)*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)(a+b\log(c(e+fx)))}\,dx$$
$$=\int \frac{1}{(h+i\,x)\,\left(d\,e+d\,f\,x\right)\,\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)}\,dx$$

```
[In] int(1/((h + i*x)*(d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
[Out] int(1/((h + i*x)*(d*e + d*f*x)*(a + b*log(c*(e + f*x)))), x)
```

# 3.197 $\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx = \frac{f\log(a+b\log(c(e+fx)))}{bd(fh-ei)^2} - \frac{i\mathrm{Int}\left(\frac{1}{(h+ix)^2(a+b\log(c(e+fx)))},x\right)}{d(fh-ei)} - \frac{fi\mathrm{Int}\left(\frac{1}{(h+ix)(a+b\log(c(e+fx)))},x\right)}{d(fh-ei)^2}$$

```
[Out] f*ln(a+b*ln(c*(f*x+e)))/b/d/(-e*i+f*h)^2-i*Unintegrable(1/(i*x+h)^2/(a+b*ln
(c*(f*x+e))),x)/d/(-e*i+f*h)-f*i*Unintegrable(1/(i*x+h)/(a+b*ln(c*(f*x+e)))
,x)/d/(-e*i+f*h)^2
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx$$

```
[In] Int[1/((d*e + d*f*x)*(h + i*x)^2*(a + b*Log[c*(e + f*x)])),x]
```

```
[Out] (f*Log[a + b*Log[c*(e + f*x)]])/(b*d*(f*h - e*i)^2) - (i*Defer[Int][1/((h +
 i*x)^2*(a + b*Log[c*(e + f*x)])), x])/(d*(f*h - e*i)) - (f*i*Defer[Int][1/
((h + i*x)*(a + b*Log[c*(e + f*x)])), x])/(d*(f*h - e*i)^2)
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{f^2}{d(fh-ei)^2(e+fx)(a+b\log(c(e+fx)))} - \frac{i}{d(fh-ei)(h+ix)^2(a+b\log(c(e+fx)))} - \frac{fi}{d(fh-ei)^2(h+ix)(a+b\log(c(e+fx)))}\right) dx \\
&= \frac{f^2\int \frac{1}{(e+fx)(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)^2} - \frac{(fi)\int \frac{1}{(h+ix)(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)^2} - \frac{i\int \frac{1}{(h+ix)^2(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)} \\
&= \frac{f\text{Subst}\left(\int \frac{1}{x(a+b\log(cx))}\,dx, x, e+fx\right)}{d(fh-ei)^2} - \frac{(fi)\int \frac{1}{(h+ix)(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)^2} - \frac{i\int \frac{1}{(h+ix)^2(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)} \\
&= \frac{f\text{Subst}\left(\int \frac{1}{x}\,dx, x, a+b\log(c(e+fx))\right)}{bd(fh-ei)^2} - \frac{(fi)\int \frac{1}{(h+ix)(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)^2} - \frac{i\int \frac{1}{(h+ix)^2(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)} \\
&= \frac{f\log(a+b\log(c(e+fx)))}{bd(fh-ei)^2} - \frac{(fi)\int \frac{1}{(h+ix)(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)^2} - \frac{i\int \frac{1}{(h+ix)^2(a+b\log(c(e+fx)))}\,dx}{d(fh-ei)}
\end{aligned}$$

## Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx$$

```
[In] Integrate[1/((d*e + d*f*x)*(h + i*x)^2*(a + b*Log[c*(e + f*x)])),x]
[Out] Integrate[1/((d*e + d*f*x)*(h + i*x)^2*(a + b*Log[c*(e + f*x)])), x]
```

## Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dfx+de)\,(ix+h)^2\,(a+b\ln(c\,(fx+e)))}dx$$

```
[In] int(1/(d*f*x+d*e)/(i*x+h)^2/(a+b*ln(c*(f*x+e))),x)
```

```
[Out] int(1/(d*f*x+d*e)/(i*x+h)^2/(a+b*ln(c*(f*x+e))),x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.81

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(dfx+de)(ix+h)^2(b\log((fx+e)c)+a)}\,dx$$

```
[In] integrate(1/(d*f*x+d*e)/(i*x+h)^2/(a+b*log(c*(f*x+e))),x, algorithm="fricas
")
```

```
[Out] integral(1/(a*d*f*i^2*x^3 + a*d*e*h^2 + (2*a*d*f*h*i + a*d*e*i^2)*x^2 + (a*
d*f*h^2 + 2*a*d*e*h*i)*x + (b*d*f*i^2*x^3 + b*d*e*h^2 + (2*b*d*f*h*i + b*d*
e*i^2)*x^2 + (b*d*f*h^2 + 2*b*d*e*h*i)*x)*log(c*f*x + c*e)), x)
```

## Sympy [N/A]

Not integrable

Time = 4.98 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.62

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx = \frac{\int \frac{1}{aeh^2+2aehix+aei^2x^2+afh^2x+2afhix^2+afi^2x^3+beh^2\log(ce+cfx)+2behix\log(ce+cfx)+bei^2x^2\log(ce+cfx)+bfh^2x\log(ce+cfx)+2bfhix}}{d}$$

```
[In] integrate(1/(d*f*x+d*e)/(i*x+h)**2/(a+b*ln(c*(f*x+e))),x)
```

```
[Out] Integral(1/(a*e*h**2 + 2*a*e*h*i*x + a*e*i**2*x**2 + a*f*h**2*x + 2*a*f*h*i
*x**2 + a*f*i**2*x**3 + b*e*h**2*log(c*e + c*f*x) + 2*b*e*h*i*x*log(c*e + c
*f*x) + b*e*i**2*x**2*log(c*e + c*f*x) + b*f*h**2*x*log(c*e + c*f*x) + 2*b*
f*h*i*x**2*log(c*e + c*f*x) + b*f*i**2*x**3*log(c*e + c*f*x)), x)/d
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(dfx+de)(ix+h)^2(b\log((fx+e)c)+a)}\,dx$$

```
[In] integrate(1/(d*f*x+d*e)/(i*x+h)^2/(a+b*log(c*(f*x+e))),x, algorithm="maxima
")
[Out] integrate(1/((d*f*x + d*e)*(i*x + h)^2*(b*log((f*x + e)*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(dfx+de)(ix+h)^2(b\log((fx+e)c)+a)}\,dx$$

```
[In] integrate(1/(d*f*x+d*e)/(i*x+h)^2/(a+b*log(c*(f*x+e))),x, algorithm="giac")
[Out] integrate(1/((d*f*x + d*e)*(i*x + h)^2*(b*log((f*x + e)*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(de+dfx)(h+ix)^2(a+b\log(c(e+fx)))}\,dx = \int \frac{1}{(h+i\,x)^2\,\left(d\,e+d\,f\,x\right)\,\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)}\,dx$$

```
[In] int(1/((h + i*x)^2*(d*e + d*f*x)*(a + b*log(c*(e + f*x)))),x)
[Out] int(1/((h + i*x)^2*(d*e + d*f*x)*(a + b*log(c*(e + f*x)))), x)
```

# 3.198 $\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex} dx$



## Optimal result

Integrand size = 31, antiderivative size = 485

$$\int \frac{(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = -\frac{92b(ef-dg)^2n\sqrt{f+gx}}{15e^3} - \frac{32b(ef-dg)n(f+gx)^{3/2}}{45e^2} - \frac{4bn(f+gx)^{5/2}}{25e} + \frac{92b(ef-dg)^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15e^{7/2}} + \frac{2b(ef-dg)^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{7/2}} + \frac{2(ef-dg)^2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^3} + \frac{2(ef-dg)(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e^2} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5e} - \frac{2(ef-dg)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{7/2}} - \frac{4b(ef-dg)^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{7/2}} - \frac{2b(ef-dg)^{5/2}n\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{7/2}}$$

```
[Out] -32/45*b*(-d*g+e*f)*n*(g*x+f)^(3/2)/e^2-4/25*b*n*(g*x+f)^(5/2)/e+92/15*b*(-
d*g+e*f)^(5/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(7/2)+2*
b*(-d*g+e*f)^(5/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2/e^(7
/2)+2/3*(-d*g+e*f)*(g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))/e^2+2/5*(g*x+f)^(5/2
```

```
)*(a+b*ln(c*(e*x+d)^n))/e-2*(-d*g+e*f)^(5/2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/
(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+d)^n))/e^(7/2)-4*b*(-d*g+e*f)^(5/2)*n*arct
anh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(
-d*g+e*f)^(1/2)))/e^(7/2)-2*b*(-d*g+e*f)^(5/2)*n*polylog(2,1-2/(1-e^(1/2)*(
g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/e^(7/2)-92/15*b*(-d*g+e*f)^2*n*(g*x+f)^(1/2
)/e^3+2*(-d*g+e*f)^2*(a+b*ln(c*(e*x+d)^n))*(g*x+f)^(1/2)/e^3
```

## Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {2458, 2388, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 52}

$$\int \frac{(f+gx)^{5/2} (a+b\log(c(d+ex)^n))}{d+ex} \, dx = -\frac{2(ef-dg)^{5/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{7/2}} + \frac{2\sqrt{f+gx}(ef-dg)^2(a+b\log(c(d+ex)^n))}{e^3} + \frac{2(f+gx)^{3/2}(ef-dg)(a+b\log(c(d+ex)^n))}{3e^2} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5e} + \frac{2bn(ef-dg)^{5/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{7/2}} + \frac{92bn(ef-dg)^{5/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15e^{7/2}} - \frac{4bn(ef-dg)^{5/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{7/2}} - \frac{2bn(ef-dg)^{5/2}\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{7/2}} - \frac{92bn\sqrt{f+gx}(ef-dg)^2}{15e^3} - \frac{32bn(f+gx)^{3/2}(ef-dg)}{45e^2} - \frac{4bn(f+gx)^{5/2}}{25e}$$

```
[In] Int[((f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n]))/(d + e*x),x]
```

```
[Out] (-92*b*(e*f - d*g)^2*n*Sqrt[f + g*x])/(15*e^3) - (32*b*(e*f - d*g)*n*(f + g
*x)^(3/2))/(45*e^2) - (4*b*n*(f + g*x)^(5/2))/(25*e) + (92*b*(e*f - d*g)^(5
/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(15*e^(7/2)) + (2*b
*(e*f - d*g)^(5/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]^2)/e^
(7/2) + (2*(e*f - d*g)^2*Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n]))/e^3 + (2
*(e*f - d*g)*(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n]))/(3*e^2) + (2*(f +
g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n]))/(5*e) - (2*(e*f - d*g)^(5/2)*ArcTanh
```

```
[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/e^(7/
2) - (4*b*(e*f - d*g)^(5/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*
g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])])/e^(7/2) - (2*b*(e
*f - d*g)^(5/2)*n*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f -
d*g])])/e^(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{5/2}(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{e}$$

$$= \frac{g\text{Subst}\left(\int \left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}(a+b\log(cx^n))\,dx, x, d+ex\right)}{e^2} + \frac{(ef-dg)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{e^2}$$

$$= \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5e} + \frac{(g(ef-dg))\text{Subst}\left(\int \sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}(a+b\log(cx^n))\,dx, x, d+ex\right)}{e^3} + \frac{(ef-dg)^2\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{e^3} - \frac{(2bn)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{5/2}}{x}\,dx, x, d+ex\right)}{5e}$$

$$= -\frac{4bn(f+gx)^{5/2}}{25e} + \frac{2(ef-dg)(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e^2} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5e} + \frac{(g(ef-dg)^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{e^4} + \frac{(ef-dg)^3\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{e^4} - \frac{(2b(ef-dg)n)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}{x}\,dx, x, d+ex\right)}{5e^2} - \frac{(2b(ef-dg)n)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}{x}\,dx, x, d+ex\right)}{3e^2}$$

$$= -\frac{32b(ef-dg)n(f+gx)^{3/2}}{45e^2} - \frac{4bn(f+gx)^{5/2}}{25e} + \frac{2(ef-dg)^2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^3} + \frac{2(ef-dg)(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e^2} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5e} - \frac{2(ef-dg)^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{7/2}} - \frac{(2b(ef-dg)^2n)\,\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}{x}\,dx, x, d+ex\right)}{5e^3} - \frac{(2b(ef-dg)^2n)\,\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}{x}\,dx, x, d+ex\right)}{3e^3} - \frac{(2b(ef-dg)^2n)\,\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}{x}\,dx, x, d+ex\right)}{e^3} - \frac{(b(ef-dg)^3n)\,\text{Subst}\left(\int -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}x}\,dx, x, d+ex\right)}{e^4}$$

$$= -\frac{92b(ef-dg)^2n\sqrt{f+gx}}{15e^3} - \frac{32b(ef-dg)n(f+gx)^{3/2}}{45e^2} - \frac{4bn(f+gx)^{5/2}}{25e} + \frac{2(ef-dg)^2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^3} + \frac{2(ef-dg)(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e^2} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5e} - \frac{2(ef-dg)^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{7/2}} + \frac{\left(2b(ef-dg)^{5/2}n\right)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{x}\,dx,x,d+ex\right)}{e^{7/2}} - \frac{(2b(ef-dg)^3n)\text{Subst}\left(\int\frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx,x,d+ex\right)}{5e^4} - \frac{(2b(ef-dg)^3n)\text{Subst}\left(\int\frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx,x,d+ex\right)}{3e^4} - \frac{(2b(ef-dg)^3n)\text{Subst}\left(\int\frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx,x,d+ex\right)}{e^4}$$

$$= -\frac{92b(ef-dg)^2n\sqrt{f+gx}}{15e^3} - \frac{32b(ef-dg)n(f+gx)^{3/2}}{45e^2} - \frac{4bn(f+gx)^{5/2}}{25e} + \frac{2(ef-dg)^2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^3} + \frac{2(ef-dg)(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e^2} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5e} - \frac{2(ef-dg)^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{7/2}} + \frac{(4b(ef-dg)^{5/2}n)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{dg+e(-f+x^2)}\,dx, x, \sqrt{f+gx}\right)}{e^{5/2}} - \frac{(4b(ef-dg)^3n)\,\text{Subst}\left(\int\frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{5e^3g} - \frac{(4b(ef-dg)^3n)\,\text{Subst}\left(\int\frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{3e^3g} - \frac{(4b(ef-dg)^3n)\,\text{Subst}\left(\int\frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{e^3g}$$

$$= -\frac{92b(ef-dg)^2n\sqrt{f+gx}}{15e^3} - \frac{32b(ef-dg)n(f+gx)^{3/2}}{45e^2} - \frac{4bn(f+gx)^{5/2}}{25e} + \frac{92b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15e^{7/2}} + \frac{2(ef-dg)^2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^3} + \frac{2(ef-dg)(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e^2} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5e} - \frac{2(ef-dg)^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{7/2}} + \frac{(4b(ef-dg)^{5/2}n)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{-ef+dg+ex^2}\,dx, x, \sqrt{f+gx}\right)}{e^{5/2}}$$

$$= -\frac{92b(ef-dg)^2n\sqrt{f+gx}}{15e^3} - \frac{32b(ef-dg)n(f+gx)^{3/2}}{45e^2} - \frac{4bn(f+gx)^{5/2}}{25e} + \frac{92b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15e^{7/2}} + \frac{2b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{7/2}} + \frac{2(ef-dg)^2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^3} + \frac{2(ef-dg)(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e^2} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5e} - \frac{2(ef-dg)^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{7/2}} - \frac{(4b(ef-dg)^2n)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\,dx, x, \sqrt{f+gx}\right)}{e^3}$$

$$= -\frac{92b(ef-dg)^2n\sqrt{f+gx}}{15e^3} - \frac{32b(ef-dg)n(f+gx)^{3/2}}{45e^2} - \frac{4bn(f+gx)^{5/2}}{25e} + \frac{92b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15e^{7/2}} + \frac{2b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{7/2}} + \frac{2(ef-dg)^2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^3} + \frac{2(ef-dg)(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e^2} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5e} - \frac{2(ef-dg)^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{7/2}} - \frac{4b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{7/2}} + \frac{(4b(ef-dg)^2n)\,\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\right)}{1-\frac{ex^2}{ef-dg}}\,dx, x, \sqrt{f+gx}\right)}{e^3}$$

$$= -\frac{92b(ef-dg)^2n\sqrt{f+gx}}{15e^3} - \frac{32b(ef-dg)n(f+gx)^{3/2}}{45e^2} - \frac{4bn(f+gx)^{5/2}}{25e} + \frac{92b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15e^{7/2}} + \frac{2b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{7/2}} + \frac{2(ef-dg)^2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^3} + \frac{2(ef-dg)(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e^2} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5e} - \frac{2(ef-dg)^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{7/2}} - \frac{4b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{7/2}} - \frac{\left(4b(ef-dg)^{5/2}n\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{7/2}}$$

$$= -\frac{92b(ef-dg)^2n\sqrt{f+gx}}{15e^3} - \frac{32b(ef-dg)n(f+gx)^{3/2}}{45e^2} - \frac{4bn(f+gx)^{5/2}}{25e} + \frac{92b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{15e^{7/2}} + \frac{2b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{7/2}} + \frac{2(ef-dg)^2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^3} + \frac{2(ef-dg)(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e^2} + \frac{2(f+gx)^{5/2}(a+b\log(c(d+ex)^n))}{5e} - \frac{2(ef-dg)^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{7/2}} - \frac{4b(ef-dg)^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{7/2}} - \frac{2b(ef-dg)^{5/2}n\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{7/2}}$$

## Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.69

$$\int \frac{(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx = \frac{900a\sqrt{e}(ef-dg)^2\sqrt{f+gx}-1800b\sqrt{e}(ef-dg)^2n\sqrt{f+gx}+18}{}$$

```
[In] Integrate[((f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n]))/(d + e*x),x]
```

```
[Out] (900*a*Sqrt[e]*(e*f - d*g)^2*Sqrt[f + g*x] - 1800*b*Sqrt[e]*(e*f - d*g)^2*n
*Sqrt[f + g*x] + 1800*b*(e*f - d*g)^(5/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])
/Sqrt[e*f - d*g]] - 200*b*(e*f - d*g)*n*(Sqrt[e]*Sqrt[f + g*x]*(4*e*f - 3*d
*g + e*g*x) - 3*(e*f - d*g)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f
- d*g]]) - 24*b*n*(3*e^(5/2)*(f + g*x)^(5/2) + 5*(e*f - d*g)*(Sqrt[e]*Sqrt[
f + g*x]*(4*e*f - 3*d*g + e*g*x) - 3*(e*f - d*g)^(3/2)*ArcTanh[(Sqrt[e]*Sqr
t[f + g*x])/Sqrt[e*f - d*g]])) + 900*b*Sqrt[e]*(e*f - d*g)^2*Sqrt[f + g*x]*
Log[c*(d + e*x)^n] + 300*e^(3/2)*(e*f - d*g)*(f + g*x)^(3/2)*(a + b*Log[c*(
d + e*x)^n]) + 180*e^(5/2)*(f + g*x)^(5/2)*(a + b*Log[c*(d + e*x)^n]) + 450
*(e*f - d*g)^(5/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]
*Sqrt[f + g*x]] - 450*(e*f - d*g)^(5/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt
[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] - 225*b*(e*f - d*g)^(5/2)*n*(Log[Sqrt[
e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f +
 g*x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) + 2*PolyLo
g[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 225*b*(e*f - d*g
)^(5/2)*n*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g
] + Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*
f - d*g])]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]
))/(450*e^(7/2))
```

## Maple [F]

$$\int \frac{(gx+f)^{\frac{5}{2}}\left(a+b\ln\left(c(ex+d)^n\right)\right)}{ex+d}dx$$

```
[In] int((g*x+f)^(5/2)*(a+b*ln(c*(e*x+d)^n))/(e*x+d),x)
```

```
[Out] int((g*x+f)^(5/2)*(a+b*ln(c*(e*x+d)^n))/(e*x+d),x)
```

## Fricas [F]

$$\int \frac{(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx=\int \frac{(gx+f)^{\frac{5}{2}}\left(b\log\left((ex+d)^n c\right)+a\right)}{ex+d}\,dx$$

```
[In] integrate((g*x+f)^(5/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="fricas
")
```

```
[Out] integral(((b*g^2*x^2 + 2*b*f*g*x + b*f^2)*sqrt(g*x + f)*log((e*x + d)^n*c)
+ (a*g^2*x^2 + 2*a*f*g*x + a*f^2)*sqrt(g*x + f))/(e*x + d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)**(5/2)*(a+b*ln(c*(e*x+d)**n))/(e*x+d),x)
```

```
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^(5/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \frac{(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx=\int \frac{(gx+f)^{\frac{5}{2}}\left(b\log\left((ex+d)^n c\right)+a\right)}{ex+d}\,dx$$

```
[In] integrate((g*x+f)^(5/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="giac")
[Out] integrate((g*x + f)^(5/2)*(b*log((e*x + d)^n*c) + a)/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx=\int \frac{{\left(f+g\,x\right)}^{5/2}\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}{d+e\,x}\,dx$$

```
[In] int(((f + g*x)^(5/2)*(a + b*log(c*(d + e*x)^n)))/(d + e*x),x)
[Out] int(((f + g*x)^(5/2)*(a + b*log(c*(d + e*x)^n)))/(d + e*x), x)
```

# 3.199 $\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 417

$$\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = -\frac{16b(ef-dg)n\sqrt{f+gx}}{3e^2} - \frac{4bn(f+gx)^{3/2}}{9e} + \frac{16b(ef-dg)^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3e^{5/2}} + \frac{2b(ef-dg)^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{5/2}} + \frac{2(ef-dg)\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^2} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e} - \frac{2(ef-dg)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{5/2}} - \frac{4b(ef-dg)^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{5/2}} - \frac{2b(ef-dg)^{3/2}n\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{5/2}}$$

```
[Out] -4/9*b*n*(g*x+f)^(3/2)/e+16/3*b*(-d*g+e*f)^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^
(1/2)/(-d*g+e*f)^(1/2))/e^(5/2)+2*b*(-d*g+e*f)^(3/2)*n*arctanh(e^(1/2)*(g*x
+f)^(1/2)/(-d*g+e*f)^(1/2))^2/e^(5/2)+2/3*(g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n
))/e-2*(-d*g+e*f)^(3/2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+
b*ln(c*(e*x+d)^n))/e^(5/2)-4*b*(-d*g+e*f)^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^(
1/2)/(-d*g+e*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/e^(
```

```
5/2)-2*b*(-d*g+e*f)^(3/2)*n*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*
f)^(1/2)))/e^(5/2)-16/3*b*(-d*g+e*f)*n*(g*x+f)^(1/2)/e^2+2*(-d*g+e*f)*(a+b*
ln(c*(e*x+d)^n))*(g*x+f)^(1/2)/e^2
```

## Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2458, 2388, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 52}

$$\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = -\frac{2(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{5/2}} + \frac{2\sqrt{f+gx}(ef-dg)(a+b\log(c(d+ex)^n))}{e^2} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e} + \frac{2bn(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{5/2}} + \frac{16bn(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3e^{5/2}} - \frac{4bn(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{5/2}} - \frac{2bn(ef-dg)^{3/2}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{5/2}} - \frac{16bn\sqrt{f+gx}(ef-dg)}{3e^2} - \frac{4bn(f+gx)^{3/2}}{9e}$$

```
[In] Int[((f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n]))/(d + e*x),x]
```

```
[Out] (-16*b*(e*f - d*g)*n*Sqrt[f + g*x])/(3*e^2) - (4*b*n*(f + g*x)^(3/2))/(9*e)
 + (16*b*(e*f - d*g)^(3/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g
]])/(3*e^(5/2)) + (2*b*(e*f - d*g)^(3/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/
Sqrt[e*f - d*g]]^2)/e^(5/2) + (2*(e*f - d*g)*Sqrt[f + g*x]*(a + b*Log[c*(d
+ e*x)^n]))/e^2 + (2*(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n]))/(3*e) - (2
*(e*f - d*g)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*(a + b*
Log[c*(d + e*x)^n]))/e^(5/2) - (4*b*(e*f - d*g)^(3/2)*n*ArcTanh[(Sqrt[e]*Sq
rt[f + g*x])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f -
 d*g])])/e^(5/2) - (2*b*(e*f - d*g)^(3/2)*n*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*
Sqrt[f + g*x])/Sqrt[e*f - d*g])])/e^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
```

```
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{e}$$

$$= \frac{g\text{Subst}\left(\int \sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}(a+b\log(cx^n))\,dx, x, d+ex\right)}{e^2} + \frac{(ef-dg)\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{e^2}$$

$$= \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e} + \frac{(g(ef-dg))\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{e^3} + \frac{(ef-dg)^2\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{e^3} - \frac{(2bn)\text{Subst}\left(\int \frac{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}{x}\,dx, x, d+ex\right)}{3e}$$

$$= -\frac{4bn(f+gx)^{3/2}}{9e} + \frac{2(ef-dg)\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^2} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e} - \frac{2(ef-dg)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{5/2}} - \frac{(2b(ef-dg)n)\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}{x}\,dx, x, d+ex\right)}{3e^2} - \frac{(2b(ef-dg)n)\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}{x}\,dx, x, d+ex\right)}{e^2} - \frac{(b(ef-dg)^2n)\,\text{Subst}\left(\int -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}x}\,dx, x, d+ex\right)}{e^3}$$

$$= -\frac{16b(ef-dg)n\sqrt{f+gx}}{3e^2} - \frac{4bn(f+gx)^{3/2}}{9e} + \frac{2(ef-dg)\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^2} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e} - \frac{2(ef-dg)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{5/2}} + \frac{\left(2b(ef-dg)^{3/2}n\right)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{x}\,dx, x, d+ex\right)}{e^{5/2}} - \frac{\left(2b(ef-dg)^2n\right)\text{Subst}\left(\int\frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{3e^3} - \frac{\left(2b(ef-dg)^2n\right)\text{Subst}\left(\int\frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{e^3}$$

$$= -\frac{16b(ef-dg)n\sqrt{f+gx}}{3e^2} - \frac{4bn(f+gx)^{3/2}}{9e} + \frac{2(ef-dg)\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^2} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e} - \frac{2(ef-dg)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{5/2}} + \frac{\left(4b(ef-dg)^{3/2}n\right)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{dg+e(-f+x^2)}\,dx, x, \sqrt{f+gx}\right)}{e^{3/2}} - \frac{\left(4b(ef-dg)^2n\right)\text{Subst}\left(\int\frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{3e^2g} - \frac{\left(4b(ef-dg)^2n\right)\text{Subst}\left(\int\frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{e^2g}$$

$$= -\frac{16b(ef-dg)n\sqrt{f+gx}}{3e^2} - \frac{4bn(f+gx)^{3/2}}{9e} + \frac{16b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3e^{5/2}} + \frac{2(ef-dg)\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^2} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e} - \frac{2(ef-dg)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{5/2}} + \frac{(4b(ef-dg)^{3/2}n)\,\mathrm{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{-ef+dg+ex^2}\,dx, x, \sqrt{f+gx}\right)}{e^{3/2}}$$

$$= -\frac{16b(ef-dg)n\sqrt{f+gx}}{3e^2} - \frac{4bn(f+gx)^{3/2}}{9e} + \frac{16b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3e^{5/2}} + \frac{2b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{5/2}} + \frac{2(ef-dg)\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^2} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e} - \frac{2(ef-dg)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{5/2}} - \frac{(4b(ef-dg)n)\mathrm{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\,dx, x, \sqrt{f+gx}\right)}{e^2}$$

$$= -\frac{16b(ef-dg)n\sqrt{f+gx}}{3e^2} - \frac{4bn(f+gx)^{3/2}}{9e} + \frac{16b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3e^{5/2}} + \frac{2b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{5/2}} + \frac{2(ef-dg)\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^2} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e} - \frac{2(ef-dg)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{5/2}} - \frac{4b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{5/2}} + \frac{(4b(ef-dg)n)\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\right)}{1-\frac{ex^2}{ef-dg}}dx, x, \sqrt{f+gx}\right)}{e^2}$$

$$= -\frac{16b(ef-dg)n\sqrt{f+gx}}{3e^2} - \frac{4bn(f+gx)^{3/2}}{9e} + \frac{16b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3e^{5/2}} + \frac{2b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{5/2}} + \frac{2(ef-dg)\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e^2} + \frac{2(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{3e} - \frac{2(ef-dg)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{5/2}} - \frac{4b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{5/2}} - \frac{\left(4b(ef-dg)^{3/2}n\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}dx, x, \frac{1}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{5/2}}$$

$$
\begin{aligned}
= & -\frac{16b(ef-dg)n\sqrt{f+gx}}{3e^2}-\frac{4bn(f+gx)^{3/2}}{9e} \\
& +\frac{16b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3e^{5/2}}+\frac{2b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{5/2}} \\
& +\frac{2(ef-dg)\sqrt{f+gx}(a+b\log\left(c(d+ex)^n\right))}{e^2} \\
& +\frac{2(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e} \\
& -\frac{2(ef-dg)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^{5/2}} \\
& -\frac{4b(ef-dg)^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{5/2}} \\
& -\frac{2b(ef-dg)^{3/2}n\operatorname{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{5/2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.54

$$
\int\frac{(f+gx)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx=\frac{36a\sqrt{e}(ef-dg)\sqrt{f+gx}-72b\sqrt{e}(ef-dg)n\sqrt{f+gx}-8b\sqrt{e}
$$

```
[In] Integrate[((f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n]))/(d + e*x),x]
```

```
[Out] (36*a*Sqrt[e]*(e*f - d*g)*Sqrt[f + g*x] - 72*b*Sqrt[e]*(e*f - d*g)*n*Sqrt[f
 + g*x] - 8*b*Sqrt[e]*n*Sqrt[f + g*x]*(4*e*f - 3*d*g + e*g*x) + 96*b*(e*f -
 d*g)^(3/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] + 36*b*Sqrt[
e]*(e*f - d*g)*Sqrt[f + g*x]*Log[c*(d + e*x)^n] + 12*e^(3/2)*(f + g*x)^(3/2
)*(a + b*Log[c*(d + e*x)^n]) + 18*(e*f - d*g)^(3/2)*(a + b*Log[c*(d + e*x)^
n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] - 18*(e*f - d*g)^(3/2)*(a
+ b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] + 9*b*
(e*f - d*g)^(3/2)*n*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[
e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(
2*Sqrt[e*f - d*g])]) - 9*b*(e*f - d*g)^(3/2)*n*Log[Sqrt[e*f - d*g] - Sqrt[e
]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] + 2*Log[(1 +
 (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) - 18*b*(e*f - d*g)^(3/2)*n*Po
lyLog[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])] + 18*b*(e*f - d
*g)^(3/2)*n*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2])/(1
8*e^(5/2))
```

## Maple [F]

$$\int \frac{(gx+f)^{\frac{3}{2}}(a+b\ln(c(ex+d)^n))}{ex+d}dx$$

```
[In] int((g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))/(e*x+d),x)
```

```
[Out] int((g*x+f)^(3/2)*(a+b*ln(c*(e*x+d)^n))/(e*x+d),x)
```

## Fricas [F]

$$\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}(b\log((ex+d)^n c)+a)}{ex+d}\,dx$$

```
[In] integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="fricas
")
```

```
[Out] integral(((b*g*x + b*f)*sqrt(g*x + f)*log((e*x + d)^n*c) + (a*g*x + a*f)*sq
rt(g*x + f))/(e*x + d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**(3/2)*(a+b*ln(c*(e*x+d)**n))/(e*x+d),x)
```

```
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^{3/2}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \frac{(f+gx)^{3/2}\,(a+b\log{(c(d+ex)^n)})}{d+ex}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}(b\log{((ex+d)^n c)}+a)}{ex+d}\,dx$$

```
[In] integrate((g*x+f)^(3/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="giac")
[Out] integrate((g*x + f)^(3/2)*(b*log((e*x + d)^n*c) + a)/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\,(a+b\log{(c(d+ex)^n)})}{d+ex}\,dx = \int \frac{{(f+g\,x)}^{3/2}\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)}{d+e\,x}\,dx$$

```
[In] int(((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n)))/(d + e*x),x)
[Out] int(((f + g*x)^(3/2)*(a + b*log(c*(d + e*x)^n)))/(d + e*x), x)
```

# 3.200 $\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 349

$$\begin{aligned}\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx &= -\frac{4bn\sqrt{f+gx}}{e}+\frac{4b\sqrt{ef-dg}\,n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}}\\&\quad+\frac{2b\sqrt{ef-dg}\,n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{3/2}}+\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e}\\&\quad-\frac{2\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{3/2}}\\&\quad-\frac{4b\sqrt{ef-dg}\,n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{3/2}}\\&\quad-\frac{2b\sqrt{ef-dg}\,n\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{3/2}}\end{aligned}$$

```
[Out] 4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(-d*g+e*f)^(1/2)/e^(3
/2)+2*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2*(-d*g+e*f)^(1/2
)/e^(3/2)-2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+
d)^n))*(-d*g+e*f)^(1/2)/e^(3/2)-4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e
*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))*(-d*g+e*f)^(1/2
)/e^(3/2)-2*b*n*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))*(
-d*g+e*f)^(1/2)/e^(3/2)-4*b*n*(g*x+f)^(1/2)/e+2*(a+b*ln(c*(e*x+d)^n))*(g*x+
f)^(1/2)/e
```

## Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2458, 2388, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 52}

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx$$
$$= -\frac{2\sqrt{ef-dg}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{3/2}}$$
$$+\frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e}+\frac{2bn\sqrt{ef-dg}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{3/2}}$$
$$+\frac{4bn\sqrt{ef-dg}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}}-\frac{4bn\sqrt{ef-dg}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{3/2}}$$
$$-\frac{2bn\sqrt{ef-dg}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{3/2}}-\frac{4bn\sqrt{f+gx}}{e}$$

```
[In] Int[(Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x),x]
```

```
[Out] (-4*b*n*Sqrt[f + g*x])/e + (4*b*Sqrt[e*f - d*g]*n*ArcTanh[(Sqrt[e]*Sqrt[f +
 g*x])/Sqrt[e*f - d*g]])/e^(3/2) + (2*b*Sqrt[e*f - d*g]*n*ArcTanh[(Sqrt[e]*
Sqrt[f + g*x])/Sqrt[e*f - d*g]]^2)/e^(3/2) + (2*Sqrt[f + g*x]*(a + b*Log[c*
(d + e*x)^n]))/e - (2*Sqrt[e*f - d*g]*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[
e*f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/e^(3/2) - (4*b*Sqrt[e*f - d*g]*n*Ar
cTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f +
 g*x])/Sqrt[e*f - d*g])])/e^(3/2) - (2*b*Sqrt[e*f - d*g]*n*PolyLog[2, 1 - 2
/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])])/e^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}(a+b\log(cx^n))}{x}\,dx, x, d+ex\right)}{e}$$

$$= \frac{g\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{e^2} + \frac{(ef-dg)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{e^2}$$

$$= \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e} - \frac{2\sqrt{ef-dg}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{3/2}} - \frac{(2bn)\text{Subst}\left(\int \frac{\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}{x}dx, x, d+ex\right)}{e} - \frac{(b(ef-dg)n)\text{Subst}\left(\int -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}x}dx, x, d+ex\right)}{e^2}$$

$$= -\frac{4bn\sqrt{f+gx}}{e} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e} - \frac{2\sqrt{ef-dg}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{3/2}} + \frac{\left(2b\sqrt{ef-dg}n\right)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{x}dx, x, d+ex\right)}{e^{3/2}} - \frac{(2b(ef-dg)n)\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}dx, x, d+ex\right)}{e^2}$$

$$= -\frac{4bn\sqrt{f+gx}}{e} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e} - \frac{2\sqrt{ef-dg}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{3/2}} + \frac{\left(4b\sqrt{ef-dg}n\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{dg+e(-f+x^2)}dx, x, \sqrt{f+gx}\right)}{\sqrt{e}} - \frac{(4b(ef-dg)n)\text{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}dx, x, \sqrt{f+gx}\right)}{eg}$$

$$= -\frac{4bn\sqrt{f+gx}}{e} + \frac{4b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e} - \frac{2\sqrt{ef-dg}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{3/2}} + \frac{\left(4b\sqrt{ef-dg}n\right)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{-ef+dg+ex^2}\,dx, x, \sqrt{f+gx}\right)}{\sqrt{e}}$$

$$= -\frac{4bn\sqrt{f+gx}}{e} + \frac{4b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}} + \frac{2b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{3/2}} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e} - \frac{2\sqrt{ef-dg}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{3/2}} - \frac{(4bn)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\,dx, x, \sqrt{f+gx}\right)}{e}$$

$$= -\frac{4bn\sqrt{f+gx}}{e} + \frac{4b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}} + \frac{2b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{3/2}} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e} - \frac{2\sqrt{ef-dg}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{3/2}} - \frac{4b\sqrt{ef-dg}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{3/2}} + \frac{(4bn)\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\right)}{1-\frac{ex^2}{ef-dg}}\,dx, x, \sqrt{f+gx}\right)}{e}$$

$$= -\frac{4bn\sqrt{f+gx}}{e} + \frac{4b\sqrt{ef-dg}\,n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}} + \frac{2b\sqrt{ef-dg}\,n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{3/2}} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e} - \frac{2\sqrt{ef-dg}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{3/2}} - \frac{4b\sqrt{ef-dg}\,n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{3/2}} - \frac{\left(4b\sqrt{ef-dg}\,n\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{3/2}}$$

$$= -\frac{4bn\sqrt{f+gx}}{e} + \frac{4b\sqrt{ef-dg}\,n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}} + \frac{2b\sqrt{ef-dg}\,n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{e^{3/2}} + \frac{2\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{e} - \frac{2\sqrt{ef-dg}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{e^{3/2}} - \frac{4b\sqrt{ef-dg}\,n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{3/2}} - \frac{2b\sqrt{ef-dg}\,n\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{e^{3/2}}$$

## Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \frac{4a\sqrt{e}\sqrt{f+gx} - 8b\sqrt{e}n\sqrt{f+gx} + 8b\sqrt{ef-dg}\,n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right) + 4b\sqrt{e}\sqrt{f+gx}\log(c(d+ex)^n) + 2\cdots}{}$$

```
[In] Integrate[(Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x),x]
```

```
[Out] (4*a*Sqrt[e]*Sqrt[f + g*x] - 8*b*Sqrt[e]*n*Sqrt[f + g*x] + 8*b*Sqrt[e*f - d
*g]*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] + 4*b*Sqrt[e]*Sqrt[f
 + g*x]*Log[c*(d + e*x)^n] + 2*Sqrt[e*f - d*g]*(a + b*Log[c*(d + e*x)^n])*L
og[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] - 2*Sqrt[e*f - d*g]*(a + b*Log[
```

```
c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] - b*Sqrt[e*f -
 d*g]*n*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g]
- Sqrt[e]*Sqrt[f + g*x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*
g])/2]) + 2*PolyLog[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])])
+ b*Sqrt[e*f - d*g]*n*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sq
rt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x]
)/(2*Sqrt[e*f - d*g])]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*
f - d*g])/2]))/(2*e^(3/2))
```

## Maple [F]

$$\int \frac{\sqrt{gx+f}\,(a+b\ln(c(ex+d)^n))}{ex+d}dx$$

```
[In] int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))/(e*x+d),x)
```

```
[Out] int((g*x+f)^(1/2)*(a+b*ln(c*(e*x+d)^n))/(e*x+d),x)
```

## Fricas [F]

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \int \frac{\sqrt{gx+f}(b\log((ex+d)^n c)+a)}{ex+d}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(g*x + f)*b*log((e*x + d)^n*c) + sqrt(g*x + f)*a)/(e*x + d),
x)
```

## Sympy [F]

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \int \frac{(a+b\log(c(d+ex)^n))\sqrt{f+gx}}{d+ex}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(a+b*ln(c*(e*x+d)**n))/(e*x+d),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))*sqrt(f + g*x)/(d + e*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \int \frac{\sqrt{gx+f}(b\log((ex+d)^n c)+a)}{ex+d}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(a+b*log(c*(e*x+d)^n))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)*(b*log((e*x + d)^n*c) + a)/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}(a+b\log(c(d+ex)^n))}{d+ex}\,dx = \int \frac{\sqrt{f+g\,x}\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}{d+e\,x}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n)))/(d + e*x),x)
```

```
[Out] int(((f + g*x)^(1/2)*(a + b*log(c*(d + e*x)^n)))/(d + e*x), x)
```

# 3.201 $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)\sqrt{f+gx}}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 256

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2bn\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}\sqrt{ef-dg}} - \frac{2\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{\sqrt{e}\sqrt{ef-dg}} - \frac{4bn\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}\sqrt{ef-dg}} - \frac{2bn\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}\sqrt{ef-dg}}$$

```
[Out] 2*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2/e^(1/2)/(-d*g+e*f)^
(1/2)-2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+d)^n
))/e^(1/2)/(-d*g+e*f)^(1/2)-4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^
(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/e^(1/2)/(-d*g+e*f)^
(1/2)-2*b*n*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/e^(1/
2)/(-d*g+e*f)^(1/2)
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2458, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{\sqrt{e}\sqrt{ef-dg}} + \frac{2bn\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}\sqrt{ef-dg}} - \frac{4bn\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}\sqrt{ef-dg}} - \frac{2bn\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}\sqrt{ef-dg}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*b*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]^2)/(Sqrt[e]*Sqrt[e*
f - d*g]) - (2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*(a + b*Log[
c*(d + e*x)^n]))/(Sqrt[e]*Sqrt[e*f - d*g]) - (4*b*n*ArcTanh[(Sqrt[e]*Sqrt[f
 + g*x])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g
])])/(Sqrt[e]*Sqrt[e*f - d*g]) - (2*b*n*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt
[f + g*x])/Sqrt[e*f - d*g])])/(Sqrt[e]*Sqrt[e*f - d*g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}\sqrt{ef-dg}} - \frac{(bn)\text{Subst}\left(\int -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}x}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}\sqrt{ef-dg}} + \frac{(2bn)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{x}\,dx, x, d+ex\right)}{\sqrt{e}\sqrt{ef-dg}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}\sqrt{ef-dg}} + \frac{\left(4b\sqrt{e}n\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{dg+e(-f+x^2)}\,dx, x, \sqrt{f+gx}\right)}{\sqrt{ef-dg}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}\sqrt{ef-dg}} + \frac{\left(4b\sqrt{e}n\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{-ef+dg+ex^2}\,dx, x, \sqrt{f+gx}\right)}{\sqrt{ef-dg}}$$

$$= \frac{2bn\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}\sqrt{ef-dg}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}\sqrt{ef-dg}} - \frac{(4bn)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\,dx, x, \sqrt{f+gx}\right)}{ef-dg}$$

$$= \frac{2bn\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}\sqrt{ef-dg}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}\sqrt{ef-dg}} - \frac{4bn\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}\sqrt{ef-dg}} + \frac{(4bn)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\right)}{1-\frac{ex^2}{ef-dg}}\,dx, x, \sqrt{f+gx}\right)}{ef-dg}$$

$$= \frac{2bn\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}\sqrt{ef-dg}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}\sqrt{ef-dg}} - \frac{4bn\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}\sqrt{ef-dg}} - \frac{(4bn)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}\sqrt{ef-dg}}$$

$$= \frac{2bn\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{\sqrt{e}\sqrt{ef-dg}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{e}\sqrt{ef-dg}} - \frac{4bn\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}\sqrt{ef-dg}} - \frac{2bn\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{\sqrt{e}\sqrt{ef-dg}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx$$
$$= \frac{-2a\sqrt{-ef+dg}\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)+2b\sqrt{ef-dg}\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)\left(in\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)+\log\left(c(d+ex\right.\right.}{\sqrt{e}\sqrt{-(ef-dg)^2}}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*a*Sqrt[-(e*f) + d*g]*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]] +
 2*b*Sqrt[e*f - d*g]*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]]*(I*
n*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]] + Log[c*(d + e*x)^n] +
 2*n*Log[(2*I)/(I - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g])]) + (2*I)*b
*Sqrt[e*f - d*g]*n*PolyLog[2, -((Sqrt[-(e*f) + d*g] - I*Sqrt[e]*Sqrt[f + g*
x])/(Sqrt[-(e*f) + d*g] + I*Sqrt[e]*Sqrt[f + g*x]))])/(Sqrt[e]*Sqrt[-(e*f -
 d*g)^2])
```

## Maple [F]

$$\int \frac{a+b\ln\left(c(ex+d)^n\right)}{(ex+d)\sqrt{gx+f}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(1/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(1/2),x)
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(ex+d)\sqrt{gx+f}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(g*x + f)*b*log((e*x + d)^n*c) + sqrt(g*x + f)*a)/(e*g*x^2 +
d*f + (e*f + d*g)*x), x)
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(e*x+d)/(g*x+f)**(1/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))/((d + e*x)*sqrt(f + g*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima
")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(ex+d)\sqrt{gx+f}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((e*x + d)*sqrt(g*x + f)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{a+b\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{\sqrt{f+g\,x}\,\left(d+e\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(1/2)*(d + e*x)),x)
[Out] int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(1/2)*(d + e*x)), x)
```

# 3.202 $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{3/2}}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 340

$$\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{4b\sqrt{e}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}}+\frac{2b\sqrt{e}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{3/2}}+\frac{2(a+b\log(c(d+ex)^n))}{(ef-dg)\sqrt{f+gx}}-\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{3/2}}-\frac{4b\sqrt{e}n\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{3/2}}-\frac{2b\sqrt{e}n\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{3/2}}$$

```
[Out] 4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*e^(1/2)/(-d*g+e*f)^(3
/2)+2*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2*e^(1/2)/(-d*g+e
*f)^(3/2)-2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+
d)^n))*e^(1/2)/(-d*g+e*f)^(3/2)-4*b*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e
*f)^(1/2))*ln(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))*e^(1/2)/(-d*g+e
*f)^(3/2)-2*b*n*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))*e
^(1/2)/(-d*g+e*f)^(3/2)+2*(a+b*ln(c*(e*x+d)^n))/(-d*g+e*f)/(g*x+f)^(1/2)
```

## Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {2458, 2389, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{3/2}}\,dx = -\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)^{3/2}} + \frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{\sqrt{f+gx}(ef-dg)} + \frac{2b\sqrt{e}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{3/2}} + \frac{4b\sqrt{e}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}} - \frac{4b\sqrt{e}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{3/2}} - \frac{2b\sqrt{e}n\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{3/2}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (4*b*Sqrt[e]*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e*f - d*g
)^(3/2) + (2*b*Sqrt[e]*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]^2
)/(e*f - d*g)^(3/2) + (2*(a + b*Log[c*(d + e*x)^n]))/((e*f - d*g)*Sqrt[f +
g*x]) - (2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*(a + b*
Log[c*(d + e*x)^n]))/(e*f - d*g)^(3/2) - (4*b*Sqrt[e]*n*ArcTanh[(Sqrt[e]*Sq
rt[f + g*x])/Sqrt[e*f - d*g]]*Log[2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f -
 d*g])])/(e*f - d*g)^(3/2) - (2*b*Sqrt[e]*n*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*
Sqrt[f + g*x])/Sqrt[e*f - d*g])])/(e*f - d*g)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{ef-dg} - \frac{g\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{e(ef-dg)}$$

$$= \frac{2(a+b\log\left(c(d+ex)^n\right))}{(ef-dg)\sqrt{f+gx}} - \frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log\left(c(d+ex)^n\right))}{(ef-dg)^{3/2}}$$

$$- \frac{(bn)\text{Subst}\left(\int -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}x}\,dx, x, d+ex\right)}{ef-dg}$$

$$- \frac{(2bn)\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{ef-dg}$$

$$= \frac{2(a+b\log(c(d+ex)^n))}{(ef-dg)\sqrt{f+gx}} - \frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{3/2}} + \frac{\left(2b\sqrt{e}n\right)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{x}\,dx,x,d+ex\right)}{(ef-dg)^{3/2}} - \frac{(4ben)\text{Subst}\left(\int\frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx,x,\sqrt{f+gx}\right)}{g(ef-dg)}$$

$$= \frac{4b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}} + \frac{2(a+b\log(c(d+ex)^n))}{(ef-dg)\sqrt{f+gx}} - \frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{3/2}} + \frac{\left(4be^{3/2}n\right)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{dg+e(-f+x^2)}\,dx,x,\sqrt{f+gx}\right)}{(ef-dg)^{3/2}}$$

$$= \frac{4b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}} + \frac{2(a+b\log(c(d+ex)^n))}{(ef-dg)\sqrt{f+gx}} - \frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{3/2}} + \frac{\left(4be^{3/2}n\right)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{-ef+dg+ex^2}\,dx,x,\sqrt{f+gx}\right)}{(ef-dg)^{3/2}}$$

$$= \frac{4b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}} + \frac{2b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{3/2}} + \frac{2(a+b\log(c(d+ex)^n))}{(ef-dg)\sqrt{f+gx}} - \frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{3/2}} - \frac{(4ben)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\,dx,x,\sqrt{f+gx}\right)}{(ef-dg)^2}$$

$$= \frac{4b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}} + \frac{2b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{3/2}} + \frac{2(a+b\log(c(d+ex)^n))}{(ef-dg)\sqrt{f+gx}} - \frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{3/2}} - \frac{4b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{3/2}} + \frac{(4ben)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\right)}{1-\frac{ex^2}{ef-dg}}\,dx, x, \sqrt{f+gx}\right)}{(ef-dg)^2}$$

$$= \frac{4b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}} + \frac{2b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{3/2}} + \frac{2(a+b\log(c(d+ex)^n))}{(ef-dg)\sqrt{f+gx}} - \frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{3/2}} - \frac{4b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{3/2}} - \frac{\left(4b\sqrt{e}n\right)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{3/2}}$$

$$= \frac{4b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}} + \frac{2b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{3/2}} + \frac{2(a+b\log(c(d+ex)^n))}{(ef-dg)\sqrt{f+gx}} - \frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{3/2}} - \frac{4b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{3/2}} - \frac{2b\sqrt{e}n\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{3/2}}$$

## Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.55

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{8b\sqrt{e}n\sqrt{f+gx}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)+4\sqrt{ef-dg}(a+b\log\left(c(d+ex)^n\right))+2\sqrt{e}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (8*b*Sqrt[e]*n*Sqrt[f + g*x]*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g
]] + 4*Sqrt[e*f - d*g]*(a + b*Log[c*(d + e*x)^n]) + 2*Sqrt[e]*Sqrt[f + g*x]
*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] -
2*Sqrt[e]*Sqrt[f + g*x]*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] + Sq
rt[e]*Sqrt[f + g*x]] - b*Sqrt[e]*n*Sqrt[f + g*x]*(Log[Sqrt[e*f - d*g] - Sqr
t[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]] + 2*Log[(
1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) + 2*PolyLog[2, 1/2 - (Sqrt
[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + b*Sqrt[e]*n*Sqrt[f + g*x]*(Log[S
qrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt
[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 2*
PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]))/(2*(e*f - d*g
)^(3/2)*Sqrt[f + g*x])
```

## Maple [F]

$$\int \frac{a+b\ln\left(c(ex+d)^n\right)}{(ex+d)\,(gx+f)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(3/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(3/2),x)
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{3/2}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(g*x + f)*b*log((e*x + d)^n*c) + sqrt(g*x + f)*a)/(e*g^2*x^3
 + d*f^2 + (2*e*f*g + d*g^2)*x^2 + (e*f^2 + 2*d*f*g)*x), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(e*x+d)/(g*x+f)**(3/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima
")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{3/2}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((e*x + d)*(g*x + f)^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{3/2}}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{{\left(f+g\,x\right)}^{3/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(3/2)*(d + e*x)),x)
[Out] int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(3/2)*(d + e*x)), x)
```

# 3.203 $\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx)^{5/2}}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 406

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx &= -\frac{4ben}{3(ef-dg)^2\sqrt{f+gx}}+\frac{16be^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3(ef-dg)^{5/2}} \\ &+\frac{2be^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{5/2}}+\frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{3(ef-dg)(f+gx)^{3/2}} \\ &+\frac{2e\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)^2\sqrt{f+gx}}-\frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)^{5/2}} \\ &-\frac{4be^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{5/2}} \\ &-\frac{2be^{3/2}n\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{5/2}}\end{aligned}$$

```
[Out] 16/3*b*e^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/(-d*g+e*f)
^(5/2)+2*b*e^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))^2/(-d*
g+e*f)^(5/2)+2/3*(a+b*ln(c*(e*x+d)^n))/(-d*g+e*f)/(g*x+f)^(3/2)-2*e^(3/2)*a
rctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*(a+b*ln(c*(e*x+d)^n))/(-d*g+
e*f)^(5/2)-4*b*e^(3/2)*n*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))*ln
(2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/(-d*g+e*f)^(5/2)-2*b*e^(3/2)
*n*polylog(2,1-2/(1-e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)))/(-d*g+e*f)^(5/
2)-4/3*b*e*n/(-d*g+e*f)^2/(g*x+f)^(1/2)+2*e*(a+b*ln(c*(e*x+d)^n))/(-d*g+e*f
)^2/(g*x+f)^(1/2)
```

## Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2458, 2389, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 53}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx = -\frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(ef-dg)^{5/2}} + \frac{2e\left(a+b\log\left(c(d+ex)^n\right)\right)}{\sqrt{f+gx}(ef-dg)^2} + \frac{2\left(a+b\log\left(c(d+ex)^n\right)\right)}{3(f+gx)^{3/2}(ef-dg)} + \frac{2be^{3/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{5/2}} + \frac{16be^{3/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3(ef-dg)^{5/2}} - \frac{4be^{3/2}n\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{5/2}} - \frac{2be^{3/2}n\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{5/2}} - \frac{4ben}{3\sqrt{f+gx}(ef-dg)^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/((d + e*x)*(f + g*x)^(5/2)),x]
```

```
[Out] (-4*b*e*n)/(3*(e*f - d*g)^2*Sqrt[f + g*x]) + (16*b*e^(3/2)*n*ArcTanh[(Sqrt[
e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(3*(e*f - d*g)^(5/2)) + (2*b*e^(3/2)*n*
ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]^2)/(e*f - d*g)^(5/2) + (2*
(a + b*Log[c*(d + e*x)^n]))/(3*(e*f - d*g)*(f + g*x)^(3/2)) + (2*e*(a + b*L
og[c*(d + e*x)^n]))/((e*f - d*g)^2*Sqrt[f + g*x]) - (2*e^(3/2)*ArcTanh[(Sqr
t[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*(a + b*Log[c*(d + e*x)^n]))/(e*f - d*g
)^(5/2) - (4*b*e^(3/2)*n*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]*L
og[2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])])/(e*f - d*g)^(5/2) - (2
*b*e^(3/2)*n*PolyLog[2, 1 - 2/(1 - (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])
])/(e*f - d*g)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
```

```
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{5/2}}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{ef-dg} - \frac{g\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{5/2}}\,dx, x, d+ex\right)}{e(ef-dg)}$$

$$= \frac{2(a+b\log(c(d+ex)^n))}{3(ef-dg)(f+gx)^{3/2}} + \frac{e\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{(ef-dg)^2}$$

$$- \frac{g\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{(ef-dg)^2}$$

$$- \frac{(2bn)\text{Subst}\left(\int \frac{1}{x\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)^{3/2}}\,dx, x, d+ex\right)}{3(ef-dg)}$$

$$= -\frac{4ben}{3(ef-dg)^2\sqrt{f+gx}} + \frac{2(a+b\log(c(d+ex)^n))}{3(ef-dg)(f+gx)^{3/2}} + \frac{2e(a+b\log(c(d+ex)^n))}{(ef-dg)^2\sqrt{f+gx}}$$

$$- \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{5/2}}$$

$$- \frac{(2ben)\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{3(ef-dg)^2}$$

$$- \frac{(ben)\text{Subst}\left(\int -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}x}\,dx, x, d+ex\right)}{(ef-dg)^2}$$

$$- \frac{(2ben)\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{ef-dg}{e}+\frac{gx}{e}}}\,dx, x, d+ex\right)}{(ef-dg)^2}$$

$$= -\frac{4ben}{3(ef-dg)^2\sqrt{f+gx}} + \frac{2(a+b\log(c(d+ex)^n))}{3(ef-dg)(f+gx)^{3/2}} + \frac{2e(a+b\log(c(d+ex)^n))}{(ef-dg)^2\sqrt{f+gx}}$$

$$- \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{5/2}}$$

$$+ \frac{\left(2be^{3/2}n\right)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}{\sqrt{ef-dg}}\right)}{x}\,dx, x, d+ex\right)}{(ef-dg)^{5/2}}$$

$$- \frac{\left(4be^2n\right)\text{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{3g(ef-dg)^2}$$

$$- \frac{\left(4be^2n\right)\text{Subst}\left(\int \frac{1}{-\frac{ef-dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{g(ef-dg)^2}$$

$$= -\frac{4ben}{3(ef-dg)^2\sqrt{f+gx}} + \frac{16be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3(ef-dg)^{5/2}} + \frac{2(a+b\log(c(d+ex)^n))}{3(ef-dg)(f+gx)^{3/2}} + \frac{2e(a+b\log(c(d+ex)^n))}{(ef-dg)^2\sqrt{f+gx}} - \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{5/2}} + \frac{(4be^{5/2}n)\,\mathrm{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{dg+e(-f+x^2)}\,dx, x, \sqrt{f+gx}\right)}{(ef-dg)^{5/2}}$$

$$= -\frac{4ben}{3(ef-dg)^2\sqrt{f+gx}} + \frac{16be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3(ef-dg)^{5/2}} + \frac{2(a+b\log(c(d+ex)^n))}{3(ef-dg)(f+gx)^{3/2}} + \frac{2e(a+b\log(c(d+ex)^n))}{(ef-dg)^2\sqrt{f+gx}} - \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{5/2}} + \frac{(4be^{5/2}n)\,\mathrm{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{-ef+dg+ex^2}\,dx, x, \sqrt{f+gx}\right)}{(ef-dg)^{5/2}}$$

$$= -\frac{4ben}{3(ef-dg)^2\sqrt{f+gx}} + \frac{16be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3(ef-dg)^{5/2}} + \frac{2be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{5/2}} + \frac{2(a+b\log(c(d+ex)^n))}{3(ef-dg)(f+gx)^{3/2}} + \frac{2e(a+b\log(c(d+ex)^n))}{(ef-dg)^2\sqrt{f+gx}} - \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{5/2}} - \frac{(4be^2n)\,\mathrm{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{ef-dg}}\right)}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\,dx, x, \sqrt{f+gx}\right)}{(ef-dg)^3}$$

$$= -\frac{4ben}{3(ef-dg)^2\sqrt{f+gx}} + \frac{16be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3(ef-dg)^{5/2}} + \frac{2be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{5/2}} + \frac{2(a+b\log(c(d+ex)^n))}{3(ef-dg)(f+gx)^{3/2}} + \frac{2e(a+b\log(c(d+ex)^n))}{(ef-dg)^2\sqrt{f+gx}} - \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{5/2}} - \frac{4be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{5/2}} + \frac{(4be^2n)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{e}x}{\sqrt{ef-dg}}}\right)}{1-\frac{ex^2}{ef-dg}}\,dx, x, \sqrt{f+gx}\right)}{(ef-dg)^3}$$

$$= -\frac{4ben}{3(ef-dg)^2\sqrt{f+gx}} + \frac{16be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3(ef-dg)^{5/2}} + \frac{2be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{5/2}} + \frac{2(a+b\log(c(d+ex)^n))}{3(ef-dg)(f+gx)^{3/2}} + \frac{2e(a+b\log(c(d+ex)^n))}{(ef-dg)^2\sqrt{f+gx}} - \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{5/2}} - \frac{4be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{5/2}} - \frac{(4be^{3/2}n)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{5/2}}$$

$$= -\frac{4ben}{3(ef-dg)^2\sqrt{f+gx}} + \frac{16be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{3(ef-dg)^{5/2}} + \frac{2be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)^2}{(ef-dg)^{5/2}} + \frac{2(a+b\log(c(d+ex)^n))}{3(ef-dg)(f+gx)^{3/2}} + \frac{2e(a+b\log(c(d+ex)^n))}{(ef-dg)^2\sqrt{f+gx}} - \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(a+b\log(c(d+ex)^n))}{(ef-dg)^{5/2}} - \frac{4be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\log\left(\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{5/2}} - \frac{2be^{3/2}n\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}}\right)}{(ef-dg)^{5/2}}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.51 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.50

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx = \frac{24be^{3/2}n(f+gx)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)-8be\sqrt{ef-dg}n(f+gx)\,\text{Hypergeom}}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/((d + e*x)*(f + g*x)^(5/2)),x]
```

```
[Out] (24*b*e^(3/2)*n*(f + g*x)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f -
d*g]] - 8*b*e*Sqrt[e*f - d*g]*n*(f + g*x)*Hypergeometric2F1[-1/2, 1, 1/2, (
e*(f + g*x))/(e*f - d*g)] + 4*(e*f - d*g)^(3/2)*(a + b*Log[c*(d + e*x)^n])
+ 12*e*Sqrt[e*f - d*g]*(f + g*x)*(a + b*Log[c*(d + e*x)^n]) + 6*e^(3/2)*(f
+ g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[
f + g*x]] - 6*e^(3/2)*(f + g*x)^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[Sqrt[e
*f - d*g] + Sqrt[e]*Sqrt[f + g*x]] - 3*b*e^(3/2)*n*(f + g*x)^(3/2)*(Log[Sqr
t[e*f - d*g] - Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f - d*g] - Sqrt[e]*Sqrt[f
 + g*x]] + 2*Log[(1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g])/2]) + 2*Poly
Log[2, 1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*Sqrt[e*f - d*g])]) + 3*b*e^(3/2)*n*
(f + g*x)^(3/2)*(Log[Sqrt[e*f - d*g] + Sqrt[e]*Sqrt[f + g*x]]*(Log[Sqrt[e*f
 - d*g] + Sqrt[e]*Sqrt[f + g*x]] + 2*Log[1/2 - (Sqrt[e]*Sqrt[f + g*x])/(2*S
qrt[e*f - d*g])]) + 2*PolyLog[2, (1 + (Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*
g])/2]))/(6*(e*f - d*g)^(5/2)*(f + g*x)^(3/2))
```

## Maple [F]

$$\int \frac{a+b\ln\left(c(ex+d)^n\right)}{(ex+d)\,(gx+f)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(5/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(5/2),x)
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(g*x + f)*b*log((e*x + d)^n*c) + sqrt(g*x + f)*a)/(e*g^3*x^4
+ d*f^3 + (3*e*f*g^2 + d*g^3)*x^3 + 3*(e*f^2*g + d*f*g^2)*x^2 + (e*f^3 + 3*
d*f^2*g)*x), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(e*x+d)/(g*x+f)**(5/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(5/2),x, algorithm="maxima
")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(e*x+d)/(g*x+f)^(5/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((e*x + d)*(g*x + f)^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(d+ex)(f+gx)^{5/2}}\,dx = \int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{(f+g\,x)^{5/2}\,(d+e\,x)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(5/2)*(d + e*x)),x)
[Out] int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(5/2)*(d + e*x)), x)
```

# 3.204 $\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 381

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx = -\frac{16(bd-ae)\sqrt{d+ex}}{3b^2} - \frac{4(d+ex)^{3/2}}{9b} + \frac{16(bd-ae)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3b^{5/2}} + \frac{2(bd-ae)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{5/2}} + \frac{2(bd-ae)\sqrt{d+ex}\log(a+bx)}{b^2} + \frac{2(d+ex)^{3/2}\log(a+bx)}{3b} - \frac{2(bd-ae)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{5/2}} - \frac{4(bd-ae)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{5/2}} - \frac{2(bd-ae)^{3/2}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{5/2}}$$

```
[Out] -4/9*(e*x+d)^(3/2)/b+16/3*(-a*e+b*d)^(3/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-
a*e+b*d)^(1/2))/b^(5/2)+2*(-a*e+b*d)^(3/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-
a*e+b*d)^(1/2))^2/b^(5/2)+2/3*(e*x+d)^(3/2)*ln(b*x+a)/b-2*(-a*e+b*d)^(3/2)*
arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(b*x+a)/b^(5/2)-4*(-a*e+b
*d)^(3/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(2/(1-b^(1/2)*(
e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))/b^(5/2)-2*(-a*e+b*d)^(3/2)*polylog(2,1-2/(1
-b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))/b^(5/2)-16/3*(-a*e+b*d)*(e*x+d)^(
1/2)/b^2+2*(-a*e+b*d)*ln(b*x+a)*(e*x+d)^(1/2)/b^2
```

## Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {2458, 2388, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 52}

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx = \frac{2(bd-ae)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{5/2}} + \frac{16(bd-ae)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3b^{5/2}} - \frac{2(bd-ae)^{3/2}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{5/2}} - \frac{4(bd-ae)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{5/2}} - \frac{2(bd-ae)^{3/2}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{5/2}} - \frac{16\sqrt{d+ex}(bd-ae)}{3b^2} + \frac{2\sqrt{d+ex}(bd-ae)\log(a+bx)}{b^2} + \frac{2(d+ex)^{3/2}\log(a+bx)}{3b} - \frac{4(d+ex)^{3/2}}{9b}$$

```
[In] Int[((d + e*x)^(3/2)*Log[a + b*x])/(a + b*x),x]
```

```
[Out] (-16*(b*d - a*e)*Sqrt[d + e*x])/(3*b^2) - (4*(d + e*x)^(3/2))/(9*b) + (16*(
b*d - a*e)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(3*b^(5/
2)) + (2*(b*d - a*e)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]
^2)/b^(5/2) + (2*(b*d - a*e)*Sqrt[d + e*x]*Log[a + b*x])/b^2 + (2*(d + e*x)
^(3/2)*Log[a + b*x])/(3*b) - (2*(b*d - a*e)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d +
 e*x])/Sqrt[b*d - a*e]]*Log[a + b*x])/b^(5/2) - (4*(b*d - a*e)^(3/2)*ArcTan
h[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]*Log[2/(1 - (Sqrt[b]*Sqrt[d + e*x
])/Sqrt[b*d - a*e])])/b^(5/2) - (2*(b*d - a*e)^(3/2)*PolyLog[2, 1 - 2/(1 -
(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])])/b^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
```

```
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{bd-ae}{b}+\frac{ex}{b}\right)^{3/2}\log(x)}{x}\,dx, x, a+bx\right)}{b}$$

$$= \frac{e\text{Subst}\left(\int \sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}\log(x)\,dx,x,a+bx\right)}{b^2} + \frac{(bd-ae)\text{Subst}\left(\int \frac{\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}\log(x)}{x}\,dx,x,a+bx\right)}{b^2}$$

$$= \frac{2(d+ex)^{3/2}\log(a+bx)}{3b} - \frac{2\text{Subst}\left(\int \frac{\left(\frac{bd-ae}{b}+\frac{ex}{b}\right)^{3/2}}{x}\,dx,x,a+bx\right)}{3b} + \frac{(e(bd-ae))\text{Subst}\left(\int \frac{\log(x)}{\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}\,dx,x,a+bx\right)}{b^3} + \frac{(bd-ae)^2\text{Subst}\left(\int \frac{\log(x)}{x\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}\,dx,x,a+bx\right)}{b^3}$$

$$= -\frac{4(d+ex)^{3/2}}{9b} + \frac{2(bd-ae)\sqrt{d+ex}\log(a+bx)}{b^2} + \frac{2(d+ex)^{3/2}\log(a+bx)}{3b} - \frac{2(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{5/2}} - \frac{(2(bd-ae))\text{Subst}\left(\int \frac{\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}{x}\,dx,x,a+bx\right)}{3b^2} - \frac{(2(bd-ae))\text{Subst}\left(\int \frac{\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}{x}\,dx,x,a+bx\right)}{b^2} - \frac{(bd-ae)^2\text{Subst}\left(\int -\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{ex}{b}}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}x}\,dx,x,a+bx\right)}{b^3}$$

$$= -\frac{16(bd-ae)\sqrt{d+ex}}{3b^2} - \frac{4(d+ex)^{3/2}}{9b} + \frac{2(bd-ae)\sqrt{d+ex}\log(a+bx)}{b^2} + \frac{2(d+ex)^{3/2}\log(a+bx)}{3b} - \frac{2(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{5/2}} + \frac{\left(2(bd-ae)^{3/2}\right)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{ex}{b}}}{\sqrt{bd-ae}}\right)}{x}\,dx, x, a+bx\right)}{b^{5/2}} - \frac{\left(2(bd-ae)^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}\,dx, x, a+bx\right)}{3b^3} - \frac{\left(2(bd-ae)^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}\,dx, x, a+bx\right)}{b^3}$$

$$= -\frac{16(bd-ae)\sqrt{d+ex}}{3b^2} - \frac{4(d+ex)^{3/2}}{9b} + \frac{2(bd-ae)\sqrt{d+ex}\log(a+bx)}{b^2} + \frac{2(d+ex)^{3/2}\log(a+bx)}{3b} - \frac{2(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{5/2}} + \frac{\left(4(bd-ae)^{3/2}\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{ae+b(-d+x^2)}\,dx, x, \sqrt{d+ex}\right)}{b^{3/2}} - \frac{\left(4(bd-ae)^2\right)\text{Subst}\left(\int \frac{1}{-\frac{bd-ae}{e}+\frac{bx^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{3b^2e} - \frac{\left(4(bd-ae)^2\right)\text{Subst}\left(\int \frac{1}{-\frac{bd-ae}{e}+\frac{bx^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{b^2e}$$

$$= -\frac{16(bd-ae)\sqrt{d+ex}}{3b^2} - \frac{4(d+ex)^{3/2}}{9b} + \frac{16(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3b^{5/2}} + \frac{2(bd-ae)\sqrt{d+ex}\log(a+bx)}{b^2} + \frac{2(d+ex)^{3/2}\log(a+bx)}{3b} - \frac{2(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{5/2}} + \frac{\left(4(bd-ae)^{3/2}\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{-bd+ae+bx^2}\,dx, x, \sqrt{d+ex}\right)}{b^{3/2}}$$

$$= -\frac{16(bd-ae)\sqrt{d+ex}}{3b^2} - \frac{4(d+ex)^{3/2}}{9b} + \frac{16(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3b^{5/2}} + \frac{2(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{5/2}} + \frac{2(bd-ae)\sqrt{d+ex}\log(a+bx)}{b^2} + \frac{2(d+ex)^{3/2}\log(a+bx)}{3b} - \frac{2(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{5/2}} - \frac{(4(bd-ae))\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{1-\frac{\sqrt{b}x}{\sqrt{bd-ae}}}\,dx, x, \sqrt{d+ex}\right)}{b^2}$$

$$= -\frac{16(bd-ae)\sqrt{d+ex}}{3b^2} - \frac{4(d+ex)^{3/2}}{9b} + \frac{16(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3b^{5/2}} + \frac{2(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{5/2}} + \frac{2(bd-ae)\sqrt{d+ex}\log(a+bx)}{b^2} + \frac{2(d+ex)^{3/2}\log(a+bx)}{3b} - \frac{2(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{5/2}} - \frac{4(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{5/2}} + \frac{(4(bd-ae))\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{b}x}{\sqrt{bd-ae}}}\right)}{1-\frac{bx^2}{bd-ae}}\,dx, x, \sqrt{d+ex}\right)}{b^2}$$

$$= -\frac{16(bd-ae)\sqrt{d+ex}}{3b^2} - \frac{4(d+ex)^{3/2}}{9b} + \frac{16(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3b^{5/2}} + \frac{2(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{5/2}} + \frac{2(bd-ae)\sqrt{d+ex}\log(a+bx)}{b^2} + \frac{2(d+ex)^{3/2}\log(a+bx)}{3b} - \frac{2(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{5/2}} - \frac{4(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{5/2}} - \frac{\left(4(bd-ae)^{3/2}\right)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{5/2}}$$

$$= -\frac{16(bd-ae)\sqrt{d+ex}}{3b^2} - \frac{4(d+ex)^{3/2}}{9b} + \frac{16(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3b^{5/2}}$$
$$+ \frac{2(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{5/2}} + \frac{2(bd-ae)\sqrt{d+ex}\log(a+bx)}{b^2}$$
$$+ \frac{2(d+ex)^{3/2}\log(a+bx)}{3b} - \frac{2(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{5/2}}$$
$$- \frac{4(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{5/2}}$$
$$- \frac{2(bd-ae)^{3/2}\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{5/2}}$$

## Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx = \frac{-72\sqrt{b}(bd-ae)\sqrt{d+ex} - 8\sqrt{b}\sqrt{d+ex}(4bd-3ae+bex) + 96(bd-ae)^{3/2}a}{}$$

```
[In] Integrate[((d + e*x)^(3/2)*Log[a + b*x])/(a + b*x),x]
```

```
[Out] (-72*Sqrt[b]*(b*d - a*e)*Sqrt[d + e*x] - 8*Sqrt[b]*Sqrt[d + e*x]*(4*b*d - 3
*a*e + b*e*x) + 96*(b*d - a*e)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b
*d - a*e]] + 36*Sqrt[b]*(b*d - a*e)*Sqrt[d + e*x]*Log[a + b*x] + 12*b^(3/2)
*(d + e*x)^(3/2)*Log[a + b*x] + 18*(b*d - a*e)^(3/2)*Log[a + b*x]*Log[Sqrt[
b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]] - 18*(b*d - a*e)^(3/2)*Log[a + b*x]*Log
[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]] - 9*(b*d - a*e)^(3/2)*(Log[Sqrt[b
*d - a*e] - Sqrt[b]*Sqrt[d + e*x]]*(Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d +
e*x]] + 2*Log[(1 + (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])/2]) + 2*PolyLog
[2, 1/2 - (Sqrt[b]*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e])]) + 9*(b*d - a*e)^(3/
2)*(Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]]*(Log[Sqrt[b*d - a*e] + Sqr
t[b]*Sqrt[d + e*x]] + 2*Log[1/2 - (Sqrt[b]*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e
])]) + 2*PolyLog[2, (1 + (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])/2]))/(18*
b^(5/2))
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.05 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2(ex+d)^{\frac{3}{2}}\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{3}-\frac{4b\left(-\frac{-\frac{b(ex+d)^{\frac{3}{2}}}{3}+\sqrt{ex+d}\,ae-\sqrt{ex+d}\,bd}{b^2}+\frac{\left(a^2e^2-2adeb+b^2d^2\right)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{b^2\sqrt{(ae-bd)b}}\right)}{3}}{b}$ — 2 |
| default | $\frac{\frac{2(ex+d)^{\frac{3}{2}}\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{3}-\frac{4b\left(-\frac{-\frac{b(ex+d)^{\frac{3}{2}}}{3}+\sqrt{ex+d}\,ae-\sqrt{ex+d}\,bd}{b^2}+\frac{\left(a^2e^2-2adeb+b^2d^2\right)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{b^2\sqrt{(ae-bd)b}}\right)}{3}}{b}$ — 2 |

```
[In] int((e*x+d)^(3/2)*ln(b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/3*(e*x+d)^(3/2)*ln(((e*x+d)*b+a*e-b*d)/e)-2/3*b*(-1/b^2*(-1/3*b*(e*x+d
)^(3/2)+(e*x+d)^(1/2)*a*e-(e*x+d)^(1/2)*b*d)+(a^2*e^2-2*a*b*d*e+b^2*d^2)/b^
2/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))))/b-2*((e
*x+d)^(1/2)*ln(((e*x+d)*b+a*e-b*d)/e)-2*b*((e*x+d)^(1/2)/b+(-a*e+b*d)/b/((a
*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))))*(a*e-b*d)/b^
2+2*Sum(1/2*(ln((e*x+d)^(1/2)-_alpha)*ln(((e*x+d)*b+a*e-b*d)/e)-2*b*(1/4/_a
lpha/b*ln((e*x+d)^(1/2)-_alpha)^2+1/2*_alpha/(a*e-b*d)*ln((e*x+d)^(1/2)-_al
pha)*ln(1/2*((e*x+d)^(1/2)+_alpha)/_alpha)+1/2*_alpha/(a*e-b*d)*dilog(1/2*(
(e*x+d)^(1/2)+_alpha)/_alpha)))*(a^2*e^2-2*a*b*d*e+b^2*d^2)/b^3/_alpha,_alp
ha=RootOf(_Z^2*b+a*e-b*d))
```

## Fricas [F]

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}\log(bx+a)}{bx+a}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*log(b*x+a)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^(3/2)*log(b*x + a)/(b*x + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)*ln(b*x+a)/(b*x+a),x)
```

```
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^(3/2)*log(b*x+a)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}\log(bx+a)}{bx+a}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*log(b*x+a)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*log(b*x + a)/(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\log(a+bx)}{a+bx}\,dx = \int \frac{\ln(a+b\,x)\,(d+e\,x)^{3/2}}{a+b\,x}\,dx$$

```
[In] int((log(a + b*x)*(d + e*x)^(3/2))/(a + b*x),x)
```

```
[Out] int((log(a + b*x)*(d + e*x)^(3/2))/(a + b*x), x)
```

# 3.205 $\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 323

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx = -\frac{4\sqrt{d+ex}}{b}+\frac{4\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} +\frac{2\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{3/2}}+\frac{2\sqrt{d+ex}\log(a+bx)}{b} -\frac{2\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{3/2}} -\frac{4\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{3/2}} -\frac{2\sqrt{bd-ae}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{3/2}}$$

```
[Out] 4*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*(-a*e+b*d)^(1/2)/b^(3/2)+
2*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))^2*(-a*e+b*d)^(1/2)/b^(3/2
)-2*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(b*x+a)*(-a*e+b*d)^(1
/2)/b^(3/2)-4*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(2/(1-b^(1/
2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))*(-a*e+b*d)^(1/2)/b^(3/2)-2*polylog(2,1-
2/(1-b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))*(-a*e+b*d)^(1/2)/b^(3/2)-4*(e
*x+d)^(1/2)/b+2*ln(b*x+a)*(e*x+d)^(1/2)/b
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {2458, 2388, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 52}

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx = \frac{2\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{3/2}} + \frac{4\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} - \frac{2\sqrt{bd-ae}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} - \frac{4\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{3/2}} - \frac{2\sqrt{bd-ae}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{3/2}} + \frac{2\sqrt{d+ex}\log(a+bx)}{b} - \frac{4\sqrt{d+ex}}{b}$$

```
[In] Int[(Sqrt[d + e*x]*Log[a + b*x])/(a + b*x),x]
```

```
[Out] (-4*Sqrt[d + e*x])/b + (4*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/S
qrt[b*d - a*e]])/b^(3/2) + (2*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x
])/Sqrt[b*d - a*e]]^2)/b^(3/2) + (2*Sqrt[d + e*x]*Log[a + b*x])/b - (2*Sqrt
[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]*Log[a + b*x])/
b^(3/2) - (4*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e
]]*Log[2/(1 - (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])])/b^(3/2) - (2*Sqrt[
b*d - a*e]*PolyLog[2, 1 - 2/(1 - (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])])
/b^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}\log(x)}{x}\,dx, x, a+bx\right)}{b}$$

$$= \frac{e\text{Subst}\left(\int \frac{\log(x)}{\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}\,dx, x, a+bx\right)}{b^2} + \frac{(bd-ae)\text{Subst}\left(\int \frac{\log(x)}{x\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}\,dx, x, a+bx\right)}{b^2}$$

$$=\frac{2\sqrt{d+ex}\log(a+bx)}{b}-\frac{2\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{3/2}}$$
$$-\frac{2\text{Subst}\left(\int\frac{\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}{x}\,dx,x,a+bx\right)}{b}$$
$$-\frac{(bd-ae)\text{Subst}\left(\int-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{ex}{b}}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}x}\,dx,x,a+bx\right)}{b^2}$$

$$=-\frac{4\sqrt{d+ex}}{b}+\frac{2\sqrt{d+ex}\log(a+bx)}{b}-\frac{2\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{3/2}}$$
$$+\frac{\left(2\sqrt{bd-ae}\right)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{ex}{b}}}{\sqrt{bd-ae}}\right)}{x}\,dx,x,a+bx\right)}{b^{3/2}}$$
$$-\frac{(2(bd-ae))\text{Subst}\left(\int\frac{1}{x\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}\,dx,x,a+bx\right)}{b^2}$$

$$=-\frac{4\sqrt{d+ex}}{b}+\frac{2\sqrt{d+ex}\log(a+bx)}{b}-\frac{2\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{3/2}}$$
$$+\frac{\left(4\sqrt{bd-ae}\right)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{ae+b(-d+x^2)}\,dx,x,\sqrt{d+ex}\right)}{\sqrt{b}}$$
$$-\frac{(4(bd-ae))\text{Subst}\left(\int\frac{1}{-\frac{bd-ae}{e}+\frac{bx^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{be}$$

$$=-\frac{4\sqrt{d+ex}}{b}+\frac{4\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}}$$
$$+\frac{2\sqrt{d+ex}\log(a+bx)}{b}-\frac{2\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{3/2}}$$
$$+\frac{\left(4\sqrt{bd-ae}\right)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{-bd+ae+bx^2}\,dx,x,\sqrt{d+ex}\right)}{\sqrt{b}}$$

$$=-\frac{4\sqrt{d+ex}}{b}+\frac{4\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}}+\frac{2\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{3/2}}+\frac{2\sqrt{d+ex}\log(a+bx)}{b}-\frac{2\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{3/2}}-\frac{4\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{1-\frac{\sqrt{b}x}{\sqrt{bd-ae}}}\,dx,x,\sqrt{d+ex}\right)}{b}$$

$$=-\frac{4\sqrt{d+ex}}{b}+\frac{4\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}}+\frac{2\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{3/2}}+\frac{2\sqrt{d+ex}\log(a+bx)}{b}-\frac{2\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{3/2}}-\frac{4\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{3/2}}+\frac{4\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{b}x}{\sqrt{bd-ae}}}\right)}{1-\frac{bx^2}{bd-ae}}\,dx,x,\sqrt{d+ex}\right)}{b}$$

$$=-\frac{4\sqrt{d+ex}}{b}+\frac{4\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}}+\frac{2\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{3/2}}+\frac{2\sqrt{d+ex}\log(a+bx)}{b}-\frac{2\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{3/2}}-\frac{4\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{3/2}}-\frac{\left(4\sqrt{bd-ae}\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{3/2}}$$

$$=-\frac{4\sqrt{d+ex}}{b}+\frac{4\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}}+\frac{2\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{b^{3/2}}+\frac{2\sqrt{d+ex}\log(a+bx)}{b}-\frac{2\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{b^{3/2}}-\frac{4\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{3/2}}-\frac{2\sqrt{bd-ae}\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{b^{3/2}}$$

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx$$
$$= \frac{-8\sqrt{b}\sqrt{d+ex}+8\sqrt{bd-ae}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)+4\sqrt{b}\sqrt{d+ex}\log(a+bx)+2\sqrt{bd-ae}\log(a+bx)\log\Big(}{}$$

```
[In] Integrate[(Sqrt[d + e*x]*Log[a + b*x])/(a + b*x),x]
```

```
[Out] (-8*Sqrt[b]*Sqrt[d + e*x] + 8*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x
])/Sqrt[b*d - a*e]] + 4*Sqrt[b]*Sqrt[d + e*x]*Log[a + b*x] + 2*Sqrt[b*d - a
*e]*Log[a + b*x]*Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]] - Sqrt[b*d -
a*e]*Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]]^2 - 2*Sqrt[b*d - a*e]*Log
[a + b*x]*Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]] + Sqrt[b*d - a*e]*Lo
g[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]]^2 + 2*Sqrt[b*d - a*e]*Log[Sqrt[b
*d - a*e] + Sqrt[b]*Sqrt[d + e*x]]*Log[1/2 - (Sqrt[b]*Sqrt[d + e*x])/(2*Sqr
t[b*d - a*e])] - 2*Sqrt[b*d - a*e]*Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e
*x]]*Log[(1 + (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])/2] - 2*Sqrt[b*d - a*
e]*PolyLog[2, 1/2 - (Sqrt[b]*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e])] + 2*Sqrt[b
*d - a*e]*PolyLog[2, (1 + (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])/2])/(2*b
^(3/2))
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| derivativedivides | $\frac{2\sqrt{ex+d}\,\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{b}-\frac{4\sqrt{ex+d}}{b}-\frac{4(-ae+bd)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{b\sqrt{(ae-bd)b}}+2\Bigg(\sum_{\_\alpha=\text{RootOf}(b\_Z^2+ae-bd)}$ |
| default | $\frac{2\sqrt{ex+d}\,\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{b}-\frac{4\sqrt{ex+d}}{b}-\frac{4(-ae+bd)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{b\sqrt{(ae-bd)b}}+2\Bigg(\sum_{\_\alpha=\text{RootOf}(b\_Z^2+ae-bd)}$ |

```
[In] int((e*x+d)^(1/2)*ln(b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(e*x+d)^(1/2)*ln(((e*x+d)*b+a*e-b*d)/e)/b-4*(e*x+d)^(1/2)/b-4*(-a*e+b*d)/
b/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))+2*Sum(1/2
```

```
*(ln((e*x+d)^(1/2)-_alpha)*ln(((e*x+d)*b+a*e-b*d)/e)-2*b*(1/4/_alpha/b*ln((
e*x+d)^(1/2)-_alpha)^2+1/2*_alpha/(a*e-b*d)*ln((e*x+d)^(1/2)-_alpha)*ln(1/2
*((e*x+d)^(1/2)+_alpha)/_alpha)+1/2*_alpha/(a*e-b*d)*dilog(1/2*((e*x+d)^(1/
2)+_alpha)/_alpha)))*(-a*e+b*d)/b^2/_alpha,_alpha=RootOf(_Z^2*b+a*e-b*d))
```

## Fricas [F]

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx = \int \frac{\sqrt{ex+d}\log\left(bx+a\right)}{bx+a}\,dx$$

```
[In] integrate((e*x+d)^(1/2)*log(b*x+a)/(b*x+a),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*log(b*x + a)/(b*x + a), x)
```

## Sympy [F]

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx = \int \frac{\sqrt{d+ex}\log\left(a+bx\right)}{a+bx}\,dx$$

```
[In] integrate((e*x+d)**(1/2)*ln(b*x+a)/(b*x+a),x)
[Out] Integral(sqrt(d + e*x)*log(a + b*x)/(a + b*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^(1/2)*log(b*x+a)/(b*x+a),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx = \int \frac{\sqrt{ex+d}\log(bx+a)}{bx+a}\,dx$$

```
[In] integrate((e*x+d)^(1/2)*log(b*x+a)/(b*x+a),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*log(b*x + a)/(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}\log(a+bx)}{a+bx}\,dx = \int \frac{\ln(a+b\,x)\,\sqrt{d+e\,x}}{a+b\,x}\,dx$$

```
[In] int((log(a + b*x)*(d + e*x)^(1/2))/(a + b*x),x)
[Out] int((log(a + b*x)*(d + e*x)^(1/2))/(a + b*x), x)
```

# 3.206 $\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 242

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{\sqrt{b}\sqrt{bd-ae}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{\sqrt{b}\sqrt{bd-ae}} - \frac{4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}\sqrt{bd-ae}} - \frac{2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}\sqrt{bd-ae}}$$

```
[Out] 2*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))^2/b^(1/2)/(-a*e+b*d)^(1/2
)-2*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(b*x+a)/b^(1/2)/(-a*e
+b*d)^(1/2)-4*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(2/(1-b^(1/
2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))/b^(1/2)/(-a*e+b*d)^(1/2)-2*polylog(2,1-
2/(1-b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))/b^(1/2)/(-a*e+b*d)^(1/2)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used

$= \{2458, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352\}$

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{\sqrt{b}\sqrt{bd-ae}} - \frac{2\log(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}\sqrt{bd-ae}} - \frac{4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}\sqrt{bd-ae}} - \frac{2\,\text{PolyLog}\left(2, 1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}\sqrt{bd-ae}}$$

```
[In] Int[Log[a + b*x]/((a + b*x)*Sqrt[d + e*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]^2)/(Sqrt[b]*Sqrt[b*d -
a*e]) - (2*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]*Log[a + b*x])/(
Sqrt[b]*Sqrt[b*d - a*e]) - (4*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*
e]]*Log[2/(1 - (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])])/(Sqrt[b]*Sqrt[b*d
 - a*e]) - (2*PolyLog[2, 1 - 2/(1 - (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]
)])/(Sqrt[b]*Sqrt[b*d - a*e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\log(x)}{x\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{\sqrt{b}\sqrt{bd-ae}} - \frac{\text{Subst}\left(\int -\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{ex}{b}}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}x}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{\sqrt{b}\sqrt{bd-ae}} + \frac{2\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{ex}{b}}}{\sqrt{bd-ae}}\right)}{x}\,dx, x, a+bx\right)}{\sqrt{b}\sqrt{bd-ae}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{\sqrt{b}\sqrt{bd-ae}} + \frac{\left(4\sqrt{b}\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{ae+b(-d+x^2)}\,dx, x, \sqrt{d+ex}\right)}{\sqrt{bd-ae}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{\sqrt{b}\sqrt{bd-ae}} + \frac{\left(4\sqrt{b}\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{-bd+ae+bx^2}\,dx, x, \sqrt{d+ex}\right)}{\sqrt{bd-ae}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{\sqrt{b}\sqrt{bd-ae}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{\sqrt{b}\sqrt{bd-ae}} - \frac{4\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{1-\frac{\sqrt{b}x}{\sqrt{bd-ae}}}\,dx, x, \sqrt{d+ex}\right)}{bd-ae}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{\sqrt{b}\sqrt{bd-ae}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{\sqrt{b}\sqrt{bd-ae}} - \frac{4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}\sqrt{bd-ae}} + \frac{4\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{b}x}{\sqrt{bd-ae}}}\right)}{1-\frac{bx^2}{bd-ae}}\,dx, x, \sqrt{d+ex}\right)}{bd-ae}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{\sqrt{b}\sqrt{bd-ae}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{\sqrt{b}\sqrt{bd-ae}} - \frac{4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}\sqrt{bd-ae}} - \frac{4\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}\sqrt{bd-ae}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{\sqrt{b}\sqrt{bd-ae}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{\sqrt{b}\sqrt{bd-ae}} - \frac{4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}\sqrt{bd-ae}} - \frac{2\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{\sqrt{b}\sqrt{bd-ae}}$$

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.52

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx$$
$$= \frac{2\log(a+bx)\log\left(\sqrt{bd-ae}-\sqrt{b}\sqrt{d+ex}\right)-\log^2\left(\sqrt{bd-ae}-\sqrt{b}\sqrt{d+ex}\right)-2\log(a+bx)\log\left(\sqrt{bd-\right.}{}$$

```
[In] Integrate[Log[a + b*x]/((a + b*x)*Sqrt[d + e*x]),x]
```

```
[Out] (2*Log[a + b*x]*Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]] - Log[Sqrt[b*d
 - a*e] - Sqrt[b]*Sqrt[d + e*x]]^2 - 2*Log[a + b*x]*Log[Sqrt[b*d - a*e] + S
qrt[b]*Sqrt[d + e*x]] + Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]]^2 + 2*
Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]]*Log[1/2 - (Sqrt[b]*Sqrt[d + e*
x])/(2*Sqrt[b*d - a*e])] - 2*Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]]*L
og[(1 + (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])/2] - 2*PolyLog[2, 1/2 - (S
qrt[b]*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e])] + 2*PolyLog[2, (1 + (Sqrt[b]*Sqr
t[d + e*x])/Sqrt[b*d - a*e])/2])/(2*Sqrt[b]*Sqrt[b*d - a*e])
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.02 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.64

| method | result |
|---|---|
| derivativedivides | $\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^2+ae-bd\right)} \frac{2\ln\left(\sqrt{ex+d}-\_\alpha\right)\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)-b\left(\frac{\ln\left(\sqrt{ex+d}-\_\alpha\right)^2}{\_\alpha b}+\frac{2\_\alpha\ln\left(\sqrt{ex+d}-\_\alpha\right)\ln\left(\frac{\sqrt{ex+d}+\_\alpha}{2\_\alpha}\right.}{ae-bd}\right.}{\_\alpha}}{2b}$ |
| default | $\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^2+ae-bd\right)} \frac{2\ln\left(\sqrt{ex+d}-\_\alpha\right)\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)-b\left(\frac{\ln\left(\sqrt{ex+d}-\_\alpha\right)^2}{\_\alpha b}+\frac{2\_\alpha\ln\left(\sqrt{ex+d}-\_\alpha\right)\ln\left(\frac{\sqrt{ex+d}+\_\alpha}{2\_\alpha}\right.}{ae-bd}\right.}{\_\alpha}}{2b}$ |

```
[In] int(ln(b*x+a)/(b*x+a)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*sum(1/_alpha*(2*ln((e*x+d)^(1/2)-_alpha)*ln(((e*x+d)*b+a*e-b*d)/e)-b*
(1/_alpha/b*ln((e*x+d)^(1/2)-_alpha)^2+2*_alpha/(a*e-b*d)*ln((e*x+d)^(1/2)-
_alpha)*ln(1/2*((e*x+d)^(1/2)+_alpha)/_alpha)+2*_alpha/(a*e-b*d)*dilog(1/2*
((e*x+d)^(1/2)+_alpha)/_alpha)),_alpha=RootOf(_Z^2*b+a*e-b*d))
```

## Fricas [F]

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx = \int \frac{\log(bx+a)}{(bx+a)\sqrt{ex+d}}\,dx$$

```
[In] integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*log(b*x + a)/(b*e*x^2 + a*d + (b*d + a*e)*x), x)
```

## Sympy [F]

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx = \int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx$$

```
[In] integrate(ln(b*x+a)/(b*x+a)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(log(a + b*x)/((a + b*x)*sqrt(d + e*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx = \int \frac{\log(bx+a)}{(bx+a)\sqrt{ex+d}}\,dx$$

```
[In] integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(log(b*x + a)/((b*x + a)*sqrt(e*x + d)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)}{(a+bx)\sqrt{d+ex}}\,dx = \int \frac{\ln(a+b\,x)}{(a+b\,x)\,\sqrt{d+e\,x}}\,dx$$

```
[In] int(log(a + b*x)/((a + b*x)*(d + e*x)^(1/2)),x)
```

```
[Out] int(log(a + b*x)/((a + b*x)*(d + e*x)^(1/2)), x)
```

# 3.207 $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 316

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = \frac{4\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} + \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{3/2}} + \frac{2\log(a+bx)}{(bd-ae)\sqrt{d+ex}} - \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{3/2}} - \frac{4\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{3/2}} - \frac{2\sqrt{b}\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{3/2}}$$

```
[Out] 4*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*b^(1/2)/(-a*e+b*d)^(3/2)+
2*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))^2*b^(1/2)/(-a*e+b*d)^(3/2
)-2*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(b*x+a)*b^(1/2)/(-a*e
+b*d)^(3/2)-4*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(2/(1-b^(1/
2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))*b^(1/2)/(-a*e+b*d)^(3/2)-2*polylog(2,1-
2/(1-b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)))*b^(1/2)/(-a*e+b*d)^(3/2)+2*ln
(b*x+a)/(-a*e+b*d)/(e*x+d)^(1/2)
```

## Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2458, 2389, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356}

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = \frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{3/2}}+\frac{4\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} - \frac{2\sqrt{b}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} - \frac{4\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{3/2}} - \frac{2\sqrt{b}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{3/2}}+\frac{2\log(a+bx)}{\sqrt{d+ex}(bd-ae)}$$

```
[In] Int[Log[a + b*x]/((a + b*x)*(d + e*x)^(3/2)),x]
```

```
[Out] (4*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(3
/2) + (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]^2)/(b*d -
 a*e)^(3/2) + (2*Log[a + b*x])/((b*d - a*e)*Sqrt[d + e*x]) - (2*Sqrt[b]*Arc
Tanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]*Log[a + b*x])/(b*d - a*e)^(3/
2) - (4*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]*Log[2/(1 -
 (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])])/(b*d - a*e)^(3/2) - (2*Sqrt[b]*
PolyLog[2, 1 - 2/(1 - (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])])/(b*d - a*e
)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bd-ae}{b}+\frac{ex}{b}\right)^{3/2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(x)}{x\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}\,dx, x, a+bx\right)}{bd-ae} - \frac{e\text{Subst}\left(\int \frac{\log(x)}{\left(\frac{bd-ae}{b}+\frac{ex}{b}\right)^{3/2}}\,dx, x, a+bx\right)}{b(bd-ae)}$$

$$= \frac{2\log(a+bx)}{(bd-ae)\sqrt{d+ex}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{3/2}} - \frac{\text{Subst}\left(\int -\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{ex}{b}}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}x}\,dx, x, a+bx\right)}{bd-ae} - \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}\,dx, x, a+bx\right)}{bd-ae}$$

$$= \frac{2\log(a+bx)}{(bd-ae)\sqrt{d+ex}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{3/2}} + \frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{ex}{b}}}{\sqrt{bd-ae}}\right)}{x}\,dx, x, a+bx\right)}{(bd-ae)^{3/2}} - \frac{(4b)\text{Subst}\left(\int \frac{1}{-\frac{bd-ae}{e}+\frac{bx^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{e(bd-ae)}$$

$$= \frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} + \frac{2\log(a+bx)}{(bd-ae)\sqrt{d+ex}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{3/2}} + \frac{\left(4b^{3/2}\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{ae+b(-d+x^2)}\,dx, x, \sqrt{d+ex}\right)}{(bd-ae)^{3/2}}$$

$$= \frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} + \frac{2\log(a+bx)}{(bd-ae)\sqrt{d+ex}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{3/2}} + \frac{\left(4b^{3/2}\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{-bd+ae+bx^2}\,dx, x, \sqrt{d+ex}\right)}{(bd-ae)^{3/2}}$$

$$= \frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} + \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{3/2}} + \frac{2\log(a+bx)}{(bd-ae)\sqrt{d+ex}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{3/2}} - \frac{(4b)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{1-\frac{\sqrt{b}x}{\sqrt{bd-ae}}}\,dx, x, \sqrt{d+ex}\right)}{(bd-ae)^2}$$

$$= \frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} + \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{3/2}} + \frac{2\log(a+bx)}{(bd-ae)\sqrt{d+ex}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{3/2}} - \frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{3/2}} + \frac{(4b)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{b}x}{\sqrt{bd-ae}}}\right)}{1-\frac{bx^2}{bd-ae}}\,dx, x, \sqrt{d+ex}\right)}{(bd-ae)^2}$$

$$= \frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} + \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{3/2}} + \frac{2\log(a+bx)}{(bd-ae)\sqrt{d+ex}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{3/2}} - \frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{3/2}} - \frac{\left(4\sqrt{b}\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{3/2}}$$

$$= \frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} + \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{3/2}} + \frac{2\log(a+bx)}{(bd-ae)\sqrt{d+ex}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{3/2}} - \frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{3/2}} - \frac{2\sqrt{b}\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{3/2}}$$

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.73

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = 2\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} + \frac{\log(a+bx)}{(bd-ae)\sqrt{d+ex}} + \frac{\sqrt{b}\log(a+bx)\log\left(\sqrt{bd-ae}-\sqrt{b}\sqrt{d+ex}\right)}{2(bd-ae)^{3/2}} - \frac{\sqrt{b}\log(a+bx)\log\left(\sqrt{bd-ae}+\sqrt{b}\sqrt{d+ex}\right)}{2(bd-ae)^{3/2}} - \frac{\sqrt{b}\left(\log^2\left(\sqrt{bd-ae}-\sqrt{b}\sqrt{d+ex}\right)+2\log\left(\sqrt{bd-ae}-\sqrt{b}\sqrt{d+ex}\right)\log\left(\frac{\sqrt{bd-ae}+\sqrt{b}\sqrt{d+ex}}{2\sqrt{bd-ae}}\right)+2\,\text{PolyLog}\left(\right.\right.}{4(bd-ae)^{3/2}} + \frac{\sqrt{b}\left(2\log\left(\frac{\sqrt{bd-ae}-\sqrt{b}\sqrt{d+ex}}{2\sqrt{bd-ae}}\right)\log\left(\sqrt{bd-ae}+\sqrt{b}\sqrt{d+ex}\right)+\log^2\left(\sqrt{bd-ae}+\sqrt{b}\sqrt{d+ex}\right)+2\,\text{PolyLog}\left(\right.\right.}{4(bd-ae)^{3/2}}$$

```
[In] Integrate[Log[a + b*x]/((a + b*x)*(d + e*x)^(3/2)),x]
```

```
[Out] 2*((2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)
^(3/2) + Log[a + b*x]/((b*d - a*e)*Sqrt[d + e*x]) + (Sqrt[b]*Log[a + b*x]*L
og[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]])/(2*(b*d - a*e)^(3/2)) - (Sqrt[
```

```
b]*Log[a + b*x]*Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]])/(2*(b*d - a*e
)^(3/2)) - (Sqrt[b]*(Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]]^2 + 2*Log
[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]]*Log[(Sqrt[b*d - a*e] + Sqrt[b]*Sq
rt[d + e*x])/(2*Sqrt[b*d - a*e])] + 2*PolyLog[2, (Sqrt[b*d - a*e] - Sqrt[b]
*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e])]))/(4*(b*d - a*e)^(3/2)) + (Sqrt[b]*(2*
Log[(Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e])]*Log[Sqrt
[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]] + Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d
 + e*x]]^2 + 2*PolyLog[2, (Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x])/(2*Sqrt
[b*d - a*e])]))/(4*(b*d - a*e)^(3/2)))
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.77

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{\sqrt{ex+d}}+\frac{4b\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{\sqrt{(ae-bd)b}}}{ae-bd}+2\left(\sum_{\_\alpha=\mathrm{RootOf}(b\_Z^2+ae-bd)}\left(-\frac{\ln(\sqrt{ex+d}-\_\alpha)\ln\left(\frac{(ex+d)}{}\right.}{}\right.\right.$ |
| default | $\frac{-\frac{2\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{\sqrt{ex+d}}+\frac{4b\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{\sqrt{(ae-bd)b}}}{ae-bd}+2\left(\sum_{\_\alpha=\mathrm{RootOf}(b\_Z^2+ae-bd)}\left(-\frac{\ln(\sqrt{ex+d}-\_\alpha)\ln\left(\frac{(ex+d)}{}\right.}{}\right.\right.$ |

```
[In] int(ln(b*x+a)/(b*x+a)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/(e*x+d)^(1/2)*ln(((e*x+d)*b+a*e-b*d)/e)+2*b/((a*e-b*d)*b)^(1/2)*arcta
n(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2)))/(a*e-b*d)+2*Sum(-1/2*(ln((e*x+d)^(1
/2)-_alpha)*ln(((e*x+d)*b+a*e-b*d)/e)-2*b*(1/4/_alpha/b*ln((e*x+d)^(1/2)-_a
lpha)^2+1/2*_alpha/(a*e-b*d)*ln((e*x+d)^(1/2)-_alpha)*ln(1/2*((e*x+d)^(1/2)
+_alpha)/_alpha)+1/2*_alpha/(a*e-b*d)*dilog(1/2*((e*x+d)^(1/2)+_alpha)/_alp
ha)))/(a*e-b*d)/_alpha,_alpha=RootOf(_Z^2*b+a*e-b*d))
```

## Fricas [F]

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx=\int \frac{\log\left(bx+a\right)}{(bx+a)(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*log(b*x + a)/(b*e^2*x^3 + a*d^2 + (2*b*d*e + a*e^2)*
x^2 + (b*d^2 + 2*a*d*e)*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(ln(b*x+a)/(b*x+a)/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = \int \frac{\log(bx+a)}{(bx+a)(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(log(b*x + a)/((b*x + a)*(e*x + d)^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{3/2}}\,dx = \int \frac{\ln(a+b\,x)}{(a+b\,x)\,(d+e\,x)^{3/2}}\,dx$$

```
[In] int(log(a + b*x)/((a + b*x)*(d + e*x)^(3/2)),x)
```

```
[Out] int(log(a + b*x)/((a + b*x)*(d + e*x)^(3/2)), x)
```

# 3.208 $\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 372

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx = -\frac{4b}{3(bd-ae)^2\sqrt{d+ex}}+\frac{16b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3(bd-ae)^{5/2}} + \frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{5/2}}+\frac{2\log(a+bx)}{3(bd-ae)(d+ex)^{3/2}} + \frac{2b\log(a+bx)}{(bd-ae)^2\sqrt{d+ex}}-\frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{5/2}} - \frac{4b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{5/2}}-\frac{2b^{3/2}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{5/2}}$$

```
[Out] 16/3*b^(3/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(5/
2)+2*b^(3/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))^2/(-a*e+b*d)^(
5/2)+2/3*ln(b*x+a)/(-a*e+b*d)/(e*x+d)^(3/2)-2*b^(3/2)*arctanh(b^(1/2)*(e*x+
d)^(1/2)/(-a*e+b*d)^(1/2))*ln(b*x+a)/(-a*e+b*d)^(5/2)-4*b^(3/2)*arctanh(b^(
1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))*ln(2/(1-b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*
d)^(1/2)))/(-a*e+b*d)^(5/2)-2*b^(3/2)*polylog(2,1-2/(1-b^(1/2)*(e*x+d)^(1/2
)/(-a*e+b*d)^(1/2)))/(-a*e+b*d)^(5/2)-4/3*b/(-a*e+b*d)^2/(e*x+d)^(1/2)+2*b*
ln(b*x+a)/(-a*e+b*d)^2/(e*x+d)^(1/2)
```

## Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {2458, 2389, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 53}

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx = \frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{5/2}} + \frac{16b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3(bd-ae)^{5/2}} - \frac{2b^{3/2}\log(a+bx)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{5/2}} - \frac{4b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{5/2}} - \frac{2b^{3/2}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{5/2}} - \frac{4b}{3\sqrt{d+ex}(bd-ae)^2} + \frac{2b\log(a+bx)}{\sqrt{d+ex}(bd-ae)^2} + \frac{2\log(a+bx)}{3(d+ex)^{3/2}(bd-ae)}$$

```
[In] Int[Log[a + b*x]/((a + b*x)*(d + e*x)^(5/2)),x]
```

```
[Out] (-4*b)/(3*(b*d - a*e)^2*Sqrt[d + e*x]) + (16*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[
d + e*x])/Sqrt[b*d - a*e]])/(3*(b*d - a*e)^(5/2)) + (2*b^(3/2)*ArcTanh[(Sqr
t[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]^2)/(b*d - a*e)^(5/2) + (2*Log[a + b*x]
)/(3*(b*d - a*e)*(d + e*x)^(3/2)) + (2*b*Log[a + b*x])/((b*d - a*e)^2*Sqrt[
d + e*x]) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]*Log
[a + b*x])/(b*d - a*e)^(5/2) - (4*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/S
qrt[b*d - a*e]]*Log[2/(1 - (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])])/(b*d
- a*e)^(5/2) - (2*b^(3/2)*PolyLog[2, 1 - 2/(1 - (Sqrt[b]*Sqrt[d + e*x])/Sqr
t[b*d - a*e])])/(b*d - a*e)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bd-ae}{b}+\frac{ex}{b}\right)^{5/2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bd-ae}{b}+\frac{ex}{b}\right)^{3/2}}\,dx, x, a+bx\right)}{bd-ae} - \frac{e\text{Subst}\left(\int \frac{\log(x)}{\left(\frac{bd-ae}{b}+\frac{ex}{b}\right)^{5/2}}\,dx, x, a+bx\right)}{b(bd-ae)}$$

$$= \frac{2\log(a+bx)}{3(bd-ae)(d+ex)^{3/2}} + \frac{b\text{Subst}\left(\int \frac{\log(x)}{x\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}\,dx, x, a+bx\right)}{(bd-ae)^2}$$
$$- \frac{e\text{Subst}\left(\int \frac{\log(x)}{\left(\frac{bd-ae}{b}+\frac{ex}{b}\right)^{3/2}}\,dx, x, a+bx\right)}{(bd-ae)^2}$$
$$- \frac{2\text{Subst}\left(\int \frac{1}{x\left(\frac{bd-ae}{b}+\frac{ex}{b}\right)^{3/2}}\,dx, x, a+bx\right)}{3(bd-ae)}$$

$$= -\frac{4b}{3(bd-ae)^2\sqrt{d+ex}} + \frac{2\log(a+bx)}{3(bd-ae)(d+ex)^{3/2}} + \frac{2b\log(a+bx)}{(bd-ae)^2\sqrt{d+ex}}$$
$$- \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{5/2}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}\,dx, x, a+bx\right)}{3(bd-ae)^2}$$
$$- \frac{b\text{Subst}\left(\int -\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{ex}{b}}}{\sqrt{bd-ae}}\right)}{\sqrt{bd-ae}x}\,dx, x, a+bx\right)}{(bd-ae)^2}$$
$$- \frac{(2b)\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{bd-ae}{b}+\frac{ex}{b}}}\,dx, x, a+bx\right)}{(bd-ae)^2}$$

$$= -\frac{4b}{3(bd-ae)^2\sqrt{d+ex}} + \frac{2\log(a+bx)}{3(bd-ae)(d+ex)^{3/2}}$$
$$+ \frac{2b\log(a+bx)}{(bd-ae)^2\sqrt{d+ex}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{5/2}}$$
$$+ \frac{\left(2b^{3/2}\right)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d-\frac{ae}{b}+\frac{ex}{b}}}{\sqrt{bd-ae}}\right)}{x}\,dx, x, a+bx\right)}{(bd-ae)^{5/2}}$$
$$- \frac{\left(4b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{bd-ae}{e}+\frac{bx^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{3e(bd-ae)^2}$$
$$- \frac{\left(4b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{bd-ae}{e}+\frac{bx^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{e(bd-ae)^2}$$

$$= -\frac{4b}{3(bd-ae)^2\sqrt{d+ex}} + \frac{16b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3(bd-ae)^{5/2}} + \frac{2\log(a+bx)}{3(bd-ae)(d+ex)^{3/2}} + \frac{2b\log(a+bx)}{(bd-ae)^2\sqrt{d+ex}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{5/2}} + \frac{(4b^{5/2})\,\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{ae+b(-d+x^2)}\,dx, x, \sqrt{d+ex}\right)}{(bd-ae)^{5/2}}$$

$$= -\frac{4b}{3(bd-ae)^2\sqrt{d+ex}} + \frac{16b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3(bd-ae)^{5/2}} + \frac{2\log(a+bx)}{3(bd-ae)(d+ex)^{3/2}} + \frac{2b\log(a+bx)}{(bd-ae)^2\sqrt{d+ex}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{5/2}} + \frac{(4b^{5/2})\,\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{-bd+ae+bx^2}\,dx, x, \sqrt{d+ex}\right)}{(bd-ae)^{5/2}}$$

$$= -\frac{4b}{3(bd-ae)^2\sqrt{d+ex}} + \frac{16b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3(bd-ae)^{5/2}} + \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{5/2}} + \frac{2\log(a+bx)}{3(bd-ae)(d+ex)^{3/2}} + \frac{2b\log(a+bx)}{(bd-ae)^2\sqrt{d+ex}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{5/2}} - \frac{(4b^2)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bd-ae}}\right)}{1-\frac{\sqrt{b}x}{\sqrt{bd-ae}}}\,dx, x, \sqrt{d+ex}\right)}{(bd-ae)^3}$$

$$= -\frac{4b}{3(bd-ae)^2\sqrt{d+ex}} + \frac{16b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3(bd-ae)^{5/2}} + \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{5/2}} + \frac{2\log(a+bx)}{3(bd-ae)(d+ex)^{3/2}} + \frac{2b\log(a+bx)}{(bd-ae)^2\sqrt{d+ex}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{5/2}} - \frac{4b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{5/2}} + \frac{(4b^2)\operatorname{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{b}x}{\sqrt{bd-ae}}}\right)}{1-\frac{bx^2}{bd-ae}}\,dx, x, \sqrt{d+ex}\right)}{(bd-ae)^3}$$

$$= -\frac{4b}{3(bd-ae)^2\sqrt{d+ex}} + \frac{16b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3(bd-ae)^{5/2}} + \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{5/2}} + \frac{2\log(a+bx)}{3(bd-ae)(d+ex)^{3/2}} + \frac{2b\log(a+bx)}{(bd-ae)^2\sqrt{d+ex}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{5/2}} - \frac{4b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{5/2}} - \frac{(4b^{3/2})\operatorname{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{5/2}}$$

$$= -\frac{4b}{3(bd-ae)^2\sqrt{d+ex}} + \frac{16b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{3(bd-ae)^{5/2}} + \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)^2}{(bd-ae)^{5/2}} + \frac{2\log(a+bx)}{3(bd-ae)(d+ex)^{3/2}} + \frac{2b\log(a+bx)}{(bd-ae)^2\sqrt{d+ex}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log(a+bx)}{(bd-ae)^{5/2}} - \frac{4b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{5/2}} - \frac{2b^{3/2}\operatorname{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}}\right)}{(bd-ae)^{5/2}}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.39 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.53

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx = \frac{24b^{3/2}(d+ex)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)-8b\sqrt{bd-ae}(d+ex)\,\text{Hypergeometric2F1}}{}$$

```
[In] Integrate[Log[a + b*x]/((a + b*x)*(d + e*x)^(5/2)),x]
```

```
[Out] (24*b^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]
] - 8*b*Sqrt[b*d - a*e]*(d + e*x)*Hypergeometric2F1[-1/2, 1, 1/2, (b*(d + e
*x))/(b*d - a*e)] + 4*(b*d - a*e)^(3/2)*Log[a + b*x] + 12*b*Sqrt[b*d - a*e]
*(d + e*x)*Log[a + b*x] + 6*b^(3/2)*(d + e*x)^(3/2)*Log[a + b*x]*Log[Sqrt[b
*d - a*e] - Sqrt[b]*Sqrt[d + e*x]] - 6*b^(3/2)*(d + e*x)^(3/2)*Log[a + b*x]
*Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]] - 3*b^(3/2)*(d + e*x)^(3/2)*(
Log[Sqrt[b*d - a*e] - Sqrt[b]*Sqrt[d + e*x]]*(Log[Sqrt[b*d - a*e] - Sqrt[b]
*Sqrt[d + e*x]] + 2*Log[(1 + (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e])/2]) +
 2*PolyLog[2, 1/2 - (Sqrt[b]*Sqrt[d + e*x])/(2*Sqrt[b*d - a*e])]) + 3*b^(3/
2)*(d + e*x)^(3/2)*(Log[Sqrt[b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]]*(Log[Sqrt[
b*d - a*e] + Sqrt[b]*Sqrt[d + e*x]] + 2*Log[1/2 - (Sqrt[b]*Sqrt[d + e*x])/(
2*Sqrt[b*d - a*e])]) + 2*PolyLog[2, (1 + (Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d -
 a*e])/2]))/(6*(b*d - a*e)^(5/2)*(d + e*x)^(3/2))
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.93 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| derivativedivides | $-\frac{2\left(-\frac{\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{\sqrt{ex+d}}+\frac{2b\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{\sqrt{(ae-bd)b}}\right)b}{a^2e^2-2adeb+b^2d^2}+2\left(\sum_{\_\alpha=\operatorname{RootOf}(b\_Z^2+ae-bd)}\frac{\left(\ln(\sqrt{ex+d}-\_\alpha)\ln\left(\frac{(ex+}{}\right.\right.}{}\right.$ |
| default | $-\frac{2\left(-\frac{\ln\left(\frac{(ex+d)b+ae-bd}{e}\right)}{\sqrt{ex+d}}+\frac{2b\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)}{\sqrt{(ae-bd)b}}\right)b}{a^2e^2-2adeb+b^2d^2}+2\left(\sum_{\_\alpha=\operatorname{RootOf}(b\_Z^2+ae-bd)}\frac{\left(\ln(\sqrt{ex+d}-\_\alpha)\ln\left(\frac{(ex+}{}\right.\right.}{}\right.$ |

```
[In] int(ln(b*x+a)/(b*x+a)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/(e*x+d)^(1/2)*ln(((e*x+d)*b+a*e-b*d)/e)+2*b/((a*e-b*d)*b)^(1/2)*arct
an(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2)))*b/(a^2*e^2-2*a*b*d*e+b^2*d^2)+2*Su
m(1/2*(ln((e*x+d)^(1/2)-_alpha)*ln(((e*x+d)*b+a*e-b*d)/e)-2*b*(1/4/_alpha/b
*ln((e*x+d)^(1/2)-_alpha)^2+1/2*_alpha/(a*e-b*d)*ln((e*x+d)^(1/2)-_alpha)*l
n(1/2*((e*x+d)^(1/2)+_alpha)/_alpha)+1/2*_alpha/(a*e-b*d)*dilog(1/2*((e*x+d
)^(1/2)+_alpha)/_alpha)))*b/(a*e-b*d)^2/_alpha,_alpha=RootOf(_Z^2*b+a*e-b*d
))+2*(-1/3/(e*x+d)^(3/2)*ln(((e*x+d)*b+a*e-b*d)/e)+2/3*b*(-1/(a*e-b*d)/(e*x
+d)^(1/2)-1/(a*e-b*d)*b/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*
d)*b)^(1/2))))/(a*e-b*d)
```

## Fricas [F]

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx=\int \frac{\log\left(bx+a\right)}{(bx+a)(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*log(b*x + a)/(b*e^3*x^4 + a*d^3 + (3*b*d*e^2 + a*e^3
)*x^3 + 3*(b*d^2*e + a*d*e^2)*x^2 + (b*d^3 + 3*a*d^2*e)*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(ln(b*x+a)/(b*x+a)/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx = \int \frac{\log(bx+a)}{(bx+a)(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(log(b*x+a)/(b*x+a)/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate(log(b*x + a)/((b*x + a)*(e*x + d)^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)}{(a+bx)(d+ex)^{5/2}}\,dx = \int \frac{\ln(a+b\,x)}{(a+b\,x)\,(d+e\,x)^{5/2}}\,dx$$

```
[In] int(log(a + b*x)/((a + b*x)*(d + e*x)^(5/2)),x)
[Out] int(log(a + b*x)/((a + b*x)*(d + e*x)^(5/2)), x)
```

# 3.209 $\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \operatorname{Int}\left(\frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx},x\right)$$

```
[Out] Unintegrable((i*x+h)^q*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
```

## Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$

```
[In] Int[((h + i*x)^q*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
[Out] Defer[Int][((h + i*x)^q*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$

```
[In] Integrate[((h + i*x)^q*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
[Out] Integrate[((h + i*x)^q*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x), x]
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(ix+h)^q\,(a+b\ln(c(fx+e)))^p}{dfx+de}dx$$

```
[In] int((i*x+h)^q*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
[Out] int((i*x+h)^q*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)^q(b\log((fx+e)c)+a)^p}{dfx+de}\,dx$$

```
[In] integrate((i*x+h)^q*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="fricas
")
[Out] integral((i*x + h)^q*(b*log(c*f*x + c*e) + a)^p/(d*f*x + d*e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \text{Timed out}$$

```
[In] integrate((i*x+h)**q*(a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e),x)
```

```
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)^q(b\log((fx+e)c)+a)^p}{dfx+de}\,dx$$

```
[In] integrate((i*x+h)^q*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="maxima
")
```

```
[Out] integrate((i*x + h)^q*(b*log((f*x + e)*c) + a)^p/(d*f*x + d*e), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((i*x+h)^q*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,0
,5,0,2,0,5,0,3,0,2,0]%%%}+%%%{5,[0,0,4,0,2,0,4,1,3,0,2,0]%%%}+%%%{10,[0,0,3
,0,2,0,3,
```

## Mupad [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(h+ix)^q(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\int \frac{(h+i\,x)^q\,(a+b\,\ln\left(c\,(e+f\,x)\right))^p}{d\,e+d\,f\,x}\,dx$$

```
[In] int(((h + i*x)^q*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x),x)
[Out] int(((h + i*x)^q*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x), x)
```

# 3.210 $\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 305

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(fh-ei)^3(a+b\log(c(e+fx)))^{1+p}}{bdf^4(1+p)} + \frac{3^{-1-p}e^{-\frac{3a}{b}}i^3\Gamma\left(1+p,-\frac{3(a+b\log(c(e+fx)))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{c^3df^4} + \frac{3\ 2^{-1-p}e^{-\frac{2a}{b}}i^2(fh-ei)\Gamma\left(1+p,-\frac{2(a+b\log(c(e+fx)))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{c^2df^4} + \frac{3e^{-\frac{a}{b}}i(fh-ei)^2\Gamma\left(1+p,-\frac{a+b\log(c(e+fx))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{cdf^4}$$

```
[Out] (-e*i+f*h)^3*(a+b*ln(c*(f*x+e)))^(p+1)/b/d/f^4/(p+1)+3^(-1-p)*i^3*GAMMA(p+1
,-3*(a+b*ln(c*(f*x+e)))/b)*(a+b*ln(c*(f*x+e)))^p/c^3/d/exp(3*a/b)/f^4/(((-a
-b*ln(c*(f*x+e)))/b)^p)+3*2^(-1-p)*i^2*(-e*i+f*h)*GAMMA(p+1,-2*(a+b*ln(c*(f
*x+e)))/b)*(a+b*ln(c*(f*x+e)))^p/c^2/d/exp(2*a/b)/f^4/(((-a-b*ln(c*(f*x+e))
)/b)^p)+3*i*(-e*i+f*h)^2*GAMMA(p+1,(-a-b*ln(c*(f*x+e)))/b)*(a+b*ln(c*(f*x+e
)))^p/c/d/exp(a/b)/f^4/(((-a-b*ln(c*(f*x+e)))/b)^p)
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2458, 12, 2395, 2336, 2212, 2339, 30, 2346}

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$
$$=\frac{i^3 3^{-p-1}e^{-\frac{3a}{b}}(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3(a+b\log(c(e+fx)))}{b}\right)}{c^3df^4}$$
$$+\frac{3i^2 2^{-p-1}e^{-\frac{2a}{b}}(fh-ei)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2(a+b\log(c(e+fx)))}{b}\right)}{c^2df^4}$$
$$+\frac{(fh-ei)^3(a+b\log(c(e+fx)))^{p+1}}{bdf^4(p+1)}$$
$$+\frac{3ie^{-\frac{a}{b}}(fh-ei)^2(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(c(e+fx))}{b}\right)}{cdf^4}$$

```
[In] Int[((h + i*x)^3*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
```

```
[Out] ((f*h - e*i)^3*(a + b*Log[c*(e + f*x)])^(1 + p))/(b*d*f^4*(1 + p)) + (3^(-1
 - p)*i^3*Gamma[1 + p, (-3*(a + b*Log[c*(e + f*x)]))/b]*(a + b*Log[c*(e + f
*x)])^p)/(c^3*d*E^((3*a)/b)*f^4*(-((a + b*Log[c*(e + f*x)])/b))^p) + (3*2^(
-1 - p)*i^2*(f*h - e*i)*Gamma[1 + p, (-2*(a + b*Log[c*(e + f*x)]))/b]*(a +
b*Log[c*(e + f*x)])^p)/(c^2*d*E^((2*a)/b)*f^4*(-((a + b*Log[c*(e + f*x)])/b
))^p) + (3*i*(f*h - e*i)^2*Gamma[1 + p, -((a + b*Log[c*(e + f*x)])/b)]*(a +
 b*Log[c*(e + f*x)])^p)/(c*d*E^(a/b)*f^4*(-((a + b*Log[c*(e + f*x)])/b))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3 (a+b\log(cx))^p}{dx}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^3 (a+b\log(cx))^p}{x}\,dx, x, e+fx\right)}{df}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3i(fh-ei)^2(a+b\log(cx))^p}{f^3}+\frac{(fh-ei)^3(a+b\log(cx))^p}{f^3x}+\frac{3i^2(fh-ei)x(a+b\log(cx))^p}{f^3}+\frac{i^3x^2(a+b\log(cx))^p}{f^3}\right)\,dx,\right.}{df}$$

$$= \frac{i^3\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,x,e+fx\right)}{df^4} + \frac{(3i^2(fh-ei))\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,e+fx\right)}{df^4} + \frac{(3i(fh-ei)^2)\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx,x,e+fx\right)}{df^4} + \frac{(fh-ei)^3\text{Subst}\left(\int \frac{(a+b\log(cx))^p}{x}\,dx,x,e+fx\right)}{df^4}$$

$$= \frac{i^3\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log(c(e+fx))\right)}{c^3df^4} + \frac{(3i^2(fh-ei))\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log(c(e+fx))\right)}{c^2df^4} + \frac{(3i(fh-ei)^2)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx,x,\log(c(e+fx))\right)}{cdf^4} + \frac{(fh-ei)^3\text{Subst}\left(\int x^p\,dx,x,a+b\log(c(e+fx))\right)}{bdf^4}$$

$$= \frac{(fh-ei)^3(a+b\log(c(e+fx)))^{1+p}}{bdf^4(1+p)} + \frac{3^{-1-p}e^{-\frac{3a}{b}}i^3\Gamma\left(1+p,-\frac{3(a+b\log(c(e+fx)))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{c^3df^4} + \frac{3\,2^{-1-p}e^{-\frac{2a}{b}}i^2(fh-ei)\Gamma\left(1+p,-\frac{2(a+b\log(c(e+fx)))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{c^2df^4} + \frac{3e^{-\frac{a}{b}}i(fh-ei)^2\Gamma\left(1+p,-\frac{a+b\log(c(e+fx))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{cdf^4}$$

## Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.79

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$
$$= \frac{6^{-1-p}e^{-\frac{3a}{b}}(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}\left(2^{1+p}bi^3\Gamma\left(2+p,-\frac{3(a+b\log(c(e+fx)))}{b}\right)+3^{1+p}ce^{a/b}\left(3\right.\right.}{}$$

```
[In] Integrate[((h + i*x)^3*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
[Out] (6^(-1 - p)*(a + b*Log[c*(e + f*x)])^p*(2^(1 + p)*b*i^3*Gamma[2 + p, (-3*(a
 + b*Log[c*(e + f*x)]))/b] + 3^(1 + p)*c*E^(a/b)*(3*b*i^2*(f*h - e*i)*Gamma
```

```
[2 + p, (-2*(a + b*Log[c*(e + f*x)]))/b] + 2^(1 + p)*c*E^(a/b)*(3*b*i*(f*h
- e*i)^2*Gamma[2 + p, -((a + b*Log[c*(e + f*x)])/b)] - b*c*E^(a/b)*f^3*(h +
 i*x)^3*(-((a + b*Log[c*(e + f*x)])/b))^(1 + p)))))/(b*c^3*d*E^((3*a)/b)*f^
4*(1 + p)*(-((a + b*Log[c*(e + f*x)])/b))^p)
```

## Maple [F]

$$\int \frac{(ix+h)^3 (a+b\ln (c(fx+e)))^p}{dfx+de}dx$$

[In] `int((i*x+h)^3*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)`

[Out] `int((i*x+h)^3*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)`

## Fricas [F]

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\int \frac{(ix+h)^3(b\log((fx+e)c)+a)^p}{dfx+de}\,dx$$

[In] `integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="fricas
")`

[Out] `integral((i^3*x^3 + 3*h*i^2*x^2 + 3*h^2*i*x + h^3)*(b*log(c*f*x + c*e) + a)
^p/(d*f*x + d*e), x)`

## Sympy [F(-1)]

Timed out.

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\text{Timed out}$$

[In] `integrate((i*x+h)**3*(a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e),x)`

[Out] `Timed out`

## Maxima [F]

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)^3(b\log\left((fx+e)c\right)+a)^p}{dfx+de}\,dx$$

```
[In] integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="maxima
")
```

```
[Out] (b*c*log(c*f*x + c*e) + a*c)*(b*log(c*f*x + c*e) + a)^p*h^3/(b*c*d*f*(p + 1
)) + integrate((i^3*x^3 + 3*h*i^2*x^2 + 3*h^2*i*x)*(b*log(f*x + e) + b*log(
c) + a)^p/(d*f*x + d*e), x)
```

## Giac [F]

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)^3(b\log\left((fx+e)c\right)+a)^p}{dfx+de}\,dx$$

```
[In] integrate((i*x+h)^3*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="giac")
```

```
[Out] integrate((i*x + h)^3*(b*log((f*x + e)*c) + a)^p/(d*f*x + d*e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^3(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(h+i\,x)^3\,(a+b\,\ln\left(c\,(e+f\,x)\right))^p}{d\,e+d\,f\,x}\,dx$$

```
[In] int(((h + i*x)^3*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x),x)
```

```
[Out] int(((h + i*x)^3*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x), x)
```

# 3.211 $\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 210

$$\begin{aligned}
&\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx \\
&= \frac{(fh-ei)^2(a+b\log(c(e+fx)))^{1+p}}{bdf^3(1+p)} \\
&\quad+ \frac{2^{-1-p}e^{-\frac{2a}{b}}i^2\Gamma\left(1+p,-\frac{2(a+b\log(c(e+fx)))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{c^2df^3} \\
&\quad+ \frac{2e^{-\frac{a}{b}}i(fh-ei)\Gamma\left(1+p,-\frac{a+b\log(c(e+fx))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{cdf^3}
\end{aligned}$$

```
[Out] (-e*i+f*h)^2*(a+b*ln(c*(f*x+e)))^(p+1)/b/d/f^3/(p+1)+2^(-1-p)*i^2*GAMMA(p+1
,-2*(a+b*ln(c*(f*x+e)))/b)*(a+b*ln(c*(f*x+e)))^p/c^2/d/exp(2*a/b)/f^3/(((-a
-b*ln(c*(f*x+e)))/b)^p)+2*i*(-e*i+f*h)*GAMMA(p+1,(-a-b*ln(c*(f*x+e)))/b)*(a
+b*ln(c*(f*x+e)))^p/c/d/exp(a/b)/f^3/(((-a-b*ln(c*(f*x+e)))/b)^p)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{2458, 12, 2395, 2336, 2212, 2339, 30, 2346\}$

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$
$$= \frac{i^2 2^{-p-1} e^{-\frac{2a}{b}}(a+b\log(c(e+fx)))^p \left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p} \Gamma\left(p+1,-\frac{2(a+b\log(c(e+fx)))}{b}\right)}{c^2 d f^3}$$
$$+ \frac{(fh-ei)^2(a+b\log(c(e+fx)))^{p+1}}{bdf^3(p+1)}$$
$$+ \frac{2ie^{-\frac{a}{b}}(fh-ei)(a+b\log(c(e+fx)))^p \left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p} \Gamma\left(p+1,-\frac{a+b\log(c(e+fx))}{b}\right)}{cdf^3}$$

```
[In] Int[((h + i*x)^2*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
```

```
[Out] ((f*h - e*i)^2*(a + b*Log[c*(e + f*x)])^(1 + p))/(b*d*f^3*(1 + p)) + (2^(-1
 - p)*i^2*Gamma[1 + p, (-2*(a + b*Log[c*(e + f*x)]))/b]*(a + b*Log[c*(e + f
*x)])^p)/(c^2*d*E^((2*a)/b)*f^3*(-((a + b*Log[c*(e + f*x)])/b))^p) + (2*i*(
f*h - e*i)*Gamma[1 + p, -((a + b*Log[c*(e + f*x)])/b)]*(a + b*Log[c*(e + f*
x)])^p)/(c*d*E^(a/b)*f^3*(-((a + b*Log[c*(e + f*x)])/b))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2(a+b\log(cx))^p}{dx}\,dx,x,e+fx\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)^2(a+b\log(cx))^p}{x}\,dx,x,e+fx\right)}{df}\\
&= \frac{\text{Subst}\left(\int \left(\frac{2i(fh-ei)(a+b\log(cx))^p}{f^2}+\frac{(fh-ei)^2(a+b\log(cx))^p}{f^2x}+\frac{i^2x(a+b\log(cx))^p}{f^2}\right)dx,x,e+fx\right)}{df}\\
&= \frac{i^2\text{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,e+fx\right)}{df^3}\\
&\quad+\frac{(2i(fh-ei))\text{Subst}\left(\int (a+b\log(cx))^p\,dx,x,e+fx\right)}{df^3}\\
&\quad+\frac{(fh-ei)^2\text{Subst}\left(\int \frac{(a+b\log(cx))^p}{x}\,dx,x,e+fx\right)}{df^3}
\end{aligned}$$

$$= \frac{i^2\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log(c(e+fx))\right)}{c^2df^3} + \frac{(2i(fh-ei))\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log(c(e+fx))\right)}{cdf^3} + \frac{(fh-ei)^2\text{Subst}\left(\int x^p\,dx, x, a+b\log(c(e+fx))\right)}{bdf^3}$$

$$= \frac{(fh-ei)^2(a+b\log(c(e+fx)))^{1+p}}{bdf^3(1+p)} + \frac{2^{-1-p}e^{-\frac{2a}{b}}i^2\Gamma\left(1+p,-\frac{2(a+b\log(c(e+fx)))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{c^2df^3} + \frac{2e^{-\frac{a}{b}}i(fh-ei)\Gamma\left(1+p,-\frac{a+b\log(c(e+fx))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{cdf^3}$$

## Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{2^{-1-p}e^{-\frac{2a}{b}}(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}\left(bi^2\Gamma\left(2+p,-\frac{2(a+b\log(c(e+fx)))}{b}\right)+2^{1+p}ce^{a/b}\left(2bi(f\right.\right.}{bc^2df^3(1+p)}$$

```
[In] Integrate[((h + i*x)^2*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
```

```
[Out] (2^(-1 - p)*(a + b*Log[c*(e + f*x)])^p*(b*i^2*Gamma[2 + p, (-2*(a + b*Log[c
*(e + f*x)]))/b] + 2^(1 + p)*c*E^(a/b)*(2*b*i*(f*h - e*i)*Gamma[2 + p, -((a
 + b*Log[c*(e + f*x)])/b)] - b*c*E^(a/b)*f^2*(h + i*x)^2*(-((a + b*Log[c*(e
 + f*x)])/b))^(1 + p))))/(b*c^2*d*E^((2*a)/b)*f^3*(1 + p)*(-((a + b*Log[c*(
e + f*x)])/b))^p)
```

## Maple [F]

$$\int \frac{(ix+h)^2\,(a+b\ln(c(fx+e)))^p}{dfx+de}dx$$

```
[In] int((i*x+h)^2*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
```

```
[Out] int((i*x+h)^2*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)
```

## Fricas [F]

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)^2(b\log\left((fx+e)c\right)+a)^p}{dfx+de}\,dx$$

```
[In] integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="fricas
")
```

```
[Out] integral((i^2*x^2 + 2*h*i*x + h^2)*(b*log(c*f*x + c*e) + a)^p/(d*f*x + d*e)
, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \text{Timed out}$$

```
[In] integrate((i*x+h)**2*(a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e),x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)^2(b\log\left((fx+e)c\right)+a)^p}{dfx+de}\,dx$$

```
[In] integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="maxima
")
```

```
[Out] (b*c*log(c*f*x + c*e) + a*c)*(b*log(c*f*x + c*e) + a)^p*h^2/(b*c*d*f*(p + 1
)) + integrate((i^2*x^2 + 2*h*i*x)*(b*log(f*x + e) + b*log(c) + a)^p/(d*f*x
 + d*e), x)
```

## Giac [F]

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)^2(b\log\left((fx+e)c\right)+a)^p}{dfx+de}\,dx$$

```
[In] integrate((i*x+h)^2*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="giac")
```

```
[Out] integrate((i*x + h)^2*(b*log((f*x + e)*c) + a)^p/(d*f*x + d*e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^2(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\int \frac{{\left(h+i\,x\right)}^2\,{\left(a+b\,\ln\left(c\,\left(e+f\,x\right)\right)\right)}^p}{d\,e+d\,f\,x} \,d x$$

```
[In] int(((h + i*x)^2*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x),x)
[Out] int(((h + i*x)^2*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x), x)
```

# 3.212 $\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 115

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(fh-ei)(a+b\log(c(e+fx)))^{1+p}}{bdf^2(1+p)} + \frac{e^{-\frac{a}{b}}i\Gamma\left(1+p,-\frac{a+b\log(c(e+fx))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{cdf^2}$$

```
[Out] (-e*i+f*h)*(a+b*ln(c*(f*x+e)))^(p+1)/b/d/f^2/(p+1)+i*GAMMA(p+1,(-a-b*ln(c*(
f*x+e)))/b)*(a+b*ln(c*(f*x+e)))^p/c/d/exp(a/b)/f^2/(((-a-b*ln(c*(f*x+e)))/b
)^p)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2458, 12, 2395, 2336, 2212, 2339, 30}

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(fh-ei)(a+b\log(c(e+fx)))^{p+1}}{bdf^2(p+1)} + \frac{ie^{-\frac{a}{b}}(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(c(e+fx))}{b}\right)}{cdf^2}$$

```
[In] Int[((h + i*x)*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
```

```
[Out] ((f*h - e*i)*(a + b*Log[c*(e + f*x)])^(1 + p))/(b*d*f^2*(1 + p)) + (i*Gamma
[1 + p, -((a + b*Log[c*(e + f*x)])/b)]*(a + b*Log[c*(e + f*x)])^p)/(c*d*E^(
a/b)*f^2*(-((a + b*Log[c*(e + f*x)])/b))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
```

```
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)(a+b\log(cx))^p}{dx}\,dx, x, e+fx\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{fh-ei}{f}+\frac{ix}{f}\right)(a+b\log(cx))^p}{x}\,dx, x, e+fx\right)}{df}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i(a+b\log(cx))^p}{f}+\frac{(fh-ei)(a+b\log(cx))^p}{fx}\right)\,dx, x, e+fx\right)}{df}$$

$$= \frac{i\text{Subst}\left(\int (a+b\log(cx))^p\,dx, x, e+fx\right)}{df^2} + \frac{(fh-ei)\text{Subst}\left(\int \frac{(a+b\log(cx))^p}{x}\,dx, x, e+fx\right)}{df^2}$$

$$= \frac{i\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log(c(e+fx))\right)}{cdf^2} + \frac{(fh-ei)\text{Subst}\left(\int x^p\,dx, x, a+b\log(c(e+fx))\right)}{bdf^2}$$

$$= \frac{(fh-ei)(a+b\log(c(e+fx)))^{1+p}}{bdf^2(1+p)} + \frac{e^{-\frac{a}{b}}i\Gamma\left(1+p,-\frac{a+b\log(c(e+fx))}{b}\right)(a+b\log(c(e+fx)))^p\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{cdf^2}$$

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = (a+b\log(c(e+fx)))^p\left(\frac{(h+ix)(a+b\log(c(e+fx)))}{bdf+bdfp} + \frac{e^{-\frac{a}{b}}i\Gamma\left(2+p,-\frac{a+b\log(c(e+fx))}{b}\right)\left(-\frac{a+b\log(c(e+fx))}{b}\right)^{-p}}{cdf^2+cdf^2p}\right)$$

```
[In] Integrate[((h + i*x)*(a + b*Log[c*(e + f*x)])^p)/(d*e + d*f*x),x]
```

[Out] (a + b*Log[c*(e + f*x)])^p*(((h + i*x)*(a + b*Log[c*(e + f*x)]))/(b*d*f + b
*d*f*p) + (i*Gamma[2 + p, -((a + b*Log[c*(e + f*x)])/b)])/(E^(a/b)*(c*d*f^2
 + c*d*f^2*p)*(-((a + b*Log[c*(e + f*x)])/b))^p))

## Maple [F]

$$\int \frac{(ix+h)\,(a+b\ln\,(c(fx+e)))^{p}}{dfx+de}dx$$

[In] int((i*x+h)*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)

[Out] int((i*x+h)*(a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x)

## Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$$
$$= \frac{(bip+bi)e^{\left(-\frac{bp\log\left(-\frac{1}{b}\right)+a}{b}\right)}\Gamma\Big(p+1,-\frac{b\log(cfx+ce)+a}{b}\Big)+(acfh-acei+(bcfh-bcei)\log\,(cfx+ce))(b\log\,(c}{bcdf^2p+bcdf^2}$$

[In] integrate((i*x+h)*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="fricas")

[Out] ((b*i*p + b*i)*e^(-(b*p*log(-1/b) + a)/b)*gamma(p + 1, -(b*log(c*f*x + c*e)
 + a)/b) + (a*c*f*h - a*c*e*i + (b*c*f*h - b*c*e*i)*log(c*f*x + c*e))*(b*lo
g(c*f*x + c*e) + a)^p)/(b*c*d*f^2*p + b*c*d*f^2)

## Sympy [F]

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{\int \frac{h(a+b\log\,(ce+cfx))^p}{e+fx}\,dx+\int \frac{ix(a+b\log\,(ce+cfx))^p}{e+fx}\,dx}{d}$$

[In] integrate((i*x+h)*(a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e),x)

[Out] (Integral(h*(a + b*log(c*e + c*f*x))**p/(e + f*x), x) + Integral(i*x*(a + b
*log(c*e + c*f*x))**p/(e + f*x), x))/d

## Maxima [F]

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)(b\log((fx+e)c)+a)^p}{dfx+de}\,dx$$

```
[In] integrate((i*x+h)*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="maxima")
[Out] i*integrate((b*log(f*x + e) + b*log(c) + a)^p*x/(d*f*x + d*e), x) + (b*c*lo
g(c*f*x + c*e) + a*c)*(b*log(c*f*x + c*e) + a)^p*h/(b*c*d*f*(p + 1))
```

## Giac [F]

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(ix+h)(b\log((fx+e)c)+a)^p}{dfx+de}\,dx$$

```
[In] integrate((i*x+h)*(a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="giac")
[Out] integrate((i*x + h)*(b*log((f*x + e)*c) + a)^p/(d*f*x + d*e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \int \frac{(h+i\,x)\,\left(a+b\,\ln\left(c\,(e+f\,x)\right)\right)^p}{d\,e+d\,f\,x}\,dx$$

```
[In] int(((h + i*x)*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x),x)
[Out] int(((h + i*x)*(a + b*log(c*(e + f*x)))^p)/(d*e + d*f*x), x)
```

# 3.213 $\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 31

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(a+b\log(c(e+fx)))^{1+p}}{bdf(1+p)}$$

```
[Out] (a+b*ln(c*(f*x+e)))^(p+1)/b/d/f/(p+1)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2437, 12, 2339, 30}

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(a+b\log(c(e+fx)))^{p+1}}{bdf(p+1)}$$

```
[In] Int[(a + b*Log[c*(e + f*x)])^p/(d*e + d*f*x),x]
```

```
[Out] (a + b*Log[c*(e + f*x)])^(1 + p)/(b*d*f*(1 + p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\log(cx))^p}{dx}\,dx, x, e+fx\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b\log(cx))^p}{x}\,dx, x, e+fx\right)}{df} \\
&= \frac{\text{Subst}\left(\int x^p\,dx, x, a+b\log(c(e+fx))\right)}{bdf} \\
&= \frac{(a+b\log(c(e+fx)))^{1+p}}{bdf(1+p)}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(a+b\log(c(e+fx)))^{1+p}}{bdf(1+p)}$$

```
[In] Integrate[(a + b*Log[c*(e + f*x)])^p/(d*e + d*f*x),x]
[Out] (a + b*Log[c*(e + f*x)])^(1 + p)/(b*d*f*(1 + p))
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

| method | result | size |
|---|---|---|
| default | $\frac{(a+b\ln(c(fx+e)))^{p+1}}{bdf(p+1)}$ | 32 |
| parallelrisch | $\frac{\ln(c(fx+e))(a+b\ln(c(fx+e)))^{p}bf+(a+b\ln(c(fx+e)))^{p}af}{d\,f^{2}b(p+1)}$ | 59 |
| norman | $\frac{\ln(c(fx+e))\mathrm{e}^{p\ln(a+b\ln(c(fx+e)))}}{df(p+1)}+\frac{a\,\mathrm{e}^{p\ln(a+b\ln(c(fx+e)))}}{bdf(p+1)}$ | 70 |

```
[In] int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e),x,method=_RETURNVERBOSE)
[Out] (a+b*ln(c*(f*x+e)))^(p+1)/b/d/f/(p+1)
```

## Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\frac{(b\log{(cfx+ce)}+a)(b\log{(cfx+ce)}+a)^p}{bdfp+bdf}$$

```
[In] integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="fricas")
[Out] (b*log(c*f*x + c*e) + a)*(b*log(c*f*x + c*e) + a)^p/(b*d*f*p + b*d*f)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx=\frac{\int \frac{(a+b\log{(ce+cfx)})^p}{e+fx}\,dx}{d}$$

```
[In] integrate((a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e),x)
[Out] Integral((a + b*log(c*e + c*f*x))**p/(e + f*x), x)/d
```

## Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(b\log(cfx+ce)+a)^{p+1}}{bdf(p+1)}$$

```
[In] integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="maxima")
[Out] (b*log(c*f*x + c*e) + a)^(p + 1)/(b*d*f*(p + 1))
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(b\log(cfx+ce)+a)^{p+1}}{bdf(p+1)}$$

```
[In] integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e),x, algorithm="giac")
[Out] (b*log(c*f*x + c*e) + a)^(p + 1)/(b*d*f*(p + 1))
```

## Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(e+fx)))^p}{de+dfx}\,dx = \frac{(a+b\,\ln(c\,(e+f\,x)))^{p+1}}{b\,d\,f\,\,(p+1)}$$

```
[In] int((a + b*log(c*(e + f*x)))^p/(d*e + d*f*x),x)
[Out] (a + b*log(c*(e + f*x)))^(p + 1)/(b*d*f*(p + 1))
```

# 3.214 $\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx = \text{Int}\left(\frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h),x)
```

## Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx = \int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx$$

```
[In] Int[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)),x]
```

```
[Out] Defer[Int][(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx = \int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx$$

```
[In] Integrate[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)),x]
[Out] Integrate[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)), x]
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(fx+e)))^p}{(dfx+de)(ix+h)}dx$$

```
[In] int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h),x)
[Out] int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h),x)
```

## Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx = \int \frac{(b\log((fx+e)c)+a)^p}{(dfx+de)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h),x, algorithm="fricas")
[Out] integral((b*log(c*f*x + c*e) + a)^p/(d*f*i*x^2 + d*e*h + (d*f*h + d*e*i)*x)
, x)
```

## Sympy [N/A]

Not integrable

Time = 154.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx = \frac{\int \frac{(a+b\log(ce+cfx))^p}{eh+eix+fhx+fix^2}\,dx}{d}$$

```
[In] integrate((a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e)/(i*x+h),x)
[Out] Integral((a + b*log(c*e + c*f*x))**p/(e*h + e*i*x + f*h*x + f*i*x**2), x)/d
```

## Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx = \int \frac{(b\log((fx+e)c)+a)^p}{(dfx+de)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h),x, algorithm="maxima")
[Out] integrate((b*log((f*x + e)*c) + a)^p/((d*f*x + d*e)*(i*x + h)), x)
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx = \int \frac{(b\log((fx+e)c)+a)^p}{(dfx+de)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h),x, algorithm="giac")
[Out] integrate((b*log((f*x + e)*c) + a)^p/((d*f*x + d*e)*(i*x + h)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)}\,dx = \int \frac{(a+b\,\ln\left(c\,(e+f\,x)\right))^p}{(h+i\,x)\,\left(d\,e+d\,f\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(e + f*x)))^p/((h + i*x)*(d*e + d*f*x)),x)
[Out] int((a + b*log(c*(e + f*x)))^p/((h + i*x)*(d*e + d*f*x)), x)
```

# 3.215 $\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx = \text{Int}\left(\frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx = \int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx$$

```
[In] Int[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^2),x]
```

```
[Out] Defer[Int][(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx = \int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^2),x]
[Out] Integrate[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^2), x]
```

## Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(fx+e)))^p}{(dfx+de)\,(ix+h)^2}dx$$

```
[In] int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^2,x)
[Out] int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.12

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^p}{(dfx+de)(ix+h)^2}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="fricas
")
[Out] integral((b*log(c*f*x + c*e) + a)^p/(d*f*i^2*x^3 + d*e*h^2 + (2*d*f*h*i + d
*e*i^2)*x^2 + (d*f*h^2 + 2*d*e*h*i)*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e)/(i*x+h)**2,x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx = \int \frac{(b\log\left((fx+e)c\right)+a)^p}{(dfx+de)(ix+h)^2}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="maxima
")
[Out] integrate((b*log((f*x + e)*c) + a)^p/((d*f*x + d*e)*(i*x + h)^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,1,0,0,0,0]%%%} / %%%{1,[0,0,1,1,1,0,0]%%%}+%%%{-1,[0,0
,0,1,0
```

## Mupad [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^2}\,dx = \int \frac{(a+b\,\ln\left(c\,(e+f\,x)\right))^p}{(h+i\,x)^2\,\left(d\,e+d\,f\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(e + f*x)))^p/((h + i*x)^2*(d*e + d*f*x)),x)
[Out] int((a + b*log(c*(e + f*x)))^p/((h + i*x)^2*(d*e + d*f*x)), x)
```

# 3.216 $\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx=\text{Int}\left(\frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^3,x)
```

## Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx=\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx$$

```
[In] Int[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^3),x]
```

```
[Out] Defer[Int][(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^3), x]
```

Rubi steps

$$\text{integral}=\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx = \int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx$$

```
[In] Integrate[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^3),x]
[Out] Integrate[(a + b*Log[c*(e + f*x)])^p/((d*e + d*f*x)*(h + i*x)^3), x]
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(fx+e)))^p}{(dfx+de)(ix+h)^3}dx$$

```
[In] int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^3,x)
[Out] int((a+b*ln(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.88

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx = \int \frac{(b\log((fx+e)c)+a)^p}{(dfx+de)(ix+h)^3}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="fricas
")
[Out] integral((b*log(c*f*x + c*e) + a)^p/(d*f*i^3*x^4 + d*e*h^3 + (3*d*f*h*i^2 +
 d*e*i^3)*x^3 + 3*(d*f*h^2*i + d*e*h*i^2)*x^2 + (d*f*h^3 + 3*d*e*h^2*i)*x),
 x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(f*x+e)))**p/(d*f*x+d*e)/(i*x+h)**3,x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx = \int \frac{(b\log((fx+e)c)+a)^p}{(dfx+de)(ix+h)^3}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="maxima
")
[Out] integrate((b*log((f*x + e)*c) + a)^p/((d*f*x + d*e)*(i*x + h)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx = \int \frac{(b\log((fx+e)c)+a)^p}{(dfx+de)(ix+h)^3}\,dx$$

```
[In] integrate((a+b*log(c*(f*x+e)))^p/(d*f*x+d*e)/(i*x+h)^3,x, algorithm="giac")
[Out] integrate((b*log((f*x + e)*c) + a)^p/((d*f*x + d*e)*(i*x + h)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(e+fx)))^p}{(de+dfx)(h+ix)^3}\,dx = \int \frac{(a+b\,\ln(c\,(e+f\,x)))^p}{(h+i\,x)^3\,(d\,e+d\,f\,x)}\,dx$$

```
[In] int((a + b*log(c*(e + f*x)))^p/((h + i*x)^3*(d*e + d*f*x)),x)
[Out] int((a + b*log(c*(e + f*x)))^p/((h + i*x)^3*(d*e + d*f*x)), x)
```

# 3.217 $\int \frac{(h+ix)^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 402

$$\int \frac{(h+ix)^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \frac{ai(gh-fi)^2x}{g^3} - \frac{bi(eh-di)^2nx}{3e^2g} - \frac{bi(eh-di)(gh-fi)nx}{2eg^2} - \frac{bi(gh-fi)^2nx}{g^3} - \frac{b(eh-di)n(h+ix)^2}{6eg} - \frac{b(gh-fi)n(h+ix)^2}{4g^2} - \frac{bn(h+ix)^3}{9g} - \frac{b(eh-di)^3n\log(d+ex)}{3e^3g} - \frac{b(eh-di)^2(gh-fi)n\log(d+ex)}{2e^2g^2} + \frac{bi(gh-fi)^2(d+ex)\log(c(d+ex)^n)}{eg^3} + \frac{(gh-fi)(h+ix)^2(a+b\log(c(d+ex)^n))}{2g^2} + \frac{(h+ix)^3(a+b\log(c(d+ex)^n))}{3g} + \frac{(gh-fi)^3(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4} + \frac{b(gh-fi)^3n\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4}$$

```
[Out] a*i*(-f*i+g*h)^2*x/g^3-1/3*b*i*(-d*i+e*h)^2*n*x/e^2/g-1/2*b*i*(-d*i+e*h)*(-
f*i+g*h)*n*x/e/g^2-b*i*(-f*i+g*h)^2*n*x/g^3-1/6*b*(-d*i+e*h)*n*(i*x+h)^2/e/
g-1/4*b*(-f*i+g*h)*n*(i*x+h)^2/g^2-1/9*b*n*(i*x+h)^3/g-1/3*b*(-d*i+e*h)^3*n
```

```
*ln(e*x+d)/e^3/g-1/2*b*(-d*i+e*h)^2*(-f*i+g*h)*n*ln(e*x+d)/e^2/g^2+b*i*(-f*
i+g*h)^2*(e*x+d)*ln(c*(e*x+d)^n)/e/g^3+1/2*(-f*i+g*h)*(i*x+h)^2*(a+b*ln(c*(
e*x+d)^n))/g^2+1/3*(i*x+h)^3*(a+b*ln(c*(e*x+d)^n))/g+(-f*i+g*h)^3*(a+b*ln(c
*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g^4+b*(-f*i+g*h)^3*n*polylog(2,-g*(e*
x+d)/(-d*g+e*f))/g^4
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2465, 2436, 2332, 2441, 2440, 2438, 2442, 45}

$$\int \frac{(h+ix)^3 (a+b\log (c(d+ex)^n))}{f+gx} \, dx = \frac{(gh-fi)^3 \log \left(\frac{e(f+gx)}{ef-dg}\right) (a+b\log (c(d+ex)^n))}{g^4} + \frac{(h+ix)^2(gh-fi) (a+b\log (c(d+ex)^n))}{2g^2} + \frac{(h+ix)^3 (a+b\log (c(d+ex)^n))}{3g} + \frac{aix(gh-fi)^2}{g^3} + \frac{bi(d+ex)(gh-fi)^2 \log (c(d+ex)^n)}{eg^3} - \frac{bn(eh-di)^3 \log(d+ex)}{3e^3g} - \frac{bn(eh-di)^2 \log(d+ex)(gh-fi)}{2e^2g^2} - \frac{binx(eh-di)^2}{3e^2g} + \frac{bn(gh-fi)^3 \operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4} - \frac{binx(eh-di)(gh-fi)}{2eg^2} - \frac{bn(h+ix)^2(eh-di)}{6eg} - \frac{binx(gh-fi)^2}{g^3} - \frac{bn(h+ix)^2(gh-fi)}{4g^2} - \frac{bn(h+ix)^3}{9g}$$

```
[In] Int[((h + i*x)^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

```
[Out] (a*i*(g*h - f*i)^2*x)/g^3 - (b*i*(e*h - d*i)^2*n*x)/(3*e^2*g) - (b*i*(e*h -
 d*i)*(g*h - f*i)*n*x)/(2*e*g^2) - (b*i*(g*h - f*i)^2*n*x)/g^3 - (b*(e*h -
d*i)*n*(h + i*x)^2)/(6*e*g) - (b*(g*h - f*i)*n*(h + i*x)^2)/(4*g^2) - (b*n*
```

```
(h + i*x)^3)/(9*g) - (b*(e*h - d*i)^3*n*Log[d + e*x])/(3*e^3*g) - (b*(e*h -
 d*i)^2*(g*h - f*i)*n*Log[d + e*x])/(2*e^2*g^2) + (b*i*(g*h - f*i)^2*(d + e
*x)*Log[c*(d + e*x)^n])/(e*g^3) + ((g*h - f*i)*(h + i*x)^2*(a + b*Log[c*(d
+ e*x)^n]))/(2*g^2) + ((h + i*x)^3*(a + b*Log[c*(d + e*x)^n]))/(3*g) + ((g*
h - f*i)^3*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^4 +
 (b*(g*h - f*i)^3*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \left(\frac{i(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{g^3}+\frac{(gh-fi)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{g^3(f+gx)}+\frac{i(gh-fi)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g^2}+\frac{i(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}\right)dx$$

$$=\frac{i\int (h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx}{g}+\frac{(i(gh-fi))\int (h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx}{g^2}+\frac{\left(i(gh-fi)^2\right)\int \left(a+b\log\left(c(d+ex)^n\right)\right)\,dx}{g^3}+\frac{(gh-fi)^3\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{g^3}$$

$$=\frac{ai(gh-fi)^2x}{g^3}+\frac{(gh-fi)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2g^2}+\frac{(h+ix)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3g}+\frac{(gh-fi)^3\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4}+\frac{\left(bi(gh-fi)^2\right)\int \log\left(c(d+ex)^n\right)\,dx}{g^3}-\frac{(ben)\int \frac{(h+ix)^3}{d+ex}\,dx}{3g}-\frac{(be(gh-fi)n)\int \frac{(h+ix)^2}{d+ex}\,dx}{2g^2}-\frac{\left(be(gh-fi)^3n\right)\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g^4}$$

$$= \frac{ai(gh-fi)^2x}{g^3} + \frac{(gh-fi)(h+ix)^2 (a+b\log(c(d+ex)^n))}{2g^2} + \frac{(h+ix)^3 (a+b\log(c(d+ex)^n))}{3g} + \frac{(gh-fi)^3 (a+b\log(c(d+ex)^n)) \log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4} + \frac{(bi(gh-fi)^2) \operatorname{Subst}\left(\int \log(cx^n)\, dx, x, d+ex\right)}{eg^3} - \frac{(ben)\int \left(\frac{i(eh-di)^2}{e^3} + \frac{(eh-di)^3}{e^3(d+ex)} + \frac{i(eh-di)(h+ix)}{e^2} + \frac{i(h+ix)^2}{e}\right) dx}{3g} - \frac{(be(gh-fi)n)\int \left(\frac{i(eh-di)}{e^2} + \frac{(eh-di)^2}{e^2(d+ex)} + \frac{i(h+ix)}{e}\right) dx}{2g^2} - \frac{(b(gh-fi)^3n) \operatorname{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x} dx, x, d+ex\right)}{g^4}$$

$$= \frac{ai(gh-fi)^2x}{g^3} - \frac{bi(eh-di)^2nx}{3e^2g} - \frac{bi(eh-di)(gh-fi)nx}{2eg^2} - \frac{bi(gh-fi)^2nx}{g^3} - \frac{b(eh-di)n(h+ix)^2}{6eg} - \frac{b(gh-fi)n(h+ix)^2}{4g^2} - \frac{bn(h+ix)^3}{9g} - \frac{b(eh-di)^3n\log(d+ex)}{3e^3g} - \frac{b(eh-di)^2(gh-fi)n\log(d+ex)}{2e^2g^2} + \frac{bi(gh-fi)^2(d+ex)\log(c(d+ex)^n)}{eg^3} + \frac{(gh-fi)(h+ix)^2 (a+b\log(c(d+ex)^n))}{2g^2} + \frac{(h+ix)^3 (a+b\log(c(d+ex)^n))}{3g} + \frac{(gh-fi)^3 (a+b\log(c(d+ex)^n)) \log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4} + \frac{b(gh-fi)^3n\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^4}$$

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.94

$$\int \frac{(h+ix)^3 (a+b\log(c(d+ex)^n))}{f+gx} dx = \frac{6bd^2g^2i^2(-9egh+3efi+2dgi)n\log(d+ex) + e\left(gix(6ae^2(6f^2i^2-3fgi(6h+ix)+g^2(18h^2+9hix+2\right.}{}$$

```
[In] Integrate[((h + i*x)^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

```
[Out] (6*b*d^2*g^2*i^2*(-9*e*g*h + 3*e*f*i + 2*d*g*i)*n*Log[d + e*x] + e*(g*i*x*(
6*a*e^2*(6*f^2*i^2 - 3*f*g*i*(6*h + i*x) + g^2*(18*h^2 + 9*h*i*x + 2*i^2*x^
2)) - b*n*(12*d^2*g^2*i^2 - 6*d*e*g*i*(9*g*h - 3*f*i + g*i*x) + e^2*(36*f^2
*i^2 - 9*f*g*i*(12*h + i*x) + g^2*(108*h^2 + 27*h*i*x + 4*i^2*x^2)))) + 36*
a*e^2*(g*h - f*i)^3*Log[(e*(f + g*x))/(e*f - d*g)] + 6*b*e*Log[c*(d + e*x)^
n]*(g*i*(6*d*(3*g^2*h^2 - 3*f*g*h*i + f^2*i^2) + e*x*(6*f^2*i^2 - 3*f*g*i*(
6*h + i*x) + g^2*(18*h^2 + 9*h*i*x + 2*i^2*x^2))) + 6*e*(g*h - f*i)^3*Log[(
e*(f + g*x))/(e*f - d*g)])) + 36*b*e^3*(g*h - f*i)^3*n*PolyLog[2, (g*(d + e
*x))/(-(e*f) + d*g)])/(36*e^3*g^4)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.03 (sec) , antiderivative size = 1208, normalized size of antiderivative = 3.00

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1208 |

```
[In] int((i*x+h)^3*(a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] -49/36*b*n/g^4*i^3*f^3+3*b*n/g^2*i^2*f*h*x-1/3*b/e^2*n/g*i^3*d^2*x+1/6*b/e*
n/g*i^3*d*x^2-3*b*ln((e*x+d)^n)*i^2/g^2*x*f*h+3*b*ln((e*x+d)^n)/g^3*ln(g*x+
f)*f^2*h*i^2-3*b*ln((e*x+d)^n)/g^2*ln(g*x+f)*f*h^2*i-3*b*n/g^3*dilog(((g*x+
f)*e+d*g-e*f)/(d*g-e*f))*f^2*h*i^2+3*b*n/g^2*dilog(((g*x+f)*e+d*g-e*f)/(d*g
-e*f))*f*h^2*i+b*n/g^4*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*f^3*i^3+
1/3*b/e^3*n/g*i^3*d^3*ln((g*x+f)*e+d*g-e*f)+15/4*b*n/g^3*i^2*f^2*h-3*b*n/g^
2*i*f*h^2+1/2*b/e^2*n/g^2*i^3*d^2*ln((g*x+f)*e+d*g-e*f)*f-3/2*b/e^2*n/g*i^2
*d^2*ln((g*x+f)*e+d*g-e*f)*h+b/e*n/g^3*i^3*d*ln((g*x+f)*e+d*g-e*f)*f^2+3*b/
e*n/g*i*d*ln((g*x+f)*e+d*g-e*f)*h^2-1/2*b/e*n/g^2*i^3*d*f*x+3/2*b/e*n/g*i^2
*d*h*x-3*b*n/g^3*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*f^2*h*i^2+3*b*
n/g^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*f*h^2*i+3/2*b/e*n/g^2*i^2
*d*f*h+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b
*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(
e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(i/g^3*(1/3*i^2*x^3
*g^2-1/2*x^2*f*g*i^2+3/2*x^2*g^2*h*i+x*f^2*i^2-3*x*f*g*h*i+3*x*g^2*h^2)+(-f
^3*i^3+3*f^2*g*h*i^2-3*f*g^2*h^2*i+g^3*h^3)/g^4*ln(g*x+f))-1/3*b/e^2*n/g^2*
i^3*d^2*f-2/3*b/e*n/g^3*i^3*d*f^2+1/4*b*n/g^2*i^3*f*x^2-b*n/g^3*i^3*f^2*x-3
/4*b*n/g*i^2*h*x^2-3*b*n/g*i*h^2*x+b*n/g^4*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e
*f))*f^3*i^3-b*n/g*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*h^3+3*b*ln((
e*x+d)^n)*i/g*x*h^2-b*ln((e*x+d)^n)/g^4*ln(g*x+f)*f^3*i^3-1/2*b*ln((e*x+d)^
n)*i^3/g^2*x^2*f+3/2*b*ln((e*x+d)^n)*i^2/g*x^2*h+b*ln((e*x+d)^n)*i^3/g^3*x*
f^2-3*b/e*n/g^2*i^2*d*ln((g*x+f)*e+d*g-e*f)*f*h-1/9*b*n/g*i^3*x^3+1/3*b*ln(
(e*x+d)^n)*i^3/g*x^3+b*ln((e*x+d)^n)/g*ln(g*x+f)*h^3-b*n/g*dilog(((g*x+f)*e
+d*g-e*f)/(d*g-e*f))*h^3
```

## Fricas [F]

$$\int \frac{(h+ix)^3 (a+b\log(c(d+ex)^n))}{f+gx} \, dx = \int \frac{(ix+h)^3 (b\log((ex+d)^n c)+a)}{gx+f} \, dx$$

[In] integrate((i*x+h)^3*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")

[Out] integral((a*i^3*x^3 + 3*a*h*i^2*x^2 + 3*a*h^2*i*x + a*h^3 + (b*i^3*x^3 + 3*b*h*i^2*x^2 + 3*b*h^2*i*x + b*h^3)*log((e*x + d)^n*c))/(g*x + f), x)

## Sympy [F]

$$\int \frac{(h+ix)^3 (a+b\log(c(d+ex)^n))}{f+gx} \, dx = \int \frac{(a+b\log(c(d+ex)^n)) (h+ix)^3}{f+gx} \, dx$$

[In] integrate((i*x+h)**3*(a+b*ln(c*(e*x+d)**n))/(g*x+f),x)

[Out] Integral((a + b*log(c*(d + e*x)**n))*(h + i*x)**3/(f + g*x), x)

## Maxima [F]

$$\int \frac{(h+ix)^3 (a+b\log(c(d+ex)^n))}{f+gx} \, dx = \int \frac{(ix+h)^3 (b\log((ex+d)^n c)+a)}{gx+f} \, dx$$

[In] integrate((i*x+h)^3*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")

[Out] 3*a*h^2*i*(x/g - f*log(g*x + f)/g^2) - 1/6*a*i^3*(6*f^3*log(g*x + f)/g^4 - (2*g^2*x^3 - 3*f*g*x^2 + 6*f^2*x)/g^3) + 3/2*a*h*i^2*(2*f^2*log(g*x + f)/g^3 + (g*x^2 - 2*f*x)/g^2) + a*h^3*log(g*x + f)/g + integrate((b*i^3*x^3*log(c) + 3*b*h*i^2*x^2*log(c) + 3*b*h^2*i*x*log(c) + b*h^3*log(c) + (b*i^3*x^3 + 3*b*h*i^2*x^2 + 3*b*h^2*i*x + b*h^3)*log((e*x + d)^n))/(g*x + f), x)

## Giac [F]

$$\int \frac{(h+ix)^3 (a+b\log(c(d+ex)^n))}{f+gx} \, dx = \int \frac{(ix+h)^3 (b\log((ex+d)^n c)+a)}{gx+f} \, dx$$

[In] integrate((i*x+h)^3*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")

[Out] integrate((i*x + h)^3*(b*log((e*x + d)^n*c) + a)/(g*x + f), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^3 (a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(h+i\,x)^3\,(a+b\,\ln(c\,(d+e\,x)^n))}{f+g\,x}\,dx$$

```
[In] int(((h + i*x)^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x),x)
[Out] int(((h + i*x)^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x), x)
```

# 3.218 $\int \frac{(h+ix)^2(a+b\log(c(d+ex)^n))}{f+gx} dx$



## Optimal result

Integrand size = 29, antiderivative size = 241

$$\begin{aligned}\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))}{f+gx} dx =& \frac{ai(gh-fi)x}{g^2} - \frac{bi(eh-di)nx}{2eg} - \frac{bi(gh-fi)nx}{g^2} \\ & - \frac{bn(h+ix)^2}{4g} - \frac{b(eh-di)^2 n\log(d+ex)}{2e^2g} \\ & + \frac{bi(gh-fi)(d+ex)\log(c(d+ex)^n)}{eg^2} \\ & + \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))}{2g} \\ & + \frac{(gh-fi)^2 (a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} \\ & + \frac{b(gh-fi)^2 n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}\end{aligned}$$

```
[Out] a*i*(-f*i+g*h)*x/g^2-1/2*b*i*(-d*i+e*h)*n*x/e/g-b*i*(-f*i+g*h)*n*x/g^2-1/4*
b*n*(i*x+h)^2/g-1/2*b*(-d*i+e*h)^2*n*ln(e*x+d)/e^2/g+b*i*(-f*i+g*h)*(e*x+d)
*ln(c*(e*x+d)^n)/e/g^2+1/2*(i*x+h)^2*(a+b*ln(c*(e*x+d)^n))/g+(-f*i+g*h)^2*(
a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g^3+b*(-f*i+g*h)^2*n*polylog(
2,-g*(e*x+d)/(-d*g+e*f))/g^3
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2465, 2436, 2332, 2441, 2440, 2438, 2442, 45}

$$\int \frac{(h+ix)^2 (a+b\log (c(d+ex)^n))}{f+gx} \, dx = \frac{(gh-fi)^2 \log \left(\frac{e(f+gx)}{ef-dg}\right) (a+b\log (c(d+ex)^n))}{g^3} + \frac{(h+ix)^2 (a+b\log (c(d+ex)^n))}{2g} + \frac{aix(gh-fi)}{g^2} + \frac{bi(d+ex)(gh-fi)\log (c(d+ex)^n)}{eg^2} - \frac{bn(eh-di)^2\log(d+ex)}{2e^2g} + \frac{bn(gh-fi)^2 \operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3} - \frac{binx(eh-di)}{2eg} - \frac{binx(gh-fi)}{g^2} - \frac{bn(h+ix)^2}{4g}$$

```
[In] Int[((h + i*x)^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

```
[Out] (a*i*(g*h - f*i)*x)/g^2 - (b*i*(e*h - d*i)*n*x)/(2*e*g) - (b*i*(g*h - f*i)*
n*x)/g^2 - (b*n*(h + i*x)^2)/(4*g) - (b*(e*h - d*i)^2*n*Log[d + e*x])/(2*e^
2*g) + (b*i*(g*h - f*i)*(d + e*x)*Log[c*(d + e*x)^n])/(e*g^2) + ((h + i*x)^
2*(a + b*Log[c*(d + e*x)^n]))/(2*g) + ((g*h - f*i)^2*(a + b*Log[c*(d + e*x)
^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^3 + (b*(g*h - f*i)^2*n*PolyLog[2, -(
(g*(d + e*x))/(e*f - d*g))])/g^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \left(\frac{i(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g^2}+\frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{g^2(f+gx)}+\frac{i(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}\right)dx$$

$$= \frac{i\int (h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx}{g} + \frac{(i(gh-fi))\int \left(a+b\log\left(c(d+ex)^n\right)\right)\,dx}{g^2} + \frac{(gh-fi)^2\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{g^2}$$

$$= \frac{ai(gh-fi)x}{g^2} + \frac{(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2g} + \frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} + \frac{(bi(gh-fi))\int \log\left(c(d+ex)^n\right)\,dx}{g^2} - \frac{(ben)\int \frac{(h+ix)^2}{d+ex}\,dx}{2g} - \frac{(be(gh-fi)^2n)\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g^3}$$

$$= \frac{ai(gh-fi)x}{g^2} + \frac{(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2g} + \frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} + \frac{(bi(gh-fi))\text{Subst}\left(\int \log\left(cx^n\right)\,dx, x, d+ex\right)}{eg^2} - \frac{(ben)\int \left(\frac{i(eh-di)}{e^2} + \frac{(eh-di)^2}{e^2(d+ex)} + \frac{i(h+ix)}{e}\right)dx}{2g} - \frac{(b(gh-fi)^2n)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{g^3}$$

$$= \frac{ai(gh-fi)x}{g^2} - \frac{bi(eh-di)nx}{2eg} - \frac{bi(gh-fi)nx}{g^2} - \frac{bn(h+ix)^2}{4g} - \frac{b(eh-di)^2n\log(d+ex)}{2e^2g} + \frac{bi(gh-fi)(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} + \frac{(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2g} + \frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} + \frac{b(gh-fi)^2n\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.93

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))}{f+gx}\,dx$$
$$= \frac{-2bd^2g^2i^2n\log(d+ex)+e\Big(gix(2ae(4gh-2fi+gix)+bn(2dgi-e(8gh-4fi+gix)))+4ae(gh-fi)}{}$$

```
[In] Integrate[((h + i*x)^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

```
[Out] (-2*b*d^2*g^2*i^2*n*Log[d + e*x] + e*(g*i*x*(2*a*e*(4*g*h - 2*f*i + g*i*x)
+ b*n*(2*d*g*i - e*(8*g*h - 4*f*i + g*i*x))) + 4*a*e*(g*h - f*i)^2*Log[(e*(
f + g*x))/(e*f - d*g)] + 2*b*Log[c*(d + e*x)^n]*(g*i*(d*(4*g*h - 2*f*i) + e
*x*(4*g*h - 2*f*i + g*i*x)) + 2*e*(g*h - f*i)^2*Log[(e*(f + g*x))/(e*f - d*
g)])) + 4*b*e^2*(g*h - f*i)^2*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)])/(
4*e^2*g^3)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.24 (sec) , antiderivative size = 721, normalized size of antiderivative = 2.99

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)i^2x^2}{2g}-\frac{b\ln((ex+d)^n)i^2xf}{g^2}+\frac{2b\ln((ex+d)^n)ixh}{g}+\frac{b\ln((ex+d)^n)\ln(gx+f)f^2i^2}{g^3}-\frac{2b\ln((ex+d)^n)\ln(gx+f)fhi}{g^2}$ |

```
[In] int((i*x+h)^2*(a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln((e*x+d)^n)*i^2/g*x^2-b*ln((e*x+d)^n)*i^2/g^2*x*f+2*b*ln((e*x+d)^n)
*i/g*x*h+b*ln((e*x+d)^n)/g^3*ln(g*x+f)*f^2*i^2-2*b*ln((e*x+d)^n)/g^2*ln(g*x
+f)*f*h*i+b*ln((e*x+d)^n)/g*ln(g*x+f)*h^2-b*n/g^3*dilog(((g*x+f)*e+d*g-e*f)
/(d*g-e*f))*f^2*i^2+2*b*n/g^2*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*f*h*i-b*
n/g*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*h^2-b*n/g^3*ln(g*x+f)*ln(((g*x+f)*
e+d*g-e*f)/(d*g-e*f))*f^2*i^2+2*b*n/g^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d
*g-e*f))*f*h*i-b*n/g*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*h^2-1/4*b*
n/g*i^2*x^2+b*n/g^2*i^2*f*x+5/4*b*n/g^3*i^2*f^2+1/2*b/e*n/g*i^2*d*x+1/2*b/e
*n/g^2*i^2*d*f-2*b*n/g*i*h*x-2*b*n/g^2*i*f*h-1/2*b/e^2*n/g*i^2*d^2*ln((g*x+
f)*e+d*g-e*f)-b/e*n/g^2*i^2*d*ln((g*x+f)*e+d*g-e*f)*f+2*b/e*n/g*i*d*ln((g*x
+f)*e+d*g-e*f)*h+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^
n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*
csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(i/g^2*(1
/2*i*x^2*g-x*f*i+2*x*g*h)+(f^2*i^2-2*f*g*h*i+g^2*h^2)/g^3*ln(g*x+f))
```

## Fricas [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))}{f+gx} \, dx = \int \frac{(ix+h)^2 (b\log((ex+d)^n c)+a)}{gx+f} \, dx$$

[In] integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")

[Out] integral((a*i^2*x^2 + 2*a*h*i*x + a*h^2 + (b*i^2*x^2 + 2*b*h*i*x + b*h^2)*log((e*x + d)^n*c))/(g*x + f), x)

## Sympy [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))}{f+gx} \, dx = \int \frac{(a+b\log(c(d+ex)^n))(h+ix)^2}{f+gx} \, dx$$

[In] integrate((i*x+h)**2*(a+b*ln(c*(e*x+d)**n))/(g*x+f),x)

[Out] Integral((a + b*log(c*(d + e*x)**n))*(h + i*x)**2/(f + g*x), x)

## Maxima [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))}{f+gx} \, dx = \int \frac{(ix+h)^2 (b\log((ex+d)^n c)+a)}{gx+f} \, dx$$

[In] integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")

[Out] 2*a*h*i*(x/g - f*log(g*x + f)/g^2) + 1/2*a*i^2*(2*f^2*log(g*x + f)/g^3 + (g*x^2 - 2*f*x)/g^2) + a*h^2*log(g*x + f)/g + integrate((b*i^2*x^2*log(c) + 2*b*h*i*x*log(c) + b*h^2*log(c) + (b*i^2*x^2 + 2*b*h*i*x + b*h^2)*log((e*x + d)^n))/(g*x + f), x)

## Giac [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))}{f+gx} \, dx = \int \frac{(ix+h)^2 (b\log((ex+d)^n c)+a)}{gx+f} \, dx$$

[In] integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")

[Out] integrate((i*x + h)^2*(b*log((e*x + d)^n*c) + a)/(g*x + f), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(h+i\,x)^2\,(a+b\,\ln(c\,(d+e\,x)^n))}{f+g\,x}\,dx$$

```
[In] int(((h + i*x)^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x),x)
[Out] int(((h + i*x)^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x), x)
```

# 3.219 $\int \frac{(h+ix)(a+b\log(c(d+ex)^n))}{f+gx}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 119

$$\int \frac{(h+ix)(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \frac{aix}{g}-\frac{binx}{g}+\frac{bi(d+ex)\log(c(d+ex)^n)}{eg}+\frac{(gh-fi)(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2}+\frac{b(gh-fi)n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}$$

```
[Out] a*i*x/g-b*i*n*x/g+b*i*(e*x+d)*ln(c*(e*x+d)^n)/e/g+(-f*i+g*h)*(a+b*ln(c*(e*x
+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g^2+b*(-f*i+g*h)*n*polylog(2,-g*(e*x+d)/(-
d*g+e*f))/g^2
```

## Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2465, 2436, 2332, 2441, 2440, 2438}

$$\int \frac{(h+ix)(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \frac{(gh-fi)\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^2}+\frac{aix}{g}+\frac{bi(d+ex)\log(c(d+ex)^n)}{eg}+\frac{bn(gh-fi)\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}-\frac{binx}{g}$$

```
[In] Int[((h + i*x)*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

```
[Out] (a*i*x)/g - (b*i*n*x)/g + (b*i*(d + e*x)*Log[c*(d + e*x)^n])/(e*g) + ((g*h
- f*i)*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^2 + (b*
(g*h - f*i)*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \left(\frac{i(a+b\log(c(d+ex)^n))}{g} + \frac{(gh-fi)\,(a+b\log(c(d+ex)^n))}{g(f+gx)}\right) dx$$

$$= \frac{i\int (a+b\log (c(d+ex)^n))\, dx}{g}+\frac{(gh-fi)\int \frac{a+b\log (c(d+ex)^n)}{f+gx}dx}{g}$$

$$= \frac{aix}{g}+\frac{(gh-fi)(a+b\log (c(d+ex)^n))\log \left(\frac{e(f+gx)}{ef-dg}\right)}{g^2}$$
$$+\frac{(bi)\int \log (c(d+ex)^n)\, dx}{g}-\frac{(be(gh-fi)n)\int \frac{\log \left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g^2}$$

$$= \frac{aix}{g}+\frac{(gh-fi)(a+b\log (c(d+ex)^n))\log \left(\frac{e(f+gx)}{ef-dg}\right)}{g^2}$$
$$+\frac{(bi)\text{Subst}\left(\int \log (cx^n)\, dx,x,d+ex\right)}{eg}$$
$$-\frac{(b(gh-fi)n)\text{Subst}\left(\int \frac{\log \left(1+\frac{gx}{ef-dg}\right)}{x}dx,x,d+ex\right)}{g^2}$$

$$= \frac{aix}{g}-\frac{binx}{g}+\frac{bi(d+ex)\log (c(d+ex)^n)}{eg}$$
$$+\frac{(gh-fi)(a+b\log (c(d+ex)^n))\log \left(\frac{e(f+gx)}{ef-dg}\right)}{g^2}+\frac{b(gh-fi)n\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^2}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{(h+ix)(a+b\log (c(d+ex)^n))}{f+gx}\, dx$$
$$= \frac{agix-bginx+\frac{bgi(d+ex)\log(c(d+ex)^n)}{e}+(gh-fi)(a+b\log (c(d+ex)^n))\log \left(\frac{e(f+gx)}{ef-dg}\right)+b(gh-fi)n\,\text{PolyLo}}{g^2}$$

```
[In] Integrate[((h + i*x)*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

```
[Out] (a*g*i*x - b*g*i*n*x + (b*g*i*(d + e*x)*Log[c*(d + e*x)^n])/e + (g*h - f*i)
*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] + b*(g*h - f*i)*
n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)])/g^2
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.78 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.31

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^{n})xi}{g}-\frac{b\ln((ex+d)^{n})\ln(gx+f)fi}{g^{2}}+\frac{b\ln((ex+d)^{n})\ln(gx+f)h}{g}-\frac{binx}{g}-\frac{bnif}{g^{2}}+\frac{bnid\ln((gx+f)e+dg-ef)}{eg}+$ |

```
[In] int((i*x+h)*(a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] b*ln((e*x+d)^n)*x*i/g-b*ln((e*x+d)^n)/g^2*ln(g*x+f)*f*i+b*ln((e*x+d)^n)/g*l
n(g*x+f)*h-b*i*n*x/g-b*n/g^2*i*f+b/e*n/g*i*d*ln((g*x+f)*e+d*g-e*f)+b*n/g^2*
dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*f*i-b*n/g*dilog(((g*x+f)*e+d*g-e*f)/(d
*g-e*f))*h+b*n/g^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*f*i-b*n/g*ln
(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))*h+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(
e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/
2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d
)^n)^3+b*ln(c)+a)*(x*i/g+(-f*i+g*h)/g^2*ln(g*x+f))
```

## Fricas [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^{n}\right)\right)}{f+gx}\,dx=\int \frac{(ix+h)(b\log\left((ex+d)^{n}c\right)+a)}{gx+f}\,dx$$

```
[In] integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((a*i*x + a*h + (b*i*x + b*h)*log((e*x + d)^n*c))/(g*x + f), x)
```

## Sympy [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^{n}\right)\right)}{f+gx}\,dx=\int \frac{\left(a+b\log\left(c(d+ex)^{n}\right)\right)(h+ix)}{f+gx}\,dx$$

```
[In] integrate((i*x+h)*(a+b*ln(c*(e*x+d)**n))/(g*x+f),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))*(h + i*x)/(f + g*x), x)
```

## Maxima [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f+gx}\,dx=\int \frac{(ix+h)(b\log\left((ex+d)^nc\right)+a)}{gx+f}\,dx$$

```
[In] integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
```

```
[Out] a*i*(x/g - f*log(g*x + f)/g^2) + a*h*log(g*x + f)/g + integrate((b*i*x*log(
c) + b*h*log(c) + (b*i*x + b*h)*log((e*x + d)^n))/(g*x + f), x)
```

## Giac [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f+gx}\,dx=\int \frac{(ix+h)(b\log\left((ex+d)^nc\right)+a)}{gx+f}\,dx$$

```
[In] integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((i*x + h)*(b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f+gx}\,dx=\int \frac{(h+i\,x)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}{f+g\,x}\,dx$$

```
[In] int(((h + i*x)*(a + b*log(c*(d + e*x)^n)))/(f + g*x),x)
```

```
[Out] int(((h + i*x)*(a + b*log(c*(d + e*x)^n)))/(f + g*x), x)
```

# 3.220 $\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[Out] (a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g+b*n*polylog(2,-g*(e*x+d)/(
-d*g+e*f))/g
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2441, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g + (b*n*PolyLo
g[2, -((g*(d + e*x))/(e*f - d*g))])/g
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} - \frac{(ben)\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} - \frac{(bn)\text{Subst}\left(\int\frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{bn\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int\frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx = \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{bn\,\text{PolyLog}\left(2,\frac{g(d+ex)}{-ef+dg}\right)}{g}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x),x]
[Out] ((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g + (b*n*PolyLo
g[2, (g*(d + e*x))/(-(e*f) + d*g)])/g
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.44

| method | result |
|---|---|
| risch | $\frac{b\ln\left((ex+d)^{n}\right)\ln(gx+f)}{g}-\frac{bn\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g}-\frac{bn\ln(gx+f)\ln\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g}+\frac{\left(-\frac{ib\pi\ \mathrm{csgn}\left(ic(ex+d)^{n}\right)\mathrm{csgn}(ic)\,\mathrm{csgn}}{2}\right.}{}$ |

[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)

[Out] b*ln((e*x+d)^n)*ln(g*x+f)/g-b/g*n*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b/g*n*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*ln(g*x+f)/g

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{f+gx}\,dx=\int \frac{b\log\left((ex+d)^{n}c\right)+a}{gx+f}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")

[Out] integral((b*log((e*x + d)^n*c) + a)/(g*x + f), x)

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{f+gx}\,dx=\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{f+gx}\,dx$$

[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f),x)

[Out] Integral((a + b*log(c*(d + e*x)**n))/(f + g*x), x)

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
[Out] b*integrate((log((e*x + d)^n) + log(c))/(g*x + f), x) + a*log(g*x + f)/g
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{f+g\,x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(f + g*x), x)
```

# 3.221 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 155

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx=\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi}-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi}-\frac{bn\,\text{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}$$

```
[Out] (a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)-(a+b*ln(c*(e*x+d)
^n))*ln(e*(i*x+h)/(-d*i+e*h))/(-f*i+g*h)+b*n*polylog(2,-g*(e*x+d)/(-d*g+e*f
))/(-f*i+g*h)-b*n*polylog(2,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2465, 2441, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx=\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{gh-fi}-\frac{\log\left(\frac{e(h+ix)}{eh-di}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{gh-fi}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi}-\frac{bn\,\text{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/((f + g*x)*(h + i*x)),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/(g*h - f*i) - (
(a + b*Log[c*(d + e*x)^n])*Log[(e*(h + i*x))/(e*h - d*i)])/(g*h - f*i) + (b
*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i) - (b*n*PolyLog[2,
-((i*(d + e*x))/(e*h - d*i))])/(g*h - f*i)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{g(a+b\log(c(d+ex)^n))}{(gh-fi)(f+gx)}-\frac{i(a+b\log(c(d+ex)^n))}{(gh-fi)(h+ix)}\right)\,dx\\
&= \frac{g\int\frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{gh-fi}-\frac{i\int\frac{a+b\log(c(d+ex)^n)}{h+ix}\,dx}{gh-fi}\\
&= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi}-\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi}\\
&\quad-\frac{(ben)\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{gh-fi}+\frac{(ben)\int\frac{\log\left(\frac{e(h+ix)}{eh-di}\right)}{d+ex}\,dx}{gh-fi}
\end{aligned}
$$

$$= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} - \frac{(bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{gh-fi} + \frac{(bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{ix}{eh-di}\right)}{x}\,dx, x, d+ex\right)}{gh-fi}$$

$$= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} + \frac{bn\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} - \frac{bn\text{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.72

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)}\,dx = \frac{(a+b\log(c(d+ex)^n))\left(\log\left(\frac{e(f+gx)}{ef-dg}\right) - \log\left(\frac{e(h+ix)}{eh-di}\right)\right) + bn\,\text{PolyLog}\left(2, \frac{g(d+ex)}{-ef+dg}\right) - bn\,\text{PolyLog}\left(2, \frac{i(d+ex)}{-eh+di}\right)}{gh-fi}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/((f + g*x)*(h + i*x)),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])*(Log[(e*(f + g*x))/(e*f - d*g)] - Log[(e*(h + i
*x))/(e*h - d*i)]) + b*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - b*n*Pol
yLog[2, (i*(d + e*x))/(-(e*h) + d*i)])/(g*h - f*i)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.12 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.44

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)\ln(ix+h)}{fi-gh} - \frac{b\ln((ex+d)^n)\ln(gx+f)}{fi-gh} - \frac{bn\,\text{dilog}\left(\frac{(ix+h)e+di-eh}{di-eh}\right)}{fi-gh} - \frac{bn\ln(ix+h)\ln\left(\frac{(ix+h)e+di-eh}{di-eh}\right)}{fi-gh} + \frac{bn\,\text{dilog}}{}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)/(i*x+h),x,method=_RETURNVERBOSE)
```

```
[Out] b*ln((e*x+d)^n)/(f*i-g*h)*ln(i*x+h)-b*ln((e*x+d)^n)/(f*i-g*h)*ln(g*x+f)-b*n
/(f*i-g*h)*dilog(((i*x+h)*e+d*i-e*h)/(d*i-e*h))-b*n/(f*i-g*h)*ln(i*x+h)*ln(
((i*x+h)*e+d*i-e*h)/(d*i-e*h))+b*n/(f*i-g*h)*dilog(((g*x+f)*e+d*g-e*f)/(d*g
-e*f))+b*n/(f*i-g*h)*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*
Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csg
```

```
n(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I
*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/(f*i-g*h)*ln(i*x+h)-1/(f*i-g*h)*l
n(g*x+f))
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h),x, algorithm="fricas")
[Out] integral((b*log((e*x + d)^n*c) + a)/(g*i*x^2 + f*h + (g*h + f*i)*x), x)
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx = \int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)\,(h+ix)}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)/(i*x+h),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))/((f + g*x)*(h + i*x)), x)
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h),x, algorithm="maxima")
[Out] a*(log(g*x + f)/(g*h - f*i) - log(i*x + h)/(g*h - f*i)) + b*integrate((log(
(e*x + d)^n) + log(c))/(g*i*x^2 + f*h + (g*h + f*i)*x), x)
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)*(i*x + h)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{\left(f+g\,x\right)\,\left(h+i\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/((f + g*x)*(h + i*x)),x)
[Out] int((a + b*log(c*(d + e*x)^n))/((f + g*x)*(h + i*x)), x)
```

# 3.222 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)^2}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 252

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^2}\,dx=-\frac{ben\log(d+ex)}{(eh-di)(gh-fi)}+\frac{a+b\log\left(c(d+ex)^n\right)}{(gh-fi)(h+ix)}+\frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2}+\frac{ben\log(h+ix)}{(eh-di)(gh-fi)}-\frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2}+\frac{bgn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2}-\frac{bgn\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$

```
[Out] -b*e*n*ln(e*x+d)/(-d*i+e*h)/(-f*i+g*h)+(a+b*ln(c*(e*x+d)^n))/(-f*i+g*h)/(i*
x+h)+g*(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)^2+b*e*n*ln
(i*x+h)/(-d*i+e*h)/(-f*i+g*h)-g*(a+b*ln(c*(e*x+d)^n))*ln(e*(i*x+h)/(-d*i+e*
h))/(-f*i+g*h)^2+b*g*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)^2-b*g*n*
polylog(2,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)^2
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used

= {2465, 2441, 2440, 2438, 2442, 36, 31}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^2}\,dx = \frac{a+b\log\left(c(d+ex)^n\right)}{(h+ix)(gh-fi)} + \frac{g\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(gh-fi)^2} - \frac{g\log\left(\frac{e(h+ix)}{eh-di}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{(gh-fi)^2} + \frac{bgn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} - \frac{bgn\,\text{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2} - \frac{ben\log(d+ex)}{(eh-di)(gh-fi)} + \frac{ben\log(h+ix)}{(eh-di)(gh-fi)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/((f + g*x)*(h + i*x)^2),x]
```

```
[Out] -((b*e*n*Log[d + e*x])/((e*h - d*i)*(g*h - f*i))) + (a + b*Log[c*(d + e*x)^
n])/((g*h - f*i)*(h + i*x)) + (g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x
))/(e*f - d*g)])/(g*h - f*i)^2 + (b*e*n*Log[h + i*x])/((e*h - d*i)*(g*h - f
*i)) - (g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(h + i*x))/(e*h - d*i)])/(g*h -
 f*i)^2 + (b*g*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i)^2 -
(b*g*n*PolyLog[2, -((i*(d + e*x))/(e*h - d*i))])/(g*h - f*i)^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \left(\frac{g^2(a+b\log(c(d+ex)^n))}{(gh-fi)^2(f+gx)} - \frac{i(a+b\log(c(d+ex)^n))}{(gh-fi)(h+ix)^2} - \frac{gi(a+b\log(c(d+ex)^n))}{(gh-fi)^2(h+ix)}\right) dx$$

$$= \frac{g^2\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{(gh-fi)^2} - \frac{(gi)\int \frac{a+b\log(c(d+ex)^n)}{h+ix}\,dx}{(gh-fi)^2} - \frac{i\int \frac{a+b\log(c(d+ex)^n)}{(h+ix)^2}\,dx}{gh-fi}$$

$$= \frac{a+b\log(c(d+ex)^n)}{(gh-fi)(h+ix)} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2} - \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2} - \frac{(begn)\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{(gh-fi)^2} + \frac{(begn)\int \frac{\log\left(\frac{e(h+ix)}{eh-di}\right)}{d+ex}\,dx}{(gh-fi)^2} - \frac{(ben)\int \frac{1}{(d+ex)(h+ix)}\,dx}{gh-fi}$$

$$= \frac{a+b\log(c(d+ex)^n)}{(gh-fi)(h+ix)} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2} - \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2} - \frac{(bgn)\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{(gh-fi)^2} + \frac{(bgn)\text{Subst}\left(\int \frac{\log\left(1+\frac{ix}{eh-di}\right)}{x}\,dx, x, d+ex\right)}{(gh-fi)^2} - \frac{(be^2n)\int \frac{1}{d+ex}\,dx}{(eh-di)(gh-fi)} + \frac{(bein)\int \frac{1}{h+ix}\,dx}{(eh-di)(gh-fi)}$$

$$= -\frac{ben\log(d+ex)}{(eh-di)(gh-fi)} + \frac{a+b\log(c(d+ex)^n)}{(gh-fi)(h+ix)} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2} + \frac{ben\log(h+ix)}{(eh-di)(gh-fi)} - \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2} + \frac{bgn\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} - \frac{bgn\text{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.78

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)^2}\,dx = \frac{\frac{(gh-fi)(a+b\log(c(d+ex)^n))}{h+ix} + g(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right) - \frac{be(gh-fi)n(\log(d+ex)-\log(h+ix))}{eh-di} - g(a+b\log \cdots}{(gh-fi)^2}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/((f + g*x)*(h + i*x)^2),x]
```

```
[Out] (((g*h - f*i)*(a + b*Log[c*(d + e*x)^n]))/(h + i*x) + g*(a + b*Log[c*(d + e
*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] - (b*e*(g*h - f*i)*n*(Log[d + e*x] -
 Log[h + i*x]))/(e*h - d*i) - g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(h + i*x)
)/(e*h - d*i)] + b*g*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - b*g*n*Pol
yLog[2, (i*(d + e*x))/(-(e*h) + d*i)])/(g*h - f*i)^2
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.72 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.96

| method | result |
|---|---|
| risch | $-\frac{b\ln((ex+d)^n)}{(fi-gh)(ix+h)}-\frac{b\ln((ex+d)^n)g\ln(ix+h)}{(fi-gh)^2}+\frac{b\ln((ex+d)^n)g\ln(gx+f)}{(fi-gh)^2}-\frac{bng\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{(fi-gh)^2}-\frac{bng\ln(gx+f)\ln\left(\frac{(g}{}\right.}{(fi-gh}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b*ln((e*x+d)^n)/(f*i-g*h)/(i*x+h)-b*ln((e*x+d)^n)*g/(f*i-g*h)^2*ln(i*x+h)+
b*ln((e*x+d)^n)*g/(f*i-g*h)^2*ln(g*x+f)-b*n*g/(f*i-g*h)^2*dilog(((g*x+f)*e+
d*g-e*f)/(d*g-e*f))-b*n*g/(f*i-g*h)^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g
-e*f))-b*e*n/(f*i-g*h)/(d*i-e*h)*ln(e*x+d)+b*e*n/(f*i-g*h)/(d*i-e*h)*ln(i*x
+h)+b*n*g/(f*i-g*h)^2*dilog(((i*x+h)*e+d*i-e*h)/(d*i-e*h))+b*n*g/(f*i-g*h)^
2*ln(i*x+h)*ln(((i*x+h)*e+d*i-e*h)/(d*i-e*h))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I
*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+
1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x
+d)^n)^3+b*ln(c)+a)*(-1/(f*i-g*h)/(i*x+h)-g/(f*i-g*h)^2*ln(i*x+h)+g/(f*i-g*
h)^2*ln(g*x+f))
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)(ix+h)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x + d)^n*c) + a)/(g*i^2*x^3 + f*h^2 + (2*g*h*i + f*i^2)*
x^2 + (g*h^2 + 2*f*h*i)*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)/(i*x+h)**2,x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^2}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)(ix+h)^2}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^2,x, algorithm="maxima")

[Out] a*(g*log(g*x + f)/(g^2*h^2 - 2*f*g*h*i + f^2*i^2) - g*log(i*x + h)/(g^2*h^2 - 2*f*g*h*i + f^2*i^2) + 1/(g*h^2 - f*h*i + (g*h*i - f*i^2)*x)) + b*integrate((log((e*x + d)^n) + log(c))/(g*i^2*x^3 + f*h^2 + (2*g*h*i + f*i^2)*x^2 + (g*h^2 + 2*f*h*i)*x), x)

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^2}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)(ix+h)^2}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^2,x, algorithm="giac")

[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)*(i*x + h)^2), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^2}\,dx = \int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{(f+g\,x)\,(h+i\,x)^2}\,dx$$

[In] int((a + b*log(c*(d + e*x)^n))/((f + g*x)*(h + i*x)^2),x)

[Out] int((a + b*log(c*(d + e*x)^n))/((f + g*x)*(h + i*x)^2), x)

# 3.223 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)^3} dx$



## Optimal result

Integrand size = 29, antiderivative size = 402

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)^3} dx = -\frac{ben}{2(eh-di)(gh-fi)(h+ix)} - \frac{begn\log(d+ex)}{(eh-di)(gh-fi)^2} - \frac{be^2n\log(d+ex)}{2(eh-di)^2(gh-fi)} + \frac{a+b\log(c(d+ex)^n)}{2(gh-fi)(h+ix)^2} + \frac{g(a+b\log(c(d+ex)^n))}{(gh-fi)^2(h+ix)} + \frac{g^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^3} + \frac{begn\log(h+ix)}{(eh-di)(gh-fi)^2} + \frac{be^2n\log(h+ix)}{2(eh-di)^2(gh-fi)} - \frac{g^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^3} + \frac{bg^2n\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^3} - \frac{bg^2n\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^3}$$

```
[Out] -1/2*b*e*n/(-d*i+e*h)/(-f*i+g*h)/(i*x+h)-b*e*g*n*ln(e*x+d)/(-d*i+e*h)/(-f*i
+g*h)^2-1/2*b*e^2*n*ln(e*x+d)/(-d*i+e*h)^2/(-f*i+g*h)+1/2*(a+b*ln(c*(e*x+d)
^n))/(-f*i+g*h)/(i*x+h)^2+g*(a+b*ln(c*(e*x+d)^n))/(-f*i+g*h)^2/(i*x+h)+g^2*
(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)^3+b*e*g*n*ln(i*x+
h)/(-d*i+e*h)/(-f*i+g*h)^2+1/2*b*e^2*n*ln(i*x+h)/(-d*i+e*h)^2/(-f*i+g*h)-g^
2*(a+b*ln(c*(e*x+d)^n))*ln(e*(i*x+h)/(-d*i+e*h))/(-f*i+g*h)^3+b*g^2*n*polyl
og(2,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)^3-b*g^2*n*polylog(2,-i*(e*x+d)/(-d*i
+e*h))/(-f*i+g*h)^3
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2465, 2441, 2440, 2438, 2442, 46, 36, 31}

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)(h+ix)^3}\,dx = \frac{g^2\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{(gh-fi)^3} - \frac{g^2\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log(c(d+ex)^n))}{(gh-fi)^3} + \frac{g(a+b\log(c(d+ex)^n))}{(h+ix)(gh-fi)^2} + \frac{a+b\log(c(d+ex)^n)}{2(h+ix)^2(gh-fi)} - \frac{be^2n\log(d+ex)}{2(eh-di)^2(gh-fi)} + \frac{be^2n\log(h+ix)}{2(eh-di)^2(gh-fi)} + \frac{bg^2n\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^3} - \frac{bg^2n\,\text{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^3} - \frac{ben}{2(h+ix)(eh-di)(gh-fi)} - \frac{begn\log(d+ex)}{(eh-di)(gh-fi)^2} + \frac{begn\log(h+ix)}{(eh-di)(gh-fi)^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/((f + g*x)*(h + i*x)^3),x]
```

```
[Out] -1/2*(b*e*n)/((e*h - d*i)*(g*h - f*i)*(h + i*x)) - (b*e*g*n*Log[d + e*x])/(
(e*h - d*i)*(g*h - f*i)^2) - (b*e^2*n*Log[d + e*x])/(2*(e*h - d*i)^2*(g*h -
 f*i)) + (a + b*Log[c*(d + e*x)^n])/(2*(g*h - f*i)*(h + i*x)^2) + (g*(a + b
*Log[c*(d + e*x)^n]))/((g*h - f*i)^2*(h + i*x)) + (g^2*(a + b*Log[c*(d + e*
x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/(g*h - f*i)^3 + (b*e*g*n*Log[h + i*x
])/((e*h - d*i)*(g*h - f*i)^2) + (b*e^2*n*Log[h + i*x])/(2*(e*h - d*i)^2*(g
*h - f*i)) - (g^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(h + i*x))/(e*h - d*i)]
)/(g*h - f*i)^3 + (b*g^2*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/(g*h -
 f*i)^3 - (b*g^2*n*PolyLog[2, -((i*(d + e*x))/(e*h - d*i))])/(g*h - f*i)^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \left(\frac{g^3(a+b\log(c(d+ex)^n))}{(gh-fi)^3(f+gx)} - \frac{i(a+b\log(c(d+ex)^n))}{(gh-fi)(h+ix)^3} - \frac{gi(a+b\log(c(d+ex)^n))}{(gh-fi)^2(h+ix)^2} - \frac{g^2i(a+b\log(c(d+ex)^n))}{(gh-fi)^3(h+ix)}\right) dx$$

$$= \frac{g^3 \int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{(gh-fi)^3} - \frac{(g^2 i)\int \frac{a+b\log(c(d+ex)^n)}{h+ix}\,dx}{(gh-fi)^3} - \frac{(gi)\int \frac{a+b\log(c(d+ex)^n)}{(h+ix)^2}\,dx}{(gh-fi)^2} - \frac{i\int \frac{a+b\log(c(d+ex)^n)}{(h+ix)^3}\,dx}{gh-fi}$$

$$= \frac{a+b\log\left(c(d+ex)^n\right)}{2(gh-fi)(h+ix)^2} + \frac{g(a+b\log\left(c(d+ex)^n\right))}{(gh-fi)^2(h+ix)} + \frac{g^2(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^3} - \frac{g^2(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^3} - \frac{(beg^2n)\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{(gh-fi)^3} + \frac{(beg^2n)\int \frac{\log\left(\frac{e(h+ix)}{eh-di}\right)}{d+ex}\,dx}{(gh-fi)^3} - \frac{(begn)\int \frac{1}{(d+ex)(h+ix)}\,dx}{(gh-fi)^2} - \frac{(ben)\int \frac{1}{(d+ex)(h+ix)^2}\,dx}{2(gh-fi)}$$

$$= \frac{a+b\log\left(c(d+ex)^n\right)}{2(gh-fi)(h+ix)^2} + \frac{g(a+b\log\left(c(d+ex)^n\right))}{(gh-fi)^2(h+ix)} + \frac{g^2(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^3} - \frac{g^2(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^3} - \frac{(bg^2n)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{(gh-fi)^3} + \frac{(bg^2n)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{ix}{eh-di}\right)}{x}\,dx, x, d+ex\right)}{(gh-fi)^3} - \frac{(be^2gn)\int \frac{1}{d+ex}\,dx}{(eh-di)(gh-fi)^2} + \frac{(begin)\int \frac{1}{h+ix}\,dx}{(eh-di)(gh-fi)^2} - \frac{(ben)\int \left(\frac{e^2}{(eh-di)^2(d+ex)} - \frac{i}{(eh-di)(h+ix)^2} - \frac{ei}{(eh-di)^2(h+ix)}\right)dx}{2(gh-fi)}$$

$$= -\frac{ben}{2(eh-di)(gh-fi)(h+ix)} - \frac{begn\log(d+ex)}{(eh-di)(gh-fi)^2} - \frac{be^2n\log(d+ex)}{2(eh-di)^2(gh-fi)} + \frac{a+b\log\left(c(d+ex)^n\right)}{2(gh-fi)(h+ix)^2} + \frac{g(a+b\log\left(c(d+ex)^n\right))}{(gh-fi)^2(h+ix)} + \frac{g^2(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^3} + \frac{begn\log(h+ix)}{(eh-di)(gh-fi)^2} + \frac{be^2n\log(h+ix)}{2(eh-di)^2(gh-fi)} - \frac{g^2(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^3} + \frac{bg^2n\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^3} - \frac{bg^2n\text{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^3}$$

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.77

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{(f+gx)(h+ix)^3}\,dx = \frac{\frac{(gh-fi)^2(a+b\log(c(d+ex)^n))}{(h+ix)^2}+\frac{2g(gh-fi)(a+b\log(c(d+ex)^n))}{h+ix}+2g^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)-\frac{2beg(gh-fi)n(\mathrm{l}}{}}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/((f + g*x)*(h + i*x)^3),x]
```

```
[Out] (((g*h - f*i)^2*(a + b*Log[c*(d + e*x)^n]))/(h + i*x)^2 + (2*g*(g*h - f*i)*
(a + b*Log[c*(d + e*x)^n]))/(h + i*x) + 2*g^2*(a + b*Log[c*(d + e*x)^n])*Lo
g[(e*(f + g*x))/(e*f - d*g)] - (2*b*e*g*(g*h - f*i)*n*(Log[d + e*x] - Log[h
 + i*x]))/(e*h - d*i) - (b*e*(g*h - f*i)^2*n*(e*h - d*i + e*(h + i*x)*Log[d
 + e*x] - e*(h + i*x)*Log[h + i*x]))/((e*h - d*i)^2*(h + i*x)) - 2*g^2*(a +
 b*Log[c*(d + e*x)^n])*Log[(e*(h + i*x))/(e*h - d*i)] + 2*b*g^2*n*PolyLog[2
, (g*(d + e*x))/(-(e*f) + d*g)] - 2*b*g^2*n*PolyLog[2, (i*(d + e*x))/(-(e*h
) + d*i)])/(2*(g*h - f*i)^3)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.17 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.92

| method | result |
|---|---|
| risch | $-\frac{b\ln((ex+d)^n)}{2(fi-gh)(ix+h)^2}+\frac{b\ln((ex+d)^n)g^2\ln(ix+h)}{(fi-gh)^3}+\frac{b\ln((ex+d)^n)g}{(fi-gh)^2(ix+h)}-\frac{b\ln((ex+d)^n)g^2\ln(gx+f)}{(fi-gh)^3}+\frac{ben\ln(ex+d)dgi}{(fi-gh)^2(di-eh)^2}+\frac{b}{2(}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*ln((e*x+d)^n)/(f*i-g*h)/(i*x+h)^2+b*ln((e*x+d)^n)*g^2/(f*i-g*h)^3*ln
(i*x+h)+b*ln((e*x+d)^n)*g/(f*i-g*h)^2/(i*x+h)-b*ln((e*x+d)^n)*g^2/(f*i-g*h)
^3*ln(g*x+f)+b*e*n/(f*i-g*h)^2/(d*i-e*h)^2*ln(e*x+d)*d*g*i+1/2*b*e^2*n/(f*i
-g*h)^2/(d*i-e*h)^2*ln(e*x+d)*f*i-3/2*b*e^2*n/(f*i-g*h)^2/(d*i-e*h)^2*ln(e*
x+d)*g*h-b*e*n/(f*i-g*h)^2/(d*i-e*h)^2*ln(i*x+h)*d*g*i-1/2*b*e^2*n/(f*i-g*h
)^2/(d*i-e*h)^2*ln(i*x+h)*f*i+3/2*b*e^2*n/(f*i-g*h)^2/(d*i-e*h)^2*ln(i*x+h)
*g*h-1/2*b*e*n/(f*i-g*h)^2/(d*i-e*h)/(i*x+h)*f*i+1/2*b*e*n/(f*i-g*h)^2/(d*i
-e*h)/(i*x+h)*g*h+b*n*g^2/(f*i-g*h)^3*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+
b*n*g^2/(f*i-g*h)^3*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b*n*g^2/(f*
i-g*h)^3*dilog(((i*x+h)*e+d*i-e*h)/(d*i-e*h))-b*n*g^2/(f*i-g*h)^3*ln(i*x+h)
*ln(((i*x+h)*e+d*i-e*h)/(d*i-e*h))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)
*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*
csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*
```

```
ln(c)+a)*(-1/2/(f*i-g*h)/(i*x+h)^2+g^2/(f*i-g*h)^3*ln(i*x+h)+g/(f*i-g*h)^2/
(i*x+h)-g^2/(f*i-g*h)^3*ln(g*x+f))
```

## Fricas [F]

$$\int \frac{a + b \log (c(d + ex)^n)}{(f + gx)(h + ix)^3} \, dx = \int \frac{b \log \left((ex + d)^n c\right) + a}{(gx + f)(ix + h)^3} \, dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^3,x, algorithm="fricas")
[Out] integral((b*log((e*x + d)^n*c) + a)/(g*i^3*x^4 + f*h^3 + (3*g*h*i^2 + f*i^3
)*x^3 + 3*(g*h^2*i + f*h*i^2)*x^2 + (g*h^3 + 3*f*h^2*i)*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \log (c(d + ex)^n)}{(f + gx)(h + ix)^3} \, dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)/(i*x+h)**3,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{a + b \log (c(d + ex)^n)}{(f + gx)(h + ix)^3} \, dx = \int \frac{b \log \left((ex + d)^n c\right) + a}{(gx + f)(ix + h)^3} \, dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^3,x, algorithm="maxima")
[Out] 1/2*(2*g^2*log(g*x + f)/(g^3*h^3 - 3*f*g^2*h^2*i + 3*f^2*g*h*i^2 - f^3*i^3)
 - 2*g^2*log(i*x + h)/(g^3*h^3 - 3*f*g^2*h^2*i + 3*f^2*g*h*i^2 - f^3*i^3) +
 (2*g*i*x + 3*g*h - f*i)/(g^2*h^4 - 2*f*g*h^3*i + f^2*h^2*i^2 + (g^2*h^2*i^
2 - 2*f*g*h*i^3 + f^2*i^4)*x^2 + 2*(g^2*h^3*i - 2*f*g*h^2*i^2 + f^2*h*i^3)*
x))*a + b*integrate((log((e*x + d)^n) + log(c))/(g*i^3*x^4 + f*h^3 + (3*g*h
*i^2 + f*i^3)*x^3 + 3*(g*h^2*i + f*h*i^2)*x^2 + (g*h^3 + 3*f*h^2*i)*x), x)
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^3}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)(ix+h)^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)/(i*x+h)^3,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)*(i*x + h)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)(h+ix)^3}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{\left(f+g\,x\right)\,{\left(h+i\,x\right)}^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/((f + g*x)*(h + i*x)^3),x)
[Out] int((a + b*log(c*(d + e*x)^n))/((f + g*x)*(h + i*x)^3), x)
```

# 3.224 $\int \frac{(h+ix)^2(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 469

$$\begin{aligned}
&\int \frac{(h+ix)^2\,(a+b\log\left(c(d+ex)^n\right))^2}{f+gx}\,dx\\
&=-\frac{2abi(eh-di)nx}{eg}-\frac{2abi(gh-fi)nx}{g^2}+\frac{2b^2i(eh-di)n^2x}{eg}+\frac{2b^2i(gh-fi)n^2x}{g^2}\\
&+\frac{b^2i^2n^2(d+ex)^2}{4e^2g}-\frac{2b^2i(eh-di)n(d+ex)\log\left(c(d+ex)^n\right)}{e^2g}\\
&-\frac{2b^2i(gh-fi)n(d+ex)\log\left(c(d+ex)^n\right)}{eg^2}-\frac{bi^2n(d+ex)^2\,(a+b\log\left(c(d+ex)^n\right))}{2e^2g}\\
&+\frac{i(eh-di)(d+ex)\,(a+b\log\left(c(d+ex)^n\right))^2}{e^2g}\\
&+\frac{i(gh-fi)(d+ex)\,(a+b\log\left(c(d+ex)^n\right))^2}{eg^2}+\frac{i^2(d+ex)^2\,(a+b\log\left(c(d+ex)^n\right))^2}{2e^2g}\\
&+\frac{(gh-fi)^2\,(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3}\\
&+\frac{2b(gh-fi)^2n(a+b\log\left(c(d+ex)^n\right))\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}\\
&-\frac{2b^2(gh-fi)^2n^2\,\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}
\end{aligned}$$

```
[Out] -2*a*b*i*(-d*i+e*h)*n*x/e/g-2*a*b*i*(-f*i+g*h)*n*x/g^2+2*b^2*i*(-d*i+e*h)*n
^2*x/e/g+2*b^2*i*(-f*i+g*h)*n^2*x/g^2+1/4*b^2*i^2*n^2*(e*x+d)^2/e^2/g-2*b^2
*i*(-d*i+e*h)*n*(e*x+d)*ln(c*(e*x+d)^n)/e^2/g-2*b^2*i*(-f*i+g*h)*n*(e*x+d)*
ln(c*(e*x+d)^n)/e/g^2-1/2*b*i^2*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2/g+i*(
```

```
-d*i+e*h)*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^2/g+i*(-f*i+g*h)*(e*x+d)*(a+b*l
n(c*(e*x+d)^n))^2/e/g^2+1/2*i^2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2/g+(-f
*i+g*h)^2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(g*x+f)/(-d*g+e*f))/g^3+2*b*(-f*i+g*
h)^2*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g^3-2*b^2*(-f
*i+g*h)^2*n^2*polylog(3,-g*(e*x+d)/(-d*g+e*f))/g^3
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {2465, 2436, 2333, 2332, 2443, 2481, 2421, 6724, 2448, 2437, 2342, 2341}

$$\begin{aligned}
&\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^2}{f+gx}\,dx \\
&= \frac{i(d+ex)(eh-di)(a+b\log(c(d+ex)^n))^2}{e^2g} \\
&\quad -\frac{bi^2n(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2g}+\frac{i^2(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^2g} \\
&\quad +\frac{2bn(gh-fi)^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^3} \\
&\quad +\frac{(gh-fi)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g^3} \\
&\quad +\frac{i(d+ex)(gh-fi)(a+b\log(c(d+ex)^n))^2}{eg^2}-\frac{2abinx(eh-di)}{eg} \\
&\quad -\frac{2abinx(gh-fi)}{g^2}-\frac{2b^2in(d+ex)(eh-di)\log(c(d+ex)^n)}{e^2g} \\
&\quad -\frac{2b^2in(d+ex)(gh-fi)\log(c(d+ex)^n)}{eg^2}+\frac{b^2i^2n^2(d+ex)^2}{4e^2g} \\
&\quad -\frac{2b^2n^2(gh-fi)^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}+\frac{2b^2in^2x(eh-di)}{eg}+\frac{2b^2in^2x(gh-fi)}{g^2}
\end{aligned}$$

```
[In] Int[((h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x),x]
```

```
[Out] (-2*a*b*i*(e*h - d*i)*n*x)/(e*g) - (2*a*b*i*(g*h - f*i)*n*x)/g^2 + (2*b^2*i
*(e*h - d*i)*n^2*x)/(e*g) + (2*b^2*i*(g*h - f*i)*n^2*x)/g^2 + (b^2*i^2*n^2*
(d + e*x)^2)/(4*e^2*g) - (2*b^2*i*(e*h - d*i)*n*(d + e*x)*Log[c*(d + e*x)^n
])/(e^2*g) - (2*b^2*i*(g*h - f*i)*n*(d + e*x)*Log[c*(d + e*x)^n])/(e*g^2) -
 (b*i^2*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^2*g) + (i*(e*h - d*i
)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e^2*g) + (i*(g*h - f*i)*(d + e*x
)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g^2) + (i^2*(d + e*x)^2*(a + b*Log[c*(d
+ e*x)^n])^2)/(2*e^2*g) + ((g*h - f*i)^2*(a + b*Log[c*(d + e*x)^n])^2*Log[(
```

```
e*(f + g*x))/(e*f - d*g)])/g^3 + (2*b*(g*h - f*i)^2*n*(a + b*Log[c*(d + e*x
)^n])*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g^3 - (2*b^2*(g*h - f*i)^2*
n^2*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))])/g^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{i(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{g^2}+\frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{g^2(f+gx)}+\frac{i(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{g}\right)dx$$

$$=\frac{i\int(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx}{g}+\frac{(i(gh-fi))\int\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx}{g^2}+\frac{(gh-fi)^2\int\frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx}{g^2}$$

$$=\frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3}+\frac{i\int\left(\frac{(eh-di)(a+b\log(c(d+ex)^n))^2}{e}+\frac{i(d+ex)(a+b\log(c(d+ex)^n))^2}{e}\right)\,dx}{g}+\frac{(i(gh-fi))\text{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+ex\right)}{eg^2}-\frac{\left(2be(gh-fi)^2n\right)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g^3}$$

$$=\frac{i(gh-fi)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2}+\frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3}+\frac{i^2\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx}{eg}+\frac{(i(eh-di))\int\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx}{eg}-\frac{(2bi(gh-fi)n)\text{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)\,dx,x,d+ex\right)}{eg^2}-\frac{\left(2b(gh-fi)^2n\right)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g^3}$$

$$= -\frac{2abi(gh-fi)nx}{g^2} + \frac{i(gh-fi)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2} + \frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} + \frac{2b(gh-fi)^2n(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3} + \frac{i^2\operatorname{Subst}\left(\int x(a+b\log\left(cx^n\right))^2\,dx,x,d+ex\right)}{e^2g} + \frac{(i(eh-di))\operatorname{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+ex\right)}{e^2g} - \frac{(2b^2i(gh-fi)n)\operatorname{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{eg^2} - \frac{(2b^2(gh-fi)^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g^3}$$

$$= -\frac{2abi(gh-fi)nx}{g^2} + \frac{2b^2i(gh-fi)n^2x}{g^2} - \frac{2b^2i(gh-fi)n(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} + \frac{i(eh-di)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g} + \frac{i(gh-fi)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2} + \frac{i^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g} + \frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} + \frac{2b(gh-fi)^2n(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3} - \frac{2b^2(gh-fi)^2n^2\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3} - \frac{(bi^2n)\operatorname{Subst}\left(\int x(a+b\log\left(cx^n\right))\,dx,x,d+ex\right)}{e^2g} - \frac{(2bi(eh-di)n)\operatorname{Subst}\left(\int(a+b\log\left(cx^n\right))\,dx,x,d+ex\right)}{e^2g}$$

$$
\begin{aligned}
&= -\frac{2abi(eh-di)nx}{eg} - \frac{2abi(gh-fi)nx}{g^2} + \frac{2b^2i(gh-fi)n^2x}{g^2} + \frac{b^2i^2n^2(d+ex)^2}{4e^2g} \\
&\quad - \frac{2b^2i(gh-fi)n(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} - \frac{bi^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2g} \\
&\quad + \frac{i(eh-di)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g} \\
&\quad + \frac{i(gh-fi)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2} \\
&\quad + \frac{i^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g} \\
&\quad + \frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} \\
&\quad + \frac{2b(gh-fi)^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3} \\
&\quad - \frac{2b^2(gh-fi)^2n^2\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3} \\
&\quad - \frac{\left(2b^2i(eh-di)n\right)\operatorname{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{e^2g}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2abi(eh-di)nx}{eg} - \frac{2abi(gh-fi)nx}{g^2} + \frac{2b^2i(eh-di)n^2x}{eg} \\
&\quad + \frac{2b^2i(gh-fi)n^2x}{g^2} + \frac{b^2i^2n^2(d+ex)^2}{4e^2g} - \frac{2b^2i(eh-di)n(d+ex)\log\left(c(d+ex)^n\right)}{e^2g} \\
&\quad - \frac{2b^2i(gh-fi)n(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} - \frac{bi^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2g} \\
&\quad + \frac{i(eh-di)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g} \\
&\quad + \frac{i(gh-fi)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2} \\
&\quad + \frac{i^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g} \\
&\quad + \frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} \\
&\quad + \frac{2b(gh-fi)^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3} \\
&\quad - \frac{2b^2(gh-fi)^2n^2\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 876, normalized size of antiderivative = 1.87

$$\int \frac{(h+ix)^2 (a+b\log (c(d+ex)^n))^2}{f+gx}\,dx$$
$$= \frac{4e^2gi(2gh-fi)x(a-bn\log(d+ex)+b\log (c(d+ex)^n))^2+2e^2g^2i^2x^2(a-bn\log(d+ex)+b\log (c(d+e$$

```
[In] Integrate[((h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x),x]
```

```
[Out] (4*e^2*g*i*(2*g*h - f*i)*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2
+ 2*e^2*g^2*i^2*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 4*e^2
*(g*h - f*i)^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x]
 + 8*b*e^2*g^2*h^2*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d +
 e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d
*g)]) + 2*b*i^2*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(e*g*(e*x*(
4*f - g*x) + 2*d*(2*f + g*x)) - 2*Log[d + e*x]*(g*(d + e*x)*(2*e*f + d*g -
e*g*x) - 2*e^2*f^2*Log[(e*(f + g*x))/(e*f - d*g)]) + 4*e^2*f^2*PolyLog[2, (
g*(d + e*x))/(-(e*f) + d*g)]) - 16*b*e*g*h*i*n*(a - b*n*Log[d + e*x] + b*Lo
g[c*(d + e*x)^n])*(-(g*(d + e*x)*(-1 + Log[d + e*x])) + e*f*(Log[d + e*x]*L
og[(e*(f + g*x))/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]))
+ 8*b^2*e*g*h*i*n^2*(g*(2*e*x - 2*(d + e*x)*Log[d + e*x] + (d + e*x)*Log[d
+ e*x]^2) - e*f*(Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d +
e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/
(-(e*f) + d*g)])) - b^2*i^2*n^2*(4*e*f*g*(2*e*x - 2*(d + e*x)*Log[d + e*x]
+ (d + e*x)*Log[d + e*x]^2) + g^2*(e*x*(6*d - e*x) + (-6*d^2 - 4*d*e*x + 2*
e^2*x^2)*Log[d + e*x] + 2*(d^2 - e^2*x^2)*Log[d + e*x]^2) - 4*e^2*f^2*(Log[
d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d
 + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)])) + 4
*b^2*e^2*g^2*h^2*n^2*(Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log
[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e
*x))/(-(e*f) + d*g)]))/(4*e^2*g^3)
```

## Maple [F]

$$\int \frac{(ix+h)^2 (a+b\ln (c(ex+d)^n))^2}{gx+f}dx$$

```
[In] int((i*x+h)^2*(a+b*ln(c*(e*x+d)^n))^2/(g*x+f),x)
```

```
[Out] int((i*x+h)^2*(a+b*ln(c*(e*x+d)^n))^2/(g*x+f),x)
```

## Fricas [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(ix+h)^2(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

[In] integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="fricas")

[Out] integral((a^2*i^2*x^2 + 2*a^2*h*i*x + a^2*h^2 + (b^2*i^2*x^2 + 2*b^2*h*i*x + b^2*h^2)*log((e*x + d)^n*c)^2 + 2*(a*b*i^2*x^2 + 2*a*b*h*i*x + a*b*h^2)*log((e*x + d)^n*c))/(g*x + f), x)

## Sympy [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2 (h+ix)^2}{f+gx}\,dx$$

[In] integrate((i*x+h)**2*(a+b*ln(c*(e*x+d)**n))**2/(g*x+f),x)

[Out] Integral((a + b*log(c*(d + e*x)**n))**2*(h + i*x)**2/(f + g*x), x)

## Maxima [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(ix+h)^2(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

[In] integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="maxima")

[Out] 2*a^2*h*i*(x/g - f*log(g*x + f)/g^2) + 1/2*a^2*i^2*(2*f^2*log(g*x + f)/g^3 + (g*x^2 - 2*f*x)/g^2) + a^2*h^2*log(g*x + f)/g + integrate((b^2*h^2*log(c)^2 + 2*a*b*h^2*log(c) + (b^2*i^2*log(c)^2 + 2*a*b*i^2*log(c))*x^2 + (b^2*i^2*x^2 + 2*b^2*h*i*x + b^2*h^2)*log((e*x + d)^n)^2 + 2*(b^2*h*i*log(c)^2 + 2*a*b*h*i*log(c))*x + 2*(b^2*h^2*log(c) + a*b*h^2 + (b^2*i^2*log(c) + a*b*i^2)*x^2 + 2*(b^2*h*i*log(c) + a*b*h*i)*x)*log((e*x + d)^n))/(g*x + f), x)

## Giac [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(ix+h)^2(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

```
[In] integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="giac")
[Out] integrate((i*x + h)^2*(b*log((e*x + d)^n*c) + a)^2/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(h+i\,x)^2\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{f+g\,x}\,dx$$

```
[In] int(((h + i*x)^2*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x),x)
[Out] int(((h + i*x)^2*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x), x)
```

# 3.225 $\int \frac{(h+ix)(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 215

$$\begin{aligned}\int \frac{(h+ix)(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx &= -\frac{2abinx}{g}+\frac{2b^2in^2x}{g}-\frac{2b^2in(d+ex)\log(c(d+ex)^n)}{eg}\\ &\quad+\frac{i(d+ex)(a+b\log(c(d+ex)^n))^2}{eg}+\frac{(gh-fi)(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2}\\ &\quad+\frac{2b(gh-fi)n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}\\ &\quad-\frac{2b^2(gh-fi)n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}\end{aligned}$$

```
[Out] -2*a*b*i*n*x/g+2*b^2*i*n^2*x/g-2*b^2*i*n*(e*x+d)*ln(c*(e*x+d)^n)/e/g+i*(e*x
+d)*(a+b*ln(c*(e*x+d)^n))^2/e/g+(-f*i+g*h)*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(g*
x+f)/(-d*g+e*f))/g^2+2*b*(-f*i+g*h)*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-g*(e
*x+d)/(-d*g+e*f))/g^2-2*b^2*(-f*i+g*h)*n^2*polylog(3,-g*(e*x+d)/(-d*g+e*f))
/g^2
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2465, 2436, 2333, 2332, 2443, 2481, 2421, 6724}

$$\begin{aligned}
&\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}dx\\
&=\frac{2bn(gh-fi)\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g^2}\\
&+\frac{(gh-fi)\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{g^2}\\
&+\frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg}-\frac{2abinx}{g}-\frac{2b^2in(d+ex)\log\left(c(d+ex)^n\right)}{eg}\\
&-\frac{2b^2n^2(gh-fi)\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}+\frac{2b^2in^2x}{g}
\end{aligned}$$

```
[In] Int[((h + i*x)*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x),x]
```

```
[Out] (-2*a*b*i*n*x)/g + (2*b^2*i*n^2*x)/g - (2*b^2*i*n*(d + e*x)*Log[c*(d + e*x)
^n])/(e*g) + (i*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g) + ((g*h - f*i
)*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(f + g*x))/(e*f - d*g)])/g^2 + (2*b*(
g*h - f*i)*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((g*(d + e*x))/(e*f - d
*g))])/g^2 - (2*b^2*(g*h - f*i)*n^2*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))
])/g^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left( \frac{i(a + b\log(c(d+ex)^n))^2}{g} + \frac{(gh - fi)(a + b\log(c(d+ex)^n))^2}{g(f+gx)} \right) dx$$

$$= \frac{i \int (a + b\log(c(d+ex)^n))^2 \, dx}{g} + \frac{(gh - fi) \int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx} \, dx}{g}$$

$$= \frac{(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} + \frac{i\text{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+ex\right)}{eg} - \frac{(2be(gh-fi)n)\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g^2}$$

$$= \frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg} + \frac{(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} - \frac{(2bin)\text{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)\,dx,x,d+ex\right)}{eg} - \frac{(2b(gh-fi)n)\text{Subst}\left(\int\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}dx,x,d+ex\right)}{g^2}$$

$$= -\frac{2abinx}{g} + \frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg} + \frac{(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} + \frac{2b(gh-fi)n\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^2} - \frac{\left(2b^2in\right)\text{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{eg} - \frac{\left(2b^2(gh-fi)n^2\right)\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}dx,x,d+ex\right)}{g^2}$$

$$= -\frac{2abinx}{g} + \frac{2b^2in^2x}{g} - \frac{2b^2in(d+ex)\log\left(c(d+ex)^n\right)}{eg} + \frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg} + \frac{(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} + \frac{2b(gh-fi)n(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^2} - \frac{2b^2(gh-fi)n^2\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^2}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 460 vs. $2(215) = 430$.

Time = 0.24 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.14

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx$$
$$= \frac{egix(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right))^2 + e(gh-fi)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2\log(}{}$$

```
[In] Integrate[((h + i*x)*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x),x]
```

```
[Out] (e*g*i*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + e*(g*h - f*i)*(a
 - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x] + 2*b*e*g*h*n*(a
 - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*Log[(e*(f + g*x))
/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) - 2*b*i*n*(a - b*
n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(-(g*(d + e*x)*(-1 + Log[d + e*x]))
+ e*f*(Log[d + e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + PolyLog[2, (g*(d + e*x
))/(-(e*f) + d*g)])) + b^2*i*n^2*(g*(2*e*x - 2*(d + e*x)*Log[d + e*x] + (d
+ e*x)*Log[d + e*x]^2) - e*f*(Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)]
 + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (
g*(d + e*x))/(-(e*f) + d*g)])) + b^2*e*g*h*n^2*(Log[d + e*x]^2*Log[(e*(f +
g*x))/(e*f - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)
] - 2*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]))/(e*g^2)
```

## Maple [F]

$$\int \frac{(ix+h)\left(a+b\ln\left(c(ex+d)^n\right)\right)^2}{gx+f}dx$$

[In] int((i*x+h)*(a+b*ln(c*(e*x+d)^n))^2/(g*x+f),x)

[Out] int((i*x+h)*(a+b*ln(c*(e*x+d)^n))^2/(g*x+f),x)

## Fricas [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx = \int \frac{(ix+h)(b\log\left((ex+d)^n c\right)+a)^2}{gx+f}\,dx$$

[In] integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="fricas")

[Out] integral((a^2*i*x + a^2*h + (b^2*i*x + b^2*h)*log((e*x + d)^n*c)^2 + 2*(a*b*i*x + a*b*h)*log((e*x + d)^n*c))/(g*x + f), x)

## Sympy [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx = \int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2(h+ix)}{f+gx}\,dx$$

[In] integrate((i*x+h)*(a+b*ln(c*(e*x+d)**n))**2/(g*x+f),x)

[Out] Integral((a + b*log(c*(d + e*x)**n))**2*(h + i*x)/(f + g*x), x)

## Maxima [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx = \int \frac{(ix+h)(b\log\left((ex+d)^n c\right)+a)^2}{gx+f}\,dx$$

[In] integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="maxima")

[Out] a^2*i*(x/g - f*log(g*x + f)/g^2) + a^2*h*log(g*x + f)/g + integrate((b^2*h*log(c)^2 + 2*a*b*h*log(c) + (b^2*i*x + b^2*h)*log((e*x + d)^n)^2 + (b^2*i*log(c)^2 + 2*a*b*i*log(c))*x + 2*(b^2*h*log(c) + a*b*h + (b^2*i*log(c) + a*b*i)*x)*log((e*x + d)^n))/(g*x + f), x)

## Giac [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx=\int \frac{(ix+h)(b\log\left((ex+d)^nc\right)+a)^2}{gx+f}\,dx$$

```
[In] integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="giac")
[Out] integrate((i*x + h)*(b*log((e*x + d)^n*c) + a)^2/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx}\,dx=\int \frac{(h+i\,x)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^2}{f+g\,x}\,dx$$

```
[In] int(((h + i*x)*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x),x)
[Out] int(((h + i*x)*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x), x)
```

# 3.226 $\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx} dx$



## Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx} dx = \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[Out] (a+b*ln(c*(e*x+d)^n))^2*ln(e*(g*x+f)/(-d*g+e*f))/g+2*b*n*(a+b*ln(c*(e*x+d)^
n))*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g-2*b^2*n^2*polylog(3,-g*(e*x+d)/(-d*g
+e*f))/g
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2443, 2481, 2421, 6724}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx} dx = \frac{2bn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g} + \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g} - \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(f + g*x))/(e*f - d*g)])/g + (2*b*n*(a
 + b*Log[c*(d + e*x)^n])*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g - (2*b
^2*n^2*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))])/g
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} - \frac{(2ben)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}\\
&\quad - \frac{(2bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{g}
\end{aligned}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{2bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{(2b^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{2b^2n^2\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.75

$$\int\frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx$$

$$= \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f+gx)+2bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))\left(\log(\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x),x]
```

```
[Out] ((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x] + 2*b*n*(a -
b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*Log[(e*(f + g*x))/(e
*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + b^2*n^2*(Log[d + e
*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*
x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]))/g
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.73 (sec) , antiderivative size = 737, normalized size of antiderivative = 6.64

| method | result |
|---|---|
| risch | $\frac{b^2\ln(g(ex+d)-dg+ef)\ln(ex+d)^2n^2}{g}-\frac{2b^2\ln(g(ex+d)-dg+ef)\ln((ex+d)^n)\ln(ex+d)n}{g}+\frac{b^2\ln(g(ex+d)-dg+ef)\ln((ex+d)^n)^2}{g}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*ln(g*(e*x+d)-d*g+e*f)/g*ln(e*x+d)^2*n^2-2*b^2*ln(g*(e*x+d)-d*g+e*f)/g*l
n((e*x+d)^n)*ln(e*x+d)*n+b^2*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)^2+b^2*n^
2/g*ln(e*x+d)^2*ln(1-g*(e*x+d)/(d*g-e*f))+2*b^2*n^2/g*ln(e*x+d)*polylog(2,g
*(e*x+d)/(d*g-e*f))-2*b^2*n^2/g*polylog(3,g*(e*x+d)/(d*g-e*f))-2*b^2*n^2*di
log((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln(e*x+d)+2*b^2*n*dilog((g*(e*x+d)-d*
g+e*f)/(-d*g+e*f))/g*ln((e*x+d)^n)-2*b^2*n^2*ln(e*x+d)^2*ln((g*(e*x+d)-d*g+
e*f)/(-d*g+e*f))/g+2*b^2*n*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*l
n((e*x+d)^n)+(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*
csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n
)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b*(ln((e*x+d)^n)*ln(g*x+f
)/g-1/g*n*e*(dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e+ln(g*x+f)*ln(((g*x+f)*e
+d*g-e*f)/(d*g-e*f))/e))+1/4*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*
(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*cs
gn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^2*ln(g*x+
f)/g
```

## Fricas [F]

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{f + gx} \, dx = \int \frac{(b\log((ex + d)^n c) + a)^2}{gx + f} \, dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="fricas")
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x +
 f), x)
```

## Sympy [F]

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{f + gx} \, dx = \int \frac{(a + b\log(c(d + ex)^n))^2}{f + gx} \, dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**2/(f + g*x), x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="maxima")

[Out] a^2*log(g*x + f)/g + integrate((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log((e*x + d)^n))/(g*x + f), x)

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{gx+f}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f),x, algorithm="giac")

[Out] integrate((b*log((e*x + d)^n*c) + a)^2/(g*x + f), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx = \int \frac{(a+b\ln(c(d+ex)^n))^2}{f+gx}\,dx$$

[In] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x),x)

[Out] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x), x)

# 3.227 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 264

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx &= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} \\ &\quad-\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} \\ &\quad+\frac{2bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} \\ &\quad-\frac{2bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi} \\ &\quad-\frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} \\ &\quad+\frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}\end{aligned}$$

```
[Out] (a+b*ln(c*(e*x+d)^n))^2*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)-(a+b*ln(c*(e*x+
d)^n))^2*ln(e*(i*x+h)/(-d*i+e*h))/(-f*i+g*h)+2*b*n*(a+b*ln(c*(e*x+d)^n))*po
lylog(2,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)-2*b*n*(a+b*ln(c*(e*x+d)^n))*polyl
og(2,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)-2*b^2*n^2*polylog(3,-g*(e*x+d)/(-d*g
+e*f))/(-f*i+g*h)+2*b^2*n^2*polylog(3,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2465, 2443, 2481, 2421, 6724}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx = \frac{2bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{gh-fi} - \frac{2bn\,\text{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)(a+b\log(c(d+ex)^n))}{gh-fi} + \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{gh-fi} - \frac{\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log(c(d+ex)^n))^2}{gh-fi} - \frac{2b^2n^2\,\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} + \frac{2b^2n^2\,\text{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/((f + g*x)*(h + i*x)),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(f + g*x))/(e*f - d*g)])/(g*h - f*i) -
 ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(h + i*x))/(e*h - d*i)])/(g*h - f*i)
+ (2*b*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))
])/(g*h - f*i) - (2*b*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((i*(d + e*x
))/(e*h - d*i))])/(g*h - f*i) - (2*b^2*n^2*PolyLog[3, -((g*(d + e*x))/(e*f
- d*g))])/(g*h - f*i) + (2*b^2*n^2*PolyLog[3, -((i*(d + e*x))/(e*h - d*i))]
)/(g*h - f*i)
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
```

```
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{g(a+b\log(c(d+ex)^n))^2}{(gh-fi)(f+gx)} - \frac{i(a+b\log(c(d+ex)^n))^2}{(gh-fi)(h+ix)}\right) dx$$

$$= \frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx}{gh-fi} - \frac{i\int \frac{(a+b\log(c(d+ex)^n))^2}{h+ix}\,dx}{gh-fi}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} - \frac{(2ben)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{gh-fi} + \frac{(2ben)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(h+ix)}{eh-di}\right)}{d+ex}\,dx}{gh-fi}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} - \frac{(2bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}dx,x,d+ex\right)}{gh-fi} + \frac{(2bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{eh-di}{e}+\frac{ix}{e}\right)}{eh-di}\right)}{x}dx,x,d+ex\right)}{gh-fi}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} + \frac{2bn(a+b\log(c(d+ex)^n))\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} - \frac{2bn(a+b\log(c(d+ex)^n))\text{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{gh-fi} - \frac{(2b^2n^2)\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}dx,x,d+ex\right)}{gh-fi} + \frac{(2b^2n^2)\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{ix}{eh-di}\right)}{x}dx,x,d+ex\right)}{gh-fi}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} + \frac{2bn(a+b\log(c(d+ex)^n))\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} - \frac{2bn(a+b\log(c(d+ex)^n))\text{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{gh-fi} - \frac{2b^2n^2\text{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} + \frac{2b^2n^2\text{Li}_3\left(-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx$$
$$= \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f+gx)-(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(h-}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/((f + g*x)*(h + i*x)),x]
```

```
[Out] ((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x] - (a - b*n*Lo
g[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[h + i*x] + 2*b*n*(a - b*n*Log[d +
e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*(Log[(e*(f + g*x))/(e*f - d*g)]
- Log[(e*(h + i*x))/(e*h - d*i)]) + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)
] - PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)]) + b^2*n^2*(Log[d + e*x]^2*Log
[(e*(f + g*x))/(e*f - d*g)] - Log[d + e*x]^2*Log[(e*(h + i*x))/(e*h - d*i)]
 + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*Log[d + e*x]
*PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)] - 2*PolyLog[3, (g*(d + e*x))/(-(e
*f) + d*g)] + 2*PolyLog[3, (i*(d + e*x))/(-(e*h) + d*i)]))/(g*h - f*i)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.90 (sec) , antiderivative size = 1427, normalized size of antiderivative = 5.41

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1427 |

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h),x,method=_RETURNVERBOSE)
```

```
[Out] b^2/(f*i-g*h)*ln(i*(e*x+d)-d*i+e*h)*ln(e*x+d)^2*n^2-2*b^2/(f*i-g*h)*ln(i*(e
*x+d)-d*i+e*h)*ln((e*x+d)^n)*ln(e*x+d)*n+b^2/(f*i-g*h)*ln(i*(e*x+d)-d*i+e*h
)*ln((e*x+d)^n)^2-b^2/(f*i-g*h)*ln(g*(e*x+d)-d*g+e*f)*ln(e*x+d)^2*n^2+2*b^2
/(f*i-g*h)*ln(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)*ln(e*x+d)*n-b^2/(f*i-g*h)*ln
(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)^2+b^2*n^2/(f*i-g*h)*ln(e*x+d)^2*ln(1+i*(e
*x+d)/(-d*i+e*h))+2*b^2*n^2/(f*i-g*h)*ln(e*x+d)*polylog(2,-i*(e*x+d)/(-d*i+
e*h))-2*b^2*n^2/(f*i-g*h)*polylog(3,-i*(e*x+d)/(-d*i+e*h))-b^2*n^2/(f*i-g*h
)*ln(e*x+d)^2*ln(1+g*(e*x+d)/(-d*g+e*f))-2*b^2*n^2/(f*i-g*h)*ln(e*x+d)*poly
log(2,-g*(e*x+d)/(-d*g+e*f))+2*b^2*n^2/(f*i-g*h)*polylog(3,-g*(e*x+d)/(-d*g
+e*f))-2*b^2*n^2/(f*i-g*h)*dilog((i*(e*x+d)-d*i+e*h)/(-d*i+e*h))*ln(e*x+d)+
2*b^2*n/(f*i-g*h)*dilog((i*(e*x+d)-d*i+e*h)/(-d*i+e*h))*ln((e*x+d)^n)-2*b^2
*n^2/(f*i-g*h)*ln(e*x+d)^2*ln((i*(e*x+d)-d*i+e*h)/(-d*i+e*h))+2*b^2*n/(f*i-
g*h)*ln(e*x+d)*ln((i*(e*x+d)-d*i+e*h)/(-d*i+e*h))*ln((e*x+d)^n)+2*b^2*n^2/(
```

```
f*i-g*h)*dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln(e*x+d)-2*b^2*n/(f*i-g*h)*
dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln((e*x+d)^n)+2*b^2*n^2/(f*i-g*h)*ln(
e*x+d)^2*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))-2*b^2*n/(f*i-g*h)*ln(e*x+d)*ln(
(g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln((e*x+d)^n)+(-I*b*Pi*csgn(I*c*(e*x+d)^n)*
csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csg
n(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(
c)+2*a)*b*(ln((e*x+d)^n)/(f*i-g*h)*ln(i*x+h)-ln((e*x+d)^n)/(f*i-g*h)*ln(g*x
+f)-e*n*(1/(f*i-g*h)*(dilog(((i*x+h)*e+d*i-e*h)/(d*i-e*h))/e+ln(i*x+h)*ln((
(i*x+h)*e+d*i-e*h)/(d*i-e*h))/e)-1/(f*i-g*h)*(dilog(((g*x+f)*e+d*g-e*f)/(d*
g-e*f))/e+ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e)))+1/4*(-I*b*Pi*csg
n(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d
)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+
d)^n)^3*b+2*b*ln(c)+2*a)^2*(1/(f*i-g*h)*ln(i*x+h)-1/(f*i-g*h)*ln(g*x+f))
```

## Fricas [F]

$$\int \frac{(a+b\log{(c(d+ex)^n)})^2}{(f+gx)(h+ix)}\,dx = \int \frac{(b\log{((ex+d)^n c)}+a)^2}{(gx+f)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h),x, algorithm="fricas")
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*i*x
^2 + f*h + (g*h + f*i)*x), x)
```

## Sympy [F]

$$\int \frac{(a+b\log{(c(d+ex)^n)})^2}{(f+gx)(h+ix)}\,dx = \int \frac{(a+b\log{(c(d+ex)^n)})^2}{(f+gx)\,(h+ix)}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)/(i*x+h),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**2/((f + g*x)*(h + i*x)), x)
```

## Maxima [F]

$$\int \frac{(a+b\log{(c(d+ex)^n)})^2}{(f+gx)(h+ix)}\,dx = \int \frac{(b\log{((ex+d)^n c)}+a)^2}{(gx+f)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h),x, algorithm="maxima")
[Out] a^2*(log(g*x + f)/(g*h - f*i) - log(i*x + h)/(g*h - f*i)) + integrate((b^2*
log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log
((e*x + d)^n))/(g*i*x^2 + f*h + (g*h + f*i)*x), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/((g*x + f)*(i*x + h)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)}\,dx = \int \frac{(a+b\ln(c\,(d+e\,x)^n))^2}{(f+g\,x)\,(h+i\,x)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/((f + g*x)*(h + i*x)),x)
[Out] int((a + b*log(c*(d + e*x)^n))^2/((f + g*x)*(h + i*x)), x)
```

# 3.228 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 427

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx = -\frac{i(d+ex)(a+b\log(c(d+ex)^n))^2}{(eh-di)(gh-fi)(h+ix)} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2} + \frac{2ben(a+b\log(c(d+ex)^n))\log\left(\frac{e(h+ix)}{eh-di}\right)}{(eh-di)(gh-fi)} - \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2} + \frac{2bgn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} + \frac{2b^2en^2\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)} - \frac{2bgn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2} - \frac{2b^2gn^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} + \frac{2b^2gn^2\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$

```
[Out] -i*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/(-d*i+e*h)/(-f*i+g*h)/(i*x+h)+g*(a+b*ln(
c*(e*x+d)^n))^2*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)^2+2*b*e*n*(a+b*ln(c*(e*
```

```
x+d)^n))*ln(e*(i*x+h)/(-d*i+e*h))/(-d*i+e*h)/(-f*i+g*h)-g*(a+b*ln(c*(e*x+d)
^n))^2*ln(e*(i*x+h)/(-d*i+e*h))/(-f*i+g*h)^2+2*b*g*n*(a+b*ln(c*(e*x+d)^n))*
polylog(2,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)^2+2*b^2*e*n^2*polylog(2,-i*(e*x
+d)/(-d*i+e*h))/(-d*i+e*h)/(-f*i+g*h)-2*b*g*n*(a+b*ln(c*(e*x+d)^n))*polylog
(2,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)^2-2*b^2*g*n^2*polylog(3,-g*(e*x+d)/(-d
*g+e*f))/(-f*i+g*h)^2+2*b^2*g*n^2*polylog(3,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*
h)^2
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2465, 2443, 2481, 2421, 6724, 2444, 2441, 2440, 2438}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx = \frac{2bgn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{(gh-fi)^2} - \frac{2bgn\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)(a+b\log(c(d+ex)^n))}{(gh-fi)^2} + \frac{2ben\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log(c(d+ex)^n))}{(eh-di)(gh-fi)} - \frac{i(d+ex)(a+b\log(c(d+ex)^n))^2}{(h+ix)(eh-di)(gh-fi)} + \frac{g\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{(gh-fi)^2} - \frac{g\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log(c(d+ex)^n))^2}{(gh-fi)^2} + \frac{2b^2en^2\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)} - \frac{2b^2gn^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} + \frac{2b^2gn^2\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/((f + g*x)*(h + i*x)^2),x]
```

```
[Out] -((i*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/((e*h - d*i)*(g*h - f*i)*(h +
i*x))) + (g*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(f + g*x))/(e*f - d*g)])/(g
*h - f*i)^2 + (2*b*e*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(h + i*x))/(e*h -
```

```
d*i)])/((e*h - d*i)*(g*h - f*i)) - (g*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(
h + i*x))/(e*h - d*i)])/(g*h - f*i)^2 + (2*b*g*n*(a + b*Log[c*(d + e*x)^n])
*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i)^2 + (2*b^2*e*n^2*Pol
yLog[2, -((i*(d + e*x))/(e*h - d*i))])/((e*h - d*i)*(g*h - f*i)) - (2*b*g*n
*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((i*(d + e*x))/(e*h - d*i))])/(g*h
- f*i)^2 - (2*b^2*g*n^2*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*
i)^2 + (2*b^2*g*n^2*PolyLog[3, -((i*(d + e*x))/(e*h - d*i))])/(g*h - f*i)^2
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
```

```
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{g^2(a+b\log(c(d+ex)^n))^2}{(gh-fi)^2(f+gx)} - \frac{i(a+b\log(c(d+ex)^n))^2}{(gh-fi)(h+ix)^2} - \frac{gi(a+b\log(c(d+ex)^n))^2}{(gh-fi)^2(h+ix)}\right) dx$$

$$= \frac{g^2 \int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx}\,dx}{(gh-fi)^2} - \frac{(gi)\int \frac{(a+b\log(c(d+ex)^n))^2}{h+ix}\,dx}{(gh-fi)^2} - \frac{i\int \frac{(a+b\log(c(d+ex)^n))^2}{(h+ix)^2}\,dx}{gh-fi}$$

$$= -\frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(eh-di)(gh-fi)(h+ix)} + \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2} - \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2} - \frac{(2begn)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{(gh-fi)^2} + \frac{(2begn)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(h+ix)}{eh-di}\right)}{d+ex}dx}{(gh-fi)^2} + \frac{(2bein)\int\frac{a+b\log(c(d+ex)^n)}{h+ix}dx}{(eh-di)(gh-fi)}$$

$$= -\frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(eh-di)(gh-fi)(h+ix)} + \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2} + \frac{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(h+ix)}{eh-di}\right)}{(eh-di)(gh-fi)} - \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2} - \frac{(2bgn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}dx,x,d+ex\right)}{(gh-fi)^2} + \frac{(2bgn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{eh-di}{e}+\frac{ix}{e}\right)}{eh-di}\right)}{x}dx,x,d+ex\right)}{(gh-fi)^2} - \frac{(2b^2e^2n^2)\int\frac{\log\left(\frac{e(h+ix)}{eh-di}\right)}{d+ex}dx}{(eh-di)(gh-fi)}$$

$$= -\frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(eh-di)(gh-fi)(h+ix)} + \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2}$$
$$+ \frac{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(h+ix)}{eh-di}\right)}{(eh-di)(gh-fi)}$$
$$- \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2}$$
$$+ \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2}$$
$$- \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$
$$- \frac{(2b^2gn^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{(gh-fi)^2}$$
$$+ \frac{(2b^2gn^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{ix}{eh-di}\right)}{x}\,dx,x,d+ex\right)}{(gh-fi)^2}$$
$$- \frac{(2b^2en^2)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{ix}{eh-di}\right)}{x}\,dx,x,d+ex\right)}{(eh-di)(gh-fi)}$$

$$= -\frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{(eh-di)(gh-fi)(h+ix)} + \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2}$$
$$+ \frac{2ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(h+ix)}{eh-di}\right)}{(eh-di)(gh-fi)}$$
$$- \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2}$$
$$+ \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2}$$
$$+ \frac{2b^2en^2\operatorname{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)} - \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$
$$- \frac{2b^2gn^2\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} + \frac{2b^2gn^2\operatorname{Li}_3\left(-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.48

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx$$
$$= \frac{(eh-di)(gh-fi)(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2+g(eh-di)(h+ix)(a-bn\log(d+ex)+b}{}$$

[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/((f + g*x)*(h + i*x)^2),x]

[Out] ((e*h - d*i)*(g*h - f*i)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + g*(e*h - d*i)*(h + i*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x] - g*(e*h - d*i)*(h + i*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[h + i*x] - 2*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((g*h - f*i)*(i*(d + e*x)*Log[d + e*x] - e*(h + i*x)*Log[h + i*x]) - g*(e*h - d*i)*(h + i*x)*(Log[d + e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + g*(e*h - d*i)*(h + i*x)*(Log[d + e*x]*Log[(e*(h + i*x))/(e*h - d*i)] + PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)])) - b^2*n^2*((g*h - f*i)*(Log[d + e*x]*(i*(d + e*x)*Log[d + e*x] - 2*e*(h + i*x)*Log[(e*(h + i*x))/(e*h - d*i)]) - 2*e*(h + i*x)*PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)]) - g*(e*h - d*i)*(h + i*x)*(Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]) + g*(e*h - d*i)*(h + i*x)*(Log[d + e*x]^2*Log[(e*(h + i*x))/(e*h - d*i)] + 2*Log[d + e*x]*PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)] - 2*PolyLog[3, (i*(d + e*x))/(-(e*h) + d*i)])))/((e*h - d*i)*(g*h - f*i)^2*(h + i*x))

## Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))^2}{(gx+f)(ix+h)^2}dx$$

[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h)^2,x)

[Out] int((a+b*ln(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h)^2,x)

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)(ix+h)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*i^2
*x^3 + f*h^2 + (2*g*h*i + f*i^2)*x^2 + (g*h^2 + 2*f*h*i)*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x+f)/(i*x+h)**2,x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)(ix+h)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h)^2,x, algorithm="maxima")
```

```
[Out] a^2*(g*log(g*x + f)/(g^2*h^2 - 2*f*g*h*i + f^2*i^2) - g*log(i*x + h)/(g^2*h
^2 - 2*f*g*h*i + f^2*i^2) + 1/(g*h^2 - f*h*i + (g*h*i - f*i^2)*x)) + integr
ate((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) +
 a*b)*log((e*x + d)^n))/(g*i^2*x^3 + f*h^2 + (2*g*h*i + f*i^2)*x^2 + (g*h^2
 + 2*f*h*i)*x), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx+f)(ix+h)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x+f)/(i*x+h)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/((g*x + f)*(i*x + h)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx)(h+ix)^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d+e\,x\right)^n\right)\right)^2}{\left(f+g\,x\right)\,\left(h+i\,x\right)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/((f + g*x)*(h + i*x)^2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^2/((f + g*x)*(h + i*x)^2), x)
```

# 3.229 $\int \frac{(h+ix)^2(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$

## Optimal result

Integrand size = 31, antiderivative size = 660

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$$
$$= \frac{6ab^2i(eh-di)n^2x}{eg}+\frac{6ab^2i(gh-fi)n^2x}{g^2}-\frac{6b^3i(eh-di)n^3x}{eg}-\frac{6b^3i(gh-fi)n^3x}{g^2}$$
$$-\frac{3b^3i^2n^3(d+ex)^2}{8e^2g}+\frac{6b^3i(eh-di)n^2(d+ex)\log(c(d+ex)^n)}{e^2g}$$
$$+\frac{6b^3i(gh-fi)n^2(d+ex)\log(c(d+ex)^n)}{eg^2}+\frac{3b^2i^2n^2(d+ex)^2(a+b\log(c(d+ex)^n))}{4e^2g}$$
$$-\frac{3bi(eh-di)n(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2g}$$
$$-\frac{3bi(gh-fi)n(d+ex)(a+b\log(c(d+ex)^n))^2}{eg^2}$$
$$-\frac{3bi^2n(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2g}+\frac{i(eh-di)(d+ex)(a+b\log(c(d+ex)^n))^3}{e^2g}$$
$$+\frac{i(gh-fi)(d+ex)(a+b\log(c(d+ex)^n))^3}{eg^2}+\frac{i^2(d+ex)^2(a+b\log(c(d+ex)^n))^3}{2e^2g}$$
$$+\frac{(gh-fi)^2(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3}$$
$$+\frac{3b(gh-fi)^2n(a+b\log(c(d+ex)^n))^2\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$
$$-\frac{6b^2(gh-fi)^2n^2(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$
$$+\frac{6b^3(gh-fi)^2n^3\,\mathrm{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$

```
[Out] 6*a*b^2*i*(-d*i+e*h)*n^2*x/e/g+6*a*b^2*i*(-f*i+g*h)*n^2*x/g^2-6*b^3*i*(-d*i
+e*h)*n^3*x/e/g-6*b^3*i*(-f*i+g*h)*n^3*x/g^2-3/8*b^3*i^2*n^3*(e*x+d)^2/e^2/
g+6*b^3*i*(-d*i+e*h)*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e^2/g+6*b^3*i*(-f*i+g*h)*n
^2*(e*x+d)*ln(c*(e*x+d)^n)/e/g^2+3/4*b^2*i^2*n^2*(e*x+d)^2*(a+b*ln(c*(e*x+d
)^n))/e^2/g-3*b*i*(-d*i+e*h)*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^2/g-3*b*i*
(-f*i+g*h)*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e/g^2-3/4*b*i^2*n*(e*x+d)^2*(a
+b*ln(c*(e*x+d)^n))^2/e^2/g+i*(-d*i+e*h)*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e^
2/g+i*(-f*i+g*h)*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e/g^2+1/2*i^2*(e*x+d)^2*(a
+b*ln(c*(e*x+d)^n))^3/e^2/g+(-f*i+g*h)^2*(a+b*ln(c*(e*x+d)^n))^3*ln(e*(g*x+
f)/(-d*g+e*f))/g^3+3*b*(-f*i+g*h)^2*n*(a+b*ln(c*(e*x+d)^n))^2*polylog(2,-g*
(e*x+d)/(-d*g+e*f))/g^3-6*b^2*(-f*i+g*h)^2*n^2*(a+b*ln(c*(e*x+d)^n))*polylo
```

```
g(3,-g*(e*x+d)/(-d*g+e*f))/g^3+6*b^3*(-f*i+g*h)^2*n^3*polylog(4,-g*(e*x+d)/
(-d*g+e*f))/g^3
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {2465, 2436, 2333, 2332, 2443, 2481, 2421, 2430, 6724, 2448, 2437, 2342, 2341}

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^3}{f+gx} \, dx$$
$$= \frac{3b^2i^2n^2(d+ex)^2 (a+b\log(c(d+ex)^n))}{4e^2g} - \frac{6b^2n^2(gh-fi)^2 \operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^3} + \frac{6ab^2in^2x(eh-di)}{eg} + \frac{6ab^2in^2x(gh-fi)}{g^2} - \frac{3bin(d+ex)(eh-di)(a+b\log(c(d+ex)^n))^2}{e^2g} + \frac{i(d+ex)(eh-di)(a+b\log(c(d+ex)^n))^3}{e^2g} - \frac{3bi^2n(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2g} + \frac{i^2(d+ex)^2(a+b\log(c(d+ex)^n))^3}{2e^2g} + \frac{3bn(gh-fi)^2 \operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g^3} + \frac{(gh-fi)^2\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g^3} - \frac{3bin(d+ex)(gh-fi)(a+b\log(c(d+ex)^n))^2}{eg^2} + \frac{i(d+ex)(gh-fi)(a+b\log(c(d+ex)^n))^3}{eg^2} + \frac{6b^3in^2(d+ex)(eh-di)\log(c(d+ex)^n)}{e^2g} + \frac{6b^3in^2(d+ex)(gh-fi)\log(c(d+ex)^n)}{eg^2} - \frac{3b^3i^2n^3(d+ex)^2}{8e^2g} + \frac{6b^3n^3(gh-fi)^2 \operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g^3} - \frac{6b^3in^3x(eh-di)}{eg} - \frac{6b^3in^3x(gh-fi)}{g^2}$$

```
[In] Int[((h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^3)/(f + g*x),x]
```

```
[Out] (6*a*b^2*i*(e*h - d*i)*n^2*x)/(e*g) + (6*a*b^2*i*(g*h - f*i)*n^2*x)/g^2 - (
6*b^3*i*(e*h - d*i)*n^3*x)/(e*g) - (6*b^3*i*(g*h - f*i)*n^3*x)/g^2 - (3*b^3
*i^2*n^3*(d + e*x)^2)/(8*e^2*g) + (6*b^3*i*(e*h - d*i)*n^2*(d + e*x)*Log[c*
(d + e*x)^n])/(e^2*g) + (6*b^3*i*(g*h - f*i)*n^2*(d + e*x)*Log[c*(d + e*x)^
```

```
n])/(e*g^2) + (3*b^2*i^2*n^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(4*e^2
*g) - (3*b*i*(e*h - d*i)*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e^2*g)
- (3*b*i*(g*h - f*i)*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g^2) - (3
*b*i^2*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(4*e^2*g) + (i*(e*h - d*
i)*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/(e^2*g) + (i*(g*h - f*i)*(d + e*
x)*(a + b*Log[c*(d + e*x)^n])^3)/(e*g^2) + (i^2*(d + e*x)^2*(a + b*Log[c*(d
 + e*x)^n])^3)/(2*e^2*g) + ((g*h - f*i)^2*(a + b*Log[c*(d + e*x)^n])^3*Log[
(e*(f + g*x))/(e*f - d*g)])/g^3 + (3*b*(g*h - f*i)^2*n*(a + b*Log[c*(d + e*
x)^n])^2*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g^3 - (6*b^2*(g*h - f*i)
^2*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))])
/g^3 + (6*b^3*(g*h - f*i)^2*n^3*PolyLog[4, -((g*(d + e*x))/(e*f - d*g))])/g
^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{i(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{g^2}+\frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{g^2(f+gx)}+\frac{i(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{g}\right)dx$$

$$=\frac{i\int(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx}{g}+\frac{(i(gh-fi))\int\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx}{g^2}+\frac{(gh-fi)^2\int\frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx}{g^2}$$

$$=\frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3}+\frac{i\int\left(\frac{(eh-di)(a+b\log(c(d+ex)^n))^3}{e}+\frac{i(d+ex)(a+b\log(c(d+ex)^n))^3}{e}\right)dx}{g}+\frac{(i(gh-fi))\text{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+ex\right)}{eg^2}-\frac{(3be(gh-fi)^2n)\int\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g^3}$$

$$= \frac{i(gh-fi)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{eg^2} + \frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} + \frac{i^2\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx}{eg} + \frac{(i(eh-di))\int\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx}{eg} - \frac{(3bi(gh-fi)n)\text{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+ex\right)}{eg^2} - \frac{(3b(gh-fi)^2n)\,\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g^3}$$

$$= -\frac{3bi(gh-fi)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2} + \frac{i(gh-fi)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{eg^2} + \frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} + \frac{3b(gh-fi)^2n(a+b\log\left(c(d+ex)^n\right))^2\,\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3} + \frac{i^2\text{Subst}\left(\int x(a+b\log\left(cx^n\right))^3\,dx,x,d+ex\right)}{e^2g} + \frac{(i(eh-di))\text{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+ex\right)}{e^2g} + \frac{(6b^2i(gh-fi)n^2)\,\text{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)\,dx,x,d+ex\right)}{eg^2} - \frac{(6b^2(gh-fi)^2n^2)\,\text{Subst}\left(\int\frac{(a+b\log(cx^n))\text{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g^3}$$

$$
\begin{aligned}
&= \frac{6ab^2i(gh-fi)n^2x}{g^2} - \frac{3bi(gh-fi)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2} \\
&+ \frac{i(eh-di)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2g} \\
&+ \frac{i(gh-fi)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{eg^2} \\
&+ \frac{i^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2e^2g} \\
&+ \frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} \\
&+ \frac{3b(gh-fi)^2n\left(a+b\log\left(c(d+ex)^n\right)\right)^2\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3} \\
&- \frac{6b^2(gh-fi)^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3} \\
&- \frac{(3bi^2n)\operatorname{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+ex\right)}{2e^2g} \\
&- \frac{(3bi(eh-di)n)\operatorname{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+ex\right)}{e^2g} \\
&+ \frac{(6b^3i(gh-fi)n^2)\operatorname{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{eg^2} \\
&+ \frac{(6b^3(gh-fi)^2n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g^3}
\end{aligned}
$$

$$= \frac{6ab^2i(gh-fi)n^2x}{g^2} - \frac{6b^3i(gh-fi)n^3x}{g^2} + \frac{6b^3i(gh-fi)n^2(d+ex)\log\left(c(d+ex)^n\right)}{eg^2}$$
$$- \frac{3bi(eh-di)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g}$$
$$- \frac{3bi(gh-fi)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2}$$
$$- \frac{3bi^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4e^2g}$$
$$+ \frac{i(eh-di)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2g}$$
$$+ \frac{i(gh-fi)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{eg^2}$$
$$+ \frac{i^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2e^2g}$$
$$+ \frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3}$$
$$+ \frac{3b(gh-fi)^2n\left(a+b\log\left(c(d+ex)^n\right)\right)^2\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$
$$- \frac{6b^2(gh-fi)^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$
$$+ \frac{6b^3(gh-fi)^2n^3\operatorname{Li}_4\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$
$$+ \frac{(3b^2i^2n^2)\operatorname{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)\,dx,x,d+ex\right)}{2e^2g}$$
$$+ \frac{(6b^2i(eh-di)n^2)\operatorname{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)\,dx,x,d+ex\right)}{e^2g}$$

$$
\begin{aligned}
&= \frac{6ab^2i(eh-di)n^2x}{eg} + \frac{6ab^2i(gh-fi)n^2x}{g^2} - \frac{6b^3i(gh-fi)n^3x}{g^2} \\
&- \frac{3b^3i^2n^3(d+ex)^2}{8e^2g} + \frac{6b^3i(gh-fi)n^2(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} \\
&+ \frac{3b^2i^2n^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{4e^2g} \\
&- \frac{3bi(eh-di)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g} \\
&- \frac{3bi(gh-fi)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2} \\
&- \frac{3bi^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4e^2g} \\
&+ \frac{i(eh-di)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2g} \\
&+ \frac{i(gh-fi)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{eg^2} \\
&+ \frac{i^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2e^2g} \\
&+ \frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} \\
&+ \frac{3b(gh-fi)^2n\left(a+b\log\left(c(d+ex)^n\right)\right)^2\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3} \\
&- \frac{6b^2(gh-fi)^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3} \\
&+ \frac{6b^3(gh-fi)^2n^3\operatorname{Li}_4\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3} \\
&+ \frac{\left(6b^3i(eh-di)n^2\right)\operatorname{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{e^2g}
\end{aligned}
$$

$$= \frac{6ab^2i(eh-di)n^2x}{eg} + \frac{6ab^2i(gh-fi)n^2x}{g^2} - \frac{6b^3i(eh-di)n^3x}{eg} - \frac{6b^3i(gh-fi)n^3x}{g^2}$$
$$- \frac{3b^3i^2n^3(d+ex)^2}{8e^2g} + \frac{6b^3i(eh-di)n^2(d+ex)\log\left(c(d+ex)^n\right)}{e^2g}$$
$$+ \frac{6b^3i(gh-fi)n^2(d+ex)\log\left(c(d+ex)^n\right)}{eg^2}$$
$$+ \frac{3b^2i^2n^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{4e^2g}$$
$$- \frac{3bi(eh-di)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g}$$
$$- \frac{3bi(gh-fi)n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2}$$
$$- \frac{3bi^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4e^2g}$$
$$+ \frac{i(eh-di)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2g}$$
$$+ \frac{i(gh-fi)(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{eg^2}$$
$$+ \frac{i^2(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2e^2g}$$
$$+ \frac{(gh-fi)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3}$$
$$+ \frac{3b(gh-fi)^2n\left(a+b\log\left(c(d+ex)^n\right)\right)^2\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$
$$- \frac{6b^2(gh-fi)^2n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$
$$+ \frac{6b^3(gh-fi)^2n^3\operatorname{Li}_4\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1521 vs. $2(660) = 1320$.

Time = 0.51 (sec) , antiderivative size = 1521, normalized size of antiderivative = 2.30

$$\int \frac{(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx$$
$$= \frac{8e^2gi(2gh-fi)x(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right))^3 + 4e^2g^2i^2x^2(a-bn\log(d+ex)+b\log\left(c(d+\right.}{}$$

```
[In] Integrate[((h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^3)/(f + g*x),x]
```

```
[Out] (8*e^2*g*i*(2*g*h - f*i)*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3
+ 4*e^2*g^2*i^2*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3 + 8*e^2
*(g*h - f*i)^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*Log[f + g*x]
 + 24*b*e^2*g^2*h^2*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(Log[
d + e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f)
+ d*g)]) + 6*b*i^2*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(e*g*(
e*x*(4*f - g*x) + 2*d*(2*f + g*x)) - 2*Log[d + e*x]*(g*(d + e*x)*(2*e*f + d
*g - e*g*x) - 2*e^2*f^2*Log[(e*(f + g*x))/(e*f - d*g)]) + 4*e^2*f^2*PolyLog
[2, (g*(d + e*x))/(-(e*f) + d*g)]) - 48*b*e*g*h*i*n*(a - b*n*Log[d + e*x] +
 b*Log[c*(d + e*x)^n])^2*(-(g*(d + e*x)*(-1 + Log[d + e*x])) + e*f*(Log[d +
 e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d
*g)])) + 48*b^2*e*g*h*i*n^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(
g*(2*e*x - 2*(d + e*x)*Log[d + e*x] + (d + e*x)*Log[d + e*x]^2) - e*f*(Log[
d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d + e*x]*PolyLog[2, (g*(d
 + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)])) - 6
*b^2*i^2*n^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(4*e*f*g*(2*e*x
- 2*(d + e*x)*Log[d + e*x] + (d + e*x)*Log[d + e*x]^2) + g^2*(e*x*(6*d - e*
x) + (-6*d^2 - 4*d*e*x + 2*e^2*x^2)*Log[d + e*x] + 2*(d^2 - e^2*x^2)*Log[d
+ e*x]^2) - 4*e^2*f^2*(Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Lo
g[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d +
e*x))/(-(e*f) + d*g)])) + 48*b^2*e^2*g^2*h^2*n^2*(a - b*n*Log[d + e*x] + b*
Log[c*(d + e*x)^n])*((Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)])/2 + Lo
g[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - PolyLog[3, (g*(d + e*
x))/(-(e*f) + d*g)]) + 8*b^3*e^2*g^2*h^2*n^3*(Log[d + e*x]^3*Log[(e*(f + g*
x))/(e*f - d*g)] + 3*Log[d + e*x]^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)
] - 6*Log[d + e*x]*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] + 6*PolyLog[4,
(g*(d + e*x))/(-(e*f) + d*g)]) - 16*b^3*e*g*h*i*n^3*(g*(6*e*x - 6*(d + e*x)
*Log[d + e*x] + 3*(d + e*x)*Log[d + e*x]^2 - (d + e*x)*Log[d + e*x]^3) + e*
f*(Log[d + e*x]^3*Log[(e*(f + g*x))/(e*f - d*g)] + 3*Log[d + e*x]^2*PolyLog
[2, (g*(d + e*x))/(-(e*f) + d*g)] - 6*Log[d + e*x]*PolyLog[3, (g*(d + e*x))
/(-(e*f) + d*g)] + 6*PolyLog[4, (g*(d + e*x))/(-(e*f) + d*g)])) + b^3*i^2*n
^3*(8*e*f*g*(6*e*x - 6*(d + e*x)*Log[d + e*x] + 3*(d + e*x)*Log[d + e*x]^2
- (d + e*x)*Log[d + e*x]^3) - g^2*(3*e*x*(-14*d + e*x) + 6*(7*d^2 + 6*d*e*x
 - e^2*x^2)*Log[d + e*x] - 6*(3*d^2 + 2*d*e*x - e^2*x^2)*Log[d + e*x]^2 + 4
*(d^2 - e^2*x^2)*Log[d + e*x]^3) + 8*e^2*f^2*(Log[d + e*x]^3*Log[(e*(f + g*
x))/(e*f - d*g)] + 3*Log[d + e*x]^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)
] - 6*Log[d + e*x]*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] + 6*PolyLog[4,
(g*(d + e*x))/(-(e*f) + d*g)])))/(8*e^2*g^3)
```

## Maple [F]

$$\int \frac{(ix+h)^2 (a+b\ln(c(ex+d)^n))^3}{gx+f}dx$$

[In] int((i*x+h)^2*(a+b*ln(c*(e*x+d)^n))^3/(g*x+f),x)

[Out] int((i*x+h)^2*(a+b*ln(c*(e*x+d)^n))^3/(g*x+f),x)

## Fricas [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(ix+h)^2(b\log((ex+d)^n c)+a)^3}{gx+f}\,dx$$

[In] integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="fricas")

[Out] integral((a^3*i^2*x^2 + 2*a^3*h*i*x + a^3*h^2 + (b^3*i^2*x^2 + 2*b^3*h*i*x + b^3*h^2)*log((e*x + d)^n*c)^3 + 3*(a*b^2*i^2*x^2 + 2*a*b^2*h*i*x + a*b^2*h^2)*log((e*x + d)^n*c)^2 + 3*(a^2*b*i^2*x^2 + 2*a^2*b*h*i*x + a^2*b*h^2)*log((e*x + d)^n*c))/(g*x + f), x)

## Sympy [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^3 (h+ix)^2}{f+gx}\,dx$$

[In] integrate((i*x+h)**2*(a+b*ln(c*(e*x+d)**n))**3/(g*x+f),x)

[Out] Integral((a + b*log(c*(d + e*x)**n))**3*(h + i*x)**2/(f + g*x), x)

## Maxima [F]

$$\int \frac{(h+ix)^2 (a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(ix+h)^2(b\log((ex+d)^n c)+a)^3}{gx+f}\,dx$$

[In] integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="maxima")

[Out] 2*a^3*h*i*(x/g - f*log(g*x + f)/g^2) + 1/2*a^3*i^2*(2*f^2*log(g*x + f)/g^3 + (g*x^2 - 2*f*x)/g^2) + a^3*h^2*log(g*x + f)/g + integrate((b^3*h^2*log(c)^3 + 3*a*b^2*h^2*log(c)^2 + 3*a^2*b*h^2*log(c) + (b^3*i^2*x^2 + 2*b^3*h*i*x + b^3*h^2)*log((e*x + d)^n)^3 + (b^3*i^2*log(c)^3 + 3*a*b^2*i^2*log(c)^2 + 3*a^2*b*i^2*log(c))*x^2 + 3*(b^3*h^2*log(c) + a*b^2*h^2 + (b^3*i^2*log(c)

```
+ a*b^2*i^2)*x^2 + 2*(b^3*h*i*log(c) + a*b^2*h*i)*x)*log((e*x + d)^n)^2 + 2
*(b^3*h*i*log(c)^3 + 3*a*b^2*h*i*log(c)^2 + 3*a^2*b*h*i*log(c))*x + 3*(b^3*
h^2*log(c)^2 + 2*a*b^2*h^2*log(c) + a^2*b*h^2 + (b^3*i^2*log(c)^2 + 2*a*b^2
*i^2*log(c) + a^2*b*i^2)*x^2 + 2*(b^3*h*i*log(c)^2 + 2*a*b^2*h*i*log(c) + a
^2*b*h*i)*x)*log((e*x + d)^n))/(g*x + f), x)
```

## Giac [F]

$$\int \frac{(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx=\int \frac{(ix+h)^2\left(b\log\left((ex+d)^nc\right)+a\right)^3}{gx+f}\,dx$$

```
[In] integrate((i*x+h)^2*(a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="giac")
[Out] integrate((i*x + h)^2*(b*log((e*x + d)^n*c) + a)^3/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx=\int \frac{\left(h+i\,x\right)^2\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^3}{f+g\,x}\,dx$$

```
[In] int(((h + i*x)^2*(a + b*log(c*(d + e*x)^n))^3)/(f + g*x),x)
[Out] int(((h + i*x)^2*(a + b*log(c*(d + e*x)^n))^3)/(f + g*x), x)
```

# 3.230 $\int \frac{(h+ix)(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 308

$$\begin{aligned}\int \frac{(h+ix)\,(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx &= \frac{6ab^2in^2x}{g}-\frac{6b^3in^3x}{g}+\frac{6b^3in^2(d+ex)\log(c(d+ex)^n)}{eg}\\&\quad-\frac{3bin(d+ex)\,(a+b\log(c(d+ex)^n))^2}{eg}+\frac{i(d+ex)\,(a+b\log(c(d+ex)^n))^3}{eg}\\&\quad+\frac{(gh-fi)\,(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2}\\&\quad+\frac{3b(gh-fi)n(a+b\log(c(d+ex)^n))^2\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}\\&\quad-\frac{6b^2(gh-fi)n^2(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}\\&\quad+\frac{6b^3(gh-fi)n^3\,\mathrm{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}\end{aligned}$$

```
[Out] 6*a*b^2*i*n^2*x/g-6*b^3*i*n^3*x/g+6*b^3*i*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e/g-3
*b*i*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e/g+i*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^
3/e/g+(-f*i+g*h)*(a+b*ln(c*(e*x+d)^n))^3*ln(e*(g*x+f)/(-d*g+e*f))/g^2+3*b*(
-f*i+g*h)*n*(a+b*ln(c*(e*x+d)^n))^2*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g^2-6*
b^2*(-f*i+g*h)*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(3,-g*(e*x+d)/(-d*g+e*f))/g
^2+6*b^3*(-f*i+g*h)*n^3*polylog(4,-g*(e*x+d)/(-d*g+e*f))/g^2
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2465, 2436, 2333, 2332, 2443, 2481, 2421, 2430, 6724}

$$\begin{aligned}&\int \frac{(h+ix)(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx\\&=-\frac{6b^2n^2(gh-fi)\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^2}+\frac{6ab^2in^2x}{g}\\&+\frac{3bn(gh-fi)\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g^2}\\&+\frac{(gh-fi)\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g^2}\\&-\frac{3bin(d+ex)(a+b\log(c(d+ex)^n))^2}{eg}+\frac{i(d+ex)(a+b\log(c(d+ex)^n))^3}{eg}\\&+\frac{6b^3in^2(d+ex)\log(c(d+ex)^n)}{eg}+\frac{6b^3n^3(gh-fi)\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}-\frac{6b^3in^3x}{g}\end{aligned}$$

```
[In] Int[((h + i*x)*(a + b*Log[c*(d + e*x)^n])^3)/(f + g*x),x]
```

```
[Out] (6*a*b^2*i*n^2*x)/g - (6*b^3*i*n^3*x)/g + (6*b^3*i*n^2*(d + e*x)*Log[c*(d +
 e*x)^n])/(e*g) - (3*b*i*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g) +
(i*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/(e*g) + ((g*h - f*i)*(a + b*Log[
c*(d + e*x)^n])^3*Log[(e*(f + g*x))/(e*f - d*g)])/g^2 + (3*b*(g*h - f*i)*n*
(a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g^2
- (6*b^2*(g*h - f*i)*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[3, -((g*(d + e*
x))/(e*f - d*g))])/g^2 + (6*b^3*(g*h - f*i)*n^3*PolyLog[4, -((g*(d + e*x))/
(e*f - d*g))])/g^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left( \frac{i(a+b\log(c(d+ex)^n))^3}{g} + \frac{(gh-fi)(a+b\log(c(d+ex)^n))^3}{g(f+gx)} \right) dx$$

$$= \frac{i\int (a+b\log(c(d+ex)^n))^3\,dx}{g} + \frac{(gh-fi)\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx}{g}$$

$$= \frac{(gh-fi)(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} + \frac{i\text{Subst}\left(\int (a+b\log(cx^n))^3\,dx, x, d+ex\right)}{eg} - \frac{(3be(gh-fi)n)\int \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g^2}$$

$$= \frac{i(d+ex)(a+b\log(c(d+ex)^n))^3}{eg} + \frac{(gh-fi)(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} - \frac{(3bin)\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx, x, d+ex\right)}{eg} - \frac{(3b(gh-fi)n)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{g^2}$$

$$= -\frac{3bin(d+ex)(a+b\log(c(d+ex)^n))^2}{eg} + \frac{i(d+ex)(a+b\log(c(d+ex)^n))^3}{eg} + \frac{(gh-fi)(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} + \frac{3b(gh-fi)n(a+b\log(c(d+ex)^n))^2\,\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^2} + \frac{(6b^2in^2)\,\text{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+ex\right)}{eg} - \frac{(6b^2(gh-fi)n^2)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\text{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{g^2}$$

$$= \frac{6ab^2in^2x}{g} - \frac{3bin(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg} + \frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{eg} + \frac{(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} + \frac{3b(gh-fi)n\left(a+b\log\left(c(d+ex)^n\right)\right)^2\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^2} - \frac{6b^2(gh-fi)n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^2} + \frac{(6b^3in^2)\operatorname{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{eg} + \frac{(6b^3(gh-fi)n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g^2}$$

$$= \frac{6ab^2in^2x}{g} - \frac{6b^3in^3x}{g} + \frac{6b^3in^2(d+ex)\log\left(c(d+ex)^n\right)}{eg} - \frac{3bin(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg} + \frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{eg} + \frac{(gh-fi)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} + \frac{3b(gh-fi)n\left(a+b\log\left(c(d+ex)^n\right)\right)^2\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^2} - \frac{6b^2(gh-fi)n^2\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^2} + \frac{6b^3(gh-fi)n^3\operatorname{Li}_4\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^2}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 799 vs. 2(308) = 616.

Time = 0.25 (sec) , antiderivative size = 799, normalized size of antiderivative = 2.59

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx$$
$$=\frac{egix(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right))^3+e(gh-fi)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^3\log(f$$

```
[In] Integrate[((h + i*x)*(a + b*Log[c*(d + e*x)^n])^3)/(f + g*x),x]
```

```
[Out] (e*g*i*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3 + e*(g*h - f*i)*(a
 - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*Log[f + g*x] + 3*b*e*g*h*n*(a
 - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(Log[d + e*x]*Log[(e*(f + g*x
))/(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) - 3*b*i*n*(a -
b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(-(g*(d + e*x)*(-1 + Log[d + e*x
])) + e*f*(Log[d + e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + PolyLog[2, (g*(d +
 e*x))/(-(e*f) + d*g)])) + 3*b^2*i*n^2*(a - b*n*Log[d + e*x] + b*Log[c*(d +
 e*x)^n])*(g*(2*e*x - 2*(d + e*x)*Log[d + e*x] + (d + e*x)*Log[d + e*x]^2)
- e*f*(Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d + e*x]*PolyL
og[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x))/(-(e*f) +
d*g)])) + 6*b^2*e*g*h*n^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((L
og[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)])/2 + Log[d + e*x]*PolyLog[2, (
g*(d + e*x))/(-(e*f) + d*g)] - PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]) +
b^3*e*g*h*n^3*(Log[d + e*x]^3*Log[(e*(f + g*x))/(e*f - d*g)] + 3*Log[d + e*
x]^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 6*Log[d + e*x]*PolyLog[3, (
g*(d + e*x))/(-(e*f) + d*g)] + 6*PolyLog[4, (g*(d + e*x))/(-(e*f) + d*g)])
- b^3*i*n^3*(g*(6*e*x - 6*(d + e*x)*Log[d + e*x] + 3*(d + e*x)*Log[d + e*x]
^2 - (d + e*x)*Log[d + e*x]^3) + e*f*(Log[d + e*x]^3*Log[(e*(f + g*x))/(e*f
 - d*g)] + 3*Log[d + e*x]^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 6*Lo
g[d + e*x]*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] + 6*PolyLog[4, (g*(d +
e*x))/(-(e*f) + d*g)])))/(e*g^2)
```

## Maple [F]

$$\int \frac{(ix+h)\left(a+b\ln\left(c(ex+d)^n\right)\right)^3}{gx+f}dx$$

```
[In] int((i*x+h)*(a+b*ln(c*(e*x+d)^n))^3/(g*x+f),x)
```

```
[Out] int((i*x+h)*(a+b*ln(c*(e*x+d)^n))^3/(g*x+f),x)
```

## Fricas [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx=\int \frac{(ix+h)(b\log\left((ex+d)^n c\right)+a)^3}{gx+f}\,dx$$

[In] integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="fricas")

[Out] integral((a^3*i*x + a^3*h + (b^3*i*x + b^3*h)*log((e*x + d)^n*c)^3 + 3*(a*b^2*i*x + a*b^2*h)*log((e*x + d)^n*c)^2 + 3*(a^2*b*i*x + a^2*b*h)*log((e*x + d)^n*c))/(g*x + f), x)

## Sympy [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx=\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3(h+ix)}{f+gx}\,dx$$

[In] integrate((i*x+h)*(a+b*ln(c*(e*x+d)**n))**3/(g*x+f),x)

[Out] Integral((a + b*log(c*(d + e*x)**n))**3*(h + i*x)/(f + g*x), x)

## Maxima [F]

$$\int \frac{(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx=\int \frac{(ix+h)(b\log\left((ex+d)^n c\right)+a)^3}{gx+f}\,dx$$

[In] integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="maxima")

[Out] a^3*i*(x/g - f*log(g*x + f)/g^2) + a^3*h*log(g*x + f)/g + integrate((b^3*h*log(c)^3 + 3*a*b^2*h*log(c)^2 + 3*a^2*b*h*log(c) + (b^3*i*x + b^3*h)*log((e*x + d)^n)^3 + 3*(b^3*h*log(c) + a*b^2*h + (b^3*i*log(c) + a*b^2*i)*x)*log((e*x + d)^n)^2 + (b^3*i*log(c)^3 + 3*a*b^2*i*log(c)^2 + 3*a^2*b*i*log(c))*x + 3*(b^3*h*log(c)^2 + 2*a*b^2*h*log(c) + a^2*b*h + (b^3*i*log(c)^2 + 2*a*b^2*i*log(c) + a^2*b*i)*x)*log((e*x + d)^n))/(g*x + f), x)

## Giac [F]

$$\int \frac{(h+ix)\,(a+b\log{(c(d+ex)^n)})^3}{f+gx}\,dx = \int \frac{(ix+h)(b\log{((ex+d)^n c)}+a)^3}{gx+f}\,dx$$

```
[In] integrate((i*x+h)*(a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="giac")
[Out] integrate((i*x + h)*(b*log((e*x + d)^n*c) + a)^3/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(h+ix)\,(a+b\log{(c(d+ex)^n)})^3}{f+gx}\,dx = \int \frac{(h+i\,x)\,\left(a+b\,\ln{\left(c\,{\left(d+e\,x\right)}^n\right)}\right)^3}{f+g\,x}\,dx$$

```
[In] int(((h + i*x)*(a + b*log(c*(d + e*x)^n))^3)/(f + g*x),x)
[Out] int(((h + i*x)*(a + b*log(c*(d + e*x)^n))^3)/(f + g*x), x)
```

# 3.231 $\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{3bn(a+b\log(c(d+ex)^n))^2\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{6b^2n^2(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{g} + \frac{6b^3n^3\,\text{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[Out] (a+b*ln(c*(e*x+d)^n))^3*ln(e*(g*x+f)/(-d*g+e*f))/g+3*b*n*(a+b*ln(c*(e*x+d)^
n))^2*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g-6*b^2*n^2*(a+b*ln(c*(e*x+d)^n))*po
lylog(3,-g*(e*x+d)/(-d*g+e*f))/g+6*b^3*n^3*polylog(4,-g*(e*x+d)/(-d*g+e*f))
/g
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used

$= \{2443, 2481, 2421, 2430, 6724\}$

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = -\frac{6b^2n^2\,\text{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g} + \frac{3bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{g} + \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{g} + \frac{6b^3n^3\,\text{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])^3*Log[(e*(f + g*x))/(e*f - d*g)])/g + (3*b*n*(a
 + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g - (6
*b^2*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))
])/g + (6*b^3*n^3*PolyLog[4, -((g*(d + e*x))/(e*f - d*g))])/g
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
```

```
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} - \frac{(3ben)\int\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g}$$

$$= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} - \frac{(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g}$$

$$= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{3bn(a+b\log(c(d+ex)^n))^2\,\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{(6b^2n^2)\,\text{Subst}\left(\int\frac{(a+b\log(cx^n))\text{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g}$$

$$= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{3bn(a+b\log(c(d+ex)^n))^2\,\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{6b^2n^2(a+b\log(c(d+ex)^n))\,\text{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} + \frac{(6b^3n^3)\,\text{Subst}\left(\int\frac{\text{Li}_3\left(-\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g}$$

$$= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{3bn(a+b\log(c(d+ex)^n))^2\,\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} - \frac{6b^2n^2(a+b\log(c(d+ex)^n))\,\text{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{g} + \frac{6b^3n^3\text{Li}_4\left(-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 335 vs. 2(158) = 316.

Time = 0.07 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.12

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx$$
$$= \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^3\log(f+gx)+3bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\left(\log\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^3/(f + g*x),x]
```

```
[Out] ((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*Log[f + g*x] + 3*b*n*(a -
b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(Log[d + e*x]*Log[(e*(f + g*x))/
(e*f - d*g)] + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + 6*b^2*n^2*(a - b
*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((Log[d + e*x]^2*Log[(e*(f + g*x))/
(e*f - d*g)])/2 + Log[d + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - P
olyLog[3, (g*(d + e*x))/(-(e*f) + d*g)]) + b^3*n^3*(Log[d + e*x]^3*Log[(e*(
f + g*x))/(e*f - d*g)] + 3*Log[d + e*x]^2*PolyLog[2, (g*(d + e*x))/(-(e*f)
+ d*g)] - 6*Log[d + e*x]*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] + 6*PolyL
og[4, (g*(d + e*x))/(-(e*f) + d*g)]))/g
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.06 (sec) , antiderivative size = 1396, normalized size of antiderivative = 8.84

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1396 |

```
[In] int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] -b^3*ln(g*(e*x+d)-d*g+e*f)/g*ln(e*x+d)^3*n^3+3*b^3*ln(g*(e*x+d)-d*g+e*f)/g*
ln((e*x+d)^n)*ln(e*x+d)^2*n^2-3*b^3*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)^2
*ln(e*x+d)*n+b^3*ln(g*(e*x+d)-d*g+e*f)/g*ln((e*x+d)^n)^3-2*b^3*n^3/g*ln(e*x
+d)^3*ln(1-g*(e*x+d)/(d*g-e*f))-3*b^3*n^3/g*ln(e*x+d)^2*polylog(2,g*(e*x+d)
/(d*g-e*f))+6*b^3*n^3/g*polylog(4,g*(e*x+d)/(d*g-e*f))+3*b^3*n^3*dilog((g*(
e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln(e*x+d)^2-6*b^3*n^2*dilog((g*(e*x+d)-d*g+e*
f)/(-d*g+e*f))/g*ln((e*x+d)^n)*ln(e*x+d)+3*b^3*n*dilog((g*(e*x+d)-d*g+e*f)/
(-d*g+e*f))/g*ln((e*x+d)^n)^2+3*b^3*n^3*ln(e*x+d)^3*ln((g*(e*x+d)-d*g+e*f)/
(-d*g+e*f))/g-6*b^3*n^2*ln(e*x+d)^2*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln
((e*x+d)^n)+3*b^3*n*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g*ln((e*x+
d)^n)^2+3*b^3*n^2/g*ln((e*x+d)^n)*ln(e*x+d)^2*ln(1-g*(e*x+d)/(d*g-e*f))+6*b
^3*n^2/g*ln((e*x+d)^n)*ln(e*x+d)*polylog(2,g*(e*x+d)/(d*g-e*f))-6*b^3*n^2/g
```

```
*ln((e*x+d)^n)*polylog(3,g*(e*x+d)/(d*g-e*f))+1/8*(-I*b*Pi*csgn(I*c*(e*x+d)
^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi
*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b
*ln(c)+2*a)^3*ln(g*x+f)/g+3/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I
*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*c
sgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b^2*((ln
((e*x+d)^n)-n*ln(e*x+d))^2*ln(g*(e*x+d)-d*g+e*f)/g+n^2/g*ln(e*x+d)^2*ln(1-g
*(e*x+d)/(d*g-e*f))+2*n^2/g*ln(e*x+d)*polylog(2,g*(e*x+d)/(d*g-e*f))-2*n^2/
g*polylog(3,g*(e*x+d)/(d*g-e*f))+2*n*(ln((e*x+d)^n)-n*ln(e*x+d))*dilog((g*(
e*x+d)-d*g+e*f)/(-d*g+e*f))/g+2*n*(ln((e*x+d)^n)-n*ln(e*x+d))*ln(e*x+d)*ln(
(g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g)+3/4*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*
c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*
x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)
^2*b*(ln((e*x+d)^n)*ln(g*x+f)/g-1/g*n*e*(dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f
))/e+ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e))
```

## Fricas [F]

$$\int \frac{\left(a+b\log \left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx=\int \frac{\left(b\log \left((ex+d)^n c\right)+a\right)^3}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*b
*log((e*x + d)^n*c) + a^3)/(g*x + f), x)
```

## Sympy [F]

$$\int \frac{\left(a+b\log \left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx=\int \frac{\left(a+b\log \left(c(d+ex)^n\right)\right)^3}{f+gx}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))**3/(f + g*x), x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="maxima")
```

```
[Out] a^3*log(g*x + f)/g + integrate((b^3*log((e*x + d)^n)^3 + b^3*log(c)^3 + 3*a
*b^2*log(c)^2 + 3*a^2*b*log(c) + 3*(b^3*log(c) + a*b^2)*log((e*x + d)^n)^2
+ 3*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log((e*x + d)^n))/(g*x + f), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{gx+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^3/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^3}{f+g\,x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^3/(f + g*x),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^3/(f + g*x), x)
```

# 3.232 $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 372

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx =& \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} \\ &- \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} \\ &+ \frac{3bn(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} \\ &- \frac{3bn(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi} \\ &- \frac{6b^2n^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} \\ &+ \frac{6b^2n^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi} \\ &+ \frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} \\ &- \frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}\end{aligned}$$

```
[Out] (a+b*ln(c*(e*x+d)^n))^3*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)-(a+b*ln(c*(e*x+
d)^n))^3*ln(e*(i*x+h)/(-d*i+e*h))/(-f*i+g*h)+3*b*n*(a+b*ln(c*(e*x+d)^n))^2*
polylog(2,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)-3*b*n*(a+b*ln(c*(e*x+d)^n))^2*p
olylog(2,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)-6*b^2*n^2*(a+b*ln(c*(e*x+d)^n))*
```

```
polylog(3,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)+6*b^2*n^2*(a+b*ln(c*(e*x+d)^n))
*polylog(3,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)+6*b^3*n^3*polylog(4,-g*(e*x+d)
/(-d*g+e*f))/(-f*i+g*h)-6*b^3*n^3*polylog(4,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*
h)
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2465, 2443, 2481, 2421, 2430, 6724}

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx = -\frac{6b^2n^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{gh-fi} + \frac{6b^2n^2\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)(a+b\log(c(d+ex)^n))}{gh-fi} + \frac{3bn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{gh-fi} - \frac{3bn\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)(a+b\log(c(d+ex)^n))^2}{gh-fi} + \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{gh-fi} - \frac{\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log(c(d+ex)^n))^3}{gh-fi} + \frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} - \frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^3/((f + g*x)*(h + i*x)),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])^3*Log[(e*(f + g*x))/(e*f - d*g)])/(g*h - f*i) -
 ((a + b*Log[c*(d + e*x)^n])^3*Log[(e*(h + i*x))/(e*h - d*i)])/(g*h - f*i)
+ (3*b*n*(a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((g*(d + e*x))/(e*f - d*g
))])/(g*h - f*i) - (3*b*n*(a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((i*(d +
 e*x))/(e*h - d*i))])/(g*h - f*i) - (6*b^2*n^2*(a + b*Log[c*(d + e*x)^n])*P
olyLog[3, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i) + (6*b^2*n^2*(a + b*Lo
g[c*(d + e*x)^n])*PolyLog[3, -((i*(d + e*x))/(e*h - d*i))])/(g*h - f*i) + (
6*b^3*n^3*PolyLog[4, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i) - (6*b^3*n^
3*PolyLog[4, -((i*(d + e*x))/(e*h - d*i))])/(g*h - f*i)
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left( \frac{g(a+b\log(c(d+ex)^n))^3}{(gh-fi)(f+gx)} - \frac{i(a+b\log(c(d+ex)^n))^3}{(gh-fi)(h+ix)} \right) dx$$

$$= \frac{g\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx}{gh-fi} - \frac{i\int \frac{(a+b\log(c(d+ex)^n))^3}{h+ix}\,dx}{gh-fi}$$

$$= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} - \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} - \frac{(3ben)\int \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{gh-fi} + \frac{(3ben)\int \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{d+ex}\,dx}{gh-fi}$$

$$= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} - \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} - \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{gh-fi} + \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\log\left(\frac{e\left(\frac{eh-di}{e}+\frac{ix}{e}\right)}{eh-di}\right)}{x}\,dx, x, d+ex\right)}{gh-fi}$$

$$= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} - \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} + \frac{3bn(a+b\log(c(d+ex)^n))^2\,\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} - \frac{3bn(a+b\log(c(d+ex)^n))^2\,\text{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{gh-fi} - \frac{(6b^2n^2)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\text{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{gh-fi} + \frac{(6b^2n^2)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\text{Li}_2\left(-\frac{ix}{eh-di}\right)}{x}\,dx, x, d+ex\right)}{gh-fi}$$

$$= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} - \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} + \frac{3bn(a+b\log(c(d+ex)^n))^2\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} - \frac{3bn(a+b\log(c(d+ex)^n))^2\operatorname{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{gh-fi} - \frac{6b^2n^2(a+b\log(c(d+ex)^n))\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} + \frac{6b^2n^2(a+b\log(c(d+ex)^n))\operatorname{Li}_3\left(-\frac{i(d+ex)}{eh-di}\right)}{gh-fi} + \frac{(6b^3n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{gh-fi} - \frac{(6b^3n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{ix}{eh-di}\right)}{x}\,dx, x, d+ex\right)}{gh-fi}$$

$$= \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{gh-fi} - \frac{(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(h+ix)}{eh-di}\right)}{gh-fi} + \frac{3bn(a+b\log(c(d+ex)^n))^2\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} - \frac{3bn(a+b\log(c(d+ex)^n))^2\operatorname{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{gh-fi} - \frac{6b^2n^2(a+b\log(c(d+ex)^n))\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} + \frac{6b^2n^2(a+b\log(c(d+ex)^n))\operatorname{Li}_3\left(-\frac{i(d+ex)}{eh-di}\right)}{gh-fi} + \frac{6b^3n^3\operatorname{Li}_4\left(-\frac{g(d+ex)}{ef-dg}\right)}{gh-fi} - \frac{6b^3n^3\operatorname{Li}_4\left(-\frac{i(d+ex)}{eh-di}\right)}{gh-fi}$$

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.61

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx$$
$$=\frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^3\log(f+gx)-(a-bn\log(d+ex)+b\log(c(d+ex)^n))^3\log(h+}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^3/((f + g*x)*(h + i*x)),x]
```

```
[Out] ((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*Log[f + g*x] - (a - b*n*Lo
g[d + e*x] + b*Log[c*(d + e*x)^n])^3*Log[h + i*x] + 3*b*n*(a - b*n*Log[d +
e*x] + b*Log[c*(d + e*x)^n])^2*(Log[d + e*x]*(Log[(e*(f + g*x))/(e*f - d*g)
] - Log[(e*(h + i*x))/(e*h - d*i)]) + PolyLog[2, (g*(d + e*x))/(-(e*f) + d*
g)] - PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)]) + 6*b^2*n^2*(a - b*n*Log[d
+ e*x] + b*Log[c*(d + e*x)^n])*((Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*
g)])/2 - (Log[d + e*x]^2*Log[(e*(h + i*x))/(e*h - d*i)])/2 + Log[d + e*x]*P
olyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - Log[d + e*x]*PolyLog[2, (i*(d + e
*x))/(-(e*h) + d*i)] - PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)] + PolyLog[3
, (i*(d + e*x))/(-(e*h) + d*i)]) + b^3*n^3*(Log[d + e*x]^3*Log[(e*(f + g*x)
)/(e*f - d*g)] - Log[d + e*x]^3*Log[(e*(h + i*x))/(e*h - d*i)] + 3*Log[d +
e*x]^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 3*Log[d + e*x]^2*PolyLog[
2, (i*(d + e*x))/(-(e*h) + d*i)] - 6*Log[d + e*x]*PolyLog[3, (g*(d + e*x))/
(-(e*f) + d*g)] + 6*Log[d + e*x]*PolyLog[3, (i*(d + e*x))/(-(e*h) + d*i)] +
 6*PolyLog[4, (g*(d + e*x))/(-(e*f) + d*g)] - 6*PolyLog[4, (i*(d + e*x))/(-
(e*h) + d*i)]))/(g*h - f*i)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.40 (sec) , antiderivative size = 2696, normalized size of antiderivative = 7.25

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 2696 |

```
[In] int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h),x,method=_RETURNVERBOSE)
```

```
[Out] 6*b^3*n^3/(f*i-g*h)*polylog(4,-i*(e*x+d)/(-d*i+e*h))-6*b^3*n^3/(f*i-g*h)*po
lylog(4,-g*(e*x+d)/(-d*g+e*f))+b^3/(f*i-g*h)*ln(i*(e*x+d)-d*i+e*h)*ln((e*x+
d)^n)^3-b^3/(f*i-g*h)*ln(g*(e*x+d)-d*g+e*f)*ln((e*x+d)^n)^3+1/8*(-I*b*Pi*cs
gn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+
d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x
+d)^n)^3*b+2*b*ln(c)+2*a)^3*(1/(f*i-g*h)*ln(i*x+h)-1/(f*i-g*h)*ln(g*x+f))+3
```

/4*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*
csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi
*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)^2*b*(ln((e*x+d)^n)/(f*i-g*h)*ln(i*x
+h)-ln((e*x+d)^n)/(f*i-g*h)*ln(g*x+f)-e*n*(1/(f*i-g*h)*(dilog(((i*x+h)*e+d*
i-e*h)/(d*i-e*h))/e+ln(i*x+h)*ln(((i*x+h)*e+d*i-e*h)/(d*i-e*h))/e)-1/(f*i-g
*h)*(dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))/e+ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f
)/(d*g-e*f))/e)))-b^3/(f*i-g*h)*ln(i*(e*x+d)-d*i+e*h)*ln(e*x+d)^3*n^3+b^3/(
f*i-g*h)*ln(g*(e*x+d)-d*g+e*f)*ln(e*x+d)^3*n^3+3*b^3*n^3/(f*i-g*h)*dilog((i
*(e*x+d)-d*i+e*h)/(-d*i+e*h))*ln(e*x+d)^2+3*b^3*n/(f*i-g*h)*dilog((i*(e*x+d
)-d*i+e*h)/(-d*i+e*h))*ln((e*x+d)^n)^2+3*b^3*n^3/(f*i-g*h)*ln(e*x+d)^3*ln((
i*(e*x+d)-d*i+e*h)/(-d*i+e*h))-3*b^3*n^3/(f*i-g*h)*dilog((g*(e*x+d)-d*g+e*f
)/(-d*g+e*f))*ln(e*x+d)^2-3*b^3*n/(f*i-g*h)*dilog((g*(e*x+d)-d*g+e*f)/(-d*g
+e*f))*ln((e*x+d)^n)^2-3*b^3*n^3/(f*i-g*h)*ln(e*x+d)^3*ln((g*(e*x+d)-d*g+e*
f)/(-d*g+e*f))-6*b^3*n^2/(f*i-g*h)*polylog(3,-i*(e*x+d)/(-d*i+e*h))*ln((e*x
+d)^n)-2*b^3*n^3/(f*i-g*h)*ln(e*x+d)^3*ln(1+i*(e*x+d)/(-d*i+e*h))-3*b^3*n^3
/(f*i-g*h)*ln(e*x+d)^2*polylog(2,-i*(e*x+d)/(-d*i+e*h))+2*b^3*n^3/(f*i-g*h)
*ln(e*x+d)^3*ln(1+g*(e*x+d)/(-d*g+e*f))+3*b^3*n^3/(f*i-g*h)*ln(e*x+d)^2*pol
ylog(2,-g*(e*x+d)/(-d*g+e*f))+6*b^3*n^2/(f*i-g*h)*polylog(3,-g*(e*x+d)/(-d*
g+e*f))*ln((e*x+d)^n)+3/2*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*
x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(
I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b^2/e*(e^2*(
ln((e*x+d)^n)-n*ln(e*x+d))^2*(1/e/(f*i-g*h)*ln(i*(e*x+d)-d*i+e*h)-1/e/(f*i-
g*h)*ln(g*(e*x+d)-d*g+e*f))+e^2*n^2*(1/e/(f*i-g*h)*(ln(e*x+d)^2*ln(1+i*(e*x
+d)/(-d*i+e*h))+2*ln(e*x+d)*polylog(2,-i*(e*x+d)/(-d*i+e*h))-2*polylog(3,-i
*(e*x+d)/(-d*i+e*h)))-1/e/(f*i-g*h)*(ln(e*x+d)^2*ln(1+g*(e*x+d)/(-d*g+e*f))
+2*ln(e*x+d)*polylog(2,-g*(e*x+d)/(-d*g+e*f))-2*polylog(3,-g*(e*x+d)/(-d*g+
e*f))))+2*e^2*n*(ln((e*x+d)^n)-n*ln(e*x+d))*(i/e/(f*i-g*h)*(dilog((i*(e*x+d
)-d*i+e*h)/(-d*i+e*h))/i+ln(e*x+d)*ln((i*(e*x+d)-d*i+e*h)/(-d*i+e*h))/i)-g/
e/(f*i-g*h)*(dilog((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))/g+ln(e*x+d)*ln((g*(e*x+d
)-d*g+e*f)/(-d*g+e*f))/g)))-6*b^3*n^2/(f*i-g*h)*dilog((i*(e*x+d)-d*i+e*h)/(
-d*i+e*h))*ln((e*x+d)^n)*ln(e*x+d)-6*b^3*n^2/(f*i-g*h)*ln(e*x+d)^2*ln((i*(e
*x+d)-d*i+e*h)/(-d*i+e*h))*ln((e*x+d)^n)+3*b^3*n/(f*i-g*h)*ln(e*x+d)*ln((i*
(e*x+d)-d*i+e*h)/(-d*i+e*h))*ln((e*x+d)^n)^2+6*b^3*n^2/(f*i-g*h)*dilog((g*(
e*x+d)-d*g+e*f)/(-d*g+e*f))*ln((e*x+d)^n)*ln(e*x+d)+6*b^3*n^2/(f*i-g*h)*ln(
e*x+d)^2*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln((e*x+d)^n)-3*b^3*n/(f*i-g*h)
*ln(e*x+d)*ln((g*(e*x+d)-d*g+e*f)/(-d*g+e*f))*ln((e*x+d)^n)^2+3*b^3*n^2/(f*
i-g*h)*ln(e*x+d)^2*ln(1+i*(e*x+d)/(-d*i+e*h))*ln((e*x+d)^n)+6*b^3*n^2/(f*i-
g*h)*ln(e*x+d)*polylog(2,-i*(e*x+d)/(-d*i+e*h))*ln((e*x+d)^n)-3*b^3*n^2/(f*
i-g*h)*ln(e*x+d)^2*ln(1+g*(e*x+d)/(-d*g+e*f))*ln((e*x+d)^n)-6*b^3*n^2/(f*i-
g*h)*ln(e*x+d)*polylog(2,-g*(e*x+d)/(-d*g+e*f))*ln((e*x+d)^n)+3*b^3/(f*i-g*
h)*ln(i*(e*x+d)-d*i+e*h)*ln((e*x+d)^n)*ln(e*x+d)^2*n^2-3*b^3/(f*i-g*h)*ln(i
*(e*x+d)-d*i+e*h)*ln((e*x+d)^n)^2*ln(e*x+d)*n-3*b^3/(f*i-g*h)*ln(g*(e*x+d)-
d*g+e*f)*ln((e*x+d)^n)*ln(e*x+d)^2*n^2+3*b^3/(f*i-g*h)*ln(g*(e*x+d)-d*g+e*f
)*ln((e*x+d)^n)^2*ln(e*x+d)*n

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h),x, algorithm="fricas")
[Out] integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*b
*log((e*x + d)^n*c) + a^3)/(g*i*x^2 + f*h + (g*h + f*i)*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f)/(i*x+h),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h),x, algorithm="maxima")
[Out] a^3*(log(g*x + f)/(g*h - f*i) - log(i*x + h)/(g*h - f*i)) + integrate((b^3*
log((e*x + d)^n)^3 + b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + 3*(
b^3*log(c) + a*b^2)*log((e*x + d)^n)^2 + 3*(b^3*log(c)^2 + 2*a*b^2*log(c) +
 a^2*b)*log((e*x + d)^n))/(g*i*x^2 + f*h + (g*h + f*i)*x), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)(ix+h)}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^3/((g*x + f)*(i*x + h)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^3}{(f+g\,x)\,(h+i\,x)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^3/((f + g*x)*(h + i*x)),x)
[Out] int((a + b*log(c*(d + e*x)^n))^3/((f + g*x)*(h + i*x)), x)
```

# 3.233 $\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx$

## Optimal result

Integrand size = 31, antiderivative size = 602

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx = -\frac{i(d+ex)(a+b\log(c(d+ex)^n))^3}{(eh-di)(gh-fi)(h+ix)} + \frac{g(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2} + \frac{3ben(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{(eh-di)(gh-fi)} - \frac{g(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2} + \frac{3bgn(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} + \frac{6b^2en^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)} - \frac{3bgn(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2} - \frac{6b^2gn^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} - \frac{6b^3en^3\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)} + \frac{6b^2gn^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2} + \frac{6b^3gn^3\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} - \frac{6b^3gn^3\operatorname{PolyLog}\left(4,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$

```
[Out] -i*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/(-d*i+e*h)/(-f*i+g*h)/(i*x+h)+g*(a+b*ln(
c*(e*x+d)^n))^3*ln(e*(g*x+f)/(-d*g+e*f))/(-f*i+g*h)^2+3*b*e*n*(a+b*ln(c*(e*
x+d)^n))^2*ln(e*(i*x+h)/(-d*i+e*h))/(-d*i+e*h)/(-f*i+g*h)-g*(a+b*ln(c*(e*x+
d)^n))^3*ln(e*(i*x+h)/(-d*i+e*h))/(-f*i+g*h)^2+3*b*g*n*(a+b*ln(c*(e*x+d)^n)
)^2*polylog(2,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)^2+6*b^2*e*n^2*(a+b*ln(c*(e*
x+d)^n))*polylog(2,-i*(e*x+d)/(-d*i+e*h))/(-d*i+e*h)/(-f*i+g*h)-3*b*g*n*(a+
b*ln(c*(e*x+d)^n))^2*polylog(2,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)^2-6*b^2*g*
```

```
n^2*(a+b*ln(c*(e*x+d)^n))*polylog(3,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)^2-6*b
^3*e*n^3*polylog(3,-i*(e*x+d)/(-d*i+e*h))/(-d*i+e*h)/(-f*i+g*h)+6*b^2*g*n^2
*(a+b*ln(c*(e*x+d)^n))*polylog(3,-i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)^2+6*b^3*
g*n^3*polylog(4,-g*(e*x+d)/(-d*g+e*f))/(-f*i+g*h)^2-6*b^3*g*n^3*polylog(4,-
i*(e*x+d)/(-d*i+e*h))/(-f*i+g*h)^2
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2465, 2443, 2481, 2421, 2430, 6724, 2444}

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx = \frac{6b^2en^2\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)(a+b\log(c(d+ex)^n))}{(eh-di)(gh-fi)} - \frac{6b^2gn^2\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{(gh-fi)^2} + \frac{6b^2gn^2\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)(a+b\log(c(d+ex)^n))}{(gh-fi)^2} + \frac{3bgn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^2}{(gh-fi)^2} - \frac{3bgn\operatorname{PolyLog}\left(2,-\frac{i(d+ex)}{eh-di}\right)(a+b\log(c(d+ex)^n))^2}{(gh-fi)^2} + \frac{3ben\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log(c(d+ex)^n))^2}{(eh-di)(gh-fi)} - \frac{i(d+ex)(a+b\log(c(d+ex)^n))^3}{(h+ix)(eh-di)(gh-fi)} + \frac{g\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))^3}{(gh-fi)^2} - \frac{g\log\left(\frac{e(h+ix)}{eh-di}\right)(a+b\log(c(d+ex)^n))^3}{(gh-fi)^2} - \frac{6b^3en^3\operatorname{PolyLog}\left(3,-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)} + \frac{6b^3gn^3\operatorname{PolyLog}\left(4,-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} - \frac{6b^3gn^3\operatorname{PolyLog}\left(4,-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^3/((f + g*x)*(h + i*x)^2),x]
```

```
[Out] -((i*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/((e*h - d*i)*(g*h - f*i)*(h +
i*x))) + (g*(a + b*Log[c*(d + e*x)^n])^3*Log[(e*(f + g*x))/(e*f - d*g)])/(g
*h - f*i)^2 + (3*b*e*n*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(h + i*x))/(e*h
- d*i)])/((e*h - d*i)*(g*h - f*i)) - (g*(a + b*Log[c*(d + e*x)^n])^3*Log[(e
*(h + i*x))/(e*h - d*i)])/(g*h - f*i)^2 + (3*b*g*n*(a + b*Log[c*(d + e*x)^n
])^2*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i)^2 + (6*b^2*e*n^2
*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((i*(d + e*x))/(e*h - d*i))])/((e*h
 - d*i)*(g*h - f*i)) - (3*b*g*n*(a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((
i*(d + e*x))/(e*h - d*i))])/(g*h - f*i)^2 - (6*b^2*g*n^2*(a + b*Log[c*(d +
e*x)^n])*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))])/(g*h - f*i)^2 - (6*b^3*e
*n^3*PolyLog[3, -((i*(d + e*x))/(e*h - d*i))])/((e*h - d*i)*(g*h - f*i)) +
(6*b^2*g*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[3, -((i*(d + e*x))/(e*h - d
*i))])/(g*h - f*i)^2 + (6*b^3*g*n^3*PolyLog[4, -((g*(d + e*x))/(e*f - d*g))
])/(g*h - f*i)^2 - (6*b^3*g*n^3*PolyLog[4, -((i*(d + e*x))/(e*h - d*i))])/(
g*h - f*i)^2
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
```

```
NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left( \frac{g^2(a+b\log(c(d+ex)^n))^3}{(gh-fi)^2(f+gx)} - \frac{i(a+b\log(c(d+ex)^n))^3}{(gh-fi)(h+ix)^2} - \frac{gi(a+b\log(c(d+ex)^n))^3}{(gh-fi)^2(h+ix)} \right) dx$$

$$= \frac{g^2\int \frac{(a+b\log(c(d+ex)^n))^3}{f+gx}\,dx}{(gh-fi)^2} - \frac{(gi)\int \frac{(a+b\log(c(d+ex)^n))^3}{h+ix}\,dx}{(gh-fi)^2} - \frac{i\int \frac{(a+b\log(c(d+ex)^n))^3}{(h+ix)^2}\,dx}{gh-fi}$$

$$= -\frac{i(d+ex)\,(a+b\log(c(d+ex)^n))^3}{(eh-di)(gh-fi)(h+ix)} + \frac{g(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2} - \frac{g(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2} - \frac{(3begn)\int \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{(gh-fi)^2} + \frac{(3begn)\int \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{d+ex}\,dx}{(gh-fi)^2} + \frac{(3bein)\int \frac{(a+b\log(c(d+ex)^n))^2}{h+ix}\,dx}{(eh-di)(gh-fi)}$$

$$= -\frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(eh-di)(gh-fi)(h+ix)} + \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^3 \log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2}$$

$$+ \frac{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2 \log\left(\frac{e(h+ix)}{eh-di}\right)}{(eh-di)(gh-fi)}$$

$$- \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^3 \log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2}$$

$$- \frac{(3bgn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2 \log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{(gh-fi)^2}$$

$$+ \frac{(3bgn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2 \log\left(\frac{e\left(\frac{eh-di}{e}+\frac{ix}{e}\right)}{eh-di}\right)}{x}\,dx, x, d+ex\right)}{(gh-fi)^2}$$

$$- \frac{(6b^2e^2n^2)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(h+ix)}{eh-di}\right)}{d+ex}\,dx}{(eh-di)(gh-fi)}$$

$$= -\frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(eh-di)(gh-fi)(h+ix)} + \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2}$$
$$+ \frac{3ben\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{(eh-di)(gh-fi)}$$
$$- \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2}$$
$$+ \frac{3bgn\left(a+b\log\left(c(d+ex)^n\right)\right)^2\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2}$$
$$- \frac{3bgn\left(a+b\log\left(c(d+ex)^n\right)\right)^2\operatorname{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$
$$- \frac{(6b^2gn^2)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{(gh-fi)^2}$$
$$+ \frac{(6b^2gn^2)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ix}{eh-di}\right)}{x}\,dx,x,d+ex\right)}{(gh-fi)^2}$$
$$- \frac{(6b^2en^2)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{eh-di}{e}+\frac{ix}{e}\right)}{eh-di}\right)}{x}\,dx,x,d+ex\right)}{(eh-di)(gh-fi)}$$

$$= -\frac{i(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{(eh-di)(gh-fi)(h+ix)} + \frac{g(a+b\log\left(c(d+ex)^n\right))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2}$$
$$+ \frac{3ben(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{(eh-di)(gh-fi)}$$
$$- \frac{g(a+b\log\left(c(d+ex)^n\right))^3\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2}$$
$$+ \frac{3bgn(a+b\log\left(c(d+ex)^n\right))^2\operatorname{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2}$$
$$+ \frac{6b^2en^2(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)}$$
$$- \frac{3bgn(a+b\log\left(c(d+ex)^n\right))^2\operatorname{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$
$$- \frac{6b^2gn^2(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2}$$
$$+ \frac{6b^2gn^2(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_3\left(-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$
$$+ \frac{(6b^3gn^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{(gh-fi)^2}$$
$$- \frac{(6b^3gn^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{ix}{eh-di}\right)}{x}\,dx, x, d+ex\right)}{(gh-fi)^2}$$
$$- \frac{(6b^3en^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{ix}{eh-di}\right)}{x}\,dx, x, d+ex\right)}{(eh-di)(gh-fi)}$$

$$= -\frac{i(d+ex)\,(a+b\log(c(d+ex)^n))^3}{(eh-di)(gh-fi)(h+ix)} + \frac{g(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(f+gx)}{ef-dg}\right)}{(gh-fi)^2}$$
$$+ \frac{3ben(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(h+ix)}{eh-di}\right)}{(eh-di)(gh-fi)}$$
$$- \frac{g(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(h+ix)}{eh-di}\right)}{(gh-fi)^2}$$
$$+ \frac{3bgn(a+b\log(c(d+ex)^n))^2\,\mathrm{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2}$$
$$+ \frac{6b^2en^2(a+b\log(c(d+ex)^n))\,\mathrm{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)}$$
$$- \frac{3bgn(a+b\log(c(d+ex)^n))^2\,\mathrm{Li}_2\left(-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$
$$- \frac{6b^2gn^2(a+b\log(c(d+ex)^n))\,\mathrm{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2}$$
$$- \frac{6b^3en^3\mathrm{Li}_3\left(-\frac{i(d+ex)}{eh-di}\right)}{(eh-di)(gh-fi)} + \frac{6b^2gn^2(a+b\log(c(d+ex)^n))\,\mathrm{Li}_3\left(-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$
$$+ \frac{6b^3gn^3\mathrm{Li}_4\left(-\frac{g(d+ex)}{ef-dg}\right)}{(gh-fi)^2} - \frac{6b^3gn^3\mathrm{Li}_4\left(-\frac{i(d+ex)}{eh-di}\right)}{(gh-fi)^2}$$

## Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 1025, normalized size of antiderivative = 1.70

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx$$
$$= \frac{(eh-di)(gh-fi)\,(a-bn\log(d+ex)+b\log(c(d+ex)^n))^3 + g(eh-di)(h+ix)\,(a-bn\log(d+ex)+b}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^3/((f + g*x)*(h + i*x)^2),x]
```

```
[Out] ((e*h - d*i)*(g*h - f*i)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3 +
g*(e*h - d*i)*(h + i*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^3*Log
[f + g*x] - g*(e*h - d*i)*(h + i*x)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*
x)^n])^3*Log[h + i*x] - 3*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])
^2*((g*h - f*i)*(i*(d + e*x)*Log[d + e*x] - e*(h + i*x)*Log[h + i*x]) - g*(
e*h - d*i)*(h + i*x)*(Log[d + e*x]*Log[(e*(f + g*x))/(e*f - d*g)] + PolyLog
[2, (g*(d + e*x))/(-(e*f) + d*g)]) + g*(e*h - d*i)*(h + i*x)*(Log[d + e*x]*
```

```
Log[(e*(h + i*x))/(e*h - d*i)] + PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)]))
 - 3*b^2*n^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((g*h - f*i)*(Lo
g[d + e*x]*(i*(d + e*x)*Log[d + e*x] - 2*e*(h + i*x)*Log[(e*(h + i*x))/(e*h
 - d*i)]) - 2*e*(h + i*x)*PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)]) - g*(e*
h - d*i)*(h + i*x)*(Log[d + e*x]^2*Log[(e*(f + g*x))/(e*f - d*g)] + 2*Log[d
 + e*x]*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*PolyLog[3, (g*(d + e*x
))/(-(e*f) + d*g)]) + g*(e*h - d*i)*(h + i*x)*(Log[d + e*x]^2*Log[(e*(h + i
*x))/(e*h - d*i)] + 2*Log[d + e*x]*PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)]
 - 2*PolyLog[3, (i*(d + e*x))/(-(e*h) + d*i)])) - b^3*n^3*((g*h - f*i)*(Log
[d + e*x]^2*(i*(d + e*x)*Log[d + e*x] - 3*e*(h + i*x)*Log[(e*(h + i*x))/(e*
h - d*i)]) - 6*e*(h + i*x)*Log[d + e*x]*PolyLog[2, (i*(d + e*x))/(-(e*h) +
d*i)] + 6*e*(h + i*x)*PolyLog[3, (i*(d + e*x))/(-(e*h) + d*i)]) - g*(e*h -
d*i)*(h + i*x)*(Log[d + e*x]^3*Log[(e*(f + g*x))/(e*f - d*g)] + 3*Log[d + e
*x]^2*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 6*Log[d + e*x]*PolyLog[3,
(g*(d + e*x))/(-(e*f) + d*g)] + 6*PolyLog[4, (g*(d + e*x))/(-(e*f) + d*g)])
 + g*(e*h - d*i)*(h + i*x)*(Log[d + e*x]^3*Log[(e*(h + i*x))/(e*h - d*i)] +
 3*Log[d + e*x]^2*PolyLog[2, (i*(d + e*x))/(-(e*h) + d*i)] - 6*Log[d + e*x]
*PolyLog[3, (i*(d + e*x))/(-(e*h) + d*i)] + 6*PolyLog[4, (i*(d + e*x))/(-(e
*h) + d*i)])))/((e*h - d*i)*(g*h - f*i)^2*(h + i*x))
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{3}}{\left(gx+f\right)\left(ix+h\right)^{2}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h)^2,x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h)^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\log\left(c(d+ex)^{n}\right))^{3}}{(f+gx)(h+ix)^2}\,dx=\int \frac{\left(b\log\left((ex+d)^{n}c\right)+a\right)^{3}}{(gx+f)(ix+h)^{2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x + d)^n*c)^3 + 3*a*b^2*log((e*x + d)^n*c)^2 + 3*a^2*b
*log((e*x + d)^n*c) + a^3)/(g*i^2*x^3 + f*h^2 + (2*g*h*i + f*i^2)*x^2 + (g*
h^2 + 2*f*h*i)*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**3/(g*x+f)/(i*x+h)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx=\int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)(ix+h)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h)^2,x, algorithm="maxima")
[Out] a^3*(g*log(g*x + f)/(g^2*h^2 - 2*f*g*h*i + f^2*i^2) - g*log(i*x + h)/(g^2*h
^2 - 2*f*g*h*i + f^2*i^2) + 1/(g*h^2 - f*h*i + (g*h*i - f*i^2)*x)) + integr
ate((b^3*log((e*x + d)^n)^3 + b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log
(c) + 3*(b^3*log(c) + a*b^2)*log((e*x + d)^n)^2 + 3*(b^3*log(c)^2 + 2*a*b^2
*log(c) + a^2*b)*log((e*x + d)^n))/(g*i^2*x^3 + f*h^2 + (2*g*h*i + f*i^2)*x
^2 + (g*h^2 + 2*f*h*i)*x), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx=\int \frac{(b\log((ex+d)^n c)+a)^3}{(gx+f)(ix+h)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3/(g*x+f)/(i*x+h)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^3/((g*x + f)*(i*x + h)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3}{(f+gx)(h+ix)^2}\,dx=\int \frac{(a+b\ln(c(d+e\,x)^n))^3}{(f+g\,x)\,(h+i\,x)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^3/((f + g*x)*(h + i*x)^2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^3/((f + g*x)*(h + i*x)^2), x)
```

# 3.234 $\int \frac{h+ix}{(f+gx)(a+b\log(c(d+ex)^n))} dx$



## Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)} dx = \frac{e^{-\frac{a}{bn}} i(d+ex)\left(c(d+ex)^n\right)^{-1/n} \operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{begn} + \frac{(gh-fi)\operatorname{Int}\left(\frac{1}{(f+gx)(a+b\log(c(d+ex)^n))},x\right)}{g}$$

```
[Out] i*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b/e/exp(a/b/n)/g/n/((c*(e*x+d)^n)^(
1/n))+(-f*i+g*h)*Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)/g
```

## Rubi [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)} dx = \int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)} dx$$

```
[In] Int[(h + i*x)/((f + g*x)*(a + b*Log[c*(d + e*x)^n])),x]
```

```
[Out] (i*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b*e*E^(a/(b*
n))*g*n*(c*(d + e*x)^n)^n^(-1)) + ((g*h - f*i)*Defer[Int][1/((f + g*x)*(a +
 b*Log[c*(d + e*x)^n])), x])/g
```

Rubi steps

$$\text{integral} = \int \left(\frac{i}{g\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{gh-fi}{g(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\right)dx$$

$$= \frac{i\int \frac{1}{a+b\log(c(d+ex)^n)}\,dx}{g}+\frac{(gh-fi)\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}\,dx}{g}$$

$$= \frac{i\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{eg}+\frac{(gh-fi)\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}\,dx}{g}$$

$$= \frac{(gh-fi)\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}\,dx}{g} + \frac{\left(i(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{egn}$$

$$= \frac{e^{-\frac{a}{bn}}i(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{begn}+\frac{(gh-fi)\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}\,dx}{g}$$

## Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

```
[In] Integrate[(h + i*x)/((f + g*x)*(a + b*Log[c*(d + e*x)^n])),x]
[Out] Integrate[(h + i*x)/((f + g*x)*(a + b*Log[c*(d + e*x)^n])), x]
```

## Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{ix+h}{(gx+f)\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)}dx$$

```
[In] int((i*x+h)/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
[Out] int((i*x+h)/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{ix+h}{(gx+f)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

```
[In] integrate((i*x+h)/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral((i*x + h)/(a*g*x + a*f + (b*g*x + b*f)*log((e*x + d)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{h+ix}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)(f+gx)}\,dx$$

```
[In] integrate((i*x+h)/(g*x+f)/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Integral((h + i*x)/((a + b*log(c*(d + e*x)**n))*(f + g*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{ix+h}{(gx+f)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

```
[In] integrate((i*x+h)/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
[Out] integrate((i*x + h)/((g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{ix+h}{(gx+f)(b\log\left((ex+d)^nc\right)+a)}\,dx$$

```
[In] integrate((i*x+h)/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] integrate((i*x + h)/((g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{h+i\,x}{(f+g\,x)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)}\,dx$$

```
[In] int((h + i*x)/((f + g*x)*(a + b*log(c*(d + e*x)^n))),x)
[Out] int((h + i*x)/((f + g*x)*(a + b*log(c*(d + e*x)^n))), x)
```

# 3.235 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])),x]
```

```
[Out] Defer[Int][1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

```
[In] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])),x]
[Out] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])), x]
```

## Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(a+b\ln\left(c\,(ex+d)^{n}\right)\right)}dx$$

```
[In] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
[Out] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^{n}c\right)+a)}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral(1/(a*g*x + a*f + (b*g*x + b*f)*log((e*x + d)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{\left(a+b\log\left(c\,(d+ex)^{n}\right)\right)(f+gx)}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Integral(1/((a + b*log(c*(d + e*x)**n))*(f + g*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx=\int \frac{1}{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))),x)
[Out] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))), x)
```

# 3.236 $\int \frac{1}{(f+gx)(h+ix)(a+b\log(c(d+ex)^n))}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \frac{g\text{Int}\left(\frac{1}{(f+gx)(a+b\log(c(d+ex)^n))},x\right)}{gh-fi} - \frac{i\text{Int}\left(\frac{1}{(h+ix)(a+b\log(c(d+ex)^n))},x\right)}{gh-fi}$$

```
[Out] g*Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)/(-f*i+g*h)-i*Unintegrable
(1/(i*x+h)/(a+b*ln(c*(e*x+d)^n)),x)/(-f*i+g*h)
```

## Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)*(h + i*x)*(a + b*Log[c*(d + e*x)^n])),x]
```

```
[Out] (g*Defer[Int][1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])), x])/(g*h - f*i) - (
i*Defer[Int][1/((h + i*x)*(a + b*Log[c*(d + e*x)^n])), x])/(g*h - f*i)
```

Rubi steps

$$\text{integral} = \int \left(\frac{g}{(gh-fi)(f+gx)\,(a+b\log\left(c(d+ex)^n\right))} - \frac{i}{(gh-fi)(h+ix)\,(a+b\log\left(c(d+ex)^n\right))}\right) dx$$

$$= \frac{g\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}\,dx}{gh-fi} - \frac{i\int \frac{1}{(h+ix)(a+b\log(c(d+ex)^n))}\,dx}{gh-fi}$$

## Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)\,(a+b\log\left(c(d+ex)^n\right))}\,dx = \int \frac{1}{(f+gx)(h+ix)\,(a+b\log\left(c(d+ex)^n\right))}\,dx$$

```
[In] Integrate[1/((f + g*x)*(h + i*x)*(a + b*Log[c*(d + e*x)^n])),x]
[Out] Integrate[1/((f + g*x)*(h + i*x)*(a + b*Log[c*(d + e*x)^n])), x]
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\,(ix+h)\,(a+b\ln\left(c\,(ex+d)^{n}\right))}dx$$

```
[In] int(1/(g*x+f)/(i*x+h)/(a+b*ln(c*(e*x+d)^n)),x)
[Out] int(1/(g*x+f)/(i*x+h)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{1}{(f+gx)(h+ix)\,(a+b\log\left(c(d+ex)^n\right))}\,dx = \int \frac{1}{(gx+f)(ix+h)(b\log\left((ex+d)^nc\right)+a)}\,dx$$

```
[In] integrate(1/(g*x+f)/(i*x+h)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral(1/(a*g*i*x^2 + a*f*h + (a*g*h + a*f*i)*x + (b*g*i*x^2 + b*f*h + (b
*g*h + b*f*i)*x)*log((e*x + d)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^{n}\right)\right)\left(f+gx\right)\left(h+ix\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(i*x+h)/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Integral(1/((a + b*log(c*(d + e*x)**n))*(f + g*x)*(h + i*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)(ix+h)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

```
[In] integrate(1/(g*x+f)/(i*x+h)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)*(i*x + h)*(b*log((e*x + d)^n*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)(ix+h)(b\log\left((ex+d)^n c\right)+a)}\,dx$$

```
[In] integrate(1/(g*x+f)/(i*x+h)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(i*x + h)*(b*log((e*x + d)^n*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$
$$=\int \frac{1}{(f+g\,x)\,\left(h+i\,x\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)*(h + i*x)*(a + b*log(c*(d + e*x)^n))),x)
[Out] int(1/((f + g*x)*(h + i*x)*(a + b*log(c*(d + e*x)^n))), x)
```

# 3.237 $\int \frac{1}{(f+gx)(h+ix)^2(a+b\log(c(d+ex)^n))}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \frac{g^2\text{Int}\left(\frac{1}{(f+gx)(a+b\log(c(d+ex)^n))},x\right)}{(gh-fi)^2} - \frac{i\text{Int}\left(\frac{1}{(h+ix)^2(a+b\log(c(d+ex)^n))},x\right)}{gh-fi} - \frac{gi\text{Int}\left(\frac{1}{(h+ix)(a+b\log(c(d+ex)^n))},x\right)}{(gh-fi)^2}$$

```
[Out] g^2*Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n)),x)/(-f*i+g*h)^2-i*Unintegr
able(1/(i*x+h)^2/(a+b*ln(c*(e*x+d)^n)),x)/(-f*i+g*h)-g*i*Unintegrable(1/(i*
x+h)/(a+b*ln(c*(e*x+d)^n)),x)/(-f*i+g*h)^2
```

## Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)*(h + i*x)^2*(a + b*Log[c*(d + e*x)^n])),x]
```

```
[Out] (g^2*Defer[Int][1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])), x])/(g*h - f*i)^2
 - (i*Defer[Int][1/((h + i*x)^2*(a + b*Log[c*(d + e*x)^n])), x])/(g*h - f*i
```

```
) - (g*i*Defer[Int][1/((h + i*x)*(a + b*Log[c*(d + e*x)^n])), x])/(g*h - f*
i)^2
```

Rubi steps

$$\text{integral} = \int \left(\frac{g^2}{(gh-fi)^2(f+gx)\,(a+b\log(c(d+ex)^n))} - \frac{i}{(gh-fi)(h+ix)^2\,(a+b\log(c(d+ex)^n))} - \frac{gi}{(gh-fi)^2(h+ix)\,(a+b\log(c(d+ex)^n))}\right)dx$$

$$= \frac{g^2\int\frac{1}{(f+gx)(a+b\log(c(d+ex)^n))}\,dx}{(gh-fi)^2} - \frac{(gi)\int\frac{1}{(h+ix)(a+b\log(c(d+ex)^n))}\,dx}{(gh-fi)^2} - \frac{i\int\frac{1}{(h+ix)^2(a+b\log(c(d+ex)^n))}\,dx}{gh-fi}$$

## Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int\frac{1}{(f+gx)(h+ix)^2\,(a+b\log(c(d+ex)^n))}\,dx = \int\frac{1}{(f+gx)(h+ix)^2\,(a+b\log(c(d+ex)^n))}\,dx$$

```
[In] Integrate[1/((f + g*x)*(h + i*x)^2*(a + b*Log[c*(d + e*x)^n])),x]
[Out] Integrate[1/((f + g*x)*(h + i*x)^2*(a + b*Log[c*(d + e*x)^n])), x]
```

## Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int\frac{1}{(gx+f)\,(ix+h)^2\,(a+b\ln(c\,(ex+d)^n))}dx$$

```
[In] int(1/(g*x+f)/(i*x+h)^2/(a+b*ln(c*(e*x+d)^n)),x)
[Out] int(1/(g*x+f)/(i*x+h)^2/(a+b*ln(c*(e*x+d)^n)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.58

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)(ix+h)^2(b\log\left((ex+d)^nc\right)+a)}\,dx$$

```
[In] integrate(1/(g*x+f)/(i*x+h)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral(1/(a*g*i^2*x^3 + a*f*h^2 + (2*a*g*h*i + a*f*i^2)*x^2 + (a*g*h^2 +
2*a*f*h*i)*x + (b*g*i^2*x^3 + b*f*h^2 + (2*b*g*h*i + b*f*i^2)*x^2 + (b*g*h^
2 + 2*b*f*h*i)*x)*log((e*x + d)^n*c)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)/(i*x+h)**2/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)(ix+h)^2(b\log\left((ex+d)^nc\right)+a)}\,dx$$

```
[In] integrate(1/(g*x+f)/(i*x+h)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)*(i*x + h)^2*(b*log((e*x + d)^n*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)^2\,(a+b\log{(c(d+ex)^n)})}\,dx$$
$$=\int \frac{1}{(gx+f)(ix+h)^2(b\log{((ex+d)^nc)}+a)}\,dx$$

```
[In] integrate(1/(g*x+f)/(i*x+h)^2/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(i*x + h)^2*(b*log((e*x + d)^n*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)^2\,(a+b\log{(c(d+ex)^n)})}\,dx$$
$$=\int \frac{1}{(f+g\,x)\,\left(h+i\,x\right)^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)*(h + i*x)^2*(a + b*log(c*(d + e*x)^n))),x)
[Out] int(1/((f + g*x)*(h + i*x)^2*(a + b*log(c*(d + e*x)^n))), x)
```

# 3.238 $\int \frac{h+ix}{(f+gx)(a+b\log(c(d+ex)^n))^2}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \frac{e^{-\frac{a}{bn}}i(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2egn^2} - \frac{i(d+ex)}{begn\left(a+b\log\left(c(d+ex)^n\right)\right)} + \frac{(gh-fi)\operatorname{Int}\left(\frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2},x\right)}{g}$$

```
[Out] i*(e*x+d)*Ei((a+b*ln(c*(e*x+d)^n))/b/n)/b^2/e/exp(a/b/n)/g/n^2/((c*(e*x+d)^
n)^(1/n))-i*(e*x+d)/b/e/g/n/(a+b*ln(c*(e*x+d)^n))+(-f*i+g*h)*Unintegrable(1
/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)/g
```

## Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

```
[In] Int[(h + i*x)/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
```

```
[Out] (i*(d + e*x)*ExpIntegralEi[(a + b*Log[c*(d + e*x)^n])/(b*n)])/(b^2*e*E^(a/(
b*n))*g*n^2*(c*(d + e*x)^n)^n^(-1)) - (i*(d + e*x))/(b*e*g*n*(a + b*Log[c*(
```

```
d + e*x)^n])) + ((g*h - f*i)*Defer[Int][1/((f + g*x)*(a + b*Log[c*(d + e*x)
^n])^2), x])/g
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{i}{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2}+\frac{gh-fi}{g(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\right)dx\\
&= \frac{i\int \frac{1}{(a+b\log(c(d+ex)^n))^2}\,dx}{g}+\frac{(gh-fi)\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2}\,dx}{g}\\
&= \frac{i\text{Subst}\left(\int \frac{1}{(a+b\log(cx^n))^2}\,dx,x,d+ex\right)}{eg}+\frac{(gh-fi)\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2}\,dx}{g}\\
&= -\frac{i(d+ex)}{begn\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{(gh-fi)\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2}\,dx}{g}\\
&\quad+\frac{i\text{Subst}\left(\int \frac{1}{a+b\log(cx^n)}\,dx,x,d+ex\right)}{begn}\\
&= -\frac{i(d+ex)}{begn\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{(gh-fi)\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2}\,dx}{g}\\
&\quad+\frac{\left(i(d+ex)\left(c(d+ex)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx,x,\log\left(c(d+ex)^n\right)\right)}{begn^2}\\
&= \frac{e^{-\frac{a}{bn}}i(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{Ei}\left(\frac{a+b\log(c(d+ex)^n)}{bn}\right)}{b^2egn^2}\\
&\quad-\frac{i(d+ex)}{begn\left(a+b\log\left(c(d+ex)^n\right)\right)}+\frac{(gh-fi)\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2}\,dx}{g}
\end{aligned}$$

## Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

```
[In] Integrate[(h + i*x)/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
[Out] Integrate[(h + i*x)/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2), x]
```

## Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{ix+h}{(gx+f)\left(a+b\ln\left(c\left(ex+d\right)^{n}\right)\right)^{2}}dx$$

[In] `int((i*x+h)/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)`

[Out] `int((i*x+h)/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)`

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.38

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}\,dx=\int \frac{ix+h}{(gx+f)(b\log\left((ex+d)^{n}c\right)+a)^{2}}\,dx$$

[In] `integrate((i*x+h)/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")`

[Out] `integral((i*x + h)/(a^2*g*x + a^2*f + (b^2*g*x + b^2*f)*log((e*x + d)^n*c)^2 + 2*(a*b*g*x + a*b*f)*log((e*x + d)^n*c)), x)`

## Sympy [N/A]

Not integrable

Time = 7.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}\,dx=\int \frac{h+ix}{\left(a+b\log\left(c\left(d+ex\right)^{n}\right)\right)^{2}(f+gx)}\,dx$$

[In] `integrate((i*x+h)/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**2,x)`

[Out] `Integral((h + i*x)/((a + b*log(c*(d + e*x)**n))**2*(f + g*x)), x)`

## Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 223, normalized size of antiderivative = 7.69

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{ix+h}{(gx+f)(b\log\left((ex+d)^nc\right)+a)^2}\,dx$$

```
[In] integrate((i*x+h)/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] -(e*i*x^2 + d*h + (e*h + d*i)*x)/(b^2*e*f*n*log(c) + a*b*e*f*n + (b^2*e*g*n
*log(c) + a*b*e*g*n)*x + (b^2*e*g*n*x + b^2*e*f*n)*log((e*x + d)^n)) + inte
grate((e*g*i*x^2 + 2*e*f*i*x + e*f*h - (g*h - f*i)*d)/(b^2*e*f^2*n*log(c) +
 a*b*e*f^2*n + (b^2*e*g^2*n*log(c) + a*b*e*g^2*n)*x^2 + 2*(b^2*e*f*g*n*log(
c) + a*b*e*f*g*n)*x + (b^2*e*g^2*n*x^2 + 2*b^2*e*f*g*n*x + b^2*e*f^2*n)*log
((e*x + d)^n)), x)
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{ix+h}{(gx+f)(b\log\left((ex+d)^nc\right)+a)^2}\,dx$$

```
[In] integrate((i*x+h)/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] integrate((i*x + h)/((g*x + f)*(b*log((e*x + d)^n*c) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{h+ix}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{h+i\,x}{\left(f+g\,x\right)\,{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}^2}\,dx$$

```
[In] int((h + i*x)/((f + g*x)*(a + b*log(c*(d + e*x)^n))^2),x)
```

```
[Out] int((h + i*x)/((f + g*x)*(a + b*log(c*(d + e*x)^n))^2), x)
```

# 3.239 $\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2} dx = \operatorname{Int}\left(\frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}, x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2} dx = \int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2} dx$$

```
[In] Int[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
```

```
[Out] Defer[Int][1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2} dx$$

## Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
[Out] Integrate[1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2), x]
```

## Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(a+b\ln\left(c\,(ex+d)^n\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
[Out] int(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)(b\log\left((ex+d)^nc\right)+a)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*g*x + a^2*f + (b^2*g*x + b^2*f)*log((e*x + d)^n*c)^2 + 2*(a
*b*g*x + a*b*f)*log((e*x + d)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)^2\left(f+gx\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*ln(c*(e*x+d)**n))**2,x)
[Out] Integral(1/((a + b*log(c*(d + e*x)**n))**2*(f + g*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 7.83

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
[Out] (e*f - d*g)*integrate(1/(b^2*e*f^2*n*log(c) + a*b*e*f^2*n + (b^2*e*g^2*n*lo
g(c) + a*b*e*g^2*n)*x^2 + 2*(b^2*e*f*g*n*log(c) + a*b*e*f*g*n)*x + (b^2*e*g
^2*n*x^2 + 2*b^2*e*f*g*n*x + b^2*e*f^2*n)*log((e*x + d)^n)), x) - (e*x + d)
/(b^2*e*f*n*log(c) + a*b*e*f*n + (b^2*e*g*n*log(c) + a*b*e*g*n)*x + (b^2*e*
g*n*x + b^2*e*f*n)*log((e*x + d)^n))
```

## Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(b*log((e*x + d)^n*c) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{1}{(f+g\,x)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^2}\,dx$$

```
[In] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^2),x)
[Out] int(1/((f + g*x)*(a + b*log(c*(d + e*x)^n))^2), x)
```

# 3.240 $\int \frac{1}{(f+gx)(h+ix)(a+b\log(c(d+ex)^n))^2}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \frac{g\text{Int}\left(\frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2},x\right)}{gh-fi} - \frac{i\text{Int}\left(\frac{1}{(h+ix)(a+b\log(c(d+ex)^n))^2},x\right)}{gh-fi}$$

```
[Out] g*Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)/(-f*i+g*h)-i*Unintegrab
le(1/(i*x+h)/(a+b*ln(c*(e*x+d)^n))^2,x)/(-f*i+g*h)
```

## Rubi [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

```
[In] Int[1/((f + g*x)*(h + i*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
```

```
[Out] (g*Defer[Int][1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2), x])/(g*h - f*i) -
 (i*Defer[Int][1/((h + i*x)*(a + b*Log[c*(d + e*x)^n])^2), x])/(g*h - f*i)
```

Rubi steps

$$\text{integral} = \int \left( \frac{g}{(gh - fi)(f + gx)(a + b\log(c(d + ex)^n))^2} - \frac{i}{(gh - fi)(h + ix)(a + b\log(c(d + ex)^n))^2} \right) dx$$
$$= \frac{g \int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2} dx}{gh - fi} - \frac{i \int \frac{1}{(h+ix)(a+b\log(c(d+ex)^n))^2} dx}{gh - fi}$$

## Mathematica [N/A]

Not integrable

Time = 4.73 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f + gx)(h + ix)(a + b\log(c(d + ex)^n))^2} dx = \int \frac{1}{(f + gx)(h + ix)(a + b\log(c(d + ex)^n))^2} dx$$

```
[In] Integrate[1/((f + g*x)*(h + i*x)*(a + b*Log[c*(d + e*x)^n])^2),x]
[Out] Integrate[1/((f + g*x)*(h + i*x)*(a + b*Log[c*(d + e*x)^n])^2), x]
```

## Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx + f)(ix + h)(a + b\ln(c(ex + d)^n))^2} dx$$

```
[In] int(1/(g*x+f)/(i*x+h)/(a+b*ln(c*(e*x+d)^n))^2,x)
[Out] int(1/(g*x+f)/(i*x+h)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.81

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(ix+h)(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

[In] integrate(1/(g*x+f)/(i*x+h)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")

[Out] integral(1/(a^2*g*i*x^2 + a^2*f*h + (b^2*g*i*x^2 + b^2*f*h + (b^2*g*h + b^2*f*i)*x)*log((e*x + d)^n*c)^2 + (a^2*g*h + a^2*f*i)*x + 2*(a*b*g*i*x^2 + a*b*f*h + (a*b*g*h + a*b*f*i)*x)*log((e*x + d)^n*c)), x)

## Sympy [N/A]

Not integrable

Time = 8.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)^2\left(f+gx\right)\left(h+ix\right)}\,dx$$

[In] integrate(1/(g*x+f)/(i*x+h)/(a+b*ln(c*(e*x+d)**n))**2,x)

[Out] Integral(1/((a + b*log(c*(d + e*x)**n))**2*(f + g*x)*(h + i*x)), x)

## Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 488, normalized size of antiderivative = 15.74

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(ix+h)(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

[In] integrate(1/(g*x+f)/(i*x+h)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")

[Out] -(e*x + d)/(b^2*e*f*h*n*log(c) + a*b*e*f*h*n + (b^2*e*g*i*n*log(c) + a*b*e*g*i*n)*x^2 + ((g*h*n + f*i*n)*b^2*e*log(c) + (g*h*n + f*i*n)*a*b*e)*x + (b^

```
2*e*g*i*n*x^2 + b^2*e*f*h*n + (g*h*n + f*i*n)*b^2*e*x)*log((e*x + d)^n)) -
integrate((e*g*i*x^2 + 2*d*g*i*x - e*f*h + (g*h + f*i)*d)/(b^2*e*f^2*h^2*n*
log(c) + a*b*e*f^2*h^2*n + (b^2*e*g^2*i^2*n*log(c) + a*b*e*g^2*i^2*n)*x^4 +
 2*((g^2*h*i*n + f*g*i^2*n)*b^2*e*log(c) + (g^2*h*i*n + f*g*i^2*n)*a*b*e)*x
^3 + ((g^2*h^2*n + 4*f*g*h*i*n + f^2*i^2*n)*b^2*e*log(c) + (g^2*h^2*n + 4*f
*g*h*i*n + f^2*i^2*n)*a*b*e)*x^2 + 2*((f*g*h^2*n + f^2*h*i*n)*b^2*e*log(c)
+ (f*g*h^2*n + f^2*h*i*n)*a*b*e)*x + (b^2*e*g^2*i^2*n*x^4 + b^2*e*f^2*h^2*n
 + 2*(g^2*h*i*n + f*g*i^2*n)*b^2*e*x^3 + (g^2*h^2*n + 4*f*g*h*i*n + f^2*i^2
*n)*b^2*e*x^2 + 2*(f*g*h^2*n + f^2*h*i*n)*b^2*e*x)*log((e*x + d)^n)), x)
```

## Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(ix+h)(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(i*x+h)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(i*x + h)*(b*log((e*x + d)^n*c) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)\,\left(h+i\,x\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2}\,dx$$

```
[In] int(1/((f + g*x)*(h + i*x)*(a + b*log(c*(d + e*x)^n))^2),x)
[Out] int(1/((f + g*x)*(h + i*x)*(a + b*log(c*(d + e*x)^n))^2), x)
```

# 3.241 $\int \frac{1}{(f+gx)(h+ix)^2(a+b\log(c(d+ex)^n))^2}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \frac{g^2\mathrm{Int}\left(\frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2},x\right)}{(gh-fi)^2} - \frac{i\mathrm{Int}\left(\frac{1}{(h+ix)^2(a+b\log(c(d+ex)^n))^2},x\right)}{gh-fi} - \frac{gi\mathrm{Int}\left(\frac{1}{(h+ix)(a+b\log(c(d+ex)^n))^2},x\right)}{(gh-fi)^2}$$

```
[Out] g^2*Unintegrable(1/(g*x+f)/(a+b*ln(c*(e*x+d)^n))^2,x)/(-f*i+g*h)^2-i*Uninte
grable(1/(i*x+h)^2/(a+b*ln(c*(e*x+d)^n))^2,x)/(-f*i+g*h)-g*i*Unintegrable(1
/(i*x+h)/(a+b*ln(c*(e*x+d)^n))^2,x)/(-f*i+g*h)^2
```

## Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

```
[In] Int[1/((f + g*x)*(h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^2),x]
```

```
[Out] (g^2*Defer[Int][1/((f + g*x)*(a + b*Log[c*(d + e*x)^n])^2), x])/(g*h - f*i)
^2 - (i*Defer[Int][1/((h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^2), x])/(g*h -
 f*i) - (g*i*Defer[Int][1/((h + i*x)*(a + b*Log[c*(d + e*x)^n])^2), x])/(g*
h - f*i)^2
```

Rubi steps

$$\text{integral} = \int \left(\frac{g^2}{(gh-fi)^2(f+gx)\left(a+b\log\left(c(d+ex)^n\right)\right)^2} - \frac{i}{(gh-fi)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2} - \frac{gi}{(gh-fi)^2(h+ix)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\right)dx$$

$$= \frac{g^2\int \frac{1}{(f+gx)(a+b\log(c(d+ex)^n))^2}\,dx}{(gh-fi)^2} - \frac{(gi)\int \frac{1}{(h+ix)(a+b\log(c(d+ex)^n))^2}\,dx}{(gh-fi)^2} - \frac{i\int \frac{1}{(h+ix)^2(a+b\log(c(d+ex)^n))^2}\,dx}{gh-fi}$$

## Mathematica [N/A]

Not integrable

Time = 9.88 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)*(h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^2),x]
[Out] Integrate[1/((f + g*x)*(h + i*x)^2*(a + b*Log[c*(d + e*x)^n])^2), x]
```

## Maple [N/A]

Not integrable

Time = 122.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\,(ix+h)^2\left(a+b\ln\left(c\,(ex+d)^n\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)/(i*x+h)^2/(a+b*ln(c*(e*x+d)^n))^2,x)
[Out] int(1/(g*x+f)/(i*x+h)^2/(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 203, normalized size of antiderivative = 6.55

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(ix+h)^2(b\log\left((ex+d)^nc\right)+a)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(i*x+h)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas
")
[Out] integral(1/(a^2*g*i^2*x^3 + a^2*f*h^2 + (2*a^2*g*h*i + a^2*f*i^2)*x^2 + (b^
2*g*i^2*x^3 + b^2*f*h^2 + (2*b^2*g*h*i + b^2*f*i^2)*x^2 + (b^2*g*h^2 + 2*b^
2*f*h*i)*x)*log((e*x + d)^n*c)^2 + (a^2*g*h^2 + 2*a^2*f*h*i)*x + 2*(a*b*g*i
^2*x^3 + a*b*f*h^2 + (2*a*b*g*h*i + a*b*f*i^2)*x^2 + (a*b*g*h^2 + 2*a*b*f*h
*i)*x)*log((e*x + d)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 42.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+ex\right)^n\right)\right)^2\left(f+gx\right)\left(h+ix\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(i*x+h)**2/(a+b*ln(c*(e*x+d)**n))**2,x)
[Out] Integral(1/((a + b*log(c*(d + e*x)**n))**2*(f + g*x)*(h + i*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 723, normalized size of antiderivative = 23.32

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(ix+h)^2(b\log\left((ex+d)^nc\right)+a)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(i*x+h)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima
")
```

```
[Out] -(e*x + d)/(b^2*e*f*h^2*n*log(c) + a*b*e*f*h^2*n + (b^2*e*g*i^2*n*log(c) +
a*b*e*g*i^2*n)*x^3 + ((2*g*h*i*n + f*i^2*n)*b^2*e*log(c) + (2*g*h*i*n + f*i
^2*n)*a*b*e)*x^2 + ((g*h^2*n + 2*f*h*i*n)*b^2*e*log(c) + (g*h^2*n + 2*f*h*i
*n)*a*b*e)*x + (b^2*e*g*i^2*n*x^3 + b^2*e*f*h^2*n + (2*g*h*i*n + f*i^2*n)*b
^2*e*x^2 + (g*h^2*n + 2*f*h*i*n)*b^2*e*x)*log((e*x + d)^n)) - integrate((2*
e*g*i*x^2 - e*f*h + (g*h + 2*f*i)*d + (e*f*i + 3*d*g*i)*x)/(b^2*e*f^2*h^3*n
*log(c) + a*b*e*f^2*h^3*n + (b^2*e*g^2*i^3*n*log(c) + a*b*e*g^2*i^3*n)*x^5
+ ((3*g^2*h*i^2*n + 2*f*g*i^3*n)*b^2*e*log(c) + (3*g^2*h*i^2*n + 2*f*g*i^3*
n)*a*b*e)*x^4 + ((3*g^2*h^2*i*n + 6*f*g*h*i^2*n + f^2*i^3*n)*b^2*e*log(c) +
 (3*g^2*h^2*i*n + 6*f*g*h*i^2*n + f^2*i^3*n)*a*b*e)*x^3 + ((g^2*h^3*n + 6*f
*g*h^2*i*n + 3*f^2*h*i^2*n)*b^2*e*log(c) + (g^2*h^3*n + 6*f*g*h^2*i*n + 3*f
^2*h*i^2*n)*a*b*e)*x^2 + ((2*f*g*h^3*n + 3*f^2*h^2*i*n)*b^2*e*log(c) + (2*f
*g*h^3*n + 3*f^2*h^2*i*n)*a*b*e)*x + (b^2*e*g^2*i^3*n*x^5 + b^2*e*f^2*h^3*n
 + (3*g^2*h*i^2*n + 2*f*g*i^3*n)*b^2*e*x^4 + (3*g^2*h^2*i*n + 6*f*g*h*i^2*n
 + f^2*i^3*n)*b^2*e*x^3 + (g^2*h^3*n + 6*f*g*h^2*i*n + 3*f^2*h*i^2*n)*b^2*e
*x^2 + (2*f*g*h^3*n + 3*f^2*h^2*i*n)*b^2*e*x)*log((e*x + d)^n)), x)
```

## Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)(ix+h)^2(b\log\left((ex+d)^n c\right)+a)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(i*x+h)^2/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)*(i*x + h)^2*(b*log((e*x + d)^n*c) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)(h+ix)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}\,dx = \int \frac{1}{\left(f+g\,x\right)\,{\left(h+i\,x\right)}^2\,{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}^2}\,dx$$

```
[In] int(1/((f + g*x)*(h + i*x)^2*(a + b*log(c*(d + e*x)^n))^2),x)
```

```
[Out] int(1/((f + g*x)*(h + i*x)^2*(a + b*log(c*(d + e*x)^n))^2), x)
```

# 3.242 $\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 281

$$\begin{aligned}\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx &= \frac{af^2x}{g^3}-\frac{bf^2nx}{g^3}-\frac{bdfnx}{2eg^2}-\frac{bd^2nx}{3e^2g}+\frac{bfnx^2}{4g^2}\\&+\frac{bdnx^2}{6eg}-\frac{bnx^3}{9g}+\frac{bd^2fn\log(d+ex)}{2e^2g^2}\\&+\frac{bd^3n\log(d+ex)}{3e^3g}+\frac{bf^2(d+ex)\log(c(d+ex)^n)}{eg^3}\\&-\frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2}+\frac{x^3(a+b\log(c(d+ex)^n))}{3g}\\&-\frac{f^3(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4}\\&-\frac{bf^3n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4}\end{aligned}$$

```
[Out] a*f^2*x/g^3-b*f^2*n*x/g^3-1/2*b*d*f*n*x/e/g^2-1/3*b*d^2*n*x/e^2/g+1/4*b*f*n
*x^2/g^2+1/6*b*d*n*x^2/e/g-1/9*b*n*x^3/g+1/2*b*d^2*f*n*ln(e*x+d)/e^2/g^2+1/
3*b*d^3*n*ln(e*x+d)/e^3/g+b*f^2*(e*x+d)*ln(c*(e*x+d)^n)/e/g^3-1/2*f*x^2*(a+
b*ln(c*(e*x+d)^n))/g^2+1/3*x^3*(a+b*ln(c*(e*x+d)^n))/g-f^3*(a+b*ln(c*(e*x+d
)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g^4-b*f^3*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))
/g^4
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {45, 2463, 2436, 2332, 2442, 2441, 2440, 2438}

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx = -\frac{f^3\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^4} - \frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2} + \frac{x^3(a+b\log(c(d+ex)^n))}{3g} + \frac{af^2x}{g^3} + \frac{bf^2(d+ex)\log(c(d+ex)^n)}{eg^3} + \frac{bd^3n\log(d+ex)}{3e^3g} + \frac{bd^2fn\log(d+ex)}{2e^2g^2} - \frac{bd^2nx}{3e^2g} - \frac{bf^3n\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4} - \frac{bdfnx}{2eg^2} + \frac{bdnx^2}{6eg} - \frac{bf^2nx}{g^3} + \frac{bfnx^2}{4g^2} - \frac{bnx^3}{9g}$$

```
[In] Int[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

```
[Out] (a*f^2*x)/g^3 - (b*f^2*n*x)/g^3 - (b*d*f*n*x)/(2*e*g^2) - (b*d^2*n*x)/(3*e^
2*g) + (b*f*n*x^2)/(4*g^2) + (b*d*n*x^2)/(6*e*g) - (b*n*x^3)/(9*g) + (b*d^2
*f*n*Log[d + e*x])/(2*e^2*g^2) + (b*d^3*n*Log[d + e*x])/(3*e^3*g) + (b*f^2*
(d + e*x)*Log[c*(d + e*x)^n])/(e*g^3) - (f*x^2*(a + b*Log[c*(d + e*x)^n]))/
(2*g^2) + (x^3*(a + b*Log[c*(d + e*x)^n]))/(3*g) - (f^3*(a + b*Log[c*(d + e
*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^4 - (b*f^3*n*PolyLog[2, -((g*(d +
 e*x))/(e*f - d*g))])/g^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{f^2(a+b\log(c(d+ex)^n))}{g^3} - \frac{fx(a+b\log(c(d+ex)^n))}{g^2} + \frac{x^2(a+b\log(c(d+ex)^n))}{g} - \frac{f^3(a+b\log(c(d+ex)^n))}{g^3(f+gx)}\right) dx$$

$$= \frac{f^2\int(a+b\log(c(d+ex)^n))\,dx}{g^3} - \frac{f^3\int\frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{g^3} - \frac{f\int x(a+b\log(c(d+ex)^n))\,dx}{g^2} + \frac{\int x^2(a+b\log(c(d+ex)^n))\,dx}{g}$$

$$
\begin{aligned}
&= \frac{af^2x}{g^3} - \frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2} + \frac{x^3(a+b\log(c(d+ex)^n))}{3g} \\
&\quad - \frac{f^3(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4} + \frac{(bf^2)\int\log(c(d+ex)^n)\,dx}{g^3} \\
&\quad + \frac{(bef^3n)\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g^4} + \frac{(befn)\int\frac{x^2}{d+ex}\,dx}{2g^2} - \frac{(ben)\int\frac{x^3}{d+ex}\,dx}{3g} \\
&= \frac{af^2x}{g^3} - \frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2} + \frac{x^3(a+b\log(c(d+ex)^n))}{3g} \\
&\quad - \frac{f^3(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4} + \frac{(bf^2)\,\text{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{eg^3} \\
&\quad + \frac{(bf^3n)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g^4} \\
&\quad + \frac{(befn)\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx}{2g^2} - \frac{(ben)\int\left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)dx}{3g} \\
&= \frac{af^2x}{g^3} - \frac{bf^2nx}{g^3} - \frac{bdfnx}{2eg^2} - \frac{bd^2nx}{3e^2g} + \frac{bfnx^2}{4g^2} + \frac{bdnx^2}{6eg} - \frac{bnx^3}{9g} \\
&\quad + \frac{bd^2fn\log(d+ex)}{2e^2g^2} + \frac{bd^3n\log(d+ex)}{3e^3g} + \frac{bf^2(d+ex)\log(c(d+ex)^n)}{eg^3} \\
&\quad - \frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2} + \frac{x^3(a+b\log(c(d+ex)^n))}{3g} \\
&\quad - \frac{f^3(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4} - \frac{bf^3n\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^4}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.86

$$
\int\frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx
$$

$$
= \frac{6bd^2g^2(3ef+2dg)n\log(d+ex)+e\Big(gx(6ae^2(6f^2-3fgx+2g^2x^2)-bn(12d^2g^2-6deg(-3f+gx)+e^2(\cdots}{\cdots}
$$

```
[In] Integrate[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

```
[Out] (6*b*d^2*g^2*(3*e*f + 2*d*g)*n*Log[d + e*x] + e*(g*x*(6*a*e^2*(6*f^2 - 3*f*
g*x + 2*g^2*x^2) - b*n*(12*d^2*g^2 - 6*d*e*g*(-3*f + g*x) + e^2*(36*f^2 - 9
*f*g*x + 4*g^2*x^2))) - 36*a*e^2*f^3*Log[(e*(f + g*x))/(e*f - d*g)] + 6*b*e
*Log[c*(d + e*x)^n]*(6*d*f^2*g + e*g*x*(6*f^2 - 3*f*g*x + 2*g^2*x^2) - 6*e*
f^3*Log[(e*(f + g*x))/(e*f - d*g)])) - 36*b*e^3*f^3*n*PolyLog[2, (g*(d + e*
x))/(-(e*f) + d*g)])/(36*e^3*g^4)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.80

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)x^3}{3g} - \frac{b\ln((ex+d)^n)f\,x^2}{2g^2} + \frac{b\ln((ex+d)^n)x\,f^2}{g^3} - \frac{b\ln((ex+d)^n)f^3\ln(gx+f)}{g^4} - \frac{bn\,x^3}{9g} + \frac{bfn\,x^2}{4g^2} - \frac{b\,f^2nx}{g^3} - \frac{49b}{36}$ |

```
[In] int(x^3*(a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*ln((e*x+d)^n)/g*x^3-1/2*b*ln((e*x+d)^n)/g^2*f*x^2+b*ln((e*x+d)^n)/g^3
*x*f^2-b*ln((e*x+d)^n)*f^3/g^4*ln(g*x+f)-1/9*b*n*x^3/g+1/4*b*f*n*x^2/g^2-b*
f^2*n*x/g^3-49/36*b*n/g^4*f^3+1/6*b*d*n*x^2/e/g-1/2*b*d*f*n*x/e/g^2-2/3*b/e
*n/g^3*d*f^2-1/3*b*d^2*n*x/e^2/g-1/3*b/e^2*n/g^2*d^2*f+1/3*b/e^3*n/g*d^3*ln
((g*x+f)*e+d*g-e*f)+1/2*b/e^2*n/g^2*d^2*ln((g*x+f)*e+d*g-e*f)*f+b/e*n/g^3*d
*ln((g*x+f)*e+d*g-e*f)*f^2+b*n/g^4*f^3*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))
+b*n/g^4*f^3*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(
I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e
*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*cs
gn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/g^3*(1/3*g^2*x^3-1/2*f*g*x^2+f^2*x)-f^3/g
^4*ln(g*x+f))
```

## Fricas [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^3}{gx+f}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*log((e*x + d)^n*c) + a*x^3)/(g*x + f), x)
```

## Sympy [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx$$

```
[In] integrate(x**3*(a+b*ln(c*(e*x+d)**n))/(g*x+f),x)
```

```
[Out] Integral(x**3*(a + b*log(c*(d + e*x)**n))/(f + g*x), x)
```

## Maxima [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^3}{gx+f}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
[Out] -1/6*a*(6*f^3*log(g*x + f)/g^4 - (2*g^2*x^3 - 3*f*g*x^2 + 6*f^2*x)/g^3) + b
*integrate((x^3*log((e*x + d)^n) + x^3*log(c))/(g*x + f), x)
```

## Giac [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^3}{gx+f}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*x^3/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,(d+e\,x)^n))}{f+g\,x}\,dx$$

```
[In] int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x),x)
[Out] int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x), x)
```

# 3.243 $\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 181

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx = -\frac{afx}{g^2}+\frac{bfnx}{g^2}+\frac{bdnx}{2eg}-\frac{bnx^2}{4g}-\frac{bd^2n\log(d+ex)}{2e^2g}-\frac{bf(d+ex)\log(c(d+ex)^n)}{eg^2}+\frac{x^2(a+b\log(c(d+ex)^n))}{2g}+\frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3}+\frac{bf^2n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$

```
[Out] -a*f*x/g^2+b*f*n*x/g^2+1/2*b*d*n*x/e/g-1/4*b*n*x^2/g-1/2*b*d^2*n*ln(e*x+d)/
e^2/g-b*f*(e*x+d)*ln(c*(e*x+d)^n)/e/g^2+1/2*x^2*(a+b*ln(c*(e*x+d)^n))/g+f^2
*(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g^3+b*f^2*n*polylog(2,-g*(e
*x+d)/(-d*g+e*f))/g^3
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

$= \{45, 2463, 2436, 2332, 2442, 2441, 2440, 2438\}$

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \frac{f^2\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^3} + \frac{x^2(a+b\log(c(d+ex)^n))}{2g} - \frac{afx}{g^2} - \frac{bf(d+ex)\log(c(d+ex)^n)}{eg^2} - \frac{bd^2n\log(d+ex)}{2e^2g} + \frac{bf^2n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3} + \frac{bdnx}{2eg} + \frac{bfnx}{g^2} - \frac{bnx^2}{4g}$$

```
[In] Int[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

```
[Out] -((a*f*x)/g^2) + (b*f*n*x)/g^2 + (b*d*n*x)/(2*e*g) - (b*n*x^2)/(4*g) - (b*d
^2*n*Log[d + e*x])/(2*e^2*g) - (b*f*(d + e*x)*Log[c*(d + e*x)^n])/(e*g^2) +
 (x^2*(a + b*Log[c*(d + e*x)^n]))/(2*g) + (f^2*(a + b*Log[c*(d + e*x)^n])*L
og[(e*(f + g*x))/(e*f - d*g)])/g^3 + (b*f^2*n*PolyLog[2, -((g*(d + e*x))/(e
*f - d*g))])/g^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{f(a+b\log(c(d+ex)^n))}{g^2}+\frac{x(a+b\log(c(d+ex)^n))}{g} +\frac{f^2(a+b\log(c(d+ex)^n))}{g^2(f+gx)}\right)dx$$

$$=-\frac{f\int(a+b\log(c(d+ex)^n))\,dx}{g^2}+\frac{f^2\int\frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{g^2}+\frac{\int x(a+b\log(c(d+ex)^n))\,dx}{g}$$

$$=-\frac{afx}{g^2}+\frac{x^2(a+b\log(c(d+ex)^n))}{2g}+\frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} -\frac{(bf)\int\log(c(d+ex)^n)\,dx}{g^2}-\frac{(bef^2n)\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g^3}-\frac{(ben)\int\frac{x^2}{d+ex}\,dx}{2g}$$

$$= -\frac{afx}{g^2} + \frac{x^2(a+b\log(c(d+ex)^n))}{2g} + \frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} - \frac{(bf)\text{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{eg^2} - \frac{(bf^2n)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g^3} - \frac{(ben)\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx}{2g}$$

$$= -\frac{afx}{g^2} + \frac{bfnx}{g^2} + \frac{bdnx}{2eg} - \frac{bnx^2}{4g} - \frac{bd^2n\log(d+ex)}{2e^2g} - \frac{bf(d+ex)\log(c(d+ex)^n)}{eg^2} + \frac{x^2(a+b\log(c(d+ex)^n))}{2g} + \frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} + \frac{bf^2n\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.94

$$\int\frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx = -\frac{afx}{g^2} + \frac{bfnx}{g^2} + \frac{bn\left(\frac{2dx}{e}-x^2-\frac{2d^2\log(d+ex)}{e^2}\right)}{4g} - \frac{bf(d+ex)\log(c(d+ex)^n)}{eg^2} + \frac{x^2(a+b\log(c(d+ex)^n))}{2g} + \frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} + \frac{bf^2n\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$

```
[In] Integrate[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

```
[Out] -((a*f*x)/g^2) + (b*f*n*x)/g^2 + (b*n*((2*d*x)/e - x^2 - (2*d^2*Log[d + e*x
])/e^2))/(4*g) - (b*f*(d + e*x)*Log[c*(d + e*x)^n])/(e*g^2) + (x^2*(a + b*L
og[c*(d + e*x)^n]))/(2*g) + (f^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x
))/(e*f - d*g)])/g^3 + (b*f^2*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/g
^3
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.80 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.14

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)x^2}{2g} - \frac{b\ln((ex+d)^n)fx}{g^2} + \frac{b\ln((ex+d)^n)f^2\ln(gx+f)}{g^3} - \frac{bn\,f^2\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g^3} - \frac{bn\,f^2\ln(gx+f)\ln\left(\frac{(gx+f)e}{dg}\right.}{g^3}$ |

```
[In] int(x^2*(a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln((e*x+d)^n)/g*x^2-b*ln((e*x+d)^n)/g^2*f*x+b*ln((e*x+d)^n)*f^2/g^3*l
n(g*x+f)-b*n/g^3*f^2*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b*n/g^3*f^2*ln(g*
x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-1/4*b*n*x^2/g+b*f*n*x/g^2+5/4*b*f^2*
n/g^3+1/2*b*d*n*x/e/g+1/2*b*d*f*n/e/g^2-1/2*b/e^2*n/g*d^2*ln((g*x+f)*e+d*g-
e*f)-b/e*n/g^2*d*ln((g*x+f)*e+d*g-e*f)*f+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x
+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I
*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n
)^3+b*ln(c)+a)*(1/g^2*(1/2*g*x^2-f*x)+f^2/g^3*ln(g*x+f))
```

## Fricas [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^2}{gx+f}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*log((e*x + d)^n*c) + a*x^2)/(g*x + f), x)
```

## Sympy [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx}\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*(e*x+d)**n))/(g*x+f),x)
```

```
[Out] Integral(x**2*(a + b*log(c*(d + e*x)**n))/(f + g*x), x)
```

## Maxima [F]

$$\int \frac{x^2(a + b\log(c(d + ex)^n))}{f + gx}\,dx = \int \frac{(b\log((ex + d)^n c) + a)x^2}{gx + f}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*f^2*log(g*x + f)/g^3 + (g*x^2 - 2*f*x)/g^2) + b*integrate((x^2*log
((e*x + d)^n) + x^2*log(c))/(g*x + f), x)
```

## Giac [F]

$$\int \frac{x^2(a + b\log(c(d + ex)^n))}{f + gx}\,dx = \int \frac{(b\log((ex + d)^n c) + a)x^2}{gx + f}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*x^2/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\log(c(d + ex)^n))}{f + gx}\,dx = \int \frac{x^2\,(a + b\,\ln(c\,(d + e\,x)^n))}{f + g\,x}\,dx$$

```
[In] int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x),x)
```

```
[Out] int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x), x)
```

# 3.244 $\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \frac{ax}{g}-\frac{bnx}{g}+\frac{b(d+ex)\log(c(d+ex)^n)}{eg} - \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} - \frac{bfn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}$$

```
[Out] a*x/g-b*n*x/g+b*(e*x+d)*ln(c*(e*x+d)^n)/e/g-f*(a+b*ln(c*(e*x+d)^n))*ln(e*(g
*x+f)/(-d*g+e*f))/g^2-b*f*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g^2
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {45, 2463, 2436, 2332, 2441, 2440, 2438}

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx = -\frac{f\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^2} + \frac{ax}{g}+\frac{b(d+ex)\log(c(d+ex)^n)}{eg} - \frac{bfn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}-\frac{bnx}{g}$$

```
[In] Int[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

```
[Out] (a*x)/g - (b*n*x)/g + (b*(d + e*x)*Log[c*(d + e*x)^n])/(e*g) - (f*(a + b*Lo
g[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^2 - (b*f*n*PolyLog[2, -
((g*(d + e*x))/(e*f - d*g))])/g^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{g}-\frac{f(a+b\log\left(c(d+ex)^n\right))}{g(f+gx)}\right)dx\\
&= \frac{\int \left(a+b\log\left(c(d+ex)^n\right)\right)dx}{g}-\frac{f\int \frac{a+b\log(c(d+ex)^n)}{f+gx}dx}{g}\\
&= \frac{ax}{g}-\frac{f(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2}\\
&\quad+\frac{b\int \log\left(c(d+ex)^n\right)dx}{g}+\frac{(befn)\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g^2}\\
&= \frac{ax}{g}-\frac{f(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2}+\frac{b\text{Subst}\left(\int \log\left(cx^n\right)dx,x,d+ex\right)}{eg}\\
&\quad+\frac{(bfn)\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}dx,x,d+ex\right)}{g^2}\\
&= \frac{ax}{g}-\frac{bnx}{g}+\frac{b(d+ex)\log\left(c(d+ex)^n\right)}{eg}\\
&\quad-\frac{f(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2}-\frac{bfn\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^2}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{x(a+b\log\left(c(d+ex)^n\right))}{f+gx}dx = \frac{agx-bgnx+\frac{bg(d+ex)\log(c(d+ex)^n)}{e}-f(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)-bfn\,\text{PolyLog}\left(2,\frac{g(d+ex)}{-ef+dg}\right)}{g^2}$$

```
[In] Integrate[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x),x]
```

```
[Out] (a*g*x - b*g*n*x + (b*g*(d + e*x)*Log[c*(d + e*x)^n])/e - f*(a + b*Log[c*(d
 + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] - b*f*n*PolyLog[2, (g*(d + e*x))
/(-(e*f) + d*g)])/g^2
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.80 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.73

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)x}{g} - \frac{b\ln((ex+d)^n)f\ln(gx+f)}{g^2} - \frac{bnx}{g} - \frac{bfn}{g^2} + \frac{bnd\ln((gx+f)e+dg-ef)}{eg} + \frac{bnf\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g^2} + \frac{bn}{}$ |

[In] int(x*(a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)

[Out] b*ln((e*x+d)^n)/g*x-b*ln((e*x+d)^n)*f/g^2*ln(g*x+f)-b*n*x/g-b*f*n/g^2+b/e*n /g*d*ln((g*x+f)*e+d*g-e*f)+b*n/g^2*f*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+b *n/g^2*f*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c) *csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d )^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I *c*(e*x+d)^n)^3+b*ln(c)+a)*(x/g-f/g^2*ln(g*x+f))

## Fricas [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{gx+f}\,dx$$

[In] integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")

[Out] integral((b*x*log((e*x + d)^n*c) + a*x)/(g*x + f), x)

## Sympy [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx$$

[In] integrate(x*(a+b*ln(c*(e*x+d)**n))/(g*x+f),x)

[Out] Integral(x*(a + b*log(c*(d + e*x)**n))/(f + g*x), x)

## Maxima [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{gx+f}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
```

```
[Out] a*(x/g - f*log(g*x + f)/g^2) + b*integrate((x*log((e*x + d)^n) + x*log(c))/
(g*x + f), x)
```

## Giac [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{gx+f}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*x/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx}\,dx = \int \frac{x\left(a+b\ln\left(c\left(d+e\,x\right)^n\right)\right)}{f+g\,x}\,dx$$

```
[In] int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x),x)
```

```
[Out] int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x), x)
```

# 3.245 $\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[Out] (a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g+b*n*polylog(2,-g*(e*x+d)/(
-d*g+e*f))/g
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2441, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx}\,dx = \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g + (b*n*PolyLo
g[2, -((g*(d + e*x))/(e*f - d*g))])/g
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} - \frac{(ben)\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} - \frac{(bn)\text{Subst}\left(\int\frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g}\\
&= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{bn\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int\frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx = \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g} + \frac{bn\,\text{PolyLog}\left(2,\frac{g(d+ex)}{-ef+dg}\right)}{g}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x),x]
[Out] ((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g + (b*n*PolyLo
g[2, (g*(d + e*x))/(-(e*f) + d*g)])/g
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.44

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^{n})\ln(gx+f)}{g}-\frac{bn\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g}-\frac{bn\ln(gx+f)\ln\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g}+\frac{\left(-\frac{ib\pi\ \mathrm{csgn}(ic(ex+d)^{n})\ \mathrm{csgn}(ic)\ \mathrm{csgn}}{2}\right.}{}$ |

[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f),x,method=_RETURNVERBOSE)

[Out] b*ln((e*x+d)^n)*ln(g*x+f)/g-b/g*n*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b/g*n*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*ln(g*x+f)/g

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{f+gx}\,dx=\int \frac{b\log\left((ex+d)^{n}c\right)+a}{gx+f}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="fricas")

[Out] integral((b*log((e*x + d)^n*c) + a)/(g*x + f), x)

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{f+gx}\,dx=\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{f+gx}\,dx$$

[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f),x)

[Out] Integral((a + b*log(c*(d + e*x)**n))/(f + g*x), x)

## Maxima [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{f + gx}\,dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{gx + f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="maxima")
[Out] b*integrate((log((e*x + d)^n) + log(c))/(g*x + f), x) + a*log(g*x + f)/g
```

## Giac [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{f + gx}\,dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{gx + f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/(g*x + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{f + gx}\,dx = \int \frac{a + b\,\ln\left(c\,(d + e\,x)^n\right)}{f + g\,x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(f + g*x), x)
```

# 3.246 $\int \frac{a+b\log(c(d+ex)^n)}{x(f+gx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 107

$$\int \frac{a+b\log(c(d+ex)^n)}{x(f+gx)}\,dx = \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f} - \frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f} + \frac{bn\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f}$$

```
[Out] ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/f-(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g
+e*f))/f-b*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/f+b*n*polylog(2,1+e*x/d)/f
```

## Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {36, 29, 31, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{a+b\log(c(d+ex)^n)}{x(f+gx)}\,dx = -\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{f} + \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f} - \frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f} + \frac{bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x)),x]
```

```
[Out] (Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f - ((a + b*Log[c*(d + e*x)^n]
)*Log[(e*(f + g*x))/(e*f - d*g)])/f - (b*n*PolyLog[2, -((g*(d + e*x))/(e*f
- d*g))])/f + (b*n*PolyLog[2, 1 + (e*x)/d])/f
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{fx}-\frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f(f+gx)}\right)\,dx\\
&= \frac{\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx}{f}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{f}\\
&= \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f}-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f}\\
&\quad -\frac{(ben)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{f}+\frac{(ben)\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{f}\\
&= \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f}-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f}\\
&\quad +\frac{bn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f}+\frac{(bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{f}\\
&= \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f}-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f}\\
&\quad -\frac{bn\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{f}+\frac{bn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)}\,dx = \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(\log\left(-\frac{ex}{d}\right)-\log\left(\frac{e(f+gx)}{ef-dg}\right)\right)-bn\,\text{PolyLog}\left(2,\frac{g(d+ex)}{-ef+dg}\right)+bn\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x)),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])*(Log[-((e*x)/d)] - Log[(e*(f + g*x))/(e*f - d*g
)]) - b*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] + b*n*PolyLog[2, 1 + (e*
x)/d])/f
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.58

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)\ln(x)}{f} - \frac{b\ln((ex+d)^n)\ln(gx+f)}{f} - \frac{bn\,\mathrm{dilog}\left(\frac{ex+d}{d}\right)}{f} - \frac{bn\ln(x)\ln\left(\frac{ex+d}{d}\right)}{f} + \frac{bn\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{f} + \frac{bn\ln(}{}$ |

[In] int((a+b*ln(c*(e*x+d)^n))/x/(g*x+f),x,method=_RETURNVERBOSE)

[Out] b*ln((e*x+d)^n)/f*ln(x)-b*ln((e*x+d)^n)/f*ln(g*x+f)-b*n/f*dilog((e*x+d)/d)-b*n/f*ln(x)*ln((e*x+d)/d)+b*n/f*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+b*n/f*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/f*ln(x)-1/f*ln(g*x+f))

## Fricas [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{x(f+gx)}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{(gx+f)x}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/x/(g*x+f),x, algorithm="fricas")

[Out] integral((b*log((e*x + d)^n*c) + a)/(g*x^2 + f*x), x)

## Sympy [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{x(f+gx)}\,dx = \int \frac{a + b\log\left(c(d+ex)^n\right)}{x\,(f+gx)}\,dx$$

[In] integrate((a+b*ln(c*(e*x+d)**n))/x/(g*x+f),x)

[Out] Integral((a + b*log(c*(d + e*x)**n))/(x*(f + g*x)), x)

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x/(g*x+f),x, algorithm="maxima")
```

```
[Out] -a*(log(g*x + f)/f - log(x)/f) + b*integrate((log((e*x + d)^n) + log(c))/(g
*x^2 + f*x), x)
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)}\,dx = \int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{x\,\left(f+g\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x)),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x)), x)
```

# 3.247 $\int \frac{a+b\log(c(d+ex)^n)}{x^2(f+gx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)}\,dx = \frac{ben\log(x)}{df} - \frac{ben\log(d+ex)}{df} - \frac{a+b\log\left(c(d+ex)^n\right)}{fx} - \frac{g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} + \frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^2} + \frac{bgn\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^2} - \frac{bgn\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^2}$$

```
[Out] b*e*n*ln(x)/d/f-b*e*n*ln(e*x+d)/d/f+(-a-b*ln(c*(e*x+d)^n))/f/x-g*ln(-e*x/d)
*(a+b*ln(c*(e*x+d)^n))/f^2+g*(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))
/f^2+b*g*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/f^2-b*g*n*polylog(2,1+e*x/d)/f^
2
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules

used = {46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)}\,dx = -\frac{g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} + \frac{g\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} - \frac{a+b\log\left(c(d+ex)^n\right)}{fx} + \frac{bgn\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^2} - \frac{bgn\,\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^2} + \frac{ben\log(x)}{df} - \frac{ben\log(d+ex)}{df}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x)),x]
```

```
[Out] (b*e*n*Log[x])/(d*f) - (b*e*n*Log[d + e*x])/(d*f) - (a + b*Log[c*(d + e*x)^
n])/(f*x) - (g*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f^2 + (g*(a + b*
Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/f^2 + (b*g*n*PolyLog[2,
 -((g*(d + e*x))/(e*f - d*g))])/f^2 - (b*g*n*PolyLog[2, 1 + (e*x)/d])/f^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{fx^2} - \frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2x} + \frac{g^2(a+b\log\left(c(d+ex)^n\right))}{f^2(f+gx)}\right) dx$$

$$= \frac{\int \frac{a+b\log(c(d+ex)^n)}{x^2}\,dx}{f} - \frac{g\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx}{f^2} + \frac{g^2\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{f^2}$$

$$= -\frac{a+b\log(c(d+ex)^n)}{fx} - \frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^2} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^2} + \frac{(ben)\int\frac{1}{x(d+ex)}\,dx}{f} + \frac{(begn)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{f^2} - \frac{(begn)\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{f^2}$$

$$= -\frac{a+b\log(c(d+ex)^n)}{fx} - \frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^2} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^2} - \frac{bgn\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} + \frac{(ben)\int\frac{1}{x}\,dx}{df} - \frac{(be^2n)\int\frac{1}{d+ex}\,dx}{df} - \frac{(bgn)\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{f^2}$$

$$= \frac{ben\log(x)}{df} - \frac{ben\log(d+ex)}{df} - \frac{a+b\log(c(d+ex)^n)}{fx} - \frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^2} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^2} + \frac{bgn\mathrm{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{f^2} - \frac{bgn\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{f^2}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.87

$$\int \frac{a+b\log(c(d+ex)^n)}{x^2(f+gx)}\,dx = \frac{\frac{befn(\log(x)-\log(d+ex))}{d} - \frac{f(a+b\log(c(d+ex)^n))}{x} - g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n)) + g(a+b\log(c(d+ex)^n))\log}{f^2}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x)),x]
[Out] ((b*e*f*n*(Log[x] - Log[d + e*x]))/d - (f*(a + b*Log[c*(d + e*x)^n]))/x - g
*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]) + g*(a + b*Log[c*(d + e*x)^n])*
Log[(e*(f + g*x))/(e*f - d*g)] + b*g*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d
*g)] - b*g*n*PolyLog[2, 1 + (e*x)/d])/f^2
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.81 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.09

| method | result |
|---|---|
| risch | $-\frac{b\ln\left((ex+d)^{n}\right)}{fx}-\frac{b\ln\left((ex+d)^{n}\right)g\ln\left(x\right)}{f^{2}}+\frac{b\ln\left((ex+d)^{n}\right)g\ln\left(gx+f\right)}{f^{2}}-\frac{bng\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{f^{2}}-\frac{bng\ln\left(gx+f\right)\ln\left(\frac{(gx+f)e}{dg-}\right.}{f^{2}}$ |

[In] int((a+b*ln(c*(e*x+d)^n))/x^2/(g*x+f),x,method=_RETURNVERBOSE)

[Out] -b*ln((e*x+d)^n)/f/x-b*ln((e*x+d)^n)/f^2*g*ln(x)+b*ln((e*x+d)^n)/f^2*g*ln(g*x+f)-b*n/f^2*g*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b*n/f^2*g*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-b*e*n*ln(e*x+d)/d/f+b*e*n*ln(x)/d/f+b*n/f^2*g*dilog((e*x+d)/d)+b*n/f^2*g*ln(x)*ln((e*x+d)/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(-1/f/x-1/f^2*g*ln(x)+1/f^2*g*ln(g*x+f))

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{x^{2}(f+gx)}\,dx=\int \frac{b\log\left((ex+d)^{n}c\right)+a}{(gx+f)x^{2}}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x+f),x, algorithm="fricas")

[Out] integral((b*log((e*x + d)^n*c) + a)/(g*x^3 + f*x^2), x)

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{x^{2}(f+gx)}\,dx=\text{Timed out}$$

[In] integrate((a+b*ln(c*(e*x+d)**n))/x**2/(g*x+f),x)

[Out] Timed out

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x+f),x, algorithm="maxima")
```

```
[Out] a*(g*log(g*x + f)/f^2 - g*log(x)/f^2 - 1/(f*x)) + b*integrate((log((e*x + d
)^n) + log(c))/(g*x^3 + f*x^2), x)
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{x^2\,\left(f+g\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x)),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x)), x)
```

# 3.248 $\int \frac{a+b\log(c(d+ex)^n)}{x^3(f+gx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 250

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx =& -\frac{ben}{2dfx}-\frac{be^2n\log(x)}{2d^2f}-\frac{begn\log(x)}{df^2}\\&+\frac{be^2n\log(d+ex)}{2d^2f}+\frac{begn\log(d+ex)}{df^2}\\&-\frac{a+b\log\left(c(d+ex)^n\right)}{2fx^2}+\frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2x}\\&+\frac{g^2\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3}\\&-\frac{g^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^3}\\&-\frac{bg^2n\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^3}+\frac{bg^2n\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^3}\end{aligned}$$

```
[Out] -1/2*b*e*n/d/f/x-1/2*b*e^2*n*ln(x)/d^2/f-b*e*g*n*ln(x)/d/f^2+1/2*b*e^2*n*ln
(e*x+d)/d^2/f+b*e*g*n*ln(e*x+d)/d/f^2+1/2*(-a-b*ln(c*(e*x+d)^n))/f/x^2+g*(a
+b*ln(c*(e*x+d)^n))/f^2/x+g^2*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/f^3-g^2*(a+b
*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/f^3-b*g^2*n*polylog(2,-g*(e*x+d)
/(-d*g+e*f))/f^3+b*g^2*n*polylog(2,1+e*x/d)/f^3
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx = \frac{g^2\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3} - \frac{g^2\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3} + \frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2x} - \frac{a+b\log\left(c(d+ex)^n\right)}{2fx^2} - \frac{be^2n\log(x)}{2d^2f} + \frac{be^2n\log(d+ex)}{2d^2f} - \frac{bg^2n\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^3} + \frac{bg^2n\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^3} - \frac{begn\log(x)}{df^2} + \frac{begn\log(d+ex)}{df^2} - \frac{ben}{2dfx}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x)),x]
```

```
[Out] -1/2*(b*e*n)/(d*f*x) - (b*e^2*n*Log[x])/(2*d^2*f) - (b*e*g*n*Log[x])/(d*f^2
) + (b*e^2*n*Log[d + e*x])/(2*d^2*f) + (b*e*g*n*Log[d + e*x])/(d*f^2) - (a
+ b*Log[c*(d + e*x)^n])/(2*f*x^2) + (g*(a + b*Log[c*(d + e*x)^n]))/(f^2*x)
+ (g^2*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f^3 - (g^2*(a + b*Log[c*
(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/f^3 - (b*g^2*n*PolyLog[2, -((
g*(d + e*x))/(e*f - d*g))])/f^3 + (b*g^2*n*PolyLog[2, 1 + (e*x)/d])/f^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{fx^3} - \frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2x^2} + \frac{g^2(a+b\log\left(c(d+ex)^n\right))}{f^3x} - \frac{g^3(a+b\log\left(c(d+ex)^n\right))}{f^3(f+gx)}\right)dx$$

$$= \frac{\int \frac{a+b\log(c(d+ex)^n)}{x^3}\,dx}{f} - \frac{g\int \frac{a+b\log(c(d+ex)^n)}{x^2}\,dx}{f^2} + \frac{g^2\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx}{f^3} - \frac{g^3\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{f^3}$$

$$= -\frac{a+b\log\left(c(d+ex)^n\right)}{2fx^2} + \frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2x} + \frac{g^2\log\left(-\frac{ex}{d}\right)(a+b\log\left(c(d+ex)^n\right))}{f^3} - \frac{g^2(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^3} + \frac{(ben)\int \frac{1}{x^2(d+ex)}\,dx}{2f} - \frac{(begn)\int \frac{1}{x(d+ex)}\,dx}{f^2} - \frac{(beg^2n)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{f^3} + \frac{(beg^2n)\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{f^3}$$

$$= -\frac{a+b\log\left(c(d+ex)^n\right)}{2fx^2} + \frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2x} + \frac{g^2\log\left(-\frac{ex}{d}\right)(a+b\log\left(c(d+ex)^n\right))}{f^3} - \frac{g^2(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^3} + \frac{bg^2n\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^3} + \frac{(ben)\int \left(\frac{1}{dx^2} - \frac{e}{d^2x} + \frac{e^2}{d^2(d+ex)}\right)dx}{2f} - \frac{(begn)\int \frac{1}{x}\,dx}{df^2} + \frac{(be^2gn)\int \frac{1}{d+ex}\,dx}{df^2} + \frac{(bg^2n)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{f^3}$$

$$= -\frac{ben}{2dfx} - \frac{be^2n\log(x)}{2d^2f} - \frac{begn\log(x)}{df^2} + \frac{be^2n\log(d+ex)}{2d^2f} + \frac{begn\log(d+ex)}{df^2} - \frac{a+b\log\left(c(d+ex)^n\right)}{2fx^2} + \frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2x} + \frac{g^2\log\left(-\frac{ex}{d}\right)(a+b\log\left(c(d+ex)^n\right))}{f^3} - \frac{g^2(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^3} - \frac{bg^2n\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{f^3} + \frac{bg^2n\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^3}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.83

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx = -\frac{\frac{2befgn(\log(x)-\log(d+ex))}{d}+\frac{bef^2n(d+ex\log(x)-ex\log(d+ex))}{d^2x}+\frac{f^2(a+b\log(c(d+ex)^n))}{x^2}-\frac{2fg(a+b\log(c(d+ex)^n))}{x}-2g^2\log\left(-\right.}{}$$

[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x)),x]

[Out] -1/2*((2*b*e*f*g*n*(Log[x] - Log[d + e*x]))/d + (b*e*f^2*n*(d + e*x*Log[x] - e*x*Log[d + e*x]))/(d^2*x) + (f^2*(a + b*Log[c*(d + e*x)^n]))/x^2 - (2*f*g*(a + b*Log[c*(d + e*x)^n]))/x - 2*g^2*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]) + 2*g^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] + 2*b*g^2*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*b*g^2*n*PolyLog[2, 1 + (e*x)/d])/f^3

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.96 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.72

| method | result |
|---|---|
| risch | $-\frac{b\ln((ex+d)^n)}{2f\,x^2}+\frac{b\ln((ex+d)^n)g^2\ln(x)}{f^3}+\frac{b\ln((ex+d)^n)g}{f^2x}-\frac{b\ln((ex+d)^n)g^2\ln(gx+f)}{f^3}+\frac{begn\ln(ex+d)}{d\,f^2}+\frac{b\,e^2n\ln(ex+d)}{2d^2f}$ |

[In] int((a+b*ln(c*(e*x+d)^n))/x^3/(g*x+f),x,method=_RETURNVERBOSE)

[Out] -1/2*b*ln((e*x+d)^n)/f/x^2+b*ln((e*x+d)^n)/f^3*g^2*ln(x)+b*ln((e*x+d)^n)/f^2*g/x-b*ln((e*x+d)^n)/f^3*g^2*ln(g*x+f)+b*e*g*n*ln(e*x+d)/d/f^2+1/2*b*e^2*n*ln(e*x+d)/d^2/f-b*e*g*n*ln(x)/d/f^2-1/2*b*e^2*n*ln(x)/d^2/f-1/2*b*e*n/d/f/x-b*n/f^3*g^2*dilog((e*x+d)/d)-b*n/f^3*g^2*ln(x)*ln((e*x+d)/d)+b*n/f^3*g^2*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+b*n/f^3*g^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(-1/2/f/x^2+1/f^3*g^2*ln(x)+1/f^2*g/x-1/f^3*g^2*ln(g*x+f))

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x+f),x, algorithm="fricas")
[Out] integral((b*log((e*x + d)^n*c) + a)/(g*x^4 + f*x^3), x)
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx\right)}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/x**3/(g*x+f),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))/(x**3*(f + g*x)), x)
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x+f),x, algorithm="maxima")
[Out] -1/2*a*(2*g^2*log(g*x + f)/f^3 - 2*g^2*log(x)/f^3 - (2*g*x - f)/(f^2*x^2))
+ b*integrate((log((e*x + d)^n) + log(c))/(g*x^4 + f*x^3), x)
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{x^3\,\left(f+g\,x\right)} \,d x$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x)),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x)), x)
```

# 3.249 $\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 265

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = -\frac{2afx}{g^3}+\frac{2bfnx}{g^3}+\frac{bdnx}{2eg^2}-\frac{bnx^2}{4g^2}-\frac{bd^2n\log(d+ex)}{2e^2g^2} -\frac{bef^3n\log(d+ex)}{g^4(ef-dg)}-\frac{2bf(d+ex)\log(c(d+ex)^n)}{eg^3} +\frac{x^2(a+b\log(c(d+ex)^n))}{2g^2} +\frac{f^3(a+b\log(c(d+ex)^n))}{g^4(f+gx)}+\frac{bef^3n\log(f+gx)}{g^4(ef-dg)} +\frac{3f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4} +\frac{3bf^2n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4}$$

```
[Out] -2*a*f*x/g^3+2*b*f*n*x/g^3+1/2*b*d*n*x/e/g^2-1/4*b*n*x^2/g^2-1/2*b*d^2*n*ln
(e*x+d)/e^2/g^2-b*e*f^3*n*ln(e*x+d)/g^4/(-d*g+e*f)-2*b*f*(e*x+d)*ln(c*(e*x+
d)^n)/e/g^3+1/2*x^2*(a+b*ln(c*(e*x+d)^n))/g^2+f^3*(a+b*ln(c*(e*x+d)^n))/g^4
/(g*x+f)+b*e*f^3*n*ln(g*x+f)/g^4/(-d*g+e*f)+3*f^2*(a+b*ln(c*(e*x+d)^n))*ln(
e*(g*x+f)/(-d*g+e*f))/g^4+3*b*f^2*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g^4
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {45, 2463, 2436, 2332, 2442, 36, 31, 2441, 2440, 2438}

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \frac{f^3(a+b\log(c(d+ex)^n))}{g^4(f+gx)} + \frac{3f^2\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^4} + \frac{x^2(a+b\log(c(d+ex)^n))}{2g^2} - \frac{2afx}{g^3} - \frac{2bf(d+ex)\log(c(d+ex)^n)}{eg^3} - \frac{bd^2n\log(d+ex)}{2e^2g^2} - \frac{bef^3n\log(d+ex)}{g^4(ef-dg)} + \frac{bef^3n\log(f+gx)}{g^4(ef-dg)} + \frac{3bf^2n\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4} + \frac{bdnx}{2eg^2} + \frac{2bfnx}{g^3} - \frac{bnx^2}{4g^2}$$

```
[In] Int[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x)^2,x]
```

```
[Out] (-2*a*f*x)/g^3 + (2*b*f*n*x)/g^3 + (b*d*n*x)/(2*e*g^2) - (b*n*x^2)/(4*g^2)
- (b*d^2*n*Log[d + e*x])/(2*e^2*g^2) - (b*e*f^3*n*Log[d + e*x])/(g^4*(e*f -
 d*g)) - (2*b*f*(d + e*x)*Log[c*(d + e*x)^n])/(e*g^3) + (x^2*(a + b*Log[c*(
d + e*x)^n]))/(2*g^2) + (f^3*(a + b*Log[c*(d + e*x)^n]))/(g^4*(f + g*x)) +
(b*e*f^3*n*Log[f + g*x])/(g^4*(e*f - d*g)) + (3*f^2*(a + b*Log[c*(d + e*x)^
n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^4 + (3*b*f^2*n*PolyLog[2, -((g*(d + e
*x))/(e*f - d*g))])/g^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{2f(a+b\log(c(d+ex)^n))}{g^3}+\frac{x(a+b\log(c(d+ex)^n))}{g^2}-\frac{f^3(a+b\log(c(d+ex)^n))}{g^3(f+gx)^2}+\frac{3f^2(a+b\log(c(d+ex)^n))}{g^3(f+gx)}\right)dx$$

$$=-\frac{(2f)\int(a+b\log(c(d+ex)^n))\,dx}{g^3}+\frac{(3f^2)\int\frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{g^3}-\frac{f^3\int\frac{a+b\log(c(d+ex)^n)}{(f+gx)^2}\,dx}{g^3}+\frac{\int x(a+b\log(c(d+ex)^n))\,dx}{g^2}$$

$$=-\frac{2afx}{g^3}+\frac{x^2(a+b\log(c(d+ex)^n))}{2g^2}+\frac{f^3(a+b\log(c(d+ex)^n))}{g^4(f+gx)}+\frac{3f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4}-\frac{(2bf)\int\log(c(d+ex)^n)\,dx}{g^3}-\frac{(3bef^2n)\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g^4}-\frac{(bef^3n)\int\frac{1}{(d+ex)(f+gx)}\,dx}{g^4}-\frac{(ben)\int\frac{x^2}{d+ex}\,dx}{2g^2}$$

$$=-\frac{2afx}{g^3}+\frac{x^2(a+b\log(c(d+ex)^n))}{2g^2}+\frac{f^3(a+b\log(c(d+ex)^n))}{g^4(f+gx)}+\frac{3f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4}-\frac{(2bf)\text{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{eg^3}-\frac{(3bf^2n)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g^4}-\frac{(ben)\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx}{2g^2}-\frac{(be^2f^3n)\int\frac{1}{d+ex}\,dx}{g^4(ef-dg)}+\frac{(bef^3n)\int\frac{1}{f+gx}\,dx}{g^3(ef-dg)}$$

$$=-\frac{2afx}{g^3}+\frac{2bfnx}{g^3}+\frac{bdnx}{2eg^2}-\frac{bnx^2}{4g^2}-\frac{bd^2n\log(d+ex)}{2e^2g^2}-\frac{bef^3n\log(d+ex)}{g^4(ef-dg)}-\frac{2bf(d+ex)\log(c(d+ex)^n)}{eg^3}+\frac{x^2(a+b\log(c(d+ex)^n))}{2g^2}+\frac{f^3(a+b\log(c(d+ex)^n))}{g^4(f+gx)}+\frac{bef^3n\log(f+gx)}{g^4(ef-dg)}+\frac{3f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^4}+\frac{3bf^2n\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^4}$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.83

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$$
$$= \frac{-8afgx+8bfgnx-\frac{bg^2n(ex(-2d+ex)+2d^2\log(d+ex))}{e^2}-\frac{8bfg(d+ex)\log(c(d+ex)^n)}{e}+2g^2x^2(a+b\log(c(d+ex)^n))+}{}$$

[In] Integrate[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x)^2,x]

[Out] (-8*a*f*g*x + 8*b*f*g*n*x - (b*g^2*n*(e*x*(-2*d + e*x) + 2*d^2*Log[d + e*x]
))/e^2 - (8*b*f*g*(d + e*x)*Log[c*(d + e*x)^n])/e + 2*g^2*x^2*(a + b*Log[c*
(d + e*x)^n]) + (4*f^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x) - (4*b*e*f^3*n
*(Log[d + e*x] - Log[f + g*x]))/(e*f - d*g) + 12*f^2*(a + b*Log[c*(d + e*x)
^n])*Log[(e*(f + g*x))/(e*f - d*g)] + 12*b*f^2*n*PolyLog[2, (g*(d + e*x))/(
-(e*f) + d*g)])/(4*g^4)

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.08

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)x^2}{2g^2}-\frac{2b\ln((ex+d)^n)xf}{g^3}+\frac{b\ln((ex+d)^n)f^3}{g^4(gx+f)}+\frac{3b\ln((ex+d)^n)f^2\ln(gx+f)}{g^4}-\frac{3bn\,f^2\,\mathrm{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g^4}-$ |

[In] int(x^3*(a+b*ln(c*(e*x+d)^n))/(g*x+f)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*b*ln((e*x+d)^n)/g^2*x^2-2*b*ln((e*x+d)^n)/g^3*x*f+b*ln((e*x+d)^n)*f^3/g
^4/(g*x+f)+3*b*ln((e*x+d)^n)/g^4*f^2*ln(g*x+f)-3*b*n/g^4*f^2*dilog(((g*x+f)
*e+d*g-e*f)/(d*g-e*f))-3*b*n/g^4*f^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-
e*f))-1/4*b*n*x^2/g^2+2*b*f*n*x/g^3+9/4*b*n/g^4*f^2+1/2*b*d*n*x/e/g^2+1/2*b
/e*n/g^3*d*f-b*e*n/g^4*f^3/(d*g-e*f)*ln(g*x+f)-1/2*b/e^2*n/g/(d*g-e*f)*ln((
g*x+f)*e+d*g-e*f)*d^3-3/2*b/e*n/g^2/(d*g-e*f)*ln((g*x+f)*e+d*g-e*f)*d^2*f+2
*b*n/g^3/(d*g-e*f)*ln((g*x+f)*e+d*g-e*f)*d*f^2+b*e*n/g^4/(d*g-e*f)*ln((g*x+
f)*e+d*g-e*f)*f^3+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)
^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)
*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/g^3*(
1/2*g*x^2-2*f*x)+f^3/g^4/(g*x+f)+3/g^4*f^2*ln(g*x+f))

## Fricas [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^3}{(gx+f)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="fricas")
[Out] integral((b*x^3*log((e*x + d)^n*c) + a*x^3)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

## Sympy [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$$

```
[In] integrate(x**3*(a+b*ln(c*(e*x+d)**n))/(g*x+f)**2,x)
[Out] Integral(x**3*(a + b*log(c*(d + e*x)**n))/(f + g*x)**2, x)
```

## Maxima [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^3}{(gx+f)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="maxima")
[Out] 1/2*(2*f^3/(g^5*x + f*g^4) + 6*f^2*log(g*x + f)/g^4 + (g*x^2 - 4*f*x)/g^3)*
a + b*integrate((x^3*log((e*x + d)^n) + x^3*log(c))/(g^2*x^2 + 2*f*g*x + f^
2), x)
```

## Giac [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^3}{(gx+f)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*x^3/(g*x + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log\left(c(d+ex)^n\right))}{(f+gx)^2}\,dx=\int \frac{x^3\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}{{\left(f+g\,x\right)}^2}\,dx$$

```
[In] int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x)^2,x)
[Out] int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x)^2, x)
```

# 3.250 $\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 186

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \frac{ax}{g^2}-\frac{bnx}{g^2}+\frac{bef^2n\log(d+ex)}{g^3(ef-dg)}+\frac{b(d+ex)\log(c(d+ex)^n)}{eg^2} - \frac{f^2(a+b\log(c(d+ex)^n))}{g^3(f+gx)}-\frac{bef^2n\log(f+gx)}{g^3(ef-dg)} - \frac{2f(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} - \frac{2bfn\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$

```
[Out] a*x/g^2-b*n*x/g^2+b*e*f^2*n*ln(e*x+d)/g^3/(-d*g+e*f)+b*(e*x+d)*ln(c*(e*x+d)
^n)/e/g^2-f^2*(a+b*ln(c*(e*x+d)^n))/g^3/(g*x+f)-b*e*f^2*n*ln(g*x+f)/g^3/(-d
*g+e*f)-2*f*(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/g^3-2*b*f*n*poly
log(2,-g*(e*x+d)/(-d*g+e*f))/g^3
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules

used = {45, 2463, 2436, 2332, 2442, 36, 31, 2441, 2440, 2438}

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = -\frac{f^2(a+b\log(c(d+ex)^n))}{g^3(f+gx)} - \frac{2f\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^3} + \frac{ax}{g^2} + \frac{b(d+ex)\log(c(d+ex)^n)}{eg^2} + \frac{bef^2n\log(d+ex)}{g^3(ef-dg)} - \frac{bef^2n\log(f+gx)}{g^3(ef-dg)} - \frac{2bfn\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^3} - \frac{bnx}{g^2}$$

```
[In] Int[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x)^2,x]
```

```
[Out] (a*x)/g^2 - (b*n*x)/g^2 + (b*e*f^2*n*Log[d + e*x])/(g^3*(e*f - d*g)) + (b*(
d + e*x)*Log[c*(d + e*x)^n])/(e*g^2) - (f^2*(a + b*Log[c*(d + e*x)^n]))/(g^
3*(f + g*x)) - (b*e*f^2*n*Log[f + g*x])/(g^3*(e*f - d*g)) - (2*f*(a + b*Log
[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^3 - (2*b*f*n*PolyLog[2,
-((g*(d + e*x))/(e*f - d*g))])/g^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{g^2}+\frac{f^2(a+b\log\left(c(d+ex)^n\right))}{g^2(f+gx)^2} - \frac{2f(a+b\log\left(c(d+ex)^n\right))}{g^2(f+gx)}\right)\,dx$$

$$=\frac{\int \left(a+b\log\left(c(d+ex)^n\right)\right)\,dx}{g^2}-\frac{(2f)\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{g^2}+\frac{f^2\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^2}\,dx}{g^2}$$

$$= \frac{ax}{g^2} - \frac{f^2(a+b\log(c(d+ex)^n))}{g^3(f+gx)} - \frac{2f(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} + \frac{b\int\log(c(d+ex)^n)\,dx}{g^2} + \frac{(2befn)\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx}{g^3} + \frac{(bef^2n)\int\frac{1}{(d+ex)(f+gx)}dx}{g^3}$$

$$= \frac{ax}{g^2} - \frac{f^2(a+b\log(c(d+ex)^n))}{g^3(f+gx)} - \frac{2f(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} + \frac{b\mathrm{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{eg^2} + \frac{(2bfn)\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}dx,x,d+ex\right)}{g^3} + \frac{(be^2f^2n)\int\frac{1}{d+ex}dx}{g^3(ef-dg)} - \frac{(bef^2n)\int\frac{1}{f+gx}dx}{g^2(ef-dg)}$$

$$= \frac{ax}{g^2} - \frac{bnx}{g^2} + \frac{bef^2n\log(d+ex)}{g^3(ef-dg)} + \frac{b(d+ex)\log(c(d+ex)^n)}{eg^2} - \frac{f^2(a+b\log(c(d+ex)^n))}{g^3(f+gx)} - \frac{bef^2n\log(f+gx)}{g^3(ef-dg)} - \frac{2f(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^3} - \frac{2bfn\mathrm{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^3}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.82

$$\int\frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \frac{agx - bgnx + \frac{bg(d+ex)\log(c(d+ex)^n)}{e} - \frac{f^2(a+b\log(c(d+ex)^n))}{f+gx} + \frac{bef^2n(\log(d+ex)-\log(f+gx))}{ef-dg} - 2f(a+b\log(c(d+ex)^n$$

```
[In] Integrate[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x)^2,x]
```

```
[Out] (a*g*x - b*g*n*x + (b*g*(d + e*x)*Log[c*(d + e*x)^n])/e - (f^2*(a + b*Log[c
*(d + e*x)^n]))/(f + g*x) + (b*e*f^2*n*(Log[d + e*x] - Log[f + g*x]))/(e*f
- d*g) - 2*f*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] - 2*
b*f*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)])/g^3
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.84 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.34

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)x}{g^2}-\frac{b\ln((ex+d)^n)f^2}{g^3(gx+f)}-\frac{2b\ln((ex+d)^n)f\ln(gx+f)}{g^3}-\frac{bnx}{g^2}-\frac{bfn}{g^3}+\frac{ben\,f^2\ln(gx+f)}{g^3(dg-ef)}+\frac{bn\ln((gx+f)e+dg-ef)}{eg(dg-ef)}$ |

[In] int(x^2*(a+b*ln(c*(e*x+d)^n))/(g*x+f)^2,x,method=_RETURNVERBOSE)

[Out] b*ln((e*x+d)^n)/g^2*x-b*ln((e*x+d)^n)/g^3*f^2/(g*x+f)-2*b*ln((e*x+d)^n)/g^3 *f*ln(g*x+f)-b*n*x/g^2-b*f*n/g^3+b*e*n/g^3*f^2/(d*g-e*f)*ln(g*x+f)+b/e*n/g/ (d*g-e*f)*ln((g*x+f)*e+d*g-e*f)*d^2-b*n/g^2/(d*g-e*f)*ln((g*x+f)*e+d*g-e*f) *d*f-b*e*n/g^3/(d*g-e*f)*ln((g*x+f)*e+d*g-e*f)*f^2+2*b*n/g^3*f*dilog(((g*x+ f)*e+d*g-e*f)/(d*g-e*f))+2*b*n/g^3*f*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g- e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b* Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e *x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(x/g^2-1/g^3*f^2/(g* x+f)-2/g^3*f*ln(g*x+f))

## Fricas [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)x^2}{(gx+f)^2}\,dx$$

[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="fricas")

[Out] integral((b*x^2*log((e*x + d)^n*c) + a*x^2)/(g^2*x^2 + 2*f*g*x + f^2), x)

## Sympy [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx=\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$$

[In] integrate(x**2*(a+b*ln(c*(e*x+d)**n))/(g*x+f)**2,x)

[Out] Integral(x**2*(a + b*log(c*(d + e*x)**n))/(f + g*x)**2, x)

## Maxima [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^2}{(gx+f)^2}\,dx$$

[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="maxima")

[Out] -a*(f^2/(g^4*x + f*g^3) - x/g^2 + 2*f*log(g*x + f)/g^3) + b*integrate((x^2*log((e*x + d)^n) + x^2*log(c))/(g^2*x^2 + 2*f*g*x + f^2), x)

## Giac [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^2}{(gx+f)^2}\,dx$$

[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="giac")

[Out] integrate((b*log((e*x + d)^n*c) + a)*x^2/(g*x + f)^2, x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,(d+e\,x)^n))}{(f+g\,x)^2}\,dx$$

[In] int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x)^2,x)

[Out] int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x)^2, x)

# 3.251 $\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = -\frac{befn\log(d+ex)}{g^2(ef-dg)}+\frac{f(a+b\log(c(d+ex)^n))}{g^2(f+gx)}+\frac{befn\log(f+gx)}{g^2(ef-dg)}+\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}$$

```
[Out] -b*e*f*n*ln(e*x+d)/g^2/(-d*g+e*f)+f*(a+b*ln(c*(e*x+d)^n))/g^2/(g*x+f)+b*e*f
*n*ln(g*x+f)/g^2/(-d*g+e*f)+(a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/
g^2+b*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/g^2
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {45, 2463, 2442, 36, 31, 2441, 2440, 2438}

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \frac{f(a+b\log(c(d+ex)^n))}{g^2(f+gx)}+\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{g^2}+\frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2}-\frac{befn\log(d+ex)}{g^2(ef-dg)}+\frac{befn\log(f+gx)}{g^2(ef-dg)}$$

```
[In] Int[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x)^2,x]
```

```
[Out] -((b*e*f*n*Log[d + e*x])/(g^2*(e*f - d*g))) + (f*(a + b*Log[c*(d + e*x)^n])
)/(g^2*(f + g*x)) + (b*e*f*n*Log[f + g*x])/(g^2*(e*f - d*g)) + ((a + b*Log[
c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/g^2 + (b*n*PolyLog[2, -((g*
(d + e*x))/(e*f - d*g))])/g^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{f(a+b\log(c(d+ex)^n))}{g(f+gx)^2}+\frac{a+b\log(c(d+ex)^n)}{g(f+gx)}\right)dx \\
&= \frac{\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{g}-\frac{f\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^2}\,dx}{g} \\
&= \frac{f(a+b\log(c(d+ex)^n))}{g^2(f+gx)}+\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} \\
&\quad -\frac{(ben)\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{g^2}-\frac{(befn)\int \frac{1}{(d+ex)(f+gx)}\,dx}{g^2} \\
&= \frac{f(a+b\log(c(d+ex)^n))}{g^2(f+gx)}+\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2} \\
&\quad -\frac{(bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex\right)}{g^2}-\frac{(be^2fn)\int \frac{1}{d+ex}\,dx}{g^2(ef-dg)}+\frac{(befn)\int \frac{1}{f+gx}\,dx}{g(ef-dg)} \\
&= -\frac{befn\log(d+ex)}{g^2(ef-dg)}+\frac{f(a+b\log(c(d+ex)^n))}{g^2(f+gx)}+\frac{befn\log(f+gx)}{g^2(ef-dg)} \\
&\quad +\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{g^2}+\frac{bn\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{g^2}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx$$
$$= \frac{\frac{f(a+b\log(c(d+ex)^n))}{f+gx}-\frac{befn(\log(d+ex)-\log(f+gx))}{ef-dg}+(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)+bn\,\text{PolyLog}\left(2,\frac{g(d+ex)}{-ef+d}\right.}{g^2}$$

```
[In] Integrate[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x)^2,x]
```

```
[Out] ((f*(a + b*Log[c*(d + e*x)^n]))/(f + g*x) - (b*e*f*n*(Log[d + e*x] - Log[f
+ g*x]))/(e*f - d*g) + (a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f -
d*g)] + b*n*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)])/g^2
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.80 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.25

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)f}{g^2(gx+f)}+\frac{b\ln((ex+d)^n)\ln(gx+f)}{g^2}-\frac{bn\,\text{dilog}\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g^2}-\frac{bn\ln(gx+f)\ln\left(\frac{(gx+f)e+dg-ef}{dg-ef}\right)}{g^2}-\frac{benf\ln(gx+f)}{g^2(dg-ef)}$ |

```
[In] int(x*(a+b*ln(c*(e*x+d)^n))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*ln((e*x+d)^n)/g^2*f/(g*x+f)+b*ln((e*x+d)^n)/g^2*ln(g*x+f)-b*n/g^2*dilog((
(g*x+f)*e+d*g-e*f)/(d*g-e*f))-b*n/g^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g
-e*f))-b*e*n/g^2*f/(d*g-e*f)*ln(g*x+f)+b*e*n/g^2*f/(d*g-e*f)*ln((g*x+f)*e+d
*g-e*f)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*
b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*
(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/g^2*f/(g*x+f)+1
/g^2*ln(g*x+f))
```

## Fricas [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{(gx+f)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="fricas")
[Out] integral((b*x*log((e*x + d)^n*c) + a*x)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(e*x+d)**n))/(g*x+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{(gx+f)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="maxima")
[Out] a*(f/(g^3*x + f*g^2) + log(g*x + f)/g^2) + b*integrate((x*log((e*x + d)^n)
+ x*log(c))/(g^2*x^2 + 2*f*g*x + f^2), x)
```

## Giac [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{(gx+f)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*x/(g*x + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx)^2}\,dx = \int \frac{x\,(a+b\,\ln(c\,(d+e\,x)^n))}{(f+g\,x)^2}\,dx$$

```
[In] int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x)^2,x)
[Out] int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x)^2, x)
```

# 3.252 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = \frac{ben\log(d+ex)}{g(ef-dg)} - \frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)} - \frac{ben\log(f+gx)}{g(ef-dg)}$$

```
[Out] b*e*n*ln(e*x+d)/g/(-d*g+e*f)+(-a-b*ln(c*(e*x+d)^n))/g/(g*x+f)-b*e*n*ln(g*x+
f)/g/(-d*g+e*f)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2442, 36, 31}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = -\frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)} + \frac{ben\log(d+ex)}{g(ef-dg)} - \frac{ben\log(f+gx)}{g(ef-dg)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^2,x]
```

```
[Out] (b*e*n*Log[d + e*x])/(g*(e*f - d*g)) - (a + b*Log[c*(d + e*x)^n])/(g*(f + g
*x)) - (b*e*n*Log[f + g*x])/(g*(e*f - d*g))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)}+\frac{(ben)\int\frac{1}{(d+ex)(f+gx)}\,dx}{g}\\ &= -\frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)}-\frac{(ben)\int\frac{1}{f+gx}\,dx}{ef-dg}+\frac{(be^2n)\int\frac{1}{d+ex}\,dx}{g(ef-dg)}\\ &= \frac{ben\log(d+ex)}{g(ef-dg)}-\frac{a+b\log\left(c(d+ex)^n\right)}{g(f+gx)}-\frac{ben\log(f+gx)}{g(ef-dg)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int\frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx=\frac{-\frac{a+b\log(c(d+ex)^n)}{f+gx}+\frac{ben(\log(d+ex)-\log(f+gx))}{ef-dg}}{g}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x)^2,x]
```

```
[Out] (-((a + b*Log[c*(d + e*x)^n])/(f + g*x)) + (b*e*n*(Log[d + e*x] - Log[f + g
*x]))/(e*f - d*g))/g
```

## Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

| method | result |
|---|---|
| parallelrisch | $-\frac{\ln(ex+d)xb\,e^2gn-\ln(gx+f)xb\,e^2gn+\ln(ex+d)b\,e^2fn-\ln(gx+f)b\,e^2fn+\ln(c(ex+d)^n)bdeg-\ln(c(ex+d)^n)b\,e^2f+adeg-a\,e^{2}}{(dg-ef)(gx+f)eg}$ |
| risch | $-\frac{b\ln((ex+d)^n)}{g(gx+f)}-\frac{i\pi bef\ \mathrm{csgn}(ic)\,\mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)+i\pi bdg\ \mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)^2-i\pi bef\ \mathrm{csgn}(}{}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(ln(e*x+d)*x*b*e^2*g*n-ln(g*x+f)*x*b*e^2*g*n+ln(e*x+d)*b*e^2*f*n-ln(g*x+f)
*b*e^2*f*n+ln(c*(e*x+d)^n)*b*d*e*g-ln(c*(e*x+d)^n)*b*e^2*f+a*d*e*g-a*e^2*f)
/(d*g-e*f)/(g*x+f)/e/g
```

## Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = -\frac{aef-adg-(begnx+bdgn)\log\left(ex+d\right)+(begnx+befn)\log\left(gx+f\right)+(bef-bdg)\log\left(c\right)}{ef^2g-dfg^2+(efg^2-dg^3)x}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] -(a*e*f - a*d*g - (b*e*g*n*x + b*d*g*n)*log(e*x + d) + (b*e*g*n*x + b*e*f*n
)*log(g*x + f) + (b*e*f - b*d*g)*log(c))/(e*f^2*g - d*f*g^2 + (e*f*g^2 - d*
g^3)*x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(61) = 122.

Time = 3.12 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.50

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = \begin{cases} \frac{ax+\frac{bd\log\left(c(d+ex)^n\right)}{e}-bnx+bx\log\left(c(d+ex)^n\right)}{f^2} \\ -\frac{a}{fg+g^2x}-\frac{bn}{fg+g^2x}-\frac{b\log\left(c\left(\frac{ef}{g}+ex\right)^n\right)}{fg+g^2x} \\ -\frac{adg}{dfg^2+dg^3x-ef^2g-efg^2x}+\frac{aef}{dfg^2+dg^3x-ef^2g-efg^2x}-\frac{bdg\log\left(c(d+ex)^n\right)}{dfg^2+dg^3x-ef^2g-efg^2x}+\frac{befn\log\left(\frac{f}{g}+x\right)}{dfg^2+dg^3x-ef^2g-efg^2x}+\frac{begnx\log\left(\frac{f}{g}+\right.}{dfg^2+dg^3x-ef^2g-} \end{cases}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x+f)**2,x)
```

```
[Out] Piecewise(((a*x + b*d*log(c*(d + e*x)**n)/e - b*n*x + b*x*log(c*(d + e*x)**
n))/f**2, Eq(g, 0)), (-a/(f*g + g**2*x) - b*n/(f*g + g**2*x) - b*log(c*(e*f
/g + e*x)**n)/(f*g + g**2*x), Eq(d, e*f/g)), (-a*d*g/(d*f*g**2 + d*g**3*x -
 e*f**2*g - e*f*g**2*x) + a*e*f/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*
x) - b*d*g*log(c*(d + e*x)**n)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x
) + b*e*f*n*log(f/g + x)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x) + b*
e*g*n*x*log(f/g + x)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x) - b*e*g*
x*log(c*(d + e*x)**n)/(d*f*g**2 + d*g**3*x - e*f**2*g - e*f*g**2*x), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = ben\left(\frac{\log\left(ex+d\right)}{efg-dg^2}-\frac{\log\left(gx+f\right)}{efg-dg^2}\right) - \frac{b\log\left((ex+d)^n c\right)}{g^2x+fg} - \frac{a}{g^2x+fg}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] b*e*n*(log(e*x + d)/(e*f*g - d*g^2) - log(g*x + f)/(e*f*g - d*g^2)) - b*log
((e*x + d)^n*c)/(g^2*x + f*g) - a/(g^2*x + f*g)
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = \frac{ben\log\left(ex+d\right)}{efg-dg^2}-\frac{ben\log\left(gx+f\right)}{efg-dg^2} - \frac{bn\log\left(ex+d\right)}{g^2x+fg} - \frac{b\log\left(c\right)+a}{g^2x+fg}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] b*e*n*log(e*x + d)/(e*f*g - d*g^2) - b*e*n*log(g*x + f)/(e*f*g - d*g^2) - b
*n*log(e*x + d)/(g^2*x + f*g) - (b*log(c) + a)/(g^2*x + f*g)
```

## Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx)^2}\,dx = -\frac{a}{x\,g^2+f\,g}-\frac{b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{g\,\left(f+g\,x\right)}+\frac{b\,e\,n\,\mathrm{atan}\left(\frac{e\,f\,2\mathrm{i}+e\,g\,x\,2\mathrm{i}}{d\,g-e\,f}+1\mathrm{i}\right)\,2\mathrm{i}}{g\,\left(d\,g-e\,f\right)}$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x)^2,x)
```

```
[Out] (b*e*n*atan((e*f*2i + e*g*x*2i)/(d*g - e*f) + 1i)*2i)/(g*(d*g - e*f)) - (b*
log(c*(d + e*x)^n))/(g*(f + g*x)) - a/(f*g + g^2*x)
```

# 3.253 $\int \frac{a+b\log(c(d+ex)^n)}{x(f+gx)^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 179

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)^2}\,dx = -\frac{ben\log(d+ex)}{f(ef-dg)}+\frac{a+b\log\left(c(d+ex)^n\right)}{f(f+gx)}+\frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2}+\frac{ben\log(f+gx)}{f(ef-dg)}-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^2}-\frac{bn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^2}+\frac{bn\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^2}$$

```
[Out] -b*e*n*ln(e*x+d)/f/(-d*g+e*f)+(a+b*ln(c*(e*x+d)^n))/f/(g*x+f)+ln(-e*x/d)*(a
+b*ln(c*(e*x+d)^n))/f^2+b*e*n*ln(g*x+f)/f/(-d*g+e*f)-(a+b*ln(c*(e*x+d)^n))*
ln(e*(g*x+f)/(-d*g+e*f))/f^2-b*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/f^2+b*n*p
olylog(2,1+e*x/d)/f^2
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

= {46, 2463, 2441, 2352, 2442, 36, 31, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x(f+gx)^2}\,dx = -\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} + \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} + \frac{a+b\log\left(c(d+ex)^n\right)}{f(f+gx)} - \frac{bn\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^2} + \frac{bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^2} - \frac{ben\log(d+ex)}{f(ef-dg)} + \frac{ben\log(f+gx)}{f(ef-dg)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x)^2),x]
```

```
[Out] -((b*e*n*Log[d + e*x])/(f*(e*f - d*g))) + (a + b*Log[c*(d + e*x)^n])/(f*(f
+ g*x)) + (Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f^2 + (b*e*n*Log[f +
 g*x])/(f*(e*f - d*g)) - ((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f
 - d*g)])/f^2 - (b*n*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/f^2 + (b*n*P
olyLog[2, 1 + (e*x)/d])/f^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x} - \frac{g(a+b\log\left(c(d+ex)^n\right))}{f(f+gx)^2} - \frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2(f+gx)}\right) dx$$

$$= \frac{\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx}{f^2} - \frac{g\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{f^2} - \frac{g\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^2}\,dx}{f}$$

$$= \frac{a+b\log\left(c(d+ex)^n\right)}{f(f+gx)} + \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^2} - \frac{(ben)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{f^2} + \frac{(ben)\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{f^2} - \frac{(ben)\int \frac{1}{(d+ex)(f+gx)}\,dx}{f}$$

$$= \frac{a+b\log(c(d+ex)^n)}{f(f+gx)} + \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^2} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^2} + \frac{bn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} + \frac{(bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{f^2} - \frac{(be^2n)\int \frac{1}{d+ex}\,dx}{f(ef-dg)} + \frac{(begn)\int \frac{1}{f+gx}\,dx}{f(ef-dg)}$$

$$= -\frac{ben\log(d+ex)}{f(ef-dg)} + \frac{a+b\log(c(d+ex)^n)}{f(f+gx)} + \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^2} + \frac{ben\log(f+gx)}{f(ef-dg)} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^2} - \frac{bn\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{f^2} + \frac{bn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^2}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log(c(d+ex)^n)}{x(f+gx)^2}\,dx = \frac{\frac{f(a+b\log(c(d+ex)^n))}{f+gx} + \log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n)) - \frac{befn(\log(d+ex)-\log(f+gx))}{ef-dg} - (a+b\log(c(d+ex)^n))\,\mathrm{l}}{f^2}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x)^2),x]
```

```
[Out] ((f*(a + b*Log[c*(d + e*x)^n]))/(f + g*x) + Log[-((e*x)/d)]*(a + b*Log[c*(d
 + e*x)^n]) - (b*e*f*n*(Log[d + e*x] - Log[f + g*x]))/(e*f - d*g) - (a + b*
Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] - b*n*PolyLog[2, (g*(d +
 e*x))/(-(e*f) + d*g)] + b*n*PolyLog[2, 1 + (e*x)/d])/f^2
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.74 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.98

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)\ln(x)}{f^2} - \frac{b\ln((ex+d)^n)\ln(gx+f)}{f^2} + \frac{b\ln((ex+d)^n)}{f(gx+f)} - \frac{bn\,\text{dilog}\left(\frac{ex+d}{d}\right)}{f^2} - \frac{bn\ln(x)\ln\left(\frac{ex+d}{d}\right)}{f^2} + \frac{ben\ln(ex+d)}{f(dg-ef)} -$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/x/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*ln((e*x+d)^n)/f^2*ln(x)-b*ln((e*x+d)^n)/f^2*ln(g*x+f)+b*ln((e*x+d)^n)/f/(
g*x+f)-b*n/f^2*dilog((e*x+d)/d)-b*n/f^2*ln(x)*ln((e*x+d)/d)+b*e*n/f/(d*g-e*
f)*ln(e*x+d)-b*e*n/f/(d*g-e*f)*ln(g*x+f)+b*n/f^2*dilog(((g*x+f)*e+d*g-e*f)/
(d*g-e*f))+b*n/f^2*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi
*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(
I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b
*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/f^2*ln(x)-1/f^2*ln(g*x+f)+1/f/(g*x+
f))
```

## Fricas [F]

$$\int \frac{a + b \log \left(c(d + ex)^n\right)}{x(f + gx)^2}\, dx = \int \frac{b \log \left((ex + d)^n c\right) + a}{(gx + f)^2 x}\, dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x + d)^n*c) + a)/(g^2*x^3 + 2*f*g*x^2 + f^2*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \log \left(c(d + ex)^n\right)}{x(f + gx)^2}\, dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/x/(g*x+f)**2,x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{a + b \log \left(c(d + ex)^n\right)}{x(f + gx)^2}\, dx = \int \frac{b \log \left((ex + d)^n c\right) + a}{(gx + f)^2 x}\, dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] a*(1/(f*g*x + f^2) - log(g*x + f)/f^2 + log(x)/f^2) + b*integrate((log((e*x
 + d)^n) + log(c))/(g^2*x^3 + 2*f*g*x^2 + f^2*x), x)
```

## Giac [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x(f + gx)^2}\,dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{(gx + f)^2 x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x/(g*x+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)^2*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x(f + gx)^2}\,dx = \int \frac{a + b\,\ln\left(c\,(d + e\,x)^n\right)}{x\,(f + g\,x)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x)^2),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x)^2), x)
```

# 3.254 $\int \frac{a+b\log(c(d+ex)^n)}{x^2(f+gx)^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 240

$$\int \frac{a+b\log(c(d+ex)^n)}{x^2(f+gx)^2}\,dx = \frac{ben\log(x)}{df^2} - \frac{ben\log(d+ex)}{df^2} + \frac{begn\log(d+ex)}{f^2(ef-dg)} - \frac{a+b\log(c(d+ex)^n)}{f^2x} - \frac{g(a+b\log(c(d+ex)^n))}{f^2(f+gx)} - \frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^3} - \frac{begn\log(f+gx)}{f^2(ef-dg)} + \frac{2g(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^3} + \frac{2bgn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^3} - \frac{2bgn\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^3}$$

```
[Out] b*e*n*ln(x)/d/f^2-b*e*n*ln(e*x+d)/d/f^2+b*e*g*n*ln(e*x+d)/f^2/(-d*g+e*f)+(-
a-b*ln(c*(e*x+d)^n))/f^2/x-g*(a+b*ln(c*(e*x+d)^n))/f^2/(g*x+f)-2*g*ln(-e*x/
d)*(a+b*ln(c*(e*x+d)^n))/f^3-b*e*g*n*ln(g*x+f)/f^2/(-d*g+e*f)+2*g*(a+b*ln(c
*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/f^3+2*b*g*n*polylog(2,-g*(e*x+d)/(-d*
g+e*f))/f^3-2*b*g*n*polylog(2,1+e*x/d)/f^3
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx=-\frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3}+\frac{2g\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3}-\frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2(f+gx)}-\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x}+\frac{2bgn\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^3}-\frac{2bgn\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^3}+\frac{begn\log(d+ex)}{f^2(ef-dg)}-\frac{begn\log(f+gx)}{f^2(ef-dg)}+\frac{ben\log(x)}{df^2}-\frac{ben\log(d+ex)}{df^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x)^2),x]
```

```
[Out] (b*e*n*Log[x])/(d*f^2) - (b*e*n*Log[d + e*x])/(d*f^2) + (b*e*g*n*Log[d + e*
x])/(f^2*(e*f - d*g)) - (a + b*Log[c*(d + e*x)^n])/(f^2*x) - (g*(a + b*Log[
c*(d + e*x)^n]))/(f^2*(f + g*x)) - (2*g*Log[-((e*x)/d)]*(a + b*Log[c*(d + e
*x)^n]))/f^3 - (b*e*g*n*Log[f + g*x])/(f^2*(e*f - d*g)) + (2*g*(a + b*Log[c
*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/f^3 + (2*b*g*n*PolyLog[2, -(
(g*(d + e*x))/(e*f - d*g))])/f^3 - (2*b*g*n*PolyLog[2, 1 + (e*x)/d])/f^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x^2} - \frac{2g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3x} + \frac{g^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2(f+gx)^2} + \frac{2g^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3(f+gx)}\right) dx$$

$$= \frac{\int \frac{a+b\log(c(d+ex)^n)}{x^2}\,dx}{f^2} - \frac{(2g)\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx}{f^3} + \frac{(2g^2)\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{f^3} + \frac{g^2\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^2}\,dx}{f^2}$$

$$= -\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x} - \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2(f+gx)} - \frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3} + \frac{2g\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^3} + \frac{(ben)\int \frac{1}{x(d+ex)}\,dx}{f^2} + \frac{(2begn)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{f^3} - \frac{(2begn)\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{f^3} + \frac{(begn)\int \frac{1}{(d+ex)(f+gx)}\,dx}{f^2}$$

$$= -\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x} - \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2(f+gx)} - \frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3} + \frac{2g\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^3} - \frac{2bgn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^3} + \frac{(ben)\int \frac{1}{x}\,dx}{df^2} - \frac{(be^2n)\int \frac{1}{d+ex}\,dx}{df^2} - \frac{(2bgn)\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{f^3} + \frac{(be^2gn)\int \frac{1}{d+ex}\,dx}{f^2(ef-dg)} - \frac{(beg^2n)\int \frac{1}{f+gx}\,dx}{f^2(ef-dg)}$$

$$= \frac{ben\log(x)}{df^2} - \frac{ben\log(d+ex)}{df^2} + \frac{begn\log(d+ex)}{f^2(ef-dg)} - \frac{a+b\log\left(c(d+ex)^n\right)}{f^2x} - \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2(f+gx)} - \frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3} - \frac{begn\log(f+gx)}{f^2(ef-dg)} + \frac{2g\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^3} + \frac{2bgn\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{f^3} - \frac{2bgn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^3}$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.83

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx$$
$$= \frac{\frac{befn(\log(x)-\log(d+ex))}{d}-\frac{f(a+b\log(c(d+ex)^n))}{x}-\frac{fg(a+b\log(c(d+ex)^n))}{f+gx}-2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+\frac{befgn(\mathrm{l}}{}}{}$$

[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x)^2),x]

[Out] ((b*e*f*n*(Log[x] - Log[d + e*x]))/d - (f*(a + b*Log[c*(d + e*x)^n]))/x - (
f*g*(a + b*Log[c*(d + e*x)^n]))/(f + g*x) - 2*g*Log[-((e*x)/d)]*(a + b*Log[
c*(d + e*x)^n]) + (b*e*f*g*n*(Log[d + e*x] - Log[f + g*x]))/(e*f - d*g) + 2
*g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)] + 2*b*g*n*Poly
Log[2, (g*(d + e*x))/(-(e*f) + d*g)] - 2*b*g*n*PolyLog[2, 1 + (e*x)/d])/f^3

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.12 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.82

| method | result |
|---|---|
| risch | $-\frac{b\ln((ex+d)^n)}{f^2x}-\frac{2b\ln((ex+d)^n)g\ln(x)}{f^3}-\frac{b\ln((ex+d)^n)g}{f^2(gx+f)}+\frac{2b\ln((ex+d)^n)g\ln(gx+f)}{f^3}+\frac{2bng\,\mathrm{dilog}\left(\frac{ex+d}{d}\right)}{f^3}+\frac{2bng\ln(x)\ln}{f^3}$ |

[In] int((a+b*ln(c*(e*x+d)^n))/x^2/(g*x+f)^2,x,method=_RETURNVERBOSE)

[Out] -b*ln((e*x+d)^n)/f^2/x-2*b*ln((e*x+d)^n)/f^3*g*ln(x)-b*ln((e*x+d)^n)/f^2*g/
(g*x+f)+2*b*ln((e*x+d)^n)/f^3*g*ln(g*x+f)+2*b*n/f^3*g*dilog((e*x+d)/d)+2*b*
n/f^3*g*ln(x)*ln((e*x+d)/d)-2*b*n/f^3*g*dilog(((g*x+f)*e+d*g-e*f)/(d*g-e*f)
)-2*b*n/f^3*g*ln(g*x+f)*ln(((g*x+f)*e+d*g-e*f)/(d*g-e*f))-2*b*e*n/f^2/(d*g-
e*f)*ln(e*x+d)*g+b*e^2*n/f/(d*g-e*f)/d*ln(e*x+d)+b*e*n*ln(x)/d/f^2+b*e*n/f^
2*g/(d*g-e*f)*ln(g*x+f)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(
e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x
+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(-
1/f^2/x-2/f^3*g*ln(x)-1/f^2*g/(g*x+f)+2/f^3*g*ln(g*x+f))

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx+f)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x+f)^2,x, algorithm="fricas")
[Out] integral((b*log((e*x + d)^n*c) + a)/(g^2*x^4 + 2*f*g*x^3 + f^2*x^2), x)
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx\right)^2}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/x**2/(g*x+f)**2,x)
[Out] Integral((a + b*log(c*(d + e*x)**n))/(x**2*(f + g*x)**2), x)
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx+f)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x+f)^2,x, algorithm="maxima")
[Out] -a*((2*g*x + f)/(f^2*g*x^2 + f^3*x) - 2*g*log(g*x + f)/f^3 + 2*g*log(x)/f^3
) + b*integrate((log((e*x + d)^n) + log(c))/(g^2*x^4 + 2*f*g*x^3 + f^2*x^2)
, x)
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx+f)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)^2*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(f+gx)^2}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{x^2\,{\left(f+g\,x\right)}^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x)^2),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x)^2), x)
```

# 3.255 $\int \frac{a+b\log(c(d+ex)^n)}{x^3(f+gx)^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 335

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx &= -\frac{ben}{2df^2x}-\frac{be^2n\log(x)}{2d^2f^2}-\frac{2begn\log(x)}{df^3}\\&\quad+\frac{be^2n\log(d+ex)}{2d^2f^2}+\frac{2begn\log(d+ex)}{df^3}\\&\quad-\frac{beg^2n\log(d+ex)}{f^3(ef-dg)}-\frac{a+b\log\left(c(d+ex)^n\right)}{2f^2x^2}\\&\quad+\frac{2g(a+b\log\left(c(d+ex)^n\right))}{f^3x}+\frac{g^2(a+b\log\left(c(d+ex)^n\right))}{f^3(f+gx)}\\&\quad+\frac{3g^2\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^4}+\frac{beg^2n\log(f+gx)}{f^3(ef-dg)}\\&\quad-\frac{3g^2(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^4}\\&\quad-\frac{3bg^2n\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^4}+\frac{3bg^2n\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^4}\end{aligned}$$

```
[Out] -1/2*b*e*n/d/f^2/x-1/2*b*e^2*n*ln(x)/d^2/f^2-2*b*e*g*n*ln(x)/d/f^3+1/2*b*e^
2*n*ln(e*x+d)/d^2/f^2+2*b*e*g*n*ln(e*x+d)/d/f^3-b*e*g^2*n*ln(e*x+d)/f^3/(-d
*g+e*f)+1/2*(-a-b*ln(c*(e*x+d)^n))/f^2/x^2+2*g*(a+b*ln(c*(e*x+d)^n))/f^3/x+
g^2*(a+b*ln(c*(e*x+d)^n))/f^3/(g*x+f)+3*g^2*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n)
)/f^4+b*e*g^2*n*ln(g*x+f)/f^3/(-d*g+e*f)-3*g^2*(a+b*ln(c*(e*x+d)^n))*ln(e*(
g*x+f)/(-d*g+e*f))/f^4-3*b*g^2*n*polylog(2,-g*(e*x+d)/(-d*g+e*f))/f^4+3*b*g
^2*n*polylog(2,1+e*x/d)/f^4
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx = \frac{3g^2\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^4} - \frac{3g^2\log\left(\frac{e(f+gx)}{ef-dg}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^4} + \frac{g^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3(f+gx)} + \frac{2g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3x} - \frac{a+b\log\left(c(d+ex)^n\right)}{2f^2x^2} - \frac{be^2n\log(x)}{2d^2f^2} + \frac{be^2n\log(d+ex)}{2d^2f^2} - \frac{3bg^2n\,\text{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{f^4} + \frac{3bg^2n\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^4} - \frac{beg^2n\log(d+ex)}{f^3(ef-dg)} + \frac{beg^2n\log(f+gx)}{f^3(ef-dg)} - \frac{2begn\log(x)}{df^3} + \frac{2begn\log(d+ex)}{df^3} - \frac{ben}{2df^2x}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x)^2),x]
```

```
[Out] -1/2*(b*e*n)/(d*f^2*x) - (b*e^2*n*Log[x])/(2*d^2*f^2) - (2*b*e*g*n*Log[x])/
(d*f^3) + (b*e^2*n*Log[d + e*x])/(2*d^2*f^2) + (2*b*e*g*n*Log[d + e*x])/(d*
f^3) - (b*e*g^2*n*Log[d + e*x])/(f^3*(e*f - d*g)) - (a + b*Log[c*(d + e*x)^
n])/(2*f^2*x^2) + (2*g*(a + b*Log[c*(d + e*x)^n]))/(f^3*x) + (g^2*(a + b*Lo
g[c*(d + e*x)^n]))/(f^3*(f + g*x)) + (3*g^2*Log[-((e*x)/d)]*(a + b*Log[c*(d
 + e*x)^n]))/f^4 + (b*e*g^2*n*Log[f + g*x])/(f^3*(e*f - d*g)) - (3*g^2*(a +
 b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/f^4 - (3*b*g^2*n*Pol
yLog[2, -((g*(d + e*x))/(e*f - d*g))])/f^4 + (3*b*g^2*n*PolyLog[2, 1 + (e*x
)/d])/f^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x^3} - \frac{2g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3x^2} + \frac{3g^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^4x} - \frac{g^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3(f+gx)^2} - \frac{3g^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^4(f+gx)}\right)dx$$

$$= \frac{\int \frac{a+b\log(c(d+ex)^n)}{x^3}\,dx}{f^2} - \frac{(2g)\int \frac{a+b\log(c(d+ex)^n)}{x^2}\,dx}{f^3} + \frac{(3g^2)\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx}{f^4} - \frac{(3g^3)\int \frac{a+b\log(c(d+ex)^n)}{f+gx}\,dx}{f^4} - \frac{g^3\int \frac{a+b\log(c(d+ex)^n)}{(f+gx)^2}\,dx}{f^3}$$

$$= -\frac{a+b\log\left(c(d+ex)^n\right)}{2f^2x^2} + \frac{2g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3x} + \frac{g^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3(f+gx)} + \frac{3g^2\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^4} - \frac{3g^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^4} + \frac{(ben)\int \frac{1}{x^2(d+ex)}\,dx}{2f^2} - \frac{(2begn)\int \frac{1}{x(d+ex)}\,dx}{f^3} - \frac{(3beg^2n)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{f^4} + \frac{(3beg^2n)\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx}{f^4} - \frac{(beg^2n)\int \frac{1}{(d+ex)(f+gx)}\,dx}{f^3}$$

$$= -\frac{a+b\log\left(c(d+ex)^n\right)}{2f^2x^2} + \frac{2g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3x} + \frac{g^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3(f+gx)} + \frac{3g^2\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^4} - \frac{3g^2\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^4} + \frac{3bg^2n\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^4} + \frac{(ben)\int\left(\frac{1}{dx^2}-\frac{e}{d^2x}+\frac{e^2}{d^2(d+ex)}\right)dx}{2f^2} - \frac{(2begn)\int\frac{1}{x}\,dx}{df^3} + \frac{(2be^2gn)\int\frac{1}{d+ex}\,dx}{df^3} + \frac{(3bg^2n)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{f^4} - \frac{(be^2g^2n)\int\frac{1}{d+ex}\,dx}{f^3(ef-dg)} + \frac{(beg^3n)\int\frac{1}{f+gx}\,dx}{f^3(ef-dg)}$$

$$= -\frac{ben}{2df^2x} - \frac{be^2n\log(x)}{2d^2f^2} - \frac{2begn\log(x)}{df^3} + \frac{be^2n\log(d+ex)}{2d^2f^2} + \frac{2begn\log(d+ex)}{df^3} - \frac{beg^2n\log(d+ex)}{f^3(ef-dg)} - \frac{a+b\log(c(d+ex)^n)}{2f^2x^2} + \frac{2g(a+b\log(c(d+ex)^n))}{f^3x} + \frac{g^2(a+b\log(c(d+ex)^n))}{f^3(f+gx)} + \frac{3g^2\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^4} + \frac{beg^2n\log(f+gx)}{f^3(ef-dg)} - \frac{3g^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{f^4} - \frac{3bg^2n\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{f^4} + \frac{3bg^2n\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^4}$$

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.80

$$\int \frac{a+b\log(c(d+ex)^n)}{x^3(f+gx)^2}\,dx = -\frac{\frac{4befgn(\log(x)-\log(d+ex))}{d} + \frac{bef^2n(d+ex\log(x)-ex\log(d+ex))}{d^2x} + \frac{f^2(a+b\log(c(d+ex)^n))}{x^2} - \frac{4fg(a+b\log(c(d+ex)^n))}{x} - \frac{2fg^2(a+b\log}{f\cdots}}{\cdots}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x)^2),x]
```

```
[Out] -1/2*((4*b*e*f*g*n*(Log[x] - Log[d + e*x]))/d + (b*e*f^2*n*(d + e*x*Log[x]
 - e*x*Log[d + e*x]))/(d^2*x) + (f^2*(a + b*Log[c*(d + e*x)^n]))/x^2 - (4*f*
g*(a + b*Log[c*(d + e*x)^n]))/x - (2*f*g^2*(a + b*Log[c*(d + e*x)^n]))/(f +
 g*x) - 6*g^2*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]) + (2*b*e*f*g^2*n*(
Log[d + e*x] - Log[f + g*x]))/(e*f - d*g) + 6*g^2*(a + b*Log[c*(d + e*x)^n]
)*Log[(e*(f + g*x))/(e*f - d*g)] + 6*b*g^2*n*PolyLog[2, (g*(d + e*x))/(-(e*
f) + d*g)] - 6*b*g^2*n*PolyLog[2, 1 + (e*x)/d])/f^4
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.66 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.64

| method | result |
|---|---|
| risch | $-\frac{b\ln((ex+d)^n)}{2f^2x^2} + \frac{3b\ln((ex+d)^n)g^2\ln(x)}{f^4} + \frac{2b\ln((ex+d)^n)g}{f^3x} - \frac{3b\ln((ex+d)^n)g^2\ln(gx+f)}{f^4} + \frac{b\ln((ex+d)^n)g^2}{f^3(gx+f)} + \frac{3ben\ln(e\cdots}{f^3(dg\cdots}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/x^3/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*ln((e*x+d)^n)/f^2/x^2+3*b*ln((e*x+d)^n)/f^4*g^2*ln(x)+2*b*ln((e*x+d)
^n)/f^3*g/x-3*b*ln((e*x+d)^n)/f^4*g^2*ln(g*x+f)+b*ln((e*x+d)^n)/f^3*g^2/(g*
x+f)+3*b*e*n/f^3/(d*g-e*f)*ln(e*x+d)*g^2-3/2*b*e^2*n/f^2/(d*g-e*f)/d*ln(e*x
+d)*g-1/2*b*e^3*n/f/(d*g-e*f)/d^2*ln(e*x+d)-2*b*e*g*n*ln(x)/d/f^3-1/2*b*e^2
*n*ln(x)/d^2/f^2-1/2*b*e*n/d/f^2/x-b*e*n/f^3*g^2/(d*g-e*f)*ln(g*x+f)-3*b*n/
f^4*g^2*dilog((e*x+d)/d)-3*b*n/f^4*g^2*ln(x)*ln((e*x+d)/d)+3*b*n/f^4*g^2*di
log(((g*x+f)*e+d*g-e*f)/(d*g-e*f))+3*b*n/f^4*g^2*ln(g*x+f)*ln(((g*x+f)*e+d*
g-e*f)/(d*g-e*f))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)
^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)
*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(-1/2/f^
2/x^2+3/f^4*g^2*ln(x)+2/f^3*g/x-3/f^4*g^2*ln(g*x+f)+1/f^3*g^2/(g*x+f))
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)^2x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x + d)^n*c) + a)/(g^2*x^5 + 2*f*g*x^4 + f^2*x^3), x)
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx\right)^2}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/x**3/(g*x+f)**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))/(x**3*(f + g*x)**2), x)
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx+f)^2x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*((6*g^2*x^2 + 3*f*g*x - f^2)/(f^3*g*x^3 + f^4*x^2) - 6*g^2*log(g*x +
f)/f^4 + 6*g^2*log(x)/f^4) + b*integrate((log((e*x + d)^n) + log(c))/(g^2*x
^5 + 2*f*g*x^4 + f^2*x^3), x)
```

## Giac [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{(gx+f)^2 x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x + f)^2*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{x^3(f+gx)^2}\,dx = \int \frac{a + b\ln\left(c\,(d+e\,x)^n\right)}{x^3\,(f+g\,x)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x)^2),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x)^2), x)
```

# 3.256 $\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 397

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = -\frac{bdfnx}{2eg^2}+\frac{bd^3nx}{4e^3g}+\frac{bfnx^2}{4g^2}-\frac{bd^2nx^2}{8e^2g}+\frac{bdnx^3}{12eg} -\frac{bnx^4}{16g}+\frac{bd^2fn\log(d+ex)}{2e^2g^2}-\frac{bd^4n\log(d+ex)}{4e^4g} -\frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2}+\frac{x^4(a+b\log(c(d+ex)^n))}{4g} +\frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3} +\frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3} +\frac{bf^2n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3} +\frac{bf^2n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3}$$

```
[Out] -1/2*b*d*f*n*x/e/g^2+1/4*b*d^3*n*x/e^3/g+1/4*b*f*n*x^2/g^2-1/8*b*d^2*n*x^2/
e^2/g+1/12*b*d*n*x^3/e/g-1/16*b*n*x^4/g+1/2*b*d^2*f*n*ln(e*x+d)/e^2/g^2-1/4
*b*d^4*n*ln(e*x+d)/e^4/g-1/2*f*x^2*(a+b*ln(c*(e*x+d)^n))/g^2+1/4*x^4*(a+b*l
n(c*(e*x+d)^n))/g+1/2*f^2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))
/(e*(-f)^(1/2)+d*g^(1/2)))/g^3+1/2*f^2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/
2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^3+1/2*b*f^2*n*polylog(2,-(e*x+d)*
g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^3+1/2*b*f^2*n*polylog(2,(e*x+d)*g^(1/2)
/(e*(-f)^(1/2)+d*g^(1/2)))/g^3
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {272, 45, 2463, 2442, 266, 2441, 2440, 2438}

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \frac{f^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g^3} + \frac{f^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g^3} - \frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2} + \frac{x^4(a+b\log(c(d+ex)^n))}{4g} - \frac{bd^4n\log(d+ex)}{4e^4g} + \frac{bd^3nx}{4e^3g} + \frac{bd^2fn\log(d+ex)}{2e^2g^2} - \frac{bd^2nx^2}{8e^2g} + \frac{bf^2n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3} + \frac{bf^2n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^3} - \frac{bdfnx}{2eg^2} + \frac{bdnx^3}{12eg} + \frac{bfnx^2}{4g^2} - \frac{bnx^4}{16g}$$

```
[In] Int[(x^5*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
[Out] -1/2*(b*d*f*n*x)/(e*g^2) + (b*d^3*n*x)/(4*e^3*g) + (b*f*n*x^2)/(4*g^2) - (b
*d^2*n*x^2)/(8*e^2*g) + (b*d*n*x^3)/(12*e*g) - (b*n*x^4)/(16*g) + (b*d^2*f*
n*Log[d + e*x])/(2*e^2*g^2) - (b*d^4*n*Log[d + e*x])/(4*e^4*g) - (f*x^2*(a
+ b*Log[c*(d + e*x)^n]))/(2*g^2) + (x^4*(a + b*Log[c*(d + e*x)^n]))/(4*g) +
 (f^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f]
 + d*Sqrt[g])])/(2*g^3) + (f^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f]
+ Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^3) + (b*f^2*n*PolyLog[2, -((S
qrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*g^3) + (b*f^2*n*PolyLog[2,
 (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{fx(a+b\log\left(c(d+ex)^n\right))}{g^2}+\frac{x^3(a+b\log\left(c(d+ex)^n\right))}{g} + \frac{f^2x(a+b\log\left(c(d+ex)^n\right))}{g^2\left(f+gx^2\right)}\right)\,dx$$

$$= -\frac{f\int x(a+b\log(c(d+ex)^n))\,dx}{g^2} + \frac{f^2\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx}{g^2} + \frac{\int x^3(a+b\log(c(d+ex)^n))\,dx}{g}$$

$$= -\frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2} + \frac{x^4(a+b\log(c(d+ex)^n))}{4g} + \frac{f^2\int \left(-\frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)} + \frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g^2} + \frac{(befn)\int \frac{x^2}{d+ex}\,dx}{2g^2} - \frac{(ben)\int \frac{x^4}{d+ex}\,dx}{4g}$$

$$= -\frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2} + \frac{x^4(a+b\log(c(d+ex)^n))}{4g} - \frac{f^2\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2g^{5/2}} + \frac{f^2\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2g^{5/2}} + \frac{(befn)\int \left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx}{2g^2} - \frac{(ben)\int \left(-\frac{d^3}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{e^2}+\frac{x^3}{e}+\frac{d^4}{e^4(d+ex)}\right)dx}{4g}$$

$$= -\frac{bdfnx}{2eg^2} + \frac{bd^3nx}{4e^3g} + \frac{bfnx^2}{4g^2} - \frac{bd^2nx^2}{8e^2g} + \frac{bdnx^3}{12eg} - \frac{bnx^4}{16g} + \frac{bd^2fn\log(d+ex)}{2e^2g^2} - \frac{bd^4n\log(d+ex)}{4e^4g} - \frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2} + \frac{x^4(a+b\log(c(d+ex)^n))}{4g} + \frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3} + \frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3} - \frac{(bef^2n)\int \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2g^3} - \frac{(bef^2n)\int \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2g^3}$$

$$= -\frac{bdfnx}{2eg^2} + \frac{bd^3nx}{4e^3g} + \frac{bfnx^2}{4g^2} - \frac{bd^2nx^2}{8e^2g} + \frac{bdnx^3}{12eg} - \frac{bnx^4}{16g} + \frac{bd^2fn\log(d+ex)}{2e^2g^2} - \frac{bd^4n\log(d+ex)}{4e^4g} - \frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2} + \frac{x^4(a+b\log(c(d+ex)^n))}{4g} + \frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3} + \frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3} - \frac{(bf^2n)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2g^3} - \frac{(bf^2n)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2g^3}$$

$$= -\frac{bdfnx}{2eg^2} + \frac{bd^3nx}{4e^3g} + \frac{bfnx^2}{4g^2} - \frac{bd^2nx^2}{8e^2g} + \frac{bdnx^3}{12eg} - \frac{bnx^4}{16g} + \frac{bd^2fn\log(d+ex)}{2e^2g^2} - \frac{bd^4n\log(d+ex)}{4e^4g} - \frac{fx^2(a+b\log(c(d+ex)^n))}{2g^2} + \frac{x^4(a+b\log(c(d+ex)^n))}{4g} + \frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3} + \frac{f^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3} + \frac{bf^2n\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3} + \frac{bf^2n\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3}$$

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.83

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \frac{\frac{12bfgn(ex(-2d+ex)+2d^2\log(d+ex))}{e^2} - \frac{bg^2n(ex(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+12d^4\log(d+ex))}{e^4} - 24fgx^2(a+b\log(c(d+ex)^n}{}$$

```
[In] Integrate[(x^5*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

```
[Out] ((12*b*f*g*n*(e*x*(-2*d + e*x) + 2*d^2*Log[d + e*x]))/e^2 - (b*g^2*n*(e*x*(
-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 12*d^4*Log[d + e*x]))/e^4
- 24*f*g*x^2*(a + b*Log[c*(d + e*x)^n]) + 12*g^2*x^4*(a + b*Log[c*(d + e*x)
```

^n]) + 24*f^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])] + 24*f^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])] + 24*b*f^2*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + 24*b*f^2*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(48*g^3)

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.70 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| risch | $\frac{b\ln\left(\left(ex+d\right)^{n}\right)x^{4}}{4g}-\frac{b\ln\left(\left(ex+d\right)^{n}\right)f\,x^{2}}{2g^{2}}+\frac{b\ln\left(\left(ex+d\right)^{n}\right)f^{2}\ln\left(g\,x^{2}+f\right)}{2g^{3}}-\frac{bn\,f^{2}\ln\left(ex+d\right)\ln\left(g\,x^{2}+f\right)}{2g^{3}}+\frac{bn\,f^{2}\ln\left(ex+d\right)\ln\left(\frac{e\sqrt{-}}{}\right.}{2g^{3}}$ |

[In] int(x^5*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f),x,method=_RETURNVERBOSE)

[Out] 1/4*b*ln((e*x+d)^n)/g*x^4-1/2*b*ln((e*x+d)^n)/g^2*f*x^2+1/2*b*ln((e*x+d)^n)*f^2/g^3*ln(g*x^2+f)-1/2*b*n*f^2/g^3*ln(e*x+d)*ln(g*x^2+f)+1/2*b*n*f^2/g^3*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n*f^2/g^3*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*n*f^2/g^3*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n*f^2/g^3*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))-1/16*b*n*x^4/g+1/12*b*d*n*x^3/e/g-1/8*b*d^2*n*x^2/e^2/g+1/4*b*f*n*x^2/g^2+1/4*b*d^3*n*x/e^3/g-1/2*b*d*f*n*x/e/g^2-1/4*b*d^4*n*ln(e*x+d)/e^4/g+1/2*b*d^2*f*n*ln(e*x+d)/e^2/g^2+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/2/g^2*(1/2*g*x^4-f*x^2)+1/2*f^2/g^3*ln(g*x^2+f))

## Fricas [F]

$$\int \frac{x^5(a+b\log\left(c(d+ex)^n\right))}{f+gx^2}\,dx=\int \frac{(b\log\left((ex+d)^nc\right)+a)x^5}{gx^2+f}\,dx$$

[In] integrate(x^5*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="fricas")

[Out] integral((b*x^5*log((e*x + d)^n*c) + a*x^5)/(g*x^2 + f), x)

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx=\text{Timed out}$$

```
[In] integrate(x**5*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)x^5}{gx^2+f}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="maxima")
[Out] 1/4*a*(2*f^2*log(g*x^2 + f)/g^3 + (g*x^4 - 2*f*x^2)/g^2) + b*integrate((x^5
*log((e*x + d)^n) + x^5*log(c))/(g*x^2 + f), x)
```

## Giac [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)x^5}{gx^2+f}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*x^5/(g*x^2 + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx=\int \frac{x^5\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}{g\,x^2+f}\,dx$$

```
[In] int((x^5*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2),x)
[Out] int((x^5*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2), x)
```

# 3.257 $\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2} dx$



## Optimal result

Integrand size = 27, antiderivative size = 278

$$\begin{aligned}\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2} dx &= \frac{bdnx}{2eg} - \frac{bnx^2}{4g} - \frac{bd^2n\log(d+ex)}{2e^2g} \\ &+ \frac{x^2(a+b\log(c(d+ex)^n))}{2g} \\ &- \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} \\ &- \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} \\ &- \frac{bfn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} \\ &- \frac{bfn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}\end{aligned}$$

```
[Out] 1/2*b*d*n*x/e/g-1/4*b*n*x^2/g-1/2*b*d^2*n*ln(e*x+d)/e^2/g+1/2*x^2*(a+b*ln(c
*(e*x+d)^n))/g-1/2*f*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(
-f)^(1/2)+d*g^(1/2)))/g^2-1/2*f*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^
(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^2-1/2*b*f*n*polylog(2,-(e*x+d)*g^(1/2)/(
e*(-f)^(1/2)-d*g^(1/2)))/g^2-1/2*b*f*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1
/2)+d*g^(1/2)))/g^2
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {272, 45, 2463, 2442, 266, 2441, 2440, 2438}

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = -\frac{f\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g^2} - \frac{f\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g^2} + \frac{x^2(a+b\log(c(d+ex)^n))}{2g} - \frac{bd^2n\log(d+ex)}{2e^2g} - \frac{bfn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} - \frac{bfn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^2} + \frac{bdnx}{2eg} - \frac{bnx^2}{4g}$$

```
[In] Int[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

```
[Out] (b*d*n*x)/(2*e*g) - (b*n*x^2)/(4*g) - (b*d^2*n*Log[d + e*x])/(2*e^2*g) + (x
^2*(a + b*Log[c*(d + e*x)^n]))/(2*g) - (f*(a + b*Log[c*(d + e*x)^n])*Log[(e
*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^2) - (f*(a + b*Log
[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/
(2*g^2) - (b*f*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))
])/(2*g^2) - (b*f*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])
])/(2*g^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x(a+b\log(c(d+ex)^n))}{g} - \frac{fx(a+b\log(c(d+ex)^n))}{g(f+gx^2)}\right) dx$$

$$= \frac{\int x(a+b\log(c(d+ex)^n))\,dx}{g} - \frac{f\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx}{g}$$

$$= \frac{x^2(a+b\log(c(d+ex)^n))}{2g} - \frac{f\int \left(-\frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)} + \frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right) dx}{g} - \frac{(ben)\int \frac{x^2}{d+ex}\,dx}{2g}$$

$$= \frac{x^2(a+b\log(c(d+ex)^n))}{2g} + \frac{f\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2g^{3/2}} - \frac{f\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2g^{3/2}} - \frac{(ben)\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx}{2g}$$

$$= \frac{bdnx}{2eg} - \frac{bnx^2}{4g} - \frac{bd^2n\log(d+ex)}{2e^2g} + \frac{x^2(a+b\log(c(d+ex)^n))}{2g} - \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} - \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} + \frac{(befn)\int\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2g^2} + \frac{(befn)\int\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2g^2}$$

$$= \frac{bdnx}{2eg} - \frac{bnx^2}{4g} - \frac{bd^2n\log(d+ex)}{2e^2g} + \frac{x^2(a+b\log(c(d+ex)^n))}{2g} - \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} - \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} + \frac{(bfn)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2g^2} + \frac{(bfn)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2g^2}$$

$$= \frac{bdnx}{2eg} - \frac{bnx^2}{4g} - \frac{bd^2n\log(d+ex)}{2e^2g} + \frac{x^2(a+b\log(c(d+ex)^n))}{2g} - \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} - \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} - \frac{bfn\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} - \frac{bfn\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.87

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = -\frac{\frac{bgn(ex(-2d+ex)+2d^2\log(d+ex))}{e^2}-2gx^2(a+b\log(c(d+ex)^n))+2f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{}$$

```
[In] Integrate[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

```
[Out] -1/4*((b*g*n*(e*x*(-2*d + e*x) + 2*d^2*Log[d + e*x]))/e^2 - 2*g*x^2*(a + b*
Log[c*(d + e*x)^n]) + 2*f*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqr
t[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])] + 2*f*(a + b*Log[c*(d + e*x)^n])*Log[(e*
(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])] + 2*b*f*n*PolyLog[2, -((S
qrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + 2*b*f*n*PolyLog[2, (Sqrt[g]*
(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^2
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.55

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)x^2}{2g}-\frac{b\ln((ex+d)^n)f\ln(gx^2+f)}{2g^2}-\frac{bnx^2}{4g}+\frac{bdnx}{2eg}-\frac{bd^2n\ln(ex+d)}{2e^2g}+\frac{bnf\ln(ex+d)\ln(gx^2+f)}{2g^2}-\frac{bnf\ln(ex}{}$ |

```
[In] int(x^3*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln((e*x+d)^n)/g*x^2-1/2*b*ln((e*x+d)^n)*f/g^2*ln(g*x^2+f)-1/4*b*n*x^2
/g+1/2*b*d*n*x/e/g-1/2*b*d^2*n*ln(e*x+d)/e^2/g+1/2*b*n*f/g^2*ln(e*x+d)*ln(g
*x^2+f)-1/2*b*n*f/g^2*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)
^(1/2)+d*g))-1/2*b*n*f/g^2*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(
-f*g)^(1/2)-d*g))-1/2*b*n*f/g^2*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f
*g)^(1/2)+d*g))-1/2*b*n*f/g^2*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g
)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+
1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csg
n(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/2*x^2/g-1
/2*f/g^2*ln(g*x^2+f))
```

## Fricas [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^3}{gx^2+f}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="fricas")
[Out] integral((b*x^3*log((e*x + d)^n*c) + a*x^3)/(g*x^2 + f), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^3}{gx^2+f}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="maxima")
[Out] 1/2*a*(x^2/g - f*log(g*x^2 + f)/g^2) + b*integrate((x^3*log((e*x + d)^n) +
x^3*log(c))/(g*x^2 + f), x)
```

## Giac [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^3}{gx^2+f}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*x^3/(g*x^2 + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log\left(c(d+ex)^n\right))}{f+gx^2}\,dx = \int \frac{x^3\left(a+b\,\ln\left(c\left(d+e\,x\right)^n\right)\right)}{g\,x^2+f}\,dx$$

```
[In] int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2),x)
[Out] int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2), x)
```

# 3.258 $\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 203

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g} + \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g} + \frac{bn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g} + \frac{bn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g}$$

```
[Out] 1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/
2)))/g+1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-
d*g^(1/2)))/g+1/2*b*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/
g+1/2*b*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{266, 2463, 2441, 2440, 2438\}$

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g} + \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g} + \frac{bn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g} + \frac{bn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g}$$

```
[In] Int[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*
Sqrt[g])])/(2*g) + ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x
))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g) + (b*n*PolyLog[2, -((Sqrt[g]*(d + e*x))
/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*g) + (b*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e
*Sqrt[-f] + d*Sqrt[g])])/(2*g)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{a+b\log\left(c(d+ex)^n\right)}{2\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{a+b\log\left(c(d+ex)^n\right)}{2\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx \\
&= -\frac{\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2\sqrt{g}}+\frac{\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2\sqrt{g}} \\
&= \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g} \\
&\quad+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g} \\
&\quad-\frac{(ben)\int \frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2g}-\frac{(ben)\int \frac{\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2g} \\
&= \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g} \\
&\quad+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g} \\
&\quad-\frac{(bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2g} \\
&\quad-\frac{(bn)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2g} \\
&= \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g} \\
&\quad+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g} \\
&\quad+\frac{bn\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g}+\frac{bn\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.85

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \frac{(a+b\log(c(d+ex)^n))\left(\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)+\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)\right)+bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)+bn\,\text{Poly}}{2g}$$

```
[In] Integrate[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])*(Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d
*Sqrt[g])] + Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])]) + b*
n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + b*n*PolyLog
[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.75

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)\ln(g\,x^2+f)}{2g}-\frac{bn\ln(ex+d)\ln(g\,x^2+f)}{2g}+\frac{bn\ln(ex+d)\ln\left(\frac{e\sqrt{-fg}-g(ex+d)+dg}{e\sqrt{-fg}+dg}\right)}{2g}+\frac{bn\ln(ex+d)\ln\left(\frac{e\sqrt{-fg}+g(ex+d}{e\sqrt{-fg}-dg}\right.}{2g}$ |

```
[In] int(x*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln((e*x+d)^n)/g*ln(g*x^2+f)-1/2*b/g*n*ln(e*x+d)*ln(g*x^2+f)+1/2*b/g*n
*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b/g*
n*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b/g
*n*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b/g*n*dil
og((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*(-1/2*I*b*Pi*cs
gn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c
*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi
*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)/g*ln(g*x^2+f)
```

## Fricas [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{gx^2+f}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="fricas")
[Out] integral((b*x*log((e*x + d)^n*c) + a*x)/(g*x^2 + f), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{gx^2+f}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="maxima")
[Out] b*integrate((x*log((e*x + d)^n) + x*log(c))/(g*x^2 + f), x) + 1/2*a*log(g*x
^2 + f)/g
```

## Giac [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x}{gx^2+f}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*x/(g*x^2 + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{x\,(a+b\,\ln(c\,(d+e\,x)^n))}{g\,x^2+f}\,dx$$

```
[In] int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2),x)
[Out] int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2), x)
```

# 3.259 $\int \frac{a+b\log(c(d+ex)^n)}{x(f+gx^2)}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 245

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx = \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} - \frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} - \frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} + \frac{bn\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f}$$

```
[Out] ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/f-1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/
2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f-1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((
-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/f+b*n*polylog(2,1+e*x/d)/f-1
/2*b*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f-1/2*b*n*polyl
og(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {272, 36, 29, 31, 2463, 2441, 2352, 266, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx=-\frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{d\sqrt{g}+e\sqrt{-f}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f}-\frac{\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f}+\frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f}-\frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f}-\frac{bn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2f}+\frac{bn\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x^2)),x]
```

```
[Out] (Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f - ((a + b*Log[c*(d + e*x)^n]
)*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f) - ((a + b
*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g]
)])/(2*f) - (b*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))
])/(2*f) - (b*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(
2*f) + (b*n*PolyLog[2, 1 + (e*x)/d])/f
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{fx}-\frac{gx(a+b\log\left(c(d+ex)^n\right))}{f\left(f+gx^2\right)}\right)\,dx\\ &= \frac{\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx}{f}-\frac{g\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx}{f}\end{aligned}$$

$$= \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f} - \frac{g\int\left(-\frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)}+\frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right)dx}{f} - \frac{(ben)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}dx}{f}$$

$$= \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f} + \frac{bn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f} + \frac{\sqrt{g}\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}dx}{2f} - \frac{\sqrt{g}\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}dx}{2f}$$

$$= \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} + \frac{bn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f} + \frac{(ben)\int\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}dx}{2f} + \frac{(ben)\int\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}dx}{2f}$$

$$= \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} + \frac{bn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f} + \frac{(bn)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2f} + \frac{(bn)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2f}$$

$$= \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} - \frac{bn\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} - \frac{bn\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} + \frac{bn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.91

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx =$$

$$-\frac{-2\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)+\left(a+b\log\left(c(d+ex)\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x^2)),x]
```

```
[Out] -1/2*(-2*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]) + (a + b*Log[c*(d + e*x
)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])] + (a + b*Log
[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])] +
 b*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + b*n*Poly
Log[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])] - 2*b*n*PolyLog[2, 1 +
 (e*x)/d])/f
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.69

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)\ln(x)}{f}-\frac{b\ln((ex+d)^n)\ln(g\,x^2+f)}{2f}-\frac{bn\,\mathrm{dilog}\left(\frac{ex+d}{d}\right)}{f}-\frac{bn\ln(x)\ln\left(\frac{ex+d}{d}\right)}{f}+\frac{bn\ln(ex+d)\ln(g\,x^2+f)}{2f}-\frac{bn\ln(e}{}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/x/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] b*ln((e*x+d)^n)/f*ln(x)-1/2*b*ln((e*x+d)^n)/f*ln(g*x^2+f)-b*n/f*dilog((e*x+
d)/d)-b*n/f*ln(x)*ln((e*x+d)/d)+1/2*b*n/f*ln(e*x+d)*ln(g*x^2+f)-1/2*b*n/f*l
n(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n/f*
ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))-1/2*b*n/f
*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n/f*dilog
((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c
)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+
d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(
I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/f*ln(x)-1/2/f*ln(g*x^2+f))
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx^2+f)x}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/x/(g*x^2+f),x, algorithm="fricas")

[Out] integral((b*log((e*x + d)^n*c) + a)/(g*x^3 + f*x), x)

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx$$

[In] integrate((a+b*ln(c*(e*x+d)**n))/x/(g*x**2+f),x)

[Out] Integral((a + b*log(c*(d + e*x)**n))/(x*(f + g*x**2)), x)

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx^2+f)x}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/x/(g*x^2+f),x, algorithm="maxima")

[Out] -1/2*a*(log(g*x^2 + f)/f - 2*log(x)/f) + b*integrate((log((e*x + d)^n) + lo
g(c))/(g*x^3 + f*x), x)

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{(gx^2+f)x}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/x/(g*x^2+f),x, algorithm="giac")

[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)*x), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)}\,dx = \int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{x\,\left(g\,x^2+f\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x^2)),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x^2)), x)
```

# 3.260 $\int \frac{a+b\log(c(d+ex)^n)}{x^3(f+gx^2)}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 331

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)}\,dx = -\frac{ben}{2dfx}-\frac{be^2n\log(x)}{2d^2f}+\frac{be^2n\log(d+ex)}{2d^2f} - \frac{a+b\log\left(c(d+ex)^n\right)}{2fx^2}-\frac{g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} + \frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} + \frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} + \frac{bgn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} + \frac{bgn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}-\frac{bgn\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^2}$$

```
[Out] -1/2*b*e*n/d/f/x-1/2*b*e^2*n*ln(x)/d^2/f+1/2*b*e^2*n*ln(e*x+d)/d^2/f+1/2*(-
a-b*ln(c*(e*x+d)^n))/f/x^2-g*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/f^2+1/2*g*(a+
b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f^
2+1/2*g*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g
^(1/2)))/f^2-b*g*n*polylog(2,1+e*x/d)/f^2+1/2*b*g*n*polylog(2,-(e*x+d)*g^(1
/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f^2+1/2*b*g*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-
f)^(1/2)+d*g^(1/2)))/f^2
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 46, 2463, 2442, 2441, 2352, 266, 2440, 2438}

$$\int \frac{a+b\log(c(d+ex)^n)}{x^3(f+gx^2)}\,dx = -\frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^2} + \frac{g\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2f^2} + \frac{g\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2f^2} - \frac{a+b\log(c(d+ex)^n)}{2fx^2} - \frac{be^2n\log(x)}{2d^2f} + \frac{be^2n\log(d+ex)}{2d^2f} + \frac{bgn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} + \frac{bgn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2f^2} - \frac{bgn\,\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^2} - \frac{ben}{2dfx}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x^2)),x]
```

```
[Out] -1/2*(b*e*n)/(d*f*x) - (b*e^2*n*Log[x])/(2*d^2*f) + (b*e^2*n*Log[d + e*x])/
(2*d^2*f) - (a + b*Log[c*(d + e*x)^n])/(2*f*x^2) - (g*Log[-((e*x)/d)]*(a +
b*Log[c*(d + e*x)^n]))/f^2 + (g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f]
 - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^2) + (g*(a + b*Log[c*(d + e*
x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*f^2) +
(b*g*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*f^2)
 + (b*g*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^2)
 - (b*g*n*PolyLog[2, 1 + (e*x)/d])/f^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{fx^3} - \frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2x} + \frac{g^2x(a+b\log\left(c(d+ex)^n\right))}{f^2\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\log(c(d+ex)^n)}{x^3}\,dx}{f} - \frac{g\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx}{f^2} + \frac{g^2\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx}{f^2}$$

$$= -\frac{a+b\log\left(c(d+ex)^n\right)}{2fx^2} - \frac{g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} + \frac{g^2\int\left(-\frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)} + \frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right)dx}{f^2} + \frac{(ben)\int \frac{1}{x^2(d+ex)}\,dx}{2f} + \frac{(begn)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{f^2}$$

$$= -\frac{a+b\log\left(c(d+ex)^n\right)}{2fx^2} - \frac{g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} - \frac{bgn\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} - \frac{g^{3/2}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2f^2} + \frac{g^{3/2}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2f^2} + \frac{(ben)\int\left(\frac{1}{dx^2} - \frac{e}{d^2x} + \frac{e^2}{d^2(d+ex)}\right)dx}{2f}$$

$$= -\frac{ben}{2dfx} - \frac{be^2n\log(x)}{2d^2f} + \frac{be^2n\log(d+ex)}{2d^2f} - \frac{a+b\log\left(c(d+ex)^n\right)}{2fx^2} - \frac{g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} + \frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} + \frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} - \frac{bgn\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} - \frac{(begn)\int \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2f^2} - \frac{(begn)\int \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2f^2}$$

$$= -\frac{ben}{2dfx} - \frac{be^2n\log(x)}{2d^2f} + \frac{be^2n\log(d+ex)}{2d^2f} - \frac{a+b\log(c(d+ex)^n)}{2fx^2} - \frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^2} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} - \frac{bgn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} - \frac{(bgn)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2f^2} - \frac{(bgn)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2f^2}$$

$$= -\frac{ben}{2dfx} - \frac{be^2n\log(x)}{2d^2f} + \frac{be^2n\log(d+ex)}{2d^2f} - \frac{a+b\log(c(d+ex)^n)}{2fx^2} - \frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^2} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} + \frac{bgn\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} + \frac{bgn\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} - \frac{bgn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^2}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.84

$$\int \frac{a+b\log(c(d+ex)^n)}{x^3(f+gx^2)}\,dx = \frac{-\frac{befn(d+ex\log(x)-ex\log(d+ex))}{d^2x} - \frac{f(a+b\log(c(d+ex)^n))}{x^2} - 2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n)) + g(a+b\log(c(d-$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x^2)),x]
```

```
[Out] (-((b*e*f*n*(d + e*x*Log[x] - e*x*Log[d + e*x]))/(d^2*x)) - (f*(a + b*Log[c
*(d + e*x)^n]))/x^2 - 2*g*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]) + g*(a
 + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqr
t[g])] + g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqr
t[-f] - d*Sqrt[g])] + b*g*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] -
d*Sqrt[g]))] + b*g*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g]
)] - 2*b*g*n*PolyLog[2, 1 + (e*x)/d])/(2*f^2)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.20 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.51

| method | result |
|---|---|
| risch | $-\frac{b\ln\left((ex+d)^{n}\right)}{2f\,x^{2}}-\frac{b\ln\left((ex+d)^{n}\right)g\ln(x)}{f^{2}}+\frac{b\ln\left((ex+d)^{n}\right)g\ln\left(g\,x^{2}+f\right)}{2f^{2}}-\frac{bng\ln(ex+d)\ln\left(g\,x^{2}+f\right)}{2f^{2}}+\frac{bng\ln(ex+d)\ln\left(\frac{e\sqrt{-fg}-g}{e\sqrt{-}}\right.}{2f^{2}}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/x^3/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*ln((e*x+d)^n)/f/x^2-b*ln((e*x+d)^n)/f^2*g*ln(x)+1/2*b*ln((e*x+d)^n)*
g/f^2*ln(g*x^2+f)-1/2*b*n/f^2*g*ln(e*x+d)*ln(g*x^2+f)+1/2*b*n/f^2*g*ln(e*x+
d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n/f^2*g*ln
(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*n/f^2
*g*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n/f^2*g
*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*e^2*n*ln(
e*x+d)/d^2/f-1/2*b*e*n/d/f/x-1/2*b*e^2*n*ln(x)/d^2/f+b*n/f^2*g*dilog((e*x+d
)/d)+b*n/f^2*g*ln(x)*ln((e*x+d)/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)
*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*
csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*
ln(c)+a)*(-1/2/f/x^2-1/f^2*g*ln(x)+1/2*g/f^2*ln(g*x^2+f))
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{x^{3}\left(f+gx^{2}\right)}\,dx=\int \frac{b\log\left((ex+d)^{n}c\right)+a}{(gx^{2}+f)x^{3}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x + d)^n*c) + a)/(g*x^5 + f*x^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{x^{3}\left(f+gx^{2}\right)}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/x**3/(g*x**2+f),x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x^3\left(f + gx^2\right)}\,dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{(gx^2 + f)x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*a*(g*log(g*x^2 + f)/f^2 - 2*g*log(x)/f^2 - 1/(f*x^2)) + b*integrate((lo
g((e*x + d)^n) + log(c))/(g*x^5 + f*x^3), x)
```

## Giac [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x^3\left(f + gx^2\right)}\,dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{(gx^2 + f)x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x^3\left(f + gx^2\right)}\,dx = \int \frac{a + b\,\ln\left(c\,{\left(d + e\,x\right)}^n\right)}{x^3\,\left(g\,x^2 + f\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x^2)),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x^2)), x)
```

# 3.261 $\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2} dx$



## Optimal result

Integrand size = 27, antiderivative size = 369

$$\begin{aligned}\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = &-\frac{afx}{g^2}+\frac{bfnx}{g^2}-\frac{bd^2nx}{3e^2g}+\frac{bdnx^2}{6eg}-\frac{bnx^3}{9g}+\frac{bd^3n\log(d+ex)}{3e^3g}\\ &-\frac{bf(d+ex)\log(c(d+ex)^n)}{eg^2}+\frac{x^3(a+b\log(c(d+ex)^n))}{3g}\\ &+\frac{(-f)^{3/2}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}}\\ &-\frac{(-f)^{3/2}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}}\\ &-\frac{b(-f)^{3/2}n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}}\\ &+\frac{b(-f)^{3/2}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}}\end{aligned}$$

```
[Out] -a*f*x/g^2+b*f*n*x/g^2-1/3*b*d^2*n*x/e^2/g+1/6*b*d*n*x^2/e/g-1/9*b*n*x^3/g+
1/3*b*d^3*n*ln(e*x+d)/e^3/g-b*f*(e*x+d)*ln(c*(e*x+d)^n)/e/g^2+1/3*x^3*(a+b*
ln(c*(e*x+d)^n))/g+1/2*(-f)^(3/2)*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*
g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/g^(5/2)-1/2*(-f)^(3/2)*(a+b*ln(c*(e*x+d)
^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^(5/2)-1/2*b*(-
f)^(3/2)*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^(5/2)+1/2
*b*(-f)^(3/2)*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^(5/2)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {308, 211, 2463, 2436, 2332, 2442, 45, 2456, 2441, 2440, 2438}

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \frac{(-f)^{3/2}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g^{5/2}} - \frac{(-f)^{3/2}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g^{5/2}} + \frac{x^3(a+b\log(c(d+ex)^n))}{3g} - \frac{afx}{g^2} - \frac{bf(d+ex)\log(c(d+ex)^n)}{eg^2} + \frac{bd^3n\log(d+ex)}{3e^3g} - \frac{bd^2nx}{3e^2g} - \frac{b(-f)^{3/2}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} + \frac{b(-f)^{3/2}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^{5/2}} + \frac{bdnx^2}{6eg} + \frac{bfnx}{g^2} - \frac{bnx^3}{9g}$$

```
[In] Int[(x^4*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

```
[Out] -((a*f*x)/g^2) + (b*f*n*x)/g^2 - (b*d^2*n*x)/(3*e^2*g) + (b*d*n*x^2)/(6*e*g
) - (b*n*x^3)/(9*g) + (b*d^3*n*Log[d + e*x])/(3*e^3*g) - (b*f*(d + e*x)*Log
[c*(d + e*x)^n])/(e*g^2) + (x^3*(a + b*Log[c*(d + e*x)^n]))/(3*g) + ((-f)^(
3/2)*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f]
+ d*Sqrt[g])])/(2*g^(5/2)) - ((-f)^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[(e*
(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^(5/2)) - (b*(-f)^(3
/2)*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*g^(5/
2)) + (b*(-f)^(3/2)*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g
])])/(2*g^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
```

```
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{f(a+b\log(c(d+ex)^n))}{g^2}+\frac{x^2(a+b\log(c(d+ex)^n))}{g}+\frac{f^2(a+b\log(c(d+ex)^n))}{g^2(f+gx^2)}\right)dx$$

$$=-\frac{f\int(a+b\log(c(d+ex)^n))\,dx}{g^2}+\frac{f^2\int\frac{a+b\log(c(d+ex)^n)}{f+gx^2}\,dx}{g^2}+\frac{\int x^2(a+b\log(c(d+ex)^n))\,dx}{g}$$

$$=-\frac{afx}{g^2}+\frac{x^3(a+b\log(c(d+ex)^n))}{3g}-\frac{(bf)\int\log(c(d+ex)^n)\,dx}{g^2}+\frac{f^2\int\left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}-\sqrt{g}x)}+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g^2}-\frac{(ben)\int\frac{x^3}{d+ex}\,dx}{3g}$$

$$=-\frac{afx}{g^2}+\frac{x^3(a+b\log(c(d+ex)^n))}{3g}-\frac{(-f)^{3/2}\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2g^2}-\frac{(-f)^{3/2}\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2g^2}-\frac{(bf)\text{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{eg^2}-\frac{(ben)\int\left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)dx}{3g}$$

$$=-\frac{afx}{g^2}+\frac{bfnx}{g^2}-\frac{bd^2nx}{3e^2g}+\frac{bdnx^2}{6eg}-\frac{bnx^3}{9g}+\frac{bd^3n\log(d+ex)}{3e^3g}-\frac{bf(d+ex)\log(c(d+ex)^n)}{eg^2}+\frac{x^3(a+b\log(c(d+ex)^n))}{3g}+\frac{(-f)^{3/2}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}}-\frac{(-f)^{3/2}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}}-\frac{\left(be(-f)^{3/2}n\right)\int\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2g^{5/2}}+\frac{\left(be(-f)^{3/2}n\right)\int\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2g^{5/2}}$$

$$= -\frac{afx}{g^2} + \frac{bfnx}{g^2} - \frac{bd^2nx}{3e^2g} + \frac{bdnx^2}{6eg} - \frac{bnx^3}{9g} + \frac{bd^3n\log(d+ex)}{3e^3g} - \frac{bf(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} + \frac{x^3(a+b\log\left(c(d+ex)^n\right))}{3g} + \frac{(-f)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}} - \frac{(-f)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} + \frac{\left(b(-f)^{3/2}n\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2g^{5/2}} - \frac{\left(b(-f)^{3/2}n\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2g^{5/2}}$$

$$= -\frac{afx}{g^2} + \frac{bfnx}{g^2} - \frac{bd^2nx}{3e^2g} + \frac{bdnx^2}{6eg} - \frac{bnx^3}{9g} + \frac{bd^3n\log(d+ex)}{3e^3g} - \frac{bf(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} + \frac{x^3(a+b\log\left(c(d+ex)^n\right))}{3g} + \frac{(-f)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}} - \frac{(-f)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} - \frac{b(-f)^{3/2}n\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} + \frac{b(-f)^{3/2}n\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}}$$

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.92

$$\int \frac{x^4(a+b\log\left(c(d+ex)^n\right))}{f+gx^2}\,dx = \frac{-18af\sqrt{g}x + 18bf\sqrt{g}nx - \frac{bg^{3/2}n\left(ex(6d^2-3dex+2e^2x^2)-6d^3\log(d+ex)\right)}{e^3} - \frac{18bf\sqrt{g}(d+ex)\log(c(d+ex)^n)}{e} + 6g^{3/2}x^3(a+b\log\cdots}{}$$

```
[In] Integrate[(x^4*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

```
[Out] (-18*a*f*Sqrt[g]*x + 18*b*f*Sqrt[g]*n*x - (b*g^(3/2)*n*(e*x*(6*d^2 - 3*d*e*
x + 2*e^2*x^2) - 6*d^3*Log[d + e*x]))/e^3 - (18*b*f*Sqrt[g]*(d + e*x)*Log[c
*(d + e*x)^n])/e + 6*g^(3/2)*x^3*(a + b*Log[c*(d + e*x)^n]) + 9*(-f)^(3/2)*
```

```
(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*S
qrt[g])] + 9*Sqrt[-f]*f*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[
g]*x))/(e*Sqrt[-f] - d*Sqrt[g])] - 9*b*(-f)^(3/2)*n*PolyLog[2, -((Sqrt[g]*(
d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + 9*b*(-f)^(3/2)*n*PolyLog[2, (Sqrt[g]
*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(18*g^(5/2))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.17 (sec) , antiderivative size = 606, normalized size of antiderivative = 1.64

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)x^3}{3g}+\frac{b\,d^3\ln((ex+d)^n)}{3e^3g}-\frac{b\ln((ex+d)^n)fx}{g^2}-\frac{bdf\ln((ex+d)^n)}{e\,g^2}-\frac{b\,f^2\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)n\ln(ex+d)}{g^2\sqrt{fg}}+\frac{b\,f^2}{}$ |

```
[In] int(x^4*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*ln((e*x+d)^n)/g*x^3+1/3*b/e^3/g*d^3*ln((e*x+d)^n)-b*ln((e*x+d)^n)/g^2
*f*x-b/e/g^2*d*f*ln((e*x+d)^n)-b*f^2/g^2/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d
)-2*d*g)/e/(f*g)^(1/2))*n*ln(e*x+d)+b*f^2/g^2/(f*g)^(1/2)*arctan(1/2*(2*g*(
e*x+d)-2*d*g)/e/(f*g)^(1/2))*ln((e*x+d)^n)-1/9*b*n*x^3/g+1/6*b*d*n*x^2/e/g-
1/3*b*d^2*n*x/e^2/g-11/18*b*d^3*n/e^3/g+b*f*n*x/g^2+b*d*f*n/e/g^2+1/2*b*n*f
^2/g^2*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(
1/2)+d*g))-1/2*b*n*f^2/g^2*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x
+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*n*f^2/g^2/(-f*g)^(1/2)*dilog((e*(-f*g)
^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n*f^2/g^2/(-f*g)^(1/2)*di
log((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(
I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e
*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*cs
gn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/g^2*(1/3*g*x^3-f*x)+f^2/g^2/(f*g)^(1/2)*a
rctan(g*x/(f*g)^(1/2)))
```

## Fricas [F]

$$\int \frac{x^4(a+b\log\left(c(d+ex)^n\right))}{f+gx^2}\,dx=\int \frac{(b\log\left((ex+d)^nc\right)+a)x^4}{gx^2+f}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*log((e*x + d)^n*c) + a*x^4)/(g*x^2 + f), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^4}{gx^2+f}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="maxima")
[Out] 1/3*a*(3*f^2*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g^2) + (g*x^3 - 3*f*x)/g^2) +
 b*integrate((x^4*log((e*x + d)^n) + x^4*log(c))/(g*x^2 + f), x)
```

## Giac [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^4}{gx^2+f}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*x^4/(g*x^2 + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{x^4\,(a+b\,\ln(c\,(d+e\,x)^n))}{g\,x^2+f}\,dx$$

```
[In] int((x^4*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2),x)
[Out] int((x^4*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2), x)
```

# 3.262 $\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 276

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \frac{ax}{g}-\frac{bnx}{g}+\frac{b(d+ex)\log(c(d+ex)^n)}{eg}+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}}-\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}}-\frac{b\sqrt{-f}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}}+\frac{b\sqrt{-f}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}}$$

```
[Out] a*x/g-b*n*x/g+b*(e*x+d)*ln(c*(e*x+d)^n)/e/g+1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*
((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(3/2)-1/2*(a+
b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))*(-
f)^(1/2)/g^(3/2)-1/2*b*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)
))*(-f)^(1/2)/g^(3/2)+1/2*b*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(
1/2)))*(-f)^(1/2)/g^(3/2)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {327, 211, 2463, 2436, 2332, 2456, 2441, 2440, 2438}

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \frac{\sqrt{-f}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g^{3/2}} - \frac{\sqrt{-f}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g^{3/2}} + \frac{ax}{g} + \frac{b(d+ex)\log(c(d+ex)^n)}{eg} - \frac{b\sqrt{-f}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}} + \frac{b\sqrt{-f}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^{3/2}} - \frac{bnx}{g}$$

```
[In] Int[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

```
[Out] (a*x)/g - (b*n*x)/g + (b*(d + e*x)*Log[c*(d + e*x)^n])/(e*g) + (Sqrt[-f]*(a
 + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqr
t[g])])/(2*g^(3/2)) - (Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f]
 + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^(3/2)) - (b*Sqrt[-f]*n*PolyL
og[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*g^(3/2)) + (b*Sq
rt[-f]*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^(3/
2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{g} - \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)}{g\left(f+gx^2\right)}\right) dx$$

$$= \frac{\int \left(a+b\log\left(c(d+ex)^n\right)\right) dx}{g} - \frac{f\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx^2}\, dx}{g}$$

$$= \frac{ax}{g} + \frac{b\int \log\left(c(d+ex)^n\right)\,dx}{g} - \frac{f\int \left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f\left(\sqrt{-f}+\sqrt{g}x\right)}\right)\,dx}{g}$$

$$= \frac{ax}{g} + \frac{b\text{Subst}\left(\int \log\left(cx^n\right)\,dx, x, d+ex\right)}{eg} - \frac{\sqrt{-f}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2g} - \frac{\sqrt{-f}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2g}$$

$$= \frac{ax}{g} - \frac{bnx}{g} + \frac{b(d+ex)\log\left(c(d+ex)^n\right)}{eg} + \frac{\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}} - \frac{\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}} - \frac{\left(be\sqrt{-f}n\right)\int \frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2g^{3/2}} + \frac{\left(be\sqrt{-f}n\right)\int \frac{\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2g^{3/2}}$$

$$= \frac{ax}{g} - \frac{bnx}{g} + \frac{b(d+ex)\log\left(c(d+ex)^n\right)}{eg} + \frac{\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}} - \frac{\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}} + \frac{\left(b\sqrt{-f}n\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2g^{3/2}} - \frac{\left(b\sqrt{-f}n\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2g^{3/2}}$$

$$= \frac{ax}{g} - \frac{bnx}{g} + \frac{b(d+ex)\log\left(c(d+ex)^n\right)}{eg} + \frac{\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}} - \frac{\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}} - \frac{b\sqrt{-f}n\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}} + \frac{b\sqrt{-f}n\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.95

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx$$
$$= \frac{2a\sqrt{g}x - 2b\sqrt{g}nx + \frac{2b\sqrt{g}(d+ex)\log(c(d+ex)^n)}{e} + \sqrt{-f}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right) - \sqrt{-f}(a+b}{2g}$$

```
[In] Integrate[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2),x]
```

```
[Out] (2*a*Sqrt[g]*x - 2*b*Sqrt[g]*n*x + (2*b*Sqrt[g]*(d + e*x)*Log[c*(d + e*x)^n
])/e + Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(
e*Sqrt[-f] + d*Sqrt[g])] - Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt
[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])] - b*Sqrt[-f]*n*PolyLog[2, -((S
qrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + b*Sqrt[-f]*n*PolyLog[2, (Sqr
t[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^(3/2))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.83 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.78

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)x}{g} + \frac{bd\ln((ex+d)^n)}{eg} + \frac{bf\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)n\ln(ex+d)}{g\sqrt{fg}} - \frac{bf\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)\ln((ex+d)^n)}{g\sqrt{fg}} - \frac{bnx}{g} -$ |

```
[In] int(x^2*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] b*ln((e*x+d)^n)/g*x+b/e/g*d*ln((e*x+d)^n)+b*f/g/(f*g)^(1/2)*arctan(1/2*(2*g
*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*n*ln(e*x+d)-b*f/g/(f*g)^(1/2)*arctan(1/2*(2*
g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*ln((e*x+d)^n)-b*n*x/g-b*d*n/e/g-1/2*b*n*f/g
*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d
*g))+1/2*b*n*f/g*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(
e*(-f*g)^(1/2)-d*g))-1/2*b*n*f/g/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)-g*(e*x+
d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n*f/g/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2
)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d
)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b
*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^
3+b*ln(c)+a)*(x/g-f/g/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2)))
```

## Fricas [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^2}{gx^2+f}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="fricas")
[Out] integral((b*x^2*log((e*x + d)^n*c) + a*x^2)/(g*x^2 + f), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^2}{gx^2+f}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="maxima")
[Out] -a*(f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g) - x/g) + b*integrate((x^2*log((e*
x + d)^n) + x^2*log(c))/(g*x^2 + f), x)
```

## Giac [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^2}{gx^2+f}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*x^2/(g*x^2 + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log\left(c(d+ex)^n\right))}{f+gx^2}\,dx=\int \frac{x^2\left(a+b\,\ln\left(c\left(d+e\,x\right)^n\right)\right)}{g\,x^2+f}\,dx$$

```
[In] int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2),x)
[Out] int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2), x)
```

# 3.263 $\int \frac{a+b\log(c(d+ex)^n)}{f+gx^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 239

$$\int \frac{a+b\log(c(d+ex)^n)}{f+gx^2}\,dx = \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

```
[Out] 1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/
2)))/(-f)^(1/2)/g^(1/2)-1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2
))/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*b*n*polylog(2,-(e*x+d)*
g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)+1/2*b*n*polylog(2,(e*x
+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/(-f)^(1/2)/g^(1/2)
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{2456, 2441, 2440, 2438\}$

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx^2}\,dx = \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2\sqrt{-f}\sqrt{g}} - \frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x^2),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*
Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[
-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (b*n*Po
lyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*Sqrt[-f]*Sqrt
[g]) + (b*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sq
rt[-f]*Sqrt[g])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f\left(\sqrt{-f}+\sqrt{g}x\right)}\right) dx$$

$$= -\frac{\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x} dx}{2\sqrt{-f}} - \frac{\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x} dx}{2\sqrt{-f}}$$

$$= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(ben)\int \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex} dx}{2\sqrt{-f}\sqrt{g}} + \frac{(ben)\int \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex} dx}{2\sqrt{-f}\sqrt{g}}$$

$$= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{(bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x} dx, x, d+ex\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(bn)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x} dx, x, d+ex\right)}{2\sqrt{-f}\sqrt{g}}$$

$$= \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{bn\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{bn\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.77

$$\int \frac{a+b\log \left(c(d+ex)^n\right)}{f+gx^2}\,dx = \frac{(a+b\log \left(c(d+ex)^n\right))\left(\log \left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)-\log \left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)\right)-bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)+bn\,\text{Pol}}{2\sqrt{-f}\sqrt{g}}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x^2),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])*(Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d
*Sqrt[g])] - Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])]) - b*
n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + b*n*PolyLog
[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.75 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.69

| method | result |
|---|---|
| risch | $-\frac{b\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)n\ln(ex+d)}{\sqrt{fg}}+\frac{b\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)\ln((ex+d)^n)}{\sqrt{fg}}+\frac{bn\ln(ex+d)\ln\left(\frac{e\sqrt{-fg}-g(ex+d)+dg}{e\sqrt{-fg}+dg}\right)}{2\sqrt{-fg}}-\frac{bn\ln}{}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] -b/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*n*ln(e*x+d)+b/
(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*ln((e*x+d)^n)+1/2
*b*n*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/
2)+d*g))-1/2*b*n*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(
e*(-f*g)^(1/2)-d*g))+1/2*b*n/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d
*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)+g*(e*x
+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csg
n(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c
)+a)/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2))
```

## Fricas [F]

$$\int \frac{a+b\log(c(d+ex)^n)}{f+gx^2}\,dx = \int \frac{b\log((ex+d)^n c)+a}{gx^2+f}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="fricas")

[Out] integral((b*log((e*x + d)^n*c) + a)/(g*x^2 + f), x)

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log(c(d+ex)^n)}{f+gx^2}\,dx = \text{Timed out}$$

[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x**2+f),x)

[Out] Timed out

## Maxima [F]

$$\int \frac{a+b\log(c(d+ex)^n)}{f+gx^2}\,dx = \int \frac{b\log((ex+d)^n c)+a}{gx^2+f}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="maxima")

[Out] b*integrate((log((e*x + d)^n) + log(c))/(g*x^2 + f), x) + a*arctan(g*x/sqrt(f*g))/sqrt(f*g)

## Giac [F]

$$\int \frac{a+b\log(c(d+ex)^n)}{f+gx^2}\,dx = \int \frac{b\log((ex+d)^n c)+a}{gx^2+f}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f),x, algorithm="giac")

[Out] integrate((b*log((e*x + d)^n*c) + a)/(g*x^2 + f), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{f+gx^2}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{g\,x^2+f}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x^2),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(f + g*x^2), x)
```

# 3.264 $\int \frac{a+b\log(c(d+ex)^n)}{x^2(f+gx^2)}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 290

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)}\,dx &= \frac{ben\log(x)}{df}-\frac{ben\log(d+ex)}{df}-\frac{a+b\log\left(c(d+ex)^n\right)}{fx}\\ &\quad+\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}}\\ &\quad-\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}}\\ &\quad-\frac{b\sqrt{g}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}}\\ &\quad+\frac{b\sqrt{g}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}}\end{aligned}$$

```
[Out] b*e*n*ln(x)/d/f-b*e*n*ln(e*x+d)/d/f+(-a-b*ln(c*(e*x+d)^n))/f/x+1/2*(a+b*ln(
c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)
/(-f)^(3/2)-1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(
1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(3/2)-1/2*b*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(
-f)^(1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(3/2)+1/2*b*n*polylog(2,(e*x+d)*g^(1/2)/
(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/(-f)^(3/2)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {331, 211, 2463, 2442, 36, 29, 31, 2456, 2441, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)}\,dx = \frac{\sqrt{g}\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{d\sqrt{g}+e\sqrt{-f}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2(-f)^{3/2}} - \frac{\sqrt{g}\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2(-f)^{3/2}} - \frac{a+b\log\left(c(d+ex)^n\right)}{fx} - \frac{b\sqrt{g}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}} + \frac{b\sqrt{g}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2(-f)^{3/2}} + \frac{ben\log(x)}{df} - \frac{ben\log(d+ex)}{df}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x^2)),x]
```

```
[Out] (b*e*n*Log[x])/(d*f) - (b*e*n*Log[d + e*x])/(d*f) - (a + b*Log[c*(d + e*x)^
n])/(f*x) + (Sqrt[g]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*
x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(-f)^(3/2)) - (Sqrt[g]*(a + b*Log[c*(d +
e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*(-f)^
(3/2)) - (b*Sqrt[g]*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt
[g]))])/(2*(-f)^(3/2)) + (b*Sqrt[g]*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqr
t[-f] + d*Sqrt[g])])/(2*(-f)^(3/2))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{fx^2}-\frac{g(a+b\log\left(c(d+ex)^n\right))}{f\left(f+gx^2\right)}\right)dx\\
&= \frac{\int \frac{a+b\log(c(d+ex)^n)}{x^2}\,dx}{f}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{f+gx^2}\,dx}{f}\\
&= -\frac{a+b\log\left(c(d+ex)^n\right)}{fx}\\
&\quad-\frac{g\int \left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx}{f}+\frac{(ben)\int \frac{1}{x(d+ex)}\,dx}{f}\\
&= -\frac{a+b\log\left(c(d+ex)^n\right)}{fx}-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2(-f)^{3/2}}\\
&\quad-\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2(-f)^{3/2}}+\frac{(ben)\int \frac{1}{x}\,dx}{df}-\frac{\left(be^2n\right)\int \frac{1}{d+ex}\,dx}{df}\\
&= \frac{ben\log(x)}{df}-\frac{ben\log(d+ex)}{df}-\frac{a+b\log\left(c(d+ex)^n\right)}{fx}\\
&\quad+\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}}\\
&\quad-\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}}\\
&\quad-\frac{\left(be\sqrt{g}n\right)\int \frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2(-f)^{3/2}}+\frac{\left(be\sqrt{g}n\right)\int \frac{\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2(-f)^{3/2}}
\end{aligned}$$

$$= \frac{ben\log(x)}{df} - \frac{ben\log(d+ex)}{df} - \frac{a+b\log(c(d+ex)^n)}{fx} + \frac{\sqrt{g}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}} - \frac{\sqrt{g}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}} + \frac{(b\sqrt{g}n)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2(-f)^{3/2}} - \frac{(b\sqrt{g}n)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2(-f)^{3/2}}$$

$$= \frac{ben\log(x)}{df} - \frac{ben\log(d+ex)}{df} - \frac{a+b\log(c(d+ex)^n)}{fx} + \frac{\sqrt{g}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}} - \frac{\sqrt{g}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}} - \frac{b\sqrt{g}n\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}} + \frac{b\sqrt{g}n\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}}$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.97

$$\int\frac{a+b\log(c(d+ex)^n)}{x^2(f+gx^2)}\,dx = \frac{f\left(2be(-f)^{3/2}nx(\log(x)-\log(d+ex))+2d\sqrt{-f}f(a+b\log(c(d+ex)^n))+df\sqrt{g}x(a+b\log(c(d+ex)^n)\right)}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x^2)),x]
```

```
[Out] (f*(2*b*e*(-f)^(3/2)*n*x*(Log[x] - Log[d + e*x]) + 2*d*Sqrt[-f]*f*(a + b*Lo
g[c*(d + e*x)^n]) + d*f*Sqrt[g]*x*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-
f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])] - d*f*Sqrt[g]*x*(a + b*Log[c*(d
+ e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])] - b*d*f
*Sqrt[g]*n*x*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] +
b*d*f*Sqrt[g]*n*x*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])
)/(2*d*(-f)^(7/2)*x)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.16 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.70

| method | result |
|---|---|
| risch | $-\frac{b\ln((ex+d)^n)}{fx}+\frac{bg\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)n\ln(ex+d)}{f\sqrt{fg}}-\frac{bg\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)\ln((ex+d)^n)}{f\sqrt{fg}}+\frac{ben\ln(ex)}{fd}-\frac{ben\ln(ex+}{df}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/x^2/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] -b*ln((e*x+d)^n)/f/x+b/f*g/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*
g)^(1/2))*n*ln(e*x+d)-b/f*g/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f
*g)^(1/2))*ln((e*x+d)^n)+b*e*n/f/d*ln(e*x)-b*e*n*ln(e*x+d)/d/f-1/2*b*n/f*g*
ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*
g))+1/2*b*n/f*g*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e
*(-f*g)^(1/2)-d*g))-1/2*b*n/f*g/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)-g*(e*x+d
)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n/f*g/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)
+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)
^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*
Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3
+b*ln(c)+a)*(-1/f/x-1/f*g/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2)))
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx^2+f)x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x + d)^n*c) + a)/(g*x^4 + f*x^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/x**2/(g*x**2+f),x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx^2+f)x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] -a*(g*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*f) + 1/(f*x)) + b*integrate((log((e*
x + d)^n) + log(c))/(g*x^4 + f*x^2), x)
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{(gx^2+f)x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{x^2\,\left(g\,x^2+f\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x^2)),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x^2)), x)
```

# 3.265 $\int \frac{a+b\log(c(d+ex)^n)}{x^4(f+gx^2)}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 388

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4\left(f+gx^2\right)}\,dx = & -\frac{ben}{6dfx^2}+\frac{be^2n}{3d^2fx}+\frac{be^3n\log(x)}{3d^3f}-\frac{begn\log(x)}{df^2} \\ & -\frac{be^3n\log(d+ex)}{3d^3f}+\frac{begn\log(d+ex)}{df^2} \\ & -\frac{a+b\log\left(c(d+ex)^n\right)}{3fx^3}+\frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2x} \\ & +\frac{g^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} \\ & -\frac{g^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} \\ & -\frac{bg^{3/2}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} \\ & +\frac{bg^{3/2}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}}\end{aligned}$$

```
[Out] -1/6*b*e*n/d/f/x^2+1/3*b*e^2*n/d^2/f/x+1/3*b*e^3*n*ln(x)/d^3/f-b*e*g*n*ln(x
)/d/f^2-1/3*b*e^3*n*ln(e*x+d)/d^3/f+b*e*g*n*ln(e*x+d)/d/f^2+1/3*(-a-b*ln(c*
(e*x+d)^n))/f/x^3+g*(a+b*ln(c*(e*x+d)^n))/f^2/x+1/2*g^(3/2)*(a+b*ln(c*(e*x+
d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/(-f)^(5/2)-1/2
*g^(3/2)*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*
g^(1/2)))/(-f)^(5/2)-1/2*b*g^(3/2)*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/
2)-d*g^(1/2)))/(-f)^(5/2)+1/2*b*g^(3/2)*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)
^(1/2)+d*g^(1/2)))/(-f)^(5/2)
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {331, 211, 2463, 2442, 46, 36, 29, 31, 2456, 2441, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4(f+gx^2)}\,dx = \frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2x} + \frac{g^{3/2}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log\left(c(d+ex)^n\right))}{2(-f)^{5/2}} - \frac{g^{3/2}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log\left(c(d+ex)^n\right))}{2(-f)^{5/2}} - \frac{a+b\log\left(c(d+ex)^n\right)}{3fx^3} + \frac{be^3n\log(x)}{3d^3f} - \frac{be^3n\log(d+ex)}{3d^3f} + \frac{be^2n}{3d^2fx} - \frac{begn\log(x)}{df^2} + \frac{begn\log(d+ex)}{df^2} - \frac{bg^{3/2}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{bg^{3/2}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2(-f)^{5/2}} - \frac{ben}{6dfx^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(x^4*(f + g*x^2)),x]
```

```
[Out] -1/6*(b*e*n)/(d*f*x^2) + (b*e^2*n)/(3*d^2*f*x) + (b*e^3*n*Log[x])/(3*d^3*f)
 - (b*e*g*n*Log[x])/(d*f^2) - (b*e^3*n*Log[d + e*x])/(3*d^3*f) + (b*e*g*n*L
og[d + e*x])/(d*f^2) - (a + b*Log[c*(d + e*x)^n])/(3*f*x^3) + (g*(a + b*Log
[c*(d + e*x)^n]))/(f^2*x) + (g^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqr
t[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(-f)^(5/2)) - (g^(3/2)*(a
 + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqr
t[g])])/(2*(-f)^(5/2)) - (b*g^(3/2)*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*S
qrt[-f] - d*Sqrt[g]))])/(2*(-f)^(5/2)) + (b*g^(3/2)*n*PolyLog[2, (Sqrt[g]*(
d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(-f)^(5/2))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{fx^4} - \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2x^2} + \frac{g^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\log(c(d+ex)^n)}{x^4}\,dx}{f} - \frac{g\int \frac{a+b\log(c(d+ex)^n)}{x^2}\,dx}{f^2} + \frac{g^2\int \frac{a+b\log(c(d+ex)^n)}{f+gx^2}\,dx}{f^2}$$

$$= -\frac{a+b\log\left(c(d+ex)^n\right)}{3fx^3} + \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2x} + \frac{g^2\int \left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}-\sqrt{g}x)} + \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}+\sqrt{g}x)}\right)dx}{f^2} + \frac{(ben)\int \frac{1}{x^3(d+ex)}\,dx}{3f} - \frac{(begn)\int \frac{1}{x(d+ex)}\,dx}{f^2}$$

$$= -\frac{a+b\log\left(c(d+ex)^n\right)}{3fx^3} + \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2x} - \frac{g^2\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2(-f)^{5/2}} - \frac{g^2\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2(-f)^{5/2}} + \frac{(ben)\int \left(\frac{1}{dx^3} - \frac{e}{d^2x^2} + \frac{e^2}{d^3x} - \frac{e^3}{d^3(d+ex)}\right)dx}{3f} - \frac{(begn)\int \frac{1}{x}\,dx}{df^2} + \frac{(be^2gn)\int \frac{1}{d+ex}\,dx}{df^2}$$

$$= -\frac{ben}{6dfx^2} + \frac{be^2n}{3d^2fx} + \frac{be^3n\log(x)}{3d^3f} - \frac{begn\log(x)}{df^2} - \frac{be^3n\log(d+ex)}{3d^3f} + \frac{begn\log(d+ex)}{df^2} - \frac{a+b\log\left(c(d+ex)^n\right)}{3fx^3} + \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2x} + \frac{g^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{g^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{\left(beg^{3/2}n\right)\int\frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}dx}{2(-f)^{5/2}} + \frac{\left(beg^{3/2}n\right)\int\frac{\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}dx}{2(-f)^{5/2}}$$

$$= -\frac{ben}{6dfx^2} + \frac{be^2n}{3d^2fx} + \frac{be^3n\log(x)}{3d^3f} - \frac{begn\log(x)}{df^2} - \frac{be^3n\log(d+ex)}{3d^3f} + \frac{begn\log(d+ex)}{df^2} - \frac{a+b\log\left(c(d+ex)^n\right)}{3fx^3} + \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2x} + \frac{g^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{g^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{\left(bg^{3/2}n\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2(-f)^{5/2}} - \frac{\left(bg^{3/2}n\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2(-f)^{5/2}}$$

$$= -\frac{ben}{6dfx^2} + \frac{be^2n}{3d^2fx} + \frac{be^3n\log(x)}{3d^3f} - \frac{begn\log(x)}{df^2} - \frac{be^3n\log(d+ex)}{3d^3f} + \frac{begn\log(d+ex)}{df^2} - \frac{a+b\log\left(c(d+ex)^n\right)}{3fx^3} + \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2x} + \frac{g^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{g^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{bg^{3/2}n\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{bg^{3/2}n\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}}$$

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.90

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4(f+gx^2)}\,dx = \frac{1}{6}\left(-\frac{6begn(\log(x)-\log(d+ex))}{df^2} - \frac{ben(d(d-2ex)-2e^2x^2\log(x)+2e^2x^2\log(d+ex))}{d^3fx^2} - \frac{2(a+b\log\left(c(d+ex)^n\right))}{fx^3} + \frac{6g(a+b\log\left(c(d+ex)^n\right))}{f^2x} + \frac{3g^{3/2}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}} - \frac{3g^{3/2}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}} - \frac{3bg^{3/2}n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}} + \frac{3bg^{3/2}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}}\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(x^4*(f + g*x^2)),x]
```

```
[Out] ((-6*b*e*g*n*(Log[x] - Log[d + e*x]))/(d*f^2) - (b*e*n*(d*(d - 2*e*x) - 2*e
^2*x^2*Log[x] + 2*e^2*x^2*Log[d + e*x]))/(d^3*f*x^2) - (2*(a + b*Log[c*(d +
 e*x)^n]))/(f*x^3) + (6*g*(a + b*Log[c*(d + e*x)^n]))/(f^2*x) + (3*g^(3/2)*
(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*S
qrt[g])])/(-f)^(5/2) - (3*g^(3/2)*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-
f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(-f)^(5/2) - (3*b*g^(3/2)*n*Pol
yLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(-f)^(5/2) + (3*b
*g^(3/2)*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(-f)^(
5/2))/6
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.75 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.54

| method | result |
|---|---|
| risch | $-\frac{b\,g^2\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)n\ln(ex+d)}{f^2\sqrt{fg}}+\frac{b\,g^2\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)\ln((ex+d)^n)}{f^2\sqrt{fg}}-\frac{b\ln((ex+d)^n)}{3f\,x^3}+\frac{b\ln((ex+d)^n)g}{f^2x}-$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/x^4/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] -b*g^2/f^2/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*n*ln(e
*x+d)+b*g^2/f^2/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*l
n((e*x+d)^n)-1/3*b*ln((e*x+d)^n)/f/x^3+b*ln((e*x+d)^n)/f^2*g/x-e*b*n/f^2*g/
d*ln(e*x)+b*e*g*n*ln(e*x+d)/d/f^2-1/6*b*e*n/d/f/x^2+1/3*b*e^2*n/d^2/f/x+1/3
*e^3*b*n/f/d^3*ln(e*x)-1/3*b*e^3*n*ln(e*x+d)/d^3/f+1/2*b*n*g^2/f^2*ln(e*x+d
)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*
b*n*g^2/f^2*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f
*g)^(1/2)-d*g))+1/2*b*n*g^2/f^2/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)-g*(e*x+d
)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n*g^2/f^2/(-f*g)^(1/2)*dilog((e*(-f*g)^(
1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*
x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*
I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^
n)^3+b*ln(c)+a)*(-1/3/f/x^3+1/f^2*g/x+g^2/f^2/(f*g)^(1/2)*arctan(g*x/(f*g)^
(1/2)))
```

## Fricas [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x^4\left(f + gx^2\right)}\,dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{(gx^2 + f)x^4}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^4/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x + d)^n*c) + a)/(g*x^6 + f*x^4), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x^4\left(f + gx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/x**4/(g*x**2+f),x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x^4\left(f + gx^2\right)}\,dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{(gx^2 + f)x^4}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^4/(g*x^2+f),x, algorithm="maxima")
[Out] 1/3*a*(3*g^2*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*f^2) + (3*g*x^2 - f)/(f^2*x^3
)) + b*integrate((log((e*x + d)^n) + log(c))/(g*x^6 + f*x^4), x)
```

## Giac [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x^4\left(f + gx^2\right)}\,dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{(gx^2 + f)x^4}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^4/(g*x^2+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{x^4\left(f + gx^2\right)}\,dx = \int \frac{a + b\,\ln\left(c\left(d + e\,x\right)^n\right)}{x^4\,\left(g\,x^2 + f\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x^4*(f + g*x^2)),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(x^4*(f + g*x^2)), x)
```

# 3.266 $\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 417

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \frac{bdnx}{2eg^2}-\frac{bnx^2}{4g^2}+\frac{bdef^{3/2}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2g^{5/2}(e^2f+d^2g)} - \frac{bd^2n\log(d+ex)}{2e^2g^2}+\frac{be^2f^2n\log(d+ex)}{2g^3(e^2f+d^2g)} + \frac{x^2(a+b\log(c(d+ex)^n))}{2g^2}-\frac{f^2(a+b\log(c(d+ex)^n))}{2g^3(f+gx^2)} - \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3} - \frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} - \frac{be^2f^2n\log(f+gx^2)}{4g^3(e^2f+d^2g)}-\frac{bfn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} - \frac{bfn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$

```
[Out] 1/2*b*d*n*x/e/g^2-1/4*b*n*x^2/g^2+1/2*b*d*e*f^(3/2)*n*arctan(x*g^(1/2)/f^(1
/2))/g^(5/2)/(d^2*g+e^2*f)-1/2*b*d^2*n*ln(e*x+d)/e^2/g^2+1/2*b*e^2*f^2*n*ln
(e*x+d)/g^3/(d^2*g+e^2*f)+1/2*x^2*(a+b*ln(c*(e*x+d)^n))/g^2-1/2*f^2*(a+b*ln
(c*(e*x+d)^n))/g^3/(g*x^2+f)-1/4*b*e^2*f^2*n*ln(g*x^2+f)/g^3/(d^2*g+e^2*f)-
f*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)
))/g^3-f*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*
g^(1/2)))/g^3-b*f*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^
3-b*f*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^3
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {272, 45, 2463, 2442, 2460, 720, 31, 649, 211, 266, 2441, 2440, 2438}

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = -\frac{f^2(a+b\log(c(d+ex)^n))}{2g^3(f+gx^2)} - \frac{f\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{g^3} - \frac{f\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{g^3} + \frac{x^2(a+b\log(c(d+ex)^n))}{2g^2} + \frac{bdef^{3/2}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2g^{5/2}(d^2g+e^2f)} - \frac{be^2f^2n\log(f+gx^2)}{4g^3(d^2g+e^2f)} + \frac{be^2f^2n\log(d+ex)}{2g^3(d^2g+e^2f)} - \frac{bd^2n\log(d+ex)}{2e^2g^2} - \frac{bfn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} - \frac{bfn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^3} + \frac{bdnx}{2eg^2} - \frac{bnx^2}{4g^2}$$

```
[In] Int[(x^5*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

```
[Out] (b*d*n*x)/(2*e*g^2) - (b*n*x^2)/(4*g^2) + (b*d*e*f^(3/2)*n*ArcTan[(Sqrt[g]*
x)/Sqrt[f]])/(2*g^(5/2)*(e^2*f + d^2*g)) - (b*d^2*n*Log[d + e*x])/(2*e^2*g^
2) + (b*e^2*f^2*n*Log[d + e*x])/(2*g^3*(e^2*f + d^2*g)) + (x^2*(a + b*Log[c
*(d + e*x)^n]))/(2*g^2) - (f^2*(a + b*Log[c*(d + e*x)^n]))/(2*g^3*(f + g*x^
2)) - (f*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[
-f] + d*Sqrt[g])])/g^3 - (f*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + S
qrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/g^3 - (b*e^2*f^2*n*Log[f + g*x^2])/(4
*g^3*(e^2*f + d^2*g)) - (b*f*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f]
 - d*Sqrt[g]))])/g^3 - (b*f*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] +
d*Sqrt[g])])/g^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x(a+b\log(c(d+ex)^n))}{g^2}+\frac{f^2x(a+b\log(c(d+ex)^n))}{g^2(f+gx^2)^2}-\frac{2fx(a+b\log(c(d+ex)^n))}{g^2(f+gx^2)}\right)dx$$

$$=\frac{\int x(a+b\log(c(d+ex)^n))\,dx}{g^2}-\frac{(2f)\int\frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx}{g^2}+\frac{f^2\int\frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx}{g^2}$$

$$=\frac{x^2(a+b\log(c(d+ex)^n))}{2g^2}-\frac{f^2(a+b\log(c(d+ex)^n))}{2g^3(f+gx^2)}-\frac{(2f)\int\left(-\frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)}+\frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g^2}+\frac{(bef^2n)\int\frac{1}{(d+ex)(f+gx^2)}\,dx}{2g^3}-\frac{(ben)\int\frac{x^2}{d+ex}\,dx}{2g^2}$$

$$=\frac{x^2(a+b\log(c(d+ex)^n))}{2g^2}-\frac{f^2(a+b\log(c(d+ex)^n))}{2g^3(f+gx^2)}+\frac{f\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}dx}{g^{5/2}}-\frac{f\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}dx}{g^{5/2}}-\frac{(ben)\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx}{2g^2}+\frac{(bef^2n)\int\frac{dg-egx}{f+gx^2}dx}{2g^3(e^2f+d^2g)}+\frac{(be^3f^2n)\int\frac{1}{d+ex}dx}{2g^3(e^2f+d^2g)}$$

$$=\frac{bdnx}{2eg^2}-\frac{bnx^2}{4g^2}-\frac{bd^2n\log(d+ex)}{2e^2g^2}+\frac{be^2f^2n\log(d+ex)}{2g^3(e^2f+d^2g)}+\frac{x^2(a+b\log(c(d+ex)^n))}{2g^2}-\frac{f^2(a+b\log(c(d+ex)^n))}{2g^3(f+gx^2)}-\frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}-\frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}+\frac{(befn)\int\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}dx}{g^3}+\frac{(befn)\int\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}dx}{g^3}+\frac{(bdef^2n)\int\frac{1}{f+gx^2}dx}{2g^2(e^2f+d^2g)}-\frac{(be^2f^2n)\int\frac{x}{f+gx^2}dx}{2g^2(e^2f+d^2g)}$$

$$=\frac{bdnx}{2eg^2}-\frac{bnx^2}{4g^2}+\frac{bdef^{3/2}n\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2g^{5/2}(e^2f+d^2g)}-\frac{bd^2n\log(d+ex)}{2e^2g^2}+\frac{be^2f^2n\log(d+ex)}{2g^3(e^2f+d^2g)}+\frac{x^2(a+b\log(c(d+ex)^n))}{2g^2}-\frac{f^2(a+b\log(c(d+ex)^n))}{2g^3(f+gx^2)}-\frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}-\frac{f(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}-\frac{be^2f^2n\log(f+gx^2)}{4g^3(e^2f+d^2g)}+\frac{(bfn)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{g^3}+\frac{(bfn)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{g^3}$$

$$
\begin{aligned}
&= \frac{bdnx}{2eg^2} - \frac{bnx^2}{4g^2} + \frac{bdef^{3/2}n\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2g^{5/2}\left(e^2f+d^2g\right)} - \frac{bd^2n\log(d+ex)}{2e^2g^2} \\
&\quad + \frac{be^2f^2n\log(d+ex)}{2g^3\left(e^2f+d^2g\right)} + \frac{x^2(a+b\log\left(c(d+ex)^n\right))}{2g^2} \\
&\quad - \frac{f^2(a+b\log\left(c(d+ex)^n\right))}{2g^3\left(f+gx^2\right)} - \frac{f(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3} \\
&\quad - \frac{f(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} - \frac{be^2f^2n\log\left(f+gx^2\right)}{4g^3\left(e^2f+d^2g\right)} \\
&\quad - \frac{bfn\mathrm{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} - \frac{bfn\mathrm{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}
\end{aligned}
$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.27

$$
\int \frac{x^5(a+b\log\left(c(d+ex)^n\right))}{\left(f+gx^2\right)^2}\,dx
$$

$$
= \frac{2gx^2(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)) - \frac{2f^2(a-bn\log(d+ex)+b\log(c(d+ex)^n))}{f+gx^2} - 4f(a-bn\log(d+ex)+b
$$

```
[In] Integrate[(x^5*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

```
[Out] (2*g*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]) - (2*f^2*(a - b*n*Lo
g[d + e*x] + b*Log[c*(d + e*x)^n]))/(f + g*x^2) - 4*f*(a - b*n*Log[d + e*x]
 + b*Log[c*(d + e*x)^n])*Log[f + g*x^2] + b*n*((g*(e*x*(2*d - e*x) - 2*(d^2
 - e^2*x^2)*Log[d + e*x]))/e^2 + (f^(3/2)*(I*Sqrt[g]*(d + e*x)*Log[d + e*x]
 - e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt[f] - I*d
*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (I*f^(3/2)*(-(Sqrt[g]*(d + e*x)*Log[d
+ e*x]) + e*(I*Sqrt[f] + Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*x]))/((e*Sqrt[f
] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) - 4*f*(Log[d + e*x]*Log[(e*(Sqrt[
f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d
 + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) - 4*f*(Log[d + e*x]*Log[(e*(Sqrt[f] -
I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x))
/(e*Sqrt[f] + I*d*Sqrt[g])]))))/(4*g^3)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.57 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.50

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)x^2}{2g^2} - \frac{b\ln((ex+d)^n)f^2}{2g^3(g\,x^2+f)} - \frac{b\ln((ex+d)^n)f\ln(g\,x^2+f)}{g^3} - \frac{bn\,x^2}{4g^2} + \frac{bdnx}{2e\,g^2} - \frac{bn\ln(ex+d)d^4}{2e^2g(d^2g+f\,e^2)} - \frac{bn\ln(ex+d)d^2f}{2g^2(d^2g+f\,e^2)}$ |

[In] int(x^5*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*b*ln((e*x+d)^n)/g^2*x^2-1/2*b*ln((e*x+d)^n)*f^2/g^3/(g*x^2+f)-b*ln((e*x+d)^n)*f/g^3*ln(g*x^2+f)-1/4*b*n*x^2/g^2+1/2*b*d*n*x/e/g^2-1/2*b/e^2*n/g/(d^2*g+e^2*f)*ln(e*x+d)*d^4-1/2*b*n/g^2/(d^2*g+e^2*f)*ln(e*x+d)*d^2*f+1/2*b*e^2*f^2*n*ln(e*x+d)/g^3/(d^2*g+e^2*f)-1/4*b*e^2*f^2*n*ln(g*x^2+f)/g^3/(d^2*g+e^2*f)+1/2*b*e*n/g^2*f^2/(d^2*g+e^2*f)*d/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2))+b*n*f/g^3*ln(e*x+d)*ln(g*x^2+f)-b*n*f/g^3*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-b*n*f/g^3*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))-b*n*f/g^3*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-b*n*f/g^3*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/2*x^2/g^2-1/2*f/g^2*(f/g/(g*x^2+f)+2/g*ln(g*x^2+f)))

## Fricas [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^5}{(gx^2+f)^2}\,dx$$

[In] integrate(x^5*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="fricas")

[Out] integral((b*x^5*log((e*x + d)^n*c) + a*x^5)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^5}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(f^2/(g^4*x^2 + f*g^3) - x^2/g^2 + 2*f*log(g*x^2 + f)/g^3) + b*integ
rate((x^5*log((e*x + d)^n) + x^5*log(c))/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

## Giac [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^5}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*x^5/(g*x^2 + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{x^5\,(a+b\,\ln(c\,(d+e\,x)^n))}{(g\,x^2+f)^2}\,dx$$

```
[In] int((x^5*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2,x)
```

```
[Out] int((x^5*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2, x)
```

# 3.267 $\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 344

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = -\frac{bde\sqrt{f}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2g^{3/2}(e^2f+d^2g)} - \frac{be^2fn\log(d+ex)}{2g^2(e^2f+d^2g)} + \frac{f(a+b\log(c(d+ex)^n))}{2g^2(f+gx^2)} + \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} + \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} + \frac{be^2fn\log(f+gx^2)}{4g^2(e^2f+d^2g)} + \frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} + \frac{bn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}$$

```
[Out] -1/2*b*e^2*f*n*ln(e*x+d)/g^2/(d^2*g+e^2*f)+1/2*f*(a+b*ln(c*(e*x+d)^n))/g^2/
(g*x^2+f)+1/4*b*e^2*f*n*ln(g*x^2+f)/g^2/(d^2*g+e^2*f)+1/2*(a+b*ln(c*(e*x+d)
^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/g^2+1/2*(a+b*ln(
c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^2+1/2
*b*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^2+1/2*b*n*polyl
og(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^2-1/2*b*d*e*n*arctan(x*g^(
1/2)/f^(1/2))*f^(1/2)/g^(3/2)/(d^2*g+e^2*f)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {272, 45, 2463, 2460, 720, 31, 649, 211, 266, 2441, 2440, 2438}

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \frac{f(a+b\log(c(d+ex)^n))}{2g^2(f+gx^2)} + \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g^2} + \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g^2} - \frac{bde\sqrt{f}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2g^{3/2}(d^2g+e^2f)} + \frac{be^2fn\log(f+gx^2)}{4g^2(d^2g+e^2f)} - \frac{be^2fn\log(d+ex)}{2g^2(d^2g+e^2f)} + \frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} + \frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^2}$$

```
[In] Int[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

```
[Out] -1/2*(b*d*e*Sqrt[f]*n*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(g^(3/2)*(e^2*f + d^2*g)
) - (b*e^2*f*n*Log[d + e*x])/(2*g^2*(e^2*f + d^2*g)) + (f*(a + b*Log[c*(d +
 e*x)^n]))/(2*g^2*(f + g*x^2)) + ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-
f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^2) + ((a + b*Log[c*(d + e*
x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^2) +
(b*e^2*f*n*Log[f + g*x^2])/(4*g^2*(e^2*f + d^2*g)) + (b*n*PolyLog[2, -((Sqr
t[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*g^2) + (b*n*PolyLog[2, (Sqrt
[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{fx(a+b\log(c(d+ex)^n))}{g(f+gx^2)^2}+\frac{x(a+b\log(c(d+ex)^n))}{g(f+gx^2)}\right)dx$$

$$=\frac{\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx}{g}-\frac{f\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx}{g}$$

$$=\frac{f(a+b\log(c(d+ex)^n))}{2g^2(f+gx^2)}+\frac{\int\left(-\frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)}+\frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g}-\frac{(befn)\int\frac{1}{(d+ex)(f+gx^2)}\,dx}{2g^2}$$

$$=\frac{f(a+b\log(c(d+ex)^n))}{2g^2(f+gx^2)}-\frac{\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2g^{3/2}}+\frac{\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2g^{3/2}}-\frac{(befn)\int\frac{dg-egx}{f+gx^2}\,dx}{2g^2(e^2f+d^2g)}-\frac{(be^3fn)\int\frac{1}{d+ex}\,dx}{2g^2(e^2f+d^2g)}$$

$$= -\frac{be^2fn\log(d+ex)}{2g^2\left(e^2f+d^2g\right)} + \frac{f(a+b\log\left(c(d+ex)^n\right))}{2g^2\left(f+gx^2\right)} + \frac{(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} + \frac{(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} - \frac{(ben)\int\frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}dx}{2g^2} - \frac{(ben)\int\frac{\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}dx}{2g^2} - \frac{(bdefn)\int\frac{1}{f+gx^2}dx}{2g\left(e^2f+d^2g\right)} + \frac{(be^2fn)\int\frac{x}{f+gx^2}dx}{2g\left(e^2f+d^2g\right)}$$

$$= -\frac{bde\sqrt{f}n\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2g^{3/2}\left(e^2f+d^2g\right)} - \frac{be^2fn\log(d+ex)}{2g^2\left(e^2f+d^2g\right)} + \frac{f(a+b\log\left(c(d+ex)^n\right))}{2g^2\left(f+gx^2\right)} + \frac{(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} + \frac{(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} + \frac{be^2fn\log\left(f+gx^2\right)}{4g^2\left(e^2f+d^2g\right)} - \frac{(bn)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2g^2} - \frac{(bn)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2g^2}$$

$$= -\frac{bde\sqrt{f}n\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2g^{3/2}\left(e^2f+d^2g\right)} - \frac{be^2fn\log(d+ex)}{2g^2\left(e^2f+d^2g\right)} + \frac{f(a+b\log\left(c(d+ex)^n\right))}{2g^2\left(f+gx^2\right)} + \frac{(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} + \frac{(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} + \frac{be^2fn\log\left(f+gx^2\right)}{4g^2\left(e^2f+d^2g\right)} + \frac{bn\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} + \frac{bn\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.32

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$$
$$=\frac{\frac{2f(a-bn\log(d+ex)+b\log(c(d+ex)^n))}{f+gx^2}+2(a-bn\log(d+ex)+b\log(c(d+ex)^n))\log(f+gx^2)+bn\left(\frac{\sqrt{f}(-i\sqrt{g}(d+ex)}{(}\right.}{}$$

[In] Integrate[(x^3*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]

[Out] ((2*f*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]))/(f + g*x^2) + 2*(a - b *n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*Log[f + g*x^2] + b*n*((Sqrt[f]*((-I )*Sqrt[g]*(d + e*x)*Log[d + e*x] + e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (Sqrt[ f]*(I*Sqrt[g]*(d + e*x)*Log[d + e*x] + e*(Sqrt[f] - I*Sqrt[g]*x)*Log[I*Sqrt [f] + Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) + 2* (Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + 2*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog [2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])])))/(4*g^2)

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.18 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.44

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)f}{2g^2(g\,x^2+f)}+\frac{b\ln((ex+d)^n)\ln(g\,x^2+f)}{2g^2}-\frac{bn\ln(ex+d)\ln(g\,x^2+f)}{2g^2}+\frac{bn\ln(ex+d)\ln\left(\frac{e\sqrt{-fg}-g(ex+d)+dg}{e\sqrt{-fg}+dg}\right)}{2g^2}+\frac{bn\ln(ex+d)\,\mathrm{l}}{}$ |

[In] int(x^3*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*b*ln((e*x+d)^n)*f/g^2/(g*x^2+f)+1/2*b*ln((e*x+d)^n)/g^2*ln(g*x^2+f)-1/2 *b*n/g^2*ln(e*x+d)*ln(g*x^2+f)+1/2*b*n/g^2*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*( e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n/g^2*ln(e*x+d)*ln((e*(-f*g)^(1/2)+ g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*n/g^2*dilog((e*(-f*g)^(1/2)-g*(e *x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/2*b*n/g^2*dilog((e*(-f*g)^(1/2)+g*(e*x+d )-d*g)/(e*(-f*g)^(1/2)-d*g))-1/2*b*e^2*f*n*ln(e*x+d)/g^2/(d^2*g+e^2*f)+1/4* b*e^2*f*n*ln(g*x^2+f)/g^2/(d^2*g+e^2*f)-1/2*b*e*n*f/g/(d^2*g+e^2*f)*d/(f*g) ^(1/2)*arctan(g*x/(f*g)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csg n(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn

```
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c
)+a)*(1/2*f/g^2/(g*x^2+f)+1/2/g^2*ln(g*x^2+f))
```

## Fricas [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^3}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral((b*x^3*log((e*x + d)^n*c) + a*x^3)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^3}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="maxima")
[Out] 1/2*a*(f/(g^3*x^2 + f*g^2) + log(g*x^2 + f)/g^2) + b*integrate((x^3*log((e*
x + d)^n) + x^3*log(c))/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

## Giac [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^3}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*x^3/(g*x^2 + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx=\int \frac{x^3\,(a+b\,\ln(c\,(d+e\,x)^n))}{(g\,x^2+f)^2}\,dx$$

```
[In] int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2,x)
[Out] int((x^3*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2, x)
```

# 3.268 $\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \frac{bden\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}\,(e^2f+d^2g)}+\frac{be^2n\log(d+ex)}{2g\,(e^2f+d^2g)} - \frac{a+b\log(c(d+ex)^n)}{2g\,(f+gx^2)}-\frac{be^2n\log(f+gx^2)}{4g\,(e^2f+d^2g)}$$

```
[Out] 1/2*b*e^2*n*ln(e*x+d)/g/(d^2*g+e^2*f)+1/2*(-a-b*ln(c*(e*x+d)^n))/g/(g*x^2+f
)-1/4*b*e^2*n*ln(g*x^2+f)/g/(d^2*g+e^2*f)+1/2*b*d*e*n*arctan(x*g^(1/2)/f^(1
/2))/(d^2*g+e^2*f)/f^(1/2)/g^(1/2)
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2460, 720, 31, 649, 211, 266}

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = -\frac{a+b\log(c(d+ex)^n)}{2g\,(f+gx^2)}+\frac{bden\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}\,(d^2g+e^2f)} - \frac{be^2n\log(f+gx^2)}{4g\,(d^2g+e^2f)}+\frac{be^2n\log(d+ex)}{2g\,(d^2g+e^2f)}$$

```
[In] Int[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

```
[Out] (b*d*e*n*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(2*Sqrt[f]*Sqrt[g]*(e^2*f + d^2*g)) +
 (b*e^2*n*Log[d + e*x])/(2*g*(e^2*f + d^2*g)) - (a + b*Log[c*(d + e*x)^n])/
(2*g*(f + g*x^2)) - (b*e^2*n*Log[f + g*x^2])/(4*g*(e^2*f + d^2*g))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{a+b\log\left(c(d+ex)^n\right)}{2g\left(f+gx^2\right)}+\frac{(ben)\int\frac{1}{(d+ex)(f+gx^2)}\,dx}{2g}$$

$$= -\frac{a+b\log\left(c(d+ex)^n\right)}{2g\left(f+gx^2\right)}+\frac{(ben)\int\frac{dg-egx}{f+gx^2}\,dx}{2g\left(e^2f+d^2g\right)}+\frac{\left(be^3n\right)\int\frac{1}{d+ex}\,dx}{2g\left(e^2f+d^2g\right)}$$

$$= \frac{be^2n\log(d+ex)}{2g\left(e^2f+d^2g\right)}-\frac{a+b\log\left(c(d+ex)^n\right)}{2g\left(f+gx^2\right)}+\frac{(bden)\int\frac{1}{f+gx^2}\,dx}{2\left(e^2f+d^2g\right)}-\frac{\left(be^2n\right)\int\frac{x}{f+gx^2}\,dx}{2\left(e^2f+d^2g\right)}$$

$$= \frac{bden \tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}\,(e^2f+d^2g)} + \frac{be^2n\log(d+ex)}{2g\,(e^2f+d^2g)} - \frac{a+b\log\left(c(d+ex)^n\right)}{2g\,(f+gx^2)} - \frac{be^2n\log\left(f+gx^2\right)}{4g\,(e^2f+d^2g)}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.19

$$\int \frac{x(a+b\log\left(c(d+ex)^n\right))}{\left(f+gx^2\right)^2}\,dx$$

$$= \frac{2bde\sqrt{g}n(f+gx^2)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right) - \sqrt{f}(2ae^2f+2ad^2g-2be^2n(f+gx^2)\log(d+ex)+2b(e^2f+d^2g)\log}{4\sqrt{f}g\,(e^2f+d^2g)\,(f+gx^2)}$$

```
[In] Integrate[(x*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

```
[Out] (2*b*d*e*Sqrt[g]*n*(f + g*x^2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]] - Sqrt[f]*(2*a*e
^2*f + 2*a*d^2*g - 2*b*e^2*n*(f + g*x^2)*Log[d + e*x] + 2*b*(e^2*f + d^2*g)
*Log[c*(d + e*x)^n] + b*e^2*f*n*Log[f + g*x^2] + b*e^2*g*n*x^2*Log[f + g*x^
2]))/(4*Sqrt[f]*g*(e^2*f + d^2*g)*(f + g*x^2))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.36 (sec) , antiderivative size = 969, normalized size of antiderivative = 6.97

| method | result |
|---|---|
| risch | $-\frac{b\ln((ex+d)^n)}{2g(g\,x^2+f)} - \frac{i\pi b\,e^2f^2\,\mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)^2+i\pi b\,e^2f^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic(ex+d)^n)^2+ifg\pi b\,d^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic(ex-}{}$ |

```
[In] int(x*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/g/(g*x^2+f)*ln((e*x+d)^n)-1/4/f*(I*Pi*b*e^2*f^2*csgn(I*(e*x+d)^n)*cs
gn(I*c*(e*x+d)^n)^2+I*Pi*b*e^2*f^2*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+I*f*g*Pi
*b*d^2*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2-I*f*g*Pi*b*d^2*csgn(I*c)*csgn(I*(e*x
+d)^n)*csgn(I*c*(e*x+d)^n)-I*f*g*Pi*b*d^2*csgn(I*c*(e*x+d)^n)^3+I*f*g*Pi*b*
d^2*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I*Pi*b*e^2*f^2*csgn(I*c)*csgn(I
*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)-I*Pi*b*e^2*f^2*csgn(I*c*(e*x+d)^n)^3-ln((-(
-f*g)^(1/2)*d^2*g+3*(-f*g)^(1/2)*e^2*f+4*d*e*f*g)*x+4*(-f*g)^(1/2)*d*e*f+d^
2*f*g-3*e^2*f^2)*(-f*g)^(1/2)*b*d*e*g*n*x^2+ln(((-f*g)^(1/2)*d^2*g-3*(-f*g)
^(1/2)*e^2*f+4*d*e*f*g)*x-4*(-f*g)^(1/2)*d*e*f+d^2*f*g-3*e^2*f^2)*(-f*g)^(1
/2)*b*d*e*g*n*x^2+ln((-(-f*g)^(1/2)*d^2*g+3*(-f*g)^(1/2)*e^2*f+4*d*e*f*g)*x
+4*(-f*g)^(1/2)*d*e*f+d^2*f*g-3*e^2*f^2)*b*e^2*f*g*n*x^2+ln(((-f*g)^(1/2)*d
^2*g-3*(-f*g)^(1/2)*e^2*f+4*d*e*f*g)*x-4*(-f*g)^(1/2)*d*e*f+d^2*f*g-3*e^2*f
^2)*b*e^2*f*g*n*x^2-2*ln(e*x+d)*b*e^2*f*g*n*x^2-ln((-(-f*g)^(1/2)*d^2*g+3*(
```

```
-f*g)^(1/2)*e^2*f+4*d*e*f*g)*x+4*(-f*g)^(1/2)*d*e*f+d^2*f*g-3*e^2*f^2)*(-f*
g)^(1/2)*b*d*e*f*n+ln(((-f*g)^(1/2)*d^2*g-3*(-f*g)^(1/2)*e^2*f+4*d*e*f*g)*x
-4*(-f*g)^(1/2)*d*e*f+d^2*f*g-3*e^2*f^2)*(-f*g)^(1/2)*b*d*e*f*n+ln((-(-f*g)
^(1/2)*d^2*g+3*(-f*g)^(1/2)*e^2*f+4*d*e*f*g)*x+4*(-f*g)^(1/2)*d*e*f+d^2*f*g
-3*e^2*f^2)*b*e^2*f^2*n+ln(((-f*g)^(1/2)*d^2*g-3*(-f*g)^(1/2)*e^2*f+4*d*e*f
*g)*x-4*(-f*g)^(1/2)*d*e*f+d^2*f*g-3*e^2*f^2)*b*e^2*f^2*n-2*b*e^2*f^2*n*ln(
e*x+d)+2*ln(c)*b*d^2*f*g+2*ln(c)*b*e^2*f^2+2*a*d^2*f*g+2*a*e^2*f^2)/(g*x^2+
f)/(d*g-e*(-f*g)^(1/2))/(e*(-f*g)^(1/2)+d*g)
```

## Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.68

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$$
$$= \left[-\frac{2\,ae^2f^2+2\,ad^2fg+(bdegnx^2+bdefn)\sqrt{-fg}\log\left(\frac{gx^2-2\sqrt{-fg}x-f}{gx^2+f}\right)+(be^2fgnx^2+be^2f^2n)\log(gx^2+f}{4\,(e^2f^3g+d^2f^2g^2+(e^2f^2g^2+d^2fg^3)x}\right.$$
$$-\frac{2\,ae^2f^2+2\,ad^2fg-2\,(bdegnx^2+bdefn)\sqrt{fg}\arctan\left(\frac{\sqrt{fg}x}{f}\right)+(be^2fgnx^2+be^2f^2n)\log(gx^2+f)-2\,(}{4\,(e^2f^3g+d^2f^2g^2+(e^2f^2g^2+d^2fg^3)x^2)}$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*e^2*f^2 + 2*a*d^2*f*g + (b*d*e*g*n*x^2 + b*d*e*f*n)*sqrt(-f*g)*l
og((g*x^2 - 2*sqrt(-f*g)*x - f)/(g*x^2 + f)) + (b*e^2*f*g*n*x^2 + b*e^2*f^2
*n)*log(g*x^2 + f) - 2*(b*e^2*f*g*n*x^2 - b*d^2*f*g*n)*log(e*x + d) + 2*(b*
e^2*f^2 + b*d^2*f*g)*log(c))/(e^2*f^3*g + d^2*f^2*g^2 + (e^2*f^2*g^2 + d^2*
f*g^3)*x^2), -1/4*(2*a*e^2*f^2 + 2*a*d^2*f*g - 2*(b*d*e*g*n*x^2 + b*d*e*f*n
)*sqrt(f*g)*arctan(sqrt(f*g)*x/f) + (b*e^2*f*g*n*x^2 + b*e^2*f^2*n)*log(g*x
^2 + f) - 2*(b*e^2*f*g*n*x^2 - b*d^2*f*g*n)*log(e*x + d) + 2*(b*e^2*f^2 + b
*d^2*f*g)*log(c))/(e^2*f^3*g + d^2*f^2*g^2 + (e^2*f^2*g^2 + d^2*f*g^3)*x^2)
]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\log (c(d+ex)^n))}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \frac{x(a+b\log (c(d+ex)^n))}{(f+gx^2)^2}\,dx = -\frac{1}{4}\,ben\left(\frac{e\log (gx^2+f)}{e^2fg+d^2g^2}-\frac{2\,e\log (ex+d)}{e^2fg+d^2g^2}-\frac{2\,d\arctan \left(\frac{gx}{\sqrt{fg}}\right)}{(e^2f+d^2g)\sqrt{fg}}\right) - \frac{b\log ((ex+d)^nc)}{2\,(g^2x^2+fg)}-\frac{a}{2\,(g^2x^2+fg)}$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="maxima")
[Out] -1/4*b*e*n*(e*log(g*x^2 + f)/(e^2*f*g + d^2*g^2) - 2*e*log(e*x + d)/(e^2*f*
g + d^2*g^2) - 2*d*arctan(g*x/sqrt(f*g))/((e^2*f + d^2*g)*sqrt(f*g))) - 1/2
*b*log((e*x + d)^n*c)/(g^2*x^2 + f*g) - 1/2*a/(g^2*x^2 + f*g)
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.40

$$\int \frac{x(a+b\log (c(d+ex)^n))}{(f+gx^2)^2}\,dx = -\frac{be^2n\log (gx^2+f)}{4\,(e^2fg+d^2g^2)}+\frac{be^2n\log (ex+d)}{2\,(e^2fg+d^2g^2)} + \frac{bden\arctan \left(\frac{gx}{\sqrt{fg}}\right)}{2\,(e^2f+d^2g)\sqrt{fg}}-\frac{bn\log (ex+d)}{2\,(g^2x^2+fg)}-\frac{b\log (c)+a}{g^2x^2+fg} - \frac{be^2f\log (c)+bd^2g\log (c)+ae^2f+ad^2g}{2\,(e^2f+d^2g)(gx^2+f)g}$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] -1/4*b*e^2*n*log(g*x^2 + f)/(e^2*f*g + d^2*g^2) + 1/2*b*e^2*n*log(e*x + d)/
(e^2*f*g + d^2*g^2) + 1/2*b*d*e*n*arctan(g*x/sqrt(f*g))/((e^2*f + d^2*g)*sq
rt(f*g)) - 1/2*b*n*log(e*x + d)/(g^2*x^2 + f*g) - (b*log(c) + a)/(g^2*x^2 +
 f*g) - 1/2*(b*e^2*f*log(c) + b*d^2*g*log(c) + a*e^2*f + a*d^2*g)/((e^2*f +
 d^2*g)*(g*x^2 + f)*g)
```

## Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.63

$$\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \frac{b\,e^2\,n\,\ln(d+e\,x)}{2\,d^2\,g^2+2\,f\,e^2\,g} - \frac{\ln\left(\frac{\left(b\,e^2\,f\,g\,n+b\,d\,e\,n\,\sqrt{-f\,g^3}\right)\left(x\left(2\,d^2\,e\,g^3-6\,e^3\,f\,g^2\right)-8\,d\,e^2\,f\,g^2\right)}{4\left(d^2\,f\,g^3+e^2\,f^2\,g^2\right)}+\frac{b\,d\,e^2\,g\,n}{2}+\frac{3\,b\,e^3\,g\,n\,x}{2}\right)\left(b\,e^2\,f\,g\,n+b\,d\,e\,n\,\sqrt{-f\,g^3}\right)}{4\left(d^2\,f\,g^3+e^2\,f^2\,g^2\right)} - \frac{\ln\left(\frac{\left(b\,e^2\,f\,g\,n-b\,d\,e\,n\,\sqrt{-f\,g^3}\right)\left(x\left(2\,d^2\,e\,g^3-6\,e^3\,f\,g^2\right)-8\,d\,e^2\,f\,g^2\right)}{4\left(d^2\,f\,g^3+e^2\,f^2\,g^2\right)}+\frac{b\,d\,e^2\,g\,n}{2}+\frac{3\,b\,e^3\,g\,n\,x}{2}\right)\left(b\,e^2\,f\,g\,n-b\,d\,e\,n\,\sqrt{-f\,g^3}\right)}{4\left(d^2\,f\,g^3+e^2\,f^2\,g^2\right)} - \frac{b\,\ln\left(c\,(d+e\,x)^n\right)}{2\,g\,\left(g\,x^2+f\right)} - \frac{a}{2\,g^2\,x^2+2\,f\,g}$$

```
[In] int((x*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2,x)
```

```
[Out] (b*e^2*n*log(d + e*x))/(2*d^2*g^2 + 2*e^2*f*g) - (log(((b*e^2*f*g*n + b*d*e
*n*(-f*g^3)^(1/2))*(x*(2*d^2*e*g^3 - 6*e^3*f*g^2) - 8*d*e^2*f*g^2))/(4*(d^2
*f*g^3 + e^2*f^2*g^2)) + (b*d*e^2*g*n)/2 + (3*b*e^3*g*n*x)/2)*(b*e^2*f*g*n
+ b*d*e*n*(-f*g^3)^(1/2)))/(4*(d^2*f*g^3 + e^2*f^2*g^2)) - (log(((b*e^2*f*g
*n - b*d*e*n*(-f*g^3)^(1/2))*(x*(2*d^2*e*g^3 - 6*e^3*f*g^2) - 8*d*e^2*f*g^2
))/(4*(d^2*f*g^3 + e^2*f^2*g^2)) + (b*d*e^2*g*n)/2 + (3*b*e^3*g*n*x)/2)*(b*
e^2*f*g*n - b*d*e*n*(-f*g^3)^(1/2)))/(4*(d^2*f*g^3 + e^2*f^2*g^2)) - (b*log
(c*(d + e*x)^n))/(2*g*(f + g*x^2)) - a/(2*f*g + 2*g^2*x^2)
```

# 3.269 $\int \frac{a+b\log(c(d+ex)^n)}{x(f+gx^2)^2}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 383

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx = -\frac{bde\sqrt{g}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2f^{3/2}\left(e^2f+d^2g\right)}-\frac{be^2n\log(d+ex)}{2f\left(e^2f+d^2g\right)}+\frac{a+b\log\left(c(d+ex)^n\right)}{2f\left(f+gx^2\right)}+\frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2}-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}+\frac{be^2n\log\left(f+gx^2\right)}{4f\left(e^2f+d^2g\right)}-\frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}-\frac{bn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}+\frac{bn\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^2}$$

```
[Out] -1/2*b*e^2*n*ln(e*x+d)/f/(d^2*g+e^2*f)+1/2*(a+b*ln(c*(e*x+d)^n))/f/(g*x^2+f
)+ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/f^2+1/4*b*e^2*n*ln(g*x^2+f)/f/(d^2*g+e^2
*f)-1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g
^(1/2)))/f^2-1/2*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^
(1/2)-d*g^(1/2)))/f^2+b*n*polylog(2,1+e*x/d)/f^2-1/2*b*n*polylog(2,-(e*x+d)
*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f^2-1/2*b*n*polylog(2,(e*x+d)*g^(1/2)/(e
*(-f)^(1/2)+d*g^(1/2)))/f^2-1/2*b*d*e*n*arctan(x*g^(1/2)/f^(1/2))*g^(1/2)/f
^(3/2)/(d^2*g+e^2*f)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {272, 46, 2463, 2441, 2352, 2460, 720, 31, 649, 211, 266, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx = -\frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{d\sqrt{g}+e\sqrt{-f}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f^2} - \frac{\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f^2} + \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} + \frac{a+b\log\left(c(d+ex)^n\right)}{2f\left(f+gx^2\right)} - \frac{bde\sqrt{g}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2f^{3/2}\left(d^2g+e^2f\right)} + \frac{be^2n\log\left(f+gx^2\right)}{4f\left(d^2g+e^2f\right)} - \frac{be^2n\log(d+ex)}{2f\left(d^2g+e^2f\right)} - \frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} - \frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2f^2} + \frac{bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x^2)^2),x]
```

```
[Out] -1/2*(b*d*e*Sqrt[g]*n*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(f^(3/2)*(e^2*f + d^2*g)
) - (b*e^2*n*Log[d + e*x])/(2*f*(e^2*f + d^2*g)) + (a + b*Log[c*(d + e*x)^n
])/(2*f*(f + g*x^2)) + (Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f^2 - (
(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*S
qrt[g])])/(2*f^2) - ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*
x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*f^2) + (b*e^2*n*Log[f + g*x^2])/(4*f*(e^2
*f + d^2*g)) - (b*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g
]))])/(2*f^2) - (b*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g]
)])/(2*f^2) + (b*n*PolyLog[2, 1 + (e*x)/d])/f^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x} - \frac{gx(a+b\log\left(c(d+ex)^n\right))}{f\left(f+gx^2\right)^2} - \frac{gx(a+b\log\left(c(d+ex)^n\right))}{f^2\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx}{f^2} - \frac{g\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx}{f^2} - \frac{g\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx}{f}$$

$$= \frac{a+b\log\left(c(d+ex)^n\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} - \frac{g\int\left(-\frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx}{f^2} - \frac{(ben)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{f^2} - \frac{(ben)\int\frac{1}{(d+ex)(f+gx^2)}\,dx}{2f}$$

$$= \frac{a+b\log\left(c(d+ex)^n\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^2} + \frac{bn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} + \frac{\sqrt{g}\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2f^2} - \frac{\sqrt{g}\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2f^2} - \frac{(ben)\int\frac{dg-egx}{f+gx^2}\,dx}{2f\left(e^2f+d^2g\right)} - \frac{\left(be^3n\right)\int\frac{1}{d+ex}\,dx}{2f\left(e^2f+d^2g\right)}$$

$$= -\frac{be^2 n\log(d+ex)}{2f(e^2f+d^2g)} + \frac{a+b\log(c(d+ex)^n)}{2f(f+gx^2)} + \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^2} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} + \frac{bn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} + \frac{(ben)\int \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2f^2} + \frac{(ben)\int \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2f^2} - \frac{(bdegn)\int \frac{1}{f+gx^2}\,dx}{2f(e^2f+d^2g)} + \frac{(be^2gn)\int \frac{x}{f+gx^2}\,dx}{2f(e^2f+d^2g)}$$

$$= -\frac{bde\sqrt{g}n\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2f^{3/2}(e^2f+d^2g)} - \frac{be^2n\log(d+ex)}{2f(e^2f+d^2g)} + \frac{a+b\log(c(d+ex)^n)}{2f(f+gx^2)} + \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^2} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} + \frac{be^2n\log(f+gx^2)}{4f(e^2f+d^2g)} + \frac{bn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} + \frac{(bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2f^2} + \frac{(bn)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2f^2}$$

$$= -\frac{bde\sqrt{g}n\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2f^{3/2}(e^2f+d^2g)} - \frac{be^2n\log(d+ex)}{2f(e^2f+d^2g)} + \frac{a+b\log(c(d+ex)^n)}{2f(f+gx^2)} + \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^2} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} - \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} + \frac{be^2n\log(f+gx^2)}{4f(e^2f+d^2g)} - \frac{bn\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} - \frac{bn\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} + \frac{bn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^2}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.36

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx$$
$$=\frac{a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)}{2f^2+2fgx^2}+\frac{\log(x)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)}{f^2}$$
$$-\frac{\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)\log\left(f+gx^2\right)}{2f^2}$$
$$+\frac{bn\left(\frac{\sqrt{f}\left(-i\sqrt{g}(d+ex)\log(d+ex)+e\left(\sqrt{f}+i\sqrt{g}x\right)\log\left(i\sqrt{f}-\sqrt{g}x\right)\right)}{\left(e\sqrt{f}-id\sqrt{g}\right)\left(\sqrt{f}+i\sqrt{g}x\right)}+\frac{\sqrt{f}\left(i\sqrt{g}(d+ex)\log(d+ex)+e\left(\sqrt{f}-i\sqrt{g}x\right)\log\left(i\sqrt{f}+\sqrt{g}x\right)\right)}{\left(e\sqrt{f}+id\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}-2\left(\log(\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(x*(f + g*x^2)^2),x]
```

```
[Out] (a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])/(2*f^2 + 2*f*g*x^2) + (Log[x]
*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]))/f^2 - ((a - b*n*Log[d + e*x
] + b*Log[c*(d + e*x)^n])*Log[f + g*x^2])/(2*f^2) + (b*n*((Sqrt[f]*((-I)*Sq
rt[g]*(d + e*x)*Log[d + e*x] + e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sq
rt[g]*x]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (Sqrt[f]*(
I*Sqrt[g]*(d + e*x)*Log[d + e*x] + e*(Sqrt[f] - I*Sqrt[g]*x)*Log[I*Sqrt[f]
+ Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) - 2*(Log
[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + Poly
Log[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) - 2*(Log[d + e*
x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2,
(I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]) + 4*(Log[-((e*x)/d)]*Log[
d + e*x] + PolyLog[2, 1 + (e*x)/d])))/(4*f^2)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.91 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.45

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)\ln(x)}{f^2}+\frac{b\ln((ex+d)^n)}{2f(g\,x^2+f)}-\frac{b\ln((ex+d)^n)\ln(g\,x^2+f)}{2f^2}-\frac{b\,e^2n\ln(ex+d)}{2f(d^2g+f\,e^2)}+\frac{b\,e^2n\ln(g\,x^2+f)}{4f(d^2g+f\,e^2)}-\frac{bengd\arctan\left(\frac{gx}{\sqrt{fg}}\right)}{2f(d^2g+f\,e^2)\sqrt{fg}}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/x/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*ln((e*x+d)^n)/f^2*ln(x)+1/2*b*ln((e*x+d)^n)/f/(g*x^2+f)-1/2*b*ln((e*x+d)^
n)/f^2*ln(g*x^2+f)-1/2*b*e^2*n*ln(e*x+d)/f/(d^2*g+e^2*f)+1/4*b*e^2*n*ln(g*x
```

```
^2+f)/f/(d^2*g+e^2*f)-1/2*b*e*n/f/(d^2*g+e^2*f)*g*d/(f*g)^(1/2)*arctan(g*x/
(f*g)^(1/2))-b*n/f^2*dilog((e*x+d)/d)-b*n/f^2*ln(x)*ln((e*x+d)/d)+1/2*b*n/f
^2*ln(e*x+d)*ln(g*x^2+f)-1/2*b*n/f^2*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)
+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n/f^2*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x
+d)-d*g)/(e*(-f*g)^(1/2)-d*g))-1/2*b*n/f^2*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+
d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n/f^2*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)
/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e
*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+
d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(1/
f^2*ln(x)-1/2*g/f^2*(-f/g/(g*x^2+f)+1/g*ln(g*x^2+f)))
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\left(gx^2+f\right)^2x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral((b*log((e*x + d)^n*c) + a)/(g^2*x^5 + 2*f*g*x^3 + f^2*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/x/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\left(gx^2+f\right)^2x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x/(g*x^2+f)^2,x, algorithm="maxima")
[Out] 1/2*a*(1/(f*g*x^2 + f^2) - log(g*x^2 + f)/f^2 + 2*log(x)/f^2) + b*integrate
((log((e*x + d)^n) + log(c))/(g^2*x^5 + 2*f*g*x^3 + f^2*x), x)
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx^2+f\right)^2 x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)^2*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x\left(f+gx^2\right)^2}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{x\,{\left(g\,x^2+f\right)}^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x^2)^2),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(x*(f + g*x^2)^2), x)
```

# 3.270 $\int \frac{a+b\log(c(d+ex)^n)}{x^3(f+gx^2)^2}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 460

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx = -\frac{ben}{2df^2x}+\frac{bdeg^{3/2}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2f^{5/2}\left(e^2f+d^2g\right)}-\frac{be^2n\log(x)}{2d^2f^2} +\frac{be^2n\log(d+ex)}{2d^2f^2}+\frac{be^2gn\log(d+ex)}{2f^2\left(e^2f+d^2g\right)} -\frac{a+b\log\left(c(d+ex)^n\right)}{2f^2x^2}-\frac{g(a+b\log\left(c(d+ex)^n\right))}{2f^2\left(f+gx^2\right)} -\frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3} +\frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} +\frac{g(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} -\frac{be^2gn\log\left(f+gx^2\right)}{4f^2\left(e^2f+d^2g\right)}+\frac{bgn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} +\frac{bgn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3}-\frac{2bgn\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^3}$$

```
[Out] -1/2*b*e*n/d/f^2/x+1/2*b*d*e*g^(3/2)*n*arctan(x*g^(1/2)/f^(1/2))/f^(5/2)/(d
^2*g+e^2*f)-1/2*b*e^2*n*ln(x)/d^2/f^2+1/2*b*e^2*n*ln(e*x+d)/d^2/f^2+1/2*b*e
^2*g*n*ln(e*x+d)/f^2/(d^2*g+e^2*f)+1/2*(-a-b*ln(c*(e*x+d)^n))/f^2/x^2-1/2*g
*(a+b*ln(c*(e*x+d)^n))/f^2/(g*x^2+f)-2*g*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/f
^3-1/4*b*e^2*g*n*ln(g*x^2+f)/f^2/(d^2*g+e^2*f)+g*(a+b*ln(c*(e*x+d)^n))*ln(e
```

```
*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f^3+g*(a+b*ln(c*(e*x+d)^n
))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/f^3-2*b*g*n*polylo
g(2,1+e*x/d)/f^3+b*g*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))
/f^3+b*g*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f^3
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {272, 46, 2463, 2442, 2441, 2352, 2460, 720, 31, 649, 211, 266, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx = -\frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3} + \frac{g\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{d\sqrt{g}+e\sqrt{-f}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3} + \frac{g\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{f^3} - \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f^2\left(f+gx^2\right)} - \frac{a+b\log\left(c(d+ex)^n\right)}{2f^2x^2} + \frac{bdeg^{3/2}n\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2f^{5/2}\left(d^2g+e^2f\right)} - \frac{be^2gn\log\left(f+gx^2\right)}{4f^2\left(d^2g+e^2f\right)} + \frac{be^2gn\log(d+ex)}{2f^2\left(d^2g+e^2f\right)} - \frac{be^2n\log(x)}{2d^2f^2} + \frac{be^2n\log(d+ex)}{2d^2f^2} + \frac{bgn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} + \frac{bgn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{f^3} - \frac{2bgn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^3} - \frac{ben}{2df^2x}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x^2)^2),x]

[Out] -1/2*(b*e*n)/(d*f^2*x) + (b*d*e*g^(3/2)*n*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(2*f
^(5/2)*(e^2*f + d^2*g)) - (b*e^2*n*Log[x])/(2*d^2*f^2) + (b*e^2*n*Log[d + e
*x])/(2*d^2*f^2) + (b*e^2*g*n*Log[d + e*x])/(2*f^2*(e^2*f + d^2*g)) - (a +
b*Log[c*(d + e*x)^n])/(2*f^2*x^2) - (g*(a + b*Log[c*(d + e*x)^n]))/(2*f^2*(
f + g*x^2)) - (2*g*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/f^3 + (g*(a
+ b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt
[g])])/f^3 + (g*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(
e*Sqrt[-f] - d*Sqrt[g])])/f^3 - (b*e^2*g*n*Log[f + g*x^2])/(4*f^2*(e^2*f +
d^2*g)) + (b*g*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))
])/f^3 + (b*g*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/f
^3 - (2*b*g*n*PolyLog[2, 1 + (e*x)/d])/f^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x^3} - \frac{2g(a+b\log\left(c(d+ex)^n\right))}{f^3x} + \frac{g^2x(a+b\log\left(c(d+ex)^n\right))}{f^2\left(f+gx^2\right)^2} + \frac{2g^2x(a+b\log\left(c(d+ex)^n\right))}{f^3\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\log(c(d+ex)^n)}{x^3}\,dx}{f^2} - \frac{(2g)\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx}{f^3} + \frac{(2g^2)\int \frac{x(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx}{f^3} + \frac{g^2\int \frac{x(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx}{f^2}$$

$$= -\frac{a+b\log(c(d+ex)^n)}{2f^2x^2} - \frac{g(a+b\log(c(d+ex)^n))}{2f^2(f+gx^2)} - \frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^3} + \frac{(2g^2)\int \left(-\frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)} + \frac{a+b\log(c(d+ex)^n)}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right)dx}{f^3} + \frac{(ben)\int \frac{1}{x^2(d+ex)}\,dx}{2f^2} + \frac{(2begn)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{f^3} + \frac{(begn)\int \frac{1}{(d+ex)(f+gx^2)}\,dx}{2f^2}$$

$$= -\frac{a+b\log(c(d+ex)^n)}{2f^2x^2} - \frac{g(a+b\log(c(d+ex)^n))}{2f^2(f+gx^2)} - \frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^3} - \frac{2bgn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^3} - \frac{g^{3/2}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{f^3} + \frac{g^{3/2}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{f^3} + \frac{(ben)\int \left(\frac{1}{dx^2} - \frac{e}{d^2x} + \frac{e^2}{d^2(d+ex)}\right)dx}{2f^2} + \frac{(begn)\int \frac{dg-egx}{f+gx^2}\,dx}{2f^2(e^2f+d^2g)} + \frac{(be^3gn)\int \frac{1}{d+ex}\,dx}{2f^2(e^2f+d^2g)}$$

$$= -\frac{ben}{2df^2x} - \frac{be^2n\log(x)}{2d^2f^2} + \frac{be^2n\log(d+ex)}{2d^2f^2} + \frac{be^2gn\log(d+ex)}{2f^2(e^2f+d^2g)} - \frac{a+b\log(c(d+ex)^n)}{2f^2x^2} - \frac{g(a+b\log(c(d+ex)^n))}{2f^2(f+gx^2)} - \frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^3} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} - \frac{2bgn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^3} - \frac{(begn)\int \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{f^3} - \frac{(begn)\int \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{f^3} + \frac{(bdeg^2n)\int \frac{1}{f+gx^2}\,dx}{2f^2(e^2f+d^2g)} - \frac{(be^2g^2n)\int \frac{x}{f+gx^2}\,dx}{2f^2(e^2f+d^2g)}$$

$$= -\frac{ben}{2df^2x} + \frac{bdeg^{3/2}n\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2f^{5/2}(e^2f+d^2g)} - \frac{be^2n\log(x)}{2d^2f^2} + \frac{be^2n\log(d+ex)}{2d^2f^2} + \frac{be^2gn\log(d+ex)}{2f^2(e^2f+d^2g)} - \frac{a+b\log(c(d+ex)^n)}{2f^2x^2} - \frac{g(a+b\log(c(d+ex)^n))}{2f^2(f+gx^2)} - \frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^3} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} - \frac{be^2gn\log(f+gx^2)}{4f^2(e^2f+d^2g)} - \frac{2bgn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^3} - \frac{(bgn)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{f^3} - \frac{(bgn)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{f^3}$$

$$= -\frac{ben}{2df^2x} + \frac{bdeg^{3/2}n\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2f^{5/2}(e^2f+d^2g)} - \frac{be^2n\log(x)}{2d^2f^2} + \frac{be^2n\log(d+ex)}{2d^2f^2} + \frac{be^2gn\log(d+ex)}{2f^2(e^2f+d^2g)} - \frac{a+b\log(c(d+ex)^n)}{2f^2x^2} - \frac{g(a+b\log(c(d+ex)^n))}{2f^2(f+gx^2)} - \frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{f^3} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} + \frac{g(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} - \frac{be^2gn\log(f+gx^2)}{4f^2(e^2f+d^2g)} + \frac{bgn\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} + \frac{bgn\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} - \frac{2bgn\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^3}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.30

$$\int\frac{a+b\log(c(d+ex)^n)}{x^3(f+gx^2)^2}dx$$
$$= \frac{-\frac{2f(a-bn\log(d+ex)+b\log(c(d+ex)^n))}{x^2} - \frac{2fg(a-bn\log(d+ex)+b\log(c(d+ex)^n))}{f+gx^2} - 8g\log(x)(a-bn\log(d+ex)+b\log(c(d}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(x^3*(f + g*x^2)^2),x]
```

```
[Out] ((-2*f*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]))/x^2 - (2*f*g*(a - b*n
*Log[d + e*x] + b*Log[c*(d + e*x)^n]))/(f + g*x^2) - 8*g*Log[x]*(a - b*n*Lo
g[d + e*x] + b*Log[c*(d + e*x)^n]) + 4*g*(a - b*n*Log[d + e*x] + b*Log[c*(d
 + e*x)^n])*Log[f + g*x^2] + b*n*((-2*f*(d*e*x + e^2*x^2*Log[x] + (d^2 - e^
2*x^2)*Log[d + e*x]))/(d^2*x^2) + (I*Sqrt[f]*g*(Sqrt[g]*(d + e*x)*Log[d + e
*x] + I*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt[f]
- I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (I*Sqrt[f]*g*(-(Sqrt[g]*(d + e*x)
*Log[d + e*x]) + e*(I*Sqrt[f] + Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*x]))/((e
*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) + 4*g*(Log[d + e*x]*Log[(e
*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqr
t[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + 4*g*(Log[d + e*x]*Log[(e*(Sqr
t[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d
+ e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]) - 8*g*(Log[-((e*x)/d)]*Log[d + e*x] + P
olyLog[2, 1 + (e*x)/d])))/(4*f^3)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.91 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| risch | $-\frac{b\ln((ex+d)^n)}{2f^2x^2}-\frac{2b\ln((ex+d)^n)g\ln(x)}{f^3}-\frac{b\ln((ex+d)^n)g}{2f^2(g\,x^2+f)}+\frac{b\ln((ex+d)^n)g\ln(g\,x^2+f)}{f^3}+\frac{2bng\operatorname{dilog}\left(\frac{ex+d}{d}\right)}{f^3}+\frac{2bng\ln(x}{}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/x^3/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*ln((e*x+d)^n)/f^2/x^2-2*b*ln((e*x+d)^n)/f^3*g*ln(x)-1/2*b*ln((e*x+d)
^n)*g/f^2/(g*x^2+f)+b*ln((e*x+d)^n)*g/f^3*ln(g*x^2+f)+2*b*n/f^3*g*dilog((e*
x+d)/d)+2*b*n/f^3*g*ln(x)*ln((e*x+d)/d)-b*n/f^3*g*ln(e*x+d)*ln(g*x^2+f)+b*n
/f^3*g*ln(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+b*
n/f^3*g*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+b
*n/f^3*g*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+b*n/f^3
*g*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+b*e^2*g*n*ln(
e*x+d)/f^2/(d^2*g+e^2*f)+1/2*b*e^4*n/f/(d^2*g+e^2*f)/d^2*ln(e*x+d)-1/2*b*e*
n/d/f^2/x-1/2*b*e^2*n*ln(x)/d^2/f^2-1/4*b*e^2*g*n*ln(g*x^2+f)/f^2/(d^2*g+e^
2*f)+1/2*b*e*n/f^2/(d^2*g+e^2*f)*g^2*d/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2))+
(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csg
n(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^
n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(-1/2/f^2/x^2-2/f^3*g*ln(x
)+1/2*g^2/f^3*(-f/g/(g*x^2+f)+2/g*ln(g*x^2+f)))
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx^2+f\right)^2 x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral((b*log((e*x + d)^n*c) + a)/(g^2*x^7 + 2*f*g*x^5 + f^2*x^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/x**3/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx^2+f\right)^2 x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x^2+f)^2,x, algorithm="maxima")
[Out] -1/2*a*((2*g*x^2 + f)/(f^2*g*x^4 + f^3*x^2) - 2*g*log(g*x^2 + f)/f^3 + 4*g*
log(x)/f^3) + b*integrate((log((e*x + d)^n) + log(c))/(g^2*x^7 + 2*f*g*x^5
+ f^2*x^3), x)
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx^2+f\right)^2 x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^3/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)^2*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3\left(f+gx^2\right)^2}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{x^3\,{\left(g\,x^2+f\right)}^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x^2)^2),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g*x^2)^2), x)
```

# 3.271 $\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2} dx$



## Optimal result

Integrand size = 27, antiderivative size = 534

$$\begin{aligned}\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx &= \frac{ax}{g^2}-\frac{bnx}{g^2}-\frac{befn\log(d+ex)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}+\frac{befn\log(d+ex)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}\\
&+\frac{b(d+ex)\log\left(c(d+ex)^n\right)}{eg^2}-\frac{f(a+b\log\left(c(d+ex)^n\right))}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)}\\
&+\frac{f(a+b\log\left(c(d+ex)^n\right))}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{befn\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}\\
&+\frac{3\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}}\\
&+\frac{befn\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}\\
&-\frac{3\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}\\
&-\frac{3b\sqrt{-f}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}\\
&+\frac{3b\sqrt{-f}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}}\end{aligned}$$

```
[Out] a*x/g^2-b*n*x/g^2+b*(e*x+d)*ln(c*(e*x+d)^n)/e/g^2+3/4*(a+b*ln(c*(e*x+d)^n))
*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(5/2)-3
/4*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2
)))*(-f)^(1/2)/g^(5/2)-3/4*b*n*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g
^(1/2)))*(-f)^(1/2)/g^(5/2)+3/4*b*n*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)
```

```
+d*g^(1/2)))*(-f)^(1/2)/g^(5/2)-1/4*b*e*f*n*ln(e*x+d)/g^(5/2)/(e*(-f)^(1/2)
-d*g^(1/2))+1/4*b*e*f*n*ln((-f)^(1/2)+x*g^(1/2))/g^(5/2)/(e*(-f)^(1/2)-d*g^
(1/2))+1/4*b*e*f*n*ln(e*x+d)/g^(5/2)/(e*(-f)^(1/2)+d*g^(1/2))-1/4*b*e*f*n*l
n((-f)^(1/2)-x*g^(1/2))/g^(5/2)/(e*(-f)^(1/2)+d*g^(1/2))-1/4*f*(a+b*ln(c*(e
*x+d)^n))/g^(5/2)/((-f)^(1/2)-x*g^(1/2))+1/4*f*(a+b*ln(c*(e*x+d)^n))/g^(5/2
)/((-f)^(1/2)+x*g^(1/2))
```

## Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {294, 327, 211, 2463, 2436, 2332, 2456, 2442, 36, 31, 2441, 2440, 2438}

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = -\frac{f(a+b\log(c(d+ex)^n))}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{f(a+b\log(c(d+ex)^n))}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{3\sqrt{-f}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{4g^{5/2}}-\frac{3\sqrt{-f}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{4g^{5/2}}+\frac{ax}{g^2}+\frac{b(d+ex)\log(c(d+ex)^n)}{eg^2}-\frac{3b\sqrt{-f}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}+\frac{3b\sqrt{-f}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4g^{5/2}}-\frac{befn\log(d+ex)}{4g^{5/2}\left(e\sqrt{-f}-d\sqrt{g}\right)}+\frac{befn\log(d+ex)}{4g^{5/2}\left(d\sqrt{g}+e\sqrt{-f}\right)}-\frac{befn\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4g^{5/2}\left(d\sqrt{g}+e\sqrt{-f}\right)}+\frac{befn\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4g^{5/2}\left(e\sqrt{-f}-d\sqrt{g}\right)}-\frac{bnx}{g^2}$$

```
[In] Int[(x^4*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

```
[Out] (a*x)/g^2 - (b*n*x)/g^2 - (b*e*f*n*Log[d + e*x])/(4*(e*Sqrt[-f] - d*Sqrt[g]
)*g^(5/2)) + (b*e*f*n*Log[d + e*x])/(4*(e*Sqrt[-f] + d*Sqrt[g])*g^(5/2)) +
(b*(d + e*x)*Log[c*(d + e*x)^n])/(e*g^2) - (f*(a + b*Log[c*(d + e*x)^n]))/(
4*g^(5/2)*(Sqrt[-f] - Sqrt[g]*x)) + (f*(a + b*Log[c*(d + e*x)^n]))/(4*g^(5/
2)*(Sqrt[-f] + Sqrt[g]*x)) - (b*e*f*n*Log[Sqrt[-f] - Sqrt[g]*x])/(4*(e*Sqrt
[-f] + d*Sqrt[g])*g^(5/2)) + (3*Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])*Log[(e*
```

```
(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*g^(5/2)) + (b*e*f*n*L
og[Sqrt[-f] + Sqrt[g]*x])/(4*(e*Sqrt[-f] - d*Sqrt[g])*g^(5/2)) - (3*Sqrt[-f
]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d
*Sqrt[g])])/(4*g^(5/2)) - (3*b*Sqrt[-f]*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/
(e*Sqrt[-f] - d*Sqrt[g]))])/(4*g^(5/2)) + (3*b*Sqrt[-f]*n*PolyLog[2, (Sqrt[
g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*g^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{g^2}+\frac{f^2(a+b\log\left(c(d+ex)^n\right))}{g^2\left(f+gx^2\right)^2}-\frac{2f(a+b\log\left(c(d+ex)^n\right))}{g^2\left(f+gx^2\right)}\right)dx$$

$$=\frac{\int \left(a+b\log\left(c(d+ex)^n\right)\right)dx}{g^2}-\frac{(2f)\int \frac{a+b\log(c(d+ex)^n)}{f+gx^2}dx}{g^2}+\frac{f^2\int \frac{a+b\log(c(d+ex)^n)}{(f+gx^2)^2}dx}{g^2}$$

$$=\frac{ax}{g^2}+\frac{b\int \log\left(c(d+ex)^n\right)dx}{g^2}-\frac{(2f)\int \left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}-\sqrt{g}x)}+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g^2}+\frac{f^2\int \left(-\frac{g(a+b\log(c(d+ex)^n))}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}-\frac{g(a+b\log(c(d+ex)^n))}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2}-\frac{g(a+b\log(c(d+ex)^n))}{2f(-fg-g^2x^2)}\right)dx}{g^2}$$

$$=\frac{ax}{g^2}+\frac{b\text{Subst}\left(\int \log\left(cx^n\right)dx,x,d+ex\right)}{eg^2}-\frac{\sqrt{-f}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}dx}{g^2}-\frac{\sqrt{-f}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}dx}{g^2}-\frac{f\int \frac{a+b\log(c(d+ex)^n)}{\left(\sqrt{-f}\sqrt{g}-gx\right)^2}dx}{4g}-\frac{f\int \frac{a+b\log(c(d+ex)^n)}{\left(\sqrt{-f}\sqrt{g}+gx\right)^2}dx}{4g}-\frac{f\int \frac{a+b\log(c(d+ex)^n)}{-fg-g^2x^2}dx}{2g}$$

$$=\frac{ax}{g^2}-\frac{bnx}{g^2}+\frac{b(d+ex)\log\left(c(d+ex)^n\right)}{eg^2}-\frac{f(a+b\log\left(c(d+ex)^n\right))}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{f(a+b\log\left(c(d+ex)^n\right))}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}}-\frac{\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}}-\frac{f\int \left(-\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2fg(\sqrt{-f}-\sqrt{g}x)}-\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2fg(\sqrt{-f}+\sqrt{g}x)}\right)dx}{2g}-\frac{\left(be\sqrt{-f}n\right)\int \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}dx}{g^{5/2}}+\frac{\left(be\sqrt{-f}n\right)\int \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}dx}{g^{5/2}}+\frac{(befn)\int \frac{1}{(d+ex)\left(\sqrt{-f}\sqrt{g}-gx\right)}dx}{4g^2}-\frac{(befn)\int \frac{1}{(d+ex)\left(\sqrt{-f}\sqrt{g}+gx\right)}dx}{4g^2}$$

$$
\begin{aligned}
&= \frac{ax}{g^2} - \frac{bnx}{g^2} + \frac{b(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} - \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)} \\
&+ \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}} \\
&- \frac{\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}} + \frac{\sqrt{-f}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{4g^2} \\
&+ \frac{\sqrt{-f}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{4g^2} + \frac{\left(b\sqrt{-f}n\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{g^{5/2}} \\
&- \frac{\left(b\sqrt{-f}n\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{g^{5/2}} - \frac{\left(be^2fn\right)\int \frac{1}{d+ex}\,dx}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}} \\
&+ \frac{\left(be^2fn\right)\int \frac{1}{d+ex}\,dx}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}} + \frac{(befn)\int \frac{1}{\sqrt{-f}\sqrt{g}+gx}\,dx}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}} + \frac{(befn)\int \frac{1}{\sqrt{-f}\sqrt{g}-gx}\,dx}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ax}{g^2} - \frac{bnx}{g^2} - \frac{befn\log(d+ex)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}} + \frac{befn\log(d+ex)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}} \\
&+ \frac{b(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} - \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&- \frac{befn\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}} + \frac{3\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}} \\
&+ \frac{befn\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}} - \frac{3\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}} \\
&- \frac{b\sqrt{-f}n\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}} + \frac{b\sqrt{-f}n\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}} \\
&+ \frac{\left(be\sqrt{-f}n\right)\int \frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{4g^{5/2}} - \frac{\left(be\sqrt{-f}n\right)\int \frac{\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{4g^{5/2}}
\end{aligned}
$$

$$=\frac{ax}{g^2}-\frac{bnx}{g^2}-\frac{befn\log(d+ex)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}+\frac{befn\log(d+ex)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}+\frac{b(d+ex)\log\left(c(d+ex)^n\right)}{eg^2}-\frac{f(a+b\log\left(c(d+ex)^n\right))}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{f(a+b\log\left(c(d+ex)^n\right))}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{befn\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}+\frac{3\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}}+\frac{befn\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}-\frac{3\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}-\frac{b\sqrt{-f}n\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}}+\frac{b\sqrt{-f}n\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}}-\frac{\left(b\sqrt{-f}n\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{4g^{5/2}}+\frac{\left(b\sqrt{-f}n\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{4g^{5/2}}$$

$$=\frac{ax}{g^2}-\frac{bnx}{g^2}-\frac{befn\log(d+ex)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}+\frac{befn\log(d+ex)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}+\frac{b(d+ex)\log\left(c(d+ex)^n\right)}{eg^2}-\frac{f(a+b\log\left(c(d+ex)^n\right))}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{f(a+b\log\left(c(d+ex)^n\right))}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{befn\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}+\frac{3\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}}+\frac{befn\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}-\frac{3\sqrt{-f}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}-\frac{3b\sqrt{-f}n\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}+\frac{3b\sqrt{-f}n\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}}$$

## Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.81

$$\int\frac{x^4(a+b\log\left(c(d+ex)^n\right))}{\left(f+gx^2\right)^2}\,dx$$
$$=\frac{4a\sqrt{g}x-4b\sqrt{g}nx+\frac{4b\sqrt{g}(d+ex)\log(c(d+ex)^n)}{e}-\frac{f(a+b\log(c(d+ex)^n))}{\sqrt{-f}-\sqrt{g}x}+\frac{f(a+b\log(c(d+ex)^n))}{\sqrt{-f}+\sqrt{g}x}+\frac{befn\left(\log(d+ex)-\log\left(\sqrt{-f}-\sqrt{\right.\right.}}{e\sqrt{-f}+d\sqrt{g}}}{}$$

```
[In] Integrate[(x^4*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

```
[Out] (4*a*Sqrt[g]*x - 4*b*Sqrt[g]*n*x + (4*b*Sqrt[g]*(d + e*x)*Log[c*(d + e*x)^n
])/e - (f*(a + b*Log[c*(d + e*x)^n]))/(Sqrt[-f] - Sqrt[g]*x) + (f*(a + b*Lo
g[c*(d + e*x)^n]))/(Sqrt[-f] + Sqrt[g]*x) + (b*e*f*n*(Log[d + e*x] - Log[Sq
rt[-f] - Sqrt[g]*x]))/(e*Sqrt[-f] + d*Sqrt[g]) + 3*Sqrt[-f]*(a + b*Log[c*(d
 + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])] + (b*e
*f*n*(Log[d + e*x] - Log[Sqrt[-f] + Sqrt[g]*x]))/(-(e*Sqrt[-f]) + d*Sqrt[g]
) - 3*Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e
*Sqrt[-f] - d*Sqrt[g])] - 3*b*Sqrt[-f]*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(
e*Sqrt[-f] - d*Sqrt[g]))] + 3*b*Sqrt[-f]*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(
e*Sqrt[-f] + d*Sqrt[g])])/(4*g^(5/2))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.32 (sec) , antiderivative size = 1619, normalized size of antiderivative = 3.03

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1619 |

```
[In] int(x^4*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b*d*n/e/g^2-1/4*b*e^4*n/g*f^2*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-
f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*x^2+1/4*
b*e^2*n*f*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*
g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*x^2*d^2-1/4*b*e^2*n*f*ln(e*x+
d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d
)-d*g)/(e*(-f*g)^(1/2)-d*g))*x^2*d^2+1/4*b*e^4*n/g*f^2*ln(e*x+d)/(d^2*g+e^2
*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f
*g)^(1/2)+d*g))*x^2-3/4*b*n/g^2*f/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)-g*(e*x
+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+3/4*b*n/g^2*f/(-f*g)^(1/2)*dilog((e*(-f*g)^(
1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))-3/2*b/g^2*f/(f*g)^(1/2)*arctan(1/
2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*ln((e*x+d)^n)+1/4*b*e^2*n/g*f^2*ln(e*x
+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+
d)+d*g)/(e*(-f*g)^(1/2)+d*g))*d^2-1/4*b*e^2*n/g*f^2*ln(e*x+d)/(d^2*g+e^2*f)
/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)
^(1/2)-d*g))*d^2+b*n/g^2*f*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x
+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*e^2/g^2*f*x/(e^2*g*x^2+e^2*f)*ln((e*x+
d)^n)+b*ln((e*x+d)^n)/g^2*x+b/e/g^2*d*ln((e*x+d)^n)-1/4*b*e*n/g^2*f/(d^2*g+
e^2*f)*d*ln(g*(e*x+d)^2-2*(e*x+d)*d*g+d^2*g+f*e^2)-1/2*b*e^2*n/g^2*f^2/(d^2
*g+e^2*f)/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))-1/2*b*e
^2/g^2*f*x/(e^2*g*x^2+e^2*f)*n*ln(e*x+d)+1/2*b*e^3*n/g^2*f^2*ln(e*x+d)/(d^2
*g+e^2*f)/(e^2*g*x^2+e^2*f)*d+1/2*b*e^4*n/g^2*f^2*ln(e*x+d)/(d^2*g+e^2*f)/(
e^2*g*x^2+e^2*f)*x+3/2*b/g^2*f/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e
/(f*g)^(1/2))*n*ln(e*x+d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c
*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e
```

```
*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*
(x/g^2-1/g^2*f*(-1/2*x/(g*x^2+f)+3/2/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2))))+
1/2*b*e^3*n/g*f*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*x^2*d+1/4*b*e^4*n
/g^2*f^3*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g
)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/4*b*e^4*n/g^2*f^3*ln(e*x+d)/
(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d
*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*e^2*n/g*f*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^
2+e^2*f)*x*d^2-b*n/g^2*f*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d
)+d*g)/(e*(-f*g)^(1/2)+d*g))-b*n*x/g^2
```

## Fricas [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^4}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral((b*x^4*log((e*x + d)^n*c) + a*x^4)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^4}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="maxima")
[Out] 1/2*a*(f*x/(g^3*x^2 + f*g^2) - 3*f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g^2) +
2*x/g^2) + b*integrate((x^4*log((e*x + d)^n) + x^4*log(c))/(g^2*x^4 + 2*f*g
*x^2 + f^2), x)
```

## Giac [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^4}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*x^4/(g*x^2 + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{x^4\,(a+b\,\ln(c\,(d+e\,x)^n))}{(g\,x^2+f)^2}\,dx$$

```
[In] int((x^4*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2,x)
[Out] int((x^4*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2, x)
```

# 3.272 $\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 491

$$\begin{aligned}\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx &= \frac{ben\log(d+ex)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}} - \frac{ben\log(d+ex)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}} \\ &+ \frac{a+b\log(c(d+ex)^n)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{a+b\log(c(d+ex)^n)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} \\ &+ \frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}} \\ &+ \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}} \\ &- \frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}} \\ &- \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}} \\ &- \frac{bn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}} \\ &+ \frac{bn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}\end{aligned}$$

```
[Out] 1/4*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/
2)))/g^(3/2)/(-f)^(1/2)-1/4*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2
))/(e*(-f)^(1/2)-d*g^(1/2)))/g^(3/2)/(-f)^(1/2)-1/4*b*n*polylog(2,-(e*x+d)*
g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^(3/2)/(-f)^(1/2)+1/4*b*n*polylog(2,(e*x
+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^(3/2)/(-f)^(1/2)+1/4*b*e*n*ln(e*x+d
```

```
)/g^(3/2)/(e*(-f)^(1/2)-d*g^(1/2))-1/4*b*e*n*ln((-f)^(1/2)+x*g^(1/2))/g^(3/
2)/(e*(-f)^(1/2)-d*g^(1/2))-1/4*b*e*n*ln(e*x+d)/g^(3/2)/(e*(-f)^(1/2)+d*g^(
1/2))+1/4*b*e*n*ln((-f)^(1/2)-x*g^(1/2))/g^(3/2)/(e*(-f)^(1/2)+d*g^(1/2))+1
/4*(a+b*ln(c*(e*x+d)^n))/g^(3/2)/((-f)^(1/2)-x*g^(1/2))+1/4*(-a-b*ln(c*(e*x
+d)^n))/g^(3/2)/((-f)^(1/2)+x*g^(1/2))
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {294, 211, 2463, 2456, 2442, 36, 31, 2441, 2440, 2438}

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \frac{a+b\log(c(d+ex)^n)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{a+b\log(c(d+ex)^n)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{4\sqrt{-f}g^{3/2}}-\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{4\sqrt{-f}g^{3/2}}-\frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}+\frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4\sqrt{-f}g^{3/2}}+\frac{ben\log(d+ex)}{4g^{3/2}\left(e\sqrt{-f}-d\sqrt{g}\right)}-\frac{ben\log(d+ex)}{4g^{3/2}\left(d\sqrt{g}+e\sqrt{-f}\right)}+\frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4g^{3/2}\left(d\sqrt{g}+e\sqrt{-f}\right)}-\frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4g^{3/2}\left(e\sqrt{-f}-d\sqrt{g}\right)}$$

```
[In] Int[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

```
[Out] (b*e*n*Log[d + e*x])/(4*(e*Sqrt[-f] - d*Sqrt[g])*g^(3/2)) - (b*e*n*Log[d +
e*x])/(4*(e*Sqrt[-f] + d*Sqrt[g])*g^(3/2)) + (a + b*Log[c*(d + e*x)^n])/(4*
g^(3/2)*(Sqrt[-f] - Sqrt[g]*x)) - (a + b*Log[c*(d + e*x)^n])/(4*g^(3/2)*(Sq
rt[-f] + Sqrt[g]*x)) + (b*e*n*Log[Sqrt[-f] - Sqrt[g]*x])/(4*(e*Sqrt[-f] + d
*Sqrt[g])*g^(3/2)) + ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]
*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*Sqrt[-f]*g^(3/2)) - (b*e*n*Log[Sqrt[-f]
+ Sqrt[g]*x])/(4*(e*Sqrt[-f] - d*Sqrt[g])*g^(3/2)) - ((a + b*Log[c*(d + e*x
)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(4*Sqrt[-f]
*g^(3/2)) - (b*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))
])/(4*Sqrt[-f]*g^(3/2)) + (b*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] +
 d*Sqrt[g])])/(4*Sqrt[-f]*g^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{f(a+b\log(c(d+ex)^n))}{g(f+gx^2)^2}+\frac{a+b\log(c(d+ex)^n)}{g(f+gx^2)}\right)dx\\
&= \frac{\int \frac{a+b\log(c(d+ex)^n)}{f+gx^2}\,dx}{g}-\frac{f\int \frac{a+b\log(c(d+ex)^n)}{(f+gx^2)^2}\,dx}{g}\\
&= \frac{\int \left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}-\sqrt{g}x)}+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g}\\
&\quad -\frac{f\int \left(-\frac{g(a+b\log(c(d+ex)^n))}{4f(\sqrt{-f}\sqrt{g}-gx)^2}-\frac{g(a+b\log(c(d+ex)^n))}{4f(\sqrt{-f}\sqrt{g}+gx)^2}-\frac{g(a+b\log(c(d+ex)^n))}{2f(-fg-g^2x^2)}\right)dx}{g}\\
&= \frac{1}{4}\int \frac{a+b\log(c(d+ex)^n)}{(\sqrt{-f}\sqrt{g}-gx)^2}\,dx+\frac{1}{4}\int \frac{a+b\log(c(d+ex)^n)}{(\sqrt{-f}\sqrt{g}+gx)^2}\,dx\\
&\quad +\frac{1}{2}\int \frac{a+b\log(c(d+ex)^n)}{-fg-g^2x^2}\,dx-\frac{\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2\sqrt{-f}g}-\frac{\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2\sqrt{-f}g}
\end{aligned}$$

$$= \frac{a+b\log\left(c(d+ex)^n\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{a+b\log\left(c(d+ex)^n\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}$$
$$-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}$$
$$+\frac{1}{2}\int\left(-\frac{\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)}{2fg\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)}{2fg\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx-\frac{(ben)\int\frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}}{2\sqrt{-f}g^{3/2}}$$

$$= \frac{a+b\log\left(c(d+ex)^n\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{a+b\log\left(c(d+ex)^n\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}$$
$$-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}+\frac{\int\frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{-f}-\sqrt{g}x}dx}{4\sqrt{-f}g}$$
$$+\frac{\int\frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{-f}+\sqrt{g}x}dx}{4\sqrt{-f}g}+\frac{(bn)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2\sqrt{-f}g^{3/2}}$$
$$-\frac{(bn)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2\sqrt{-f}g^{3/2}}+\frac{\left(be^2n\right)\int\frac{1}{d+ex}dx}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}$$
$$-\frac{\left(be^2n\right)\int\frac{1}{d+ex}dx}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}-\frac{(ben)\int\frac{1}{\sqrt{-f}\sqrt{g}+gx}dx}{4\left(e\sqrt{-f}-d\sqrt{g}\right)\sqrt{g}}-\frac{(ben)\int\frac{1}{\sqrt{-f}\sqrt{g}-gx}dx}{4\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}}$$

$$= \frac{ben\log(d+ex)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}-\frac{ben\log(d+ex)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}+\frac{a+b\log\left(c(d+ex)^n\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)}$$
$$-\frac{a+b\log\left(c(d+ex)^n\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}$$
$$+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}-\frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}$$
$$-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}-\frac{bn\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}$$
$$+\frac{bn\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}+\frac{(ben)\int\frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}dx}{4\sqrt{-f}g^{3/2}}-\frac{(ben)\int\frac{\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}dx}{4\sqrt{-f}g^{3/2}}$$

$$= \frac{ben\log(d+ex)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}} - \frac{ben\log(d+ex)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}} + \frac{a+b\log\left(c(d+ex)^n\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{a+b\log\left(c(d+ex)^n\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}} + \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}} - \frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}} - \frac{bn\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}} + \frac{bn\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}} - \frac{(bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{4\sqrt{-f}g^{3/2}} + \frac{(bn)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{4\sqrt{-f}g^{3/2}}$$

$$= \frac{ben\log(d+ex)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}} - \frac{ben\log(d+ex)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}} + \frac{a+b\log\left(c(d+ex)^n\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{a+b\log\left(c(d+ex)^n\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}} + \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}} - \frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}} - \frac{bn\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}} + \frac{bn\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}$$

## Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.78

$$\int \frac{x^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{\left(f+gx^2\right)^2}\,dx$$

$$= \frac{\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x} - \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x} + \frac{ben\left(-\log(d+ex)+\log\left(\sqrt{-f}-\sqrt{g}x\right)\right)}{e\sqrt{-f}+d\sqrt{g}} + \frac{\left(a+b\log(c(d+ex)^n)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{\sqrt{-f}} + \frac{ben(\log(d\cdots}{}}{4g^{3/2}\cdots}$$

```
[In] Integrate[(x^2*(a + b*Log[c*(d + e*x)^n]))/(f + g*x^2)^2,x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])/(Sqrt[-f] - Sqrt[g]*x) - (a + b*Log[c*(d + e*x)
^n])/(Sqrt[-f] + Sqrt[g]*x) + (b*e*n*(-Log[d + e*x] + Log[Sqrt[-f] - Sqrt[g
```

```
]*x]))/(e*Sqrt[-f] + d*Sqrt[g]) + ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[
-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/Sqrt[-f] + (b*e*n*(Log[d + e*x
] - Log[Sqrt[-f] + Sqrt[g]*x]))/(e*Sqrt[-f] - d*Sqrt[g]) + (f*(a + b*Log[c*
(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(-f
)^(3/2) + (b*f*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))
])/(-f)^(3/2) + (b*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g]
)])/Sqrt[-f])/(4*g^(3/2))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.31 (sec) , antiderivative size = 1521, normalized size of antiderivative = 3.10

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1521 |

```
[In] int(x^2*(a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*e^2/(e^2*g*x^2+e^2*f)/g*x*n*ln(e*x+d)-1/2*b*e^2/(e^2*g*x^2+e^2*f)/g*x
*ln((e*x+d)^n)-1/2*b/g/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(
1/2))*n*ln(e*x+d)+1/2*b/g/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g
)^(1/2))*ln((e*x+d)^n)+1/4*b*e*n/g/(d^2*g+e^2*f)*d*ln(g*(e*x+d)^2-2*(e*x+d)
*d*g+d^2*g+f*e^2)+1/2*b*e^2*n*f/g/(d^2*g+e^2*f)/(f*g)^(1/2)*arctan(1/2*(2*g
*(e*x+d)-2*d*g)/e/(f*g)^(1/2))-1/4*b*e^2*n*g*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g
*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+
d*g))*x^2*d^2-1/4*b*e^4*n*f*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g
)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*x^2+1/4*b*e
^2*n*g*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^
(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*x^2*d^2+1/4*b*e^4*n*f*ln(e*x+d)/
(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d
*g)/(e*(-f*g)^(1/2)-d*g))*x^2-1/2*b*e^3*n*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^
2+e^2*f)*x^2*d-1/4*b*e^2*n*f*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*
g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*d^2-1/4*b*
e^4*n*f^2/g*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-
f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/4*b*e^2*n*f*ln(e*x+d)/(d^
2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)
/(e*(-f*g)^(1/2)-d*g))*d^2+1/4*b*e^4*n*f^2/g*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g
*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-
d*g))-1/2*b*e^2*n*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*x*d^2-1/2*b*e^4
*n*f/g*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*x-1/2*b*e^3*n*f/g*ln(e*x+d
)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*d+1/4*b*n/g/(-f*g)^(1/2)*dilog((e*(-f*g)^
(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/4*b*n/g/(-f*g)^(1/2)*dilog((e*
(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b*n/g*ln(e*x+d)/(-f*g
)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b*n/g*l
n(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g
```

```
))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*
csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+
d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(-1/2/g*x/(g*x^2+f)+1/2
/g/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2)))
```

## Fricas [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^2}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral((b*x^2*log((e*x + d)^n*c) + a*x^2)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)x^2}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="maxima")
[Out] -1/2*a*(x/(g^2*x^2 + f*g) - arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g)) + b*integr
ate((x^2*log((e*x + d)^n) + x^2*log(c))/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

## Giac [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)x^2}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*x^2/(g*x^2 + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))}{(f+gx^2)^2}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,(d+e\,x)^n))}{(g\,x^2+f)^2}\,dx$$

```
[In] int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2,x)
[Out] int((x^2*(a + b*log(c*(d + e*x)^n)))/(f + g*x^2)^2, x)
```

# 3.273 $\int \frac{a+b\log(c(d+ex)^n)}{(f+gx^2)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 503

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{(f+gx^2)^2}\,dx &= \frac{ben\log(d+ex)}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}}+\frac{ben\log(d+ex)}{4\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}}\\ &-\frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}\\ &-\frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}}\\ &-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}\\ &-\frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}}\\ &+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}\\ &+\frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}-\frac{bn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}\end{aligned}$$

```
[Out] -1/4*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1
/2)))/(-f)^(3/2)/g^(1/2)+1/4*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/
2))/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(3/2)/g^(1/2)+1/4*b*n*polylog(2,-(e*x+d)
*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(3/2)/g^(1/2)-1/4*b*n*polylog(2,(e*
x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/(-f)^(3/2)/g^(1/2)+1/4*b*e*n*ln(e*x+
d)/f/g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2))-1/4*b*e*n*ln((-f)^(1/2)-x*g^(1/2))/f/
g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2))+1/4*b*e*n*ln(e*x+d)/g^(1/2)/(e*(-f)^(3/2)+
d*f*g^(1/2))-1/4*b*e*n*ln((-f)^(1/2)+x*g^(1/2))/g^(1/2)/(e*(-f)^(3/2)+d*f*g
```

```
^(1/2))+1/4*(-a-b*ln(c*(e*x+d)^n))/f/g^(1/2)/((-f)^(1/2)-x*g^(1/2))+1/4*(a+
b*ln(c*(e*x+d)^n))/f/g^(1/2)/((-f)^(1/2)+x*g^(1/2))
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2456, 2442, 36, 31, 2441, 2440, 2438}

$$\int \frac{a+b\log(c(d+ex)^n)}{(f+gx^2)^2}\,dx = -\frac{a+b\log(c(d+ex)^n)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{a+b\log(c(d+ex)^n)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{4(-f)^{3/2}\sqrt{g}} + \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{4(-f)^{3/2}\sqrt{g}} + \frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} - \frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4(-f)^{3/2}\sqrt{g}} + \frac{ben\log(d+ex)}{4f\sqrt{g}\left(d\sqrt{g}+e\sqrt{-f}\right)} + \frac{ben\log(d+ex)}{4\sqrt{g}\left(df\sqrt{g}+e(-f)^{3/2}\right)} - \frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f\sqrt{g}\left(d\sqrt{g}+e\sqrt{-f}\right)} - \frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\sqrt{g}\left(df\sqrt{g}+e(-f)^{3/2}\right)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(f + g*x^2)^2,x]
```

```
[Out] (b*e*n*Log[d + e*x])/(4*f*(e*Sqrt[-f] + d*Sqrt[g])*Sqrt[g]) + (b*e*n*Log[d
+ e*x])/(4*(e*(-f)^(3/2) + d*f*Sqrt[g])*Sqrt[g]) - (a + b*Log[c*(d + e*x)^n
])/(4*f*Sqrt[g]*(Sqrt[-f] - Sqrt[g]*x)) + (a + b*Log[c*(d + e*x)^n])/(4*f*S
qrt[g]*(Sqrt[-f] + Sqrt[g]*x)) - (b*e*n*Log[Sqrt[-f] - Sqrt[g]*x])/(4*f*(e*
Sqrt[-f] + d*Sqrt[g])*Sqrt[g]) - ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-
f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*(-f)^(3/2)*Sqrt[g]) - (b*e*n
*Log[Sqrt[-f] + Sqrt[g]*x])/(4*(e*(-f)^(3/2) + d*f*Sqrt[g])*Sqrt[g]) + ((a
+ b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt
[g])])/(4*(-f)^(3/2)*Sqrt[g]) + (b*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sq
rt[-f] - d*Sqrt[g]))])/(4*(-f)^(3/2)*Sqrt[g]) - (b*n*PolyLog[2, (Sqrt[g]*(d
 + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*(-f)^(3/2)*Sqrt[g])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rubi steps

$$\text{integral} = \int \left(-\frac{g(a+b\log(c(d+ex)^n))}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2} - \frac{g(a+b\log(c(d+ex)^n))}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2} - \frac{g(a+b\log(c(d+ex)^n))}{2f\left(-fg-g^2x^2\right)}\right) dx$$

$$= -\frac{g\int \frac{a+b\log(c(d+ex)^n)}{\left(\sqrt{-f}\sqrt{g}-gx\right)^2}\,dx}{4f} - \frac{g\int \frac{a+b\log(c(d+ex)^n)}{\left(\sqrt{-f}\sqrt{g}+gx\right)^2}\,dx}{4f} - \frac{g\int \frac{a+b\log(c(d+ex)^n)}{-fg-g^2x^2}\,dx}{2f}$$

$$= -\frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{g\int\left(-\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2fg(\sqrt{-f}-\sqrt{g}x)} - \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2fg(\sqrt{-f}+\sqrt{g}x)}\right)dx}{2f} + \frac{(ben)\int \frac{1}{(d+ex)(\sqrt{-f}\sqrt{g}-gx)}\,dx}{4f} - \frac{(ben)\int \frac{1}{(d+ex)(\sqrt{-f}\sqrt{g}+gx)}\,dx}{4f}$$

$$= -\frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{4(-f)^{3/2}} + \frac{\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{4(-f)^{3/2}} + \frac{(be^2n)\int \frac{1}{d+ex}\,dx}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} + \frac{(be^2n)\int \frac{1}{d+ex}\,dx}{4\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} + \frac{\left(be\sqrt{g}n\right)\int \frac{1}{\sqrt{-f}\sqrt{g}+gx}\,dx}{4f\left(e\sqrt{-f}-d\sqrt{g}\right)} + \frac{\left(be\sqrt{g}n\right)\int \frac{1}{\sqrt{-f}\sqrt{g}-gx}\,dx}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)}$$

$$= \frac{ben\log(d+ex)}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} + \frac{ben\log(d+ex)}{4\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} - \frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} - \frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} + \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} + \frac{(ben)\int \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{4(-f)^{3/2}\sqrt{g}} - \frac{(ben)\int \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{4(-f)^{3/2}\sqrt{g}}$$

$$= \frac{ben\log(d+ex)}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} + \frac{ben\log(d+ex)}{4\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} - \frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} - \frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} + \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} - \frac{(bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{4(-f)^{3/2}\sqrt{g}} + \frac{(bn)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{4(-f)^{3/2}\sqrt{g}}$$

$$= \frac{ben\log(d+ex)}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} + \frac{ben\log(d+ex)}{4\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} - \frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{a+b\log\left(c(d+ex)^n\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{ben\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} - \frac{ben\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} + \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} + \frac{bn\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} - \frac{bn\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}$$

## Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.81

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+gx^2\right)^2}\,dx = \frac{1}{4}\left(\frac{a+b\log\left(c(d+ex)^n\right)}{f\left(\sqrt{-f}\sqrt{g}+gx\right)}+\frac{a+b\log\left(c(d+ex)^n\right)}{(-f)^{3/2}\sqrt{g}+fgx}+\frac{ben\left(\log(d+ex)-\log\left(\sqrt{-f}-\sqrt{g}x\right)\right)}{e\sqrt{-f}f\sqrt{g}+dfg}+\frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}\sqrt{g}}+\frac{ben\left(\log(d+ex)-\log\left(\sqrt{-f}+\sqrt{g}x\right)\right)}{e(-f)^{3/2}\sqrt{g}+dfg}+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{3/2}\sqrt{g}}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{3/2}\sqrt{g}}+\frac{bfn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}\sqrt{g}}\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(f + g*x^2)^2,x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])/(f*(Sqrt[-f]*Sqrt[g] + g*x)) + (a + b*Log[c*(d
+ e*x)^n])/((-f)^(3/2)*Sqrt[g] + f*g*x) + (b*e*n*(Log[d + e*x] - Log[Sqrt[-
f] - Sqrt[g]*x]))/(e*Sqrt[-f]*f*Sqrt[g] + d*f*g) + (f*(a + b*Log[c*(d + e*x
)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/((-f)^(5/2)
*Sqrt[g]) + (b*e*n*(Log[d + e*x] - Log[Sqrt[-f] + Sqrt[g]*x]))/(e*(-f)^(3/2
)*Sqrt[g] + d*f*g) + ((a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]
*x))/(e*Sqrt[-f] - d*Sqrt[g])])/((-f)^(3/2)*Sqrt[g]) + (b*n*PolyLog[2, -((S
qrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/((-f)^(3/2)*Sqrt[g]) + (b*f*n
*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/((-f)^(5/2)*Sqrt
[g]))/4
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.31 (sec) , antiderivative size = 1406, normalized size of antiderivative = 2.80

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1406 |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*e^2/f/(e^2*g*x^2+e^2*f)*x*n*ln(e*x+d)+1/2*b*e^2/f/(e^2*g*x^2+e^2*f)*
x*ln((e*x+d)^n)-1/2*b/f/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^
(1/2))*n*ln(e*x+d)+1/2*b/f/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*
g)^(1/2))*ln((e*x+d)^n)-1/4*b*e*n/f/(d^2*g+e^2*f)*d*ln(g*(e*x+d)^2-2*(e*x+d
)*d*g+d^2*g+f*e^2)-1/2*b*e^2*n/(d^2*g+e^2*f)/(f*g)^(1/2)*arctan(1/2*(2*g*(e
*x+d)-2*d*g)/e/(f*g)^(1/2))+1/4*b*e^2*n*ln(e*x+d)/f/(d^2*g+e^2*f)/(e^2*g*x^
2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g
))*x^2*d^2*g^2+1/4*b*e^4*n*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)
^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*x^2*g-1/4*b*
e^2*n*ln(e*x+d)/f/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)
^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*x^2*d^2*g^2-1/4*b*e^4*n*ln(e*x+
d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d
)-d*g)/(e*(-f*g)^(1/2)-d*g))*x^2*g+1/2*b*e^3*n*ln(e*x+d)/f/(d^2*g+e^2*f)/(e
^2*g*x^2+e^2*f)*x^2*d*g+1/4*b*e^2*n*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*
f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*d^2
*g+1/4*b*e^4*n*ln(e*x+d)*f/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln(
(e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/4*b*e^2*n*ln(e*x+d)/
(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d
*g)/(e*(-f*g)^(1/2)-d*g))*d^2*g-1/4*b*e^4*n*ln(e*x+d)*f/(d^2*g+e^2*f)/(e^2*
g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)
-d*g))+1/2*b*e^2*n*ln(e*x+d)/f/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*x*d^2*g+1/2*
b*e^4*n*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*x+1/2*b*e^3*n*ln(e*x+d)/(
d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*d+1/4*b*n/f/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/
2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/4*b*n/f/(-f*g)^(1/2)*dilog((e*(-f
*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(
I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2
+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*
x+d)^n)^3+b*ln(c)+a)*(1/2*x/f/(g*x^2+f)+1/2/f/(f*g)^(1/2)*arctan(g*x/(f*g)^
(1/2)))
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\left(gx^2+f\right)^2}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="fricas")

[Out] integral((b*log((e*x + d)^n*c) + a)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+gx^2\right)^2}\,dx=\text{Timed out}$$

[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**2,x)

[Out] Timed out

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\left(gx^2+f\right)^2}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="maxima")

[Out] 1/2*a*(x/(f*g*x^2 + f^2) + arctan(g*x/sqrt(f*g))/(sqrt(f*g)*f)) + b*integrate((log((e*x + d)^n) + log(c))/(g^2*x^4 + 2*f*g*x^2 + f^2), x)

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\left(gx^2+f\right)^2}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f)^2,x, algorithm="giac")

[Out] integrate((b*log((e*x + d)^n*c) + a)/(g*x^2 + f)^2, x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+gx^2\right)^2}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x^2)^2,x)
[Out] int((a + b*log(c*(d + e*x)^n))/(f + g*x^2)^2, x)
```

# 3.274 $\int \frac{a+b\log(c(d+ex)^n)}{x^2(f+gx^2)^2}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 560

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx &= \frac{ben\log(x)}{df^2}-\frac{ben\log(d+ex)}{df^2}-\frac{be\sqrt{g}n\log(d+ex)}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)}\\
&-\frac{be\sqrt{g}n\log(d+ex)}{4f\left(e(-f)^{3/2}+df\sqrt{g}\right)}-\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x}\\
&+\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)}\\
&-\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{be\sqrt{g}n\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)}\\
&-\frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}}\\
&+\frac{be\sqrt{g}n\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4f\left(e(-f)^{3/2}+df\sqrt{g}\right)}\\
&+\frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}}\\
&+\frac{3b\sqrt{g}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}}\\
&-\frac{3b\sqrt{g}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}}\end{aligned}$$

```
[Out] b*e*n*ln(x)/d/f^2-b*e*n*ln(e*x+d)/d/f^2+(-a-b*ln(c*(e*x+d)^n))/f^2/x-3/4*(a
+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*g
^(1/2)/(-f)^(5/2)+3/4*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*
```

```
(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(5/2)+3/4*b*n*polylog(2,-(e*x+d)*g^(1/2
)/(e*(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(5/2)-3/4*b*n*polylog(2,(e*x+d)*g^
(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/(-f)^(5/2)-1/4*b*e*n*ln(e*x+d)*g^(1
/2)/f^2/(e*(-f)^(1/2)+d*g^(1/2))+1/4*b*e*n*ln((-f)^(1/2)-x*g^(1/2))*g^(1/2)
/f^2/(e*(-f)^(1/2)+d*g^(1/2))-1/4*b*e*n*ln(e*x+d)*g^(1/2)/f/(e*(-f)^(3/2)+d
*f*g^(1/2))+1/4*b*e*n*ln((-f)^(1/2)+x*g^(1/2))*g^(1/2)/f/(e*(-f)^(3/2)+d*f*
g^(1/2))+1/4*(a+b*ln(c*(e*x+d)^n))*g^(1/2)/f^2/((-f)^(1/2)-x*g^(1/2))-1/4*(
a+b*ln(c*(e*x+d)^n))*g^(1/2)/f^2/((-f)^(1/2)+x*g^(1/2))
```

## Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {296, 331, 211, 2463, 2442, 36, 29, 31, 2456, 2441, 2440, 2438}

$$\begin{aligned}\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}dx = &\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)}\\ &-\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x}\\ &-\frac{3\sqrt{g}\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log\left(c(d+ex)^n\right))}{4(-f)^{5/2}}\\ &+\frac{3\sqrt{g}\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log\left(c(d+ex)^n\right))}{4(-f)^{5/2}}\\ &-\frac{be\sqrt{g}n\log(d+ex)}{4f^2\left(d\sqrt{g}+e\sqrt{-f}\right)}+\frac{be\sqrt{g}n\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f^2\left(d\sqrt{g}+e\sqrt{-f}\right)}\\ &+\frac{ben\log(x)}{df^2}-\frac{ben\log(d+ex)}{df^2}\\ &+\frac{3b\sqrt{g}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}}\\ &-\frac{3b\sqrt{g}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4(-f)^{5/2}}\\ &-\frac{be\sqrt{g}n\log(d+ex)}{4f\left(df\sqrt{g}+e(-f)^{3/2}\right)}+\frac{be\sqrt{g}n\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4f\left(df\sqrt{g}+e(-f)^{3/2}\right)}\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x^2)^2),x]
```

```
[Out] (b*e*n*Log[x])/(d*f^2) - (b*e*n*Log[d + e*x])/(d*f^2) - (b*e*Sqrt[g]*n*Log[
d + e*x])/(4*f^2*(e*Sqrt[-f] + d*Sqrt[g])) - (b*e*Sqrt[g]*n*Log[d + e*x])/(
4*f*(e*(-f)^(3/2) + d*f*Sqrt[g])) - (a + b*Log[c*(d + e*x)^n])/(f^2*x) + (S
qrt[g]*(a + b*Log[c*(d + e*x)^n]))/(4*f^2*(Sqrt[-f] - Sqrt[g]*x)) - (Sqrt[g
```

```
]*(a + b*Log[c*(d + e*x)^n]))/(4*f^2*(Sqrt[-f] + Sqrt[g]*x)) + (b*e*Sqrt[g]
*n*Log[Sqrt[-f] - Sqrt[g]*x])/(4*f^2*(e*Sqrt[-f] + d*Sqrt[g])) - (3*Sqrt[g]
*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*
Sqrt[g])])/(4*(-f)^(5/2)) + (b*e*Sqrt[g]*n*Log[Sqrt[-f] + Sqrt[g]*x])/(4*f*
(e*(-f)^(3/2) + d*f*Sqrt[g])) + (3*Sqrt[g]*(a + b*Log[c*(d + e*x)^n])*Log[(
e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(4*(-f)^(5/2)) + (3*b*
Sqrt[g]*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(4*(
-f)^(5/2)) - (3*b*Sqrt[g]*n*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*
Sqrt[g])])/(4*(-f)^(5/2))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x^2} - \frac{g(a+b\log\left(c(d+ex)^n\right))}{f\left(f+gx^2\right)^2} - \frac{g(a+b\log\left(c(d+ex)^n\right))}{f^2\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\log(c(d+ex)^n)}{x^2}\,dx}{f^2} - \frac{g\int \frac{a+b\log(c(d+ex)^n)}{f+gx^2}\,dx}{f^2} - \frac{g\int \frac{a+b\log(c(d+ex)^n)}{(f+gx^2)^2}\,dx}{f}$$

$$= -\frac{a+b\log(c(d+ex)^n)}{f^2x} - \frac{g\int \left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}-\sqrt{g}x)} + \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}+\sqrt{g}x)}\right)dx}{f^2}$$
$$- \frac{g\int \left(-\frac{g(a+b\log(c(d+ex)^n))}{4f(\sqrt{-f}\sqrt{g}-gx)^2} - \frac{g(a+b\log(c(d+ex)^n))}{4f(\sqrt{-f}\sqrt{g}+gx)^2} - \frac{g(a+b\log(c(d+ex)^n))}{2f(-fg-g^2x^2)}\right)dx}{f}$$
$$+ \frac{(ben)\int \frac{1}{x(d+ex)}\,dx}{f^2}$$

$$= -\frac{a+b\log(c(d+ex)^n)}{f^2x} + \frac{g\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2(-f)^{5/2}} + \frac{g\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2(-f)^{5/2}}$$
$$+ \frac{g^2\int \frac{a+b\log(c(d+ex)^n)}{(\sqrt{-f}\sqrt{g}-gx)^2}\,dx}{4f^2} + \frac{g^2\int \frac{a+b\log(c(d+ex)^n)}{(\sqrt{-f}\sqrt{g}+gx)^2}\,dx}{4f^2}$$
$$+ \frac{g^2\int \frac{a+b\log(c(d+ex)^n)}{-fg-g^2x^2}\,dx}{2f^2} + \frac{(ben)\int \frac{1}{x}\,dx}{df^2} - \frac{(be^2n)\int \frac{1}{d+ex}\,dx}{df^2}$$

$$= \frac{ben\log(x)}{df^2} - \frac{ben\log(d+ex)}{df^2} - \frac{a+b\log(c(d+ex)^n)}{f^2x} + \frac{\sqrt{g}(a+b\log(c(d+ex)^n))}{4f^2(\sqrt{-f}-\sqrt{g}x)}$$
$$- \frac{\sqrt{g}(a+b\log(c(d+ex)^n))}{4f^2(\sqrt{-f}+\sqrt{g}x)} - \frac{\sqrt{g}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}}$$
$$+ \frac{\sqrt{g}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}}$$
$$+ \frac{g^2\int \left(-\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2fg(\sqrt{-f}-\sqrt{g}x)} - \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))}{2fg(\sqrt{-f}+\sqrt{g}x)}\right)dx}{2f^2}$$
$$+ \frac{(be\sqrt{g}n)\int \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2(-f)^{5/2}} - \frac{(be\sqrt{g}n)\int \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2(-f)^{5/2}}$$
$$- \frac{(begn)\int \frac{1}{(d+ex)(\sqrt{-f}\sqrt{g}-gx)}\,dx}{4f^2} + \frac{(begn)\int \frac{1}{(d+ex)(\sqrt{-f}\sqrt{g}+gx)}\,dx}{4f^2}$$

$$=\frac{ben\log(x)}{df^2}-\frac{ben\log(d+ex)}{df^2}-\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x}+\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)}$$
$$-\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}}$$
$$+\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}}+\frac{g\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{4(-f)^{5/2}}$$
$$+\frac{g\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{4(-f)^{5/2}}-\frac{\left(b\sqrt{g}n\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2(-f)^{5/2}}$$
$$+\frac{\left(b\sqrt{g}n\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2(-f)^{5/2}}+\frac{\left(be^2\sqrt{g}n\right)\int\frac{1}{d+ex}\,dx}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)}$$
$$-\frac{\left(be^2\sqrt{g}n\right)\int\frac{1}{d+ex}\,dx}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)}-\frac{\left(beg^{3/2}n\right)\int\frac{1}{\sqrt{-f}\sqrt{g}+gx}\,dx}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)}-\frac{\left(beg^{3/2}n\right)\int\frac{1}{\sqrt{-f}\sqrt{g}-gx}\,dx}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)}$$

$$=\frac{ben\log(x)}{df^2}-\frac{ben\log(d+ex)}{df^2}+\frac{be\sqrt{g}n\log(d+ex)}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)}-\frac{be\sqrt{g}n\log(d+ex)}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)}$$
$$-\frac{a+b\log\left(c(d+ex)^n\right)}{f^2x}+\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$+\frac{be\sqrt{g}n\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)}-\frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}}$$
$$-\frac{be\sqrt{g}n\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)}+\frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}}$$
$$+\frac{b\sqrt{g}n\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}}-\frac{b\sqrt{g}n\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}}$$
$$+\frac{\left(be\sqrt{g}n\right)\int\frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{4(-f)^{5/2}}-\frac{\left(be\sqrt{g}n\right)\int\frac{\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{4(-f)^{5/2}}$$

$$= \frac{ben\log(x)}{df^2} - \frac{ben\log(d+ex)}{df^2} + \frac{be\sqrt{g}n\log(d+ex)}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)} - \frac{be\sqrt{g}n\log(d+ex)}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} - \frac{a+b\log\left(c(d+ex)^n\right)}{f^2x} + \frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{be\sqrt{g}n\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} - \frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}} - \frac{be\sqrt{g}n\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)} + \frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}} + \frac{b\sqrt{g}n\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{b\sqrt{g}n\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{\left(b\sqrt{g}n\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{4(-f)^{5/2}} + \frac{\left(b\sqrt{g}n\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{4(-f)^{5/2}}$$

$$= \frac{ben\log(x)}{df^2} - \frac{ben\log(d+ex)}{df^2} + \frac{be\sqrt{g}n\log(d+ex)}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)} - \frac{be\sqrt{g}n\log(d+ex)}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} - \frac{a+b\log\left(c(d+ex)^n\right)}{f^2x} + \frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{be\sqrt{g}n\log\left(\sqrt{-f}-\sqrt{g}x\right)}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} - \frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}} - \frac{be\sqrt{g}n\log\left(\sqrt{-f}+\sqrt{g}x\right)}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)} + \frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}} + \frac{3b\sqrt{g}n\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}} - \frac{3b\sqrt{g}n\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}}$$

## Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 475, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx=\frac{1}{4}\left(\frac{4ben(\log(x)-\log(d+ex))}{df^2}-\frac{4(a+b\log\left(c(d+ex)^n\right))}{f^2x}+\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{f^2\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{\sqrt{g}(a+b\log\left(c(d+ex)^n\right))}{f^2\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{be\sqrt{g}n\left(-\log(d+ex)+\log\left(\sqrt{-f}-\sqrt{g}x\right)\right)}{f^2\left(e\sqrt{-f}+d\sqrt{g}\right)}-\frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}}+\frac{be\sqrt{g}n\left(\log(d+ex)-\log\left(\sqrt{-f}+\sqrt{g}x\right)\right)}{f^2\left(e\sqrt{-f}-d\sqrt{g}\right)}+\frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}}+\frac{3b\sqrt{g}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}}-\frac{3b\sqrt{g}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}}\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(x^2*(f + g*x^2)^2),x]
```

```
[Out] ((4*b*e*n*(Log[x] - Log[d + e*x]))/(d*f^2) - (4*(a + b*Log[c*(d + e*x)^n]))
/(f^2*x) + (Sqrt[g]*(a + b*Log[c*(d + e*x)^n]))/(f^2*(Sqrt[-f] - Sqrt[g]*x)
) - (Sqrt[g]*(a + b*Log[c*(d + e*x)^n]))/(f^2*(Sqrt[-f] + Sqrt[g]*x)) + (b*
e*Sqrt[g]*n*(-Log[d + e*x] + Log[Sqrt[-f] - Sqrt[g]*x]))/(f^2*(e*Sqrt[-f] +
 d*Sqrt[g])) - (3*Sqrt[g]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqr
t[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(-f)^(5/2) + (b*e*Sqrt[g]*n*(Log[d + e*
x] - Log[Sqrt[-f] + Sqrt[g]*x]))/(f^2*(e*Sqrt[-f] - d*Sqrt[g])) + (3*Sqrt[g
]*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d
*Sqrt[g])])/(-f)^(5/2) + (3*b*Sqrt[g]*n*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e
*Sqrt[-f] - d*Sqrt[g]))])/(-f)^(5/2) - (3*b*Sqrt[g]*n*PolyLog[2, (Sqrt[g]*(
d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(-f)^(5/2))/4
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.25 (sec) , antiderivative size = 1619, normalized size of antiderivative = 2.89

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1619 |

```
[In] int((a+b*ln(c*(e*x+d)^n))/x^2/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*b/f^2*g/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*n*ln(
e*x+d)-1/2*b*e^2/f^2*g*x/(e^2*g*x^2+e^2*f)*ln((e*x+d)^n)-b*ln((e*x+d)^n)/f^
2/x-1/2*b*e^3*n/f^2*g^2*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*x^2*d-1/2
*b*e^2*n/f^2*g^2*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*x*d^2-1/2*b*n/f^
2*g*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2
)+d*g))+1/2*b*n/f^2*g*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d
*g)/(e*(-f*g)^(1/2)-d*g))+1/4*b*e*n/f^2*g/(d^2*g+e^2*f)*d*ln(g*(e*x+d)^2-2*
(e*x+d)*d*g+d^2*g+f*e^2)+1/2*b*e^2*n/f*g/(d^2*g+e^2*f)/(f*g)^(1/2)*arctan(1
/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))+1/2*b*e^2/f^2*g*x/(e^2*g*x^2+e^2*f)*n
*ln(e*x+d)-1/2*b*e^3*n/f*g*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*d-1/2*
b*e^4*n/f*g*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)*x-1/4*b*e^4*n*g*ln(e*
x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x
+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+1/4*b*e^4*n*g*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g
*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-
d*g))-1/4*b*e^4*n/f*g^2*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1
/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*x^2+1/4*b*e^4*n
/f*g^2*ln(e*x+d)/(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^
(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))*x^2-1/4*b*e^2*n/f*g^2*ln(e*x+d)/
(d^2*g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d
*g)/(e*(-f*g)^(1/2)+d*g))*d^2+1/4*b*e^2*n/f*g^2*ln(e*x+d)/(d^2*g+e^2*f)/(e^
2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/
2)-d*g))*d^2-3/2*b/f^2*g/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)
^(1/2))*ln((e*x+d)^n)+b*e*n/f^2/d*ln(e*x)-3/4*b*n/f^2*g/(-f*g)^(1/2)*dilog(
(e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))+3/4*b*n/f^2*g/(-f*g)^(
1/2)*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+(-1/2*I*b*P
i*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn
(I*c*(e*x+d)^n)^2+1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*
b*Pi*csgn(I*c*(e*x+d)^n)^3+b*ln(c)+a)*(-1/f^2/x-1/f^2*g*(1/2*x/(g*x^2+f)+3/
2/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2))))-1/4*b*e^2*n/f^2*g^3*ln(e*x+d)/(d^2*
g+e^2*f)/(e^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(
e*(-f*g)^(1/2)+d*g))*x^2*d^2+1/4*b*e^2*n/f^2*g^3*ln(e*x+d)/(d^2*g+e^2*f)/(e
^2*g*x^2+e^2*f)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1
/2)-d*g))*x^2*d^2-b*e*n*ln(e*x+d)/d/f^2
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx^2+f\right)^2 x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral((b*log((e*x + d)^n*c) + a)/(g^2*x^6 + 2*f*g*x^4 + f^2*x^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/x**2/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx^2+f\right)^2 x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x^2+f)^2,x, algorithm="maxima")
[Out] -1/2*a*((3*g*x^2 + 2*f)/(f^2*g*x^3 + f^3*x) + 3*g*arctan(g*x/sqrt(f*g))/(sq
rt(f*g)*f^2)) + b*integrate((log((e*x + d)^n) + log(c))/(g^2*x^6 + 2*f*g*x^
4 + f^2*x^2), x)
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\left(gx^2+f\right)^2 x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/x^2/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((g*x^2 + f)^2*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2\left(f+gx^2\right)^2}\,dx=\int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{x^2\,\left(g\,x^2+f\right)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x^2)^2),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g*x^2)^2), x)
```

# 3.275 $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{2+gx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 326

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2+gx^2}}\,dx = \frac{bn\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)^2}{2\sqrt{g}} - \frac{bn\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}} - \frac{bn\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}+\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}} + \frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{\sqrt{g}} - \frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}} - \frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}+\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}}$$

```
[Out] 1/2*b*n*arcsinh(1/2*x*g^(1/2)*2^(1/2))^2/g^(1/2)+arcsinh(1/2*x*g^(1/2)*2^(1
/2))*(a+b*ln(c*(e*x+d)^n))/g^(1/2)-b*n*arcsinh(1/2*x*g^(1/2)*2^(1/2))*ln(1+
e*(1/2*x*g^(1/2)*2^(1/2)+1/2*(2*g*x^2+4)^(1/2))*2^(1/2)/(d*g^(1/2)-(d^2*g+2
*e^2)^(1/2)))/g^(1/2)-b*n*arcsinh(1/2*x*g^(1/2)*2^(1/2))*ln(1+e*(1/2*x*g^(1
/2)*2^(1/2)+1/2*(2*g*x^2+4)^(1/2))*2^(1/2)/(d*g^(1/2)+(d^2*g+2*e^2)^(1/2)))
```

```
/g^(1/2)-b*n*polylog(2,-e*(1/2*x*g^(1/2)*2^(1/2)+1/2*(2*g*x^2+4)^(1/2))*2^(
1/2)/(d*g^(1/2)-(d^2*g+2*e^2)^(1/2)))/g^(1/2)-b*n*polylog(2,-e*(1/2*x*g^(1/
2)*2^(1/2)+1/2*(2*g*x^2+4)^(1/2))*2^(1/2)/(d*g^(1/2)+(d^2*g+2*e^2)^(1/2)))/
g^(1/2)
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {221, 2451, 12, 5827, 5680, 2221, 2317, 2438}

$$\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{2+gx^2}}\,dx = \frac{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}} - \frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{2}ee^{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{gd^2+2e^2}}\right)}{\sqrt{g}} - \frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{2}ee^{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{\sqrt{g}d+\sqrt{gd^2+2e^2}}\right)}{\sqrt{g}} - \frac{bn\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(\frac{\sqrt{2}ee^{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{d^2g+2e^2}}+1\right)}{\sqrt{g}} - \frac{bn\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(\frac{\sqrt{2}ee^{\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{\sqrt{d^2g+2e^2}+d\sqrt{g}}+1\right)}{\sqrt{g}} + \frac{bn\text{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)^2}{2\sqrt{g}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/Sqrt[2 + g*x^2],x]
```

```
[Out] (b*n*ArcSinh[(Sqrt[g]*x)/Sqrt[2]]^2)/(2*Sqrt[g]) - (b*n*ArcSinh[(Sqrt[g]*x)
/Sqrt[2]]*Log[1 + (Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2]])/(d*Sqrt[g] - S
qrt[2*e^2 + d^2*g])])/Sqrt[g] - (b*n*ArcSinh[(Sqrt[g]*x)/Sqrt[2]]*Log[1 + (
Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2]])/(d*Sqrt[g] + Sqrt[2*e^2 + d^2*g])
])/Sqrt[g] + (ArcSinh[(Sqrt[g]*x)/Sqrt[2]]*(a + b*Log[c*(d + e*x)^n]))/Sqrt
[g] - (b*n*PolyLog[2, -((Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2]])/(d*Sqrt[
g] - Sqrt[2*e^2 + d^2*g]))])/Sqrt[g] - (b*n*PolyLog[2, -((Sqrt[2]*e*E^ArcSi
nh[(Sqrt[g]*x)/Sqrt[2]])/(d*Sqrt[g] + Sqrt[2*e^2 + d^2*g]))])/Sqrt[g]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
```

```
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}} - (ben)\int \frac{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}{\sqrt{g}(d+ex)}\,dx$$

$$= \frac{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}} - \frac{(ben)\int \frac{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}{d+ex}\,dx}{\sqrt{g}}$$

$$= \frac{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}} - \frac{(ben)\text{Subst}\left(\int \frac{x\cosh(x)}{\frac{d\sqrt{g}}{\sqrt{2}}+e\sinh(x)}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\right)}{\sqrt{g}}$$

$$= \frac{bn\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)^2}{2\sqrt{g}} + \frac{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}} - \frac{(ben)\text{Subst}\left(\int \frac{e^x x}{ee^x+\frac{d\sqrt{g}}{\sqrt{2}}-\frac{\sqrt{2e^2+d^2g}}{\sqrt{2}}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\right)}{\sqrt{g}} - \frac{(ben)\text{Subst}\left(\int \frac{e^x x}{ee^x+\frac{d\sqrt{g}}{\sqrt{2}}+\frac{\sqrt{2e^2+d^2g}}{\sqrt{2}}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\right)}{\sqrt{g}}$$

$$= \frac{bn\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)^2}{2\sqrt{g}} - \frac{bn\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}} - \frac{bn\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}+\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}} + \frac{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}} + \frac{(bn)\text{Subst}\left(\int \log\left(1+\frac{ee^x}{\frac{d\sqrt{g}}{\sqrt{2}}-\frac{\sqrt{2e^2+d^2g}}{\sqrt{2}}}\right)dx, x, \sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\right)}{\sqrt{g}} + \frac{(bn)\text{Subst}\left(\int \log\left(1+\frac{ee^x}{\frac{d\sqrt{g}}{\sqrt{2}}+\frac{\sqrt{2e^2+d^2g}}{\sqrt{2}}}\right)dx, x, \sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\right)}{\sqrt{g}}$$

$$= \frac{bn\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)^2}{2\sqrt{g}} - \frac{bn\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}}$$
$$- \frac{bn\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}+\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}}$$
$$+ \frac{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}}$$
$$+ \frac{(bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{\frac{d\sqrt{g}}{\sqrt{2}}-\frac{\sqrt{2e^2+d^2g}}{\sqrt{2}}}\right)}{x}dx, x, e^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}\right)}{\sqrt{g}}$$
$$+ \frac{(bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{\frac{d\sqrt{g}}{\sqrt{2}}+\frac{\sqrt{2e^2+d^2g}}{\sqrt{2}}}\right)}{x}dx, x, e^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}\right)}{\sqrt{g}}$$

$$= \frac{bn\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)^2}{2\sqrt{g}} - \frac{bn\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}}$$
$$- \frac{bn\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}+\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}}$$
$$+ \frac{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}}$$
$$- \frac{bn\text{Li}_2\left(-\frac{\sqrt{2}ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}} - \frac{bn\text{Li}_2\left(-\frac{\sqrt{2}ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}+\sqrt{2e^2+d^2g}}\right)}{\sqrt{g}}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.84

$$\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{2+gx^2}}\,dx = \frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)\left(2a+bn\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)-2bn\log\left(1+\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}-\sqrt{2e^2+d^2g}}\right)-2bn\log\left(1+\frac{\sqrt{2}ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{2}}\right)}}{d\sqrt{g}+\sqrt{2e^2+d^2g}}\right)+\cdots}{2\sqrt{g}}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/Sqrt[2 + g*x^2],x]
```

```
[Out] (ArcSinh[(Sqrt[g]*x)/Sqrt[2]]*(2*a + b*n*ArcSinh[(Sqrt[g]*x)/Sqrt[2]] - 2*b
*n*Log[1 + (Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2]])/(d*Sqrt[g] - Sqrt[2*e
^2 + d^2*g])] - 2*b*n*Log[1 + (Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2]])/(d
*Sqrt[g] + Sqrt[2*e^2 + d^2*g])] + 2*b*Log[c*(d + e*x)^n]) - 2*b*n*PolyLog[
2, (Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2]])/(-(d*Sqrt[g]) + Sqrt[2*e^2 +
d^2*g])] - 2*b*n*PolyLog[2, -((Sqrt[2]*e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[2]])/(d
*Sqrt[g] + Sqrt[2*e^2 + d^2*g]))])/(2*Sqrt[g])
```

## Maple [F]

$$\int \frac{a+b\ln(c(ex+d)^n)}{\sqrt{g\,x^2+2}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x^2+2)^(1/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))/(g*x^2+2)^(1/2),x)
```

## Fricas [F]

$$\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{2+gx^2}}\,dx = \int \frac{b\log((ex+d)^n c)+a}{\sqrt{gx^2+2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(g*x^2 + 2)*b*log((e*x + d)^n*c) + sqrt(g*x^2 + 2)*a)/(g*x^2
+ 2), x)
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2+gx^2}}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{gx^2+2}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x**2+2)**(1/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))/sqrt(g*x**2 + 2), x)
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2+gx^2}}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\sqrt{gx^2+2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+2)^(1/2),x, algorithm="maxima")
[Out] b*integrate((log((e*x + d)^n) + log(c))/sqrt(g*x^2 + 2), x) + a*arcsinh(1/2
*sqrt(2)*sqrt(g)*x)/sqrt(g)
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2+gx^2}}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\sqrt{gx^2+2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/sqrt(g*x^2 + 2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{\sqrt{g\,x^2+2}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(g*x^2 + 2)^(1/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(g*x^2 + 2)^(1/2), x)
```

# 3.276 $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f+gx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 506

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx = \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)^2}{2\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}-\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}+\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}} + \frac{\sqrt{f}\sqrt{1+\frac{gx^2}{f}}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}-\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}+\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}}$$

```
[Out] 1/2*b*n*arcsinh(x*g^(1/2)/f^(1/2))^2*f^(1/2)*(1+g*x^2/f)^(1/2)/g^(1/2)/(g*x
^2+f)^(1/2)+arcsinh(x*g^(1/2)/f^(1/2))*(a+b*ln(c*(e*x+d)^n))*f^(1/2)*(1+g*x
^2/f)^(1/2)/g^(1/2)/(g*x^2+f)^(1/2)-b*n*arcsinh(x*g^(1/2)/f^(1/2))*ln(1+e*(
x*g^(1/2)/f^(1/2)+(1+g*x^2/f)^(1/2))*f^(1/2)/(d*g^(1/2)-(d^2*g+e^2*f)^(1/2)
```

```
))*f^(1/2)*(1+g*x^2/f)^(1/2)/g^(1/2)/(g*x^2+f)^(1/2)-b*n*arcsinh(x*g^(1/2)/
f^(1/2))*ln(1+e*(x*g^(1/2)/f^(1/2)+(1+g*x^2/f)^(1/2))*f^(1/2)/(d*g^(1/2)+(d
^2*g+e^2*f)^(1/2)))*f^(1/2)*(1+g*x^2/f)^(1/2)/g^(1/2)/(g*x^2+f)^(1/2)-b*n*p
olylog(2,-e*(x*g^(1/2)/f^(1/2)+(1+g*x^2/f)^(1/2))*f^(1/2)/(d*g^(1/2)-(d^2*g
+e^2*f)^(1/2)))*f^(1/2)*(1+g*x^2/f)^(1/2)/g^(1/2)/(g*x^2+f)^(1/2)-b*n*polyl
og(2,-e*(x*g^(1/2)/f^(1/2)+(1+g*x^2/f)^(1/2))*f^(1/2)/(d*g^(1/2)+(d^2*g+e^2
*f)^(1/2)))*f^(1/2)*(1+g*x^2/f)^(1/2)/g^(1/2)/(g*x^2+f)^(1/2)
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2453, 221, 2451, 12, 5827, 5680, 2221, 2317, 2438}

$$\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f+gx^2}}\,dx = \frac{\sqrt{f}\sqrt{\frac{gx^2}{f}+1}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{\frac{gx^2}{f}+1}\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}-\sqrt{gd^2+e^2f}}\right)}{\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{\frac{gx^2}{f}+1}\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{\sqrt{g}d+\sqrt{gd^2+e^2f}}\right)}{\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{\frac{gx^2}{f}+1}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{e\sqrt{f}e^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}}{d\sqrt{g}-\sqrt{d^2g+e^2f}}+1\right)}{\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{\frac{gx^2}{f}+1}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{e\sqrt{f}e^{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}}{\sqrt{d^2g+e^2f}+d\sqrt{g}}+1\right)}{\sqrt{g}\sqrt{f+gx^2}} + \frac{b\sqrt{f}n\sqrt{\frac{gx^2}{f}+1}\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)^2}{2\sqrt{g}\sqrt{f+gx^2}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/Sqrt[f + g*x^2],x]
```

```
[Out] (b*Sqrt[f]*n*Sqrt[1 + (g*x^2)/f]*ArcSinh[(Sqrt[g]*x)/Sqrt[f]]^2)/(2*Sqrt[g]
*Sqrt[f + g*x^2]) - (b*Sqrt[f]*n*Sqrt[1 + (g*x^2)/f]*ArcSinh[(Sqrt[g]*x)/Sq
rt[f]]*Log[1 + (e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[f]]*Sqrt[f])/(d*Sqrt[g] - Sqrt
[e^2*f + d^2*g])])/(Sqrt[g]*Sqrt[f + g*x^2]) - (b*Sqrt[f]*n*Sqrt[1 + (g*x^2
)/f]*ArcSinh[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + (e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[f]]
*Sqrt[f])/(d*Sqrt[g] + Sqrt[e^2*f + d^2*g])])/(Sqrt[g]*Sqrt[f + g*x^2]) + (
```

```
Sqrt[f]*Sqrt[1 + (g*x^2)/f]*ArcSinh[(Sqrt[g]*x)/Sqrt[f]]*(a + b*Log[c*(d +
e*x)^n]))/(Sqrt[g]*Sqrt[f + g*x^2]) - (b*Sqrt[f]*n*Sqrt[1 + (g*x^2)/f]*Poly
Log[2, -((e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[f]]*Sqrt[f])/(d*Sqrt[g] - Sqrt[e^2*f
 + d^2*g]))])/(Sqrt[g]*Sqrt[f + g*x^2]) - (b*Sqrt[f]*n*Sqrt[1 + (g*x^2)/f]*
PolyLog[2, -((e*E^ArcSinh[(Sqrt[g]*x)/Sqrt[f]]*Sqrt[f])/(d*Sqrt[g] + Sqrt[e
^2*f + d^2*g]))])/(Sqrt[g]*Sqrt[f + g*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 2453

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (g/f)*x^2]/Sqrt[f + g*x^2], Int[(a + b*
```

```
Log[c*(d + e*x)^n])/Sqrt[1 + (g/f)*x^2], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, n}, x] &&  !GtQ[f, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{gx^2}{f}}\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{1+\frac{gx^2}{f}}}\,dx}{\sqrt{f+gx^2}}$$

$$= \frac{\sqrt{f}\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}\sqrt{f+gx^2}} - \frac{\left(ben\sqrt{1+\frac{gx^2}{f}}\right)\int \frac{\sqrt{f}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}(d+ex)}\,dx}{\sqrt{f+gx^2}}$$

$$= \frac{\sqrt{f}\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}\sqrt{f+gx^2}} - \frac{\left(be\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\right)\int \frac{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex}\,dx}{\sqrt{g}\sqrt{f+gx^2}}$$

$$= \frac{\sqrt{f}\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}\sqrt{f+gx^2}}$$
$$- \frac{\left(be\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\right)\text{Subst}\left(\int \frac{x\cosh(x)}{\frac{d\sqrt{g}}{\sqrt{f}}+e\sinh(x)}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\right)}{\sqrt{g}\sqrt{f+gx^2}}$$

$$=\frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)^2}{2\sqrt{g}\sqrt{f+gx^2}}+\frac{\sqrt{f}\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}\sqrt{f+gx^2}}$$
$$-\frac{\left(be\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\right)\text{Subst}\left(\int\frac{e^xx}{ee^x+\frac{d\sqrt{g}}{\sqrt{f}}-\frac{\sqrt{e^2f+d^2g}}{\sqrt{f}}}dx,x,\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\right)}{\sqrt{g}\sqrt{f+gx^2}}$$
$$-\frac{\left(be\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\right)\text{Subst}\left(\int\frac{e^xx}{ee^x+\frac{d\sqrt{g}}{\sqrt{f}}+\frac{\sqrt{e^2f+d^2g}}{\sqrt{f}}}dx,x,\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\right)}{\sqrt{g}\sqrt{f+gx^2}}$$

$$=\frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)^2}{2\sqrt{g}\sqrt{f+gx^2}}$$
$$-\frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}-\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}}$$
$$-\frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}+\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}}$$
$$+\frac{\sqrt{f}\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}\sqrt{f+gx^2}}$$
$$+\frac{\left(b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\right)\text{Subst}\left(\int\log\left(1+\frac{ee^x}{\frac{d\sqrt{g}}{\sqrt{f}}-\frac{\sqrt{e^2f+d^2g}}{\sqrt{f}}}\right)dx,x,\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\right)}{\sqrt{g}\sqrt{f+gx^2}}$$
$$+\frac{\left(b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\right)\text{Subst}\left(\int\log\left(1+\frac{ee^x}{\frac{d\sqrt{g}}{\sqrt{f}}+\frac{\sqrt{e^2f+d^2g}}{\sqrt{f}}}\right)dx,x,\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\right)}{\sqrt{g}\sqrt{f+gx^2}}$$

$$
\begin{aligned}
&= \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)^2}{2\sqrt{g}\sqrt{f+gx^2}} \\
&\quad - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}-\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}} \\
&\quad - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}+\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}} \\
&\quad + \frac{\sqrt{f}\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}\sqrt{f+gx^2}} \\
&\quad + \frac{\left(b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{ex}{\frac{d\sqrt{g}}{\sqrt{f}}-\frac{\sqrt{e^2f+d^2g}}{\sqrt{f}}}\right)}{x}dx,x,e^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\right)}{\sqrt{g}\sqrt{f+gx^2}} \\
&\quad + \frac{\left(b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{ex}{\frac{d\sqrt{g}}{\sqrt{f}}+\frac{\sqrt{e^2f+d^2g}}{\sqrt{f}}}\right)}{x}dx,x,e^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\right)}{\sqrt{g}\sqrt{f+gx^2}}
\end{aligned}
$$

$$= \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)^2}{2\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}-\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}+\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}} + \frac{\sqrt{f}\sqrt{1+\frac{gx^2}{f}}\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\mathrm{Li}_2\left(-\frac{ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}-\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}} - \frac{b\sqrt{f}n\sqrt{1+\frac{gx^2}{f}}\mathrm{Li}_2\left(-\frac{ee^{\sinh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}\sqrt{f}}{d\sqrt{g}+\sqrt{e^2f+d^2g}}\right)}{\sqrt{g}\sqrt{f+gx^2}}$$

## Mathematica [F]

$$\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f+gx^2}}\,dx = \int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f+gx^2}}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/Sqrt[f + g*x^2],x]
[Out] Integrate[(a + b*Log[c*(d + e*x)^n])/Sqrt[f + g*x^2], x]
```

## Maple [F]

$$\int \frac{a+b\ln(c(ex+d)^n)}{\sqrt{g\,x^2+f}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^(1/2),x)
[Out] int((a+b*ln(c*(e*x+d)^n))/(g*x^2+f)^(1/2),x)
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\sqrt{gx^2+f}}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f)^(1/2),x, algorithm="fricas")

[Out] integral((sqrt(g*x^2 + f)*b*log((e*x + d)^n*c) + sqrt(g*x^2 + f)*a)/(g*x^2 + f), x)

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx$$

[In] integrate((a+b*ln(c*(e*x+d)**n))/(g*x**2+f)**(1/2),x)

[Out] Integral((a + b*log(c*(d + e*x)**n))/sqrt(f + g*x**2), x)

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\sqrt{gx^2+f}}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f)^(1/2),x, algorithm="maxima")

[Out] b*integrate((log((e*x + d)^n) + log(c))/sqrt(g*x^2 + f), x) + a*arcsinh(g*x/sqrt(f*g))/sqrt(g)

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{b\log\left((ex+d)^n c\right)+a}{\sqrt{gx^2+f}}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))/(g*x^2+f)^(1/2),x, algorithm="giac")

[Out] integrate((b*log((e*x + d)^n*c) + a)/sqrt(g*x^2 + f), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f+gx^2}}\,dx = \int \frac{a+b\,\ln\left(c\,(d+e\,x)^n\right)}{\sqrt{g\,x^2+f}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(f + g*x^2)^(1/2),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(f + g*x^2)^(1/2), x)
```

# 3.277 $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx$



## Optimal result

Integrand size = 34, antiderivative size = 278

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx = \frac{ibn\arcsin\left(\frac{gx}{2}\right)^2}{2g} - \frac{bn\arcsin\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{idg-\sqrt{4e^2-d^2g^2}}\right)}{g} - \frac{bn\arcsin\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{idg+\sqrt{4e^2-d^2g^2}}\right)}{g} + \frac{\arcsin\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g} + \frac{ibn\operatorname{PolyLog}\left(2,-\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{idg-\sqrt{4e^2-d^2g^2}}\right)}{g} + \frac{ibn\operatorname{PolyLog}\left(2,-\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{idg+\sqrt{4e^2-d^2g^2}}\right)}{g}$$

```
[Out] 1/2*I*b*n*arcsin(1/2*g*x)^2/g+arcsin(1/2*g*x)*(a+b*ln(c*(e*x+d)^n))/g-b*n*a
rcsin(1/2*g*x)*ln(1+2*e*(1/2*I*g*x+1/2*(-g^2*x^2+4)^(1/2))/(I*d*g-(-d^2*g^2
+4*e^2)^(1/2)))/g-b*n*arcsin(1/2*g*x)*ln(1+2*e*(1/2*I*g*x+1/2*(-g^2*x^2+4)^
(1/2))/(I*d*g+(-d^2*g^2+4*e^2)^(1/2)))/g+I*b*n*polylog(2,-2*e*(1/2*I*g*x+1/
2*(-g^2*x^2+4)^(1/2))/(I*d*g-(-d^2*g^2+4*e^2)^(1/2)))/g+I*b*n*polylog(2,-2*
e*(1/2*I*g*x+1/2*(-g^2*x^2+4)^(1/2))/(I*d*g+(-d^2*g^2+4*e^2)^(1/2)))/g
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {222, 2452, 4825, 4617, 2221, 2317, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx = \frac{\arcsin\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g} + \frac{ibn\,\text{PolyLog}\left(2,-\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{idg-\sqrt{4e^2-d^2g^2}}\right)}{g} + \frac{ibn\,\text{PolyLog}\left(2,-\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{idg+\sqrt{4e^2-d^2g^2}}\right)}{g} - \frac{bn\arcsin\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{-\sqrt{4e^2-d^2g^2}+idg}\right)}{g} - \frac{bn\arcsin\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\arcsin\left(\frac{gx}{2}\right)}}{\sqrt{4e^2-d^2g^2}+idg}\right)}{g} + \frac{ibn\arcsin\left(\frac{gx}{2}\right)^2}{2g}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(Sqrt[2 - g*x]*Sqrt[2 + g*x]),x]
```

```
[Out] ((I/2)*b*n*ArcSin[(g*x)/2]^2)/g - (b*n*ArcSin[(g*x)/2]*Log[1 + (2*e*E^(I*Ar
cSin[(g*x)/2]))/(I*d*g - Sqrt[4*e^2 - d^2*g^2])])/g - (b*n*ArcSin[(g*x)/2]*
Log[1 + (2*e*E^(I*ArcSin[(g*x)/2]))/(I*d*g + Sqrt[4*e^2 - d^2*g^2])])/g + (
ArcSin[(g*x)/2]*(a + b*Log[c*(d + e*x)^n]))/g + (I*b*n*PolyLog[2, (-2*e*E^(
I*ArcSin[(g*x)/2]))/(I*d*g - Sqrt[4*e^2 - d^2*g^2])])/g + (I*b*n*PolyLog[2,
 (-2*e*E^(I*ArcSin[(g*x)/2]))/(I*d*g + Sqrt[4*e^2 - d^2*g^2])])/g
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2452

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/(Sqrt[(f1_) + (g1_
.)*(x_)]*Sqrt[(f2_) + (g2_.)*(x_)]), x_Symbol] :> With[{u = IntHide[1/Sqrt[
f1*f2 + g1*g2*x^2], x]}, Simp[u*(a + b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n
, Int[SimplifyIntegrand[u/(d + e*x), x], x], x]] /; FreeQ[{a, b, c, d, e, f
1, g1, f2, g2, n}, x] && EqQ[f2*g1 + f1*g2, 0] && GtQ[f1, 0] && GtQ[f2, 0]
```

Rule 4617

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2
] + b*E^(I*(c + d*x)))), x], x] + Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/
(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sin^{-1}\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-(ben)\int\frac{\sin^{-1}\left(\frac{gx}{2}\right)}{dg+egx}\,dx\\ &= \frac{\sin^{-1}\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g}-(ben)\text{Subst}\left(\int\frac{x\cos(x)}{\frac{dg^2}{2}+eg\sin(x)}\,dx,x,\sin^{-1}\left(\frac{gx}{2}\right)\right)\end{aligned}$$

$$= \frac{ibn\sin^{-1}\left(\frac{gx}{2}\right)^2}{2g} + \frac{\sin^{-1}\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g} - (iben)\text{Subst}\left(\int \frac{e^{ix}x}{ee^{ix}g+\frac{1}{2}idg^2-\frac{1}{2}g\sqrt{4e^2-d^2g^2}}\,dx,x,\sin^{-1}\left(\frac{gx}{2}\right)\right) - (iben)\text{Subst}\left(\int \frac{e^{ix}x}{ee^{ix}g+\frac{1}{2}idg^2+\frac{1}{2}g\sqrt{4e^2-d^2g^2}}\,dx,x,\sin^{-1}\left(\frac{gx}{2}\right)\right)$$

$$= \frac{ibn\sin^{-1}\left(\frac{gx}{2}\right)^2}{2g} - \frac{bn\sin^{-1}\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\sin^{-1}\left(\frac{gx}{2}\right)}}{idg-\sqrt{4e^2-d^2g^2}}\right)}{g} - \frac{bn\sin^{-1}\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\sin^{-1}\left(\frac{gx}{2}\right)}}{idg+\sqrt{4e^2-d^2g^2}}\right)}{g} + \frac{\sin^{-1}\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g} + \frac{(bn)\text{Subst}\left(\int\log\left(1+\frac{ee^{ix}g}{\frac{1}{2}idg^2-\frac{1}{2}g\sqrt{4e^2-d^2g^2}}\right)dx,x,\sin^{-1}\left(\frac{gx}{2}\right)\right)}{g} + \frac{(bn)\text{Subst}\left(\int\log\left(1+\frac{ee^{ix}g}{\frac{1}{2}idg^2+\frac{1}{2}g\sqrt{4e^2-d^2g^2}}\right)dx,x,\sin^{-1}\left(\frac{gx}{2}\right)\right)}{g}$$

$$= \frac{ibn\sin^{-1}\left(\frac{gx}{2}\right)^2}{2g} - \frac{bn\sin^{-1}\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\sin^{-1}\left(\frac{gx}{2}\right)}}{idg-\sqrt{4e^2-d^2g^2}}\right)}{g} - \frac{bn\sin^{-1}\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\sin^{-1}\left(\frac{gx}{2}\right)}}{idg+\sqrt{4e^2-d^2g^2}}\right)}{g} + \frac{\sin^{-1}\left(\frac{gx}{2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g} - \frac{(ibn)\text{Subst}\left(\int\frac{\log\left(1+\frac{egx}{\frac{1}{2}idg^2-\frac{1}{2}g\sqrt{4e^2-d^2g^2}}\right)}{x}dx,x,e^{i\sin^{-1}\left(\frac{gx}{2}\right)}\right)}{g} - \frac{(ibn)\text{Subst}\left(\int\frac{\log\left(1+\frac{egx}{\frac{1}{2}idg^2+\frac{1}{2}g\sqrt{4e^2-d^2g^2}}\right)}{x}dx,x,e^{i\sin^{-1}\left(\frac{gx}{2}\right)}\right)}{g}$$

$$= \frac{ibn\sin^{-1}\left(\frac{gx}{2}\right)^2}{2g} - \frac{bn\sin^{-1}\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\sin^{-1}\left(\frac{gx}{2}\right)}}{idg-\sqrt{4e^2-d^2g^2}}\right)}{g} - \frac{bn\sin^{-1}\left(\frac{gx}{2}\right)\log\left(1+\frac{2ee^{i\sin^{-1}\left(\frac{gx}{2}\right)}}{idg+\sqrt{4e^2-d^2g^2}}\right)}{g} + \frac{\sin^{-1}\left(\frac{gx}{2}\right)(a+b\log(c(d+ex)^n))}{g} + \frac{ibn\text{Li}_2\left(-\frac{2ee^{i\sin^{-1}\left(\frac{gx}{2}\right)}}{idg-\sqrt{4e^2-d^2g^2}}\right)}{g} + \frac{ibn\text{Li}_2\left(-\frac{2ee^{i\sin^{-1}\left(\frac{gx}{2}\right)}}{idg+\sqrt{4e^2-d^2g^2}}\right)}{g}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx = \frac{a\arcsin\left(\frac{gx}{2}\right)}{g} + \frac{ibn\arcsin\left(\frac{gx}{2}\right)^2}{2g} - \frac{bn\arcsin\left(\frac{gx}{2}\right)\log\left(1+\frac{ee^{i\arcsin\left(\frac{gx}{2}\right)}g}{\frac{1}{2}idg^2-\frac{1}{2}g\sqrt{4e^2-d^2g^2}}\right)}{g} - \frac{bn\arcsin\left(\frac{gx}{2}\right)\log\left(1+\frac{ee^{i\arcsin\left(\frac{gx}{2}\right)}g}{\frac{1}{2}idg^2+\frac{1}{2}g\sqrt{4e^2-d^2g^2}}\right)}{g} + \frac{b\arcsin\left(\frac{gx}{2}\right)\log(c(d+ex)^n)}{g} + \frac{ibn\,\text{PolyLog}\left(2,\frac{2iee^{i\arcsin\left(\frac{gx}{2}\right)}}{dg-i\sqrt{4e^2-d^2g^2}}\right)}{g} + \frac{ibn\,\text{PolyLog}\left(2,\frac{2iee^{i\arcsin\left(\frac{gx}{2}\right)}}{dg+i\sqrt{4e^2-d^2g^2}}\right)}{g}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(Sqrt[2 - g*x]*Sqrt[2 + g*x]),x]
```

```
[Out] (a*ArcSin[(g*x)/2])/g + ((I/2)*b*n*ArcSin[(g*x)/2]^2)/g - (b*n*ArcSin[(g*x)
/2]*Log[1 + (e*E^(I*ArcSin[(g*x)/2])*g)/((I/2)*d*g^2 - (g*Sqrt[4*e^2 - d^2*
g^2])/2)])/g - (b*n*ArcSin[(g*x)/2]*Log[1 + (e*E^(I*ArcSin[(g*x)/2])*g)/((I
/2)*d*g^2 + (g*Sqrt[4*e^2 - d^2*g^2])/2)])/g + (b*ArcSin[(g*x)/2]*Log[c*(d
+ e*x)^n])/g + (I*b*n*PolyLog[2, ((2*I)*e*E^(I*ArcSin[(g*x)/2]))/(d*g - I*S
qrt[4*e^2 - d^2*g^2])])/g + (I*b*n*PolyLog[2, ((2*I)*e*E^(I*ArcSin[(g*x)/2]
))/(d*g + I*Sqrt[4*e^2 - d^2*g^2])])/g
```

## Maple [F]

$$\int \frac{a + b\ln\left(c(ex+d)^n\right)}{\sqrt{-gx+2}\,\sqrt{gx+2}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))/(-g*x+2)^(1/2)/(g*x+2)^(1/2),x)
[Out] int((a+b*ln(c*(e*x+d)^n))/(-g*x+2)^(1/2)/(g*x+2)^(1/2),x)
```

## Fricas [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{\sqrt{gx+2}\sqrt{-gx+2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(-g*x+2)^(1/2)/(g*x+2)^(1/2),x, algorithm=
"fricas")
[Out] integral(-(sqrt(g*x + 2)*sqrt(-g*x + 2)*b*log((e*x + d)^n*c) + sqrt(g*x + 2
)*sqrt(-g*x + 2)*a)/(g^2*x^2 - 4), x)
```

## Sympy [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx = \int \frac{a + b\log\left(c(d+ex)^n\right)}{\sqrt{-gx+2}\sqrt{gx+2}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(-g*x+2)**(1/2)/(g*x+2)**(1/2),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))/(sqrt(-g*x + 2)*sqrt(g*x + 2)), x)
```

## Maxima [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{\sqrt{2-gx}\sqrt{2+gx}}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{\sqrt{gx+2}\sqrt{-gx+2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(-g*x+2)^(1/2)/(g*x+2)^(1/2),x, algorithm=
"maxima")
[Out] b*integrate((log((e*x + d)^n) + log(c))/(sqrt(g*x + 2)*sqrt(-g*x + 2)), x)
+ a*arcsin(1/2*g*x)/g
```

## Giac [F]

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{\sqrt{2 - gx}\sqrt{2 + gx}}\, dx = \int \frac{b\log\left((ex + d)^n c\right) + a}{\sqrt{gx + 2}\sqrt{-gx + 2}}\, dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(-g*x+2)^(1/2)/(g*x+2)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)/(sqrt(g*x + 2)*sqrt(-g*x + 2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex)^n\right)}{\sqrt{2 - gx}\sqrt{2 + gx}}\, dx = \int \frac{a + b\,\ln\left(c\,{\left(d + e\,x\right)}^n\right)}{\sqrt{2 - g\,x}\,\sqrt{g\,x + 2}}\, dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/((2 - g*x)^(1/2)*(g*x + 2)^(1/2)),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))/((2 - g*x)^(1/2)*(g*x + 2)^(1/2)), x)
```

# 3.278 $\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx$



## Optimal result

Integrand size = 34, antiderivative size = 510

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx = \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\arcsin\left(\frac{gx}{f}\right)^2}{2g\sqrt{f-gx}\sqrt{f+gx}} - \frac{bfn\sqrt{1-\frac{g^2x^2}{f^2}}\arcsin\left(\frac{gx}{f}\right)\log\left(1+\frac{ee^{i\arcsin\left(\frac{gx}{f}\right)}f}{idg-\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}} - \frac{bfn\sqrt{1-\frac{g^2x^2}{f^2}}\arcsin\left(\frac{gx}{f}\right)\log\left(1+\frac{ee^{i\arcsin\left(\frac{gx}{f}\right)}f}{idg+\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}} + \frac{f\sqrt{1-\frac{g^2x^2}{f^2}}\arcsin\left(\frac{gx}{f}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g\sqrt{f-gx}\sqrt{f+gx}} + \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\operatorname{PolyLog}\left(2,-\frac{ee^{i\arcsin\left(\frac{gx}{f}\right)}f}{idg-\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}} + \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\operatorname{PolyLog}\left(2,-\frac{ee^{i\arcsin\left(\frac{gx}{f}\right)}f}{idg+\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}}$$

```
[Out] 1/2*I*b*f*n*arcsin(g*x/f)^2*(1-g^2*x^2/f^2)^(1/2)/g/(-g*x+f)^(1/2)/(g*x+f)^
(1/2)+f*arcsin(g*x/f)*(a+b*ln(c*(e*x+d)^n))*(1-g^2*x^2/f^2)^(1/2)/g/(-g*x+f
)^(1/2)/(g*x+f)^(1/2)-b*f*n*arcsin(g*x/f)*ln(1+e*(I*g*x/f+(1-g^2*x^2/f^2)^(
1/2))*f/(I*d*g-(-d^2*g^2+e^2*f^2)^(1/2)))*(1-g^2*x^2/f^2)^(1/2)/g/(-g*x+f)^
(1/2)/(g*x+f)^(1/2)-b*f*n*arcsin(g*x/f)*ln(1+e*(I*g*x/f+(1-g^2*x^2/f^2)^(1/
2))*f/(I*d*g+(-d^2*g^2+e^2*f^2)^(1/2)))*(1-g^2*x^2/f^2)^(1/2)/g/(-g*x+f)^(1
/2)/(g*x+f)^(1/2)+I*b*f*n*polylog(2,-e*(I*g*x/f+(1-g^2*x^2/f^2)^(1/2))*f/(I
```

```
*d*g-(-d^2*g^2+e^2*f^2)^(1/2)))*(1-g^2*x^2/f^2)^(1/2)/g/(-g*x+f)^(1/2)/(g*x
+f)^(1/2)+I*b*f*n*polylog(2,-e*(I*g*x/f+(1-g^2*x^2/f^2)^(1/2))*f/(I*d*g+(-d
^2*g^2+e^2*f^2)^(1/2)))*(1-g^2*x^2/f^2)^(1/2)/g/(-g*x+f)^(1/2)/(g*x+f)^(1/2
)
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {2454, 222, 2451, 12, 4825, 4617, 2221, 2317, 2438}

$$\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx = \frac{f\sqrt{1-\frac{g^2x^2}{f^2}}\arcsin\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{f-gx}\sqrt{f+gx}} + \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\,\text{PolyLog}\left(2,-\frac{ee^{i\arcsin\left(\frac{gx}{f}\right)}f}{idg-\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}} + \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\,\text{PolyLog}\left(2,-\frac{ee^{i\arcsin\left(\frac{gx}{f}\right)}f}{idg+\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}} - \frac{bfn\sqrt{1-\frac{g^2x^2}{f^2}}\arcsin\left(\frac{gx}{f}\right)\log\left(1+\frac{efe^{i\arcsin\left(\frac{gx}{f}\right)}}{-\sqrt{e^2f^2-d^2g^2}+idg}\right)}{g\sqrt{f-gx}\sqrt{f+gx}} - \frac{bfn\sqrt{1-\frac{g^2x^2}{f^2}}\arcsin\left(\frac{gx}{f}\right)\log\left(1+\frac{efe^{i\arcsin\left(\frac{gx}{f}\right)}}{\sqrt{e^2f^2-d^2g^2}+idg}\right)}{g\sqrt{f-gx}\sqrt{f+gx}} + \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\arcsin\left(\frac{gx}{f}\right)^2}{2g\sqrt{f-gx}\sqrt{f+gx}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/(Sqrt[f - g*x]*Sqrt[f + g*x]),x]
```

```
[Out] ((I/2)*b*f*n*Sqrt[1 - (g^2*x^2)/f^2]*ArcSin[(g*x)/f]^2)/(g*Sqrt[f - g*x]*Sq
rt[f + g*x]) - (b*f*n*Sqrt[1 - (g^2*x^2)/f^2]*ArcSin[(g*x)/f]*Log[1 + (e*E^
(I*ArcSin[(g*x)/f])*f)/(I*d*g - Sqrt[e^2*f^2 - d^2*g^2])])/(g*Sqrt[f - g*x]
*Sqrt[f + g*x]) - (b*f*n*Sqrt[1 - (g^2*x^2)/f^2]*ArcSin[(g*x)/f]*Log[1 + (e
*E^(I*ArcSin[(g*x)/f])*f)/(I*d*g + Sqrt[e^2*f^2 - d^2*g^2])])/(g*Sqrt[f - g
*x]*Sqrt[f + g*x]) + (f*Sqrt[1 - (g^2*x^2)/f^2]*ArcSin[(g*x)/f]*(a + b*Log[
c*(d + e*x)^n]))/(g*Sqrt[f - g*x]*Sqrt[f + g*x]) + (I*b*f*n*Sqrt[1 - (g^2*x
^2)/f^2]*PolyLog[2, -((e*E^(I*ArcSin[(g*x)/f])*f)/(I*d*g - Sqrt[e^2*f^2 - d
^2*g^2]))])/(g*Sqrt[f - g*x]*Sqrt[f + g*x]) + (I*b*f*n*Sqrt[1 - (g^2*x^2)/f
^2]*PolyLog[2, -((e*E^(I*ArcSin[(g*x)/f])*f)/(I*d*g + Sqrt[e^2*f^2 - d^2*g^
2]))])/(g*Sqrt[f - g*x]*Sqrt[f + g*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/(Sqrt[(f1_) + (g1_
.)*(x_)]*Sqrt[(f2_) + (g2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + g1*(g2/(f1*
f2))*x^2]/(Sqrt[f1 + g1*x]*Sqrt[f2 + g2*x]), Int[(a + b*Log[c*(d + e*x)^n])
/Sqrt[1 + g1*(g2/(f1*f2))*x^2], x], x] /; FreeQ[{a, b, c, d, e, f1, g1, f2,
 g2, n}, x] && EqQ[f2*g1 + f1*g2, 0]
```

Rule 4617

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
```

```
))), x] + (Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2
] + b*E^(I*(c + d*x)))), x], x] + Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/
(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{g^2x^2}{f^2}}\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{1-\frac{g^2x^2}{f^2}}}\,dx}{\sqrt{f-gx}\sqrt{f+gx}}$$

$$= \frac{f\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{f-gx}\sqrt{f+gx}} - \frac{\left(ben\sqrt{1-\frac{g^2x^2}{f^2}}\right)\int\frac{f\sin^{-1}\left(\frac{gx}{f}\right)}{dg+egx}\,dx}{\sqrt{f-gx}\sqrt{f+gx}}$$

$$= \frac{f\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{f-gx}\sqrt{f+gx}} - \frac{\left(befn\sqrt{1-\frac{g^2x^2}{f^2}}\right)\int\frac{\sin^{-1}\left(\frac{gx}{f}\right)}{dg+egx}\,dx}{\sqrt{f-gx}\sqrt{f+gx}}$$

$$= \frac{f\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{f-gx}\sqrt{f+gx}} - \frac{\left(befn\sqrt{1-\frac{g^2x^2}{f^2}}\right)\text{Subst}\left(\int\frac{x\cos(x)}{\frac{dg^2}{f}+eg\sin(x)}\,dx,x,\sin^{-1}\left(\frac{gx}{f}\right)\right)}{\sqrt{f-gx}\sqrt{f+gx}}$$

$$= \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)^2}{2g\sqrt{f-gx}\sqrt{f+gx}} + \frac{f\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{f-gx}\sqrt{f+gx}} - \frac{\left(ibefn\sqrt{1-\frac{g^2x^2}{f^2}}\right)\text{Subst}\left(\int\frac{e^{ix}x}{ee^{ix}g+\frac{idg^2}{f}-\frac{g\sqrt{e^2f^2-d^2g^2}}{f}}\,dx,x,\sin^{-1}\left(\frac{gx}{f}\right)\right)}{\sqrt{f-gx}\sqrt{f+gx}} - \frac{\left(ibefn\sqrt{1-\frac{g^2x^2}{f^2}}\right)\text{Subst}\left(\int\frac{e^{ix}x}{ee^{ix}g+\frac{idg^2}{f}+\frac{g\sqrt{e^2f^2-d^2g^2}}{f}}\,dx,x,\sin^{-1}\left(\frac{gx}{f}\right)\right)}{\sqrt{f-gx}\sqrt{f+gx}}$$

$$= \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)^2}{2g\sqrt{f-gx}\sqrt{f+gx}} - \frac{bfn\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)\log\left(1+\frac{ee^{i\sin^{-1}\left(\frac{gx}{f}\right)}f}{idg-\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}}$$
$$- \frac{bfn\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)\log\left(1+\frac{ee^{i\sin^{-1}\left(\frac{gx}{f}\right)}f}{idg+\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}}$$
$$+ \frac{f\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{f-gx}\sqrt{f+gx}}$$
$$+ \frac{\left(bfn\sqrt{1-\frac{g^2x^2}{f^2}}\right)\text{Subst}\left(\int\log\left(1+\frac{ee^{ix}g}{\frac{idg^2}{f}-\frac{g\sqrt{e^2f^2-d^2g^2}}{f}}\right)dx, x, \sin^{-1}\left(\frac{gx}{f}\right)\right)}{g\sqrt{f-gx}\sqrt{f+gx}}$$
$$+ \frac{\left(bfn\sqrt{1-\frac{g^2x^2}{f^2}}\right)\text{Subst}\left(\int\log\left(1+\frac{ee^{ix}g}{\frac{idg^2}{f}+\frac{g\sqrt{e^2f^2-d^2g^2}}{f}}\right)dx, x, \sin^{-1}\left(\frac{gx}{f}\right)\right)}{g\sqrt{f-gx}\sqrt{f+gx}}$$

$$= \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)^2}{2g\sqrt{f-gx}\sqrt{f+gx}} - \frac{bfn\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)\log\left(1+\frac{ee^{i\sin^{-1}\left(\frac{gx}{f}\right)}f}{idg-\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}}$$
$$- \frac{bfn\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)\log\left(1+\frac{ee^{i\sin^{-1}\left(\frac{gx}{f}\right)}f}{idg+\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}}$$
$$+ \frac{f\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{f-gx}\sqrt{f+gx}}$$
$$- \frac{\left(ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{egx}{\frac{idg^2}{f}-\frac{g\sqrt{e^2f^2-d^2g^2}}{f}}\right)}{x}dx, x, e^{i\sin^{-1}\left(\frac{gx}{f}\right)}\right)}{g\sqrt{f-gx}\sqrt{f+gx}}$$
$$- \frac{\left(ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{egx}{\frac{idg^2}{f}+\frac{g\sqrt{e^2f^2-d^2g^2}}{f}}\right)}{x}dx, x, e^{i\sin^{-1}\left(\frac{gx}{f}\right)}\right)}{g\sqrt{f-gx}\sqrt{f+gx}}$$

$$= \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)^2}{2g\sqrt{f-gx}\sqrt{f+gx}} - \frac{bfn\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)\log\left(1+\frac{ee^{i\sin^{-1}\left(\frac{gx}{f}\right)}f}{idg-\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}}$$
$$- \frac{bfn\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)\log\left(1+\frac{ee^{i\sin^{-1}\left(\frac{gx}{f}\right)}f}{idg+\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}}$$
$$+ \frac{f\sqrt{1-\frac{g^2x^2}{f^2}}\sin^{-1}\left(\frac{gx}{f}\right)(a+b\log(c(d+ex)^n))}{g\sqrt{f-gx}\sqrt{f+gx}}$$
$$+ \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\operatorname{Li}_2\left(-\frac{ee^{i\sin^{-1}\left(\frac{gx}{f}\right)}f}{idg-\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}} + \frac{ibfn\sqrt{1-\frac{g^2x^2}{f^2}}\operatorname{Li}_2\left(-\frac{ee^{i\sin^{-1}\left(\frac{gx}{f}\right)}f}{idg+\sqrt{e^2f^2-d^2g^2}}\right)}{g\sqrt{f-gx}\sqrt{f+gx}}$$

## Mathematica [A] (warning: unable to verify)

Time = 10.18 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.24

$$\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx = \frac{\arctan\left(\frac{gx}{\sqrt{f-gx}\sqrt{f+gx}}\right)(a-bn\log(d+ex)+b\log(c(d+ex)^n))}{g}$$
$$- \frac{bn\sqrt{f-gx}\left(2fg(d+ex)\sqrt{\frac{f+gx}{f-gx}}\arctan\left(\frac{1}{\sqrt{\frac{f+gx}{f-gx}}}\right)\log(d+ex)+(f+gx)\left(dg+ef\cos\left(2\arctan\left(\frac{1}{\sqrt{\frac{f}{f}}}\right.\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/(Sqrt[f - g*x]*Sqrt[f + g*x]),x]
```

```
[Out] (ArcTan[(g*x)/(Sqrt[f - g*x]*Sqrt[f + g*x])]*(a - b*n*Log[d + e*x] + b*Log[
c*(d + e*x)^n]))/g - (b*n*Sqrt[f - g*x]*(2*f*g*(d + e*x)*Sqrt[(f + g*x)/(f
- g*x)]*ArcTan[1/Sqrt[(f + g*x)/(f - g*x)]]*Log[d + e*x] + (f + g*x)*(d*g +
 e*f*Cos[2*ArcTan[1/Sqrt[(f + g*x)/(f - g*x)]]])*Csc[2*ArcTan[1/Sqrt[(f + g
*x)/(f - g*x)]]]*((2*I)*ArcTan[1/Sqrt[(f + g*x)/(f - g*x)]]^2 - (4*I)*ArcSi
n[Sqrt[1 + (d*g)/(e*f)]/Sqrt[2]]*ArcTan[(-(e*f) + d*g)/(Sqrt[-(e^2*f^2) + d
^2*g^2]*Sqrt[(f + g*x)/(f - g*x)])] - 2*(ArcSin[Sqrt[1 + (d*g)/(e*f)]/Sqrt[
2]] + ArcTan[1/Sqrt[(f + g*x)/(f - g*x)]])*Log[1 + (E^((2*I)*ArcTan[1/Sqrt[
(f + g*x)/(f - g*x)]])*(d*g - Sqrt[-(e^2*f^2) + d^2*g^2]))/(e*f)] + 2*(ArcS
in[Sqrt[1 + (d*g)/(e*f)]/Sqrt[2]] - ArcTan[1/Sqrt[(f + g*x)/(f - g*x)]])*Lo
g[1 + (E^((2*I)*ArcTan[1/Sqrt[(f + g*x)/(f - g*x)]])*(d*g + Sqrt[-(e^2*f^2)
 + d^2*g^2]))/(e*f)] + I*(PolyLog[2, (E^((2*I)*ArcTan[1/Sqrt[(f + g*x)/(f -
 g*x)]])*(-(d*g) + Sqrt[-(e^2*f^2) + d^2*g^2]))/(e*f)] + PolyLog[2, -((E^((
2*I)*ArcTan[1/Sqrt[(f + g*x)/(f - g*x)]])*(d*g + Sqrt[-(e^2*f^2) + d^2*g^2]
))/(e*f))]))))/(f*g^2*(d + e*x)*Sqrt[f + g*x])
```

## Maple [F]

$$\int \frac{a+b\ln\left(c(ex+d)^{n}\right)}{\sqrt{-gx+f}\,\sqrt{gx+f}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))/(-g*x+f)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))/(-g*x+f)^(1/2)/(g*x+f)^(1/2),x)
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx=\int \frac{b\log\left((ex+d)^{n}c\right)+a}{\sqrt{gx+f}\sqrt{-gx+f}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(-g*x+f)^(1/2)/(g*x+f)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral(-(sqrt(g*x + f)*sqrt(-g*x + f)*b*log((e*x + d)^n*c) + sqrt(g*x + f
)*sqrt(-g*x + f)*a)/(g^2*x^2 - f^2), x)
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx=\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(-g*x+f)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))/(sqrt(f - g*x)*sqrt(f + g*x)), x)
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex)^{n}\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx=\int \frac{b\log\left((ex+d)^{n}c\right)+a}{\sqrt{gx+f}\sqrt{-gx+f}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(-g*x+f)^(1/2)/(g*x+f)^(1/2),x, algorithm=
"maxima")
```

```
[Out] b*integrate((log((e*x + d)^n) + log(c))/(sqrt(g*x + f)*sqrt(-g*x + f)), x)
+ a*arcsin(g*x/f)/g
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx = \int \frac{b\log\left((ex+d)^n c\right)+a}{\sqrt{gx+f}\sqrt{-gx+f}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(-g*x+f)^(1/2)/(g*x+f)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)/(sqrt(g*x + f)*sqrt(-g*x + f)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{f-gx}\sqrt{f+gx}}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{\sqrt{f+g\,x}\,\sqrt{f-g\,x}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(1/2)*(f - g*x)^(1/2)),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))/((f + g*x)^(1/2)*(f - g*x)^(1/2)), x)
```

# 3.279 $\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 24

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

```
[Out] 1/2*polylog(2,1-2*e/(f*x+e))/e/f
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2449, 2352}

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

```
[In] Int[Log[(2*e)/(e + f*x)]/(e^2 - f^2*x^2),x]
[Out] PolyLog[2, 1 - (2*e)/(e + f*x)]/(2*e*f)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\log(2ex)}{1-2ex}\,dx, x, \frac{1}{e+fx}\right)}{f}$$
$$= \frac{\text{Li}_2\left(1-\frac{2e}{e+fx}\right)}{2ef}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{\text{PolyLog}\left(2, \frac{-e+fx}{e+fx}\right)}{2ef}$$

```
[In] Integrate[Log[(2*e)/(e + f*x)]/(e^2 - f^2*x^2),x]
[Out] PolyLog[2, (-e + f*x)/(e + f*x)]/(2*e*f)
```

## Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$ | 20 |
| default | $\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$ | 20 |
| risch | $\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$ | 20 |
| parts | $\frac{\ln\left(\frac{2e}{fx+e}\right)\ln(fx+e)}{2ef}-\frac{\ln\left(\frac{2e}{fx+e}\right)\ln(-fx+e)}{2ef}+\frac{f\left(\frac{\ln(fx+e)^2}{2e\,f^2}+\frac{-\operatorname{dilog}\left(-\frac{-fx-e}{2e}\right)-\ln(-fx+e)\ln\left(-\frac{-fx-e}{2e}\right)}{e\,f^2}\right)}{2}$ | 120 |

```
[In] int(ln(2*e/(f*x+e))/(-f^2*x^2+e^2),x,method=_RETURNVERBOSE)
[Out] 1/2/f/e*dilog(2*e/(f*x+e))
```

## Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{\operatorname{Li}_2\left(-\frac{2\,e}{fx+e}+1\right)}{2\,ef}$$

```
[In] integrate(log(2*e/(f*x+e))/(-f^2*x^2+e^2),x, algorithm="fricas")
[Out] 1/2*dilog(-2*e/(f*x + e) + 1)/(e*f)
```

## Sympy [F]

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = -\int \frac{\log(2)}{-e^2+f^2x^2}\,dx - \int \frac{\log\left(\frac{e}{e+fx}\right)}{-e^2+f^2x^2}\,dx$$

```
[In] integrate(ln(2*e/(f*x+e))/(-f**2*x**2+e**2),x)
[Out] -Integral(log(2)/(-e**2 + f**2*x**2), x) - Integral(log(e/(e + f*x))/(-e**2
 + f**2*x**2), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(21) = 42.

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.00

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{1}{4}\,f\left(\frac{\log(fx+e)^2-2\,\log(fx+e)\log(fx-e)}{ef^2}+\frac{2\left(\log(fx+e)\log\left(-\frac{fx+e}{2\,e}+1\right)+\operatorname{Li}_2\left(\frac{fx+e}{2\,e}\right)\right)}{ef^2}\right) + \frac{1}{2}\left(\frac{\log(fx+e)}{ef}-\frac{\log(fx-e)}{ef}\right)\log\left(\frac{2\,e}{fx+e}\right)$$

```
[In] integrate(log(2*e/(f*x+e))/(-f^2*x^2+e^2),x, algorithm="maxima")
[Out] 1/4*f*((log(f*x + e)^2 - 2*log(f*x + e)*log(f*x - e))/(e*f^2) + 2*(log(f*x
+ e)*log(-1/2*(f*x + e)/e + 1) + dilog(1/2*(f*x + e)/e))/(e*f^2)) + 1/2*(lo
g(f*x + e)/(e*f) - log(f*x - e)/(e*f))*log(2*e/(f*x + e))
```

## Giac [F]

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2 - f^2x^2}\,dx = \int -\frac{\log\left(\frac{2\,e}{fx+e}\right)}{f^2x^2 - e^2}\,dx$$

```
[In] integrate(log(2*e/(f*x+e))/(-f^2*x^2+e^2),x, algorithm="giac")
[Out] integrate(-log(2*e/(f*x + e))/(f^2*x^2 - e^2), x)
```

## Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2 - f^2x^2}\,dx = \frac{\mathrm{Li}_2\left(\frac{2\,e}{e+f\,x}\right)}{2\,e\,f}$$

```
[In] int(log((2*e)/(e + f*x))/(e^2 - f^2*x^2),x)
[Out] dilog((2*e)/(e + f*x))/(2*e*f)
```

# 3.280 $\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 42

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=-\frac{\operatorname{arctanh}\left(\frac{fx}{e}\right)\log(2)}{ef}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

```
[Out] -arctanh(f*x/e)*ln(2)/e/f+1/2*polylog(2,1-2*e/(f*x+e))/e/f
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2450, 214, 2449, 2352}

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\frac{\operatorname{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}-\frac{\log(2)\operatorname{arctanh}\left(\frac{fx}{e}\right)}{ef}$$

```
[In] Int[Log[e/(e + f*x)]/(e^2 - f^2*x^2),x]
```

```
[Out] -((ArcTanh[(f*x)/e]*Log[2])/(e*f)) + PolyLog[2, 1 - (2*e)/(e + f*x)]/(2*e*f
)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)/((d_) + (e_.)*(x_))]*(b_.))/((f_) + (g_.)*(x_)^2), x
_Symbol] :> Dist[a + b*Log[c/(2*d)], Int[1/(f + g*x^2), x], x] + Dist[b, In
t[Log[2*(d/(d + e*x))]/(f + g*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && EqQ[e^2*f + d^2*g, 0] && GtQ[c/(2*d), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\log(2)\int \frac{1}{e^2-f^2x^2}\,dx\right)+\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx\\
&= -\frac{\tanh^{-1}\left(\frac{fx}{e}\right)\log(2)}{ef}+\frac{\text{Subst}\left(\int \frac{\log(2ex)}{1-2ex}\,dx, x, \frac{1}{e+fx}\right)}{f}\\
&= -\frac{\tanh^{-1}\left(\frac{fx}{e}\right)\log(2)}{ef}+\frac{\text{Li}_2\left(1-\frac{2e}{e+fx}\right)}{2ef}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.93

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx = -\frac{\log\left(\frac{e-fx}{2e}\right)\log\left(\frac{e}{e+fx}\right)}{2ef}-\frac{\log^2\left(\frac{e}{e+fx}\right)}{4ef}+\frac{\text{PolyLog}\left(2,\frac{e+fx}{2e}\right)}{2ef}$$

```
[In] Integrate[Log[e/(e + f*x)]/(e^2 - f^2*x^2),x]
```

```
[Out] -1/2*(Log[(e - f*x)/(2*e)]*Log[e/(e + f*x)])/(e*f) - Log[e/(e + f*x)]^2/(4*
e*f) + PolyLog[2, (e + f*x)/(2*e)]/(2*e*f)
```

## Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\frac{\left(\ln\left(\frac{e}{fx+e}\right)-\ln\left(\frac{2e}{fx+e}\right)\right)\ln\left(1-\frac{2e}{fx+e}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2}}{fe}$ | 62 |
| default | $-\frac{\frac{\left(\ln\left(\frac{e}{fx+e}\right)-\ln\left(\frac{2e}{fx+e}\right)\right)\ln\left(1-\frac{2e}{fx+e}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2}}{fe}$ | 62 |
| risch | $-\frac{\ln\left(1-\frac{2e}{fx+e}\right)\ln\left(\frac{e}{fx+e}\right)}{2ef}+\frac{\ln\left(1-\frac{2e}{fx+e}\right)\ln\left(\frac{2e}{fx+e}\right)}{2ef}+\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$ | 84 |
| parts | $\frac{\ln\left(\frac{e}{fx+e}\right)\ln(fx+e)}{2ef}-\frac{\ln\left(\frac{e}{fx+e}\right)\ln(-fx+e)}{2ef}+\frac{f\left(\frac{\ln(fx+e)^2}{2e\,f^2}+\frac{-\operatorname{dilog}\left(-\frac{-fx-e}{2e}\right)-\ln(-fx+e)\ln\left(-\frac{-fx-e}{2e}\right)}{e\,f^2}\right)}{2}$ | 118 |

[In] int(ln(e/(f*x+e))/(-f^2*x^2+e^2),x,method=_RETURNVERBOSE)

[Out] -1/f/e*(1/2*(ln(e/(f*x+e))-ln(2*e/(f*x+e)))*ln(1-2*e/(f*x+e))-1/2*dilog(2*e /(f*x+e)))

## Fricas [F]

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\int -\frac{\log\left(\frac{e}{fx+e}\right)}{f^2x^2-e^2}\,dx$$

[In] integrate(log(e/(f*x+e))/(-f^2*x^2+e^2),x, algorithm="fricas")

[Out] integral(-log(e/(f*x + e))/(f^2*x^2 - e^2), x)

## Sympy [F]

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=-\int \frac{\log\left(\frac{e}{e+fx}\right)}{-e^2+f^2x^2}\,dx$$

[In] integrate(ln(e/(f*x+e))/(-f**2*x**2+e**2),x)

[Out] -Integral(log(e/(e + f*x))/(-e**2 + f**2*x**2), x)

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(39) = 78.

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.83

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{1}{4}\,f\left(\frac{\log\left(fx+e\right)^2-2\,\log\left(fx+e\right)\log\left(fx-e\right)}{ef^2}+\frac{2\,\left(\log\left(fx+e\right)\log\left(-\frac{fx+e}{2\,e}+1\right)+\mathrm{Li}_2\left(\frac{fx+e}{2\,e}\right)\right)}{ef^2}\right) + \frac{1}{2}\left(\frac{\log\left(fx+e\right)}{ef}-\frac{\log\left(fx-e\right)}{ef}\right)\log\left(\frac{e}{fx+e}\right)$$

```
[In] integrate(log(e/(f*x+e))/(-f^2*x^2+e^2),x, algorithm="maxima")
[Out] 1/4*f*((log(f*x + e)^2 - 2*log(f*x + e)*log(f*x - e))/(e*f^2) + 2*(log(f*x
+ e)*log(-1/2*(f*x + e)/e + 1) + dilog(1/2*(f*x + e)/e))/(e*f^2)) + 1/2*(lo
g(f*x + e)/(e*f) - log(f*x - e)/(e*f))*log(e/(f*x + e))
```

## Giac [F]

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \int -\frac{\log\left(\frac{e}{fx+e}\right)}{f^2x^2-e^2}\,dx$$

```
[In] integrate(log(e/(f*x+e))/(-f^2*x^2+e^2),x, algorithm="giac")
[Out] integrate(-log(e/(f*x + e))/(f^2*x^2 - e^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \int \frac{\ln\left(\frac{e}{e+f\,x}\right)}{e^2-f^2\,x^2}\,dx$$

```
[In] int(log(e/(e + f*x))/(e^2 - f^2*x^2),x)
[Out] int(log(e/(e + f*x))/(e^2 - f^2*x^2), x)
```

# 3.281 $\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 41

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\frac{a\text{arctanh}\left(\frac{fx}{e}\right)}{ef}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

```
[Out] a*arctanh(f*x/e)/e/f+1/2*b*polylog(2,1-2*e/(f*x+e))/e/f
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2450, 214, 2449, 2352}

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\frac{a\text{arctanh}\left(\frac{fx}{e}\right)}{ef}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

```
[In] Int[(a + b*Log[(2*e)/(e + f*x)])/(e^2 - f^2*x^2),x]
[Out] (a*ArcTanh[(f*x)/e])/(e*f) + (b*PolyLog[2, 1 - (2*e)/(e + f*x)])/(2*e*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)/((d_) + (e_.)*(x_))]*(b_.))/((f_) + (g_.)*(x_)^2), x
_Symbol] :> Dist[a + b*Log[c/(2*d)], Int[1/(f + g*x^2), x], x] + Dist[b, In
t[Log[2*(d/(d + e*x))]/(f + g*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && EqQ[e^2*f + d^2*g, 0] && GtQ[c/(2*d), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= a\int \frac{1}{e^2-f^2x^2}\,dx+b\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx\\ &= \frac{a\tanh^{-1}\left(\frac{fx}{e}\right)}{ef}+\frac{b\text{Subst}\left(\int \frac{\log(2ex)}{1-2ex}\,dx,x,\frac{1}{e+fx}\right)}{f}\\ &= \frac{a\tanh^{-1}\left(\frac{fx}{e}\right)}{ef}+\frac{b\text{Li}_2\left(1-\frac{2e}{e+fx}\right)}{2ef}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{-\left(\left(a+b\log\left(\frac{2e}{e+fx}\right)\right)\left(a+2b\log\left(\frac{e-fx}{2e}\right)+b\log\left(\frac{2e}{e+fx}\right)\right)\right)+2b^2\,\text{PolyLog}\left(2,\frac{e+fx}{2e}\right)}{4bef}$$

```
[In] Integrate[(a + b*Log[(2*e)/(e + f*x)])/(e^2 - f^2*x^2),x]
[Out] (-((a + b*Log[(2*e)/(e + f*x)])*(a + 2*b*Log[(e - f*x)/(2*e)] + b*Log[(2*e)
/(e + f*x)])) + 2*b^2*PolyLog[2, (e + f*x)/(2*e)])/(4*b*e*f)
```

## Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{2e\left(\frac{a\ln\left(\frac{2e}{fx+e}-1\right)}{4e^2}-\frac{b\,\mathrm{dilog}\left(\frac{2e}{fx+e}\right)}{4e^2}\right)}{f}$ | 44 |
| default | $-\frac{2e\left(\frac{a\ln\left(\frac{2e}{fx+e}-1\right)}{4e^2}-\frac{b\,\mathrm{dilog}\left(\frac{2e}{fx+e}\right)}{4e^2}\right)}{f}$ | 44 |
| risch | $-\frac{a\ln(fx-e)}{2ef}+\frac{a\ln(fx+e)}{2ef}+\frac{b\,\mathrm{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$ | 54 |
| parts | $-\frac{a\ln(fx-e)}{2ef}+\frac{a\ln(fx+e)}{2ef}+\frac{b\,\mathrm{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$ | 54 |

```
[In] int((a+b*ln(2*e/(f*x+e)))/(-f^2*x^2+e^2),x,method=_RETURNVERBOSE)
[Out] -2/f*e*(1/4/e^2*a*ln(2*e/(f*x+e)-1)-1/4/e^2*b*dilog(2*e/(f*x+e)))
```

## Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\frac{b\mathrm{Li}_2\left(-\frac{2\,e}{fx+e}+1\right)+a\log\left(fx+e\right)-a\log\left(fx-e\right)}{2\,ef}$$

```
[In] integrate((a+b*log(2*e/(f*x+e)))/(-f^2*x^2+e^2),x, algorithm="fricas")
[Out] 1/2*(b*dilog(-2*e/(f*x + e) + 1) + a*log(f*x + e) - a*log(f*x - e))/(e*f)
```

## Sympy [F]

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx=-\int \frac{a}{-e^2+f^2x^2}\,dx-\int \frac{b\log\left(2\right)}{-e^2+f^2x^2}\,dx-\int \frac{b\log\left(\frac{e}{e+fx}\right)}{-e^2+f^2x^2}\,dx$$

```
[In] integrate((a+b*ln(2*e/(f*x+e)))/(-f**2*x**2+e**2),x)
[Out] -Integral(a/(-e**2 + f**2*x**2), x) - Integral(b*log(2)/(-e**2 + f**2*x**2)
, x) - Integral(b*log(e/(e + f*x))/(-e**2 + f**2*x**2), x)
```

## Maxima [F]

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\int -\frac{b\log\left(\frac{2\,e}{fx+e}\right)+a}{f^2x^2-e^2}\,dx$$

```
[In] integrate((a+b*log(2*e/(f*x+e)))/(-f^2*x^2+e^2),x, algorithm="maxima")
[Out] 1/2*a*(log(f*x + e)/(e*f) - log(f*x - e)/(e*f)) + b*integrate(-(log(2) - lo
g(f*x + e) + log(e))/(f^2*x^2 - e^2), x)
```

## Giac [F]

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\int -\frac{b\log\left(\frac{2\,e}{fx+e}\right)+a}{f^2x^2-e^2}\,dx$$

```
[In] integrate((a+b*log(2*e/(f*x+e)))/(-f^2*x^2+e^2),x, algorithm="giac")
[Out] integrate(-(b*log(2*e/(f*x + e)) + a)/(f^2*x^2 - e^2), x)
```

## Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx=-\frac{a\,\ln\left(f\,x-e\right)-b\,\mathrm{Li}_2\left(\frac{2\,e}{e+f\,x}\right)+a\,\ln\left(\frac{1}{e+f\,x}\right)}{2\,e\,f}$$

```
[In] int((a + b*log((2*e)/(e + f*x)))/(e^2 - f^2*x^2),x)
[Out] -(a*log(f*x - e) - b*dilog((2*e)/(e + f*x)) + a*log(1/(e + f*x)))/(2*e*f)
```

# 3.282 $\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 47

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{fx}{e}\right)(a-b\log(2))}{ef}+\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

```
[Out] arctanh(f*x/e)*(a-b*ln(2))/e/f+1/2*b*polylog(2,1-2*e/(f*x+e))/e/f
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2450, 214, 2449, 2352}

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\frac{(a-b\log(2))\operatorname{arctanh}\left(\frac{fx}{e}\right)}{ef}+\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2e}{e+fx}\right)}{2ef}$$

```
[In] Int[(a + b*Log[e/(e + f*x)])/(e^2 - f^2*x^2),x]
```

```
[Out] (ArcTanh[(f*x)/e]*(a - b*Log[2]))/(e*f) + (b*PolyLog[2, 1 - (2*e)/(e + f*x)
])/(2*e*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)/((d_) + (e_.)*(x_))]*(b_.))/((f_) + (g_.)*(x_)^2), x
_Symbol] :> Dist[a + b*Log[c/(2*d)], Int[1/(f + g*x^2), x], x] + Dist[b, In
t[Log[2*(d/(d + e*x))]/(f + g*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && EqQ[e^2*f + d^2*g, 0] && GtQ[c/(2*d), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b\int \frac{\log\left(\frac{2e}{e+fx}\right)}{e^2-f^2x^2}\,dx + (a-b\log(2))\int \frac{1}{e^2-f^2x^2}\,dx \\
&= \frac{\tanh^{-1}\left(\frac{fx}{e}\right)(a-b\log(2))}{ef} + \frac{b\text{Subst}\left(\int \frac{\log(2ex)}{1-2ex}\,dx, x, \frac{1}{e+fx}\right)}{f} \\
&= \frac{\tanh^{-1}\left(\frac{fx}{e}\right)(a-b\log(2))}{ef} + \frac{b\text{Li}_2\left(1-\frac{2e}{e+fx}\right)}{2ef}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \frac{-\left(\left(a+b\log\left(\frac{e}{e+fx}\right)\right)\left(a+2b\log\left(\frac{e-fx}{2e}\right)+b\log\left(\frac{e}{e+fx}\right)\right)\right)+2b^2\,\text{PolyLog}\left(2,\frac{e+fx}{2e}\right)}{4bef}$$

```
[In] Integrate[(a + b*Log[e/(e + f*x)])/(e^2 - f^2*x^2),x]
[Out] (-((a + b*Log[e/(e + f*x)])*(a + 2*b*Log[(e - f*x)/(2*e)] + b*Log[e/(e + f*
x)])) + 2*b^2*PolyLog[2, (e + f*x)/(2*e)])/(4*b*e*f)
```

## Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.81

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{e\left(\frac{a\ln\left(\frac{2e}{fx+e}-1\right)}{2e^2}+\frac{b\left(\frac{\left(\ln\left(\frac{e}{fx+e}\right)-\ln\left(\frac{2e}{fx+e}\right)\right)\ln\left(1-\frac{2e}{fx+e}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2}\right)}{e^2}\right)}{f}$ | 85 |
| default | $-\frac{e\left(\frac{a\ln\left(\frac{2e}{fx+e}-1\right)}{2e^2}+\frac{b\left(\frac{\left(\ln\left(\frac{e}{fx+e}\right)-\ln\left(\frac{2e}{fx+e}\right)\right)\ln\left(1-\frac{2e}{fx+e}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2}\right)}{e^2}\right)}{f}$ | 85 |
| parts | $-\frac{a\ln(fx-e)}{2ef}+\frac{a\ln(fx+e)}{2ef}-\frac{b\left(\frac{\left(\ln\left(\frac{e}{fx+e}\right)-\ln\left(\frac{2e}{fx+e}\right)\right)\ln\left(1-\frac{2e}{fx+e}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2}\right)}{fe}$ | 96 |
| risch | $-\frac{a\ln(fx-e)}{2ef}+\frac{a\ln(fx+e)}{2ef}-\frac{b\ln\left(1-\frac{2e}{fx+e}\right)\ln\left(\frac{e}{fx+e}\right)}{2ef}+\frac{b\ln\left(1-\frac{2e}{fx+e}\right)\ln\left(\frac{2e}{fx+e}\right)}{2ef}+\frac{b\operatorname{dilog}\left(\frac{2e}{fx+e}\right)}{2fe}$ | 119 |

```
[In] int((a+b*ln(e/(f*x+e)))/(-f^2*x^2+e^2),x,method=_RETURNVERBOSE)
[Out] -1/f*e*(1/2/e^2*a*ln(2*e/(f*x+e)-1)+1/e^2*b*(1/2*(ln(e/(f*x+e))-ln(2*e/(f*x
+e)))*ln(1-2*e/(f*x+e))-1/2*dilog(2*e/(f*x+e))))
```

## Fricas [F]

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx = \int -\frac{b\log\left(\frac{e}{fx+e}\right)+a}{f^2x^2-e^2}\,dx$$

```
[In] integrate((a+b*log(e/(f*x+e)))/(-f^2*x^2+e^2),x, algorithm="fricas")
[Out] integral(-(b*log(e/(f*x + e)) + a)/(f^2*x^2 - e^2), x)
```

## Sympy [F]

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx = -\int \frac{a}{-e^2+f^2x^2}\,dx - \int \frac{b\log\left(\frac{e}{e+fx}\right)}{-e^2+f^2x^2}\,dx$$

```
[In] integrate((a+b*ln(e/(f*x+e)))/(-f**2*x**2+e**2),x)
[Out] -Integral(a/(-e**2 + f**2*x**2), x) - Integral(b*log(e/(e + f*x))/(-e**2 +
f**2*x**2), x)
```

## Maxima [F]

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\int -\frac{b\log\left(\frac{e}{fx+e}\right)+a}{f^2x^2-e^2}\,dx$$

```
[In] integrate((a+b*log(e/(f*x+e)))/(-f^2*x^2+e^2),x, algorithm="maxima")
[Out] 1/2*a*(log(f*x + e)/(e*f) - log(f*x - e)/(e*f)) + b*integrate((log(f*x + e)
 - log(e))/(f^2*x^2 - e^2), x)
```

## Giac [F]

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\int -\frac{b\log\left(\frac{e}{fx+e}\right)+a}{f^2x^2-e^2}\,dx$$

```
[In] integrate((a+b*log(e/(f*x+e)))/(-f^2*x^2+e^2),x, algorithm="giac")
[Out] integrate(-(b*log(e/(f*x + e)) + a)/(f^2*x^2 - e^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(\frac{e}{e+fx}\right)}{e^2-f^2x^2}\,dx=\int \frac{a+b\,\ln\left(\frac{e}{e+f\,x}\right)}{e^2-f^2\,x^2}\,dx$$

```
[In] int((a + b*log(e/(e + f*x)))/(e^2 - f^2*x^2),x)
[Out] int((a + b*log(e/(e + f*x)))/(e^2 - f^2*x^2), x)
```

# 3.283 $\int \frac{x^5 \log(c+dx)}{a+bx^3} \, dx$



## Optimal result

Integrand size = 19, antiderivative size = 371

$$\int \frac{x^5 \log(c+dx)}{a+bx^3} \, dx = -\frac{c^2x}{3bd^2} + \frac{cx^2}{6bd} - \frac{x^3}{9b} + \frac{c^3 \log(c+dx)}{3bd^3} + \frac{x^3 \log(c+dx)}{3b} - \frac{a \log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right) \log(c+dx)}{3b^2} - \frac{a \log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right) \log(c+dx)}{3b^2} - \frac{a \log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right) \log(c+dx)}{3b^2} - \frac{a \operatorname{PolyLog}\left(2, \frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^2} - \frac{a \operatorname{PolyLog}\left(2, \frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^2} - \frac{a \operatorname{PolyLog}\left(2, \frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^2}$$

```
[Out] -1/3*c^2*x/b/d^2+1/6*c*x^2/b/d-1/9*x^3/b+1/3*c^3*ln(d*x+c)/b/d^3+1/3*x^3*ln
(d*x+c)/b-1/3*a*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/
b^2-1/3*a*ln(-d*((-1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3
)*d))*ln(d*x+c)/b^2-1/3*a*ln((-1)^(1/3)*d*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b
^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))*ln(d*x+c)/b^2-1/3*a*polylog(2,b^(1/3)*(d*x+
```

```
c)/(b^(1/3)*c-a^(1/3)*d))/b^2-1/3*a*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c+(-
1)^(1/3)*a^(1/3)*d))/b^2-1/3*a*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-(-1)^(2
/3)*a^(1/3)*d))/b^2
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {272, 45, 2463, 2442, 266, 2441, 2440, 2438}

$$\int \frac{x^5\log(c+dx)}{a+bx^3}\,dx = -\frac{a\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^2}-\frac{a\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^2}-\frac{a\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^2}-\frac{a\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^2}-\frac{a\log(c+dx)\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^2}-\frac{a\log(c+dx)\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c}\right)}{3b^2}+\frac{c^3\log(c+dx)}{3bd^3}-\frac{c^2x}{3bd^2}+\frac{x^3\log(c+dx)}{3b}+\frac{cx^2}{6bd}-\frac{x^3}{9b}$$

```
[In] Int[(x^5*Log[c + d*x])/(a + b*x^3),x]
```

```
[Out] -1/3*(c^2*x)/(b*d^2) + (c*x^2)/(6*b*d) - x^3/(9*b) + (c^3*Log[c + d*x])/(3*
b*d^3) + (x^3*Log[c + d*x])/(3*b) - (a*Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(
1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*b^2) - (a*Log[-((d*((-1)^(2/3)*a^(1/
3) + b^(1/3)*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d))]*Log[c + d*x])/(3*b^2)
 - (a*Log[((-1)^(1/3)*d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*c + (-1)
^(1/3)*a^(1/3)*d)]*Log[c + d*x])/(3*b^2) - (a*PolyLog[2, (b^(1/3)*(c + d*x)
)/(b^(1/3)*c - a^(1/3)*d)])/(3*b^2) - (a*PolyLog[2, (b^(1/3)*(c + d*x))/(b^
(1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3*b^2) - (a*PolyLog[2, (b^(1/3)*(c + d*x
))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x^2\log(c+dx)}{b} - \frac{ax^2\log(c+dx)}{b(a+bx^3)}\right) dx$$

$$= \frac{\int x^2\log(c+dx)\,dx}{b} - \frac{a\int \frac{x^2\log(c+dx)}{a+bx^3}\,dx}{b}$$

$$= \frac{x^3\log(c+dx)}{3b} - \frac{a\int \left(\frac{\log(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right) dx}{b} - \frac{d\int \frac{x^3}{c+dx}\,dx}{3b}$$

$$= \frac{x^3\log(c+dx)}{3b} - \frac{a\int \frac{\log(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{5/3}} - \frac{a\int \frac{\log(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{5/3}} - \frac{a\int \frac{\log(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{5/3}} - \frac{d\int \left(\frac{c^2}{d^3} - \frac{cx}{d^2} + \frac{x^2}{d} - \frac{c^3}{d^3(c+dx)}\right) dx}{3b}$$

$$= -\frac{c^2x}{3bd^2} + \frac{cx^2}{6bd} - \frac{x^3}{9b} + \frac{c^3\log(c+dx)}{3bd^3} + \frac{x^3\log(c+dx)}{3b} - \frac{a\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} - \frac{a\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2}$$

$$- \frac{a\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} + \frac{(ad)\int \frac{\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3b^2}$$

$$+ \frac{(ad)\int \frac{\log\left(\frac{d\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c-\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3b^2} + \frac{(ad)\int \frac{\log\left(\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+(-1)^{2/3}\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3b^2}$$

$$= -\frac{c^2x}{3bd^2} + \frac{cx^2}{6bd} - \frac{x^3}{9b} + \frac{c^3\log(c+dx)}{3bd^3} + \frac{x^3\log(c+dx)}{3b} - \frac{a\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} - \frac{a\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} - \frac{a\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} + \frac{a\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3b^2} + \frac{a\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c-\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3b^2} + \frac{a\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c+(-1)^{2/3}\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3b^2}$$

$$= -\frac{c^2x}{3bd^2} + \frac{cx^2}{6bd} - \frac{x^3}{9b} + \frac{c^3\log(c+dx)}{3bd^3} + \frac{x^3\log(c+dx)}{3b} - \frac{a\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} - \frac{a\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} - \frac{a\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} - \frac{a\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^2} - \frac{a\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^2} - \frac{a\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^2}$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.01

$$\int \frac{x^5 \log(c+dx)}{a+bx^3}\,dx = -\frac{c^2x}{3bd^2}+\frac{cx^2}{6bd}-\frac{x^3}{9b}+\frac{c^3\log(c+dx)}{3bd^3}+\frac{x^3\log(c+dx)}{3b} - \frac{a\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} - \frac{a\log\left(-\frac{(-1)^{2/3}d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} - \frac{a\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3b^2} - \frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^2} - \frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^2} - \frac{a\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^2}$$

```
[In] Integrate[(x^5*Log[c + d*x])/(a + b*x^3),x]
```

```
[Out] -1/3*(c^2*x)/(b*d^2) + (c*x^2)/(6*b*d) - x^3/(9*b) + (c^3*Log[c + d*x])/(3*
b*d^3) + (x^3*Log[c + d*x])/(3*b) - (a*Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(
1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*b^2) - (a*Log[-(((-1)^(2/3)*d*(a^(1/
3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d))]*Log[c + d*
x])/(3*b^2) - (a*Log[((-1)^(1/3)*d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/
3)*c + (-1)^(1/3)*a^(1/3)*d)]*Log[c + d*x])/(3*b^2) - (a*PolyLog[2, (b^(1/3
)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)])/(3*b^2) - (a*PolyLog[2, (b^(1/3)*(c
+ d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3*b^2) - (a*PolyLog[2, (b^(1/
3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*b^2)
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.41

| method | result |
|---|---|
| risch | $-\frac{c^2 x}{3b\,d^2}-\frac{11c^3}{18d^3 b}+\frac{c\,x^2}{6bd}+\frac{x^3\ln(dx+c)}{3b}+\frac{c^3\ln(dx+c)}{3b\,d^3}-\frac{x^3}{9b}-\frac{a\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\right.}\right.}{}$ |
| derivativedivides | $\frac{\frac{d^3\left(c^2((dx+c)\ln(dx+c)-dx-c)-2c\left(\frac{(dx+c)^2\ln(dx+c)}{2}-\frac{(dx+c)^2}{4}\right)+\frac{(dx+c)^3\ln(dx+c)}{3}-\frac{(dx+c)^3}{9}\right)}{b}-\frac{a\,d^6\left(\_R1=\mathrm{RootOf}\left(b\_Z^3-3\right.\right.}{}}{d^6}$ |
| default | $\frac{\frac{d^3\left(c^2((dx+c)\ln(dx+c)-dx-c)-2c\left(\frac{(dx+c)^2\ln(dx+c)}{2}-\frac{(dx+c)^2}{4}\right)+\frac{(dx+c)^3\ln(dx+c)}{3}-\frac{(dx+c)^3}{9}\right)}{b}-\frac{a\,d^6\left(\_R1=\mathrm{RootOf}\left(b\_Z^3-3\right.\right.}{}}{d^6}$ |
| parts | $\frac{x^3\ln(dx+c)}{3b}-\frac{\ln(dx+c)a\ln\left(b\,x^3+a\right)}{3b^2}-\frac{d\left(\frac{x^3}{3bd}-\frac{x^2c}{2b\,d^2}+\frac{x\,c^2}{b\,d^3}-\frac{c^3\ln(dx+c)}{b\,d^4}-\frac{a\ln(dx+c)\ln\left(b\,x^3+a\right)}{b^2 d}+\frac{a\left(\_R1=\mathrm{RootOf}\left(b\_\right.\right.}{}\right.}{}$ |

```
[In] int(x^5*ln(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c^2*x/b/d^2-11/18/d^3/b*c^3+1/6*c*x^2/b/d+1/3*x^3*ln(d*x+c)/b+1/3*c^3*
ln(d*x+c)/b/d^3-1/9*x^3/b-1/3*a/b^2*sum(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilo
g((-d*x+_R1-c)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

## Fricas [F]

$$\int \frac{x^5\log(c+dx)}{a+bx^3}\,dx=\int \frac{x^5\log\left(dx+c\right)}{bx^3+a}\,dx$$

```
[In] integrate(x^5*log(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral(x^5*log(d*x + c)/(b*x^3 + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 \log(c+dx)}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*ln(d*x+c)/(b*x**3+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^5 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^5 \log(dx+c)}{bx^3+a}\,dx$$

```
[In] integrate(x^5*log(d*x+c)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(x^5*log(d*x + c)/(b*x^3 + a), x)
```

## Giac [F]

$$\int \frac{x^5 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^5 \log(dx+c)}{bx^3+a}\,dx$$

```
[In] integrate(x^5*log(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(x^5*log(d*x + c)/(b*x^3 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^5 \ln(c+d\,x)}{b\,x^3+a}\,dx$$

```
[In] int((x^5*log(c + d*x))/(a + b*x^3),x)
[Out] int((x^5*log(c + d*x))/(a + b*x^3), x)
```

# 3.284 $\int \frac{x^2\log(c+dx)}{a+bx^3}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 292

$$\int \frac{x^2\log(c+dx)}{a+bx^3}\,dx = \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b} + \frac{\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3b} + \frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3b} + \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b} + \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b} + \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b}$$

```
[Out] 1/3*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/b+1/3*ln(-d*
((-1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3)*d))*ln(d*x+c)/
b+1/3*ln((-1)^(1/3)*d*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*c+(-1)^(1/3)*
a^(1/3)*d))*ln(d*x+c)/b+1/3*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1/3)*d)
)/b+1/3*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))/b+1/3*p
olylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3)*d))/b
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {266, 2463, 2441, 2440, 2438}

$$\int \frac{x^2\log(c+dx)}{a+bx^3}\,dx = \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b}+\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b}+\frac{\log(c+dx)\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b}+\frac{\log(c+dx)\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c}\right)}{3b}$$

```
[In] Int[(x^2*Log[c + d*x])/(a + b*x^3),x]
```

```
[Out] (Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3
*b) + (Log[-((d*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - (-1)^(2/3)*a
^(1/3)*d))]*Log[c + d*x])/(3*b) + (Log[((-1)^(1/3)*d*(a^(1/3) + (-1)^(2/3)*
b^(1/3)*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]*Log[c + d*x])/(3*b) + PolyL
og[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)]/(3*b) + PolyLog[2, (b^(1
/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]/(3*b) + PolyLog[2, (b^(1
/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)]/(3*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\log(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\log(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx$$

$$=\frac{\int \frac{\log(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}+\frac{\int \frac{\log(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}+\frac{\int \frac{\log(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$=\frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b}+\frac{\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3b}+\frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3b}-\frac{d\int \frac{\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3b}-\frac{d\int \frac{\log\left(\frac{d\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c-\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3b}-\frac{d\int \frac{\log\left(\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+(-1)^{2/3}\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3b}$$

$$= \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b} + \frac{\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3b}$$

$$+ \frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3b}$$

$$- \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3b}$$

$$- \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c-\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3b}$$

$$- \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c+(-1)^{2/3}\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3b}$$

$$= \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b} + \frac{\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3b}$$

$$+ \frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3b} + \frac{\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b}$$

$$+ \frac{\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b} + \frac{\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.02

$$\int \frac{x^2 \log(c+dx)}{a+bx^3}\,dx = \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b} + \frac{\log\left(-\frac{(-1)^{2/3}d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3b} + \frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b}$$

```
[In] Integrate[(x^2*Log[c + d*x])/(a + b*x^3),x]
```

```
[Out] (Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3
*b) + (Log[-(((-1)^(2/3)*d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*c - (
-1)^(2/3)*a^(1/3)*d))]*Log[c + d*x])/(3*b) + (Log[((-1)^(1/3)*d*(a^(1/3) +
(-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]*Log[c + d*x])/(3
*b) + PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)]/(3*b) + PolyL
og[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]/(3*b) + PolyL
og[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)]/(3*b)
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.26

| method | result |
|---|---|
| derivativedivides | $\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3b\,c^{2}\_Z+a\,d^{3}-b\,c^{3}\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)\right)}{3b}$ |
| default | $\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3b\,c^{2}\_Z+a\,d^{3}-b\,c^{3}\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)\right)}{3b}$ |
| risch | $\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3b\,c^{2}\_Z+a\,d^{3}-b\,c^{3}\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)\right)}{3b}$ |
| parts | $\frac{\ln(dx+c)\ln\left(b\,x^{3}+a\right)}{3b}-\frac{d\left(\frac{\ln(dx+c)\ln\left(b\,x^{3}+a\right)}{d}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3b\,c^{2}\_Z+a\,d^{3}-b\,c^{3}\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_}{\_}\right.\right.}{d}\right.}{3b}$ |

[In] int(x^2*ln(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/3/b*sum(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1),_R1=RootOf
(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))

## Fricas [F]

$$\int \frac{x^2\log(c+dx)}{a+bx^3}\,dx=\int \frac{x^2\log\left(dx+c\right)}{bx^3+a}\,dx$$

[In] integrate(x^2*log(d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] integral(x^2*log(d*x + c)/(b*x^3 + a), x)

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2\log(c+dx)}{a+bx^3}\,dx=\text{Timed out}$$

[In] integrate(x**2*ln(d*x+c)/(b*x**3+a),x)

[Out] Timed out

## Maxima [F]

$$\int \frac{x^2 \log(c+dx)}{a+bx^3} \, dx = \int \frac{x^2 \log(dx+c)}{bx^3+a} \, dx$$

```
[In] integrate(x^2*log(d*x+c)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(x^2*log(d*x + c)/(b*x^3 + a), x)
```

## Giac [F]

$$\int \frac{x^2 \log(c+dx)}{a+bx^3} \, dx = \int \frac{x^2 \log(dx+c)}{bx^3+a} \, dx$$

```
[In] integrate(x^2*log(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(x^2*log(d*x + c)/(b*x^3 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log(c+dx)}{a+bx^3} \, dx = \int \frac{x^2 \ln(c+d\,x)}{b\,x^3+a} \, dx$$

```
[In] int((x^2*log(c + d*x))/(a + b*x^3),x)
[Out] int((x^2*log(c + d*x))/(a + b*x^3), x)
```

# 3.285 $\int \frac{\log(c+dx)}{x(a+bx^3)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 324

$$\int \frac{\log(c+dx)}{x(a+bx^3)}\,dx = \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a} - \frac{\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a} - \frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a} + \frac{\operatorname{PolyLog}\left(2,1+\frac{dx}{c}\right)}{a}$$

```
[Out] ln(-d*x/c)*ln(d*x+c)/a-1/3*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d))
*ln(d*x+c)/a-1/3*ln(-d*((-1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-(-1)^(2/3)
*a^(1/3)*d))*ln(d*x+c)/a-1/3*ln((-1)^(1/3)*d*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)
/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))*ln(d*x+c)/a-1/3*polylog(2,b^(1/3)*(d*x+c
)/(b^(1/3)*c-a^(1/3)*d))/a-1/3*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c+(-1)^(1
/3)*a^(1/3)*d))/a-1/3*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-(-1)^(2/3)*a^(1/
3)*d))/a+polylog(2,1+d*x/c)/a
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {272, 36, 29, 31, 2463, 2441, 2352, 266, 2440, 2438}

$$\int \frac{\log(c+dx)}{x(a+bx^3)}\,dx = -\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a}-\frac{\log(c+dx)\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a}-\frac{\log(c+dx)\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c}\right)}{3a}+\frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{a}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a}$$

```
[In] Int[Log[c + d*x]/(x*(a + b*x^3)),x]
```

```
[Out] (Log[-((d*x)/c)]*Log[c + d*x])/a - (Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3
)*c - a^(1/3)*d))]*Log[c + d*x])/(3*a) - (Log[-((d*((-1)^(2/3)*a^(1/3) + b^
(1/3)*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d))]*Log[c + d*x])/(3*a) - (Log[(
(-1)^(1/3)*d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1
/3)*d)]*Log[c + d*x])/(3*a) - PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a
^(1/3)*d)]/(3*a) - PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a
^(1/3)*d)]/(3*a) - PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a
^(1/3)*d)]/(3*a) + PolyLog[2, 1 + (d*x)/c]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log(c+dx)}{ax} - \frac{bx^2\log(c+dx)}{a(a+bx^3)}\right)dx$$

$$= \frac{\int \frac{\log(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{x^2\log(c+dx)}{a+bx^3}\,dx}{a}$$

$$= \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{b\int \left(\frac{\log(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{a} - \frac{d\int \frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\,dx}{a}$$

$$= \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} + \frac{\text{Li}_2\left(1+\frac{dx}{c}\right)}{a} - \frac{\sqrt[3]{b}\int \frac{\log(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a} - \frac{\sqrt[3]{b}\int \frac{\log(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a} - \frac{\sqrt[3]{b}\int \frac{\log(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$= \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a} - \frac{\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a} - \frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a} + \frac{\text{Li}_2\left(1+\frac{dx}{c}\right)}{a} + \frac{d\int \frac{\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3a} + \frac{d\int \frac{\log\left(\frac{d\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c-\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3a} + \frac{d\int \frac{\log\left(\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+(-1)^{2/3}\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3a}$$

$$=\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a}-\frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a}$$
$$-\frac{\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a}$$
$$-\frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a}+\frac{\text{Li}_2\left(1+\frac{dx}{c}\right)}{a}$$
$$+\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{x}dx,x,c+dx\right)}{3a}$$
$$+\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c-\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{x}dx,x,c+dx\right)}{3a}$$
$$+\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c+(-1)^{2/3}\sqrt[3]{a}d}\right)}{x}dx,x,c+dx\right)}{3a}$$

$$=\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a}-\frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a}$$
$$-\frac{\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a}$$
$$-\frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a}-\frac{\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a}$$
$$-\frac{\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a}-\frac{\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a}+\frac{\text{Li}_2\left(1+\frac{dx}{c}\right)}{a}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.02

$$\int \frac{\log(c+dx)}{x(a+bx^3)}\,dx = \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a} - \frac{\log\left(-\frac{(-1)^{2/3}d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a} - \frac{\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a} + \frac{\text{PolyLog}\left(2,\frac{c+dx}{c}\right)}{a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a}$$

```
[In] Integrate[Log[c + d*x]/(x*(a + b*x^3)),x]
```

```
[Out] (Log[-((d*x)/c)]*Log[c + d*x])/a - (Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3
)*c - a^(1/3)*d))]*Log[c + d*x])/(3*a) - (Log[-(((-1)^(2/3)*d*(a^(1/3) - (-
1)^(1/3)*b^(1/3)*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d))]*Log[c + d*x])/(3*
a) - (Log[((-1)^(1/3)*d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*c + (-1)
^(1/3)*a^(1/3)*d)]*Log[c + d*x])/(3*a) + PolyLog[2, (c + d*x)/c]/a - PolyLo
g[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)]/(3*a) - PolyLog[2, (b^(1/
3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]/(3*a) - PolyLog[2, (b^(1/
3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)]/(3*a)
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.67 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.33

| method | result |
|---|---|
| derivativedivides | $\frac{\operatorname{dilog}\left(-\frac{xd}{c}\right)+\ln(dx+c)\ln\left(-\frac{xd}{c}\right)}{a}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)\right.}{3a}$ |
| default | $\frac{\operatorname{dilog}\left(-\frac{xd}{c}\right)+\ln(dx+c)\ln\left(-\frac{xd}{c}\right)}{a}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)\right.}{3a}$ |
| risch | $\frac{\ln\left(-\frac{xd}{c}\right)\ln(dx+c)}{a}+\frac{\operatorname{dilog}\left(-\frac{xd}{c}\right)}{a}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)\right.}{3a}$ |
| parts | $\frac{\ln(dx+c)\ln(x)}{a}-\frac{\ln(dx+c)\ln(b\,x^3+a)}{3a}-\frac{d\left(\frac{3\operatorname{dilog}\left(\frac{dx+c}{c}\right)}{ad}+\frac{3\ln(x)\ln\left(\frac{dx+c}{c}\right)}{ad}-\frac{\ln(dx+c)\ln\left(b\,x^3+a\right)}{ad}+\frac{\_R1=\operatorname{RootOf}(b}{}\right.}{}$ |

```
[In] int(ln(d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(dilog(-x*d/c)+ln(d*x+c)*ln(-x*d/c))-1/3/a*sum(ln(d*x+c)*ln((-d*x+_R1-c
)/_R1)+dilog((-d*x+_R1-c)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^
3-b*c^3))
```

## Fricas [F]

$$\int \frac{\log(c+dx)}{x\,(a+bx^3)}\,dx=\int \frac{\log\left(dx+c\right)}{(bx^3+a)x}\,dx$$

```
[In] integrate(log(d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral(log(d*x + c)/(b*x^4 + a*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x\,(a+bx^3)}\,dx=\text{Timed out}$$

```
[In] integrate(ln(d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^3+a)x}\,dx$$

[In] integrate(log(d*x+c)/x/(b*x^3+a),x, algorithm="maxima")

[Out] integrate(log(d*x + c)/((b*x^3 + a)*x), x)

## Giac [F]

$$\int \frac{\log(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^3+a)x}\,dx$$

[In] integrate(log(d*x+c)/x/(b*x^3+a),x, algorithm="giac")

[Out] integrate(log(d*x + c)/((b*x^3 + a)*x), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\ln\left(c+d\,x\right)}{x\,\left(b\,x^3+a\right)}\,dx$$

[In] int(log(c + d*x)/(x*(a + b*x^3)),x)

[Out] int(log(c + d*x)/(x*(a + b*x^3)), x)

# 3.286 $\int \frac{\log(c+dx)}{x^4(a+bx^3)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 414

$$\int \frac{\log(c+dx)}{x^4(a+bx^3)}\,dx = -\frac{d}{6acx^2}+\frac{d^2}{3ac^2x}+\frac{d^3\log(x)}{3ac^3}-\frac{d^3\log(c+dx)}{3ac^3}-\frac{\log(c+dx)}{3ax^3}-\frac{b\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a^2}+\frac{b\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2}+\frac{b\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2}+\frac{b\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2}+\frac{b\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^2}+\frac{b\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a^2}+\frac{b\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a^2}-\frac{b\,\mathrm{PolyLog}\left(2,1+\frac{dx}{c}\right)}{a^2}$$

```
[Out] -1/6*d/a/c/x^2+1/3*d^2/a/c^2/x+1/3*d^3*ln(x)/a/c^3-1/3*d^3*ln(d*x+c)/a/c^3-
1/3*ln(d*x+c)/a/x^3-b*ln(-d*x/c)*ln(d*x+c)/a^2+1/3*b*ln(-d*(a^(1/3)+b^(1/3)
*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^2+1/3*b*ln(-d*((-1)^(2/3)*a^(1/3)+b^
(1/3)*x)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3)*d))*ln(d*x+c)/a^2+1/3*b*ln((-1)^(1/3
)*d*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))*ln(d*x
```

```
+c)/a^2+1/3*b*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1/3)*d))/a^2+1/3*b*po
lylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))/a^2+1/3*b*polylog
(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3)*d))/a^2-b*polylog(2,1+d*x/
c)/a^2
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {272, 46, 2463, 2442, 2441, 2352, 266, 2440, 2438}

$$
\begin{aligned}
\int \frac{\log(c+dx)}{x^4(a+bx^3)}\,dx = {} & \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^2}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a^2} \\
& +\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a^2}-\frac{b\,\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{a^2} \\
& -\frac{b\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a^2}+\frac{b\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^2} \\
& +\frac{b\log(c+dx)\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a^2} \\
& +\frac{b\log(c+dx)\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c}\right)}{3a^2}+\frac{d^3\log(x)}{3ac^3} \\
& -\frac{d^3\log(c+dx)}{3ac^3}+\frac{d^2}{3ac^2x}-\frac{\log(c+dx)}{3ax^3}-\frac{d}{6acx^2}
\end{aligned}
$$

```
[In] Int[Log[c + d*x]/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/6*d/(a*c*x^2) + d^2/(3*a*c^2*x) + (d^3*Log[x])/(3*a*c^3) - (d^3*Log[c +
d*x])/(3*a*c^3) - Log[c + d*x]/(3*a*x^3) - (b*Log[-((d*x)/c)]*Log[c + d*x])
/a^2 + (b*Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c +
 d*x])/(3*a^2) + (b*Log[-((d*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*c -
 (-1)^(2/3)*a^(1/3)*d))]*Log[c + d*x])/(3*a^2) + (b*Log[((-1)^(1/3)*d*(a^(1
/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]*Log[c + d*
x])/(3*a^2) + (b*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)])/(
3*a^2) + (b*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*
d)])/(3*a^2) + (b*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^
(1/3)*d)])/(3*a^2) - (b*PolyLog[2, 1 + (d*x)/c])/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\log(c+dx)}{ax^4} - \frac{b\log(c+dx)}{a^2x} + \frac{b^2x^2\log(c+dx)}{a^2(a+bx^3)}\right) dx \\
&= \frac{\int \frac{\log(c+dx)}{x^4}\,dx}{a} - \frac{b\int \frac{\log(c+dx)}{x}\,dx}{a^2} + \frac{b^2\int \frac{x^2\log(c+dx)}{a+bx^3}\,dx}{a^2} \\
&= -\frac{\log(c+dx)}{3ax^3} - \frac{b\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a^2} \\
&\quad + \frac{b^2\int \left(\frac{\log(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right) dx}{a^2} \\
&\quad + \frac{d\int \frac{1}{x^3(c+dx)}\,dx}{3a} + \frac{(bd)\int \frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\,dx}{a^2} \\
&= -\frac{\log(c+dx)}{3ax^3} - \frac{b\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a^2} - \frac{b\text{Li}_2\left(1+\frac{dx}{c}\right)}{a^2} \\
&\quad + \frac{b^{4/3}\int \frac{\log(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2} + \frac{b^{4/3}\int \frac{\log(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2} \\
&\quad + \frac{b^{4/3}\int \frac{\log(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2} + \frac{d\int \left(\frac{1}{cx^3} - \frac{d}{c^2x^2} + \frac{d^2}{c^3x} - \frac{d^3}{c^3(c+dx)}\right) dx}{3a}
\end{aligned}
$$

$$= -\frac{d}{6acx^2} + \frac{d^2}{3ac^2x} + \frac{d^3\log(x)}{3ac^3} - \frac{d^3\log(c+dx)}{3ac^3} - \frac{\log(c+dx)}{3ax^3} - \frac{b\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a^2} + \frac{b\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2} + \frac{b\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2} + \frac{b\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2} - \frac{b\text{Li}_2\left(1+\frac{dx}{c}\right)}{a^2} - \frac{(bd)\int\frac{\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{c+dx}dx}{3a^2} - \frac{(bd)\int\frac{\log\left(\frac{d\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c-\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{c+dx}dx}{3a^2} - \frac{(bd)\int\frac{\log\left(\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+(-1)^{2/3}\sqrt[3]{a}d}\right)}{c+dx}dx}{3a^2}$$

$$= -\frac{d}{6acx^2} + \frac{d^2}{3ac^2x} + \frac{d^3\log(x)}{3ac^3} - \frac{d^3\log(c+dx)}{3ac^3} - \frac{\log(c+dx)}{3ax^3}$$

$$- \frac{b\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a^2} + \frac{b\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2}$$

$$+ \frac{b\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2}$$

$$+ \frac{b\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2}$$

$$- \frac{b\text{Li}_2\left(1+\frac{dx}{c}\right)}{a^2} - \frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3a^2}$$

$$- \frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c-\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3a^2}$$

$$- \frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c+(-1)^{2/3}\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3a^2}$$

$$= -\frac{d}{6acx^2} + \frac{d^2}{3ac^2x} + \frac{d^3\log(x)}{3ac^3} - \frac{d^3\log(c+dx)}{3ac^3} - \frac{\log(c+dx)}{3ax^3} - \frac{b\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a^2} + \frac{b\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2} + \frac{b\log\left(-\frac{d\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2} + \frac{b\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2} + \frac{b\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^2} + \frac{b\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a^2} + \frac{b\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a^2} - \frac{b\text{Li}_2\left(1+\frac{dx}{c}\right)}{a^2}$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.98

$$\int \frac{\log(c+dx)}{x^4(a+bx^3)}\,dx = -\frac{\log(c+dx)}{3ax^3} - \frac{b\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a^2} + \frac{b\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2} + \frac{b\log\left(-\frac{(-1)^{2/3}d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2} + \frac{b\log\left(\frac{\sqrt[3]{-1}d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)\log(c+dx)}{3a^2} - \frac{d\left(\frac{1}{cx^2}-\frac{2d}{c^2x}-\frac{2d^2\log(x)}{c^3}+\frac{2d^2\log(c+dx)}{c^3}\right)}{6a} - \frac{b\,\text{PolyLog}\left(2,\frac{c+dx}{c}\right)}{a^2} + \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^2} + \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3a^2} + \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3a^2}$$

[In] Integrate[Log[c + d*x]/(x^4*(a + b*x^3)),x]

[Out] -1/3*Log[c + d*x]/(a*x^3) - (b*Log[-((d*x)/c)]*Log[c + d*x])/a^2 + (b*Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*a^2) + (b*Log[-(((-1)^(2/3)*d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d))]*Log[c + d*x])/(3*a^2) + (b*Log[((-1)^(1/3)*d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]*Log[c + d*x])/(3*a^2) - (d*(1/(c*x^2) - (2*d)/(c^2*x) - (2*d^2*Log[x])/c^3 + (2*d^2*Log[c + d*x])/c^3))/(6*a) - (b*PolyLog[2, (c + d*x)/c])/a^2 + (b*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)])/(3*a^2) + (b*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3*a^2) + (b*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*a^2)

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.73 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.45

| method | result |
|---|---|
| risch | $-\frac{d}{6ac\,x^{2}}+\frac{d^{2}}{3a\,c^{2}x}+\frac{d^{3}\ln(-dx)}{3a\,c^{3}}-\frac{d^{3}\ln(dx+c)}{3a\,c^{3}}-\frac{\ln(dx+c)}{3a\,x^{3}}+\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3b\,c^{2}\_Z+a\,d^{3}-b\,c\right.}\right.}{}$ |
| derivativedivides | $d^{3}\left(\frac{-\frac{1}{6c\,d^{2}x^{2}}+\frac{1}{3c^{2}dx}+\frac{\ln(-dx)}{3c^{3}}-\frac{\ln(dx+c)(dx+c)\left(3c^{2}-3c(dx+c)+(dx+c)^{2}\right)}{3c^{3}d^{3}x^{3}}}{a}+\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3b\,c^{2}\_Z+\right.}\right.}{}\right.$ |
| default | $d^{3}\left(\frac{-\frac{1}{6c\,d^{2}x^{2}}+\frac{1}{3c^{2}dx}+\frac{\ln(-dx)}{3c^{3}}-\frac{\ln(dx+c)(dx+c)\left(3c^{2}-3c(dx+c)+(dx+c)^{2}\right)}{3c^{3}d^{3}x^{3}}}{a}+\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3b\,c^{2}\_Z+\right.}\right.}{}\right.$ |
| parts | $-\frac{\ln(dx+c)}{3a\,x^{3}}-\frac{\ln(dx+c)b\ln(x)}{a^{2}}+\frac{\ln(dx+c)b\ln\left(b\,x^{3}+a\right)}{3a^{2}}-\frac{d\left(\frac{b\left(\frac{\ln(dx+c)\ln\left(b\,x^{3}+a\right)}{d}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+\right.}}{}\right.}{}\right.}{}$ |

[In] int(ln(d*x+c)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/6*d/a/c/x^2+1/3*d^2/a/c^2/x+1/3*d^3/a/c^3*ln(-d*x)-1/3*d^3*ln(d*x+c)/a/c^3-1/3*ln(d*x+c)/a/x^3+1/3*b/a^2*sum(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((

```
-d*x+_R1-c)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-b*ln
(-x*d/c)*ln(d*x+c)/a^2-b/a^2*dilog(-x*d/c)
```

## Fricas [F]

$$\int \frac{\log(c+dx)}{x^4\left(a+bx^3\right)}\,dx=\int \frac{\log\left(dx+c\right)}{(bx^3+a)x^4}\,dx$$

```
[In] integrate(log(d*x+c)/x^4/(b*x^3+a),x, algorithm="fricas")
[Out] integral(log(d*x + c)/(b*x^7 + a*x^4), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^4\left(a+bx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate(ln(d*x+c)/x**4/(b*x**3+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log(c+dx)}{x^4\left(a+bx^3\right)}\,dx=\int \frac{\log\left(dx+c\right)}{(bx^3+a)x^4}\,dx$$

```
[In] integrate(log(d*x+c)/x^4/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(log(d*x + c)/((b*x^3 + a)*x^4), x)
```

## Giac [F]

$$\int \frac{\log(c+dx)}{x^4\left(a+bx^3\right)}\,dx=\int \frac{\log\left(dx+c\right)}{(bx^3+a)x^4}\,dx$$

```
[In] integrate(log(d*x+c)/x^4/(b*x^3+a),x, algorithm="giac")
[Out] integrate(log(d*x + c)/((b*x^3 + a)*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^4\,(a+bx^3)}\,dx = \int \frac{\ln\left(c+d\,x\right)}{x^4\,\left(b\,x^3+a\right)}\,dx$$

```
[In] int(log(c + d*x)/(x^4*(a + b*x^3)),x)
[Out] int(log(c + d*x)/(x^4*(a + b*x^3)), x)
```

# 3.287 $\int \frac{x^4 \log(c+dx)}{a+bx^3} dx$



## Optimal result

Integrand size = 19, antiderivative size = 416

$$\int \frac{x^4 \log(c+dx)}{a+bx^3} dx = \frac{cx}{2bd} - \frac{x^2}{4b} - \frac{c^2 \log(c+dx)}{2bd^2} + \frac{x^2 \log(c+dx)}{2b} + \frac{a^{2/3} \log\left(-\frac{d(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right) \log(c+dx)}{3b^{5/3}} - \frac{\sqrt[3]{-1}a^{2/3} \log\left(\frac{d(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right) \log(c+dx)}{3b^{5/3}} + \frac{(-1)^{2/3}a^{2/3} \log\left(-\frac{d(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right) \log(c+dx)}{3b^{5/3}} + \frac{a^{2/3} \operatorname{PolyLog}\left(2, \frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}} + \frac{(-1)^{2/3}a^{2/3} \operatorname{PolyLog}\left(2, \frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1}a^{2/3} \operatorname{PolyLog}\left(2, \frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3b^{5/3}}$$

```
[Out] 1/2*c*x/b/d-1/4*x^2/b-1/2*c^2*ln(d*x+c)/b/d^2+1/2*x^2*ln(d*x+c)/b+1/3*a^(2/
3)*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/b^(5/3)-1/3*(
```

```
-1)^(1/3)*a^(2/3)*ln(d*(a^(1/3)-(-1)^(1/3)*b^(1/3)*x)/((-1)^(1/3)*b^(1/3)*c
+a^(1/3)*d))*ln(d*x+c)/b^(5/3)+1/3*(-1)^(2/3)*a^(2/3)*ln(-d*(a^(1/3)+(-1)^(
2/3)*b^(1/3)*x)/((-1)^(2/3)*b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/b^(5/3)+1/3*a^(
2/3)*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1/3)*d))/b^(5/3)+1/3*(-1)^(2/3
)*a^(2/3)*polylog(2,(-1)^(2/3)*b^(1/3)*(d*x+c)/((-1)^(2/3)*b^(1/3)*c-a^(1/3
)*d))/b^(5/3)-1/3*(-1)^(1/3)*a^(2/3)*polylog(2,(-1)^(1/3)*b^(1/3)*(d*x+c)/(
(-1)^(1/3)*b^(1/3)*c+a^(1/3)*d))/b^(5/3)
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {327, 298, 31, 648, 631, 210, 642, 2463, 2442, 45, 2441, 2440, 2438}

$$\int \frac{x^4 \log(c+dx)}{a+bx^3}\,dx = \frac{a^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}} + \frac{(-1)^{2/3}a^{2/3}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1}a^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3b^{5/3}} + \frac{a^{2/3}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1}a^{2/3}\log(c+dx)\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}c}\right)}{3b^{5/3}} + \frac{(-1)^{2/3}a^{2/3}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}} - \frac{c^2\log(c+dx)}{2bd^2} + \frac{x^2\log(c+dx)}{2b} + \frac{cx}{2bd} - \frac{x^2}{4b}$$

```
[In] Int[(x^4*Log[c + d*x])/(a + b*x^3),x]
```

```
[Out] (c*x)/(2*b*d) - x^2/(4*b) - (c^2*Log[c + d*x])/(2*b*d^2) + (x^2*Log[c + d*x
])/(2*b) + (a^(2/3)*Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d)
)]*Log[c + d*x])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*Log[(d*(a^(1/3) - (-1)^(
1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*x])/(3*b^(5/
```

```
3)) + ((-1)^(2/3)*a^(2/3)*Log[-((d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/((-1)^
(2/3)*b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*b^(5/3)) + (a^(2/3)*PolyLog
[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)])/(3*b^(5/3)) + ((-1)^(2/3)
*a^(2/3)*PolyLog[2, ((-1)^(2/3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c -
a^(1/3)*d)])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*PolyLog[2, ((-1)^(1/3)*b^(1/
3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(3*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x\log(c+dx)}{b} - \frac{ax\log(c+dx)}{b\,(a+bx^3)}\right)\,dx$$

$$= \frac{\int x\log(c+dx)\,dx}{b} - \frac{a\int \frac{x\log(c+dx)}{a+bx^3}\,dx}{b}$$

$$= \frac{x^2\log(c+dx)}{2b} - \frac{a\int \left(-\frac{\log(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\log(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\log(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)\,dx}{b} - \frac{d\int \frac{x^2}{c+dx}\,dx}{2b}$$

$$= \frac{x^2\log(c+dx)}{2b} + \frac{a^{2/3}\int \frac{\log(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}a^{2/3}\right)\int \frac{\log(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b^{4/3}} + \frac{\left((-1)^{2/3}a^{2/3}\right)\int \frac{\log(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3b^{4/3}} - \frac{d\int \left(-\frac{c}{d^2}+\frac{x}{d}+\frac{c^2}{d^2(c+dx)}\right)\,dx}{2b}$$

$$= \frac{cx}{2bd} - \frac{x^2}{4b} - \frac{c^2\log(c+dx)}{2bd^2} + \frac{x^2\log(c+dx)}{2b} + \frac{a^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{5/3}} - \frac{\sqrt[3]{-1}a^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{5/3}} + \frac{(-1)^{2/3}a^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{5/3}} - \frac{\left(a^{2/3}d\right)\int \frac{\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3b^{5/3}} + \frac{\left(\sqrt[3]{-1}a^{2/3}d\right)\int \frac{\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3b^{5/3}} - \frac{\left((-1)^{2/3}a^{2/3}d\right)\int \frac{\log\left(\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{-(-1)^{2/3}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3b^{5/3}}$$

$$=\frac{cx}{2bd}-\frac{x^2}{4b}-\frac{c^2\log(c+dx)}{2bd^2}+\frac{x^2\log(c+dx)}{2b}+\frac{a^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{5/3}}-\frac{\sqrt[3]{-1}a^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{5/3}}+\frac{(-1)^{2/3}a^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{5/3}}-\frac{a^{2/3}\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{x}dx,x,c+dx\right)}{3b^{5/3}}+\frac{\left(\sqrt[3]{-1}a^{2/3}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{x}dx,x,c+dx\right)}{3b^{5/3}}-\frac{\left((-1)^{2/3}a^{2/3}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{b}x}{-(-1)^{2/3}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{x}dx,x,c+dx\right)}{3b^{5/3}}$$

$$=\frac{cx}{2bd}-\frac{x^2}{4b}-\frac{c^2\log(c+dx)}{2bd^2}+\frac{x^2\log(c+dx)}{2b}+\frac{a^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{5/3}}-\frac{\sqrt[3]{-1}a^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{5/3}}+\frac{(-1)^{2/3}a^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{5/3}}+\frac{a^{2/3}\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}}+\frac{(-1)^{2/3}a^{2/3}\text{Li}_2\left(\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{5/3}}-\frac{\sqrt[3]{-1}a^{2/3}\text{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3b^{5/3}}$$

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.97

$$\int \frac{x^4 \log(c+dx)}{a+bx^3} \, dx$$

$$= \frac{6b^{2/3}cdx - 3b^{2/3}d^2x^2 - 6b^{2/3}c^2\log(c+dx) + 6b^{2/3}d^2x^2\log(c+dx) + 4a^{2/3}d^2\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+}{}$$

```
[In] Integrate[(x^4*Log[c + d*x])/(a + b*x^3),x]
```

```
[Out] (6*b^(2/3)*c*d*x - 3*b^(2/3)*d^2*x^2 - 6*b^(2/3)*c^2*Log[c + d*x] + 6*b^(2/
3)*d^2*x^2*Log[c + d*x] + 4*a^(2/3)*d^2*Log[(d*(a^(1/3) + b^(1/3)*x))/(-(b^
(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] - 4*(-1)^(1/3)*a^(2/3)*d^2*Log[(d*(a^(1
/3) - (-1)^(1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*
x] + 4*(-1)^(2/3)*a^(2/3)*d^2*Log[(d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(-((
-1)^(2/3)*b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] + 4*a^(2/3)*d^2*PolyLog[2,
(b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)] + 4*(-1)^(2/3)*a^(2/3)*d^2*Pol
yLog[2, ((-1)^(2/3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)]
- 4*(-1)^(1/3)*a^(2/3)*d^2*PolyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^
(1/3)*b^(1/3)*c + a^(1/3)*d)])/(12*b^(5/3)*d^2)
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.36

| method | result |
|---|---|
| risch | $\frac{x^2\ln(dx+c)}{2b} - \frac{c^2\ln(dx+c)}{2b\,d^2} - \frac{x^2}{4b} + \frac{cx}{2db} + \frac{3c^2}{4d^2b} + \frac{da\left(\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)} \frac{\ln(dx+}{}\right.}{3b^2}$ |
| derivativedivides | $\frac{-\frac{d^3\left(-\frac{(dx+c)^2\ln(dx+c)}{2}+\frac{(dx+c)^2}{4}+c((dx+c)\ln(dx+c)-dx-c)\right)}{b}+\frac{a\,d^6\left(\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\right.}{3b^2}}{d^5}$ |
| default | $\frac{-\frac{d^3\left(-\frac{(dx+c)^2\ln(dx+c)}{2}+\frac{(dx+c)^2}{4}+c((dx+c)\ln(dx+c)-dx-c)\right)}{b}+\frac{a\,d^6\left(\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\right.}{3b^2}}{d^5}$ |

```
[In] int(x^4*ln(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*ln(d*x+c)/b-1/2*c^2*ln(d*x+c)/b/d^2-1/4*x^2/b+1/2/d/b*c*x+3/4/d^2/b
*c^2+1/3*d*a/b^2*sum(1/(-_R1+c)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x
+_R1-c)/_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

## Fricas [F]

$$\int \frac{x^4 \log(c+dx)}{a+bx^3} \, dx = \int \frac{x^4 \log (dx+c)}{bx^3+a} \, dx$$

```
[In] integrate(x^4*log(d*x+c)/(b*x^3+a),x, algorithm="fricas")
[Out] integral(x^4*log(d*x + c)/(b*x^3 + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \log(c+dx)}{a+bx^3} \, dx = \text{Timed out}$$

```
[In] integrate(x**4*ln(d*x+c)/(b*x**3+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^4 \log(c+dx)}{a+bx^3} \, dx = \int \frac{x^4 \log (dx+c)}{bx^3+a} \, dx$$

```
[In] integrate(x^4*log(d*x+c)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(x^4*log(d*x + c)/(b*x^3 + a), x)
```

## Giac [F]

$$\int \frac{x^4 \log(c+dx)}{a+bx^3} \, dx = \int \frac{x^4 \log (dx+c)}{bx^3+a} \, dx$$

```
[In] integrate(x^4*log(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(x^4*log(d*x + c)/(b*x^3 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^4\,\ln(c+d\,x)}{b\,x^3+a}\,dx$$

```
[In] int((x^4*log(c + d*x))/(a + b*x^3),x)
[Out] int((x^4*log(c + d*x))/(a + b*x^3), x)
```

# 3.288 $\int \frac{x^3 \log(c+dx)}{a+bx^3} dx$



## Optimal result

Integrand size = 19, antiderivative size = 383

$$\int \frac{x^3 \log(c+dx)}{a+bx^3} dx = -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} - \frac{\sqrt[3]{a}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}} + \frac{\sqrt[3]{-1}\sqrt[3]{a}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}} - \frac{\sqrt[3]{a}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}} + \frac{\sqrt[3]{-1}\sqrt[3]{a}\,\text{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3b^{4/3}}$$

```
[Out] -x/b+(d*x+c)*ln(d*x+c)/b/d-1/3*a^(1/3)*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c
-a^(1/3)*d))*ln(d*x+c)/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*ln(d*(a^(1/3)-(-1)^(1
/3)*b^(1/3)*x)/((-1)^(1/3)*b^(1/3)*c+a^(1/3)*d))*ln(d*x+c)/b^(4/3)+1/3*(-1)
^(1/3)*a^(1/3)*ln(-d*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/((-1)^(2/3)*b^(1/3)*c-a
```

```
^(1/3)*d))*ln(d*x+c)/b^(4/3)-1/3*a^(1/3)*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)
*c-a^(1/3)*d))/b^(4/3)+1/3*(-1)^(1/3)*a^(1/3)*polylog(2,(-1)^(2/3)*b^(1/3)*
(d*x+c)/((-1)^(2/3)*b^(1/3)*c-a^(1/3)*d))/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*po
lylog(2,(-1)^(1/3)*b^(1/3)*(d*x+c)/((-1)^(1/3)*b^(1/3)*c+a^(1/3)*d))/b^(4/3
)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {327, 206, 31, 648, 631, 210, 642, 2463, 2436, 2332, 2456, 2441, 2440, 2438}

$$\begin{aligned}\int \frac{x^3\log(c+dx)}{a+bx^3}\,dx = &-\frac{\sqrt[3]{a}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}}\\ &+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,\text{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}}\\ &-\frac{(-1)^{2/3}\sqrt[3]{a}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3b^{4/3}}\\ &-\frac{\sqrt[3]{a}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}}\\ &-\frac{(-1)^{2/3}\sqrt[3]{a}\log(c+dx)\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}c}\right)}{3b^{4/3}}\\ &+\frac{\sqrt[3]{-1}\sqrt[3]{a}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}}\\ &+\frac{(c+dx)\log(c+dx)}{bd}-\frac{x}{b}\end{aligned}$$

```
[In] Int[(x^3*Log[c + d*x])/(a + b*x^3),x]
```

```
[Out] -(x/b) + ((c + d*x)*Log[c + d*x])/(b*d) - (a^(1/3)*Log[-((d*(a^(1/3) + b^(1
/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*b^(4/3)) - ((-1)^(2/3)*a
^(1/3)*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)*c + a^(
1/3)*d)]*Log[c + d*x])/(3*b^(4/3)) + ((-1)^(1/3)*a^(1/3)*Log[-((d*(a^(1/3)
+ (-1)^(2/3)*b^(1/3)*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])
/(3*b^(4/3)) - (a^(1/3)*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)
```

```
*d)])/(3*b^(4/3)) + ((-1)^(1/3)*a^(1/3)*PolyLog[2, ((-1)^(2/3)*b^(1/3)*(c +
 d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)])/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/
3)*PolyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3
)*d)])/(3*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log(c+dx)}{b} - \frac{a\log(c+dx)}{b(a+bx^3)}\right) dx$$

$$= \frac{\int \log(c+dx)\,dx}{b} - \frac{a\int \frac{\log(c+dx)}{a+bx^3}\,dx}{b}$$

$$= -\frac{a\int \left(-\frac{\log(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\log(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\log(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx}{b} + \frac{\text{Subst}\left(\int \log(x)\,dx, x, c+dx\right)}{bd}$$

$$= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} + \frac{\sqrt[3]{a}\int \frac{\log(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{\log(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{\log(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b}$$

$$= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} - \frac{\sqrt[3]{a}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}} + \frac{\sqrt[3]{-1}\sqrt[3]{a}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}} + \frac{\left(\sqrt[3]{a}d\right)\int \frac{\log\left(\frac{d\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}\sqrt[3]{a}d\right)\int \frac{\log\left(\frac{d\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3b^{4/3}} + \frac{\left((-1)^{2/3}\sqrt[3]{a}d\right)\int \frac{\log\left(\frac{d\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{-\sqrt[3]{-1}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3b^{4/3}}$$

$$= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} - \frac{\sqrt[3]{a}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}} + \frac{\sqrt[3]{-1}\sqrt[3]{a}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}} + \frac{\sqrt[3]{a}\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{(-1)^{2/3}\sqrt[3]{b}x}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3b^{4/3}} + \frac{\left((-1)^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{-\sqrt[3]{-1}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3b^{4/3}}$$

$$= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} - \frac{\sqrt[3]{a}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}} + \frac{\sqrt[3]{-1}\sqrt[3]{a}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3b^{4/3}} - \frac{\sqrt[3]{a}\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}} + \frac{\sqrt[3]{-1}\sqrt[3]{a}\text{Li}_2\left(\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\text{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3b^{4/3}}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.96

$$\int \frac{x^3 \log(c+dx)}{a+bx^3}\,dx$$

$$= \frac{-3\sqrt[3]{b}dx + 3\sqrt[3]{b}c\log(c+dx) + 3\sqrt[3]{b}dx\log(c+dx) - \sqrt[3]{a}d\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx) - (-1)^{2/3}\sqrt[3]{a}d\,\mathrm{l}}{}$$

```
[In] Integrate[(x^3*Log[c + d*x])/(a + b*x^3),x]
```

```
[Out] (-3*b^(1/3)*d*x + 3*b^(1/3)*c*Log[c + d*x] + 3*b^(1/3)*d*x*Log[c + d*x] - a
^(1/3)*d*Log[(d*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*c) + a^(1/3)*d)]*Log[c +
d*x] - (-1)^(2/3)*a^(1/3)*d*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((-1)^
(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*x] + (-1)^(1/3)*a^(1/3)*d*Log[(d*(a
^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(-((-1)^(2/3)*b^(1/3)*c) + a^(1/3)*d)]*Log[
c + d*x] - a^(1/3)*d*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)
] + (-1)^(1/3)*a^(1/3)*d*PolyLog[2, ((-1)^(2/3)*b^(1/3)*(c + d*x))/((-1)^(2
/3)*b^(1/3)*c - a^(1/3)*d)] - (-1)^(2/3)*a^(1/3)*d*PolyLog[2, ((-1)^(1/3)*b
^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(3*b^(4/3)*d)
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.33

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d^3((dx+c)\ln(dx+c)-dx-c)}{b} - \frac{a d^6 \left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+a d^3-b c^3\right)} \frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-}{}\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b^2}}{d^4}$ |
| default | $\frac{\frac{d^3((dx+c)\ln(dx+c)-dx-c)}{b} - \frac{a d^6 \left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+a d^3-b c^3\right)} \frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-}{}\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b^2}}{d^4}$ |
| risch | $\frac{x\ln(dx+c)}{b} + \frac{\ln(dx+c)c}{db} - \frac{x}{b} - \frac{c}{db} - \frac{d^2 a \left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+a d^3-b c^3\right)} \frac{\ln(dx+c)\ln\left(\frac{-dx+\_R}{\_R}\right.}{\_R1}\right.}{3b^2}$ |

```
[In] int(x^3*ln(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(d^3/b*((d*x+c)*ln(d*x+c)-d*x-c)-1/3*a*d^6/b^2*sum(1/(_R1^2-2*_R1*c+c
^2)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

## Fricas [F]

$$\int \frac{x^3\log(c+dx)}{a+bx^3}\,dx = \int \frac{x^3\log\left(dx+c\right)}{bx^3+a}\,dx$$

```
[In] integrate(x^3*log(d*x+c)/(b*x^3+a),x, algorithm="fricas")
[Out] integral(x^3*log(d*x + c)/(b*x^3 + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3\log(c+dx)}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*ln(d*x+c)/(b*x**3+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^3\log(c+dx)}{a+bx^3}\,dx = \int \frac{x^3\log\left(dx+c\right)}{bx^3+a}\,dx$$

```
[In] integrate(x^3*log(d*x+c)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(x^3*log(d*x + c)/(b*x^3 + a), x)
```

## Giac [F]

$$\int \frac{x^3\log(c+dx)}{a+bx^3}\,dx = \int \frac{x^3\log\left(dx+c\right)}{bx^3+a}\,dx$$

```
[In] integrate(x^3*log(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(x^3*log(d*x + c)/(b*x^3 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log(c+dx)}{a+bx^3}\,dx = \int \frac{x^3\,\ln(c+d\,x)}{b\,x^3+a}\,dx$$

```
[In] int((x^3*log(c + d*x))/(a + b*x^3),x)
[Out] int((x^3*log(c + d*x))/(a + b*x^3), x)
```

# 3.289 $\int \frac{x \log(c+dx)}{a+bx^3} dx$



## Optimal result

Integrand size = 17, antiderivative size = 359

$$\int \frac{x\log(c+dx)}{a+bx^3}\,dx = -\frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}} - \frac{(-1)^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{(-1)^{2/3}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}}$$

```
[Out] -1/3*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^(1/3)/b^(
2/3)+1/3*(-1)^(1/3)*ln(d*(a^(1/3)-(-1)^(1/3)*b^(1/3)*x)/((-1)^(1/3)*b^(1/3)
*c+a^(1/3)*d))*ln(d*x+c)/a^(1/3)/b^(2/3)-1/3*(-1)^(2/3)*ln(-d*(a^(1/3)+(-1)
^(2/3)*b^(1/3)*x)/((-1)^(2/3)*b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^(1/3)/b^(2/
3)-1/3*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1/3)*d))/a^(1/3)/b^(2/3)-1/3
*(-1)^(2/3)*polylog(2,(-1)^(2/3)*b^(1/3)*(d*x+c)/((-1)^(2/3)*b^(1/3)*c-a^(1
/3)*d))/a^(1/3)/b^(2/3)+1/3*(-1)^(1/3)*polylog(2,(-1)^(1/3)*b^(1/3)*(d*x+c)
/((-1)^(1/3)*b^(1/3)*c+a^(1/3)*d))/a^(1/3)/b^(2/3)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {298, 31, 648, 631, 210, 642, 2463, 2441, 2440, 2438}

$$\int \frac{x\log(c+dx)}{a+bx^3}\,dx = -\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{(-1)^{2/3}\,\text{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\log(c+dx)\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}c}\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{(-1)^{2/3}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}}$$

```
[In] Int[(x*Log[c + d*x])/(a + b*x^3),x]
```

```
[Out] -1/3*(Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x
])/(a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))
/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*x])/(3*a^(1/3)*b^(2/3)) - ((
-1)^(2/3)*Log[-((d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/((-1)^(2/3)*b^(1/3)*c
- a^(1/3)*d))]*Log[c + d*x])/(3*a^(1/3)*b^(2/3)) - PolyLog[2, (b^(1/3)*(c +
 d*x))/(b^(1/3)*c - a^(1/3)*d)]/(3*a^(1/3)*b^(2/3)) - ((-1)^(2/3)*PolyLog[2
, ((-1)^(2/3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)])/(3*a^
(1/3)*b^(2/3)) + ((-1)^(1/3)*PolyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1
)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(3*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{\log(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\log(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\log(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx$$

$$= -\frac{\int \frac{\log(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{\log(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{\log(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$= -\frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}} - \frac{(-1)^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}} + \frac{d\int \frac{\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3\sqrt[3]{a}b^{2/3}} - \frac{\left(\sqrt[3]{-1}d\right)\int \frac{\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3\sqrt[3]{a}b^{2/3}} + \frac{\left((-1)^{2/3}d\right)\int \frac{\log\left(\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{-(-1)^{2/3}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3\sqrt[3]{a}b^{2/3}}$$

$$= -\frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}}$$

$$- \frac{(-1)^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}}$$

$$+ \frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$- \frac{\sqrt[3]{-1}\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$+ \frac{(-1)^{2/3}\text{Subst}\left(\int\frac{\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{b}x}{-(-1)^{2/3}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$= -\frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}}$$

$$- \frac{(-1)^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3\sqrt[3]{a}b^{2/3}} - \frac{\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$- \frac{(-1)^{2/3}\text{Li}_2\left(\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\text{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3\sqrt[3]{a}b^{2/3}}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.83

$$\int \frac{x \log(c+dx)}{a+bx^3} dx = \frac{-\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)+\sqrt[3]{-1}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)-(-1)^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}+(-\right.}{-(-1)^{2/3}\sqrt[3]{}}\right.}{}$$

```
[In] Integrate[(x*Log[c + d*x])/(a + b*x^3),x]
```

```
[Out] (-(Log[(d*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x])
+ (-1)^(1/3)*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)*c
 + a^(1/3)*d)]*Log[c + d*x] - (-1)^(2/3)*Log[(d*(a^(1/3) + (-1)^(2/3)*b^(1/
3)*x))/(-((-1)^(2/3)*b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] - PolyLog[2, (b^
(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)] - (-1)^(2/3)*PolyLog[2, ((-1)^(2/
3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)] + (-1)^(1/3)*Poly
Log[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/
(3*a^(1/3)*b^(2/3))
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.24

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{3b}$ | 86 |
| default | $-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{3b}$ | 86 |
| risch | $-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{3b}$ | 86 |

```
[In] int(x*ln(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*d/b*sum(1/(-_R1+c)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/
_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

## Fricas [F]

$$\int \frac{x \log(c+dx)}{a+bx^3}\,dx = \int \frac{x \log\left(dx+c\right)}{bx^3+a}\,dx$$

```
[In] integrate(x*log(d*x+c)/(b*x^3+a),x, algorithm="fricas")
[Out] integral(x*log(d*x + c)/(b*x^3 + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x \log(c+dx)}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*ln(d*x+c)/(b*x**3+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x \log(c+dx)}{a+bx^3}\,dx = \int \frac{x \log\left(dx+c\right)}{bx^3+a}\,dx$$

```
[In] integrate(x*log(d*x+c)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(x*log(d*x + c)/(b*x^3 + a), x)
```

## Giac [F]

$$\int \frac{x \log(c+dx)}{a+bx^3}\,dx = \int \frac{x \log\left(dx+c\right)}{bx^3+a}\,dx$$

```
[In] integrate(x*log(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(x*log(d*x + c)/(b*x^3 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x \log(c+dx)}{a+bx^3} \, dx = \int \frac{x \ln(c+d\,x)}{b\,x^3+a} \, dx$$

```
[In] int((x*log(c + d*x))/(a + b*x^3),x)
[Out] int((x*log(c + d*x))/(a + b*x^3), x)
```

# 3.290 $\int \frac{\log(c+dx)}{a+bx^3}\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 359

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx = \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}} - \frac{\sqrt[3]{-1}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\sqrt[3]{-1}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^(2/3)/b^(1
/3)+1/3*(-1)^(2/3)*ln(d*(a^(1/3)-(-1)^(1/3)*b^(1/3)*x)/((-1)^(1/3)*b^(1/3)*
c+a^(1/3)*d))*ln(d*x+c)/a^(2/3)/b^(1/3)-1/3*(-1)^(1/3)*ln(-d*(a^(1/3)+(-1)^
(2/3)*b^(1/3)*x)/((-1)^(2/3)*b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^(2/3)/b^(1/3
)+1/3*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1/3)*d))/a^(2/3)/b^(1/3)-1/3*
(-1)^(1/3)*polylog(2,(-1)^(2/3)*b^(1/3)*(d*x+c)/((-1)^(2/3)*b^(1/3)*c-a^(1/
3)*d))/a^(2/3)/b^(1/3)+1/3*(-1)^(2/3)*polylog(2,(-1)^(1/3)*b^(1/3)*(d*x+c)/
((-1)^(1/3)*b^(1/3)*c+a^(1/3)*d))/a^(2/3)/b^(1/3)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2456, 2441, 2440, 2438}

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx = \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\sqrt[3]{-1}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\log(c+dx)\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}c}\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\sqrt[3]{-1}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}}$$

```
[In] Int[Log[c + d*x]/(a + b*x^3),x]
```

```
[Out] (Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3
*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((
-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*x])/(3*a^(2/3)*b^(1/3)) - ((-1)
^(1/3)*Log[-((d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/((-1)^(2/3)*b^(1/3)*c - a
^(1/3)*d))]*Log[c + d*x])/(3*a^(2/3)*b^(1/3)) + PolyLog[2, (b^(1/3)*(c + d*
x))/(b^(1/3)*c - a^(1/3)*d)]/(3*a^(2/3)*b^(1/3)) - ((-1)^(1/3)*PolyLog[2, (
(-1)^(2/3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)])/(3*a^(2/
3)*b^(1/3)) + ((-1)^(2/3)*PolyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^(
1/3)*b^(1/3)*c + a^(1/3)*d)])/(3*a^(2/3)*b^(1/3))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rubi steps

$$\text{integral} = \int \left(-\frac{\log(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\log(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\log(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx$$

$$= -\frac{\int \frac{\log(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{\log(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{\log(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$= \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}} - \frac{\sqrt[3]{-1}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}} - \frac{d\int \frac{\log\left(\frac{d\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3a^{2/3}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{-1}d\right)\int \frac{\log\left(\frac{d\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3a^{2/3}\sqrt[3]{b}} - \frac{\left((-1)^{2/3}d\right)\int \frac{\log\left(\frac{d\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{-\sqrt[3]{-1}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{c+dx}\,dx}{3a^{2/3}\sqrt[3]{b}}$$

$$= \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}}$$

$$- \frac{\sqrt[3]{-1}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}}$$

$$- \frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$+ \frac{\sqrt[3]{-1}\text{Subst}\left(\int \frac{\log\left(1-\frac{(-1)^{2/3}\sqrt[3]{b}x}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$- \frac{(-1)^{2/3}\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{-\sqrt[3]{-1}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$= \frac{\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}}$$

$$- \frac{\sqrt[3]{-1}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{2/3}\sqrt[3]{b}} + \frac{\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$- \frac{\sqrt[3]{-1}\text{Li}_2\left(\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\text{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{2/3}\sqrt[3]{b}}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.82

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx$$
$$= \frac{\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)+(-1)^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)-\sqrt[3]{-1}\log\left(\frac{d\left(\sqrt[3]{a}+(-1\right.}{-(-1)^{2/3}\sqrt[3]{}}\right.}{}$$

```
[In] Integrate[Log[c + d*x]/(a + b*x^3),x]
```

```
[Out] (Log[(d*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] + (
-1)^(2/3)*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)*c +
a^(1/3)*d)]*Log[c + d*x] - (-1)^(1/3)*Log[(d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*
x))/(-((-1)^(2/3)*b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] + PolyLog[2, (b^(1/
3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)] - (-1)^(1/3)*PolyLog[2, ((-1)^(2/3)*
b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)] + (-1)^(2/3)*PolyLog
[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(3*
a^(2/3)*b^(1/3))
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.26

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{\_R1^2-2\_R1c+c^2}\right)}{3b}$ | 94 |
| default | $\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{\_R1^2-2\_R1c+c^2}\right)}{3b}$ | 94 |
| risch | $\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{\_R1^2-2\_R1c+c^2}\right)}{3b}$ | 94 |

```
[In] int(ln(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*d^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((
-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

## Fricas [F]

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx = \int \frac{\log\left(dx+c\right)}{bx^3+a}\,dx$$

```
[In] integrate(log(d*x+c)/(b*x^3+a),x, algorithm="fricas")
[Out] integral(log(d*x + c)/(b*x^3 + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(d*x+c)/(b*x**3+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx = \int \frac{\log\left(dx+c\right)}{bx^3+a}\,dx$$

```
[In] integrate(log(d*x+c)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(log(d*x + c)/(b*x^3 + a), x)
```

## Giac [F]

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx = \int \frac{\log\left(dx+c\right)}{bx^3+a}\,dx$$

```
[In] integrate(log(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(log(d*x + c)/(b*x^3 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{a+bx^3}\,dx = \int \frac{\ln(c+d\,x)}{b\,x^3+a}\,dx$$

```
[In] int(log(c + d*x)/(a + b*x^3),x)
[Out] int(log(c + d*x)/(a + b*x^3), x)
```

# 3.291 $\int \frac{\log(c+dx)}{x^2(a+bx^3)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 398

$$\int \frac{\log(c+dx)}{x^2(a+bx^3)}\,dx = \frac{d\log(x)}{ac} - \frac{d\log(c+dx)}{ac} - \frac{\log(c+dx)}{ax} + \frac{\sqrt[3]{b}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}} - \frac{\sqrt[3]{-1}\sqrt[3]{b}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}} + \frac{(-1)^{2/3}\sqrt[3]{b}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}} + \frac{\sqrt[3]{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}} + \frac{(-1)^{2/3}\sqrt[3]{b}\,\mathrm{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}} - \frac{\sqrt[3]{-1}\sqrt[3]{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{4/3}}$$

```
[Out] d*ln(x)/a/c-d*ln(d*x+c)/a/c-ln(d*x+c)/a/x+1/3*b^(1/3)*ln(-d*(a^(1/3)+b^(1/3
)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^(4/3)-1/3*(-1)^(1/3)*b^(1/3)*ln(d*(
```

```
a^(1/3)-(-1)^(1/3)*b^(1/3)*x)/((-1)^(1/3)*b^(1/3)*c+a^(1/3)*d))*ln(d*x+c)/a
^(4/3)+1/3*(-1)^(2/3)*b^(1/3)*ln(-d*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/((-1)^(2
/3)*b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^(4/3)+1/3*b^(1/3)*polylog(2,b^(1/3)*(
d*x+c)/(b^(1/3)*c-a^(1/3)*d))/a^(4/3)+1/3*(-1)^(2/3)*b^(1/3)*polylog(2,(-1)
^(2/3)*b^(1/3)*(d*x+c)/((-1)^(2/3)*b^(1/3)*c-a^(1/3)*d))/a^(4/3)-1/3*(-1)^(
1/3)*b^(1/3)*polylog(2,(-1)^(1/3)*b^(1/3)*(d*x+c)/((-1)^(1/3)*b^(1/3)*c+a^(
1/3)*d))/a^(4/3)
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {331, 298, 31, 648, 631, 210, 642, 2463, 2442, 36, 29, 2441, 2440, 2438}

$$\int \frac{\log(c+dx)}{x^2(a+bx^3)}\,dx = \frac{\sqrt[3]{b}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}} + \frac{(-1)^{2/3}\sqrt[3]{b}\,\text{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}} - \frac{\sqrt[3]{-1}\sqrt[3]{b}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{4/3}} + \frac{\sqrt[3]{b}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}} - \frac{\sqrt[3]{-1}\sqrt[3]{b}\log(c+dx)\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}c}\right)}{3a^{4/3}} + \frac{(-1)^{2/3}\sqrt[3]{b}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}} + \frac{d\log(x)}{ac} - \frac{d\log(c+dx)}{ac} - \frac{\log(c+dx)}{ax}$$

```
[In] Int[Log[c + d*x]/(x^2*(a + b*x^3)),x]
```

```
[Out] (d*Log[x])/(a*c) - (d*Log[c + d*x])/(a*c) - Log[c + d*x]/(a*x) + (b^(1/3)*L
og[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*a
^(4/3)) - ((-1)^(1/3)*b^(1/3)*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/((-1
)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*x])/(3*a^(4/3)) + ((-1)^(2/3)*b^(
```

```
1/3)*Log[-((d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/((-1)^(2/3)*b^(1/3)*c - a^(
1/3)*d))]*Log[c + d*x])/(3*a^(4/3)) + (b^(1/3)*PolyLog[2, (b^(1/3)*(c + d*x
))/(b^(1/3)*c - a^(1/3)*d)])/(3*a^(4/3)) + ((-1)^(2/3)*b^(1/3)*PolyLog[2, (
(-1)^(2/3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)])/(3*a^(4/
3)) - ((-1)^(1/3)*b^(1/3)*PolyLog[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^(
1/3)*b^(1/3)*c + a^(1/3)*d)])/(3*a^(4/3))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log(c+dx)}{ax^2} - \frac{bx\log(c+dx)}{a\left(a+bx^3\right)}\right)dx$$

$$= \frac{\int \frac{\log(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{x\log(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{\log(c+dx)}{ax} - \frac{b\int \left(-\frac{\log(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\log(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\log(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{a} + \frac{d\int \frac{1}{x(c+dx)}\,dx}{a}$$

$$= -\frac{\log(c+dx)}{ax} + \frac{b^{2/3}\int \frac{\log(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}b^{2/3}\right)\int \frac{\log(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{4/3}} + \frac{\left((-1)^{2/3}b^{2/3}\right)\int \frac{\log(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{4/3}} + \frac{d\int \frac{1}{x}\,dx}{ac} - \frac{d^2\int \frac{1}{c+dx}\,dx}{ac}$$

$$= \frac{d\log(x)}{ac} - \frac{d\log(c+dx)}{ac} - \frac{\log(c+dx)}{ax} + \frac{\sqrt[3]{b}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}}$$

$$- \frac{\sqrt[3]{-1}\sqrt[3]{b}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}}$$

$$+ \frac{(-1)^{2/3}\sqrt[3]{b}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}}$$

$$- \frac{\left(\sqrt[3]{b}d\right)\int \frac{\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{c+dx}dx}{3a^{4/3}} + \frac{\left(\sqrt[3]{-1}\sqrt[3]{b}d\right)\int \frac{\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{c+dx}dx}{3a^{4/3}}$$

$$- \frac{\left((-1)^{2/3}\sqrt[3]{b}d\right)\int \frac{\log\left(\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{-(-1)^{2/3}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{c+dx}dx}{3a^{4/3}}$$

$$= \frac{d\log(x)}{ac} - \frac{d\log(c+dx)}{ac} - \frac{\log(c+dx)}{ax} + \frac{\sqrt[3]{b}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}} - \frac{\sqrt[3]{-1}\sqrt[3]{b}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}} + \frac{(-1)^{2/3}\sqrt[3]{b}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}} - \frac{\sqrt[3]{b}\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{x}dx,x,c+dx\right)}{3a^{4/3}} + \frac{\left(\sqrt[3]{-1}\sqrt[3]{b}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{x}dx,x,c+dx\right)}{3a^{4/3}} - \frac{\left((-1)^{2/3}\sqrt[3]{b}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{b}x}{-(-1)^{2/3}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{x}dx,x,c+dx\right)}{3a^{4/3}}$$

$$= \frac{d\log(x)}{ac} - \frac{d\log(c+dx)}{ac} - \frac{\log(c+dx)}{ax} + \frac{\sqrt[3]{b}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}} - \frac{\sqrt[3]{-1}\sqrt[3]{b}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}} + \frac{(-1)^{2/3}\sqrt[3]{b}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{4/3}} + \frac{\sqrt[3]{b}\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}} + \frac{(-1)^{2/3}\sqrt[3]{b}\text{Li}_2\left(\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{4/3}} - \frac{\sqrt[3]{-1}\sqrt[3]{b}\text{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{4/3}}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.95

$$\int \frac{\log(c+dx)}{x^2(a+bx^3)}\,dx$$
$$= \frac{3\sqrt[3]{a}dx\log(x) - 3\sqrt[3]{a}c\log(c+dx) - 3\sqrt[3]{a}dx\log(c+dx) + \sqrt[3]{b}cx\log\left(\frac{d(\sqrt[3]{a}+\sqrt[3]{b}x)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx) - \sqrt[3]{-1}}{}$$

```
[In] Integrate[Log[c + d*x]/(x^2*(a + b*x^3)),x]
```

```
[Out] (3*a^(1/3)*d*x*Log[x] - 3*a^(1/3)*c*Log[c + d*x] - 3*a^(1/3)*d*x*Log[c + d*
x] + b^(1/3)*c*x*Log[(d*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*c) + a^(1/3)*d)]*
Log[c + d*x] - (-1)^(1/3)*b^(1/3)*c*x*Log[(d*(a^(1/3) - (-1)^(1/3)*b^(1/3)*
x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*x] + (-1)^(2/3)*b^(1/3)*c
*x*Log[(d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(-((-1)^(2/3)*b^(1/3)*c) + a^(1
/3)*d)]*Log[c + d*x] + b^(1/3)*c*x*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*
c - a^(1/3)*d)] + (-1)^(2/3)*b^(1/3)*c*x*PolyLog[2, ((-1)^(2/3)*b^(1/3)*(c
+ d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)] - (-1)^(1/3)*b^(1/3)*c*x*PolyLo
g[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(3
*a^(4/3)*c*x)
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.31

| method | result |
|---|---|
| derivativedivides | $d\left(\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}}{3a}+\frac{\frac{\ln(-dx)}{c}}{}\right.$ |
| default | $d\left(\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}}{3a}+\frac{\frac{\ln(-dx)}{c}}{}\right.$ |
| risch | $\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{3a}+\frac{d\ln(-d}{ac}$ |

[In] int(ln(d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] d*(1/3/a*sum(1/(-_R1+c)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a*(1/c*ln(-d*x)-ln(d*x+c)*(d*x+c)/c/d/x))

## Fricas [F]

$$\int \frac{\log(c+dx)}{x^2\,(a+bx^3)}\,dx=\int \frac{\log\left(dx+c\right)}{(bx^3+a)x^2}\,dx$$

[In] integrate(log(d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")

[Out] integral(log(d*x + c)/(b*x^5 + a*x^2), x)

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^2\,(a+bx^3)}\,dx=\text{Timed out}$$

[In] integrate(ln(d*x+c)/x**2/(b*x**3+a),x)

[Out] Timed out

## Maxima [F]

$$\int \frac{\log(c+dx)}{x^2(a+bx^3)}\,dx = \int \frac{\log(dx+c)}{(bx^3+a)x^2}\,dx$$

```
[In] integrate(log(d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(log(d*x + c)/((b*x^3 + a)*x^2), x)
```

## Giac [F]

$$\int \frac{\log(c+dx)}{x^2(a+bx^3)}\,dx = \int \frac{\log(dx+c)}{(bx^3+a)x^2}\,dx$$

```
[In] integrate(log(d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
[Out] integrate(log(d*x + c)/((b*x^3 + a)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^2(a+bx^3)}\,dx = \int \frac{\ln(c+d\,x)}{x^2\,(b\,x^3+a)}\,dx$$

```
[In] int(log(c + d*x)/(x^2*(a + b*x^3)),x)
[Out] int(log(c + d*x)/(x^2*(a + b*x^3)), x)
```

# 3.292 $\int \frac{\log(c+dx)}{x^3(a+bx^3)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 423

$$\int \frac{\log(c+dx)}{x^3(a+bx^3)}\,dx = -\frac{d}{2acx}-\frac{d^2\log(x)}{2ac^2}+\frac{d^2\log(c+dx)}{2ac^2}-\frac{\log(c+dx)}{2ax^2} - \frac{b^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}} - \frac{(-1)^{2/3}b^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}} + \frac{\sqrt[3]{-1}b^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}} - \frac{b^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}} + \frac{\sqrt[3]{-1}b^{2/3}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}} - \frac{(-1)^{2/3}b^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{5/3}}$$

```
[Out] -1/2*d/a/c/x-1/2*d^2*ln(x)/a/c^2+1/2*d^2*ln(d*x+c)/a/c^2-1/2*ln(d*x+c)/a/x^
2-1/3*b^(2/3)*ln(-d*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^
```

```
(5/3)-1/3*(-1)^(2/3)*b^(2/3)*ln(d*(a^(1/3)-(-1)^(1/3)*b^(1/3)*x)/((-1)^(1/3
)*b^(1/3)*c+a^(1/3)*d))*ln(d*x+c)/a^(5/3)+1/3*(-1)^(1/3)*b^(2/3)*ln(-d*(a^(
1/3)+(-1)^(2/3)*b^(1/3)*x)/((-1)^(2/3)*b^(1/3)*c-a^(1/3)*d))*ln(d*x+c)/a^(5
/3)-1/3*b^(2/3)*polylog(2,b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1/3)*d))/a^(5/3)+1/
3*(-1)^(1/3)*b^(2/3)*polylog(2,(-1)^(2/3)*b^(1/3)*(d*x+c)/((-1)^(2/3)*b^(1/
3)*c-a^(1/3)*d))/a^(5/3)-1/3*(-1)^(2/3)*b^(2/3)*polylog(2,(-1)^(1/3)*b^(1/3
)*(d*x+c)/((-1)^(1/3)*b^(1/3)*c+a^(1/3)*d))/a^(5/3)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {331, 206, 31, 648, 631, 210, 642, 2463, 2442, 46, 2456, 2441, 2440, 2438}

$$\begin{aligned}\int \frac{\log(c+dx)}{x^3(a+bx^3)}\,dx = &-\frac{b^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}}\\&+\frac{\sqrt[3]{-1}b^{2/3}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}}\\&-\frac{(-1)^{2/3}b^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{5/3}}\\&-\frac{b^{2/3}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}}\\&-\frac{(-1)^{2/3}b^{2/3}\log(c+dx)\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}c}\right)}{3a^{5/3}}\\&+\frac{\sqrt[3]{-1}b^{2/3}\log(c+dx)\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}}\\&-\frac{d^2\log(x)}{2ac^2}+\frac{d^2\log(c+dx)}{2ac^2}-\frac{\log(c+dx)}{2ax^2}-\frac{d}{2acx}\end{aligned}$$

```
[In] Int[Log[c + d*x]/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*d/(a*c*x) - (d^2*Log[x])/(2*a*c^2) + (d^2*Log[c + d*x])/(2*a*c^2) - Lo
g[c + d*x]/(2*a*x^2) - (b^(2/3)*Log[-((d*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*c
- a^(1/3)*d))]*Log[c + d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Log[(d*(a^(1
/3) - (-1)^(1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*
```

```
x])/(3*a^(5/3)) + ((-1)^(1/3)*b^(2/3)*Log[-((d*(a^(1/3) + (-1)^(2/3)*b^(1/3
)*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d))]*Log[c + d*x])/(3*a^(5/3)) - (b^(
2/3)*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(1/3)*c - a^(1/3)*d)])/(3*a^(5/3)) +
 ((-1)^(1/3)*b^(2/3)*PolyLog[2, ((-1)^(2/3)*b^(1/3)*(c + d*x))/((-1)^(2/3)*
b^(1/3)*c - a^(1/3)*d)])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*PolyLog[2, ((-1)
^(1/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(3*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log(c+dx)}{ax^3} - \frac{b\log(c+dx)}{a(a+bx^3)}\right) dx$$

$$= \frac{\int \frac{\log(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\log(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{\log(c+dx)}{2ax^2} - \frac{b\int \left(-\frac{\log(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\log(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\log(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx}{a} + \frac{d\int \frac{1}{x^2(c+dx)}\,dx}{2a}$$

$$= -\frac{\log(c+dx)}{2ax^2} + \frac{b\int \frac{\log(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{b\int \frac{\log(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{b\int \frac{\log(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{d\int \left(\frac{1}{cx^2} - \frac{d}{c^2x} + \frac{d^2}{c^2(c+dx)}\right) dx}{2a}$$

$$= -\frac{d}{2acx} - \frac{d^2\log(x)}{2ac^2} + \frac{d^2\log(c+dx)}{2ac^2} - \frac{\log(c+dx)}{2ax^2}$$

$$- \frac{b^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}}$$

$$- \frac{(-1)^{2/3}b^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}}$$

$$+ \frac{\sqrt[3]{-1}b^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}}$$

$$+ \frac{\left(b^{2/3}d\right)\int\frac{\log\left(\frac{d\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{c+dx}dx}{3a^{5/3}} - \frac{\left(\sqrt[3]{-1}b^{2/3}d\right)\int\frac{\log\left(\frac{d\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{c+dx}dx}{3a^{5/3}}$$

$$+ \frac{\left((-1)^{2/3}b^{2/3}d\right)\int\frac{\log\left(\frac{d\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{-\sqrt[3]{-1}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{c+dx}dx}{3a^{5/3}}$$

$$= -\frac{d}{2acx} - \frac{d^2\log(x)}{2ac^2} + \frac{d^2\log(c+dx)}{2ac^2} - \frac{\log(c+dx)}{2ax^2}$$

$$- \frac{b^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}}$$

$$- \frac{(-1)^{2/3}b^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}}$$

$$+ \frac{\sqrt[3]{-1}b^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}}$$

$$+ \frac{b^{2/3}\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3a^{5/3}}$$

$$- \frac{\left(\sqrt[3]{-1}b^{2/3}\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{(-1)^{2/3}\sqrt[3]{b}x}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3a^{5/3}}$$

$$+ \frac{\left((-1)^{2/3}b^{2/3}\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{-\sqrt[3]{-1}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{x}dx, x, c+dx\right)}{3a^{5/3}}$$

$$= -\frac{d}{2acx} - \frac{d^2\log(x)}{2ac^2} + \frac{d^2\log(c+dx)}{2ac^2} - \frac{\log(c+dx)}{2ax^2} - \frac{b^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}} - \frac{(-1)^{2/3}b^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}} + \frac{\sqrt[3]{-1}b^{2/3}\log\left(-\frac{d\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)\log(c+dx)}{3a^{5/3}} - \frac{b^{2/3}\text{Li}_2\left(\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}} + \frac{\sqrt[3]{-1}b^{2/3}\text{Li}_2\left(\frac{(-1)^{2/3}\sqrt[3]{b}(c+dx)}{(-1)^{2/3}\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3a^{5/3}} - \frac{(-1)^{2/3}b^{2/3}\text{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)}{3a^{5/3}}$$

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.88

$$\int \frac{\log(c+dx)}{x^3(a+bx^3)}\,dx$$

$$= \frac{-\frac{3a^{2/3}\log(c+dx)}{x^2} - 2b^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+dx) - 2(-1)^{2/3}b^{2/3}\log\left(\frac{d\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{b}c+\sqrt[3]{a}d}\right)\log(c+}{}$$

```
[In] Integrate[Log[c + d*x]/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(2/3)*Log[c + d*x])/x^2 - 2*b^(2/3)*Log[(d*(a^(1/3) + b^(1/3)*x))/(-
(b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] - 2*(-1)^(2/3)*b^(2/3)*Log[(d*(a^(1/
3) - (-1)^(1/3)*b^(1/3)*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)]*Log[c + d*x
] + 2*(-1)^(1/3)*b^(2/3)*Log[(d*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(-((-1)^(
2/3)*b^(1/3)*c) + a^(1/3)*d)]*Log[c + d*x] - (3*a^(2/3)*d*(c + d*x*Log[x] -
 d*x*Log[c + d*x]))/(c^2*x) - 2*b^(2/3)*PolyLog[2, (b^(1/3)*(c + d*x))/(b^(
1/3)*c - a^(1/3)*d)] + 2*(-1)^(1/3)*b^(2/3)*PolyLog[2, ((-1)^(2/3)*b^(1/3)*
(c + d*x))/((-1)^(2/3)*b^(1/3)*c - a^(1/3)*d)] - 2*(-1)^(2/3)*b^(2/3)*PolyL
og[2, ((-1)^(1/3)*b^(1/3)*(c + d*x))/((-1)^(1/3)*b^(1/3)*c + a^(1/3)*d)])/(
6*a^(5/3))
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.35

| method | result |
|---|---|
| derivativedivides | $d^{2}\left(\frac{-\frac{\ln(-dx)}{2c^{2}}-\frac{1}{2cdx}-\frac{\ln(dx+c)(dx+c)(-dx+c)}{2c^{2}d^{2}x^{2}}}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3bc^{2}\_Z+ad^{3}-bc^{3}\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_}{\_}\right)}{\_}}{3a}\right.$ |
| default | $d^{2}\left(\frac{-\frac{\ln(-dx)}{2c^{2}}-\frac{1}{2cdx}-\frac{\ln(dx+c)(dx+c)(-dx+c)}{2c^{2}d^{2}x^{2}}}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3bc^{2}\_Z+ad^{3}-bc^{3}\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_}{\_}\right)}{\_}}{3a}\right.$ |
| risch | $-\frac{d^{2}\ln(-dx)}{2a\,c^{2}}-\frac{d}{2acx}+\frac{d^{2}\ln(dx+c)}{2a\,c^{2}}-\frac{\ln(dx+c)}{2a\,x^{2}}-\frac{d^{2}\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3bc^{2}\_Z+ad^{3}-bc^{3}\right)}\frac{\ln(dx+c)}{}\right.}{3a}$ |

[In] int(ln(d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] d^2*(1/a*(-1/2/c^2*ln(-d*x)-1/2/c/d/x-1/2*ln(d*x+c)*(d*x+c)*(-d*x+c)/c^2/d^2/x^2)-1/3/a*sum(1/(_R1^2-2*_R1*c+c^2)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))

## Fricas [F]

$$\int \frac{\log(c+dx)}{x^{3}\left(a+bx^{3}\right)}\,dx=\int \frac{\log\left(dx+c\right)}{(bx^{3}+a)x^{3}}\,dx$$

[In] integrate(log(d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")

[Out] integral(log(d*x + c)/(b*x^6 + a*x^3), x)

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^3\left(a+bx^3\right)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(d*x+c)/x**3/(b*x**3+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log(c+dx)}{x^3\left(a+bx^3\right)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^3+a)x^3}\,dx$$

```
[In] integrate(log(d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(log(d*x + c)/((b*x^3 + a)*x^3), x)
```

## Giac [F]

$$\int \frac{\log(c+dx)}{x^3\left(a+bx^3\right)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^3+a)x^3}\,dx$$

```
[In] integrate(log(d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
[Out] integrate(log(d*x + c)/((b*x^3 + a)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^3\left(a+bx^3\right)}\,dx = \int \frac{\ln\left(c+d\,x\right)}{x^3\,\left(b\,x^3+a\right)}\,dx$$

```
[In] int(log(c + d*x)/(x^3*(a + b*x^3)),x)
[Out] int(log(c + d*x)/(x^3*(a + b*x^3)), x)
```

# 3.293 $\int \frac{x^7 \log(c+dx)}{a+bx^4} dx$



## Optimal result

Integrand size = 19, antiderivative size = 498

$$\int \frac{x^7 \log(c+dx)}{a+bx^4} dx = \frac{c^3x}{4bd^3} - \frac{c^2x^2}{8bd^2} + \frac{cx^3}{12bd} - \frac{x^4}{16b} - \frac{c^4\log(c+dx)}{4bd^4} + \frac{x^4\log(c+dx)}{4b} - \frac{a\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^2} - \frac{a\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^2} - \frac{a\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^2} - \frac{a\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^2} - \frac{a\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^2} - \frac{a\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^2} - \frac{a\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^2} - \frac{a\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^2}$$

```
[Out] 1/4*c^3*x/b/d^3-1/8*c^2*x^2/b/d^2+1/12*c*x^3/b/d-1/16*x^4/b-1/4*c^4*ln(d*x+
c)/b/d^4+1/4*x^4*ln(d*x+c)/b-1/4*a*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(
```

```
-a)^(1/4)*d))*ln(d*x+c)/b^2-1/4*a*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(
-a)^(1/4)*d))*ln(d*x+c)/b^2-1/4*a*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^
(1/2))/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^2-1/4*a*ln(d*x+c)*ln(d*(-b^(1/4
)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^2-1/4*a*polyl
og(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))/b^2-1/4*a*polylog(2,b^(1/4)*
(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^2-1/4*a*polylog(2,b^(1/4)*(d*x+c)/(b^(1
/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^2-1/4*a*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*
c+d*(-(-a)^(1/2))^(1/2)))/b^2
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {272, 45, 2463, 2442, 266, 2441, 2440, 2438}

$$\int \frac{x^7 \log(c+dx)}{a+bx^4} dx = -\frac{a \operatorname{PolyLog}\left(2, \frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^2} - \frac{a \operatorname{PolyLog}\left(2, \frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^2} - \frac{a \operatorname{PolyLog}\left(2, \frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^2} - \frac{a \operatorname{PolyLog}\left(2, \frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^2} - \frac{a \log(c+dx) \log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4b^2} - \frac{a \log(c+dx) \log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4b^2} - \frac{a \log(c+dx) \log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^2} - \frac{a \log(c+dx) \log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^2} - \frac{c^4 \log(c+dx)}{4bd^4} + \frac{c^3 x}{4bd^3} - \frac{c^2 x^2}{8bd^2} + \frac{x^4 \log(c+dx)}{4b} + \frac{cx^3}{12bd} - \frac{x^4}{16b}$$

```
[In] Int[(x^7*Log[c + d*x])/(a + b*x^4),x]
```

```
[Out] (c^3*x)/(4*b*d^3) - (c^2*x^2)/(8*b*d^2) + (c*x^3)/(12*b*d) - x^4/(16*b) - (
c^4*Log[c + d*x])/(4*b*d^4) + (x^4*Log[c + d*x])/(4*b) - (a*Log[(d*(Sqrt[-S
```

```
qrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Log[c + d*x])/(4*b^
2) - (a*Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c
+ d*x])/(4*b^2) - (a*Log[-((d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - S
qrt[-Sqrt[-a]]*d))]*Log[c + d*x])/(4*b^2) - (a*Log[-((d*((-a)^(1/4) + b^(1/
4)*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*b^2) - (a*PolyLog[2, (
b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^2) - (a*PolyLog[2
, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^2) - (a*PolyLo
g[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^2) - (a*PolyLog[
2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x^3\log(c+dx)}{b} - \frac{ax^3\log(c+dx)}{b\left(a+bx^4\right)}\right)dx$$

$$= \frac{\int x^3\log(c+dx)\,dx}{b} - \frac{a\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx}{b}$$

$$= \frac{x^4\log(c+dx)}{4b} - \frac{a\int \left(\frac{x\log(c+dx)}{2\left(-\sqrt{-a}\sqrt{b}+bx^2\right)} + \frac{x\log(c+dx)}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx}{b} - \frac{d\int \frac{x^4}{c+dx}\,dx}{4b}$$

$$= \frac{x^4\log(c+dx)}{4b} - \frac{a\int \frac{x\log(c+dx)}{-\sqrt{-a}\sqrt{b}+bx^2}\,dx}{2b} - \frac{a\int \frac{x\log(c+dx)}{\sqrt{-a}\sqrt{b}+bx^2}\,dx}{2b} - \frac{d\int \left(-\frac{c^3}{d^4} + \frac{c^2x}{d^3} - \frac{cx^2}{d^2} + \frac{x^3}{d} + \frac{c^4}{d^4(c+dx)}\right)dx}{4b}$$

$$= \frac{c^3x}{4bd^3} - \frac{c^2x^2}{8bd^2} + \frac{cx^3}{12bd} - \frac{x^4}{16b} - \frac{c^4\log(c+dx)}{4bd^4} + \frac{x^4\log(c+dx)}{4b} - \frac{a\int \left(-\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)} + \frac{\log(c+dx)}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}\right)dx}{2b} - \frac{a\int \left(-\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)} + \frac{\log(c+dx)}{2b^{3/4}\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}\right)dx}{2b}$$

$$= \frac{c^3x}{4bd^3} - \frac{c^2x^2}{8bd^2} + \frac{cx^3}{12bd} - \frac{x^4}{16b} - \frac{c^4\log(c+dx)}{4bd^4} + \frac{x^4\log(c+dx)}{4b} + \frac{a\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}\,dx}{4b^{7/4}} + \frac{a\int \frac{\log(c+dx)}{\sqrt[4]{-a}-\sqrt[4]{b}x}\,dx}{4b^{7/4}} - \frac{a\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}\,dx}{4b^{7/4}} - \frac{a\int \frac{\log(c+dx)}{\sqrt[4]{-a}+\sqrt[4]{b}x}\,dx}{4b^{7/4}}$$

$$= \frac{c^3 x}{4bd^3} - \frac{c^2 x^2}{8bd^2} + \frac{cx^3}{12bd} - \frac{x^4}{16b} - \frac{c^4 \log(c+dx)}{4bd^4} + \frac{x^4 \log(c+dx)}{4b} - \frac{a \log\left(\frac{d\left(\sqrt{-\sqrt{-a}} - \sqrt[4]{b}x\right)}{\sqrt[4]{b}c + \sqrt{-\sqrt{-a}}d}\right) \log(c+dx)}{4b^2} - \frac{a \log\left(\frac{d\left(\sqrt[4]{-a} - \sqrt[4]{b}x\right)}{\sqrt[4]{b}c + \sqrt[4]{-a}d}\right) \log(c+dx)}{4b^2} - \frac{a \log\left(-\frac{d\left(\sqrt{-\sqrt{-a}} + \sqrt[4]{b}x\right)}{\sqrt[4]{b}c - \sqrt{-\sqrt{-a}}d}\right) \log(c+dx)}{4b^2} - \frac{a \log\left(-\frac{d\left(\sqrt[4]{-a} + \sqrt[4]{b}x\right)}{\sqrt[4]{b}c - \sqrt[4]{-a}d}\right) \log(c+dx)}{4b^2} + \frac{(ad) \int \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}} - \sqrt[4]{b}x\right)}{\sqrt[4]{b}c + \sqrt{-\sqrt{-a}}d}\right)}{c+dx} dx}{4b^2} + \frac{(ad) \int \frac{\log\left(\frac{d\left(\sqrt[4]{-a} - \sqrt[4]{b}x\right)}{\sqrt[4]{b}c + \sqrt[4]{-a}d}\right)}{c+dx} dx}{4b^2} + \frac{(ad) \int \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}} + \sqrt[4]{b}x\right)}{-\sqrt[4]{b}c + \sqrt{-\sqrt{-a}}d}\right)}{c+dx} dx}{4b^2} + \frac{(ad) \int \frac{\log\left(\frac{d\left(\sqrt[4]{-a} + \sqrt[4]{b}x\right)}{-\sqrt[4]{b}c + \sqrt[4]{-a}d}\right)}{c+dx} dx}{4b^2}$$

$$= \frac{c^3x}{4bd^3} - \frac{c^2x^2}{8bd^2} + \frac{cx^3}{12bd} - \frac{x^4}{16b} - \frac{c^4\log(c+dx)}{4bd^4} + \frac{x^4\log(c+dx)}{4b}$$
$$- \frac{a\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^2} - \frac{a\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^2}$$
$$- \frac{a\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^2} - \frac{a\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^2}$$
$$+ \frac{a\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4b^2}$$
$$+ \frac{a\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4b^2}$$
$$+ \frac{a\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4b^2}$$
$$+ \frac{a\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4b^2}$$

$$= \frac{c^3x}{4bd^3} - \frac{c^2x^2}{8bd^2} + \frac{cx^3}{12bd} - \frac{x^4}{16b} - \frac{c^4\log(c+dx)}{4bd^4} + \frac{x^4\log(c+dx)}{4b} - \frac{a\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^2} - \frac{a\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^2} - \frac{a\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^2} - \frac{a\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^2} - \frac{a\mathrm{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^2} - \frac{a\mathrm{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^2} - \frac{a\mathrm{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^2} - \frac{a\mathrm{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^2}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.96

$$\int \frac{x^7 \log(c+dx)}{a+bx^4} dx = \frac{c^3 x}{4bd^3} - \frac{c^2 x^2}{8bd^2} + \frac{cx^3}{12bd} - \frac{x^4}{16b} - \frac{c^4 \log(c+dx)}{4bd^4} + \frac{x^4 \log(c+dx)}{4b} - \frac{a \log\left(\frac{d\left(i\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right) \log(c+dx)}{4b^2} - \frac{a \log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right) \log(c+dx)}{4b^2} - \frac{a \log\left(-\frac{d\left(i\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right) \log(c+dx)}{4b^2} - \frac{a \log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right) \log(c+dx)}{4b^2} - \frac{a \operatorname{PolyLog}\left(2, \frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^2} - \frac{a \operatorname{PolyLog}\left(2, \frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{4b^2} - \frac{a \operatorname{PolyLog}\left(2, \frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)}{4b^2} - \frac{a \operatorname{PolyLog}\left(2, \frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^2}$$

```
[In] Integrate[(x^7*Log[c + d*x])/(a + b*x^4),x]
```

```
[Out] (c^3*x)/(4*b*d^3) - (c^2*x^2)/(8*b*d^2) + (c*x^3)/(12*b*d) - x^4/(16*b) - (
c^4*Log[c + d*x])/(4*b*d^4) + (x^4*Log[c + d*x])/(4*b) - (a*Log[(d*(I*(-a)^
(1/4) - b^(1/4)*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]*Log[c + d*x])/(4*b^2) - (
a*Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x]
)/(4*b^2) - (a*Log[-((d*(I*(-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - I*(-a)^(1/
4)*d))]*Log[c + d*x])/(4*b^2) - (a*Log[-((d*((-a)^(1/4) + b^(1/4)*x))/(b^(1
/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*b^2) - (a*PolyLog[2, (b^(1/4)*(c +
 d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^2) - (a*PolyLog[2, (b^(1/4)*(c + d
*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)])/(4*b^2) - (a*PolyLog[2, (b^(1/4)*(c + d
*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)])/(4*b^2) - (a*PolyLog[2, (b^(1/4)*(c + d
*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^2)
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.75 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.35

| method | result |
|---|---|
| risch | $-\frac{c^4 \ln(dx+c)}{4b\,d^4}+\frac{c^3 x}{4b\,d^3}+\frac{25c^4}{48d^4 b}-\frac{c^2 x^2}{8b\,d^2}+\frac{c\,x^3}{12bd}+\frac{x^4 \ln(dx+c)}{4b}-\frac{x^4}{16b}-\frac{a\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\right.}\right.}{}$ |
| parts | $\frac{x^4 \ln(dx+c)}{4b}-\frac{\ln(dx+c)a\ln\left(b\,x^4+a\right)}{4b^2}-\frac{d\left(\frac{\frac{\frac{1}{4}d^3x^4-\frac{1}{3}c\,x^3d^2+\frac{1}{2}d\,x^2c^2-x\,c^3}{d^4}+\frac{c^4\ln(dx+c)}{d^5}}{b}-\frac{a\ln(dx+c)\ln\left(b\,x^4+a\right)}{b^2 d}+\frac{a\left(\_R\right.}{}\right.}{}$ |
| derivativedivides | $-\frac{d^4\left(c^3((dx+c)\ln(dx+c)-dx-c)-3c^2\left(\frac{(dx+c)^2\ln(dx+c)}{2}-\frac{(dx+c)^2}{4}\right)+3c\left(\frac{(dx+c)^3\ln(dx+c)}{3}-\frac{(dx+c)^3}{9}\right)-\frac{(dx+c)^4\ln(dx+c)}{4}+\frac{(dx-}{1}\right.}{b}$ |
| default | $-\frac{d^4\left(c^3((dx+c)\ln(dx+c)-dx-c)-3c^2\left(\frac{(dx+c)^2\ln(dx+c)}{2}-\frac{(dx+c)^2}{4}\right)+3c\left(\frac{(dx+c)^3\ln(dx+c)}{3}-\frac{(dx+c)^3}{9}\right)-\frac{(dx+c)^4\ln(dx+c)}{4}+\frac{(dx-}{1}\right.}{b}$ |

```
[In] int(x^7*ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*c^4*ln(d*x+c)/b/d^4+1/4*c^3*x/b/d^3+25/48/d^4/b*c^4-1/8*c^2*x^2/b/d^2+
1/12*c*x^3/b/d+1/4*x^4*ln(d*x+c)/b-1/16*x^4/b-1/4*a/b^2*sum(ln(d*x+c)*ln((-
d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2
*b*c^2-4*_Z*b*c^3+a*d^4+b*c^4))
```

## Fricas [F]

$$\int \frac{x^7 \log(c+dx)}{a+bx^4}\,dx=\int \frac{x^7 \log(dx+c)}{bx^4+a}\,dx$$

```
[In] integrate(x^7*log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral(x^7*log(d*x + c)/(b*x^4 + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^7 \log(c+dx)}{a+bx^4}\, dx = \text{Timed out}$$

```
[In] integrate(x**7*ln(d*x+c)/(b*x**4+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^7 \log(c+dx)}{a+bx^4}\, dx = \int \frac{x^7 \log(dx+c)}{bx^4+a}\, dx$$

```
[In] integrate(x^7*log(d*x+c)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(x^7*log(d*x + c)/(b*x^4 + a), x)
```

## Giac [F]

$$\int \frac{x^7 \log(c+dx)}{a+bx^4}\, dx = \int \frac{x^7 \log(dx+c)}{bx^4+a}\, dx$$

```
[In] integrate(x^7*log(d*x+c)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(x^7*log(d*x + c)/(b*x^4 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 \log(c+dx)}{a+bx^4}\, dx = \int \frac{x^7\, \ln(c+d\,x)}{b\,x^4+a}\, dx$$

```
[In] int((x^7*log(c + d*x))/(a + b*x^4),x)
[Out] int((x^7*log(c + d*x))/(a + b*x^4), x)
```

# 3.294 $\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 401

$$\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx = \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b} + \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b} + \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b}$$

```
[Out] 1/4*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)/b+1/4*l
n(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln(d*x+c)/b+1/4*ln(d*
x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)
```

```
))/b+1/4*ln(d*x+c)*ln(d*(-b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a
)^(1/2))^(1/2)))/b+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))/
b+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b+1/4*polylog(2,b
^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b+1/4*polylog(2,b^(1/4)*(
d*x+c)/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {266, 2463, 2441, 2440, 2438}

$$\int \frac{x^3 \log(c+dx)}{a+bx^4} dx = \frac{\operatorname{PolyLog}\left(2, \frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2, \frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2, \frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2, \frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b} + \frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4b} + \frac{\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4b} + \frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b} + \frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b}$$

```
[In] Int[(x^3*Log[c + d*x])/(a + b*x^4),x]
```

```
[Out] (Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Log
[c + d*x])/(4*b) + (Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4
)*d)]*Log[c + d*x])/(4*b) + (Log[-((d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/
4)*c - Sqrt[-Sqrt[-a]]*d))]*Log[c + d*x])/(4*b) + (Log[-((d*((-a)^(1/4) + b
^(1/4)*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*b) + PolyLog[2, (b
^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)]/(4*b) + PolyLog[2, (b^(1
/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]/(4*b) + PolyLog[2, (b^(1/4)
```

```
*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)]/(4*b) + PolyLog[2, (b^(1/4)*(c + d*
x))/(b^(1/4)*c + (-a)^(1/4)*d)]/(4*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{x\log(c+dx)}{2\left(-\sqrt{-a}\sqrt{b}+bx^2\right)}+\frac{x\log(c+dx)}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx\\ &= \frac{1}{2}\int \frac{x\log(c+dx)}{-\sqrt{-a}\sqrt{b}+bx^2}\,dx+\frac{1}{2}\int \frac{x\log(c+dx)}{\sqrt{-a}\sqrt{b}+bx^2}\,dx\end{aligned}$$

$$= \frac{1}{2}\int \left(-\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}+\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}\right)dx$$

$$+\frac{1}{2}\int \left(-\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}+\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}\right)dx$$

$$= -\frac{\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}dx}{4b^{3/4}}-\frac{\int \frac{\log(c+dx)}{\sqrt[4]{-a}-\sqrt[4]{b}x}dx}{4b^{3/4}}+\frac{\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}dx}{4b^{3/4}}+\frac{\int \frac{\log(c+dx)}{\sqrt[4]{-a}+\sqrt[4]{b}x}dx}{4b^{3/4}}$$

$$= \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b}+\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b}$$

$$+\frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b}+\frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b}$$

$$-\frac{d\int \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4b}-\frac{d\int \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4b}$$

$$-\frac{d\int \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4b}-\frac{d\int \frac{\log\left(\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4b}$$

$$= \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b} + \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b} + \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} - \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4b} - \frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4b} - \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4b} - \frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4b}$$

$$= \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b} + \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b} + \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\text{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b} + \frac{\text{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b} + \frac{\text{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b} + \frac{\text{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.96

$$\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx = \frac{\log\left(\frac{d\left(i\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\log\left(-\frac{d\left(i\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b}$$

```
[In] Integrate[(x^3*Log[c + d*x])/(a + b*x^4),x]
```

```
[Out] (Log[(d*(I*(-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]*Log[c + d
*x])/(4*b) + (Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*
Log[c + d*x])/(4*b) + (Log[-((d*(I*(-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - I*
(-a)^(1/4)*d))]*Log[c + d*x])/(4*b) + (Log[-((d*((-a)^(1/4) + b^(1/4)*x))/(
b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*b) + PolyLog[2, (b^(1/4)*(c +
d*x))/(b^(1/4)*c - (-a)^(1/4)*d)]/(4*b) + PolyLog[2, (b^(1/4)*(c + d*x))/(b
^(1/4)*c - I*(-a)^(1/4)*d)]/(4*b) + PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)
*c + I*(-a)^(1/4)*d)]/(4*b) + PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (
-a)^(1/4)*d)]/(4*b)
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.21

| method | result |
|---|---|
| derivativedivides | $\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)\right)}{4b}$ |
| default | $\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)\right)}{4b}$ |
| risch | $\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)\right)}{4b}$ |
| parts | $\frac{\ln(dx+c)\ln\left(b\,x^4+a\right)}{4b}-\frac{d\left(\frac{\ln(dx+c)\ln\left(b\,x^4+a\right)}{d}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\left(\ln(dx+c)\,\mathrm{l}\right.}{d}\right.}{4b}$ |

```
[In] int(x^3*ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4/b*sum(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1),_R1=RootOf
(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-4*_Z*b*c^3+a*d^4+b*c^4))
```

## Fricas [F]

$$\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx=\int \frac{x^3\log(dx+c)}{bx^4+a}\,dx$$

```
[In] integrate(x^3*log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
[Out] integral(x^3*log(d*x + c)/(b*x^4 + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx=\text{Timed out}$$

```
[In] integrate(x**3*ln(d*x+c)/(b*x**4+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^3 \log(c+dx)}{a+bx^4} \, dx = \int \frac{x^3 \log(dx+c)}{bx^4+a} \, dx$$

```
[In] integrate(x^3*log(d*x+c)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(x^3*log(d*x + c)/(b*x^4 + a), x)
```

## Giac [F]

$$\int \frac{x^3 \log(c+dx)}{a+bx^4} \, dx = \int \frac{x^3 \log(dx+c)}{bx^4+a} \, dx$$

```
[In] integrate(x^3*log(d*x+c)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(x^3*log(d*x + c)/(b*x^4 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log(c+dx)}{a+bx^4} \, dx = \int \frac{x^3 \ln(c+d\,x)}{b\,x^4+a} \, dx$$

```
[In] int((x^3*log(c + d*x))/(a + b*x^4),x)
[Out] int((x^3*log(c + d*x))/(a + b*x^4), x)
```

# 3.295 $\int \frac{\log(c+dx)}{x(a+bx^4)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 433

$$\int \frac{\log(c+dx)}{x(a+bx^4)}\,dx = \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4a} - \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} - \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4a} - \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4a} - \frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4a} + \frac{\text{PolyLog}\left(2,1+\frac{dx}{c}\right)}{a}$$

```
[Out] ln(-d*x/c)*ln(d*x+c)/a-1/4*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/4
)*d))*ln(d*x+c)/a-1/4*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d)
)*ln(d*x+c)/a-1/4*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*
```

```
c-d*(-(-a)^(1/2))^(1/2)))/a-1/4*ln(d*x+c)*ln(d*(-b^(1/4)*x+(-(-a)^(1/2))^(1
/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/a-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^
(1/4)*c-(-a)^(1/4)*d))/a-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4
)*d))/a+polylog(2,1+d*x/c)/a-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-(
-a)^(1/2))^(1/2)))/a-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+d*(-(-a)^(1/2
))^(1/2)))/a
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {272, 36, 29, 31, 2463, 2441, 2352, 266, 2440, 2438}

$$\int \frac{\log(c+dx)}{x(a+bx^4)}\,dx = -\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4a}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4a}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4a}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4a}-\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4a}-\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4a}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4a}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4a}+\frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{a}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a}$$

```
[In] Int[Log[c + d*x]/(x*(a + b*x^4)),x]
```

```
[Out] (Log[-((d*x)/c)]*Log[c + d*x])/a - (Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(
b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Log[c + d*x])/(4*a) - (Log[(d*((-a)^(1/4) -
 b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*a) - (Log[-((d*(S
qrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d))]*Log[c + d*x]
```

```
)/(4*a) - (Log[-((d*((-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*
Log[c + d*x])/(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqr
t[-a]]*d)]/(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-
a]]*d)]/(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)]/
(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]/(4*a) +
PolyLog[2, 1 + (d*x)/c]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log(c+dx)}{ax} - \frac{bx^3\log(c+dx)}{a(a+bx^4)}\right)dx$$

$$= \frac{\int \frac{\log(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{x^3\log(c+dx)}{a+bx^4}\,dx}{a}$$

$$= \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{b\int \left(\frac{x\log(c+dx)}{2\left(-\sqrt{-a}\sqrt{b}+bx^2\right)} + \frac{x\log(c+dx)}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx}{a} - \frac{d\int \frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\,dx}{a}$$

$$= \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} + \frac{\text{Li}_2\left(1+\frac{dx}{c}\right)}{a} - \frac{b\int \frac{x\log(c+dx)}{-\sqrt{-a}\sqrt{b}+bx^2}\,dx}{2a} - \frac{b\int \frac{x\log(c+dx)}{\sqrt{-a}\sqrt{b}+bx^2}\,dx}{2a}$$

$$= \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} + \frac{\text{Li}_2\left(1+\frac{dx}{c}\right)}{a} - \frac{b\int \left(-\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)} + \frac{\log(c+dx)}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}\right)dx}{2a} - \frac{b\int \left(-\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)} + \frac{\log(c+dx)}{2b^{3/4}\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}\right)dx}{2a}$$

$$= \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} + \frac{\text{Li}_2\left(1+\frac{dx}{c}\right)}{a} + \frac{\sqrt[4]{b}\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}\,dx}{4a} + \frac{\sqrt[4]{b}\int \frac{\log(c+dx)}{\sqrt[4]{-a}-\sqrt[4]{b}x}\,dx}{4a} - \frac{\sqrt[4]{b}\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}\,dx}{4a} - \frac{\sqrt[4]{b}\int \frac{\log(c+dx)}{\sqrt[4]{-a}+\sqrt[4]{b}x}\,dx}{4a}$$

$$= \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4a} - \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} - \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4a} - \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} + \frac{\text{Li}_2\left(1+\frac{dx}{c}\right)}{a} + \frac{d\int\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4a} + \frac{d\int\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4a} + \frac{d\int\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4a} + \frac{d\int\frac{\log\left(\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4a}$$

$$= \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4a} - \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} - \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4a} - \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} + \frac{\text{Li}_2\left(1+\frac{dx}{c}\right)}{a} + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4a} + \frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4a} + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4a} + \frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4a}$$

$$= \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4a} - \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} - \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4a} - \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} - \frac{\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4a} - \frac{\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4a} - \frac{\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4a} - \frac{\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4a} + \frac{\operatorname{Li}_2\left(1+\frac{dx}{c}\right)}{a}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.96

$$\int \frac{\log(c+dx)}{x\left(a+bx^4\right)}\,dx = \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a} - \frac{\log\left(\frac{d\left(i\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} - \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} - \frac{\log\left(-\frac{d\left(i\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} - \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4a} + \frac{\operatorname{PolyLog}\left(2,\frac{c+dx}{c}\right)}{a} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4a} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{4a} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)}{4a} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4a}$$

```
[In] Integrate[Log[c + d*x]/(x*(a + b*x^4)),x]
```

```
[Out] (Log[-((d*x)/c)]*Log[c + d*x])/a - (Log[(d*(I*(-a)^(1/4) - b^(1/4)*x))/(b^(
1/4)*c + I*(-a)^(1/4)*d)]*Log[c + d*x])/(4*a) - (Log[(d*((-a)^(1/4) - b^(1/
4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*a) - (Log[-((d*(I*(-a)^
(1/4) + b^(1/4)*x))/(b^(1/4)*c - I*(-a)^(1/4)*d))]*Log[c + d*x])/(4*a) - (L
og[-((d*((-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x]
)/(4*a) + PolyLog[2, (c + d*x)/c]/a - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/
4)*c - (-a)^(1/4)*d)]/(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I
*(-a)^(1/4)*d)]/(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + I*(-a)^
(1/4)*d)]/(4*a) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)
]/(4*a)
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.26

| method | result |
|---|---|
| derivativedivides | $\frac{\operatorname{dilog}\left(-\frac{xd}{c}\right)+\ln(dx+c)\ln\left(-\frac{xd}{c}\right)}{a}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^4-4cb\_Z^3+6b c^2\_Z^2-4b c^3\_Z+a d^4+b c^4\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\ldots}{\ldots}\right.\right.}{4a}$ |
| default | $\frac{\operatorname{dilog}\left(-\frac{xd}{c}\right)+\ln(dx+c)\ln\left(-\frac{xd}{c}\right)}{a}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^4-4cb\_Z^3+6b c^2\_Z^2-4b c^3\_Z+a d^4+b c^4\right)}\left(\ln(dx+c)\ln\left(\frac{-dx+\ldots}{\ldots}\right.\right.}{4a}$ |
| risch | $\frac{\ln\left(-\frac{xd}{c}\right)\ln(dx+c)}{a}+\frac{\operatorname{dilog}\left(-\frac{xd}{c}\right)}{a}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^4-4cb\_Z^3+6b c^2\_Z^2-4b c^3\_Z+a d^4+b c^4\right)}\left(\ln(dx+c)\ln\left(\frac{-d\ldots}{\ldots}\right.\right.}{4a}$ |
| parts | $\frac{\ln(dx+c)\ln(x)}{a}-\frac{\ln(dx+c)\ln(b x^4+a)}{4a}-\frac{d\left(\frac{4\operatorname{dilog}\left(\frac{dx+c}{c}\right)}{ad}+\frac{4\ln(x)\ln\left(\frac{dx+c}{c}\right)}{ad}-\frac{\ln(dx+c)\ln\left(b x^4+a\right)}{ad}+\frac{\_R1=\operatorname{RootOf}\left(b\ldots\right.}{\ldots}\right.}{\ldots}$ |

```
[In] int(ln(d*x+c)/x/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(dilog(-x*d/c)+ln(d*x+c)*ln(-x*d/c))-1/4/a*sum(ln(d*x+c)*ln((-d*x+_R1-c
)/_R1)+dilog((-d*x+_R1-c)/_R1),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-4*
_Z*b*c^3+a*d^4+b*c^4))
```

## Fricas [F]

$$\int \frac{\log(c+dx)}{x(a+bx^4)}\,dx = \int \frac{\log(dx+c)}{(bx^4+a)x}\,dx$$

```
[In] integrate(log(d*x+c)/x/(b*x^4+a),x, algorithm="fricas")
[Out] integral(log(d*x + c)/(b*x^5 + a*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x(a+bx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(d*x+c)/x/(b*x**4+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log(c+dx)}{x(a+bx^4)}\,dx = \int \frac{\log(dx+c)}{(bx^4+a)x}\,dx$$

```
[In] integrate(log(d*x+c)/x/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(log(d*x + c)/((b*x^4 + a)*x), x)
```

## Giac [F]

$$\int \frac{\log(c+dx)}{x(a+bx^4)}\,dx = \int \frac{\log(dx+c)}{(bx^4+a)x}\,dx$$

```
[In] integrate(log(d*x+c)/x/(b*x^4+a),x, algorithm="giac")
[Out] integrate(log(d*x + c)/((b*x^4 + a)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x\,(a+bx^4)}\,dx = \int \frac{\ln\left(c+d\,x\right)}{x\,\left(b\,x^4+a\right)}\,dx$$

```
[In] int(log(c + d*x)/(x*(a + b*x^4)),x)
[Out] int(log(c + d*x)/(x*(a + b*x^4)), x)
```

# 3.296 $\int \frac{x^5 \log(c+dx)}{a+bx^4} \, dx$

## Optimal result

Integrand size = 19, antiderivative size = 530

$$\int \frac{x^5 \log(c+dx)}{a+bx^4}\,dx = \frac{cx}{2bd} - \frac{x^2}{4b} - \frac{c^2\log(c+dx)}{2bd^2} + \frac{x^2\log(c+dx)}{2b} - \frac{\sqrt{-a}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{3/2}} + \frac{\sqrt{-a}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{3/2}} - \frac{\sqrt{-a}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{3/2}} + \frac{\sqrt{-a}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{3/2}} - \frac{\sqrt{-a}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/2}} - \frac{\sqrt{-a}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/2}} + \frac{\sqrt{-a}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/2}} + \frac{\sqrt{-a}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/2}}$$

```
[Out] 1/2*c*x/b/d-1/4*x^2/b-1/2*c^2*ln(d*x+c)/b/d^2+1/2*x^2*ln(d*x+c)/b+1/4*ln(d*
((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)*(-a)^(1/2)/b^(3/
2)+1/4*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln(d*x+c)*(-a
)^(1/2)/b^(3/2)-1/4*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4
)*c-d*(-(-a)^(1/2))^(1/2)))*(-a)^(1/2)/b^(3/2)-1/4*ln(d*x+c)*ln(d*(-b^(1/4)
*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))*(-a)^(1/2)/b^(3/
2)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))*(-a)^(1/2)/b^(3/
2)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))*(-a)^(1/2)/b^(3/
2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))*(-a)^(1
```

```
/2)/b^(3/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2))
)*(-a)^(1/2)/b^(3/2)
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {281, 327, 211, 2463, 2442, 45, 266, 2441, 2440, 2438}

$$\int \frac{x^5 \log(c+dx)}{a+bx^4}\,dx = -\frac{\sqrt{-a}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/2}} - \frac{\sqrt{-a}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/2}} + \frac{\sqrt{-a}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/2}} + \frac{\sqrt{-a}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/2}} - \frac{\sqrt{-a}\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4b^{3/2}} + \frac{\sqrt{-a}\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4b^{3/2}} - \frac{\sqrt{-a}\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/2}} + \frac{\sqrt{-a}\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/2}} - \frac{c^2\log(c+dx)}{2bd^2} + \frac{x^2\log(c+dx)}{2b} + \frac{cx}{2bd} - \frac{x^2}{4b}$$

```
[In] Int[(x^5*Log[c + d*x])/(a + b*x^4),x]
```

```
[Out] (c*x)/(2*b*d) - x^2/(4*b) - (c^2*Log[c + d*x])/(2*b*d^2) + (x^2*Log[c + d*x
])/(2*b) - (Sqrt[-a]*Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqr
```

```
t[-Sqrt[-a]]*d)]*Log[c + d*x])/(4*b^(3/2)) + (Sqrt[-a]*Log[(d*((-a)^(1/4) -
 b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*b^(3/2)) - (Sqrt[
-a]*Log[-((d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)
)]*Log[c + d*x])/(4*b^(3/2)) + (Sqrt[-a]*Log[-((d*((-a)^(1/4) + b^(1/4)*x))
/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*b^(3/2)) - (Sqrt[-a]*PolyLog
[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/2)) - (Sq
rt[-a]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*
b^(3/2)) + (Sqrt[-a]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)
*d)])/(4*b^(3/2)) + (Sqrt[-a]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (
-a)^(1/4)*d)])/(4*b^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x\log(c+dx)}{b} - \frac{ax\log(c+dx)}{b\,(a+bx^4)}\right)dx$$

$$= \frac{\int x\log(c+dx)\,dx}{b} - \frac{a\int \frac{x\log(c+dx)}{a+bx^4}\,dx}{b}$$

$$= \frac{x^2\log(c+dx)}{2b} - \frac{a\int \left(-\frac{\sqrt{b}x\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-bx^2\right)} - \frac{\sqrt{b}x\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx}{b} - \frac{d\int \frac{x^2}{c+dx}\,dx}{2b}$$

$$= \frac{x^2\log(c+dx)}{2b} - \frac{\sqrt{-a}\int \frac{x\log(c+dx)}{\sqrt{-a}\sqrt{b}-bx^2}\,dx}{2\sqrt{b}} - \frac{\sqrt{-a}\int \frac{x\log(c+dx)}{\sqrt{-a}\sqrt{b}+bx^2}\,dx}{2\sqrt{b}} - \frac{d\int \left(-\frac{c}{d^2}+\frac{x}{d}+\frac{c^2}{d^2(c+dx)}\right)dx}{2b}$$

$$= \frac{cx}{2bd} - \frac{x^2}{4b} - \frac{c^2\log(c+dx)}{2bd^2} + \frac{x^2\log(c+dx)}{2b} - \frac{\sqrt{-a}\int\left(-\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)} + \frac{\log(c+dx)}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}\right)dx}{2\sqrt{b}} - \frac{\sqrt{-a}\int\left(\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)} - \frac{\log(c+dx)}{2b^{3/4}\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}\right)dx}{2\sqrt{b}}$$

$$= \frac{cx}{2bd} - \frac{x^2}{4b} - \frac{c^2\log(c+dx)}{2bd^2} + \frac{x^2\log(c+dx)}{2b} + \frac{\sqrt{-a}\int\frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}dx}{4b^{5/4}} - \frac{\sqrt{-a}\int\frac{\log(c+dx)}{\sqrt[4]{-a}-\sqrt[4]{b}x}dx}{4b^{5/4}} - \frac{\sqrt{-a}\int\frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}dx}{4b^{5/4}} + \frac{\sqrt{-a}\int\frac{\log(c+dx)}{\sqrt[4]{-a}+\sqrt[4]{b}x}dx}{4b^{5/4}}$$

$$= \frac{cx}{2bd} - \frac{x^2}{4b} - \frac{c^2\log(c+dx)}{2bd^2} + \frac{x^2\log(c+dx)}{2b} - \frac{\sqrt{-a}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{3/2}} + \frac{\sqrt{-a}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{3/2}} - \frac{\sqrt{-a}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{3/2}} + \frac{\sqrt{-a}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{3/2}} + \frac{\left(\sqrt{-a}d\right)\int\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4b^{3/2}} - \frac{\left(\sqrt{-a}d\right)\int\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4b^{3/2}} + \frac{\left(\sqrt{-a}d\right)\int\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4b^{3/2}} - \frac{\left(\sqrt{-a}d\right)\int\frac{\log\left(\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4b^{3/2}}$$

$$= \frac{cx}{2bd} - \frac{x^2}{4b} - \frac{c^2\log(c+dx)}{2bd^2} + \frac{x^2\log(c+dx)}{2b}$$

$$- \frac{\sqrt{-a}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{3/2}}$$

$$+ \frac{\sqrt{-a}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{3/2}}$$

$$- \frac{\sqrt{-a}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{3/2}}$$

$$+ \frac{\sqrt{-a}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{3/2}}$$

$$+ \frac{\sqrt{-a}\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx,x,c+dx\right)}{4b^{3/2}}$$

$$+ \frac{\sqrt{-a}\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx,x,c+dx\right)}{4b^{3/2}}$$

$$- \frac{\sqrt{-a}\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx,x,c+dx\right)}{4b^{3/2}}$$

$$- \frac{\sqrt{-a}\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx,x,c+dx\right)}{4b^{3/2}}$$

$$= \frac{cx}{2bd} - \frac{x^2}{4b} - \frac{c^2\log(c+dx)}{2bd^2} + \frac{x^2\log(c+dx)}{2b} - \frac{\sqrt{-a}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{3/2}} + \frac{\sqrt{-a}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{3/2}} - \frac{\sqrt{-a}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{3/2}} + \frac{\sqrt{-a}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{3/2}} - \frac{\sqrt{-a}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/2}} - \frac{\sqrt{-a}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/2}} + \frac{\sqrt{-a}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/2}} + \frac{\sqrt{-a}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/2}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.91

$$\int \frac{x^5\log(c+dx)}{a+bx^4}\,dx$$

$$= \frac{2\sqrt{b}cdx - \sqrt{b}d^2x^2 - 2\sqrt{b}c^2\log(c+dx) + 2\sqrt{b}d^2x^2\log(c+dx) + \sqrt{-a}d^2\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+d}{}$$

```
[In] Integrate[(x^5*Log[c + d*x])/(a + b*x^4),x]
```

```
[Out] (2*Sqrt[b]*c*d*x - Sqrt[b]*d^2*x^2 - 2*Sqrt[b]*c^2*Log[c + d*x] + 2*Sqrt[b]
*d^2*x^2*Log[c + d*x] + Sqrt[-a]*d^2*Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1
/4)*c + (-a)^(1/4)*d)]*Log[c + d*x] - Sqrt[-a]*d^2*Log[(d*((-a)^(1/4) - I*b
^(1/4)*x))/(I*b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x] - Sqrt[-a]*d^2*Log[(d
*((-a)^(1/4) + I*b^(1/4)*x))/((-I)*b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x]
+ Sqrt[-a]*d^2*Log[(d*((-a)^(1/4) + b^(1/4)*x))/(-(b^(1/4)*c) + (-a)^(1/4)*
d)]*Log[c + d*x] + Sqrt[-a]*d^2*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c -
 (-a)^(1/4)*d)] - Sqrt[-a]*d^2*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c -
I*(-a)^(1/4)*d)] - Sqrt[-a]*d^2*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c +
 I*(-a)^(1/4)*d)] + Sqrt[-a]*d^2*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c
+ (-a)^(1/4)*d)])/(4*b^(3/2)*d^2)
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.31

| method | result |
|---|---|
| derivativedivides | $-\frac{\frac{d^4\left(-\frac{(dx+c)^2\ln(dx+c)}{2}+\frac{(dx+c)^2}{4}+c((dx+c)\ln(dx+c)-dx-c)\right)}{b}-\frac{a\,d^8\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4\right.}\right.}{}}{d^6}$ |
| default | $-\frac{\frac{d^4\left(-\frac{(dx+c)^2\ln(dx+c)}{2}+\frac{(dx+c)^2}{4}+c((dx+c)\ln(dx+c)-dx-c)\right)}{b}-\frac{a\,d^8\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4\right.}\right.}{}}{d^6}$ |
| risch | $\frac{x^2\ln(dx+c)}{2b}-\frac{c^2\ln(dx+c)}{2b\,d^2}-\frac{x^2}{4b}+\frac{cx}{2db}+\frac{3c^2}{4d^2b}-\frac{d^2a\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c\right.}\right.}{4b^2}$ |

```
[In] int(x^5*ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^6*(-d^4/b*(-1/2*(d*x+c)^2*ln(d*x+c)+1/4*(d*x+c)^2+c*((d*x+c)*ln(d*x+c)-
d*x-c))-1/4*a*d^8/b^2*sum(1/(_R1^2-2*_R1*c+c^2)*(ln(d*x+c)*ln((-d*x+_R1-c)/
_R1)+dilog((-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-4*_
Z*b*c^3+a*d^4+b*c^4)))
```

## Fricas [F]

$$\int \frac{x^5\log(c+dx)}{a+bx^4}\,dx=\int \frac{x^5\log(dx+c)}{bx^4+a}\,dx$$

```
[In] integrate(x^5*log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral(x^5*log(d*x + c)/(b*x^4 + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 \log(c+dx)}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*ln(d*x+c)/(b*x**4+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^5 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^5 \log(dx+c)}{bx^4+a}\,dx$$

```
[In] integrate(x^5*log(d*x+c)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(x^5*log(d*x + c)/(b*x^4 + a), x)
```

## Giac [F]

$$\int \frac{x^5 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^5 \log(dx+c)}{bx^4+a}\,dx$$

```
[In] integrate(x^5*log(d*x+c)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(x^5*log(d*x + c)/(b*x^4 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^5 \ln(c+d\,x)}{b\,x^4+a}\,dx$$

```
[In] int((x^5*log(c + d*x))/(a + b*x^4),x)
[Out] int((x^5*log(c + d*x))/(a + b*x^4), x)
```

# 3.297 $\int \frac{x \log(c+dx)}{a+bx^4} dx$



## Optimal result

Integrand size = 17, antiderivative size = 473

$$\int \frac{x\log(c+dx)}{a+bx^4}\,dx = -\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}} + \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}} - \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}} + \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-a}\sqrt{b}} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-a}\sqrt{b}} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt{-a}\sqrt{b}} + \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4\sqrt{-a}\sqrt{b}}$$

```
[Out] 1/4*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)/(-a)^(1
/2)/b^(1/2)+1/4*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln(d
```

```
*x+c)/(-a)^(1/2)/b^(1/2)-1/4*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^(1/2)
)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/(-a)^(1/2)/b^(1/2)-1/4*ln(d*x+c)*ln(d*
(-b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/(-a)^(1
/2)/b^(1/2)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))/(-a)^(1
/2)/b^(1/2)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/(-a)^(1
/2)/b^(1/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2))
)/(-a)^(1/2)/b^(1/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+d*(-(-a)^(1/2
))^(1/2)))/(-a)^(1/2)/b^(1/2)
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {281, 211, 2463, 266, 2441, 2440, 2438}

$$\int \frac{x\log(c+dx)}{a+bx^4}\,dx = -\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt{-a}\sqrt{b}}$$

```
[In] Int[(x*Log[c + d*x])/(a + b*x^4),x]
```

```
[Out] -1/4*(Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)
]*Log[c + d*x])/(Sqrt[-a]*Sqrt[b]) + (Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(
1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*Sqrt[-a]*Sqrt[b]) - (Log[-((d*(Sqr
```

```
t[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d))]*Log[c + d*x])/
(4*Sqrt[-a]*Sqrt[b]) + (Log[-((d*((-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - (-a
)^(1/4)*d))]*Log[c + d*x])/(4*Sqrt[-a]*Sqrt[b]) - PolyLog[2, (b^(1/4)*(c +
d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)]/(4*Sqrt[-a]*Sqrt[b]) - PolyLog[2, (b
^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]/(4*Sqrt[-a]*Sqrt[b]) + P
olyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)]/(4*Sqrt[-a]*Sqrt[
b]) + PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]/(4*Sqrt[-a
]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{\sqrt{b}x\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-bx^2\right)}-\frac{\sqrt{b}x\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx$$

$$= -\frac{\sqrt{b}\int \frac{x\log(c+dx)}{\sqrt{-a}\sqrt{b}-bx^2}\,dx}{2\sqrt{-a}}-\frac{\sqrt{b}\int \frac{x\log(c+dx)}{\sqrt{-a}\sqrt{b}+bx^2}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\sqrt{b}\int \left(-\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}+\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}\right)dx}{2\sqrt{-a}}$$

$$-\frac{\sqrt{b}\int \left(\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}-\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}\right)dx}{2\sqrt{-a}}$$

$$= \frac{\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}\,dx}{4\sqrt{-a}\sqrt[4]{b}}-\frac{\int \frac{\log(c+dx)}{\sqrt[4]{-a}-\sqrt[4]{b}x}\,dx}{4\sqrt{-a}\sqrt[4]{b}}-\frac{\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}\,dx}{4\sqrt{-a}\sqrt[4]{b}}+\frac{\int \frac{\log(c+dx)}{\sqrt[4]{-a}+\sqrt[4]{b}x}\,dx}{4\sqrt{-a}\sqrt[4]{b}}$$

$$= -\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}}+\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}}$$

$$-\frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}}+\frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}}$$

$$+\frac{d\int \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}\,dx}{4\sqrt{-a}\sqrt{b}}-\frac{d\int \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}\,dx}{4\sqrt{-a}\sqrt{b}}$$

$$+\frac{d\int \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}\,dx}{4\sqrt{-a}\sqrt{b}}-\frac{d\int \frac{\log\left(\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}\,dx}{4\sqrt{-a}\sqrt{b}}$$

$$= -\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}}+\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}}$$

$$-\frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}}+\frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}}$$

$$+\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx,x,c+dx\right)}{4\sqrt{-a}\sqrt{b}}$$

$$+\frac{\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx,x,c+dx\right)}{4\sqrt{-a}\sqrt{b}}$$

$$-\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx,x,c+dx\right)}{4\sqrt{-a}\sqrt{b}}$$

$$-\frac{\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx,x,c+dx\right)}{4\sqrt{-a}\sqrt{b}}$$

$$
\begin{aligned}
&= -\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}}+\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}} \\
&\quad-\frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}}+\frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt{-a}\sqrt{b}} \\
&\quad-\frac{\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt{-a}\sqrt{b}} \\
&\quad+\frac{\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4\sqrt{-a}\sqrt{b}}
\end{aligned}
$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.74

$$
\int \frac{x\log(c+dx)}{a+bx^4}\,dx
$$

$$
= \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)-\log\left(\frac{d\left(\sqrt[4]{-a}-i\sqrt[4]{b}x\right)}{i\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)-\log\left(\frac{d\left(\sqrt[4]{-a}+i\sqrt[4]{b}x\right)}{-i\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+}{}
$$

```
[In] Integrate[(x*Log[c + d*x])/(a + b*x^4),x]
```

```
[Out] (Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x]
- Log[(d*((-a)^(1/4) - I*b^(1/4)*x))/(I*b^(1/4)*c + (-a)^(1/4)*d)]*Log[c +
d*x] - Log[(d*((-a)^(1/4) + I*b^(1/4)*x))/((-I)*b^(1/4)*c + (-a)^(1/4)*d)]*
Log[c + d*x] + Log[(d*((-a)^(1/4) + b^(1/4)*x))/(-(b^(1/4)*c) + (-a)^(1/4)*
d)]*Log[c + d*x] + PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d
)] - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)] - PolyLog
[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)] + PolyLog[2, (b^(1/4)
*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*Sqrt[-a]*Sqrt[b])
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.61 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.22

| method | result |
|---|---|
| derivativedivides | $\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{\_R1^2-2\_R1c+c^2}\right)}{4b}$ |
| default | $\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{\_R1^2-2\_R1c+c^2}\right)}{4b}$ |
| risch | $\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{\_R1^2-2\_R1c+c^2}\right)}{4b}$ |

```
[In] int(x*ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4*d^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((
-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-4*_Z*b*c^3+a*d^
4+b*c^4))
```

## Fricas [F]

$$\int \frac{x\log(c+dx)}{a+bx^4}\,dx = \int \frac{x\log(dx+c)}{bx^4+a}\,dx$$

```
[In] integrate(x*log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
[Out] integral(x*log(d*x + c)/(b*x^4 + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x\log(c+dx)}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x*ln(d*x+c)/(b*x**4+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x\log(c+dx)}{a+bx^4}\,dx = \int \frac{x\log(dx+c)}{bx^4+a}\,dx$$

[In] integrate(x*log(d*x+c)/(b*x^4+a),x, algorithm="maxima")

[Out] integrate(x*log(d*x + c)/(b*x^4 + a), x)

## Giac [F]

$$\int \frac{x\log(c+dx)}{a+bx^4}\,dx = \int \frac{x\log(dx+c)}{bx^4+a}\,dx$$

[In] integrate(x*log(d*x+c)/(b*x^4+a),x, algorithm="giac")

[Out] integrate(x*log(d*x + c)/(b*x^4 + a), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\log(c+dx)}{a+bx^4}\,dx = \int \frac{x\,\ln(c+d\,x)}{b\,x^4+a}\,dx$$

[In] int((x*log(c + d*x))/(a + b*x^4),x)

[Out] int((x*log(c + d*x))/(a + b*x^4), x)

# 3.298 $\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 537

$$\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx = -\frac{d}{2acx} - \frac{d^2\log(x)}{2ac^2} + \frac{d^2\log(c+dx)}{2ac^2} - \frac{\log(c+dx)}{2ax^2} - \frac{\sqrt{b}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4(-a)^{3/2}} + \frac{\sqrt{b}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} - \frac{\sqrt{b}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4(-a)^{3/2}} + \frac{\sqrt{b}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} - \frac{\sqrt{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4(-a)^{3/2}} - \frac{\sqrt{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4(-a)^{3/2}} + \frac{\sqrt{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}} + \frac{\sqrt{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}}$$

```
[Out] -1/2*d/a/c/x-1/2*d^2*ln(x)/a/c^2+1/2*d^2*ln(d*x+c)/a/c^2-1/2*ln(d*x+c)/a/x^
2+1/4*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)*b^(1/
2)/(-a)^(3/2)+1/4*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln
(d*x+c)*b^(1/2)/(-a)^(3/2)-1/4*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^(1/
2))/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))*b^(1/2)/(-a)^(3/2)-1/4*ln(d*x+c)*ln(
d*(-b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))*b^(1/
2)/(-a)^(3/2)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))*b^(1/
2)/(-a)^(3/2)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))*b^(1/
2)/(-a)^(3/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2
)))*b^(1/2)/(-a)^(3/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+d*(-(-a)^(1
/2))^(1/2)))*b^(1/2)/(-a)^(3/2)
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {281, 331, 211, 2463, 2442, 46, 266, 2441, 2440, 2438}

$$\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx = -\frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4(-a)^{3/2}}-\frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4(-a)^{3/2}}+\frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}}+\frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}}-\frac{\sqrt{b}\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4(-a)^{3/2}}+\frac{\sqrt{b}\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4(-a)^{3/2}}-\frac{\sqrt{b}\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4(-a)^{3/2}}+\frac{\sqrt{b}\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}}-\frac{d^2\log(x)}{2ac^2}+\frac{d^2\log(c+dx)}{2ac^2}-\frac{\log(c+dx)}{2ax^2}-\frac{d}{2acx}$$

```
[In] Int[Log[c + d*x]/(x^3*(a + b*x^4)),x]
```

```
[Out] -1/2*d/(a*c*x) - (d^2*Log[x])/(2*a*c^2) + (d^2*Log[c + d*x])/(2*a*c^2) - Lo
g[c + d*x]/(2*a*x^2) - (Sqrt[b]*Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1
/4)*c + Sqrt[-Sqrt[-a]]*d)]*Log[c + d*x])/(4*(-a)^(3/2)) + (Sqrt[b]*Log[(d*
((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*(-a)
^(3/2)) - (Sqrt[b]*Log[-((d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqr
t[-Sqrt[-a]]*d))]*Log[c + d*x])/(4*(-a)^(3/2)) + (Sqrt[b]*Log[-((d*((-a)^(1
/4) + b^(1/4)*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*(-a)^(3/2))
 - (Sqrt[b]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)]
)/(4*(-a)^(3/2)) - (Sqrt[b]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqr
t[-Sqrt[-a]]*d)])/(4*(-a)^(3/2)) + (Sqrt[b]*PolyLog[2, (b^(1/4)*(c + d*x))/
(b^(1/4)*c - (-a)^(1/4)*d)])/(4*(-a)^(3/2)) + (Sqrt[b]*PolyLog[2, (b^(1/4)*
(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*(-a)^(3/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log(c+dx)}{ax^3} - \frac{bx\log(c+dx)}{a(a+bx^4)}\right) dx$$

$$= \frac{\int \frac{\log(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{x\log(c+dx)}{a+bx^4}\,dx}{a}$$

$$= -\frac{\log(c+dx)}{2ax^2} - \frac{b\int \left(-\frac{\sqrt{b}x\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-bx^2\right)} - \frac{\sqrt{b}x\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right) dx}{a} + \frac{d\int \frac{1}{x^2(c+dx)}\,dx}{2a}$$

$$= -\frac{\log(c+dx)}{2ax^2} - \frac{b^{3/2}\int \frac{x\log(c+dx)}{\sqrt{-a}\sqrt{b}-bx^2}\,dx}{2(-a)^{3/2}} - \frac{b^{3/2}\int \frac{x\log(c+dx)}{\sqrt{-a}\sqrt{b}+bx^2}\,dx}{2(-a)^{3/2}} + \frac{d\int \left(\frac{1}{cx^2} - \frac{d}{c^2x} + \frac{d^2}{c^2(c+dx)}\right) dx}{2a}$$

$$= -\frac{d}{2acx} - \frac{d^2\log(x)}{2ac^2} + \frac{d^2\log(c+dx)}{2ac^2} - \frac{\log(c+dx)}{2ax^2} - \frac{b^{3/2}\int\left(-\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}+\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}\right)dx}{2(-a)^{3/2}} - \frac{b^{3/2}\int\left(\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}-\frac{\log(c+dx)}{2b^{3/4}\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}\right)dx}{2(-a)^{3/2}}$$

$$= -\frac{d}{2acx} - \frac{d^2\log(x)}{2ac^2} + \frac{d^2\log(c+dx)}{2ac^2} - \frac{\log(c+dx)}{2ax^2} + \frac{b^{3/4}\int\frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}dx}{4(-a)^{3/2}} - \frac{b^{3/4}\int\frac{\log(c+dx)}{\sqrt[4]{-a}-\sqrt[4]{b}x}dx}{4(-a)^{3/2}} - \frac{b^{3/4}\int\frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}dx}{4(-a)^{3/2}} + \frac{b^{3/4}\int\frac{\log(c+dx)}{\sqrt[4]{-a}+\sqrt[4]{b}x}dx}{4(-a)^{3/2}}$$

$$= -\frac{d}{2acx} - \frac{d^2\log(x)}{2ac^2} + \frac{d^2\log(c+dx)}{2ac^2} - \frac{\log(c+dx)}{2ax^2} - \frac{\sqrt{b}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4(-a)^{3/2}} + \frac{\sqrt{b}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} - \frac{\sqrt{b}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4(-a)^{3/2}} + \frac{\sqrt{b}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} + \frac{\left(\sqrt{b}d\right)\int\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4(-a)^{3/2}} - \frac{\left(\sqrt{b}d\right)\int\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4(-a)^{3/2}} + \frac{\left(\sqrt{b}d\right)\int\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4(-a)^{3/2}} - \frac{\left(\sqrt{b}d\right)\int\frac{\log\left(\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4(-a)^{3/2}}$$

$$= -\frac{d}{2acx} - \frac{d^2\log(x)}{2ac^2} + \frac{d^2\log(c+dx)}{2ac^2} - \frac{\log(c+dx)}{2ax^2} - \frac{\sqrt{b}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4(-a)^{3/2}} + \frac{\sqrt{b}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} - \frac{\sqrt{b}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4(-a)^{3/2}} + \frac{\sqrt{b}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} + \frac{\sqrt{b}\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4(-a)^{3/2}} + \frac{\sqrt{b}\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4(-a)^{3/2}} - \frac{\sqrt{b}\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4(-a)^{3/2}} - \frac{\sqrt{b}\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4(-a)^{3/2}}$$

$$= -\frac{d}{2acx} - \frac{d^2\log(x)}{2ac^2} + \frac{d^2\log(c+dx)}{2ac^2} - \frac{\log(c+dx)}{2ax^2}$$
$$- \frac{\sqrt{b}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4(-a)^{3/2}} + \frac{\sqrt{b}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}}$$
$$- \frac{\sqrt{b}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4(-a)^{3/2}}$$
$$+ \frac{\sqrt{b}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} - \frac{\sqrt{b}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4(-a)^{3/2}}$$
$$- \frac{\sqrt{b}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4(-a)^{3/2}} + \frac{\sqrt{b}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}} + \frac{\sqrt{b}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 506, normalized size of antiderivative = 0.94

$$\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx = -\frac{\log(c+dx)}{2ax^2} - \frac{\sqrt{b}\log\left(\frac{d\left(i\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} + \frac{\sqrt{b}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} - \frac{\sqrt{b}\log\left(-\frac{d\left(i\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} + \frac{\sqrt{b}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/2}} - \frac{d\left(\frac{1}{cx}+\frac{d\log(x)}{c^2}-\frac{d\log(c+dx)}{c^2}\right)}{2a} + \frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}} - \frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}} - \frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}} + \frac{\sqrt{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{3/2}}$$

```
[In] Integrate[Log[c + d*x]/(x^3*(a + b*x^4)),x]
```

```
[Out] -1/2*Log[c + d*x]/(a*x^2) - (Sqrt[b]*Log[(d*(I*(-a)^(1/4) - b^(1/4)*x))/(b^
(1/4)*c + I*(-a)^(1/4)*d)]*Log[c + d*x])/(4*(-a)^(3/2)) + (Sqrt[b]*Log[(d*(
(-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*(-a)^
(3/2)) - (Sqrt[b]*Log[-((d*(I*(-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - I*(-a)^
(1/4)*d))]*Log[c + d*x])/(4*(-a)^(3/2)) + (Sqrt[b]*Log[-((d*((-a)^(1/4) + b
^(1/4)*x))/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*(-a)^(3/2)) - (d*(
1/(c*x) + (d*Log[x])/c^2 - (d*Log[c + d*x])/c^2))/(2*a) + (Sqrt[b]*PolyLog[
2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*(-a)^(3/2)) - (Sqrt[
b]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)])/(4*(-a)^(3
/2)) - (Sqrt[b]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)
])/(4*(-a)^(3/2)) + (Sqrt[b]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-
a)^(1/4)*d)])/(4*(-a)^(3/2))
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.29

| method | result |
|---|---|
| derivativedivides | $d^{2}\left(\frac{-\frac{\ln(-dx)}{2c^{2}}-\frac{1}{2cdx}-\frac{\ln(dx+c)(dx+c)(-dx+c)}{2c^{2}d^{2}x^{2}}}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{4}-4cb\_Z^{3}+6b\,c^{2}\_Z^{2}-4b\,c^{3}\_Z+a\,d^{4}+b\,c^{4}\right)} \frac{\ln(dx+c)\ln\ldots}{\ldots}}{4a}\ldots\right.$ |
| default | $d^{2}\left(\frac{-\frac{\ln(-dx)}{2c^{2}}-\frac{1}{2cdx}-\frac{\ln(dx+c)(dx+c)(-dx+c)}{2c^{2}d^{2}x^{2}}}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{4}-4cb\_Z^{3}+6b\,c^{2}\_Z^{2}-4b\,c^{3}\_Z+a\,d^{4}+b\,c^{4}\right)} \frac{\ln(dx+c)\ln\ldots}{\ldots}}{4a}\ldots\right.$ |
| risch | $-\frac{d^{2}\ln(-dx)}{2a\,c^{2}}-\frac{d}{2acx}+\frac{d^{2}\ln(dx+c)}{2a\,c^{2}}-\frac{\ln(dx+c)}{2a\,x^{2}}-\frac{d^{2}\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{4}-4cb\_Z^{3}+6b\,c^{2}\_Z^{2}-4b\,c^{3}\_Z+a\,d^{4}+b\,c^{4}\ldots\right.}\ldots\right.}{4a}$ |

```
[In] int(ln(d*x+c)/x^3/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(1/a*(-1/2/c^2*ln(-d*x)-1/2/c/d/x-1/2*ln(d*x+c)*(d*x+c)*(-d*x+c)/c^2/d^
2/x^2)-1/4/a*sum(1/(_R1^2-2*_R1*c+c^2)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilo
g((-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-4*_Z*b*c^3+a
*d^4+b*c^4)))
```

## Fricas [F]

$$\int \frac{\log(c+dx)}{x^3\,(a+bx^4)}\,dx = \int \frac{\log(dx+c)}{(bx^4+a)x^3}\,dx$$

```
[In] integrate(log(d*x+c)/x^3/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral(log(d*x + c)/(b*x^7 + a*x^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(d*x+c)/x**3/(b*x**4+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx = \int \frac{\log(dx+c)}{(bx^4+a)x^3}\,dx$$

```
[In] integrate(log(d*x+c)/x^3/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(log(d*x + c)/((b*x^4 + a)*x^3), x)
```

## Giac [F]

$$\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx = \int \frac{\log(dx+c)}{(bx^4+a)x^3}\,dx$$

```
[In] integrate(log(d*x+c)/x^3/(b*x^4+a),x, algorithm="giac")
[Out] integrate(log(d*x + c)/((b*x^4 + a)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^3(a+bx^4)}\,dx = \int \frac{\ln(c+d\,x)}{x^3\,(b\,x^4+a)}\,dx$$

```
[In] int(log(c + d*x)/(x^3*(a + b*x^4)),x)
[Out] int(log(c + d*x)/(x^3*(a + b*x^4)), x)
```

# 3.299 $\int \frac{x^4 \log(c+dx)}{a+bx^4} dx$

## Optimal result

Integrand size = 19, antiderivative size = 521

$$\int \frac{x^4 \log(c+dx)}{a+bx^4} dx = -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} + \frac{\sqrt{-\sqrt{-a}}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{5/4}} + \frac{\sqrt[4]{-a}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{5/4}} - \frac{\sqrt{-\sqrt{-a}}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{5/4}} - \frac{\sqrt[4]{-a}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{5/4}} - \frac{\sqrt{-\sqrt{-a}}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{5/4}} + \frac{\sqrt{-\sqrt{-a}}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{5/4}} - \frac{\sqrt[4]{-a}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{5/4}} + \frac{\sqrt[4]{-a}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{5/4}}$$

```
[Out] -x/b+(d*x+c)*ln(d*x+c)/b/d+1/4*(-a)^(1/4)*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1
/4)*c+(-a)^(1/4)*d))*ln(d*x+c)/b^(5/4)-1/4*(-a)^(1/4)*ln(-d*((-a)^(1/4)+b^(
1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln(d*x+c)/b^(5/4)-1/4*(-a)^(1/4)*polylog(
2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))/b^(5/4)+1/4*(-a)^(1/4)*polylog(
2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^(5/4)-1/4*ln(d*x+c)*ln(-d*(b^
(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))*(-(-a)^(1/2
))^(1/2)/b^(5/4)+1/4*ln(d*x+c)*ln(d*(-b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/
4)*c+d*(-(-a)^(1/2))^(1/2)))*(-(-a)^(1/2))^(1/2)/b^(5/4)-1/4*polylog(2,b^(1
/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))*(-(-a)^(1/2))^(1/2)/b^(5/4)+
```

```
1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))*(-(-a)^(1/
2))^(1/2)/b^(5/4)
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {327, 217, 1179, 642, 1176, 631, 210, 2463, 2436, 2332, 2456, 2441, 2440, 2438}

$$\begin{aligned}\int \frac{x^4\log(c+dx)}{a+bx^4}\,dx = &-\frac{\sqrt{-\sqrt{-a}}\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{5/4}}\\&+\frac{\sqrt{-\sqrt{-a}}\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{5/4}}\\&-\frac{\sqrt[4]{-a}\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{5/4}}\\&+\frac{\sqrt[4]{-a}\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{5/4}}\\&+\frac{\sqrt{-\sqrt{-a}}\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4b^{5/4}}\\&+\frac{\sqrt[4]{-a}\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4b^{5/4}}\\&-\frac{\sqrt{-\sqrt{-a}}\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{5/4}}\\&-\frac{\sqrt[4]{-a}\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{5/4}}\\&+\frac{(c+dx)\log(c+dx)}{bd}-\frac{x}{b}\end{aligned}$$

```
[In] Int[(x^4*Log[c + d*x])/(a + b*x^4),x]
```

```
[Out] -(x/b) + ((c + d*x)*Log[c + d*x])/(b*d) + (Sqrt[-Sqrt[-a]]*Log[(d*(Sqrt[-Sq
rt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Log[c + d*x])/(4*b^(
```

```
5/4)) + ((-a)^(1/4)*Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4
)*d)]*Log[c + d*x])/(4*b^(5/4)) - (Sqrt[-Sqrt[-a]]*Log[-((d*(Sqrt[-Sqrt[-a]
] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d))]*Log[c + d*x])/(4*b^(5/4))
 - ((-a)^(1/4)*Log[-((d*((-a)^(1/4) + b^(1/4)*x))/(b^(1/4)*c - (-a)^(1/4)*d
))]*Log[c + d*x])/(4*b^(5/4)) - (Sqrt[-Sqrt[-a]]*PolyLog[2, (b^(1/4)*(c + d
*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(5/4)) + (Sqrt[-Sqrt[-a]]*PolyL
og[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(5/4)) - (
(-a)^(1/4)*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b
^(5/4)) + ((-a)^(1/4)*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4
)*d)])/(4*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
```

```
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\log(c+dx)}{b} - \frac{a\log(c+dx)}{b\,(a+bx^4)}\right) dx \\
&= \frac{\int \log(c+dx)\,dx}{b} - \frac{a\int \frac{\log(c+dx)}{a+bx^4}\,dx}{b} \\
&= -\frac{a\int \left(\frac{\sqrt{-a}\log(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x^2\right)} + \frac{\sqrt{-a}\log(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x^2\right)}\right) dx}{b} + \frac{\text{Subst}\left(\int \log(x)\,dx, x, c+dx\right)}{bd} \\
&= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} - \frac{\sqrt{-a}\int \frac{\log(c+dx)}{\sqrt{-a}-\sqrt{b}x^2}\,dx}{2b} - \frac{\sqrt{-a}\int \frac{\log(c+dx)}{\sqrt{-a}+\sqrt{b}x^2}\,dx}{2b} \\
&= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} \\
&\quad - \frac{\sqrt{-a}\int \left(\frac{\sqrt{-\sqrt{-a}}\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)} + \frac{\sqrt{-\sqrt{-a}}\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}\right) dx}{2b} \\
&\quad - \frac{\sqrt{-a}\int \left(\frac{\log(c+dx)}{2\sqrt[4]{-a}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)} + \frac{\log(c+dx)}{2\sqrt[4]{-a}\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}\right) dx}{2b} \\
&= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} - \frac{\sqrt{-\sqrt{-a}}\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}\,dx}{4b} \\
&\quad - \frac{\sqrt{-\sqrt{-a}}\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}\,dx}{4b} - \frac{\sqrt[4]{-a}\int \frac{\log(c+dx)}{\sqrt[4]{-a}-\sqrt[4]{b}x}\,dx}{4b} - \frac{\sqrt[4]{-a}\int \frac{\log(c+dx)}{\sqrt[4]{-a}+\sqrt[4]{b}x}\,dx}{4b}
\end{aligned}$$

$$= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} + \frac{\sqrt{-\sqrt{-a}}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{5/4}}$$

$$+ \frac{\sqrt[4]{-a}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{5/4}}$$

$$- \frac{\sqrt{-\sqrt{-a}}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{5/4}}$$

$$- \frac{\sqrt[4]{-a}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{5/4}}$$

$$- \frac{\left(\sqrt{-\sqrt{-a}}d\right)\int\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4b^{5/4}}$$

$$+ \frac{\left(\sqrt{-\sqrt{-a}}d\right)\int\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4b^{5/4}}$$

$$- \frac{\left(\sqrt[4]{-a}d\right)\int\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4b^{5/4}} + \frac{\left(\sqrt[4]{-a}d\right)\int\frac{\log\left(\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4b^{5/4}}$$

$$= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} + \frac{\sqrt{-\sqrt{-a}}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{5/4}}$$

$$+ \frac{\sqrt[4]{-a}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{5/4}}$$

$$- \frac{\sqrt{-\sqrt{-a}}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{5/4}}$$

$$- \frac{\sqrt[4]{-a}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{5/4}}$$

$$+ \frac{\sqrt{-\sqrt{-a}}\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4b^{5/4}}$$

$$- \frac{\sqrt{-\sqrt{-a}}\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4b^{5/4}}$$

$$+ \frac{\sqrt[4]{-a}\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4b^{5/4}}$$

$$- \frac{\sqrt[4]{-a}\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4b^{5/4}}$$

$$= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} + \frac{\sqrt{-\sqrt{-a}}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{5/4}}$$
$$+ \frac{\sqrt[4]{-a}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{5/4}}$$
$$- \frac{\sqrt{-\sqrt{-a}}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4b^{5/4}}$$
$$- \frac{\sqrt[4]{-a}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4b^{5/4}}$$
$$- \frac{\sqrt{-\sqrt{-a}}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{5/4}} + \frac{\sqrt{-\sqrt{-a}}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{5/4}}$$
$$- \frac{\sqrt[4]{-a}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{5/4}} + \frac{\sqrt[4]{-a}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{5/4}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.88

$$\int \frac{x^4\log(c+dx)}{a+bx^4}\,dx$$
$$= \frac{-4\sqrt[4]{b}dx + 4\sqrt[4]{b}c\log(c+dx) + 4\sqrt[4]{b}dx\log(c+dx) + \sqrt[4]{-a}d\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx) - i\sqrt[4]{-a}d\log}{}$$

```
[In] Integrate[(x^4*Log[c + d*x])/(a + b*x^4),x]
```

```
[Out] (-4*b^(1/4)*d*x + 4*b^(1/4)*c*Log[c + d*x] + 4*b^(1/4)*d*x*Log[c + d*x] + (
-a)^(1/4)*d*Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Lo
g[c + d*x] - I*(-a)^(1/4)*d*Log[(d*((-a)^(1/4) - I*b^(1/4)*x))/(I*b^(1/4)*c
 + (-a)^(1/4)*d)]*Log[c + d*x] + I*(-a)^(1/4)*d*Log[(d*((-a)^(1/4) + I*b^(1
/4)*x))/((-I)*b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x] - (-a)^(1/4)*d*Log[(d
*((-a)^(1/4) + b^(1/4)*x))/(-(b^(1/4)*c) + (-a)^(1/4)*d)]*Log[c + d*x] - (-
a)^(1/4)*d*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)] - I*(
```

```
-a)^(1/4)*d*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)] +
I*(-a)^(1/4)*d*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]
 + (-a)^(1/4)*d*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])
/(4*b^(5/4)*d)
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.28

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d^4((dx+c)\ln(dx+c)-dx-c)}{b}+\frac{a\,d^8\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-}{\_R1}\right.}{-\_R1^3+3c\_R}\right.}{4b^2}}{d^5}$ |
| default | $\frac{\frac{d^4((dx+c)\ln(dx+c)-dx-c)}{b}+\frac{a\,d^8\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-}{\_R1}\right.}{-\_R1^3+3c\_R}\right.}{4b^2}}{d^5}$ |
| risch | $\frac{x\ln(dx+c)}{b}+\frac{\ln(dx+c)c}{db}-\frac{x}{b}-\frac{c}{db}+\frac{d^3a\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln}{-}\right.}{4b^2}$ |

```
[In] int(x^4*ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^5*(d^4/b*((d*x+c)*ln(d*x+c)-d*x-c)+1/4*a*d^8/b^2*sum(1/(-_R1^3+3*_R1^2*
c-3*_R1*c^2+c^3)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1)),_
R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-4*_Z*b*c^3+a*d^4+b*c^4)))
```

## Fricas [F]

$$\int \frac{x^4\log(c+dx)}{a+bx^4}\,dx=\int \frac{x^4\log(dx+c)}{bx^4+a}\,dx$$

```
[In] integrate(x^4*log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral(x^4*log(d*x + c)/(b*x^4 + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \log(c+dx)}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*ln(d*x+c)/(b*x**4+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^4 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^4 \log\left(dx+c\right)}{bx^4+a}\,dx$$

```
[In] integrate(x^4*log(d*x+c)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(x^4*log(d*x + c)/(b*x^4 + a), x)
```

## Giac [F]

$$\int \frac{x^4 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^4 \log\left(dx+c\right)}{bx^4+a}\,dx$$

```
[In] integrate(x^4*log(d*x+c)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(x^4*log(d*x + c)/(b*x^4 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^4\,\ln\left(c+d\,x\right)}{b\,x^4+a}\,dx$$

```
[In] int((x^4*log(c + d*x))/(a + b*x^4),x)
[Out] int((x^4*log(c + d*x))/(a + b*x^4), x)
```

# 3.300 $\int \frac{x^2\log(c+dx)}{a+bx^4}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 497

$$\begin{aligned}\int \frac{x^2\log(c+dx)}{a+bx^4}\,dx &= \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-\sqrt{-a}}b^{3/4}} \\ &+\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt[4]{-a}b^{3/4}} \\ &-\frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-\sqrt{-a}}b^{3/4}} \\ &-\frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt[4]{-a}b^{3/4}} \\ &-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}} \\ &-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt[4]{-a}b^{3/4}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4\sqrt[4]{-a}b^{3/4}}\end{aligned}$$

```
[Out] 1/4*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)/(-a)^(1
/4)/b^(3/4)-1/4*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln(d
```

```
*x+c)/(-a)^(1/4)/b^(3/4)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4
)*d))/(-a)^(1/4)/b^(3/4)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4
)*d))/(-a)^(1/4)/b^(3/4)-1/4*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^(1/2)
)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(-(-a)^(1/2))^(1/2)+1/4*ln(d*x
+c)*ln(d*(-b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2))
)/b^(3/4)/(-(-a)^(1/2))^(1/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-
(-a)^(1/2))^(1/2)))/b^(3/4)/(-(-a)^(1/2))^(1/2)+1/4*polylog(2,b^(1/4)*(d*x+
c)/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(-(-a)^(1/2))^(1/2)
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {303, 1176, 631, 210, 1179, 642, 2463, 2456, 2441, 2440, 2438}

$$\begin{aligned}\int \frac{x^2\log(c+dx)}{a+bx^4}\,dx = &-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}}\\ &-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt[4]{-a}b^{3/4}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4\sqrt[4]{-a}b^{3/4}}\\ &+\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}}\\ &+\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4\sqrt[4]{-a}b^{3/4}}\\ &-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}}\\ &-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt[4]{-a}b^{3/4}}\end{aligned}$$

```
[In] Int[(x^2*Log[c + d*x])/(a + b*x^4),x]
```

```
[Out] (Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Log
[c + d*x])/(4*Sqrt[-Sqrt[-a]]*b^(3/4)) + (Log[(d*((-a)^(1/4) - b^(1/4)*x))/
(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*(-a)^(1/4)*b^(3/4)) - (Log[-((
```

```
d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d))]*Log[c +
d*x])/(4*Sqrt[-Sqrt[-a]]*b^(3/4)) - (Log[-((d*((-a)^(1/4) + b^(1/4)*x))/(b^
(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*(-a)^(1/4)*b^(3/4)) - PolyLog[2,
 (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)]/(4*Sqrt[-Sqrt[-a]]*b^
(3/4)) + PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]/(4
*Sqrt[-Sqrt[-a]]*b^(3/4)) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a
)^(1/4)*d)]/(4*(-a)^(1/4)*b^(3/4)) + PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4
)*c + (-a)^(1/4)*d)]/(4*(-a)^(1/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{\log(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x^2\right)}+\frac{\log(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x^2\right)}\right) dx$$

$$= -\frac{\int \frac{\log(c+dx)}{\sqrt{-a}-\sqrt{b}x^2}\,dx}{2\sqrt{b}}+\frac{\int \frac{\log(c+dx)}{\sqrt{-a}+\sqrt{b}x^2}\,dx}{2\sqrt{b}}$$

$$= \frac{\int \left( \frac{\sqrt{-\sqrt{-a}}\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)} + \frac{\sqrt{-\sqrt{-a}}\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)} \right) dx}{2\sqrt{b}} - \frac{\int \left( \frac{\log(c+dx)}{2\sqrt[4]{-a}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)} + \frac{\log(c+dx)}{2\sqrt[4]{-a}\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)} \right) dx}{2\sqrt{b}}$$

$$= -\frac{\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}dx}{4\sqrt{-\sqrt{-a}}\sqrt{b}} - \frac{\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}dx}{4\sqrt{-\sqrt{-a}}\sqrt{b}} - \frac{\int \frac{\log(c+dx)}{\sqrt[4]{-a}-\sqrt[4]{b}x}dx}{4\sqrt[4]{-a}\sqrt{b}} - \frac{\int \frac{\log(c+dx)}{\sqrt[4]{-a}+\sqrt[4]{b}x}dx}{4\sqrt[4]{-a}\sqrt{b}}$$

$$= \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-\sqrt{-a}}b^{3/4}} + \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt[4]{-a}b^{3/4}} - \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-\sqrt{-a}}b^{3/4}} - \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt[4]{-a}b^{3/4}} - \frac{d\int \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4\sqrt{-\sqrt{-a}}b^{3/4}} + \frac{d\int \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4\sqrt{-\sqrt{-a}}b^{3/4}} - \frac{d\int \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4\sqrt[4]{-a}b^{3/4}} + \frac{d\int \frac{\log\left(\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4\sqrt[4]{-a}b^{3/4}}$$

$$= \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-\sqrt{-a}}b^{3/4}}+\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt[4]{-a}b^{3/4}}$$

$$-\frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-\sqrt{-a}}b^{3/4}}-\frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt[4]{-a}b^{3/4}}$$

$$+\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx,x,c+dx\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}}$$

$$-\frac{\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx,x,c+dx\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}}$$

$$+\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx,x,c+dx\right)}{4\sqrt[4]{-a}b^{3/4}}$$

$$-\frac{\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx,x,c+dx\right)}{4\sqrt[4]{-a}b^{3/4}}$$

$$= \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-\sqrt{-a}}b^{3/4}} + \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt[4]{-a}b^{3/4}}$$

$$- \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\sqrt{-\sqrt{-a}}b^{3/4}} - \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4\sqrt[4]{-a}b^{3/4}}$$

$$- \frac{\mathrm{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}} + \frac{\mathrm{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\sqrt{-\sqrt{-a}}b^{3/4}}$$

$$- \frac{\mathrm{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4\sqrt[4]{-a}b^{3/4}} + \frac{\mathrm{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4\sqrt[4]{-a}b^{3/4}}$$

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.93

$$\int \frac{x^2\log(c+dx)}{a+bx^4}\,dx$$

$$= \frac{\sqrt[4]{-a}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)+\sqrt{-\sqrt{-a}}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)-\sqrt[4]{-a}\log\left(\frac{d(\cdots}{-\cdots}\right.}{\cdots}$$

```
[In] Integrate[(x^2*Log[c + d*x])/(a + b*x^4),x]
```

```
[Out] ((-a)^(1/4)*Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-
a]]*d)]*Log[c + d*x] + Sqrt[-Sqrt[-a]]*Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^
(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x] - (-a)^(1/4)*Log[(d*(Sqrt[-Sqrt[-a]]
+ b^(1/4)*x))/(-(b^(1/4)*c) + Sqrt[-Sqrt[-a]]*d)]*Log[c + d*x] - Sqrt[-Sqrt
[-a]]*Log[(d*((-a)^(1/4) + b^(1/4)*x))/(-(b^(1/4)*c) + (-a)^(1/4)*d)]*Log[c
 + d*x] - (-a)^(1/4)*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt
[-a]]*d)] + (-a)^(1/4)*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sq
rt[-a]]*d)] - Sqrt[-Sqrt[-a]]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (
-a)^(1/4)*d)] + Sqrt[-Sqrt[-a]]*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c +
 (-a)^(1/4)*d)])/(4*Sqrt[-Sqrt[-a]]*(-a)^(1/4)*b^(3/4))
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.19

| method | result |
|---|---|
| derivativedivides | $-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{4b}$ |
| default | $-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{4b}$ |
| risch | $-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1+c}\right)}{4b}$ |

```
[In] int(x^2*ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] -1/4*d/b*sum(1/(-_R1+c)*(ln(d*x+c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/
_R1)),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-4*_Z*b*c^3+a*d^4+b*c^4))
```

## Fricas [F]

$$\int \frac{x^2 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^2 \log\left(dx+c\right)}{bx^4+a}\,dx$$

```
[In] integrate(x^2*log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
[Out] integral(x^2*log(d*x + c)/(b*x^4 + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log(c+dx)}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*ln(d*x+c)/(b*x**4+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^2 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^2 \log{(dx+c)}}{bx^4+a}\,dx$$

```
[In] integrate(x^2*log(d*x+c)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(x^2*log(d*x + c)/(b*x^4 + a), x)
```

## Giac [F]

$$\int \frac{x^2 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^2 \log{(dx+c)}}{bx^4+a}\,dx$$

```
[In] integrate(x^2*log(d*x+c)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(x^2*log(d*x + c)/(b*x^4 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log(c+dx)}{a+bx^4}\,dx = \int \frac{x^2\,\ln{(c+d\,x)}}{b\,x^4+a}\,dx$$

```
[In] int((x^2*log(c + d*x))/(a + b*x^4),x)
[Out] int((x^2*log(c + d*x))/(a + b*x^4), x)
```

# 3.301 $\int \frac{\log(c+dx)}{a+bx^4}\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 497

$$\begin{aligned}\int \frac{\log(c+dx)}{a+bx^4}\,dx &= \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}} \\ &+\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/4}\sqrt[4]{b}} \\ &-\frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}} \\ &-\frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/4}\sqrt[4]{b}} \\ &-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}} \\ &-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/4}\sqrt[4]{b}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{3/4}\sqrt[4]{b}}\end{aligned}$$

```
[Out] 1/4*ln(d*((-a)^(1/4)-b^(1/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)/(-a)^(3
/4)/b^(1/4)-1/4*ln(-d*((-a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln(d
```

```
*x+c)/(-a)^(3/4)/b^(1/4)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4
)*d))/(-a)^(3/4)/b^(1/4)+1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4
)*d))/(-a)^(3/4)/b^(1/4)-1/4*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^(1/2)
)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^(1/4)/(-(-a)^(1/2))^(3/2)+1/4*ln(d*x
+c)*ln(d*(-b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2))
)/b^(1/4)/(-(-a)^(1/2))^(3/2)-1/4*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-
(-a)^(1/2))^(1/2)))/b^(1/4)/(-(-a)^(1/2))^(3/2)+1/4*polylog(2,b^(1/4)*(d*x+
c)/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^(1/4)/(-(-a)^(1/2))^(3/2)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2456, 2441, 2440, 2438}

$$\int \frac{\log(c+dx)}{a+bx^4}\,dx = -\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/4}\sqrt[4]{b}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{3/4}\sqrt[4]{b}}+\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}+\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4(-a)^{3/4}\sqrt[4]{b}}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/4}\sqrt[4]{b}}$$

```
[In] Int[Log[c + d*x]/(a + b*x^4),x]
```

```
[Out] (Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Log
[c + d*x])/(4*(-Sqrt[-a])^(3/2)*b^(1/4)) + (Log[(d*((-a)^(1/4) - b^(1/4)*x)
```

```
)/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*(-a)^(3/4)*b^(1/4)) - (Log[-
((d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d))]*Log[c
+ d*x])/(4*(-Sqrt[-a])^(3/2)*b^(1/4)) - (Log[-((d*((-a)^(1/4) + b^(1/4)*x))
/(b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*(-a)^(3/4)*b^(1/4)) - PolyLo
g[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)]/(4*(-Sqrt[-a])^(3
/2)*b^(1/4)) + PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*
d)]/(4*(-Sqrt[-a])^(3/2)*b^(1/4)) - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)
*c - (-a)^(1/4)*d)]/(4*(-a)^(3/4)*b^(1/4)) + PolyLog[2, (b^(1/4)*(c + d*x))
/(b^(1/4)*c + (-a)^(1/4)*d)]/(4*(-a)^(3/4)*b^(1/4))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rubi steps

$$\text{integral} = \int \left( \frac{\sqrt{-a}\log(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x^2\right)} + \frac{\sqrt{-a}\log(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x^2\right)} \right) dx$$

$$= -\frac{\int \frac{\log(c+dx)}{\sqrt{-a}-\sqrt{b}x^2}\,dx}{2\sqrt{-a}} - \frac{\int \frac{\log(c+dx)}{\sqrt{-a}+\sqrt{b}x^2}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\int \left(\frac{\sqrt{-\sqrt{-a}}\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}+\frac{\sqrt{-\sqrt{-a}}\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}\right)dx}{2\sqrt{-a}}$$
$$-\frac{\int \left(\frac{\log(c+dx)}{2\sqrt[4]{-a}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}+\frac{\log(c+dx)}{2\sqrt[4]{-a}\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}\right)dx}{2\sqrt{-a}}$$

$$= -\frac{\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}dx}{4\left(-\sqrt{-a}\right)^{3/2}}-\frac{\int \frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}dx}{4\left(-\sqrt{-a}\right)^{3/2}}-\frac{\int \frac{\log(c+dx)}{\sqrt[4]{-a}-\sqrt[4]{b}x}dx}{4(-a)^{3/4}}-\frac{\int \frac{\log(c+dx)}{\sqrt[4]{-a}+\sqrt[4]{b}x}dx}{4(-a)^{3/4}}$$

$$= \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}+\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/4}\sqrt[4]{b}}$$
$$-\frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}-\frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/4}\sqrt[4]{b}}$$
$$-\frac{d\int \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}+\frac{d\int \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}$$
$$-\frac{d\int \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4(-a)^{3/4}\sqrt[4]{b}}+\frac{d\int \frac{\log\left(\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4(-a)^{3/4}\sqrt[4]{b}}$$

$$= \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}} + \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/4}\sqrt[4]{b}}$$

$$- \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}} - \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/4}\sqrt[4]{b}}$$

$$+ \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}$$

$$- \frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}}$$

$$+ \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4(-a)^{3/4}\sqrt[4]{b}}$$

$$- \frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4(-a)^{3/4}\sqrt[4]{b}}$$

$$= \frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}} + \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/4}\sqrt[4]{b}} - \frac{\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}} - \frac{\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{3/4}\sqrt[4]{b}} - \frac{\text{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}} + \frac{\text{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{3/2}\sqrt[4]{b}} - \frac{\text{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{3/4}\sqrt[4]{b}} + \frac{\text{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{3/4}\sqrt[4]{b}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.72

$$\int \frac{\log(c+dx)}{a+bx^4}\,dx = \frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx) - i\log\left(\frac{d\left(\sqrt[4]{-a}-i\sqrt[4]{b}x\right)}{i\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx) + i\log\left(\frac{d\left(\sqrt[4]{-a}+i\sqrt[4]{b}x\right)}{-i\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(\ldots}{\ldots}$$

```
[In] Integrate[Log[c + d*x]/(a + b*x^4),x]
```

```
[Out] (Log[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x]
- I*Log[(d*((-a)^(1/4) - I*b^(1/4)*x))/(I*b^(1/4)*c + (-a)^(1/4)*d)]*Log[c
+ d*x] + I*Log[(d*((-a)^(1/4) + I*b^(1/4)*x))/((-I)*b^(1/4)*c + (-a)^(1/4)*
d)]*Log[c + d*x] - Log[(d*((-a)^(1/4) + b^(1/4)*x))/(-(b^(1/4)*c) + (-a)^(1
/4)*d)]*Log[c + d*x] - PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/
4)*d)] - I*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)] + I
*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)] + PolyLog[2,
(b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*(-a)^(3/4)*b^(1/4))
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.23

| method | result |
|---|---|
| derivativedivides | $-\frac{d^3\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1^3+3c\_R1^2-3c^2\_R1+c^3}\right.}{4b}$ |
| default | $-\frac{d^3\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1^3+3c\_R1^2-3c^2\_R1+c^3}\right.}{4b}$ |
| risch | $-\frac{d^3\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)+\mathrm{dilog}\left(\frac{-dx+\_R1-c}{\_R1}\right)}{-\_R1^3+3c\_R1^2-3c^2\_R1+c^3}\right.}{4b}$ |

```
[In] int(ln(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] -1/4*d^3/b*sum(1/(-_R1^3+3*_R1^2*c-3*_R1*c^2+c^3)*(ln(d*x+c)*ln((-d*x+_R1-c
)/_R1)+dilog((-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^4*b-4*_Z^3*b*c+6*_Z^2*b*c^2-4
*_Z*b*c^3+a*d^4+b*c^4))
```

## Fricas [F]

$$\int \frac{\log(c+dx)}{a+bx^4}\,dx = \int \frac{\log(dx+c)}{bx^4+a}\,dx$$

```
[In] integrate(log(d*x+c)/(b*x^4+a),x, algorithm="fricas")
[Out] integral(log(d*x + c)/(b*x^4 + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate(ln(d*x+c)/(b*x**4+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log(c+dx)}{a+bx^4}\,dx = \int \frac{\log(dx+c)}{bx^4+a}\,dx$$

```
[In] integrate(log(d*x+c)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(log(d*x + c)/(b*x^4 + a), x)
```

## Giac [F]

$$\int \frac{\log(c+dx)}{a+bx^4}\,dx = \int \frac{\log(dx+c)}{bx^4+a}\,dx$$

```
[In] integrate(log(d*x+c)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(log(d*x + c)/(b*x^4 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{a+bx^4}\,dx = \int \frac{\ln(c+d\,x)}{b\,x^4+a}\,dx$$

```
[In] int(log(c + d*x)/(a + b*x^4),x)
[Out] int(log(c + d*x)/(a + b*x^4), x)
```

# 3.302 $\int \frac{\log(c+dx)}{x^2(a+bx^4)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 536

$$\int \frac{\log(c+dx)}{x^2(a+bx^4)}\,dx = \frac{d\log(x)}{ac} - \frac{d\log(c+dx)}{ac} - \frac{\log(c+dx)}{ax} + \frac{\sqrt[4]{b}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{5/2}} + \frac{\sqrt[4]{b}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{5/4}} - \frac{\sqrt[4]{b}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{5/2}} - \frac{\sqrt[4]{b}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{5/4}} - \frac{\sqrt[4]{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{5/2}} + \frac{\sqrt[4]{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{5/2}} - \frac{\sqrt[4]{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{5/4}} + \frac{\sqrt[4]{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{5/4}}$$

```
[Out] d*ln(x)/a/c-d*ln(d*x+c)/a/c-ln(d*x+c)/a/x+1/4*b^(1/4)*ln(d*((-a)^(1/4)-b^(1
/4)*x)/(b^(1/4)*c+(-a)^(1/4)*d))*ln(d*x+c)/(-a)^(5/4)-1/4*b^(1/4)*ln(-d*((-
a)^(1/4)+b^(1/4)*x)/(b^(1/4)*c-(-a)^(1/4)*d))*ln(d*x+c)/(-a)^(5/4)-1/4*b^(1
/4)*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))/(-a)^(5/4)+1/4*b^(1
/4)*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/(-a)^(5/4)-1/4*b^(1
/4)*ln(d*x+c)*ln(-d*(b^(1/4)*x+(-(-a)^(1/2))^(1/2))/(b^(1/4)*c-d*(-(-a)^(1/
2))^(1/2)))/(-(-a)^(1/2))^(5/2)+1/4*b^(1/4)*ln(d*x+c)*ln(d*(-b^(1/4)*x+(-(-
a)^(1/2))^(1/2))/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/(-(-a)^(1/2))^(5/2)-1/4
*b^(1/4)*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/(-(-a
)^(1/2))^(5/2)+1/4*b^(1/4)*polylog(2,b^(1/4)*(d*x+c)/(b^(1/4)*c+d*(-(-a)^(1
/2))^(1/2)))/(-(-a)^(1/2))^(5/2)
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.842$, Rules used = {331, 303, 1176, 631, 210, 1179, 642, 2463, 2442, 36, 29, 31, 2456, 2441, 2440, 2438}

$$\int \frac{\log(c+dx)}{x^2(a+bx^4)}\,dx = -\frac{\sqrt[4]{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{5/2}}+\frac{\sqrt[4]{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{5/2}} -\frac{\sqrt[4]{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{5/4}}+\frac{\sqrt[4]{b}\,\text{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{5/4}} +\frac{\sqrt[4]{b}\log(c+dx)\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c}\right)}{4\left(-\sqrt{-a}\right)^{5/2}} +\frac{\sqrt[4]{b}\log(c+dx)\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4(-a)^{5/4}} -\frac{\sqrt[4]{b}\log(c+dx)\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{5/2}} -\frac{\sqrt[4]{b}\log(c+dx)\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{5/4}} +\frac{d\log(x)}{ac}-\frac{d\log(c+dx)}{ac}-\frac{\log(c+dx)}{ax}$$

```
[In] Int[Log[c + d*x]/(x^2*(a + b*x^4)),x]
```

```
[Out] (d*Log[x])/(a*c) - (d*Log[c + d*x])/(a*c) - Log[c + d*x]/(a*x) + (b^(1/4)*L
og[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)]*Log[c
 + d*x])/(4*(-Sqrt[-a])^(5/2)) + (b^(1/4)*Log[(d*((-a)^(1/4) - b^(1/4)*x))/
(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x])/(4*(-a)^(5/4)) - (b^(1/4)*Log[-((
d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d))]*Log[c +
d*x])/(4*(-Sqrt[-a])^(5/2)) - (b^(1/4)*Log[-((d*((-a)^(1/4) + b^(1/4)*x))/(
b^(1/4)*c - (-a)^(1/4)*d))]*Log[c + d*x])/(4*(-a)^(5/4)) - (b^(1/4)*PolyLog
[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*(-Sqrt[-a])^(5
/2)) + (b^(1/4)*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]
*d)])/(4*(-Sqrt[-a])^(5/2)) - (b^(1/4)*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1
/4)*c - (-a)^(1/4)*d)])/(4*(-a)^(5/4)) + (b^(1/4)*PolyLog[2, (b^(1/4)*(c +
d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*(-a)^(5/4))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\log(c+dx)}{ax^2} - \frac{bx^2\log(c+dx)}{a\,(a+bx^4)}\right)dx \\
&= \frac{\int \frac{\log(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{x^2\log(c+dx)}{a+bx^4}\,dx}{a} \\
&= -\frac{\log(c+dx)}{ax} - \frac{b\int \left(-\frac{\log(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x^2\right)} + \frac{\log(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x^2\right)}\right)dx}{a} + \frac{d\int \frac{1}{x(c+dx)}\,dx}{a} \\
&= -\frac{\log(c+dx)}{ax} + \frac{\sqrt{b}\int \frac{\log(c+dx)}{\sqrt{-a}-\sqrt{b}x^2}\,dx}{2a} - \frac{\sqrt{b}\int \frac{\log(c+dx)}{\sqrt{-a}+\sqrt{b}x^2}\,dx}{2a} + \frac{d\int \frac{1}{x}\,dx}{ac} - \frac{d^2\int \frac{1}{c+dx}\,dx}{ac}
\end{aligned}
$$

$$= \frac{d\log(x)}{ac} - \frac{d\log(c+dx)}{ac} - \frac{\log(c+dx)}{ax} - \frac{\sqrt{b}\int\left(\frac{\sqrt{-\sqrt{-a}}\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)} + \frac{\sqrt{-\sqrt{-a}}\log(c+dx)}{2\sqrt{-a}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}\right)dx}{2a} + \frac{\sqrt{b}\int\left(\frac{\log(c+dx)}{2\sqrt[4]{-a}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)} + \frac{\log(c+dx)}{2\sqrt[4]{-a}\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}\right)dx}{2a}$$

$$= \frac{d\log(x)}{ac} - \frac{d\log(c+dx)}{ac} - \frac{\log(c+dx)}{ax} - \frac{\sqrt{b}\int\frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}dx}{4\left(-\sqrt{-a}\right)^{5/2}} - \frac{\sqrt{b}\int\frac{\log(c+dx)}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}dx}{4\left(-\sqrt{-a}\right)^{5/2}} - \frac{\sqrt{b}\int\frac{\log(c+dx)}{\sqrt[4]{-a}-\sqrt[4]{b}x}dx}{4(-a)^{5/4}} - \frac{\sqrt{b}\int\frac{\log(c+dx)}{\sqrt[4]{-a}+\sqrt[4]{b}x}dx}{4(-a)^{5/4}}$$

$$= \frac{d\log(x)}{ac} - \frac{d\log(c+dx)}{ac} - \frac{\log(c+dx)}{ax} + \frac{\sqrt[4]{b}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{5/2}} + \frac{\sqrt[4]{b}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{5/4}} - \frac{\sqrt[4]{b}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{5/2}} - \frac{\sqrt[4]{b}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{5/4}} - \frac{\left(\sqrt[4]{b}d\right)\int\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4\left(-\sqrt{-a}\right)^{5/2}} + \frac{\left(\sqrt[4]{b}d\right)\int\frac{\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{c+dx}dx}{4\left(-\sqrt{-a}\right)^{5/2}} - \frac{\left(\sqrt[4]{b}d\right)\int\frac{\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4(-a)^{5/4}} + \frac{\left(\sqrt[4]{b}d\right)\int\frac{\log\left(\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{c+dx}dx}{4(-a)^{5/4}}$$

$$= \frac{d\log(x)}{ac} - \frac{d\log(c+dx)}{ac} - \frac{\log(c+dx)}{ax} + \frac{\sqrt[4]{b}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{5/2}}$$

$$+ \frac{\sqrt[4]{b}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{5/4}}$$

$$- \frac{\sqrt[4]{b}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{5/2}}$$

$$- \frac{\sqrt[4]{b}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{5/4}}$$

$$+ \frac{\sqrt[4]{b}\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4\left(-\sqrt{-a}\right)^{5/2}}$$

$$- \frac{\sqrt[4]{b}\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{x}dx, x, c+dx\right)}{4\left(-\sqrt{-a}\right)^{5/2}}$$

$$+ \frac{\sqrt[4]{b}\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[4]{b}x}{-\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4(-a)^{5/4}}$$

$$- \frac{\sqrt[4]{b}\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{x}dx, x, c+dx\right)}{4(-a)^{5/4}}$$

$$
=\frac{d\log(x)}{ac}-\frac{d\log(c+dx)}{ac}-\frac{\log(c+dx)}{ax}+\frac{\sqrt[4]{b}\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{5/2}}
+\frac{\sqrt[4]{b}\log\left(\frac{d\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{5/4}}
-\frac{\sqrt[4]{b}\log\left(-\frac{d\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{4\left(-\sqrt{-a}\right)^{5/2}}
-\frac{\sqrt[4]{b}\log\left(-\frac{d\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)\log(c+dx)}{4(-a)^{5/4}}-\frac{\sqrt[4]{b}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{5/2}}
+\frac{\sqrt[4]{b}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4\left(-\sqrt{-a}\right)^{5/2}}-\frac{\sqrt[4]{b}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4(-a)^{5/4}}+\frac{\sqrt[4]{b}\operatorname{Li}_2\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4(-a)^{5/4}}
$$

## Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.00

$$
\int \frac{\log(c+dx)}{x^2\left(a+bx^4\right)}\,dx
$$

$$
=\frac{\frac{a\sqrt[4]{b}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{\sqrt{-\sqrt{-a}}}-\frac{-4adx\log(x)+4ac\log(c+dx)+4adx\log(c+dx)+\frac{a\sqrt[4]{b}cx\log\left(\frac{d\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)\log(c+dx)}{\sqrt{-\sqrt{-a}}}-(-a)^{3/4}}{}}{}
$$

```
[In] Integrate[Log[c + d*x]/(x^2*(a + b*x^4)),x]
```

```
[Out] ((a*b^(1/4)*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)]
)/Sqrt[-Sqrt[-a]] - (-4*a*d*x*Log[x] + 4*a*c*Log[c + d*x] + 4*a*d*x*Log[c +
 d*x] + (a*b^(1/4)*c*x*Log[(d*(Sqrt[-Sqrt[-a]] - b^(1/4)*x))/(b^(1/4)*c + S
qrt[-Sqrt[-a]]*d)]*Log[c + d*x])/Sqrt[-Sqrt[-a]] - (-a)^(3/4)*b^(1/4)*c*x*L
og[(d*((-a)^(1/4) - b^(1/4)*x))/(b^(1/4)*c + (-a)^(1/4)*d)]*Log[c + d*x] -
(a*b^(1/4)*c*x*Log[(d*(Sqrt[-Sqrt[-a]] + b^(1/4)*x))/(-(b^(1/4)*c) + Sqrt[-
Sqrt[-a]]*d)]*Log[c + d*x])/Sqrt[-Sqrt[-a]] + (-a)^(3/4)*b^(1/4)*c*x*Log[(d
```

```
*((-a)^(1/4) + b^(1/4)*x))/(-(b^(1/4)*c) + (-a)^(1/4)*d)]*Log[c + d*x] + (a
*b^(1/4)*c*x*PolyLog[2, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)
])/Sqrt[-Sqrt[-a]] + (-a)^(3/4)*b^(1/4)*c*x*PolyLog[2, (b^(1/4)*(c + d*x))/
(b^(1/4)*c - (-a)^(1/4)*d)] - (-a)^(3/4)*b^(1/4)*c*x*PolyLog[2, (b^(1/4)*(c
 + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(c*x))/(4*a^2)
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.25

| method | result |
|---|---|
| derivativedivides | $d\left(\frac{\frac{\ln(-dx)}{c}-\frac{\ln(dx+c)(dx+c)}{cdx}}{a}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)\cdots}{-\_R\cdots}}{4a}\cdots\right.$ |
| default | $d\left(\frac{\frac{\ln(-dx)}{c}-\frac{\ln(dx+c)(dx+c)}{cdx}}{a}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\frac{-dx+\_R1-c}{\_R1}\right)\cdots}{-\_R\cdots}}{4a}\cdots\right.$ |
| risch | $\frac{d\ln(-dx)}{ac}-\frac{d\ln(dx+c)}{ac}-\frac{\ln(dx+c)}{ax}+\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^4-4cb\_Z^3+6b\,c^2\_Z^2-4b\,c^3\_Z+a\,d^4+b\,c^4\right)}\frac{\ln(dx+c)\ln\left(\cdots\right.}{\cdots}\right.}{4a}\cdots$ |

```
[In] int(ln(d*x+c)/x^2/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] d*(1/a*(1/c*ln(-d*x)-ln(d*x+c)*(d*x+c)/c/d/x)+1/4/a*sum(1/(-_R1+c)*(ln(d*x+
c)*ln((-d*x+_R1-c)/_R1)+dilog((-d*x+_R1-c)/_R1)),_R1=RootOf(_Z^4*b-4*_Z^3*b
*c+6*_Z^2*b*c^2-4*_Z*b*c^3+a*d^4+b*c^4)))
```

## Fricas [F]

$$\int \frac{\log(c+dx)}{x^2\left(a+bx^4\right)}\,dx=\int \frac{\log\left(dx+c\right)}{(bx^4+a)x^2}\,dx$$

```
[In] integrate(log(d*x+c)/x^2/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral(log(d*x + c)/(b*x^6 + a*x^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^2\,(a+bx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(d*x+c)/x**2/(b*x**4+a),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log(c+dx)}{x^2\,(a+bx^4)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^4+a)x^2}\,dx$$

```
[In] integrate(log(d*x+c)/x^2/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(log(d*x + c)/((b*x^4 + a)*x^2), x)
```

## Giac [F]

$$\int \frac{\log(c+dx)}{x^2\,(a+bx^4)}\,dx = \int \frac{\log\left(dx+c\right)}{(bx^4+a)x^2}\,dx$$

```
[In] integrate(log(d*x+c)/x^2/(b*x^4+a),x, algorithm="giac")
[Out] integrate(log(d*x + c)/((b*x^4 + a)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c+dx)}{x^2\,(a+bx^4)}\,dx = \int \frac{\ln\left(c+d\,x\right)}{x^2\,\left(b\,x^4+a\right)}\,dx$$

```
[In] int(log(c + d*x)/(x^2*(a + b*x^4)),x)
[Out] int(log(c + d*x)/(x^2*(a + b*x^4)), x)
```

# 3.303 $\int \left(f+\frac{g}{x}\right) x(a+b\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \left(f+\frac{g}{x}\right) x(a+b\log(c(d+ex)^n))\,dx=\frac{b(df-eg)nx}{2e}-\frac{bn(g+fx)^2}{4f}-\frac{b(df-eg)^2n\log(d+ex)}{2e^2f}+\frac{(g+fx)^2(a+b\log(c(d+ex)^n))}{2f}$$

```
[Out] 1/2*b*(d*f-e*g)*n*x/e-1/4*b*n*(f*x+g)^2/f-1/2*b*(d*f-e*g)^2*n*ln(e*x+d)/e^2
/f+1/2*(f*x+g)^2*(a+b*ln(c*(e*x+d)^n))/f
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2459, 2442, 45}

$$\int \left(f+\frac{g}{x}\right) x(a+b\log(c(d+ex)^n))\,dx=\frac{(fx+g)^2(a+b\log(c(d+ex)^n))}{2f}-\frac{bn(df-eg)^2\log(d+ex)}{2e^2f}+\frac{bnx(df-eg)}{2e}-\frac{bn(fx+g)^2}{4f}$$

```
[In] Int[(f + g/x)*x*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (b*(d*f - e*g)*n*x)/(2*e) - (b*n*(g + f*x)^2)/(4*f) - (b*(d*f - e*g)^2*n*Lo
g[d + e*x])/(2*e^2*f) + ((g + f*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (g+fx)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx\\
&= \frac{(g+fx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f}-\frac{(ben)\int\frac{(g+fx)^2}{d+ex}\,dx}{2f}\\
&= \frac{(g+fx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f}-\frac{(ben)\int\left(\frac{f(-df+eg)}{e^2}+\frac{(-df+eg)^2}{e^2(d+ex)}+\frac{f(g+fx)}{e}\right)dx}{2f}\\
&= \frac{b(df-eg)nx}{2e}-\frac{bn(g+fx)^2}{4f}-\frac{b(df-eg)^2n\log(d+ex)}{2e^2f}+\frac{(g+fx)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04

$$
\begin{aligned}
\int\left(f+\frac{g}{x}\right)x\left(a+b\log\left(c(d+ex)^n\right)\right)dx &= agx-bgnx+\frac{1}{2}afx^2\\
&\quad-\frac{1}{2}bfn\left(-\frac{dx}{e}+\frac{x^2}{2}+\frac{d^2\log(d+ex)}{e^2}\right)\\
&\quad+\frac{1}{2}bfx^2\log\left(c(d+ex)^n\right)\\
&\quad+\frac{bg(d+ex)\log\left(c(d+ex)^n\right)}{e}
\end{aligned}
$$

[In] Integrate[(f + g/x)*x*(a + b*Log[c*(d + e*x)^n]),x]

[Out] a*g*x - b*g*n*x + (a*f*x^2)/2 - (b*f*n*(-((d*x)/e) + x^2/2 + (d^2*Log[d + e*x])/e^2))/2 + (b*f*x^2*Log[c*(d + e*x)^n])/2 + (b*g*(d + e*x)*Log[c*(d + e*x)^n])/e

## Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

| method | result |
|---|---|
| parts | $a\left(\frac{1}{2}f\,x^2+gx\right)+b\Big(g\ln\left(c(ex+d)^n\right)x-gnx+\frac{gnd\ln(ex+d)}{e}+\frac{f\,x^2\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)}{2}-\frac{fn\,x^2}{4}-\frac{n\,d^2f\ln}{2e}$ |
| default | $agx+\frac{af\,x^2}{2}+bg\ln\left(c(ex+d)^n\right)x-bgnx+\frac{bgnd\ln(ex+d)}{e}+\frac{bf\,x^2\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)}{2}-\frac{nbf\,x^2}{4}-\frac{nb\,d^2f\ln(}{2e^2}$ |
| parallelrisch | $-\frac{-2x^2\ln(c(ex+d)^n)b\,e^2f+b\,e^2fn\,x^2+2\ln(ex+d)b\,d^2fn-8\ln(ex+d)bdegn-2a\,e^2f\,x^2-4x\ln(c(ex+d)^n)b\,e^2g-2bdefnx+4b\,e^2}{4e^2}$ |
| risch | $\frac{bx(fx+2g)\ln((ex+d)^n)}{2}-\frac{i\pi bf\,x^2\,\mathrm{csgn}(ic(ex+d)^n)^3}{4}+\frac{i\pi bf\,x^2\,\mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)^2}{4}+\frac{i\pi bgx\,\mathrm{csgn}(ic)\,\mathrm{csgn}(i}{2}$ |

[In] int((f+g/x)*x*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)

[Out] a*(1/2*f*x^2+g*x)+b*(g*ln(c*(e*x+d)^n)*x-g*n*x+g/e*n*d*ln(e*x+d)+1/2*f*x^2*ln(c*exp(n*ln(e*x+d)))-1/4*f*n*x^2-1/2*n*d^2*f/e^2*ln(e*x+d)+1/2*d*f*n/e*x)

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.29

$$\int\left(f+\frac{g}{x}\right)x(a+b\log\left(c(d+ex)^n\right))\,dx=$$
$$-\frac{(be^2fn-2\,ae^2f)x^2-2\,(2\,ae^2g+(bdef-2\,be^2g)n)x-2\,(be^2fnx^2+2\,be^2gnx-(bd^2f-2\,bdeg)n)\log\left(e\right.}{4\,e^2}$$

[In] integrate((f+g/x)*x*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")

[Out] -1/4*((b*e^2*f*n - 2*a*e^2*f)*x^2 - 2*(2*a*e^2*g + (b*d*e*f - 2*b*e^2*g)*n)*x - 2*(b*e^2*f*n*x^2 + 2*b*e^2*g*n*x - (b*d^2*f - 2*b*d*e*g)*n)*log(e*x + d) - 2*(b*e^2*f*x^2 + 2*b*e^2*g*x)*log(c))/e^2

## Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

$$\int \left(f+\frac{g}{x}\right)x(a+b\log\left(c(d+ex)^n\right))\,dx$$
$$=\begin{cases}\frac{afx^2}{2}+agx-\frac{bd^2f\log\left(c(d+ex)^n\right)}{2e^2}+\frac{bdfnx}{2e}+\frac{bdg\log\left(c(d+ex)^n\right)}{e}-\frac{bfnx^2}{4}+\frac{bfx^2\log\left(c(d+ex)^n\right)}{2}-bgnx+bgx\log\left(c(d\right. \\ (a+b\log\left(cd^n\right))\left(\frac{fx^2}{2}+gx\right)\end{cases}$$

```
[In] integrate((f+g/x)*x*(a+b*ln(c*(e*x+d)**n)),x)
```

```
[Out] Piecewise((a*f*x**2/2 + a*g*x - b*d**2*f*log(c*(d + e*x)**n)/(2*e**2) + b*d
*f*n*x/(2*e) + b*d*g*log(c*(d + e*x)**n)/e - b*f*n*x**2/4 + b*f*x**2*log(c*
(d + e*x)**n)/2 - b*g*n*x + b*g*x*log(c*(d + e*x)**n), Ne(e, 0)), ((a + b*l
og(c*d**n))*(f*x**2/2 + g*x), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \left(f+\frac{g}{x}\right)x(a+b\log\left(c(d+ex)^n\right))\,dx &= -begn\left(\frac{x}{e}-\frac{d\log\left(ex+d\right)}{e^2}\right) \\ &\quad -\frac{1}{4}\,befn\left(\frac{2\,d^2\log\left(ex+d\right)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right) \\ &\quad +\frac{1}{2}\,bfx^2\log\left((ex+d)^nc\right)+\frac{1}{2}\,afx^2 \\ &\quad +bgx\log\left((ex+d)^nc\right)+agx\end{aligned}$$

```
[In] integrate((f+g/x)*x*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] -b*e*g*n*(x/e - d*log(e*x + d)/e^2) - 1/4*b*e*f*n*(2*d^2*log(e*x + d)/e^3 +
 (e*x^2 - 2*d*x)/e^2) + 1/2*b*f*x^2*log((e*x + d)^n*c) + 1/2*a*f*x^2 + b*g*
x*log((e*x + d)^n*c) + a*g*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(83) = 166$.

Time = 0.30 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.01

$$\int \left(f+\frac{g}{x}\right)x(a+b\log\left(c(d+ex)^n\right))\,dx = \frac{(ex+d)^2bfn\log(ex+d)}{2\,e^2} - \frac{(ex+d)bdfn\log(ex+d)}{e^2} + \frac{(ex+d)bgn\log(ex+d)}{e} - \frac{(ex+d)^2bfn}{4\,e^2} + \frac{(ex+d)bdfn}{e^2} - \frac{(ex+d)bgn}{e} + \frac{(ex+d)^2bf\log(c)}{2\,e^2} - \frac{(ex+d)bdf\log(c)}{e^2} + \frac{(ex+d)bg\log(c)}{e} + \frac{(ex+d)^2af}{2\,e^2} - \frac{(ex+d)adf}{e^2} + \frac{(ex+d)ag}{e}$$

```
[In] integrate((f+g/x)*x*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] 1/2*(e*x + d)^2*b*f*n*log(e*x + d)/e^2 - (e*x + d)*b*d*f*n*log(e*x + d)/e^2
 + (e*x + d)*b*g*n*log(e*x + d)/e - 1/4*(e*x + d)^2*b*f*n/e^2 + (e*x + d)*b
*d*f*n/e^2 - (e*x + d)*b*g*n/e + 1/2*(e*x + d)^2*b*f*log(c)/e^2 - (e*x + d)
*b*d*f*log(c)/e^2 + (e*x + d)*b*g*log(c)/e + 1/2*(e*x + d)^2*a*f/e^2 - (e*x
 + d)*a*d*f/e^2 + (e*x + d)*a*g/e
```

## Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \left(f+\frac{g}{x}\right)x(a+b\log\left(c(d+ex)^n\right))\,dx = x\left(\frac{2\,a\,d\,f+2\,a\,e\,g-2\,b\,e\,g\,n}{2\,e} - \frac{d\,f\,(2\,a-b\,n)}{2\,e}\right) + \ln\left(c\,(d+e\,x)^n\right)\left(\frac{b\,f\,x^2}{2}+b\,g\,x\right) - \frac{\ln(d+e\,x)\,\left(b\,d^2\,f\,n-2\,b\,d\,e\,g\,n\right)}{2\,e^2} + \frac{f\,x^2\,(2\,a-b\,n)}{4}$$

```
[In] int(x*(f + g/x)*(a + b*log(c*(d + e*x)^n)),x)
```

```
[Out] x*((2*a*d*f + 2*a*e*g - 2*b*e*g*n)/(2*e) - (d*f*(2*a - b*n))/(2*e)) + log(c
*(d + e*x)^n)*(b*g*x + (b*f*x^2)/2) - (log(d + e*x)*(b*d^2*f*n - 2*b*d*e*g*
n))/(2*e^2) + (f*x^2*(2*a - b*n))/4
```

# 3.304 $\int \left(f+\frac{g}{x}\right)^2 x^2 (a+b\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 120

$$\int \left(f+\frac{g}{x}\right)^2 x^2 (a+b\log(c(d+ex)^n))\,dx = -\frac{b(df-eg)^2nx}{3e^2}+\frac{b(df-eg)n(g+fx)^2}{6ef}-\frac{bn(g+fx)^3}{9f}+\frac{b(df-eg)^3n\log(d+ex)}{3e^3f}+\frac{(g+fx)^3(a+b\log(c(d+ex)^n))}{3f}$$

```
[Out] -1/3*b*(d*f-e*g)^2*n*x/e^2+1/6*b*(d*f-e*g)*n*(f*x+g)^2/e/f-1/9*b*n*(f*x+g)^
3/f+1/3*b*(d*f-e*g)^3*n*ln(e*x+d)/e^3/f+1/3*(f*x+g)^3*(a+b*ln(c*(e*x+d)^n))
/f
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2459, 2442, 45}

$$\int \left(f+\frac{g}{x}\right)^2 x^2 (a+b\log(c(d+ex)^n))\,dx = \frac{(fx+g)^3(a+b\log(c(d+ex)^n))}{3f}+\frac{bn(df-eg)^3\log(d+ex)}{3e^3f}-\frac{bnx(df-eg)^2}{3e^2}+\frac{bn(fx+g)^2(df-eg)}{6ef}-\frac{bn(fx+g)^3}{9f}$$

```
[In] Int[(f + g/x)^2*x^2*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] -1/3*(b*(d*f - e*g)^2*n*x)/e^2 + (b*(d*f - e*g)*n*(g + f*x)^2)/(6*e*f) - (b
*n*(g + f*x)^3)/(9*f) + (b*(d*f - e*g)^3*n*Log[d + e*x])/(3*e^3*f) + ((g +
f*x)^3*(a + b*Log[c*(d + e*x)^n]))/(3*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (g+fx)^2 \left(a+b\log\left(c(d+ex)^n\right)\right) dx \\
&= \frac{(g+fx)^3 \left(a+b\log\left(c(d+ex)^n\right)\right)}{3f} - \frac{(ben)\int \frac{(g+fx)^3}{d+ex}\,dx}{3f} \\
&= \frac{(g+fx)^3 \left(a+b\log\left(c(d+ex)^n\right)\right)}{3f} \\
&\quad - \frac{(ben)\int \left(\frac{f(-df+eg)^2}{e^3}+\frac{(-df+eg)^3}{e^3(d+ex)}+\frac{f(-df+eg)(g+fx)}{e^2}+\frac{f(g+fx)^2}{e}\right)dx}{3f} \\
&= -\frac{b(df-eg)^2nx}{3e^2}+\frac{b(df-eg)n(g+fx)^2}{6ef}-\frac{bn(g+fx)^3}{9f} \\
&\quad +\frac{b(df-eg)^3n\log(d+ex)}{3e^3f}+\frac{(g+fx)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3f}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25

$$\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log(c(d+ex)^n))\,dx$$
$$=\frac{6bd^2f(df-3eg)n\log(d+ex)+e(x(6ae^2(3g^2+3fgx+f^2x^2)-bn(6d^2f^2-3def(6g+fx)+e^2(18g^2+}{18e^3}$$

```
[In] Integrate[(f + g/x)^2*x^2*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (6*b*d^2*f*(d*f - 3*e*g)*n*Log[d + e*x] + e*(x*(6*a*e^2*(3*g^2 + 3*f*g*x +
f^2*x^2) - b*n*(6*d^2*f^2 - 3*d*e*f*(6*g + f*x) + e^2*(18*g^2 + 9*f*g*x + 2
*f^2*x^2))) + 6*b*e*(3*d*g^2 + e*x*(3*g^2 + 3*f*g*x + f^2*x^2))*Log[c*(d +
e*x)^n]))/(18*e^3)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(110) = 220.

Time = 0.46 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.21

| method | result |
|---|---|
| parallelrisch | $6x^3\ln(c(ex+d)^n)bd\,e^3f^2-2x^3bd\,e^3f^2n+6x^3ad\,e^3f^2+18x^2\ln(c(ex+d)^n)bd\,e^3fg+3x^2b\,d^2e^2f^2n-9x^2bd\,e^3fgn+6\ln(ex+d)b\,d$ |
| risch | $-\frac{f\ln(ex+d)b\,d^2gn}{e^2}-\frac{fbgn\,x^2}{2}-\frac{f^2b\,d^2nx}{3e^2}+\frac{f^2\ln(ex+d)b\,d^3n}{3e^3}-\frac{if\pi bg\,x^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)}{2}$ |

```
[In] int((f+g/x)^2*x^2*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(6*x^3*ln(c*(e*x+d)^n)*b*d*e^3*f^2-2*x^3*b*d*e^3*f^2*n+6*x^3*a*d*e^3*f
^2+18*x^2*ln(c*(e*x+d)^n)*b*d*e^3*f*g+3*x^2*b*d^2*e^2*f^2*n-9*x^2*b*d*e^3*f
*g*n+6*ln(e*x+d)*b*d^4*f^2*n-18*ln(e*x+d)*b*d^3*e*f*g*n+36*ln(e*x+d)*b*d^2*
e^2*g^2*n+18*x^2*a*d*e^3*f*g+18*x*ln(c*(e*x+d)^n)*b*d*e^3*g^2-6*x*b*d^3*e*f
^2*n+18*x*b*d^2*e^2*f*g*n-18*x*b*d*e^3*g^2*n+18*x*a*d*e^3*g^2-18*ln(c*(e*x+
d)^n)*b*d^2*e^2*g^2)/d/e^3
```

## Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.82

$$\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log(c(d+ex)^n))\,dx = -\frac{2\,(be^3f^2n-3\,ae^3f^2)x^3-3\,(6\,ae^3fg+(bde^2f^2-3\,be^3fg)n)x^2-6\,(3\,ae^3g^2-(bd^2ef^2-3\,bde^2fg+3\,be^3}{}$$

```
[In] integrate((f+g/x)^2*x^2*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

```
[Out] -1/18*(2*(b*e^3*f^2*n - 3*a*e^3*f^2)*x^3 - 3*(6*a*e^3*f*g + (b*d*e^2*f^2 -
3*b*e^3*f*g)*n)*x^2 - 6*(3*a*e^3*g^2 - (b*d^2*e*f^2 - 3*b*d*e^2*f*g + 3*b*e
^3*g^2)*n)*x - 6*(b*e^3*f^2*n*x^3 + 3*b*e^3*f*g*n*x^2 + 3*b*e^3*g^2*n*x + (
b*d^3*f^2 - 3*b*d^2*e*f*g + 3*b*d*e^2*g^2)*n)*log(e*x + d) - 6*(b*e^3*f^2*x
^3 + 3*b*e^3*f*g*x^2 + 3*b*e^3*g^2*x)*log(c))/e^3
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(102) = 204$.

Time = 4.50 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.10

$$\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log(c(d+ex)^n))\,dx = \begin{cases} \frac{af^2x^3}{3}+afgx^2+ag^2x+\frac{bd^3f^2\log(c(d+ex)^n)}{3e^3}-\frac{bd^2f^2nx}{3e^2}-\frac{bd^2fg\log(c(d+ex)^n)}{e^2}+\frac{bdf^2nx^2}{6e}+\frac{bdfgnx}{e}+\frac{bdg^2\log(c(d+ex)^n)}{e} \\ (a+b\log(cd^n))\left(\frac{f^2x^3}{3}+fgx^2+g^2x\right) \end{cases}$$

```
[In] integrate((f+g/x)**2*x**2*(a+b*ln(c*(e*x+d)**n)),x)
```

```
[Out] Piecewise((a*f**2*x**3/3 + a*f*g*x**2 + a*g**2*x + b*d**3*f**2*log(c*(d + e
*x)**n)/(3*e**3) - b*d**2*f**2*n*x/(3*e**2) - b*d**2*f*g*log(c*(d + e*x)**n
)/e**2 + b*d*f**2*n*x**2/(6*e) + b*d*f*g*n*x/e + b*d*g**2*log(c*(d + e*x)**
n)/e - b*f**2*n*x**3/9 + b*f**2*x**3*log(c*(d + e*x)**n)/3 - b*f*g*n*x**2/2
 + b*f*g*x**2*log(c*(d + e*x)**n) - b*g**2*n*x + b*g**2*x*log(c*(d + e*x)**
n), Ne(e, 0)), ((a + b*log(c*d**n))*(f**2*x**3/3 + f*g*x**2 + g**2*x), True
))
```

## Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.56

$$\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log(c(d+ex)^n))\,dx$$
$$=\frac{1}{3}bf^2x^3\log\left((ex+d)^nc\right)+\frac{1}{3}af^2x^3-beg^2n\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)$$
$$+\frac{1}{18}bef^2n\left(\frac{6\,d^3\log(ex+d)}{e^4}-\frac{2\,e^2x^3-3\,dex^2+6\,d^2x}{e^3}\right)$$
$$-\frac{1}{2}befgn\left(\frac{2\,d^2\log(ex+d)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right)$$
$$+bfgx^2\log\left((ex+d)^nc\right)+afgx^2+bg^2x\log\left((ex+d)^nc\right)+ag^2x$$

```
[In] integrate((f+g/x)^2*x^2*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*f^2*x^3*log((e*x + d)^n*c) + 1/3*a*f^2*x^3 - b*e*g^2*n*(x/e - d*log(e
*x + d)/e^2) + 1/18*b*e*f^2*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*
x^2 + 6*d^2*x)/e^3) - 1/2*b*e*f*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*
x)/e^2) + b*f*g*x^2*log((e*x + d)^n*c) + a*f*g*x^2 + b*g^2*x*log((e*x + d)^
n*c) + a*g^2*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(110) = 220.

Time = 0.31 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.53

$$\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log(c(d+ex)^n))\,dx$$
$$=\frac{(ex+d)^3bf^2n\log(ex+d)}{3\,e^3}-\frac{(ex+d)^2bdf^2n\log(ex+d)}{e^3}+\frac{(ex+d)bd^2f^2n\log(ex+d)}{e^3}$$
$$+\frac{(ex+d)^2bfgn\log(ex+d)}{e^2}-\frac{2\,(ex+d)bdfgn\log(ex+d)}{e^2}+\frac{(ex+d)bg^2n\log(ex+d)}{e}$$
$$-\frac{(ex+d)^3bf^2n}{9\,e^3}+\frac{(ex+d)^2bdf^2n}{2\,e^3}-\frac{(ex+d)bd^2f^2n}{e^3}-\frac{(ex+d)^2bfgn}{2\,e^2}+\frac{2\,(ex+d)bdfgn}{e^2}$$
$$-\frac{(ex+d)bg^2n}{e}+\frac{(ex+d)^3bf^2\log(c)}{3\,e^3}-\frac{(ex+d)^2bdf^2\log(c)}{e^3}+\frac{(ex+d)bd^2f^2\log(c)}{e^3}$$
$$+\frac{(ex+d)^2bfg\log(c)}{e^2}-\frac{2\,(ex+d)bdfg\log(c)}{e^2}+\frac{(ex+d)bg^2\log(c)}{e}+\frac{(ex+d)^3af^2}{3\,e^3}$$
$$-\frac{(ex+d)^2adf^2}{e^3}+\frac{(ex+d)ad^2f^2}{e^3}+\frac{(ex+d)^2afg}{e^2}-\frac{2\,(ex+d)adfg}{e^2}+\frac{(ex+d)ag^2}{e}$$

```
[In] integrate((f+g/x)^2*x^2*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] 1/3*(e*x + d)^3*b*f^2*n*log(e*x + d)/e^3 - (e*x + d)^2*b*d*f^2*n*log(e*x +
d)/e^3 + (e*x + d)*b*d^2*f^2*n*log(e*x + d)/e^3 + (e*x + d)^2*b*f*g*n*log(e
*x + d)/e^2 - 2*(e*x + d)*b*d*f*g*n*log(e*x + d)/e^2 + (e*x + d)*b*g^2*n*lo
g(e*x + d)/e - 1/9*(e*x + d)^3*b*f^2*n/e^3 + 1/2*(e*x + d)^2*b*d*f^2*n/e^3
- (e*x + d)*b*d^2*f^2*n/e^3 - 1/2*(e*x + d)^2*b*f*g*n/e^2 + 2*(e*x + d)*b*d
*f*g*n/e^2 - (e*x + d)*b*g^2*n/e + 1/3*(e*x + d)^3*b*f^2*log(c)/e^3 - (e*x
+ d)^2*b*d*f^2*log(c)/e^3 + (e*x + d)*b*d^2*f^2*log(c)/e^3 + (e*x + d)^2*b*
f*g*log(c)/e^2 - 2*(e*x + d)*b*d*f*g*log(c)/e^2 + (e*x + d)*b*g^2*log(c)/e
+ 1/3*(e*x + d)^3*a*f^2/e^3 - (e*x + d)^2*a*d*f^2/e^3 + (e*x + d)*a*d^2*f^2
/e^3 + (e*x + d)^2*a*f*g/e^2 - 2*(e*x + d)*a*d*f*g/e^2 + (e*x + d)*a*g^2/e
```

## Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.77

$$\int \left(f+\frac{g}{x}\right)^2 x^2(a+b\log(c(d+ex)^n))\,dx = x^2\left(\frac{f\,(a\,d\,f+2\,a\,e\,g-b\,e\,g\,n)}{2\,e}-\frac{d\,f^2\,(3\,a-b\,n)}{6\,e}\right) + x\left(\frac{3\,a\,e\,g^2-3\,b\,e\,g^2\,n+6\,a\,d\,f\,g}{3\,e}-\frac{d\left(\frac{f\,(a\,d\,f+2\,a\,e\,g-b\,e\,g\,n)}{e}-\frac{d\,f^2\,(3\,a-b\,n)}{3\,e}\right)}{e}\right) + \ln\left(c\,(d+e\,x)^n\right)\,\left(\frac{b\,f^2\,x^3}{3}+b\,f\,g\,x^2+b\,g^2\,x\right)+\frac{f^2\,x^3\,(3\,a-b\,n)}{9} + \frac{\ln\left(d+e\,x\right)\,\left(b\,n\,d^3\,f^2-3\,b\,n\,d^2\,e\,f\,g+3\,b\,n\,d\,e^2\,g^2\right)}{3\,e^3}$$

```
[In] int(x^2*(f + g/x)^2*(a + b*log(c*(d + e*x)^n)),x)
```

```
[Out] x^2*((f*(a*d*f + 2*a*e*g - b*e*g*n))/(2*e) - (d*f^2*(3*a - b*n))/(6*e)) + x
*((3*a*e*g^2 - 3*b*e*g^2*n + 6*a*d*f*g)/(3*e) - (d*((f*(a*d*f + 2*a*e*g - b
*e*g*n))/e - (d*f^2*(3*a - b*n))/(3*e)))/e) + log(c*(d + e*x)^n)*((b*f^2*x^
3)/3 + b*g^2*x + b*f*g*x^2) + (f^2*x^3*(3*a - b*n))/9 + (log(d + e*x)*(b*d^
3*f^2*n + 3*b*d*e^2*g^2*n - 3*b*d^2*e*f*g*n))/(3*e^3)
```

# 3.305 $\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 149

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log(c(d+ex)^n))\,dx = \frac{b(df-eg)^3nx}{4e^3}-\frac{b(df-eg)^2n(g+fx)^2}{8e^2f}+\frac{b(df-eg)n(g+fx)^3}{12ef}-\frac{bn(g+fx)^4}{16f}-\frac{b(df-eg)^4n\log(d+ex)}{4e^4f}+\frac{(g+fx)^4(a+b\log(c(d+ex)^n))}{4f}$$

```
[Out] 1/4*b*(d*f-e*g)^3*n*x/e^3-1/8*b*(d*f-e*g)^2*n*(f*x+g)^2/e^2/f+1/12*b*(d*f-e
*g)*n*(f*x+g)^3/e/f-1/16*b*n*(f*x+g)^4/f-1/4*b*(d*f-e*g)^4*n*ln(e*x+d)/e^4/
f+1/4*(f*x+g)^4*(a+b*ln(c*(e*x+d)^n))/f
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2459, 2442, 45}

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log(c(d+ex)^n))\,dx = \frac{(fx+g)^4(a+b\log(c(d+ex)^n))}{4f}-\frac{bn(df-eg)^4\log(d+ex)}{4e^4f}+\frac{bnx(df-eg)^3}{4e^3}-\frac{bn(fx+g)^2(df-eg)^2}{8e^2f}+\frac{bn(fx+g)^3(df-eg)}{12ef}-\frac{bn(fx+g)^4}{16f}$$

```
[In] Int[(f + g/x)^3*x^3*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (b*(d*f - e*g)^3*n*x)/(4*e^3) - (b*(d*f - e*g)^2*n*(g + f*x)^2)/(8*e^2*f) +
 (b*(d*f - e*g)*n*(g + f*x)^3)/(12*e*f) - (b*n*(g + f*x)^4)/(16*f) - (b*(d*
f - e*g)^4*n*Log[d + e*x])/(4*e^4*f) + ((g + f*x)^4*(a + b*Log[c*(d + e*x)^
n]))/(4*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (g+fx)^3 \left(a+b\log\left(c(d+ex)^n\right)\right) \, dx \\
&= \frac{(g+fx)^4 \left(a+b\log\left(c(d+ex)^n\right)\right)}{4f} - \frac{(ben)\int \frac{(g+fx)^4}{d+ex} \, dx}{4f} \\
&= \frac{(g+fx)^4 \left(a+b\log\left(c(d+ex)^n\right)\right)}{4f} \\
&\quad - \frac{(ben)\int \left(\frac{f(-df+eg)^3}{e^4} + \frac{(-df+eg)^4}{e^4(d+ex)} + \frac{f(-df+eg)^2(g+fx)}{e^3} + \frac{f(-df+eg)(g+fx)^2}{e^2} + \frac{f(g+fx)^3}{e}\right) dx}{4f} \\
&= \frac{b(df-eg)^3nx}{4e^3} - \frac{b(df-eg)^2n(g+fx)^2}{8e^2f} + \frac{b(df-eg)n(g+fx)^3}{12ef} \\
&\quad - \frac{bn(g+fx)^4}{16f} - \frac{b(df-eg)^4n\log(d+ex)}{4e^4f} + \frac{(g+fx)^4 \left(a+b\log\left(c(d+ex)^n\right)\right)}{4f}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.52

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log\left(c(d+ex)^n\right))\,dx$$
$$=\frac{ex(12ae^3(4g^3+6fg^2x+4f^2gx^2+f^3x^3)+bn(12d^3f^3-6d^2ef^2(8g+fx)+4de^2f(18g^2+6fgx+f^2x^2)}{}$$

```
[In] Integrate[(f + g/x)^3*x^3*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (e*x*(12*a*e^3*(4*g^3 + 6*f*g^2*x + 4*f^2*g*x^2 + f^3*x^3) + b*n*(12*d^3*f^
3 - 6*d^2*e*f^2*(8*g + f*x) + 4*d*e^2*f*(18*g^2 + 6*f*g*x + f^2*x^2) - e^3*
(48*g^3 + 36*f*g^2*x + 16*f^2*g*x^2 + 3*f^3*x^3))) - 12*b*d^2*f*(d^2*f^2 -
4*d*e*f*g + 6*e^2*g^2)*n*Log[d + e*x] + 12*b*e^3*(4*d*g^3 + e*x*(4*g^3 + 6*
f*g^2*x + 4*f^2*g*x^2 + f^3*x^3))*Log[c*(d + e*x)^n])/(48*e^4)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(137) = 274.

Time = 0.72 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.87

| method | result |
|---|---|
| parallelrisch | $-\frac{-48bd\,e^3g^3n+12b\,d^4f^3n+36x^2b\,e^4f\,g^2n-48x^3\ln(c(ex+d)^n)b\,e^4f^2g-4x^3bd\,e^3f^3n+16x^3b\,e^4f^2gn+6x^2b\,d^2e^2f^3n-12xb\,d^3}{}$ |
| risch | Expression too large to display |

```
[In] int((f+g/x)^3*x^3*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(-48*b*d*e^3*g^3*n+12*b*d^4*f^3*n+36*x^2*b*e^4*f*g^2*n-48*x^3*ln(c*(e
*x+d)^n)*b*e^4*f^2*g-4*x^3*b*d*e^3*f^3*n+16*x^3*b*e^4*f^2*g*n+6*x^2*b*d^2*e
^2*f^3*n-12*x*b*d^3*e*f^3*n-96*ln(e*x+d)*b*d*e^3*g^3*n+72*b*d^2*e^2*f*g^2*n
-72*x^2*a*e^4*f*g^2-12*x^4*a*e^4*f^3-48*x*a*e^4*g^3-48*b*d^3*e*f^2*g*n+48*a
*d*g^3*e^3-24*x^2*b*d*e^3*f^2*g*n+48*x*b*d^2*e^2*f^2*g*n-48*ln(e*x+d)*b*d^3
*e*f^2*g*n+72*ln(e*x+d)*b*d^2*e^2*f*g^2*n-72*x^2*ln(c*(e*x+d)^n)*b*e^4*f*g^
2-72*x*b*d*e^3*f*g^2*n-48*x^3*a*e^4*f^2*g+12*ln(e*x+d)*b*d^4*f^3*n-12*x^4*l
n(c*(e*x+d)^n)*b*e^4*f^3+48*ln(c*(e*x+d)^n)*b*d*e^3*g^3+48*x*b*e^4*g^3*n+3*
x^4*b*e^4*f^3*n-48*x*ln(c*(e*x+d)^n)*b*e^4*g^3)/e^4
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(137) = 274.

Time = 0.31 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.26

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log\left(c(d+ex)^n\right))\,dx = -\frac{3\,(be^4f^3n-4\,ae^4f^3)x^4-4\,(12\,ae^4f^2g+(bde^3f^3-4\,be^4f^2g)n)x^3-6\,(12\,ae^4fg^2-(bd^2e^2f^3-4\,bde^3f^2g}{}$$

```
[In] integrate((f+g/x)^3*x^3*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

```
[Out] -1/48*(3*(b*e^4*f^3*n - 4*a*e^4*f^3)*x^4 - 4*(12*a*e^4*f^2*g + (b*d*e^3*f^3
 - 4*b*e^4*f^2*g)*n)*x^3 - 6*(12*a*e^4*f*g^2 - (b*d^2*e^2*f^3 - 4*b*d*e^3*f
^2*g + 6*b*e^4*f*g^2)*n)*x^2 - 12*(4*a*e^4*g^3 + (b*d^3*e*f^3 - 4*b*d^2*e^2
*f^2*g + 6*b*d*e^3*f*g^2 - 4*b*e^4*g^3)*n)*x - 12*(b*e^4*f^3*n*x^4 + 4*b*e^
4*f^2*g*n*x^3 + 6*b*e^4*f*g^2*n*x^2 + 4*b*e^4*g^3*n*x - (b*d^4*f^3 - 4*b*d^
3*e*f^2*g + 6*b*d^2*e^2*f*g^2 - 4*b*d*e^3*g^3)*n)*log(e*x + d) - 12*(b*e^4*
f^3*x^4 + 4*b*e^4*f^2*g*x^3 + 6*b*e^4*f*g^2*x^2 + 4*b*e^4*g^3*x)*log(c))/e^
4
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(128) = 256.

Time = 19.60 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.75

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log\left(c(d+ex)^n\right))\,dx = \begin{cases} \frac{af^3x^4}{4}+af^2gx^3+\frac{3afg^2x^2}{2}+ag^3x-\frac{bd^4f^3\log\left(c(d+ex)^n\right)}{4e^4}+\frac{bd^3f^3nx}{4e^3}+\frac{bd^3f^2g\log\left(c(d+ex)^n\right)}{e^3}-\frac{bd^2f^3nx^2}{8e^2}-\frac{bd^2f^2gnx}{e^2}- \\ (a+b\log\left(cd^n\right))\left(\frac{f^3x^4}{4}+f^2gx^3+\frac{3fg^2x^2}{2}+g^3x\right) \end{cases}$$

```
[In] integrate((f+g/x)**3*x**3*(a+b*ln(c*(e*x+d)**n)),x)
```

```
[Out] Piecewise((a*f**3*x**4/4 + a*f**2*g*x**3 + 3*a*f*g**2*x**2/2 + a*g**3*x - b
*d**4*f**3*log(c*(d + e*x)**n)/(4*e**4) + b*d**3*f**3*n*x/(4*e**3) + b*d**3
*f**2*g*log(c*(d + e*x)**n)/e**3 - b*d**2*f**3*n*x**2/(8*e**2) - b*d**2*f**
2*g*n*x/e**2 - 3*b*d**2*f*g**2*log(c*(d + e*x)**n)/(2*e**2) + b*d*f**3*n*x*
*3/(12*e) + b*d*f**2*g*n*x**2/(2*e) + 3*b*d*f*g**2*n*x/(2*e) + b*d*g**3*log
(c*(d + e*x)**n)/e - b*f**3*n*x**4/16 + b*f**3*x**4*log(c*(d + e*x)**n)/4 -
 b*f**2*g*n*x**3/3 + b*f**2*g*x**3*log(c*(d + e*x)**n) - 3*b*f*g**2*n*x**2/
4 + 3*b*f*g**2*x**2*log(c*(d + e*x)**n)/2 - b*g**3*n*x + b*g**3*x*log(c*(d
+ e*x)**n), Ne(e, 0)), ((a + b*log(c*d**n))*(f**3*x**4/4 + f**2*g*x**3 + 3*
f*g**2*x**2/2 + g**3*x), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(137) = 274$.

Time = 0.21 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.91

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log\left(c(d+ex)^n\right))\,dx$$
$$= \frac{1}{4}\,bf^3x^4\log\left((ex+d)^n c\right) + \frac{1}{4}\,af^3x^4 + bf^2gx^3\log\left((ex+d)^n c\right) + af^2gx^3 - beg^3n\left(\frac{x}{e} - \frac{d\log\left(ex+d\right)}{e^2}\right) - \frac{1}{48}\,bef^3n\left(\frac{12\,d^4\log\left(ex+d\right)}{e^5} + \frac{3\,e^3x^4 - 4\,de^2x^3 + 6\,d^2ex^2 - 12\,d^3x}{e^4}\right) + \frac{1}{6}\,bef^2gn\left(\frac{6\,d^3\log\left(ex+d\right)}{e^4} - \frac{2\,e^2x^3 - 3\,dex^2 + 6\,d^2x}{e^3}\right) - \frac{3}{4}\,befg^2n\left(\frac{2\,d^2\log\left(ex+d\right)}{e^3} + \frac{ex^2 - 2\,dx}{e^2}\right) + \frac{3}{2}\,bfg^2x^2\log\left((ex+d)^n c\right) + \frac{3}{2}\,afg^2x^2 + bg^3x\log\left((ex+d)^n c\right) + ag^3x$$

```
[In] integrate((f+g/x)^3*x^3*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*f^3*x^4*log((e*x + d)^n*c) + 1/4*a*f^3*x^4 + b*f^2*g*x^3*log((e*x + d
)^n*c) + a*f^2*g*x^3 - b*e*g^3*n*(x/e - d*log(e*x + d)/e^2) - 1/48*b*e*f^3*
n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e^2*x^3 + 6*d^2*e*x^2 - 12*d^
3*x)/e^4) + 1/6*b*e*f^2*g*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^
2 + 6*d^2*x)/e^3) - 3/4*b*e*f*g^2*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*
x)/e^2) + 3/2*b*f*g^2*x^2*log((e*x + d)^n*c) + 3/2*a*f*g^2*x^2 + b*g^3*x*lo
g((e*x + d)^n*c) + a*g^3*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(137) = 274$.

Time = 0.33 (sec) , antiderivative size = 770, normalized size of antiderivative = 5.17

$$
\begin{aligned}
\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log(c(d+ex)^n))\,dx = {} & \frac{(ex+d)^4 bf^3 n\log(ex+d)}{4e^4} \\
& - \frac{(ex+d)^3 bdf^3 n\log(ex+d)}{e^4} \\
& + \frac{3(ex+d)^2 bd^2 f^3 n\log(ex+d)}{2e^4} \\
& - \frac{(ex+d)bd^3 f^3 n\log(ex+d)}{e^4} \\
& + \frac{(ex+d)^3 bf^2 gn\log(ex+d)}{e^3} \\
& - \frac{3(ex+d)^2 bdf^2 gn\log(ex+d)}{e^3} \\
& + \frac{3(ex+d)bd^2 f^2 gn\log(ex+d)}{e^3} \\
& + \frac{3(ex+d)^2 bfg^2 n\log(ex+d)}{2e^2} \\
& - \frac{3(ex+d)bdfg^2 n\log(ex+d)}{e^2} \\
& + \frac{(ex+d)bg^3 n\log(ex+d)}{e} \\
& - \frac{(ex+d)^4 bf^3 n}{16e^4} + \frac{(ex+d)^3 bdf^3 n}{3e^4} \\
& - \frac{3(ex+d)^2 bd^2 f^3 n}{4e^4} + \frac{(ex+d)bd^3 f^3 n}{e^4} \\
& - \frac{(ex+d)^3 bf^2 gn}{3e^3} + \frac{3(ex+d)^2 bdf^2 gn}{2e^3} \\
& - \frac{3(ex+d)bd^2 f^2 gn}{e^3} - \frac{3(ex+d)^2 bfg^2 n}{4e^2} \\
& + \frac{3(ex+d)bdfg^2 n}{e^2} - \frac{(ex+d)bg^3 n}{e} \\
& + \frac{(ex+d)^4 bf^3\log(c)}{4e^4} - \frac{(ex+d)^3 bdf^3\log(c)}{e^4} \\
& + \frac{3(ex+d)^2 bd^2 f^3\log(c)}{2e^4} \\
& - \frac{(ex+d)bd^3 f^3\log(c)}{e^4} + \frac{(ex+d)^3 bf^2 g\log(c)}{e^3} \\
& - \frac{3(ex+d)^2 bdf^2 g\log(c)}{e^3} \\
& + \frac{3(ex+d)bd^2 f^2 g\log(c)}{e^3} \\
& + \frac{3(ex+d)^2 bfg^2\log(c)}{2e^2} \\
& - \frac{3(ex+d)bdfg^2\log(c)}{e^2} + \frac{(ex+d)bg^3\log(c)}{e} \\
& + \frac{(ex+d)^4 af^3}{4e^4} - \frac{(ex+d)^3 adf^3}{e^4} \\
& + \frac{3(ex+d)^2 ad^2 f^3}{\ldots} - \frac{(ex+d)ad^3 f^3}{\ldots}
\end{aligned}
$$

```
[In] integrate((f+g/x)^3*x^3*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] 1/4*(e*x + d)^4*b*f^3*n*log(e*x + d)/e^4 - (e*x + d)^3*b*d*f^3*n*log(e*x +
d)/e^4 + 3/2*(e*x + d)^2*b*d^2*f^3*n*log(e*x + d)/e^4 - (e*x + d)*b*d^3*f^3
*n*log(e*x + d)/e^4 + (e*x + d)^3*b*f^2*g*n*log(e*x + d)/e^3 - 3*(e*x + d)^
2*b*d*f^2*g*n*log(e*x + d)/e^3 + 3*(e*x + d)*b*d^2*f^2*g*n*log(e*x + d)/e^3
 + 3/2*(e*x + d)^2*b*f*g^2*n*log(e*x + d)/e^2 - 3*(e*x + d)*b*d*f*g^2*n*log
(e*x + d)/e^2 + (e*x + d)*b*g^3*n*log(e*x + d)/e - 1/16*(e*x + d)^4*b*f^3*n
/e^4 + 1/3*(e*x + d)^3*b*d*f^3*n/e^4 - 3/4*(e*x + d)^2*b*d^2*f^3*n/e^4 + (e
*x + d)*b*d^3*f^3*n/e^4 - 1/3*(e*x + d)^3*b*f^2*g*n/e^3 + 3/2*(e*x + d)^2*b
*d*f^2*g*n/e^3 - 3*(e*x + d)*b*d^2*f^2*g*n/e^3 - 3/4*(e*x + d)^2*b*f*g^2*n/
e^2 + 3*(e*x + d)*b*d*f*g^2*n/e^2 - (e*x + d)*b*g^3*n/e + 1/4*(e*x + d)^4*b
*f^3*log(c)/e^4 - (e*x + d)^3*b*d*f^3*log(c)/e^4 + 3/2*(e*x + d)^2*b*d^2*f^
3*log(c)/e^4 - (e*x + d)*b*d^3*f^3*log(c)/e^4 + (e*x + d)^3*b*f^2*g*log(c)/
e^3 - 3*(e*x + d)^2*b*d*f^2*g*log(c)/e^3 + 3*(e*x + d)*b*d^2*f^2*g*log(c)/e
^3 + 3/2*(e*x + d)^2*b*f*g^2*log(c)/e^2 - 3*(e*x + d)*b*d*f*g^2*log(c)/e^2
+ (e*x + d)*b*g^3*log(c)/e + 1/4*(e*x + d)^4*a*f^3/e^4 - (e*x + d)^3*a*d*f^
3/e^4 + 3/2*(e*x + d)^2*a*d^2*f^3/e^4 - (e*x + d)*a*d^3*f^3/e^4 + (e*x + d)
^3*a*f^2*g/e^3 - 3*(e*x + d)^2*a*d*f^2*g/e^3 + 3*(e*x + d)*a*d^2*f^2*g/e^3
+ 3/2*(e*x + d)^2*a*f*g^2/e^2 - 3*(e*x + d)*a*d*f*g^2/e^2 + (e*x + d)*a*g^3
/e
```

## Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.36

$$\int \left(f+\frac{g}{x}\right)^3 x^3(a+b\log\left(c(d+ex)^n\right))\,dx$$
$$=x\left(\frac{4\,a\,e\,g^3+12\,a\,d\,f\,g^2-4\,b\,e\,g^3\,n}{4\,e}+\frac{d\left(\frac{d\left(\frac{f^2\,(a\,d\,f+3\,a\,e\,g-b\,e\,g\,n)}{e}-\frac{d\,f^3\,(4\,a-b\,n)}{4\,e}\right)}{e}-\frac{3\,f\,g\,(2\,a\,d\,f+2\,a\,e\,g-b\,e\,g\,n)}{2\,e}\right)}{e}\right)$$
$$+x^3\left(\frac{f^2\,(a\,d\,f+3\,a\,e\,g-b\,e\,g\,n)}{3\,e}-\frac{d\,f^3\,(4\,a-b\,n)}{12\,e}\right)$$
$$+\ln\left(c\,(d+e\,x)^n\right)\,\left(\frac{b\,f^3\,x^4}{4}+b\,f^2\,g\,x^3+\frac{3\,b\,f\,g^2\,x^2}{2}+b\,g^3\,x\right)$$
$$-x^2\left(\frac{d\left(\frac{f^2\,(a\,d\,f+3\,a\,e\,g-b\,e\,g\,n)}{e}-\frac{d\,f^3\,(4\,a-b\,n)}{4\,e}\right)}{2\,e}-\frac{3\,f\,g\,(2\,a\,d\,f+2\,a\,e\,g-b\,e\,g\,n)}{4\,e}\right)$$
$$-\frac{\ln\left(d+e\,x\right)\,\left(b\,n\,d^4\,f^3-4\,b\,n\,d^3\,e\,f^2\,g+6\,b\,n\,d^2\,e^2\,f\,g^2-4\,b\,n\,d\,e^3\,g^3\right)}{4\,e^4}$$
$$+\frac{f^3\,x^4\,(4\,a-b\,n)}{16}$$

```
[In] int(x^3*(f + g/x)^3*(a + b*log(c*(d + e*x)^n)),x)
```

```
[Out] x*((4*a*e*g^3 + 12*a*d*f*g^2 - 4*b*e*g^3*n)/(4*e) + (d*((d*((f^2*(a*d*f + 3
*a*e*g - b*e*g*n))/e - (d*f^3*(4*a - b*n))/(4*e)))/e - (3*f*g*(2*a*d*f + 2*
a*e*g - b*e*g*n))/(2*e)))/e) + x^3*((f^2*(a*d*f + 3*a*e*g - b*e*g*n))/(3*e)
 - (d*f^3*(4*a - b*n))/(12*e)) + log(c*(d + e*x)^n)*((b*f^3*x^4)/4 + b*g^3*
x + (3*b*f*g^2*x^2)/2 + b*f^2*g*x^3) - x^2*((d*((f^2*(a*d*f + 3*a*e*g - b*e
*g*n))/e - (d*f^3*(4*a - b*n))/(4*e)))/(2*e) - (3*f*g*(2*a*d*f + 2*a*e*g -
b*e*g*n))/(4*e)) - (log(d + e*x)*(b*d^4*f^3*n - 4*b*d*e^3*g^3*n - 4*b*d^3*e
*f^2*g*n + 6*b*d^2*e^2*f*g^2*n))/(4*e^4) + (f^3*x^4*(4*a - b*n))/16
```

# 3.306 $\int \frac{a+b\log(c(d+ex)^n)}{(f+\frac{g}{x})x}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 63

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)x}\,dx = \frac{(a+b\log\left(c(d+ex)^n\right))\log\left(-\frac{e(g+fx)}{df-eg}\right)}{f}+\frac{bn\,\mathrm{PolyLog}\left(2,\frac{f(d+ex)}{df-eg}\right)}{f}$$

```
[Out] (a+b*ln(c*(e*x+d)^n))*ln(-e*(f*x+g)/(d*f-e*g))/f+b*n*polylog(2,f*(e*x+d)/(d
*f-e*g))/f
```

## Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2459, 2441, 2440, 2438}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)x}\,dx = \frac{\log\left(-\frac{e(fx+g)}{df-eg}\right)(a+b\log\left(c(d+ex)^n\right))}{f}+\frac{bn\,\mathrm{PolyLog}\left(2,\frac{f(d+ex)}{df-eg}\right)}{f}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/((f + g/x)*x),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])*Log[-((e*(g + f*x))/(d*f - e*g))])/f + (b*n*Pol
yLog[2, (f*(d + e*x))/(d*f - e*g)])/f
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{a+b\log\left(c(d+ex)^n\right)}{g+fx}\,dx\\
&= \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(-\frac{e(g+fx)}{df-eg}\right)}{f}-\frac{(ben)\int \frac{\log\left(\frac{e(g+fx)}{-df+eg}\right)}{d+ex}\,dx}{f}\\
&= \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(-\frac{e(g+fx)}{df-eg}\right)}{f}-\frac{(bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{fx}{-df+eg}\right)}{x}\,dx,x,d+ex\right)}{f}\\
&= \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(-\frac{e(g+fx)}{df-eg}\right)}{f}+\frac{bn\text{Li}_2\left(\frac{f(d+ex)}{df-eg}\right)}{f}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)x}\,dx=\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(g+fx)}{-df+eg}\right)}{f}+\frac{bn\,\mathrm{PolyLog}\left(2,\frac{f(d+ex)}{df-eg}\right)}{f}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/((f + g/x)*x),x]
[Out] ((a + b*Log[c*(d + e*x)^n])*Log[(e*(g + f*x))/(-(d*f) + e*g)])/f + (b*n*Pol
yLog[2, (f*(d + e*x))/(d*f - e*g)])/f
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.44

| method | result |
|---|---|
| risch | $\frac{b\ln((ex+d)^n)\ln(fx+g)}{f}-\frac{bn\,\mathrm{dilog}\left(\frac{(fx+g)e+df-eg}{df-eg}\right)}{f}-\frac{bn\ln(fx+g)\ln\left(\frac{(fx+g)e+df-eg}{df-eg}\right)}{f}+\frac{\left(-\frac{ib\pi\,\mathrm{csgn}(ic(ex+d)^n)\,\mathrm{csgn}(ic)\,\mathrm{csgn}(i(}{2}\right.}{}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(f+g/x)/x,x,method=_RETURNVERBOSE)
[Out] b*ln((e*x+d)^n)*ln(f*x+g)/f-b/f*n*dilog(((f*x+g)*e+d*f-e*g)/(d*f-e*g))-b/f*
n*ln(f*x+g)*ln(((f*x+g)*e+d*f-e*g)/(d*f-e*g))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I
*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+
1/2*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/2*I*b*Pi*csgn(I*c*(e*x
+d)^n)^3+b*ln(c)+a)*ln(f*x+g)/f
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)x}\,dx=\int \frac{b\log\left((ex+d)^nc\right)+a}{\left(f+\frac{g}{x}\right)x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)/x,x, algorithm="fricas")
[Out] integral((b*log((e*x + d)^n*c) + a)/(f*x + g), x)
```

## Sympy [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{\left(f + \frac{g}{x}\right)x}\,dx = \int \frac{a + b\log\left(c(d+ex)^n\right)}{fx+g}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(f+g/x)/x,x)
[Out] Integral((a + b*log(c*(d + e*x)**n))/(f*x + g), x)
```

## Maxima [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{\left(f + \frac{g}{x}\right)x}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{\left(f + \frac{g}{x}\right)x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)/x,x, algorithm="maxima")
[Out] b*integrate((log((e*x + d)^n) + log(c))/(f*x + g), x) + a*log(f*x + g)/f
```

## Giac [F]

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{\left(f + \frac{g}{x}\right)x}\,dx = \int \frac{b\log\left((ex+d)^n c\right) + a}{\left(f + \frac{g}{x}\right)x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)/x,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)/((f + g/x)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{\left(f + \frac{g}{x}\right)x}\,dx = \int \frac{a + b\,\ln\left(c\,(d+e\,x)^n\right)}{x\,\left(f + \frac{g}{x}\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x*(f + g/x)),x)
[Out] int((a + b*log(c*(d + e*x)^n))/(x*(f + g/x)), x)
```

# 3.307 $\int \frac{a+b\log(c(d+ex)^n)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 74

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx=-\frac{ben\log(d+ex)}{f(df-eg)}-\frac{a+b\log\left(c(d+ex)^n\right)}{f(g+fx)}+\frac{ben\log(g+fx)}{f(df-eg)}$$

```
[Out] -b*e*n*ln(e*x+d)/f/(d*f-e*g)+(-a-b*ln(c*(e*x+d)^n))/f/(f*x+g)+b*e*n*ln(f*x+
g)/f/(d*f-e*g)
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2459, 2442, 36, 31}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx=-\frac{a+b\log\left(c(d+ex)^n\right)}{f(fx+g)}-\frac{ben\log(d+ex)}{f(df-eg)}+\frac{ben\log(fx+g)}{f(df-eg)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/((f + g/x)^2*x^2),x]
```

```
[Out] -((b*e*n*Log[d + e*x])/(f*(d*f - e*g))) - (a + b*Log[c*(d + e*x)^n])/(f*(g
+ f*x)) + (b*e*n*Log[g + f*x])/(f*(d*f - e*g))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{a+b\log\left(c(d+ex)^n\right)}{(g+fx)^2}\,dx \\
&= -\frac{a+b\log\left(c(d+ex)^n\right)}{f(g+fx)}+\frac{(ben)\int\frac{1}{(d+ex)(g+fx)}\,dx}{f} \\
&= -\frac{a+b\log\left(c(d+ex)^n\right)}{f(g+fx)}+\frac{(ben)\int\frac{1}{g+fx}\,dx}{df-eg}-\frac{(be^2n)\int\frac{1}{d+ex}\,dx}{f(df-eg)} \\
&= -\frac{ben\log(d+ex)}{f(df-eg)}-\frac{a+b\log\left(c(d+ex)^n\right)}{f(g+fx)}+\frac{ben\log(g+fx)}{f(df-eg)}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$
\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx=\frac{-\frac{a+b\log(c(d+ex)^n)}{g+fx}+\frac{ben(\log(d+ex)-\log(g+fx))}{-df+eg}}{f}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/((f + g/x)^2*x^2),x]
[Out] (-((a + b*Log[c*(d + e*x)^n])/(g + f*x)) + (b*e*n*(Log[d + e*x] - Log[g + f
*x]))/(-(d*f) + e*g))/f
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

| method | result |
|---|---|
| parallelrisch | $-\frac{\ln(ex+d)xb\,e^2fn-\ln(fx+g)xb\,e^2fn+\ln(ex+d)b\,e^2gn-\ln(fx+g)b\,e^2gn+\ln(c(ex+d)^n)bdef-\ln(c(ex+d)^n)b\,e^2g+adef-a\,e^2g}{(df-eg)(fx+g)ef}$ |
| risch | $-\frac{b\ln((ex+d)^n)}{f(fx+g)}-\frac{-i\pi\,\mathrm{csgn}(ic(ex+d)^n)^3bdf-i\pi beg\,\mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)^2+i\pi\,\mathrm{csgn}(i(ex+d)^n)\,\mathrm{csgn}(ic(ex+d)^n)^2$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(f+g/x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(ln(e*x+d)*x*b*e^2*f*n-ln(f*x+g)*x*b*e^2*f*n+ln(e*x+d)*b*e^2*g*n-ln(f*x+g)
*b*e^2*g*n+ln(c*(e*x+d)^n)*b*d*e*f-ln(c*(e*x+d)^n)*b*e^2*g+a*d*e*f-a*e^2*g)
/(d*f-e*g)/(f*x+g)/e/f
```

## Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{a+b\log(c(d+ex)^n)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx = -\frac{adf-aeg+(befnx+bdfn)\log(ex+d)-(befnx+begn)\log(fx+g)+(bdf-beg)\log(c)}{df^2g-efg^2+(df^3-ef^2g)x}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(a*d*f - a*e*g + (b*e*f*n*x + b*d*f*n)*log(e*x + d) - (b*e*f*n*x + b*e*g*n
)*log(f*x + g) + (b*d*f - b*e*g)*log(c))/(d*f^2*g - e*f*g^2 + (d*f^3 - e*f^
2*g)*x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(61) = 122.

Time = 34.40 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.50

$$\int \frac{a+b\log(c(d+ex)^n)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx = \begin{cases} \frac{ax+\frac{bd\log(c(d+ex)^n)}{e}-bnx+bx\log(c(d+ex)^n)}{g^2} \\ -\frac{a}{f^2x+fg}-\frac{bn}{f^2x+fg}-\frac{b\log\left(c\left(ex+\frac{eg}{f}\right)^n\right)}{f^2x+fg} \\ -\frac{adf}{df^3x+df^2g-ef^2gx-efg^2}+\frac{aeg}{df^3x+df^2g-ef^2gx-efg^2}-\frac{bdf\log(c(d+ex)^n)}{df^3x+df^2g-ef^2gx-efg^2}+\frac{befnx\log\left(x+\frac{g}{f}\right)}{df^3x+df^2g-ef^2gx-efg^2}-\frac{befx\log(c(d+ex)}{df^3x+df^2g-ef^2gx-}\end{cases}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(f+g/x)**2/x**2,x)
```

```
[Out] Piecewise(((a*x + b*d*log(c*(d + e*x)**n)/e - b*n*x + b*x*log(c*(d + e*x)**
n))/g**2, Eq(f, 0)), (-a/(f**2*x + f*g) - b*n/(f**2*x + f*g) - b*log(c*(e*x
 + e*g/f)**n)/(f**2*x + f*g), Eq(d, e*g/f)), (-a*d*f/(d*f**3*x + d*f**2*g -
 e*f**2*g*x - e*f*g**2) + a*e*g/(d*f**3*x + d*f**2*g - e*f**2*g*x - e*f*g**
2) - b*d*f*log(c*(d + e*x)**n)/(d*f**3*x + d*f**2*g - e*f**2*g*x - e*f*g**2
) + b*e*f*n*x*log(x + g/f)/(d*f**3*x + d*f**2*g - e*f**2*g*x - e*f*g**2) -
b*e*f*x*log(c*(d + e*x)**n)/(d*f**3*x + d*f**2*g - e*f**2*g*x - e*f*g**2) +
 b*e*g*n*log(x + g/f)/(d*f**3*x + d*f**2*g - e*f**2*g*x - e*f*g**2), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^2 x^2}\,dx = -ben\left(\frac{\log\left(ex+d\right)}{df^2-efg}-\frac{\log\left(fx+g\right)}{df^2-efg}\right) - \frac{b\log\left((ex+d)^n c\right)}{f^2x+fg}-\frac{a}{f^2x+fg}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)^2/x^2,x, algorithm="maxima")
```

```
[Out] -b*e*n*(log(e*x + d)/(d*f^2 - e*f*g) - log(f*x + g)/(d*f^2 - e*f*g)) - b*lo
g((e*x + d)^n*c)/(f^2*x + f*g) - a/(f^2*x + f*g)
```

## Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^2 x^2}\,dx = -\frac{ben\log\left(ex+d\right)}{df^2-efg}+\frac{ben\log\left(fx+g\right)}{df^2-efg} - \frac{bn\log\left(ex+d\right)}{f^2x+fg}-\frac{b\log\left(c\right)+a}{f^2x+fg}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)^2/x^2,x, algorithm="giac")
```

```
[Out] -b*e*n*log(e*x + d)/(d*f^2 - e*f*g) + b*e*n*log(f*x + g)/(d*f^2 - e*f*g) -
b*n*log(e*x + d)/(f^2*x + f*g) - (b*log(c) + a)/(f^2*x + f*g)
```

## Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^2x^2}\,dx = -\frac{a}{x\,f^2+g\,f}-\frac{b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{f\,\left(g+f\,x\right)}+\frac{b\,e\,n\,\mathrm{atan}\left(\frac{e\,g\,2{}\mathrm{i}+e\,f\,x\,2{}\mathrm{i}}{d\,f-e\,g}+1{}\mathrm{i}\right)\,2{}\mathrm{i}}{f\,\left(d\,f-e\,g\right)}$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x^2*(f + g/x)^2),x)
[Out] (b*e*n*atan((e*g*2i + e*f*x*2i)/(d*f - e*g) + 1i)*2i)/(f*(d*f - e*g)) - (b*
log(c*(d + e*x)^n))/(f*(g + f*x)) - a/(f*g + f^2*x)
```

# 3.308 $\int \frac{a+b\log(c(d+ex)^n)}{\left(f+\frac{g}{x}\right)^3 x^3}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 112

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3 x^3}\,dx = -\frac{ben}{2f(df-eg)(g+fx)} + \frac{be^2n\log(d+ex)}{2f(df-eg)^2} - \frac{a+b\log\left(c(d+ex)^n\right)}{2f(g+fx)^2} - \frac{be^2n\log(g+fx)}{2f(df-eg)^2}$$

```
[Out] -1/2*b*e*n/f/(d*f-e*g)/(f*x+g)+1/2*b*e^2*n*ln(e*x+d)/f/(d*f-e*g)^2+1/2*(-a-b*ln(c*(e*x+d)^n))/f/(f*x+g)^2-1/2*b*e^2*n*ln(f*x+g)/f/(d*f-e*g)^2
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2459, 2442, 46}

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3 x^3}\,dx = -\frac{a+b\log\left(c(d+ex)^n\right)}{2f(fx+g)^2} + \frac{be^2n\log(d+ex)}{2f(df-eg)^2} - \frac{be^2n\log(fx+g)}{2f(df-eg)^2} - \frac{ben}{2f(fx+g)(df-eg)}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])/((f + g/x)^3*x^3),x]
```

```
[Out] -1/2*(b*e*n)/(f*(d*f - e*g)*(g + f*x)) + (b*e^2*n*Log[d + e*x])/(2*f*(d*f - e*g)^2) - (a + b*Log[c*(d + e*x)^n])/(2*f*(g + f*x)^2) - (b*e^2*n*Log[g + f*x])/(2*f*(d*f - e*g)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{a+b\log\left(c(d+ex)^n\right)}{(g+fx)^3}\,dx \\
&= -\frac{a+b\log\left(c(d+ex)^n\right)}{2f(g+fx)^2}+\frac{(ben)\int \frac{1}{(d+ex)(g+fx)^2}\,dx}{2f} \\
&= -\frac{a+b\log\left(c(d+ex)^n\right)}{2f(g+fx)^2}+\frac{(ben)\int \left(\frac{e^2}{(df-eg)^2(d+ex)}+\frac{f}{(df-eg)(g+fx)^2}-\frac{ef}{(df-eg)^2(g+fx)}\right)dx}{2f} \\
&= -\frac{ben}{2f(df-eg)(g+fx)}+\frac{be^2n\log(d+ex)}{2f(df-eg)^2}-\frac{a+b\log\left(c(d+ex)^n\right)}{2f(g+fx)^2}-\frac{be^2n\log(g+fx)}{2f(df-eg)^2}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx = -\frac{a+b\log\left(c(d+ex)^n\right)-\frac{ben(g+fx)(-df+eg+e(g+fx)\log(d+ex)-e(g+fx)\log(g+fx))}{(df-eg)^2}}{2f(g+fx)^2}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])/((f + g/x)^3*x^3),x]
```

```
[Out] -1/2*(a + b*Log[c*(d + e*x)^n] - (b*e*n*(g + f*x)*(-(d*f) + e*g + e*(g + f*
x)*Log[d + e*x] - e*(g + f*x)*Log[g + f*x]))/(d*f - e*g)^2)/(f*(g + f*x)^2)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(107) = 214$.

Time = 0.61 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.53

| method | result |
|---|---|
| parallelrisch | $\frac{-a d^2 e f^3 - a e^3 f g^2 - \ln(c(ex+d)^n) b d^2 e f^3 - \ln(c(ex+d)^n) b e^3 f g^2 - \ln(fx+g) b e^3 f g^2 n - xbd e^2 f^3 n + 2\ln(c(ex+d)^n) bd e^2 f^2 g -}{2(f}$ |
| risch | $-\frac{b\ln((ex+d)^n)}{2f(fx+g)^2} - \frac{2\ln(c) b d^2 f^2 + 2\ln(c) b e^2 g^2 - 4\ln(c) bdefg + 2i\pi bdefg \operatorname{csgn}(ic(ex+d)^n)^3 - i\pi b e^2 g^2 \operatorname{csgn}(ic) \operatorname{csgn}(i(ex+d)^n}{}$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))/(f+g/x)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-a*d^2*e*f^3-a*e^3*f*g^2-ln(c*(e*x+d)^n)*b*d^2*e*f^3-ln(c*(e*x+d)^n)*b
*e^3*f*g^2-ln(f*x+g)*b*e^3*f*g^2*n-x*b*d*e^2*f^3*n+2*ln(c*(e*x+d)^n)*b*d*e^
2*f^2*g-b*d*e^2*f^2*n*g+b*e^3*f^2*g*n*x+2*a*d*e^2*f^2*g+b*e^3*f*g^2*n+2*ln(
e*x+d)*b*e^3*f^2*g*n*x+ln(e*x+d)*x^2*b*e^3*f^3*n-ln(f*x+g)*x^2*b*e^3*f^3*n-
2*ln(f*x+g)*x*b*e^3*f^2*g*n+ln(e*x+d)*b*e^3*f*g^2*n)/(d^2*f^2-2*d*e*f*g+e^2
*g^2)/(f*x+g)^2/e/f^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(104) = 208$.

Time = 0.33 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.43

$$\int \frac{a + b\log\left(c(d+ex)^n\right)}{\left(f + \frac{g}{x}\right)^3 x^3}\,dx = -\frac{ad^2f^2 - 2\,adefg + ae^2g^2 + (bdef^2 - be^2fg)nx + (bdefg - be^2g^2)n - (be^2f^2nx^2 + 2\,be^2fgnx - (bd^2f^2}{2\left(d^2f^3g^2 - 2\,def^2g^3 + e^2fg^4 + (d^2f^5 - 2\,de\right.}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*d^2*f^2 - 2*a*d*e*f*g + a*e^2*g^2 + (b*d*e*f^2 - b*e^2*f*g)*n*x + (
b*d*e*f*g - b*e^2*g^2)*n - (b*e^2*f^2*n*x^2 + 2*b*e^2*f*g*n*x - (b*d^2*f^2
- 2*b*d*e*f*g)*n)*log(e*x + d) + (b*e^2*f^2*n*x^2 + 2*b*e^2*f*g*n*x + b*e^2
*g^2*n)*log(f*x + g) + (b*d^2*f^2 - 2*b*d*e*f*g + b*e^2*g^2)*log(c))/(d^2*f
^3*g^2 - 2*d*e*f^2*g^3 + e^2*f*g^4 + (d^2*f^5 - 2*d*e*f^4*g + e^2*f^3*g^2)*
x^2 + 2*(d^2*f^4*g - 2*d*e*f^3*g^2 + e^2*f^2*g^3)*x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))/(f+g/x)**3/x**3,x)
```

```
[Out] Timed out
```

## Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.51

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx$$
$$= \frac{1}{2}\,ben\left(\frac{e\log\left(ex+d\right)}{d^2f^3-2\,def^2g+e^2fg^2}-\frac{e\log\left(fx+g\right)}{d^2f^3-2\,def^2g+e^2fg^2}-\frac{1}{df^2g-efg^2+(df^3-ef^2g)x}\right)$$
$$-\frac{b\log\left((ex+d)^nc\right)}{2\,(f^3x^2+2\,f^2gx+fg^2)}-\frac{a}{2\,(f^3x^2+2\,f^2gx+fg^2)}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*e*n*(e*log(e*x + d)/(d^2*f^3 - 2*d*e*f^2*g + e^2*f*g^2) - e*log(f*x +
 g)/(d^2*f^3 - 2*d*e*f^2*g + e^2*f*g^2) - 1/(d*f^2*g - e*f*g^2 + (d*f^3 - e
*f^2*g)*x)) - 1/2*b*log((e*x + d)^n*c)/(f^3*x^2 + 2*f^2*g*x + f*g^2) - 1/2*
a/(f^3*x^2 + 2*f^2*g*x + f*g^2)
```

## Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.79

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx = \frac{be^2n\log\left(ex+d\right)}{2\,(d^2f^3-2\,def^2g+e^2fg^2)}$$
$$-\frac{be^2n\log\left(fx+g\right)}{2\,(d^2f^3-2\,def^2g+e^2fg^2)}-\frac{bn\log\left(ex+d\right)}{2\,(f^3x^2+2\,f^2gx+fg^2)}$$
$$-\frac{befnx+begn+bdf\log\left(c\right)-beg\log\left(c\right)+adf-aeg}{2\,(df^4x^2-ef^3gx^2+2\,df^3gx-2\,ef^2g^2x+df^2g^2-efg^3)}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))/(f+g/x)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*e^2*n*log(e*x + d)/(d^2*f^3 - 2*d*e*f^2*g + e^2*f*g^2) - 1/2*b*e^2*n*
log(f*x + g)/(d^2*f^3 - 2*d*e*f^2*g + e^2*f*g^2) - 1/2*b*n*log(e*x + d)/(f^
3*x^2 + 2*f^2*g*x + f*g^2) - 1/2*(b*e*f*n*x + b*e*g*n + b*d*f*log(c) - b*e*
g*log(c) + a*d*f - a*e*g)/(d*f^4*x^2 - e*f^3*g*x^2 + 2*d*f^3*g*x - 2*e*f^2*
g^2*x + d*f^2*g^2 - e*f*g^3)
```

## Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.54

$$\int \frac{a+b\log\left(c(d+ex)^n\right)}{\left(f+\frac{g}{x}\right)^3x^3}\,dx=\frac{b\,e^2\,n\,\mathrm{atanh}\left(\frac{2\,d^2\,f^3-2\,e^2\,f\,g^2}{2\,f\,{\left(d\,f-e\,g\right)}^2}+\frac{2\,e\,f\,x}{d\,f-e\,g}\right)}{f\,{\left(d\,f-e\,g\right)}^2}-\frac{b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)}{2\,f\,\left(f^2\,x^2+2\,f\,g\,x+g^2\right)}-\frac{\frac{a\,d\,f-a\,e\,g+b\,e\,g\,n}{d\,f-e\,g}+\frac{b\,e\,f\,n\,x}{d\,f-e\,g}}{2\,f^3\,x^2+4\,f^2\,g\,x+2\,f\,g^2}$$

```
[In] int((a + b*log(c*(d + e*x)^n))/(x^3*(f + g/x)^3),x)
```

```
[Out] (b*e^2*n*atanh((2*d^2*f^3 - 2*e^2*f*g^2)/(2*f*(d*f - e*g)^2) + (2*e*f*x)/(d
*f - e*g)))/(f*(d*f - e*g)^2) - (b*log(c*(d + e*x)^n))/(2*f*(g^2 + f^2*x^2
+ 2*f*g*x)) - ((a*d*f - a*e*g + b*e*g*n)/(d*f - e*g) + (b*e*f*n*x)/(d*f - e
*g))/(2*f*g^2 + 2*f^3*x^2 + 4*f^2*g*x)
```

# 3.309 $\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 247

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx = -\frac{x}{c}+\frac{(a+bx)\log(a+bx)}{bc}-\frac{\sqrt{d}\log(a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}}+\frac{\sqrt{d}\log(a+bx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}}+\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}}-\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}}$$

```
[Out] -x/c+(b*x+a)*ln(b*x+a)/b/c+1/2*ln(b*x+a)*ln(-b*(x*(-c)^(1/2)+d^(1/2))/(a*(-
c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/2*ln(b*x+a)*ln(b*(-x*(-c)^(1/2)+d
^(1/2))/(a*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/2*polylog(2,(b*x+a)*
(-c)^(1/2)/(a*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/2*polylog(2,(b*x+
a)*(-c)^(1/2)/(a*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used

= {2456, 2436, 2332, 2441, 2440, 2438}

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx = \frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}} - \frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx)}{\sqrt{-c}a+b\sqrt{d}}\right)}{2(-c)^{3/2}} - \frac{\sqrt{d}\log(a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}} + \frac{\sqrt{d}\log(a+bx)\log\left(-\frac{b\left(\sqrt{-c}x+\sqrt{d}\right)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}} + \frac{(a+bx)\log(a+bx)}{bc} - \frac{x}{c}$$

```
[In] Int[Log[a + b*x]/(c + d/x^2),x]
```

```
[Out] -(x/c) + ((a + b*x)*Log[a + b*x])/(b*c) - (Sqrt[d]*Log[a + b*x]*Log[(b*(Sqr
t[d] - Sqrt[-c]*x))/(a*Sqrt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2)) + (Sqrt[d]*Lo
g[a + b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/(a*Sqrt[-c] - b*Sqrt[d]))])/(2*
(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sqrt[-c] - b*Sqrt
[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sqrt[-c
] + b*Sqrt[d])])/(2*(-c)^(3/2))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\log(a+bx)}{c} - \frac{d\log(a+bx)}{c\,(d+cx^2)}\right) dx \\
&= \frac{\int \log(a+bx)\,dx}{c} - \frac{d\int \frac{\log(a+bx)}{d+cx^2}\,dx}{c} \\
&= \frac{\text{Subst}(\int \log(x)\,dx, x, a+bx)}{bc} - \frac{d\int \left(\frac{\log(a+bx)}{2\sqrt{d}\left(\sqrt{d}-\sqrt{-c}x\right)} + \frac{\log(a+bx)}{2\sqrt{d}\left(\sqrt{d}+\sqrt{-c}x\right)}\right) dx}{c} \\
&= -\frac{x}{c} + \frac{(a+bx)\log(a+bx)}{bc} - \frac{\sqrt{d}\int \frac{\log(a+bx)}{\sqrt{d}-\sqrt{-c}x}\,dx}{2c} - \frac{\sqrt{d}\int \frac{\log(a+bx)}{\sqrt{d}+\sqrt{-c}x}\,dx}{2c} \\
&= -\frac{x}{c} + \frac{(a+bx)\log(a+bx)}{bc} - \frac{\sqrt{d}\log(a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}} \\
&\quad + \frac{\sqrt{d}\log(a+bx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}} \\
&\quad + \frac{\left(b\sqrt{d}\right)\int \frac{\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{a\sqrt{-c}+b\sqrt{d}}\right)}{a+bx}\,dx}{2(-c)^{3/2}} - \frac{\left(b\sqrt{d}\right)\int \frac{\log\left(\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{-a\sqrt{-c}+b\sqrt{d}}\right)}{a+bx}\,dx}{2(-c)^{3/2}}
\end{aligned}
$$

$$= -\frac{x}{c} + \frac{(a+bx)\log(a+bx)}{bc} - \frac{\sqrt{d}\log(a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}} + \frac{\sqrt{d}\log(a+bx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-c}x}{-a\sqrt{-c}+b\sqrt{d}}\right)}{x}\,dx, x, a+bx\right)}{2(-c)^{3/2}} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-c}x}{a\sqrt{-c}+b\sqrt{d}}\right)}{x}\,dx, x, a+bx\right)}{2(-c)^{3/2}}$$

$$= -\frac{x}{c} + \frac{(a+bx)\log(a+bx)}{bc} - \frac{\sqrt{d}\log(a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}} + \frac{\sqrt{d}\log(a+bx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}} + \frac{\sqrt{d}\text{Li}_2\left(\frac{\sqrt{-c}(a+bx)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}} - \frac{\sqrt{d}\text{Li}_2\left(\frac{\sqrt{-c}(a+bx)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}}$$

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx = -\frac{x}{c} + \frac{(a+bx)\log(a+bx)}{bc} - \frac{\sqrt{d}\log(a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}} + \frac{\sqrt{d}\log(a+bx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}} + \frac{\sqrt{d}\,\text{PolyLog}\left(2, \frac{\sqrt{-c}(a+bx)}{a\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}} - \frac{\sqrt{d}\,\text{PolyLog}\left(2, \frac{\sqrt{-c}(a+bx)}{a\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}}$$

```
[In] Integrate[Log[a + b*x]/(c + d/x^2),x]
```

```
[Out] -(x/c) + ((a + b*x)*Log[a + b*x])/(b*c) - (Sqrt[d]*Log[a + b*x]*Log[(b*(Sqr
t[d] - Sqrt[-c]*x))/(a*Sqrt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2)) + (Sqrt[d]*Lo
g[a + b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/(a*Sqrt[-c] - b*Sqrt[d]))])/(2*
(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sqrt[-c] - b*Sqrt
[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sqrt[-c
] + b*Sqrt[d])])/(2*(-c)^(3/2))
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{(bx+a)\ln(bx+a)-bx-a}{c}-\frac{d\,b^2\left(-\frac{\ln(bx+a)\left(-\ln\left(\frac{b\sqrt{-cd}+ca-c(bx+a)}{b\sqrt{-cd}+ca}\right)+\ln\left(\frac{b\sqrt{-cd}-ca+c(bx+a)}{b\sqrt{-cd}-ca}\right)\right)}{2b\sqrt{-cd}}+\frac{\operatorname{dilog}\left(\frac{b\sqrt{-cd}+ca-c(bx+a)}{b\sqrt{-cd}+ca}\right)}{2b}\right.}{c}}{b}$ |
| default | $\frac{\frac{(bx+a)\ln(bx+a)-bx-a}{c}-\frac{d\,b^2\left(-\frac{\ln(bx+a)\left(-\ln\left(\frac{b\sqrt{-cd}+ca-c(bx+a)}{b\sqrt{-cd}+ca}\right)+\ln\left(\frac{b\sqrt{-cd}-ca+c(bx+a)}{b\sqrt{-cd}-ca}\right)\right)}{2b\sqrt{-cd}}+\frac{\operatorname{dilog}\left(\frac{b\sqrt{-cd}+ca-c(bx+a)}{b\sqrt{-cd}+ca}\right)}{2b}\right.}{c}}{b}$ |
| risch | $\frac{\ln(bx+a)x}{c}+\frac{\ln(bx+a)a}{bc}-\frac{x}{c}-\frac{a}{bc}-\frac{d\ln(bx+a)\ln\left(\frac{b\sqrt{-cd}+ca-c(bx+a)}{b\sqrt{-cd}+ca}\right)}{2c\sqrt{-cd}}+\frac{d\ln(bx+a)\ln\left(\frac{b\sqrt{-cd}-ca+c(bx+a)}{b\sqrt{-cd}-ca}\right)}{2c\sqrt{-cd}}-$ |

```
[In] int(ln(b*x+a)/(c+d/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/c*((b*x+a)*ln(b*x+a)-b*x-a)-d*b^2/c*(-1/2*ln(b*x+a)*(-ln((b*(-c*d)^(
1/2)+c*a-c*(b*x+a))/(b*(-c*d)^(1/2)+c*a))+ln((b*(-c*d)^(1/2)-c*a+c*(b*x+a))
/(b*(-c*d)^(1/2)-c*a)))/b/(-c*d)^(1/2)+1/2*(dilog((b*(-c*d)^(1/2)+c*a-c*(b*
x+a))/(b*(-c*d)^(1/2)+c*a))-dilog((b*(-c*d)^(1/2)-c*a+c*(b*x+a))/(b*(-c*d)^
(1/2)-c*a)))/b/(-c*d)^(1/2)))
```

## Fricas [F]

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\log(bx+a)}{c+\frac{d}{x^2}}\,dx$$

```
[In] integrate(log(b*x+a)/(c+d/x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(b*x + a)/(c*x^2 + d), x)
```

## Sympy [F]

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{x^2\log(a+bx)}{cx^2+d}\,dx$$

```
[In] integrate(ln(b*x+a)/(c+d/x**2),x)
```

```
[Out] Integral(x**2*log(a + b*x)/(c*x**2 + d), x)
```

## Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.21

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx = -\left(\frac{d\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c} - \frac{x}{c}\right)\log(bx+a)$$
$$-\frac{2\,bcx - 2\,ac\log(bx+a) + \left(b\arctan\left(\frac{(b^2x+ab)\sqrt{c}\sqrt{d}}{a^2c+b^2d}, \frac{abcx+a^2c}{a^2c+b^2d}\right)\log(cx^2+d) - b\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\log\left(\frac{b^2cx^2+2\cdots}{a^2c\cdots}\right.\right.}{2\,bc^2}$$

```
[In] integrate(log(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

```
[Out] -(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*log(b*x + a) - 1/2*(2*b*c*x
- 2*a*c*log(b*x + a) + (b*arctan2((b^2*x + a*b)*sqrt(c)*sqrt(d)/(a^2*c + b^
2*d), (a*b*c*x + a^2*c)/(a^2*c + b^2*d))*log(c*x^2 + d) - b*arctan(sqrt(c)*
x/sqrt(d))*log((b^2*c*x^2 + 2*a*b*c*x + a^2*c)/(a^2*c + b^2*d)) + I*b*dilog
(-(a*b*c*x + b^2*d + (I*b^2*x - I*a*b)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*b*sq
rt(c)*sqrt(d) - b^2*d)) - I*b*dilog(-(a*b*c*x + b^2*d - (I*b^2*x - I*a*b)*s
qrt(c)*sqrt(d))/(a^2*c - 2*I*a*b*sqrt(c)*sqrt(d) - b^2*d)))*sqrt(c)*sqrt(d)
)/(b*c^2)
```

## Giac [F]

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx = \int \frac{\log(bx+a)}{c+\frac{d}{x^2}}\,dx$$

```
[In] integrate(log(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

```
[Out] integrate(log(b*x + a)/(c + d/x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)}{c+\frac{d}{x^2}}\,dx = \int \frac{\ln(a+b\,x)}{c+\frac{d}{x^2}}\,dx$$

```
[In] int(log(a + b*x)/(c + d/x^2),x)
```

```
[Out] int(log(a + b*x)/(c + d/x^2), x)
```

# 3.310 $\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$

## Optimal result

Integrand size = 29, antiderivative size = 831

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = -\frac{2abdfnx}{eg^2}+\frac{2b^2dfn^2x}{eg^2}-\frac{2b^2d^3n^2x}{e^3g}-\frac{b^2fn^2(d+ex)^2}{4e^2g^2}+\frac{3b^2d^2n^2(d+ex)^2}{4e^4g}-\frac{2b^2dn^2(d+ex)^3}{9e^4g}+\frac{b^2n^2(d+ex)^4}{32e^4g}+\frac{b^2d^4n^2\log^2(d+ex)}{4e^4g}-\frac{2b^2dfn(d+ex)\log(c(d+ex)^n)}{e^2g^2}+\frac{2bd^3n(d+ex)(a+b\log(c(d+ex)^n))}{e^4g}+\frac{bfn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2g^2}-\frac{3bd^2n(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^4g}+\frac{2bdn(d+ex)^3(a+b\log(c(d+ex)^n))}{3e^4g}-\frac{bn(d+ex)^4(a+b\log(c(d+ex)^n))}{8e^4g}-\frac{bd^4n\log(d+ex)(a+b\log(c(d+ex)^n))}{2e^4g}+\frac{x^4(a+b\log(c(d+ex)^n))^2}{4g}+\frac{df(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2g^2}-\frac{f(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^2g^2}+\frac{f^2(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3}+\frac{f^2(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3}+\frac{bf^2n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}+\frac{bf^2n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}-\frac{b^2f^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}-\frac{b^2f^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$

```
[Out] -2*a*b*d*f*n*x/e/g^2+2*b^2*d*f*n^2*x/e/g^2-2*b^2*d^3*n^2*x/e^3/g-1/4*b^2*f*
n^2*(e*x+d)^2/e^2/g^2+3/4*b^2*d^2*n^2*(e*x+d)^2/e^4/g-2/9*b^2*d*n^2*(e*x+d)
^3/e^4/g+1/32*b^2*n^2*(e*x+d)^4/e^4/g+1/4*b^2*d^4*n^2*ln(e*x+d)^2/e^4/g-2*b
^2*d*f*n*(e*x+d)*ln(c*(e*x+d)^n)/e^2/g^2+2*b*d^3*n*(e*x+d)*(a+b*ln(c*(e*x+d
)^n))/e^4/g+1/2*b*f*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2/g^2-3/2*b*d^2*n*(
e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^4/g+2/3*b*d*n*(e*x+d)^3*(a+b*ln(c*(e*x+d)^
n))/e^4/g-1/8*b*n*(e*x+d)^4*(a+b*ln(c*(e*x+d)^n))/e^4/g-1/2*b*d^4*n*ln(e*x+
d)*(a+b*ln(c*(e*x+d)^n))/e^4/g+1/4*x^4*(a+b*ln(c*(e*x+d)^n))^2/g+d*f*(e*x+d
)*(a+b*ln(c*(e*x+d)^n))^2/e^2/g^2-1/2*f*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e
^2/g^2+1/2*f^2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^
(1/2)+d*g^(1/2)))/g^3+1/2*f^2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^
(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^3+b*f^2*n*(a+b*ln(c*(e*x+d)^n))*polylog(
2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^3+b*f^2*n*(a+b*ln(c*(e*x+d)^
n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^3-b^2*f^2*n^2*pol
ylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^3-b^2*f^2*n^2*polylog(3
,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^3
```

## Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.655$, Rules used = {2463, 2448, 2436, 2333, 2332, 2437, 2342, 2341, 2445, 2458, 45, 2372, 12, 14, 2338,

2443, 2481, 2421, 6724}

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \frac{b^2n^2\log^2(d+ex)d^4}{4e^4g} - \frac{bn\log(d+ex)(a+b\log(c(d+ex)^n))d^4}{2e^4g} - \frac{2b^2n^2xd^3}{e^3g} + \frac{2bn(d+ex)(a+b\log(c(d+ex)^n))d^3}{e^4g} + \frac{3b^2n^2(d+ex)^2d^2}{4e^4g} - \frac{3bn(d+ex)^2(a+b\log(c(d+ex)^n))d^2}{2e^4g} - \frac{2b^2n^2(d+ex)^3d}{9e^4g} + \frac{f(d+ex)(a+b\log(c(d+ex)^n))^2d}{e^2g^2} + \frac{2b^2fn^2xd}{eg^2} - \frac{2abfnxd}{eg^2} - \frac{2b^2fn(d+ex)\log(c(d+ex)^n)d}{e^2g^2} + \frac{2bn(d+ex)^3(a+b\log(c(d+ex)^n))d}{3e^4g} + \frac{b^2n^2(d+ex)^4}{32e^4g} - \frac{b^2fn^2(d+ex)^2}{4e^2g^2} + \frac{x^4(a+b\log(c(d+ex)^n))^2}{4g} - \frac{f(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^2g^2} - \frac{bn(d+ex)^4(a+b\log(c(d+ex)^n))}{8e^4g} + \frac{bfn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2g^2} + \frac{f^2(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^3} + \frac{f^2(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3} + \frac{bf^2n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} + \frac{bf^2n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^3} - \frac{b^2f^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} - \frac{b^2f^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^3}$$

```
[In] Int[(x^5*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

```
[Out] (-2*a*b*d*f*n*x)/(e*g^2) + (2*b^2*d*f*n^2*x)/(e*g^2) - (2*b^2*d^3*n^2*x)/(e
^3*g) - (b^2*f*n^2*(d + e*x)^2)/(4*e^2*g^2) + (3*b^2*d^2*n^2*(d + e*x)^2)/(
4*e^4*g) - (2*b^2*d*n^2*(d + e*x)^3)/(9*e^4*g) + (b^2*n^2*(d + e*x)^4)/(32*
e^4*g) + (b^2*d^4*n^2*Log[d + e*x]^2)/(4*e^4*g) - (2*b^2*d*f*n*(d + e*x)*Lo
g[c*(d + e*x)^n])/(e^2*g^2) + (2*b*d^3*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n
]))/(e^4*g) + (b*f*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^2*g^2) -
(3*b*d^2*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^4*g) + (2*b*d*n*(d
+ e*x)^3*(a + b*Log[c*(d + e*x)^n]))/(3*e^4*g) - (b*n*(d + e*x)^4*(a + b*Lo
g[c*(d + e*x)^n]))/(8*e^4*g) - (b*d^4*n*Log[d + e*x]*(a + b*Log[c*(d + e*x)
^n]))/(2*e^4*g) + (x^4*(a + b*Log[c*(d + e*x)^n])^2)/(4*g) + (d*f*(d + e*x)
*(a + b*Log[c*(d + e*x)^n])^2)/(e^2*g^2) - (f*(d + e*x)^2*(a + b*Log[c*(d +
 e*x)^n])^2)/(2*e^2*g^2) + (f^2*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-
f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^3) + (f^2*(a + b*Log[c*(d
+ e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g
^3) + (b*f^2*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/
(e*Sqrt[-f] - d*Sqrt[g]))])/g^3 + (b*f^2*n*(a + b*Log[c*(d + e*x)^n])*PolyL
og[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^3 - (b^2*f^2*n^2*Pol
yLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/g^3 - (b^2*f^2*n^
2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{fx(a+b\log(c(d+ex)^n))^2}{g^2}+\frac{x^3(a+b\log(c(d+ex)^n))^2}{g}+\frac{f^2x(a+b\log(c(d+ex)^n))^2}{g^2(f+gx^2)}\right)dx$$

$$=-\frac{f\int x(a+b\log(c(d+ex)^n))^2\,dx}{g^2}+\frac{f^2\int\frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx}{g^2}+\frac{\int x^3(a+b\log(c(d+ex)^n))^2\,dx}{g}$$

$$=\frac{x^4(a+b\log(c(d+ex)^n))^2}{4g}-\frac{f\int\left(-\frac{d(a+b\log(c(d+ex)^n))^2}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{e}\right)dx}{g^2}+\frac{f^2\int\left(-\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)}+\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g^2}-\frac{(ben)\int\frac{x^4(a+b\log(c(d+ex)^n))}{d+ex}\,dx}{2g}$$

$$=\frac{x^4(a+b\log(c(d+ex)^n))^2}{4g}-\frac{f^2\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{2g^{5/2}}+\frac{f^2\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{2g^{5/2}}-\frac{f\int(d+ex)(a+b\log(c(d+ex)^n))^2\,dx}{eg^2}+\frac{(df)\int(a+b\log(c(d+ex)^n))^2\,dx}{eg^2}-\frac{(bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^4(a+b\log(cx^n))}{x}\,dx,x,d+ex\right)}{2g}$$

$$= \frac{2bd^3n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^4g} - \frac{3bd^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4g}$$

$$+ \frac{2bdn(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^4g} - \frac{bn(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{8e^4g}$$

$$- \frac{bd^4n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4g} + \frac{x^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4g}$$

$$+ \frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3}$$

$$+ \frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3}$$

$$- \frac{f\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^2\,dx, x, d+ex\right)}{e^2g^2}$$

$$+ \frac{(df)\text{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)^2\,dx, x, d+ex\right)}{e^2g^2}$$

$$- \frac{(bef^2n)\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{g^3}$$

$$- \frac{(bef^2n)\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{g^3}$$

$$+ \frac{(b^2n^2)\,\text{Subst}\left(\int \frac{x\left(-48d^3+36d^2x-16dx^2+3x^3\right)+12d^4\log(x)}{12e^4x}\,dx, x, d+ex\right)}{2g}$$

$$= \frac{2bd^3n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^4g} - \frac{3bd^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4g} + \frac{2bdn(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^4g} - \frac{bn(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{8e^4g} - \frac{bd^4n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4g} + \frac{x^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4g} + \frac{df(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g^2} - \frac{f(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g^2} + \frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3} + \frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3} - \frac{(bf^2n)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{g^3} - \frac{(bf^2n)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{g^3} + \frac{(bfn)\operatorname{Subst}\left(\int x(a+b\log(cx^n))\,dx,x,d+ex\right)}{e^2g^2} - \frac{(2bdfn)\operatorname{Subst}\left(\int(a+b\log(cx^n))\,dx,x,d+ex\right)}{e^2g^2} + \frac{(b^2n^2)\operatorname{Subst}\left(\int\frac{x(-48d^3+36d^2x-16dx^2+3x^3)+12d^4\log(x)}{x}dx,x,d+ex\right)}{24e^4g}$$

$$= -\frac{2abdfnx}{eg^2} - \frac{b^2fn^2(d+ex)^2}{4e^2g^2} + \frac{2bd^3n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^4g}$$
$$+ \frac{bfn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2g^2} - \frac{3bd^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4g}$$
$$+ \frac{2bdn(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^4g} - \frac{bn(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{8e^4g}$$
$$- \frac{bd^4n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4g} + \frac{x^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4g}$$
$$+ \frac{df(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g^2} - \frac{f(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g^2}$$
$$+ \frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3}$$
$$+ \frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3}$$
$$+ \frac{bf^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}$$
$$+ \frac{bf^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$
$$- \frac{(2b^2dfn)\operatorname{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{e^2g^2}$$
$$- \frac{(b^2f^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{g^3}$$
$$- \frac{(b^2f^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{g^3}$$
$$+ \frac{(b^2n^2)\operatorname{Subst}\left(\int\left(-48d^3+36d^2x-16dx^2+3x^3+\frac{12d^4\log(x)}{x}\right)\,dx,x,d+ex\right)}{24e^4g}$$

$$
\begin{aligned}
&= -\frac{2abdfnx}{eg^2}+\frac{2b^2dfn^2x}{eg^2}-\frac{2b^2d^3n^2x}{e^3g}-\frac{b^2fn^2(d+ex)^2}{4e^2g^2} \\
&+\frac{3b^2d^2n^2(d+ex)^2}{4e^4g}-\frac{2b^2dn^2(d+ex)^3}{9e^4g}+\frac{b^2n^2(d+ex)^4}{32e^4g} \\
&-\frac{2b^2dfn(d+ex)\log\left(c(d+ex)^n\right)}{e^2g^2}+\frac{2bd^3n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^4g} \\
&+\frac{bfn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2g^2}-\frac{3bd^2n(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4g} \\
&+\frac{2bdn(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^4g}-\frac{bn(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{8e^4g} \\
&-\frac{bd^4n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^4g}+\frac{x^4\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4g} \\
&+\frac{df(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g^2}-\frac{f(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g^2} \\
&+\frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3} \\
&+\frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3} \\
&+\frac{bf^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} \\
&+\frac{bf^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}-\frac{b^2f^2n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} \\
&-\frac{b^2f^2n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}+\frac{(b^2d^4n^2)\operatorname{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,d+ex\right)}{2e^4g}
\end{aligned}
$$

$$= -\frac{2abdfnx}{eg^2} + \frac{2b^2dfn^2x}{eg^2} - \frac{2b^2d^3n^2x}{e^3g} - \frac{b^2fn^2(d+ex)^2}{4e^2g^2} + \frac{3b^2d^2n^2(d+ex)^2}{4e^4g} - \frac{2b^2dn^2(d+ex)^3}{9e^4g} + \frac{b^2n^2(d+ex)^4}{32e^4g} + \frac{b^2d^4n^2\log^2(d+ex)}{4e^4g} - \frac{2b^2dfn(d+ex)\log(c(d+ex)^n)}{e^2g^2} + \frac{2bd^3n(d+ex)(a+b\log(c(d+ex)^n))}{e^4g} + \frac{bfn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2g^2} - \frac{3bd^2n(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^4g} + \frac{2bdn(d+ex)^3(a+b\log(c(d+ex)^n))}{3e^4g} - \frac{bn(d+ex)^4(a+b\log(c(d+ex)^n))}{8e^4g} - \frac{bd^4n\log(d+ex)(a+b\log(c(d+ex)^n))}{2e^4g} + \frac{x^4(a+b\log(c(d+ex)^n))^2}{4g} + \frac{df(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2g^2} - \frac{f(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^2g^2} + \frac{f^2(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3} + \frac{f^2(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3} + \frac{bf^2n(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} + \frac{bf^2n(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3} - \frac{b^2f^2n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} - \frac{b^2f^2n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 862, normalized size of antiderivative = 1.04

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \frac{-144e^4fgx^2(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2+72e^4g^2x^4(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{}$$

```
[In] Integrate[(x^5*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

```
[Out] (-144*e^4*f*g*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 72*e^4*
g^2*x^4*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 144*e^4*f^2*(a -
```

```
b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] - 12*b*n*(a - b*n
*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(12*e^2*f*g*(e*x*(2*d - e*x) - 2*(d^2
 - e^2*x^2)*Log[d + e*x]) + g^2*(e*x*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3
*e^3*x^3) + 12*(d^4 - e^4*x^4)*Log[d + e*x]) - 24*e^4*f^2*(Log[d + e*x]*Log
[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g
]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) - 24*e^4*f^2*(Log[d + e*x]*Log[
1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d
 + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])) + b^2*n^2*(-72*e^2*f*g*(e*x*(-6*d + e
*x) + (6*d^2 + 4*d*e*x - 2*e^2*x^2)*Log[d + e*x] - 2*(d^2 - e^2*x^2)*Log[d
+ e*x]^2) - g^2*(e*x*(300*d^3 - 78*d^2*e*x + 28*d*e^2*x^2 - 9*e^3*x^3) - 12
*(25*d^4 + 12*d^3*e*x - 6*d^2*e^2*x^2 + 4*d*e^3*x^3 - 3*e^4*x^4)*Log[d + e*
x] + 72*(d^4 - e^4*x^4)*Log[d + e*x]^2) + 144*e^4*f^2*(Log[d + e*x]^2*Log[1
 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyL
og[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqr
t[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) + 144*e^4*f^2*(Log[d + e*x]^
2*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*P
olyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sq
rt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])))/(288*e^4*g^3)
```

## Maple [F]

$$\int \frac{x^5(a+b\ln(c(ex+d)^n))^2}{g\,x^2+f}dx$$

```
[In] int(x^5*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
[Out] int(x^5*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

## Fricas [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2x^5}{gx^2+f}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="fricas")
[Out] integral((b^2*x^5*log((e*x + d)^n*c)^2 + 2*a*b*x^5*log((e*x + d)^n*c) + a^2
*x^5)/(g*x^2 + f), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx=\text{Timed out}$$

```
[In] integrate(x**5*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f),x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2x^5}{gx^2+f}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/4*a^2*(2*f^2*log(g*x^2 + f)/g^3 + (g*x^4 - 2*f*x^2)/g^2) + integrate((b^2
*x^5*log((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x^5*log((e*x + d)^n) + (b^2*
log(c)^2 + 2*a*b*log(c))*x^5)/(g*x^2 + f), x)
```

## Giac [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2x^5}{gx^2+f}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*x^5/(g*x^2 + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx=\int \frac{x^5\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2}{g\,x^2+f}\,dx$$

```
[In] int((x^5*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2),x)
```

```
[Out] int((x^5*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2), x)
```

# 3.311 $\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 499

$$\begin{aligned}\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx &= \frac{2abdnx}{eg}-\frac{2b^2dn^2x}{eg}+\frac{b^2n^2(d+ex)^2}{4e^2g}\\
&+\frac{2b^2dn(d+ex)\log(c(d+ex)^n)}{e^2g}\\
&-\frac{bn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2g}\\
&-\frac{d(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2g}\\
&+\frac{(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^2g}\\
&-\frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}\\
&-\frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2}\\
&-\frac{bfn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2}\\
&-\frac{bfn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2}\\
&+\frac{b^2fn^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2}\\
&+\frac{b^2fn^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2}\end{aligned}$$

```
[Out] 2*a*b*d*n*x/e/g-2*b^2*d*n^2*x/e/g+1/4*b^2*n^2*(e*x+d)^2/e^2/g+2*b^2*d*n*(e*
x+d)*ln(c*(e*x+d)^n)/e^2/g-1/2*b*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2/g-d*
(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^2/g+1/2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2
/e^2/g-1/2*f*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1
/2)+d*g^(1/2)))/g^2-1/2*f*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2
))/(e*(-f)^(1/2)-d*g^(1/2)))/g^2-b*f*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*
x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^2-b*f*n*(a+b*ln(c*(e*x+d)^n))*poly
log(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^2+b^2*f*n^2*polylog(3,-(e
*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^2+b^2*f*n^2*polylog(3,(e*x+d)*g^(
1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^2
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {2463, 2448, 2436, 2333, 2332, 2437, 2342, 2341, 2443, 2481, 2421, 6724}

$$\begin{aligned}\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = &-\frac{bn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2g}\\&+\frac{(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g}\\&-\frac{d(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g}\\&-\frac{bfn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g^2}\\&-\frac{bfn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{g^2}\\&-\frac{f\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g^2}\\&-\frac{f\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g^2}\\&+\frac{2abdnx}{eg}+\frac{2b^2dn(d+ex)\log\left(c(d+ex)^n\right)}{e^2g}\\&+\frac{b^2n^2(d+ex)^2}{4e^2g}+\frac{b^2fn^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2}\\&+\frac{b^2fn^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^2}-\frac{2b^2dn^2x}{eg}\end{aligned}$$

```
[In] Int[(x^3*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

```
[Out] (2*a*b*d*n*x)/(e*g) - (2*b^2*d*n^2*x)/(e*g) + (b^2*n^2*(d + e*x)^2)/(4*e^2*
g) + (2*b^2*d*n*(d + e*x)*Log[c*(d + e*x)^n])/(e^2*g) - (b*n*(d + e*x)^2*(a
 + b*Log[c*(d + e*x)^n]))/(2*e^2*g) - (d*(d + e*x)*(a + b*Log[c*(d + e*x)^n
])^2)/(e^2*g) + ((d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^2*g) - (f*(
a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*
Sqrt[g])])/(2*g^2) - (f*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqr
t[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^2) - (b*f*n*(a + b*Log[c*(d + e*x)
^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/g^2 - (b*
f*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] +
 d*Sqrt[g])])/g^2 + (b^2*f*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f]
 - d*Sqrt[g]))])/g^2 + (b^2*f*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f
] + d*Sqrt[g])])/g^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x(a+b\log(c(d+ex)^n))^2}{g} - \frac{fx(a+b\log(c(d+ex)^n))^2}{g(f+gx^2)}\right)dx$$

$$= \frac{\int x(a+b\log(c(d+ex)^n))^2\,dx}{g} - \frac{f\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx}{g}$$

$$= \frac{\int \left(-\frac{d(a+b\log(c(d+ex)^n))^2}{e} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{e}\right)dx}{g} - \frac{f\int \left(-\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)} + \frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g}$$

$$= \frac{f\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{2g^{3/2}} - \frac{f\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{2g^{3/2}} + \frac{\int (d+ex)(a+b\log(c(d+ex)^n))^2\,dx}{eg} - \frac{d\int (a+b\log(c(d+ex)^n))^2\,dx}{eg}$$

$$= -\frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} - \frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} + \frac{\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx, x, d+ex\right)}{e^2g} - \frac{d\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx, x, d+ex\right)}{e^2g} + \frac{(befn)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{g^2} + \frac{(befn)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{g^2}$$

$$= -\frac{d(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g} + \frac{(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g}$$
$$- \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}$$
$$- \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2}$$
$$+ \frac{(bfn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g^2}$$
$$+ \frac{(bfn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g^2}$$
$$- \frac{(bn)\text{Subst}\left(\int x(a+b\log(cx^n))\,dx, x, d+ex\right)}{e^2g}$$
$$+ \frac{(2bdn)\text{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+ex\right)}{e^2g}$$

$$= \frac{2abdnx}{eg} + \frac{b^2n^2(d+ex)^2}{4e^2g} - \frac{bn(d+ex)^2\,(a+b\log\left(c(d+ex)^n\right))}{2e^2g} - \frac{d(d+ex)\,(a+b\log\left(c(d+ex)^n\right))^2}{e^2g} + \frac{(d+ex)^2\,(a+b\log\left(c(d+ex)^n\right))^2}{2e^2g} - \frac{f(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} - \frac{f(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} - \frac{bfn(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2} - \frac{bfn(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2} + \frac{(2b^2dn)\operatorname{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{e^2g} + \frac{(b^2fn^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{g^2} + \frac{(b^2fn^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{g^2}$$

$$= \frac{2abdnx}{eg} - \frac{2b^2dn^2x}{eg} + \frac{b^2n^2(d+ex)^2}{4e^2g} + \frac{2b^2dn(d+ex)\log\left(c(d+ex)^n\right)}{e^2g} - \frac{bn(d+ex)^2\,(a+b\log\left(c(d+ex)^n\right))}{2e^2g} - \frac{d(d+ex)\,(a+b\log\left(c(d+ex)^n\right))^2}{e^2g} + \frac{(d+ex)^2\,(a+b\log\left(c(d+ex)^n\right))^2}{2e^2g} - \frac{f(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} - \frac{f(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} - \frac{bfn(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2} - \frac{bfn(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2} + \frac{b^2fn^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2} + \frac{b^2fn^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.28

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$$
$$= \frac{2e^2gx^2(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2-2e^2f(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f+$$

```
[In] Integrate[(x^3*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

```
[Out] (2*e^2*g*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - 2*e^2*f*(a -
 b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a - b*n
*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(e*g*x*(2*d - e*x) - 2*g*(d^2 - e^2*x
^2)*Log[d + e*x] - 2*e^2*f*(Log[d + e*x]*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*
e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] +
d*Sqrt[g])]) - 2*e^2*f*(Log[d + e*x]*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[
f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])
])) - b^2*n^2*(g*(e*x*(6*d - e*x) + (-6*d^2 - 4*d*e*x + 2*e^2*x^2)*Log[d +
e*x] + 2*(d^2 - e^2*x^2)*Log[d + e*x]^2) + 2*e^2*f*(Log[d + e*x]^2*Log[1 -
(Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[
2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g
]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) + 2*e^2*f*(Log[d + e*x]^2*Log[1
 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[
2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(
d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])))/(4*e^2*g^2)
```

## Maple [F]

$$\int \frac{x^3(a+b\ln{(c(ex+d)^n)})^2}{g\,x^2+f}dx$$

[In] int(x^3*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)

[Out] int(x^3*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)

## Fricas [F]

$$\int \frac{x^3(a+b\log{(c(d+ex)^n)})^2}{f+gx^2}\,dx = \int \frac{(b\log{((ex+d)^n c)}+a)^2x^3}{gx^2+f}\,dx$$

[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="fricas")

[Out] integral((b^2*x^3*log((e*x + d)^n*c)^2 + 2*a*b*x^3*log((e*x + d)^n*c) + a^2*x^3)/(g*x^2 + f), x)

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log{(c(d+ex)^n)})^2}{f+gx^2}\,dx = \text{Timed out}$$

[In] integrate(x**3*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f),x)

[Out] Timed out

## Maxima [F]

$$\int \frac{x^3(a+b\log{(c(d+ex)^n)})^2}{f+gx^2}\,dx = \int \frac{(b\log{((ex+d)^n c)}+a)^2x^3}{gx^2+f}\,dx$$

[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="maxima")

[Out] 1/2*a^2*(x^2/g - f*log(g*x^2 + f)/g^2) + integrate((b^2*x^3*log((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x^3*log((e*x + d)^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x^3)/(g*x^2 + f), x)

## Giac [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2x^3}{gx^2+f}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*x^3/(g*x^2 + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{g\,x^2+f}\,dx$$

```
[In] int((x^3*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2),x)
[Out] int((x^3*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2), x)
```

# 3.312 $\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 317

$$\begin{aligned}\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx &= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g} \\ &\quad+\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g} \\ &\quad+\frac{bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g} \\ &\quad+\frac{bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g} \\ &\quad-\frac{b^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g} \\ &\quad-\frac{b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g}\end{aligned}$$

```
[Out] 1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(
1/2)))/g+1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1
/2)-d*g^(1/2)))/g+b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(
-f)^(1/2)-d*g^(1/2)))/g+b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)
/(e*(-f)^(1/2)+d*g^(1/2)))/g-b^2*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/
2)-d*g^(1/2)))/g-b^2*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2))
)/g
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2463, 2443, 2481, 2421, 6724}

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{g} + \frac{bn\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{g} + \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2g} + \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2g} - \frac{b^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g} - \frac{b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g}$$

```
[In] Int[(x*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] +
d*Sqrt[g])])/(2*g) + ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[
g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g) + (b*n*(a + b*Log[c*(d + e*x)^n])*P
olyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/g + (b*n*(a + b
*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g]
)])/g - (b^2*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))
])/g - (b^2*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
```

```
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx$$

$$= -\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{2\sqrt{g}}+\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{2\sqrt{g}}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g}+\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g}$$

$$-\frac{(ben)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{g}-\frac{(ben)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{g}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2 \log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g} + \frac{(a+b\log(c(d+ex)^n))^2 \log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g} - \frac{(bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g} - \frac{(bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2 \log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g} + \frac{(a+b\log(c(d+ex)^n))^2 \log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g} + \frac{bn(a+b\log(c(d+ex)^n))\,\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g} + \frac{bn(a+b\log(c(d+ex)^n))\,\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g} - \frac{(b^2n^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g} - \frac{(b^2n^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g} + \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g} + \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g} + \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g} - \frac{b^2n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g} - \frac{b^2n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.46

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \frac{(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f+gx^2)+2bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))\left(\mathrm{lo}\right.}{}$$

```
[In] Integrate[(x*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

```
[Out] ((a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a
- b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*(Log[1 - (Sqrt[g]*
(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + Log[1 - (Sqrt[g]*(d + e*x))/(I*e
*Sqrt[f] + d*Sqrt[g])]) + PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] +
d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]) +
b^2*n^2*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqr
t[g])] + Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g
])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqr
t[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqr
t[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2
*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]))/(2*g)
```

## Maple [F]

$$\int \frac{x(a + b\ln(c(ex+d)^n))^2}{g\,x^2 + f}dx$$

```
[In] int(x*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
[Out] int(x*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

## Fricas [F]

$$\int \frac{x(a + b\log(c(d+ex)^n))^2}{f + gx^2}\,dx = \int \frac{(b\log((ex+d)^n c) + a)^2 x}{gx^2 + f}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="fricas")
[Out] integral((b^2*x*log((e*x + d)^n*c)^2 + 2*a*b*x*log((e*x + d)^n*c) + a^2*x)/
(g*x^2 + f), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(a + b\log(c(d+ex)^n))^2}{f + gx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x(a + b\log(c(d+ex)^n))^2}{f + gx^2}\,dx = \int \frac{(b\log((ex+d)^n c) + a)^2 x}{gx^2 + f}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="maxima")
[Out] 1/2*a^2*log(g*x^2 + f)/g + integrate((b^2*x*log((e*x + d)^n)^2 + 2*(b^2*log
(c) + a*b)*x*log((e*x + d)^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x)/(g*x^2 + f
), x)
```

## Giac [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x}{gx^2+f}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*x/(g*x^2 + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{x\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{g\,x^2+f}\,dx$$

```
[In] int((x*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2),x)
[Out] int((x*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2), x)
```

# 3.313 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 397

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx = \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} - \frac{bn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f} - \frac{bn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f} + \frac{2bn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f} + \frac{b^2n^2\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f} + \frac{b^2n^2\,\mathrm{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f} - \frac{2b^2n^2\,\mathrm{PolyLog}\left(3,1+\frac{ex}{d}\right)}{f}$$

```
[Out] ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))^2/f-1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)
^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f-1/2*(a+b*ln(c*(e*x+d)^n))^2*l
n(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/f+2*b*n*(a+b*ln(c*(e*x
+d)^n))*polylog(2,1+e*x/d)/f-b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g
^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f-b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x
```

```
+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f-2*b^2*n^2*polylog(3,1+e*x/d)/f+b^2*
n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f+b^2*n^2*polylog(
3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2463, 2443, 2481, 2421, 6724}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx = -\frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{f} - \frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{f} - \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2f} - \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2f} + \frac{2bn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)(a+b\log(c(d+ex)^n))}{f} + \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f} + \frac{b^2n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f} + \frac{b^2n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{f} - \frac{2b^2n^2\,\text{PolyLog}\left(3,\frac{ex}{d}+1\right)}{f}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(x*(f + g*x^2)),x]
```

```
[Out] (Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n])^2)/f - ((a + b*Log[c*(d + e*x)^
n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f) - ((a
 + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*S
qrt[g])])/(2*f) - (b*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d
+ e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/f - (b*n*(a + b*Log[c*(d + e*x)^n])*Pol
yLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/f + (2*b*n*(a + b*Lo
g[c*(d + e*x)^n])*PolyLog[2, 1 + (e*x)/d])/f + (b^2*n^2*PolyLog[3, -((Sqrt[
g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/f + (b^2*n^2*PolyLog[3, (Sqrt[g]*
(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/f - (2*b^2*n^2*PolyLog[3, 1 + (e*x)/d
])/f
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left( \frac{(a + b \log (c(d+ex)^n))^2}{fx} - \frac{gx(a + b \log (c(d+ex)^n))^2}{f (f + gx^2)} \right) dx$$

$$= \frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{x} \, dx}{f} - \frac{g \int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2} \, dx}{f}$$

$$= \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f} - \frac{g\int\left(-\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx}{f} - \frac{(2ben)\int\frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{d+ex}dx}{f}$$

$$= \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f} + \frac{\sqrt{g}\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}dx}{2f} - \frac{\sqrt{g}\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}dx}{2f} - \frac{(2bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}dx,x,d+ex\right)}{f}$$

$$= \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} + \frac{2bn\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(1+\frac{ex}{d}\right)}{f} + \frac{(ben)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}dx}{f} + \frac{(ben)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}dx}{f} - \frac{(2b^2n^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}dx,x,d+ex\right)}{f}$$

$$= \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} + \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{f} - \frac{2b^2n^2\operatorname{Li}_3\left(1+\frac{ex}{d}\right)}{f} + \frac{(bn)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{f} + \frac{(bn)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{f}$$

$$= \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} - \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f} - \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f} + \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{f} - \frac{2b^2n^2\operatorname{Li}_3\left(1+\frac{ex}{d}\right)}{f} + \frac{(b^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{f} + \frac{(b^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{f}$$

$$
\begin{aligned}
&= \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f} \\
&- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f} \\
&- \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f} \\
&- \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f} \\
&+ \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{f} + \frac{b^2n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f} \\
&+ \frac{b^2n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f} - \frac{2b^2n^2\operatorname{Li}_3\left(1+\frac{ex}{d}\right)}{f}
\end{aligned}
$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.45

$$
\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx =
$$

$$
-\frac{-2\log(x)(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2+(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x*(f + g*x^2)),x]
```

```
[Out] -1/2*(-2*Log[x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + (a - b*n*
Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a - b*n*Log[
d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*Log[1 - (Sqrt[g]*(d + e*x))/
((-I)*e*Sqrt[f] + d*Sqrt[g])] + Log[d + e*x]*Log[1 - (Sqrt[g]*(d + e*x))/(I
*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] +
 d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] -
2*(Log[-((e*x)/d)]*Log[d + e*x] + PolyLog[2, 1 + (e*x)/d])) + b^2*n^2*(-2*L
og[-((e*x)/d)]*Log[d + e*x]^2 + Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/
((-I)*e*Sqrt[f] + d*Sqrt[g])] + Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/
(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/
((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x
))/(I*e*Sqrt[f] + d*Sqrt[g])] - 4*Log[d + e*x]*PolyLog[2, 1 + (e*x)/d] - 2*
PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3,
 (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 4*PolyLog[3, 1 + (e*x)/d]
))/f
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}{x\left(g\,x^{2}+f\right)}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/x/(g*x^2+f),x)
[Out] int((a+b*ln(c*(e*x+d)^n))^2/x/(g*x^2+f),x)
```

## Fricas [F]

$$\int \frac{\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{x\left(f+gx^{2}\right)}\,dx = \int \frac{\left(b\log\left((ex+d)^{n}c\right)+a\right)^{2}}{(gx^{2}+f)x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x/(g*x^2+f),x, algorithm="fricas")
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x^3
 + f*x), x)
```

## Sympy [F]

$$\int \frac{\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{x\left(f+gx^{2}\right)}\,dx = \int \frac{\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{x\left(f+gx^{2}\right)}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/x/(g*x**2+f),x)
[Out] Integral((a + b*log(c*(d + e*x)**n))**2/(x*(f + g*x**2)), x)
```

## Maxima [F]

$$\int \frac{\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{x\left(f+gx^{2}\right)}\,dx = \int \frac{\left(b\log\left((ex+d)^{n}c\right)+a\right)^{2}}{(gx^{2}+f)x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x/(g*x^2+f),x, algorithm="maxima")
[Out] -1/2*a^2*(log(g*x^2 + f)/f - 2*log(x)/f) + integrate((b^2*log((e*x + d)^n)^
2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log((e*x + d)^n))/(g
*x^3 + f*x), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2}{(gx^2+f)x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x/(g*x^2+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)}\,dx=\int \frac{(a+b\ln(c(d+e\,x)^n))^2}{x\,(g\,x^2+f)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(x*(f + g*x^2)),x)
[Out] int((a + b*log(c*(d + e*x)^n))^2/(x*(f + g*x^2)), x)
```

# 3.314 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx$

## Optimal result

Integrand size = 29, antiderivative size = 551

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx = \frac{b^2e^2n^2\log(x)}{d^2f} - \frac{ben(d+ex)(a+b\log(c(d+ex)^n))}{d^2fx} - \frac{(a+b\log(c(d+ex)^n))^2}{2fx^2} - \frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} - \frac{be^2n(a+b\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{d^2f} + \frac{b^2e^2n^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d^2f} + \frac{bgn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2} + \frac{bgn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2} - \frac{2bgn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^2} - \frac{b^2gn^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2} - \frac{b^2gn^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2} + \frac{2b^2gn^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{f^2}$$

```
[Out] b^2*e^2*n^2*ln(x)/d^2/f-b*e*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))/d^2/f/x-1/2*(a+
b*ln(c*(e*x+d)^n))^2/f/x^2-g*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))^2/f^2-b*e^2*n
*(a+b*ln(c*(e*x+d)^n))*ln(1-d/(e*x+d))/d^2/f+1/2*g*(a+b*ln(c*(e*x+d)^n))^2*
ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f^2+1/2*g*(a+b*ln(c*(
e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/f^2+b^2*
e^2*n^2*polylog(2,d/(e*x+d))/d^2/f-2*b*g*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,
1+e*x/d)/f^2+b*g*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)
^(1/2)-d*g^(1/2)))/f^2+b*g*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2
```

```
)/(e*(-f)^(1/2)+d*g^(1/2)))/f^2+2*b^2*g*n^2*polylog(3,1+e*x/d)/f^2-b^2*g*n^
2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f^2-b^2*g*n^2*polylo
g(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f^2
```

## Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2463, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2443, 2481, 2421, 6724}

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx = &-\frac{be^2n\log\left(1-\frac{d}{d+ex}\right)(a+b\log(c(d+ex)^n))}{d^2f}\\ &-\frac{ben(d+ex)(a+b\log(c(d+ex)^n))}{d^2fx}\\ &+\frac{bgn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{f^2}\\ &+\frac{bgn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{f^2}\\ &-\frac{2bgn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)(a+b\log(c(d+ex)^n))}{f^2}\\ &-\frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2}\\ &+\frac{g\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2f^2}\\ &+\frac{g\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2f^2}\\ &-\frac{(a+b\log(c(d+ex)^n))^2}{2fx^2}+\frac{b^2e^2n^2\,\text{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d^2f}\\ &+\frac{b^2e^2n^2\log(x)}{d^2f}-\frac{b^2gn^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2}\\ &-\frac{b^2gn^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{f^2}\\ &+\frac{2b^2gn^2\,\text{PolyLog}\left(3,\frac{ex}{d}+1\right)}{f^2}\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(x^3*(f + g*x^2)),x]
```

```
[Out] (b^2*e^2*n^2*Log[x])/(d^2*f) - (b*e*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n]))
/(d^2*f*x) - (a + b*Log[c*(d + e*x)^n])^2/(2*f*x^2) - (g*Log[-((e*x)/d)]*(a
```

```
 + b*Log[c*(d + e*x)^n])^2)/f^2 + (g*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(S
qrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^2) + (g*(a + b*Log[c*
(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(
2*f^2) - (b*e^2*n*(a + b*Log[c*(d + e*x)^n])*Log[1 - d/(d + e*x)])/(d^2*f)
+ (b^2*e^2*n^2*PolyLog[2, d/(d + e*x)])/(d^2*f) + (b*g*n*(a + b*Log[c*(d +
e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/f^2 +
 (b*g*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-
f] + d*Sqrt[g])])/f^2 - (2*b*g*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, 1 +
(e*x)/d])/f^2 - (b^2*g*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d
*Sqrt[g]))])/f^2 - (b^2*g*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] +
d*Sqrt[g])])/f^2 + (2*b^2*g*n^2*PolyLog[3, 1 + (e*x)/d])/f^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+b\log(c(d+ex)^n))^2}{fx^3} - \frac{g(a+b\log(c(d+ex)^n))^2}{f^2x} + \frac{g^2x(a+b\log(c(d+ex)^n))^2}{f^2(f+gx^2)}\right)dx$$

$$= \frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3}\,dx}{f} - \frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx}{f^2} + \frac{g^2\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx}{f^2}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^2}{2fx^2} - \frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2} + \frac{g^2\int\left(-\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)} + \frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right)dx}{f^2} + \frac{(ben)\int \frac{a+b\log(c(d+ex)^n)}{x^2(d+ex)}\,dx}{f} + \frac{(2begn)\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{d+ex}\,dx}{f^2}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^2}{2fx^2} - \frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2} - \frac{g^{3/2}\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{2f^2} + \frac{g^{3/2}\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{2f^2} + \frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex\right)}{f} + \frac{(2bgn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+ex\right)}{f^2}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^2}{2fx^2} - \frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} - \frac{2bgn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} + \frac{(bn)\operatorname{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex\right)}{df} - \frac{(ben)\operatorname{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+ex\right)}{df} - \frac{(begn)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{f^2} - \frac{(begn)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{f^2} + \frac{(2b^2gn^2)\operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(\frac{x}{d}\right)}{x}\,dx, x, d+ex\right)}{f^2}$$

$$= -\frac{ben(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{d^2fx} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2fx^2}$$
$$- \frac{g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f^2}$$
$$+ \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}$$
$$+ \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}$$
$$- \frac{be^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(1-\frac{d}{d+ex}\right)}{d^2f}$$
$$- \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} + \frac{2b^2gn^2\mathrm{Li}_3\left(1+\frac{ex}{d}\right)}{f^2}$$
$$- \frac{(bgn)\mathrm{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{f^2}$$
$$- \frac{(bgn)\mathrm{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{f^2}$$
$$+ \frac{(b^2en^2)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex\right)}{d^2f}$$
$$+ \frac{(b^2e^2n^2)\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx, x, d+ex\right)}{d^2f}$$

$$= \frac{b^2e^2n^2\log(x)}{d^2f} - \frac{ben(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{d^2fx}$$
$$- \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2fx^2} - \frac{g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f^2}$$
$$+ \frac{g(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}$$
$$+ \frac{g(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}$$
$$- \frac{be^2n(a+b\log\left(c(d+ex)^n\right))\log\left(1-\frac{d}{d+ex}\right)}{d^2f} + \frac{b^2e^2n^2\text{Li}_2\left(\frac{d}{d+ex}\right)}{d^2f}$$
$$+ \frac{bgn(a+b\log\left(c(d+ex)^n\right))\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2}$$
$$+ \frac{bgn(a+b\log\left(c(d+ex)^n\right))\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2}$$
$$- \frac{2bgn(a+b\log\left(c(d+ex)^n\right))\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} + \frac{2b^2gn^2\text{Li}_3\left(1+\frac{ex}{d}\right)}{f^2}$$
$$- \frac{(b^2gn^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{f^2}$$
$$- \frac{(b^2gn^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{f^2}$$

$$= \frac{b^2e^2n^2\log(x)}{d^2f} - \frac{ben(d+ex)(a+b\log(c(d+ex)^n))}{d^2fx} - \frac{(a+b\log(c(d+ex)^n))^2}{2fx^2} - \frac{g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} - \frac{be^2n(a+b\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{d^2f} + \frac{b^2e^2n^2\operatorname{Li}_2\left(\frac{d}{d+ex}\right)}{d^2f} + \frac{bgn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2} + \frac{bgn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2} - \frac{2bgn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} - \frac{b^2gn^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2} - \frac{b^2gn^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2} + \frac{2b^2gn^2\operatorname{Li}_3\left(1+\frac{ex}{d}\right)}{f^2}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 811, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx = \frac{-d^2f(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2 - 2d^2gx^2\log(x)(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x^3*(f + g*x^2)),x]
```

```
[Out] (-(d^2*f*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2) - 2*d^2*g*x^2*Log
[x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + d^2*g*x^2*(a - b*n*Lo
g[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] - 2*b*n*(a - b*n*Log[d
+ e*x] + b*Log[c*(d + e*x)^n])*(f*(d*e*x + e^2*x^2*Log[x] + (d^2 - e^2*x^2)
*Log[d + e*x]) - d^2*g*x^2*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e
*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] -
 I*d*Sqrt[g])]) - d^2*g*x^2*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(
e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I
```

```
*d*Sqrt[g])]) + 2*d^2*g*x^2*(Log[-((e*x)/d)]*Log[d + e*x] + PolyLog[2, 1 +
(e*x)/d])) + b^2*n^2*(f*(2*e^2*x^2*Log[x] - Log[d + e*x]*(2*e^2*x^2*Log[-((
e*x)/d)] + (d + e*x)*(2*e*x + (d - e*x)*Log[d + e*x])) - 2*e^2*x^2*PolyLog[
2, 1 + (e*x)/d]) + d^2*g*x^2*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((
-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))
/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*S
qrt[f] + d*Sqrt[g])]) + d^2*g*x^2*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x
))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x
))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[
f] + d*Sqrt[g])]) - 2*d^2*g*x^2*(Log[-((e*x)/d)]*Log[d + e*x]^2 + 2*Log[d +
 e*x]*PolyLog[2, 1 + (e*x)/d] - 2*PolyLog[3, 1 + (e*x)/d])))/(2*d^2*f^2*x^2
)
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}{x^{3}\left(g\,x^{2}+f\right)}dx$$

[In] int((a+b*ln(c*(e*x+d)^n))^2/x^3/(g*x^2+f),x)

[Out] int((a+b*ln(c*(e*x+d)^n))^2/x^3/(g*x^2+f),x)

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{(gx^2+f)x^3}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^3/(g*x^2+f),x, algorithm="fricas")

[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x^5
 + f*x^3), x)

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)}\,dx = \text{Timed out}$$

[In] integrate((a+b*ln(c*(e*x+d)**n))**2/x**3/(g*x**2+f),x)

[Out] Timed out

## Maxima [F]

$$\int \frac{(a + b\log (c(d + ex)^n))^2}{x^3 (f + gx^2)} dx = \int \frac{(b\log ((ex + d)^n c) + a)^2}{(gx^2 + f)x^3} dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^3/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(g*log(g*x^2 + f)/f^2 - 2*g*log(x)/f^2 - 1/(f*x^2)) + integrate((b^
2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*l
og((e*x + d)^n))/(g*x^5 + f*x^3), x)
```

## Giac [F]

$$\int \frac{(a + b\log (c(d + ex)^n))^2}{x^3 (f + gx^2)} dx = \int \frac{(b\log ((ex + d)^n c) + a)^2}{(gx^2 + f)x^3} dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^3/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log (c(d + ex)^n))^2}{x^3 (f + gx^2)} dx = \int \frac{(a + b\ln (c(d + e\,x)^n))^2}{x^3\,(g\,x^2 + f)} dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(x^3*(f + g*x^2)),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^2/(x^3*(f + g*x^2)), x)
```

# 3.315 $\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 701

$$\begin{aligned}
&\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx\\
&= \frac{2abfnx}{g^2}-\frac{2b^2fn^2x}{g^2}+\frac{2b^2d^2n^2x}{e^2g}-\frac{b^2dn^2(d+ex)^2}{2e^3g}+\frac{2b^2n^2(d+ex)^3}{27e^3g}\\
&\quad-\frac{b^2d^3n^2\log^2(d+ex)}{3e^3g}+\frac{2b^2fn(d+ex)\log(c(d+ex)^n)}{eg^2}\\
&\quad-\frac{2bd^2n(d+ex)\left(a+b\log(c(d+ex)^n)\right)}{e^3g}+\frac{bdn(d+ex)^2\left(a+b\log(c(d+ex)^n)\right)}{e^3g}\\
&\quad-\frac{2bn(d+ex)^3\left(a+b\log(c(d+ex)^n)\right)}{9e^3g}+\frac{2bd^3n\log(d+ex)\left(a+b\log(c(d+ex)^n)\right)}{3e^3g}\\
&\quad+\frac{x^3\left(a+b\log(c(d+ex)^n)\right)^2}{3g}-\frac{f(d+ex)\left(a+b\log(c(d+ex)^n)\right)^2}{eg^2}\\
&\quad+\frac{(-f)^{3/2}\left(a+b\log(c(d+ex)^n)\right)^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}}\\
&\quad-\frac{(-f)^{3/2}\left(a+b\log(c(d+ex)^n)\right)^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}}\\
&\quad-\frac{b(-f)^{3/2}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}}\\
&\quad+\frac{b(-f)^{3/2}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}}\\
&\quad+\frac{b^2(-f)^{3/2}n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}}-\frac{b^2(-f)^{3/2}n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}}
\end{aligned}$$

```
[Out] 2*a*b*f*n*x/g^2-2*b^2*f*n^2*x/g^2+2*b^2*d^2*n^2*x/e^2/g-1/2*b^2*d*n^2*(e*x+
d)^2/e^3/g+2/27*b^2*n^2*(e*x+d)^3/e^3/g-1/3*b^2*d^3*n^2*ln(e*x+d)^2/e^3/g+2
*b^2*f*n*(e*x+d)*ln(c*(e*x+d)^n)/e/g^2-2*b*d^2*n*(e*x+d)*(a+b*ln(c*(e*x+d)^
n))/e^3/g+b*d*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^3/g-2/9*b*n*(e*x+d)^3*(a+
b*ln(c*(e*x+d)^n))/e^3/g+2/3*b*d^3*n*ln(e*x+d)*(a+b*ln(c*(e*x+d)^n))/e^3/g+
1/3*x^3*(a+b*ln(c*(e*x+d)^n))^2/g-f*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e/g^2+1
/2*(-f)^(3/2)*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(
1/2)+d*g^(1/2)))/g^(5/2)-1/2*(-f)^(3/2)*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^
(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^(5/2)-b*(-f)^(3/2)*n*(a+b*ln(c
*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^(5/2)+b
*(-f)^(3/2)*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)
+d*g^(1/2)))/g^(5/2)+b^2*(-f)^(3/2)*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^
(1/2)-d*g^(1/2)))/g^(5/2)-b^2*(-f)^(3/2)*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(
-f)^(1/2)+d*g^(1/2)))/g^(5/2)
```

## Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.552$, Rules used = {2463, 2436, 2333, 2332, 2445, 2458, 45, 2372, 12, 14, 2338, 2456, 2443, 2481, 2421,

6724}

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$$
$$= \frac{2bd^3n\log(d+ex)(a+b\log(c(d+ex)^n))}{3e^3g} - \frac{2bd^2n(d+ex)(a+b\log(c(d+ex)^n))}{e^3g}$$
$$+ \frac{bdn(d+ex)^2(a+b\log(c(d+ex)^n))}{e^3g} - \frac{2bn(d+ex)^3(a+b\log(c(d+ex)^n))}{9e^3g}$$
$$- \frac{b(-f)^{3/2}n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{g^{5/2}}$$
$$+ \frac{b(-f)^{3/2}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{g^{5/2}}$$
$$+ \frac{(-f)^{3/2}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2g^{5/2}}$$
$$- \frac{(-f)^{3/2}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2g^{5/2}}$$
$$- \frac{f(d+ex)(a+b\log(c(d+ex)^n))^2}{eg^2} + \frac{x^3(a+b\log(c(d+ex)^n))^2}{3g} + \frac{2abfnx}{g^2}$$
$$+ \frac{2b^2fn(d+ex)\log(c(d+ex)^n)}{eg^2} - \frac{b^2d^3n^2\log^2(d+ex)}{3e^3g} + \frac{2b^2d^2n^2x}{e^2g}$$
$$- \frac{b^2dn^2(d+ex)^2}{2e^3g} + \frac{2b^2n^2(d+ex)^3}{27e^3g} + \frac{b^2(-f)^{3/2}n^2\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}}$$
$$- \frac{b^2(-f)^{3/2}n^2\,\mathrm{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^{5/2}} - \frac{2b^2fn^2x}{g^2}$$

```
[In] Int[(x^4*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

```
[Out] (2*a*b*f*n*x)/g^2 - (2*b^2*f*n^2*x)/g^2 + (2*b^2*d^2*n^2*x)/(e^2*g) - (b^2*
d*n^2*(d + e*x)^2)/(2*e^3*g) + (2*b^2*n^2*(d + e*x)^3)/(27*e^3*g) - (b^2*d^
3*n^2*Log[d + e*x]^2)/(3*e^3*g) + (2*b^2*f*n*(d + e*x)*Log[c*(d + e*x)^n])/
(e*g^2) - (2*b*d^2*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n]))/(e^3*g) + (b*d*n
*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(e^3*g) - (2*b*n*(d + e*x)^3*(a +
b*Log[c*(d + e*x)^n]))/(9*e^3*g) + (2*b*d^3*n*Log[d + e*x]*(a + b*Log[c*(d
+ e*x)^n]))/(3*e^3*g) + (x^3*(a + b*Log[c*(d + e*x)^n])^2)/(3*g) - (f*(d +
e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g^2) + ((-f)^(3/2)*(a + b*Log[c*(d +
e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^(
5/2)) - ((-f)^(3/2)*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]
*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^(5/2)) - (b*(-f)^(3/2)*n*(a + b*Log[c*
(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/
g^(5/2) + (b*(-f)^(3/2)*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d
```

```
 + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^(5/2) + (b^2*(-f)^(3/2)*n^2*PolyLog[3
, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/g^(5/2) - (b^2*(-f)^(3/
2)*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{f(a+b\log(c(d+ex)^n))^2}{g^2}+\frac{x^2(a+b\log(c(d+ex)^n))^2}{g}+\frac{f^2(a+b\log(c(d+ex)^n))^2}{g^2(f+gx^2)}\right)dx \\
&= -\frac{f\int(a+b\log(c(d+ex)^n))^2\,dx}{g^2}+\frac{f^2\int\frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx}{g^2}+\frac{\int x^2(a+b\log(c(d+ex)^n))^2\,dx}{g} \\
&= \frac{x^3(a+b\log(c(d+ex)^n))^2}{3g}-\frac{f\text{Subst}\left(\int(a+b\log(cx^n))^2\,dx,x,d+ex\right)}{eg^2} \\
&\quad+\frac{f^2\int\left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}-\sqrt{g}x)}+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g^2} \\
&\quad-\frac{(2ben)\int\frac{x^3(a+b\log(c(d+ex)^n))}{d+ex}\,dx}{3g}
\end{aligned}
$$

$$
= \frac{x^3(a+b\log(c(d+ex)^n))^2}{3g} - \frac{f(d+ex)(a+b\log(c(d+ex)^n))^2}{eg^2} - \frac{(-f)^{3/2}\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{2g^2} - \frac{(-f)^{3/2}\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{2g^2} + \frac{(2bfn)\text{Subst}\left(\int(a+b\log(cx^n))\,dx,x,d+ex\right)}{eg^2} - \frac{(2bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3(a+b\log(cx^n))}{x}\,dx,x,d+ex\right)}{3g}
$$

$$
= \frac{2abfnx}{g^2} - \frac{2bd^2n(d+ex)(a+b\log(c(d+ex)^n))}{e^3g} + \frac{bdn(d+ex)^2(a+b\log(c(d+ex)^n))}{e^3g} - \frac{2bn(d+ex)^3(a+b\log(c(d+ex)^n))}{9e^3g} + \frac{2bd^3n\log(d+ex)(a+b\log(c(d+ex)^n))}{3e^3g} + \frac{x^3(a+b\log(c(d+ex)^n))^2}{3g} - \frac{f(d+ex)(a+b\log(c(d+ex)^n))^2}{eg^2} + \frac{(-f)^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}} - \frac{(-f)^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} - \frac{\left(be(-f)^{3/2}n\right)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{g^{5/2}} + \frac{\left(be(-f)^{3/2}n\right)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{g^{5/2}} + \frac{(2b^2fn)\,\text{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{eg^2} + \frac{(2b^2n^2)\,\text{Subst}\left(\int\frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{6e^3x}\,dx,x,d+ex\right)}{3g}
$$

$$
\begin{aligned}
&= \frac{2abfnx}{g^2} - \frac{2b^2fn^2x}{g^2} + \frac{2b^2fn(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} \\
&- \frac{2bd^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3g} \\
&+ \frac{bdn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3g} - \frac{2bn(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3g} \\
&+ \frac{2bd^3n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^3g} \\
&+ \frac{x^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g} - \frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2} \\
&+ \frac{(-f)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}} \\
&- \frac{(-f)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} \\
&- \frac{\left(b(-f)^{3/2}n\right)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g^{5/2}} \\
&+ \frac{\left(b(-f)^{3/2}n\right)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g^{5/2}} \\
&+ \frac{(b^2n^2)\,\text{Subst}\left(\int \frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{x}dx, x, d+ex\right)}{9e^3g}
\end{aligned}
$$

$$= \frac{2abfnx}{g^2} - \frac{2b^2fn^2x}{g^2} + \frac{2b^2fn(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} - \frac{2bd^2n(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3g} + \frac{bdn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3g} - \frac{2bn(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3g} + \frac{2bd^3n\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^3g} + \frac{x^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{3g} - \frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2} + \frac{(-f)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}} - \frac{(-f)^{3/2}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} - \frac{b(-f)^{3/2}n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}} + \frac{b(-f)^{3/2}n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}} + \frac{\left(b^2(-f)^{3/2}n^2\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{g^{5/2}} - \frac{\left(b^2(-f)^{3/2}n^2\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{g^{5/2}} + \frac{\left(b^2n^2\right)\operatorname{Subst}\left(\int\left(18d^2-9dx+2x^2-\frac{6d^3\log(x)}{x}\right)dx,x,d+ex\right)}{9e^3g}$$

$$= \frac{2abfnx}{g^2} - \frac{2b^2fn^2x}{g^2} + \frac{2b^2d^2n^2x}{e^2g} - \frac{b^2dn^2(d+ex)^2}{2e^3g} + \frac{2b^2n^2(d+ex)^3}{27e^3g}$$
$$+ \frac{2b^2fn(d+ex)\log(c(d+ex)^n)}{eg^2} - \frac{2bd^2n(d+ex)(a+b\log(c(d+ex)^n))}{e^3g}$$
$$+ \frac{bdn(d+ex)^2(a+b\log(c(d+ex)^n))}{e^3g} - \frac{2bn(d+ex)^3(a+b\log(c(d+ex)^n))}{9e^3g}$$
$$+ \frac{2bd^3n\log(d+ex)(a+b\log(c(d+ex)^n))}{3e^3g}$$
$$+ \frac{x^3(a+b\log(c(d+ex)^n))^2}{3g} - \frac{f(d+ex)(a+b\log(c(d+ex)^n))^2}{eg^2}$$
$$+ \frac{(-f)^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}}$$
$$- \frac{(-f)^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}}$$
$$- \frac{b(-f)^{3/2}n(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}}$$
$$+ \frac{b(-f)^{3/2}n(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}}$$
$$+ \frac{b^2(-f)^{3/2}n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}} - \frac{b^2(-f)^{3/2}n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}}$$
$$- \frac{(2b^2d^3n^2)\operatorname{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,d+ex\right)}{3e^3g}$$

$$= \frac{2abfnx}{g^2} - \frac{2b^2fn^2x}{g^2} + \frac{2b^2d^2n^2x}{e^2g} - \frac{b^2dn^2(d+ex)^2}{2e^3g} + \frac{2b^2n^2(d+ex)^3}{27e^3g} - \frac{b^2d^3n^2\log^2(d+ex)}{3e^3g} + \frac{2b^2fn(d+ex)\log(c(d+ex)^n)}{eg^2} - \frac{2bd^2n(d+ex)(a+b\log(c(d+ex)^n))}{e^3g} + \frac{bdn(d+ex)^2(a+b\log(c(d+ex)^n))}{e^3g} - \frac{2bn(d+ex)^3(a+b\log(c(d+ex)^n))}{9e^3g} + \frac{2bd^3n\log(d+ex)(a+b\log(c(d+ex)^n))}{3e^3g} + \frac{x^3(a+b\log(c(d+ex)^n))^2}{3g} - \frac{f(d+ex)(a+b\log(c(d+ex)^n))^2}{eg^2} + \frac{(-f)^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}} - \frac{(-f)^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} - \frac{b(-f)^{3/2}n(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}} + \frac{b(-f)^{3/2}n(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}} + \frac{b^2(-f)^{3/2}n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}} - \frac{b^2(-f)^{3/2}n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$$

$$= \frac{-54e^3f\sqrt{g}x(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2 + 18e^3g^{3/2}x^3(a-bn\log(d+ex)+b\log(c(d+ex)^n)}{}$$

```
[In] Integrate[(x^4*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

```
[Out] (-54*e^3*f*Sqrt[g]*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 18*e
^3*g^(3/2)*x^3*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 54*e^3*f^(
3/2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^
n])^2 + 6*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(-18*e^2*f*Sqrt
```

```
[g]*(d + e*x)*(-1 + Log[d + e*x]) + g^(3/2)*(e*x*(-6*d^2 + 3*d*e*x - 2*e^2*
x^2) + 6*(d^3 + e^3*x^3)*Log[d + e*x]) + (9*I)*e^3*f^(3/2)*(Log[d + e*x]*Lo
g[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[
g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) - (9*I)*e^3*f^(3/2)*(Log[d + e
*x]*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (Sq
rt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])) + b^2*n^2*(-54*e^2*f*Sqrt[g]*
(2*e*x - 2*(d + e*x)*Log[d + e*x] + (d + e*x)*Log[d + e*x]^2) + g^(3/2)*(e*
x*(66*d^2 - 15*d*e*x + 4*e^2*x^2) - 6*(11*d^3 + 6*d^2*e*x - 3*d*e^2*x^2 + 2
*e^3*x^3)*Log[d + e*x] + 18*(d^3 + e^3*x^3)*Log[d + e*x]^2) + (27*I)*e^3*f^
(3/2)*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[
g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sq
rt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) -
 (27*I)*e^3*f^(3/2)*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f
] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f
] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g]
)])))/(54*e^3*g^(5/2))
```

## Maple [F]

$$\int \frac{x^4(a+b\ln(c(ex+d)^n))^2}{g\,x^2+f}dx$$

[In] int(x^4*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)

[Out] int(x^4*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)

## Fricas [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^4}{gx^2+f}\,dx$$

[In] integrate(x^4*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="fricas")

[Out] integral((b^2*x^4*log((e*x + d)^n*c)^2 + 2*a*b*x^4*log((e*x + d)^n*c) + a^2
*x^4)/(g*x^2 + f), x)

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx=\text{Timed out}$$

```
[In] integrate(x**4*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2x^4}{gx^2+f}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="maxima")
[Out] 1/3*a^2*(3*f^2*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g^2) + (g*x^3 - 3*f*x)/g^2)
 + integrate((b^2*x^4*log((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x^4*log((e*
x + d)^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x^4)/(g*x^2 + f), x)
```

## Giac [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2x^4}{gx^2+f}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*x^4/(g*x^2 + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx=\int \frac{x^4\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2}{g\,x^2+f}\,dx$$

```
[In] int((x^4*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2),x)
[Out] int((x^4*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2), x)
```

# 3.316 $\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 447

$$\begin{aligned}\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx &= -\frac{2abnx}{g}+\frac{2b^2n^2x}{g}-\frac{2b^2n(d+ex)\log(c(d+ex)^n)}{eg}\\
&\quad+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{eg}+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}}\\
&\quad-\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}}\\
&\quad-\frac{b\sqrt{-f}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{3/2}}\\
&\quad+\frac{b\sqrt{-f}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{3/2}}\\
&\quad+\frac{b^2\sqrt{-f}n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{3/2}}-\frac{b^2\sqrt{-f}n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{3/2}}\end{aligned}$$

```
[Out] -2*a*b*n*x/g+2*b^2*n^2*x/g-2*b^2*n*(e*x+d)*ln(c*(e*x+d)^n)/e/g+(e*x+d)*(a+b
*ln(c*(e*x+d)^n))^2/e/g+1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1
/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(3/2)-1/2*(a+b*ln(c*(e*x+d)^n))
^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))*(-f)^(1/2)/g^(3/2)
-b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/
2)))*(-f)^(1/2)/g^(3/2)+b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)
/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(3/2)+b^2*n^2*polylog(3,-(e*x+d)*g^
(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*(-f)^(1/2)/g^(3/2)-b^2*n^2*polylog(3,(e*x+d
)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(3/2)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2463, 2436, 2333, 2332, 2456, 2443, 2481, 2421, 6724}

$$\begin{aligned}
&\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx \\
&= -\frac{b\sqrt{-f}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{g^{3/2}} \\
&+\frac{b\sqrt{-f}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{g^{3/2}} \\
&+\frac{\sqrt{-f}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2g^{3/2}} \\
&-\frac{\sqrt{-f}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2g^{3/2}} \\
&+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{eg}-\frac{2abnx}{g}-\frac{2b^2n(d+ex)\log(c(d+ex)^n)}{eg} \\
&+\frac{b^2\sqrt{-f}n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{3/2}}-\frac{b^2\sqrt{-f}n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^{3/2}}+\frac{2b^2n^2x}{g}
\end{aligned}$$

```
[In] Int[(x^2*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

```
[Out] (-2*a*b*n*x)/g + (2*b^2*n^2*x)/g - (2*b^2*n*(d + e*x)*Log[c*(d + e*x)^n])/(
e*g) + ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g) + (Sqrt[-f]*(a + b*Lo
g[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])
])/(2*g^(3/2)) - (Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] +
Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^(3/2)) - (b*Sqrt[-f]*n*(a + b*L
og[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]
))])/g^(3/2) + (b*Sqrt[-f]*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]
*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^(3/2) + (b^2*Sqrt[-f]*n^2*PolyLog[
3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/g^(3/2) - (b^2*Sqrt[-f
]*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^(3/2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
```

```
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(a+b\log(c(d+ex)^n))^2}{g}-\frac{f(a+b\log(c(d+ex)^n))^2}{g(f+gx^2)}\right)dx\\
&= \frac{\int (a+b\log(c(d+ex)^n))^2\,dx}{g}-\frac{f\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx}{g}\\
&= \frac{\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx,x,d+ex\right)}{eg}\\
&\quad -\frac{f\int \left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}-\sqrt{g}x)}+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g}\\
&= \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{eg}-\frac{\sqrt{-f}\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{2g}\\
&\quad -\frac{\sqrt{-f}\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{2g}-\frac{(2bn)\text{Subst}\left(\int (a+b\log(cx^n))\,dx,x,d+ex\right)}{eg}\\
&= -\frac{2abnx}{g}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{eg}\\
&\quad +\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}}\\
&\quad -\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}}\\
&\quad -\frac{\left(be\sqrt{-f}n\right)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{g^{3/2}}\\
&\quad +\frac{\left(be\sqrt{-f}n\right)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{g^{3/2}}\\
&\quad -\frac{(2b^2n)\,\text{Subst}\left(\int \log(cx^n)\,dx,x,d+ex\right)}{eg}
\end{aligned}$$

$$= -\frac{2abnx}{g} + \frac{2b^2n^2x}{g} - \frac{2b^2n(d+ex)\log(c(d+ex)^n)}{eg} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{eg} + \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}} - \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}} - \frac{(b\sqrt{-f}n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g^{3/2}} + \frac{(b\sqrt{-f}n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g^{3/2}}$$

$$= -\frac{2abnx}{g} + \frac{2b^2n^2x}{g} - \frac{2b^2n(d+ex)\log(c(d+ex)^n)}{eg} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{eg} + \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}} - \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}} - \frac{b\sqrt{-f}n(a+b\log(c(d+ex)^n))\,\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{3/2}} + \frac{b\sqrt{-f}n(a+b\log(c(d+ex)^n))\,\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{3/2}} + \frac{(b^2\sqrt{-f}n^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g^{3/2}} - \frac{(b^2\sqrt{-f}n^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g^{3/2}}$$

$$
\begin{aligned}
&= -\frac{2abnx}{g} + \frac{2b^2n^2x}{g} - \frac{2b^2n(d+ex)\log\left(c(d+ex)^n\right)}{eg} \\
&+ \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg} \\
&+ \frac{\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{3/2}} \\
&- \frac{\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{3/2}} \\
&- \frac{b\sqrt{-f}n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{3/2}} \\
&+ \frac{b\sqrt{-f}n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{3/2}} \\
&+ \frac{b^2\sqrt{-f}n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{3/2}} - \frac{b^2\sqrt{-f}n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{3/2}}
\end{aligned}
$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.39

$$
\int \frac{x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx^2}\,dx
$$

$$
= \frac{e\sqrt{g}x\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2 - e\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)\right.\right.}{}
$$

```
[In] Integrate[(x^2*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2),x]
```

```
[Out] (e*Sqrt[g]*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - e*Sqrt[f]*Ar
cTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 +
 I*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((-2*I)*Sqrt[g]*(d + e
*x)*(-1 + Log[d + e*x]) - e*Sqrt[f]*(Log[d + e*x]*Log[1 - (Sqrt[g]*(d + e*x
))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*S
qrt[f] + d*Sqrt[g])]) + e*Sqrt[f]*(Log[d + e*x]*Log[1 - (Sqrt[g]*(d + e*x))
/(I*e*Sqrt[f] + d*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] +
 d*Sqrt[g])])) + b^2*n^2*(Sqrt[g]*(2*e*x - 2*(d + e*x)*Log[d + e*x] + (d +
e*x)*Log[d + e*x]^2) - (I/2)*e*Sqrt[f]*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d
+ e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*
(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))
/((-I)*e*Sqrt[f] + d*Sqrt[g])]) + (I/2)*e*Sqrt[f]*(Log[d + e*x]^2*Log[1 - (
Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (
```

```
Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d +
e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]))/(e*g^(3/2))
```

## Maple [F]

$$\int \frac{x^2(a+b\ln(c(ex+d)^n))^2}{g\,x^2+f}\,dx$$

```
[In] int(x^2*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
[Out] int(x^2*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

## Fricas [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^2}{gx^2+f}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="fricas")
[Out] integral((b^2*x^2*log((e*x + d)^n*c)^2 + 2*a*b*x^2*log((e*x + d)^n*c) + a^2
*x^2)/(g*x^2 + f), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^2}{gx^2+f}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="maxima")
[Out] -a^2*(f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g) - x/g) + integrate((b^2*x^2*log
((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x^2*log((e*x + d)^n) + (b^2*log(c)^2
 + 2*a*b*log(c))*x^2)/(g*x^2 + f), x)
```

## Giac [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^2}{gx^2+f}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*x^2/(g*x^2 + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{g\,x^2+f}\,dx$$

```
[In] int((x^2*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2),x)
[Out] int((x^2*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2), x)
```

# 3.317 $\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 371

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{bn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}} + \frac{bn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}} + \frac{b^2n^2\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}} - \frac{b^2n^2\,\mathrm{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}}$$

```
[Out] 1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(
1/2)))/(-f)^(1/2)/g^(1/2)-1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^
(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)-b*n*(a+b*ln(c*(e*x+d)^n
))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)+
b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)
))/(-f)^(1/2)/g^(1/2)+b^2*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^
(1/2)))/(-f)^(1/2)/g^(1/2)-b^2*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+
d*g^(1/2)))/(-f)^(1/2)/g^(1/2)
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2456, 2443, 2481, 2421, 6724}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = -\frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{-f}\sqrt{g}} + \frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{\sqrt{-f}\sqrt{g}} + \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2\sqrt{-f}\sqrt{g}} + \frac{b^2n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}} - \frac{b^2n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{\sqrt{-f}\sqrt{g}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x^2),x]
```

```
[Out] ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] +
d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(S
qrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (b*
n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] -
 d*Sqrt[g]))])/(Sqrt[-f]*Sqrt[g]) + (b*n*(a + b*Log[c*(d + e*x)^n])*PolyLog
[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(Sqrt[-f]*Sqrt[g]) + (b^
2*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(Sqrt[-f
]*Sqrt[g]) - (b^2*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g
])])/(Sqrt[-f]*Sqrt[g])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx$$

$$= -\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{2\sqrt{-f}} - \frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{2\sqrt{-f}}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

$$- \frac{(ben)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{\sqrt{-f}\sqrt{g}} + \frac{(ben)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{\sqrt{-f}\sqrt{g}}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{\sqrt{-f}\sqrt{g}} + \frac{(bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{\sqrt{-f}\sqrt{g}}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{bn(a+b\log(c(d+ex)^n))\,\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}} + \frac{bn(a+b\log(c(d+ex)^n))\,\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}} + \frac{(b^2n^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{\sqrt{-f}\sqrt{g}} - \frac{(b^2n^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{\sqrt{-f}\sqrt{g}}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}} + \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}} + \frac{b^2n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}} - \frac{b^2n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{\sqrt{-f}\sqrt{g}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx = \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2+ibn(a-bn\log(d+ex)+b\log(c(d+ex)^n))\left(\mathrm{lo}\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^
2 + I*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(Log[d + e*x]*(Log[
1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - Log[1 - (Sqrt[g]*(d
 + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]) + PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)
*e*Sqrt[f] + d*Sqrt[g])] - PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*
Sqrt[g])]) + (I/2)*b^2*n^2*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I
)*e*Sqrt[f] + d*Sqrt[g])] - Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/(I*e
*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/((-I
)*e*Sqrt[f] + d*Sqrt[g])] - 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/(
I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f
] + d*Sqrt[g])] + 2*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g]
)]))/(Sqrt[f]*Sqrt[g])
```

## Maple [F]

$$\int \frac{\left(a+b\ln \left(c(ex+d)^{n}\right)\right)^{2}}{g\,x^{2}+f}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
[Out] int((a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f),x)
```

## Fricas [F]

$$\int \frac{(a+b\log{(c(d+ex)^{n})})^{2}}{f+gx^{2}}\,dx=\int \frac{(b\log{((ex+d)^{n}c)}+a)^{2}}{gx^{2}+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="fricas")
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x^2
 + f), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(c(d+ex)^{n})})^{2}}{f+gx^{2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(a+b\log{(c(d+ex)^{n})})^{2}}{f+gx^{2}}\,dx=\int \frac{(b\log{((ex+d)^{n}c)}+a)^{2}}{gx^{2}+f}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="maxima")
[Out] a^2*arctan(g*x/sqrt(f*g))/sqrt(f*g) + integrate((b^2*log((e*x + d)^n)^2 + b
^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log((e*x + d)^n))/(g*x^2
+ f), x)
```

## Giac [F]

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{f + gx^2}\,dx = \int \frac{(b\log((ex + d)^n c) + a)^2}{gx^2 + f}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x^2+f),x, algorithm="giac")

[Out] integrate((b*log((e*x + d)^n*c) + a)^2/(g*x^2 + f), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d + ex)^n))^2}{f + gx^2}\,dx = \int \frac{(a + b\,\ln(c\,(d + e\,x)^n))^2}{g\,x^2 + f}\,dx$$

[In] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x^2),x)

[Out] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x^2), x)

# 3.318 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 461

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx &= \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df} \\ &- \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{dfx} \\ &+ \frac{\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}} \\ &- \frac{\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}} \\ &- \frac{b\sqrt{g}n(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{3/2}} \\ &+ \frac{b\sqrt{g}n(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{3/2}} \\ &+ \frac{2b^2en^2\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{df} \\ &+ \frac{b^2\sqrt{g}n^2\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{3/2}} \\ &- \frac{b^2\sqrt{g}n^2\,\mathrm{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{3/2}}\end{aligned}$$

```
[Out] 2*b*e*n*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/d/f-(e*x+d)*(a+b*ln(c*(e*x+d)^n))^
2/d/f/x+2*b^2*e*n^2*polylog(2,1+e*x/d)/d/f+1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e
```

```
*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/(-f)^(3/2)-1/2*(a
+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))
*g^(1/2)/(-f)^(3/2)-b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e
*(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(3/2)+b*n*(a+b*ln(c*(e*x+d)^n))*polylo
g(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/(-f)^(3/2)+b^2*n^2*po
lylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(3/2)-b^2*n
^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/(-f)^(3/2)
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2463, 2444, 2441, 2352, 2456, 2443, 2481, 2421, 6724}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx = -\frac{b\sqrt{g}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{(-f)^{3/2}} + \frac{b\sqrt{g}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{(-f)^{3/2}} + \frac{\sqrt{g}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2(-f)^{3/2}} - \frac{\sqrt{g}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2(-f)^{3/2}} + \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{dfx} + \frac{b^2\sqrt{g}n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{3/2}} - \frac{b^2\sqrt{g}n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{(-f)^{3/2}} + \frac{2b^2en^2\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{df}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(x^2*(f + g*x^2)),x]
```

```
[Out] (2*b*e*n*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/(d*f) - ((d + e*x)*(a
+ b*Log[c*(d + e*x)^n])^2)/(d*f*x) + (Sqrt[g]*(a + b*Log[c*(d + e*x)^n])^2*
Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(-f)^(3/2)) -
(Sqrt[g]*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqr
t[-f] - d*Sqrt[g])])/(2*(-f)^(3/2)) - (b*Sqrt[g]*n*(a + b*Log[c*(d + e*x)^n
```

```
])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(-f)^(3/2)
+ (b*Sqrt[g]*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e
*Sqrt[-f] + d*Sqrt[g])])/(-f)^(3/2) + (2*b^2*e*n^2*PolyLog[2, 1 + (e*x)/d])
/(d*f) + (b^2*Sqrt[g]*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*
Sqrt[g]))])/(-f)^(3/2) - (b^2*Sqrt[g]*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e
*Sqrt[-f] + d*Sqrt[g])])/(-f)^(3/2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
```

```
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left( \frac{(a+b\log(c(d+ex)^n))^2}{fx^2} - \frac{g(a+b\log(c(d+ex)^n))^2}{f(f+gx^2)} \right) dx$$

$$= \frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2}\,dx}{f} - \frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx}{f}$$

$$= -\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{dfx} - \frac{g\int \left( \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}-\sqrt{g}x)} + \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}+\sqrt{g}x)} \right) dx}{f} + \frac{(2ben)\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx}{df}$$

$$= \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{dfx} - \frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{2(-f)^{3/2}} - \frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{2(-f)^{3/2}} - \frac{(2b^2e^2n^2)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{df}$$

$$= \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{dfx} + \frac{\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}} - \frac{\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}} + \frac{2b^2en^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{df} - \frac{\left(be\sqrt{g}n\right)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}dx}{(-f)^{3/2}} + \frac{\left(be\sqrt{g}n\right)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}dx}{(-f)^{3/2}}$$

$$= \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{dfx} + \frac{\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}} - \frac{\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}} + \frac{2b^2en^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{df} - \frac{\left(b\sqrt{g}n\right)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{(-f)^{3/2}} + \frac{\left(b\sqrt{g}n\right)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{(-f)^{3/2}}$$

$$= \frac{2ben\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{df} - \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{dfx} + \frac{\sqrt{g}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}} - \frac{\sqrt{g}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}} - \frac{b\sqrt{g}n\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{3/2}} + \frac{b\sqrt{g}n\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{3/2}} + \frac{2b^2en^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{df} + \frac{\left(b^2\sqrt{g}n^2\right)\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{(-f)^{3/2}} - \frac{\left(b^2\sqrt{g}n^2\right)\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{(-f)^{3/2}}$$

$$= \frac{2ben\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{df} - \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{dfx} + \frac{\sqrt{g}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}} - \frac{\sqrt{g}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}} - \frac{b\sqrt{g}n\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{3/2}} + \frac{b\sqrt{g}n\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{3/2}} + \frac{2b^2en^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{df} + \frac{b^2\sqrt{g}n^2\text{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{3/2}} - \frac{b^2\sqrt{g}n^2\text{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{3/2}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.45

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx$$
$$= \frac{-2d\sqrt{f}(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2-2d\sqrt{g}x\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a-bn\log(d+ex)+b\log(c(d+$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x^2*(f + g*x^2)),x]
```

```
[Out] (-2*d*Sqrt[f]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - 2*d*Sqrt[g]
*x*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]
)^2 + 2*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(2*Sqrt[f]*(e*x*L
og[x] - (d + e*x)*Log[d + e*x]) + I*d*Sqrt[g]*x*(Log[d + e*x]*Log[(e*(Sqrt[
f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d
 + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) - I*d*Sqrt[g]*x*(Log[d + e*x]*Log[(e*(
Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*
(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])])) + b^2*n^2*(2*Sqrt[f]*(2*e*x*Log[-((
e*x)/d)]*Log[d + e*x] - (d + e*x)*Log[d + e*x]^2 + 2*e*x*PolyLog[2, 1 + (e*
x)/d]) - I*d*Sqrt[g]*x*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*
Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)
*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f]
 + d*Sqrt[g])]) + I*d*Sqrt[g]*x*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))
/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))
/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f]
 + d*Sqrt[g])])))/(2*d*f^(3/2)*x)
```

## Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))^2}{x^2(g\,x^2+f)}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/x^2/(g*x^2+f),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))^2/x^2/(g*x^2+f),x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2}{(gx^2+f)x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^2/(g*x^2+f),x, algorithm="fricas")
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x^4
 + f*x^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/x**2/(g*x**2+f),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2}{(gx^2+f)x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^2/(g*x^2+f),x, algorithm="maxima")
[Out] -a^2*(g*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*f) + 1/(f*x)) + integrate((b^2*log
((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log((e
*x + d)^n))/(g*x^4 + f*x^2), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2}{(gx^2+f)x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^2/(g*x^2+f),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log (c(d+ex)^n))^2}{x^2(f+gx^2)}\,dx = \int \frac{\left(a+b\,\ln \left(c\,{\left(d+e\,x\right)}^n\right)\right)^2}{x^2\,\left(g\,x^2+f\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(x^2*(f + g*x^2)),x)
[Out] int((a + b*log(c*(d + e*x)^n))^2/(x^2*(f + g*x^2)), x)
```

# 3.319 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx$

## Optimal result

Integrand size = 29, antiderivative size = 694

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx = -\frac{b^2e^2n^2}{3d^2fx}-\frac{b^2e^3n^2\log(x)}{d^3f}+\frac{b^2e^3n^2\log(d+ex)}{3d^3f}-\frac{ben(a+b\log(c(d+ex)^n))}{3dfx^2}+\frac{2be^2n(d+ex)(a+b\log(c(d+ex)^n))}{3d^3fx}-\frac{2begn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2}-\frac{(a+b\log(c(d+ex)^n))^2}{3fx^3}+\frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x}+\frac{g^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}}-\frac{g^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}}+\frac{2be^3n(a+b\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{3d^3f}-\frac{2b^2e^3n^2\,\text{PolyLog}\left(2,\frac{d}{d+ex}\right)}{3d^3f}-\frac{bg^{3/2}n(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}}+\frac{bg^{3/2}n(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}}-\frac{2b^2egn^2\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{df^2}+\frac{b^2g^{3/2}n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}}-\frac{b^2g^{3/2}n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}}$$

```
[Out] -1/3*b^2*e^2*n^2/d^2/f/x-b^2*e^3*n^2*ln(x)/d^3/f+1/3*b^2*e^3*n^2*ln(e*x+d)/
d^3/f-1/3*b*e*n*(a+b*ln(c*(e*x+d)^n))/d/f/x^2+2/3*b*e^2*n*(e*x+d)*(a+b*ln(c
*(e*x+d)^n))/d^3/f/x-2*b*e*g*n*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/d/f^2-1/3*(
```

```
a+b*ln(c*(e*x+d)^n))^2/f/x^3+g*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/d/f^2/x+2/3*
b*e^3*n*(a+b*ln(c*(e*x+d)^n))*ln(1-d/(e*x+d))/d^3/f+1/2*g^(3/2)*(a+b*ln(c*(
e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/(-f)^(5/
2)-1/2*g^(3/2)*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^
(1/2)-d*g^(1/2)))/(-f)^(5/2)-2/3*b^2*e^3*n^2*polylog(2,d/(e*x+d))/d^3/f-2*b
^2*e*g*n^2*polylog(2,1+e*x/d)/d/f^2-b*g^(3/2)*n*(a+b*ln(c*(e*x+d)^n))*polyl
og(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(5/2)+b*g^(3/2)*n*(a+b
*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/(-f)^
(5/2)+b^2*g^(3/2)*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/
(-f)^(5/2)-b^2*g^(3/2)*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2
)))/(-f)^(5/2)
```

## Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.621$, Rules used = {2463, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46, 2444, 2441, 2352, 2456,

2443, 2481, 2421, 6724}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx = \frac{2be^3n\log\left(1-\frac{d}{d+ex}\right)(a+b\log(c(d+ex)^n))}{3d^3f} + \frac{2be^2n(d+ex)(a+b\log(c(d+ex)^n))}{3d^3fx} - \frac{2begn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} - \frac{bg^{3/2}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{(-f)^{5/2}} + \frac{bg^{3/2}n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{(-f)^{5/2}} + \frac{g^{3/2}\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2(-f)^{5/2}} - \frac{g^{3/2}\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2(-f)^{5/2}} - \frac{(a+b\log(c(d+ex)^n))^2}{3fx^3} - \frac{ben(a+b\log(c(d+ex)^n))}{3dfx^2} - \frac{2b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{3d^3f} - \frac{b^2e^3n^2\log(x)}{d^3f} + \frac{b^2e^3n^2\log(d+ex)}{3d^3f} - \frac{b^2e^2n^2}{3d^2fx} - \frac{2b^2egn^2\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{df^2} + \frac{b^2g^{3/2}n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}} - \frac{b^2g^{3/2}n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{(-f)^{5/2}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(x^4*(f + g*x^2)),x]
```

```
[Out] -1/3*(b^2*e^2*n^2)/(d^2*f*x) - (b^2*e^3*n^2*Log[x])/(d^3*f) + (b^2*e^3*n^2*
Log[d + e*x])/(3*d^3*f) - (b*e*n*(a + b*Log[c*(d + e*x)^n]))/(3*d*f*x^2) +
(2*b*e^2*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n]))/(3*d^3*f*x) - (2*b*e*g*n*L
og[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/(d*f^2) - (a + b*Log[c*(d + e*x)
^n])^2/(3*f*x^3) + (g*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(d*f^2*x) + (
g^(3/2)*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt
```

```
[-f] + d*Sqrt[g])])/(2*(-f)^(5/2)) - (g^(3/2)*(a + b*Log[c*(d + e*x)^n])^2*
Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*(-f)^(5/2)) +
(2*b*e^3*n*(a + b*Log[c*(d + e*x)^n])*Log[1 - d/(d + e*x)])/(3*d^3*f) - (2*
b^2*e^3*n^2*PolyLog[2, d/(d + e*x)])/(3*d^3*f) - (b*g^(3/2)*n*(a + b*Log[c*
(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/
(-f)^(5/2) + (b*g^(3/2)*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d
 + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(-f)^(5/2) - (2*b^2*e*g*n^2*PolyLog[2,
1 + (e*x)/d])/(d*f^2) + (b^2*g^(3/2)*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(
e*Sqrt[-f] - d*Sqrt[g]))])/(-f)^(5/2) - (b^2*g^(3/2)*n^2*PolyLog[3, (Sqrt[g
]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(-f)^(5/2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
```

```
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left( \frac{(a+b\log(c(d+ex)^n))^2}{fx^4} - \frac{g(a+b\log(c(d+ex)^n))^2}{f^2x^2} + \frac{g^2(a+b\log(c(d+ex)^n))^2}{f^2(f+gx^2)} \right) dx$$

$$= \frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4} dx}{f} - \frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2} dx}{f^2} + \frac{g^2\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2} dx}{f^2}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^2}{3fx^3} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} + \frac{g^2\int \left( \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}-\sqrt{g}x)} + \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}+\sqrt{g}x)} \right) dx}{f^2} + \frac{(2ben)\int \frac{a+b\log(c(d+ex)^n)}{x^3(d+ex)} dx}{3f} - \frac{(2begn)\int \frac{a+b\log(c(d+ex)^n)}{x} dx}{df^2}$$

$$= -\frac{2begn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} - \frac{(a+b\log(c(d+ex)^n))^2}{3fx^3} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} - \frac{g^2\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x} dx}{2(-f)^{5/2}} - \frac{g^2\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x} dx}{2(-f)^{5/2}} + \frac{(2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3} dx, x, d+ex\right)}{3f} + \frac{(2b^2e^2gn^2)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex} dx}{df^2}$$

$$= -\frac{2begn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} - \frac{(a+b\log(c(d+ex)^n))^2}{3fx^3} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} + \frac{g^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{g^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{2b^2egn^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{df^2} + \frac{(2bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+ex\right)}{3df} - \frac{(2ben)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex\right)}{3df} - \frac{(beg^{3/2}n)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{(-f)^{5/2}} + \frac{(beg^{3/2}n)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{(-f)^{5/2}}$$

$$= -\frac{ben(a+b\log(c(d+ex)^n))}{3dfx^2} - \frac{2begn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} - \frac{(a+b\log(c(d+ex)^n))^2}{3fx^3} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} + \frac{g^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{g^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{2b^2egn^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{df^2} - \frac{(2ben)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx,x,d+ex\right)}{3d^2f} + \frac{(2be^2n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}dx,x,d+ex\right)}{3d^2f} - \frac{(bg^{3/2}n)\,\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{(-f)^{5/2}} + \frac{(bg^{3/2}n)\,\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{(-f)^{5/2}} + \frac{(b^2en^2)\,\text{Subst}\left(\int\frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx,x,d+ex\right)}{3df}$$

$$= -\frac{ben(a+b\log(c(d+ex)^n))}{3dfx^2} + \frac{2be^2n(d+ex)(a+b\log(c(d+ex)^n))}{3d^3fx}$$
$$- \frac{2begn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2}$$
$$- \frac{(a+b\log(c(d+ex)^n))^2}{3fx^3} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x}$$
$$+ \frac{g^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}}$$
$$- \frac{g^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}}$$
$$+ \frac{2be^3n(a+b\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{3d^3f}$$
$$- \frac{bg^{3/2}n(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}}$$
$$+ \frac{bg^{3/2}n(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}} - \frac{2b^2egn^2\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{df^2}$$
$$+ \frac{(b^2en^2)\operatorname{Subst}\left(\int\left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)dx, x, d+ex\right)}{3df}$$
$$- \frac{(2b^2e^2n^2)\operatorname{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}dx, x, d+ex\right)}{3d^3f}$$
$$- \frac{(2b^2e^3n^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}dx, x, d+ex\right)}{3d^3f}$$
$$+ \frac{(b^2g^{3/2}n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{(-f)^{5/2}}$$
$$- \frac{(b^2g^{3/2}n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{(-f)^{5/2}}$$

$$= -\frac{b^2e^2n^2}{3d^2fx} - \frac{b^2e^3n^2\log(x)}{d^3f} + \frac{b^2e^3n^2\log(d+ex)}{3d^3f} - \frac{ben(a+b\log(c(d+ex)^n))}{3dfx^2} + \frac{2be^2n(d+ex)(a+b\log(c(d+ex)^n))}{3d^3fx} - \frac{2begn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} - \frac{(a+b\log(c(d+ex)^n))^2}{3fx^3} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} + \frac{g^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{g^{3/2}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{2be^3n(a+b\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{3d^3f} - \frac{2b^2e^3n^2\text{Li}_2\left(\frac{d}{d+ex}\right)}{3d^3f} - \frac{bg^{3/2}n(a+b\log(c(d+ex)^n))\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}} + \frac{bg^{3/2}n(a+b\log(c(d+ex)^n))\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}} - \frac{2b^2egn^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{df^2} + \frac{b^2g^{3/2}n^2\text{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}} - \frac{b^2g^{3/2}n^2\text{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx = \frac{-2d^3f^{3/2}(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2 + 6d^3\sqrt{f}gx^2(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2 +}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x^4*(f + g*x^2)),x]
```

```
[Out] (-2*d^3*f^(3/2)*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 6*d^3*Sqr
t[f]*g*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 6*d^3*g^(3/2)*
x^3*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n
])^2 + (2*I)*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((6*I)*d^2*S
qrt[f]*g*x^2*(e*x*Log[x] - (d + e*x)*Log[d + e*x]) + I*f^(3/2)*(d*e*x*(d -
2*e*x) - 2*e^3*x^3*Log[x] + 2*(d^3 + e^3*x^3)*Log[d + e*x]) - 3*d^3*g^(3/2)
*x^3*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g]
)] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + 3*d^
```

```
3*g^(3/2)*x^3*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*
d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))
 + I*b^2*n^2*((6*I)*d^2*Sqrt[f]*g*x^2*(2*e*x*Log[-((e*x)/d)]*Log[d + e*x] -
 (d + e*x)*Log[d + e*x]^2 + 2*e*x*PolyLog[2, 1 + (e*x)/d]) + (2*I)*f^(3/2)*
(d*e^2*x^2 + 3*e^3*x^3*Log[x] + d^2*e*x*Log[d + e*x] - 2*d*e^2*x^2*Log[d +
e*x] - 3*e^3*x^3*Log[d + e*x] - 2*e^3*x^3*Log[-((e*x)/d)]*Log[d + e*x] + d^
3*Log[d + e*x]^2 + e^3*x^3*Log[d + e*x]^2 - 2*e^3*x^3*PolyLog[2, 1 + (e*x)/
d]) + 3*d^3*g^(3/2)*x^3*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e
*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/((-I
)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f
] + d*Sqrt[g])]) - 3*d^3*g^(3/2)*x^3*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d +
e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d +
e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sq
rt[f] + d*Sqrt[g])])))/(6*d^3*f^(5/2)*x^3)
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}{x^{4}\left(g\,x^{2}+f\right)}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/x^4/(g*x^2+f),x)
[Out] int((a+b*ln(c*(e*x+d)^n))^2/x^4/(g*x^2+f),x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2}{(gx^2+f)x^4}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^4/(g*x^2+f),x, algorithm="fricas")
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g*x^6
 + f*x^4), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/x**4/(g*x**2+f),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{(gx^2+f)x^4}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^4/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3*g^2*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*f^2) + (3*g*x^2 - f)/(f^2*x
^3)) + integrate((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*
(b^2*log(c) + a*b)*log((e*x + d)^n))/(g*x^6 + f*x^4), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{(gx^2+f)x^4}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^4/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(f+gx^2)}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2}{x^4\,(g\,x^2+f)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(x^4*(f + g*x^2)),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^2/(x^4*(f + g*x^2)), x)
```

# 3.320 $\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

## Optimal result

Integrand size = 29, antiderivative size = 936

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$
$$= \frac{2abdnx}{eg^2} - \frac{2b^2dn^2x}{eg^2} + \frac{b^2n^2(d+ex)^2}{4e^2g^2} + \frac{2b^2dn(d+ex)\log(c(d+ex)^n)}{e^2g^2} - \frac{bn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2g^2} + \frac{e^2f^2(a+b\log(c(d+ex)^n))^2}{2g^3(e^2f+d^2g)} - \frac{d(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2g^2} + \frac{(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^2g^2} - \frac{f^2(a+b\log(c(d+ex)^n))^2}{2g^3(f+gx^2)} - \frac{bef\left(ef+d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3(e^2f+d^2g)} - \frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3} - \frac{be(-f)^{3/2}\left(e\sqrt{-f}+d\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3(e^2f+d^2g)} - \frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} - \frac{b^2e(-f)^{3/2}\left(e\sqrt{-f}+d\sqrt{g}\right)n^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3(e^2f+d^2g)} - \frac{2bfn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} - \frac{b^2e(-f)^{3/2}\left(e\sqrt{-f}-d\sqrt{g}\right)n^2\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3(e^2f+d^2g)} - \frac{2bfn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3} + \frac{2b^2fn^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} + \frac{2b^2fn^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$

```
[Out] 2*a*b*d*n*x/e/g^2-2*b^2*d*n^2*x/e/g^2+1/4*b^2*n^2*(e*x+d)^2/e^2/g^2+2*b^2*d
*n*(e*x+d)*ln(c*(e*x+d)^n)/e^2/g^2-1/2*b*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/
e^2/g^2+1/2*e^2*f^2*(a+b*ln(c*(e*x+d)^n))^2/g^3/(d^2*g+e^2*f)-d*(e*x+d)*(a+
b*ln(c*(e*x+d)^n))^2/e^2/g^2+1/2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2/g^2-
```

```
1/2*f^2*(a+b*ln(c*(e*x+d)^n))^2/g^3/(g*x^2+f)-f*(a+b*ln(c*(e*x+d)^n))^2*ln(
e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/g^3-f*(a+b*ln(c*(e*x+d)^
n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^3-2*b*f*n*(a+
b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^3
-2*b*f*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^
(1/2)))/g^3+2*b^2*f*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2))
)/g^3+2*b^2*f*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^3-1
/2*b^2*e*(-f)^(3/2)*n^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))
*(e*(-f)^(1/2)-d*g^(1/2))/g^3/(d^2*g+e^2*f)-1/2*b*e*(-f)^(3/2)*n*(a+b*ln(c*
(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))*(e*(-f)^(
1/2)+d*g^(1/2))/g^3/(d^2*g+e^2*f)-1/2*b^2*e*(-f)^(3/2)*n^2*polylog(2,-(e*x+
d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*(e*(-f)^(1/2)+d*g^(1/2))/g^3/(d^2*g+e^
2*f)-1/2*b*e*f*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^
(1/2)+d*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2))/g^3/(d^2*g+e^2*f)
```

## Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 936, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.621$, Rules used = {2463, 2448, 2436, 2333, 2332, 2437, 2342, 2341, 2460, 2465, 2338, 2441, 2440, 2438,

2443, 2481, 2421, 6724}

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$
$$= \frac{n^2(d+ex)^2b^2}{4e^2g^2} - \frac{2dn^2xb^2}{eg^2} + \frac{2dn(d+ex)\log(c(d+ex)^n)\,b^2}{e^2g^2}$$
$$- \frac{e(-f)^{3/2}\left(\sqrt{g}d+e\sqrt{-f}\right)n^2\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)b^2}{2g^3\,(gd^2+e^2f)}$$
$$- \frac{e(-f)^{3/2}\left(e\sqrt{-f}-d\sqrt{g}\right)n^2\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)b^2}{2g^3\,(gd^2+e^2f)}$$
$$+ \frac{2fn^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)b^2}{g^3} + \frac{2fn^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)b^2}{g^3}$$
$$+ \frac{2adnxb}{eg^2} - \frac{n(d+ex)^2\,(a+b\log(c(d+ex)^n))\,b}{2e^2g^2}$$
$$- \frac{ef\left(\sqrt{-f}\sqrt{g}d+ef\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)b}{2g^3\,(gd^2+e^2f)}$$
$$- \frac{e(-f)^{3/2}\left(\sqrt{g}d+e\sqrt{-f}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)b}{2g^3\,(gd^2+e^2f)}$$
$$- \frac{2fn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)b}{g^3}$$
$$- \frac{2fn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)b}{g^3}$$
$$+ \frac{(d+ex)^2\,(a+b\log(c(d+ex)^n))^2}{2e^2g^2} - \frac{d(d+ex)\,(a+b\log(c(d+ex)^n))^2}{e^2g^2}$$
$$+ \frac{e^2f^2(a+b\log(c(d+ex)^n))^2}{2g^3\,(gd^2+e^2f)} - \frac{f^2(a+b\log(c(d+ex)^n))^2}{2g^3\,(gx^2+f)}$$
$$- \frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^3}$$
$$- \frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}$$

```
[In] Int[(x^5*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

```
[Out] (2*a*b*d*n*x)/(e*g^2) - (2*b^2*d*n^2*x)/(e*g^2) + (b^2*n^2*(d + e*x)^2)/(4*
e^2*g^2) + (2*b^2*d*n*(d + e*x)*Log[c*(d + e*x)^n])/(e^2*g^2) - (b*n*(d + e
*x)^2*(a + b*Log[c*(d + e*x)^n]))/(2*e^2*g^2) + (e^2*f^2*(a + b*Log[c*(d +
e*x)^n])^2)/(2*g^3*(e^2*f + d^2*g)) - (d*(d + e*x)*(a + b*Log[c*(d + e*x)^n
```

```
])^2)/(e^2*g^2) + ((d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^2*g^2) -
(f^2*(a + b*Log[c*(d + e*x)^n])^2)/(2*g^3*(f + g*x^2)) - (b*e*f*(e*f + d*Sq
rt[-f]*Sqrt[g])*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))
/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^3*(e^2*f + d^2*g)) - (f*(a + b*Log[c*(d +
e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^3 -
(b*e*(-f)^(3/2)*(e*Sqrt[-f] + d*Sqrt[g])*n*(a + b*Log[c*(d + e*x)^n])*Log[(
e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^3*(e^2*f + d^2*g)
) - (f*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[
-f] - d*Sqrt[g])])/g^3 - (b^2*e*(-f)^(3/2)*(e*Sqrt[-f] + d*Sqrt[g])*n^2*Pol
yLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*g^3*(e^2*f + d
^2*g)) - (2*b*f*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x
))/(e*Sqrt[-f] - d*Sqrt[g]))])/g^3 - (b^2*e*(-f)^(3/2)*(e*Sqrt[-f] - d*Sqrt
[g])*n^2*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^3*(
e^2*f + d^2*g)) - (2*b*f*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(
d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^3 + (2*b^2*f*n^2*PolyLog[3, -((Sqrt[
g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/g^3 + (2*b^2*f*n^2*PolyLog[3, (Sq
rt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x(a+b\log(c(d+ex)^n))^2}{g^2}+\frac{f^2x(a+b\log(c(d+ex)^n))^2}{g^2(f+gx^2)^2}-\frac{2fx(a+b\log(c(d+ex)^n))^2}{g^2(f+gx^2)}\right)dx$$

$$=\frac{\int x(a+b\log(c(d+ex)^n))^2\,dx}{g^2}-\frac{(2f)\int\frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx}{g^2}+\frac{f^2\int\frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx}{g^2}$$

$$=-\frac{f^2(a+b\log(c(d+ex)^n))^2}{2g^3(f+gx^2)}+\frac{\int\left(-\frac{d(a+b\log(c(d+ex)^n))^2}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{e}\right)dx}{g^2}-\frac{(2f)\int\left(-\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)}+\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g^2}+\frac{(bef^2n)\int\frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx^2)}\,dx}{g^3}$$

$$=-\frac{f^2(a+b\log(c(d+ex)^n))^2}{2g^3(f+gx^2)}+\frac{f\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{g^{5/2}}-\frac{f\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{g^{5/2}}+\frac{\int(d+ex)(a+b\log(c(d+ex)^n))^2\,dx}{eg^2}-\frac{d\int(a+b\log(c(d+ex)^n))^2\,dx}{eg^2}+\frac{(bef^2n)\int\left(\frac{e^2(a+b\log(c(d+ex)^n))}{(e^2f+d^2g)(d+ex)}-\frac{g(-d+ex)(a+b\log(c(d+ex)^n))}{(e^2f+d^2g)(f+gx^2)}\right)dx}{g^3}$$

$$=-\frac{f^2(a+b\log(c(d+ex)^n))^2}{2g^3(f+gx^2)}-\frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}-\frac{f(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}+\frac{\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx,x,d+ex\right)}{e^2g^2}-\frac{d\text{Subst}\left(\int(a+b\log(cx^n))^2\,dx,x,d+ex\right)}{e^2g^2}+\frac{(2befn)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{g^3}+\frac{(2befn)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{g^3}+\frac{(be^3f^2n)\int\frac{a+b\log(c(d+ex)^n)}{d+ex}\,dx}{g^3(e^2f+d^2g)}-\frac{(bef^2n)\int\frac{(-d+ex)(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx}{g^2(e^2f+d^2g)}$$

$$= -\frac{d(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g^2} + \frac{(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g^2}$$

$$- \frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g^3\left(f+gx^2\right)} - \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$

$$- \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}$$

$$+ \frac{(2bfn)\text{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{g^3}$$

$$+ \frac{(2bfn)\text{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{g^3}$$

$$- \frac{(bn)\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)\,dx, x, d+ex\right)}{e^2g^2}$$

$$+ \frac{(2bdn)\text{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)\,dx, x, d+ex\right)}{e^2g^2}$$

$$+ \frac{\left(be^2f^2n\right)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x}\,dx, x, d+ex\right)}{g^3\left(e^2f+d^2g\right)}$$

$$- \frac{\left(bef^2n\right)\int \left(\frac{\left(-d\sqrt{-f}-\frac{ef}{\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\left(-d\sqrt{-f}+\frac{ef}{\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx}{g^2\left(e^2f+d^2g\right)}$$

$$= \frac{2abdnx}{eg^2} + \frac{b^2n^2(d+ex)^2}{4e^2g^2} - \frac{bn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2g^2}$$
$$+ \frac{e^2f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g^3\left(e^2f+d^2g\right)} - \frac{d(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g^2}$$
$$+ \frac{(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g^2} - \frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g^3\left(f+gx^2\right)}$$
$$- \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$
$$- \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}$$
$$- \frac{2bfn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}$$
$$- \frac{2bfn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$
$$+ \frac{\left(2b^2dn\right)\operatorname{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{e^2g^2}$$
$$- \frac{\left(bef^2\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)n\right)\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2g^2\left(e^2f+d^2g\right)}$$
$$+ \frac{\left(bef^2\left(\frac{df}{(-f)^{3/2}}+\frac{e}{\sqrt{g}}\right)n\right)\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2g^2\left(e^2f+d^2g\right)}$$
$$+ \frac{\left(2b^2fn^2\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{g^3}$$
$$+ \frac{\left(2b^2fn^2\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{g^3}$$

$$
\begin{aligned}
&= \frac{2abdnx}{eg^2} - \frac{2b^2dn^2x}{eg^2} + \frac{b^2n^2(d+ex)^2}{4e^2g^2} + \frac{2b^2dn(d+ex)\log\left(c(d+ex)^n\right)}{e^2g^2} \\
&- \frac{bn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2g^2} \\
&+ \frac{e^2f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g^3\left(e^2f+d^2g\right)} - \frac{d(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g^2} \\
&+ \frac{(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g^2} - \frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g^3\left(f+gx^2\right)} \\
&- \frac{bef\left(ef+d\sqrt{-f}\sqrt{g}\right)n\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3\left(e^2f+d^2g\right)} \\
&- \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3} \\
&- \frac{bef\left(ef-d\sqrt{-f}\sqrt{g}\right)n\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3\left(e^2f+d^2g\right)} \\
&- \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} \\
&- \frac{2bfn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} \\
&- \frac{2bfn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3} + \frac{2b^2fn^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} \\
&+ \frac{2b^2fn^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3} + \frac{\left(b^2e^2f^2\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)n^2\right)\int\frac{\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2g^{5/2}\left(e^2f+d^2g\right)} \\
&+ \frac{\left(b^2e^2f^2\left(\frac{df}{(-f)^{3/2}}+\frac{e}{\sqrt{g}}\right)n^2\right)\int\frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2g^{5/2}\left(e^2f+d^2g\right)}
\end{aligned}
$$

$$= \frac{2abdnx}{eg^2} - \frac{2b^2dn^2x}{eg^2} + \frac{b^2n^2(d+ex)^2}{4e^2g^2} + \frac{2b^2dn(d+ex)\log\left(c(d+ex)^n\right)}{e^2g^2}$$
$$- \frac{bn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2g^2}$$
$$+ \frac{e^2f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g^3\left(e^2f+d^2g\right)} - \frac{d(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g^2}$$
$$+ \frac{(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g^2} - \frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g^3\left(f+gx^2\right)}$$
$$- \frac{bef\left(ef+d\sqrt{-f}\sqrt{g}\right)n\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3\left(e^2f+d^2g\right)}$$
$$- \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$
$$- \frac{bef\left(ef-d\sqrt{-f}\sqrt{g}\right)n\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3\left(e^2f+d^2g\right)}$$
$$- \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}$$
$$- \frac{2bfn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3}$$
$$- \frac{2bfn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$
$$+ \frac{2b^2fn^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} + \frac{2b^2fn^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}$$
$$+ \frac{\left(b^2ef^2\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)n^2\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2g^{5/2}\left(e^2f+d^2g\right)}$$
$$+ \frac{\left(b^2ef^2\left(\frac{df}{(-f)^{3/2}}+\frac{e}{\sqrt{g}}\right)n^2\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2g^{5/2}\left(e^2f+d^2g\right)}$$

$$
\begin{aligned}
&= \frac{2abdnx}{eg^2} - \frac{2b^2dn^2x}{eg^2} + \frac{b^2n^2(d+ex)^2}{4e^2g^2} + \frac{2b^2dn(d+ex)\log\left(c(d+ex)^n\right)}{e^2g^2} \\
&- \frac{bn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2g^2} \\
&+ \frac{e^2f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g^3\left(e^2f+d^2g\right)} - \frac{d(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e^2g^2} \\
&+ \frac{(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2g^2} - \frac{f^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2g^3\left(f+gx^2\right)} \\
&- \frac{bef\left(ef+d\sqrt{-f}\sqrt{g}\right)n\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3\left(e^2f+d^2g\right)} \\
&- \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3} \\
&- \frac{bef\left(ef-d\sqrt{-f}\sqrt{g}\right)n\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3\left(e^2f+d^2g\right)} \\
&- \frac{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} \\
&- \frac{b^2ef\left(ef-d\sqrt{-f}\sqrt{g}\right)n^2\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^3\left(e^2f+d^2g\right)} \\
&- \frac{2bfn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} \\
&- \frac{b^2ef\left(ef+d\sqrt{-f}\sqrt{g}\right)n^2\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^3\left(e^2f+d^2g\right)} \\
&- \frac{2bfn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3} \\
&+ \frac{2b^2fn^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^3} + \frac{2b^2fn^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^3}
\end{aligned}
$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 1254, normalized size of antiderivative = 1.34

$$
\int \frac{x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{\left(f+gx^2\right)^2}\,dx
$$

$$
= \frac{2gx^2\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2 - \frac{2f^2\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx^2} - 4f\left(a-bn\log(d+ex)\right.}{}
$$

```
[In] Integrate[(x^5*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

```
[Out] (2*g*x^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - (2*f^2*(a - b*n*
Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) - 4*f*(a - b*n*Log[d +
e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a - b*n*Log[d + e*x]
 + b*Log[c*(d + e*x)^n])*((g*(e*x*(2*d - e*x) - 2*(d^2 - e^2*x^2)*Log[d + e
*x]))/e^2 + (f^(3/2)*(I*Sqrt[g]*(d + e*x)*Log[d + e*x] - e*(Sqrt[f] + I*Sqr
t[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] +
I*Sqrt[g]*x)) + (I*f^(3/2)*(-(Sqrt[g]*(d + e*x)*Log[d + e*x]) + e*(I*Sqrt[f
] + Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqr
t[f] - I*Sqrt[g]*x)) - 4*f*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e
*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] -
 I*d*Sqrt[g])]) - 4*f*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt
[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqr
t[g])])) + b^2*n^2*((g*(e*x*(-6*d + e*x) + (6*d^2 + 4*d*e*x - 2*e^2*x^2)*Lo
g[d + e*x] - 2*(d^2 - e^2*x^2)*Log[d + e*x]^2))/e^2 + (I*f^(3/2)*(-(Sqrt[g]
*(d + e*x)*Log[d + e*x]^2) + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*Log[d + e*x]*Log[(
e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + 2*e*(I*Sqrt[f] + Sq
rt[g]*x)*PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/((e*
Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) - (f^(3/2)*(Log[d + e*x]*((
-I)*Sqrt[g]*(d + e*x)*Log[d + e*x] + 2*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[(e*(Sq
rt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + 2*e*(Sqrt[f] + I*Sqrt[g
]*x)*PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]))/((e*Sqrt[f
] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) - 4*f*(Log[d + e*x]^2*Log[1 - (Sq
rt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2,
(Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(
d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) - 4*f*(Log[d + e*x]^2*Log[1 - (Sqr
t[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqr
t[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x
))/(I*e*Sqrt[f] + d*Sqrt[g])])))/(4*g^3)
```

## Maple [F]

$$\int \frac{x^5 (a + b\ln (c(ex + d)^n))^2}{(g\,x^2 + f)^2} dx$$

```
[In] int(x^5*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

```
[Out] int(x^5*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

## Fricas [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^5}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral((b^2*x^5*log((e*x + d)^n*c)^2 + 2*a*b*x^5*log((e*x + d)^n*c) + a^2
*x^5)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^5}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="maxima")
[Out] -1/2*a^2*(f^2/(g^4*x^2 + f*g^3) - x^2/g^2 + 2*f*log(g*x^2 + f)/g^3) + integ
rate((b^2*x^5*log((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x^5*log((e*x + d)^n
) + (b^2*log(c)^2 + 2*a*b*log(c))*x^5)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

## Giac [F]

$$\int \frac{x^5(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^5}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*x^5/(g*x^2 + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log{(c(d+ex)^n)})^2}{(f+gx^2)^2}\,dx = \int \frac{x^5\,{\left(a+b\,\ln{\left(c\,{\left(d+e\,x\right)}^n\right)}\right)}^2}{{\left(g\,x^2+f\right)}^2}\,dx$$

```
[In] int((x^5*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2,x)
[Out] int((x^5*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2, x)
```

# 3.321 $\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

## Optimal result

Integrand size = 29, antiderivative size = 739

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = -\frac{e^2f(a+b\log(c(d+ex)^n))^2}{2g^2(e^2f+d^2g)}+\frac{f(a+b\log(c(d+ex)^n))^2}{2g^2(f+gx^2)} + \frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)} + \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} + \frac{be\left(ef-d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)} + \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} - \frac{b^2e\sqrt{-f}\left(e\sqrt{-f}+d\sqrt{g}\right)n^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)} + \frac{bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2} + \frac{b^2e\left(ef+d\sqrt{-f}\sqrt{g}\right)n^2\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)} + \frac{bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2} - \frac{b^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2} - \frac{b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2}$$

```
[Out] -1/2*e^2*f*(a+b*ln(c*(e*x+d)^n))^2/g^2/(d^2*g+e^2*f)+1/2*f*(a+b*ln(c*(e*x+d
)^n))^2/g^2/(g*x^2+f)+1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2
))/(e*(-f)^(1/2)+d*g^(1/2)))/g^2+1/2*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/
2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^2+b*n*(a+b*ln(c*(e*x+d)^n))*polyl
og(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^2+b*n*(a+b*ln(c*(e*x+d)^n
))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^2-b^2*n^2*polylog(
3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^2-b^2*n^2*polylog(3,(e*x+d)*
g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^2-1/2*b^2*e*n^2*polylog(2,-(e*x+d)*g^(1
/2)/(e*(-f)^(1/2)-d*g^(1/2)))*(-f)^(1/2)*(e*(-f)^(1/2)+d*g^(1/2))/g^2/(d^2*
g+e^2*f)+1/2*b*e*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f
)^(1/2)-d*g^(1/2)))*(e*f-d*(-f)^(1/2)*g^(1/2))/g^2/(d^2*g+e^2*f)+1/2*b*e*n*
```

```
(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))
*(e*f+d*(-f)^(1/2)*g^(1/2))/g^2/(d^2*g+e^2*f)+1/2*b^2*e*n^2*polylog(2,(e*x+
d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2))/g^2/(d^2*g+
e^2*f)
```

## Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {2463, 2460, 2465, 2437, 2338, 2441, 2440, 2438, 2443, 2481, 2421, 6724}

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$
$$= \frac{ben\left(d\sqrt{-f}\sqrt{g}+ef\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2g^2(d^2g+e^2f)}$$
$$+\frac{ben\left(ef-d\sqrt{-f}\sqrt{g}\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2g^2(d^2g+e^2f)}$$
$$-\frac{e^2f(a+b\log(c(d+ex)^n))^2}{2g^2(d^2g+e^2f)}+\frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{g^2}$$
$$+\frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{g^2}$$
$$+\frac{f(a+b\log(c(d+ex)^n))^2}{2g^2(f+gx^2)}+\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))^2}{2g^2}$$
$$+\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))^2}{2g^2}$$
$$-\frac{b^2e\sqrt{-f}n^2\left(d\sqrt{g}+e\sqrt{-f}\right)\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2(d^2g+e^2f)}$$
$$+\frac{b^2en^2\left(d\sqrt{-f}\sqrt{g}+ef\right)\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2g^2(d^2g+e^2f)}$$
$$-\frac{b^2n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2}-\frac{b^2n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{g^2}$$

```
[In] Int[(x^3*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

```
[Out] -1/2*(e^2*f*(a + b*Log[c*(d + e*x)^n])^2)/(g^2*(e^2*f + d^2*g)) + (f*(a + b
*Log[c*(d + e*x)^n])^2)/(2*g^2*(f + g*x^2)) + (b*e*(e*f + d*Sqrt[-f]*Sqrt[g
])*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f]
+ d*Sqrt[g])])/(2*g^2*(e^2*f + d^2*g)) + ((a + b*Log[c*(d + e*x)^n])^2*Log[
```

```
(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^2) + (b*e*(e*f -
 d*Sqrt[-f]*Sqrt[g])*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g
]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*g^2*(e^2*f + d^2*g)) + ((a + b*Log[c*(d
 + e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*
g^2) - (b^2*e*Sqrt[-f]*(e*Sqrt[-f] + d*Sqrt[g])*n^2*PolyLog[2, -((Sqrt[g]*(
d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*g^2*(e^2*f + d^2*g)) + (b*n*(a + b
*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[
g]))])/g^2 + (b^2*e*(e*f + d*Sqrt[-f]*Sqrt[g])*n^2*PolyLog[2, (Sqrt[g]*(d +
 e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^2*(e^2*f + d^2*g)) + (b*n*(a + b*Log
[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/
g^2 - (b^2*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])
/g^2 - (b^2*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/g
^2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{fx(a+b\log(c(d+ex)^n))^2}{g(f+gx^2)^2}+\frac{x(a+b\log(c(d+ex)^n))^2}{g(f+gx^2)}\right)dx$$

$$=\frac{\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx}{g}-\frac{f\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx}{g}$$

$$=\frac{f(a+b\log(c(d+ex)^n))^2}{2g^2(f+gx^2)}+\frac{\int \left(-\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)}+\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g}-\frac{(befn)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx^2)}\,dx}{g^2}$$

$$=\frac{f(a+b\log(c(d+ex)^n))^2}{2g^2(f+gx^2)}-\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{2g^{3/2}}+\frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{2g^{3/2}}-\frac{(befn)\int \left(\frac{e^2(a+b\log(c(d+ex)^n))}{(e^2f+d^2g)(d+ex)}-\frac{g(-d+ex)(a+b\log(c(d+ex)^n))}{(e^2f+d^2g)(f+gx^2)}\right)dx}{g^2}$$

$$=\frac{f(a+b\log(c(d+ex)^n))^2}{2g^2(f+gx^2)}+\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}+\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2}-\frac{(ben)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{g^2}-\frac{(ben)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{g^2}-\frac{(be^3fn)\int \frac{a+b\log(c(d+ex)^n)}{d+ex}\,dx}{g^2(e^2f+d^2g)}+\frac{(befn)\int \frac{(-d+ex)(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx}{g(e^2f+d^2g)}$$

$$= \frac{f(a+b\log(c(d+ex)^n))^2}{2g^2(f+gx^2)} + \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}$$
$$+ \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2}$$
$$- \frac{(bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g^2}$$
$$- \frac{(bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{g^2}$$
$$- \frac{(be^2fn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x}dx, x, d+ex\right)}{g^2(e^2f+d^2g)}$$
$$+ \frac{(befn)\int \left(\frac{\left(-d\sqrt{-f}-\frac{ef}{\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}-\sqrt{g}x)} + \frac{\left(-d\sqrt{-f}+\frac{ef}{\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g(e^2f+d^2g)}$$

$$= -\frac{e^2 f (a + b\log(c(d+ex)^n))^2}{2g^2(e^2f + d^2g)} + \frac{f(a + b\log(c(d+ex)^n))^2}{2g^2(f+gx^2)}$$
$$+ \frac{(a + b\log(c(d+ex)^n))^2 \log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}$$
$$+ \frac{(a + b\log(c(d+ex)^n))^2 \log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2}$$
$$+ \frac{bn(a + b\log(c(d+ex)^n)) \operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2}$$
$$+ \frac{bn(a + b\log(c(d+ex)^n)) \operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2}$$
$$+ \frac{\left(bef\left(\frac{d}{\sqrt{-f}} + \frac{e}{\sqrt{g}}\right)n\right)\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2g(e^2f + d^2g)}$$
$$- \frac{\left(bef\left(\frac{df}{(-f)^{3/2}} + \frac{e}{\sqrt{g}}\right)n\right)\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2g(e^2f + d^2g)}$$
$$- \frac{(b^2n^2)\operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{g^2}$$
$$- \frac{(b^2n^2)\operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{g^2}$$

$$
\begin{aligned}
&= -\frac{e^2 f (a + b\log(c(d+ex)^n))^2}{2g^2(e^2 f + d^2 g)} + \frac{f(a + b\log(c(d+ex)^n))^2}{2g^2(f+gx^2)} \\
&+ \frac{be\left(ef + d\sqrt{-f}\sqrt{g}\right) n(a + b\log(c(d+ex)^n)) \log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2(e^2 f + d^2 g)} \\
&+ \frac{(a + b\log(c(d+ex)^n))^2 \log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} \\
&+ \frac{be\left(ef - d\sqrt{-f}\sqrt{g}\right) n(a + b\log(c(d+ex)^n)) \log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2(e^2 f + d^2 g)} \\
&+ \frac{(a + b\log(c(d+ex)^n))^2 \log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} \\
&+ \frac{bn(a + b\log(c(d+ex)^n)) \operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2} \\
&+ \frac{bn(a + b\log(c(d+ex)^n)) \operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2} - \frac{b^2 n^2 \operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2} \\
&- \frac{b^2 n^2 \operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2} - \frac{\left(b^2 e^2 \left(ef + d\sqrt{-f}\sqrt{g}\right) n^2\right) \int \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex} \, dx}{2g^2(e^2 f + d^2 g)} \\
&- \frac{\left(b^2 e^2 f \left(\frac{d}{\sqrt{-f}} + \frac{e}{\sqrt{g}}\right) n^2\right) \int \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex} \, dx}{2g^{3/2}(e^2 f + d^2 g)}
\end{aligned}
$$

$$= -\frac{e^2 f (a + b\log(c(d+ex)^n))^2}{2g^2(e^2f + d^2g)} + \frac{f(a + b\log(c(d+ex)^n))^2}{2g^2(f+gx^2)}$$
$$+ \frac{be\left(ef + d\sqrt{-f}\sqrt{g}\right) n(a + b\log(c(d+ex)^n)) \log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)}$$
$$+ \frac{(a + b\log(c(d+ex)^n))^2 \log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2}$$
$$+ \frac{be\left(ef - d\sqrt{-f}\sqrt{g}\right) n(a + b\log(c(d+ex)^n)) \log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)}$$
$$+ \frac{(a + b\log(c(d+ex)^n))^2 \log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2}$$
$$+ \frac{bn(a + b\log(c(d+ex)^n)) \operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2}$$
$$+ \frac{bn(a + b\log(c(d+ex)^n)) \operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2}$$
$$- \frac{b^2n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2} - \frac{b^2n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2}$$
$$- \frac{\left(b^2e\left(ef + d\sqrt{-f}\sqrt{g}\right)n^2\right) \operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2g^2(e^2f+d^2g)}$$
$$- \frac{\left(b^2ef\left(\frac{d}{\sqrt{-f}} + \frac{e}{\sqrt{g}}\right)n^2\right) \operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2g^{3/2}(e^2f+d^2g)}$$

$$= -\frac{e^2f(a+b\log(c(d+ex)^n))^2}{2g^2(e^2f+d^2g)} + \frac{f(a+b\log(c(d+ex)^n))^2}{2g^2(f+gx^2)} + \frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)} + \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2} + \frac{be\left(ef-d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)} + \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2} + \frac{b^2e\left(ef-d\sqrt{-f}\sqrt{g}\right)n^2\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)} + \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2} + \frac{b^2e\left(ef+d\sqrt{-f}\sqrt{g}\right)n^2\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^2(e^2f+d^2g)} + \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2} - \frac{b^2n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^2} - \frac{b^2n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^2}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 1103, normalized size of antiderivative = 1.49

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$

$$= \frac{\frac{2f(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2} + 2(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2\log(f+gx^2) + 2bn(a-bn\log}{}$$

```
[In] Integrate[(x^3*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

```
[Out] ((2*f*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) + 2*(a -
 b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a - b*n
*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((Sqrt[f]*((-I)*Sqrt[g]*(d + e*x)*Log
```

```
[d + e*x] + e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt
[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (Sqrt[f]*(I*Sqrt[g]*(d + e*x)
*Log[d + e*x] + e*(Sqrt[f] - I*Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*x]))/((e*
Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) + 2*(Log[d + e*x]*Log[(e*(S
qrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g
]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + 2*(Log[d + e*x]*Log[(e*(Sqrt[f]
- I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x
))/(e*Sqrt[f] + I*d*Sqrt[g])])) + b^2*n^2*(2*Log[d + e*x]^2*Log[1 - (Sqrt[g
]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]^2*Log[1 - (Sqrt
[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + (Sqrt[f]*(Log[d + e*x]*(I*Sqrt[
g]*(d + e*x)*Log[d + e*x] + 2*e*(Sqrt[f] - I*Sqrt[g]*x)*Log[(e*(Sqrt[f] - I
*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])]) + 2*e*(Sqrt[f] - I*Sqrt[g]*x)*Poly
Log[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/((e*Sqrt[f] + I*d
*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) + 4*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d
+ e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 4*Log[d + e*x]*PolyLog[2, (Sqrt[g]*
(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + (Sqrt[f]*(Log[d + e*x]*((-I)*Sqrt[g
]*(d + e*x)*Log[d + e*x] + 2*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[(e*(Sqrt[f] + I*
Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + 2*e*(Sqrt[f] + I*Sqrt[g]*x)*PolyL
og[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]))/((e*Sqrt[f] - I*d*Sq
rt[g])*(Sqrt[f] + I*Sqrt[g]*x)) - 4*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*
Sqrt[f] + d*Sqrt[g])] - 4*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*S
qrt[g])]))/(4*g^2)
```

## Maple [F]

$$\int \frac{x^3(a + b\ln(c(ex + d)^n))^2}{(g\,x^2 + f)^2}dx$$

[In] `int(x^3*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)`

[Out] `int(x^3*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)`

## Fricas [F]

$$\int \frac{x^3(a + b\log(c(d + ex)^n))^2}{(f + gx^2)^2}\,dx = \int \frac{(b\log((ex + d)^n c) + a)^2 x^3}{(gx^2 + f)^2}\,dx$$

[In] `integrate(x^3*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="fricas")`

[Out] `integral((b^2*x^3*log((e*x + d)^n*c)^2 + 2*a*b*x^3*log((e*x + d)^n*c) + a^2*x^3)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)`

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^3}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="maxima")
[Out] 1/2*a^2*(f/(g^3*x^2 + f*g^2) + log(g*x^2 + f)/g^2) + integrate((b^2*x^3*log
((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x^3*log((e*x + d)^n) + (b^2*log(c)^2
 + 2*a*b*log(c))*x^3)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

## Giac [F]

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x^3}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*x^3/(g*x^2 + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{x^3\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2}{{\left(g\,x^2+f\right)}^2}\,dx$$

```
[In] int((x^3*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2,x)
[Out] int((x^3*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2, x)
```

# 3.322 $\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 430

$$\begin{aligned}\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx &= \frac{e^2(a+b\log(c(d+ex)^n))^2}{2g(e^2f+d^2g)}-\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)}\\ &\quad-\frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fg(e^2f+d^2g)}\\ &\quad-\frac{be\left(ef-d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fg(e^2f+d^2g)}\\ &\quad-\frac{b^2e\left(e\sqrt{-f}+d\sqrt{g}\right)n^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g(e^2f+d^2g)}\\ &\quad-\frac{b^2e\left(ef+d\sqrt{-f}\sqrt{g}\right)n^2\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fg(e^2f+d^2g)}\end{aligned}$$

```
[Out] 1/2*e^2*(a+b*ln(c*(e*x+d)^n))^2/g/(d^2*g+e^2*f)-1/2*(a+b*ln(c*(e*x+d)^n))^2
/g/(g*x^2+f)-1/2*b^2*e*n^2*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/
2)))*(e*(-f)^(1/2)+d*g^(1/2))/g/(d^2*g+e^2*f)/(-f)^(1/2)-1/2*b*e*n*(a+b*ln(
c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))*(e*f-d*
(-f)^(1/2)*g^(1/2))/f/g/(d^2*g+e^2*f)-1/2*b*e*n*(a+b*ln(c*(e*x+d)^n))*ln(e*
((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2))
/f/g/(d^2*g+e^2*f)-1/2*b^2*e*n^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*
g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2))/f/g/(d^2*g+e^2*f)
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2460, 2465, 2437, 2338, 2441, 2440, 2438}

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$
$$= -\frac{ben\left(d\sqrt{-f}\sqrt{g}+ef\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)(a+b\log(c(d+ex)^n))}{2fg(d^2g+e^2f)}$$
$$-\frac{ben\left(ef-d\sqrt{-f}\sqrt{g}\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2fg(d^2g+e^2f)}$$
$$+\frac{e^2(a+b\log(c(d+ex)^n))^2}{2g(d^2g+e^2f)}-\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)}$$
$$-\frac{b^2en^2\left(d\sqrt{g}+e\sqrt{-f}\right)\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g(d^2g+e^2f)}$$
$$-\frac{b^2en^2\left(d\sqrt{-f}\sqrt{g}+ef\right)\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2fg(d^2g+e^2f)}$$

```
[In] Int[(x*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

```
[Out] (e^2*(a + b*Log[c*(d + e*x)^n])^2)/(2*g*(e^2*f + d^2*g)) - (a + b*Log[c*(d
+ e*x)^n])^2/(2*g*(f + g*x^2)) - (b*e*(e*f + d*Sqrt[-f]*Sqrt[g])*n*(a + b*L
og[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])]
)/(2*f*g*(e^2*f + d^2*g)) - (b*e*(e*f - d*Sqrt[-f]*Sqrt[g])*n*(a + b*Log[c*
(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*
f*g*(e^2*f + d^2*g)) - (b^2*e*(e*Sqrt[-f] + d*Sqrt[g])*n^2*PolyLog[2, -((Sq
rt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*Sqrt[-f]*g*(e^2*f + d^2*g))
 - (b^2*e*(e*f + d*Sqrt[-f]*Sqrt[g])*n^2*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*
Sqrt[-f] + d*Sqrt[g])])/(2*f*g*(e^2*f + d^2*g))
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)} + \frac{(ben)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx^2)}\,dx}{g}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)} + \frac{(ben)\int \left(\frac{e^2(a+b\log(c(d+ex)^n))}{(e^2f+d^2g)(d+ex)} - \frac{g(-d+ex)(a+b\log(c(d+ex)^n))}{(e^2f+d^2g)(f+gx^2)}\right)dx}{g}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)} - \frac{(ben)\int \frac{(-d+ex)(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx}{e^2f+d^2g} + \frac{(be^3n)\int \frac{a+b\log(c(d+ex)^n)}{d+ex}\,dx}{g(e^2f+d^2g)}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)} - \frac{(ben)\int\left(\frac{\left(-d\sqrt{-f}-\frac{ef}{\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}-\sqrt{g}x)}+\frac{\left(-d\sqrt{-f}+\frac{ef}{\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}+\sqrt{g}x)}\right)dx}{e^2f+d^2g} + \frac{(be^2n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x}\,dx,x,d+ex\right)}{g(e^2f+d^2g)}$$

$$= \frac{e^2(a+b\log(c(d+ex)^n))^2}{2g(e^2f+d^2g)} - \frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)} - \frac{\left(be\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)n\right)\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2(e^2f+d^2g)} + \frac{\left(be\left(\frac{df}{(-f)^{3/2}}+\frac{e}{\sqrt{g}}\right)n\right)\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2(e^2f+d^2g)}$$

$$= \frac{e^2(a+b\log(c(d+ex)^n))^2}{2g(e^2f+d^2g)} - \frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)} - \frac{be\left(\frac{df}{(-f)^{3/2}}+\frac{e}{\sqrt{g}}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{g}(e^2f+d^2g)} - \frac{be\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{g}(e^2f+d^2g)} + \frac{\left(b^2e^2\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)n^2\right)\int\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2\sqrt{g}(e^2f+d^2g)} + \frac{\left(b^2e^2\left(\frac{df}{(-f)^{3/2}}+\frac{e}{\sqrt{g}}\right)n^2\right)\int\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2\sqrt{g}(e^2f+d^2g)}$$

$$= \frac{e^2(a+b\log(c(d+ex)^n))^2}{2g(e^2f+d^2g)} - \frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)} - \frac{be\left(\frac{df}{(-f)^{3/2}}+\frac{e}{\sqrt{g}}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{g}(e^2f+d^2g)} - \frac{be\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{g}(e^2f+d^2g)} + \frac{\left(b^2e\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)n^2\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2\sqrt{g}(e^2f+d^2g)} + \frac{\left(b^2e\left(\frac{df}{(-f)^{3/2}}+\frac{e}{\sqrt{g}}\right)n^2\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2\sqrt{g}(e^2f+d^2g)}$$

$$= \frac{e^2(a+b\log(c(d+ex)^n))^2}{2g(e^2f+d^2g)} - \frac{(a+b\log(c(d+ex)^n))^2}{2g(f+gx^2)} - \frac{be\left(\frac{df}{(-f)^{3/2}}+\frac{e}{\sqrt{g}}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{g}(e^2f+d^2g)} - \frac{be\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{g}(e^2f+d^2g)} - \frac{b^2e\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)n^2\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{g}(e^2f+d^2g)} - \frac{b^2e\left(\frac{df}{(-f)^{3/2}}+\frac{e}{\sqrt{g}}\right)n^2\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{g}(e^2f+d^2g)}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.37

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$

$$= \frac{-\frac{2(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2} + \frac{2bn(-a+bn\log(d+ex)-b\log(c(d+ex)^n))(2\sqrt{f}g(d^2-e^2x^2)\log(d+ex)+e(f+gx^2)((e\sqrt{f}+id\sqrt{g})\log(}{\sqrt{f}(e^2f+d^2g)(f+gx^2)}}{}$$

```
[In] Integrate[(x*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

```
[Out] ((-2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) + (2*b*n*
(-a + b*n*Log[d + e*x] - b*Log[c*(d + e*x)^n])*(2*Sqrt[f]*g*(d^2 - e^2*x^2)
*Log[d + e*x] + e*(f + g*x^2)*((e*Sqrt[f] + I*d*Sqrt[g])*Log[I*Sqrt[f] - Sq
```

```
rt[g]*x] + (e*Sqrt[f] - I*d*Sqrt[g])*Log[I*Sqrt[f] + Sqrt[g]*x])))/(Sqrt[f]
*(e^2*f + d^2*g)*(f + g*x^2)) + (I*b^2*n^2*((-(Sqrt[g]*(d + e*x)*Log[d + e*
x]^2) + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g
]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*PolyLog[2, (
I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])])/((e*Sqrt[f] + I*d*Sqrt[g])
*(Sqrt[f] - I*Sqrt[g]*x)) + (Log[d + e*x]*(Sqrt[g]*(d + e*x)*Log[d + e*x] +
 (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f]
 - I*d*Sqrt[g])]) + (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*PolyLog[2, (Sqrt[g]*(d
+ e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I
*Sqrt[g]*x))))/Sqrt[f])/(4*g)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.54 (sec) , antiderivative size = 1231, normalized size of antiderivative = 2.86

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1231 |

```
[In] int(x*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2*ln((e*x+d)^n)^2/g/(g*x^2+f)-1/2*b^2/g*n^2*e^2/(d^2*g+e^2*f)*ln(e*x
+d)^2+b^2/g*n*e^2/(d^2*g+e^2*f)*ln(e*x+d)*ln((e*x+d)^n)+1/2*b^2/g*n^2*e^2/(
d^2*g+e^2*f)*ln(g*(e*x+d)^2-2*(e*x+d)*d*g+d^2*g+f*e^2)*ln(e*x+d)-1/2*b^2/g*
n*e^2/(d^2*g+e^2*f)*ln(g*(e*x+d)^2-2*(e*x+d)*d*g+d^2*g+f*e^2)*ln((e*x+d)^n)
-b^2*n^2*e/(d^2*g+e^2*f)*d/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*
g)^(1/2))*ln(e*x+d)+b^2*n*e/(d^2*g+e^2*f)*d/(f*g)^(1/2)*arctan(1/2*(2*g*(e*
x+d)-2*d*g)/e/(f*g)^(1/2))*ln((e*x+d)^n)-1/2*b^2/g*n^2*e^2/(d^2*g+e^2*f)*ln
(e*x+d)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*b^2/g*n
^2*e^2/(d^2*g+e^2*f)*ln(e*x+d)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^
(1/2)-d*g))+1/2*b^2*n^2*e/(d^2*g+e^2*f)*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)
^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*d-1/2*b^2*n^2*e/(d^2*g+e^2*f)*l
n(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g
))*d-1/2*b^2/g*n^2*e^2/(d^2*g+e^2*f)*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(
e*(-f*g)^(1/2)+d*g))-1/2*b^2/g*n^2*e^2/(d^2*g+e^2*f)*dilog((e*(-f*g)^(1/2)+
g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*b^2*n^2*e/(d^2*g+e^2*f)/(-f*g)^(1/
2)*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))*d-1/2*b^2*n^2
*e/(d^2*g+e^2*f)/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-f*g
)^(1/2)-d*g))*d+(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*
Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d
)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*b*(-1/2*ln((e*x+d)^n)/
g/(g*x^2+f)+1/2/g*n*e*(e/(d^2*g+e^2*f)*ln(e*x+d)-1/(d^2*g+e^2*f)*g*(1/2*e/g
*ln(g*x^2+f)-d/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2)))))-1/8*(-I*b*Pi*csgn(I*c
*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^
```

```
2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)
^3*b+2*b*ln(c)+2*a)^2/g/(g*x^2+f)
```

## Fricas [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral((b^2*x*log((e*x + d)^n*c)^2 + 2*a*b*x*log((e*x + d)^n*c) + a^2*x)/
(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2 x}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="maxima")
[Out] -1/2*a*b*e*n*(e*log(g*x^2 + f)/(e^2*f*g + d^2*g^2) - 2*e*log(e*x + d)/(e^2*
f*g + d^2*g^2) - 2*d*arctan(g*x/sqrt(f*g))/((e^2*f + d^2*g)*sqrt(f*g))) - 1
/2*b^2*(log((e*x + d)^n)^2/(g^2*x^2 + f*g) - 2*integrate((e*g*x^2*log(c)^2
+ d*g*x*log(c)^2 + (2*d*g*x*log(c) + e*f*n + (e*g*n + 2*e*g*log(c))*x^2)*lo
g((e*x + d)^n))/(e*g^3*x^5 + d*g^3*x^4 + 2*e*f*g^2*x^3 + 2*d*f*g^2*x^2 + e*
f^2*g*x + d*f^2*g), x)) - a*b*log((e*x + d)^n*c)/(g^2*x^2 + f*g) - 1/2*a^2/
(g^2*x^2 + f*g)
```

## Giac [F]

$$\int \frac{x(a+b\log\left(c(d+ex)^n\right))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log\left((ex+d)^n c\right)+a)^2 x}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*x/(g*x^2 + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log\left(c(d+ex)^n\right))^2}{(f+gx^2)^2}\,dx = \int \frac{x\,\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)^2}{(g\,x^2+f)^2}\,dx$$

```
[In] int((x*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2,x)
[Out] int((x*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2, x)
```

# 3.323 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx$

## Optimal result

Integrand size = 29, antiderivative size = 814

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx$$
$$= -\frac{e^2(a+b\log(c(d+ex)^n))^2}{2f(e^2f+d^2g)} + \frac{(a+b\log(c(d+ex)^n))^2}{2f(f+gx^2)}$$
$$+ \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2}$$
$$+ \frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)}$$
$$- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}$$
$$+ \frac{be\left(ef-d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)}$$
$$- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}$$
$$- \frac{b^2e\left(e\sqrt{-f}+d\sqrt{g}\right)n^2\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}(e^2f+d^2g)}$$
$$- \frac{bn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2}$$
$$+ \frac{b^2e\left(ef+d\sqrt{-f}\sqrt{g}\right)n^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)}$$
$$- \frac{bn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2}$$
$$+ \frac{2bn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^2} + \frac{b^2n^2\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2}$$
$$+ \frac{b^2n^2\,\mathrm{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2} - \frac{2b^2n^2\,\mathrm{PolyLog}\left(3,1+\frac{ex}{d}\right)}{f^2}$$

```
[Out] -1/2*e^2*(a+b*ln(c*(e*x+d)^n))^2/f/(d^2*g+e^2*f)+1/2*(a+b*ln(c*(e*x+d)^n))^
2/f/(g*x^2+f)+ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))^2/f^2-1/2*(a+b*ln(c*(e*x+d)^
n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f^2-1/2*(a+b*ln
(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/f^2+
2*b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,1+e*x/d)/f^2-b*n*(a+b*ln(c*(e*x+d)^n)
)*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f^2-b*n*(a+b*ln(c*(e
```

```
*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f^2-2*b^2*n^2
*polylog(3,1+e*x/d)/f^2+b^2*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*
g^(1/2)))/f^2+b^2*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f
^2-1/2*b^2*e*n^2*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*(e*(-
f)^(1/2)+d*g^(1/2))/(-f)^(3/2)/(d^2*g+e^2*f)+1/2*b*e*n*(a+b*ln(c*(e*x+d)^n)
)*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))*(e*f-d*(-f)^(1/2)*g
^(1/2))/f^2/(d^2*g+e^2*f)+1/2*b*e*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-
x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2))/f^2/(d^2*g+
e^2*f)+1/2*b^2*e*n^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*(e
*f+d*(-f)^(1/2)*g^(1/2))/f^2/(d^2*g+e^2*f)
```

## Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules

used = {2463, 2443, 2481, 2421, 6724, 2460, 2465, 2437, 2338, 2441, 2440, 2438}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx$$
$$= -\frac{b^2e\left(\sqrt{g}d+e\sqrt{-f}\right)\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2(-f)^{3/2}(gd^2+e^2f)}$$
$$+\frac{b^2e\left(\sqrt{-f}\sqrt{g}d+ef\right)\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2f^2(gd^2+e^2f)}+\frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{f^2}$$
$$+\frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{f^2}-\frac{2b^2\,\text{PolyLog}\left(3,\frac{ex}{d}+1\right)n^2}{f^2}$$
$$+\frac{be\left(\sqrt{-f}\sqrt{g}d+ef\right)(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2f^2(gd^2+e^2f)}$$
$$+\frac{be\left(ef-d\sqrt{-f}\sqrt{g}\right)(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2f^2(gd^2+e^2f)}$$
$$-\frac{b(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n}{f^2}$$
$$-\frac{b(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{f^2}$$
$$+\frac{2b(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)n}{f^2}$$
$$+\frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2}-\frac{e^2(a+b\log(c(d+ex)^n))^2}{2f(gd^2+e^2f)}$$
$$+\frac{(a+b\log(c(d+ex)^n))^2}{2f(gx^2+f)}-\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2f^2}$$
$$-\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(x*(f + g*x^2)^2),x]
```

```
[Out] -1/2*(e^2*(a + b*Log[c*(d + e*x)^n])^2)/(f*(e^2*f + d^2*g)) + (a + b*Log[c*
(d + e*x)^n])^2/(2*f*(f + g*x^2)) + (Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)
^n])^2)/f^2 + (b*e*(e*f + d*Sqrt[-f]*Sqrt[g])*n*(a + b*Log[c*(d + e*x)^n])*
Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^2*(e^2*f + d
^2*g)) - ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sq
rt[-f] + d*Sqrt[g])])/(2*f^2) + (b*e*(e*f - d*Sqrt[-f]*Sqrt[g])*n*(a + b*Lo
g[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])
/(2*f^2*(e^2*f + d^2*g)) - ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] +
```

```
Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*f^2) - (b^2*e*(e*Sqrt[-f] + d*Sq
rt[g])*n^2*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*
(-f)^(3/2)*(e^2*f + d^2*g)) - (b*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -(
(Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/f^2 + (b^2*e*(e*f + d*Sqrt[
-f]*Sqrt[g])*n^2*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/
(2*f^2*(e^2*f + d^2*g)) - (b*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[
g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/f^2 + (2*b*n*(a + b*Log[c*(d + e*x
)^n])*PolyLog[2, 1 + (e*x)/d])/f^2 + (b^2*n^2*PolyLog[3, -((Sqrt[g]*(d + e*
x))/(e*Sqrt[-f] - d*Sqrt[g]))])/f^2 + (b^2*n^2*PolyLog[3, (Sqrt[g]*(d + e*x
))/(e*Sqrt[-f] + d*Sqrt[g])])/f^2 - (2*b^2*n^2*PolyLog[3, 1 + (e*x)/d])/f^2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+b\log(c(d+ex)^n))^2}{f^2x} - \frac{gx(a+b\log(c(d+ex)^n))^2}{f(f+gx^2)^2} - \frac{gx(a+b\log(c(d+ex)^n))^2}{f^2(f+gx^2)}\right)dx$$

$$= \frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx}{f^2} - \frac{g\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx}{f^2} - \frac{g\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx}{f}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2}{2f(f+gx^2)} + \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2} - \frac{g\int\left(-\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)} + \frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right)dx}{f^2} - \frac{(2ben)\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{d+ex}\,dx}{f^2} - \frac{(ben)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx^2)}\,dx}{f}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2}{2f(f+gx^2)} + \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2} + \frac{\sqrt{g}\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{2f^2} - \frac{\sqrt{g}\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{2f^2} - \frac{(2bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+ex\right)}{f^2} - \frac{(ben)\int\left(\frac{e^2(a+b\log(c(d+ex)^n))}{(e^2f+d^2g)(d+ex)} - \frac{g(-d+ex)(a+b\log(c(d+ex)^n))}{(e^2f+d^2g)(f+gx^2)}\right)dx}{f}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2}{2f(f+gx^2)} + \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} + \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} + \frac{(ben)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{f^2} + \frac{(ben)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{f^2} - \frac{(be^3n)\int\frac{a+b\log(c(d+ex)^n)}{d+ex}\,dx}{f(e^2f+d^2g)} + \frac{(begn)\int\frac{(-d+ex)(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx}{f(e^2f+d^2g)} - \frac{(2b^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{x}{d}\right)}{x}\,dx, x, d+ex\right)}{f^2}$$

$$= \frac{(a+b\log(c(d+ex)^n))^2}{2f(f+gx^2)} + \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2}$$
$$- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}$$
$$- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}$$
$$+ \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} - \frac{2b^2n^2\operatorname{Li}_3\left(1+\frac{ex}{d}\right)}{f^2}$$
$$+ \frac{(bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{f^2}$$
$$+ \frac{(bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{f^2}$$
$$- \frac{(be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x}dx, x, d+ex\right)}{f(e^2f+d^2g)}$$
$$+ \frac{(begn)\int \left(\frac{\left(-d\sqrt{-f}-\frac{ef}{\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}-\sqrt{g}x)} + \frac{\left(-d\sqrt{-f}+\frac{ef}{\sqrt{g}}\right)(a+b\log(c(d+ex)^n))}{2f(\sqrt{-f}+\sqrt{g}x)}\right)dx}{f(e^2f+d^2g)}$$

$$= -\frac{e^2(a+b\log(c(d+ex)^n))^2}{2f(e^2f+d^2g)} + \frac{(a+b\log(c(d+ex)^n))^2}{2f(f+gx^2)} + \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} - \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2} - \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2} + \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} - \frac{2b^2n^2\operatorname{Li}_3\left(1+\frac{ex}{d}\right)}{f^2} + \frac{\left(be\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)gn\right)\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2f(e^2f+d^2g)} - \frac{\left(be\left(\frac{df}{(-f)^{3/2}}+\frac{e}{\sqrt{g}}\right)gn\right)\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2f(e^2f+d^2g)} + \frac{(b^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{f^2} + \frac{(b^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{f^2}$$

$$
\begin{aligned}
&= -\frac{e^2(a+b\log(c(d+ex)^n))^2}{2f(e^2f+d^2g)} + \frac{(a+b\log(c(d+ex)^n))^2}{2f(f+gx^2)} \\
&+ \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2} \\
&+ \frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)} \\
&- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2} \\
&+ \frac{be\left(ef-d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)} \\
&- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} \\
&- \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2} \\
&- \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2} \\
&+ \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} + \frac{b^2n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2} \\
&+ \frac{b^2n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2} - \frac{2b^2n^2\operatorname{Li}_3\left(1+\frac{ex}{d}\right)}{f^2} \\
&- \frac{\left(b^2e^2\left(ef+d\sqrt{-f}\sqrt{g}\right)n^2\right)\int\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2f^2(e^2f+d^2g)} \\
&- \frac{\left(b^2e^2\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)\sqrt{g}n^2\right)\int\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2f(e^2f+d^2g)}
\end{aligned}
$$

$$= -\frac{e^2(a+b\log(c(d+ex)^n))^2}{2f(e^2f+d^2g)} + \frac{(a+b\log(c(d+ex)^n))^2}{2f(f+gx^2)}$$
$$+ \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2}$$
$$+ \frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)}$$
$$- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}$$
$$+ \frac{be\left(ef-d\sqrt{-f}\sqrt{g}\right)n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)}$$
$$- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}$$
$$- \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2}$$
$$- \frac{bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2}$$
$$+ \frac{2bn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} + \frac{b^2n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2}$$
$$+ \frac{b^2n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2} - \frac{2b^2n^2\operatorname{Li}_3\left(1+\frac{ex}{d}\right)}{f^2}$$
$$- \frac{\left(b^2e\left(ef+d\sqrt{-f}\sqrt{g}\right)n^2\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{2f^2(e^2f+d^2g)}$$
$$- \frac{\left(b^2e\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)\sqrt{g}n^2\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{2f(e^2f+d^2g)}$$

$$= -\frac{e^2(a+b\log(c(d+ex)^n))^2}{2f(e^2f+d^2g)} + \frac{(a+b\log(c(d+ex)^n))^2}{2f(f+gx^2)}$$
$$+ \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^2}$$
$$+ \frac{be(ef+d\sqrt{-f}\sqrt{g})\,n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)}$$
$$- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2}$$
$$+ \frac{be(ef-d\sqrt{-f}\sqrt{g})\,n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)}$$
$$- \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2}$$
$$+ \frac{b^2e(ef-d\sqrt{-f}\sqrt{g})\,n^2\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)}$$
$$- \frac{bn(a+b\log(c(d+ex)^n))\,\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2}$$
$$+ \frac{b^2e(ef+d\sqrt{-f}\sqrt{g})\,n^2\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2(e^2f+d^2g)}$$
$$- \frac{bn(a+b\log(c(d+ex)^n))\,\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2}$$
$$+ \frac{2bn(a+b\log(c(d+ex)^n))\,\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^2} + \frac{b^2n^2\text{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^2}$$
$$+ \frac{b^2n^2\text{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^2} - \frac{2b^2n^2\text{Li}_3\left(1+\frac{ex}{d}\right)}{f^2}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 1209, normalized size of antiderivative = 1.49

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx$$

$$= \frac{\frac{2f(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2} + 4\log(x)(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2 - 2(a-bn\log(d+ex)\ -}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x*(f + g*x^2)^2),x]
```

```
[Out] ((2*f*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) + 4*Log[
x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - 2*(a - b*n*Log[d + e*x
] + b*Log[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a - b*n*Log[d + e*x] +
b*Log[c*(d + e*x)^n])*((Sqrt[f]*((-I)*Sqrt[g]*(d + e*x)*Log[d + e*x] + e*(S
qrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt[f] - I*d*Sqrt[g
])*(Sqrt[f] + I*Sqrt[g]*x)) + (Sqrt[f]*(I*Sqrt[g]*(d + e*x)*Log[d + e*x] +
e*(Sqrt[f] - I*Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x)) - 2*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[
g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*
Sqrt[f] - I*d*Sqrt[g])]) - 2*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/
(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] +
I*d*Sqrt[g])]) + 4*(Log[-((e*x)/d)]*Log[d + e*x] + PolyLog[2, 1 + (e*x)/d])
) + b^2*n^2*(4*Log[-((e*x)/d)]*Log[d + e*x]^2 - 2*Log[d + e*x]^2*Log[1 - (S
qrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*Log[d + e*x]^2*Log[1 -
(Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + (Sqrt[f]*(Log[d + e*x]*(I*
Sqrt[g]*(d + e*x)*Log[d + e*x] + 2*e*(Sqrt[f] - I*Sqrt[g]*x)*Log[(e*(Sqrt[f
] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])]) + 2*e*(Sqrt[f] - I*Sqrt[g]*x)
*PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/((e*Sqrt[f]
+ I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) - 4*Log[d + e*x]*PolyLog[2, (Sqrt[g
]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 4*Log[d + e*x]*PolyLog[2, (Sqr
t[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + (Sqrt[f]*(Log[d + e*x]*((-I)*S
qrt[g]*(d + e*x)*Log[d + e*x] + 2*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[(e*(Sqrt[f]
 + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + 2*e*(Sqrt[f] + I*Sqrt[g]*x)*
PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]))/((e*Sqrt[f] - I
*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + 8*Log[d + e*x]*PolyLog[2, 1 + (e*x)/
d] + 4*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 4*Pol
yLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 8*PolyLog[3, 1 + (
e*x)/d]))/(4*f^2)
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}{x\left(g\,x^{2}+f\right)^{2}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/x/(g*x^2+f)^2,x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))^2/x/(g*x^2+f)^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2 x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^2*x
^5 + 2*f*g*x^3 + f^2*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/x/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2 x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x/(g*x^2+f)^2,x, algorithm="maxima")
[Out] 1/2*a^2*(1/(f*g*x^2 + f^2) - log(g*x^2 + f)/f^2 + 2*log(x)/f^2) + integrate
((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*
b)*log((e*x + d)^n))/(g^2*x^5 + 2*f*g*x^3 + f^2*x), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2 x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)^2*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x(f+gx^2)^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2}{x\,(g\,x^2+f)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(x*(f + g*x^2)^2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^2/(x*(f + g*x^2)^2), x)
```

# 3.324 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2} \, dx$

## Optimal result

Integrand size = 29, antiderivative size = 970

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx$$
$$= \frac{b^2e^2n^2\log(x)}{d^2f^2} - \frac{ben(d+ex)(a+b\log(c(d+ex)^n))}{d^2f^2x} + \frac{e^2g(a+b\log(c(d+ex)^n))^2}{2f^2(e^2f+d^2g)} - \frac{(a+b\log(c(d+ex)^n))^2}{2f^2x^2} - \frac{g(a+b\log(c(d+ex)^n))^2}{2f^2(f+gx^2)} - \frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^3} - \frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)gn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^3(e^2f+d^2g)} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} - \frac{be\left(ef-d\sqrt{-f}\sqrt{g}\right)gn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^3(e^2f+d^2g)} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} - \frac{be^2n(a+b\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{d^2f^2} + \frac{b^2e^2n^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d^2f^2} - \frac{b^2e\left(e\sqrt{-f}+d\sqrt{g}\right)gn^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}(e^2f+d^2g)} + \frac{2bgn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} - \frac{b^2e\left(ef+d\sqrt{-f}\sqrt{g}\right)gn^2\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^3(e^2f+d^2g)} + \frac{2bgn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} - \frac{4bgn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{f^3} - \frac{2b^2gn^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} - \frac{2b^2gn^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} + \frac{4b^2gn^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{f^3}$$

```
[Out] b^2*e^2*n^2*ln(x)/d^2/f^2-b*e*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))/d^2/f^2/x+1/2
*e^2*g*(a+b*ln(c*(e*x+d)^n))^2/f^2/(d^2*g+e^2*f)-1/2*(a+b*ln(c*(e*x+d)^n))^
```

```
2/f^2/x^2-1/2*g*(a+b*ln(c*(e*x+d)^n))^2/f^2/(g*x^2+f)-2*g*ln(-e*x/d)*(a+b*l
n(c*(e*x+d)^n))^2/f^3-b*e^2*n*(a+b*ln(c*(e*x+d)^n))*ln(1-d/(e*x+d))/d^2/f^2
+g*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1
/2)))/f^3+g*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/
2)-d*g^(1/2)))/f^3+b^2*e^2*n^2*polylog(2,d/(e*x+d))/d^2/f^2-4*b*g*n*(a+b*ln
(c*(e*x+d)^n))*polylog(2,1+e*x/d)/f^3+2*b*g*n*(a+b*ln(c*(e*x+d)^n))*polylog
(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f^3+2*b*g*n*(a+b*ln(c*(e*x+d)
^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f^3+4*b^2*g*n^2*po
lylog(3,1+e*x/d)/f^3-2*b^2*g*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d
*g^(1/2)))/f^3-2*b^2*g*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2
)))/f^3-1/2*b^2*e*g*n^2*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2))
)*(e*(-f)^(1/2)+d*g^(1/2))/(-f)^(5/2)/(d^2*g+e^2*f)-1/2*b*e*g*n*(a+b*ln(c*(
e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))*(e*f-d*(-f
)^(1/2)*g^(1/2))/f^3/(d^2*g+e^2*f)-1/2*b*e*g*n*(a+b*ln(c*(e*x+d)^n))*ln(e*(
(-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2))/
f^3/(d^2*g+e^2*f)-1/2*b^2*e*g*n^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d
*g^(1/2)))*(e*f+d*(-f)^(1/2)*g^(1/2))/f^3/(d^2*g+e^2*f)
```

## Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 970, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.621$, Rules used = {2463, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2443, 2481, 2421, 6724, 2460, 2465,

2437, 2338, 2441, 2440}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx$$

$$= \frac{g(a+b\log(c(d+ex)^n))^2 e^2}{2f^2(gd^2+e^2f)} + \frac{b^2n^2\log(x)e^2}{d^2f^2}$$
$$- \frac{bn(a+b\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)e^2}{d^2f^2}$$
$$+ \frac{b^2n^2\,\text{PolyLog}\left(2,\frac{d}{d+ex}\right)e^2}{d^2f^2} - \frac{bn(d+ex)(a+b\log(c(d+ex)^n))e}{d^2f^2x}$$
$$- \frac{b\left(\sqrt{-f}\sqrt{g}d+ef\right)gn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)e}{2f^3(gd^2+e^2f)}$$
$$- \frac{b\left(ef-d\sqrt{-f}\sqrt{g}\right)gn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)e}{2f^3(gd^2+e^2f)}$$
$$- \frac{b^2\left(\sqrt{g}d+e\sqrt{-f}\right)gn^2\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)e}{2(-f)^{5/2}(gd^2+e^2f)}$$
$$- \frac{b^2\left(\sqrt{-f}\sqrt{g}d+ef\right)gn^2\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)e}{2f^3(gd^2+e^2f)}$$
$$- \frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^3} - \frac{g(a+b\log(c(d+ex)^n))^2}{2f^2(gx^2+f)}$$
$$- \frac{(a+b\log(c(d+ex)^n))^2}{2f^2x^2} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)}{f^3}$$
$$+ \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}$$
$$+ \frac{2bgn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}$$
$$+ \frac{2bgn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{f^3}$$
$$- \frac{4bgn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{f^3} - \frac{2b^2gn^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}$$
$$- \frac{2b^2gn^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{f^3} + \frac{4b^2gn^2\,\text{PolyLog}\left(3,\frac{ex}{d}+1\right)}{f^3}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(x^3*(f + g*x^2)^2),x]
```

```
[Out] (b^2*e^2*n^2*Log[x])/(d^2*f^2) - (b*e*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n]
))/(d^2*f^2*x) + (e^2*g*(a + b*Log[c*(d + e*x)^n])^2)/(2*f^2*(e^2*f + d^2*g
```

```
)) - (a + b*Log[c*(d + e*x)^n])^2/(2*f^2*x^2) - (g*(a + b*Log[c*(d + e*x)^n
])^2)/(2*f^2*(f + g*x^2)) - (2*g*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n])
^2)/f^3 - (b*e*(e*f + d*Sqrt[-f]*Sqrt[g])*g*n*(a + b*Log[c*(d + e*x)^n])*Lo
g[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^3*(e^2*f + d^2
*g)) + (g*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sq
rt[-f] + d*Sqrt[g])])/f^3 - (b*e*(e*f - d*Sqrt[-f]*Sqrt[g])*g*n*(a + b*Log[
c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(
2*f^3*(e^2*f + d^2*g)) + (g*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] +
 Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/f^3 - (b*e^2*n*(a + b*Log[c*(d + e*
x)^n])*Log[1 - d/(d + e*x)])/(d^2*f^2) + (b^2*e^2*n^2*PolyLog[2, d/(d + e*x
)])/(d^2*f^2) - (b^2*e*(e*Sqrt[-f] + d*Sqrt[g])*g*n^2*PolyLog[2, -((Sqrt[g]
*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*(-f)^(5/2)*(e^2*f + d^2*g)) + (2
*b*g*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[
-f] - d*Sqrt[g]))])/f^3 - (b^2*e*(e*f + d*Sqrt[-f]*Sqrt[g])*g*n^2*PolyLog[2
, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^3*(e^2*f + d^2*g)) +
(2*b*g*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[
-f] + d*Sqrt[g])])/f^3 - (4*b*g*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, 1 +
 (e*x)/d])/f^3 - (2*b^2*g*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f]
- d*Sqrt[g]))])/f^3 - (2*b^2*g*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-
f] + d*Sqrt[g])])/f^3 + (4*b^2*g*n^2*PolyLog[3, 1 + (e*x)/d])/f^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left( \frac{(a+b\log(c(d+ex)^n))^2}{f^2x^3} - \frac{2g(a+b\log(c(d+ex)^n))^2}{f^3x} + \frac{g^2x(a+b\log(c(d+ex)^n))^2}{f^2(f+gx^2)^2} + \frac{2g^2x(a+b\log(c(d+ex)^n))^2}{f^3(f+gx^2)} \right) dx$$

$$= \frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3}\,dx}{f^2} - \frac{(2g)\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx}{f^3} + \frac{(2g^2)\int \frac{x(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx}{f^3} + \frac{g^2\int \frac{x(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx}{f^2}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^2}{2f^2x^2} - \frac{g(a+b\log(c(d+ex)^n))^2}{2f^2(f+gx^2)} - \frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^3} + \frac{(2g^2)\int \left(-\frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)} + \frac{(a+b\log(c(d+ex)^n))^2}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right) dx}{f^3} + \frac{(ben)\int \frac{a+b\log(c(d+ex)^n)}{x^2(d+ex)}\,dx}{f^2} + \frac{(4begn)\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{d+ex}\,dx}{f^3} + \frac{(begn)\int \frac{a+b\log(c(d+ex)^n)}{(d+ex)(f+gx^2)}\,dx}{f^2}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^2}{2f^2x^2} - \frac{g(a+b\log(c(d+ex)^n))^2}{2f^2(f+gx^2)} - \frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^3} - \frac{g^{3/2}\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{f^3} + \frac{g^{3/2}\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{f^3} + \frac{(bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+ex\right)}{f^2} + \frac{(4bgn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx,x,d+ex\right)}{f^3} + \frac{(begn)\int\left(\frac{e^2(a+b\log(c(d+ex)^n))}{(e^2f+d^2g)(d+ex)} - \frac{g(-d+ex)(a+b\log(c(d+ex)^n))}{(e^2f+d^2g)(f+gx^2)}\right)dx}{f^2}$$

$$= -\frac{(a+b\log(c(d+ex)^n))^2}{2f^2x^2} - \frac{g(a+b\log(c(d+ex)^n))^2}{2f^2(f+gx^2)} - \frac{2g\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{f^3} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} + \frac{g(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} - \frac{4bgn(a+b\log(c(d+ex)^n))\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^3} + \frac{(bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+ex\right)}{df^2} - \frac{(ben)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx,x,d+ex\right)}{df^2} - \frac{(2begn)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{f^3} - \frac{(2begn)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{f^3} + \frac{(be^3gn)\int\frac{a+b\log(c(d+ex)^n)}{d+ex}\,dx}{f^2(e^2f+d^2g)} - \frac{(beg^2n)\int\frac{(-d+ex)(a+b\log(c(d+ex)^n))}{f+gx^2}\,dx}{f^2(e^2f+d^2g)} + \frac{(4b^2gn^2)\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx,x,d+ex\right)}{f^3}$$

$$= -\frac{ben(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{d^2f^2x} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2x^2}$$

$$- \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2\left(f+gx^2\right)} - \frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f^3}$$

$$+ \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3}$$

$$+ \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}$$

$$- \frac{be^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(1-\frac{d}{d+ex}\right)}{d^2f^2}$$

$$- \frac{4bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{f^3} + \frac{4b^2gn^2\mathrm{Li}_3\left(1+\frac{ex}{d}\right)}{f^3}$$

$$- \frac{(2bgn)\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{f^3}$$

$$- \frac{(2bgn)\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{f^3}$$

$$+ \frac{\left(be^2gn\right)\mathrm{Subst}\left(\int \frac{a+b\log\left(cx^n\right)}{x}\,dx, x, d+ex\right)}{f^2\left(e^2f+d^2g\right)}$$

$$- \frac{\left(beg^2n\right)\int\left(\frac{\left(-d\sqrt{-f}-\frac{ef}{\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\left(-d\sqrt{-f}+\frac{ef}{\sqrt{g}}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2f\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx}{f^2\left(e^2f+d^2g\right)}$$

$$+ \frac{\left(b^2en^2\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex\right)}{d^2f^2}$$

$$+ \frac{\left(b^2e^2n^2\right)\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx, x, d+ex\right)}{d^2f^2}$$

$$= \frac{b^2e^2n^2\log(x)}{d^2f^2} - \frac{ben(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{d^2f^2x}$$
$$+ \frac{e^2g\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2\left(e^2f+d^2g\right)} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2x^2}$$
$$- \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2\left(f+gx^2\right)} - \frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f^3}$$
$$+ \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3}$$
$$+ \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}$$
$$- \frac{be^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(1-\frac{d}{d+ex}\right)}{d^2f^2} + \frac{b^2e^2n^2\operatorname{Li}_2\left(\frac{d}{d+ex}\right)}{d^2f^2}$$
$$+ \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3}$$
$$+ \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3}$$
$$- \frac{4bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{f^3} + \frac{4b^2gn^2\operatorname{Li}_3\left(1+\frac{ex}{d}\right)}{f^3}$$
$$- \frac{\left(be\left(\frac{d}{\sqrt{-f}}+\frac{e}{\sqrt{g}}\right)g^2n\right)\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2f^2\left(e^2f+d^2g\right)}$$
$$+ \frac{\left(be\left(\frac{df}{(-f)^{3/2}}+\frac{e}{\sqrt{g}}\right)g^2n\right)\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2f^2\left(e^2f+d^2g\right)}$$
$$- \frac{\left(2b^2gn^2\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{f^3}$$
$$- \frac{\left(2b^2gn^2\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{f^3}$$

$$
\begin{aligned}
&= \frac{b^2e^2n^2\log(x)}{d^2f^2} - \frac{ben(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{d^2f^2x} \\
&+ \frac{e^2g\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2\left(e^2f+d^2g\right)} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2x^2} \\
&- \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2\left(f+gx^2\right)} - \frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f^3} \\
&- \frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)gn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^3\left(e^2f+d^2g\right)} \\
&+ \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} \\
&- \frac{be\left(e\sqrt{-f}+d\sqrt{g}\right)gn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}\left(e^2f+d^2g\right)} \\
&+ \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} \\
&- \frac{be^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(1-\frac{d}{d+ex}\right)}{d^2f^2} + \frac{b^2e^2n^2\operatorname{Li}_2\left(\frac{d}{d+ex}\right)}{d^2f^2} \\
&+ \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} \\
&+ \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} \\
&- \frac{4bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{f^3} - \frac{2b^2gn^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} \\
&- \frac{2b^2gn^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} + \frac{4b^2gn^2\operatorname{Li}_3\left(1+\frac{ex}{d}\right)}{f^3} \\
&+ \frac{\left(b^2e^2\left(e\sqrt{-f}+d\sqrt{g}\right)gn^2\right)\int\frac{\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2(-f)^{5/2}\left(e^2f+d^2g\right)} \\
&+ \frac{\left(b^2e^2\left(ef+d\sqrt{-f}\sqrt{g}\right)gn^2\right)\int\frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2f^3\left(e^2f+d^2g\right)}
\end{aligned}
$$

$$= \frac{b^2e^2n^2\log(x)}{d^2f^2} - \frac{ben(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{d^2f^2x} + \frac{e^2g\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2\left(e^2f+d^2g\right)} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2x^2} - \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2\left(f+gx^2\right)} - \frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f^3} - \frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)gn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^3\left(e^2f+d^2g\right)} + \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} - \frac{be\left(e\sqrt{-f}+d\sqrt{g}\right)gn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}\left(e^2f+d^2g\right)} + \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} - \frac{be^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(1-\frac{d}{d+ex}\right)}{d^2f^2} + \frac{b^2e^2n^2\text{Li}_2\left(\frac{d}{d+ex}\right)}{d^2f^2} + \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} + \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} - \frac{4bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^3} - \frac{2b^2gn^2\text{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} - \frac{2b^2gn^2\text{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} + \frac{4b^2gn^2\text{Li}_3\left(1+\frac{ex}{d}\right)}{f^3} + \frac{\left(b^2e\left(e\sqrt{-f}+d\sqrt{g}\right)gn^2\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2(-f)^{5/2}\left(e^2f+d^2g\right)} + \frac{\left(b^2e\left(ef+d\sqrt{-f}\sqrt{g}\right)gn^2\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2f^3\left(e^2f+d^2g\right)}$$

$$
\begin{aligned}
&= \frac{b^2e^2n^2\log(x)}{d^2f^2} - \frac{ben(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{d^2f^2x} \\
&+ \frac{e^2g\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2\left(e^2f+d^2g\right)} - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2x^2} \\
&- \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2f^2\left(f+gx^2\right)} - \frac{2g\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{f^3} \\
&- \frac{be\left(ef+d\sqrt{-f}\sqrt{g}\right)gn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^3\left(e^2f+d^2g\right)} \\
&+ \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} \\
&- \frac{be\left(e\sqrt{-f}+d\sqrt{g}\right)gn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}\left(e^2f+d^2g\right)} \\
&+ \frac{g\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} \\
&- \frac{be^2n\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(1-\frac{d}{d+ex}\right)}{d^2f^2} \\
&+ \frac{b^2e^2n^2\text{Li}_2\left(\frac{d}{d+ex}\right)}{d^2f^2} - \frac{b^2e\left(e\sqrt{-f}+d\sqrt{g}\right)gn^2\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}\left(e^2f+d^2g\right)} \\
&+ \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} \\
&- \frac{b^2e\left(ef+d\sqrt{-f}\sqrt{g}\right)gn^2\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^3\left(e^2f+d^2g\right)} \\
&+ \frac{2bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} \\
&- \frac{4bgn\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(1+\frac{ex}{d}\right)}{f^3} - \frac{2b^2gn^2\text{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{f^3} \\
&- \frac{2b^2gn^2\text{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{f^3} + \frac{4b^2gn^2\text{Li}_3\left(1+\frac{ex}{d}\right)}{f^3}
\end{aligned}
$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 1391, normalized size of antiderivative = 1.43

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx$$

$$= \frac{-\frac{2f(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{x^2}-\frac{2fg(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2}-8g\log(x)\left(a-bn\log(d+ex)+b\log\left(c(\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x^3*(f + g*x^2)^2),x]
```

```
[Out] ((-2*f*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/x^2 - (2*f*g*(a - b
*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) - 8*g*Log[x]*(a - b*
n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 4*g*(a - b*n*Log[d + e*x] + b*Lo
g[c*(d + e*x)^n])^2*Log[f + g*x^2] + 2*b*n*(a - b*n*Log[d + e*x] + b*Log[c*
(d + e*x)^n])*((-2*f*(d*e*x + e^2*x^2*Log[x] + (d^2 - e^2*x^2)*Log[d + e*x]
))/(d^2*x^2) + (I*Sqrt[f]*g*(Sqrt[g]*(d + e*x)*Log[d + e*x] + I*e*(Sqrt[f]
+ I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqr
t[f] + I*Sqrt[g]*x)) + (I*Sqrt[f]*g*(-(Sqrt[g]*(d + e*x)*Log[d + e*x]) + e*
(I*Sqrt[f] + Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sqrt
[g])*(Sqrt[f] - I*Sqrt[g]*x)) + 4*g*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[
g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*
Sqrt[f] - I*d*Sqrt[g])]) + 4*g*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x)
)/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f]
+ I*d*Sqrt[g])]) - 8*g*(Log[-((e*x)/d)]*Log[d + e*x] + PolyLog[2, 1 + (e*x)
/d])) + b^2*n^2*((I*Sqrt[f]*g*(-(Sqrt[g]*(d + e*x)*Log[d + e*x]^2) + 2*e*(I
*Sqrt[f] + Sqrt[g]*x)*Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[
f] + I*d*Sqrt[g])] + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*PolyLog[2, (I*Sqrt[g]*(d +
 e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I
*Sqrt[g]*x)) - (Sqrt[f]*g*(Log[d + e*x]*((-I)*Sqrt[g]*(d + e*x)*Log[d + e*x
] + 2*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f]
- I*d*Sqrt[g])]) + 2*e*(Sqrt[f] + I*Sqrt[g]*x)*PolyLog[2, (Sqrt[g]*(d + e*x
))/(I*e*Sqrt[f] + d*Sqrt[g])]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqr
t[g]*x)) - (2*f*(-2*e^2*Log[x] + (Log[d + e*x]*(2*e^2*x^2*Log[-((e*x)/d)] +
 (d + e*x)*(2*e*x + (d - e*x)*Log[d + e*x])))/x^2 + 2*e^2*PolyLog[2, 1 + (e
*x)/d]))/d^2 + 4*g*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt
[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*S
qrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d
*Sqrt[g])]) + 4*g*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f]
+ d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f]
+ d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]
```

```
) - 8*g*(Log[-((e*x)/d)]*Log[d + e*x]^2 + 2*Log[d + e*x]*PolyLog[2, 1 + (e*
x)/d] - 2*PolyLog[3, 1 + (e*x)/d])))/(4*f^3)
```

## Maple [F]

$$\int \frac{(a+b\ln (c(ex+d)^{n}))^{2}}{x^{3} (g\,x^{2}+f)^{2}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/x^3/(g*x^2+f)^2,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^2/x^3/(g*x^2+f)^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\log (c(d+ex)^{n}))^{2}}{x^{3} (f+gx^{2})^{2}}\,dx = \int \frac{(b\log ((ex+d)^{n}c)+a)^{2}}{(gx^{2}+f)^{2}x^{3}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^3/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^2*x
^7 + 2*f*g*x^5 + f^2*x^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log (c(d+ex)^{n}))^{2}}{x^{3} (f+gx^{2})^{2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/x**3/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(a+b\log (c(d+ex)^{n}))^{2}}{x^{3} (f+gx^{2})^{2}}\,dx = \int \frac{(b\log ((ex+d)^{n}c)+a)^{2}}{(gx^{2}+f)^{2}x^{3}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^3/(g*x^2+f)^2,x, algorithm="maxima")
[Out] -1/2*a^2*((2*g*x^2 + f)/(f^2*g*x^4 + f^3*x^2) - 2*g*log(g*x^2 + f)/f^3 + 4*
g*log(x)/f^3) + integrate((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*lo
g(c) + 2*(b^2*log(c) + a*b)*log((e*x + d)^n))/(g^2*x^7 + 2*f*g*x^5 + f^2*x^
3), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2}{(gx^2+f)^2x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^3/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)^2*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(f+gx^2)^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2}{x^3\,(g\,x^2+f)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(x^3*(f + g*x^2)^2),x)
[Out] int((a + b*log(c*(d + e*x)^n))^2/(x^3*(f + g*x^2)^2), x)
```

# 3.325 $\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

## Optimal result

Integrand size = 29, antiderivative size = 897

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$
$$= -\frac{2abnx}{g^2}+\frac{2b^2n^2x}{g^2}-\frac{2b^2n(d+ex)\log(c(d+ex)^n)}{eg^2}$$
$$+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{eg^2}-\frac{f(d+ex)(a+b\log(c(d+ex)^n))^2}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^2\left(\sqrt{-f}-\sqrt{g}x\right)}$$
$$-\frac{f(d+ex)(a+b\log(c(d+ex)^n))^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^2\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{befn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}$$
$$+\frac{3\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}}$$
$$+\frac{befn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}$$
$$-\frac{3\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}+\frac{b^2efn^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}$$
$$-\frac{3b\sqrt{-f}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}}$$
$$-\frac{b^2efn^2\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}$$
$$+\frac{3b\sqrt{-f}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}}$$
$$+\frac{3b^2\sqrt{-f}n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}}-\frac{3b^2\sqrt{-f}n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}}$$

```
[Out] -2*a*b*n*x/g^2+2*b^2*n^2*x/g^2-2*b^2*n*(e*x+d)*ln(c*(e*x+d)^n)/e/g^2+(e*x+d
)*(a+b*ln(c*(e*x+d)^n))^2/e/g^2+3/4*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2
)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(5/2)-3/4*(a+b*ln(c*(e*
x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))*(-f)^(1/2)
/g^(5/2)-3/2*b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(
1/2)-d*g^(1/2)))*(-f)^(1/2)/g^(5/2)+3/2*b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2
,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(5/2)+3/2*b^2*n^2*p
olylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*(-f)^(1/2)/g^(5/2)-3/2*
b^2*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*(-f)^(1/2)/g^(5
/2)+1/2*b*e*f*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(
1/2)-d*g^(1/2)))/g^(5/2)/(e*(-f)^(1/2)-d*g^(1/2))+1/2*b^2*e*f*n^2*polylog(2
```

```
,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^(5/2)/(e*(-f)^(1/2)-d*g^(1/2)
)-1/2*b*e*f*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/
2)+d*g^(1/2)))/g^(5/2)/(e*(-f)^(1/2)+d*g^(1/2))-1/2*b^2*e*f*n^2*polylog(2,(
e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/g^(5/2)/(e*(-f)^(1/2)+d*g^(1/2))-1
/4*f*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/g^2/(e*(-f)^(1/2)+d*g^(1/2))/((-f)^(1/
2)-x*g^(1/2))-1/4*f*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/g^2/(e*(-f)^(1/2)-d*g^(
1/2))/((-f)^(1/2)+x*g^(1/2))
```

## Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 897, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules

used = {2463, 2436, 2333, 2332, 2456, 2444, 2441, 2440, 2438, 2443, 2481, 2421, 6724}

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$
$$= \frac{2n^2xb^2}{g^2} - \frac{2n(d+ex)\log(c(d+ex)^n)\,b^2}{eg^2} + \frac{efn^2\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)b^2}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}$$
$$- \frac{efn^2\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)b^2}{2\left(\sqrt{g}d+e\sqrt{-f}\right)g^{5/2}} + \frac{3\sqrt{-f}n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)b^2}{2g^{5/2}}$$
$$- \frac{3\sqrt{-f}n^2\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)b^2}{2g^{5/2}} - \frac{2anxb}{g^2}$$
$$- \frac{efn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)b}{2\left(\sqrt{g}d+e\sqrt{-f}\right)g^{5/2}}$$
$$+ \frac{efn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)b}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}$$
$$- \frac{3\sqrt{-f}n(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)b}{2g^{5/2}}$$
$$+ \frac{3\sqrt{-f}n(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)b}{2g^{5/2}}$$
$$+ \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{eg^2} - \frac{f(d+ex)(a+b\log(c(d+ex)^n))^2}{4\left(\sqrt{g}d+e\sqrt{-f}\right)g^2\left(\sqrt{-f}-\sqrt{g}x\right)}$$
$$- \frac{f(d+ex)(a+b\log(c(d+ex)^n))^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^2\left(\sqrt{g}x+\sqrt{-f}\right)}$$
$$+ \frac{3\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4g^{5/2}}$$
$$- \frac{3\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}$$

```
[In] Int[(x^4*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

```
[Out] (-2*a*b*n*x)/g^2 + (2*b^2*n^2*x)/g^2 - (2*b^2*n*(d + e*x)*Log[c*(d + e*x)^n
])/(e*g^2) + ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(e*g^2) - (f*(d + e*x
)*(a + b*Log[c*(d + e*x)^n])^2)/(4*(e*Sqrt[-f] + d*Sqrt[g])*g^2*(Sqrt[-f] -
 Sqrt[g]*x)) - (f*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(4*(e*Sqrt[-f] -
d*Sqrt[g])*g^2*(Sqrt[-f] + Sqrt[g]*x)) - (b*e*f*n*(a + b*Log[c*(d + e*x)^n]
)*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(e*Sqrt[-f]
+ d*Sqrt[g])*g^(5/2)) + (3*Sqrt[-f]*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sq
rt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*g^(5/2)) + (b*e*f*n*(a +
```

```
 b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[
g])])/(2*(e*Sqrt[-f] - d*Sqrt[g])*g^(5/2)) - (3*Sqrt[-f]*(a + b*Log[c*(d +
e*x)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(4*g^(
5/2)) + (b^2*e*f*n^2*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[
g]))])/(2*(e*Sqrt[-f] - d*Sqrt[g])*g^(5/2)) - (3*b*Sqrt[-f]*n*(a + b*Log[c*
(d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/
(2*g^(5/2)) - (b^2*e*f*n^2*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*S
qrt[g])])/(2*(e*Sqrt[-f] + d*Sqrt[g])*g^(5/2)) + (3*b*Sqrt[-f]*n*(a + b*Log
[c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/
(2*g^(5/2)) + (3*b^2*Sqrt[-f]*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[
-f] - d*Sqrt[g]))])/(2*g^(5/2)) - (3*b^2*Sqrt[-f]*n^2*PolyLog[3, (Sqrt[g]*(
d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*g^(5/2))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left( \frac{(a+b\log(c(d+ex)^n))^2}{g^2} + \frac{f^2(a+b\log(c(d+ex)^n))^2}{g^2(f+gx^2)^2} - \frac{2f(a+b\log(c(d+ex)^n))^2}{g^2(f+gx^2)} \right) dx$$

$$= \frac{\int (a+b\log(c(d+ex)^n))^2\,dx}{g^2} - \frac{(2f)\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx}{g^2} + \frac{f^2\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx}{g^2}$$

$$= \frac{\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx, x, d+ex\right)}{eg^2} - \frac{(2f)\int \left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}-\sqrt{g}x)} + \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g^2} + \frac{f^2\int \left(-\frac{g(a+b\log(c(d+ex)^n))^2}{4f(\sqrt{-f}\sqrt{g}-gx)^2} - \frac{g(a+b\log(c(d+ex)^n))^2}{4f(\sqrt{-f}\sqrt{g}+gx)^2} - \frac{g(a+b\log(c(d+ex)^n))^2}{2f(-fg-g^2x^2)}\right)dx}{g^2}$$

$$= \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{eg^2} - \frac{\sqrt{-f}\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{g^2} - \frac{\sqrt{-f}\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{g^2} - \frac{f\int \frac{(a+b\log(c(d+ex)^n))^2}{(\sqrt{-f}\sqrt{g}-gx)^2}\,dx}{4g} - \frac{f\int \frac{(a+b\log(c(d+ex)^n))^2}{(\sqrt{-f}\sqrt{g}+gx)^2}\,dx}{4g} - \frac{f\int \frac{(a+b\log(c(d+ex)^n))^2}{-fg-g^2x^2}\,dx}{2g} - \frac{(2bn)\text{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+ex\right)}{eg^2}$$

$$
\begin{aligned}
&= -\frac{2abnx}{g^2} + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2} \\
&\quad - \frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^2\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&\quad + \frac{\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}} \\
&\quad - \frac{\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}} \\
&\quad - \frac{f\int\left(-\frac{\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2fg\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2fg\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx}{2g} \\
&\quad - \frac{\left(2be\sqrt{-f}n\right)\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}dx}{g^{5/2}} \\
&\quad + \frac{\left(2be\sqrt{-f}n\right)\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}dx}{g^{5/2}} \\
&\quad - \frac{\left(2b^2n\right)\operatorname{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{eg^2} \\
&\quad + \frac{(befn)\int\frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{-f}\sqrt{g}+gx}dx}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}} + \frac{(befn)\int\frac{a+b\log\left(c(d+ex)^n\right)}{\sqrt{-f}\sqrt{g}-gx}dx}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}
\end{aligned}
$$

$$= -\frac{2abnx}{g^2}+\frac{2b^2n^2x}{g^2}-\frac{2b^2n(d+ex)\log\left(c(d+ex)^n\right)}{eg^2}$$
$$+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2}-\frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^2\left(\sqrt{-f}-\sqrt{g}x\right)}$$
$$-\frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^2\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$-\frac{befn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}$$
$$+\frac{\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}}$$
$$+\frac{befn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}$$
$$-\frac{\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}}$$
$$+\frac{\sqrt{-f}\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{4g^2}+\frac{\sqrt{-f}\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{4g^2}$$
$$-\frac{\left(2b\sqrt{-f}n\right)\text{Subst}\left(\int\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{g^{5/2}}$$
$$+\frac{\left(2b\sqrt{-f}n\right)\text{Subst}\left(\int\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{g^{5/2}}$$
$$-\frac{\left(b^2e^2fn^2\right)\int\frac{\log\left(\frac{e\left(\sqrt{-f}\sqrt{g}+gx\right)}{e\sqrt{-f}\sqrt{g}-dg}\right)}{d+ex}\,dx}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}}+\frac{\left(b^2e^2fn^2\right)\int\frac{\log\left(\frac{e\left(\sqrt{-f}\sqrt{g}-gx\right)}{e\sqrt{-f}\sqrt{g}+dg}\right)}{d+ex}\,dx}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}$$

$$= -\frac{2abnx}{g^2} + \frac{2b^2n^2x}{g^2} - \frac{2b^2n(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2} - \frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^2\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{befn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}} + \frac{3\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}} + \frac{befn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}} - \frac{3\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}} - \frac{2b\sqrt{-f}n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}} + \frac{2b\sqrt{-f}n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}} + \frac{\left(be\sqrt{-f}n\right)\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2g^{5/2}} - \frac{\left(be\sqrt{-f}n\right)\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2g^{5/2}} + \frac{\left(2b^2\sqrt{-f}n^2\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{g^{5/2}} - \frac{\left(2b^2\sqrt{-f}n^2\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{g^{5/2}} - \frac{\left(b^2efn^2\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{gx}{e\sqrt{-f}\sqrt{g}-dg}\right)}{x}\,dx,x,d+ex\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}} + \frac{\left(b^2efn^2\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{gx}{e\sqrt{-f}\sqrt{g}+dg}\right)}{x}\,dx,x,d+ex\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}$$

$$
\begin{aligned}
&= -\frac{2abnx}{g^2}+\frac{2b^2n^2x}{g^2}-\frac{2b^2n(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} \\
&+ \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2}-\frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^2\left(\sqrt{-f}-\sqrt{g}x\right)} \\
&- \frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^2\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&- \frac{befn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}} \\
&+ \frac{3\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}} \\
&+ \frac{befn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}} \\
&- \frac{3\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}}+\frac{b^2efn^2\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}} \\
&- \frac{2b\sqrt{-f}n\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}} \\
&- \frac{b^2efn^2\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}}+\frac{2b\sqrt{-f}n\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}} \\
&+ \frac{2b^2\sqrt{-f}n^2\text{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}}-\frac{2b^2\sqrt{-f}n^2\text{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}} \\
&+ \frac{\left(b\sqrt{-f}n\right)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2g^{5/2}} \\
&- \frac{\left(b\sqrt{-f}n\right)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2g^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2abnx}{g^2} + \frac{2b^2n^2x}{g^2} - \frac{2b^2n(d+ex)\log\left(c(d+ex)^n\right)}{eg^2} \\
&+ \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{eg^2} - \frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g^2\left(\sqrt{-f}-\sqrt{g}x\right)} \\
&- \frac{f(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g^2\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&- \frac{befn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}} \\
&+ \frac{3\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}} \\
&+ \frac{befn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}} \\
&- \frac{3\sqrt{-f}\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}} + \frac{b^2efn^2\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{5/2}} \\
&- \frac{3b\sqrt{-f}n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} \\
&- \frac{b^2efn^2\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{5/2}} + \frac{3b\sqrt{-f}n\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}} \\
&+ \frac{2b^2\sqrt{-f}n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{g^{5/2}} - \frac{2b^2\sqrt{-f}n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{g^{5/2}} \\
&- \frac{\left(b^2\sqrt{-f}n^2\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2g^{5/2}} \\
&+ \frac{\left(b^2\sqrt{-f}n^2\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2g^{5/2}}
\end{aligned}
$$

$$= -\frac{2abnx}{g^2} + \frac{2b^2n^2x}{g^2} - \frac{2b^2n(d+ex)\log(c(d+ex)^n)}{eg^2} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{eg^2} - \frac{f(d+ex)(a+b\log(c(d+ex)^n))^2}{4(e\sqrt{-f}+d\sqrt{g})g^2(\sqrt{-f}-\sqrt{g}x)} - \frac{f(d+ex)(a+b\log(c(d+ex)^n))^2}{4(e\sqrt{-f}-d\sqrt{g})g^2(\sqrt{-f}+\sqrt{g}x)} - \frac{befn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(e\sqrt{-f}+d\sqrt{g})g^{5/2}} + \frac{3\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4g^{5/2}} + \frac{befn(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(e\sqrt{-f}-d\sqrt{g})g^{5/2}} - \frac{3\sqrt{-f}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4g^{5/2}} + \frac{b^2efn^2\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(e\sqrt{-f}-d\sqrt{g})g^{5/2}} - \frac{3b\sqrt{-f}n(a+b\log(c(d+ex)^n))\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} - \frac{b^2efn^2\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(e\sqrt{-f}+d\sqrt{g})g^{5/2}} + \frac{3b\sqrt{-f}n(a+b\log(c(d+ex)^n))\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}} + \frac{3b^2\sqrt{-f}n^2\text{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2g^{5/2}} - \frac{3b^2\sqrt{-f}n^2\text{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2g^{5/2}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.56 (sec) , antiderivative size = 1237, normalized size of antiderivative = 1.38

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$$

$$= \frac{4\sqrt{g}x(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2 + \frac{2f\sqrt{g}x(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2} - 6\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(\cdots}{\cdots}$$

```
[In] Integrate[(x^4*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

```
[Out] (4*Sqrt[g]*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + (2*f*Sqrt[g]
*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) - 6*Sqrt[f]
*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^
```

```
2 + 2*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((4*Sqrt[g]*(d + e*
x)*(-1 + Log[d + e*x]))/e + (f*(Sqrt[g]*(d + e*x)*Log[d + e*x] + I*e*(Sqrt[
f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))/((e*Sqrt[f] - I*d*Sqrt[g])*(
Sqrt[f] + I*Sqrt[g]*x)) + (f*(Sqrt[g]*(d + e*x)*Log[d + e*x] + e*((-I)*Sqrt
[f] - Sqrt[g]*x)*Log[I*Sqrt[f] + Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sqrt[g])*(S
qrt[f] - I*Sqrt[g]*x)) + (3*I)*Sqrt[f]*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sq
rt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/
(e*Sqrt[f] - I*d*Sqrt[g])]) - (3*I)*Sqrt[f]*(Log[d + e*x]*Log[(e*(Sqrt[f] -
 I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x)
)/(e*Sqrt[f] + I*d*Sqrt[g])])) + b^2*n^2*((4*Sqrt[g]*(2*e*x - 2*(d + e*x)*L
og[d + e*x] + (d + e*x)*Log[d + e*x]^2))/e - (f*(-(Sqrt[g]*(d + e*x)*Log[d
+ e*x]^2) + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sq
rt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*PolyLog[
2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/((e*Sqrt[f] + I*d*Sqr
t[g])*(Sqrt[f] - I*Sqrt[g]*x)) + (f*(Log[d + e*x]*(Sqrt[g]*(d + e*x)*Log[d
+ e*x] + (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e
*Sqrt[f] - I*d*Sqrt[g])]) + (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*PolyLog[2, (Sqr
t[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sq
rt[f] + I*Sqrt[g]*x)) - (3*I)*Sqrt[f]*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d +
 e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(
d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/
((-I)*e*Sqrt[f] + d*Sqrt[g])]) + (3*I)*Sqrt[f]*(Log[d + e*x]^2*Log[1 - (Sqr
t[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[2, (Sqr
t[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x
))/(I*e*Sqrt[f] + d*Sqrt[g])])))/(4*g^(5/2))
```

## Maple [F]

$$\int \frac{x^4(a+b\ln(c(ex+d)^n))^2}{(g\,x^2+f)^2}dx$$

```
[In] int(x^4*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

```
[Out] int(x^4*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

## Fricas [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx=\int \frac{(b\log((ex+d)^nc)+a)^2x^4}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*log((e*x + d)^n*c)^2 + 2*a*b*x^4*log((e*x + d)^n*c) + a^2
*x^4)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2x^4}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="maxima")
[Out] 1/2*a^2*(f*x/(g^3*x^2 + f*g^2) - 3*f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g^2)
+ 2*x/g^2) + integrate((b^2*x^4*log((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x
^4*log((e*x + d)^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x^4)/(g^2*x^4 + 2*f*g*x
^2 + f^2), x)
```

## Giac [F]

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2x^4}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*x^4/(g*x^2 + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{x^4\left(a+b\ln\left(c\left(d+e\,x\right)^n\right)\right)^2}{\left(g\,x^2+f\right)^2}\,dx$$

```
[In] int((x^4*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2,x)
[Out] int((x^4*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2, x)
```

# 3.326 $\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

## Optimal result

Integrand size = 29, antiderivative size = 815

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4(e\sqrt{-f}+d\sqrt{g})\,g\,(\sqrt{-f}-\sqrt{g}x)} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4(e\sqrt{-f}-d\sqrt{g})\,g\,(\sqrt{-f}+\sqrt{g}x)} + \frac{ben(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(e\sqrt{-f}+d\sqrt{g})\,g^{3/2}} + \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}} - \frac{ben(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(e\sqrt{-f}-d\sqrt{g})\,g^{3/2}} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}} - \frac{b^2en^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(e\sqrt{-f}-d\sqrt{g})\,g^{3/2}} - \frac{bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}} + \frac{b^2en^2\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(e\sqrt{-f}+d\sqrt{g})\,g^{3/2}} + \frac{bn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}} + \frac{b^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}} - \frac{b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}$$

```
[Out] 1/4*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(
1/2)))/g^(3/2)/(-f)^(1/2)-1/4*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^
(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^(3/2)/(-f)^(1/2)-1/2*b*n*(a+b*ln(c*(e*x+
d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/g^(3/2)/(-f)^(1
/2)+1/2*b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d
*g^(1/2)))/g^(3/2)/(-f)^(1/2)+1/2*b^2*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f
)^(1/2)-d*g^(1/2)))/g^(3/2)/(-f)^(1/2)-1/2*b^2*n^2*polylog(3,(e*x+d)*g^(1/2
```

```
)/(e*(-f)^(1/2)+d*g^(1/2)))/g^(3/2)/(-f)^(1/2)-1/2*b*e*n*(a+b*ln(c*(e*x+d)^
n))*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^(3/2)/(e*(-f)^(
1/2)-d*g^(1/2))-1/2*b^2*e*n^2*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^
(1/2)))/g^(3/2)/(e*(-f)^(1/2)-d*g^(1/2))+1/2*b*e*n*(a+b*ln(c*(e*x+d)^n))*ln
(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/g^(3/2)/(e*(-f)^(1/2)+d
*g^(1/2))+1/2*b^2*e*n^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))
/g^(3/2)/(e*(-f)^(1/2)+d*g^(1/2))+1/4*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/g/(e*
(-f)^(1/2)+d*g^(1/2))/((-f)^(1/2)-x*g^(1/2))+1/4*(e*x+d)*(a+b*ln(c*(e*x+d)^
n))^2/g/(e*(-f)^(1/2)-d*g^(1/2))/((-f)^(1/2)+x*g^(1/2))
```

## Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 815, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules

used = {2463, 2456, 2444, 2441, 2440, 2438, 2443, 2481, 2421, 6724}

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = -\frac{b^2e\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}} + \frac{b^2e\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2\left(\sqrt{g}d+e\sqrt{-f}\right)g^{3/2}} + \frac{b^2\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2\sqrt{-f}g^{3/2}} - \frac{b^2\,\mathrm{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2\sqrt{-f}g^{3/2}} + \frac{be(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2\left(\sqrt{g}d+e\sqrt{-f}\right)g^{3/2}} - \frac{be(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}} - \frac{b(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2\sqrt{-f}g^{3/2}} + \frac{b(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2\sqrt{-f}g^{3/2}} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4\left(\sqrt{g}d+e\sqrt{-f}\right)g\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g\left(\sqrt{g}x+\sqrt{-f}\right)} + \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4\sqrt{-f}g^{3/2}} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}$$

```
[In] Int[(x^2*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

```
[Out] ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(4*(e*Sqrt[-f] + d*Sqrt[g])*g*(Sqr
t[-f] - Sqrt[g]*x)) + ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(4*(e*Sqrt[-
f] - d*Sqrt[g])*g*(Sqrt[-f] + Sqrt[g]*x)) + (b*e*n*(a + b*Log[c*(d + e*x)^n
])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(e*Sqrt[-f]
 + d*Sqrt[g])*g^(3/2)) + ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - S
qrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*Sqrt[-f]*g^(3/2)) - (b*e*n*(a + b*
Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])
```

```
])/(2*(e*Sqrt[-f] - d*Sqrt[g])*g^(3/2)) - ((a + b*Log[c*(d + e*x)^n])^2*Log
[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(4*Sqrt[-f]*g^(3/2))
 - (b^2*e*n^2*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/
(2*(e*Sqrt[-f] - d*Sqrt[g])*g^(3/2)) - (b*n*(a + b*Log[c*(d + e*x)^n])*Poly
Log[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*Sqrt[-f]*g^(3/2
)) + (b^2*e*n^2*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(
2*(e*Sqrt[-f] + d*Sqrt[g])*g^(3/2)) + (b*n*(a + b*Log[c*(d + e*x)^n])*PolyL
og[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*g^(3/2)) +
 (b^2*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*S
qrt[-f]*g^(3/2)) - (b^2*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*
Sqrt[g])])/(2*Sqrt[-f]*g^(3/2))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{f(a+b\log(c(d+ex)^n))^2}{g(f+gx^2)^2}+\frac{(a+b\log(c(d+ex)^n))^2}{g(f+gx^2)}\right)dx$$

$$= \frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx}{g}-\frac{f\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx}{g}$$

$$= \frac{\int \left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}-\sqrt{g}x)}+\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}+\sqrt{g}x)}\right)dx}{g} - \frac{f\int \left(-\frac{g(a+b\log(c(d+ex)^n))^2}{4f(\sqrt{-f}\sqrt{g}-gx)^2}-\frac{g(a+b\log(c(d+ex)^n))^2}{4f(\sqrt{-f}\sqrt{g}+gx)^2}-\frac{g(a+b\log(c(d+ex)^n))^2}{2f(-fg-g^2x^2)}\right)dx}{g}$$

$$= \frac{1}{4}\int \frac{(a+b\log(c(d+ex)^n))^2}{(\sqrt{-f}\sqrt{g}-gx)^2}dx + \frac{1}{4}\int \frac{(a+b\log(c(d+ex)^n))^2}{(\sqrt{-f}\sqrt{g}+gx)^2}dx + \frac{1}{2}\int \frac{(a+b\log(c(d+ex)^n))^2}{-fg-g^2x^2}dx - \frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}dx}{2\sqrt{-f}g} - \frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}dx}{2\sqrt{-f}g}$$

$$= \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4(e\sqrt{-f}+d\sqrt{g})g(\sqrt{-f}-\sqrt{g}x)} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4(e\sqrt{-f}-d\sqrt{g})g(\sqrt{-f}+\sqrt{g}x)} + \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}} + \frac{1}{2}\int \left(-\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2fg(\sqrt{-f}-\sqrt{g}x)}-\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2fg(\sqrt{-f}+\sqrt{g}x)}\right)dx - \frac{(ben)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}dx}{\sqrt{-f}g^{3/2}} + \frac{(ben)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}dx}{\sqrt{-f}g^{3/2}} - \frac{(ben)\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}\sqrt{g}+gx}dx}{2(e\sqrt{-f}-d\sqrt{g})\sqrt{g}} - \frac{(ben)\int \frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}\sqrt{g}-gx}dx}{2(e\sqrt{-f}+d\sqrt{g})\sqrt{g}}$$

$$=\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$+\frac{ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}$$
$$+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}$$
$$-\frac{ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}$$
$$-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}$$
$$+\frac{\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{4\sqrt{-f}g}+\frac{\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{4\sqrt{-f}g}$$
$$-\frac{(bn)\text{Subst}\left(\int\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{\sqrt{-f}g^{3/2}}$$
$$+\frac{(bn)\text{Subst}\left(\int\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{\sqrt{-f}g^{3/2}}$$
$$+\frac{\left(b^2e^2n^2\right)\int\frac{\log\left(\frac{e\left(\sqrt{-f}\sqrt{g}+gx\right)}{e\sqrt{-f}\sqrt{g}-dg}\right)}{d+ex}dx}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}-\frac{\left(b^2e^2n^2\right)\int\frac{\log\left(\frac{e\left(\sqrt{-f}\sqrt{g}-gx\right)}{e\sqrt{-f}\sqrt{g}+dg}\right)}{d+ex}dx}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}$$

$$=\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$+\frac{ben(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}$$
$$+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}$$
$$-\frac{ben(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}$$
$$-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}$$
$$-\frac{bn(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{\sqrt{-f}g^{3/2}}$$
$$+\frac{bn(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{\sqrt{-f}g^{3/2}}$$
$$+\frac{(ben)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2\sqrt{-f}g^{3/2}}$$
$$-\frac{(ben)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2\sqrt{-f}g^{3/2}}$$
$$+\frac{(b^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{\sqrt{-f}g^{3/2}}$$
$$-\frac{(b^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{\sqrt{-f}g^{3/2}}$$
$$+\frac{(b^2en^2)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{gx}{e\sqrt{-f}\sqrt{g}-dg}\right)}{x}\,dx,x,d+ex\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}$$
$$-\frac{(b^2en^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{gx}{e\sqrt{-f}\sqrt{g}+dg}\right)}{x}\,dx,x,d+ex\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}$$

$$= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$+\frac{ben(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}$$
$$+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}$$
$$-\frac{ben(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}$$
$$-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}-\frac{b^2en^2\mathrm{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}$$
$$-\frac{bn(a+b\log\left(c(d+ex)^n\right))\mathrm{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{\sqrt{-f}g^{3/2}}$$
$$+\frac{b^2en^2\mathrm{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}+\frac{bn(a+b\log\left(c(d+ex)^n\right))\mathrm{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{\sqrt{-f}g^{3/2}}$$
$$+\frac{b^2n^2\mathrm{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{\sqrt{-f}g^{3/2}}-\frac{b^2n^2\mathrm{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{\sqrt{-f}g^{3/2}}$$
$$+\frac{(bn)\mathrm{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2\sqrt{-f}g^{3/2}}$$
$$-\frac{(bn)\mathrm{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2\sqrt{-f}g^{3/2}}$$

$$= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g\left(\sqrt{-f}+\sqrt{g}x\right)}$$

$$+\frac{ben(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}$$

$$+\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}$$

$$-\frac{ben(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}$$

$$-\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}-\frac{b^2en^2\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}$$

$$-\frac{bn(a+b\log\left(c(d+ex)^n\right))\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}+\frac{b^2en^2\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}$$

$$+\frac{bn(a+b\log\left(c(d+ex)^n\right))\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}+\frac{b^2n^2\text{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{\sqrt{-f}g^{3/2}}$$

$$-\frac{b^2n^2\text{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{\sqrt{-f}g^{3/2}}-\frac{(b^2n^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2\sqrt{-f}g^{3/2}}$$

$$+\frac{(b^2n^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2\sqrt{-f}g^{3/2}}$$

$$= \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}+d\sqrt{g}\right)g\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4\left(e\sqrt{-f}-d\sqrt{g}\right)g\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$+ \frac{ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}}$$
$$+ \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}}$$
$$- \frac{ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}$$
$$- \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4\sqrt{-f}g^{3/2}} - \frac{b^2en^2\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e\sqrt{-f}-d\sqrt{g}\right)g^{3/2}}$$
$$- \frac{bn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}$$
$$+ \frac{b^2en^2\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\left(e\sqrt{-f}+d\sqrt{g}\right)g^{3/2}} + \frac{bn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}$$
$$+ \frac{b^2n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}} - \frac{b^2n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}g^{3/2}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.07 (sec) , antiderivative size = 1132, normalized size of antiderivative = 1.39

$$\int \frac{x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{\left(f+gx^2\right)^2}\,dx$$
$$= \frac{-\frac{2\sqrt{g}x\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2}{f+gx^2} + \frac{2\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2}{\sqrt{f}} + 2bn(a-bn\log(d+ex)+b}{}$$

```
[In] Integrate[(x^2*(a + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)^2,x]
```

```
[Out] ((-2*Sqrt[g]*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)
 + (2*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)
^n])^2)/Sqrt[f] + 2*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((-(S
qrt[g]*(d + e*x)*Log[d + e*x]) + e*((-I)*Sqrt[f] + Sqrt[g]*x)*Log[I*Sqrt[f]
 - Sqrt[g]*x])/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (-(Sqr
t[g]*(d + e*x)*Log[d + e*x]) + e*(I*Sqrt[f] + Sqrt[g]*x)*Log[I*Sqrt[f] + Sq
rt[g]*x])/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) - (I*(Log[d +
```

```
 e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])] + PolyLog[
2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]))/Sqrt[f] + (I*(Log[
d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + PolyL
og[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/Sqrt[f]) + b^2*n^2
*((-(Sqrt[g]*(d + e*x)*Log[d + e*x]^2) + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*Log[d
+ e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] + 2*e*(I*
Sqrt[f] + Sqrt[g]*x)*PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt
[g])])/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) - (Log[d + e*x]*
(Sqrt[g]*(d + e*x)*Log[d + e*x] + (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[(e*(S
qrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + (2*I)*e*(Sqrt[f] + I*S
qrt[g]*x)*PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])/((e*Sq
rt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (I*(Log[d + e*x]^2*Log[1 -
(Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*PolyLog[
2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[g
]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]))/Sqrt[f] - (I*(Log[d + e*x]^2*L
og[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2*Log[d + e*x]*Poly
Log[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2*PolyLog[3, (Sqrt[
g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]))/Sqrt[f]))/(4*g^(3/2))
```

## Maple [F]

$$\int \frac{x^2(a+b\ln(c(ex+d)^n))^2}{(g\,x^2+f)^2}dx$$

[In] `int(x^2*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)`

[Out] `int(x^2*(a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)`

## Fricas [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2x^2}{(gx^2+f)^2}\,dx$$

[In] `integrate(x^2*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="fricas")`

[Out] `integral((b^2*x^2*log((e*x + d)^n*c)^2 + 2*a*b*x^2*log((e*x + d)^n*c) + a^2*x^2)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)`

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2x^2}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="maxima")
[Out] -1/2*a^2*(x/(g^2*x^2 + f*g) - arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g)) + integr
ate((b^2*x^2*log((e*x + d)^n)^2 + 2*(b^2*log(c) + a*b)*x^2*log((e*x + d)^n)
 + (b^2*log(c)^2 + 2*a*b*log(c))*x^2)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

## Giac [F]

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2x^2}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*x^2/(g*x^2 + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,(d+e\,x)^n))^2}{(g\,x^2+f)^2}\,dx$$

```
[In] int((x^2*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2,x)
[Out] int((x^2*(a + b*log(c*(d + e*x)^n))^2)/(f + g*x^2)^2, x)
```

# 3.327 $\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx$

## Optimal result

Integrand size = 26, antiderivative size = 821

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = -\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4f\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{ben(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} - \frac{ben(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} + \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} - \frac{b^2en^2\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} + \frac{bn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}} - \frac{b^2en^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} - \frac{bn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}} - \frac{b^2n^2\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}} + \frac{b^2n^2\,\mathrm{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}}$$

```
[Out] -1/4*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^
(1/2)))/(-f)^(3/2)/g^(1/2)+1/4*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g
^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(3/2)/g^(1/2)+1/2*b*n*(a+b*ln(c*(e*x
+d)^n))*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/(-f)^(3/2)/g^(
1/2)-1/2*b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+
d*g^(1/2)))/(-f)^(3/2)/g^(1/2)-1/2*b^2*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-
f)^(1/2)-d*g^(1/2)))/(-f)^(3/2)/g^(1/2)+1/2*b^2*n^2*polylog(3,(e*x+d)*g^(1/
```

```
2)/(e*(-f)^(1/2)+d*g^(1/2)))/(-f)^(3/2)/g^(1/2)-1/2*b*e*n*(a+b*ln(c*(e*x+d)
^n))*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f/g^(1/2)/(e*(-f
)^(1/2)+d*g^(1/2))-1/2*b^2*e*n^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*
g^(1/2)))/f/g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2))-1/2*b*e*n*(a+b*ln(c*(e*x+d)^n)
)*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/g^(1/2)/(e*(-f)^(3/
2)+d*f*g^(1/2))-1/2*b^2*e*n^2*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^
(1/2)))/g^(1/2)/(e*(-f)^(3/2)+d*f*g^(1/2))-1/4*(e*x+d)*(a+b*ln(c*(e*x+d)^n)
)^2/f/(e*(-f)^(1/2)+d*g^(1/2))/((-f)^(1/2)-x*g^(1/2))-1/4*(e*x+d)*(a+b*ln(c
*(e*x+d)^n))^2/f/(e*(-f)^(1/2)-d*g^(1/2))/((-f)^(1/2)+x*g^(1/2))
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used

$= \{2456, 2444, 2441, 2440, 2438, 2443, 2481, 2421, 6724\}$

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = -\frac{b^2e\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} - \frac{b^2e\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2f\left(\sqrt{g}d+e\sqrt{-f}\right)\sqrt{g}} - \frac{b^2\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2(-f)^{3/2}\sqrt{g}} + \frac{b^2\,\mathrm{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2(-f)^{3/2}\sqrt{g}} - \frac{be(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2f\left(\sqrt{g}d+e\sqrt{-f}\right)\sqrt{g}} - \frac{be(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} + \frac{b(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2(-f)^{3/2}\sqrt{g}} - \frac{b(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2(-f)^{3/2}\sqrt{g}} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4f\left(\sqrt{g}d+e\sqrt{-f}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{4f\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{g}x+\sqrt{-f}\right)} - \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4(-f)^{3/2}\sqrt{g}} + \frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x^2)^2,x]
```

```
[Out] -1/4*((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(f*(e*Sqrt[-f] + d*Sqrt[g])*(
Sqrt[-f] - Sqrt[g]*x)) - ((d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(4*f*(e*S
qrt[-f] - d*Sqrt[g])*(Sqrt[-f] + Sqrt[g]*x)) - (b*e*n*(a + b*Log[c*(d + e*x
)^n])*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f*(e*Sqr
t[-f] + d*Sqrt[g])*Sqrt[g]) - ((a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f
] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(4*(-f)^(3/2)*Sqrt[g]) - (b*e*n*
(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*S
```

```
qrt[g])])/(2*(e*(-f)^(3/2) + d*f*Sqrt[g])*Sqrt[g]) + ((a + b*Log[c*(d + e*x
)^n])^2*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(4*(-f)^(
3/2)*Sqrt[g]) - (b^2*e*n^2*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d
*Sqrt[g]))])/(2*(e*(-f)^(3/2) + d*f*Sqrt[g])*Sqrt[g]) + (b*n*(a + b*Log[c*(
d + e*x)^n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(
2*(-f)^(3/2)*Sqrt[g]) - (b^2*e*n^2*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-
f] + d*Sqrt[g])])/(2*f*(e*Sqrt[-f] + d*Sqrt[g])*Sqrt[g]) - (b*n*(a + b*Log[
c*(d + e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(
2*(-f)^(3/2)*Sqrt[g]) - (b^2*n^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-
f] - d*Sqrt[g]))])/(2*(-f)^(3/2)*Sqrt[g]) + (b^2*n^2*PolyLog[3, (Sqrt[g]*(d
 + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(-f)^(3/2)*Sqrt[g])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{g(a+b\log(c(d+ex)^n))^2}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}-\frac{g(a+b\log(c(d+ex)^n))^2}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2}-\frac{g(a+b\log(c(d+ex)^n))^2}{2f(-fg-g^2x^2)}\right)dx$$

$$=-\frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{\left(\sqrt{-f}\sqrt{g}-gx\right)^2}\,dx}{4f}-\frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{\left(\sqrt{-f}\sqrt{g}+gx\right)^2}\,dx}{4f}-\frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{-fg-g^2x^2}\,dx}{2f}$$

$$= -\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$- \frac{g\int\left(-\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2fg(\sqrt{-f}-\sqrt{g}x)} - \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2fg(\sqrt{-f}+\sqrt{g}x)}\right)dx}{2f}$$
$$+ \frac{\left(be\sqrt{g}n\right)\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}\sqrt{g}+gx}dx}{2f\left(e\sqrt{-f}-d\sqrt{g}\right)} + \frac{\left(be\sqrt{g}n\right)\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}\sqrt{g}-gx}dx}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)}$$

$$= -\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$- \frac{ben(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}}$$
$$- \frac{ben(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}}$$
$$+ \frac{\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}dx}{4(-f)^{3/2}} + \frac{\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}dx}{4(-f)^{3/2}}$$
$$+ \frac{\left(b^2e^2n^2\right)\int\frac{\log\left(\frac{e(\sqrt{-f}\sqrt{g}-gx)}{e\sqrt{-f}\sqrt{g}+dg}\right)}{d+ex}dx}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} + \frac{\left(b^2e^2n^2\right)\int\frac{\log\left(\frac{e(\sqrt{-f}\sqrt{g}+gx)}{e\sqrt{-f}\sqrt{g}-dg}\right)}{d+ex}dx}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}}$$

$$
\begin{aligned}
&= -\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&\quad - \frac{ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} \\
&\quad - \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} \\
&\quad - \frac{ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} \\
&\quad + \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} \\
&\quad + \frac{(ben)\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2(-f)^{3/2}\sqrt{g}} \\
&\quad - \frac{(ben)\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2(-f)^{3/2}\sqrt{g}} \\
&\quad + \frac{\left(b^2en^2\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{gx}{e\sqrt{-f}\sqrt{g}+dg}\right)}{x}\,dx, x, d+ex\right)}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} \\
&\quad + \frac{\left(b^2en^2\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{gx}{e\sqrt{-f}\sqrt{g}-dg}\right)}{x}\,dx, x, d+ex\right)}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}}
\end{aligned}
$$

$$= -\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$- \frac{ben(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}}$$
$$- \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}$$
$$- \frac{ben(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}}$$
$$+ \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}}$$
$$- \frac{b^2en^2\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} - \frac{b^2en^2\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}}$$
$$+ \frac{(bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{2(-f)^{3/2}\sqrt{g}}$$
$$- \frac{(bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx, x, d+ex\right)}{2(-f)^{3/2}\sqrt{g}}$$

$$
\begin{aligned}
&= -\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&- \frac{ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} \\
&- \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} \\
&- \frac{ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} \\
&+ \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} - \frac{b^2en^2\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} \\
&+ \frac{bn\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}} \\
&- \frac{b^2en^2\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} - \frac{bn\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}} \\
&- \frac{(b^2n^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2(-f)^{3/2}\sqrt{g}} \\
&+ \frac{(b^2n^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2(-f)^{3/2}\sqrt{g}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&- \frac{ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} \\
&- \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} \\
&- \frac{ben\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} \\
&+ \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{3/2}\sqrt{g}} - \frac{b^2en^2\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\left(e(-f)^{3/2}+df\sqrt{g}\right)\sqrt{g}} \\
&+ \frac{bn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}} \\
&- \frac{b^2en^2\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f\left(e\sqrt{-f}+d\sqrt{g}\right)\sqrt{g}} - \frac{bn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}} \\
&- \frac{b^2n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}} + \frac{b^2n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{3/2}\sqrt{g}}
\end{aligned}
$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.37 (sec) , antiderivative size = 1143, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{\left(f+gx^2\right)^2}\,dx$$

$$= \frac{\frac{2\sqrt{f}x(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2} + \frac{2\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{\sqrt{g}} + \frac{2bn(a-bn\log(d+ex)+b\log(c(d+ex)^n))\left(\cdots\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(f + g*x^2)^2,x]
```

```
[Out] ((2*Sqrt[f]*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2)
+ (2*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^
n])^2)/Sqrt[g] + (2*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((Sqr
t[f]*(Sqrt[g]*(d + e*x)*Log[d + e*x] + I*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sq
rt[f] - Sqrt[g]*x]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) +
(Sqrt[f]*(Sqrt[g]*(d + e*x)*Log[d + e*x] + e*((-I)*Sqrt[f] - Sqrt[g]*x)*Log
[I*Sqrt[f] + Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x
```

```
)) - I*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[
g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) + I*
(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sqrt[g])] +
PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])])))/Sqrt[g] + (b
^2*n^2*(-((Sqrt[f]*(-(Sqrt[g]*(d + e*x)*Log[d + e*x]^2) + 2*e*(I*Sqrt[f] +
Sqrt[g]*x)*Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*Sq
rt[g])] + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*S
qrt[f] + I*d*Sqrt[g])]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)
)) + (Sqrt[f]*(Log[d + e*x]*(Sqrt[g]*(d + e*x)*Log[d + e*x] + (2*I)*e*(Sqrt
[f] + I*Sqrt[g]*x)*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[g]
)]) + (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*S
qrt[f] + d*Sqrt[g])]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x))
+ I*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g]
)] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt
[g])] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) - I
*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] + 2
*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])] - 2
*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])))/Sqrt[g])/(4*f
^(3/2))
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}{\left(g\,x^{2}+f\right)^{2}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^2/(g*x^2+f)^2,x)
```

## Fricas [F]

$$\int \frac{\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{\left(f+gx^{2}\right)^{2}}\,dx=\int \frac{\left(b\log\left((ex+d)^{n}c\right)+a\right)^{2}}{\left(gx^{2}+f\right)^{2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^2*x
^4 + 2*f*g*x^2 + f^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/(g*x**2+f)**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{(gx^2+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="maxima")
[Out] 1/2*a^2*(x/(f*g*x^2 + f^2) + arctan(g*x/sqrt(f*g))/(sqrt(f*g)*f)) + integra
te((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*log(c) + 2*(b^2*log(c) +
a*b)*log((e*x + d)^n))/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{(gx^2+f)^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/(g*x^2 + f)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2}{(g\,x^2+f)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x^2)^2,x)
[Out] int((a + b*log(c*(d + e*x)^n))^2/(f + g*x^2)^2, x)
```

# 3.328 $\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx$

## Optimal result

Integrand size = 29, antiderivative size = 919

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx = \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{be\sqrt{g}n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} - \frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}} + \frac{be\sqrt{g}n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f\left(e(-f)^{3/2}+df\sqrt{g}\right)} + \frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}} + \frac{b^2e\sqrt{g}n^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f\left(e(-f)^{3/2}+df\sqrt{g}\right)} + \frac{3b\sqrt{g}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{b^2e\sqrt{g}n^2\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} - \frac{3b\sqrt{g}n(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{2b^2en^2\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{df^2} - \frac{3b^2\sqrt{g}n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{3b^2\sqrt{g}n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}}$$

```
[Out] 2*b*e*n*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/d/f^2-(e*x+d)*(a+b*ln(c*(e*x+d)^n)
)^2/d/f^2/x+2*b^2*e*n^2*polylog(2,1+e*x/d)/d/f^2-3/4*(a+b*ln(c*(e*x+d)^n))^
2*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/(-f)^(5/2)+
3/4*(a+b*ln(c*(e*x+d)^n))^2*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(1/2)-d*g^(
1/2)))*g^(1/2)/(-f)^(5/2)+3/2*b*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-(e*x+d)*
g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/(-f)^(5/2)-3/2*b*n*(a+b*ln(c*(e*x
+d)^n))*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/(-f)^(5
/2)-3/2*b^2*n^2*polylog(3,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*g^(1/2
)/(-f)^(5/2)+3/2*b^2*n^2*polylog(3,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2))
)*g^(1/2)/(-f)^(5/2)+1/2*b*e*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)-x*g^(
1/2))/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/f^2/(e*(-f)^(1/2)+d*g^(1/2))+1/2*b^
2*e*n^2*polylog(2,(e*x+d)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))*g^(1/2)/f^2/(e*
(-f)^(1/2)+d*g^(1/2))+1/2*b*e*n*(a+b*ln(c*(e*x+d)^n))*ln(e*((-f)^(1/2)+x*g^
(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/f/(e*(-f)^(3/2)+d*f*g^(1/2))+1/2*b
^2*e*n^2*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))*g^(1/2)/f/(e*
(-f)^(3/2)+d*f*g^(1/2))+1/4*g*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/f^2/(e*(-f)^(
1/2)+d*g^(1/2))/((-f)^(1/2)-x*g^(1/2))+1/4*g*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^
2/f^2/(e*(-f)^(1/2)-d*g^(1/2))/((-f)^(1/2)+x*g^(1/2))
```

## Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules

used = {2463, 2444, 2441, 2352, 2456, 2440, 2438, 2443, 2481, 2421, 6724}

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx = \frac{b^2e\sqrt{g}\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2f\left(e(-f)^{3/2}+df\sqrt{g}\right)} + \frac{b^2e\sqrt{g}\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2f^2\left(\sqrt{g}d+e\sqrt{-f}\right)} + \frac{2b^2e\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)n^2}{df^2} - \frac{3b^2\sqrt{g}\,\text{PolyLog}\left(3,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n^2}{2(-f)^{5/2}} + \frac{3b^2\sqrt{g}\,\text{PolyLog}\left(3,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n^2}{2(-f)^{5/2}} + \frac{2be\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))n}{df^2} + \frac{be\sqrt{g}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2f^2\left(\sqrt{g}d+e\sqrt{-f}\right)} + \frac{be\sqrt{g}(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2f\left(e(-f)^{3/2}+df\sqrt{g}\right)} + \frac{3b\sqrt{g}(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)n}{2(-f)^{5/2}} - \frac{3b\sqrt{g}(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)n}{2(-f)^{5/2}} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(\sqrt{g}d+e\sqrt{-f}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{g}x+\sqrt{-f}\right)} - \frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}d+e\sqrt{-f}}\right)}{4(-f)^{5/2}} + \frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{g}x+\sqrt{-f})}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2/(x^2*(f + g*x^2)^2),x]
```

```
[Out] (2*b*e*n*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/(d*f^2) - ((d + e*x)*(
a + b*Log[c*(d + e*x)^n])^2)/(d*f^2*x) + (g*(d + e*x)*(a + b*Log[c*(d + e*x
)^n])^2)/(4*f^2*(e*Sqrt[-f] + d*Sqrt[g])*(Sqrt[-f] - Sqrt[g]*x)) + (g*(d +
e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(4*f^2*(e*Sqrt[-f] - d*Sqrt[g])*(Sqrt[-f
] + Sqrt[g]*x)) + (b*e*Sqrt[g]*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(Sqrt[-f
] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f^2*(e*Sqrt[-f] + d*Sqrt[g]))
 - (3*Sqrt[g]*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(
e*Sqrt[-f] + d*Sqrt[g])])/(4*(-f)^(5/2)) + (b*e*Sqrt[g]*n*(a + b*Log[c*(d +
 e*x)^n])*Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*f*(e
*(-f)^(3/2) + d*f*Sqrt[g])) + (3*Sqrt[g]*(a + b*Log[c*(d + e*x)^n])^2*Log[(
e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])])/(4*(-f)^(5/2)) + (b^2*
e*Sqrt[g]*n^2*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/
(2*f*(e*(-f)^(3/2) + d*f*Sqrt[g])) + (3*b*Sqrt[g]*n*(a + b*Log[c*(d + e*x)^
n])*PolyLog[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*(-f)^(5
/2)) + (b^2*e*Sqrt[g]*n^2*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sq
rt[g])])/(2*f^2*(e*Sqrt[-f] + d*Sqrt[g])) - (3*b*Sqrt[g]*n*(a + b*Log[c*(d
+ e*x)^n])*PolyLog[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*(-f
)^(5/2)) + (2*b^2*e*n^2*PolyLog[2, 1 + (e*x)/d])/(d*f^2) - (3*b^2*Sqrt[g]*n
^2*PolyLog[3, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*(-f)^(5/
2)) + (3*b^2*Sqrt[g]*n^2*PolyLog[3, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqr
t[g])])/(2*(-f)^(5/2))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+b\log(c(d+ex)^n))^2}{f^2x^2} - \frac{g(a+b\log(c(d+ex)^n))^2}{f(f+gx^2)^2} - \frac{g(a+b\log(c(d+ex)^n))^2}{f^2(f+gx^2)}\right) dx$$

$$= \frac{\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2}\,dx}{f^2} - \frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{f+gx^2}\,dx}{f^2} - \frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{(f+gx^2)^2}\,dx}{f}$$

$$= -\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} - \frac{g\int \left(\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}-\sqrt{g}x)} + \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2f(\sqrt{-f}+\sqrt{g}x)}\right)dx}{f^2} - \frac{g\int \left(-\frac{g(a+b\log(c(d+ex)^n))^2}{4f(\sqrt{-f}\sqrt{g}-gx)^2} - \frac{g(a+b\log(c(d+ex)^n))^2}{4f(\sqrt{-f}\sqrt{g}+gx)^2} - \frac{g(a+b\log(c(d+ex)^n))^2}{2f(-fg-g^2x^2)}\right)dx}{f} + \frac{(2ben)\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx}{df^2}$$

$$= \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} + \frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}\,dx}{2(-f)^{5/2}} + \frac{g\int \frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}\,dx}{2(-f)^{5/2}} + \frac{g^2\int \frac{(a+b\log(c(d+ex)^n))^2}{(\sqrt{-f}\sqrt{g}-gx)^2}\,dx}{4f^2} + \frac{g^2\int \frac{(a+b\log(c(d+ex)^n))^2}{(\sqrt{-f}\sqrt{g}+gx)^2}\,dx}{4f^2} + \frac{g^2\int \frac{(a+b\log(c(d+ex)^n))^2}{-fg-g^2x^2}\,dx}{2f^2} - \frac{(2b^2e^2n^2)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{df^2}$$

$$= \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2(e\sqrt{-f}+d\sqrt{g})(\sqrt{-f}-\sqrt{g}x)} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2(e\sqrt{-f}-d\sqrt{g})(\sqrt{-f}+\sqrt{g}x)} - \frac{\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{2b^2en^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{df^2} + \frac{g^2\int\left(-\frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2fg(\sqrt{-f}-\sqrt{g}x)} - \frac{\sqrt{-f}(a+b\log(c(d+ex)^n))^2}{2fg(\sqrt{-f}+\sqrt{g}x)}\right)dx}{2f^2} + \frac{\left(be\sqrt{g}n\right)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}dx}{(-f)^{5/2}} - \frac{\left(be\sqrt{g}n\right)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}dx}{(-f)^{5/2}} - \frac{\left(beg^{3/2}n\right)\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}\sqrt{g}+gx}dx}{2f^2(e\sqrt{-f}-d\sqrt{g})} - \frac{\left(beg^{3/2}n\right)\int\frac{a+b\log(c(d+ex)^n)}{\sqrt{-f}\sqrt{g}-gx}dx}{2f^2(e\sqrt{-f}+d\sqrt{g})}$$

$$= \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{be\sqrt{g}n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} - \frac{\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} - \frac{be\sqrt{g}n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}-d\sqrt{g}\right)} + \frac{\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{2b^2en^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{df^2} + \frac{g\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}-\sqrt{g}x}dx}{4(-f)^{5/2}} + \frac{g\int\frac{(a+b\log(c(d+ex)^n))^2}{\sqrt{-f}+\sqrt{g}x}dx}{4(-f)^{5/2}} + \frac{\left(b\sqrt{g}n\right)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{(-f)^{5/2}} - \frac{\left(b\sqrt{g}n\right)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{(-f)^{5/2}} + \frac{\left(b^2e^2\sqrt{g}n^2\right)\int\frac{\log\left(\frac{e(\sqrt{-f}\sqrt{g}+gx)}{e\sqrt{-f}\sqrt{g}-dg}\right)}{d+ex}dx}{2f^2\left(e\sqrt{-f}-d\sqrt{g}\right)} - \frac{\left(b^2e^2\sqrt{g}n^2\right)\int\frac{\log\left(\frac{e(\sqrt{-f}\sqrt{g}-gx)}{e\sqrt{-f}\sqrt{g}+dg}\right)}{d+ex}dx}{2f^2\left(e\sqrt{-f}+d\sqrt{g}\right)}$$

$$
\begin{aligned}
&= \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} \\
&+ \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&+ \frac{be\sqrt{g}n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} \\
&- \frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}} \\
&- \frac{be\sqrt{g}n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}-d\sqrt{g}\right)} \\
&+ \frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}} \\
&+ \frac{b\sqrt{g}n(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}} \\
&- \frac{b\sqrt{g}n(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}} + \frac{2b^2en^2\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{df^2} \\
&+ \frac{\left(be\sqrt{g}n\right)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2(-f)^{5/2}} \\
&- \frac{\left(be\sqrt{g}n\right)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2(-f)^{5/2}} \\
&- \frac{\left(b^2\sqrt{g}n^2\right)\operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{(-f)^{5/2}} \\
&+ \frac{\left(b^2\sqrt{g}n^2\right)\operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{(-f)^{5/2}} \\
&+ \frac{\left(b^2e\sqrt{g}n^2\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{gx}{e\sqrt{-f}\sqrt{g}-dg}\right)}{x}\,dx, x, d+ex\right)}{2f^2\left(e\sqrt{-f}-d\sqrt{g}\right)} \\
&- \frac{\left(b^2e\sqrt{g}n^2\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{gx}{e\sqrt{-f}\sqrt{g}+dg}\right)}{x}\,dx, x, d+ex\right)}{2f^2\left(e\sqrt{-f}+d\sqrt{g}\right)}
\end{aligned}
$$

$$= \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{be\sqrt{g}n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} - \frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}} - \frac{be\sqrt{g}n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}-d\sqrt{g}\right)} + \frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}} - \frac{b^2e\sqrt{g}n^2\mathrm{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}-d\sqrt{g}\right)} + \frac{b\sqrt{g}n(a+b\log(c(d+ex)^n))\mathrm{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}} + \frac{b^2e\sqrt{g}n^2\mathrm{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} - \frac{b\sqrt{g}n(a+b\log(c(d+ex)^n))\mathrm{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}} + \frac{2b^2en^2\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{df^2} - \frac{b^2\sqrt{g}n^2\mathrm{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}} + \frac{b^2\sqrt{g}n^2\mathrm{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}} + \frac{\left(b\sqrt{g}n\right)\mathrm{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}+d\sqrt{g}}{e}-\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2(-f)^{5/2}} - \frac{\left(b\sqrt{g}n\right)\mathrm{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{e\sqrt{-f}-d\sqrt{g}}{e}+\frac{\sqrt{g}x}{e}\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex\right)}{2(-f)^{5/2}}$$

$$
\begin{aligned}
&= \frac{2ben\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{df^2} - \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{df^2x} \\
&+ \frac{g(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f^2\left(e\sqrt{-f}+d\sqrt{g}\right)\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{g(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4f^2\left(e\sqrt{-f}-d\sqrt{g}\right)\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&+ \frac{be\sqrt{g}n(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} \\
&- \frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}} \\
&- \frac{be\sqrt{g}n(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}-d\sqrt{g}\right)} \\
&+ \frac{3\sqrt{g}(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}} - \frac{b^2e\sqrt{g}n^2\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}-d\sqrt{g}\right)} \\
&+ \frac{3b\sqrt{g}n(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{b^2e\sqrt{g}n^2\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2\left(e\sqrt{-f}+d\sqrt{g}\right)} \\
&- \frac{3b\sqrt{g}n(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{2b^2en^2\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{df^2} \\
&- \frac{b^2\sqrt{g}n^2\operatorname{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{(-f)^{5/2}} + \frac{b^2\sqrt{g}n^2\operatorname{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{(-f)^{5/2}} \\
&- \frac{\left(b^2\sqrt{g}n^2\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2(-f)^{5/2}} \\
&+ \frac{\left(b^2\sqrt{g}n^2\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex\right)}{2(-f)^{5/2}}
\end{aligned}
$$

$$= \frac{2ben\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{df^2} - \frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{df^2x} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2(e\sqrt{-f}+d\sqrt{g})(\sqrt{-f}-\sqrt{g}x)} + \frac{g(d+ex)(a+b\log(c(d+ex)^n))^2}{4f^2(e\sqrt{-f}-d\sqrt{g})(\sqrt{-f}+\sqrt{g}x)} + \frac{be\sqrt{g}n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2(e\sqrt{-f}+d\sqrt{g})} - \frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{4(-f)^{5/2}} - \frac{be\sqrt{g}n(a+b\log(c(d+ex)^n))\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2(e\sqrt{-f}-d\sqrt{g})} + \frac{3\sqrt{g}(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{4(-f)^{5/2}} - \frac{b^2e\sqrt{g}n^2\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2(e\sqrt{-f}-d\sqrt{g})} + \frac{3b\sqrt{g}n(a+b\log(c(d+ex)^n))\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{b^2e\sqrt{g}n^2\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2(e\sqrt{-f}+d\sqrt{g})} - \frac{3b\sqrt{g}n(a+b\log(c(d+ex)^n))\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{2b^2en^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{df^2} - \frac{3b^2\sqrt{g}n^2\text{Li}_3\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2(-f)^{5/2}} + \frac{3b^2\sqrt{g}n^2\text{Li}_3\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2(-f)^{5/2}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.68 (sec) , antiderivative size = 1304, normalized size of antiderivative = 1.42

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx$$

$$= \frac{-\frac{4\sqrt{f}(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{x} - \frac{2\sqrt{f}gx(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{f+gx^2} - 6\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)(a-bn\log(d+}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2/(x^2*(f + g*x^2)^2),x]
```

```
[Out] ((-4*Sqrt[f]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/x - (2*Sqrt[f
]*g*x*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/(f + g*x^2) - 6*Sqrt
[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n
])^2 + 2*b*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((4*Sqrt[f]*(e*x
*Log[x] - (d + e*x)*Log[d + e*x]))/(d*x) - (Sqrt[f]*Sqrt[g]*(Sqrt[g]*(d + e
```

```
*x)*Log[d + e*x] + I*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[I*Sqrt[f] - Sqrt[g]*x]))
/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] + I*Sqrt[g]*x)) + (Sqrt[f]*Sqrt[g]*(-(
Sqrt[g]*(d + e*x)*Log[d + e*x]) + e*(I*Sqrt[f] + Sqrt[g]*x)*Log[I*Sqrt[f] +
 Sqrt[g]*x]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)) + (3*I)*S
qrt[g]*(Log[d + e*x]*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f] - I*d*Sqrt[
g])] + PolyLog[2, ((-I)*Sqrt[g]*(d + e*x))/(e*Sqrt[f] - I*d*Sqrt[g])]) - (3
*I)*Sqrt[g]*(Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f] + I*d*
Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(d + e*x))/(e*Sqrt[f] + I*d*Sqrt[g])])) +
 b^2*n^2*((Sqrt[f]*Sqrt[g]*(-(Sqrt[g]*(d + e*x)*Log[d + e*x]^2) + 2*e*(I*Sq
rt[f] + Sqrt[g]*x)*Log[d + e*x]*Log[(e*(Sqrt[f] - I*Sqrt[g]*x))/(e*Sqrt[f]
+ I*d*Sqrt[g])] + 2*e*(I*Sqrt[f] + Sqrt[g]*x)*PolyLog[2, (I*Sqrt[g]*(d + e*
x))/(e*Sqrt[f] + I*d*Sqrt[g])]))/((e*Sqrt[f] + I*d*Sqrt[g])*(Sqrt[f] - I*Sq
rt[g]*x)) - (Sqrt[f]*Sqrt[g]*(Log[d + e*x]*(Sqrt[g]*(d + e*x)*Log[d + e*x]
+ (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*Log[(e*(Sqrt[f] + I*Sqrt[g]*x))/(e*Sqrt[f
] - I*d*Sqrt[g])]) + (2*I)*e*(Sqrt[f] + I*Sqrt[g]*x)*PolyLog[2, (Sqrt[g]*(d
 + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])]))/((e*Sqrt[f] - I*d*Sqrt[g])*(Sqrt[f] +
 I*Sqrt[g]*x)) + (4*Sqrt[f]*(2*e*x*Log[-((e*x)/d)]*Log[d + e*x] - (d + e*x)
*Log[d + e*x]^2 + 2*e*x*PolyLog[2, 1 + (e*x)/d]))/(d*x) - (3*I)*Sqrt[g]*(Lo
g[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] + 2*
Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])] -
 2*PolyLog[3, (Sqrt[g]*(d + e*x))/((-I)*e*Sqrt[f] + d*Sqrt[g])]) + (3*I)*Sq
rt[g]*(Log[d + e*x]^2*Log[1 - (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])
] + 2*Log[d + e*x]*PolyLog[2, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])
] - 2*PolyLog[3, (Sqrt[g]*(d + e*x))/(I*e*Sqrt[f] + d*Sqrt[g])])))/(4*f^(5/
2))
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}{x^{2}\left(g\,x^{2}+f\right)^{2}}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2/x^2/(g*x^2+f)^2,x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))^2/x^2/(g*x^2+f)^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^{n}))^{2}}{x^{2}\left(f+gx^{2}\right)^{2}}\,dx=\int \frac{(b\log\left((ex+d)^{n}c\right)+a)^{2}}{(gx^{2}+f)^{2}x^{2}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^2/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) + a^2)/(g^2*x
^6 + 2*f*g*x^4 + f^2*x^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2/x**2/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{(gx^2+f)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^2/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((3*g*x^2 + 2*f)/(f^2*g*x^3 + f^3*x) + 3*g*arctan(g*x/sqrt(f*g))/(
sqrt(f*g)*f^2)) + integrate((b^2*log((e*x + d)^n)^2 + b^2*log(c)^2 + 2*a*b*
log(c) + 2*(b^2*log(c) + a*b)*log((e*x + d)^n))/(g^2*x^6 + 2*f*g*x^4 + f^2*
x^2), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^2}{(gx^2+f)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2/x^2/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2/((g*x^2 + f)^2*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(f+gx^2)^2}\,dx = \int \frac{(a+b\ln(c(d+e\,x)^n))^2}{x^2\,(g\,x^2+f)^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2/(x^2*(f + g*x^2)^2),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^2/(x^2*(f + g*x^2)^2), x)
```

# 3.329 $\int \frac{\log^3(c(a+bx)^n)}{d+ex^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 477

$$\begin{aligned}\int \frac{\log^3(c(a+bx)^n)}{d+ex^2}\,dx = {} & \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & - \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & - \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & + \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & + \frac{3n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} \\ & - \frac{3n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} \\ & - \frac{3n^3\operatorname{PolyLog}\left(4,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{3n^3\operatorname{PolyLog}\left(4,\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}}\end{aligned}$$

```
[Out] 1/2*ln(c*(b*x+a)^n)^3*ln(b*((-d)^(1/2)-x*e^(1/2))/(b*(-d)^(1/2)+a*e^(1/2)))
/(-d)^(1/2)/e^(1/2)-1/2*ln(c*(b*x+a)^n)^3*ln(b*((-d)^(1/2)+x*e^(1/2))/(b*(-
d)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)-3/2*n*ln(c*(b*x+a)^n)^2*polylog(2,-
(b*x+a)*e^(1/2)/(b*(-d)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)+3/2*n*ln(c*(b*
x+a)^n)^2*polylog(2,(b*x+a)*e^(1/2)/(b*(-d)^(1/2)+a*e^(1/2)))/(-d)^(1/2)/e^
(1/2)+3*n^2*ln(c*(b*x+a)^n)*polylog(3,-(b*x+a)*e^(1/2)/(b*(-d)^(1/2)-a*e^(1
```

```
/2)))/(-d)^(1/2)/e^(1/2)-3*n^2*ln(c*(b*x+a)^n)*polylog(3,(b*x+a)*e^(1/2)/(b
*(-d)^(1/2)+a*e^(1/2)))/(-d)^(1/2)/e^(1/2)-3*n^3*polylog(4,-(b*x+a)*e^(1/2)
/(b*(-d)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)+3*n^3*polylog(4,(b*x+a)*e^(1/
2)/(b*(-d)^(1/2)+a*e^(1/2)))/(-d)^(1/2)/e^(1/2)
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2456, 2443, 2481, 2421, 2430, 6724}

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex^2}\,dx = \frac{3n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{3n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,\frac{\sqrt{e}(a+bx)}{\sqrt{e}a+b\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(a+bx)}{\sqrt{e}a+b\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{a\sqrt{e}+b\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{3n^3\operatorname{PolyLog}\left(4,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{3n^3\operatorname{PolyLog}\left(4,\frac{\sqrt{e}(a+bx)}{\sqrt{e}a+b\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}}$$

```
[In] Int[Log[c*(a + b*x)^n]^3/(d + e*x^2),x]
```

```
[Out] (Log[c*(a + b*x)^n]^3*Log[(b*(Sqrt[-d] - Sqrt[e]*x))/(b*Sqrt[-d] + a*Sqrt[e
])])/(2*Sqrt[-d]*Sqrt[e]) - (Log[c*(a + b*x)^n]^3*Log[(b*(Sqrt[-d] + Sqrt[e
]*x))/(b*Sqrt[-d] - a*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - (3*n*Log[c*(a + b*x
)^n]^2*PolyLog[2, -((Sqrt[e]*(a + b*x))/(b*Sqrt[-d] - a*Sqrt[e]))])/(2*Sqrt
[-d]*Sqrt[e]) + (3*n*Log[c*(a + b*x)^n]^2*PolyLog[2, (Sqrt[e]*(a + b*x))/(b
*Sqrt[-d] + a*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) + (3*n^2*Log[c*(a + b*x)^n]*P
olyLog[3, -((Sqrt[e]*(a + b*x))/(b*Sqrt[-d] - a*Sqrt[e]))])/(Sqrt[-d]*Sqrt[
e]) - (3*n^2*Log[c*(a + b*x)^n]*PolyLog[3, (Sqrt[e]*(a + b*x))/(b*Sqrt[-d]
+ a*Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) - (3*n^3*PolyLog[4, -((Sqrt[e]*(a + b*x))
/(b*Sqrt[-d] - a*Sqrt[e]))])/(Sqrt[-d]*Sqrt[e]) + (3*n^3*PolyLog[4, (Sqrt[e
]*(a + b*x))/(b*Sqrt[-d] + a*Sqrt[e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{-d}\log^3(c(a+bx)^n)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\log^3(c(a+bx)^n)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right) dx$$

$$= -\frac{\int \frac{\log^3(c(a+bx)^n)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{-d}} - \frac{\int \frac{\log^3(c(a+bx)^n)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}}$$

$$= \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(3bn)\int \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{a+bx}\,dx}{2\sqrt{-d}\sqrt{e}} + \frac{(3bn)\int \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{a+bx}\,dx}{2\sqrt{-d}\sqrt{e}}$$

$$= \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(3n)\text{Subst}\left(\int \frac{\log^2(cx^n)\log\left(\frac{b\left(\frac{b\sqrt{-d}+a\sqrt{e}}{b}-\frac{\sqrt{e}x}{b}\right)}{b\sqrt{-d}+a\sqrt{e}}\right)}{x}\,dx, x, a+bx\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(3n)\text{Subst}\left(\int \frac{\log^2(cx^n)\log\left(\frac{b\left(\frac{b\sqrt{-d}-a\sqrt{e}}{b}+\frac{\sqrt{e}x}{b}\right)}{b\sqrt{-d}-a\sqrt{e}}\right)}{x}\,dx, x, a+bx\right)}{2\sqrt{-d}\sqrt{e}}$$

$$= \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{3n\log^2(c(a+bx)^n)\,\text{Li}_2\left(-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{3n\log^2(c(a+bx)^n)\,\text{Li}_2\left(\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(3n^2)\,\text{Subst}\left(\int \frac{\log(cx^n)\text{Li}_2\left(-\frac{\sqrt{e}x}{b\sqrt{-d}-a\sqrt{e}}\right)}{x}\,dx, x, a+bx\right)}{\sqrt{-d}\sqrt{e}} - \frac{(3n^2)\,\text{Subst}\left(\int \frac{\log(cx^n)\text{Li}_2\left(\frac{\sqrt{e}x}{b\sqrt{-d}+a\sqrt{e}}\right)}{x}\,dx, x, a+bx\right)}{\sqrt{-d}\sqrt{e}}$$

$$= \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{3n\log^2(c(a+bx)^n)\operatorname{Li}_2\left(-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{3n\log^2(c(a+bx)^n)\operatorname{Li}_2\left(\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{3n^2\log(c(a+bx)^n)\operatorname{Li}_3\left(-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{3n^2\log(c(a+bx)^n)\operatorname{Li}_3\left(\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{(3n^3)\operatorname{Subst}\left(\int \frac{\operatorname{Li}_3\left(-\frac{\sqrt{e}x}{b\sqrt{-d}-a\sqrt{e}}\right)}{x}dx, x, a+bx\right)}{\sqrt{-d}\sqrt{e}} + \frac{(3n^3)\operatorname{Subst}\left(\int \frac{\operatorname{Li}_3\left(\frac{\sqrt{e}x}{b\sqrt{-d}+a\sqrt{e}}\right)}{x}dx, x, a+bx\right)}{\sqrt{-d}\sqrt{e}}$$

$$= \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{3n\log^2(c(a+bx)^n)\operatorname{Li}_2\left(-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{3n\log^2(c(a+bx)^n)\operatorname{Li}_2\left(\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{3n^2\log(c(a+bx)^n)\operatorname{Li}_3\left(-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{3n^2\log(c(a+bx)^n)\operatorname{Li}_3\left(\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{3n^3\operatorname{Li}_4\left(-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{3n^3\operatorname{Li}_4\left(\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.58

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex^2}\,dx = \frac{-2n^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log^3(a+bx) + 6n^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log^2(a+bx)\log(c(a+bx)^n) - 6n\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(a}{}$$

```
[In] Integrate[Log[c*(a + b*x)^n]^3/(d + e*x^2),x]
```

```
[Out] (-2*n^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[a + b*x]^3 + 6*n^2*ArcTan[(Sqrt[e]*
x)/Sqrt[d]]*Log[a + b*x]^2*Log[c*(a + b*x)^n] - 6*n*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]*Log[a + b*x]*Log[c*(a + b*x)^n]^2 + 2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[
c*(a + b*x)^n]^3 + I*n^3*Log[a + b*x]^3*Log[1 - (Sqrt[e]*(a + b*x))/((-I)*b
```

```
*Sqrt[d] + a*Sqrt[e])] - (3*I)*n^2*Log[a + b*x]^2*Log[c*(a + b*x)^n]*Log[1
- (Sqrt[e]*(a + b*x))/((-I)*b*Sqrt[d] + a*Sqrt[e])] + (3*I)*n*Log[a + b*x]*
Log[c*(a + b*x)^n]^2*Log[1 - (Sqrt[e]*(a + b*x))/((-I)*b*Sqrt[d] + a*Sqrt[e
])] - I*n^3*Log[a + b*x]^3*Log[1 - (Sqrt[e]*(a + b*x))/(I*b*Sqrt[d] + a*Sqr
t[e])] + (3*I)*n^2*Log[a + b*x]^2*Log[c*(a + b*x)^n]*Log[1 - (Sqrt[e]*(a +
b*x))/(I*b*Sqrt[d] + a*Sqrt[e])] - (3*I)*n*Log[a + b*x]*Log[c*(a + b*x)^n]^
2*Log[1 - (Sqrt[e]*(a + b*x))/(I*b*Sqrt[d] + a*Sqrt[e])] + (3*I)*n*Log[c*(a
 + b*x)^n]^2*PolyLog[2, (Sqrt[e]*(a + b*x))/((-I)*b*Sqrt[d] + a*Sqrt[e])] -
 (3*I)*n*Log[c*(a + b*x)^n]^2*PolyLog[2, (Sqrt[e]*(a + b*x))/(I*b*Sqrt[d] +
 a*Sqrt[e])] - (6*I)*n^2*Log[c*(a + b*x)^n]*PolyLog[3, (Sqrt[e]*(a + b*x))/
((-I)*b*Sqrt[d] + a*Sqrt[e])] + (6*I)*n^2*Log[c*(a + b*x)^n]*PolyLog[3, (Sq
rt[e]*(a + b*x))/(I*b*Sqrt[d] + a*Sqrt[e])] + (6*I)*n^3*PolyLog[4, (Sqrt[e]
*(a + b*x))/((-I)*b*Sqrt[d] + a*Sqrt[e])] - (6*I)*n^3*PolyLog[4, (Sqrt[e]*(
a + b*x))/(I*b*Sqrt[d] + a*Sqrt[e])])/(2*Sqrt[d]*Sqrt[e])
```

## Maple [F]

$$\int \frac{\ln \left(c(bx+a)^n\right)^3}{e\,x^2+d}dx$$

[In] int(ln(c*(b*x+a)^n)^3/(e*x^2+d),x)

[Out] int(ln(c*(b*x+a)^n)^3/(e*x^2+d),x)

## Fricas [F]

$$\int \frac{\log^3\left(c(a+bx)^n\right)}{d+ex^2}\,dx=\int \frac{\log\left((bx+a)^nc\right)^3}{ex^2+d}\,dx$$

[In] integrate(log(c*(b*x+a)^n)^3/(e*x^2+d),x, algorithm="fricas")

[Out] integral(log((b*x + a)^n*c)^3/(e*x^2 + d), x)

## Sympy [F]

$$\int \frac{\log^3\left(c(a+bx)^n\right)}{d+ex^2}\,dx=\int \frac{\log\left(c(a+bx)^n\right)^3}{d+ex^2}\,dx$$

[In] integrate(ln(c*(b*x+a)**n)**3/(e*x**2+d),x)

[Out] Integral(log(c*(a + b*x)**n)**3/(d + e*x**2), x)

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(b*x+a)^n)^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^3}{ex^2+d}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^n*c)^3/(e*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)^3}{e\,x^2+d}\,dx$$

```
[In] int(log(c*(a + b*x)^n)^3/(d + e*x^2),x)
```

```
[Out] int(log(c*(a + b*x)^n)^3/(d + e*x^2), x)
```

# 3.330 $\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 347

$$\begin{aligned}\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx &= \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &- \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &- \frac{n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} \\ &+ \frac{n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} \\ &+ \frac{n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}}\end{aligned}$$

```
[Out] 1/2*ln(c*(b*x+a)^n)^2*ln(b*((-d)^(1/2)-x*e^(1/2))/(b*(-d)^(1/2)+a*e^(1/2)))
/(-d)^(1/2)/e^(1/2)-1/2*ln(c*(b*x+a)^n)^2*ln(b*((-d)^(1/2)+x*e^(1/2))/(b*(-
d)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)-n*ln(c*(b*x+a)^n)*polylog(2,-(b*x+a
)*e^(1/2)/(b*(-d)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)+n*ln(c*(b*x+a)^n)*po
lylog(2,(b*x+a)*e^(1/2)/(b*(-d)^(1/2)+a*e^(1/2)))/(-d)^(1/2)/e^(1/2)+n^2*po
lylog(3,-(b*x+a)*e^(1/2)/(b*(-d)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)-n^2*p
olylog(3,(b*x+a)*e^(1/2)/(b*(-d)^(1/2)+a*e^(1/2)))/(-d)^(1/2)/e^(1/2)
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2456, 2443, 2481, 2421, 6724}

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx = -\frac{n\log(c(a+bx)^n)\,\text{PolyLog}\left(2,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{n\log(c(a+bx)^n)\,\text{PolyLog}\left(2,\frac{\sqrt{e}(a+bx)}{\sqrt{e}a+b\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{a\sqrt{e}+b\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{n^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}(a+bx)}{\sqrt{e}a+b\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}}$$

```
[In] Int[Log[c*(a + b*x)^n]^2/(d + e*x^2),x]
```

```
[Out] (Log[c*(a + b*x)^n]^2*Log[(b*(Sqrt[-d] - Sqrt[e]*x))/(b*Sqrt[-d] + a*Sqrt[e
])])/(2*Sqrt[-d]*Sqrt[e]) - (Log[c*(a + b*x)^n]^2*Log[(b*(Sqrt[-d] + Sqrt[e
]*x))/(b*Sqrt[-d] - a*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - (n*Log[c*(a + b*x)^
n]*PolyLog[2, -((Sqrt[e]*(a + b*x))/(b*Sqrt[-d] - a*Sqrt[e]))])/(Sqrt[-d]*S
qrt[e]) + (n*Log[c*(a + b*x)^n]*PolyLog[2, (Sqrt[e]*(a + b*x))/(b*Sqrt[-d]
+ a*Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) + (n^2*PolyLog[3, -((Sqrt[e]*(a + b*x))/(
b*Sqrt[-d] - a*Sqrt[e]))])/(Sqrt[-d]*Sqrt[e]) - (n^2*PolyLog[3, (Sqrt[e]*(a
 + b*x))/(b*Sqrt[-d] + a*Sqrt[e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sqrt{-d}\log^2\left(c(a+bx)^n\right)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\log^2\left(c(a+bx)^n\right)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)\,dx\\
&= -\frac{\int \frac{\log^2(c(a+bx)^n)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{-d}}-\frac{\int \frac{\log^2(c(a+bx)^n)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}}\\
&= \frac{\log^2\left(c(a+bx)^n\right)\log\left(\frac{b\left(\sqrt{-d}-\sqrt{e}x\right)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\log^2\left(c(a+bx)^n\right)\log\left(\frac{b\left(\sqrt{-d}+\sqrt{e}x\right)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}\\
&\quad -\frac{(bn)\int \frac{\log(c(a+bx)^n)\log\left(\frac{b\left(\sqrt{-d}-\sqrt{e}x\right)}{b\sqrt{-d}+a\sqrt{e}}\right)}{a+bx}\,dx}{\sqrt{-d}\sqrt{e}}+\frac{(bn)\int \frac{\log(c(a+bx)^n)\log\left(\frac{b\left(\sqrt{-d}+\sqrt{e}x\right)}{b\sqrt{-d}-a\sqrt{e}}\right)}{a+bx}\,dx}{\sqrt{-d}\sqrt{e}}
\end{aligned}$$

$$= \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\text{Subst}\left(\int \frac{\log(cx^n)\log\left(\frac{b\left(\frac{b\sqrt{-d}+a\sqrt{e}}{b}-\frac{\sqrt{e}x}{b}\right)}{b\sqrt{-d}+a\sqrt{e}}\right)}{x}dx, x, a+bx\right)}{\sqrt{-d}\sqrt{e}} + \frac{n\text{Subst}\left(\int \frac{\log(cx^n)\log\left(\frac{b\left(\frac{b\sqrt{-d}-a\sqrt{e}}{b}+\frac{\sqrt{e}x}{b}\right)}{b\sqrt{-d}-a\sqrt{e}}\right)}{x}dx, x, a+bx\right)}{\sqrt{-d}\sqrt{e}}$$

$$= \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\log(c(a+bx)^n)\operatorname{Li}_2\left(-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{n\log(c(a+bx)^n)\operatorname{Li}_2\left(\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{n^2\text{Subst}\left(\int \frac{\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{b\sqrt{-d}-a\sqrt{e}}\right)}{x}dx, x, a+bx\right)}{\sqrt{-d}\sqrt{e}} - \frac{n^2\text{Subst}\left(\int \frac{\operatorname{Li}_2\left(\frac{\sqrt{e}x}{b\sqrt{-d}+a\sqrt{e}}\right)}{x}dx, x, a+bx\right)}{\sqrt{-d}\sqrt{e}}$$

$$= \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\log(c(a+bx)^n)\operatorname{Li}_2\left(-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{n\log(c(a+bx)^n)\operatorname{Li}_2\left(\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{n^2\operatorname{Li}_3\left(-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{n^2\operatorname{Li}_3\left(\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{\sqrt{-d}\sqrt{e}}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.41

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx = \frac{2n^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log^2(a+bx)-4n\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(a+bx)\log(c(a+bx)^n)+2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log^2(c(a}{}$$

[In] Integrate[Log[c*(a + b*x)^n]^2/(d + e*x^2),x]

[Out] (2*n^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[a + b*x]^2 - 4*n*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[a + b*x]*Log[c*(a + b*x)^n] + 2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(a + b*x)^n]^2 - I*n^2*Log[a + b*x]^2*Log[1 - (Sqrt[e]*(a + b*x))/((-I)*b*Sqrt[d] + a*Sqrt[e])] + (2*I)*n*Log[a + b*x]*Log[c*(a + b*x)^n]*Log[1 - (Sqrt[e]*(a + b*x))/((-I)*b*Sqrt[d] + a*Sqrt[e])] + I*n^2*Log[a + b*x]^2*Log[1 - (Sqrt[e]*(a + b*x))/(I*b*Sqrt[d] + a*Sqrt[e])] - (2*I)*n*Log[a + b*x]*Log[c*(a + b*x)^n]*Log[1 - (Sqrt[e]*(a + b*x))/(I*b*Sqrt[d] + a*Sqrt[e])] + (2*I)*n*Log[c*(a + b*x)^n]*PolyLog[2, (Sqrt[e]*(a + b*x))/((-I)*b*Sqrt[d] + a*Sqrt[e])] - (2*I)*n*Log[c*(a + b*x)^n]*PolyLog[2, (Sqrt[e]*(a + b*x))/(I*b*Sqrt[d] + a*Sqrt[e])] - (2*I)*n^2*PolyLog[3, (Sqrt[e]*(a + b*x))/((-I)*b*Sqrt[d] + a*Sqrt[e])] + (2*I)*n^2*PolyLog[3, (Sqrt[e]*(a + b*x))/(I*b*Sqrt[d] + a*Sqrt[e])])/(2*Sqrt[d]*Sqrt[e])

## Maple [F]

$$\int \frac{\ln\left(c(bx+a)^n\right)^2}{e\,x^2+d}dx$$

[In] int(ln(c*(b*x+a)^n)^2/(e*x^2+d),x)

[Out] int(ln(c*(b*x+a)^n)^2/(e*x^2+d),x)

## Fricas [F]

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^2}{ex^2+d}\,dx$$

[In] integrate(log(c*(b*x+a)^n)^2/(e*x^2+d),x, algorithm="fricas")

[Out] integral(log((b*x + a)^n*c)^2/(e*x^2 + d), x)

## Sympy [F]

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx = \int \frac{\log(c(a+bx)^n)^2}{d+ex^2}\,dx$$

```
[In] integrate(ln(c*(b*x+a)**n)**2/(e*x**2+d),x)
```

```
[Out] Integral(log(c*(a + b*x)**n)**2/(d + e*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(b*x+a)^n)^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx = \int \frac{\log((bx+a)^n c)^2}{ex^2+d}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^n*c)^2/(e*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)^2}{e\,x^2+d}\,dx$$

```
[In] int(log(c*(a + b*x)^n)^2/(d + e*x^2),x)
```

```
[Out] int(log(c*(a + b*x)^n)^2/(d + e*x^2), x)
```

# 3.331 $\int \frac{\log(c(a+bx)^n)}{d+ex^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 229

$$\int \frac{\log(c(a+bx)^n)}{d+ex^2} dx = \frac{\log(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[Out] 1/2*ln(c*(b*x+a)^n)*ln(b*((-d)^(1/2)-x*e^(1/2))/(b*(-d)^(1/2)+a*e^(1/2)))/(
-d)^(1/2)/e^(1/2)-1/2*ln(c*(b*x+a)^n)*ln(b*((-d)^(1/2)+x*e^(1/2))/(b*(-d)^(
1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*polylog(2,-(b*x+a)*e^(1/2)/(b*(-d
)^(1/2)-a*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*n*polylog(2,(b*x+a)*e^(1/2)/(b*(
-d)^(1/2)+a*e^(1/2)))/(-d)^(1/2)/e^(1/2)
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{2456, 2441, 2440, 2438\}$

$$\int \frac{\log(c(a+bx)^n)}{d+ex^2}\,dx = \frac{\log(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{a\sqrt{e}+b\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(a+bx)}{\sqrt{e}a+b\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[In] Int[Log[c*(a + b*x)^n]/(d + e*x^2),x]
```

```
[Out] (Log[c*(a + b*x)^n]*Log[(b*(Sqrt[-d] - Sqrt[e]*x))/(b*Sqrt[-d] + a*Sqrt[e])
])/(2*Sqrt[-d]*Sqrt[e]) - (Log[c*(a + b*x)^n]*Log[(b*(Sqrt[-d] + Sqrt[e]*x)
)/(b*Sqrt[-d] - a*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - (n*PolyLog[2, -((Sqrt[e
]*(a + b*x))/(b*Sqrt[-d] - a*Sqrt[e]))])/(2*Sqrt[-d]*Sqrt[e]) + (n*PolyLog[
2, (Sqrt[e]*(a + b*x))/(b*Sqrt[-d] + a*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rubi steps

$$\text{integral} = \int \left( \frac{\sqrt{-d}\log(c(a+bx)^n)}{2d(\sqrt{-d}-\sqrt{e}x)} + \frac{\sqrt{-d}\log(c(a+bx)^n)}{2d(\sqrt{-d}+\sqrt{e}x)} \right) dx$$

$$= -\frac{\int \frac{\log(c(a+bx)^n)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{-d}} - \frac{\int \frac{\log(c(a+bx)^n)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}}$$

$$= \frac{\log(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(bn)\int \frac{\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{a+bx}\,dx}{2\sqrt{-d}\sqrt{e}} + \frac{(bn)\int \frac{\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{a+bx}\,dx}{2\sqrt{-d}\sqrt{e}}$$

$$= \frac{\log(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{e}x}{b\sqrt{-d}-a\sqrt{e}}\right)}{x}\,dx, x, a+bx\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{e}x}{b\sqrt{-d}+a\sqrt{e}}\right)}{x}\,dx, x, a+bx\right)}{2\sqrt{-d}\sqrt{e}}$$

$$= \frac{\log(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log(c(a+bx)^n)\log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\text{Li}_2\left(-\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\text{Li}_2\left(\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.78

$$\int \frac{\log(c(a+bx)^n)}{d+ex^2}\,dx = \frac{\log(c(a+bx)^n)\left(\log\left(\frac{b(\sqrt{-d}-\sqrt{e}x)}{b\sqrt{-d}+a\sqrt{e}}\right) - \log\left(\frac{b(\sqrt{-d}+\sqrt{e}x)}{b\sqrt{-d}-a\sqrt{e}}\right)\right) - n\,\text{PolyLog}\left(2, -\frac{\sqrt{e}(a+bx)}{b\sqrt{-d}-a\sqrt{e}}\right) + n\,\text{PolyLog}\left(2, \frac{\sqrt{e}(a+bx)}{b\sqrt{-d}+a\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[In] Integrate[Log[c*(a + b*x)^n]/(d + e*x^2),x]
[Out] (Log[c*(a + b*x)^n]*(Log[(b*(Sqrt[-d] - Sqrt[e]*x))/(b*Sqrt[-d] + a*Sqrt[e]
)] - Log[(b*(Sqrt[-d] + Sqrt[e]*x))/(b*Sqrt[-d] - a*Sqrt[e])]) - n*PolyLog[
2, -((Sqrt[e]*(a + b*x))/(b*Sqrt[-d] - a*Sqrt[e]))] + n*PolyLog[2, (Sqrt[e]
*(a + b*x))/(b*Sqrt[-d] + a*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.75 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.71

| method | result |
|---|---|
| risch | $-\frac{\arctan\left(\frac{2e(bx+a)-2ae}{2\sqrt{de}\,b}\right)n\ln(bx+a)}{\sqrt{de}}+\frac{\arctan\left(\frac{2e(bx+a)-2ae}{2\sqrt{de}\,b}\right)\ln\left((bx+a)^{n}\right)}{\sqrt{de}}+\frac{n\ln(bx+a)\ln\left(\frac{b\sqrt{-de}-e(bx+a)+ae}{b\sqrt{-de}+ae}\right)}{2\sqrt{-de}}-\frac{n\ln(bx+a)}{}$ |

[In] int(ln(c*(b*x+a)^n)/(e*x^2+d),x,method=_RETURNVERBOSE)

[Out] -1/(d*e)^(1/2)*arctan(1/2*(2*e*(b*x+a)-2*a*e)/(d*e)^(1/2)/b)*n*ln(b*x+a)+1/(d*e)^(1/2)*arctan(1/2*(2*e*(b*x+a)-2*a*e)/(d*e)^(1/2)/b)*ln((b*x+a)^n)+1/2*n*ln(b*x+a)/(-d*e)^(1/2)*ln((b*(-d*e)^(1/2)-e*(b*x+a)+a*e)/(b*(-d*e)^(1/2)+a*e))-1/2*n*ln(b*x+a)/(-d*e)^(1/2)*ln((b*(-d*e)^(1/2)+e*(b*x+a)-a*e)/(b*(-d*e)^(1/2)-a*e))+1/2*n/(-d*e)^(1/2)*dilog((b*(-d*e)^(1/2)-e*(b*x+a)+a*e)/(b*(-d*e)^(1/2)+a*e))-1/2*n/(-d*e)^(1/2)*dilog((b*(-d*e)^(1/2)+e*(b*x+a)-a*e)/(b*(-d*e)^(1/2)-a*e))+(-1/2*I*Pi*csgn(I*c*(b*x+a)^n)^3+1/2*I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+1/2*I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*c)-1/2*I*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^n)*csgn(I*c)+ln(c))/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))

## Fricas [F]

$$\int \frac{\log\left(c(a+bx)^n\right)}{d+ex^2}\,dx=\int \frac{\log\left((bx+a)^n c\right)}{ex^2+d}\,dx$$

[In] integrate(log(c*(b*x+a)^n)/(e*x^2+d),x, algorithm="fricas")

[Out] integral(log((b*x + a)^n*c)/(e*x^2 + d), x)

## Sympy [F]

$$\int \frac{\log\left(c(a+bx)^n\right)}{d+ex^2}\,dx=\int \frac{\log\left(c(a+bx)^n\right)}{d+ex^2}\,dx$$

[In] integrate(ln(c*(b*x+a)**n)/(e*x**2+d),x)

[Out] Integral(log(c*(a + b*x)**n)/(d + e*x**2), x)

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(a+bx)^n\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(b*x+a)^n)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{\log\left(c(a+bx)^n\right)}{d+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)}{ex^2+d}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^n*c)/(e*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx)^n\right)}{d+ex^2}\,dx = \int \frac{\ln\left(c\,{(a+b\,x)}^n\right)}{e\,x^2+d}\,dx$$

```
[In] int(log(c*(a + b*x)^n)/(d + e*x^2),x)
```

```
[Out] int(log(c*(a + b*x)^n)/(d + e*x^2), x)
```

# 3.332 $\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx = -\frac{\text{Int}\left(\frac{1}{(\sqrt{-d}-\sqrt{e}x)\log(c(a+bx)^n)},x\right)}{2\sqrt{-d}} - \frac{\text{Int}\left(\frac{1}{(\sqrt{-d}+\sqrt{e}x)\log(c(a+bx)^n)},x\right)}{2\sqrt{-d}}$$

```
[Out] -1/2*Unintegrable(1/ln(c*(b*x+a)^n)/((-d)^(1/2)-x*e^(1/2)),x)/(-d)^(1/2)-1/
2*Unintegrable(1/ln(c*(b*x+a)^n)/((-d)^(1/2)+x*e^(1/2)),x)/(-d)^(1/2)
```

## Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx$$

```
[In] Int[1/((d + e*x^2)*Log[c*(a + b*x)^n]),x]
```

```
[Out] -1/2*Defer[Int][1/((Sqrt[-d] - Sqrt[e]*x)*Log[c*(a + b*x)^n]), x]/Sqrt[-d]
- Defer[Int][1/((Sqrt[-d] + Sqrt[e]*x)*Log[c*(a + b*x)^n]), x]/(2*Sqrt[-d])
```

Rubi steps

$$\text{integral} = \int \left( \frac{\sqrt{-d}}{2d\left(\sqrt{-d} - \sqrt{e}x\right)\log\left(c(a+bx)^n\right)} + \frac{\sqrt{-d}}{2d\left(\sqrt{-d} + \sqrt{e}x\right)\log\left(c(a+bx)^n\right)} \right) dx$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-d}-\sqrt{e}x\right)\log(c(a+bx)^n)}\,dx}{2\sqrt{-d}} - \frac{\int \frac{1}{\left(\sqrt{-d}+\sqrt{e}x\right)\log(c(a+bx)^n)}\,dx}{2\sqrt{-d}}$$

## Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\log\left(c(a+bx)^n\right)}\,dx = \int \frac{1}{(d+ex^2)\log\left(c(a+bx)^n\right)}\,dx$$

```
[In] Integrate[1/((d + e*x^2)*Log[c*(a + b*x)^n]),x]
[Out] Integrate[1/((d + e*x^2)*Log[c*(a + b*x)^n]), x]
```

## Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)\ln\left(c\,(bx+a)^n\right)}dx$$

```
[In] int(1/(e*x^2+d)/ln(c*(b*x+a)^n),x)
[Out] int(1/(e*x^2+d)/ln(c*(b*x+a)^n),x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\log\left(c(a+bx)^n\right)}\,dx = \int \frac{1}{(ex^2+d)\log\left((bx+a)^n c\right)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/log(c*(b*x+a)^n),x, algorithm="fricas")
[Out] integral(1/((e*x^2 + d)*log((b*x + a)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 21.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx$$

```
[In] integrate(1/(e*x**2+d)/ln(c*(b*x+a)**n),x)
[Out] Integral(1/((d + e*x**2)*log(c*(a + b*x)**n)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(ex^2+d)\log((bx+a)^n c)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/log(c*(b*x+a)^n),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)*log((b*x + a)^n*c)), x)
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(ex^2+d)\log((bx+a)^n c)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/log(c*(b*x+a)^n),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)*log((b*x + a)^n*c)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{\ln\left(c\,(a+b\,x)^n\right)\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int(1/(log(c*(a + b*x)^n)*(d + e*x^2)),x)
[Out] int(1/(log(c*(a + b*x)^n)*(d + e*x^2)), x)
```

# 3.333 $\int \frac{\log\left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x(a+bx^m)}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 27

$$\int \frac{\log\left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x\left(a+bx^m\right)}\,dx=\frac{\text{PolyLog}\left(2,\frac{(1-c)\left(b+ax^{-m}\right)}{b}\right)}{am}$$

```
[Out] polylog(2,(1-c)*(b+a/(x^m))/b)/a/m
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2525, 2459, 2440, 2438}

$$\int \frac{\log\left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x\left(a+bx^m\right)}\,dx=\frac{\text{PolyLog}\left(2,\frac{(1-c)\left(ax^{-m}+b\right)}{b}\right)}{am}$$

```
[In] Int[Log[c - (a*(1 - c))/(b*x^m)]/(x*(a + b*x^m)),x]
```

```
[Out] PolyLog[2, ((1 - c)*(b + a/x^m))/b]/(a*m)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\log\left(c-\frac{a(1-c)x}{b}\right)}{\left(a+\frac{b}{x}\right)x}\,dx, x, x^{-m}\right)}{m}\\
&= -\frac{\text{Subst}\left(\int \frac{\log\left(c-\frac{a(1-c)x}{b}\right)}{b+ax}\,dx, x, x^{-m}\right)}{m}\\
&= -\frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{(1-c)x}{b}\right)}{x}\,dx, x, b+ax^{-m}\right)}{am}\\
&= \frac{\text{Li}_2\left(\frac{(1-c)(b+ax^{-m})}{b}\right)}{am}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x(a+bx^m)}\,dx = \frac{\text{PolyLog}\left(2,-\frac{(-1+c)x^{-m}(a+bx^m)}{b}\right)}{am}$$

```
[In] Integrate[Log[c - (a*(1 - c))/(b*x^m)]/(x*(a + b*x^m)),x]
[Out] PolyLog[2, -(((-1 + c)*(a + b*x^m))/(b*x^m))]/(a*m)
```

## Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\operatorname{dilog}\left(\frac{(ca-a)x^{-m}}{b}+c\right)}{ma}$ | 27 |
| default | $\frac{\operatorname{dilog}\left(\frac{(ca-a)x^{-m}}{b}+c\right)}{ma}$ | 27 |
| risch | Expression too large to display | 1267 |

```
[In] int(ln(c-a*(1-c)/b/(x^m))/x/(a+b*x^m),x,method=_RETURNVERBOSE)
[Out] 1/m/a*dilog(1/b*(a*c-a)/(x^m)+c)
```

## Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{\log\left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x(a+bx^m)}\,dx = \frac{\text{Li}_2\left(-\frac{bcx^m+ac-a}{bx^m}+1\right)}{am}$$

```
[In] integrate(log(c-a*(1-c)/b/(x^m))/x/(a+b*x^m),x, algorithm="fricas")
[Out] dilog(-(b*c*x^m + a*c - a)/(b*x^m) + 1)/(a*m)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x\,(a+bx^m)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c-a*(1-c)/b/(x**m))/x/(a+b*x**m),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log \left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x\,(a+bx^m)}\,dx = \int \frac{\log \left(c+\frac{a(c-1)}{bx^m}\right)}{(bx^m+a)x}\,dx$$

```
[In] integrate(log(c-a*(1-c)/b/(x^m))/x/(a+b*x^m),x, algorithm="maxima")
[Out] (c*m - m)*integrate(log(x)/(b*c*x*x^m + a*(c - 1)*x), x) + (log(b*c*x^m + a
*c - a)*log(x) - log(b)*log(x) - log(x)*log(x^m))/a + log(b)*log((b*x^m + a
)/b)/(a*m) + (log(x^m)*log(b*x^m/a + 1) + dilog(-b*x^m/a))/(a*m) - (log(b*c
*x^m + a*c - a)*log((b*c*x^m + a*(c - 1))/a + 1) + dilog(-(b*c*x^m + a*(c -
 1))/a))/(a*m)
```

## Giac [F]

$$\int \frac{\log \left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x\,(a+bx^m)}\,dx = \int \frac{\log \left(c+\frac{a(c-1)}{bx^m}\right)}{(bx^m+a)x}\,dx$$

```
[In] integrate(log(c-a*(1-c)/b/(x^m))/x/(a+b*x^m),x, algorithm="giac")
[Out] integrate(log(c + a*(c - 1)/(b*x^m))/((b*x^m + a)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c-\frac{a(1-c)x^{-m}}{b}\right)}{x\,(a+bx^m)}\,dx = \int \frac{\ln \left(c+\frac{a\,(c-1)}{b\,x^m}\right)}{x\,\left(a+b\,x^m\right)}\,dx$$

```
[In] int(log(c + (a*(c - 1))/(b*x^m))/(x*(a + b*x^m)),x)
[Out] int(log(c + (a*(c - 1))/(b*x^m))/(x*(a + b*x^m)), x)
```

# 3.334 $\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x(a+bx^m)}\,dx$



## Optimal result

Integrand size = 36, antiderivative size = 27

$$\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x(a+bx^m)}\,dx = \frac{\text{PolyLog}\left(2,\frac{(1-c)(b+ax^{-m})}{b}\right)}{am}$$

```
[Out] polylog(2,(1-c)*(b+a/(x^m))/b)/a/m
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2530, 2525, 2459, 2440, 2438}

$$\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x(a+bx^m)}\,dx = \frac{\text{PolyLog}\left(2,\frac{(1-c)(ax^{-m}+b)}{b}\right)}{am}$$

```
[In] Int[Log[(-a + a*c + b*c*x^m)/(b*x^m)]/(x*(a + b*x^m)),x]
```

```
[Out] PolyLog[2, ((1 - c)*(b + a/x^m))/b]/(a*m)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2530

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.)*((h_.)*(x_))^(m_
.), x_Symbol] :> Int[(h*x)^m*ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v
, x]^p])^q, x] /; FreeQ[{a, b, c, h, m, p, q, r}, x] && BinomialQ[{u, v}, x
] &&  !BinomialMatchQ[{u, v}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\log\left(c+\frac{(-a+ac)x^{-m}}{b}\right)}{x\left(a+bx^m\right)}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{\log\left(c+\frac{(-a+ac)x}{b}\right)}{\left(a+\frac{b}{x}\right)x}\,dx,x,x^{-m}\right)}{m}\\
&= -\frac{\text{Subst}\left(\int \frac{\log\left(c+\frac{(-a+ac)x}{b}\right)}{b+ax}\,dx,x,x^{-m}\right)}{m}\\
&= -\frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{(-a+ac)x}{ab}\right)}{x}\,dx,x,b+ax^{-m}\right)}{am}\\
&= \frac{\text{Li}_2\left(\frac{(1-c)\left(b+ax^{-m}\right)}{b}\right)}{am}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\log \left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x\,(a+bx^m)}\,dx = \frac{\text{PolyLog}\left(2,-\frac{(-1+c)x^{-m}(a+bx^m)}{b}\right)}{am}$$

```
[In] Integrate[Log[(-a + a*c + b*c*x^m)/(b*x^m)]/(x*(a + b*x^m)),x]
[Out] PolyLog[2, -(((-1 + c)*(a + b*x^m))/(b*x^m))]/(a*m)
```

## Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\text{dilog}\left(\frac{(ca-a)x^{-m}}{b}+c\right)}{ma}$ | 27 |
| default | $\frac{\text{dilog}\left(\frac{(ca-a)x^{-m}}{b}+c\right)}{ma}$ | 27 |
| risch | Expression too large to display | 1267 |

```
[In] int(ln((-a+c*a+b*c*x^m)/b/(x^m))/x/(a+b*x^m),x,method=_RETURNVERBOSE)
[Out] 1/m/a*dilog(1/b*(a*c-a)/(x^m)+c)
```

## Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{\log \left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x\,(a+bx^m)}\,dx = \frac{\text{Li}_2\left(-\frac{bcx^m+ac-a}{bx^m}+1\right)}{am}$$

```
[In] integrate(log((-a+a*c+b*c*x^m)/b/(x^m))/x/(a+b*x^m),x, algorithm="fricas")
[Out] dilog(-(b*c*x^m + a*c - a)/(b*x^m) + 1)/(a*m)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x\left(a+bx^m\right)}\,dx = \text{Timed out}$$

```
[In] integrate(ln((-a+a*c+b*c*x**m)/b/(x**m))/x/(a+b*x**m),x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x\left(a+bx^m\right)}\,dx = \int \frac{\log\left(\frac{bcx^m+ac-a}{bx^m}\right)}{(bx^m+a)x}\,dx$$

```
[In] integrate(log((-a+a*c+b*c*x^m)/b/(x^m))/x/(a+b*x^m),x, algorithm="maxima")
```

```
[Out] (c*m - m)*integrate(log(x)/(b*c*x*x^m + a*(c - 1)*x), x) + (log(b*c*x^m + a
*c - a)*log(x) - log(b)*log(x) - log(x)*log(x^m))/a + log(b)*log((b*x^m + a
)/b)/(a*m) + (log(x^m)*log(b*x^m/a + 1) + dilog(-b*x^m/a))/(a*m) - (log(b*c
*x^m + a*c - a)*log((b*c*x^m + a*(c - 1))/a + 1) + dilog(-(b*c*x^m + a*(c -
 1))/a))/(a*m)
```

## Giac [F]

$$\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x\left(a+bx^m\right)}\,dx = \int \frac{\log\left(\frac{bcx^m+ac-a}{bx^m}\right)}{(bx^m+a)x}\,dx$$

```
[In] integrate(log((-a+a*c+b*c*x^m)/b/(x^m))/x/(a+b*x^m),x, algorithm="giac")
```

```
[Out] integrate(log((b*c*x^m + a*c - a)/(b*x^m))/((b*x^m + a)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{x^{-m}(-a+ac+bcx^m)}{b}\right)}{x\left(a+bx^m\right)}\,dx = \int \frac{\ln\left(\frac{a\,c-a+b\,c\,x^m}{b\,x^m}\right)}{x\,\left(a+b\,x^m\right)}\,dx$$

```
[In] int(log((a*c - a + b*c*x^m)/(b*x^m))/(x*(a + b*x^m)),x)
```

```
[Out] int(log((a*c - a + b*c*x^m)/(b*x^m))/(x*(a + b*x^m)), x)
```

# 3.335 $\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x(d+ex^m)}\,dx$



## Optimal result

Integrand size = 38, antiderivative size = 28

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\,(d+ex^m)}\,dx=\frac{\operatorname{PolyLog}\left(2,\frac{(1-ac)(e+dx^{-m})}{e}\right)}{dm}$$

```
[Out] polylog(2,(-a*c+1)*(e+d/(x^m))/e)/d/m
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2525, 2459, 2440, 2438}

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\,(d+ex^m)}\,dx=\frac{\operatorname{PolyLog}\left(2,\frac{(1-ac)(dx^{-m}+e)}{e}\right)}{dm}$$

```
[In] Int[Log[c*(a - (d - a*c*d)/(c*e*x^m))]/(x*(d + e*x^m)),x]
```

```
[Out] PolyLog[2, ((1 - a*c)*(e + d/x^m))/e]/(d*m)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\log\left(c\left(a-\frac{(d-acd)x}{ce}\right)\right)}{\left(d+\frac{e}{x}\right)x}\,dx, x, x^{-m}\right)}{m} \\
&= -\frac{\text{Subst}\left(\int \frac{\log\left(c\left(a-\frac{(d-acd)x}{ce}\right)\right)}{e+dx}\,dx, x, x^{-m}\right)}{m} \\
&= -\frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{(d-acd)x}{de}\right)}{x}\,dx, x, e+dx^{-m}\right)}{dm} \\
&= \frac{\text{Li}_2\left(\frac{(1-ac)(e+dx^{-m})}{e}\right)}{dm}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\,(d+ex^m)}\,dx=\frac{\text{PolyLog}\left(2,-\frac{(-1+ac)x^{-m}(d+ex^m)}{e}\right)}{dm}$$

```
[In] Integrate[Log[c*(a - (d - a*c*d)/(c*e*x^m))]/(x*(d + e*x^m)),x]
[Out] PolyLog[2, -(((-1 + a*c)*(d + e*x^m))/(e*x^m))]/(d*m)
```

## Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\operatorname{dilog}\left(\frac{(acd-d)x^{-m}}{e}+ca\right)}{md}$ | 30 |
| default | $\frac{\operatorname{dilog}\left(\frac{(acd-d)x^{-m}}{e}+ca\right)}{md}$ | 30 |
| risch | Expression too large to display | 1200 |

```
[In] int(ln(c*(a+(a*c*d-d)/c/e/(x^m)))/x/(d+e*x^m),x,method=_RETURNVERBOSE)
[Out] 1/m/d*dilog(1/e*(a*c*d-d)/(x^m)+c*a)
```

## Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\,(d+ex^m)}\,dx=\frac{\text{Li}_2\left(-\frac{acex^m+(ac-1)d}{ex^m}+1\right)}{dm}$$

```
[In] integrate(log(c*(a+(a*c*d-d)/c/e/(x^m)))/x/(d+e*x^m),x, algorithm="fricas")
[Out] dilog(-(a*c*e*x^m + (a*c - 1)*d)/(e*x^m) + 1)/(d*m)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\left(d+ex^m\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

[In] integrate(ln(c*(a+(a*c*d-d)/c/e/(x**m)))/x/(d+e*x**m),x)

[Out] Exception raised: HeuristicGCDFailed >> no luck

## Maxima [F]

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\left(d+ex^m\right)}\,dx = \int \frac{\log\left(\left(a+\frac{acd-d}{cex^m}\right)c\right)}{(ex^m+d)x}\,dx$$

[In] integrate(log(c*(a+(a*c*d-d)/c/e/(x^m)))/x/(d+e*x^m),x, algorithm="maxima")

[Out] (a*c*m - m)*integrate(log(x)/(a*c*e*x*x^m + (a*c*d - d)*x), x) + (log(a*c*e*x^m + (a*c - 1)*d)*log(x) - log(e)*log(x) - log(x)*log(x^m))/d + log(e)*log((e*x^m + d)/e)/(d*m) + (log(x^m)*log(e*x^m/d + 1) + dilog(-e*x^m/d))/(d*m) - (log(a*c*e*x^m + (a*c - 1)*d)*log((a*c*e*x^m + a*c*d - d)/d + 1) + dilog(-(a*c*e*x^m + a*c*d - d)/d))/(d*m)

## Giac [F]

$$\int \frac{\log\left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\left(d+ex^m\right)}\,dx = \int \frac{\log\left(\left(a+\frac{acd-d}{cex^m}\right)c\right)}{(ex^m+d)x}\,dx$$

[In] integrate(log(c*(a+(a*c*d-d)/c/e/(x^m)))/x/(d+e*x^m),x, algorithm="giac")

[Out] integrate(log((a + (a*c*d - d)/(c*e*x^m))*c)/((e*x^m + d)*x), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a-\frac{(d-acd)x^{-m}}{ce}\right)\right)}{x\left(d+ex^m\right)}\,dx=\int \frac{\ln \left(c\left(a-\frac{d-a\,c\,d}{c\,e\,x^m}\right)\right)}{x\,\left(d+e\,x^m\right)}\,dx$$

```
[In] int(log(c*(a - (d - a*c*d)/(c*e*x^m)))/(x*(d + e*x^m)),x)
[Out] int(log(c*(a - (d - a*c*d)/(c*e*x^m)))/(x*(d + e*x^m)), x)
```

# 3.336 $\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x(d+ex^m)}\,dx$



## Optimal result

Integrand size = 38, antiderivative size = 28

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x\,(d+ex^m)}\,dx=\frac{\text{PolyLog}\left(2,\frac{(1-ac)(e+dx^{-m})}{e}\right)}{dm}$$

```
[Out] polylog(2,(-a*c+1)*(e+d/(x^m))/e)/d/m
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2530, 2525, 2459, 2440, 2438}

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x\,(d+ex^m)}\,dx=\frac{\text{PolyLog}\left(2,\frac{(1-ac)(dx^{-m}+e)}{e}\right)}{dm}$$

```
[In] Int[Log[(-d + a*c*d + a*c*e*x^m)/(e*x^m)]/(x*(d + e*x^m)),x]
[Out] PolyLog[2, ((1 - a*c)*(e + d/x^m))/e]/(d*m)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2530

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.)*((h_.)*(x_))^(m_
.), x_Symbol] :> Int[(h*x)^m*ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v
, x]^p])^q, x] /; FreeQ[{a, b, c, h, m, p, q, r}, x] && BinomialQ[{u, v}, x
] &&  !BinomialMatchQ[{u, v}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\log\left(ac+\frac{(-d+acd)x^{-m}}{e}\right)}{x\left(d+ex^m\right)}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{\log\left(ac+\frac{(-d+acd)x}{e}\right)}{\left(d+\frac{e}{x}\right)x}\,dx, x, x^{-m}\right)}{m} \\
&= -\frac{\text{Subst}\left(\int \frac{\log\left(ac+\frac{(-d+acd)x}{e}\right)}{e+dx}\,dx, x, x^{-m}\right)}{m} \\
&= -\frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{(-d+acd)x}{de}\right)}{x}\,dx, x, e+dx^{-m}\right)}{dm} \\
&= \frac{\text{Li}_2\left(\frac{(1-ac)\left(e+dx^{-m}\right)}{e}\right)}{dm}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x\left(d+ex^m\right)}\,dx = \frac{\text{PolyLog}\left(2,-\frac{(-1+ac)x^{-m}(d+ex^m)}{e}\right)}{dm}$$

```
[In] Integrate[Log[(-d + a*c*d + a*c*e*x^m)/(e*x^m)]/(x*(d + e*x^m)),x]
[Out] PolyLog[2, -(((-1 + a*c)*(d + e*x^m))/(e*x^m))]/(d*m)
```

## Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\text{dilog}\left(\frac{(acd-d)x^{-m}}{e}+ca\right)}{md}$ | 30 |
| default | $\frac{\text{dilog}\left(\frac{(acd-d)x^{-m}}{e}+ca\right)}{md}$ | 30 |
| risch | Expression too large to display | 1200 |

```
[In] int(ln((-d+a*c*d+a*c*e*x^m)/e/(x^m))/x/(d+e*x^m),x,method=_RETURNVERBOSE)
[Out] 1/m/d*dilog(1/e*(a*c*d-d)/(x^m)+c*a)
```

## Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x\left(d+ex^m\right)}\,dx = \frac{\text{Li}_2\left(-\frac{acex^m+(ac-1)d}{ex^m}+1\right)}{dm}$$

```
[In] integrate(log((-d+a*c*d+a*c*e*x^m)/e/(x^m))/x/(d+e*x^m),x, algorithm="frica
s")
[Out] dilog(-(a*c*e*x^m + (a*c - 1)*d)/(e*x^m) + 1)/(d*m)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x\left(d+ex^m\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(ln((-d+a*c*d+a*c*e*x**m)/e/(x**m))/x/(d+e*x**m),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [F]

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x\left(d+ex^m\right)}\,dx = \int \frac{\log\left(\frac{acex^m+acd-d}{ex^m}\right)}{(ex^m+d)x}\,dx$$

```
[In] integrate(log((-d+a*c*d+a*c*e*x^m)/e/(x^m))/x/(d+e*x^m),x, algorithm="maxim
a")
```

```
[Out] (a*c*m - m)*integrate(log(x)/(a*c*e*x*x^m + (a*c*d - d)*x), x) + (log(a*c*e
*x^m + (a*c - 1)*d)*log(x) - log(e)*log(x) - log(x)*log(x^m))/d + log(e)*lo
g((e*x^m + d)/e)/(d*m) + (log(x^m)*log(e*x^m/d + 1) + dilog(-e*x^m/d))/(d*m
) - (log(a*c*e*x^m + (a*c - 1)*d)*log((a*c*e*x^m + a*c*d - d)/d + 1) + dilo
g(-(a*c*e*x^m + a*c*d - d)/d))/(d*m)
```

## Giac [F]

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x\left(d+ex^m\right)}\,dx = \int \frac{\log\left(\frac{acex^m+acd-d}{ex^m}\right)}{(ex^m+d)x}\,dx$$

```
[In] integrate(log((-d+a*c*d+a*c*e*x^m)/e/(x^m))/x/(d+e*x^m),x, algorithm="giac"
)
```

```
[Out] integrate(log((a*c*e*x^m + a*c*d - d)/(e*x^m))/((e*x^m + d)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{x^{-m}(-d+acd+acex^m)}{e}\right)}{x\,(d+ex^m)}\,dx=\int \frac{\ln\left(\frac{a\,c\,d-d+a\,c\,e\,x^m}{e\,x^m}\right)}{x\,\left(d+e\,x^m\right)}\,dx$$

```
[In] int(log((a*c*d - d + a*c*e*x^m)/(e*x^m))/(x*(d + e*x^m)),x)
[Out] int(log((a*c*d - d + a*c*e*x^m)/(e*x^m))/(x*(d + e*x^m)), x)
```

# 3.337 $\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 24

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{2a}{a+bx}\right)}{2ab}$$

```
[Out] 1/2*polylog(2,1-2*a/(b*x+a))/a/b
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2449, 2352}

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{2a}{a+bx}\right)}{2ab}$$

```
[In] Int[Log[(2*a)/(a + b*x)]/(a^2 - b^2*x^2),x]
```

```
[Out] PolyLog[2, 1 - (2*a)/(a + b*x)]/(2*a*b)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\log(2ax)}{1-2ax}\,dx, x, \frac{1}{a+bx}\right)}{b}$$
$$= \frac{\text{Li}_2\left(1-\frac{2a}{a+bx}\right)}{2ab}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\text{PolyLog}\left(2, \frac{-a+bx}{a+bx}\right)}{2ab}$$

```
[In] Integrate[Log[(2*a)/(a + b*x)]/(a^2 - b^2*x^2),x]
[Out] PolyLog[2, (-a + b*x)/(a + b*x)]/(2*a*b)
```

## Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\text{dilog}\left(\frac{2a}{bx+a}\right)}{2ba}$ | 20 |
| default | $\frac{\text{dilog}\left(\frac{2a}{bx+a}\right)}{2ba}$ | 20 |
| risch | $\frac{\text{dilog}\left(\frac{2a}{bx+a}\right)}{2ba}$ | 20 |
| parts | $\frac{\ln\left(\frac{2a}{bx+a}\right)\ln(bx+a)}{2ab}-\frac{\ln\left(\frac{2a}{bx+a}\right)\ln(-bx+a)}{2ab}+\frac{b\left(\frac{\ln(bx+a)^2}{2a\,b^2}+\frac{-\text{dilog}\left(-\frac{-bx-a}{2a}\right)-\ln(-bx+a)\ln\left(-\frac{-bx-a}{2a}\right)}{a\,b^2}\right)}{2}$ | 120 |

```
[In] int(ln(2*a/(b*x+a))/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
[Out] 1/2/b/a*dilog(2*a/(b*x+a))
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\mathrm{Li}_2\left(-\frac{2\,a}{bx+a}+1\right)}{2\,ab}$$

```
[In] integrate(log(2*a/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="fricas")
[Out] 1/2*dilog(-2*a/(b*x + a) + 1)/(a*b)
```

## Sympy [F]

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx = -\int \frac{\log(2)}{-a^2+b^2x^2}\,dx - \int \frac{\log\left(\frac{a}{a+bx}\right)}{-a^2+b^2x^2}\,dx$$

```
[In] integrate(ln(2*a/(b*x+a))/(-b**2*x**2+a**2),x)
[Out] -Integral(log(2)/(-a**2 + b**2*x**2), x) - Integral(log(a/(a + b*x))/(-a**2
 + b**2*x**2), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(21) = 42.

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.00

$$\begin{aligned}
&\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx \\
&= \frac{1}{4}\,b\left(\frac{\log(bx+a)^2-2\,\log(bx+a)\log(bx-a)}{ab^2}+\frac{2\left(\log(bx+a)\log\left(-\frac{bx+a}{2\,a}+1\right)+\mathrm{Li}_2\left(\frac{bx+a}{2\,a}\right)\right)}{ab^2}\right) \\
&\quad+\frac{1}{2}\left(\frac{\log(bx+a)}{ab}-\frac{\log(bx-a)}{ab}\right)\log\left(\frac{2\,a}{bx+a}\right)
\end{aligned}$$

```
[In] integrate(log(2*a/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="maxima")
[Out] 1/4*b*((log(b*x + a)^2 - 2*log(b*x + a)*log(b*x - a))/(a*b^2) + 2*(log(b*x
+ a)*log(-1/2*(b*x + a)/a + 1) + dilog(1/2*(b*x + a)/a))/(a*b^2)) + 1/2*(lo
g(b*x + a)/(a*b) - log(b*x - a)/(a*b))*log(2*a/(b*x + a))
```

## Giac [F]

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx = \int -\frac{\log\left(\frac{2\,a}{bx+a}\right)}{b^2x^2-a^2}\,dx$$

```
[In] integrate(log(2*a/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="giac")
[Out] integrate(-log(2*a/(b*x + a))/(b^2*x^2 - a^2), x)
```

## Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\mathrm{Li}_2\left(\frac{2\,a}{a+b\,x}\right)}{2\,a\,b}$$

```
[In] int(log((2*a)/(a + b*x))/(a^2 - b^2*x^2),x)
[Out] dilog((2*a)/(a + b*x))/(2*a*b)
```

# 3.338 $\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 24

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \frac{\text{PolyLog}\left(2, 1-\frac{2a}{a+bx}\right)}{2ab}$$

```
[Out] 1/2*polylog(2,1-2*a/(b*x+a))/a/b
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2458, 2378, 2370, 2352}

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \frac{\text{PolyLog}\left(2, 1-\frac{2a}{a+bx}\right)}{2ab}$$

```
[In] Int[Log[(2*a)/(a + b*x)]/((a - b*x)*(a + b*x)),x]
[Out] PolyLog[2, 1 - (2*a)/(a + b*x)]/(2*a*b)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
```

```
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log\left(\frac{2a}{x}\right)}{(2a-x)x}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{\log(2ax)}{\left(2a-\frac{1}{x}\right)x}\,dx, x, \frac{1}{a+bx}\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{\log(2ax)}{-1+2ax}\,dx, x, \frac{1}{a+bx}\right)}{b}\\
&= \frac{\text{Li}_2\left(1-\frac{2a}{a+bx}\right)}{2ab}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$
\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \frac{\text{PolyLog}\left(2, \frac{-a+bx}{a+bx}\right)}{2ab}
$$

```
[In] Integrate[Log[(2*a)/(a + b*x)]/((a - b*x)*(a + b*x)),x]
[Out] PolyLog[2, (-a + b*x)/(a + b*x)]/(2*a*b)
```

## Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\operatorname{dilog}\left(\frac{2a}{bx+a}\right)}{2ba}$ | 20 |
| default | $\frac{\operatorname{dilog}\left(\frac{2a}{bx+a}\right)}{2ba}$ | 20 |
| risch | $\frac{\operatorname{dilog}\left(\frac{2a}{bx+a}\right)}{2ba}$ | 20 |
| parts | $\frac{\ln\left(\frac{2a}{bx+a}\right)\ln(bx+a)}{2ab}-\frac{\ln\left(\frac{2a}{bx+a}\right)\ln(-bx+a)}{2ab}+\frac{b\left(\frac{\ln(bx+a)^{2}}{2a\,b^{2}}+\frac{-\operatorname{dilog}\left(-\frac{-bx-a}{2a}\right)-\ln(-bx+a)\ln\left(-\frac{-bx-a}{2a}\right)}{a\,b^{2}}\right)}{2}$ | 120 |

```
[In] int(ln(2*a/(b*x+a))/(-b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2/b/a*dilog(2*a/(b*x+a))
```

## Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\frac{\operatorname{Li}_2\left(-\frac{2\,a}{bx+a}+1\right)}{2\,ab}$$

```
[In] integrate(log(2*a/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="fricas")
[Out] 1/2*dilog(-2*a/(b*x + a) + 1)/(a*b)
```

## Sympy [F]

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=-\int \frac{\log(2)}{-a^2+b^2x^2}\,dx-\int \frac{\log\left(\frac{a}{a+bx}\right)}{-a^2+b^2x^2}\,dx$$

```
[In] integrate(ln(2*a/(b*x+a))/(-b*x+a)/(b*x+a),x)
[Out] -Integral(log(2)/(-a**2 + b**2*x**2), x) - Integral(log(a/(a + b*x))/(-a**2
 + b**2*x**2), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(21) = 42.

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.00

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx$$
$$=\frac{1}{4}b\left(\frac{\log(bx+a)^2-2\log(bx+a)\log(bx-a)}{ab^2}+\frac{2\left(\log(bx+a)\log\left(-\frac{bx+a}{2a}+1\right)+\mathrm{Li}_2\left(\frac{bx+a}{2a}\right)\right)}{ab^2}\right)$$
$$+\frac{1}{2}\left(\frac{\log(bx+a)}{ab}-\frac{\log(bx-a)}{ab}\right)\log\left(\frac{2a}{bx+a}\right)$$

```
[In] integrate(log(2*a/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*b*((log(b*x + a)^2 - 2*log(b*x + a)*log(b*x - a))/(a*b^2) + 2*(log(b*x
+ a)*log(-1/2*(b*x + a)/a + 1) + dilog(1/2*(b*x + a)/a))/(a*b^2)) + 1/2*(lo
g(b*x + a)/(a*b) - log(b*x - a)/(a*b))*log(2*a/(b*x + a))
```

## Giac [F]

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\int -\frac{\log\left(\frac{2a}{bx+a}\right)}{(bx+a)(bx-a)}\,dx$$

```
[In] integrate(log(2*a/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(-log(2*a/(b*x + a))/((b*x + a)*(b*x - a)), x)
```

## Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{\log\left(\frac{2a}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\frac{\mathrm{Li}_2\left(\frac{2a}{a+bx}\right)}{2ab}$$

```
[In] int(log((2*a)/(a + b*x))/((a + b*x)*(a - b*x)),x)
```

```
[Out] dilog((2*a)/(a + b*x))/(2*a*b)
```

# 3.339 $\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx$



## Optimal result

Integrand size = 38, antiderivative size = 37

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{2ab}$$

```
[Out] 1/2*polylog(2,1+(-a*(1-c)-b*(1+c)*x)/(b*x+a))/a/b
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2497}

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{a(1-c)+b(c+1)x}{a+bx}\right)}{2ab}$$

```
[In] Int[Log[(a*(1 - c) + b*(1 + c)*x)/(a + b*x)]/(a^2 - b^2*x^2),x]
```

```
[Out] PolyLog[2, 1 - (a*(1 - c) + b*(1 + c)*x)/(a + b*x)]/(2*a*b)
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\text{integral} = \frac{\text{Li}_2\left(1-\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{2ab}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 252 vs. $2(37) = 74$.

Time = 0.14 (sec) , antiderivative size = 252, normalized size of antiderivative = 6.81

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\log^2\left(\frac{2ac}{(1+c)(a+bx)}\right) - 2\log(a-bx)\log\left(\frac{a+bx}{2a}\right) + 2\log(a-bx)\log\left(\frac{a-ac+b(1+c)x}{2a}\right) + 2\log\left(\frac{2ac}{(1+c)(a+bx)}\right)\log\left(-\right.}{}$$

```
[In] Integrate[Log[(a*(1 - c) + b*(1 + c)*x)/(a + b*x)]/(a^2 - b^2*x^2),x]
```

```
[Out] (Log[(2*a*c)/((1 + c)*(a + b*x))]^2 - 2*Log[a - b*x]*Log[(a + b*x)/(2*a)] +
 2*Log[a - b*x]*Log[(a - a*c + b*(1 + c)*x)/(2*a)] + 2*Log[(2*a*c)/((1 + c)
*(a + b*x))]*Log[-1/2*(a - a*c + b*(1 + c)*x)/(a*c)] - 2*Log[a - b*x]*Log[(
a - a*c + b*(1 + c)*x)/(a + b*x)] - 2*Log[(2*a*c)/((1 + c)*(a + b*x))]*Log[
(a - a*c + b*(1 + c)*x)/(a + b*x)] - 2*PolyLog[2, (a - b*x)/(2*a)] + 2*Poly
Log[2, ((1 + c)*(a - b*x))/(2*a)] - 2*PolyLog[2, ((1 + c)*(a + b*x))/(2*a*c
)])/(4*a*b)
```

## Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

| method | result |
|---|---|
| derivativedivides | $\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$ |
| default | $\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$ |
| risch | $\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$ |
| parts | $\frac{\ln\left(\frac{a(1-c)+b(1+c)x}{bx+a}\right)\ln(bx+a)}{2ab} - \frac{\ln\left(\frac{a(1-c)+b(1+c)x}{bx+a}\right)\ln(-bx+a)}{2ab} + \frac{c\left(\frac{(1+c)\left(\frac{\operatorname{dilog}\left(-\frac{(1+c)(-bx+a)-2a}{2a}\right)}{1+c}+\frac{\ln(-bx+a}{}\right.}{2ca}\right.}{}$ |

```
[In] int(ln((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b/a*dilog(1+c-2*c*a/(b*x+a))
```

## Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\mathrm{Li}_2\left(\frac{ac-(bc+b)x-a}{bx+a}+1\right)}{2\,ab}$$

```
[In] integrate(log((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="fri
cas")
[Out] 1/2*dilog((a*c - (b*c + b)*x - a)/(b*x + a) + 1)/(a*b)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b**2*x**2+a**2),x)
[Out] Timed out
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(33) = 66.

Time = 0.21 (sec) , antiderivative size = 246, normalized size of antiderivative = 6.65

$$\begin{aligned}\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx &= \frac{1}{2}\left(\frac{\log(bx+a)}{ab}-\frac{\log(bx-a)}{ab}\right)\log\left(\frac{b(c+1)x-a(c-1)}{bx+a}\right)\\ &\quad+\frac{\log(bx+a)^2-2\,\log(bx+a)\log(bx-a)}{4\,ab}\\ &\quad+\frac{\log(bx-a)\log\left(\frac{b(c+1)x-a(c+1)}{2\,a}+1\right)+\mathrm{Li}_2\left(-\frac{b(c+1)x-a(c+1)}{2\,a}\right)}{2\,ab}\\ &\quad+\frac{\log(bx+a)\log\left(-\frac{bx+a}{2\,a}+1\right)+\mathrm{Li}_2\left(\frac{bx+a}{2\,a}\right)}{2\,ab}\\ &\quad-\frac{\log(bx+a)\log\left(-\frac{b(c+1)x+a(c+1)}{2\,ac}+1\right)+\mathrm{Li}_2\left(\frac{b(c+1)x+a(c+1)}{2\,ac}\right)}{2\,ab}\end{aligned}$$

```
[In] integrate(log((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="max
ima")
```

```
[Out] 1/2*(log(b*x + a)/(a*b) - log(b*x - a)/(a*b))*log((b*(c + 1)*x - a*(c - 1))
/(b*x + a)) + 1/4*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x - a))/(a*b) + 1/
2*(log(b*x - a)*log(1/2*(b*(c + 1)*x - a*(c + 1))/a + 1) + dilog(-1/2*(b*(c
 + 1)*x - a*(c + 1))/a))/(a*b) + 1/2*(log(b*x + a)*log(-1/2*(b*x + a)/a + 1
) + dilog(1/2*(b*x + a)/a))/(a*b) - 1/2*(log(b*x + a)*log(-1/2*(b*(c + 1)*x
 + a*(c + 1))/(a*c) + 1) + dilog(1/2*(b*(c + 1)*x + a*(c + 1))/(a*c)))/(a*b
)
```

## Giac [F]

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx = \int -\frac{\log\left(\frac{b(c+1)x-a(c-1)}{bx+a}\right)}{b^2x^2-a^2}\,dx$$

```
[In] integrate(log((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="gia
c")
```

```
[Out] integrate(-log((b*(c + 1)*x - a*(c - 1))/(b*x + a))/(b^2*x^2 - a^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{a^2-b^2x^2}\,dx = \int \frac{\ln\left(-\frac{a\,(c-1)-b\,x\,(c+1)}{a+b\,x}\right)}{a^2-b^2\,x^2}\,dx$$

```
[In] int(log(-(a*(c - 1) - b*x*(c + 1))/(a + b*x))/(a^2 - b^2*x^2),x)
```

```
[Out] int(log(-(a*(c - 1) - b*x*(c + 1))/(a + b*x))/(a^2 - b^2*x^2), x)
```

# 3.340 $\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx$



## Optimal result

Integrand size = 39, antiderivative size = 27

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\frac{\text{PolyLog}\left(2,\frac{c(a-bx)}{a+bx}\right)}{2ab}$$

```
[Out] 1/2*polylog(2,c*(-b*x+a)/(b*x+a))/a/b
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2565, 2352}

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\frac{\text{PolyLog}\left(2,\frac{c(a-bx)}{a+bx}\right)}{2ab}$$

```
[In] Int[Log[(a*(1 - c) + b*(1 + c)*x)/(a + b*x)]/((a - b*x)*(a + b*x)),x]
```

```
[Out] PolyLog[2, (c*(a - b*x))/(a + b*x)]/(2*a*b)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2565

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
```

```
] :> Dist[(b*c - a*d)^(q + 1)*(i/d)^q, Subst[Int[(b*f - a*g - (d*f - c*g)*x
)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x
)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{\log(x)}{ab(1-c)+ab(1+c)-2abx}\,dx, x, \frac{a(1-c)+b(1+c)x}{a+bx}\right)\\ &= \frac{\text{Li}_2\left(\frac{c(a-bx)}{a+bx}\right)}{2ab}\end{aligned}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 252 vs. 2(27) = 54.

Time = 0.12 (sec) , antiderivative size = 252, normalized size of antiderivative = 9.33

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \frac{\log^2\left(\frac{2ac}{(1+c)(a+bx)}\right) - 2\log(a-bx)\log\left(\frac{a+bx}{2a}\right) + 2\log(a-bx)\log\left(\frac{a-ac+b(1+c)x}{2a}\right) + 2\log\left(\frac{2ac}{(1+c)(a+bx)}\right)\log\left(-\right.}{}$$

```
[In] Integrate[Log[(a*(1 - c) + b*(1 + c)*x)/(a + b*x)]/((a - b*x)*(a + b*x)),x]
```

```
[Out] (Log[(2*a*c)/((1 + c)*(a + b*x))]^2 - 2*Log[a - b*x]*Log[(a + b*x)/(2*a)] +
 2*Log[a - b*x]*Log[(a - a*c + b*(1 + c)*x)/(2*a)] + 2*Log[(2*a*c)/((1 + c)
*(a + b*x))]*Log[-1/2*(a - a*c + b*(1 + c)*x)/(a*c)] - 2*Log[a - b*x]*Log[(
a - a*c + b*(1 + c)*x)/(a + b*x)] - 2*Log[(2*a*c)/((1 + c)*(a + b*x))]*Log[
(a - a*c + b*(1 + c)*x)/(a + b*x)] - 2*PolyLog[2, (a - b*x)/(2*a)] + 2*Poly
Log[2, ((1 + c)*(a - b*x))/(2*a)] - 2*PolyLog[2, ((1 + c)*(a + b*x))/(2*a*c
)])/(4*a*b)
```

## Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| derivativedivides | $\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$ |
| default | $\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$ |
| risch | $\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$ |
| parts | $\frac{\ln\left(\frac{a(1-c)+b(1+c)x}{bx+a}\right)\ln(bx+a)}{2ab}-\frac{\ln\left(\frac{a(1-c)+b(1+c)x}{bx+a}\right)\ln(-bx+a)}{2ab}+\frac{c\left(\frac{(1+c)\left(\frac{\operatorname{dilog}\left(-\frac{(1+c)(-bx+a)-2a}{2a}\right)}{1+c}+\frac{\ln(-bx+a)\,\mathrm{l}}{}\right.}{2ca}\right.}{}$ |

```
[In] int(ln((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b*x+a)/(b*x+a),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/b/a*dilog(1+c-2*c*a/(b*x+a))
```

## Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\frac{\operatorname{Li}_2\left(\frac{ac-(bc+b)x-a}{bx+a}+1\right)}{2\,ab}$$

```
[In] integrate(log((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="f
ricas")
```

```
[Out] 1/2*dilog((a*c - (b*c + b)*x - a)/(b*x + a) + 1)/(a*b)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\text{Timed out}$$

```
[In] integrate(ln((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b*x+a)/(b*x+a),x)
```

```
[Out] Timed out
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(26) = 52$.

Time = 0.21 (sec) , antiderivative size = 246, normalized size of antiderivative = 9.11

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \frac{1}{2}\left(\frac{\log(bx+a)}{ab} - \frac{\log(bx-a)}{ab}\right)\log\left(\frac{b(c+1)x-a(c-1)}{bx+a}\right) + \frac{\log(bx+a)^2 - 2\log(bx+a)\log(bx-a)}{4\,ab} + \frac{\log(bx-a)\log\left(\frac{b(c+1)x-a(c+1)}{2\,a}+1\right) + \mathrm{Li}_2\left(-\frac{b(c+1)x-a(c+1)}{2\,a}\right)}{2\,ab} + \frac{\log(bx+a)\log\left(-\frac{bx+a}{2\,a}+1\right) + \mathrm{Li}_2\left(\frac{bx+a}{2\,a}\right)}{2\,ab} - \frac{\log(bx+a)\log\left(-\frac{b(c+1)x+a(c+1)}{2\,ac}+1\right) + \mathrm{Li}_2\left(\frac{b(c+1)x+a(c+1)}{2\,ac}\right)}{2\,ab}$$

```
[In] integrate(log((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="m
axima")
```

```
[Out] 1/2*(log(b*x + a)/(a*b) - log(b*x - a)/(a*b))*log((b*(c + 1)*x - a*(c - 1))
/(b*x + a)) + 1/4*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x - a))/(a*b) + 1/
2*(log(b*x - a)*log(1/2*(b*(c + 1)*x - a*(c + 1))/a + 1) + dilog(-1/2*(b*(c
 + 1)*x - a*(c + 1))/a))/(a*b) + 1/2*(log(b*x + a)*log(-1/2*(b*x + a)/a + 1
) + dilog(1/2*(b*x + a)/a))/(a*b) - 1/2*(log(b*x + a)*log(-1/2*(b*(c + 1)*x
 + a*(c + 1))/(a*c) + 1) + dilog(1/2*(b*(c + 1)*x + a*(c + 1))/(a*c)))/(a*b
)
```

## Giac [F]

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \int -\frac{\log\left(\frac{b(c+1)x-a(c-1)}{bx+a}\right)}{(bx+a)(bx-a)}\,dx$$

```
[In] integrate(log((a*(1-c)+b*(1+c)*x)/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="g
iac")
```

```
[Out] integrate(-log((b*(c + 1)*x - a*(c - 1))/(b*x + a))/((b*x + a)*(b*x - a)),
x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \int \frac{\ln\left(-\frac{a\,(c-1)-b\,x\,(c+1)}{a+b\,x}\right)}{(a+b\,x)\,\,(a-b\,x)}\,dx$$

```
[In] int(log(-(a*(c - 1) - b*x*(c + 1))/(a + b*x))/((a + b*x)*(a - b*x)),x)
[Out] int(log(-(a*(c - 1) - b*x*(c + 1))/(a + b*x))/((a + b*x)*(a - b*x)), x)
```

# 3.341 $\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx$



## Optimal result

Integrand size = 34, antiderivative size = 27

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\text{PolyLog}\left(2,\frac{c(a-bx)}{a+bx}\right)}{2ab}$$

```
[Out] 1/2*polylog(2,c*(-b*x+a)/(b*x+a))/a/b
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {2497}

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\text{PolyLog}\left(2,\frac{c(a-bx)}{a+bx}\right)}{2ab}$$

```
[In] Int[Log[1 - (c*(a - b*x))/(a + b*x)]/(a^2 - b^2*x^2),x]
[Out] PolyLog[2, (c*(a - b*x))/(a + b*x)]/(2*a*b)
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\text{integral} = \frac{\text{Li}_2\left(\frac{c(a-bx)}{a+bx}\right)}{2ab}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(27) = 54.

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 9.59

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx$$
$$= \frac{4\text{arctanh}\left(\frac{bx}{a}\right)\log\left(\frac{a}{b}+x\right)-\log^2\left(\frac{a}{b}+x\right)-4\text{arctanh}\left(\frac{bx}{a}\right)\log\left(\frac{a-ac}{b+bc}+x\right)+2\log\left(\frac{a}{b}+x\right)\log\left(\frac{a-bx}{2a}\right)-2\log\left(\right.}{}$$

```
[In] Integrate[Log[1 - (c*(a - b*x))/(a + b*x)]/(a^2 - b^2*x^2),x]
```

```
[Out] (4*ArcTanh[(b*x)/a]*Log[a/b + x] - Log[a/b + x]^2 - 4*ArcTanh[(b*x)/a]*Log[
(a - a*c)/(b + b*c) + x] + 2*Log[a/b + x]*Log[(a - b*x)/(2*a)] - 2*Log[(a -
 a*c)/(b + b*c) + x]*Log[((1 + c)*(a - b*x))/(2*a)] + 2*Log[(a - a*c)/(b +
b*c) + x]*Log[((1 + c)*(a + b*x))/(2*a*c)] + 4*ArcTanh[(b*x)/a]*Log[(a - a*
c + b*(1 + c)*x)/(a + b*x)] + 2*PolyLog[2, (a + b*x)/(2*a)] - 2*PolyLog[2,
(a - a*c + b*(1 + c)*x)/(2*a)] + 2*PolyLog[2, -1/2*(a - a*c + b*(1 + c)*x)/
(a*c)])/(4*a*b)
```

## Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| derivativedivides | $\frac{\text{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$ |
| default | $\frac{\text{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$ |
| risch | $\frac{\text{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$ |
| parts | $\frac{\ln\left(1-\frac{c(-bx+a)}{bx+a}\right)\ln(bx+a)}{2ab}-\frac{\ln\left(1-\frac{c(-bx+a)}{bx+a}\right)\ln(-bx+a)}{2ab}+\frac{c\left(\frac{(1+c)\left(\frac{\text{dilog}\left(-\frac{(1+c)(-bx+a)-2a}{2a}\right)}{1+c}+\frac{\ln(-bx+a)\ln\left(-\frac{(1}{}\right.}{1+}\right.}{2ca}\right.}{}$ |

```
[In] int(ln(1-c*(-b*x+a)/(b*x+a))/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b/a*dilog(1+c-2*c*a/(b*x+a))
```

## Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx = \frac{\mathrm{Li}_2\left(\frac{ac-(bc+b)x-a}{bx+a}+1\right)}{2\,ab}$$

```
[In] integrate(log(1-c*(-b*x+a)/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="fricas")
[Out] 1/2*dilog((a*c - (b*c + b)*x - a)/(b*x + a) + 1)/(a*b)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(1-c*(-b*x+a)/(b*x+a))/(-b**2*x**2+a**2),x)
[Out] Timed out
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(26) = 52.

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 9.00

$$\begin{aligned}\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx &= \frac{1}{2}\left(\frac{\log(bx+a)}{ab}-\frac{\log(bx-a)}{ab}\right)\log\left(\frac{(bx-a)c}{bx+a}+1\right)\\ &+ \frac{\log(bx+a)^2-2\,\log(bx+a)\log(bx-a)}{4\,ab}\\ &+ \frac{\log(bx-a)\log\left(\frac{b(c+1)x-a(c+1)}{2\,a}+1\right)+\mathrm{Li}_2\left(-\frac{b(c+1)x-a(c+1)}{2\,a}\right)}{2\,ab}\\ &+ \frac{\log(bx+a)\log\left(-\frac{bx+a}{2\,a}+1\right)+\mathrm{Li}_2\left(\frac{bx+a}{2\,a}\right)}{2\,ab}\\ &- \frac{\log(bx+a)\log\left(-\frac{b(c+1)x+a(c+1)}{2\,ac}+1\right)+\mathrm{Li}_2\left(\frac{b(c+1)x+a(c+1)}{2\,ac}\right)}{2\,ab}\end{aligned}$$

```
[In] integrate(log(1-c*(-b*x+a)/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="maxima")
[Out] 1/2*(log(b*x + a)/(a*b) - log(b*x - a)/(a*b))*log((b*x - a)*c/(b*x + a) + 1
) + 1/4*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x - a))/(a*b) + 1/2*(log(b*x
```

```
 - a)*log(1/2*(b*(c + 1)*x - a*(c + 1))/a + 1) + dilog(-1/2*(b*(c + 1)*x -
a*(c + 1))/a))/(a*b) + 1/2*(log(b*x + a)*log(-1/2*(b*x + a)/a + 1) + dilog(
1/2*(b*x + a)/a))/(a*b) - 1/2*(log(b*x + a)*log(-1/2*(b*(c + 1)*x + a*(c +
1))/(a*c) + 1) + dilog(1/2*(b*(c + 1)*x + a*(c + 1))/(a*c)))/(a*b)
```

## Giac [F]

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx = \int -\frac{\log\left(\frac{(bx-a)c}{bx+a}+1\right)}{b^2x^2-a^2}\,dx$$

```
[In] integrate(log(1-c*(-b*x+a)/(b*x+a))/(-b^2*x^2+a^2),x, algorithm="giac")
[Out] integrate(-log((b*x - a)*c/(b*x + a) + 1)/(b^2*x^2 - a^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{a^2-b^2x^2}\,dx = \int \frac{\ln\left(1-\frac{c\,(a-b\,x)}{a+b\,x}\right)}{a^2-b^2\,x^2}\,dx$$

```
[In] int(log(1 - (c*(a - b*x))/(a + b*x))/(a^2 - b^2*x^2),x)
[Out] int(log(1 - (c*(a - b*x))/(a + b*x))/(a^2 - b^2*x^2), x)
```

# 3.342 $\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx$



## Optimal result

Integrand size = 35, antiderivative size = 27

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\frac{\text{PolyLog}\left(2,\frac{c(a-bx)}{a+bx}\right)}{2ab}$$

```
[Out] 1/2*polylog(2,c*(-b*x+a)/(b*x+a))/a/b
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2597, 2565, 2352}

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\frac{\text{PolyLog}\left(2,\frac{c(a-bx)}{a+bx}\right)}{2ab}$$

```
[In] Int[Log[1 - (c*(a - b*x))/(a + b*x)]/((a - b*x)*(a + b*x)),x]
[Out] PolyLog[2, (c*(a - b*x))/(a + b*x)]/(2*a*b)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2565

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
```

```
] :> Dist[(b*c - a*d)^(q + 1)*(i/d)^q, Subst[Int[(b*f - a*g - (d*f - c*g)*x
)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x
)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

Rule 2597

```
Int[Log[(e_.)*((f_.)*((g_) + (v_.)/(w_)))^(r_.)]^(s_.)*(u_.), x_Symbol] :>
Int[u*Log[e*(f*(ExpandToSum[v + g*w, x]/ExpandToSum[w, x]))^r]^s, x] /; Fre
eQ[{e, f, g, r, s}, x] && LinearQ[w, x] && (FreeQ[v, x] || LinearQ[v, x]) &
& AlgebraicFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log\left(\frac{a(1-c)+b(1+c)x}{a+bx}\right)}{(a-bx)(a+bx)}\,dx$$
$$= \text{Subst}\left(\int \frac{\log(x)}{ab(1-c)+ab(1+c)-2abx}\,dx, x, \frac{a(1-c)+b(1+c)x}{a+bx}\right)$$
$$= \frac{\text{Li}_2\left(\frac{c(a-bx)}{a+bx}\right)}{2ab}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 259 vs. $2(27) = 54$.

Time = 0.13 (sec) , antiderivative size = 259, normalized size of antiderivative = 9.59

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx$$
$$= \frac{4\text{arctanh}\left(\frac{bx}{a}\right)\log\left(\frac{a}{b}+x\right)-\log^2\left(\frac{a}{b}+x\right)-4\text{arctanh}\left(\frac{bx}{a}\right)\log\left(\frac{a-ac}{b+bc}+x\right)+2\log\left(\frac{a}{b}+x\right)\log\left(\frac{a-bx}{2a}\right)-2\log\left(\right.}{}$$

```
[In] Integrate[Log[1 - (c*(a - b*x))/(a + b*x)]/((a - b*x)*(a + b*x)),x]
```

```
[Out] (4*ArcTanh[(b*x)/a]*Log[a/b + x] - Log[a/b + x]^2 - 4*ArcTanh[(b*x)/a]*Log[
(a - a*c)/(b + b*c) + x] + 2*Log[a/b + x]*Log[(a - b*x)/(2*a)] - 2*Log[(a -
 a*c)/(b + b*c) + x]*Log[((1 + c)*(a - b*x))/(2*a)] + 2*Log[(a - a*c)/(b +
b*c) + x]*Log[((1 + c)*(a + b*x))/(2*a*c)] + 4*ArcTanh[(b*x)/a]*Log[(a - a*
c + b*(1 + c)*x)/(a + b*x)] + 2*PolyLog[2, (a + b*x)/(2*a)] - 2*PolyLog[2,
(a - a*c + b*(1 + c)*x)/(2*a)] + 2*PolyLog[2, -1/2*(a - a*c + b*(1 + c)*x)/
(a*c)])/(4*a*b)
```

## Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| derivativedivides | $\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$ |
| default | $\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$ |
| risch | $\frac{\operatorname{dilog}\left(1+c-\frac{2ca}{bx+a}\right)}{2ba}$ |
| parts | $\frac{\ln\left(1-\frac{c(-bx+a)}{bx+a}\right)\ln(bx+a)}{2ab}-\frac{\ln\left(1-\frac{c(-bx+a)}{bx+a}\right)\ln(-bx+a)}{2ab}+\frac{c\left(\frac{(1+c)\left(\frac{\operatorname{dilog}\left(-\frac{(1+c)(-bx+a)-2a}{2a}\right)}{1+c}+\frac{\ln(-bx+a)\ln\left(-\right.}{}\right.}{2ca}\right.}{}$ |

[In] int(ln(1-c*(-b*x+a)/(b*x+a))/(-b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/2/b/a*dilog(1+c-2*c*a/(b*x+a))

## Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\frac{\operatorname{Li}_2\left(\frac{ac-(bc+b)x-a}{bx+a}+1\right)}{2\,ab}$$

[In] integrate(log(1-c*(-b*x+a)/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="fricas")

[Out] 1/2*dilog((a*c - (b*c + b)*x - a)/(b*x + a) + 1)/(a*b)

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx=\text{Timed out}$$

[In] integrate(ln(1-c*(-b*x+a)/(b*x+a))/(-b*x+a)/(b*x+a),x)

[Out] Timed out

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(26) = 52.

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 9.00

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \frac{1}{2}\left(\frac{\log(bx+a)}{ab}-\frac{\log(bx-a)}{ab}\right)\log\left(\frac{(bx-a)c}{bx+a}+1\right) + \frac{\log(bx+a)^2-2\log(bx+a)\log(bx-a)}{4ab} + \frac{\log(bx-a)\log\left(\frac{b(c+1)x-a(c+1)}{2a}+1\right)+\mathrm{Li}_2\left(-\frac{b(c+1)x-a(c+1)}{2a}\right)}{2ab} + \frac{\log(bx+a)\log\left(-\frac{bx+a}{2a}+1\right)+\mathrm{Li}_2\left(\frac{bx+a}{2a}\right)}{2ab} - \frac{\log(bx+a)\log\left(-\frac{b(c+1)x+a(c+1)}{2ac}+1\right)+\mathrm{Li}_2\left(\frac{b(c+1)x+a(c+1)}{2ac}\right)}{2ab}$$

```
[In] integrate(log(1-c*(-b*x+a)/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(log(b*x + a)/(a*b) - log(b*x - a)/(a*b))*log((b*x - a)*c/(b*x + a) + 1
) + 1/4*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x - a))/(a*b) + 1/2*(log(b*x
 - a)*log(1/2*(b*(c + 1)*x - a*(c + 1))/a + 1) + dilog(-1/2*(b*(c + 1)*x -
a*(c + 1))/a))/(a*b) + 1/2*(log(b*x + a)*log(-1/2*(b*x + a)/a + 1) + dilog(
1/2*(b*x + a)/a))/(a*b) - 1/2*(log(b*x + a)*log(-1/2*(b*(c + 1)*x + a*(c +
1))/(a*c) + 1) + dilog(1/2*(b*(c + 1)*x + a*(c + 1))/(a*c)))/(a*b)
```

## Giac [F]

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \int -\frac{\log\left(\frac{(bx-a)c}{bx+a}+1\right)}{(bx+a)(bx-a)}\,dx$$

```
[In] integrate(log(1-c*(-b*x+a)/(b*x+a))/(-b*x+a)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(-log((b*x - a)*c/(b*x + a) + 1)/((b*x + a)*(b*x - a)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{c(a-bx)}{a+bx}\right)}{(a-bx)(a+bx)}\,dx = \int \frac{\ln\left(1-\frac{c\,(a-b\,x)}{a+b\,x}\right)}{(a+b\,x)\,(a-b\,x)}\,dx$$

```
[In] int(log(1 - (c*(a - b*x))/(a + b*x))/((a + b*x)*(a - b*x)),x)
[Out] int(log(1 - (c*(a - b*x))/(a + b*x))/((a + b*x)*(a - b*x)), x)
```

# 3.343 $\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 238

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2} dx = \frac{\log\left(-\frac{bx}{a}\right)\log^3(c(a+bx)^n)}{d} - \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right)}{d} + \frac{6n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,-\frac{e(a+bx)}{bd-ae}\right)}{d} - \frac{6n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,1+\frac{bx}{a}\right)}{d} - \frac{6n^3\operatorname{PolyLog}\left(4,-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{6n^3\operatorname{PolyLog}\left(4,1+\frac{bx}{a}\right)}{d}$$

```
[Out] ln(-b*x/a)*ln(c*(b*x+a)^n)^3/d-ln(c*(b*x+a)^n)^3*ln(b*(e*x+d)/(-a*e+b*d))/d
-3*n*ln(c*(b*x+a)^n)^2*polylog(2,-e*(b*x+a)/(-a*e+b*d))/d+3*n*ln(c*(b*x+a)^
n)^2*polylog(2,1+b*x/a)/d+6*n^2*ln(c*(b*x+a)^n)*polylog(3,-e*(b*x+a)/(-a*e+
b*d))/d-6*n^2*ln(c*(b*x+a)^n)*polylog(3,1+b*x/a)/d-6*n^3*polylog(4,-e*(b*x+
a)/(-a*e+b*d))/d+6*n^3*polylog(4,1+b*x/a)/d
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1607, 2463, 2443, 2481, 2421, 2430, 6724}

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx = \frac{6n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,-\frac{e(a+bx)}{bd-ae}\right)}{d} - \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d} - \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{6n^2\operatorname{PolyLog}\left(3,\frac{bx}{a}+1\right)\log(c(a+bx)^n)}{d} + \frac{3n\operatorname{PolyLog}\left(2,\frac{bx}{a}+1\right)\log^2(c(a+bx)^n)}{d} + \frac{\log\left(-\frac{bx}{a}\right)\log^3(c(a+bx)^n)}{d} - \frac{6n^3\operatorname{PolyLog}\left(4,-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{6n^3\operatorname{PolyLog}\left(4,\frac{bx}{a}+1\right)}{d}$$

```
[In] Int[Log[c*(a + b*x)^n]^3/(d*x + e*x^2),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[c*(a + b*x)^n]^3)/d - (Log[c*(a + b*x)^n]^3*Log[(b*(d
 + e*x))/(b*d - a*e)])/d - (3*n*Log[c*(a + b*x)^n]^2*PolyLog[2, -((e*(a + b*
x))/(b*d - a*e))])/d + (3*n*Log[c*(a + b*x)^n]^2*PolyLog[2, 1 + (b*x)/a])/d
 + (6*n^2*Log[c*(a + b*x)^n]*PolyLog[3, -((e*(a + b*x))/(b*d - a*e))])/d -
(6*n^2*Log[c*(a + b*x)^n]*PolyLog[3, 1 + (b*x)/a])/d - (6*n^3*PolyLog[4, -(
(e*(a + b*x))/(b*d - a*e))])/d + (6*n^3*PolyLog[4, 1 + (b*x)/a])/d
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\log^3\left(c(a+bx)^n\right)}{x(d+ex)}\,dx\\
&= \int \left(\frac{\log^3\left(c(a+bx)^n\right)}{dx} - \frac{e\log^3\left(c(a+bx)^n\right)}{d(d+ex)}\right)\,dx\\
&= \frac{\int \frac{\log^3(c(a+bx)^n)}{x}\,dx}{d} - \frac{e\int \frac{\log^3(c(a+bx)^n)}{d+ex}\,dx}{d}
\end{aligned}$$

$$= \frac{\log\left(-\frac{bx}{a}\right)\log^3\left(c(a+bx)^n\right)}{d} - \frac{\log^3\left(c(a+bx)^n\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{(3bn)\int \frac{\log\left(-\frac{bx}{a}\right)\log^2(c(a+bx)^n)}{a+bx}\,dx}{d} + \frac{(3bn)\int \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{d}$$

$$= \frac{\log\left(-\frac{bx}{a}\right)\log^3\left(c(a+bx)^n\right)}{d} - \frac{\log^3\left(c(a+bx)^n\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{(3n)\text{Subst}\left(\int \frac{\log^2(cx^n)\log\left(-\frac{b\left(-\frac{a}{b}+\frac{x}{b}\right)}{a}\right)}{x}\,dx, x, a+bx\right)}{d} + \frac{(3n)\text{Subst}\left(\int \frac{\log^2(cx^n)\log\left(\frac{b\left(\frac{bd-ae}{b}+\frac{ex}{b}\right)}{bd-ae}\right)}{x}\,dx, x, a+bx\right)}{d}$$

$$= \frac{\log\left(-\frac{bx}{a}\right)\log^3\left(c(a+bx)^n\right)}{d} - \frac{\log^3\left(c(a+bx)^n\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{3n\log^2\left(c(a+bx)^n\right)\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{3n\log^2\left(c(a+bx)^n\right)\text{Li}_2\left(1+\frac{bx}{a}\right)}{d} - \frac{(6n^2)\,\text{Subst}\left(\int \frac{\log(cx^n)\text{Li}_2\left(\frac{x}{a}\right)}{x}\,dx, x, a+bx\right)}{d} + \frac{(6n^2)\,\text{Subst}\left(\int \frac{\log(cx^n)\text{Li}_2\left(-\frac{ex}{bd-ae}\right)}{x}\,dx, x, a+bx\right)}{d}$$

$$= \frac{\log\left(-\frac{bx}{a}\right)\log^3\left(c(a+bx)^n\right)}{d} - \frac{\log^3\left(c(a+bx)^n\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{3n\log^2\left(c(a+bx)^n\right)\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{3n\log^2\left(c(a+bx)^n\right)\text{Li}_2\left(1+\frac{bx}{a}\right)}{d} + \frac{6n^2\log\left(c(a+bx)^n\right)\text{Li}_3\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} - \frac{6n^2\log\left(c(a+bx)^n\right)\text{Li}_3\left(1+\frac{bx}{a}\right)}{d} + \frac{(6n^3)\,\text{Subst}\left(\int \frac{\text{Li}_3\left(\frac{x}{a}\right)}{x}\,dx, x, a+bx\right)}{d} - \frac{(6n^3)\,\text{Subst}\left(\int \frac{\text{Li}_3\left(-\frac{ex}{bd-ae}\right)}{x}\,dx, x, a+bx\right)}{d}$$

$$= \frac{\log\left(-\frac{bx}{a}\right)\log^3(c(a+bx)^n)}{d} - \frac{\log^3(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{3n\log^2(c(a+bx)^n)\operatorname{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{3n\log^2(c(a+bx)^n)\operatorname{Li}_2\left(1+\frac{bx}{a}\right)}{d} + \frac{6n^2\log(c(a+bx)^n)\operatorname{Li}_3\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} - \frac{6n^2\log(c(a+bx)^n)\operatorname{Li}_3\left(1+\frac{bx}{a}\right)}{d} - \frac{6n^3\operatorname{Li}_4\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{6n^3\operatorname{Li}_4\left(1+\frac{bx}{a}\right)}{d}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 494 vs. $2(238) = 476$.

Time = 0.14 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.08

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx$$

$$= \frac{-\log(x)\left(n\log(a+bx)-\log(c(a+bx)^n)\right)^3 + \left(n\log(a+bx)-\log(c(a+bx)^n)\right)^3\log(d+ex) + 3n(-n\log$$

```
[In] Integrate[Log[c*(a + b*x)^n]^3/(d*x + e*x^2),x]
```

```
[Out] (-(Log[x]*(n*Log[a + b*x] - Log[c*(a + b*x)^n])^3) + (n*Log[a + b*x] - Log[
c*(a + b*x)^n])^3*Log[d + e*x] + 3*n*(-(n*Log[a + b*x]) + Log[c*(a + b*x)^n
])^2*(Log[x]*(Log[a + b*x] - Log[1 + (b*x)/a]) - Log[a + b*x]*Log[(b*(d + e
*x))/(b*d - a*e)] - PolyLog[2, -((b*x)/a)] - PolyLog[2, (e*(a + b*x))/(-(b*
d) + a*e)]) - 3*n^2*(n*Log[a + b*x] - Log[c*(a + b*x)^n])*(Log[-((b*x)/a)]*
Log[a + b*x]^2 - Log[a + b*x]^2*Log[(b*(d + e*x))/(b*d - a*e)] - 2*Log[a +
b*x]*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)] + 2*Log[a + b*x]*PolyLog[2, 1
 + (b*x)/a] + 2*PolyLog[3, (e*(a + b*x))/(-(b*d) + a*e)] - 2*PolyLog[3, 1 +
 (b*x)/a]) + n^3*(Log[-((b*x)/a)]*Log[a + b*x]^3 - Log[a + b*x]^3*Log[(b*(d
 + e*x))/(b*d - a*e)] - 3*Log[a + b*x]^2*PolyLog[2, (e*(a + b*x))/(-(b*d) +
 a*e)] + 3*Log[a + b*x]^2*PolyLog[2, 1 + (b*x)/a] + 6*Log[a + b*x]*PolyLog[
3, (e*(a + b*x))/(-(b*d) + a*e)] - 6*Log[a + b*x]*PolyLog[3, 1 + (b*x)/a] -
 6*PolyLog[4, (e*(a + b*x))/(-(b*d) + a*e)] + 6*PolyLog[4, 1 + (b*x)/a]))/d
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.02 (sec) , antiderivative size = 1756, normalized size of antiderivative = 7.38

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1756 |

```
[In] int(ln(c*(b*x+a)^n)^3/(e*x^2+d*x),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((b*x+a)^n)-n*ln(b*x+a))^3/d*ln(b*x)-(ln((b*x+a)^n)-n*ln(b*x+a))^3/d*ln(
e*(b*x+a)-a*e+b*d)+n^3/d*ln(b*x+a)^3*ln(1-(b*x+a)/a)+3*n^3/d*ln(b*x+a)^2*po
lylog(2,(b*x+a)/a)-6*n^3/d*ln(b*x+a)*polylog(3,(b*x+a)/a)+6*n^3/d*polylog(4
,(b*x+a)/a)-n^3/d*ln(b*x+a)^3*ln(1+e*(b*x+a)/(-a*e+b*d))-3*n^3/d*ln(b*x+a)^
2*polylog(2,-e*(b*x+a)/(-a*e+b*d))+6*n^3/d*ln(b*x+a)*polylog(3,-e*(b*x+a)/(
-a*e+b*d))-6*n^3*polylog(4,-e*(b*x+a)/(-a*e+b*d))/d+3*b*n*(ln((b*x+a)^n)-n*
ln(b*x+a))^2*(1/b/d*(dilog(-x/a*b)+ln(b*x+a)*ln(-x/a*b))-e/b/d*(dilog((e*(b
*x+a)-a*e+b*d)/(-a*e+b*d))/e+ln(b*x+a)*ln((e*(b*x+a)-a*e+b*d)/(-a*e+b*d))/e
))+3*b*n^2*(ln((b*x+a)^n)-n*ln(b*x+a))*(1/b/d*(ln(b*x+a)^2*ln(1-(b*x+a)/a)+
2*ln(b*x+a)*polylog(2,(b*x+a)/a)-2*polylog(3,(b*x+a)/a))-1/b/d*(ln(b*x+a)^2
*ln(1+e*(b*x+a)/(-a*e+b*d))+2*ln(b*x+a)*polylog(2,-e*(b*x+a)/(-a*e+b*d))-2*
polylog(3,-e*(b*x+a)/(-a*e+b*d))))+1/8*(-I*Pi*csgn(I*c*(b*x+a)^n)^3+I*Pi*cs
gn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*c)-
I*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^n)*csgn(I*c)+2*ln(c))^3*(-1/d*ln(e*
x+d)+1/d*ln(x))+(-3/2*I*Pi*csgn(I*c*(b*x+a)^n)^3+3/2*I*Pi*csgn(I*c*(b*x+a)^
n)^2*csgn(I*(b*x+a)^n)+3/2*I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*c)-3/2*I*Pi*cs
gn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^n)*csgn(I*c)+3*ln(c))*((ln((b*x+a)^n)-n*ln
(b*x+a))^2/d*ln(b*x)-(ln((b*x+a)^n)-n*ln(b*x+a))^2/d*ln(e*(b*x+a)-a*e+b*d)+
b*n^2*(1/b/d*(ln(b*x+a)^2*ln(1-(b*x+a)/a)+2*ln(b*x+a)*polylog(2,(b*x+a)/a)-
2*polylog(3,(b*x+a)/a))-1/b/d*(ln(b*x+a)^2*ln(1+e*(b*x+a)/(-a*e+b*d))+2*ln(
b*x+a)*polylog(2,-e*(b*x+a)/(-a*e+b*d))-2*polylog(3,-e*(b*x+a)/(-a*e+b*d)))
)+2*b*n*(ln((b*x+a)^n)-n*ln(b*x+a))*(1/b/d*(dilog(-x/a*b)+ln(b*x+a)*ln(-x/a
*b))-e/b/d*(dilog((e*(b*x+a)-a*e+b*d)/(-a*e+b*d))/e+ln(b*x+a)*ln((e*(b*x+a)
-a*e+b*d)/(-a*e+b*d))/e)))+(-3/4*Pi^2*csgn(I*c*(b*x+a)^n)^6+3/2*Pi^2*csgn(I
*c*(b*x+a)^n)^5*csgn(I*(b*x+a)^n)+3/2*Pi^2*csgn(I*c*(b*x+a)^n)^5*csgn(I*c)-
3/4*Pi^2*csgn(I*c*(b*x+a)^n)^4*csgn(I*(b*x+a)^n)^2-3*Pi^2*csgn(I*c*(b*x+a)^
n)^4*csgn(I*(b*x+a)^n)*csgn(I*c)-3/4*Pi^2*csgn(I*c*(b*x+a)^n)^4*csgn(I*c)^2
+3/2*Pi^2*csgn(I*c*(b*x+a)^n)^3*csgn(I*(b*x+a)^n)^2*csgn(I*c)+3/2*Pi^2*csgn
(I*c*(b*x+a)^n)^3*csgn(I*(b*x+a)^n)*csgn(I*c)^2-3/4*Pi^2*csgn(I*c*(b*x+a)^n
)^2*csgn(I*(b*x+a)^n)^2*csgn(I*c)^2-3*I*ln(c)*Pi*csgn(I*c*(b*x+a)^n)^3+3*I*
ln(c)*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+3*I*ln(c)*Pi*csgn(I*c*(b*x
+a)^n)^2*csgn(I*c)-3*I*ln(c)*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^n)*csgn(
I*c)+3*ln(c)^2)*(-1/d*ln(e*x+d)*ln((b*x+a)^n)+ln((b*x+a)^n)/d*ln(x)-b*n*(1/
d*dilog((b*x+a)/a)/b+1/d*ln(x)*ln((b*x+a)/a)/b-1/d*dilog(((e*x+d)*b+a*e-b*d
)/(a*e-b*d))/b-1/d*ln(e*x+d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b))
```

## Fricas [F]

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx=\int \frac{\log\left((bx+a)^n c\right)^3}{ex^2+dx}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)^3/(e*x^2+d*x),x, algorithm="fricas")
[Out] integral(log((b*x + a)^n*c)^3/(e*x^2 + d*x), x)
```

## Sympy [F]

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx=\int \frac{\log\left(c(a+bx)^n\right)^3}{x\,(d+ex)}\,dx$$

```
[In] integrate(ln(c*(b*x+a)**n)**3/(e*x**2+d*x),x)
[Out] Integral(log(c*(a + b*x)**n)**3/(x*(d + e*x)), x)
```

## Maxima [F]

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx=\int \frac{\log\left((bx+a)^n c\right)^3}{ex^2+dx}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)^3/(e*x^2+d*x),x, algorithm="maxima")
[Out] integrate(log((b*x + a)^n*c)^3/(e*x^2 + d*x), x)
```

## Giac [F]

$$\int \frac{\log^3(c(a+bx)^n)}{dx+ex^2}\,dx=\int \frac{\log\left((bx+a)^n c\right)^3}{ex^2+dx}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)^3/(e*x^2+d*x),x, algorithm="giac")
[Out] integrate(log((b*x + a)^n*c)^3/(e*x^2 + d*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3 (c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\ln \left(c\,{\left(a+b\,x\right)}^n\right)^3}{e\,x^2+d\,x}\,dx$$

```
[In] int(log(c*(a + b*x)^n)^3/(d*x + e*x^2),x)
[Out] int(log(c*(a + b*x)^n)^3/(d*x + e*x^2), x)
```

# 3.344 $\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2} dx = \frac{\log\left(-\frac{bx}{a}\right)\log^2(c(a+bx)^n)}{d} - \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{2n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{2n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right)}{d} + \frac{2n^2\operatorname{PolyLog}\left(3,-\frac{e(a+bx)}{bd-ae}\right)}{d} - \frac{2n^2\operatorname{PolyLog}\left(3,1+\frac{bx}{a}\right)}{d}$$

```
[Out] ln(-b*x/a)*ln(c*(b*x+a)^n)^2/d-ln(c*(b*x+a)^n)^2*ln(b*(e*x+d)/(-a*e+b*d))/d
-2*n*ln(c*(b*x+a)^n)*polylog(2,-e*(b*x+a)/(-a*e+b*d))/d+2*n*ln(c*(b*x+a)^n)
*polylog(2,1+b*x/a)/d+2*n^2*polylog(3,-e*(b*x+a)/(-a*e+b*d))/d-2*n^2*polylo
g(3,1+b*x/a)/d
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{1607, 2463, 2443, 2481, 2421, 6724\}$

$$\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2}\,dx = -\frac{2n\log(c(a+bx)^n)\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d} - \frac{\log^2(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} + \frac{2n\,\text{PolyLog}\left(2,\frac{bx}{a}+1\right)\log(c(a+bx)^n)}{d} + \frac{\log\left(-\frac{bx}{a}\right)\log^2(c(a+bx)^n)}{d} + \frac{2n^2\,\text{PolyLog}\left(3,-\frac{e(a+bx)}{bd-ae}\right)}{d} - \frac{2n^2\,\text{PolyLog}\left(3,\frac{bx}{a}+1\right)}{d}$$

```
[In] Int[Log[c*(a + b*x)^n]^2/(d*x + e*x^2),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[c*(a + b*x)^n]^2)/d - (Log[c*(a + b*x)^n]^2*Log[(b*(d
+ e*x))/(b*d - a*e)])/d - (2*n*Log[c*(a + b*x)^n]*PolyLog[2, -((e*(a + b*x)
)/(b*d - a*e))])/d + (2*n*Log[c*(a + b*x)^n]*PolyLog[2, 1 + (b*x)/a])/d + (
2*n^2*PolyLog[3, -((e*(a + b*x))/(b*d - a*e))])/d - (2*n^2*PolyLog[3, 1 + (
b*x)/a])/d
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \frac{\log^2\left(c(a+bx)^n\right)}{x(d+ex)}\,dx$$

$$= \int \left(\frac{\log^2\left(c(a+bx)^n\right)}{dx} - \frac{e\log^2\left(c(a+bx)^n\right)}{d(d+ex)}\right) dx$$

$$= \frac{\int \frac{\log^2(c(a+bx)^n)}{x}\,dx}{d} - \frac{e\int \frac{\log^2(c(a+bx)^n)}{d+ex}\,dx}{d}$$

$$= \frac{\log\left(-\frac{bx}{a}\right)\log^2\left(c(a+bx)^n\right)}{d} - \frac{\log^2\left(c(a+bx)^n\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{(2bn)\int \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^n)}{a+bx}\,dx}{d} + \frac{(2bn)\int \frac{\log(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{d}$$

$$= \frac{\log\left(-\frac{bx}{a}\right)\log^2\left(c(a+bx)^n\right)}{d} - \frac{\log^2\left(c(a+bx)^n\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{(2n)\text{Subst}\left(\int \frac{\log(cx^n)\log\left(-\frac{b\left(-\frac{a}{b}+\frac{x}{b}\right)}{a}\right)}{x}\,dx, x, a+bx\right)}{d} + \frac{(2n)\text{Subst}\left(\int \frac{\log(cx^n)\log\left(\frac{b\left(\frac{bd-ae}{b}+\frac{ex}{b}\right)}{bd-ae}\right)}{x}\,dx, x, a+bx\right)}{d}$$

$$= \frac{\log\left(-\frac{bx}{a}\right)\log^2\left(c(a+bx)^n\right)}{d} - \frac{\log^2\left(c(a+bx)^n\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{2n\log\left(c(a+bx)^n\right)\operatorname{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{2n\log\left(c(a+bx)^n\right)\operatorname{Li}_2\left(1+\frac{bx}{a}\right)}{d} - \frac{(2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{x}{a}\right)}{x}\,dx, x, a+bx\right)}{d} + \frac{(2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{ex}{bd-ae}\right)}{x}\,dx, x, a+bx\right)}{d}$$

$$= \frac{\log\left(-\frac{bx}{a}\right)\log^2\left(c(a+bx)^n\right)}{d} - \frac{\log^2\left(c(a+bx)^n\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{2n\log\left(c(a+bx)^n\right)\operatorname{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{2n\log\left(c(a+bx)^n\right)\operatorname{Li}_2\left(1+\frac{bx}{a}\right)}{d} + \frac{2n^2\operatorname{Li}_3\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} - \frac{2n^2\operatorname{Li}_3\left(1+\frac{bx}{a}\right)}{d}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.74

$$\int\frac{\log^2\left(c(a+bx)^n\right)}{dx+ex^2}\,dx$$

$$= \frac{\log(x)\left(-n\log(a+bx)+\log\left(c(a+bx)^n\right)\right)^2 - \left(-n\log(a+bx)+\log\left(c(a+bx)^n\right)\right)^2\log(d+ex) - 2n(n\log$$

```
[In] Integrate[Log[c*(a + b*x)^n]^2/(d*x + e*x^2),x]
```

```
[Out] (Log[x]*(-(n*Log[a + b*x]) + Log[c*(a + b*x)^n])^2 - (-(n*Log[a + b*x]) + L
og[c*(a + b*x)^n])^2*Log[d + e*x] - 2*n*(n*Log[a + b*x] - Log[c*(a + b*x)^n
])*(Log[x]*(Log[a + b*x] - Log[1 + (b*x)/a]) - Log[a + b*x]*Log[(b*(d + e*x
))/(b*d - a*e)] - PolyLog[2, -((b*x)/a)] - PolyLog[2, (e*(a + b*x))/(-(b*d)
 + a*e)]) + n^2*(Log[-((b*x)/a)]*Log[a + b*x]^2 - Log[a + b*x]^2*Log[(b*(d
+ e*x))/(b*d - a*e)] - 2*Log[a + b*x]*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*
e)] + 2*Log[a + b*x]*PolyLog[2, 1 + (b*x)/a] + 2*PolyLog[3, (e*(a + b*x))/(
-(b*d) + a*e)] - 2*PolyLog[3, 1 + (b*x)/a]))/d
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.28 (sec) , antiderivative size = 761, normalized size of antiderivative = 4.53

| method | result |
|---|---|
| risch | $\frac{(\ln((bx+a)^n)-n\ln(bx+a))^2\ln(bx)}{d}-\frac{(\ln((bx+a)^n)-n\ln(bx+a))^2\ln(e(bx+a)-ae+bd)}{d}+\frac{n^2\ln(bx+a)^2\ln\left(1-\frac{bx+a}{a}\right)}{d}+\frac{2n^2\ln(bx$ |

```
[In] int(ln(c*(b*x+a)^n)^2/(e*x^2+d*x),x,method=_RETURNVERBOSE)
[Out] (ln((b*x+a)^n)-n*ln(b*x+a))^2/d*ln(b*x)-(ln((b*x+a)^n)-n*ln(b*x+a))^2/d*ln(
e*(b*x+a)-a*e+b*d)+n^2/d*ln(b*x+a)^2*ln(1-(b*x+a)/a)+2*n^2/d*ln(b*x+a)*poly
log(2,(b*x+a)/a)-2*n^2/d*polylog(3,(b*x+a)/a)-n^2/d*ln(b*x+a)^2*ln(1+e*(b*x
+a)/(-a*e+b*d))-2*n^2/d*ln(b*x+a)*polylog(2,-e*(b*x+a)/(-a*e+b*d))+2*n^2*po
lylog(3,-e*(b*x+a)/(-a*e+b*d))/d+2*b*n*(ln((b*x+a)^n)-n*ln(b*x+a))*(1/b/d*(
dilog(-x/a*b)+ln(b*x+a)*ln(-x/a*b))-e/b/d*(dilog((e*(b*x+a)-a*e+b*d)/(-a*e+
b*d))/e+ln(b*x+a)*ln((e*(b*x+a)-a*e+b*d)/(-a*e+b*d))/e))+(-I*Pi*csgn(I*c*(b
*x+a)^n)^3+I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+I*Pi*csgn(I*c*(b*x+
a)^n)^2*csgn(I*c)-I*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^n)*csgn(I*c)+2*ln
(c))*(-1/d*ln(e*x+d)*ln((b*x+a)^n)+ln((b*x+a)^n)/d*ln(x)-b*n*(1/d*dilog((b*
x+a)/a)/b+1/d*ln(x)*ln((b*x+a)/a)/b-1/d*dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d)
)/b-1/d*ln(e*x+d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b))+1/4*(-I*Pi*csgn(I*c
*(b*x+a)^n)^3+I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+I*Pi*csgn(I*c*(b
*x+a)^n)^2*csgn(I*c)-I*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a)^n)*csgn(I*c)+2
*ln(c))^2*(-1/d*ln(e*x+d)+1/d*ln(x))
```

## Fricas [F]

$$\int \frac{\log^2\left(c(a+bx)^n\right)}{dx+ex^2}\,dx=\int \frac{\log\left((bx+a)^n c\right)^2}{ex^2+dx}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)^2/(e*x^2+d*x),x, algorithm="fricas")
[Out] integral(log((b*x + a)^n*c)^2/(e*x^2 + d*x), x)
```

## Sympy [F]

$$\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\log(c(a+bx)^n)^2}{x(d+ex)}\,dx$$

```
[In] integrate(ln(c*(b*x+a)**n)**2/(e*x**2+d*x),x)
[Out] Integral(log(c*(a + b*x)**n)**2/(x*(d + e*x)), x)
```

## Maxima [F]

$$\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\log((bx+a)^n c)^2}{ex^2+dx}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)^2/(e*x^2+d*x),x, algorithm="maxima")
[Out] integrate(log((b*x + a)^n*c)^2/(e*x^2 + d*x), x)
```

## Giac [F]

$$\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\log((bx+a)^n c)^2}{ex^2+dx}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)^2/(e*x^2+d*x),x, algorithm="giac")
[Out] integrate(log((b*x + a)^n*c)^2/(e*x^2 + d*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)^2}{e\,x^2+d\,x}\,dx$$

```
[In] int(log(c*(a + b*x)^n)^2/(d*x + e*x^2),x)
[Out] int(log(c*(a + b*x)^n)^2/(d*x + e*x^2), x)
```

# 3.345 $\int \frac{\log(c(a+bx)^n)}{dx+ex^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{\log(c(a+bx)^n)}{dx+ex^2}\,dx = \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^n)}{d} - \frac{\log(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{n\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{n\,\text{PolyLog}\left(2,1+\frac{bx}{a}\right)}{d}$$

```
[Out] ln(-b*x/a)*ln(c*(b*x+a)^n)/d-ln(c*(b*x+a)^n)*ln(b*(e*x+d)/(-a*e+b*d))/d-n*p
olylog(2,-e*(b*x+a)/(-a*e+b*d))/d+n*polylog(2,1+b*x/a)/d
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1607, 36, 29, 31, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{\log(c(a+bx)^n)}{dx+ex^2}\,dx = -\frac{\log(c(a+bx)^n)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} + \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^n)}{d} - \frac{n\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{n\,\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d}$$

```
[In] Int[Log[c*(a + b*x)^n]/(d*x + e*x^2),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[c*(a + b*x)^n])/d - (Log[c*(a + b*x)^n]*Log[(b*(d + e*
x))/(b*d - a*e)])/d - (n*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d + (n*P
olyLog[2, 1 + (b*x)/a])/d
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\log\left(c(a+bx)^n\right)}{x(d+ex)}\,dx \\
&= \int \left(\frac{\log\left(c(a+bx)^n\right)}{dx} - \frac{e\log\left(c(a+bx)^n\right)}{d(d+ex)}\right)dx \\
&= \frac{\int \frac{\log(c(a+bx)^n)}{x}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx)^n)}{d+ex}\,dx}{d} \\
&= \frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^n\right)}{d} - \frac{\log\left(c(a+bx)^n\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} \\
&\quad - \frac{(bn)\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx}{d} + \frac{(bn)\int \frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{d} \\
&= \frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^n\right)}{d} - \frac{\log\left(c(a+bx)^n\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} \\
&\quad + \frac{n\text{Li}_2\left(1+\frac{bx}{a}\right)}{d} + \frac{n\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}\,dx, x, a+bx\right)}{d} \\
&= \frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^n\right)}{d} - \frac{\log\left(c(a+bx)^n\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{n\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{n\text{Li}_2\left(1+\frac{bx}{a}\right)}{d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$
\begin{aligned}
\int \frac{\log\left(c(a+bx)^n\right)}{dx+ex^2}\,dx &= \frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^n\right)}{d} - \frac{\log\left(c(a+bx)^n\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} \\
&\quad + \frac{n\,\text{PolyLog}\left(2,\frac{a+bx}{a}\right)}{d} - \frac{n\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}
\end{aligned}
$$

```
[In] Integrate[Log[c*(a + b*x)^n]/(d*x + e*x^2),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[c*(a + b*x)^n])/d - (Log[c*(a + b*x)^n]*Log[(b*(d + e*
x))/(b*d - a*e)])/d + (n*PolyLog[2, (a + b*x)/a])/d - (n*PolyLog[2, -((e*(a
 + b*x))/(b*d - a*e))])/d
```

## Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.61

| method | result |
|---|---|
| parts | $-\frac{\ln(c(bx+a)^n)\ln(ex+d)}{d}+\frac{\ln(c(bx+a)^n)\ln(x)}{d}-bn\left(\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{db}+\frac{\ln(x)\ln\left(\frac{bx+a}{a}\right)}{db}-\frac{\operatorname{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{db}-\frac{\ln(ex-$ |
| risch | $-\frac{\ln(ex+d)\ln((bx+a)^n)}{d}+\frac{\ln((bx+a)^n)\ln(x)}{d}-\frac{n\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{d}-\frac{n\ln(x)\ln\left(\frac{bx+a}{a}\right)}{d}+\frac{n\operatorname{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{d}+\frac{n\ln(ex+$ |

```
[In] int(ln(c*(b*x+a)^n)/(e*x^2+d*x),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(c*(b*x+a)^n)/d*ln(e*x+d)+ln(c*(b*x+a)^n)/d*ln(x)-b*n*(1/d*dilog((b*x+a)
/a)/b+1/d*ln(x)*ln((b*x+a)/a)/b-1/d*dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b-
1/d*ln(e*x+d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b)
```

## Fricas [F]

$$\int \frac{\log(c(a+bx)^n)}{dx+ex^2}\,dx=\int \frac{\log((bx+a)^nc)}{ex^2+dx}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)/(e*x^2+d*x),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^n*c)/(e*x^2 + d*x), x)
```

## Sympy [F]

$$\int \frac{\log(c(a+bx)^n)}{dx+ex^2}\,dx=\int \frac{\log(c(a+bx)^n)}{x(d+ex)}\,dx$$

```
[In] integrate(ln(c*(b*x+a)**n)/(e*x**2+d*x),x)
```

```
[Out] Integral(log(c*(a + b*x)**n)/(x*(d + e*x)), x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.27

$$\int \frac{\log(c(a+bx)^n)}{dx+ex^2}\,dx = -bn\left(\frac{\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{bd}-\frac{\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{bd}\right) - \left(\frac{\log(ex+d)}{d}-\frac{\log(x)}{d}\right)\log\left((bx+a)^n c\right)$$

```
[In] integrate(log(c*(b*x+a)^n)/(e*x^2+d*x),x, algorithm="maxima")
```

```
[Out] -b*n*((log(b*x/a + 1)*log(x) + dilog(-b*x/a))/(b*d) - (log(e*x + d)*log(-(b
*e*x + b*d)/(b*d - a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/(b*d)) - (
log(e*x + d)/d - log(x)/d)*log((b*x + a)^n*c)
```

## Giac [F]

$$\int \frac{\log(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)}{ex^2+dx}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)/(e*x^2+d*x),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^n*c)/(e*x^2 + d*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(a+bx)^n)}{dx+ex^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)}{e\,x^2+d\,x}\,dx$$

```
[In] int(log(c*(a + b*x)^n)/(d*x + e*x^2),x)
```

```
[Out] int(log(c*(a + b*x)^n)/(d*x + e*x^2), x)
```

# 3.346 $\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx = \frac{\text{Int}\left(\frac{1}{x\log(c(a+bx)^n)},x\right)}{d} - \frac{e\text{Int}\left(\frac{1}{(d+ex)\log(c(a+bx)^n)},x\right)}{d}$$

```
[Out] Unintegrable(1/x/ln(c*(b*x+a)^n),x)/d-e*Unintegrable(1/(e*x+d)/ln(c*(b*x+a)
^n),x)/d
```

## Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx$$

```
[In] Int[1/((d*x + e*x^2)*Log[c*(a + b*x)^n]),x]
[Out] Defer[Int][1/(x*Log[c*(a + b*x)^n]), x]/d - (e*Defer[Int][1/((d + e*x)*Log[
c*(a + b*x)^n]), x])/d
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x(d+ex)\log(c(a+bx)^n)}\,dx \\
&= \int \left(\frac{1}{dx\log(c(a+bx)^n)} - \frac{e}{d(d+ex)\log(c(a+bx)^n)}\right)dx \\
&= \frac{\int \frac{1}{x\log(c(a+bx)^n)}\,dx}{d} - \frac{e\int \frac{1}{(d+ex)\log(c(a+bx)^n)}\,dx}{d}
\end{aligned}$$

## Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx$$

```
[In] Integrate[1/((d*x + e*x^2)*Log[c*(a + b*x)^n]),x]
[Out] Integrate[1/((d*x + e*x^2)*Log[c*(a + b*x)^n]), x]
```

## Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+dx)\ln\left(c\,(bx+a)^n\right)}dx$$

```
[In] int(1/(e*x^2+d*x)/ln(c*(b*x+a)^n),x)
[Out] int(1/(e*x^2+d*x)/ln(c*(b*x+a)^n),x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(ex^2+dx)\log\left((bx+a)^n c\right)}\,dx$$

```
[In] integrate(1/(e*x^2+d*x)/log(c*(b*x+a)^n),x, algorithm="fricas")
[Out] integral(1/((e*x^2 + d*x)*log((b*x + a)^n*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{x\,(d+ex)\log\left(c\,(a+bx)^n\right)}\,dx$$

```
[In] integrate(1/(e*x**2+d*x)/ln(c*(b*x+a)**n),x)
[Out] Integral(1/(x*(d + e*x)*log(c*(a + b*x)**n)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(ex^2+dx)\log((bx+a)^n c)}\,dx$$

```
[In] integrate(1/(e*x^2+d*x)/log(c*(b*x+a)^n),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d*x)*log((b*x + a)^n*c)), x)
```

## Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(ex^2+dx)\log((bx+a)^n c)}\,dx$$

```
[In] integrate(1/(e*x^2+d*x)/log(c*(b*x+a)^n),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d*x)*log((b*x + a)^n*c)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(dx+ex^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{\ln\left(c\,(a+b\,x)^n\right)\,\left(e\,x^2+d\,x\right)}\,dx$$

```
[In] int(1/(log(c*(a + b*x)^n)*(d*x + e*x^2)),x)
[Out] int(1/(log(c*(a + b*x)^n)*(d*x + e*x^2)), x)
```

# 3.347 $\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 500

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx = \frac{\log^3(c(a+bx)^n)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log^3(c(a+bx)^n)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{3n\log^2(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{6n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{6n^2\log(c(a+bx)^n)\operatorname{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{6n^3\operatorname{PolyLog}\left(4,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{6n^3\operatorname{PolyLog}\left(4,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

```
[Out] ln(c*(b*x+a)^n)^3*ln(-b*(e+2*f*x-(-4*d*f+e^2)^(1/2))/(2*a*f-b*(e-(-4*d*f+e^
2)^(1/2))))/(-4*d*f+e^2)^(1/2)-ln(c*(b*x+a)^n)^3*ln(-b*(e+2*f*x+(-4*d*f+e^2
)^(1/2))/(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+3*n*ln(c*(b*x
+a)^n)^2*polylog(2,2*f*(b*x+a)/(2*a*f-b*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^
2)^(1/2)-3*n*ln(c*(b*x+a)^n)^2*polylog(2,2*f*(b*x+a)/(2*a*f-b*(e+(-4*d*f+e^
2)^(1/2))))/(-4*d*f+e^2)^(1/2)-6*n^2*ln(c*(b*x+a)^n)*polylog(3,2*f*(b*x+a)/
(2*a*f-b*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+6*n^2*ln(c*(b*x+a)^n)*
polylog(3,2*f*(b*x+a)/(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+
6*n^3*polylog(4,2*f*(b*x+a)/(2*a*f-b*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^
(1/2)-6*n^3*polylog(4,2*f*(b*x+a)/(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f
+e^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2465, 2443, 2481, 2421, 2430, 6724}

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx = -\frac{6n^2\log(c(a+bx)^n)\,\text{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{6n^2\log(c(a+bx)^n)\,\text{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{3n\log^2(c(a+bx)^n)\,\text{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{3n\log^2(c(a+bx)^n)\,\text{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{\log^3(c(a+bx)^n)\log\left(-\frac{b\left(-\sqrt{e^2-4df}+e+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log^3(c(a+bx)^n)\log\left(-\frac{b\left(\sqrt{e^2-4df}+e+2fx\right)}{2af-b\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}} + \frac{6n^3\,\text{PolyLog}\left(4,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{6n^3\,\text{PolyLog}\left(4,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

```
[In] Int[Log[c*(a + b*x)^n]^3/(d + e*x + f*x^2),x]
```

```
[Out] (Log[c*(a + b*x)^n]^3*Log[-((b*(e - Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*a*f - b*
(e - Sqrt[e^2 - 4*d*f]))))])/Sqrt[e^2 - 4*d*f] - (Log[c*(a + b*x)^n]^3*Log[-
((b*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))))])
/Sqrt[e^2 - 4*d*f] + (3*n*Log[c*(a + b*x)^n]^2*PolyLog[2, (2*f*(a + b*x))/(
2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f] - (3*n*Log[c*(a + b*
x)^n]^2*PolyLog[2, (2*f*(a + b*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])/Sq
rt[e^2 - 4*d*f] - (6*n^2*Log[c*(a + b*x)^n]*PolyLog[3, (2*f*(a + b*x))/(2*a
*f - b*(e - Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f] + (6*n^2*Log[c*(a + b*x
)^n]*PolyLog[3, (2*f*(a + b*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt[
e^2 - 4*d*f] + (6*n^3*PolyLog[4, (2*f*(a + b*x))/(2*a*f - b*(e - Sqrt[e^2 -
 4*d*f]))])/Sqrt[e^2 - 4*d*f] - (6*n^3*PolyLog[4, (2*f*(a + b*x))/(2*a*f -
b*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2f\log^3\left(c(a+bx)^n\right)}{\sqrt{e^2-4df}\left(e-\sqrt{e^2-4df}+2fx\right)}-\frac{2f\log^3\left(c(a+bx)^n\right)}{\sqrt{e^2-4df}\left(e+\sqrt{e^2-4df}+2fx\right)}\right)dx \\
&= \frac{(2f)\int\frac{\log^3(c(a+bx)^n)}{e-\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}}-\frac{(2f)\int\frac{\log^3(c(a+bx)^n)}{e+\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}} \\
&= \frac{\log^3\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\
&\quad -\frac{\log^3\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\
&\quad -\frac{(3bn)\int\frac{\log^2(c(a+bx)^n)\log\left(\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{-2af+b\left(e-\sqrt{e^2-4df}\right)}\right)}{a+bx}\,dx}{\sqrt{e^2-4df}} \\
&\quad +\frac{(3bn)\int\frac{\log^2(c(a+bx)^n)\log\left(\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{-2af+b\left(e+\sqrt{e^2-4df}\right)}\right)}{a+bx}\,dx}{\sqrt{e^2-4df}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\log^3\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\
&\quad - \frac{\log^3\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\
&\quad - \frac{(3n)\text{Subst}\left(\int \frac{\log^2(cx^n)\log\left(\frac{b\left(\frac{-2af+b\left(e-\sqrt{e^2-4df}\right)}{b}+\frac{2fx}{b}\right)}{-2af+b\left(e-\sqrt{e^2-4df}\right)}\right)}{x}\,dx, x, a+bx\right)}{\sqrt{e^2-4df}} \\
&\quad + \frac{(3n)\text{Subst}\left(\int \frac{\log^2(cx^n)\log\left(\frac{b\left(\frac{-2af+b\left(e+\sqrt{e^2-4df}\right)}{b}+\frac{2fx}{b}\right)}{-2af+b\left(e+\sqrt{e^2-4df}\right)}\right)}{x}\,dx, x, a+bx\right)}{\sqrt{e^2-4df}}
\end{aligned}
$$

$$= \frac{\log^3\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log^3\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{3n\log^2\left(c(a+bx)^n\right)\operatorname{Li}_2\left(\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{3n\log^2\left(c(a+bx)^n\right)\operatorname{Li}_2\left(\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{(6n^2)\operatorname{Subst}\left(\int\frac{\log(cx^n)\operatorname{Li}_2\left(-\frac{2fx}{-2af+b\left(e-\sqrt{e^2-4df}\right)}\right)}{x}\,dx, x, a+bx\right)}{\sqrt{e^2-4df}} + \frac{(6n^2)\operatorname{Subst}\left(\int\frac{\log(cx^n)\operatorname{Li}_2\left(-\frac{2fx}{-2af+b\left(e+\sqrt{e^2-4df}\right)}\right)}{x}\,dx, x, a+bx\right)}{\sqrt{e^2-4df}}$$

$$= \frac{\log^3\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log^3\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{3n\log^2\left(c(a+bx)^n\right)\text{Li}_2\left(\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{3n\log^2\left(c(a+bx)^n\right)\text{Li}_2\left(\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{6n^2\log\left(c(a+bx)^n\right)\text{Li}_3\left(\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{6n^2\log\left(c(a+bx)^n\right)\text{Li}_3\left(\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{(6n^3)\,\text{Subst}\left(\int\frac{\text{Li}_3\left(-\frac{2fx}{-2af+b\left(e-\sqrt{e^2-4df}\right)}\right)}{x}\,dx,x,a+bx\right)}{\sqrt{e^2-4df}} - \frac{(6n^3)\,\text{Subst}\left(\int\frac{\text{Li}_3\left(-\frac{2fx}{-2af+b\left(e+\sqrt{e^2-4df}\right)}\right)}{x}\,dx,x,a+bx\right)}{\sqrt{e^2-4df}}$$

$$= \frac{\log^3(c(a+bx)^n)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log^3(c(a+bx)^n)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{3n\log^2(c(a+bx)^n)\operatorname{Li}_2\left(\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{3n\log^2(c(a+bx)^n)\operatorname{Li}_2\left(\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{6n^2\log(c(a+bx)^n)\operatorname{Li}_3\left(\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{6n^2\log(c(a+bx)^n)\operatorname{Li}_3\left(\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{6n^3\operatorname{Li}_4\left(\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{6n^3\operatorname{Li}_4\left(\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

## Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 993, normalized size of antiderivative = 1.99

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx = \frac{-2\sqrt{e^2-4df}n^3\arctan\left(\frac{e+2fx}{\sqrt{-e^2+4df}}\right)\log^3(a+bx) + 6\sqrt{e^2-4df}n^2\arctan\left(\frac{e+2fx}{\sqrt{-e^2+4df}}\right)\log^2(a+bx)\log(c(a}{}$$

```
[In] Integrate[Log[c*(a + b*x)^n]^3/(d + e*x + f*x^2),x]
```

```
[Out] (-2*Sqrt[e^2 - 4*d*f]*n^3*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]*Log[a + b*
x]^3 + 6*Sqrt[e^2 - 4*d*f]*n^2*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]*Log[a
 + b*x]^2*Log[c*(a + b*x)^n] - 6*Sqrt[e^2 - 4*d*f]*n*ArcTan[(e + 2*f*x)/Sqr
t[-e^2 + 4*d*f]]*Log[a + b*x]*Log[c*(a + b*x)^n]^2 + 2*Sqrt[e^2 - 4*d*f]*Ar
cTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]*Log[c*(a + b*x)^n]^3 + Sqrt[-e^2 + 4*d
*f]*n^3*Log[a + b*x]^3*Log[1 - (2*f*(a + b*x))/(-(b*e) + 2*a*f + b*Sqrt[e^2
 - 4*d*f])] - 3*Sqrt[-e^2 + 4*d*f]*n^2*Log[a + b*x]^2*Log[c*(a + b*x)^n]*Lo
g[1 - (2*f*(a + b*x))/(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f])] + 3*Sqrt[-e^2
```

```
 + 4*d*f]*n*Log[a + b*x]*Log[c*(a + b*x)^n]^2*Log[1 - (2*f*(a + b*x))/(-(b*
e) + 2*a*f + b*Sqrt[e^2 - 4*d*f])] - Sqrt[-e^2 + 4*d*f]*n^3*Log[a + b*x]^3*
Log[1 + (2*f*(a + b*x))/(-2*a*f + b*(e + Sqrt[e^2 - 4*d*f]))] + 3*Sqrt[-e^2
 + 4*d*f]*n^2*Log[a + b*x]^2*Log[c*(a + b*x)^n]*Log[1 + (2*f*(a + b*x))/(-2
*a*f + b*(e + Sqrt[e^2 - 4*d*f]))] - 3*Sqrt[-e^2 + 4*d*f]*n*Log[a + b*x]*Lo
g[c*(a + b*x)^n]^2*Log[1 + (2*f*(a + b*x))/(-2*a*f + b*(e + Sqrt[e^2 - 4*d*
f]))] + 3*Sqrt[-e^2 + 4*d*f]*n*Log[c*(a + b*x)^n]^2*PolyLog[2, (2*f*(a + b*
x))/(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))] - 3*Sqrt[-e^2 + 4*d*f]*n*Log[c*(a
 + b*x)^n]^2*PolyLog[2, (2*f*(a + b*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))
] - 6*Sqrt[-e^2 + 4*d*f]*n^2*Log[c*(a + b*x)^n]*PolyLog[3, (2*f*(a + b*x))/
(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f])] + 6*Sqrt[-e^2 + 4*d*f]*n^2*Log[c*(a
 + b*x)^n]*PolyLog[3, (2*f*(a + b*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]
+ 6*Sqrt[-e^2 + 4*d*f]*n^3*PolyLog[4, (2*f*(a + b*x))/(-(b*e) + 2*a*f + b*S
qrt[e^2 - 4*d*f])] - 6*Sqrt[-e^2 + 4*d*f]*n^3*PolyLog[4, (2*f*(a + b*x))/(2
*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt[-(e^2 - 4*d*f)^2]
```

## Maple [F]

$$\int \frac{\ln \left(c(bx+a)^{n}\right)^{3}}{f\,x^{2}+ex+d}dx$$

```
[In] int(ln(c*(b*x+a)^n)^3/(f*x^2+e*x+d),x)
[Out] int(ln(c*(b*x+a)^n)^3/(f*x^2+e*x+d),x)
```

## Fricas [F]

$$\int \frac{\log^3\left(c(a+bx)^n\right)}{d+ex+fx^2}\,dx=\int \frac{\log\left((bx+a)^n c\right)^3}{fx^2+ex+d}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)^3/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x + a)^n*c)^3/(f*x^2 + e*x + d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(a+bx)^n\right)}{d+ex+fx^2}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*(b*x+a)**n)**3/(f*x**2+e*x+d),x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(b*x+a)^n)^3/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

## Giac [F]

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^3}{fx^2+ex+d}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)^3/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^n*c)^3/(f*x^2 + e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3(c(a+bx)^n)}{d+ex+fx^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)^3}{f\,x^2+e\,x+d}\,dx$$

```
[In] int(log(c*(a + b*x)^n)^3/(d + e*x + f*x^2),x)
```

```
[Out] int(log(c*(a + b*x)^n)^3/(d + e*x + f*x^2), x)
```

# 3.348 $\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 372

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2}\,dx = \frac{\log^2(c(a+bx)^n)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log^2(c(a+bx)^n)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{2n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{2n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{2n^2\operatorname{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{2n^2\operatorname{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

```
[Out] ln(c*(b*x+a)^n)^2*ln(-b*(e+2*f*x-(-4*d*f+e^2)^(1/2))/(2*a*f-b*(e-(-4*d*f+e^
2)^(1/2))))/(-4*d*f+e^2)^(1/2)-ln(c*(b*x+a)^n)^2*ln(-b*(e+2*f*x+(-4*d*f+e^2
)^(1/2))/(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+2*n*ln(c*(b*x
+a)^n)*polylog(2,2*f*(b*x+a)/(2*a*f-b*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)
^(1/2)-2*n*ln(c*(b*x+a)^n)*polylog(2,2*f*(b*x+a)/(2*a*f-b*(e+(-4*d*f+e^2)^(
1/2))))/(-4*d*f+e^2)^(1/2)-2*n^2*polylog(3,2*f*(b*x+a)/(2*a*f-b*(e-(-4*d*f+
```

```
e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+2*n^2*polylog(3,2*f*(b*x+a)/(2*a*f-b*(e+(-
4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2465, 2443, 2481, 2421, 6724}

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2}\,dx = \frac{2n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{2n\log(c(a+bx)^n)\operatorname{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{\log^2(c(a+bx)^n)\log\left(-\frac{b\left(-\sqrt{e^2-4df}+e+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log^2(c(a+bx)^n)\log\left(-\frac{b\left(\sqrt{e^2-4df}+e+2fx\right)}{2af-b\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}} - \frac{2n^2\operatorname{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{2n^2\operatorname{PolyLog}\left(3,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

```
[In] Int[Log[c*(a + b*x)^n]^2/(d + e*x + f*x^2),x]
```

```
[Out] (Log[c*(a + b*x)^n]^2*Log[-((b*(e - Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*a*f - b*
(e - Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 - 4*d*f] - (Log[c*(a + b*x)^n]^2*Log[-
((b*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))])
/Sqrt[e^2 - 4*d*f] + (2*n*Log[c*(a + b*x)^n]*PolyLog[2, (2*f*(a + b*x))/(2*
a*f - b*(e - Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f] - (2*n*Log[c*(a + b*x)
^n]*PolyLog[2, (2*f*(a + b*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt[e
^2 - 4*d*f] - (2*n^2*PolyLog[3, (2*f*(a + b*x))/(2*a*f - b*(e - Sqrt[e^2 -
4*d*f]))])/Sqrt[e^2 - 4*d*f] + (2*n^2*PolyLog[3, (2*f*(a + b*x))/(2*a*f - b
*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
```

```
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{2f\log^2\left(c(a+bx)^n\right)}{\sqrt{e^2-4df}\left(e-\sqrt{e^2-4df}+2fx\right)}-\frac{2f\log^2\left(c(a+bx)^n\right)}{\sqrt{e^2-4df}\left(e+\sqrt{e^2-4df}+2fx\right)}\right)dx$$

$$=\frac{(2f)\int\frac{\log^2(c(a+bx)^n)}{e-\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}}-\frac{(2f)\int\frac{\log^2(c(a+bx)^n)}{e+\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}}$$

$$= \frac{\log^2\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log^2\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{(2bn)\int\frac{\log(c(a+bx)^n)\log\left(\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{-2af+b\left(e-\sqrt{e^2-4df}\right)}\right)}{a+bx}dx}{\sqrt{e^2-4df}} + \frac{(2bn)\int\frac{\log(c(a+bx)^n)\log\left(\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{-2af+b\left(e+\sqrt{e^2-4df}\right)}\right)}{a+bx}dx}{\sqrt{e^2-4df}}$$

$$= \frac{\log^2\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log^2\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{(2n)\text{Subst}\left(\int\frac{\log(cx^n)\log\left(\frac{b\left(\frac{-2af+b\left(e-\sqrt{e^2-4df}\right)}{b}+\frac{2fx}{b}\right)}{-2af+b\left(e-\sqrt{e^2-4df}\right)}\right)}{x}dx,x,a+bx\right)}{\sqrt{e^2-4df}} + \frac{(2n)\text{Subst}\left(\int\frac{\log(cx^n)\log\left(\frac{b\left(\frac{-2af+b\left(e+\sqrt{e^2-4df}\right)}{b}+\frac{2fx}{b}\right)}{-2af+b\left(e+\sqrt{e^2-4df}\right)}\right)}{x}dx,x,a+bx\right)}{\sqrt{e^2-4df}}$$

$$= \frac{\log^2(c(a+bx)^n)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log^2(c(a+bx)^n)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{2n\log(c(a+bx)^n)\operatorname{Li}_2\left(\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{2n\log(c(a+bx)^n)\operatorname{Li}_2\left(\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{(2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{2fx}{-2af+b\left(e-\sqrt{e^2-4df}\right)}\right)}{x}dx,x,a+bx\right)}{\sqrt{e^2-4df}} + \frac{(2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{2fx}{-2af+b\left(e+\sqrt{e^2-4df}\right)}\right)}{x}dx,x,a+bx\right)}{\sqrt{e^2-4df}}$$

$$= \frac{\log^2(c(a+bx)^n)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log^2(c(a+bx)^n)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{2n\log(c(a+bx)^n)\operatorname{Li}_2\left(\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{2n\log(c(a+bx)^n)\operatorname{Li}_2\left(\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{2n^2\operatorname{Li}_3\left(\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{2n^2\operatorname{Li}_3\left(\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

## Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.76

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2}\,dx$$

$$= \frac{2\sqrt{e^2-4df}n^2\arctan\left(\frac{e+2fx}{\sqrt{-e^2+4df}}\right)\log^2(a+bx)-4\sqrt{e^2-4df}n\arctan\left(\frac{e+2fx}{\sqrt{-e^2+4df}}\right)\log(a+bx)\log(c(a+b}{}$$

```
[In] Integrate[Log[c*(a + b*x)^n]^2/(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[e^2 - 4*d*f]*n^2*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]*Log[a + b*x
]^2 - 4*Sqrt[e^2 - 4*d*f]*n*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]*Log[a +
b*x]*Log[c*(a + b*x)^n] + 2*Sqrt[e^2 - 4*d*f]*ArcTan[(e + 2*f*x)/Sqrt[-e^2
+ 4*d*f]]*Log[c*(a + b*x)^n]^2 - Sqrt[-e^2 + 4*d*f]*n^2*Log[a + b*x]^2*Log[
1 - (2*f*(a + b*x))/(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f])] + 2*Sqrt[-e^2 +
 4*d*f]*n*Log[a + b*x]*Log[c*(a + b*x)^n]*Log[1 - (2*f*(a + b*x))/(-(b*e) +
 2*a*f + b*Sqrt[e^2 - 4*d*f])] + Sqrt[-e^2 + 4*d*f]*n^2*Log[a + b*x]^2*Log[
1 + (2*f*(a + b*x))/(-2*a*f + b*(e + Sqrt[e^2 - 4*d*f]))] - 2*Sqrt[-e^2 + 4
*d*f]*n*Log[a + b*x]*Log[c*(a + b*x)^n]*Log[1 + (2*f*(a + b*x))/(-2*a*f + b
*(e + Sqrt[e^2 - 4*d*f]))] + 2*Sqrt[-e^2 + 4*d*f]*n*Log[c*(a + b*x)^n]*Poly
Log[2, (2*f*(a + b*x))/(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))] - 2*Sqrt[-e^2
+ 4*d*f]*n*Log[c*(a + b*x)^n]*PolyLog[2, (2*f*(a + b*x))/(2*a*f - b*(e + Sq
rt[e^2 - 4*d*f]))] - 2*Sqrt[-e^2 + 4*d*f]*n^2*PolyLog[3, (2*f*(a + b*x))/(-
(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f])] + 2*Sqrt[-e^2 + 4*d*f]*n^2*PolyLog[3,
 (2*f*(a + b*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt[-(e^2 - 4*d*f)^
2]
```

## Maple [F]

$$\int \frac{\ln \left(c(bx+a)^{n}\right)^{2}}{f\,x^{2}+ex+d}dx$$

```
[In] int(ln(c*(b*x+a)^n)^2/(f*x^2+e*x+d),x)
[Out] int(ln(c*(b*x+a)^n)^2/(f*x^2+e*x+d),x)
```

## Fricas [F]

$$\int \frac{\log^{2}\left(c(a+bx)^{n}\right)}{d+ex+fx^{2}}\,dx = \int \frac{\log\left((bx+a)^{n}c\right)^{2}}{fx^{2}+ex+d}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)^2/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x + a)^n*c)^2/(f*x^2 + e*x + d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log^{2}\left(c(a+bx)^{n}\right)}{d+ex+fx^{2}}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x+a)**n)**2/(f*x**2+e*x+d),x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^{2}\left(c(a+bx)^{n}\right)}{d+ex+fx^{2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(b*x+a)^n)^2/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

## Giac [F]

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2}\,dx = \int \frac{\log\left((bx+a)^n c\right)^2}{fx^2+ex+d}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)^2/(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x + a)^n*c)^2/(f*x^2 + e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(c(a+bx)^n)}{d+ex+fx^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)^2}{f\,x^2+e\,x+d}\,dx$$

```
[In] int(log(c*(a + b*x)^n)^2/(d + e*x + f*x^2),x)
[Out] int(log(c*(a + b*x)^n)^2/(d + e*x + f*x^2), x)
```

# 3.349 $\int \frac{\log(c(a+bx)^n)}{d+ex+fx^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 243

$$\int \frac{\log\left(c(a+bx)^n\right)}{d+ex+fx^2}\,dx = \frac{\log\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{n\,\text{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{n\,\text{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

```
[Out] ln(c*(b*x+a)^n)*ln(-b*(e+2*f*x-(-4*d*f+e^2)^(1/2))/(2*a*f-b*(e-(-4*d*f+e^2)
^(1/2))))/(-4*d*f+e^2)^(1/2)-ln(c*(b*x+a)^n)*ln(-b*(e+2*f*x+(-4*d*f+e^2)^(1
/2))/(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+n*polylog(2,2*f*(
b*x+a)/(2*a*f-b*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)-n*polylog(2,2*f
*(b*x+a)/(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2465, 2441, 2440, 2438}

$$\int \frac{\log\left(c(a+bx)^n\right)}{d+ex+fx^2}\,dx = \frac{\log\left(c(a+bx)^n\right)\log\left(-\frac{b\left(-\sqrt{e^2-4df}+e+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log\left(c(a+bx)^n\right)\log\left(-\frac{b\left(\sqrt{e^2-4df}+e+2fx\right)}{2af-b\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}} + \frac{n\,\text{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{n\,\text{PolyLog}\left(2,\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

```
[In] Int[Log[c*(a + b*x)^n]/(d + e*x + f*x^2),x]
```

```
[Out] (Log[c*(a + b*x)^n]*Log[-((b*(e - Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*a*f - b*(e
 - Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 - 4*d*f] - (Log[c*(a + b*x)^n]*Log[-((b*
(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))])/Sqr
t[e^2 - 4*d*f] + (n*PolyLog[2, (2*f*(a + b*x))/(2*a*f - b*(e - Sqrt[e^2 - 4
*d*f]))])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, (2*f*(a + b*x))/(2*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left( \frac{2f\log\left(c(a+bx)^n\right)}{\sqrt{e^2-4df}\left(e-\sqrt{e^2-4df}+2fx\right)} - \frac{2f\log\left(c(a+bx)^n\right)}{\sqrt{e^2-4df}\left(e+\sqrt{e^2-4df}+2fx\right)} \right) dx \\
&= \frac{(2f)\int \frac{\log(c(a+bx)^n)}{e-\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{\log(c(a+bx)^n)}{e+\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}} \\
&= \frac{\log\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\
&\quad - \frac{\log\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\
&\quad - \frac{(bn)\int \frac{\log\left(\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{-2af+b\left(e-\sqrt{e^2-4df}\right)}\right)}{a+bx}\,dx}{\sqrt{e^2-4df}} + \frac{(bn)\int \frac{\log\left(\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{-2af+b\left(e+\sqrt{e^2-4df}\right)}\right)}{a+bx}\,dx}{\sqrt{e^2-4df}} \\
&= \frac{\log\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\
&\quad - \frac{\log\left(c(a+bx)^n\right)\log\left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\
&\quad - \frac{n\text{Subst}\left(\int \frac{\log\left(1+\frac{2fx}{-2af+b\left(e-\sqrt{e^2-4df}\right)}\right)}{x}\,dx, x, a+bx\right)}{\sqrt{e^2-4df}} \\
&\quad + \frac{n\text{Subst}\left(\int \frac{\log\left(1+\frac{2fx}{-2af+b\left(e+\sqrt{e^2-4df}\right)}\right)}{x}\,dx, x, a+bx\right)}{\sqrt{e^2-4df}}
\end{aligned}$$

$$= \frac{\log (c(a+bx)^n) \log \left(-\frac{b\left(e-\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{\log (c(a+bx)^n) \log \left(-\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{n\mathrm{Li}_2\left(\frac{2f(a+bx)}{2af-b\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{n\mathrm{Li}_2\left(\frac{2f(a+bx)}{2af-b\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.80

$$\int \frac{\log (c(a+bx)^n)}{d+ex+fx^2}\,dx = \frac{\log (c(a+bx)^n) \left(\log \left(\frac{b\left(-e+\sqrt{e^2-4df}-2fx\right)}{-be+2af+b\sqrt{e^2-4df}}\right) - \log \left(\frac{b\left(e+\sqrt{e^2-4df}+2fx\right)}{-2af+b\left(e+\sqrt{e^2-4df}\right)}\right)\right) + n\,\mathrm{PolyLog}\left(2, \frac{2f(a+bx)}{2af+b\left(-e+\sqrt{e^2-4\cdots}\right.}\right.}{\sqrt{e^2-4df}}$$

```
[In] Integrate[Log[c*(a + b*x)^n]/(d + e*x + f*x^2),x]
```

```
[Out] (Log[c*(a + b*x)^n]*(Log[(b*(-e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(-(b*e) + 2*a
*f + b*Sqrt[e^2 - 4*d*f])] - Log[(b*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(-2*a*
f + b*(e + Sqrt[e^2 - 4*d*f]))]) + n*PolyLog[2, (2*f*(a + b*x))/(2*a*f + b*
(-e + Sqrt[e^2 - 4*d*f]))] - n*PolyLog[2, (2*f*(a + b*x))/(2*a*f - b*(e + S
qrt[e^2 - 4*d*f]))])/Sqrt[e^2 - 4*d*f]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.29 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.53

| method | result |
|---|---|
| risch | $-\frac{2b\arctan\left(\frac{2(bx+a)f-2af+be}{\sqrt{4b^2df-e^2b^2}}\right)n\ln(bx+a)}{\sqrt{4b^2df-e^2b^2}} + \frac{2b\arctan\left(\frac{2(bx+a)f-2af+be}{\sqrt{4b^2df-e^2b^2}}\right)\ln((bx+a)^n)}{\sqrt{4b^2df-e^2b^2}} + \frac{bn\ln(bx+a)\ln\left(\frac{-2(bx+a)f+2af-be\cdots}{2af-be+\sqrt{-4\cdots}}\right.}{\sqrt{-4b^2df+e^2b^2}}$ |

```
[In] int(ln(c*(b*x+a)^n)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/(4*b^2*d*f-b^2*e^2)^(1/2)*arctan((2*(b*x+a)*f-2*a*f+b*e)/(4*b^2*d*f-b^
2*e^2)^(1/2))*n*ln(b*x+a)+2*b/(4*b^2*d*f-b^2*e^2)^(1/2)*arctan((2*(b*x+a)*f
-2*a*f+b*e)/(4*b^2*d*f-b^2*e^2)^(1/2))*ln((b*x+a)^n)+b*n*ln(b*x+a)/(-4*b^2*
```

```
d*f+b^2*e^2)^(1/2)*ln((-2*(b*x+a)*f+2*a*f-b*e+(-4*b^2*d*f+b^2*e^2)^(1/2))/(
2*a*f-b*e+(-4*b^2*d*f+b^2*e^2)^(1/2)))-b*n*ln(b*x+a)/(-4*b^2*d*f+b^2*e^2)^(
1/2)*ln((2*(b*x+a)*f-2*a*f+b*e+(-4*b^2*d*f+b^2*e^2)^(1/2))/(-2*a*f+b*e+(-4*
b^2*d*f+b^2*e^2)^(1/2)))+b*n/(-4*b^2*d*f+b^2*e^2)^(1/2)*dilog((-2*(b*x+a)*f
+2*a*f-b*e+(-4*b^2*d*f+b^2*e^2)^(1/2))/(2*a*f-b*e+(-4*b^2*d*f+b^2*e^2)^(1/2
)))-b*n/(-4*b^2*d*f+b^2*e^2)^(1/2)*dilog((2*(b*x+a)*f-2*a*f+b*e+(-4*b^2*d*f
+b^2*e^2)^(1/2))/(-2*a*f+b*e+(-4*b^2*d*f+b^2*e^2)^(1/2)))+2*(-1/2*I*Pi*csgn
(I*c*(b*x+a)^n)^3+1/2*I*Pi*csgn(I*c*(b*x+a)^n)^2*csgn(I*(b*x+a)^n)+1/2*I*Pi
*csgn(I*c*(b*x+a)^n)^2*csgn(I*c)-1/2*I*Pi*csgn(I*c*(b*x+a)^n)*csgn(I*(b*x+a
)^n)*csgn(I*c)+ln(c))/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))
```

## Fricas [F]

$$\int \frac{\log(c(a+bx)^n)}{d+ex+fx^2}\,dx = \int \frac{\log((bx+a)^n c)}{fx^2+ex+d}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x + a)^n*c)/(f*x^2 + e*x + d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c(a+bx)^n)}{d+ex+fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x+a)**n)/(f*x**2+e*x+d),x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(c(a+bx)^n)}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

## Giac [F]

$$\int \frac{\log(c(a+bx)^n)}{d+ex+fx^2}\,dx = \int \frac{\log((bx+a)^n c)}{fx^2+ex+d}\,dx$$

```
[In] integrate(log(c*(b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x + a)^n*c)/(f*x^2 + e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(a+bx)^n)}{d+ex+fx^2}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^n\right)}{f\,x^2+e\,x+d}\,dx$$

```
[In] int(log(c*(a + b*x)^n)/(d + e*x + f*x^2),x)
[Out] int(log(c*(a + b*x)^n)/(d + e*x + f*x^2), x)
```

# 3.350 $\int \frac{1}{(d+ex+fx^2)\log(c(a+bx)^n)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(d+ex+fx^2)\log(c(a+bx)^n)}\,dx = \frac{2f\text{Int}\left(\frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\log(c(a+bx)^n)},x\right)}{\sqrt{e^2-4df}} - \frac{2f\text{Int}\left(\frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\log(c(a+bx)^n)},x\right)}{\sqrt{e^2-4df}}$$

```
[Out] 2*f*Unintegrable(1/ln(c*(b*x+a)^n)/(e+2*f*x-(-4*d*f+e^2)^(1/2)),x)/(-4*d*f+
e^2)^(1/2)-2*f*Unintegrable(1/ln(c*(b*x+a)^n)/(e+2*f*x+(-4*d*f+e^2)^(1/2)),
x)/(-4*d*f+e^2)^(1/2)
```

## Rubi [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex+fx^2)\log(c(a+bx)^n)}\,dx = \int \frac{1}{(d+ex+fx^2)\log(c(a+bx)^n)}\,dx$$

```
[In] Int[1/((d + e*x + f*x^2)*Log[c*(a + b*x)^n]),x]
```

```
[Out] (2*f*Defer[Int][1/((e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Log[c*(a + b*x)^n]), x])
/Sqrt[e^2 - 4*d*f] - (2*f*Defer[Int][1/((e + Sqrt[e^2 - 4*d*f] + 2*f*x)*Log
[c*(a + b*x)^n]), x])/Sqrt[e^2 - 4*d*f]
```

Rubi steps

$$\text{integral} = \int \left(\frac{2f}{\sqrt{e^2-4df}\left(e-\sqrt{e^2-4df}+2fx\right)\log\left(c(a+bx)^n\right)} - \frac{2f}{\sqrt{e^2-4df}\left(e+\sqrt{e^2-4df}+2fx\right)\log\left(c(a+bx)^n\right)}\right)dx$$

$$= \frac{(2f)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\log(c(a+bx)^n)}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\log(c(a+bx)^n)}\,dx}{\sqrt{e^2-4df}}$$

## Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex+fx^2)\log\left(c(a+bx)^n\right)}\,dx = \int \frac{1}{(d+ex+fx^2)\log\left(c(a+bx)^n\right)}\,dx$$

```
[In] Integrate[1/((d + e*x + f*x^2)*Log[c*(a + b*x)^n]),x]
[Out] Integrate[1/((d + e*x + f*x^2)*Log[c*(a + b*x)^n]), x]
```

## Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(f\,x^2+ex+d\right)\ln\left(c\left(bx+a\right)^n\right)}dx$$

```
[In] int(1/(f*x^2+e*x+d)/ln(c*(b*x+a)^n),x)
[Out] int(1/(f*x^2+e*x+d)/ln(c*(b*x+a)^n),x)
```

## Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex+fx^2)\log\left(c(a+bx)^n\right)}\,dx = \int \frac{1}{(fx^2+ex+d)\log\left((bx+a)^n c\right)}\,dx$$

```
[In] integrate(1/(f*x^2+e*x+d)/log(c*(b*x+a)^n),x, algorithm="fricas")
[Out] integral(1/((f*x^2 + e*x + d)*log((b*x + a)^n*c)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex+fx^2)\log\left(c(a+bx)^n\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(f*x**2+e*x+d)/ln(c*(b*x+a)**n),x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex+fx^2)\log\left(c(a+bx)^n\right)}\,dx = \int \frac{1}{(fx^2+ex+d)\log\left((bx+a)^n c\right)}\,dx$$

```
[In] integrate(1/(f*x^2+e*x+d)/log(c*(b*x+a)^n),x, algorithm="maxima")
[Out] integrate(1/((f*x^2 + e*x + d)*log((b*x + a)^n*c)), x)
```

## Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex+fx^2)\log\left(c(a+bx)^n\right)}\,dx = \int \frac{1}{(fx^2+ex+d)\log\left((bx+a)^n c\right)}\,dx$$

```
[In] integrate(1/(f*x^2+e*x+d)/log(c*(b*x+a)^n),x, algorithm="giac")
[Out] integrate(1/((f*x^2 + e*x + d)*log((b*x + a)^n*c)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex+fx^2)\log\left(c(a+bx)^n\right)}\,dx = \int \frac{1}{\ln\left(c\,{\left(a+b\,x\right)}^n\right)\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(1/(log(c*(a + b*x)^n)*(d + e*x + f*x^2)),x)
[Out] int(1/(log(c*(a + b*x)^n)*(d + e*x + f*x^2)), x)
```

# 3.351 $\int \frac{x^3 \log(x)}{a+bx+cx^2} dx$



## Optimal result

Integrand size = 18, antiderivative size = 286

$$\begin{aligned}\int \frac{x^3\log(x)}{a+bx+cx^2}\,dx &= \frac{bx}{c^2}-\frac{x^2}{4c}-\frac{bx\log(x)}{c^2}+\frac{x^2\log(x)}{2c} \\ &+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^3} \\ &+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^3} \\ &+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^3} \\ &+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^3}\end{aligned}$$

```
[Out] b*x/c^2-1/4*x^2/c-b*x*ln(x)/c^2+1/2*x^2*ln(x)/c+1/2*ln(x)*ln(1+2*c*x/(b-(-4
*a*c+b^2)^(1/2)))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^3+1/2*polyl
og(2,-2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1
/2))/c^3+1/2*ln(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(b^2-a*c+b*(-3*a*c+b^
2)/(-4*a*c+b^2)^(1/2))/c^3+1/2*polylog(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(b^
2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^3
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2404, 2332, 2341, 2354, 2438}

$$\int \frac{x^3 \log(x)}{a+bx+cx^2}\,dx = \frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\operatorname{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^3} + \frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\operatorname{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^3} + \frac{\log(x)\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{2c^3} + \frac{\log(x)\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{2c^3} + \frac{bx}{c^2} - \frac{bx\log(x)}{c^2} - \frac{x^2}{4c} + \frac{x^2\log(x)}{2c}$$

```
[In] Int[(x^3*Log[x])/(a + b*x + c*x^2),x]
```

```
[Out] (b*x)/c^2 - x^2/(4*c) - (b*x*Log[x])/c^2 + (x^2*Log[x])/(2*c) + ((b^2 - a*c
 - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^
2 - 4*a*c])])/(2*c^3) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*
Log[x]*Log[1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*c^3) + ((b^2 - a*c - (b
*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*
c])])/(2*c^3) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*PolyLog[
2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*c^3)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{b\log(x)}{c^2}+\frac{x\log(x)}{c}+\frac{(ab+(b^2-ac)\,x)\log(x)}{c^2\,(a+bx+cx^2)}\right)dx \\
&= \frac{\int \frac{(ab+(b^2-ac)x)\log(x)}{a+bx+cx^2}\,dx}{c^2}-\frac{b\int \log(x)\,dx}{c^2}+\frac{\int x\log(x)\,dx}{c} \\
&= \frac{bx}{c^2}-\frac{x^2}{4c}-\frac{bx\log(x)}{c^2}+\frac{x^2\log(x)}{2c}+\frac{\int \left(\frac{\left(b^2-ac+\frac{b(-b^2+3ac)}{\sqrt{b^2-4ac}}\right)\log(x)}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(b^2-ac-\frac{b(-b^2+3ac)}{\sqrt{b^2-4ac}}\right)\log(x)}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{c^2} \\
&= \frac{bx}{c^2}-\frac{x^2}{4c}-\frac{bx\log(x)}{c^2}+\frac{x^2\log(x)}{2c}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{\log(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{c^2} \\
&\quad+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{\log(x)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{c^2} \\
&= \frac{bx}{c^2}-\frac{x^2}{4c}-\frac{bx\log(x)}{c^2}+\frac{x^2\log(x)}{2c}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^3} \\
&\quad+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^3} \\
&\quad-\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{x}\,dx}{2c^3} \\
&\quad-\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{x}\,dx}{2c^3}
\end{aligned}
$$

$$=\frac{bx}{c^2}-\frac{x^2}{4c}-\frac{bx\log(x)}{c^2}+\frac{x^2\log(x)}{2c}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^3}$$
$$+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^3}$$
$$+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\operatorname{Li}_2\left(-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^3}+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\operatorname{Li}_2\left(-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^3}$$

## Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.62

$$\int\frac{x^3\log(x)}{a+bx+cx^2}\,dx$$
$$=\frac{4bcx-c^2x^2-4bcx\log(x)+2c^2x^2\log(x)+\frac{4abc\log(x)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}+2(b^2-ac)\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log$$

```
[In] Integrate[(x^3*Log[x])/(a + b*x + c*x^2),x]
```

```
[Out] (4*b*c*x - c^2*x^2 - 4*b*c*x*Log[x] + 2*c^2*x^2*Log[x] + (4*a*b*c*Log[x]*Lo
g[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[b^2 - 4*a*
c] + 2*(b^2 - a*c)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])] - (4*a*b*c*Log[x]*Log[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[b^2 - 4*a*c] + 2*(b^2 - a*c
)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b +
 Sqrt[b^2 - 4*a*c])] + (4*a*b*c*PolyLog[2, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])
])/Sqrt[b^2 - 4*a*c] + 2*(b^2 - a*c)*(1 - b/Sqrt[b^2 - 4*a*c])*PolyLog[2, (
2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] - (4*a*b*c*PolyLog[2, (-2*c*x)/(b + Sqrt[b
^2 - 4*a*c])])/Sqrt[b^2 - 4*a*c] + 2*(b^2 - a*c)*(1 + b/Sqrt[b^2 - 4*a*c])*
PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(4*c^3)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(258) = 516$.

Time = 1.21 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.68

| method | result |
|---|---|
| default | $\frac{\frac{x^2\ln(x)}{2}-\frac{x^2}{4}}{c}-\frac{b(\ln(x)x-x)}{c^2}+\frac{-\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}\,ac-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}\,b^2-3\ln\left(\frac{-2xc+\sqrt{-}}{-b+\sqrt{-}}\right.\right.}{}$ |
| risch | $\frac{x^2\ln(x)}{2c}-\frac{x^2}{4c}-\frac{bx\ln(x)}{c^2}+\frac{bx}{c^2}-\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)a}{2c^2}+\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b^2}{2c^3}+\frac{3\ln(x)\ln\left(\frac{-2xc+\sqrt{-}}{-b+\sqrt{-}}\right)}{2c^2\sqrt{-4ca}}$ |

```
[In] int(x^3*ln(x)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*x^2*ln(x)-1/4*x^2)-b/c^2*(ln(x)*x-x)+1/c^2*(-1/2*ln(x)*(ln((-2*x*c
+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-ln((
-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b^
2-3*ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*a*b*c+ln((-2*
x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*b^3+ln((2*x*c+(-4*a*c+b^
2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-ln((2*x*c+(-4*a*
c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b^2+3*ln((2*x*c+
(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*a*b*c-ln((2*x*c+(-4*a*c+b^2)^
(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^3)/c/(-4*a*c+b^2)^(1/2)-1/2*(dilog((-2*x
*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-di
log((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/
2)*b^2-3*dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*a*b*c
+dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*b^3+dilog((2*
x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-di
log((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)
*b^2+3*dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*a*b*c-dil
og((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^3)/c/(-4*a*c+b^2)
^(1/2))
```

## Fricas [F]

$$\int \frac{x^3 \log(x)}{a+bx+cx^2}\,dx = \int \frac{x^3 \log(x)}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^3*log(x)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(x)/(c*x^2 + b*x + a), x)
```

## Sympy [F]

$$\int \frac{x^3 \log(x)}{a+bx+cx^2}\,dx = \int \frac{x^3 \log(x)}{a+bx+cx^2}\,dx$$

```
[In] integrate(x**3*ln(x)/(c*x**2+b*x+a),x)
```

```
[Out] Integral(x**3*log(x)/(a + b*x + c*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \log(x)}{a + bx + cx^2} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*log(x)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

## Giac [F]

$$\int \frac{x^3 \log(x)}{a + bx + cx^2} \, dx = \int \frac{x^3 \log(x)}{cx^2 + bx + a} \, dx$$

```
[In] integrate(x^3*log(x)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(x)/(c*x^2 + b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log(x)}{a + bx + cx^2} \, dx = \int \frac{x^3 \ln(x)}{c\,x^2 + b\,x + a} \, dx$$

```
[In] int((x^3*log(x))/(a + b*x + c*x^2),x)
```

```
[Out] int((x^3*log(x))/(a + b*x + c*x^2), x)
```

# 3.352 $\int \frac{x^2 \log(x)}{a+bx+cx^2} dx$



## Optimal result

Integrand size = 18, antiderivative size = 234

$$\int \frac{x^2 \log(x)}{a+bx+cx^2} dx = -\frac{x}{c} + \frac{x\log(x)}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^2}$$

```
[Out] -x/c+x*ln(x)/c-1/2*ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(b+(2*a*c-b^2)/
(-4*a*c+b^2)^(1/2))/c^2-1/2*polylog(2,-2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(b+(2*
a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^2-1/2*ln(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)
))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2-1/2*polylog(2,-2*c*x/(b+(-4*a*c+
b^2)^(1/2)))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used

$= \{2404, 2332, 2354, 2438\}$

$$\int \frac{x^2 \log(x)}{a+bx+cx^2}\,dx = -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{\log(x)\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{2c^2} - \frac{\log(x)\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{2c^2} - \frac{x}{c} + \frac{x\log(x)}{c}$$

```
[In] Int[(x^2*Log[x])/(a + b*x + c*x^2),x]
```

```
[Out] -(x/c) + (x*Log[x])/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1
 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*c^2) - ((b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*c^2) - ((b -
 (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c
])])/(2*c^2) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(
b + Sqrt[b^2 - 4*a*c])])/(2*c^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log(x)}{c} - \frac{(a+bx)\log(x)}{c\,(a+bx+cx^2)}\right) dx$$

$$= \frac{\int \log(x)\,dx}{c} - \frac{\int \frac{(a+bx)\log(x)}{a+bx+cx^2}\,dx}{c}$$

$$= -\frac{x}{c} + \frac{x\log(x)}{c} - \frac{\int \left(\frac{\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\log(x)}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(b-\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\log(x)}{b+\sqrt{b^2-4ac}+2cx}\right) dx}{c}$$

$$= -\frac{x}{c} + \frac{x\log(x)}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{\log(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{\log(x)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{c}$$

$$= -\frac{x}{c} + \frac{x\log(x)}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^2} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{x}\,dx}{2c^2} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{x}\,dx}{2c^2}$$

$$= -\frac{x}{c} + \frac{x\log(x)}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Li}_2\left(-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Li}_2\left(-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^2}$$

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.85

$$\int \frac{x^2\log(x)}{a+bx+cx^2}\,dx = -\frac{x}{c}+\frac{x\log(x)}{c}-\frac{a\log(x)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} - \frac{b\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^2} + \frac{a\log(x)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} - \frac{b\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{a\,\mathrm{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} - \frac{b\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\mathrm{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c^2} + \frac{a\,\mathrm{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} - \frac{b\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\mathrm{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c^2}$$

```
[In] Integrate[(x^2*Log[x])/(a + b*x + c*x^2),x]
```

```
[Out] -(x/c) + (x*Log[x])/c - (a*Log[x]*Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b -
Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]) - (b*(1 - b/Sqrt[b^2 - 4*a*c])*L
og[x]*Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*c^2)
 + (a*Log[x]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/
(c*Sqrt[b^2 - 4*a*c]) - (b*(1 + b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*c^2) - (a*PolyLog[2, (-2*
c*x)/(b - Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]) - (b*(1 - b/Sqrt[b^2 -
 4*a*c])*PolyLog[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*c^2) + (a*PolyLog
[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]) - (b*(1 + b/Sq
rt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*c^2)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(212) = 424$.

Time = 1.10 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.32

| method | result |
|---|---|
| default | $\frac{\ln(x)x-x}{c}+\frac{-\frac{\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}\,b+2\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)ac-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b^2+\ln\left(\frac{2xc+\sqrt{-4ca+b^2}}{b+\sqrt{-4ca+b^2}}\right.}{2c\sqrt{-4ca+b^2}}}{}$ |
| risch | $\frac{x\ln(x)}{c}-\frac{x}{c}-\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b}{2c^2}-\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)a}{c\sqrt{-4ca+b^2}}+\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b^2}{2c^2\sqrt{-4ca+b^2}}-\ln(x$ |

```
[In] int(x^2*ln(x)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(ln(x)*x-x)+1/c*(-1/2*ln(x)*(ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a
*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b+2*ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b
+(-4*a*c+b^2)^(1/2)))*a*c-ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)
^(1/2)))*b^2+ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*
c+b^2)^(1/2)*b-2*ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*a*
c+ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^2)/c/(-4*a*c+b^
2)^(1/2)-1/2*(dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*
(-4*a*c+b^2)^(1/2)*b+2*dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)
^(1/2)))*a*c-dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*b
^2+dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^
(1/2)*b-2*dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*a*c+di
log((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^2)/c/(-4*a*c+b^2
)^(1/2))
```

## Fricas [F]

$$\int \frac{x^2\log(x)}{a+bx+cx^2}\,dx = \int \frac{x^2\log{(x)}}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^2*log(x)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(x)/(c*x^2 + b*x + a), x)
```

## Sympy [F]

$$\int \frac{x^2 \log(x)}{a + bx + cx^2} \, dx = \int \frac{x^2 \log{(x)}}{a + bx + cx^2} \, dx$$

```
[In] integrate(x**2*ln(x)/(c*x**2+b*x+a),x)
```

```
[Out] Integral(x**2*log(x)/(a + b*x + c*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \log(x)}{a + bx + cx^2} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*log(x)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

## Giac [F]

$$\int \frac{x^2 \log(x)}{a + bx + cx^2} \, dx = \int \frac{x^2 \log{(x)}}{cx^2 + bx + a} \, dx$$

```
[In] integrate(x^2*log(x)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(x)/(c*x^2 + b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log(x)}{a + bx + cx^2} \, dx = \int \frac{x^2 \ln{(x)}}{c\,x^2 + b\,x + a} \, dx$$

```
[In] int((x^2*log(x))/(a + b*x + c*x^2),x)
```

```
[Out] int((x^2*log(x))/(a + b*x + c*x^2), x)
```

# 3.353 $\int \frac{x \log(x)}{a+bx+cx^2} \, dx$



## Optimal result

Integrand size = 16, antiderivative size = 193

$$\int \frac{x \log(x)}{a+bx+cx^2} \, dx = \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right) \log(x) \log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right) \log(x) \log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c} + \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right) \text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right) \text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c}$$

```
[Out] 1/2*ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(1-b/(-4*a*c+b^2)^(1/2))/c+1/2
*polylog(2,-2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(1-b/(-4*a*c+b^2)^(1/2))/c+1/2*ln
(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(1+b/(-4*a*c+b^2)^(1/2))/c+1/2*polyl
og(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(1+b/(-4*a*c+b^2)^(1/2))/c
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used

$= \{2404, 2354, 2438\}$

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c} + \frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c} + \frac{\log(x)\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{2c} + \frac{\log(x)\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{2c}$$

```
[In] Int[(x*Log[x])/(a + b*x + c*x^2),x]
```

```
[Out] ((1 - b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])])
/(2*c) + ((1 + b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b + Sqrt[b^2 -
4*a*c])])/(2*c) + ((1 - b/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b - Sqrt[
b^2 - 4*a*c])])/(2*c) + ((1 + b/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b +
 Sqrt[b^2 - 4*a*c])])/(2*c)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)}{b+\sqrt{b^2-4ac}+2cx}\right)dx$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{\log(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx + \left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{\log(x)}{b+\sqrt{b^2-4ac}+2cx}\,dx$$

$$= \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c} - \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{x}dx}{2c} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{x}dx}{2c}$$

$$= \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c} + \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\mathrm{Li}_2\left(-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2c} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\mathrm{Li}_2\left(-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2c}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.09

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \frac{\log(x)\left(\left(-b+\sqrt{b^2-4ac}\right)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)+\left(b+\sqrt{b^2-4ac}\right)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)\right)+\left(-b+\sqrt{b^2-4\right.}{2c\sqrt{b^2-4ac}}$$

```
[In] Integrate[(x*Log[x])/(a + b*x + c*x^2),x]
```

```
[Out] (Log[x]*((-b + Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b -
Sqrt[b^2 - 4*a*c])] + (b + Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/(b + Sqrt[b^2 - 4*a*c])]) + (-b + Sqrt[b^2 - 4*a*c])*PolyLog[2, (2*c
*x)/(-b + Sqrt[b^2 - 4*a*c])] + (b + Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)
/(b + Sqrt[b^2 - 4*a*c])])/(2*c*Sqrt[b^2 - 4*a*c])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(169) = 338.

Time = 1.05 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.87

| method | result |
|---|---|
| risch | $\frac{\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b+\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}+\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right.\right.}{2c\sqrt{-4ca+b^2}}$ |
| default | $\frac{\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b+\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}+\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right.\right.}{2c\sqrt{-4ca+b^2}}$ |

```
[In] int(x*ln(x)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x)*(ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*
c+b^2)^(1/2)-ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*b+ln
((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)+ln
((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b)/c/(-4*a*c+b^2)^(1/
2)+1/2/c*dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))-1/2/c
/(-4*a*c+b^2)^(1/2)*dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1
/2)))*b+1/2/c*dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))+1/
2/c/(-4*a*c+b^2)^(1/2)*dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(
1/2)))*b
```

## Fricas [F]

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \int \frac{x\log\left(x\right)}{cx^2+bx+a}\,dx$$

```
[In] integrate(x*log(x)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*log(x)/(c*x^2 + b*x + a), x)
```

## Sympy [F]

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \int \frac{x\log\left(x\right)}{a+bx+cx^2}\,dx$$

```
[In] integrate(x*ln(x)/(c*x**2+b*x+a),x)
```

```
[Out] Integral(x*log(x)/(a + b*x + c*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*log(x)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

## Giac [F]

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \int \frac{x\log\left(x\right)}{cx^2+bx+a}\,dx$$

```
[In] integrate(x*log(x)/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate(x*log(x)/(c*x^2 + b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\log(x)}{a+bx+cx^2}\,dx = \int \frac{x\,\ln\left(x\right)}{c\,x^2+b\,x+a}\,dx$$

```
[In] int((x*log(x))/(a + b*x + c*x^2),x)
[Out] int((x*log(x))/(a + b*x + c*x^2), x)
```

# 3.354 $\int \frac{\log(x)}{a+bx+cx^2} dx$



## Optimal result

Integrand size = 15, antiderivative size = 153

$$\int \frac{\log(x)}{a+bx+cx^2} dx = \frac{\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} - \frac{\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} + \frac{\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} - \frac{\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)-ln(x)*ln(1+2*c*
x/(b+(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+polylog(2,-2*c*x/(b-(-4*a*c+b^
2)^(1/2)))/(-4*a*c+b^2)^(1/2)-polylog(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/(-4*
a*c+b^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2404, 2354, 2438}

$$\int \frac{\log(x)}{a+bx+cx^2} dx = \frac{\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} - \frac{\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} + \frac{\log(x)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{\sqrt{b^2-4ac}} - \frac{\log(x)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{\sqrt{b^2-4ac}}$$

```
[In] Int[Log[x]/(a + b*x + c*x^2),x]
```

```
[Out] (Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[b^2 - 4*a*c] - (Log[
x]*Log[1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[b^2 - 4*a*c] + PolyLog[2,
```

```
 (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[b^2 - 4*a*c] - PolyLog[2, (-2*c*x)/
(b + Sqrt[b^2 - 4*a*c])]/Sqrt[b^2 - 4*a*c]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2c\log(x)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)}-\frac{2c\log(x)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)dx\\
&= \frac{(2c)\int \frac{\log(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{\log(x)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\\
&\quad -\frac{\int \frac{\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{x}\,dx}{\sqrt{b^2-4ac}}+\frac{\int \frac{\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{x}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\\
&\quad +\frac{\text{Li}_2\left(-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{\text{Li}_2\left(-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.94

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx = \frac{\log(x)\left(\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)-\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)\right)+\text{PolyLog}\left(2,\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)-\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Integrate[Log[x]/(a + b*x + c*x^2),x]
```

```
[Out] (Log[x]*(Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])] - Log
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])]) + PolyLog[2, (2*
c*x)/(-b + Sqrt[b^2 - 4*a*c])] - PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]
)])/Sqrt[b^2 - 4*a*c]
```

## Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10

| method | result | size |
|---|---|---|
| default | $-\frac{\ln(x)\left(\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\right)}{\sqrt{-4ca+b^2}}+\frac{\operatorname{dilog}\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)-\operatorname{dilog}\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)}{\sqrt{-4ca+b^2}}$ | 169 |
| risch | $-\frac{\ln(x)\left(\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\right)}{\sqrt{-4ca+b^2}}+\frac{\operatorname{dilog}\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)}{\sqrt{-4ca+b^2}}-\frac{\operatorname{dilog}\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)}{\sqrt{-4ca+b^2}}$ | 178 |

```
[In] int(ln(x)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)*(ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))-ln((-2*x*c+
(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)+(dilog((
-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))-dilog((2*x*c+(-4*a*c+
b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)
```

## Fricas [F]

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx = \int \frac{\log(x)}{cx^2+bx+a}\,dx$$

```
[In] integrate(log(x)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(x)/(c*x^2 + b*x + a), x)
```

## Sympy [F]

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx=\int \frac{\log{(x)}}{a+bx+cx^2}\,dx$$

```
[In] integrate(ln(x)/(c*x**2+b*x+a),x)
```

```
[Out] Integral(log(x)/(a + b*x + c*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(log(x)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

## Giac [F]

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx=\int \frac{\log{(x)}}{cx^2+bx+a}\,dx$$

```
[In] integrate(log(x)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(x)/(c*x^2 + b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{a+bx+cx^2}\,dx=\int \frac{\ln{(x)}}{c\,x^2+b\,x+a}\,dx$$

```
[In] int(log(x)/(a + b*x + c*x^2),x)
```

```
[Out] int(log(x)/(a + b*x + c*x^2), x)
```

# 3.355 $\int \frac{\log(x)}{x(a+bx+cx^2)}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 204

$$\int \frac{\log(x)}{x(a+bx+cx^2)}\,dx = \frac{\log^2(x)}{2a} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a} - \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a} - \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a}$$

```
[Out] 1/2*ln(x)^2/a-1/2*ln(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(1-b/(-4*a*c+b^2
)^(1/2))/a-1/2*polylog(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(1-b/(-4*a*c+b^2)^(
1/2))/a-1/2*ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(1+b/(-4*a*c+b^2)^(1/2
))/a-1/2*polylog(2,-2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(1+b/(-4*a*c+b^2)^(1/2))/
a
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used

= {2404, 2338, 2354, 2438}

$$\int \frac{\log(x)}{x(a+bx+cx^2)}\,dx = -\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a} - \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a} - \frac{\log(x)\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{2a} - \frac{\log(x)\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{2a} + \frac{\log^2(x)}{2a}$$

```
[In] Int[Log[x]/(x*(a + b*x + c*x^2)),x]
```

```
[Out] Log[x]^2/(2*a) - ((1 + b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b - Sqr
t[b^2 - 4*a*c])])/(2*a) - ((1 - b/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)
/(b + Sqrt[b^2 - 4*a*c])])/(2*a) - ((1 + b/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-
2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*a) - ((1 - b/Sqrt[b^2 - 4*a*c])*PolyLog
[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*a)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log(x)}{ax} + \frac{(-b-cx)\log(x)}{a(a+bx+cx^2)}\right) dx$$

$$= \frac{\int \frac{\log(x)}{x}\,dx}{a} + \frac{\int \frac{(-b-cx)\log(x)}{a+bx+cx^2}\,dx}{a}$$

$$= \frac{\log^2(x)}{2a} + \frac{\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)\log(x)}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(-c+\frac{bc}{\sqrt{b^2-4ac}}\right)\log(x)}{b+\sqrt{b^2-4ac}+2cx}\right) dx}{a}$$

$$= \frac{\log^2(x)}{2a} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\log(x)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\log(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{a}$$

$$= \frac{\log^2(x)}{2a} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a} - \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a} + \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{x}\,dx}{2a} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{x}\,dx}{2a}$$

$$= \frac{\log^2(x)}{2a} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a} - \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Li}_2\left(-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a} - \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Li}_2\left(-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a}$$

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.11

$$\int \frac{\log(x)}{x(a+bx+cx^2)}\,dx = \frac{\log(x)\left(\sqrt{b^2-4ac}\log(x) - \left(b+\sqrt{b^2-4ac}\right)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right) + \left(b-\sqrt{b^2-4ac}\right)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)\right) -}{2a\sqrt{b^2-4ac}}$$

```
[In] Integrate[Log[x]/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (Log[x]*(Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b
^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Log
```

```
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])]) - (b + Sqrt[b^2
- 4*a*c])*PolyLog[2, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*
a*c])*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[b^2 - 4*a*c])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(178) = 356$.

Time = 1.07 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.81

| method | result |
|---|---|
| default | $\frac{\ln(x)^2}{2a} + \frac{-\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}+\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b+\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}-\ln\left(\frac{2xc+\sqrt{-4ca+}}{b+\sqrt{-4ca+}}\right.}{2\sqrt{-4ca+b^2}}$ |
| risch | $\frac{\ln(x)^2}{2a} - \frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)}{2a} - \frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b}{2a\sqrt{-4ca+b^2}} - \frac{\ln(x)\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)}{2a} + \frac{\ln(x)\ln\left(\frac{2xc+\sqrt{}}{b+\sqrt{}}\right.}{2a\sqrt{-4c}}$ |

```
[In] int(ln(x)/x/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*ln(x)^2+1/a*(-1/2*ln(x)*(ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c
+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)+ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a
*c+b^2)^(1/2)))*b+ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(
-4*a*c+b^2)^(1/2)-ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b
)/(-4*a*c+b^2)^(1/2)-1/2*(dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b
^2)^(1/2)))*(-4*a*c+b^2)^(1/2)+dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*
a*c+b^2)^(1/2)))*b+dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)
))*(-4*a*c+b^2)^(1/2)-dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1
/2)))*b)/(-4*a*c+b^2)^(1/2))
```

## Fricas [F]

$$\int \frac{\log(x)}{x\left(a+bx+cx^2\right)}\,dx = \int \frac{\log\left(x\right)}{(cx^2+bx+a)x}\,dx$$

```
[In] integrate(log(x)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(x)/(c*x^3 + b*x^2 + a*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(x)}{x\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(x)/x/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(x)}{x\,(a+bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(x)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

## Giac [F]

$$\int \frac{\log(x)}{x\,(a+bx+cx^2)}\,dx = \int \frac{\log(x)}{(cx^2+bx+a)x}\,dx$$

```
[In] integrate(log(x)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(x)/((c*x^2 + b*x + a)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{x\,(a+bx+cx^2)}\,dx = \int \frac{\ln(x)}{x\,(c\,x^2+b\,x+a)}\,dx$$

```
[In] int(log(x)/(x*(a + b*x + c*x^2)),x)
```

```
[Out] int(log(x)/(x*(a + b*x + c*x^2)), x)
```

# 3.356 $\int \frac{\log(x)}{x^2(a+bx+cx^2)}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 251

$$\int \frac{\log(x)}{x^2(a+bx+cx^2)}\,dx = -\frac{1}{ax}-\frac{\log(x)}{ax}-\frac{b\log^2(x)}{2a^2} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^2} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^2} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^2} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^2}$$

```
[Out] -1/a/x-ln(x)/a/x-1/2*b*ln(x)^2/a^2+1/2*ln(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1/
2)))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2+1/2*polylog(2,-2*c*x/(b+(-4*a*c
+b^2)^(1/2)))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2+1/2*ln(x)*ln(1+2*c*x/(
b-(-4*a*c+b^2)^(1/2)))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2+1/2*polylog(
2,-2*c*x/(b-(-4*a*c+b^2)^(1/2)))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2404, 2341, 2338, 2354, 2438}

$$\int \frac{\log(x)}{x^2(a+bx+cx^2)}\,dx = \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^2} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^2} + \frac{\log(x)\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{2a^2} + \frac{\log(x)\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{2a^2} - \frac{b\log^2(x)}{2a^2} - \frac{1}{ax} - \frac{\log(x)}{ax}$$

```
[In] Int[Log[x]/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -(1/(a*x)) - Log[x]/(a*x) - (b*Log[x]^2)/(2*a^2) + ((b + (b^2 - 2*a*c)/Sqrt
[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2) + (
(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b + Sqrt[b^2
- 4*a*c])])/(2*a^2) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2
*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*
a*c])*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\log(x)}{ax^2}-\frac{b\log(x)}{a^2x}+\frac{(b^2-ac+bcx)\log(x)}{a^2(a+bx+cx^2)}\right)dx\\
&= \frac{\int \frac{(b^2-ac+bcx)\log(x)}{a+bx+cx^2}\,dx}{a^2}+\frac{\int \frac{\log(x)}{x^2}\,dx}{a}-\frac{b\int \frac{\log(x)}{x}\,dx}{a^2}\\
&= -\frac{1}{ax}-\frac{\log(x)}{ax}-\frac{b\log^2(x)}{2a^2}+\frac{\int \left(\frac{\left(bc+\frac{c(b^2-2ac)}{\sqrt{b^2-4ac}}\right)\log(x)}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(bc-\frac{c(b^2-2ac)}{\sqrt{b^2-4ac}}\right)\log(x)}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{a^2}\\
&= -\frac{1}{ax}-\frac{\log(x)}{ax}-\frac{b\log^2(x)}{2a^2}+\frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\log(x)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{a^2}\\
&\quad+\frac{\left(c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\log(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{a^2}\\
&= -\frac{1}{ax}-\frac{\log(x)}{ax}-\frac{b\log^2(x)}{2a^2}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^2}\\
&\quad+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^2}\\
&\quad-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{x}\,dx}{2a^2}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{x}\,dx}{2a^2}\\
&= -\frac{1}{ax}-\frac{\log(x)}{ax}-\frac{b\log^2(x)}{2a^2}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^2}\\
&\quad+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^2}\\
&\quad+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Li}_2\left(-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^2}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Li}_2\left(-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^2}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.02

$$\int \frac{\log(x)}{x^2\left(a+bx+cx^2\right)}\,dx = \frac{-\frac{2a}{x}-\frac{2a\log(x)}{x}-b\log^2(x)+\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)+\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(\frac{b+\sqrt{b^2-4a}}{b+\sqrt{b^2-}}\right.}{2a^2}$$

```
[In] Integrate[Log[x]/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] ((-2*a)/x - (2*a*Log[x])/x - b*Log[x]^2 + (b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a
*c])*Log[x]*Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])] +
(b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[x]*Log[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/(b + Sqrt[b^2 - 4*a*c])] + (b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Pol
yLog[2, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (b + (-b^2 + 2*a*c)/Sqrt[b^2 -
4*a*c])*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(227) = 454.

Time = 1.18 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.20

| method | result |
|---|---|
| default | $-\frac{b\ln(x)^2}{2a^2}+\frac{\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}\,b-2\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)ac+\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b^2+\ln\left(\frac{2xc+\sqrt{-4ca+b^2}}{b+\sqrt{-4ca+b^2}}\right.\right.}{2\sqrt{-4ca+b^2}}$ |
| risch | $-\frac{b\ln(x)^2}{2a^2}+\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b}{2a^2}-\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)c}{a\sqrt{-4ca+b^2}}+\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b^2}{2a^2\sqrt{-4ca+b^2}}+\frac{\ln(x)\ln\left(\right.}{}$ |

```
[In] int(ln(x)/x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*ln(x)^2/a^2+1/a^2*(1/2*ln(x)*(ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(
-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b-2*ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)
/(-b+(-4*a*c+b^2)^(1/2)))*a*c+ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+
b^2)^(1/2)))*b^2+ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-
4*a*c+b^2)^(1/2)*b+2*ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))
)*a*c-ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^2)/(-4*a*c+
b^2)^(1/2)+1/2*(dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2))
)*(-4*a*c+b^2)^(1/2)*b-2*dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^
2)^(1/2)))*a*c+dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))
*b^2+dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2
)^(1/2)*b+2*dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*a*c-
```

```
dilog((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^2)/(-4*a*c+b^2
)^(1/2))+1/a*(-1/x*ln(x)-1/x)
```

## Fricas [F]

$$\int \frac{\log(x)}{x^2(a+bx+cx^2)}\,dx = \int \frac{\log(x)}{(cx^2+bx+a)x^2}\,dx$$

```
[In] integrate(log(x)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral(log(x)/(c*x^4 + b*x^3 + a*x^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(x)}{x^2(a+bx+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(x)/x**2/(c*x**2+b*x+a),x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(x)}{x^2(a+bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(x)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

## Giac [F]

$$\int \frac{\log(x)}{x^2 (a + bx + cx^2)} \, dx = \int \frac{\log(x)}{(cx^2 + bx + a)x^2} \, dx$$

```
[In] integrate(log(x)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate(log(x)/((c*x^2 + b*x + a)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{x^2 (a + bx + cx^2)} \, dx = \int \frac{\ln(x)}{x^2 \left(c\,x^2 + b\,x + a\right)} \, dx$$

```
[In] int(log(x)/(x^2*(a + b*x + c*x^2)),x)
[Out] int(log(x)/(x^2*(a + b*x + c*x^2)), x)
```

# 3.357 $\int \frac{\log(x)}{x^3(a+bx+cx^2)}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 308

$$\int \frac{\log(x)}{x^3(a+bx+cx^2)}\,dx = -\frac{1}{4ax^2}+\frac{b}{a^2x}-\frac{\log(x)}{2ax^2}+\frac{b\log(x)}{a^2x}+\frac{(b^2-ac)\log^2(x)}{2a^3} - \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^3} - \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^3} - \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\operatorname{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^3} - \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\operatorname{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^3}$$

```
[Out] -1/4/a/x^2+b/a^2/x-1/2*ln(x)/a/x^2+b*ln(x)/a^2/x+1/2*(-a*c+b^2)*ln(x)^2/a^3
-1/2*ln(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a
*c+b^2)^(1/2))/a^3-1/2*polylog(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2)))*(b^2-a*c-b*
(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^3-1/2*ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^2)^(
1/2)))*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^3-1/2*polylog(2,-2*c*x
/(b-(-4*a*c+b^2)^(1/2)))*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^3
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2404, 2341, 2338, 2354, 2438}

$$\int \frac{\log(x)}{x^3(a+bx+cx^2)}\,dx = -\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^3} - \frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^3} + \frac{\log^2(x)\,(b^2-ac)}{2a^3} - \frac{\log(x)\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)}{2a^3} - \frac{\log(x)\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)}{2a^3} + \frac{b}{a^2x} + \frac{b\log(x)}{a^2x} - \frac{1}{4ax^2} - \frac{\log(x)}{2ax^2}$$

```
[In] Int[Log[x]/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/4*1/(a*x^2) + b/(a^2*x) - Log[x]/(2*a*x^2) + (b*Log[x])/(a^2*x) + ((b^2
- a*c)*Log[x]^2)/(2*a^3) - ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c
])*Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^3) - ((b^2 - a*c -
 (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Log[x]*Log[1 + (2*c*x)/(b + Sqrt[b^2
- 4*a*c])])/(2*a^3) - ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Po
lyLog[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^3) - ((b^2 - a*c - (b*(b^2
 - 3*a*c))/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])
/(2*a^3)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
```

```
Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

integral

$$= \int \left(\frac{\log(x)}{ax^3} - \frac{b\log(x)}{a^2x^2} + \frac{(b^2-ac)\log(x)}{a^3x} + \frac{(-b(b^2-2ac)-c(b^2-ac)x)\log(x)}{a^3(a+bx+cx^2)}\right)dx$$

$$= \frac{\int \frac{(-b(b^2-2ac)-c(b^2-ac)x)\log(x)}{a+bx+cx^2}\,dx}{a^3} + \frac{\int \frac{\log(x)}{x^3}\,dx}{a} - \frac{b\int \frac{\log(x)}{x^2}\,dx}{a^2} + \frac{(b^2-ac)\int \frac{\log(x)}{x}\,dx}{a^3}$$

$$= -\frac{1}{4ax^2} + \frac{b}{a^2x} - \frac{\log(x)}{2ax^2} + \frac{b\log(x)}{a^2x} + \frac{(b^2-ac)\log^2(x)}{2a^3} + \frac{\int \left(\frac{\left(-\frac{bc(b^2-3ac)}{\sqrt{b^2-4ac}}-c(b^2-ac)\right)\log(x)}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(\frac{bc(b^2-3ac)}{\sqrt{b^2-4ac}}-c(b^2-ac)\right)\log(x)}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{a^3}$$

$$= -\frac{1}{4ax^2} + \frac{b}{a^2x} - \frac{\log(x)}{2ax^2} + \frac{b\log(x)}{a^2x} + \frac{(b^2-ac)\log^2(x)}{2a^3} - \frac{\left(c\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\log(x)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{a^3} - \frac{\left(c\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\log(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{a^3}$$

$$= -\frac{1}{4ax^2} + \frac{b}{a^2x} - \frac{\log(x)}{2ax^2} + \frac{b\log(x)}{a^2x} + \frac{(b^2-ac)\log^2(x)}{2a^3} - \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^3} - \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^3} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int\frac{\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{x}dx}{2a^3} + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int\frac{\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{x}dx}{2a^3}$$

$$= -\frac{1}{4ax^2} + \frac{b}{a^2x} - \frac{\log(x)}{2ax^2} + \frac{b\log(x)}{a^2x} + \frac{(b^2-ac)\log^2(x)}{2a^3} - \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^3} - \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^3} - \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\operatorname{Li}_2\left(-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{2a^3} - \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\operatorname{Li}_2\left(-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{2a^3}$$

## Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.01

$$\int \frac{\log(x)}{x^3(a+bx+cx^2)}\,dx = -\frac{\frac{a^2}{x^2} - \frac{4ab}{x} + \frac{2a^2\log(x)}{x^2} - \frac{4ab\log(x)}{x} - 2(b^2-ac)\log^2(x) + 2\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\log(x)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right) +$$

```
[In] Integrate[Log[x]/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/4*(a^2/x^2 - (4*a*b)/x + (2*a^2*Log[x])/x^2 - (4*a*b*Log[x])/x - 2*(b^2
- a*c)*Log[x]^2 + 2*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Log[x
]*Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])] + 2*(b^2 - a
*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Log[x]*Log[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])] + 2*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[
b^2 - 4*a*c])*PolyLog[2, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(b^2 - a*c -
 (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4
*a*c])])/a^3
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(278) = 556$.

Time = 1.16 (sec) , antiderivative size = 782, normalized size of antiderivative = 2.54

| method | result |
|---|---|
| default | $\frac{(-ca+b^2)\ln(x)^2}{2a^3}+\frac{\ln(x)\left(\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}\,ac-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\sqrt{-4ca+b^2}\,b^2+3\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)ab}{}$ |
| risch | $-\frac{\ln(x)^2c}{2a^2}+\frac{\ln(x)^2b^2}{2a^3}+\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)c}{2a^2}-\frac{\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)b^2}{2a^3}+\frac{3\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right.}{2a^2\sqrt{-4ca+b^2}}$ |

```
[In] int(ln(x)/x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-a*c+b^2)*ln(x)^2/a^3+1/a^3*(1/2*ln(x)*(ln((-2*x*c+(-4*a*c+b^2)^(1/2)-
b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-ln((-2*x*c+(-4*a*c+b^2)^
(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b^2+3*ln((-2*x*c+(-4*a
*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*a*b*c-ln((-2*x*c+(-4*a*c+b^2)^(1/
2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*b^3+ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a
*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(
-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b^2-3*ln((2*x*c+(-4*a*c+b^2)^(1/2)+b
)/(b+(-4*a*c+b^2)^(1/2)))*a*b*c+ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+
b^2)^(1/2)))*b^3)/(-4*a*c+b^2)^(1/2)+1/2*(dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-
b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-dilog((-2*x*c+(-4*a*c+b^
2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b^2+3*dilog((-2*x*c
+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*a*b*c-dilog((-2*x*c+(-4*a*c
+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*b^3+dilog((2*x*c+(-4*a*c+b^2)^(1/2)
+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*a*c-dilog((2*x*c+(-4*a*c+b^2
)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)*b^2-3*dilog((2*x*c+(-
4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*a*b*c+dilog((2*x*c+(-4*a*c+b^2)
^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b^3)/(-4*a*c+b^2)^(1/2))+1/a*(-1/2/x^2*ln
(x)-1/4/x^2)-1/a^2*b*(-1/x*ln(x)-1/x)
```

## Fricas [F]

$$\int \frac{\log(x)}{x^3\,(a+bx+cx^2)}\,dx=\int \frac{\log(x)}{(cx^2+bx+a)x^3}\,dx$$

```
[In] integrate(log(x)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(x)/(c*x^5 + b*x^4 + a*x^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(x)}{x^3 (a + bx + cx^2)} dx = \text{Timed out}$$

```
[In] integrate(ln(x)/x**3/(c*x**2+b*x+a),x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(x)}{x^3 (a + bx + cx^2)} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(x)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

## Giac [F]

$$\int \frac{\log(x)}{x^3 (a + bx + cx^2)} dx = \int \frac{\log(x)}{(cx^2 + bx + a)x^3} dx$$

```
[In] integrate(log(x)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate(log(x)/((c*x^2 + b*x + a)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{x^3 (a + bx + cx^2)} dx = \int \frac{\ln(x)}{x^3 (c x^2 + b x + a)} dx$$

```
[In] int(log(x)/(x^3*(a + b*x + c*x^2)),x)
[Out] int(log(x)/(x^3*(a + b*x + c*x^2)), x)
```

# 3.358 $\int x^3 \log(fx^m)(a+b\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 232

$$\int x^3\log(fx^m)(a+b\log(c(d+ex)^n))\,dx = -\frac{5bd^3mnx}{16e^3}+\frac{3bd^2mnx^2}{32e^2}-\frac{7bdmnx^3}{144e}+\frac{1}{32}bmnx^4+\frac{bd^3nx\log(fx^m)}{4e^3}-\frac{bd^2nx^2\log(fx^m)}{8e^2}+\frac{bdnx^3\log(fx^m)}{12e}-\frac{1}{16}bnx^4\log(fx^m)+\frac{bd^4mn\log(d+ex)}{16e^4}-\frac{1}{16}\left(mx^4-4x^4\log(fx^m)\right)(a+b\log(c(d+ex)^n))-\frac{bd^4n\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{4e^4}-\frac{bd^4mn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{4e^4}$$

```
[Out] -5/16*b*d^3*m*n*x/e^3+3/32*b*d^2*m*n*x^2/e^2-7/144*b*d*m*n*x^3/e+1/32*b*m*n
*x^4+1/4*b*d^3*n*x*ln(f*x^m)/e^3-1/8*b*d^2*n*x^2*ln(f*x^m)/e^2+1/12*b*d*n*x
^3*ln(f*x^m)/e-1/16*b*n*x^4*ln(f*x^m)+1/16*b*d^4*m*n*ln(e*x+d)/e^4-1/16*(m*
x^4-4*x^4*ln(f*x^m))*(a+b*ln(c*(e*x+d)^n))-1/4*b*d^4*n*ln(f*x^m)*ln(1+e*x/d
)/e^4-1/4*b*d^4*m*n*polylog(2,-e*x/d)/e^4
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2473, 45, 2393, 2332, 2341, 2354, 2438}

$$\int x^3\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx=-\frac{1}{16}\left(mx^4-4x^4\log\left(fx^m\right)\right)\left(a+b\log\left(c(d+ex)^n\right)\right)-\frac{bd^4n\log\left(\frac{ex}{d}+1\right)\log\left(fx^m\right)}{4e^4}-\frac{bd^4mn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{4e^4}+\frac{bd^4mn\log(d+ex)}{16e^4}+\frac{bd^3nx\log\left(fx^m\right)}{4e^3}-\frac{5bd^3mnx}{16e^3}-\frac{bd^2nx^2\log\left(fx^m\right)}{8e^2}+\frac{3bd^2mnx^2}{32e^2}+\frac{bdnx^3\log\left(fx^m\right)}{12e}-\frac{7bdmnx^3}{144e}-\frac{1}{16}bnx^4\log\left(fx^m\right)+\frac{1}{32}bmnx^4$$

```
[In] Int[x^3*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (-5*b*d^3*m*n*x)/(16*e^3) + (3*b*d^2*m*n*x^2)/(32*e^2) - (7*b*d*m*n*x^3)/(1
44*e) + (b*m*n*x^4)/32 + (b*d^3*n*x*Log[f*x^m])/(4*e^3) - (b*d^2*n*x^2*Log[
f*x^m])/(8*e^2) + (b*d*n*x^3*Log[f*x^m])/(12*e) - (b*n*x^4*Log[f*x^m])/16 +
 (b*d^4*m*n*Log[d + e*x])/(16*e^4) - ((m*x^4 - 4*x^4*Log[f*x^m])*(a + b*Log
[c*(d + e*x)^n]))/16 - (b*d^4*n*Log[f*x^m]*Log[1 + (e*x)/d])/(4*e^4) - (b*d
^4*m*n*PolyLog[2, -((e*x)/d)])/(4*e^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2473

```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Dist[b*e*(n/(g*(q + 1))), Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x], x
] + Dist[b*e*m*(n/(g*(q + 1)^2)), Int[(g*x)^(q + 1)/(d + e*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{16}\left(mx^4-4x^4\log\left(fx^m\right)\right)\left(a+b\log\left(c(d+ex)^n\right)\right) \\
&\quad -\frac{1}{4}(ben)\int\frac{x^4\log\left(fx^m\right)}{d+ex}\,dx+\frac{1}{16}(bemn)\int\frac{x^4}{d+ex}\,dx \\
&= -\frac{1}{16}\left(mx^4-4x^4\log\left(fx^m\right)\right)\left(a+b\log\left(c(d+ex)^n\right)\right)-\frac{1}{4}(ben)\int\left(-\frac{d^3\log\left(fx^m\right)}{e^4}\right. \\
&\quad \left.+\frac{d^2x\log\left(fx^m\right)}{e^3}-\frac{dx^2\log\left(fx^m\right)}{e^2}+\frac{x^3\log\left(fx^m\right)}{e}+\frac{d^4\log\left(fx^m\right)}{e^4(d+ex)}\right)dx \\
&\quad +\frac{1}{16}(bemn)\int\left(-\frac{d^3}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{e^2}+\frac{x^3}{e}+\frac{d^4}{e^4(d+ex)}\right)dx
\end{aligned}
$$

$$= -\frac{bd^3mnx}{16e^3} + \frac{bd^2mnx^2}{32e^2} - \frac{bdmnx^3}{48e} + \frac{1}{64}bmnx^4 + \frac{bd^4mn\log(d+ex)}{16e^4} - \frac{1}{16}(mx^4 - 4x^4\log(fx^m))(a + b\log(c(d+ex)^n)) - \frac{1}{4}(bn)\int x^3\log(fx^m)\,dx + \frac{(bd^3n)\int\log(fx^m)\,dx}{4e^3} - \frac{(bd^4n)\int\frac{\log(fx^m)}{d+ex}\,dx}{4e^3} - \frac{(bd^2n)\int x\log(fx^m)\,dx}{4e^2} + \frac{(bdn)\int x^2\log(fx^m)\,dx}{4e}$$

$$= -\frac{5bd^3mnx}{16e^3} + \frac{3bd^2mnx^2}{32e^2} - \frac{7bdmnx^3}{144e} + \frac{1}{32}bmnx^4 + \frac{bd^3nx\log(fx^m)}{4e^3} - \frac{bd^2nx^2\log(fx^m)}{8e^2} + \frac{bdnx^3\log(fx^m)}{12e} - \frac{1}{16}bnx^4\log(fx^m) + \frac{bd^4mn\log(d+ex)}{16e^4} - \frac{1}{16}(mx^4 - 4x^4\log(fx^m))(a + b\log(c(d+ex)^n)) - \frac{bd^4n\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{4e^4} + \frac{(bd^4mn)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{4e^4}$$

$$= -\frac{5bd^3mnx}{16e^3} + \frac{3bd^2mnx^2}{32e^2} - \frac{7bdmnx^3}{144e} + \frac{1}{32}bmnx^4 + \frac{bd^3nx\log(fx^m)}{4e^3} - \frac{bd^2nx^2\log(fx^m)}{8e^2} + \frac{bdnx^3\log(fx^m)}{12e} - \frac{1}{16}bnx^4\log(fx^m) + \frac{bd^4mn\log(d+ex)}{16e^4} - \frac{1}{16}(mx^4 - 4x^4\log(fx^m))(a + b\log(c(d+ex)^n)) - \frac{bd^4n\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{4e^4} - \frac{bd^4mn\text{Li}_2\left(-\frac{ex}{d}\right)}{4e^4}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.95

$$\int x^3\log(fx^m)(a + b\log(c(d+ex)^n))\,dx = \frac{-6\log(fx^m)(-12ae^4x^4 + benx(-12d^3 + 6d^2ex - 4de^2x^2 + 3e^3x^3) + 12bd^4n\log(d+ex) - 12be^4x^4\log(c(d+ex)^n)) + \cdots}{\cdots}$$

```
[In] Integrate[x^3*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (-6*Log[f*x^m]*(-12*a*e^4*x^4 + b*e*n*x*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2
+ 3*e^3*x^3) + 12*b*d^4*n*Log[d + e*x] - 12*b*e^4*x^4*Log[c*(d + e*x)^n]) +
 m*(-90*b*d^3*e*n*x + 27*b*d^2*e^2*n*x^2 - 14*b*d*e^3*n*x^3 - 18*a*e^4*x^4
+ 9*b*e^4*n*x^4 + 18*b*d^4*n*(1 + 4*Log[x])*Log[d + e*x] - 18*b*e^4*x^4*Log
[c*(d + e*x)^n] - 72*b*d^4*n*Log[x]*Log[1 + (e*x)/d]) - 72*b*d^4*m*n*PolyLo
g[2, -((e*x)/d)])/(288*e^4)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 78.15 (sec) , antiderivative size = 1180, normalized size of antiderivative = 5.09

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1180 |

```
[In] int(x^3*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -205/576*b*d^4*m*n/e^4-1/32*I*n*b*x^4*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1/32*I*n
*b*x^4*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)
^n)*csgn(I*c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*
Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3
+1/2*b*ln(c)+1/2*a)*(1/4*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*cs
gn(I*f)*csgn(I*f*x^m)^2+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*csgn(I*f*x^m)
^3+2*ln(f))*x^4+1/2*x^4*ln(x^m)-1/8*m*x^4)-1/8/e^2*n*b*ln(x^m)*x^2*d^2+1/4/
e^3*n*b*ln(x^m)*x*d^3-1/4/e^4*n*b*ln(x^m)*d^4*ln(e*x+d)+1/12/e*n*b*d*x^3*ln
(f)-1/8/e^2*n*b*d^2*x^2*ln(f)+1/4/e^3*n*b*d^3*x*ln(f)-1/4/e^4*n*b*d^4*ln(e*
x+d)*ln(f)+1/32*I*n*b*x^4*Pi*csgn(I*f*x^m)^3+1/4*m/e^4*b*d^4*n*dilog(-e*x/d
)-1/16*n*b*ln(x^m)*x^4-1/16*n*b*x^4*ln(f)-1/16*I/e^2*n*b*d^2*x^2*Pi*csgn(I*
x^m)*csgn(I*f*x^m)^2+1/8*I/e^3*n*b*d^3*x*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/8*I
/e^3*n*b*d^3*x*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/8*I/e^4*n*b*d^4*ln(e*x+d)*P
i*csgn(I*f)*csgn(I*f*x^m)^2+(1/4*b*x^4*ln(x^m)+1/16*b*x^4*(-2*I*Pi*csgn(I*f
)*csgn(I*x^m)*csgn(I*f*x^m)+2*I*Pi*csgn(I*f)*csgn(I*f*x^m)^2+2*I*Pi*csgn(I*
x^m)*csgn(I*f*x^m)^2-2*I*Pi*csgn(I*f*x^m)^3+4*ln(f)-m))*ln((e*x+d)^n)-1/24*
I/e*n*b*d*x^3*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/16*I/e^2*n*b*d^2*x^2
*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/8*I/e^3*n*b*d^3*x*Pi*csgn(I*f)*cs
gn(I*x^m)*csgn(I*f*x^m)+1/8*I/e^4*n*b*d^4*ln(e*x+d)*Pi*csgn(I*f)*csgn(I*x^m
)*csgn(I*f*x^m)-1/8*I/e^4*n*b*d^4*ln(e*x+d)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+
1/24*I/e*n*b*d*x^3*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/24*I/e*n*b*d*x^3*Pi*csgn(
I*x^m)*csgn(I*f*x^m)^2-1/16*I/e^2*n*b*d^2*x^2*Pi*csgn(I*f)*csgn(I*f*x^m)^2+
1/4*m/e^4*b*d^4*n*ln(e*x+d)*ln(-e*x/d)+1/32*b*m*n*x^4+1/12/e*n*b*ln(x^m)*d*
x^3-5/16*b*d^3*m*n*x/e^3+1/32*I*n*b*x^4*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x
^m)-1/24*I/e*n*b*d*x^3*Pi*csgn(I*f*x^m)^3+1/16*I/e^2*n*b*d^2*x^2*Pi*csgn(I*
f*x^m)^3-1/8*I/e^3*n*b*d^3*x*Pi*csgn(I*f*x^m)^3+1/8*I/e^4*n*b*d^4*ln(e*x+d)
*Pi*csgn(I*f*x^m)^3+3/32*b*d^2*m*n*x^2/e^2-7/144*b*d*m*n*x^3/e+1/16*b*d^4*m
*n*ln(e*x+d)/e^4
```

## Fricas [F]

$$\int x^3 \log(fx^m)\left(a + b\log\left(c(d+ex)^n\right)\right) dx = \int (b\log\left((ex+d)^n c\right) + a)x^3 \log(fx^m)\, dx$$

```
[In] integrate(x^3*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x^3*log((e*x + d)^n*c)*log(f*x^m) + a*x^3*log(f*x^m), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^3 \log(fx^m)\left(a + b\log\left(c(d+ex)^n\right)\right) dx = \text{Timed out}$$

```
[In] integrate(x**3*ln(f*x**m)*(a+b*ln(c*(e*x+d)**n)),x)
```

```
[Out] Timed out
```

## Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00

$$\int x^3 \log(fx^m)\left(a + b\log\left(c(d+ex)^n\right)\right) dx$$
$$= \frac{1}{288}\left(\frac{72\left(\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right) + \text{Li}_2\left(\frac{ex+d}{d}\right)\right)bd^4n}{e^4} - \frac{18\,be^4x^4\log\left((ex+d)^n\right) + 14\,bde^3nx^3 - 27\,bd^2 \cdots}{\cdots}\right.$$
$$\left. + \frac{1}{48}\left(12\,bx^4\log\left((ex+d)^n c\right) + 12\,ax^4 - ben\left(\frac{12\,d^4\log(ex+d)}{e^5} + \frac{3\,e^3x^4 - 4\,de^2x^3 + 6\,d^2ex^2 - 12\,d^3x}{e^4}\right)\right)\right.$$

```
[In] integrate(x^3*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] 1/288*(72*(log(e*x + d)*log(-(e*x + d)/d + 1) + dilog((e*x + d)/d))*b*d^4*n
/e^4 - (18*b*e^4*x^4*log((e*x + d)^n) + 14*b*d*e^3*n*x^3 - 27*b*d^2*e^2*n*x
^2 + 90*b*d^3*e*n*x - 18*b*d^4*n*log(e*x + d) + 9*(2*a*e^4 - (e^4*n - 2*e^4
*log(c))*b)*x^4)/e^4)*m + 1/48*(12*b*x^4*log((e*x + d)^n*c) + 12*a*x^4 - b*
e*n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e^2*x^3 + 6*d^2*e*x^2 - 12*
d^3*x)/e^4))*log(f*x^m)
```

## Giac [F]

$$\int x^3 \log(fx^m)\left(a + b\log\left(c(d+ex)^n\right)\right)\,dx = \int (b\log\left((ex+d)^n c\right) + a)x^3 \log(fx^m)\,dx$$

```
[In] integrate(x^3*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*x^3*log(f*x^m), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^3 \log(fx^m)\left(a + b\log\left(c(d+ex)^n\right)\right)\,dx = \int x^3\,\ln\left(f\,x^m\right)\,\left(a + b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)\,dx$$

```
[In] int(x^3*log(f*x^m)*(a + b*log(c*(d + e*x)^n)),x)
[Out] int(x^3*log(f*x^m)*(a + b*log(c*(d + e*x)^n)), x)
```

# 3.359 $\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 195

$$\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = \frac{4bd^2mnx}{9e^2} - \frac{5bdmnx^2}{36e} + \frac{2}{27}bmnx^3 - \frac{bd^2nx\log(fx^m)}{3e^2} + \frac{bdnx^2\log(fx^m)}{6e} - \frac{1}{9}bnx^3\log(fx^m) - \frac{bd^3mn\log(d+ex)}{9e^3} - \frac{1}{9}\left(mx^3 - 3x^3\log(fx^m)\right)(a + b\log(c(d+ex)^n)) + \frac{bd^3n\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{3e^3} + \frac{bd^3mn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{3e^3}$$

```
[Out] 4/9*b*d^2*m*n*x/e^2-5/36*b*d*m*n*x^2/e+2/27*b*m*n*x^3-1/3*b*d^2*n*x*ln(f*x^
m)/e^2+1/6*b*d*n*x^2*ln(f*x^m)/e-1/9*b*n*x^3*ln(f*x^m)-1/9*b*d^3*m*n*ln(e*x
+d)/e^3-1/9*(m*x^3-3*x^3*ln(f*x^m))*(a+b*ln(c*(e*x+d)^n))+1/3*b*d^3*n*ln(f*
x^m)*ln(1+e*x/d)/e^3+1/3*b*d^3*m*n*polylog(2,-e*x/d)/e^3
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2473, 45, 2393, 2332, 2341, 2354, 2438}

$$\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = -\frac{1}{9}\left(mx^3-3x^3\log(fx^m)\right)(a+b\log(c(d+ex)^n)) + \frac{bd^3n\log\left(\frac{ex}{d}+1\right)\log(fx^m)}{3e^3} + \frac{bd^3mn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{3e^3} - \frac{bd^3mn\log(d+ex)}{9e^3} - \frac{bd^2nx\log(fx^m)}{3e^2} + \frac{4bd^2mnx}{9e^2} + \frac{bdnx^2\log(fx^m)}{6e} - \frac{5bdmnx^2}{36e} - \frac{1}{9}bnx^3\log(fx^m) + \frac{2}{27}bmnx^3$$

```
[In] Int[x^2*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (4*b*d^2*m*n*x)/(9*e^2) - (5*b*d*m*n*x^2)/(36*e) + (2*b*m*n*x^3)/27 - (b*d^
2*n*x*Log[f*x^m])/(3*e^2) + (b*d*n*x^2*Log[f*x^m])/(6*e) - (b*n*x^3*Log[f*x
^m])/9 - (b*d^3*m*n*Log[d + e*x])/(9*e^3) - ((m*x^3 - 3*x^3*Log[f*x^m])*(a
+ b*Log[c*(d + e*x)^n]))/9 + (b*d^3*n*Log[f*x^m]*Log[1 + (e*x)/d])/(3*e^3)
+ (b*d^3*m*n*PolyLog[2, -((e*x)/d)])/(3*e^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2473

```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Dist[b*e*(n/(g*(q + 1))), Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x], x
] + Dist[b*e*m*(n/(g*(q + 1)^2)), Int[(g*x)^(q + 1)/(d + e*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{9}\left(mx^3-3x^3\log\left(fx^m\right)\right)\left(a+b\log\left(c(d+ex)^n\right)\right) \\
&\quad -\frac{1}{3}(ben)\int\frac{x^3\log\left(fx^m\right)}{d+ex}\,dx+\frac{1}{9}(bemn)\int\frac{x^3}{d+ex}\,dx \\
&= -\frac{1}{9}\left(mx^3-3x^3\log\left(fx^m\right)\right)\left(a+b\log\left(c(d+ex)^n\right)\right) \\
&\quad -\frac{1}{3}(ben)\int\left(\frac{d^2\log\left(fx^m\right)}{e^3}-\frac{dx\log\left(fx^m\right)}{e^2}+\frac{x^2\log\left(fx^m\right)}{e}-\frac{d^3\log\left(fx^m\right)}{e^3(d+ex)}\right)dx \\
&\quad +\frac{1}{9}(bemn)\int\left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)dx \\
&= \frac{bd^2mnx}{9e^2}-\frac{bdmnx^2}{18e}+\frac{1}{27}bmnx^3-\frac{bd^3mn\log(d+ex)}{9e^3} \\
&\quad -\frac{1}{9}\left(mx^3-3x^3\log\left(fx^m\right)\right)\left(a+b\log\left(c(d+ex)^n\right)\right)-\frac{1}{3}(bn)\int x^2\log\left(fx^m\right)\,dx \\
&\quad -\frac{\left(bd^2n\right)\int\log\left(fx^m\right)\,dx}{3e^2}+\frac{\left(bd^3n\right)\int\frac{\log\left(fx^m\right)}{d+ex}\,dx}{3e^2}+\frac{(bdn)\int x\log\left(fx^m\right)\,dx}{3e}
\end{aligned}
$$

$$= \frac{4bd^2mnx}{9e^2} - \frac{5bdmnx^2}{36e} + \frac{2}{27}bmnx^3 - \frac{bd^2nx\log(fx^m)}{3e^2} + \frac{bdnx^2\log(fx^m)}{6e} - \frac{1}{9}bnx^3\log(fx^m) - \frac{bd^3mn\log(d+ex)}{9e^3} - \frac{1}{9}(mx^3 - 3x^3\log(fx^m))(a + b\log(c(d+ex)^n)) + \frac{bd^3n\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{3e^3} - \frac{(bd^3mn)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}dx}{3e^3}$$

$$= \frac{4bd^2mnx}{9e^2} - \frac{5bdmnx^2}{36e} + \frac{2}{27}bmnx^3 - \frac{bd^2nx\log(fx^m)}{3e^2} + \frac{bdnx^2\log(fx^m)}{6e} - \frac{1}{9}bnx^3\log(fx^m) - \frac{bd^3mn\log(d+ex)}{9e^3} - \frac{1}{9}(mx^3 - 3x^3\log(fx^m))(a + b\log(c(d+ex)^n)) + \frac{bd^3n\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{3e^3} + \frac{bd^3mn\text{Li}_2\left(-\frac{ex}{d}\right)}{3e^3}$$

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.01

$$\int x^2\log(fx^m)(a + b\log(c(d+ex)^n))\,dx$$

$$= \frac{6\log(fx^m)(6ae^3x^3 + benx(-6d^2 + 3dex - 2e^2x^2) + 6bd^3n\log(d+ex) + 6be^3x^3\log(c(d+ex)^n)) + m(4\ldots}{}$$

```
[In] Integrate[x^2*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (6*Log[f*x^m]*(6*a*e^3*x^3 + b*e*n*x*(-6*d^2 + 3*d*e*x - 2*e^2*x^2) + 6*b*d
^3*n*Log[d + e*x] + 6*b*e^3*x^3*Log[c*(d + e*x)^n]) + m*(48*b*d^2*e*n*x - 1
5*b*d*e^2*n*x^2 - 12*a*e^3*x^3 + 8*b*e^3*n*x^3 - 12*b*d^3*n*(1 + 3*Log[x])*
Log[d + e*x] - 12*b*e^3*x^3*Log[c*(d + e*x)^n] + 36*b*d^3*n*Log[x]*Log[1 +
(e*x)/d]) + 36*b*d^3*m*n*PolyLog[2, -((e*x)/d)])/(108*e^3)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 32.94 (sec) , antiderivative size = 1012, normalized size of antiderivative = 5.19

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1012 |

```
[In] int(x^2*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I/e*n*b*d*x^2*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/18*I*n*b*x^3*P
i*csgn(I*f)*csgn(I*f*x^m)^2-1/18*I*n*b*x^3*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1
/6*I/e^2*n*b*d^2*x*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/6*I/e^3*n*b*d^3
*ln(e*x+d)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/3*m/e^3*b*d^3*n*dilog(-
e*x/d)-1/12*I/e*n*b*d*x^2*Pi*csgn(I*f*x^m)^3+1/6*I/e^2*n*b*d^2*x*Pi*csgn(I*
f*x^m)^3+1/18*I*n*b*x^3*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/3/e^2*n*b*
ln(x^m)*x*d^2+49/108/e^3*n*b*m*d^3+(1/3*b*x^3*ln(x^m)+1/18*b*x^3*(-3*I*Pi*c
sgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+3*I*Pi*csgn(I*f)*csgn(I*f*x^m)^2+3*I*Pi*
csgn(I*x^m)*csgn(I*f*x^m)^2-3*I*Pi*csgn(I*f*x^m)^3+6*ln(f)-2*m))*ln((e*x+d)
^n)-1/3*m/e^3*b*d^3*n*ln(e*x+d)*ln(-e*x/d)-1/6*I/e^3*n*b*d^3*ln(e*x+d)*Pi*c
sgn(I*f*x^m)^3+2/27*b*m*n*x^3+1/6*I/e^3*n*b*d^3*ln(e*x+d)*Pi*csgn(I*f)*csgn
(I*f*x^m)^2+1/6*I/e^3*n*b*d^3*ln(e*x+d)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+(-1/
4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*
c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2
-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*a)*(1/3*(-I*Pi*csgn(I*f)*
csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2+I*Pi*csgn(I*x^m)*c
sgn(I*f*x^m)^2-I*Pi*csgn(I*f*x^m)^3+2*ln(f))*x^3+2/3*x^3*ln(x^m)-2/9*m*x^3)
+1/3/e^3*n*b*d^3*ln(e*x+d)*ln(f)+1/18*I*n*b*x^3*Pi*csgn(I*f*x^m)^3+1/3/e^3*
n*b*ln(x^m)*d^3*ln(e*x+d)+1/6/e*n*b*ln(x^m)*d*x^2+1/6/e*n*b*d*x^2*ln(f)-1/3
/e^2*n*b*d^2*x*ln(f)-1/9*n*b*x^3*ln(f)+1/12*I/e*n*b*d*x^2*Pi*csgn(I*f)*csgn
(I*f*x^m)^2+1/12*I/e*n*b*d*x^2*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/6*I/e^2*n*b
*d^2*x*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1/6*I/e^2*n*b*d^2*x*Pi*csgn(I*x^m)*csgn
(I*f*x^m)^2-1/9*n*b*ln(x^m)*x^3+4/9*b*d^2*m*n*x/e^2-5/36*b*d*m*n*x^2/e-1/9*
b*d^3*m*n*ln(e*x+d)/e^3
```

## Fricas [F]

$$\int x^2 \log(fx^m) (a + b \log(c(d + ex)^n)) \, dx = \int (b \log((ex + d)^n c) + a)x^2 \log(fx^m) \, dx$$

```
[In] integrate(x^2*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral(b*x^2*log((e*x + d)^n*c)*log(f*x^m) + a*x^2*log(f*x^m), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2 \log(fx^m) (a + b \log(c(d + ex)^n)) \, dx = \text{Timed out}$$

```
[In] integrate(x**2*ln(f*x**m)*(a+b*ln(c*(e*x+d)**n)),x)
[Out] Timed out
```

## Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.06

$$\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))\,dx =$$
$$-\frac{1}{108}\left(\frac{36\left(\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex+d}{d}\right)\right)bd^3n}{e^3}+\frac{12\,be^3x^3\log((ex+d)^n)+15\,bde^2nx^2-48}{}\right.$$
$$+\frac{1}{18}\left(6\,bx^3\log((ex+d)^nc)+6\,ax^3+ben\left(\frac{6\,d^3\log(ex+d)}{e^4}-\frac{2\,e^2x^3-3\,dex^2+6\,d^2x}{e^3}\right)\right)\log(fx^m)$$

```
[In] integrate(x^2*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] -1/108*(36*(log(e*x + d)*log(-(e*x + d)/d + 1) + dilog((e*x + d)/d))*b*d^3*
n/e^3 + (12*b*e^3*x^3*log((e*x + d)^n) + 15*b*d*e^2*n*x^2 - 48*b*d^2*e*n*x
+ 12*b*d^3*n*log(e*x + d) + 4*(3*a*e^3 - (2*e^3*n - 3*e^3*log(c))*b)*x^3)/e
^3)*m + 1/18*(6*b*x^3*log((e*x + d)^n*c) + 6*a*x^3 + b*e*n*(6*d^3*log(e*x +
 d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3))*log(f*x^m)
```

## Giac [F]

$$\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = \int (b\log((ex+d)^nc)+a)x^2\log(fx^m)\,dx$$

```
[In] integrate(x^2*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*x^2*log(f*x^m), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = \int x^2\,\ln(f\,x^m)\,\left(a+b\,\ln\left(c\,(d+e\,x)^n\right)\right)\,dx$$

```
[In] int(x^2*log(f*x^m)*(a + b*log(c*(d + e*x)^n)),x)
```

```
[Out] int(x^2*log(f*x^m)*(a + b*log(c*(d + e*x)^n)), x)
```

# 3.360 $\int x \log(fx^m)(a+b\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 158

$$\int x\log(fx^m)(a+b\log(c(d+ex)^n))\,dx = -\frac{3bdmnx}{4e}+\frac{1}{4}bmnx^2+\frac{bdnx\log(fx^m)}{2e} -\frac{1}{4}bnx^2\log(fx^m)+\frac{bd^2mn\log(d+ex)}{4e^2} -\frac{1}{4}\left(mx^2-2x^2\log(fx^m)\right)(a+b\log(c(d+ex)^n)) -\frac{bd^2n\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{2e^2} -\frac{bd^2mn\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{2e^2}$$

```
[Out] -3/4*b*d*m*n*x/e+1/4*b*m*n*x^2+1/2*b*d*n*x*ln(f*x^m)/e-1/4*b*n*x^2*ln(f*x^m
)+1/4*b*d^2*m*n*ln(e*x+d)/e^2-1/4*(m*x^2-2*x^2*ln(f*x^m))*(a+b*ln(c*(e*x+d)
^n))-1/2*b*d^2*n*ln(f*x^m)*ln(1+e*x/d)/e^2-1/2*b*d^2*m*n*polylog(2,-e*x/d)/
e^2
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used

$= \{2473, 45, 2393, 2332, 2341, 2354, 2438\}$

$$\int x\log(fx^m)(a+b\log(c(d+ex)^n))\,dx = -\frac{1}{4}(mx^2-2x^2\log(fx^m))(a+b\log(c(d+ex)^n)) - \frac{bd^2n\log\left(\frac{ex}{d}+1\right)\log(fx^m)}{2e^2} - \frac{bd^2mn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{2e^2} + \frac{bd^2mn\log(d+ex)}{4e^2} + \frac{bdnx\log(fx^m)}{2e} - \frac{3bdmnx}{4e} - \frac{1}{4}bnx^2\log(fx^m) + \frac{1}{4}bmnx^2$$

```
[In] Int[x*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (-3*b*d*m*n*x)/(4*e) + (b*m*n*x^2)/4 + (b*d*n*x*Log[f*x^m])/(2*e) - (b*n*x^
2*Log[f*x^m])/4 + (b*d^2*m*n*Log[d + e*x])/(4*e^2) - ((m*x^2 - 2*x^2*Log[f*
x^m])*(a + b*Log[c*(d + e*x)^n]))/4 - (b*d^2*n*Log[f*x^m]*Log[1 + (e*x)/d])
/(2*e^2) - (b*d^2*m*n*PolyLog[2, -((e*x)/d)])/(2*e^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2473

```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Dist[b*e*(n/(g*(q + 1))), Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x], x
] + Dist[b*e*m*(n/(g*(q + 1)^2)), Int[(g*x)^(q + 1)/(d + e*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{4}\left(mx^2-2x^2\log\left(fx^m\right)\right)\left(a+b\log\left(c(d+ex)^n\right)\right) \\
&\quad -\frac{1}{2}(ben)\int\frac{x^2\log\left(fx^m\right)}{d+ex}\,dx+\frac{1}{4}(bemn)\int\frac{x^2}{d+ex}\,dx \\
&= -\frac{1}{4}\left(mx^2-2x^2\log\left(fx^m\right)\right)\left(a+b\log\left(c(d+ex)^n\right)\right) \\
&\quad -\frac{1}{2}(ben)\int\left(-\frac{d\log\left(fx^m\right)}{e^2}+\frac{x\log\left(fx^m\right)}{e}+\frac{d^2\log\left(fx^m\right)}{e^2(d+ex)}\right)dx \\
&\quad +\frac{1}{4}(bemn)\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx \\
&= -\frac{bdmnx}{4e}+\frac{1}{8}bmnx^2+\frac{bd^2mn\log(d+ex)}{4e^2}-\frac{1}{4}\left(mx^2-2x^2\log\left(fx^m\right)\right)(a \\
&\qquad +b\log\left(c(d+ex)^n\right)) \\
&\quad -\frac{1}{2}(bn)\int x\log\left(fx^m\right)\,dx+\frac{(bdn)\int\log\left(fx^m\right)\,dx}{2e}-\frac{(bd^2n)\int\frac{\log(fx^m)}{d+ex}\,dx}{2e} \\
&= -\frac{3bdmnx}{4e}+\frac{1}{4}bmnx^2+\frac{bdnx\log\left(fx^m\right)}{2e}-\frac{1}{4}bnx^2\log\left(fx^m\right) \\
&\quad +\frac{bd^2mn\log(d+ex)}{4e^2}-\frac{1}{4}\left(mx^2-2x^2\log\left(fx^m\right)\right)\left(a+b\log\left(c(d+ex)^n\right)\right) \\
&\quad -\frac{bd^2n\log\left(fx^m\right)\log\left(1+\frac{ex}{d}\right)}{2e^2}+\frac{(bd^2mn)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{2e^2}
\end{aligned}$$

$$= -\frac{3bdmnx}{4e} + \frac{1}{4}bmnx^2 + \frac{bdnx\log(fx^m)}{2e} - \frac{1}{4}bnx^2\log(fx^m) + \frac{bd^2mn\log(d+ex)}{4e^2} - \frac{1}{4}(mx^2 - 2x^2\log(fx^m))(a + b\log(c(d+ex)^n)) - \frac{bd^2n\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{2e^2} - \frac{bd^2mn\text{Li}_2\left(-\frac{ex}{d}\right)}{2e^2}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04

$$\int x\log(fx^m)(a + b\log(c(d+ex)^n))\,dx = \frac{\log(fx^m)(-2bd^2n\log(d+ex) + ex(2bdn + 2aex - benx + 2bex\log(c(d+ex)^n))) + m(-3bdenx - ae^2x}{}$$

```
[In] Integrate[x*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] (Log[f*x^m]*(-2*b*d^2*n*Log[d + e*x] + e*x*(2*b*d*n + 2*a*e*x - b*e*n*x + 2
*b*e*x*Log[c*(d + e*x)^n])) + m*(-3*b*d*e*n*x - a*e^2*x^2 + b*e^2*n*x^2 + b
*d^2*n*(1 + 2*Log[x])*Log[d + e*x] - b*e^2*x^2*Log[c*(d + e*x)^n] - 2*b*d^2
*n*Log[x]*Log[1 + (e*x)/d]) - 2*b*d^2*m*n*PolyLog[2, -((e*x)/d)])/(4*e^2)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.04 (sec) , antiderivative size = 843, normalized size of antiderivative = 5.34

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 843 |

```
[In] int(x*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/e*n*b*d*x*Pi*csgn(I*f*x^m)^3+1/4*I/e^2*n*b*d^2*ln(e*x+d)*Pi*csgn(I*f
*x^m)^3+1/8*I*n*b*x^2*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2/e^2*n*b*ln
(x^m)*d^2*ln(e*x+d)+1/2/e^2*n*b*m*d^2*dilog(-e*x/d)-1/2/e^2*n*b*d^2*ln(e*x+
d)*ln(f)+1/8*I*n*b*x^2*Pi*csgn(I*f*x^m)^3-1/4*I/e^2*n*b*d^2*ln(e*x+d)*Pi*cs
gn(I*f)*csgn(I*f*x^m)^2-1/4*I/e^2*n*b*d^2*ln(e*x+d)*Pi*csgn(I*x^m)*csgn(I*f
*x^m)^2-1/8*I*n*b*x^2*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4*I/e*n*b*d*x*Pi*csg
n(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I/e^2*n*b*d^2*ln(e*x+d)*Pi*csgn(I*f)*c
sgn(I*x^m)*csgn(I*f*x^m)+1/4*I/e*n*b*d*x*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/4*I
/e*n*b*d*x*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2/e^2*n*b*m*d^2*ln(e*x+d)*ln(-e
*x/d)+1/4*b*m*n*x^2+(1/2*b*x^2*ln(x^m)+1/4*b*x^2*(-I*Pi*csgn(I*f)*csgn(I*x^
m)*csgn(I*f*x^m)+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2+I*Pi*csgn(I*x^m)*csgn(I*f*x
^m)^2-I*Pi*csgn(I*f*x^m)^3+2*ln(f)-m))*ln((e*x+d)^n)-1/8*I*n*b*x^2*Pi*csgn(
```

```
I*f)*csgn(I*f*x^m)^2+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x
+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)
^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*
a)*(1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*csgn(I*f)*csgn(I*f*
x^m)^2+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*csgn(I*f*x^m)^3+2*ln(f))*x^2+x
^2*ln(x^m)-1/2*m*x^2)-1/4*n*b*x^2*ln(f)-1/4*n*b*ln(x^m)*x^2-5/8/e^2*n*b*m*d
^2+1/2/e*n*b*d*x*ln(f)+1/2/e*n*b*ln(x^m)*d*x-3/4*b*d*m*n*x/e+1/4*b*d^2*m*n*
ln(e*x+d)/e^2
```

## Fricas [F]

$$\int x \log(fx^m)(a + b\log(c(d+ex)^n))\,dx = \int (b\log((ex+d)^n c) + a)x\log(fx^m)\,dx$$

```
[In] integrate(x*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral(b*x*log((e*x + d)^n*c)*log(f*x^m) + a*x*log(f*x^m), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x \log(fx^m)(a + b\log(c(d+ex)^n))\,dx = \text{Timed out}$$

```
[In] integrate(x*ln(f*x**m)*(a+b*ln(c*(e*x+d)**n)),x)
[Out] Timed out
```

## Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.13

$$\int x \log(fx^m)(a + b\log(c(d+ex)^n))\,dx = \frac{1}{4}\left(\frac{2\left(\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right) + \mathrm{Li}_2\left(\frac{ex+d}{d}\right)\right)bd^2n}{e^2} - \frac{be^2x^2\log((ex+d)^n) + 3\,bdenx - bd^2n\log(ex+d}{e^2}\right. - \frac{1}{4}\left(ben\left(\frac{2\,d^2\log(ex+d)}{e^3} + \frac{ex^2 - 2\,dx}{e^2}\right) - 2\,bx^2\log((ex+d)^n c) - 2\,ax^2\right)\log(fx^m)$$

```
[In] integrate(x*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(log(e*x + d)*log(-(e*x + d)/d + 1) + dilog((e*x + d)/d))*b*d^2*n/e^
2 - (b*e^2*x^2*log((e*x + d)^n) + 3*b*d*e*n*x - b*d^2*n*log(e*x + d) + (a*e
^2 - (e^2*n - e^2*log(c))*b)*x^2)/e^2)*m - 1/4*(b*e*n*(2*d^2*log(e*x + d)/e
^3 + (e*x^2 - 2*d*x)/e^2) - 2*b*x^2*log((e*x + d)^n*c) - 2*a*x^2)*log(f*x^m
)
```

## Giac [F]

$$\int x\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx=\int(b\log\left((ex+d)^nc\right)+a)x\log\left(fx^m\right)\,dx$$

```
[In] integrate(x*log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*x*log(f*x^m), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx=\int x\,\ln\left(f\,x^m\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)\,dx$$

```
[In] int(x*log(f*x^m)*(a + b*log(c*(d + e*x)^n)),x)
[Out] int(x*log(f*x^m)*(a + b*log(c*(d + e*x)^n)), x)
```

# 3.361 $\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = 2bmnx - bnx\log(fx^m) - \frac{bdmn\log(d+ex)}{e} - x(m-\log(fx^m))(a+b\log(c(d+ex)^n)) + \frac{bdn\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{e} + \frac{bdmn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e}$$

```
[Out] 2*b*m*n*x-b*n*x*ln(f*x^m)-b*d*m*n*ln(e*x+d)/e-x*(m-ln(f*x^m))*(a+b*ln(c*(e*
x+d)^n))+b*d*n*ln(f*x^m)*ln(1+e*x/d)/e+b*d*m*n*polylog(2,-e*x/d)/e
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2469, 45, 2393, 2332, 2354, 2438}

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = -x(m-\log(fx^m))(a+b\log(c(d+ex)^n)) + \frac{bdn\log\left(\frac{ex}{d}+1\right)\log(fx^m)}{e} + \frac{bdmn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e} - \frac{bdmn\log(d+ex)}{e} - bnx\log(fx^m) + 2bmnx$$

```
[In] Int[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
```

```
[Out] 2*b*m*n*x - b*n*x*Log[f*x^m] - (b*d*m*n*Log[d + e*x])/e - x*(m - Log[f*x^m]
)*(a + b*Log[c*(d + e*x)^n]) + (b*d*n*Log[f*x^m]*Log[1 + (e*x)/d])/e + (b*d
*m*n*PolyLog[2, -((e*x)/d)])/e
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2469

```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.)), x_Symbol] :> Simp[(-x)*(m - Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x]
 + (-Dist[b*e*n, Int[(x*Log[f*x^m])/(d + e*x), x], x] + Dist[b*e*m*n, Int[x
/(d + e*x), x], x]) /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -x(m-\log(fx^m))\,(a+b\log(c(d+ex)^n))\\ &\quad -(ben)\int\frac{x\log(fx^m)}{d+ex}\,dx+(bemn)\int\frac{x}{d+ex}\,dx\end{aligned}$$

$$= -x(m - \log(fx^m))(a + b\log(c(d+ex)^n)) - (ben)\int \left(\frac{\log(fx^m)}{e} - \frac{d\log(fx^m)}{e(d+ex)}\right)dx + (bemn)\int \left(\frac{1}{e} - \frac{d}{e(d+ex)}\right)dx$$

$$= bmnx - \frac{bdmn\log(d+ex)}{e} - x(m - \log(fx^m))(a + b\log(c(d+ex)^n)) - (bn)\int \log(fx^m)\,dx + (bdn)\int \frac{\log(fx^m)}{d+ex}dx$$

$$= 2bmnx - bnx\log(fx^m) - \frac{bdmn\log(d+ex)}{e} - x(m - \log(fx^m))(a + b\log(c(d+ex)^n)) + \frac{bdn\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{e} - \frac{(bdmn)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}dx}{e}$$

$$= 2bmnx - bnx\log(fx^m) - \frac{bdmn\log(d+ex)}{e} - x(m - \log(fx^m))(a + b\log(c(d+ex)^n)) + \frac{bdn\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{e} + \frac{bdmn\text{Li}_2\left(-\frac{ex}{d}\right)}{e}$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \log(fx^m)(a + b\log(c(d+ex)^n))\,dx = \frac{\log(fx^m)(bdn\log(d+ex) + ex(a - bn + b\log(c(d+ex)^n))) - m\big(aex - 2benx + bdn(1+\log(x))\log(d+ex)\cdots}{e}$$

```
[In] Integrate[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]),x]
[Out] (Log[f*x^m]*(b*d*n*Log[d + e*x] + e*x*(a - b*n + b*Log[c*(d + e*x)^n])) - m
*(a*e*x - 2*b*e*n*x + b*d*n*(1 + Log[x])*Log[d + e*x] + b*e*x*Log[c*(d + e*
x)^n] - b*d*n*Log[x]*Log[1 + (e*x)/d]) + b*d*m*n*PolyLog[2, -((e*x)/d)])/e
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.25 (sec) , antiderivative size = 657, normalized size of antiderivative = 6.64

| method | result |
|---|---|
| risch | $\left(bx\ln(x^m) + \frac{xb\left(-i\pi\ \text{csgn}(if)\ \text{csgn}(ix^m)\ \text{csgn}(ifx^m) + i\pi\ \text{csgn}(if)\ \text{csgn}(ifx^m)^2 + i\pi\ \text{csgn}(ix^m)\ \text{csgn}(ifx^m)^2 - i\pi\ \text{csgn}(ifx^m)^3\cdots\right.}{2}\right.$ |

```
[In] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x*ln(x^m)+1/2*x*b*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*csgn(I
*f)*csgn(I*f*x^m)^2+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*csgn(I*f*x^m)^3+2
*ln(f)-2*m))*ln((e*x+d)^n)+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*
c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(
e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c
)+1/2*a)*(I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*x+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2
*x+2*x*ln(f)-I*Pi*csgn(I*f*x^m)^3*x-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m
)*x+2*ln(x^m)*x-2*m*x)+1/2*I*n*b*x*Pi*csgn(I*f*x^m)^3-1/2*I*n*b*x*Pi*csgn(I
*x^m)*csgn(I*f*x^m)^2+1/2*I*n*b*x*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/
2*I/e*n*b*d*ln(e*x+d)*Pi*csgn(I*f*x^m)^3-n*b*x*ln(f)+2*b*m*n*x-1/2*I*n*b*x*
Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I/e*n*b*d*ln(e*x+d)*Pi*csgn(I*x^m)*csgn(I*
f*x^m)^2+1/2*I/e*n*b*d*ln(e*x+d)*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1/2*I/e*n*b*d
*ln(e*x+d)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/e*n*b*d*ln(e*x+d)*ln(f)
-b*d*m*n*ln(e*x+d)/e-n*b*ln(x^m)*x+1/e*n*b*ln(x^m)*d*ln(e*x+d)+1/e*n*b*m*d-
1/e*n*b*m*d*ln(e*x+d)*ln(-e*x/d)-1/e*n*b*m*d*dilog(-e*x/d)
```

## Fricas [F]

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx=\int \left(b\log\left((ex+d)^n c\right)+a\right)\log(fx^m)\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
```

```
[Out] integral(b*log((e*x + d)^n*c)*log(f*x^m) + a*log(f*x^m), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n)),x)
```

```
[Out] Timed out
```

## Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.40

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = -\left(\frac{\left(\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex+d}{d}\right)\right)bdn}{e}+\frac{bdn\log(ex+d)+bex\log((ex+d)^n)-((2\,en-e\log ...}{e} \right. \\ -\left(ben\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)-bx\log((ex+d)^nc)-ax\right)\log(fx^m)$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
```

```
[Out] -((log(e*x + d)*log(-(e*x + d)/d + 1) + dilog((e*x + d)/d))*b*d*n/e + (b*d*
n*log(e*x + d) + b*e*x*log((e*x + d)^n) - ((2*e*n - e*log(c))*b - a*e)*x)/e
)*m - (b*e*n*(x/e - d*log(e*x + d)/e^2) - b*x*log((e*x + d)^n*c) - a*x)*log
(f*x^m)
```

## Giac [F]

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = \int (b\log((ex+d)^nc)+a)\log(fx^m)\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*log(f*x^m), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))\,dx = \int \ln(f\,x^m)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)\,dx$$

```
[In] int(log(f*x^m)*(a + b*log(c*(d + e*x)^n)),x)
```

```
[Out] int(log(f*x^m)*(a + b*log(c*(d + e*x)^n)), x)
```

# 3.362 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \frac{\log^2(fx^m)(a+b\log(c(d+ex)^n))}{2m} - \frac{bn\log^2(fx^m)\log\left(1+\frac{ex}{d}\right)}{2m} - bn\log(fx^m)\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right) + bmn\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)$$

```
[Out] 1/2*ln(f*x^m)^2*(a+b*ln(c*(e*x+d)^n))/m-1/2*b*n*ln(f*x^m)^2*ln(1+e*x/d)/m-b
*n*ln(f*x^m)*polylog(2,-e*x/d)+b*m*n*polylog(3,-e*x/d)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2472, 2354, 2421, 6724}

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \frac{\log^2(fx^m)(a+b\log(c(d+ex)^n))}{2m} - bn\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)\log(fx^m) - \frac{bn\log\left(\frac{ex}{d}+1\right)\log^2(fx^m)}{2m} + bmn\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)$$

```
[In] Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x,x]
```

```
[Out] (Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*m) - (b*n*Log[f*x^m]^2*Log[1 +
 (e*x)/d])/(2*m) - b*n*Log[f*x^m]*PolyLog[2, -((e*x)/d)] + b*m*n*PolyLog[3,
 -((e*x)/d)]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2472

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[Log[f*x^m]^2*((a + b*Log[c*(d + e*x)^n])/(2*m
)), x] - Dist[b*e*(n/(2*m)), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\log^2(fx^m)\,(a+b\log(c(d+ex)^n))}{2m} - \frac{(ben)\int \frac{\log^2(fx^m)}{d+ex}\,dx}{2m} \\
&= \frac{\log^2(fx^m)\,(a+b\log(c(d+ex)^n))}{2m} - \frac{bn\log^2(fx^m)\log\left(1+\frac{ex}{d}\right)}{2m} \\
&\quad + (bn)\int \frac{\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{x}\,dx \\
&= \frac{\log^2(fx^m)\,(a+b\log(c(d+ex)^n))}{2m} - \frac{bn\log^2(fx^m)\log\left(1+\frac{ex}{d}\right)}{2m} \\
&\quad - bn\log(fx^m)\,\mathrm{Li}_2\left(-\frac{ex}{d}\right) + (bmn)\int \frac{\mathrm{Li}_2\left(-\frac{ex}{d}\right)}{x}\,dx
\end{aligned}
$$

$$= \frac{\log^2(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)}{2m} - \frac{bn\log^2(fx^m)\log\left(1+\frac{ex}{d}\right)}{2m} - bn\log(fx^m)\,\text{Li}_2\left(-\frac{ex}{d}\right) + bmn\text{Li}_3\left(-\frac{ex}{d}\right)$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.45

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)}{x}\,dx = \frac{1}{2}\left(\frac{a\log^2(fx^m)}{m} - bm\log^2(x)\log\left(c(d+ex)^n\right) + 2b\log(x)\log(fx^m)\log\left(c(d+ex)^n\right) + bmn\log^2(x)\log\left(1+\frac{ex}{d}\right) - 2bn\log(x)\log(fx^m)\log\left(1+\frac{ex}{d}\right) - 2bn\log(fx^m)\,\text{PolyLog}\left(2,-\frac{ex}{d}\right) + 2bmn\,\text{PolyLog}\left(3,-\frac{ex}{d}\right)\right)$$

```
[In] Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x,x]
```

```
[Out] ((a*Log[f*x^m]^2)/m - b*m*Log[x]^2*Log[c*(d + e*x)^n] + 2*b*Log[x]*Log[f*x^
m]*Log[c*(d + e*x)^n] + b*m*n*Log[x]^2*Log[1 + (e*x)/d] - 2*b*n*Log[x]*Log[
f*x^m]*Log[1 + (e*x)/d] - 2*b*n*Log[f*x^m]*PolyLog[2, -((e*x)/d)] + 2*b*m*n
*PolyLog[3, -((e*x)/d)])/2
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.60 (sec) , antiderivative size = 756, normalized size of antiderivative = 8.59

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 756 |

```
[In] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] (b*ln(x)*ln(x^m)-1/2*b*m*ln(x)^2-1/2*I*Pi*ln(x)*b*csgn(I*f)*csgn(I*x^m)*csg
n(I*f*x^m)+1/2*I*Pi*ln(x)*b*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I*Pi*ln(x)*b*csgn
(I*x^m)*csgn(I*f*x^m)^2-1/2*I*Pi*ln(x)*b*csgn(I*f*x^m)^3+ln(f)*ln(x)*b)*ln(
(e*x+d)^n)+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/4
*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I
*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*a)*(I*Pi*c
sgn(I*f)*csgn(I*f*x^m)^2*ln(x)+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*ln(x)+2*ln(
```

```
x)*ln(f)-I*Pi*csgn(I*f*x^m)^3*ln(x)-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m
)*ln(x)+1/m*ln(x^m)^2)-1/2*I*n*b*ln(x)*ln((e*x+d)/d)*Pi*csgn(I*x^m)*csgn(I*
f*x^m)^2-1/2*I*n*b*dilog((e*x+d)/d)*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1/2*I*n*b*
dilog((e*x+d)/d)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2*I*n*b*dilog((e*x+d)/d)*
Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/2*I*n*b*ln(x)*ln((e*x+d)/d)*Pi*csg
n(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2*I*n*b*ln(x)*ln((e*x+d)/d)*Pi*csgn(I*f)
*csgn(I*f*x^m)^2+1/2*I*n*b*dilog((e*x+d)/d)*Pi*csgn(I*f*x^m)^3+1/2*I*n*b*ln
(x)*ln((e*x+d)/d)*Pi*csgn(I*f*x^m)^3-1/2*n*b*m*ln(x)^2*ln(1+e*x/d)+n*b*ln(x
)^2*ln((e*x+d)/d)*m-n*b*m*ln(x)*polylog(2,-e*x/d)+n*b*dilog((e*x+d)/d)*m*ln
(x)-n*b*ln(x)*ln((e*x+d)/d)*ln(x^m)-n*b*ln(x)*ln((e*x+d)/d)*ln(f)+b*m*n*pol
ylog(3,-e*x/d)-n*b*dilog((e*x+d)/d)*ln(x^m)-n*b*dilog((e*x+d)/d)*ln(f)
```

## Fricas [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \int \frac{(b\log((ex+d)^nc)+a)\log(fx^m)}{x}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x,x, algorithm="fricas")
[Out] integral((b*log((e*x + d)^n*c)*log(f*x^m) + a*log(f*x^m))/x, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))/x,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \int \frac{(b\log((ex+d)^nc)+a)\log(fx^m)}{x}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x,x, algorithm="maxima")
[Out] -1/2*(b*m*log(x)^2 - 2*b*log(f)*log(x) - 2*b*log(x)*log(x^m))*log((e*x + d)
^n) - integrate(-1/2*(b*e*m*n*x*log(x)^2 - 2*b*e*n*x*log(f)*log(x) + 2*b*d*
log(c)*log(f) + 2*a*d*log(f) + 2*(b*e*log(c)*log(f) + a*e*log(f))*x - 2*(b*
e*n*x*log(x) - b*d*log(c) - a*d - (b*e*log(c) + a*e)*x)*log(x^m))/(e*x^2 +
d*x), x)
```

## Giac [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \int \frac{(b\log((ex+d)^n c)+a)\log(fx^m)}{x}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*log(f*x^m)/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x}\,dx = \int \frac{\ln(f\,x^m)\,(a+b\,\ln(c\,(d+e\,x)^n))}{x}\,dx$$

```
[In] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x,x)
[Out] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x, x)
```

# 3.363 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^2}\,dx = \frac{bemn\log(x)}{d} - \frac{ben\log\left(1+\frac{d}{ex}\right)\log(fx^m)}{d} - \frac{bemn\log(d+ex)}{d} - \left(\frac{m}{x}+\frac{\log(fx^m)}{x}\right)(a+b\log(c(d+ex)^n)) + \frac{bemn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d}$$

```
[Out] b*e*m*n*ln(x)/d-b*e*n*ln(1+d/e/x)*ln(f*x^m)/d-b*e*m*n*ln(e*x+d)/d-(m/x+ln(f
*x^m)/x)*(a+b*ln(c*(e*x+d)^n))+b*e*m*n*polylog(2,-d/e/x)/d
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2473, 2379, 2438, 36, 29, 31}

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^2}\,dx = -\left(\frac{\log(fx^m)}{x}+\frac{m}{x}\right)(a+b\log(c(d+ex)^n)) - \frac{ben\log\left(\frac{d}{ex}+1\right)\log(fx^m)}{d} + \frac{bemn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d} + \frac{bemn\log(x)}{d} - \frac{bemn\log(d+ex)}{d}$$

```
[In] Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^2,x]
```

```
[Out] (b*e*m*n*Log[x])/d - (b*e*n*Log[1 + d/(e*x)]*Log[f*x^m])/d - (b*e*m*n*Log[d
 + e*x])/d - (m/x + Log[f*x^m]/x)*(a + b*Log[c*(d + e*x)^n]) + (b*e*m*n*Pol
yLog[2, -(d/(e*x))])/d
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2473

```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Dist[b*e*(n/(g*(q + 1))), Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x], x
] + Dist[b*e*m*(n/(g*(q + 1)^2)), Int[(g*x)^(q + 1)/(d + e*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\left(\frac{m}{x}+\frac{\log\left(fx^m\right)}{x}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)\right)\\ &\quad +(ben)\int\frac{\log\left(fx^m\right)}{x(d+ex)}\,dx+(bemn)\int\frac{1}{x(d+ex)}\,dx\end{aligned}$$

$$= -\frac{ben\log\left(1+\frac{d}{ex}\right)\log\left(fx^m\right)}{d} - \left(\frac{m}{x}+\frac{\log\left(fx^m\right)}{x}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)$$
$$+\frac{(bemn)\int\frac{1}{x}\,dx}{d}+\frac{(bemn)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d}-\frac{(be^2mn)\int\frac{1}{d+ex}\,dx}{d}$$
$$=\frac{bemn\log(x)}{d}-\frac{ben\log\left(1+\frac{d}{ex}\right)\log\left(fx^m\right)}{d}-\frac{bemn\log(d+ex)}{d}$$
$$-\left(\frac{m}{x}+\frac{\log\left(fx^m\right)}{x}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+\frac{bemn\text{Li}_2\left(-\frac{d}{ex}\right)}{d}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

$$\int\frac{\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{x^2}\,dx=$$
$$-\frac{bemnx\log^2(x)+2(m+\log\left(fx^m\right))\left(ad+benx\log(d+ex)+bd\log\left(c(d+ex)^n\right)\right)-2benx\log(x)\left(m+\text{lo}\right.}{2dx}$$

```
[In] Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^2,x]
```

```
[Out] -1/2*(b*e*m*n*x*Log[x]^2 + 2*(m + Log[f*x^m])*(a*d + b*e*n*x*Log[d + e*x] +
 b*d*Log[c*(d + e*x)^n]) - 2*b*e*n*x*Log[x]*(m + Log[f*x^m] + m*Log[d + e*x
] - m*Log[1 + (e*x)/d]) + 2*b*e*m*n*x*PolyLog[2, -((e*x)/d)])/(d*x)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.36 (sec) , antiderivative size = 709, normalized size of antiderivative = 6.95

| method | result |
|---|---|
| risch | $\left(-\frac{b\ln(x^m)}{x}-\frac{-i\pi b\,\text{csgn}(if)\,\text{csgn}(ix^m)\,\text{csgn}(if\,x^m)+i\pi b\,\text{csgn}(if)\,\text{csgn}(if\,x^m)^2+i\pi b\,\text{csgn}(ix^m)\,\text{csgn}(if\,x^m)^2-i\pi b\,\text{csgn}(if\,x^m)^3+}{2x}\right.$ |

```
[In] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-b/x*ln(x^m)-1/2*(-I*Pi*b*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*b*csgn(
I*f)*csgn(I*f*x^m)^2+I*Pi*b*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*b*csgn(I*f*x^m
)^3+2*b*ln(f)+2*b*m)/x)*ln((e*x+d)^n)+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)
^n)*csgn(I*c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*
Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3
+1/2*b*ln(c)+1/2*a)*(-(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*csgn(
I*f)*csgn(I*f*x^m)^2+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*csgn(I*f*x^m)^3+
2*ln(f))/x-2*ln(x^m)/x-2*m/x)-e*n*b*ln(x^m)/d*ln(e*x+d)+e*n*b*ln(x^m)/d*ln(
```

```
x)-1/2*e*n*b*m/d*ln(x)^2+e*n*b*m/d*ln(e*x+d)*ln(-e*x/d)+e*n*b*m/d*dilog(-e*
x/d)-1/2*I*e*n*b/d*ln(e*x+d)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/2*I*e*n*b/d*l
n(x)*Pi*csgn(I*f*x^m)^3-1/2*I*e*n*b/d*ln(e*x+d)*Pi*csgn(I*f)*csgn(I*f*x^m)^
2-1/2*I*e*n*b/d*ln(x)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*n*b/d*ln(e*x
+d)*ln(f)-b*e*m*n*ln(e*x+d)/d+1/2*I*e*n*b/d*ln(e*x+d)*Pi*csgn(I*f)*csgn(I*x
^m)*csgn(I*f*x^m)+1/2*I*e*n*b/d*ln(x)*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I*e*
n*b/d*ln(e*x+d)*Pi*csgn(I*f*x^m)^3+1/2*I*e*n*b/d*ln(x)*Pi*csgn(I*x^m)*csgn(
I*f*x^m)^2+e*n*b/d*ln(x)*ln(f)+b*e*m*n*ln(x)/d
```

## Fricas [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)\log(fx^m)}{x^2}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^2,x, algorithm="fricas")
[Out] integral((b*log((e*x + d)^n*c)*log(f*x^m) + a*log(f*x^m))/x^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))/x**2,x)
[Out] Timed out
```

## Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.59

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^2}\,dx = -\frac{1}{2}\left(\frac{2\left(\log\left(\frac{ex}{d}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ex}{d}\right)\right)ben}{d}+\frac{2\,ben\log(ex+d)}{d}-\frac{2\,benx\log(ex+d)\log(x)-benx\log\right.$$
$$-\left(ben\left(\frac{\log(ex+d)}{d}-\frac{\log(x)}{d}\right)+\frac{b\log((ex+d)^n c)}{x}+\frac{a}{x}\right)\log(fx^m)$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^2,x, algorithm="maxima")
[Out] -1/2*(2*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))*b*e*n/d + 2*b*e*n*log(e*x +
 d)/d - (2*b*e*n*x*log(e*x + d)*log(x) - b*e*n*x*log(x)^2 + 2*b*e*n*x*log(x
) - 2*b*d*log((e*x + d)^n) - 2*b*d*log(c) - 2*a*d)/(d*x))*m - (b*e*n*(log(e
*x + d)/d - log(x)/d) + b*log((e*x + d)^n*c)/x + a/x)*log(f*x^m)
```

## Giac [F]

$$\int \frac{\log (fx^m) (a + b\log (c(d + ex)^n))}{x^2} \, dx = \int \frac{(b\log ((ex + d)^n c) + a) \log (fx^m)}{x^2} \, dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*log(f*x^m)/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log (fx^m) (a + b\log (c(d + ex)^n))}{x^2} \, dx = \int \frac{\ln (f\,x^m)\,\left(a + b\,\ln \left(c\,{\left(d + e\,x\right)}^n\right)\right)}{x^2} \, dx$$

```
[In] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^2,x)
[Out] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^2, x)
```

# 3.364 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^3}\,dx = -\frac{3bemn}{4dx}-\frac{be^2mn\log(x)}{4d^2}-\frac{ben\log(fx^m)}{2dx}+\frac{be^2n\log\left(1+\frac{d}{ex}\right)\log(fx^m)}{2d^2}+\frac{be^2mn\log(d+ex)}{4d^2}-\frac{1}{4}\left(\frac{m}{x^2}+\frac{2\log(fx^m)}{x^2}\right)(a+b\log(c(d+ex)^n))-\frac{be^2mn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{2d^2}$$

```
[Out] -3/4*b*e*m*n/d/x-1/4*b*e^2*m*n*ln(x)/d^2-1/2*b*e*n*ln(f*x^m)/d/x+1/2*b*e^2*
n*ln(1+d/e/x)*ln(f*x^m)/d^2+1/4*b*e^2*m*n*ln(e*x+d)/d^2-1/4*(m/x^2+2*ln(f*x
^m)/x^2)*(a+b*ln(c*(e*x+d)^n))-1/2*b*e^2*m*n*polylog(2,-d/e/x)/d^2
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{2473, 2380, 2341, 2379, 2438, 46\}$

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^3}\,dx = -\frac{1}{4}\left(\frac{2\log(fx^m)}{x^2}+\frac{m}{x^2}\right)(a+b\log(c(d+ex)^n)) + \frac{be^2n\log\left(\frac{d}{ex}+1\right)\log(fx^m)}{2d^2} - \frac{be^2mn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{2d^2} - \frac{be^2mn\log(x)}{4d^2} + \frac{be^2mn\log(d+ex)}{4d^2} - \frac{ben\log(fx^m)}{2dx} - \frac{3bemn}{4dx}$$

```
[In] Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^3,x]
```

```
[Out] (-3*b*e*m*n)/(4*d*x) - (b*e^2*m*n*Log[x])/(4*d^2) - (b*e*n*Log[f*x^m])/(2*d
*x) + (b*e^2*n*Log[1 + d/(e*x)]*Log[f*x^m])/(2*d^2) + (b*e^2*m*n*Log[d + e*
x])/(4*d^2) - ((m/x^2 + (2*Log[f*x^m])/x^2)*(a + b*Log[c*(d + e*x)^n]))/4 -
 (b*e^2*m*n*PolyLog[2, -(d/(e*x))])/(2*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2473

```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Dist[b*e*(n/(g*(q + 1))), Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x], x
] + Dist[b*e*m*(n/(g*(q + 1)^2)), Int[(g*x)^(q + 1)/(d + e*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{4}\left(\frac{m}{x^2}+\frac{2\log\left(fx^m\right)}{x^2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right) \\
&\quad +\frac{1}{2}(ben)\int\frac{\log\left(fx^m\right)}{x^2(d+ex)}\,dx+\frac{1}{4}(bemn)\int\frac{1}{x^2(d+ex)}\,dx \\
&= -\frac{1}{4}\left(\frac{m}{x^2}+\frac{2\log\left(fx^m\right)}{x^2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+\frac{(ben)\int\frac{\log(fx^m)}{x^2}\,dx}{2d} \\
&\quad -\frac{\left(be^2n\right)\int\frac{\log(fx^m)}{x(d+ex)}\,dx}{2d}+\frac{1}{4}(bemn)\int\left(\frac{1}{dx^2}-\frac{e}{d^2x}+\frac{e^2}{d^2(d+ex)}\right)dx \\
&= -\frac{3bemn}{4dx}-\frac{be^2mn\log(x)}{4d^2}-\frac{ben\log\left(fx^m\right)}{2dx}+\frac{be^2n\log\left(1+\frac{d}{ex}\right)\log\left(fx^m\right)}{2d^2} \\
&\quad +\frac{be^2mn\log(d+ex)}{4d^2}-\frac{1}{4}\left(\frac{m}{x^2}+\frac{2\log\left(fx^m\right)}{x^2}\right)\left(a+b\log\left(c(d+ex)^n\right)\right) \\
&\quad -\frac{\left(be^2mn\right)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{2d^2} \\
&= -\frac{3bemn}{4dx}-\frac{be^2mn\log(x)}{4d^2}-\frac{ben\log\left(fx^m\right)}{2dx}+\frac{be^2n\log\left(1+\frac{d}{ex}\right)\log\left(fx^m\right)}{2d^2} \\
&\quad +\frac{be^2mn\log(d+ex)}{4d^2}-\frac{1}{4}\left(\frac{m}{x^2}+\frac{2\log\left(fx^m\right)}{x^2}\right)\bigg(a \\
&\quad +b\log\left(c(d+ex)^n\right)\bigg)-\frac{be^2mn\text{Li}_2\left(-\frac{d}{ex}\right)}{2d^2}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.31

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^3}\,dx = -\frac{ad^2m+3bdemnx-be^2mnx^2\log^2(x)+2ad^2\log(fx^m)+2bdenx\log(fx^m)-be^2mnx^2\log(d+ex)-2b\cdots}{\cdots}$$

```
[In] Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^3,x]
```

```
[Out] -1/4*(a*d^2*m + 3*b*d*e*m*n*x - b*e^2*m*n*x^2*Log[x]^2 + 2*a*d^2*Log[f*x^m]
 + 2*b*d*e*n*x*Log[f*x^m] - b*e^2*m*n*x^2*Log[d + e*x] - 2*b*e^2*n*x^2*Log[
f*x^m]*Log[d + e*x] + b*d^2*m*Log[c*(d + e*x)^n] + 2*b*d^2*Log[f*x^m]*Log[c
*(d + e*x)^n] + b*e^2*n*x^2*Log[x]*(m + 2*Log[f*x^m] + 2*m*Log[d + e*x] - 2
*m*Log[1 + (e*x)/d]) - 2*b*e^2*m*n*x^2*PolyLog[2, -((e*x)/d)])/(d^2*x^2)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.39 (sec) , antiderivative size = 901, normalized size of antiderivative = 5.78

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 901 |

```
[In] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e^2*n*b*m/d^2*ln(e*x+d)*ln(-e*x/d)+1/2*e^2*n*b*ln(x^m)/d^2*ln(e*x+d)-1
/2*e^2*n*b*ln(x^m)/d^2*ln(x)-1/2*e^2*n*b*m/d^2*dilog(-e*x/d)+1/4*e^2*n*b*m/
d^2*ln(x)^2+1/2*e^2*n*b/d^2*ln(e*x+d)*ln(f)-1/2*e^2*n*b/d^2*ln(x)*ln(f)+1/4
*I*e*n*b/d/x*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I*e^2*n*b/d^2*ln(x)
*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/4*I*e^2*n*b/d^2*ln(e*x+d)*Pi*csgn
(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/4*I*e^2*n*b/d^2*ln(e*x+d)*Pi*csgn(I*f*x^m
)^3+1/4*I*e^2*n*b/d^2*ln(x)*Pi*csgn(I*f*x^m)^3+1/4*I*e*n*b/d/x*Pi*csgn(I*f*
x^m)^3-1/2*e*n*b*ln(x^m)/d/x-1/4*I*e*n*b/d/x*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1
/4*I*e*n*b/d/x*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1/4*I*e^2*n*b/d^2*ln(e*x+d)*P
i*csgn(I*f)*csgn(I*f*x^m)^2+1/4*I*e^2*n*b/d^2*ln(e*x+d)*Pi*csgn(I*x^m)*csgn
(I*f*x^m)^2-1/4*I*e^2*n*b/d^2*ln(x)*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1/4*I*e^2*
n*b/d^2*ln(x)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*
(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1
/4*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+
d)^n)^3+1/2*b*ln(c)+1/2*a)*(-ln(x^m)/x^2-1/2*m/x^2-1/2*(-I*Pi*csgn(I*f)*csg
n(I*x^m)*csgn(I*f*x^m)+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2+I*Pi*csgn(I*x^m)*csgn
(I*f*x^m)^2-I*Pi*csgn(I*f*x^m)^3+2*ln(f))/x^2)+(-1/2*b/x^2*ln(x^m)-1/4*(-I*
Pi*b*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*b*csgn(I*f)*csgn(I*f*x^m)^2+I
```

```
*Pi*b*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*b*csgn(I*f*x^m)^3+2*b*ln(f)+b*m)/x^2
)*ln((e*x+d)^n)-1/2*e*n*b/d/x*ln(f)-3/4*b*e*m*n/d/x-1/4*b*e^2*m*n*ln(x)/d^2
+1/4*b*e^2*m*n*ln(e*x+d)/d^2
```

## Fricas [F]

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)}{x^3}\,dx=\int \frac{\left(b\log\left((ex+d)^n c\right)+a\right)\log(fx^m)}{x^3}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x + d)^n*c)*log(f*x^m) + a*log(f*x^m))/x^3, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)}{x^3}\,dx=\text{Timed out}$$

```
[In] integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))/x**3,x)
```

```
[Out] Timed out
```

## Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.27

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)}{x^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\left(\log\left(\frac{ex}{d}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ex}{d}\right)\right)be^2n}{d^2}+\frac{be^2n\log(ex+d)}{d^2}-\frac{2\,be^2nx^2\log(ex+d)\log(x)-be^2nx^2\ldots}{\ldots}\right.$$
$$\left.+\frac{1}{2}\left(ben\left(\frac{e\log(ex+d)}{d^2}-\frac{e\log(x)}{d^2}-\frac{1}{dx}\right)-\frac{b\log\left((ex+d)^n c\right)}{x^2}-\frac{a}{x^2}\right)\log(fx^m)\right.$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))*b*e^2*n/d^2 + b*e^2*n*log(e*
x + d)/d^2 - (2*b*e^2*n*x^2*log(e*x + d)*log(x) - b*e^2*n*x^2*log(x)^2 + b*
e^2*n*x^2*log(x) + 3*b*d*e*n*x + b*d^2*log((e*x + d)^n) + b*d^2*log(c) + a*
d^2)/(d^2*x^2))*m + 1/2*(b*e*n*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)
) - b*log((e*x + d)^n*c)/x^2 - a/x^2)*log(f*x^m)
```

## Giac [F]

$$\int \frac{\log (fx^m) (a + b\log (c(d + ex)^n))}{x^3} dx = \int \frac{(b\log ((ex + d)^n c) + a) \log (fx^m)}{x^3} dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^3,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*log(f*x^m)/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log (fx^m) (a + b\log (c(d + ex)^n))}{x^3} dx = \int \frac{\ln (f\,x^m)\,\left(a + b\,\ln \left(c\,{\left(d + e\,x\right)}^n\right)\right)}{x^3} dx$$

```
[In] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^3,x)
[Out] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^3, x)
```

# 3.365 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 193

$$\begin{aligned}\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx = &-\frac{5bemn}{36dx^2}+\frac{4be^2mn}{9d^2x}+\frac{be^3mn\log(x)}{9d^3}\\ &-\frac{ben\log(fx^m)}{6dx^2}+\frac{be^2n\log(fx^m)}{3d^2x}\\ &-\frac{be^3n\log\left(1+\frac{d}{ex}\right)\log(fx^m)}{3d^3}\\ &-\frac{be^3mn\log(d+ex)}{9d^3}\\ &-\frac{1}{9}\left(\frac{m}{x^3}+\frac{3\log(fx^m)}{x^3}\right)(a+b\log(c(d+ex)^n))\\ &+\frac{be^3mn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{3d^3}\end{aligned}$$

```
[Out] -5/36*b*e*m*n/d/x^2+4/9*b*e^2*m*n/d^2/x+1/9*b*e^3*m*n*ln(x)/d^3-1/6*b*e*n*l
n(f*x^m)/d/x^2+1/3*b*e^2*n*ln(f*x^m)/d^2/x-1/3*b*e^3*n*ln(1+d/e/x)*ln(f*x^m
)/d^3-1/9*b*e^3*m*n*ln(e*x+d)/d^3-1/9*(m/x^3+3*ln(f*x^m)/x^3)*(a+b*ln(c*(e*
x+d)^n))+1/3*b*e^3*m*n*polylog(2,-d/e/x)/d^3
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2473, 2380, 2341, 2379, 2438, 46}

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx = -\frac{1}{9}\left(\frac{3\log(fx^m)}{x^3}+\frac{m}{x^3}\right)(a+b\log(c(d+ex)^n)) - \frac{be^3n\log\left(\frac{d}{ex}+1\right)\log(fx^m)}{3d^3} + \frac{be^3mn\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{3d^3} + \frac{be^3mn\log(x)}{9d^3} - \frac{be^3mn\log(d+ex)}{9d^3} + \frac{be^2n\log(fx^m)}{3d^2x} + \frac{4be^2mn}{9d^2x} - \frac{ben\log(fx^m)}{6dx^2} - \frac{5bemn}{36dx^2}$$

```
[In] Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^4,x]
```

```
[Out] (-5*b*e*m*n)/(36*d*x^2) + (4*b*e^2*m*n)/(9*d^2*x) + (b*e^3*m*n*Log[x])/(9*d
^3) - (b*e*n*Log[f*x^m])/(6*d*x^2) + (b*e^2*n*Log[f*x^m])/(3*d^2*x) - (b*e^
3*n*Log[1 + d/(e*x)]*Log[f*x^m])/(3*d^3) - (b*e^3*m*n*Log[d + e*x])/(9*d^3)
 - ((m/x^3 + (3*Log[f*x^m])/x^3)*(a + b*Log[c*(d + e*x)^n]))/9 + (b*e^3*m*n
*PolyLog[2, -(d/(e*x))])/(3*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2473

```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Dist[b*e*(n/(g*(q + 1))), Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x], x
] + Dist[b*e*m*(n/(g*(q + 1)^2)), Int[(g*x)^(q + 1)/(d + e*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{9}\left(\frac{m}{x^3}+\frac{3\log\left(fx^m\right)}{x^3}\right)\left(a+b\log\left(c(d+ex)^n\right)\right) \\
&\quad +\frac{1}{3}(ben)\int\frac{\log\left(fx^m\right)}{x^3(d+ex)}\,dx+\frac{1}{9}(bemn)\int\frac{1}{x^3(d+ex)}\,dx \\
&= -\frac{1}{9}\left(\frac{m}{x^3}+\frac{3\log\left(fx^m\right)}{x^3}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+\frac{(ben)\int\frac{\log(fx^m)}{x^3}\,dx}{3d} \\
&\quad -\frac{(be^2n)\int\frac{\log(fx^m)}{x^2(d+ex)}\,dx}{3d}+\frac{1}{9}(bemn)\int\left(\frac{1}{dx^3}-\frac{e}{d^2x^2}+\frac{e^2}{d^3x}-\frac{e^3}{d^3(d+ex)}\right)dx \\
&= -\frac{5bemn}{36dx^2}+\frac{be^2mn}{9d^2x}+\frac{be^3mn\log(x)}{9d^3}-\frac{ben\log\left(fx^m\right)}{6dx^2}-\frac{be^3mn\log(d+ex)}{9d^3}-\frac{1}{9}\left(\frac{m}{x^3}\right. \\
&\quad \left.+\frac{3\log\left(fx^m\right)}{x^3}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)-\frac{(be^2n)\int\frac{\log(fx^m)}{x^2}\,dx}{3d^2}+\frac{(be^3n)\int\frac{\log(fx^m)}{x(d+ex)}\,dx}{3d^2} \\
&= -\frac{5bemn}{36dx^2}+\frac{4be^2mn}{9d^2x}+\frac{be^3mn\log(x)}{9d^3}-\frac{ben\log\left(fx^m\right)}{6dx^2} \\
&\quad +\frac{be^2n\log\left(fx^m\right)}{3d^2x}-\frac{be^3n\log\left(1+\frac{d}{ex}\right)\log\left(fx^m\right)}{3d^3}-\frac{be^3mn\log(d+ex)}{9d^3} \\
&\quad -\frac{1}{9}\left(\frac{m}{x^3}+\frac{3\log\left(fx^m\right)}{x^3}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+\frac{(be^3mn)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{3d^3}
\end{aligned}$$

$$= -\frac{5bemn}{36dx^2} + \frac{4be^2mn}{9d^2x} + \frac{be^3mn\log(x)}{9d^3} - \frac{ben\log(fx^m)}{6dx^2} + \frac{be^2n\log(fx^m)}{3d^2x} - \frac{be^3n\log\left(1+\frac{d}{ex}\right)\log(fx^m)}{3d^3} - \frac{be^3mn\log(d+ex)}{9d^3} - \frac{1}{9}\left(\frac{m}{x^3} + \frac{3\log(fx^m)}{x^3}\right)(a+b\log(c(d+ex)^n)) + \frac{be^3mn\mathrm{Li}_2\left(-\frac{d}{ex}\right)}{3d^3}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.24

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx = -\frac{4ad^3m + 5bd^2emnx - 16bde^2mnx^2 + 6be^3mnx^3\log^2(x) + 12ad^3\log(fx^m) + 6bd^2enx\log(fx^m) - 12bde}{}$$

```
[In] Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^4,x]
```

```
[Out] -1/36*(4*a*d^3*m + 5*b*d^2*e*m*n*x - 16*b*d*e^2*m*n*x^2 + 6*b*e^3*m*n*x^3*L
og[x]^2 + 12*a*d^3*Log[f*x^m] + 6*b*d^2*e*n*x*Log[f*x^m] - 12*b*d*e^2*n*x^2
*Log[f*x^m] + 4*b*e^3*m*n*x^3*Log[d + e*x] + 12*b*e^3*n*x^3*Log[f*x^m]*Log[
d + e*x] + 4*b*d^3*m*Log[c*(d + e*x)^n] + 12*b*d^3*Log[f*x^m]*Log[c*(d + e*
x)^n] - 4*b*e^3*n*x^3*Log[x]*(m + 3*Log[f*x^m] + 3*m*Log[d + e*x] - 3*m*Log
[1 + (e*x)/d]) + 12*b*e^3*m*n*x^3*PolyLog[2, -((e*x)/d)])/(d^3*x^3)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.38 (sec) , antiderivative size = 1070, normalized size of antiderivative = 5.54

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1070 |

```
[In] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*b/x^3*ln(x^m)-1/18*(-3*I*Pi*b*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+3*I
*Pi*b*csgn(I*f)*csgn(I*f*x^m)^2+3*I*Pi*b*csgn(I*x^m)*csgn(I*f*x^m)^2-3*I*Pi
*b*csgn(I*f*x^m)^3+6*b*ln(f)+2*b*m)/x^3)*ln((e*x+d)^n)-1/6*I*e^3*n*b/d^3*ln
(x)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/6*I*e^2*n*b/d^2/x*Pi*csgn(I*f)
*csgn(I*x^m)*csgn(I*f*x^m)-1/3*e^3*n*b*ln(x^m)/d^3*ln(e*x+d)+1/3*e^3*n*b*ln
(x^m)/d^3*ln(x)-1/6*e^3*n*b*m/d^3*ln(x)^2+1/3*e^3*n*b*m/d^3*dilog(-e*x/d)-1
/3*e^3*n*b/d^3*ln(e*x+d)*ln(f)+1/3*e^3*n*b/d^3*ln(x)*ln(f)-1/6*e*n*b/d/x^2*
ln(f)+1/3*e^3*n*b*m/d^3*ln(e*x+d)*ln(-e*x/d)+1/6*I*e^3*n*b/d^3*ln(e*x+d)*Pi
*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/12*I*e*n*b/d/x^2*Pi*csgn(I*f)*csgn(I
```

```
*x^m)*csgn(I*f*x^m)-1/12*I*e*n*b/d/x^2*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1/12*I*
e*n*b/d/x^2*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/6*I*e^3*n*b/d^3*ln(e*x+d)*Pi*c
sgn(I*f)*csgn(I*f*x^m)^2-1/6*I*e^3*n*b/d^3*ln(e*x+d)*Pi*csgn(I*x^m)*csgn(I*
f*x^m)^2+1/6*I*e^3*n*b/d^3*ln(x)*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/6*I*e^3*n*b
/d^3*ln(x)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1/6*I*e^3*n*b/d^3*ln(e*x+d)*Pi*cs
gn(I*f*x^m)^3-1/6*I*e^3*n*b/d^3*ln(x)*Pi*csgn(I*f*x^m)^3-1/6*I*e^2*n*b/d^2/
x*Pi*csgn(I*f*x^m)^3+1/12*I*e*n*b/d/x^2*Pi*csgn(I*f*x^m)^3+1/6*I*e^2*n*b/d^
2/x*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/6*I*e^2*n*b/d^2/x*Pi*csgn(I*x^m)*csgn(I*
f*x^m)^2+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/4*I
*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c
*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*a)*(-1/3*(-I
*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2+I*Pi
*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*csgn(I*f*x^m)^3+2*ln(f))/x^3-2/3*ln(x^m)/
x^3-2/9*m/x^3)+1/3*e^2*n*b/d^2/x*ln(f)+1/3*e^2*n*b*ln(x^m)/d^2/x-1/6*e*n*b*
ln(x^m)/d/x^2-5/36*b*e*m*n/d/x^2+4/9*b*e^2*m*n/d^2/x+1/9*b*e^3*m*n*ln(x)/d^
3-1/9*b*e^3*m*n*ln(e*x+d)/d^3
```

## Fricas [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx=\int \frac{(b\log((ex+d)^nc)+a)\log(fx^m)}{x^4}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^4,x, algorithm="fricas")
[Out] integral((b*log((e*x + d)^n*c)*log(f*x^m) + a*log(f*x^m))/x^4, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx=\text{Timed out}$$

```
[In] integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))/x**4,x)
[Out] Timed out
```

## Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.19

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx = -\frac{1}{36}\left(\frac{12\left(\log\left(\frac{ex}{d}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ex}{d}\right)\right)be^3n}{d^3}+\frac{4\,be^3n\log(ex+d)}{d^3}-\frac{12\,be^3nx^3\log(ex+d)\log(x)-6}{}\right. -\frac{1}{6}\left(ben\left(\frac{2\,e^2\log(ex+d)}{d^3}-\frac{2\,e^2\log(x)}{d^3}-\frac{2\,ex-d}{d^2x^2}\right)+\frac{2\,b\log((ex+d)^nc)}{x^3}+\frac{2\,a}{x^3}\right)\log(fx^m)$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^4,x, algorithm="maxima")
```

```
[Out] -1/36*(12*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))*b*e^3*n/d^3 + 4*b*e^3*n*l
og(e*x + d)/d^3 - (12*b*e^3*n*x^3*log(e*x + d)*log(x) - 6*b*e^3*n*x^3*log(x
)^2 + 4*b*e^3*n*x^3*log(x) + 16*b*d*e^2*n*x^2 - 5*b*d^2*e*n*x - 4*b*d^3*log
((e*x + d)^n) - 4*b*d^3*log(c) - 4*a*d^3)/(d^3*x^3))*m - 1/6*(b*e*n*(2*e^2*
log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2)) + 2*b*log((e*x
 + d)^n*c)/x^3 + 2*a/x^3)*log(f*x^m)
```

## Giac [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx = \int \frac{(b\log((ex+d)^nc)+a)\log(fx^m)}{x^4}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*log(f*x^m)/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^4}\,dx = \int \frac{\ln(f\,x^m)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)}{x^4}\,dx$$

```
[In] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^4,x)
```

```
[Out] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^4, x)
```

# 3.366 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 230

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx = -\frac{7bemn}{144dx^3}+\frac{3be^2mn}{32d^2x^2}-\frac{5be^3mn}{16d^3x}-\frac{be^4mn\log(x)}{16d^4} -\frac{ben\log(fx^m)}{12dx^3}+\frac{be^2n\log(fx^m)}{8d^2x^2} -\frac{be^3n\log(fx^m)}{4d^3x}+\frac{be^4n\log\left(1+\frac{d}{ex}\right)\log(fx^m)}{4d^4} +\frac{be^4mn\log(d+ex)}{16d^4} -\frac{1}{16}\left(\frac{m}{x^4}+\frac{4\log(fx^m)}{x^4}\right)(a+b\log(c(d+ex)^n)) -\frac{be^4mn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{4d^4}$$

```
[Out] -7/144*b*e*m*n/d/x^3+3/32*b*e^2*m*n/d^2/x^2-5/16*b*e^3*m*n/d^3/x-1/16*b*e^4
*m*n*ln(x)/d^4-1/12*b*e*n*ln(f*x^m)/d/x^3+1/8*b*e^2*n*ln(f*x^m)/d^2/x^2-1/4
*b*e^3*n*ln(f*x^m)/d^3/x+1/4*b*e^4*n*ln(1+d/e/x)*ln(f*x^m)/d^4+1/16*b*e^4*m
*n*ln(e*x+d)/d^4-1/16*(m/x^4+4*ln(f*x^m)/x^4)*(a+b*ln(c*(e*x+d)^n))-1/4*b*e
^4*m*n*polylog(2,-d/e/x)/d^4
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2473, 2380, 2341, 2379, 2438, 46}

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx = -\frac{1}{16}\left(\frac{4\log(fx^m)}{x^4}+\frac{m}{x^4}\right)(a+b\log(c(d+ex)^n)) + \frac{be^4n\log\left(\frac{d}{ex}+1\right)\log(fx^m)}{4d^4} - \frac{be^4mn\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{4d^4} - \frac{be^4mn\log(x)}{16d^4} + \frac{be^4mn\log(d+ex)}{16d^4} - \frac{be^3n\log(fx^m)}{4d^3x} - \frac{5be^3mn}{16d^3x} + \frac{be^2n\log(fx^m)}{8d^2x^2} + \frac{3be^2mn}{32d^2x^2} - \frac{ben\log(fx^m)}{12dx^3} - \frac{7bemn}{144dx^3}$$

```
[In] Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^5,x]
```

```
[Out] (-7*b*e*m*n)/(144*d*x^3) + (3*b*e^2*m*n)/(32*d^2*x^2) - (5*b*e^3*m*n)/(16*d
^3*x) - (b*e^4*m*n*Log[x])/(16*d^4) - (b*e*n*Log[f*x^m])/(12*d*x^3) + (b*e^
2*n*Log[f*x^m])/(8*d^2*x^2) - (b*e^3*n*Log[f*x^m])/(4*d^3*x) + (b*e^4*n*Log
[1 + d/(e*x)]*Log[f*x^m])/(4*d^4) + (b*e^4*m*n*Log[d + e*x])/(16*d^4) - ((m
/x^4 + (4*Log[f*x^m])/x^4)*(a + b*Log[c*(d + e*x)^n]))/16 - (b*e^4*m*n*Poly
Log[2, -(d/(e*x))])/(4*d^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
```

```
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2473

```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))*((g_.)*(x_))^(q_.), x_Symbol] :> Simp[(-(g*(q + 1))^(-1))*(m*((g*x)^(q
+ 1)/(q + 1)) - (g*x)^(q + 1)*Log[f*x^m])*(a + b*Log[c*(d + e*x)^n]), x] +
(-Dist[b*e*(n/(g*(q + 1))), Int[(g*x)^(q + 1)*(Log[f*x^m]/(d + e*x)), x], x
] + Dist[b*e*m*(n/(g*(q + 1)^2)), Int[(g*x)^(q + 1)/(d + e*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{16}\left(\frac{m}{x^4}+\frac{4\log\left(fx^m\right)}{x^4}\right)\left(a+b\log\left(c(d+ex)^n\right)\right) \\
&\quad+\frac{1}{4}(ben)\int\frac{\log\left(fx^m\right)}{x^4(d+ex)}\,dx+\frac{1}{16}(bemn)\int\frac{1}{x^4(d+ex)}\,dx \\
&= -\frac{1}{16}\left(\frac{m}{x^4}+\frac{4\log\left(fx^m\right)}{x^4}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+\frac{(ben)\int\frac{\log(fx^m)}{x^4}\,dx}{4d} \\
&\quad-\frac{(be^2n)\int\frac{\log(fx^m)}{x^3(d+ex)}\,dx}{4d}+\frac{1}{16}(bemn)\int\left(\frac{1}{dx^4}-\frac{e}{d^2x^3}+\frac{e^2}{d^3x^2}-\frac{e^3}{d^4x}+\frac{e^4}{d^4(d+ex)}\right)\,dx \\
&= -\frac{7bemn}{144dx^3}+\frac{be^2mn}{32d^2x^2}-\frac{be^3mn}{16d^3x}-\frac{be^4mn\log(x)}{16d^4}-\frac{ben\log\left(fx^m\right)}{12dx^3}+\frac{be^4mn\log(d+ex)}{16d^4} \\
&\quad-\frac{1}{16}\left(\frac{m}{x^4}+\frac{4\log\left(fx^m\right)}{x^4}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)-\frac{(be^2n)\int\frac{\log(fx^m)}{x^3}\,dx}{4d^2}+\frac{(be^3n)\int\frac{\log(fx^m)}{x^2(d+ex)}\,dx}{4d^2} \\
&= -\frac{7bemn}{144dx^3}+\frac{3be^2mn}{32d^2x^2}-\frac{be^3mn}{16d^3x}-\frac{be^4mn\log(x)}{16d^4}-\frac{ben\log\left(fx^m\right)}{12dx^3}+\frac{be^2n\log\left(fx^m\right)}{8d^2x^2} \\
&\quad+\frac{be^4mn\log(d+ex)}{16d^4}-\frac{1}{16}\left(\frac{m}{x^4}+\frac{4\log\left(fx^m\right)}{x^4}\right)\left(a+b\log\left(c(d+ex)^n\right)\right) \\
&\quad+\frac{(be^3n)\int\frac{\log(fx^m)}{x^2}\,dx}{4d^3}-\frac{(be^4n)\int\frac{\log(fx^m)}{x(d+ex)}\,dx}{4d^3}
\end{aligned}$$

$$= -\frac{7bemn}{144dx^3} + \frac{3be^2mn}{32d^2x^2} - \frac{5be^3mn}{16d^3x} - \frac{be^4mn\log(x)}{16d^4} - \frac{ben\log(fx^m)}{12dx^3} + \frac{be^2n\log(fx^m)}{8d^2x^2} - \frac{be^3n\log(fx^m)}{4d^3x} + \frac{be^4n\log\left(1+\frac{d}{ex}\right)\log(fx^m)}{4d^4} + \frac{be^4mn\log(d+ex)}{16d^4} - \frac{1}{16}\left(\frac{m}{x^4} + \frac{4\log(fx^m)}{x^4}\right)(a+b\log(c(d+ex)^n)) - \frac{(be^4mn)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}dx}{4d^4}$$

$$= -\frac{7bemn}{144dx^3} + \frac{3be^2mn}{32d^2x^2} - \frac{5be^3mn}{16d^3x} - \frac{be^4mn\log(x)}{16d^4} - \frac{ben\log(fx^m)}{12dx^3} + \frac{be^2n\log(fx^m)}{8d^2x^2} - \frac{be^3n\log(fx^m)}{4d^3x} + \frac{be^4n\log\left(1+\frac{d}{ex}\right)\log(fx^m)}{4d^4} + \frac{be^4mn\log(d+ex)}{16d^4} - \frac{1}{16}\left(\frac{m}{x^4} + \frac{4\log(fx^m)}{x^4}\right)(a+b\log(c(d+ex)^n)) - \frac{be^4mn\text{Li}_2\left(-\frac{d}{ex}\right)}{4d^4}$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.19

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}dx = -\frac{18ad^4m + 14bd^3emnx - 27bd^2e^2mnx^2 + 90bde^3mnx^3 - 36be^4mnx^4\log^2(x) + 72ad^4\log(fx^m) + 24bd^3e}{}$$

```
[In] Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n]))/x^5,x]
```

```
[Out] -1/288*(18*a*d^4*m + 14*b*d^3*e*m*n*x - 27*b*d^2*e^2*m*n*x^2 + 90*b*d*e^3*m
*n*x^3 - 36*b*e^4*m*n*x^4*Log[x]^2 + 72*a*d^4*Log[f*x^m] + 24*b*d^3*e*n*x*L
og[f*x^m] - 36*b*d^2*e^2*n*x^2*Log[f*x^m] + 72*b*d*e^3*n*x^3*Log[f*x^m] - 1
8*b*e^4*m*n*x^4*Log[d + e*x] - 72*b*e^4*n*x^4*Log[f*x^m]*Log[d + e*x] + 18*
b*d^4*m*Log[c*(d + e*x)^n] + 72*b*d^4*Log[f*x^m]*Log[c*(d + e*x)^n] + 18*b*
e^4*n*x^4*Log[x]*(m + 4*Log[f*x^m] + 4*m*Log[d + e*x] - 4*m*Log[1 + (e*x)/d
]) - 72*b*e^4*m*n*x^4*PolyLog[2, -((e*x)/d)])/(d^4*x^4)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 32.96 (sec) , antiderivative size = 1237, normalized size of antiderivative = 5.38

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1237 |

```
[In] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*I*e*n*b/d/x^3*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1/8*I*e^4*n*b/d^4*ln(e*x
+d)*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/8*I*e^4*n*b/d^4*ln(e*x+d)*Pi*csgn(I*x^m)
*csgn(I*f*x^m)^2-1/8*I*e^3*n*b/d^3/x*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1/8*I*e^3
*n*b/d^3/x*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+1/16*I*e^2*n*b/d^2/x^2*Pi*csgn(I*
f)*csgn(I*f*x^m)^2+1/16*I*e^2*n*b/d^2/x^2*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/
8*I*e^4*n*b/d^4*ln(x)*Pi*csgn(I*f)*csgn(I*f*x^m)^2-1/8*I*e^4*n*b/d^4*ln(x)*
Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/24*I*e*n*b/d/x^3*Pi*csgn(I*f)*csgn(I*f*x^m
)^2+1/24*I*e*n*b/d/x^3*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/24*I*e*n*b/
d/x^3*Pi*csgn(I*f*x^m)^3+1/8*I*e^3*n*b/d^3/x*Pi*csgn(I*f*x^m)^3-1/16*I*e^2*
n*b/d^2/x^2*Pi*csgn(I*f*x^m)^3+1/8*I*e^4*n*b/d^4*ln(x)*Pi*csgn(I*f*x^m)^3-1
/8*I*e^4*n*b/d^4*ln(e*x+d)*Pi*csgn(I*f*x^m)^3-1/16*I*e^2*n*b/d^2/x^2*Pi*csg
n(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/8*I*e^4*n*b/d^4*ln(x)*Pi*csgn(I*f)*csgn(
I*x^m)*csgn(I*f*x^m)-1/8*I*e^4*n*b/d^4*ln(e*x+d)*Pi*csgn(I*f)*csgn(I*x^m)*c
sgn(I*f*x^m)+1/8*I*e^3*n*b/d^3/x*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/4
*e^3*n*b/d^3/x*ln(f)-1/12*e*n*b/d/x^3*ln(f)+1/8*e^2*n*b/d^2/x^2*ln(f)-1/4*e
^4*n*b*m/d^4*ln(e*x+d)*ln(-e*x/d)+(-1/4*b/x^4*ln(x^m)-1/16*(-2*I*Pi*b*csgn(
I*f)*csgn(I*x^m)*csgn(I*f*x^m)+2*I*Pi*b*csgn(I*f)*csgn(I*f*x^m)^2+2*I*Pi*b*
csgn(I*x^m)*csgn(I*f*x^m)^2-2*I*Pi*b*csgn(I*f*x^m)^3+4*b*ln(f)+b*m)/x^4)*ln
((e*x+d)^n)+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/
4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(
I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*a)*(-1/4*
(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2+I
*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-I*Pi*csgn(I*f*x^m)^3+2*ln(f))/x^4-1/2*ln(x^
m)/x^4-1/8*m/x^4)+1/4*e^4*n*b*ln(x^m)/d^4*ln(e*x+d)-1/4*e^4*n*b*ln(x^m)/d^4
*ln(x)+1/8*e^4*n*b*m/d^4*ln(x)^2-1/4*e^4*n*b*m/d^4*dilog(-e*x/d)+1/4*e^4*n*
b/d^4*ln(e*x+d)*ln(f)-1/4*e^4*n*b/d^4*ln(x)*ln(f)+1/8*e^2*n*b*ln(x^m)/d^2/x
^2-1/12*e*n*b*ln(x^m)/d/x^3-1/4*e^3*n*b*ln(x^m)/d^3/x+1/16*b*e^4*m*n*ln(e*x
+d)/d^4-7/144*b*e*m*n/d/x^3+3/32*b*e^2*m*n/d^2/x^2-5/16*b*e^3*m*n/d^3/x-1/1
6*b*e^4*m*n*ln(x)/d^4
```

## Fricas [F]

$$\int \frac{\log(fx^m)\,(a+b\log(c(d+ex)^n))}{x^5}\,dx = \int \frac{(b\log((ex+d)^n c)+a)\log(fx^m)}{x^5}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x + d)^n*c)*log(f*x^m) + a*log(f*x^m))/x^5, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))/x**5,x)
[Out] Timed out
```

## Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.10

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx$$
$$= \frac{1}{288}\left(\frac{72\left(\log\left(\frac{ex}{d}+1\right)\log(x)+\text{Li}_2\left(-\frac{ex}{d}\right)\right)be^4n}{d^4}+\frac{18\,be^4n\log(ex+d)}{d^4}-\frac{72\,be^4nx^4\log(ex+d)\log(x)-3}{}\right.$$
$$+\frac{1}{24}\left(ben\left(\frac{6\,e^3\log(ex+d)}{d^4}-\frac{6\,e^3\log(x)}{d^4}-\frac{6\,e^2x^2-3\,dex+2\,d^2}{d^3x^3}\right)-\frac{6\,b\log((ex+d)^nc)}{x^4}-\frac{6\,a}{x^4}\right)\log(fx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^5,x, algorithm="maxima")
[Out] 1/288*(72*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))*b*e^4*n/d^4 + 18*b*e^4*n*
log(e*x + d)/d^4 - (72*b*e^4*n*x^4*log(e*x + d)*log(x) - 36*b*e^4*n*x^4*log
(x)^2 + 18*b*e^4*n*x^4*log(x) + 90*b*d*e^3*n*x^3 - 27*b*d^2*e^2*n*x^2 + 14*
b*d^3*e*n*x + 18*b*d^4*log((e*x + d)^n) + 18*b*d^4*log(c) + 18*a*d^4)/(d^4*
x^4))*m + 1/24*(b*e*n*(6*e^3*log(e*x + d)/d^4 - 6*e^3*log(x)/d^4 - (6*e^2*x
^2 - 3*d*e*x + 2*d^2)/(d^3*x^3)) - 6*b*log((e*x + d)^n*c)/x^4 - 6*a/x^4)*lo
g(f*x^m)
```

## Giac [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))}{x^5}\,dx = \int \frac{(b\log((ex+d)^nc)+a)\log(fx^m)}{x^5}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))/x^5,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*log(f*x^m)/x^5, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log (fx^m) (a + b \log (c(d + ex)^n))}{x^5} \, dx = \int \frac{\ln (f\, x^m)\; (a + b\, \ln (c\, (d + e\, x)^n))}{x^5} \, dx$$

```
[In] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^5,x)
[Out] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n)))/x^5, x)
```

# 3.367 $\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 705

$$\begin{aligned}
&\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx\\
&= \frac{2abd^2mnx}{9e^2} - \frac{71b^2d^2mn^2x}{54e^2} + \frac{bd^2mn(6a-11bn)x}{9e^2} + \frac{19b^2dmn^2x^2}{54e} - \frac{2}{27}b^2mn^2x^3\\
&\quad - \frac{2abd^2nx\log(fx^m)}{3e^2} + \frac{11b^2d^2n^2x\log(fx^m)}{9e^2} - \frac{5b^2dn^2x^2\log(fx^m)}{18e}\\
&\quad + \frac{2}{27}b^2n^2x^3\log(fx^m) + \frac{23b^2d^3mn^2\log(d+ex)}{54e^3} + \frac{5b^2d^3mn^2\log\left(-\frac{ex}{d}\right)\log(d+ex)}{9e^3}\\
&\quad - \frac{5b^2d^3n^2\log(fx^m)\log(d+ex)}{9e^3} + \frac{8b^2d^2mn(d+ex)\log(c(d+ex)^n)}{9e^3}\\
&\quad + \frac{2b^2d^3mn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{3e^3} - \frac{2b^2d^2n(d+ex)\log(fx^m)\log(c(d+ex)^n)}{3e^3}\\
&\quad - \frac{5bdmnx^2(a+b\log(c(d+ex)^n))}{18e} + \frac{4}{27}bmnx^3(a+b\log(c(d+ex)^n))\\
&\quad + \frac{bdnx^2\log(fx^m)(a+b\log(c(d+ex)^n))}{3e} - \frac{2}{9}bnx^3\log(fx^m)(a+b\log(c(d+ex)^n))\\
&\quad - \frac{d^3m(a+b\log(c(d+ex)^n))^2}{9e^3} - \frac{1}{9}mx^3(a+b\log(c(d+ex)^n))^2\\
&\quad - \frac{d^3m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{3e^3} + \frac{d^3\log(fx^m)(a+b\log(c(d+ex)^n))^2}{3e^3}\\
&\quad + \frac{1}{3}x^3\log(fx^m)(a+b\log(c(d+ex)^n))^2 + \frac{11b^2d^3mn^2\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{9e^3}\\
&\quad - \frac{2bd^3mn(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{3e^3} + \frac{2b^2d^3mn^2\,\mathrm{PolyLog}\left(3,1+\frac{ex}{d}\right)}{3e^3}
\end{aligned}$$

```
[Out] 2/9*a*b*d^2*m*n*x/e^2+1/9*b*d^2*m*n*(-11*b*n+6*a)*x/e^2-2/3*a*b*d^2*n*x*ln(
f*x^m)/e^2+5/9*b^2*d^3*m*n^2*ln(-e*x/d)*ln(e*x+d)/e^3+8/9*b^2*d^2*m*n*(e*x+
```

```
d)*ln(c*(e*x+d)^n)/e^3+2/3*b^2*d^3*m*n*ln(-e*x/d)*ln(c*(e*x+d)^n)/e^3-2/3*b
^2*d^2*n*(e*x+d)*ln(f*x^m)*ln(c*(e*x+d)^n)/e^3-5/18*b*d*m*n*x^2*(a+b*ln(c*(
e*x+d)^n))/e+1/3*b*d*n*x^2*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/e-2/3*b*d^3*m*n*
(a+b*ln(c*(e*x+d)^n))*polylog(2,1+e*x/d)/e^3-2/9*b*n*x^3*ln(f*x^m)*(a+b*ln(
c*(e*x+d)^n))-1/3*d^3*m*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))^2/e^3+4/27*b*m*n*x
^3*(a+b*ln(c*(e*x+d)^n))+11/9*b^2*d^3*m*n^2*polylog(2,1+e*x/d)/e^3+2/3*b^2*
d^3*m*n^2*polylog(3,1+e*x/d)/e^3+11/9*b^2*d^2*n^2*x*ln(f*x^m)/e^2-5/18*b^2*
d*n^2*x^2*ln(f*x^m)/e+23/54*b^2*d^3*m*n^2*ln(e*x+d)/e^3-5/9*b^2*d^3*n^2*ln(
f*x^m)*ln(e*x+d)/e^3-71/54*b^2*d^2*m*n^2*x/e^2+19/54*b^2*d*m*n^2*x^2/e-2/27
*b^2*m*n^2*x^3+2/27*b^2*n^2*x^3*ln(f*x^m)-1/9*d^3*m*(a+b*ln(c*(e*x+d)^n))^2
/e^3+1/3*d^3*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/e^3-1/9*m*x^3*(a+b*ln(c*(e*x
+d)^n))^2+1/3*x^3*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2
```

## Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.22, number of steps used = 52, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.731$, Rules used = {2445, 2458, 45, 2372, 12, 14, 2338, 2475, 2443, 2481, 2421, 6724, 2393, 2332, 2354,

2438, 2341, 2484, 2422}

$$\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx$$
$$= \frac{b^2mn^2\log^2(d+ex)d^3}{9e^3} + \frac{b^2mn^2\log(x)\log^2(d+ex)d^3}{3e^3} - \frac{b^2n^2\log(fx^m)\log^2(d+ex)d^3}{3e^3} + \frac{m\log(x)(a+b\log(c(d+ex)^n))^2d^3}{3e^3} - \frac{m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2d^3}{3e^3} + \frac{23b^2mn^2\log(x)d^3}{54e^3} + \frac{11bmn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))d^3}{9e^3} - \frac{2bmn\log(d+ex)(a+b\log(c(d+ex)^n))d^3}{9e^3} - \frac{2bmn\log(x)\log(d+ex)(a+b\log(c(d+ex)^n))d^3}{3e^3} + \frac{2bn\log(fx^m)\log(d+ex)(a+b\log(c(d+ex)^n))d^3}{3e^3} + \frac{11b^2mn^2\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)d^3}{9e^3} - \frac{2bmn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)d^3}{3e^3} + \frac{2b^2mn^2\operatorname{PolyLog}\left(3,\frac{ex}{d}+1\right)d^3}{3e^3} - \frac{28b^2mn^2xd^2}{9e^2} + \frac{11abmnxd^2}{9e^2} + \frac{2b^2n^2x\log(fx^m)d^2}{e^2} + \frac{11b^2mn(d+ex)\log(c(d+ex)^n)d^2}{9e^3} + \frac{2bmn(d+ex)(a+b\log(c(d+ex)^n))d^2}{3e^3} - \frac{2bn(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))d^2}{e^3} + \frac{5b^2mn^2x^2d}{36e} + \frac{13b^2mn^2(d+ex)^2d}{36e^3} - \frac{b^2n^2(d+ex)^2\log(fx^m)d}{2e^3} - \frac{13bmn(d+ex)^2(a+b\log(c(d+ex)^n))d}{18e^3} + \frac{bn(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))d}{e^3} - \frac{2}{81}b^2mn^2x^3 - \frac{4b^2mn^2(d+ex)^3}{81e^3} - \frac{1}{9}mx^3(a+b\log(c(d+ex)^n))^2 + \frac{1}{3}x^3\log(fx^m)(a+b\log(c(d+ex)^n))^2 + \frac{2b^2n^2(d+ex)^3\log(fx^m)}{27e^3} + \frac{4bmn(d+ex)^3(a+b\log(c(d+ex)^n))}{27e^3} - \frac{2bn(d+ex)^3\log(fx^m)(a+b\log(c(d+ex)^n))}{9e^3}$$

```
[In] Int[x^2*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (11*a*b*d^2*m*n*x)/(9*e^2) - (28*b^2*d^2*m*n^2*x)/(9*e^2) + (5*b^2*d*m*n^2*
x^2)/(36*e) - (2*b^2*m*n^2*x^3)/81 + (13*b^2*d*m*n^2*(d + e*x)^2)/(36*e^3)
- (4*b^2*m*n^2*(d + e*x)^3)/(81*e^3) + (23*b^2*d^3*m*n^2*Log[x])/(54*e^3) +
 (2*b^2*d^2*n^2*x*Log[f*x^m])/e^2 - (b^2*d*n^2*(d + e*x)^2*Log[f*x^m])/(2*e
^3) + (2*b^2*n^2*(d + e*x)^3*Log[f*x^m])/(27*e^3) + (b^2*d^3*m*n^2*Log[d +
e*x]^2)/(9*e^3) + (b^2*d^3*m*n^2*Log[x]*Log[d + e*x]^2)/(3*e^3) - (b^2*d^3*
n^2*Log[f*x^m]*Log[d + e*x]^2)/(3*e^3) + (11*b^2*d^2*m*n*(d + e*x)*Log[c*(d
 + e*x)^n])/(9*e^3) + (2*b*d^2*m*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n]))/(3
```

```
*e^3) - (13*b*d*m*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(18*e^3) + (4*b
*m*n*(d + e*x)^3*(a + b*Log[c*(d + e*x)^n]))/(27*e^3) + (11*b*d^3*m*n*Log[-
((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/(9*e^3) - (2*b*d^2*n*(d + e*x)*Log[f
*x^m]*(a + b*Log[c*(d + e*x)^n]))/e^3 + (b*d*n*(d + e*x)^2*Log[f*x^m]*(a +
b*Log[c*(d + e*x)^n]))/e^3 - (2*b*n*(d + e*x)^3*Log[f*x^m]*(a + b*Log[c*(d
+ e*x)^n]))/(9*e^3) - (2*b*d^3*m*n*Log[d + e*x]*(a + b*Log[c*(d + e*x)^n]))
/(9*e^3) - (2*b*d^3*m*n*Log[x]*Log[d + e*x]*(a + b*Log[c*(d + e*x)^n]))/(3*
e^3) + (2*b*d^3*n*Log[f*x^m]*Log[d + e*x]*(a + b*Log[c*(d + e*x)^n]))/(3*e^
3) - (m*x^3*(a + b*Log[c*(d + e*x)^n])^2)/9 + (d^3*m*Log[x]*(a + b*Log[c*(d
 + e*x)^n])^2)/(3*e^3) - (d^3*m*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n])^
2)/(3*e^3) + (x^3*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/3 + (11*b^2*d^3*
m*n^2*PolyLog[2, 1 + (e*x)/d])/(9*e^3) - (2*b*d^3*m*n*(a + b*Log[c*(d + e*x
)^n])*PolyLog[2, 1 + (e*x)/d])/(3*e^3) + (2*b^2*d^3*m*n^2*PolyLog[3, 1 + (e
*x)/d])/(3*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2475

```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))^(p_)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*(a + b*
Log[c*(d + e*x)^n])^p, x]}, Dist[Log[f*x^m], u, x] - Dist[m, Int[Dist[1/x,
u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && IGtQ[p, 1] &&
 IGtQ[q, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2484

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.
)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)))/(x_), x_Symbol] :> Simp[Log[x]*(a + b
*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[e*g*m, Int[Log
```

```
[x]*((a + b*Log[c*(d + e*x)^n])/(d + e*x)), x], x] - Dist[b*j*n, Int[Log[x]
*((f + g*Log[h*(i + j*x)^m])/(i + j*x)), x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, i, j, m, n}, x] && EqQ[e*i - d*j, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} = & \frac{2b^2d^2n^2x\log(fx^m)}{e^2} - \frac{b^2dn^2(d+ex)^2\log(fx^m)}{2e^3} + \frac{2b^2n^2(d+ex)^3\log(fx^m)}{27e^3} \\
& - \frac{b^2d^3n^2\log(fx^m)\log^2(d+ex)}{3e^3} - \frac{2bd^2n(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))}{e^3} \\
& + \frac{bdn(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))}{e^3} \\
& - \frac{2bn(d+ex)^3\log(fx^m)(a+b\log(c(d+ex)^n))}{9e^3} \\
& + \frac{2bd^3n\log(fx^m)\log(d+ex)(a+b\log(c(d+ex)^n))}{3e^3} \\
& + \frac{1}{3}x^3\log(fx^m)(a+b\log(c(d+ex)^n))^2 - m\int\left(\frac{2b^2d^2n^2}{e^2} - \frac{b^2dn^2(d+ex)^2}{2e^3x}\right. \\
& + \frac{2b^2n^2(d+ex)^3}{27e^3x} - \frac{b^2d^3n^2\log^2(d+ex)}{3e^3x} - \frac{2bd^2n(d+ex)(a+b\log(c(d+ex)^n))}{e^3x} \\
& + \frac{bdn(d+ex)^2(a+b\log(c(d+ex)^n))}{e^3x} - \frac{2bn(d+ex)^3(a+b\log(c(d+ex)^n))}{9e^3x} \\
& \left. + \frac{2bd^3n\log(d+ex)(a+b\log(c(d+ex)^n))}{3e^3x} + \frac{1}{3}x^2(a+b\log(c(d+ex)^n))^2\right)dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2b^2d^2mn^2x}{e^2} + \frac{2b^2d^2n^2x\log(fx^m)}{e^2} - \frac{b^2dn^2(d+ex)^2\log(fx^m)}{2e^3} \\
&\quad + \frac{2b^2n^2(d+ex)^3\log(fx^m)}{27e^3} - \frac{b^2d^3n^2\log(fx^m)\log^2(d+ex)}{3e^3} \\
&\quad - \frac{2bd^2n(d+ex)\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} \\
&\quad + \frac{bdn(d+ex)^2\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} \\
&\quad - \frac{2bn(d+ex)^3\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3} \\
&\quad + \frac{2bd^3n\log(fx^m)\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^3} \\
&\quad + \frac{1}{3}x^3\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2 - \frac{1}{3}m\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx \\
&\quad + \frac{(2bmn)\int\frac{(d+ex)^3(a+b\log(c(d+ex)^n))}{x}\,dx}{9e^3} - \frac{(bdmn)\int\frac{(d+ex)^2(a+b\log(c(d+ex)^n))}{x}\,dx}{e^3} \\
&\quad + \frac{(2bd^2mn)\int\frac{(d+ex)(a+b\log(c(d+ex)^n))}{x}\,dx}{e^3} - \frac{(2bd^3mn)\int\frac{\log(d+ex)(a+b\log(c(d+ex)^n))}{x}\,dx}{3e^3} \\
&\quad - \frac{(2b^2mn^2)\int\frac{(d+ex)^3}{x}\,dx}{27e^3} + \frac{(b^2dmn^2)\int\frac{(d+ex)^2}{x}\,dx}{2e^3} + \frac{(b^2d^3mn^2)\int\frac{\log^2(d+ex)}{x}\,dx}{3e^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2b^2d^2mn^2x}{e^2} + \frac{2b^2d^2n^2x\log(fx^m)}{e^2} - \frac{b^2dn^2(d+ex)^2\log(fx^m)}{2e^3} \\
&+ \frac{2b^2n^2(d+ex)^3\log(fx^m)}{27e^3} + \frac{b^2d^3mn^2\log\left(-\frac{ex}{d}\right)\log^2(d+ex)}{3e^3} \\
&- \frac{b^2d^3n^2\log(fx^m)\log^2(d+ex)}{3e^3} - \frac{2bd^2n(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))}{e^3} \\
&+ \frac{bdn(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))}{e^3} \\
&- \frac{2bn(d+ex)^3\log(fx^m)(a+b\log(c(d+ex)^n))}{9e^3} \\
&- \frac{2bd^3mn\log(x)\log(d+ex)(a+b\log(c(d+ex)^n))}{3e^3} \\
&+ \frac{2bd^3n\log(fx^m)\log(d+ex)(a+b\log(c(d+ex)^n))}{3e^3} \\
&- \frac{1}{9}mx^3(a+b\log(c(d+ex)^n))^2 + \frac{1}{3}x^3\log(fx^m)(a+b\log(c(d+ex)^n))^2 \\
&+ \frac{(2bmn)\text{Subst}\left(\int \frac{x^3(a+b\log(cx^n))}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex\right)}{9e^4} \\
&- \frac{(bdmn)\text{Subst}\left(\int \frac{x^2(a+b\log(cx^n))}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex\right)}{e^4} \\
&+ \frac{(2bd^2mn)\,\text{Subst}\left(\int \frac{x(a+b\log(cx^n))}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex\right)}{e^4} \\
&+ \frac{(2bd^3mn)\int \frac{\log(x)(a+b\log(c(d+ex)^n))}{d+ex}\,dx}{3e^2} + \frac{1}{9}(2bemn)\int \frac{x^3(a+b\log(c(d+ex)^n))}{d+ex}\,dx \\
&- \frac{(2b^2mn^2)\int\left(3d^2e+\frac{d^3}{x}+3de^2x+e^3x^2\right)dx}{27e^3} + \frac{(b^2dmn^2)\int\left(2de+\frac{d^2}{x}+e^2x\right)dx}{2e^3} \\
&+ \frac{(2b^2d^3mn^2)\int \frac{\log(x)\log(d+ex)}{d+ex}\,dx}{3e^2} - \frac{(2b^2d^3mn^2)\int \frac{\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d+ex}\,dx}{3e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{11b^2d^2mn^2x}{9e^2} + \frac{5b^2dmn^2x^2}{36e} - \frac{2}{81}b^2mn^2x^3 + \frac{23b^2d^3mn^2\log(x)}{54e^3} \\
&\quad + \frac{2b^2d^2n^2x\log(fx^m)}{e^2} - \frac{b^2dn^2(d+ex)^2\log(fx^m)}{2e^3} + \frac{2b^2n^2(d+ex)^3\log(fx^m)}{27e^3} \\
&\quad + \frac{b^2d^3mn^2\log\left(-\frac{ex}{d}\right)\log^2(d+ex)}{3e^3} - \frac{b^2d^3n^2\log(fx^m)\log^2(d+ex)}{3e^3} \\
&\quad - \frac{2bd^2n(d+ex)\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} \\
&\quad + \frac{bdn(d+ex)^2\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^3} \\
&\quad - \frac{2bn(d+ex)^3\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)}{9e^3} \\
&\quad - \frac{2bd^3mn\log(x)\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^3} \\
&\quad + \frac{2bd^3n\log(fx^m)\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{3e^3} \\
&\quad - \frac{1}{9}mx^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2 + \frac{1}{3}x^3\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2 \\
&\quad + \frac{1}{9}(2bmn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3\left(a+b\log(cx^n)\right)}{x}\,dx,x,d+ex\right) \\
&\quad + \frac{(2bmn)\text{Subst}\left(\int\left(d^2e(a+b\log(cx^n)) - \frac{d^3e(a+b\log(cx^n))}{d-x} + dex(a+b\log(cx^n)) + ex^2(a+b\log(c\right.\right.}{9e^4} \\
&\quad - \frac{(bdmn)\text{Subst}\left(\int\left(de(a+b\log(cx^n)) - \frac{d^2e(a+b\log(cx^n))}{d-x} + ex(a+b\log(cx^n))\right)dx,x,d+ex\right)}{e^4} \\
&\quad + \frac{(2bd^2mn)\,\text{Subst}\left(\int\left(e(a+b\log(cx^n)) - \frac{de(a+b\log(cx^n))}{d-x}\right)dx,x,d+ex\right)}{e^4} \\
&\quad + \frac{(2bd^3mn)\,\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(-\frac{d}{e}+\frac{x}{e}\right)}{x}\,dx,x,d+ex\right)}{3e^3} \\
&\quad + \frac{(2b^2d^3mn^2)\,\text{Subst}\left(\int\frac{\log(x)\log\left(-\frac{d}{e}+\frac{x}{e}\right)}{x}\,dx,x,d+ex\right)}{3e^3} \\
&\quad - \frac{(2b^2d^3mn^2)\,\text{Subst}\left(\int\frac{\log(x)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx,x,d+ex\right)}{3e^3}
\end{aligned}
$$

= Too large to display

## Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.38

$$\int x^2 \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx = \frac{48abd^2emnx-137b^2d^2emn^2x-15abde^2mnx^2+19b^2de^2mn^2x^2-6a^2e^3mx^3+8abe^3mnx^3-4b^2e^3mn^2x^3}{}$$

```
[In] Integrate[x^2*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (48*a*b*d^2*e*m*n*x - 137*b^2*d^2*e*m*n^2*x - 15*a*b*d*e^2*m*n*x^2 + 19*b^2
*d*e^2*m*n^2*x^2 - 6*a^2*e^3*m*x^3 + 8*a*b*e^3*m*n*x^3 - 4*b^2*e^3*m*n^2*x^
3 - 36*a*b*d^2*e*n*x*Log[f*x^m] + 66*b^2*d^2*e*n^2*x*Log[f*x^m] + 18*a*b*d*
e^2*n*x^2*Log[f*x^m] - 15*b^2*d*e^2*n^2*x^2*Log[f*x^m] + 18*a^2*e^3*x^3*Log
[f*x^m] - 12*a*b*e^3*n*x^3*Log[f*x^m] + 4*b^2*e^3*n^2*x^3*Log[f*x^m] - 12*a
*b*d^3*m*n*Log[d + e*x] + 71*b^2*d^3*m*n^2*Log[d + e*x] - 36*a*b*d^3*m*n*Lo
g[x]*Log[d + e*x] + 66*b^2*d^3*m*n^2*Log[x]*Log[d + e*x] + 36*a*b*d^3*n*Log
[f*x^m]*Log[d + e*x] - 66*b^2*d^3*n^2*Log[f*x^m]*Log[d + e*x] + 6*b^2*d^3*m
*n^2*Log[d + e*x]^2 + 36*b^2*d^3*m*n^2*Log[x]*Log[d + e*x]^2 - 18*b^2*d^3*m
*n^2*Log[-((e*x)/d)]*Log[d + e*x]^2 - 18*b^2*d^3*n^2*Log[f*x^m]*Log[d + e*x
]^2 + 48*b^2*d^2*e*m*n*x*Log[c*(d + e*x)^n] - 15*b^2*d*e^2*m*n*x^2*Log[c*(d
 + e*x)^n] - 12*a*b*e^3*m*x^3*Log[c*(d + e*x)^n] + 8*b^2*e^3*m*n*x^3*Log[c*
(d + e*x)^n] - 36*b^2*d^2*e*n*x*Log[f*x^m]*Log[c*(d + e*x)^n] + 18*b^2*d*e^
2*n*x^2*Log[f*x^m]*Log[c*(d + e*x)^n] + 36*a*b*e^3*x^3*Log[f*x^m]*Log[c*(d
+ e*x)^n] - 12*b^2*e^3*n*x^3*Log[f*x^m]*Log[c*(d + e*x)^n] - 12*b^2*d^3*m*n
*Log[d + e*x]*Log[c*(d + e*x)^n] - 36*b^2*d^3*m*n*Log[x]*Log[d + e*x]*Log[c
*(d + e*x)^n] + 36*b^2*d^3*n*Log[f*x^m]*Log[d + e*x]*Log[c*(d + e*x)^n] - 6
*b^2*e^3*m*x^3*Log[c*(d + e*x)^n]^2 + 18*b^2*e^3*x^3*Log[f*x^m]*Log[c*(d +
e*x)^n]^2 + 36*a*b*d^3*m*n*Log[x]*Log[1 + (e*x)/d] - 66*b^2*d^3*m*n^2*Log[x
]*Log[1 + (e*x)/d] - 36*b^2*d^3*m*n^2*Log[x]*Log[d + e*x]*Log[1 + (e*x)/d]
+ 36*b^2*d^3*m*n*Log[x]*Log[c*(d + e*x)^n]*Log[1 + (e*x)/d] + 6*b*d^3*m*n*(
6*a - 11*b*n - 6*b*n*Log[d + e*x] + 6*b*Log[c*(d + e*x)^n])*PolyLog[2, -((e
*x)/d)] - 36*b^2*d^3*m*n^2*Log[d + e*x]*PolyLog[2, 1 + (e*x)/d] + 36*b^2*d^
3*m*n^2*PolyLog[3, 1 + (e*x)/d])/(54*e^3)
```

## Maple [F]

$$\int x^2 \ln(f\,x^m)(a+b\ln(c(ex+d)^n))^2dx$$

```
[In] int(x^2*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

```
[Out] int(x^2*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Fricas [F]

$$\int x^2 \log(fx^m) (a + b \log(c(d + ex)^n))^2 \, dx = \int (b \log((ex + d)^n c) + a)^2 x^2 \log(fx^m) \, dx$$

```
[In] integrate(x^2*log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log((e*x + d)^n*c)^2*log(f*x^m) + 2*a*b*x^2*log((e*x + d)^
n*c)*log(f*x^m) + a^2*x^2*log(f*x^m), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2 \log(fx^m) (a + b \log(c(d + ex)^n))^2 \, dx = \text{Timed out}$$

```
[In] integrate(x**2*ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int x^2 \log(fx^m) (a + b \log(c(d + ex)^n))^2 \, dx = \int (b \log((ex + d)^n c) + a)^2 x^2 \log(fx^m) \, dx$$

```
[In] integrate(x^2*log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] -1/9*(b^2*(m - 3*log(f))*x^3 - 3*b^2*x^3*log(x^m))*log((e*x + d)^n)^2 + int
egrate(1/9*(9*(b^2*e*log(c)^2*log(f) + 2*a*b*e*log(c)*log(f) + a^2*e*log(f)
)*x^3 + 9*(b^2*d*log(c)^2*log(f) + 2*a*b*d*log(c)*log(f) + a^2*d*log(f))*x^
2 + 2*((9*a*b*e*log(f) + (9*e*log(c)*log(f) + (m*n - 3*n*log(f))*e)*b^2)*x^
3 + 9*(b^2*d*log(c)*log(f) + a*b*d*log(f))*x^2 - 3*(((e*n - 3*e*log(c))*b^2
 - 3*a*b*e)*x^3 - 3*(b^2*d*log(c) + a*b*d)*x^2)*log(x^m))*log((e*x + d)^n)
+ 9*((b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^3 + (b^2*d*log(c)^2 + 2*a*
b*d*log(c) + a^2*d)*x^2)*log(x^m))/(e*x + d), x)
```

## Giac [F]

$$\int x^2 \log(fx^m) (a + b \log(c(d + ex)^n))^2 \, dx = \int (b \log((ex + d)^n c) + a)^2 x^2 \log(fx^m) \, dx$$

```
[In] integrate(x^2*log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*x^2*log(f*x^m), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \log(fx^m) (a + b \log(c(d + ex)^n))^2 \, dx = \int x^2 \ln(f x^m) (a + b \ln(c (d + e x)^n))^2 \, dx$$

```
[In] int(x^2*log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2,x)
[Out] int(x^2*log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2, x)
```

# 3.368 $\int x \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 602

$$
\begin{aligned}
&\int x \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx \\
&= -\frac{abdmnx}{2e}+\frac{2b^2dmn^2x}{e}-\frac{2bdmn(a-bn)x}{e}-\frac{1}{8}b^2mn^2x^2-\frac{b^2mn^2(d+ex)^2}{4e^2} \\
&\quad -\frac{b^2d^2mn^2\log(x)}{4e^2}+\frac{2abdnx\log(fx^m)}{e}-\frac{2b^2dn^2x\log(fx^m)}{e} \\
&\quad +\frac{b^2n^2(d+ex)^2\log(fx^m)}{4e^2}-\frac{5b^2dmn(d+ex)\log(c(d+ex)^n)}{2e^2} \\
&\quad -\frac{2b^2d^2mn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e^2}+\frac{2b^2dn(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e^2} \\
&\quad +\frac{bmn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2}+\frac{bd^2mn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{2e^2} \\
&\quad -\frac{bn(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))}{2e^2}+\frac{dm(d+ex)(a+b\log(c(d+ex)^n))^2}{2e^2} \\
&\quad -\frac{m(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2}+\frac{d^2m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{2e^2} \\
&\quad -\frac{d(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e^2} \\
&\quad +\frac{(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))^2}{2e^2}-\frac{3b^2d^2mn^2\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{2e^2} \\
&\quad +\frac{bd^2mn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^2}-\frac{b^2d^2mn^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{e^2}
\end{aligned}
$$

```
[Out] -1/2*a*b*d*m*n*x/e+2*b^2*d*m*n^2*x/e-2*b*d*m*n*(-b*n+a)*x/e-1/8*b^2*m*n^2*x
^2-1/4*b^2*m*n^2*(e*x+d)^2/e^2-1/4*b^2*d^2*m*n^2*ln(x)/e^2+2*a*b*d*n*x*ln(f
*x^m)/e-2*b^2*d*n^2*x*ln(f*x^m)/e+1/4*b^2*n^2*(e*x+d)^2*ln(f*x^m)/e^2-5/2*b
^2*d*m*n*(e*x+d)*ln(c*(e*x+d)^n)/e^2-2*b^2*d^2*m*n*ln(-e*x/d)*ln(c*(e*x+d)^
```

```
n)/e^2+2*b^2*d*n*(e*x+d)*ln(f*x^m)*ln(c*(e*x+d)^n)/e^2+1/2*b*m*n*(e*x+d)^2*
(a+b*ln(c*(e*x+d)^n))/e^2+1/2*b*d^2*m*n*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/e^
2-1/2*b*n*(e*x+d)^2*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))/e^2+1/2*d*m*(e*x+d)*(a+
b*ln(c*(e*x+d)^n))^2/e^2-1/4*m*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2+1/2*d^
2*m*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))^2/e^2-d*(e*x+d)*ln(f*x^m)*(a+b*ln(c*(e
*x+d)^n))^2/e^2+1/2*(e*x+d)^2*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/e^2-3/2*b^2
*d^2*m*n^2*polylog(2,1+e*x/d)/e^2+b*d^2*m*n*(a+b*ln(c*(e*x+d)^n))*polylog(2
,1+e*x/d)/e^2-b^2*d^2*m*n^2*polylog(3,1+e*x/d)/e^2
```

## Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2448, 2436, 2333, 2332, 2437, 2342, 2341, 2475, 45, 2458, 2393, 2354, 2438, 2395, 2421, 6724}

$$\int x\log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx$$
$$=\frac{bd^2mn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)(a+b\log(c(d+ex)^n))}{e^2}+\frac{bd^2mn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{2e^2}+\frac{d^2m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{2e^2}-\frac{bn(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))}{2e^2}+\frac{(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))^2}{2e^2}-\frac{d(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e^2}+\frac{bmn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2}-\frac{m(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2}+\frac{dm(d+ex)(a+b\log(c(d+ex)^n))^2}{2e^2}+\frac{2abdnx\log(fx^m)}{e}-\frac{abdmnx}{2e}-\frac{2bdmnx(a-bn)}{e}-\frac{2b^2d^2mn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e^2}+\frac{2b^2dn(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e^2}-\frac{5b^2dmn(d+ex)\log(c(d+ex)^n)}{2e^2}-\frac{3b^2d^2mn^2\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{2e^2}-\frac{b^2d^2mn^2\,\text{PolyLog}\left(3,\frac{ex}{d}+1\right)}{e^2}-\frac{b^2d^2mn^2\log(x)}{4e^2}+\frac{b^2n^2(d+ex)^2\log(fx^m)}{4e^2}-\frac{b^2mn^2(d+ex)^2}{4e^2}-\frac{2b^2dn^2x\log(fx^m)}{e}+\frac{2b^2dmn^2x}{e}-\frac{1}{8}b^2mn^2x^2$$

```
[In] Int[x*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] -1/2*(a*b*d*m*n*x)/e + (2*b^2*d*m*n^2*x)/e - (2*b*d*m*n*(a - b*n)*x)/e - (b
^2*m*n^2*x^2)/8 - (b^2*m*n^2*(d + e*x)^2)/(4*e^2) - (b^2*d^2*m*n^2*Log[x])/
```

```
(4*e^2) + (2*a*b*d*n*x*Log[f*x^m])/e - (2*b^2*d*n^2*x*Log[f*x^m])/e + (b^2*
n^2*(d + e*x)^2*Log[f*x^m])/(4*e^2) - (5*b^2*d*m*n*(d + e*x)*Log[c*(d + e*x
)^n])/(2*e^2) - (2*b^2*d^2*m*n*Log[-((e*x)/d)]*Log[c*(d + e*x)^n])/e^2 + (2
*b^2*d*n*(d + e*x)*Log[f*x^m]*Log[c*(d + e*x)^n])/e^2 + (b*m*n*(d + e*x)^2*
(a + b*Log[c*(d + e*x)^n]))/(2*e^2) + (b*d^2*m*n*Log[-((e*x)/d)]*(a + b*Log
[c*(d + e*x)^n]))/(2*e^2) - (b*n*(d + e*x)^2*Log[f*x^m]*(a + b*Log[c*(d + e
*x)^n]))/(2*e^2) + (d*m*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^2) - (
m*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(4*e^2) + (d^2*m*Log[-((e*x)/d)
]*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^2) - (d*(d + e*x)*Log[f*x^m]*(a + b*Lo
g[c*(d + e*x)^n])^2)/e^2 + ((d + e*x)^2*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n
])^2)/(2*e^2) - (3*b^2*d^2*m*n^2*PolyLog[2, 1 + (e*x)/d])/(2*e^2) + (b*d^2*
m*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, 1 + (e*x)/d])/e^2 - (b^2*d^2*m*n^
2*PolyLog[3, 1 + (e*x)/d])/e^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2475

```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))^(p_)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*(a + b*
Log[c*(d + e*x)^n])^p, x]}, Dist[Log[f*x^m], u, x] - Dist[m, Int[Dist[1/x,
u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && IGtQ[p, 1] &&
 IGtQ[q, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{2abdnx\log(fx^m)}{e} - \frac{2b^2dn^2x\log(fx^m)}{e} + \frac{b^2n^2(d+ex)^2\log(fx^m)}{4e^2} + \frac{2b^2dn(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e^2} - \frac{bn(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))}{2e^2} - \frac{d(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e^2} + \frac{(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))^2}{2e^2} - m\int\left(\frac{2abdn}{e} - \frac{2b^2dn^2}{e} + \frac{b^2n^2(d+ex)^2}{4e^2x} + \frac{2b^2dn(d+ex)\log(c(d+ex)^n)}{e^2x} - \frac{bn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2x} - \frac{d(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2x} + \frac{(d+ex)^2(a+b\log(c(d+ex)^n))^2}{2e^2x}\right)dx$$

$$= -\frac{2bdmn(a-bn)x}{e} + \frac{2abdnx\log(fx^m)}{e} - \frac{2b^2dn^2x\log(fx^m)}{e} + \frac{b^2n^2(d+ex)^2\log(fx^m)}{4e^2} + \frac{2b^2dn(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e^2} - \frac{bn(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))}{2e^2} - \frac{d(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e^2} + \frac{(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))^2}{2e^2} - \frac{m\int\frac{(d+ex)^2(a+b\log(c(d+ex)^n))^2}{x}dx}{2e^2} + \frac{(dm)\int\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{x}dx}{e^2} + \frac{(bmn)\int\frac{(d+ex)^2(a+b\log(c(d+ex)^n))}{x}dx}{2e^2} - \frac{(2b^2dmn)\int\frac{(d+ex)\log(c(d+ex)^n)}{x}dx}{e^2} - \frac{(b^2mn^2)\int\frac{(d+ex)^2}{x}dx}{4e^2}$$

$$= -\frac{2bdmn(a-bn)x}{e} + \frac{2abdnx\log(fx^m)}{e} - \frac{2b^2dn^2x\log(fx^m)}{e} + \frac{b^2n^2(d+ex)^2\log(fx^m)}{4e^2} + \frac{2b^2dn(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e^2} - \frac{bn(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))}{2e^2} - \frac{d(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e^2} + \frac{(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))^2}{2e^2} - \frac{m\text{Subst}\left(\int \frac{x^2(a+b\log(cx^n))^2}{-\frac{d}{e}+\frac{x}{e}}dx, x, d+ex\right)}{2e^3} + \frac{(dm)\text{Subst}\left(\int \frac{x(a+b\log(cx^n))^2}{-\frac{d}{e}+\frac{x}{e}}dx, x, d+ex\right)}{e^3} + \frac{(bmn)\text{Subst}\left(\int \frac{x^2(a+b\log(cx^n))}{-\frac{d}{e}+\frac{x}{e}}dx, x, d+ex\right)}{2e^3} - \frac{(2b^2dmn)\text{Subst}\left(\int \frac{x\log(cx^n)}{-\frac{d}{e}+\frac{x}{e}}dx, x, d+ex\right)}{e^3} - \frac{(b^2mn^2)\int\left(2de+\frac{d^2}{x}+e^2x\right)dx}{4e^2}$$

$$= -\frac{b^2dmn^2x}{2e} - \frac{2bdmn(a-bn)x}{e} - \frac{1}{8}b^2mn^2x^2 - \frac{b^2d^2mn^2\log(x)}{4e^2} + \frac{2abdnx\log(fx^m)}{e} - \frac{2b^2dn^2x\log(fx^m)}{e} + \frac{b^2n^2(d+ex)^2\log(fx^m)}{4e^2} + \frac{2b^2dn(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e^2} - \frac{bn(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))}{2e^2} - \frac{d(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e^2} + \frac{(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))^2}{2e^2} - \frac{m\text{Subst}\left(\int\left(de(a+b\log(cx^n))^2 - \frac{d^2e(a+b\log(cx^n))^2}{d-x} + ex(a+b\log(cx^n))^2\right)dx, x, d+ex\right)}{2e^3} + \frac{(dm)\text{Subst}\left(\int\left(e(a+b\log(cx^n))^2 - \frac{de(a+b\log(cx^n))^2}{d-x}\right)dx, x, d+ex\right)}{e^3} + \frac{(bmn)\text{Subst}\left(\int\left(de(a+b\log(cx^n)) - \frac{d^2e(a+b\log(cx^n))}{d-x} + ex(a+b\log(cx^n))\right)dx, x, d+ex\right)}{2e^3} - \frac{(2b^2dmn)\text{Subst}\left(\int\left(e\log(cx^n) - \frac{de\log(cx^n)}{d-x}\right)dx, x, d+ex\right)}{e^3}$$

$$
\begin{aligned}
&= -\frac{b^2dmn^2x}{2e} - \frac{2bdmn(a-bn)x}{e} - \frac{1}{8}b^2mn^2x^2 \\
&- \frac{b^2d^2mn^2\log(x)}{4e^2} + \frac{2abdnx\log(fx^m)}{e} - \frac{2b^2dn^2x\log(fx^m)}{e} \\
&+ \frac{b^2n^2(d+ex)^2\log(fx^m)}{4e^2} + \frac{2b^2dn(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e^2} \\
&- \frac{bn(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))}{2e^2} \\
&- \frac{d(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e^2} \\
&+ \frac{(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))^2}{2e^2} \\
&- \frac{m\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx,x,d+ex\right)}{2e^2} \\
&- \frac{(dm)\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx,x,d+ex\right)}{2e^2} \\
&+ \frac{(dm)\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx,x,d+ex\right)}{e^2} \\
&+ \frac{(d^2m)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{d-x}\,dx,x,d+ex\right)}{2e^2} \\
&- \frac{(d^2m)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{d-x}\,dx,x,d+ex\right)}{e^2} \\
&+ \frac{(bmn)\text{Subst}\left(\int x(a+b\log(cx^n))\,dx,x,d+ex\right)}{2e^2} \\
&+ \frac{(bdmn)\text{Subst}\left(\int (a+b\log(cx^n))\,dx,x,d+ex\right)}{2e^2} \\
&- \frac{(2b^2dmn)\,\text{Subst}\left(\int \log(cx^n)\,dx,x,d+ex\right)}{e^2} \\
&- \frac{(bd^2mn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{d-x}\,dx,x,d+ex\right)}{2e^2} \\
&+ \frac{(2b^2d^2mn)\,\text{Subst}\left(\int \frac{\log(cx^n)}{d-x}\,dx,x,d+ex\right)}{e^2}
\end{aligned}
$$

$$= \frac{abdmnx}{2e} + \frac{3b^2dmn^2x}{2e} - \frac{2bdmn(a-bn)x}{e} - \frac{1}{8}b^2mn^2x^2 - \frac{b^2mn^2(d+ex)^2}{8e^2} - \frac{b^2d^2mn^2\log(x)}{4e^2} + \frac{2abdnx\log(fx^m)}{e} - \frac{2b^2dn^2x\log(fx^m)}{e} + \frac{b^2n^2(d+ex)^2\log(fx^m)}{4e^2} - \frac{2b^2dmn(d+ex)\log(c(d+ex)^n)}{e^2} - \frac{2b^2d^2mn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e^2} + \frac{2b^2dn(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e^2} + \frac{bmn(d+ex)^2(a+b\log(c(d+ex)^n))}{4e^2} + \frac{bd^2mn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{2e^2} - \frac{bn(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))}{2e^2} + \frac{dm(d+ex)(a+b\log(c(d+ex)^n))^2}{2e^2} - \frac{m(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2} + \frac{d^2m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{2e^2} - \frac{d(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e^2} + \frac{(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))^2}{2e^2} + \frac{(bmn)\text{Subst}\left(\int x(a+b\log(cx^n))\,dx,x,d+ex\right)}{2e^2} + \frac{(bdmn)\text{Subst}\left(\int (a+b\log(cx^n))\,dx,x,d+ex\right)}{e^2} - \frac{(2bdmn)\text{Subst}\left(\int (a+b\log(cx^n))\,dx,x,d+ex\right)}{e^2} + \frac{(b^2dmn)\,\text{Subst}\left(\int \log(cx^n)\,dx,x,d+ex\right)}{2e^2} + \frac{(bd^2mn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(1-\frac{x}{d}\right)}{x}\,dx,x,d+ex\right)}{e^2} - \frac{(2bd^2mn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(1-\frac{x}{d}\right)}{x}\,dx,x,d+ex\right)}{e^2} - \frac{(b^2d^2mn^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{x}{d}\right)}{x}\,dx,x,d+ex\right)}{2e^2} + \frac{(2b^2d^2mn^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{x}{d}\right)}{x}\,dx,x,d+ex\right)}{e^2}$$

$$= -\frac{abdmnx}{2e} + \frac{b^2dmn^2x}{e} - \frac{2bdmn(a-bn)x}{e} - \frac{1}{8}b^2mn^2x^2 - \frac{b^2mn^2(d+ex)^2}{4e^2} - \frac{b^2d^2mn^2\log(x)}{4e^2} + \frac{2abdnx\log(fx^m)}{e} - \frac{2b^2dn^2x\log(fx^m)}{e} + \frac{b^2n^2(d+ex)^2\log(fx^m)}{4e^2} - \frac{3b^2dmn(d+ex)\log(c(d+ex)^n)}{2e^2} - \frac{2b^2d^2mn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e^2} + \frac{2b^2dn(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e^2} + \frac{bmn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2} + \frac{bd^2mn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{2e^2} - \frac{bn(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))}{2e^2} + \frac{dm(d+ex)(a+b\log(c(d+ex)^n))^2}{2e^2} - \frac{m(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2} + \frac{d^2m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{2e^2} - \frac{d(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e^2} + \frac{(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))^2}{2e^2} - \frac{3b^2d^2mn^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{2e^2} + \frac{bd^2mn(a+b\log(c(d+ex)^n))\text{Li}_2\left(1+\frac{ex}{d}\right)}{e^2} + \frac{(b^2dmn)\,\text{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{e^2} - \frac{(2b^2dmn)\,\text{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{e^2} + \frac{(b^2d^2mn^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx,x,d+ex\right)}{e^2} - \frac{(2b^2d^2mn^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx,x,d+ex\right)}{e^2}$$

$$
\begin{aligned}
&= -\frac{abdmnx}{2e} + \frac{2b^2dmn^2x}{e} - \frac{2bdmn(a-bn)x}{e} - \frac{1}{8}b^2mn^2x^2 - \frac{b^2mn^2(d+ex)^2}{4e^2} \\
&\quad - \frac{b^2d^2mn^2\log(x)}{4e^2} + \frac{2abdnx\log(fx^m)}{e} - \frac{2b^2dn^2x\log(fx^m)}{e} \\
&\quad + \frac{b^2n^2(d+ex)^2\log(fx^m)}{4e^2} - \frac{5b^2dmn(d+ex)\log(c(d+ex)^n)}{2e^2} \\
&\quad - \frac{2b^2d^2mn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e^2} + \frac{2b^2dn(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e^2} \\
&\quad + \frac{bmn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2} \\
&\quad + \frac{bd^2mn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{2e^2} \\
&\quad - \frac{bn(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))}{2e^2} \\
&\quad + \frac{dm(d+ex)(a+b\log(c(d+ex)^n))^2}{2e^2} - \frac{m(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2} \\
&\quad + \frac{d^2m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{2e^2} \\
&\quad - \frac{d(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e^2} \\
&\quad + \frac{(d+ex)^2\log(fx^m)(a+b\log(c(d+ex)^n))^2}{2e^2} - \frac{3b^2d^2mn^2\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{2e^2} \\
&\quad + \frac{bd^2mn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^2} - \frac{b^2d^2mn^2\operatorname{Li}_3\left(1+\frac{ex}{d}\right)}{e^2}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.37

$$\int x\log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx = b^2n^2(-m\log(x)+\log(fx^m))\left(\frac{1}{2}x^2\log^2(d+ex) - e\left(\frac{3dx}{2e^2}-\frac{x^2}{4e}-\frac{3d^2\log(d+ex)}{2e^3}-\frac{dx\log(d+ex)}{e^2}+\frac{x^2\log(d+ex)}{2e}+\frac{d^2\log^2(d+ex)}{2e^3}\right)\right) + 2bn(-m\log(x)+\log(fx^m))\left(\frac{1}{2}x^2\log(d+ex)-\frac{1}{2}e\left(-\frac{dx}{e^2}+\frac{x^2}{2e}+\frac{d^2\log(d+ex)}{e^3}\right)\right)(a + b(-n\log(d+ex)+\log(c(d+ex)^n))) + \frac{1}{2}mx^2\log(x)(a+b(-n\log(d+ex)+\log(c(d+ex)^n)))^2 + \frac{1}{4}x^2(-m+2(-m\log(x)+\log(fx^m)))(a+b(-n\log(d+ex)+\log(c(d+ex)^n)))^2 + bmn(a+b(-n\log(d+ex)+\log(c(d+ex)^n)))\left(-\frac{1}{2}x^2\log(d+ex) + x^2\log(x)\log(d+ex)+\frac{1}{2}e\left(-\frac{dx}{e^2}+\frac{x^2}{2e}+\frac{d^2\log(d+ex)}{e^3}\right) - e\left(-\frac{dx(-1+\log(x))}{e^2}+\frac{-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)}{e}+\frac{d^2\left(\frac{\log(x)\log\left(\frac{d+ex}{d}\right)}{e}+\frac{\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e}\right)}{e^2}\right)\right) + \frac{1}{2}b^2mn^2\left(-\frac{1}{2}x^2\log^2(d+ex)+x^2\log(x)\log^2(d+ex) + e\left(\frac{3dx}{2e^2}-\frac{x^2}{4e}-\frac{3d^2\log(d+ex)}{2e^3}-\frac{dx\log(d+ex)}{e^2}+\frac{x^2\log(d+ex)}{2e}+\frac{d^2\log^2(d+ex)}{2e^3}\right) - 2e\left(-\frac{d\left(2ex-d\log(d+ex)-ex\log(d+ex)+\log(x)\left(-ex+ex\log(d+ex)+d\log\left(1+\frac{ex}{d}\right)\right)+d\,\text{Poly}\cdots\right.}{e^3} + \frac{-3dex+e^2x^2+d^2\log(d+ex)-e^2x^2\log(d+ex)+\log(x)\left(ex(2d-ex)+2e^2x^2\log(d+ex)-2d^2\log\cdots\right.}{4e^3} + \frac{d^2\left(\frac{1}{2}\left(\log(x)-\log\left(-\frac{ex}{d}\right)\right)\log^2(d+ex)-\log(d+ex)\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)+\text{PolyLog}\left(3,\frac{d+ex}{d}\right)\right)}{e^3}\right)\right)$$

```
[In] Integrate[x*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] b^2*n^2*(-(m*Log[x]) + Log[f*x^m])*((x^2*Log[d + e*x]^2)/2 - e*((3*d*x)/(2*
e^2) - x^2/(4*e) - (3*d^2*Log[d + e*x])/(2*e^3) - (d*x*Log[d + e*x])/e^2 +
(x^2*Log[d + e*x])/(2*e) + (d^2*Log[d + e*x]^2)/(2*e^3))) + 2*b*n*(-(m*Log[
x]) + Log[f*x^m])*((x^2*Log[d + e*x])/2 - (e*(-((d*x)/e^2) + x^2/(2*e) + (d
```

```
^2*Log[d + e*x])/e^3))/2)*(a + b*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n]))
+ (m*x^2*Log[x]*(a + b*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n]))^2)/2 + (x^
2*(-m + 2*(-(m*Log[x]) + Log[f*x^m]))*(a + b*(-(n*Log[d + e*x]) + Log[c*(d
+ e*x)^n]))^2)/4 + b*m*n*(a + b*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n]))*(
-1/2*(x^2*Log[d + e*x]) + x^2*Log[x]*Log[d + e*x] + (e*(-((d*x)/e^2) + x^2/
(2*e) + (d^2*Log[d + e*x])/e^3))/2 - e*(-((d*x*(-1 + Log[x]))/e^2) + (-1/4*
x^2 + (x^2*Log[x])/2)/e + (d^2*((Log[x]*Log[(d + e*x)/d])/e + PolyLog[2, -(
(e*x)/d)]/e))/e^2)) + (b^2*m*n^2*(-1/2*(x^2*Log[d + e*x]^2) + x^2*Log[x]*Lo
g[d + e*x]^2 + e*((3*d*x)/(2*e^2) - x^2/(4*e) - (3*d^2*Log[d + e*x])/(2*e^3
) - (d*x*Log[d + e*x])/e^2 + (x^2*Log[d + e*x])/(2*e) + (d^2*Log[d + e*x]^2
)/(2*e^3)) - 2*e*(-((d*(2*e*x - d*Log[d + e*x] - e*x*Log[d + e*x] + Log[x]*
(-(e*x) + e*x*Log[d + e*x] + d*Log[1 + (e*x)/d]) + d*PolyLog[2, -((e*x)/d)]
))/e^3) + (-3*d*e*x + e^2*x^2 + d^2*Log[d + e*x] - e^2*x^2*Log[d + e*x] + L
og[x]*(e*x*(2*d - e*x) + 2*e^2*x^2*Log[d + e*x] - 2*d^2*Log[1 + (e*x)/d]) -
 2*d^2*PolyLog[2, -((e*x)/d)])/(4*e^3) + (d^2*(((Log[x] - Log[-((e*x)/d)])*
Log[d + e*x]^2)/2 - Log[d + e*x]*PolyLog[2, (d + e*x)/d] + PolyLog[3, (d +
e*x)/d]))/e^3)))/2
```

## Maple [F]

$$\int x \ln \left(f\,x^{m}\right) \left(a +b \ln \left(c (e x +d)^{n}\right)\right)^{2}dx$$

[In] `int(x*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2,x)`

[Out] `int(x*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2,x)`

## Fricas [F]

$$\int x \log \left(f x^m\right) \left(a+b \log \left(c(d+e x)^n\right)\right)^2 \, dx=\int { {\left(b \log \left({\left(e x + d\right)}^{n} c\right) + a\right)}^{2} x \log \left(f x^{m}\right) \,d x}$$

[In] `integrate(x*log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")`

[Out] `integral(b^2*x*log((e*x + d)^n*c)^2*log(f*x^m) + 2*a*b*x*log((e*x + d)^n*c)*log(f*x^m) + a^2*x*log(f*x^m), x)`

## Sympy [F(-1)]

Timed out.

$$\int x\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate(x*ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int x\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\int\left(b\log\left((ex+d)^nc\right)+a\right)^2x\log\left(fx^m\right)\,dx$$

```
[In] integrate(x*log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(m - 2*log(f))*x^2 - 2*b^2*x^2*log(x^m))*log((e*x + d)^n)^2 + int
egrate(1/2*(2*(b^2*e*log(c)^2*log(f) + 2*a*b*e*log(c)*log(f) + a^2*e*log(f)
)*x^2 + 2*(b^2*d*log(c)^2*log(f) + 2*a*b*d*log(c)*log(f) + a^2*d*log(f))*x
+ ((4*a*b*e*log(f) + (4*e*log(c)*log(f) + (m*n - 2*n*log(f))*e)*b^2)*x^2 +
4*(b^2*d*log(c)*log(f) + a*b*d*log(f))*x - 2*(((e*n - 2*e*log(c))*b^2 - 2*a
*b*e)*x^2 - 2*(b^2*d*log(c) + a*b*d)*x)*log(x^m))*log((e*x + d)^n) + 2*((b^
2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^2 + (b^2*d*log(c)^2 + 2*a*b*d*log(
c) + a^2*d)*x)*log(x^m))/(e*x + d), x)
```

## Giac [F]

$$\int x\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\int\left(b\log\left((ex+d)^nc\right)+a\right)^2x\log\left(fx^m\right)\,dx$$

```
[In] integrate(x*log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*x*log(f*x^m), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\int x\,\ln\left(f\,x^m\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2\,dx$$

```
[In] int(x*log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2,x)
[Out] int(x*log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2, x)
```

# 3.369 $\int \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 309

$$\begin{aligned}
&\int \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx\\
&= 2abmnx-4b^2mn^2x+2bmn(a-bn)x-2abnx\log(fx^m)\\
&\quad+2b^2n^2x\log(fx^m)+\frac{4b^2mn(d+ex)\log(c(d+ex)^n)}{e}\\
&\quad+\frac{2b^2dmn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e}-\frac{2b^2n(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e}\\
&\quad-\frac{m(d+ex)(a+b\log(c(d+ex)^n))^2}{e}-\frac{dm\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{e}\\
&\quad+\frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e}+\frac{2b^2dmn^2\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}\\
&\quad-\frac{2bdmn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}+\frac{2b^2dmn^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{e}
\end{aligned}$$

```
[Out] 2*a*b*m*n*x-4*b^2*m*n^2*x+2*b*m*n*(-b*n+a)*x-2*a*b*n*x*ln(f*x^m)+2*b^2*n^2*
x*ln(f*x^m)+4*b^2*m*n*(e*x+d)*ln(c*(e*x+d)^n)/e+2*b^2*d*m*n*ln(-e*x/d)*ln(c
*(e*x+d)^n)/e-2*b^2*n*(e*x+d)*ln(f*x^m)*ln(c*(e*x+d)^n)/e-m*(e*x+d)*(a+b*ln
(c*(e*x+d)^n))^2/e-d*m*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))^2/e+(e*x+d)*ln(f*x^
m)*(a+b*ln(c*(e*x+d)^n))^2/e+2*b^2*d*m*n^2*polylog(2,1+e*x/d)/e-2*b*d*m*n*(
a+b*ln(c*(e*x+d)^n))*polylog(2,1+e*x/d)/e+2*b^2*d*m*n^2*polylog(3,1+e*x/d)/
e
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {2436, 2333, 2332, 2470, 2458, 45, 2393, 2354, 2438, 2395, 2421, 6724}

$$\begin{aligned}
&\int \log \left(fx^m\right) \left(a+b\log \left(c(d+ex)^n\right)\right)^2 \, dx \\
&= \frac{(d+ex)\log \left(fx^m\right) \left(a+b\log \left(c(d+ex)^n\right)\right)^2}{e} \\
&\quad - \frac{2bdmn \operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right) \left(a+b\log \left(c(d+ex)^n\right)\right)}{e} \\
&\quad - \frac{m(d+ex)\left(a+b\log \left(c(d+ex)^n\right)\right)^2}{e} - \frac{dm\log \left(-\frac{ex}{d}\right) \left(a+b\log \left(c(d+ex)^n\right)\right)^2}{e} \\
&\quad - 2abnx\log \left(fx^m\right) + 2abmnx + 2bmnx(a-bn) \\
&\quad - \frac{2b^2n(d+ex)\log \left(fx^m\right) \log \left(c(d+ex)^n\right)}{e} + \frac{4b^2mn(d+ex)\log \left(c(d+ex)^n\right)}{e} \\
&\quad + \frac{2b^2dmn\log \left(-\frac{ex}{d}\right) \log \left(c(d+ex)^n\right)}{e} + \frac{2b^2dmn^2 \operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e} \\
&\quad + \frac{2b^2dmn^2 \operatorname{PolyLog}\left(3,\frac{ex}{d}+1\right)}{e} + 2b^2n^2x\log \left(fx^m\right) - 4b^2mn^2x
\end{aligned}$$

```
[In] Int[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] 2*a*b*m*n*x - 4*b^2*m*n^2*x + 2*b*m*n*(a - b*n)*x - 2*a*b*n*x*Log[f*x^m] +
2*b^2*n^2*x*Log[f*x^m] + (4*b^2*m*n*(d + e*x)*Log[c*(d + e*x)^n])/e + (2*b^
2*d*m*n*Log[-((e*x)/d)]*Log[c*(d + e*x)^n])/e - (2*b^2*n*(d + e*x)*Log[f*x^
m]*Log[c*(d + e*x)^n])/e - (m*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e - (
d*m*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n])^2)/e + ((d + e*x)*Log[f*x^m]
*(a + b*Log[c*(d + e*x)^n])^2)/e + (2*b^2*d*m*n^2*PolyLog[2, 1 + (e*x)/d])/
e - (2*b*d*m*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, 1 + (e*x)/d])/e + (2*b
^2*d*m*n^2*PolyLog[3, 1 + (e*x)/d])/e
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2470

```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))^(p_), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*(d + e*x)^n])^p, x]},
 Dist[Log[f*x^m], u, x] - Dist[m, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, m, n}, x] && IGtQ[p, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -2abnx\log(fx^m)+2b^2n^2x\log(fx^m)-\frac{2b^2n(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e}\\&\quad+\frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e}-m\int\left(-2abn+2b^2n^2\right.\\&\quad\left.-\frac{2b^2n(d+ex)\log(c(d+ex)^n)}{ex}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{ex}\right)dx\\&=2bmn(a-bn)x-2abnx\log(fx^m)+2b^2n^2x\log(fx^m)\\&\quad-\frac{2b^2n(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e}\\&\quad+\frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e}\\&\quad-\frac{m\int\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{x}dx}{e}+\frac{(2b^2mn)\int\frac{(d+ex)\log(c(d+ex)^n)}{x}dx}{e}\end{aligned}$$

$$= 2bmn(a-bn)x - 2abnx\log(fx^m) + 2b^2n^2x\log(fx^m) - \frac{2b^2n(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} + \frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e} - \frac{m\text{Subst}\left(\int \frac{x(a+b\log(cx^n))^2}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex\right)}{e^2} + \frac{(2b^2mn)\,\text{Subst}\left(\int \frac{x\log(cx^n)}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex\right)}{e^2}$$

$$= 2bmn(a-bn)x - 2abnx\log(fx^m) + 2b^2n^2x\log(fx^m) - \frac{2b^2n(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} + \frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e} - \frac{m\text{Subst}\left(\int \left(e(a+b\log(cx^n))^2 - \frac{de(a+b\log(cx^n))^2}{d-x}\right)\,dx, x, d+ex\right)}{e^2} + \frac{(2b^2mn)\,\text{Subst}\left(\int \left(e\log(cx^n) - \frac{de\log(cx^n)}{d-x}\right)\,dx, x, d+ex\right)}{e^2}$$

$$= 2bmn(a-bn)x - 2abnx\log(fx^m) + 2b^2n^2x\log(fx^m) - \frac{2b^2n(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} + \frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e} - \frac{m\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx, x, d+ex\right)}{e} + \frac{(dm)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{d-x}\,dx, x, d+ex\right)}{e} + \frac{(2b^2mn)\,\text{Subst}\left(\int \log(cx^n)\,dx, x, d+ex\right)}{e} - \frac{(2b^2dmn)\,\text{Subst}\left(\int \frac{\log(cx^n)}{d-x}\,dx, x, d+ex\right)}{e}$$

$$= -2b^2mn^2x + 2bmn(a-bn)x - 2abnx\log(fx^m) + 2b^2n^2x\log(fx^m) + \frac{2b^2mn(d+ex)\log(c(d+ex)^n)}{e} + \frac{2b^2dmn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e} - \frac{2b^2n(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} - \frac{m(d+ex)\left(a+b\log(c(d+ex)^n)\right)^2}{e} - \frac{dm\log\left(-\frac{ex}{d}\right)\left(a+b\log(c(d+ex)^n)\right)^2}{e} + \frac{(d+ex)\log(fx^m)\left(a+b\log(c(d+ex)^n)\right)^2}{e} + \frac{(2bmn)\text{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+ex\right)}{e} + \frac{(2bdmn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(1-\frac{x}{d}\right)}{x}\,dx, x, d+ex\right)}{e} - \frac{(2b^2dmn^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{x}{d}\right)}{x}\,dx, x, d+ex\right)}{e}$$

$$= 2abmnx - 2b^2mn^2x + 2bmn(a-bn)x - 2abnx\log(fx^m) + 2b^2n^2x\log(fx^m) + \frac{2b^2mn(d+ex)\log(c(d+ex)^n)}{e} + \frac{2b^2dmn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e} - \frac{2b^2n(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} - \frac{m(d+ex)\left(a+b\log(c(d+ex)^n)\right)^2}{e} - \frac{dm\log\left(-\frac{ex}{d}\right)\left(a+b\log(c(d+ex)^n)\right)^2}{e} + \frac{(d+ex)\log(fx^m)\left(a+b\log(c(d+ex)^n)\right)^2}{e} + \frac{2b^2dmn^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{e} - \frac{2bdmn(a+b\log(c(d+ex)^n))\,\text{Li}_2\left(1+\frac{ex}{d}\right)}{e} + \frac{(2b^2mn)\,\text{Subst}\left(\int \log(cx^n)\,dx, x, d+ex\right)}{e} + \frac{(2b^2dmn^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx, x, d+ex\right)}{e}$$

$$
\begin{aligned}
&= 2abmnx - 4b^2mn^2x + 2bmn(a-bn)x - 2abnx\log(fx^m) \\
&+ 2b^2n^2x\log(fx^m) + \frac{4b^2mn(d+ex)\log(c(d+ex)^n)}{e} \\
&+ \frac{2b^2dmn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e} - \frac{2b^2n(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} \\
&- \frac{m(d+ex)(a+b\log(c(d+ex)^n))^2}{e} - \frac{dm\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{e} \\
&+ \frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e} + \frac{2b^2dmn^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{e} \\
&- \frac{2bdmn(a+b\log(c(d+ex)^n))\text{Li}_2\left(1+\frac{ex}{d}\right)}{e} + \frac{2b^2dmn^2\text{Li}_3\left(1+\frac{ex}{d}\right)}{e}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.48

$$
\int \log(fx^m)(a+b\log(c(d+ex)^n))^2\,dx
= \frac{-b^2n^2(m\log(x)-\log(fx^m))\left(2ex-2(d+ex)\log(d+ex)+(d+ex)\log^2(d+ex)\right)+2bn(m\log(x)-\log
$$

```
[In] Integrate[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2,x]
```

```
[Out] (-(b^2*n^2*(m*Log[x] - Log[f*x^m])*(2*e*x - 2*(d + e*x)*Log[d + e*x] + (d +
 e*x)*Log[d + e*x]^2)) + 2*b*n*(m*Log[x] - Log[f*x^m])*(e*x - (d + e*x)*Log
[d + e*x])*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]) + e*m*x*Log[x]*(a
- b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - e*x*(m + m*Log[x] - Log[f*x^
m])*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 2*b*m*n*(a - b*n*Log[
d + e*x] + b*Log[c*(d + e*x)^n])*(d + e*x - e*x*(-1 + Log[x]) - (d + e*x)*L
og[d + e*x] + e*x*Log[x]*Log[d + e*x] + d*(Log[x]*Log[1 + (e*x)/d] + PolyLo
g[2, -((e*x)/d)])) + b^2*m*n^2*(-6*e*x + 2*e*x*Log[x] + 4*d*Log[d + e*x] +
4*e*x*Log[d + e*x] - 2*e*x*Log[x]*Log[d + e*x] - d*Log[d + e*x]^2 - e*x*Log
[d + e*x]^2 + d*Log[x]*Log[d + e*x]^2 + e*x*Log[x]*Log[d + e*x]^2 - d*Log[-
((e*x)/d)]*Log[d + e*x]^2 - 2*d*Log[x]*Log[1 + (e*x)/d] - 2*d*PolyLog[2, -(
(e*x)/d)] - 2*d*Log[d + e*x]*PolyLog[2, 1 + (e*x)/d] + 2*d*PolyLog[3, 1 + (
e*x)/d]))/e
```

## Maple [F]

$$\int \ln\left(f\,x^{m}\right)\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}dx$$

```
[In] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2,x)
[Out] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Fricas [F]

$$\int \log\left(fx^{m}\right)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}\,dx=\int \left(b\log\left((ex+d)^{n}c\right)+a\right)^{2}\log\left(fx^{m}\right)\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
[Out] integral(b^2*log((e*x + d)^n*c)^2*log(f*x^m) + 2*a*b*log((e*x + d)^n*c)*log
(f*x^m) + a^2*log(f*x^m), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \log\left(fx^{m}\right)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}\,dx=\text{Timed out}$$

```
[In] integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**2,x)
[Out] Timed out
```

## Maxima [F]

$$\int \log\left(fx^{m}\right)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}\,dx=\int \left(b\log\left((ex+d)^{n}c\right)+a\right)^{2}\log\left(fx^{m}\right)\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
[Out] -(b^2*(m - log(f))*x - b^2*x*log(x^m))*log((e*x + d)^n)^2 + integrate((b^2*
d*log(c)^2*log(f) + 2*a*b*d*log(c)*log(f) + a^2*d*log(f) + (b^2*e*log(c)^2*
log(f) + 2*a*b*e*log(c)*log(f) + a^2*e*log(f))*x + 2*(b^2*d*log(c)*log(f) +
 a*b*d*log(f) + (a*b*e*log(f) + (e*log(c)*log(f) + (m*n - n*log(f))*e)*b^2)
*x + (b^2*d*log(c) + a*b*d - ((e*n - e*log(c))*b^2 - a*b*e)*x)*log(x^m))*lo
g((e*x + d)^n) + (b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d + (b^2*e*log(c)^2
 + 2*a*b*e*log(c) + a^2*e)*x)*log(x^m))/(e*x + d), x)
```

## Giac [F]

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\int \left(b\log\left((ex+d)^nc\right)+a\right)^2\log(fx^m)\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*log(f*x^m), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx=\int \ln\left(f\,x^m\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2\,dx$$

```
[In] int(log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2,x)
[Out] int(log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2, x)
```

# 3.370 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x}\,dx$

## Optimal result

Integrand size = 26, antiderivative size = 823

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x}\,dx$$
$$= \frac{1}{2}m\log^2(x)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2 + \log(x)\left(-m\log(x)+\log\left(fx^m\right)\right)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2 + 2bn(-m\log(x)+\log\left(fx^m\right))\left(a-bn\log(d+ex) + b\log\left(c(d+ex)^n\right)\right)\left(\log(x)\left(\log(d+ex)-\log\left(1+\frac{ex}{d}\right)\right)-\text{PolyLog}\left(2,-\frac{ex}{d}\right)\right) + 2bmn\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)\left(\frac{1}{2}\log^2(x)\left(\log(d+ex)-\log\left(1+\frac{ex}{d}\right)\right) - \log(x)\text{PolyLog}\left(2,-\frac{ex}{d}\right)+\text{PolyLog}\left(3,-\frac{ex}{d}\right)\right) - b^2n^2(m\log(x)-\log\left(fx^m\right))\left(\log\left(-\frac{ex}{d}\right)\log^2(d+ex)+2\log(d+ex)\text{PolyLog}\left(2,1+\frac{ex}{d}\right) - 2\text{PolyLog}\left(3,1+\frac{ex}{d}\right)\right)+\frac{1}{12}b^2mn^2\left(\log^4\left(-\frac{ex}{d}\right)+6\log^2\left(-\frac{ex}{d}\right)\log^2\left(-\frac{ex}{d+ex}\right) - 4\left(\log\left(-\frac{ex}{d}\right)+\log\left(\frac{d}{d+ex}\right)\right)\log^3\left(-\frac{ex}{d+ex}\right)+\log^4\left(-\frac{ex}{d+ex}\right) + 6\log^2(x)\log^2(d+ex)+4\left(2\log^3\left(-\frac{ex}{d}\right)-3\log^2(x)\log(d+ex)\right)\log\left(1+\frac{ex}{d}\right) + 6\left(\log(x)-\log\left(-\frac{ex}{d}\right)\right)\left(\log(x)+3\log\left(-\frac{ex}{d}\right)\right)\log^2\left(1+\frac{ex}{d}\right) - 4\log^2\left(-\frac{ex}{d}\right)\log\left(-\frac{ex}{d+ex}\right)\left(\log\left(-\frac{ex}{d}\right)+3\log\left(1+\frac{ex}{d}\right)\right) + 12\left(\log^2\left(-\frac{ex}{d}\right)-2\log\left(-\frac{ex}{d}\right)\left(\log\left(-\frac{ex}{d+ex}\right)+\log\left(1+\frac{ex}{d}\right)\right) + 2\log(x)\left(-\log(d+ex)+\log\left(1+\frac{ex}{d}\right)\right)\right)\text{PolyLog}\left(2,-\frac{ex}{d}\right) - 12\log^2\left(-\frac{ex}{d+ex}\right)\text{PolyLog}\left(2,\frac{ex}{d+ex}\right) + 12\left(\log\left(-\frac{ex}{d}\right)-\log\left(-\frac{ex}{d+ex}\right)\right)^2\text{PolyLog}\left(2,1+\frac{ex}{d}\right) + 24\left(\log(x)-\log\left(-\frac{ex}{d}\right)\right)\log\left(1+\frac{ex}{d}\right)\text{PolyLog}\left(2,1+\frac{ex}{d}\right) + 24\left(\log\left(-\frac{ex}{d+ex}\right)+\log(d+ex)\right)\text{PolyLog}\left(3,-\frac{ex}{d}\right) + 24\log\left(-\frac{ex}{d+ex}\right)\text{PolyLog}\left(3,\frac{ex}{d+ex}\right) + 24\left(-\log(x)+\log\left(-\frac{ex}{d+ex}\right)\right)\text{PolyLog}\left(3,1+\frac{ex}{d}\right) - 24\left(\text{PolyLog}\left(4,-\frac{ex}{d}\right)+\text{PolyLog}\left(4,\frac{ex}{d+ex}\right)-\text{PolyLog}\left(4,1+\frac{ex}{d}\right)\right)\right)$$

```
[Out] 1/2*m*ln(x)^2*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))^2+ln(x)*(-m*ln(x)+ln(f*x^
m))*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))^2+2*b*n*(-m*ln(x)+ln(f*x^m))*(a-b*n
*ln(e*x+d)+b*ln(c*(e*x+d)^n))*(ln(x)*(ln(e*x+d)-ln(1+e*x/d))-polylog(2,-e*x
/d))+2*b*m*n*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))*(1/2*ln(x)^2*(ln(e*x+d)-ln
(1+e*x/d))-ln(x)*polylog(2,-e*x/d)+polylog(3,-e*x/d))-b^2*n^2*(m*ln(x)-ln(f
*x^m))*(ln(-e*x/d)*ln(e*x+d)^2+2*ln(e*x+d)*polylog(2,1+e*x/d)-2*polylog(3,1
+e*x/d))+1/12*b^2*m*n^2*(ln(-e*x/d)^4+6*ln(-e*x/d)^2*ln(-e*x/(e*x+d))^2-4*(
ln(-e*x/d)+ln(d/(e*x+d)))*ln(-e*x/(e*x+d))^3+ln(-e*x/(e*x+d))^4+6*ln(x)^2*l
n(e*x+d)^2+4*(2*ln(-e*x/d)^3-3*ln(x)^2*ln(e*x+d))*ln(1+e*x/d)+6*(ln(x)-ln(-
e*x/d))*(ln(x)+3*ln(-e*x/d))*ln(1+e*x/d)^2-4*ln(-e*x/d)^2*ln(-e*x/(e*x+d))*
(ln(-e*x/d)+3*ln(1+e*x/d))+12*(ln(-e*x/d)^2-2*ln(-e*x/d)*(ln(-e*x/(e*x+d))+
ln(1+e*x/d))+2*ln(x)*(-ln(e*x+d)+ln(1+e*x/d)))*polylog(2,-e*x/d)-12*ln(-e*x
/(e*x+d))^2*polylog(2,e*x/(e*x+d))+12*(ln(-e*x/d)-ln(-e*x/(e*x+d)))^2*polyl
og(2,1+e*x/d)+24*(ln(x)-ln(-e*x/d))*ln(1+e*x/d)*polylog(2,1+e*x/d)+24*(ln(-
e*x/(e*x+d))+ln(e*x+d))*polylog(3,-e*x/d)+24*ln(-e*x/(e*x+d))*polylog(3,e*x
/(e*x+d))+24*(-ln(x)+ln(-e*x/(e*x+d)))*polylog(3,1+e*x/d)-24*polylog(4,-e*x
/d)-24*polylog(4,e*x/(e*x+d))+24*polylog(4,1+e*x/d))
```

## Rubi [F]

$$\int \frac{\log (fx^m) (a+b\log (c(d+ex)^n))^2}{x} \, dx = \int \frac{\log (fx^m) (a+b\log (c(d+ex)^n))^2}{x} \, dx$$

```
[In] Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x,x]
```

```
[Out] (Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n])^2)/(2*m) - (b*e*n*Defer[Int][(Log[
f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(d + e*x), x])/m
```

Rubi steps

$$\text{integral} = \frac{\log^2 (fx^m) (a+b\log (c(d+ex)^n))^2}{2m} - \frac{(ben)\int \frac{\log^2(fx^m)(a+b\log(c(d+ex)^n))}{d+ex}\,dx}{m}$$

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.00

$$\int \frac{\log (fx^m) (a+b\log (c(d+ex)^n))^2}{x}\,dx$$
$$= \frac{1}{2}m\log^2(x)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2 + \log(x)\left(-m\log(x)+\log\left(fx^m\right)\right)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right)\right)^2 + 2bn(-m\log(x)+\log\left(fx^m\right))\left(a-bn\log(d+ex) + b\log\left(c(d+ex)^n\right)\right)\left(\log(x)\left(\log(d+ex)-\log\left(1+\frac{ex}{d}\right)\right)-\text{PolyLog}\left(2,-\frac{ex}{d}\right)\right) + 2bmn(a-bn\log(d+ex)+b\log\left(c(d+ex)^n\right))\left(\frac{1}{2}\log^2(x)\left(\log(d+ex)-\log\left(1+\frac{ex}{d}\right)\right) - \log(x)\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)+\text{PolyLog}\left(3,-\frac{ex}{d}\right)\right) - b^2n^2(m\log(x)-\log\left(fx^m\right))\left(\log\left(-\frac{ex}{d}\right)\log^2(d+ex)+2\log(d+ex)\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right) - 2\,\text{PolyLog}\left(3,1+\frac{ex}{d}\right)\right)+\frac{1}{12}b^2mn^2\left(\log^4\left(-\frac{ex}{d}\right)+6\log^2\left(-\frac{ex}{d}\right)\log^2\left(-\frac{ex}{d+ex}\right) - 4\left(\log\left(-\frac{ex}{d}\right)+\log\left(\frac{d}{d+ex}\right)\right)\log^3\left(-\frac{ex}{d+ex}\right)+\log^4\left(-\frac{ex}{d+ex}\right) + 6\log^2(x)\log^2(d+ex)+4\left(2\log^3\left(-\frac{ex}{d}\right)-3\log^2(x)\log(d+ex)\right)\log\left(1+\frac{ex}{d}\right) + 6\left(\log(x)-\log\left(-\frac{ex}{d}\right)\right)\left(\log(x)+3\log\left(-\frac{ex}{d}\right)\right)\log^2\left(1+\frac{ex}{d}\right) - 4\log^2\left(-\frac{ex}{d}\right)\log\left(-\frac{ex}{d+ex}\right)\left(\log\left(-\frac{ex}{d}\right)+3\log\left(1+\frac{ex}{d}\right)\right) + 12\left(\log^2\left(-\frac{ex}{d}\right)-2\log\left(-\frac{ex}{d}\right)\left(\log\left(-\frac{ex}{d+ex}\right)+\log\left(1+\frac{ex}{d}\right)\right) + 2\log(x)\left(-\log(d+ex)+\log\left(1+\frac{ex}{d}\right)\right)\right)\text{PolyLog}\left(2,-\frac{ex}{d}\right) - 12\log^2\left(-\frac{ex}{d+ex}\right)\text{PolyLog}\left(2,\frac{ex}{d+ex}\right) + 12\left(\log\left(-\frac{ex}{d}\right)-\log\left(-\frac{ex}{d+ex}\right)\right)^2\text{PolyLog}\left(2,1+\frac{ex}{d}\right) + 24\left(\log(x)-\log\left(-\frac{ex}{d}\right)\right)\log\left(1+\frac{ex}{d}\right)\text{PolyLog}\left(2,1+\frac{ex}{d}\right) + 24\left(\log\left(-\frac{ex}{d+ex}\right)+\log(d+ex)\right)\text{PolyLog}\left(3,-\frac{ex}{d}\right) + 24\log\left(-\frac{ex}{d+ex}\right)\text{PolyLog}\left(3,\frac{ex}{d+ex}\right) + 24\left(-\log(x)+\log\left(-\frac{ex}{d+ex}\right)\right)\text{PolyLog}\left(3,1+\frac{ex}{d}\right) - 24\left(\text{PolyLog}\left(4,-\frac{ex}{d}\right)+\text{PolyLog}\left(4,\frac{ex}{d+ex}\right)-\text{PolyLog}\left(4,1+\frac{ex}{d}\right)\right)\right)$$

```
[In] Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x,x]
```

```
[Out] (m*Log[x]^2*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2)/2 + Log[x]*(-(
m*Log[x]) + Log[f*x^m])*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + 2
*b*n*(-(m*Log[x]) + Log[f*x^m])*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n
])*(Log[x]*(Log[d + e*x] - Log[1 + (e*x)/d]) - PolyLog[2, -((e*x)/d)]) + 2*
b*m*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*((Log[x]^2*(Log[d + e*x
] - Log[1 + (e*x)/d]))/2 - Log[x]*PolyLog[2, -((e*x)/d)] + PolyLog[3, -((e*
x)/d)]) - b^2*n^2*(m*Log[x] - Log[f*x^m])*(Log[-((e*x)/d)]*Log[d + e*x]^2 +
 2*Log[d + e*x]*PolyLog[2, 1 + (e*x)/d] - 2*PolyLog[3, 1 + (e*x)/d]) + (b^2
*m*n^2*(Log[-((e*x)/d)]^4 + 6*Log[-((e*x)/d)]^2*Log[-((e*x)/(d + e*x))]^2 -
 4*(Log[-((e*x)/d)] + Log[d/(d + e*x)])*Log[-((e*x)/(d + e*x))]^3 + Log[-((
e*x)/(d + e*x))]^4 + 6*Log[x]^2*Log[d + e*x]^2 + 4*(2*Log[-((e*x)/d)]^3 - 3
*Log[x]^2*Log[d + e*x])*Log[1 + (e*x)/d] + 6*(Log[x] - Log[-((e*x)/d)])*(Lo
g[x] + 3*Log[-((e*x)/d)])*Log[1 + (e*x)/d]^2 - 4*Log[-((e*x)/d)]^2*Log[-((e
*x)/(d + e*x))]*(Log[-((e*x)/d)] + 3*Log[1 + (e*x)/d]) + 12*(Log[-((e*x)/d)
]^2 - 2*Log[-((e*x)/d)]*(Log[-((e*x)/(d + e*x))] + Log[1 + (e*x)/d]) + 2*Lo
g[x]*(-Log[d + e*x] + Log[1 + (e*x)/d]))*PolyLog[2, -((e*x)/d)] - 12*Log[-(
(e*x)/(d + e*x))]^2*PolyLog[2, (e*x)/(d + e*x)] + 12*(Log[-((e*x)/d)] - Log
[-((e*x)/(d + e*x))])^2*PolyLog[2, 1 + (e*x)/d] + 24*(Log[x] - Log[-((e*x)/
d)])*Log[1 + (e*x)/d]*PolyLog[2, 1 + (e*x)/d] + 24*(Log[-((e*x)/(d + e*x))]
 + Log[d + e*x])*PolyLog[3, -((e*x)/d)] + 24*Log[-((e*x)/(d + e*x))]*PolyLo
g[3, (e*x)/(d + e*x)] + 24*(-Log[x] + Log[-((e*x)/(d + e*x))])*PolyLog[3, 1
 + (e*x)/d] - 24*(PolyLog[4, -((e*x)/d)] + PolyLog[4, (e*x)/(d + e*x)] - Po
lyLog[4, 1 + (e*x)/d])))/12
```

## Maple [F]

$$\int \frac{\ln\left(f\,x^{m}\right)\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}{x}dx$$

```
[In] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/x,x)
```

```
[Out] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/x,x)
```

## Fricas [F]

$$\int \frac{\log\left(fx^{m}\right)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left((ex+d)^{n}c\right)+a\right)^{2}\log\left(fx^{m}\right)}{x}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x + d)^n*c)^2*log(f*x^m) + 2*a*b*log((e*x + d)^n*c)*lo
g(f*x^m) + a^2*log(f*x^m))/x, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x}\,dx=\text{Timed out}$$

```
[In] integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**2/x,x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left((ex+d)^nc\right)+a\right)^2\log(fx^m)}{x}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*m*log(x)^2 - 2*b^2*log(f)*log(x) - 2*b^2*log(x)*log(x^m))*log((e*
x + d)^n)^2 - integrate(-(b^2*d*log(c)^2*log(f) + 2*a*b*d*log(c)*log(f) + a
^2*d*log(f) + (b^2*e*log(c)^2*log(f) + 2*a*b*e*log(c)*log(f) + a^2*e*log(f)
)*x + (b^2*e*m*n*x*log(x)^2 - 2*b^2*e*n*x*log(f)*log(x) + 2*b^2*d*log(c)*lo
g(f) + 2*a*b*d*log(f) + 2*(b^2*e*log(c)*log(f) + a*b*e*log(f))*x - 2*(b^2*e
*n*x*log(x) - b^2*d*log(c) - a*b*d - (b^2*e*log(c) + a*b*e)*x)*log(x^m))*lo
g((e*x + d)^n) + (b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d + (b^2*e*log(c)^2
 + 2*a*b*e*log(c) + a^2*e)*x)*log(x^m))/(e*x^2 + d*x), x)
```

## Giac [F]

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left((ex+d)^nc\right)+a\right)^2\log(fx^m)}{x}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*log(f*x^m)/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x}\,dx = \int \frac{\ln\left(f\,x^m\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2}{x}\,dx$$

```
[In] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2)/x,x)
[Out] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2)/x, x)
```

# 3.371 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 607

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^2}\,dx = -\frac{b^2emn^2\log^2(x)\log(d+ex)}{d}$$
$$+\frac{2b^2emn^2\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}+\frac{2b^2en^2\log(x)\log(fx^m)\log(d+ex)}{d}$$
$$-\frac{b^2emn^2\log^2(d+ex)}{d}-\frac{b^2mn^2\log^2(d+ex)}{x}+\frac{b^2emn^2\log\left(-\frac{ex}{d}\right)\log^2(d+ex)}{d}$$
$$-\frac{b^2en^2\log(fx^m)\log^2(d+ex)}{d}-\frac{b^2n^2\log(fx^m)\log^2(d+ex)}{x}$$
$$-\frac{2bn(m\log(x)-\log(fx^m))\left(ex\log\left(-\frac{ex}{d}\right)-(d+ex)\log(d+ex)\right)(a-bn\log(d+ex)+b\log(c(d+ex)^n}{dx}$$
$$-\frac{m\log(x)(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{x}$$
$$-\frac{(m-m\log(x)+\log(fx^m))(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{x}$$
$$+\frac{b^2emn^2\log^2(x)\log\left(1+\frac{ex}{d}\right)}{d}-\frac{2b^2en^2\log(x)\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{d}$$
$$-\frac{2b^2en^2\log(fx^m)\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d}$$
$$+\frac{bmn(a-bn\log(d+ex)+b\log(c(d+ex)^n))\left(2ex\log\left(-\frac{ex}{d}\right)-2(d+ex)\log(d+ex)-2d\log(x)\log(d+\right.}{dx}$$
$$+\frac{2b^2emn^2(1+\log(d+ex))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d}$$
$$+\frac{2b^2emn^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{d}-\frac{2b^2emn^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{d}$$

```
[Out] -b^2*e*m*n^2*ln(x)^2*ln(e*x+d)/d+2*b^2*e*m*n^2*ln(-e*x/d)*ln(e*x+d)/d+2*b^2
*e*n^2*ln(x)*ln(f*x^m)*ln(e*x+d)/d-b^2*e*m*n^2*ln(e*x+d)^2/d-b^2*m*n^2*ln(e
```

```
*x+d)^2/x+b^2*e*m*n^2*ln(-e*x/d)*ln(e*x+d)^2/d-b^2*e*n^2*ln(f*x^m)*ln(e*x+d
)^2/d-b^2*n^2*ln(f*x^m)*ln(e*x+d)^2/x-2*b*n*(m*ln(x)-ln(f*x^m))*(e*x*ln(-e*
x/d)-(e*x+d)*ln(e*x+d))*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))/d/x-m*ln(x)*(a-
b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))^2/x-(m-m*ln(x)+ln(f*x^m))*(a-b*n*ln(e*x+d)
+b*ln(c*(e*x+d)^n))^2/x+b^2*e*m*n^2*ln(x)^2*ln(1+e*x/d)/d-2*b^2*e*n^2*ln(x)
*ln(f*x^m)*ln(1+e*x/d)/d-2*b^2*e*n^2*ln(f*x^m)*polylog(2,-e*x/d)/d+b*m*n*(a
-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))*(2*e*x*ln(-e*x/d)-2*(e*x+d)*ln(e*x+d)-2*d
*ln(x)*ln(e*x+d)+e*x*(ln(x)^2-2*ln(x)*ln(1+e*x/d)-2*polylog(2,-e*x/d)))/d/x
+2*b^2*e*m*n^2*(1+ln(e*x+d))*polylog(2,1+e*x/d)/d+2*b^2*e*m*n^2*polylog(3,-
e*x/d)/d-2*b^2*e*m*n^2*polylog(3,1+e*x/d)/d
```

## Rubi [F]

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^2}\,dx=\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^2}\,dx$$

```
[In] Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x^2,x]
[Out] Defer[Int][(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^2}\,dx$$

## Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.85

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^2}\,dx$$
$$=\frac{2bn(m\log(x)-\log(fx^m))\left(-ex\log\left(-\frac{ex}{d}\right)+(d+ex)\log(d+ex)\right)\left(a-bn\log(d+ex)+b\log\left(c(d+ex)\right.\right.}{}$$

```
[In] Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x^2,x]
[Out] (2*b*n*(m*Log[x] - Log[f*x^m])*(-(e*x*Log[-((e*x)/d)]) + (d + e*x)*Log[d +
e*x])*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]) - d*m*Log[x]*(a - b*n*L
og[d + e*x] + b*Log[c*(d + e*x)^n])^2 + d*(-m + m*Log[x] - Log[f*x^m])*(a -
 b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 - b*m*n*(a - b*n*Log[d + e*x] +
 b*Log[c*(d + e*x)^n])*(-2*e*x*Log[-((e*x)/d)] + 2*(d + e*x)*Log[d + e*x] +
 2*d*Log[x]*Log[d + e*x] - e*x*(Log[x]^2 - 2*(Log[x]*Log[1 + (e*x)/d] + Pol
yLog[2, -((e*x)/d)]))) + b^2*n^2*(e*m*x*Log[x]^2*Log[d + e*x] + 2*e*m*x*Log
[-((e*x)/d)]*Log[d + e*x] - 2*e*m*x*Log[x]*Log[-((e*x)/d)]*Log[d + e*x] + 2
*e*x*Log[-((e*x)/d)]*Log[f*x^m]*Log[d + e*x] - d*m*Log[d + e*x]^2 - e*m*x*L
```

```
og[d + e*x]^2 + e*m*x*Log[-((e*x)/d)]*Log[d + e*x]^2 - d*Log[f*x^m]*Log[d +
 e*x]^2 - e*x*Log[f*x^m]*Log[d + e*x]^2 - e*m*x*Log[x]^2*Log[1 + (e*x)/d] -
 2*e*m*x*Log[x]*PolyLog[2, -((e*x)/d)] + 2*e*x*(m - m*Log[x] + Log[f*x^m] +
 m*Log[d + e*x])*PolyLog[2, 1 + (e*x)/d] + 2*e*m*x*PolyLog[3, -((e*x)/d)] -
 2*e*m*x*PolyLog[3, 1 + (e*x)/d]))/(d*x)
```

## Maple [F]

$$\int \frac{\ln \left(f\,x^{m}\right) \left(a+b\ln \left(c(ex+d)^{n}\right)\right)^{2}}{x^{2}}dx$$

```
[In] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/x^2,x)
```

```
[Out] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/x^2,x)
```

## Fricas [F]

$$\int \frac{\log \left(fx^{m}\right) \left(a+b\log \left(c(d+ex)^{n}\right)\right)^{2}}{x^{2}}\,dx=\int \frac{\left(b\log \left(\left(ex+d\right)^{n}c\right)+a\right)^{2}\log \left(fx^{m}\right)}{x^{2}}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x + d)^n*c)^2*log(f*x^m) + 2*a*b*log((e*x + d)^n*c)*lo
g(f*x^m) + a^2*log(f*x^m))/x^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(fx^{m}\right) \left(a+b\log \left(c(d+ex)^{n}\right)\right)^{2}}{x^{2}}\,dx=\text{Timed out}$$

```
[In] integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**2/x**2,x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^2}\,dx = \int \frac{\left(b\log\left((ex+d)^n c\right)+a\right)^2\log(fx^m)}{x^2}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -(b^2*(m + log(f)) + b^2*log(x^m))*log((e*x + d)^n)^2/x + integrate((b^2*d*
log(c)^2*log(f) + 2*a*b*d*log(c)*log(f) + a^2*d*log(f) + (b^2*e*log(c)^2*lo
g(f) + 2*a*b*e*log(c)*log(f) + a^2*e*log(f))*x + 2*(b^2*d*log(c)*log(f) + a
*b*d*log(f) + (a*b*e*log(f) + (e*log(c)*log(f) + (m*n + n*log(f))*e)*b^2)*x
 + (b^2*d*log(c) + a*b*d + ((e*n + e*log(c))*b^2 + a*b*e)*x)*log(x^m))*log(
(e*x + d)^n) + (b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d + (b^2*e*log(c)^2 +
 2*a*b*e*log(c) + a^2*e)*x)*log(x^m))/(e*x^3 + d*x^2), x)
```

## Giac [F]

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^2}\,dx = \int \frac{\left(b\log\left((ex+d)^n c\right)+a\right)^2\log(fx^m)}{x^2}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*log(f*x^m)/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^2}\,dx = \int \frac{\ln\left(f\,x^m\right)\,{\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}^2}{x^2}\,dx$$

```
[In] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2)/x^2,x)
```

```
[Out] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2)/x^2, x)
```

# 3.372 $\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^3}\,dx$

## Optimal result

Integrand size = 26, antiderivative size = 939

$$
\begin{aligned}
\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^3}\,dx &= \frac{b^2e^2mn^2\log(x)}{d^2} - \frac{b^2e^2mn^2\log^2(x)}{2d^2} \\
&+ \frac{b^2e^2mn^2\log\left(-\frac{ex}{d}\right)}{2d^2} + \frac{b^2e^2n^2\log(x)\log(fx^m)}{d^2} - \frac{3b^2e^2mn^2\log(d+ex)}{2d^2} \\
&- \frac{3b^2emn^2\log(d+ex)}{2dx} + \frac{b^2e^2mn^2\log(x)\log(d+ex)}{d^2} + \frac{b^2e^2mn^2\log^2(x)\log(d+ex)}{2d^2} \\
&- \frac{b^2e^2mn^2\log\left(-\frac{ex}{d}\right)\log(d+ex)}{2d^2} - \frac{b^2e^2n^2\log(fx^m)\log(d+ex)}{d^2} \\
&- \frac{b^2en^2\log(fx^m)\log(d+ex)}{dx} - \frac{b^2e^2n^2\log(x)\log(fx^m)\log(d+ex)}{d^2} \\
&+ \frac{b^2e^2mn^2\log^2(d+ex)}{4d^2} - \frac{b^2mn^2\log^2(d+ex)}{4x^2} - \frac{b^2e^2mn^2\log\left(-\frac{ex}{d}\right)\log^2(d+ex)}{2d^2} \\
&+ \frac{b^2e^2n^2\log(fx^m)\log^2(d+ex)}{2d^2} - \frac{b^2n^2\log(fx^m)\log^2(d+ex)}{2x^2} \\
&+ \frac{bn(m\log(x)-\log(fx^m))\left(e^2x^2\log\left(-\frac{ex}{d}\right)+(d+ex)(ex+(d-ex)\log(d+ex))\right)(a-bn\log(d+ex)+}{d^2x^2} \\
&- \frac{m\log(x)(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{2x^2} \\
&- \frac{(m-2m\log(x)+2\log(fx^m))(a-bn\log(d+ex)+b\log(c(d+ex)^n))^2}{4x^2} \\
&- \frac{b^2e^2mn^2\log(x)\log\left(1+\frac{ex}{d}\right)}{d^2} - \frac{b^2e^2mn^2\log^2(x)\log\left(1+\frac{ex}{d}\right)}{2d^2} \\
&+ \frac{b^2e^2n^2\log(x)\log(fx^m)\log\left(1+\frac{ex}{d}\right)}{d^2} - \frac{b^2e^2n^2(m-\log(fx^m))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^2} \\
&- \frac{bmn(a-bn\log(d+ex)+b\log(c(d+ex)^n))\left(ex(d+ex)+e^2x^2\log\left(-\frac{ex}{d}\right)+(d^2-e^2x^2)\log(d+ex)+\right.}{2} \\
&- \frac{b^2e^2mn^2(1+2\log(d+ex))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{2d^2} \\
&- \frac{b^2e^2mn^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{d^2} + \frac{b^2e^2mn^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{d^2}
\end{aligned}
$$

```
[Out] -3/2*b^2*e*m*n^2*ln(e*x+d)/d/x+1/2*b^2*e^2*m*n^2*ln(x)^2*ln(e*x+d)/d^2-1/2*
b^2*e^2*m*n^2*ln(-e*x/d)*ln(e*x+d)/d^2-1/2*b^2*e^2*m*n^2*ln(-e*x/d)*ln(e*x+
d)^2/d^2-1/2*b^2*e^2*m*n^2*ln(x)^2*ln(1+e*x/d)/d^2-1/2*b*m*n*(a-b*n*ln(e*x+
d)+b*ln(c*(e*x+d)^n))*(e*x*(e*x+d)+e^2*x^2*ln(-e*x/d)+(-e^2*x^2+d^2)*ln(e*x
+d)+2*d^2*ln(x)*ln(e*x+d)+e*x*(e*x*ln(x)^2+2*d*(1+ln(x))-2*e*x*(ln(x)*ln(1+
e*x/d)+polylog(2,-e*x/d))))/d^2/x^2-1/2*b^2*e^2*m*n^2*(1+2*ln(e*x+d))*polyl
og(2,1+e*x/d)/d^2-1/4*b^2*m*n^2*ln(e*x+d)^2/x^2-1/2*b^2*n^2*ln(f*x^m)*ln(e*
x+d)^2/x^2-1/2*b^2*e^2*m*n^2*ln(x)^2/d^2+1/2*b^2*e^2*m*n^2*ln(-e*x/d)/d^2-3
/2*b^2*e^2*m*n^2*ln(e*x+d)/d^2+1/4*b^2*e^2*m*n^2*ln(e*x+d)^2/d^2+1/2*b^2*e^
```

```
2*n^2*ln(f*x^m)*ln(e*x+d)^2/d^2+b^2*e^2*m*n^2*ln(x)*ln(e*x+d)/d^2-b^2*e^2*n
^2*ln(x)*ln(f*x^m)*ln(e*x+d)/d^2-b^2*e^2*m*n^2*ln(x)*ln(1+e*x/d)/d^2+b^2*e^
2*n^2*ln(x)*ln(f*x^m)*ln(1+e*x/d)/d^2+b^2*e^2*m*n^2*polylog(3,1+e*x/d)/d^2-
b^2*e^2*n^2*ln(f*x^m)*ln(e*x+d)/d^2+b^2*e^2*m*n^2*ln(x)/d^2+b^2*e^2*n^2*ln(
x)*ln(f*x^m)/d^2-b^2*e^2*n^2*(m-ln(f*x^m))*polylog(2,-e*x/d)/d^2-b^2*e*n^2*
ln(f*x^m)*ln(e*x+d)/d/x+b*n*(m*ln(x)-ln(f*x^m))*(e^2*x^2*ln(-e*x/d)+(e*x+d)
*(e*x+(-e*x+d)*ln(e*x+d)))*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))/d^2/x^2-1/2*
m*ln(x)*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))^2/x^2-1/4*(m-2*m*ln(x)+2*ln(f*x
^m))*(a-b*n*ln(e*x+d)+b*ln(c*(e*x+d)^n))^2/x^2-b^2*e^2*m*n^2*polylog(3,-e*x
/d)/d^2
```

## Rubi [F]

$$\int \frac{\log (f x^m) (a+b \log (c(d+e x)^n))^2}{x^3} \, dx = \int \frac{\log (f x^m) (a+b \log (c(d+e x)^n))^2}{x^3} \, dx$$

```
[In] Int[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x^3,x]
[Out] Defer[Int][(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x^3, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log (f x^m) (a+b \log (c(d+e x)^n))^2}{x^3} \, dx$$

## Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 781, normalized size of antiderivative = 0.83

$$\int \frac{\log (f x^m) (a+b \log (c(d+e x)^n))^2}{x^3} \, dx = \frac{4bn(m \log(x) - \log (f x^m)) \left(e^2 x^2 \log \left(-\frac{ex}{d}\right) + (d+ex)(ex + (d-ex) \log(d+ex))\right) (a - bn \log(d+ex) + b}{}$$

```
[In] Integrate[(Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/x^3,x]
[Out] (4*b*n*(m*Log[x] - Log[f*x^m])*(e^2*x^2*Log[-((e*x)/d)] + (d + e*x)*(e*x +
(d - e*x)*Log[d + e*x]))*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n]) - 2*
d^2*m*Log[x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + d^2*(-m + 2*
m*Log[x] - 2*Log[f*x^m])*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 -
2*b*m*n*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(e*x*(d + e*x) + e^2*
x^2*Log[-((e*x)/d)] + (d^2 - e^2*x^2)*Log[d + e*x] + 2*d^2*Log[x]*Log[d + e
*x] + e*x*(e*x*Log[x]^2 + 2*d*(1 + Log[x]) - 2*e*x*(Log[x]*Log[1 + (e*x)/d]
 + PolyLog[2, -((e*x)/d)]))) + b^2*n^2*(4*e^2*m*x^2*Log[x] - 2*e^2*m*x^2*Lo
g[x]^2 + 2*e^2*m*x^2*Log[-((e*x)/d)] + 4*e^2*x^2*Log[x]*Log[f*x^m] - 6*d*e*
```

```
m*x*Log[d + e*x] - 6*e^2*m*x^2*Log[d + e*x] + 4*e^2*m*x^2*Log[x]*Log[d + e*
x] - 2*e^2*m*x^2*Log[x]^2*Log[d + e*x] - 2*e^2*m*x^2*Log[-((e*x)/d)]*Log[d
+ e*x] + 4*e^2*m*x^2*Log[x]*Log[-((e*x)/d)]*Log[d + e*x] - 4*d*e*x*Log[f*x^
m]*Log[d + e*x] - 4*e^2*x^2*Log[f*x^m]*Log[d + e*x] - 4*e^2*x^2*Log[-((e*x)
/d)]*Log[f*x^m]*Log[d + e*x] - d^2*m*Log[d + e*x]^2 + e^2*m*x^2*Log[d + e*x
]^2 - 2*e^2*m*x^2*Log[-((e*x)/d)]*Log[d + e*x]^2 - 2*d^2*Log[f*x^m]*Log[d +
 e*x]^2 + 2*e^2*x^2*Log[f*x^m]*Log[d + e*x]^2 - 4*e^2*m*x^2*Log[x]*Log[1 +
(e*x)/d] + 2*e^2*m*x^2*Log[x]^2*Log[1 + (e*x)/d] + 4*e^2*m*x^2*(-1 + Log[x]
)*PolyLog[2, -((e*x)/d)] - 2*e^2*x^2*(m - 2*m*Log[x] + 2*Log[f*x^m] + 2*m*L
og[d + e*x])*PolyLog[2, 1 + (e*x)/d] - 4*e^2*m*x^2*PolyLog[3, -((e*x)/d)] +
 4*e^2*m*x^2*PolyLog[3, 1 + (e*x)/d]))/(4*d^2*x^2)
```

## Maple [F]

$$\int \frac{\ln\left(f\,x^{m}\right)\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}{x^{3}}dx$$

```
[In] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/x^3,x)
[Out] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/x^3,x)
```

## Fricas [F]

$$\int \frac{\log\left(fx^{m}\right)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{x^{3}}\,dx=\int \frac{\left(b\log\left((ex+d)^{n}c\right)+a\right)^{2}\log\left(fx^{m}\right)}{x^{3}}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x^3,x, algorithm="fricas")
[Out] integral((b^2*log((e*x + d)^n*c)^2*log(f*x^m) + 2*a*b*log((e*x + d)^n*c)*lo
g(f*x^m) + a^2*log(f*x^m))/x^3, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(fx^{m}\right)\left(a+b\log\left(c(d+ex)^{n}\right)\right)^{2}}{x^{3}}\,dx=\text{Timed out}$$

```
[In] integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**2/x**3,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2\log(fx^m)}{x^3}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(m + 2*log(f)) + 2*b^2*log(x^m))*log((e*x + d)^n)^2/x^2 + integra
te(1/2*(2*b^2*d*log(c)^2*log(f) + 4*a*b*d*log(c)*log(f) + 2*a^2*d*log(f) +
2*(b^2*e*log(c)^2*log(f) + 2*a*b*e*log(c)*log(f) + a^2*e*log(f))*x + (4*b^2
*d*log(c)*log(f) + 4*a*b*d*log(f) + (4*a*b*e*log(f) + (4*e*log(c)*log(f) +
(m*n + 2*n*log(f))*e)*b^2)*x + 2*(2*b^2*d*log(c) + 2*a*b*d + ((e*n + 2*e*lo
g(c))*b^2 + 2*a*b*e)*x)*log(x^m))*log((e*x + d)^n) + 2*(b^2*d*log(c)^2 + 2*
a*b*d*log(c) + a^2*d + (b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x)*log(x^m
))/(e*x^4 + d*x^3), x)
```

## Giac [F]

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2\log(fx^m)}{x^3}\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*log(f*x^m)/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^m)(a+b\log(c(d+ex)^n))^2}{x^3}\,dx = \int \frac{\ln(f\,x^m)\,\left(a+b\,\ln\left(c\,{(d+e\,x)}^n\right)\right)^2}{x^3}\,dx$$

```
[In] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2)/x^3,x)
```

```
[Out] int((log(f*x^m)*(a + b*log(c*(d + e*x)^n))^2)/x^3, x)
```

# 3.373 $\int \log(fx^m)(a+b\log(c(d+ex)^n))^3\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 522

$$\begin{aligned}
&\int \log(fx^m)(a+b\log(c(d+ex)^n))^3\,dx \\
&= -12ab^2mn^2x+18b^3mn^3x-6b^2mn^2(a-bn)x+6ab^2n^2x\log(fx^m) \\
&\quad -6b^3n^3x\log(fx^m)-\frac{18b^3mn^2(d+ex)\log(c(d+ex)^n)}{e} \\
&\quad -\frac{6b^3dmn^2\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e}+\frac{6b^3n^2(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} \\
&\quad +\frac{6bmn(d+ex)(a+b\log(c(d+ex)^n))^2}{e}+\frac{3bdmn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{e} \\
&\quad -\frac{3bn(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e}-\frac{m(d+ex)(a+b\log(c(d+ex)^n))^3}{e} \\
&\quad -\frac{dm\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^3}{e}+\frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^3}{e} \\
&\quad -\frac{6b^3dmn^3\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}+\frac{6b^2dmn^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e} \\
&\quad -\frac{3bdmn(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}-\frac{6b^3dmn^3\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{e} \\
&\quad +\frac{6b^2dmn^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)}{e}-\frac{6b^3dmn^3\operatorname{PolyLog}\left(4,1+\frac{ex}{d}\right)}{e}
\end{aligned}$$

```
[Out] -12*a*b^2*m*n^2*x+18*b^3*m*n^3*x-6*b^2*m*n^2*(-b*n+a)*x+6*a*b^2*n^2*x*ln(f*
x^m)-6*b^3*n^3*x*ln(f*x^m)-18*b^3*m*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e-6*b^3*d*m
*n^2*ln(-e*x/d)*ln(c*(e*x+d)^n)/e+6*b^3*n^2*(e*x+d)*ln(f*x^m)*ln(c*(e*x+d)^
n)/e+6*b*m*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e+3*b*d*m*n*ln(-e*x/d)*(a+b*ln
(c*(e*x+d)^n))^2/e-3*b*n*(e*x+d)*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^2/e-m*(e*x
+d)*(a+b*ln(c*(e*x+d)^n))^3/e-d*m*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))^3/e+(e*x
```

```
+d)*ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^3/e-6*b^3*d*m*n^3*polylog(2,1+e*x/d)/e+
6*b^2*d*m*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(2,1+e*x/d)/e-3*b*d*m*n*(a+b*ln(
c*(e*x+d)^n))^2*polylog(2,1+e*x/d)/e-6*b^3*d*m*n^3*polylog(3,1+e*x/d)/e+6*b
^2*d*m*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(3,1+e*x/d)/e-6*b^3*d*m*n^3*polylog
(4,1+e*x/d)/e
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {2436, 2333, 2332, 2470, 2458, 45, 2393, 2354, 2438, 2395, 2421, 6724, 2430}

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^3\,dx$$
$$= \frac{6b^2dmn^2\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)(a+b\log(c(d+ex)^n))}{e} + \frac{6b^2dmn^2\operatorname{PolyLog}\left(3,\frac{ex}{d}+1\right)(a+b\log(c(d+ex)^n))}{e} + 6ab^2n^2x\log(fx^m) - 12ab^2mn^2x - 6b^2mn^2x(a-bn) - \frac{3bn(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e} + \frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^3}{e} - \frac{3bdmn\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)(a+b\log(c(d+ex)^n))^2}{e} + \frac{6bmn(d+ex)(a+b\log(c(d+ex)^n))^2}{e} + \frac{3bdmn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{e} - \frac{m(d+ex)(a+b\log(c(d+ex)^n))^3}{e} - \frac{dm\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^3}{e} + \frac{6b^3n^2(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} - \frac{18b^3mn^2(d+ex)\log(c(d+ex)^n)}{e} - \frac{6b^3dmn^2\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e} - \frac{6b^3dmn^3\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e} - \frac{6b^3dmn^3\operatorname{PolyLog}\left(3,\frac{ex}{d}+1\right)}{e} - \frac{6b^3dmn^3\operatorname{PolyLog}\left(4,\frac{ex}{d}+1\right)}{e} - 6b^3n^3x\log(fx^m) + 18b^3mn^3x$$

```
[In] Int[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] -12*a*b^2*m*n^2*x + 18*b^3*m*n^3*x - 6*b^2*m*n^2*(a - b*n)*x + 6*a*b^2*n^2*
x*Log[f*x^m] - 6*b^3*n^3*x*Log[f*x^m] - (18*b^3*m*n^2*(d + e*x)*Log[c*(d +
e*x)^n])/e - (6*b^3*d*m*n^2*Log[-((e*x)/d)]*Log[c*(d + e*x)^n])/e + (6*b^3*
n^2*(d + e*x)*Log[f*x^m]*Log[c*(d + e*x)^n])/e + (6*b*m*n*(d + e*x)*(a + b*
Log[c*(d + e*x)^n])^2)/e + (3*b*d*m*n*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x
```

```
)^n])^2)/e - (3*b*n*(d + e*x)*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^2)/e -
(m*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/e - (d*m*Log[-((e*x)/d)]*(a + b*
Log[c*(d + e*x)^n])^3)/e + ((d + e*x)*Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])
^3)/e - (6*b^3*d*m*n^3*PolyLog[2, 1 + (e*x)/d])/e + (6*b^2*d*m*n^2*(a + b*L
og[c*(d + e*x)^n])*PolyLog[2, 1 + (e*x)/d])/e - (3*b*d*m*n*(a + b*Log[c*(d
+ e*x)^n])^2*PolyLog[2, 1 + (e*x)/d])/e - (6*b^3*d*m*n^3*PolyLog[3, 1 + (e*
x)/d])/e + (6*b^2*d*m*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[3, 1 + (e*x)/d
])/e - (6*b^3*d*m*n^3*PolyLog[4, 1 + (e*x)/d])/e
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
```

```
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2470

```
Int[Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_
.))^(p_), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*(d + e*x)^n])^p, x]},
 Dist[Log[f*x^m], u, x] - Dist[m, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, m, n}, x] && IGtQ[p, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 6ab^2n^2x\log(fx^m) - 6b^3n^3x\log(fx^m) + \frac{6b^3n^2(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} \\
&\quad - \frac{3bn(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e} \\
&\quad + \frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^3}{e} \\
&\quad - m\int\left(6ab^2n^2 - 6b^3n^3 + \frac{6b^3n^2(d+ex)\log(c(d+ex)^n)}{ex}\right. \\
&\quad \left. - \frac{3bn(d+ex)(a+b\log(c(d+ex)^n))^2}{ex} + \frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{ex}\right)dx \\
&= -6b^2mn^2(a-bn)x + 6ab^2n^2x\log(fx^m) - 6b^3n^3x\log(fx^m) \\
&\quad + \frac{6b^3n^2(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} \\
&\quad - \frac{3bn(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e} \\
&\quad + \frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^3}{e} - \frac{m\int\frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{x}\,dx}{e} \\
&\quad + \frac{(3bmn)\int\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{x}\,dx}{e} - \frac{(6b^3mn^2)\int\frac{(d+ex)\log(c(d+ex)^n)}{x}\,dx}{e} \\
&= -6b^2mn^2(a-bn)x + 6ab^2n^2x\log(fx^m) - 6b^3n^3x\log(fx^m) \\
&\quad + \frac{6b^3n^2(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} \\
&\quad - \frac{3bn(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e} \\
&\quad + \frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^3}{e} \\
&\quad - \frac{m\text{Subst}\left(\int\frac{x(a+b\log(cx^n))^3}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex\right)}{e^2} \\
&\quad + \frac{(3bmn)\text{Subst}\left(\int\frac{x(a+b\log(cx^n))^2}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex\right)}{e^2} \\
&\quad - \frac{(6b^3mn^2)\,\text{Subst}\left(\int\frac{x\log(cx^n)}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex\right)}{e^2}
\end{aligned}$$

$$= -6b^2mn^2(a-bn)x + 6ab^2n^2x\log(fx^m) - 6b^3n^3x\log(fx^m) + \frac{6b^3n^2(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} - \frac{3bn(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e} + \frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^3}{e} - \frac{m\text{Subst}\left(\int\left(e(a+b\log(cx^n))^3 - \frac{de(a+b\log(cx^n))^3}{d-x}\right)dx, x, d+ex\right)}{e^2} + \frac{(3bmn)\text{Subst}\left(\int\left(e(a+b\log(cx^n))^2 - \frac{de(a+b\log(cx^n))^2}{d-x}\right)dx, x, d+ex\right)}{e^2} - \frac{(6b^3mn^2)\,\text{Subst}\left(\int\left(e\log(cx^n) - \frac{de\log(cx^n)}{d-x}\right)dx, x, d+ex\right)}{e^2}$$

$$= -6b^2mn^2(a-bn)x + 6ab^2n^2x\log(fx^m) - 6b^3n^3x\log(fx^m) + \frac{6b^3n^2(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} - \frac{3bn(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e} + \frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^3}{e} - \frac{m\text{Subst}\left(\int(a+b\log(cx^n))^3\,dx, x, d+ex\right)}{e} + \frac{(dm)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^3}{d-x}dx, x, d+ex\right)}{e} + \frac{(3bmn)\text{Subst}\left(\int(a+b\log(cx^n))^2\,dx, x, d+ex\right)}{e} - \frac{(3bdmn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{d-x}dx, x, d+ex\right)}{e} - \frac{(6b^3mn^2)\,\text{Subst}\left(\int\log(cx^n)\,dx, x, d+ex\right)}{e} + \frac{(6b^3dmn^2)\,\text{Subst}\left(\int\frac{\log(cx^n)}{d-x}dx, x, d+ex\right)}{e}$$

$$
\begin{aligned}
&= 6b^3mn^3x - 6b^2mn^2(a-bn)x + 6ab^2n^2x\log(fx^m) \\
&\quad - 6b^3n^3x\log(fx^m) - \frac{6b^3mn^2(d+ex)\log(c(d+ex)^n)}{e} \\
&\quad - \frac{6b^3dmn^2\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e} + \frac{6b^3n^2(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} \\
&\quad + \frac{3bmn(d+ex)\left(a+b\log(c(d+ex)^n)\right)^2}{e} \\
&\quad + \frac{3bdmn\log\left(-\frac{ex}{d}\right)\left(a+b\log(c(d+ex)^n)\right)^2}{e} \\
&\quad - \frac{3bn(d+ex)\log(fx^m)\left(a+b\log(c(d+ex)^n)\right)^2}{e} \\
&\quad - \frac{m(d+ex)\left(a+b\log(c(d+ex)^n)\right)^3}{e} - \frac{dm\log\left(-\frac{ex}{d}\right)\left(a+b\log(c(d+ex)^n)\right)^3}{e} \\
&\quad + \frac{(d+ex)\log(fx^m)\left(a+b\log(c(d+ex)^n)\right)^3}{e} \\
&\quad + \frac{(3bmn)\text{Subst}\left(\int(a+b\log(cx^n))^2\,dx, x, d+ex\right)}{e} \\
&\quad + \frac{(3bdmn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2\log\left(1-\frac{x}{d}\right)}{x}\,dx, x, d+ex\right)}{e} \\
&\quad - \frac{(6b^2mn^2)\,\text{Subst}\left(\int(a+b\log(cx^n))\,dx, x, d+ex\right)}{e} \\
&\quad - \frac{(6b^2dmn^2)\,\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(1-\frac{x}{d}\right)}{x}\,dx, x, d+ex\right)}{e} \\
&\quad + \frac{(6b^3dmn^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{x}{d}\right)}{x}\,dx, x, d+ex\right)}{e}
\end{aligned}
$$

$$= -6ab^2mn^2x + 6b^3mn^3x - 6b^2mn^2(a - bn)x + 6ab^2n^2x\log(fx^m) - 6b^3n^3x\log(fx^m) - \frac{6b^3mn^2(d+ex)\log(c(d+ex)^n)}{e} - \frac{6b^3dmn^2\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e} + \frac{6b^3n^2(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} + \frac{6bmn(d+ex)\left(a+b\log(c(d+ex)^n)\right)^2}{e} + \frac{3bdmn\log\left(-\frac{ex}{d}\right)\left(a+b\log(c(d+ex)^n)\right)^2}{e} - \frac{3bn(d+ex)\log(fx^m)\left(a+b\log(c(d+ex)^n)\right)^2}{e} - \frac{m(d+ex)\left(a+b\log(c(d+ex)^n)\right)^3}{e} - \frac{dm\log\left(-\frac{ex}{d}\right)\left(a+b\log(c(d+ex)^n)\right)^3}{e} + \frac{(d+ex)\log(fx^m)\left(a+b\log(c(d+ex)^n)\right)^3}{e} - \frac{6b^3dmn^3\text{Li}_2\left(1+\frac{ex}{d}\right)}{e} + \frac{6b^2dmn^2(a+b\log(c(d+ex)^n))\text{Li}_2\left(1+\frac{ex}{d}\right)}{e} - \frac{3bdmn(a+b\log(c(d+ex)^n))^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{e} - \frac{(6b^2mn^2)\,\text{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+ex\right)}{e} - \frac{(6b^3mn^2)\,\text{Subst}\left(\int \log(cx^n)\,dx, x, d+ex\right)}{e} + \frac{(6b^2dmn^2)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx, x, d+ex\right)}{e} - \frac{(6b^3dmn^3)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx, x, d+ex\right)}{e}$$

$$
\begin{aligned}
&= -12ab^2mn^2x + 12b^3mn^3x - 6b^2mn^2(a-bn)x + 6ab^2n^2x\log\left(fx^m\right) \\
&\quad - 6b^3n^3x\log\left(fx^m\right) - \frac{12b^3mn^2(d+ex)\log\left(c(d+ex)^n\right)}{e} \\
&\quad - \frac{6b^3dmn^2\log\left(-\frac{ex}{d}\right)\log\left(c(d+ex)^n\right)}{e} + \frac{6b^3n^2(d+ex)\log\left(fx^m\right)\log\left(c(d+ex)^n\right)}{e} \\
&\quad + \frac{6bmn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} \\
&\quad + \frac{3bdmn\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} \\
&\quad - \frac{3bn(d+ex)\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} \\
&\quad - \frac{m(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e} - \frac{dm\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e} \\
&\quad + \frac{(d+ex)\log\left(fx^m\right)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e} - \frac{6b^3dmn^3\text{Li}_2\left(1+\frac{ex}{d}\right)}{e} \\
&\quad + \frac{6b^2dmn^2\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(1+\frac{ex}{d}\right)}{e} \\
&\quad - \frac{3bdmn\left(a+b\log\left(c(d+ex)^n\right)\right)^2\text{Li}_2\left(1+\frac{ex}{d}\right)}{e} - \frac{6b^3dmn^3\text{Li}_3\left(1+\frac{ex}{d}\right)}{e} \\
&\quad + \frac{6b^2dmn^2\left(a+b\log\left(c(d+ex)^n\right)\right)\text{Li}_3\left(1+\frac{ex}{d}\right)}{e} \\
&\quad - \frac{\left(6b^3mn^2\right)\text{Subst}\left(\int\log\left(cx^n\right)\,dx, x, d+ex\right)}{e} \\
&\quad - \frac{\left(6b^3dmn^3\right)\text{Subst}\left(\int\frac{\text{Li}_3\left(\frac{x}{d}\right)}{x}\,dx, x, d+ex\right)}{e}
\end{aligned}
$$

$$= -12ab^2mn^2x + 18b^3mn^3x - 6b^2mn^2(a-bn)x + 6ab^2n^2x\log(fx^m) - 6b^3n^3x\log(fx^m) - \frac{18b^3mn^2(d+ex)\log(c(d+ex)^n)}{e} - \frac{6b^3dmn^2\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{e} + \frac{6b^3n^2(d+ex)\log(fx^m)\log(c(d+ex)^n)}{e} + \frac{6bmn(d+ex)(a+b\log(c(d+ex)^n))^2}{e} + \frac{3bdmn\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{e} - \frac{3bn(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^2}{e} - \frac{m(d+ex)(a+b\log(c(d+ex)^n))^3}{e} - \frac{dm\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^3}{e} + \frac{(d+ex)\log(fx^m)(a+b\log(c(d+ex)^n))^3}{e} - \frac{6b^3dmn^3\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e} + \frac{6b^2dmn^2(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e} - \frac{3bdmn(a+b\log(c(d+ex)^n))^2\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e} - \frac{6b^3dmn^3\operatorname{Li}_3\left(1+\frac{ex}{d}\right)}{e} + \frac{6b^2dmn^2(a+b\log(c(d+ex)^n))\operatorname{Li}_3\left(1+\frac{ex}{d}\right)}{e} - \frac{6b^3dmn^3\operatorname{Li}_4\left(1+\frac{ex}{d}\right)}{e}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1163 vs. 2(522) = 1044.

Time = 0.46 (sec) , antiderivative size = 1163, normalized size of antiderivative = 2.23

$$\int \log(fx^m)(a+b\log(c(d+ex)^n))^3\,dx = \frac{-b^3n^3(d+ex)(m\log(x)-\log(fx^m))\left(-6+6\log(d+ex)-3\log^2(d+ex)+\log^3(d+ex)\right)-3b^2n^2(m\log}{}$$

```
[In] Integrate[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^3,x]
```

```
[Out] (-(b^3*n^3*(d + e*x)*(m*Log[x] - Log[f*x^m])*(-6 + 6*Log[d + e*x] - 3*Log[d
 + e*x]^2 + Log[d + e*x]^3)) - 3*b^2*n^2*(m*Log[x] - Log[f*x^m])*(2*e*x - 2
*(d + e*x)*Log[d + e*x] + (d + e*x)*Log[d + e*x]^2)*(a - b*n*Log[d + e*x] +
 b*Log[c*(d + e*x)^n]) - 3*b*e*n*x*(m - Log[f*x^m])*Log[d + e*x]*(a - b*n*L
og[d + e*x] + b*Log[c*(d + e*x)^n])^2 - 3*b*d*n*(m + m*Log[x] - Log[f*x^m])
*Log[d + e*x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2 + e*x*(a - b*
n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(3*b*m*n + 3*b*n*(m*Log[x] - Log[f
*x^m]) + a*(-(m*Log[x]) + Log[f*x^m]) + b*(-(m*Log[x]) + Log[f*x^m])*(-(n*L
og[d + e*x]) + Log[c*(d + e*x)^n])) + a*d*m*(a - b*n*Log[d + e*x] + b*Log[c
*(d + e*x)^n])^2*(Log[x]*Log[1 + (e*x)/d] + PolyLog[2, -((e*x)/d)]) - b*d*m
```

```
*(n*Log[d + e*x] - Log[c*(d + e*x)^n])*(a - b*n*Log[d + e*x] + b*Log[c*(d +
 e*x)^n])^2*(Log[x]*Log[1 + (e*x)/d] + PolyLog[2, -((e*x)/d)]) - a*m*(a - b
*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])^2*(e*x + Log[x]*(-(e*x) + d*Log[1 +
 (e*x)/d]) + d*PolyLog[2, -((e*x)/d)]) + 3*b*m*n*(a - b*n*Log[d + e*x] + b*
Log[c*(d + e*x)^n])^2*(e*x + Log[x]*(-(e*x) + d*Log[1 + (e*x)/d]) + d*PolyL
og[2, -((e*x)/d)]) + b*m*(n*Log[d + e*x] - Log[c*(d + e*x)^n])*(a - b*n*Log
[d + e*x] + b*Log[c*(d + e*x)^n])^2*(e*x + Log[x]*(-(e*x) + d*Log[1 + (e*x)
/d]) + d*PolyLog[2, -((e*x)/d)]) - 3*b^2*m*n^2*(-a + b*n*Log[d + e*x] - b*L
og[c*(d + e*x)^n])*(-6*e*x + 2*e*x*Log[x] + 4*d*Log[d + e*x] + 4*e*x*Log[d
+ e*x] - 2*e*x*Log[x]*Log[d + e*x] - d*Log[d + e*x]^2 - e*x*Log[d + e*x]^2
+ d*Log[x]*Log[d + e*x]^2 + e*x*Log[x]*Log[d + e*x]^2 - d*Log[-((e*x)/d)]*L
og[d + e*x]^2 - 2*d*Log[x]*Log[1 + (e*x)/d] - 2*d*PolyLog[2, -((e*x)/d)] -
2*d*Log[d + e*x]*PolyLog[2, 1 + (e*x)/d] + 2*d*PolyLog[3, 1 + (e*x)/d]) + b
^3*m*n^3*(6*d + 24*e*x - 6*e*x*Log[x] - 18*d*Log[d + e*x] - 18*e*x*Log[d +
e*x] + 6*e*x*Log[x]*Log[d + e*x] + 6*d*Log[d + e*x]^2 + 6*e*x*Log[d + e*x]^
2 - 3*d*Log[x]*Log[d + e*x]^2 - 3*e*x*Log[x]*Log[d + e*x]^2 + 3*d*Log[-((e*
x)/d)]*Log[d + e*x]^2 - d*Log[d + e*x]^3 - e*x*Log[d + e*x]^3 + d*Log[x]*Lo
g[d + e*x]^3 + e*x*Log[x]*Log[d + e*x]^3 - d*Log[-((e*x)/d)]*Log[d + e*x]^3
 + 6*d*Log[x]*Log[1 + (e*x)/d] + 6*d*PolyLog[2, -((e*x)/d)] - 3*d*(-2 + Log
[d + e*x])*Log[d + e*x]*PolyLog[2, 1 + (e*x)/d] - 6*d*PolyLog[3, 1 + (e*x)/
d] + 6*d*Log[d + e*x]*PolyLog[3, 1 + (e*x)/d] - 6*d*PolyLog[4, 1 + (e*x)/d]
))/e
```

## Maple [F]

$$\int \ln \left(f\,x^{m}\right) \left(a +b \ln \left(c (e x +d)^{n}\right)\right)^{3}d x$$

```
[In] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^3,x)
```

```
[Out] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^3,x)
```

## Fricas [F]

$$\int \log \left(fx^m\right) \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \, dx = \int \left(b\log \left((ex+d)^n c\right)+a\right)^3 \log \left(fx^m\right) \, dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log((e*x + d)^n*c)^3*log(f*x^m) + 3*a*b^2*log((e*x + d)^n*c)^2
*log(f*x^m) + 3*a^2*b*log((e*x + d)^n*c)*log(f*x^m) + a^3*log(f*x^m), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx=\text{Timed out}$$

```
[In] integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**3,x)
[Out] Timed out
```

## Maxima [F]

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx=\int (b\log\left((ex+d)^n c\right)+a)^3\log(fx^m)\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="maxima")
[Out] -(b^3*(m - log(f))*x - b^3*x*log(x^m))*log((e*x + d)^n)^3 + integrate((b^3*
d*log(c)^3*log(f) + 3*a*b^2*d*log(c)^2*log(f) + 3*a^2*b*d*log(c)*log(f) + a
^3*d*log(f) + 3*(b^3*d*log(c)*log(f) + a*b^2*d*log(f) + (a*b^2*e*log(f) + (
e*log(c)*log(f) + (m*n - n*log(f))*e)*b^3)*x + (b^3*d*log(c) + a*b^2*d - ((
e*n - e*log(c))*b^3 - a*b^2*e)*x)*log(x^m))*log((e*x + d)^n)^2 + (b^3*e*log
(c)^3*log(f) + 3*a*b^2*e*log(c)^2*log(f) + 3*a^2*b*e*log(c)*log(f) + a^3*e*
log(f))*x + 3*(b^3*d*log(c)^2*log(f) + 2*a*b^2*d*log(c)*log(f) + a^2*b*d*lo
g(f) + (b^3*e*log(c)^2*log(f) + 2*a*b^2*e*log(c)*log(f) + a^2*b*e*log(f))*x
 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d + (b^3*e*log(c)^2 + 2*a*b^2
*e*log(c) + a^2*b*e)*x)*log(x^m))*log((e*x + d)^n) + (b^3*d*log(c)^3 + 3*a*
b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d + (b^3*e*log(c)^3 + 3*a*b^2*e*log
(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x)*log(x^m))/(e*x + d), x)
```

## Giac [F]

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx=\int (b\log\left((ex+d)^n c\right)+a)^3\log(fx^m)\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^3,x, algorithm="giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^3*log(f*x^m), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx=\int \ln\left(f\,x^m\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^3\,dx$$

```
[In] int(log(f*x^m)*(a + b*log(c*(d + e*x)^n))^3,x)
[Out] int(log(f*x^m)*(a + b*log(c*(d + e*x)^n))^3, x)
```

# 3.374 $\int \frac{\log(x)\log^2(a+bx)}{x}\,dx$

## Optimal result

Integrand size = 14, antiderivative size = 519

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = \frac{1}{12}\Bigg(\log^4\left(-\frac{bx}{a}\right)+6\log^2\left(-\frac{bx}{a}\right)\log^2\left(-\frac{bx}{a+bx}\right) -4\left(\log\left(-\frac{bx}{a}\right)+\log\left(\frac{a}{a+bx}\right)\right)\log^3\left(-\frac{bx}{a+bx}\right) +\log^4\left(-\frac{bx}{a+bx}\right)+6\log^2(x)\log^2(a+bx) +4\left(2\log^3\left(-\frac{bx}{a}\right)-3\log^2(x)\log(a+bx)\right)\log\left(1+\frac{bx}{a}\right) +6\left(\log(x)-\log\left(-\frac{bx}{a}\right)\right)\left(\log(x)+3\log\left(-\frac{bx}{a}\right)\right)\log^2\left(1+\frac{bx}{a}\right)-4\log^2\left(-\frac{bx}{a}\right)\log\left(-\frac{bx}{a+bx}\right)\left(\log\left(-\frac{bx}{a}\right)+3\log\left(1+\frac{bx}{a}\right)\right)+12\left(\log^2\left(-\frac{bx}{a}\right)-2\log\left(-\frac{bx}{a}\right)\left(\log\left(-\frac{bx}{a+bx}\right)+\log\left(1+\frac{bx}{a}\right)\right)+2\log(x)\left(-\log(a+bx)+\log\left(1+\frac{bx}{a}\right)\right)\right)\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right) -12\log^2\left(-\frac{bx}{a+bx}\right)\operatorname{PolyLog}\left(2,\frac{bx}{a+bx}\right) +12\left(\log\left(-\frac{bx}{a}\right)-\log\left(-\frac{bx}{a+bx}\right)\right)^2\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right) +24\left(\log(x)-\log\left(-\frac{bx}{a}\right)\right)\log\left(1+\frac{bx}{a}\right)\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right) +24\left(\log\left(-\frac{bx}{a+bx}\right)+\log(a+bx)\right)\operatorname{PolyLog}\left(3,-\frac{bx}{a}\right) +24\log\left(-\frac{bx}{a+bx}\right)\operatorname{PolyLog}\left(3,\frac{bx}{a+bx}\right) +24\left(-\log(x)+\log\left(-\frac{bx}{a+bx}\right)\right)\operatorname{PolyLog}\left(3,1+\frac{bx}{a}\right) -24\left(\operatorname{PolyLog}\left(4,-\frac{bx}{a}\right)+\operatorname{PolyLog}\left(4,\frac{bx}{a+bx}\right) -\operatorname{PolyLog}\left(4,1+\frac{bx}{a}\right)\right)\Bigg)$$

```
[Out] 1/12*ln(-b*x/a)^4+1/2*ln(-b*x/a)^2*ln(-b*x/(b*x+a))^2-1/3*(ln(-b*x/a)+ln(a/
(b*x+a)))*ln(-b*x/(b*x+a))^3+1/12*ln(-b*x/(b*x+a))^4+1/2*ln(x)^2*ln(b*x+a)^
```

```
2+1/3*(2*ln(-b*x/a)^3-3*ln(x)^2*ln(b*x+a))*ln(1+b*x/a)+1/2*(ln(x)-ln(-b*x/a
))*(ln(x)+3*ln(-b*x/a))*ln(1+b*x/a)^2-1/3*ln(-b*x/a)^2*ln(-b*x/(b*x+a))*(ln
(-b*x/a)+3*ln(1+b*x/a))+(ln(-b*x/a)^2-2*ln(-b*x/a)*(ln(-b*x/(b*x+a))+ln(1+b
*x/a))+2*ln(x)*(-ln(b*x+a)+ln(1+b*x/a)))*polylog(2,-b*x/a)-ln(-b*x/(b*x+a))
^2*polylog(2,b*x/(b*x+a))+(ln(-b*x/a)-ln(-b*x/(b*x+a)))^2*polylog(2,1+b*x/a
)+2*(ln(x)-ln(-b*x/a))*ln(1+b*x/a)*polylog(2,1+b*x/a)+2*(ln(-b*x/(b*x+a))+l
n(b*x+a))*polylog(3,-b*x/a)+2*ln(-b*x/(b*x+a))*polylog(3,b*x/(b*x+a))+2*(-l
n(x)+ln(-b*x/(b*x+a)))*polylog(3,1+b*x/a)-2*polylog(4,-b*x/a)-2*polylog(4,b
*x/(b*x+a))+2*polylog(4,1+b*x/a)
```

## Rubi [F]

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = \int \frac{\log(x)\log^2(a+bx)}{x}\,dx$$

```
[In] Int[(Log[x]*Log[a + b*x]^2)/x,x]
```

```
[Out] (Log[x]^2*Log[a + b*x]^2)/2 - b*Defer[Int][(Log[x]^2*Log[a + b*x])/(a + b*x
), x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\log^2(x)\log^2(a+bx) - b\int \frac{\log^2(x)\log(a+bx)}{a+bx}\,dx$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = \frac{1}{12}\left(\log^4\left(-\frac{bx}{a}\right)+6\log^2\left(-\frac{bx}{a}\right)\log^2\left(-\frac{bx}{a+bx}\right)-4\left(\log\left(-\frac{bx}{a}\right)+\log\left(\frac{a}{a+bx}\right)\right)\log^3\left(-\frac{bx}{a+bx}\right)+\log^4\left(-\frac{bx}{a+bx}\right)+6\log^2(x)\log^2(a+bx)+4\left(2\log^3\left(-\frac{bx}{a}\right)-3\log^2(x)\log(a+bx)\right)\log\left(1+\frac{bx}{a}\right)+6\left(\log(x)-\log\left(-\frac{bx}{a}\right)\right)\left(\log(x)+3\log\left(-\frac{bx}{a}\right)\right)\log^2\left(1+\frac{bx}{a}\right)-4\log^2\left(-\frac{bx}{a}\right)\log\left(-\frac{bx}{a+bx}\right)\left(\log\left(-\frac{bx}{a}\right)+3\log\left(1+\frac{bx}{a}\right)\right)+12\left(\log^2\left(-\frac{bx}{a}\right)-2\log\left(-\frac{bx}{a}\right)\left(\log\left(-\frac{bx}{a+bx}\right)+\log\left(1+\frac{bx}{a}\right)\right)+2\log(x)\left(-\log(a+bx)+\log\left(1+\frac{bx}{a}\right)\right)\right)\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right)-12\log^2\left(-\frac{bx}{a+bx}\right)\operatorname{PolyLog}\left(2,\frac{bx}{a+bx}\right)+12\left(\log\left(-\frac{bx}{a}\right)-\log\left(-\frac{bx}{a+bx}\right)\right)^2\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right)+24\left(\log(x)-\log\left(-\frac{bx}{a}\right)\right)\log\left(1+\frac{bx}{a}\right)\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right)+24\left(\log\left(-\frac{bx}{a+bx}\right)+\log(a+bx)\right)\operatorname{PolyLog}\left(3,-\frac{bx}{a}\right)+24\log\left(-\frac{bx}{a+bx}\right)\operatorname{PolyLog}\left(3,\frac{bx}{a+bx}\right)+24\left(-\log(x)+\log\left(-\frac{bx}{a+bx}\right)\right)\operatorname{PolyLog}\left(3,1+\frac{bx}{a}\right)-24\left(\operatorname{PolyLog}\left(4,-\frac{bx}{a}\right)+\operatorname{PolyLog}\left(4,\frac{bx}{a+bx}\right)-\operatorname{PolyLog}\left(4,1+\frac{bx}{a}\right)\right)\right)$$

```
[In] Integrate[(Log[x]*Log[a + b*x]^2)/x,x]
```

```
[Out] (Log[-((b*x)/a)]^4 + 6*Log[-((b*x)/a)]^2*Log[-((b*x)/(a + b*x))]^2 - 4*(Log
[-((b*x)/a)] + Log[a/(a + b*x)])*Log[-((b*x)/(a + b*x))]^3 + Log[-((b*x)/(a
 + b*x))]^4 + 6*Log[x]^2*Log[a + b*x]^2 + 4*(2*Log[-((b*x)/a)]^3 - 3*Log[x]
^2*Log[a + b*x])*Log[1 + (b*x)/a] + 6*(Log[x] - Log[-((b*x)/a)])*(Log[x] +
3*Log[-((b*x)/a)])*Log[1 + (b*x)/a]^2 - 4*Log[-((b*x)/a)]^2*Log[-((b*x)/(a
+ b*x))]*(Log[-((b*x)/a)] + 3*Log[1 + (b*x)/a]) + 12*(Log[-((b*x)/a)]^2 - 2
*Log[-((b*x)/a)]*(Log[-((b*x)/(a + b*x))] + Log[1 + (b*x)/a]) + 2*Log[x]*(-
Log[a + b*x] + Log[1 + (b*x)/a]))*PolyLog[2, -((b*x)/a)] - 12*Log[-((b*x)/(
a + b*x))]^2*PolyLog[2, (b*x)/(a + b*x)] + 12*(Log[-((b*x)/a)] - Log[-((b*x
)/(a + b*x))])^2*PolyLog[2, 1 + (b*x)/a] + 24*(Log[x] - Log[-((b*x)/a)])*Lo
g[1 + (b*x)/a]*PolyLog[2, 1 + (b*x)/a] + 24*(Log[-((b*x)/(a + b*x))] + Log[
a + b*x])*PolyLog[3, -((b*x)/a)] + 24*Log[-((b*x)/(a + b*x))]*PolyLog[3, (b
*x)/(a + b*x)] + 24*(-Log[x] + Log[-((b*x)/(a + b*x))])*PolyLog[3, 1 + (b*x
)/a] - 24*(PolyLog[4, -((b*x)/a)] + PolyLog[4, (b*x)/(a + b*x)] - PolyLog[4
, 1 + (b*x)/a]))/12
```

## Maple [F]

$$\int \frac{\ln \left(x\right) \ln \left(bx+a\right)^{2}}{x}dx$$

```
[In] int(ln(x)/x*ln(b*x+a)^2,x)
```

```
[Out] int(ln(x)/x*ln(b*x+a)^2,x)
```

## Fricas [F]

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = \int \frac{\log\left(bx+a\right)^{2}\log\left(x\right)}{x}\,dx$$

```
[In] integrate(log(x)*log(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(log(b*x + a)^2*log(x)/x, x)
```

## Sympy [F]

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = -b\int \frac{\log\left(x\right)^{2}\log\left(a+bx\right)}{a+bx}\,dx + \frac{\log\left(x\right)^{2}\log\left(a+bx\right)^{2}}{2}$$

```
[In] integrate(ln(x)*ln(b*x+a)**2/x,x)
```

```
[Out] -b*Integral(log(x)**2*log(a + b*x)/(a + b*x), x) + log(x)**2*log(a + b*x)**
2/2
```

## Maxima [F]

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = \int \frac{\log\left(bx+a\right)^2\log\left(x\right)}{x}\,dx$$

[In] integrate(log(x)*log(b*x+a)^2/x,x, algorithm="maxima")

[Out] 1/2*log(b*x + a)^2*log(x)^2 - b*integrate(log(b*x + a)*log(x)^2/(b*x + a), x)

## Giac [F]

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = \int \frac{\log\left(bx+a\right)^2\log\left(x\right)}{x}\,dx$$

[In] integrate(log(x)*log(b*x+a)^2/x,x, algorithm="giac")

[Out] integrate(log(b*x + a)^2*log(x)/x, x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)\log^2(a+bx)}{x}\,dx = \int \frac{\ln\left(a+b\,x\right)^2\,\ln\left(x\right)}{x}\,dx$$

[In] int((log(a + b*x)^2*log(x))/x,x)

[Out] int((log(a + b*x)^2*log(x))/x, x)

# 3.375 $\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx = \text{Int}\left(\frac{\log(fx^m)}{a+b\log(c(d+ex)^n)},x\right)$$

```
[Out] Unintegrable(ln(f*x^m)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx$$

```
[In] Int[Log[f*x^m]/(a + b*Log[c*(d + e*x)^n]),x]
[Out] Defer[Int][Log[f*x^m]/(a + b*Log[c*(d + e*x)^n]), x]
```

Rubi steps

$$\text{integral} = \int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx=\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx$$

```
[In] Integrate[Log[f*x^m]/(a + b*Log[c*(d + e*x)^n]),x]
[Out] Integrate[Log[f*x^m]/(a + b*Log[c*(d + e*x)^n]), x]
```

## Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\ln(f\,x^m)}{a+b\ln(c\,(ex+d)^n)}dx$$

```
[In] int(ln(f*x^m)/(a+b*ln(c*(e*x+d)^n)),x)
[Out] int(ln(f*x^m)/(a+b*ln(c*(e*x+d)^n)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx=\int \frac{\log(fx^m)}{b\log((ex+d)^nc)+a}\,dx$$

```
[In] integrate(log(f*x^m)/(a+b*log(c*(e*x+d)^n)),x, algorithm="fricas")
[Out] integral(log(f*x^m)/(b*log((e*x + d)^n*c) + a), x)
```

## Sympy [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx=\int \frac{\log(fx^m)}{a+b\log(c\,(d+ex)^n)}\,dx$$

```
[In] integrate(ln(f*x**m)/(a+b*ln(c*(e*x+d)**n)),x)
[Out] Integral(log(f*x**m)/(a + b*log(c*(d + e*x)**n)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{\log(fx^m)}{b\log((ex+d)^n c)+a}\,dx$$

```
[In] integrate(log(f*x^m)/(a+b*log(c*(e*x+d)^n)),x, algorithm="maxima")
[Out] integrate(log(f*x^m)/(b*log((e*x + d)^n*c) + a), x)
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{\log(fx^m)}{b\log((ex+d)^n c)+a}\,dx$$

```
[In] integrate(log(f*x^m)/(a+b*log(c*(e*x+d)^n)),x, algorithm="giac")
[Out] integrate(log(f*x^m)/(b*log((e*x + d)^n*c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{a+b\log(c(d+ex)^n)}\,dx = \int \frac{\ln(f\,x^m)}{a+b\,\ln(c\,(d+e\,x)^n)}\,dx$$

```
[In] int(log(f*x^m)/(a + b*log(c*(d + e*x)^n)),x)
[Out] int(log(f*x^m)/(a + b*log(c*(d + e*x)^n)), x)
```

# 3.376 $\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx = \text{Int}\left(\frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2},x\right)$$

```
[Out] Unintegrable(ln(f*x^m)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx$$

```
[In] Int[Log[f*x^m]/(a + b*Log[c*(d + e*x)^n])^2,x]
[Out] Defer[Int][Log[f*x^m]/(a + b*Log[c*(d + e*x)^n])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx$$

```
[In] Integrate[Log[f*x^m]/(a + b*Log[c*(d + e*x)^n])^2,x]
[Out] Integrate[Log[f*x^m]/(a + b*Log[c*(d + e*x)^n])^2, x]
```

## Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\ln(f\,x^m)}{(a+b\ln(c\,(ex+d)^n))^2}dx$$

```
[In] int(ln(f*x^m)/(a+b*ln(c*(e*x+d)^n))^2,x)
[Out] int(ln(f*x^m)/(a+b*ln(c*(e*x+d)^n))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int \frac{\log(fx^m)}{(a+b\log(c(d+ex)^n))^2}\,dx = \int \frac{\log(fx^m)}{(b\log((ex+d)^n c)+a)^2}\,dx$$

```
[In] integrate(log(f*x^m)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="fricas")
[Out] integral(log(f*x^m)/(b^2*log((e*x + d)^n*c)^2 + 2*a*b*log((e*x + d)^n*c) +
a^2), x)
```

## Sympy [N/A]

Not integrable

Time = 31.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\log (fx^m)}{(a+b\log (c(d+ex)^n))^2}\,dx=\int \frac{\log (fx^m)}{(a+b\log (c\,(d+ex)^n))^2}\,dx$$

```
[In] integrate(ln(f*x**m)/(a+b*ln(c*(e*x+d)**n))**2,x)
```

```
[Out] Integral(log(f*x**m)/(a + b*log(c*(d + e*x)**n))**2, x)
```

## Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.70

$$\int \frac{\log (fx^m)}{(a+b\log (c(d+ex)^n))^2}\,dx=\int \frac{\log (fx^m)}{(b\log ((ex+d)^nc)+a)^2}\,dx$$

```
[In] integrate(log(f*x^m)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="maxima")
```

```
[Out] -(e*x*log(f) + d*log(f) + (e*x + d)*log(x^m))/(b^2*e*n*log((e*x + d)^n) + b
^2*e*n*log(c) + a*b*e*n) + integrate((e*(m + log(f))*x + e*x*log(x^m) + d*m
)/(b^2*e*n*x*log((e*x + d)^n) + (b^2*e*n*log(c) + a*b*e*n)*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log (fx^m)}{(a+b\log (c(d+ex)^n))^2}\,dx=\int \frac{\log (fx^m)}{(b\log ((ex+d)^nc)+a)^2}\,dx$$

```
[In] integrate(log(f*x^m)/(a+b*log(c*(e*x+d)^n))^2,x, algorithm="giac")
```

```
[Out] integrate(log(f*x^m)/(b*log((e*x + d)^n*c) + a)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log (fx^m)}{(a+b\log (c(d+ex)^n))^2}\,dx=\int \frac{\ln \left(f\,x^m\right)}{{\left(a+b\,\ln \left(c\,{\left(d+e\,x\right)}^n\right)\right)}^2}\,dx$$

```
[In] int(log(f*x^m)/(a + b*log(c*(d + e*x)^n))^2,x)
[Out] int(log(f*x^m)/(a + b*log(c*(d + e*x)^n))^2, x)
```

# 3.377 $\int \log(fx^m)\,(a+b\log(c(d+ex)^n))^p\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \log(fx^m)\,(a+b\log(c(d+ex)^n))^p\,dx=\text{Int}(\log(fx^m)\,(a+b\log(c(d+ex)^n))^p\,,x)$$

```
[Out] Unintegrable(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^p,x)
```

## Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \log(fx^m)\,(a+b\log(c(d+ex)^n))^p\,dx=\int \log(fx^m)\,(a+b\log(c(d+ex)^n))^p\,dx$$

```
[In] Int[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^p,x]
```

```
[Out] Defer[Int][Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^p, x]
```

Rubi steps

$$\text{integral}=\int \log(fx^m)\,(a+b\log(c(d+ex)^n))^p\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^p\,dx=\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^p\,dx$$

```
[In] Integrate[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^p,x]
[Out] Integrate[Log[f*x^m]*(a + b*Log[c*(d + e*x)^n])^p, x]
```

## Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \ln\left(f\,x^m\right)\left(a+b\ln\left(c(ex+d)^n\right)\right)^p dx$$

```
[In] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^p,x)
[Out] int(ln(f*x^m)*(a+b*ln(c*(e*x+d)^n))^p,x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^p\,dx=\int \left(b\log\left((ex+d)^nc\right)+a\right)^p\log(fx^m)\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^p,x, algorithm="fricas")
[Out] integral((b*log((e*x + d)^n*c) + a)^p*log(f*x^m), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^p\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(ln(f*x**m)*(a+b*ln(c*(e*x+d)**n))**p,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [F(-2)]

Exception generated.

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^p\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^p,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

## Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^p\,dx = \int \left(b\log\left((ex+d)^n c\right)+a\right)^p \log(fx^m)\,dx$$

```
[In] integrate(log(f*x^m)*(a+b*log(c*(e*x+d)^n))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^p*log(f*x^m), x)
```

## Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \log(fx^m)\left(a+b\log\left(c(d+ex)^n\right)\right)^p\,dx = \int \ln\left(f\,x^m\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^p\,dx$$

```
[In] int(log(f*x^m)*(a + b*log(c*(d + e*x)^n))^p,x)
```

```
[Out] int(log(f*x^m)*(a + b*log(c*(d + e*x)^n))^p, x)
```

# 3.378 $\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 364

$$\begin{aligned}\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx &= \log\left(-\frac{bx}{a}\right)\log(a+bx)\log(c+dx)\\
&\quad+\frac{1}{2}\left(\log\left(-\frac{bx}{a}\right)+\log\left(\frac{bc-ad}{b(c+dx)}\right)\right.\\
&\quad\left.-\log\left(-\frac{(bc-ad)x}{a(c+dx)}\right)\right)\log^2\left(\frac{a(c+dx)}{c(a+bx)}\right)-\frac{1}{2}\left(\log\left(-\frac{bx}{a}\right)\right.\\
&\quad\left.-\log\left(-\frac{dx}{c}\right)\right)\left(\log(a+bx)+\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)^2\\
&\quad+\left(\log(c+dx)-\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right)\\
&\quad+\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{c(a+bx)}{a(c+dx)}\right)\\
&\quad-\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\\
&\quad+\left(\log(a+bx)+\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)\operatorname{PolyLog}\left(2,1+\frac{dx}{c}\right)\\
&\quad-\operatorname{PolyLog}\left(3,1+\frac{bx}{a}\right)+\operatorname{PolyLog}\left(3,\frac{c(a+bx)}{a(c+dx)}\right)\\
&\quad-\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)-\operatorname{PolyLog}\left(3,1+\frac{dx}{c}\right)\end{aligned}$$

```
[Out] ln(-b*x/a)*ln(b*x+a)*ln(d*x+c)+1/2*(ln(-b*x/a)+ln((-a*d+b*c)/b/(d*x+c))-ln(
-(-a*d+b*c)*x/a/(d*x+c)))*ln(a*(d*x+c)/c/(b*x+a))^2-1/2*(ln(-b*x/a)-ln(-d*x
/c))*(ln(b*x+a)+ln(a*(d*x+c)/c/(b*x+a)))^2+(ln(d*x+c)-ln(a*(d*x+c)/c/(b*x+a
)))*polylog(2,1+b*x/a)+ln(a*(d*x+c)/c/(b*x+a))*polylog(2,c*(b*x+a)/a/(d*x+c
```

```
))-ln(a*(d*x+c)/c/(b*x+a))*polylog(2,d*(b*x+a)/b/(d*x+c))+(ln(b*x+a)+ln(a*(
d*x+c)/c/(b*x+a)))*polylog(2,1+d*x/c)-polylog(3,1+b*x/a)+polylog(3,c*(b*x+a
)/a/(d*x+c))-polylog(3,d*(b*x+a)/b/(d*x+c))-polylog(3,1+d*x/c)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2485}

$$\begin{aligned}\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx &= \text{PolyLog}\left(3,\frac{c(a+bx)}{a(c+dx)}\right)-\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)\\ &+\text{PolyLog}\left(2,\frac{c(a+bx)}{a(c+dx)}\right)\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\\ &-\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\\ &+\text{PolyLog}\left(2,\frac{bx}{a}+1\right)\left(\log(c+dx)-\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)\\ &+\text{PolyLog}\left(2,\frac{dx}{c}+1\right)\left(\log\left(\frac{a(c+dx)}{c(a+bx)}\right)+\log(a+bx)\right)\\ &+\frac{1}{2}\left(\log\left(\frac{bc-ad}{b(c+dx)}\right)-\log\left(-\frac{x(bc-ad)}{a(c+dx)}\right)\right.\\ &\qquad\left.+\log\left(-\frac{bx}{a}\right)\right)\log^2\left(\frac{a(c+dx)}{c(a+bx)}\right)\\ &-\frac{1}{2}\left(\log\left(-\frac{bx}{a}\right)-\log\left(-\frac{dx}{c}\right)\right)\left(\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right.\\ &\qquad\left.+\log(a+bx)\right)^2+\log\left(-\frac{bx}{a}\right)\log(a+bx)\log(c+dx)\\ &-\text{PolyLog}\left(3,\frac{bx}{a}+1\right)-\text{PolyLog}\left(3,\frac{dx}{c}+1\right)\end{aligned}$$

```
[In] Int[(Log[a + b*x]*Log[c + d*x])/x,x]
```

```
[Out] Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x] + ((Log[-((b*x)/a)] + Log[(b*c -
a*d)/(b*(c + d*x))] - Log[-(((b*c - a*d)*x)/(a*(c + d*x)))])*Log[(a*(c + d*
x))/(c*(a + b*x))]^2)/2 - ((Log[-((b*x)/a)] - Log[-((d*x)/c)])*(Log[a + b*x
] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2 + (Log[c + d*x] - Log[(a*(c + d*
x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a] + Log[(a*(c + d*x))/(c*(a + b*x
))]*PolyLog[2, (c*(a + b*x))/(a*(c + d*x))] - Log[(a*(c + d*x))/(c*(a + b*x
))]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] + (Log[a + b*x] + Log[(a*(c + d
*x))/(c*(a + b*x))])*PolyLog[2, 1 + (d*x)/c] - PolyLog[3, 1 + (b*x)/a] + Po
lyLog[3, (c*(a + b*x))/(a*(c + d*x))] - PolyLog[3, (d*(a + b*x))/(b*(c + d*
x))] - PolyLog[3, 1 + (d*x)/c]
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} = {} & \log\left(-\frac{bx}{a}\right)\log(a+bx)\log(c+dx) \\
& + \frac{1}{2}\left(\log\left(-\frac{bx}{a}\right)+\log\left(\frac{bc-ad}{b(c+dx)}\right)-\log\left(-\frac{(bc-ad)x}{a(c+dx)}\right)\right)\log^2\left(\frac{a(c+dx)}{c(a+bx)}\right) \\
& - \frac{1}{2}\left(\log\left(-\frac{bx}{a}\right)-\log\left(-\frac{dx}{c}\right)\right)\left(\log(a+bx)+\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)^2 \\
& + \left(\log(c+dx)-\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)\mathrm{Li}_2\left(1+\frac{bx}{a}\right) \\
& + \log\left(\frac{a(c+dx)}{c(a+bx)}\right)\mathrm{Li}_2\left(\frac{c(a+bx)}{a(c+dx)}\right)-\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right) \\
& + \left(\log(a+bx)+\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)\mathrm{Li}_2\left(1+\frac{dx}{c}\right)-\mathrm{Li}_3\left(1+\frac{bx}{a}\right) \\
& + \mathrm{Li}_3\left(\frac{c(a+bx)}{a(c+dx)}\right)-\mathrm{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)-\mathrm{Li}_3\left(1+\frac{dx}{c}\right)
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.08

$$\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx = \log\left(-\frac{bx}{a}\right)\log(a+bx)\log(c+dx) + \frac{1}{2}\log^2\left(\frac{a(c+dx)}{c(a+bx)}\right)\left(\log\left(-\frac{bx}{a}\right)+\log\left(\frac{-bc+ad}{d(a+bx)}\right)-\log\left(\frac{bcx-adx}{ac+bcx}\right)\right) + \left(-\log\left(-\frac{bx}{a}\right)+\log\left(-\frac{dx}{c}\right)\right)\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\log\left(1+\frac{dx}{c}\right) + \frac{1}{2}\left(\log\left(-\frac{bx}{a}\right)-\log\left(-\frac{dx}{c}\right)\right)\log\left(1+\frac{dx}{c}\right)\left(-2\log(a+bx)+\log\left(1+\frac{dx}{c}\right)\right) + \left(\log(c+dx)-\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)\mathrm{PolyLog}\left(2,1+\frac{bx}{a}\right) + \log\left(\frac{a(c+dx)}{c(a+bx)}\right)\left(-\mathrm{PolyLog}\left(2,\frac{a(c+dx)}{c(a+bx)}\right)+\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\right) + \left(\log(a+bx)+\log\left(\frac{a(c+dx)}{c(a+bx)}\right)\right)\mathrm{PolyLog}\left(2,1+\frac{dx}{c}\right) - \mathrm{PolyLog}\left(3,1+\frac{bx}{a}\right)+\mathrm{PolyLog}\left(3,\frac{a(c+dx)}{c(a+bx)}\right) - \mathrm{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)-\mathrm{PolyLog}\left(3,1+\frac{dx}{c}\right)$$

```
[In] Integrate[(Log[a + b*x]*Log[c + d*x])/x,x]
```

```
[Out] Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x] + (Log[(a*(c + d*x))/(c*(a + b*x)
)]^2*(Log[-((b*x)/a)] + Log[(-(b*c) + a*d)/(d*(a + b*x))] - Log[(b*c*x - a*
d*x)/(a*c + b*c*x)]))/2 + (-Log[-((b*x)/a)] + Log[-((d*x)/c)])*Log[(a*(c +
d*x))/(c*(a + b*x))]*Log[1 + (d*x)/c] + ((Log[-((b*x)/a)] - Log[-((d*x)/c)]
)*Log[1 + (d*x)/c]*(-2*Log[a + b*x] + Log[1 + (d*x)/c]))/2 + (Log[c + d*x]
- Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a] + Log[(a*(c + d
*x))/(c*(a + b*x))]*(-PolyLog[2, (a*(c + d*x))/(c*(a + b*x))] + PolyLog[2,
(b*(c + d*x))/(d*(a + b*x))]) + (Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b
*x))])*PolyLog[2, 1 + (d*x)/c] - PolyLog[3, 1 + (b*x)/a] + PolyLog[3, (a*(c
 + d*x))/(c*(a + b*x))] - PolyLog[3, (b*(c + d*x))/(d*(a + b*x))] - PolyLog
[3, 1 + (d*x)/c]
```

## Maple [F]

$$\int \frac{\ln(bx+a)\ln(dx+c)}{x}dx$$

```
[In] int(ln(b*x+a)*ln(d*x+c)/x,x)
[Out] int(ln(b*x+a)*ln(d*x+c)/x,x)
```

## Fricas [F]

$$\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx = \int \frac{\log(bx+a)\log(dx+c)}{x}\,dx$$

```
[In] integrate(log(b*x+a)*log(d*x+c)/x,x, algorithm="fricas")
[Out] integral(log(b*x + a)*log(d*x + c)/x, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx = \text{Timed out}$$

```
[In] integrate(ln(b*x+a)*ln(d*x+c)/x,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx = \int \frac{\log(bx+a)\log(dx+c)}{x}\,dx$$

```
[In] integrate(log(b*x+a)*log(d*x+c)/x,x, algorithm="maxima")
[Out] integrate(log(b*x + a)*log(d*x + c)/x, x)
```

## Giac [F]

$$\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx = \int \frac{\log\left(bx+a\right)\log\left(dx+c\right)}{x}\,dx$$

```
[In] integrate(log(b*x+a)*log(d*x+c)/x,x, algorithm="giac")
[Out] integrate(log(b*x + a)*log(d*x + c)/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(a+bx)\log(c+dx)}{x}\,dx = \int \frac{\ln\left(a+b\,x\right)\,\ln\left(c+d\,x\right)}{x}\,dx$$

```
[In] int((log(a + b*x)*log(c + d*x))/x,x)
[Out] int((log(a + b*x)*log(c + d*x))/x, x)
```

# 3.379 $\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,d$



## Optimal result

Integrand size = 32, antiderivative size = 258

$$
\begin{aligned}
&\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx\\
&=\frac{2bd^2gn^2x}{e^2}-\frac{bdgn^2(d+ex)^2}{2e^3}+\frac{2bgn^2(d+ex)^3}{27e^3}-\frac{bd^3gn^2\log^2(d+ex)}{3e^3}\\
&\quad+\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\\
&\quad-\frac{d^2n(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{e^3}\\
&\quad+\frac{dn(d+ex)^2(bf+ag+2bg\log(c(d+ex)^n))}{2e^3}\\
&\quad-\frac{n(d+ex)^3(bf+ag+2bg\log(c(d+ex)^n))}{9e^3}\\
&\quad+\frac{d^3n\log(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{3e^3}
\end{aligned}
$$

```
[Out] 2*b*d^2*g*n^2*x/e^2-1/2*b*d*g*n^2*(e*x+d)^2/e^3+2/27*b*g*n^2*(e*x+d)^3/e^3-
1/3*b*d^3*g*n^2*ln(e*x+d)^2/e^3+1/3*x^3*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*
x+d)^n))-d^2*n*(e*x+d)*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))/e^3+1/2*d*n*(e*x+d)^
2*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))/e^3-1/9*n*(e*x+d)^3*(b*f+a*g+2*b*g*ln(c*(
e*x+d)^n))/e^3+1/3*d^3*n*ln(e*x+d)*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))/e^3
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2483, 2458, 45, 2372, 12, 14, 2338}

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$=\frac{d^3n\log(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)}{3e^3}-\frac{d^2n(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)}{e^3}+\frac{dn(d+ex)^2(ag+2bg\log(c(d+ex)^n)+bf)}{2e^3}-\frac{n(d+ex)^3(ag+2bg\log(c(d+ex)^n)+bf)}{9e^3}+\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)-\frac{bd^3gn^2\log^2(d+ex)}{3e^3}+\frac{2bd^2gn^2x}{e^2}-\frac{bdgn^2(d+ex)^2}{2e^3}+\frac{2bgn^2(d+ex)^3}{27e^3}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]),x]
```

```
[Out] (2*b*d^2*g*n^2*x)/e^2 - (b*d*g*n^2*(d + e*x)^2)/(2*e^3) + (2*b*g*n^2*(d + e
*x)^3)/(27*e^3) - (b*d^3*g*n^2*Log[d + e*x]^2)/(3*e^3) + (x^3*(a + b*Log[c*
(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/3 - (d^2*n*(d + e*x)*(b*f + a*g +
 2*b*g*Log[c*(d + e*x)^n]))/e^3 + (d*n*(d + e*x)^2*(b*f + a*g + 2*b*g*Log[c
*(d + e*x)^n]))/(2*e^3) - (n*(d + e*x)^3*(b*f + a*g + 2*b*g*Log[c*(d + e*x)
^n]))/(9*e^3) + (d^3*n*Log[d + e*x]*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))
/(3*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Dist[
e*(n/(m + 1)), Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) \\
&\quad -\frac{1}{3}(en)\int\frac{x^3(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}\,dx \\
&= \frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) \\
&\quad -\frac{1}{3}n\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3(bf+ag+2bg\log(cx^n))}{x}\,dx,x,d+ex\right)
\end{aligned}$$

$$= \frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) - \frac{d^2n(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{e^3} + \frac{dn(d+ex)^2(bf+ag+2bg\log(c(d+ex)^n))}{2e^3} - \frac{n(d+ex)^3(bf+ag+2bg\log(c(d+ex)^n))}{9e^3} + \frac{d^3n\log(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{3e^3} + \frac{1}{3}(2bgn^2)\operatorname{Subst}\left(\int\frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{6e^3x}\,dx,x,d+ex\right)$$

$$= \frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) - \frac{d^2n(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{e^3} + \frac{dn(d+ex)^2(bf+ag+2bg\log(c(d+ex)^n))}{2e^3} - \frac{n(d+ex)^3(bf+ag+2bg\log(c(d+ex)^n))}{9e^3} + \frac{d^3n\log(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{3e^3} + \frac{(bgn^2)\operatorname{Subst}\left(\int\frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{x}\,dx,x,d+ex\right)}{9e^3}$$

$$= \frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) - \frac{d^2n(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{e^3} + \frac{dn(d+ex)^2(bf+ag+2bg\log(c(d+ex)^n))}{2e^3} - \frac{n(d+ex)^3(bf+ag+2bg\log(c(d+ex)^n))}{9e^3} + \frac{d^3n\log(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{3e^3} + \frac{(bgn^2)\operatorname{Subst}\left(\int\left(18d^2-9dx+2x^2-\frac{6d^3\log(x)}{x}\right)dx,x,d+ex\right)}{9e^3}$$

$$= \frac{2bd^2gn^2x}{e^2} - \frac{bdgn^2(d+ex)^2}{2e^3} + \frac{2bgn^2(d+ex)^3}{27e^3} + \frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) - \frac{d^2n(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{e^3} + \frac{dn(d+ex)^2(bf+ag+2bg\log(c(d+ex)^n))}{2e^3} - \frac{n(d+ex)^3(bf+ag+2bg\log(c(d+ex)^n))}{9e^3} + \frac{d^3n\log(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{3e^3} - \frac{(2bd^3gn^2)\,\mathrm{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, d+ex\right)}{3e^3}$$

$$= \frac{2bd^2gn^2x}{e^2} - \frac{bdgn^2(d+ex)^2}{2e^3} + \frac{2bgn^2(d+ex)^3}{27e^3} - \frac{bd^3gn^2\log^2(d+ex)}{3e^3} + \frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) - \frac{d^2n(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{e^3} + \frac{dn(d+ex)^2(bf+ag+2bg\log(c(d+ex)^n))}{2e^3} - \frac{n(d+ex)^3(bf+ag+2bg\log(c(d+ex)^n))}{9e^3} + \frac{d^3n\log(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{3e^3}$$

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.79

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= \frac{ex(3a(-6d^2gn+3degnx+2e^2(3f-gn)x^2)+bn(d^2(-18f+66gn)+3de(3f-5gn)x+2e^2(-3f+2gn)x}{}$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]),x]
```

```
[Out] (e*x*(3*a*(-6*d^2*g*n + 3*d*e*g*n*x + 2*e^2*(3*f - g*n)*x^2) + b*n*(d^2*(-1
8*f + 66*g*n) + 3*d*e*(3*f - 5*g*n)*x + 2*e^2*(-3*f + 2*g*n)*x^2)) + 18*d^3
*(b*f + a*g)*n*Log[d + e*x] - 6*(-3*a*e^3*g*x^3 + b*(11*d^3*g*n + 6*d^2*e*g
*n*x - 3*d*e^2*g*n*x^2 + e^3*(-3*f + 2*g*n)*x^3))*Log[c*(d + e*x)^n] + 18*b
*g*(d^3 + e^3*x^3)*Log[c*(d + e*x)^n]^2)/(54*e^3)
```

## Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.37

| method | result |
|---|---|
| parallelrisch | $\frac{18a\,e^3 f\,x^3-66b\,d^3 g\,n^2-12x^3\ln(c(ex+d)^n)b\,e^3 gn-102\ln(ex+d)b\,d^3 g\,n^2+18\ln(ex+d)a\,d^3 gn+18\ln(ex+d)b\,d^3 fn-15bd\,e^2 g\,n}{}$ |
| risch | Expression too large to display |

```
[In] int(x^2*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/54*(18*a*e^3*f*x^3-66*b*d^3*g*n^2-12*x^3*ln(c*(e*x+d)^n)*b*e^3*g*n-102*ln
(e*x+d)*b*d^3*g*n^2+18*ln(e*x+d)*a*d^3*g*n+18*ln(e*x+d)*b*d^3*f*n-15*b*d*e^
2*g*n^2*x^2+66*b*d^2*e*g*n^2*x+18*a*d^3*g*n+18*b*d^3*f*n+4*b*e^3*g*n^2*x^3-
6*n*a*e^3*g*x^3-6*n*b*e^3*f*x^3-36*x*ln(c*(e*x+d)^n)*b*d^2*e*g*n+18*x^2*ln(
c*(e*x+d)^n)*b*d*e^2*g*n+18*x^3*ln(c*(e*x+d)^n)^2*b*e^3*g+18*x^3*ln(c*(e*x+
d)^n)*a*e^3*g+18*x^3*ln(c*(e*x+d)^n)*b*e^3*f+36*ln(c*(e*x+d)^n)*b*d^3*g*n+1
8*ln(c*(e*x+d)^n)^2*b*d^3*g-18*a*d^2*e*g*n*x-18*b*d^2*e*f*n*x+9*a*d*e^2*g*n
*x^2+9*b*d*e^2*f*n*x^2)/e^3
```

## Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.28

$$\int x^2(a+b\log(c(d+ex)^n))\,(f+g\log(c(d+ex)^n))\;dx = \frac{18\,be^3gx^3\log(c)^2+2\,(2\,be^3gn^2+9\,ae^3f-3\,(be^3f+ae^3g)n)x^3-3\,(5\,bde^2gn^2-3\,(bde^2f+ade^2g)n)x^2+}{}$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="f
ricas")
```

```
[Out] 1/54*(18*b*e^3*g*x^3*log(c)^2 + 2*(2*b*e^3*g*n^2 + 9*a*e^3*f - 3*(b*e^3*f +
 a*e^3*g)*n)*x^3 - 3*(5*b*d*e^2*g*n^2 - 3*(b*d*e^2*f + a*d*e^2*g)*n)*x^2 +
18*(b*e^3*g*n^2*x^3 + b*d^3*g*n^2)*log(e*x + d)^2 + 6*(11*b*d^2*e*g*n^2 - 3
*(b*d^2*e*f + a*d^2*e*g)*n)*x + 6*(3*b*d*e^2*g*n^2*x^2 - 6*b*d^2*e*g*n^2*x
- 11*b*d^3*g*n^2 - (2*b*e^3*g*n^2 - 3*(b*e^3*f + a*e^3*g)*n)*x^3 + 3*(b*d^3
*f + a*d^3*g)*n + 6*(b*e^3*g*n*x^3 + b*d^3*g*n)*log(c))*log(e*x + d) + 6*(3
*b*d*e^2*g*n*x^2 - 6*b*d^2*e*g*n*x - (2*b*e^3*g*n - 3*b*e^3*f - 3*a*e^3*g)*
x^3)*log(c))/e^3
```

## Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.49

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$=\begin{cases}\frac{ad^3g\log(c(d+ex)^n)}{3e^3}-\frac{ad^2gnx}{3e^2}+\frac{adgnx^2}{6e}+\frac{afx^3}{3}-\frac{agnx^3}{9}+\frac{agx^3\log(c(d+ex)^n)}{3}+\frac{bd^3f\log(c(d+ex)^n)}{3e^3}-\frac{11bd^3gn\log(c(d+ex)^n)}{9e^3}\\ \frac{x^3(a+b\log(cd^n))(f+g\log(cd^n))}{3}\end{cases}$$

```
[In] integrate(x**2*(a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n)),x)
```

```
[Out] Piecewise((a*d**3*g*log(c*(d + e*x)**n)/(3*e**3) - a*d**2*g*n*x/(3*e**2) +
a*d*g*n*x**2/(6*e) + a*f*x**3/3 - a*g*n*x**3/9 + a*g*x**3*log(c*(d + e*x)**
n)/3 + b*d**3*f*log(c*(d + e*x)**n)/(3*e**3) - 11*b*d**3*g*n*log(c*(d + e*x
)**n)/(9*e**3) + b*d**3*g*log(c*(d + e*x)**n)**2/(3*e**3) - b*d**2*f*n*x/(3
*e**2) + 11*b*d**2*g*n**2*x/(9*e**2) - 2*b*d**2*g*n*x*log(c*(d + e*x)**n)/(
3*e**2) + b*d*f*n*x**2/(6*e) - 5*b*d*g*n**2*x**2/(18*e) + b*d*g*n*x**2*log(
c*(d + e*x)**n)/(3*e) - b*f*n*x**3/9 + b*f*x**3*log(c*(d + e*x)**n)/3 + 2*b
*g*n**2*x**3/27 - 2*b*g*n*x**3*log(c*(d + e*x)**n)/9 + b*g*x**3*log(c*(d +
e*x)**n)**2/3, Ne(e, 0)), (x**3*(a + b*log(c*d**n))*(f + g*log(c*d**n))/3,
True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.06

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$=\frac{1}{3}bgx^3\log((ex+d)^nc)^2+\frac{1}{3}bfx^3\log((ex+d)^nc)+\frac{1}{3}agx^3\log((ex+d)^nc)$$
$$+\frac{1}{3}afx^3+\frac{1}{18}befn\left(\frac{6d^3\log(ex+d)}{e^4}-\frac{2e^2x^3-3dex^2+6d^2x}{e^3}\right)$$
$$+\frac{1}{18}aegn\left(\frac{6d^3\log(ex+d)}{e^4}-\frac{2e^2x^3-3dex^2+6d^2x}{e^3}\right)$$
$$+\frac{1}{54}\left(6en\left(\frac{6d^3\log(ex+d)}{e^4}-\frac{2e^2x^3-3dex^2+6d^2x}{e^3}\right)\log((ex+d)^nc)+\frac{(4e^3x^3-15de^2x^2-18d^3\log}{}\right.$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="m
axima")
```

```
[Out] 1/3*b*g*x^3*log((e*x + d)^n*c)^2 + 1/3*b*f*x^3*log((e*x + d)^n*c) + 1/3*a*g
*x^3*log((e*x + d)^n*c) + 1/3*a*f*x^3 + 1/18*b*e*f*n*(6*d^3*log(e*x + d)/e^
```

```
4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) + 1/18*a*e*g*n*(6*d^3*log(e*x +
d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) + 1/54*(6*e*n*(6*d^3*log(e*
x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3)*log((e*x + d)^n*c) + (4
*e^3*x^3 - 15*d*e^2*x^2 - 18*d^3*log(e*x + d)^2 + 66*d^2*e*x - 66*d^3*log(e
*x + d))*n^2/e^3)*b*g
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(244) = 488$.

Time = 0.31 (sec) , antiderivative size = 741, normalized size of antiderivative = 2.87

$$\begin{aligned}
&\int x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx \\
&= \frac{(ex+d)^3bgn^2\log(ex+d)^2}{3e^3} - \frac{(ex+d)^2bdgn^2\log(ex+d)^2}{e^3} \\
&+ \frac{(ex+d)bd^2gn^2\log(ex+d)^2}{e^3} - \frac{2(ex+d)^3bgn^2\log(ex+d)}{9e^3} \\
&+ \frac{(ex+d)^2bdgn^2\log(ex+d)}{e^3} - \frac{2(ex+d)bd^2gn^2\log(ex+d)}{e^3} \\
&+ \frac{2(ex+d)^3bgn\log(ex+d)\log(c)}{3e^3} - \frac{2(ex+d)^2bdgn\log(ex+d)\log(c)}{e^3} \\
&+ \frac{2(ex+d)bd^2gn\log(ex+d)\log(c)}{e^3} + \frac{2(ex+d)^3bgn^2}{27e^3} - \frac{(ex+d)^2bdgn^2}{2e^3} \\
&+ \frac{2(ex+d)bd^2gn^2}{e^3} + \frac{(ex+d)^3bfn\log(ex+d)}{3e^3} - \frac{(ex+d)^2bdfn\log(ex+d)}{e^3} \\
&+ \frac{(ex+d)bd^2fn\log(ex+d)}{e^3} + \frac{(ex+d)^3agn\log(ex+d)}{3e^3} - \frac{(ex+d)^2adgn\log(ex+d)}{e^3} \\
&+ \frac{(ex+d)ad^2gn\log(ex+d)}{e^3} - \frac{2(ex+d)^3bgn\log(c)}{9e^3} + \frac{(ex+d)^2bdgn\log(c)}{e^3} \\
&- \frac{2(ex+d)bd^2gn\log(c)}{e^3} + \frac{(ex+d)^3bg\log(c)^2}{3e^3} - \frac{(ex+d)^2bdg\log(c)^2}{e^3} \\
&+ \frac{(ex+d)bd^2g\log(c)^2}{e^3} - \frac{(ex+d)^3bfn}{9e^3} + \frac{(ex+d)^2bdfn}{2e^3} - \frac{(ex+d)bd^2fn}{e^3} \\
&- \frac{(ex+d)^3agn}{9e^3} + \frac{(ex+d)^2adgn}{2e^3} - \frac{(ex+d)ad^2gn}{e^3} + \frac{(ex+d)^3bf\log(c)}{3e^3} \\
&- \frac{(ex+d)^2bdf\log(c)}{e^3} + \frac{(ex+d)bd^2f\log(c)}{e^3} + \frac{(ex+d)^3ag\log(c)}{3e^3} - \frac{(ex+d)^2adg\log(c)}{e^3} \\
&+ \frac{(ex+d)ad^2g\log(c)}{e^3} + \frac{(ex+d)^3af}{3e^3} - \frac{(ex+d)^2adf}{e^3} + \frac{(ex+d)ad^2f}{e^3}
\end{aligned}$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="g
iac")
```

```
[Out] 1/3*(e*x + d)^3*b*g*n^2*log(e*x + d)^2/e^3 - (e*x + d)^2*b*d*g*n^2*log(e*x
+ d)^2/e^3 + (e*x + d)*b*d^2*g*n^2*log(e*x + d)^2/e^3 - 2/9*(e*x + d)^3*b*g
*n^2*log(e*x + d)/e^3 + (e*x + d)^2*b*d*g*n^2*log(e*x + d)/e^3 - 2*(e*x + d
)*b*d^2*g*n^2*log(e*x + d)/e^3 + 2/3*(e*x + d)^3*b*g*n*log(e*x + d)*log(c)/
e^3 - 2*(e*x + d)^2*b*d*g*n*log(e*x + d)*log(c)/e^3 + 2*(e*x + d)*b*d^2*g*n
*log(e*x + d)*log(c)/e^3 + 2/27*(e*x + d)^3*b*g*n^2/e^3 - 1/2*(e*x + d)^2*b
*d*g*n^2/e^3 + 2*(e*x + d)*b*d^2*g*n^2/e^3 + 1/3*(e*x + d)^3*b*f*n*log(e*x
+ d)/e^3 - (e*x + d)^2*b*d*f*n*log(e*x + d)/e^3 + (e*x + d)*b*d^2*f*n*log(e
*x + d)/e^3 + 1/3*(e*x + d)^3*a*g*n*log(e*x + d)/e^3 - (e*x + d)^2*a*d*g*n*
log(e*x + d)/e^3 + (e*x + d)*a*d^2*g*n*log(e*x + d)/e^3 - 2/9*(e*x + d)^3*b
*g*n*log(c)/e^3 + (e*x + d)^2*b*d*g*n*log(c)/e^3 - 2*(e*x + d)*b*d^2*g*n*lo
g(c)/e^3 + 1/3*(e*x + d)^3*b*g*log(c)^2/e^3 - (e*x + d)^2*b*d*g*log(c)^2/e^
3 + (e*x + d)*b*d^2*g*log(c)^2/e^3 - 1/9*(e*x + d)^3*b*f*n/e^3 + 1/2*(e*x +
 d)^2*b*d*f*n/e^3 - (e*x + d)*b*d^2*f*n/e^3 - 1/9*(e*x + d)^3*a*g*n/e^3 + 1
/2*(e*x + d)^2*a*d*g*n/e^3 - (e*x + d)*a*d^2*g*n/e^3 + 1/3*(e*x + d)^3*b*f*
log(c)/e^3 - (e*x + d)^2*b*d*f*log(c)/e^3 + (e*x + d)*b*d^2*f*log(c)/e^3 +
1/3*(e*x + d)^3*a*g*log(c)/e^3 - (e*x + d)^2*a*d*g*log(c)/e^3 + (e*x + d)*a
*d^2*g*log(c)/e^3 + 1/3*(e*x + d)^3*a*f/e^3 - (e*x + d)^2*a*d*f/e^3 + (e*x
+ d)*a*d^2*f/e^3
```

## Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.25

$$\int x^2(a+b\log(c(d+ex)^n))\,(f+g\log(c(d+ex)^n))\;dx$$
$$=\ln\left(c\,(d+e\,x)^n\right)\left(\frac{x^3\left(a\,g+b\,f-\frac{2\,b\,g\,n}{3}\right)}{3}+\frac{x^2\left(\frac{3\,d\,(a\,g+b\,f)}{2\,e}-\frac{d\,(9\,a\,g+9\,b\,f-6\,b\,g\,n)}{6\,e}\right)}{3}-\frac{d\,x\left(\frac{9\,d\,(a\,g+b\,f)}{e}-\frac{d\,(9\,a\,g+9\,b\,f-6\,b\,g\,n)}{e}\right)}{9\,e}\right)+x^2\left(\frac{d\,(3\,a\,f-b\,g\,n^2)}{6\,e}-\frac{d\left(a\,f-\frac{a\,g\,n}{3}-\frac{b\,f\,n}{3}+\frac{2\,b\,g\,n^2}{9}\right)}{2\,e}\right)+\ln\left(c\,(d+e\,x)^n\right)^2\left(\frac{b\,g\,x^3}{3}+\frac{b\,d^3\,g}{3\,e^3}\right)-x\left(\frac{d\left(\frac{d\,(3\,a\,f-b\,g\,n^2)}{3\,e}-\frac{d\left(a\,f-\frac{a\,g\,n}{3}-\frac{b\,f\,n}{3}+\frac{2\,b\,g\,n^2}{9}\right)}{e}\right)}{e}-\frac{2\,b\,d^2\,g\,n^2}{3\,e^2}\right)+x^3\left(\frac{a\,f}{3}-\frac{a\,g\,n}{9}-\frac{b\,f\,n}{9}+\frac{2\,b\,g\,n^2}{27}\right)+\frac{\ln\left(d+e\,x\right)\,\left(3\,a\,d^3\,g\,n+3\,b\,d^3\,f\,n-11\,b\,d^3\,g\,n^2\right)}{9\,e^3}$$

```
[In] int(x^2*(a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)),x)
```

```
[Out] log(c*(d + e*x)^n)*((x^3*(a*g + b*f - (2*b*g*n)/3))/3 + (x^2*((3*d*(a*g + b
*f))/(2*e) - (d*(9*a*g + 9*b*f - 6*b*g*n))/(6*e)))/3 - (d*x*((9*d*(a*g + b*
f))/e - (d*(9*a*g + 9*b*f - 6*b*g*n))/e))/(9*e)) + x^2*((d*(3*a*f - b*g*n^2
))/(6*e) - (d*(a*f - (a*g*n)/3 - (b*f*n)/3 + (2*b*g*n^2)/9))/(2*e)) + log(c
*(d + e*x)^n)^2*((b*g*x^3)/3 + (b*d^3*g)/(3*e^3)) - x*((d*((d*(3*a*f - b*g*
n^2))/(3*e) - (d*(a*f - (a*g*n)/3 - (b*f*n)/3 + (2*b*g*n^2)/9))/e))/e - (2*
b*d^2*g*n^2)/(3*e^2)) + x^3*((a*f)/3 - (a*g*n)/9 - (b*f*n)/9 + (2*b*g*n^2)/
27) + (log(d + e*x)*(3*a*d^3*g*n + 3*b*d^3*f*n - 11*b*d^3*g*n^2))/(9*e^3)
```

# 3.380 $\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 196

$$\begin{aligned}&\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx\\&=-\frac{2bdgn^2x}{e}+\frac{bgn^2(d+ex)^2}{4e^2}+\frac{bd^2gn^2\log^2(d+ex)}{2e^2}\\&\quad+\frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\\&\quad+\frac{dn(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{e^2}\\&\quad-\frac{n(d+ex)^2(bf+ag+2bg\log(c(d+ex)^n))}{4e^2}\\&\quad-\frac{d^2n\log(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{2e^2}\end{aligned}$$

```
[Out] -2*b*d*g*n^2*x/e+1/4*b*g*n^2*(e*x+d)^2/e^2+1/2*b*d^2*g*n^2*ln(e*x+d)^2/e^2+
1/2*x^2*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))+d*n*(e*x+d)*(b*f+a*g+2*
b*g*ln(c*(e*x+d)^n))/e^2-1/4*n*(e*x+d)^2*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))/e^
2-1/2*d^2*n*ln(e*x+d)*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))/e^2
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used

= {2483, 2458, 45, 2372, 12, 14, 2338}

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= -\frac{d^2n\log(d+ex)\,(ag+2bg\log(c(d+ex)^n)+bf)}{2e^2} + \frac{dn(d+ex)\,(ag+2bg\log(c(d+ex)^n)+bf)}{e^2} - \frac{n(d+ex)^2\,(ag+2bg\log(c(d+ex)^n)+bf)}{4e^2} + \frac{1}{2}x^2(a+b\log(c(d+ex)^n))\,(g\log(c(d+ex)^n)+f) + \frac{bd^2gn^2\log^2(d+ex)}{2e^2} + \frac{bgn^2(d+ex)^2}{4e^2} - \frac{2bdgn^2x}{e}$$

```
[In] Int[x*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]),x]
```

```
[Out] (-2*b*d*g*n^2*x)/e + (b*g*n^2*(d + e*x)^2)/(4*e^2) + (b*d^2*g*n^2*Log[d + e
*x]^2)/(2*e^2) + (x^2*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n])
)/2 + (d*n*(d + e*x)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/e^2 - (n*(d +
e*x)^2*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(4*e^2) - (d^2*n*Log[d + e*x
]*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(2*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Dist[
e*(n/(m + 1)), Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) \\
&\quad -\frac{1}{2}(en)\int\frac{x^2(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}\,dx \\
&= \frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) \\
&\quad -\frac{1}{2}n\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^2(bf+ag+2bg\log(cx^n))}{x}\,dx,x,d+ex\right) \\
&= \frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) \\
&\quad +\frac{dn(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{e^2} \\
&\quad -\frac{n(d+ex)^2(bf+ag+2bg\log(c(d+ex)^n))}{4e^2} \\
&\quad -\frac{d^2n\log(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{2e^2} \\
&\quad +(bgn^2)\,\text{Subst}\left(\int\frac{x(-4d+x)+2d^2\log(x)}{2e^2x}\,dx,x,d+ex\right)
\end{aligned}$$

$$= \frac{1}{2}x^2(a + b\log(c(d + ex)^n))(f + g\log(c(d + ex)^n)) + \frac{dn(d + ex)(bf + ag + 2bg\log(c(d + ex)^n))}{e^2} - \frac{n(d + ex)^2(bf + ag + 2bg\log(c(d + ex)^n))}{4e^2} - \frac{d^2n\log(d + ex)(bf + ag + 2bg\log(c(d + ex)^n))}{2e^2} + \frac{(bgn^2)\operatorname{Subst}\left(\int \frac{x(-4d+x)+2d^2\log(x)}{x}\,dx, x, d + ex\right)}{2e^2}$$

$$= \frac{1}{2}x^2(a + b\log(c(d + ex)^n))(f + g\log(c(d + ex)^n)) + \frac{dn(d + ex)(bf + ag + 2bg\log(c(d + ex)^n))}{e^2} - \frac{n(d + ex)^2(bf + ag + 2bg\log(c(d + ex)^n))}{4e^2} - \frac{d^2n\log(d + ex)(bf + ag + 2bg\log(c(d + ex)^n))}{2e^2} + \frac{(bgn^2)\operatorname{Subst}\left(\int \left(-4d + x + \frac{2d^2\log(x)}{x}\right)dx, x, d + ex\right)}{2e^2}$$

$$= -\frac{2bdgn^2x}{e} + \frac{bgn^2(d + ex)^2}{4e^2} + \frac{1}{2}x^2(a + b\log(c(d + ex)^n))(f + g\log(c(d + ex)^n)) + \frac{dn(d + ex)(bf + ag + 2bg\log(c(d + ex)^n))}{e^2} - \frac{n(d + ex)^2(bf + ag + 2bg\log(c(d + ex)^n))}{4e^2} - \frac{d^2n\log(d + ex)(bf + ag + 2bg\log(c(d + ex)^n))}{2e^2} + \frac{(bd^2gn^2)\operatorname{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, d + ex\right)}{e^2}$$

$$= -\frac{2bdgn^2x}{e} + \frac{bgn^2(d + ex)^2}{4e^2} + \frac{bd^2gn^2\log^2(d + ex)}{2e^2} + \frac{1}{2}x^2(a + b\log(c(d + ex)^n))(f + g\log(c(d + ex)^n)) + \frac{dn(d + ex)(bf + ag + 2bg\log(c(d + ex)^n))}{e^2} - \frac{n(d + ex)^2(bf + ag + 2bg\log(c(d + ex)^n))}{4e^2} - \frac{d^2n\log(d + ex)(bf + ag + 2bg\log(c(d + ex)^n))}{2e^2}$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.78

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$=\frac{ex(2adgn+2bdn(f-3gn)+ae(2f-gn)x+ben(-f+gn)x)-2d^2(bf+ag)n\log(d+ex)+2(ae^2gx^2+}{4e^2}$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]),x]
```

```
[Out] (e*x*(2*a*d*g*n + 2*b*d*n*(f - 3*g*n) + a*e*(2*f - g*n)*x + b*e*n*(-f + g*n
)*x) - 2*d^2*(b*f + a*g)*n*Log[d + e*x] + 2*(a*e^2*g*x^2 + b*(3*d^2*g*n + 2
*d*e*g*n*x + e^2*(f - g*n)*x^2))*Log[c*(d + e*x)^n] - 2*b*g*(d^2 - e^2*x^2)
*Log[c*(d + e*x)^n]^2)/(4*e^2)
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.45

| method | result |
|---|---|
| parallelrisch | $-\frac{-2x^2\ln(c(ex+d)^n)^2 b e^2 g+2x^2\ln(c(ex+d)^n) b e^2 gn-x^2e^2bg n^2-10\ln(ex+d)b d^2 g n^2-2x^2\ln(c(ex+d)^n) a e^2 g-2x^2\ln(c(ex+d}{}$ |
| risch | Expression too large to display |

```
[In] int(x*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*x^2*ln(c*(e*x+d)^n)^2*b*e^2*g+2*x^2*ln(c*(e*x+d)^n)*b*e^2*g*n-x^2*
e^2*b*g*n^2-10*ln(e*x+d)*b*d^2*g*n^2-2*x^2*ln(c*(e*x+d)^n)*a*e^2*g-2*x^2*ln
(c*(e*x+d)^n)*b*e^2*f+x^2*e^2*n*a*g+b*e^2*f*n*x^2-4*x*ln(c*(e*x+d)^n)*b*d*e
*g*n+6*x*e*b*d*g*n^2+2*ln(e*x+d)*a*d^2*g*n+2*ln(e*x+d)*b*d^2*f*n-2*a*e^2*f*
x^2-2*a*d*e*g*n*x-2*b*d*e*f*n*x+2*ln(c*(e*x+d)^n)^2*b*d^2*g+4*ln(c*(e*x+d)^
n)*b*d^2*g*n-6*b*d^2*g*n^2+2*a*d^2*g*n+2*d^2*b*f*n)/e^2
```

## Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.31

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$=\frac{2be^2gx^2\log(c)^2+(be^2gn^2+2ae^2f-(be^2f+ae^2g)n)x^2+2(be^2gn^2x^2-bd^2gn^2)\log(ex+d)^2-2(3bdeg}{}$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="fri
cas")
```

```
[Out] 1/4*(2*b*e^2*g*x^2*log(c)^2 + (b*e^2*g*n^2 + 2*a*e^2*f - (b*e^2*f + a*e^2*g
)*n)*x^2 + 2*(b*e^2*g*n^2*x^2 - b*d^2*g*n^2)*log(e*x + d)^2 - 2*(3*b*d*e*g*
n^2 - (b*d*e*f + a*d*e*g)*n)*x + 2*(2*b*d*e*g*n^2*x + 3*b*d^2*g*n^2 - (b*e^
2*g*n^2 - (b*e^2*f + a*e^2*g)*n)*x^2 - (b*d^2*f + a*d^2*g)*n + 2*(b*e^2*g*n
*x^2 - b*d^2*g*n)*log(c))*log(e*x + d) + 2*(2*b*d*e*g*n*x - (b*e^2*g*n - b*
e^2*f - a*e^2*g)*x^2)*log(c))/e^2
```

## Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.51

$$\int x(a+b\log{(c(d+ex)^n)})\,(f+g\log{(c(d+ex)^n)})\,\,dx$$
$$=\begin{cases} -\frac{ad^2g\log{(c(d+ex)^n)}}{2e^2}+\frac{adgnx}{2e}+\frac{afx^2}{2}-\frac{agnx^2}{4}+\frac{agx^2\log{(c(d+ex)^n)}}{2}-\frac{bd^2f\log{(c(d+ex)^n)}}{2e^2}+\frac{3bd^2gn\log{(c(d+ex)^n)}}{2e^2}-\frac{bd^2g\,l}{} \\ \frac{x^2(a+b\log{(cd^n)})(f+g\log{(cd^n)})}{2} \end{cases}$$

```
[In] integrate(x*(a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n)),x)
```

```
[Out] Piecewise((-a*d**2*g*log(c*(d + e*x)**n)/(2*e**2) + a*d*g*n*x/(2*e) + a*f*x
**2/2 - a*g*n*x**2/4 + a*g*x**2*log(c*(d + e*x)**n)/2 - b*d**2*f*log(c*(d +
 e*x)**n)/(2*e**2) + 3*b*d**2*g*n*log(c*(d + e*x)**n)/(2*e**2) - b*d**2*g*l
og(c*(d + e*x)**n)**2/(2*e**2) + b*d*f*n*x/(2*e) - 3*b*d*g*n**2*x/(2*e) + b
*d*g*n*x*log(c*(d + e*x)**n)/e - b*f*n*x**2/4 + b*f*x**2*log(c*(d + e*x)**n
)/2 + b*g*n**2*x**2/4 - b*g*n*x**2*log(c*(d + e*x)**n)/2 + b*g*x**2*log(c*(
d + e*x)**n)**2/2, Ne(e, 0)), (x**2*(a + b*log(c*d**n))*(f + g*log(c*d**n))
/2, True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.14

$$\int x(a+b\log{(c(d+ex)^n)})\,(f+g\log{(c(d+ex)^n)})\,\,dx=\frac{1}{2}\,bgx^2\log{((ex+d)^nc)}^2$$
$$-\frac{1}{4}\,befn\left(\frac{2\,d^2\log{(ex+d)}}{e^3}+\frac{ex^2-2\,dx}{e^2}\right)-\frac{1}{4}\,aegn\left(\frac{2\,d^2\log{(ex+d)}}{e^3}+\frac{ex^2-2\,dx}{e^2}\right)$$
$$+\frac{1}{2}\,bfx^2\log{((ex+d)^nc)}+\frac{1}{2}\,agx^2\log{((ex+d)^nc)}+\frac{1}{2}\,afx^2$$
$$-\frac{1}{4}\left(2\,en\left(\frac{2\,d^2\log{(ex+d)}}{e^3}+\frac{ex^2-2\,dx}{e^2}\right)\log{((ex+d)^nc)}-\frac{\left(e^2x^2+2\,d^2\log{(ex+d)}^2-6\,dex+6\,d^2\,l\right.}{e^2}\right.$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="max
ima")
```

```
[Out] 1/2*b*g*x^2*log((e*x + d)^n*c)^2 - 1/4*b*e*f*n*(2*d^2*log(e*x + d)/e^3 + (e
*x^2 - 2*d*x)/e^2) - 1/4*a*e*g*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/
e^2) + 1/2*b*f*x^2*log((e*x + d)^n*c) + 1/2*a*g*x^2*log((e*x + d)^n*c) + 1/
2*a*f*x^2 - 1/4*(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((
e*x + d)^n*c) - (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x +
 d))*n^2/e^2)*b*g
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(186) = 372$.

Time = 0.32 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.38

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx = \frac{(ex+d)^2bgn^2\log(ex+d)^2}{2e^2} - \frac{(ex+d)bdgn^2\log(ex+d)^2}{e^2} - \frac{(ex+d)^2bgn^2\log(ex+d)}{2e^2} + \frac{2(ex+d)bdgn^2\log(ex+d)}{e^2} + \frac{(ex+d)^2bgn\log(ex+d)\log(c)}{e^2} - \frac{2(ex+d)bdgn\log(ex+d)\log(c)}{e^2} + \frac{(ex+d)^2bgn^2}{4e^2} - \frac{2(ex+d)bdgn^2}{e^2} + \frac{(ex+d)^2bfn\log(ex+d)}{2e^2} - \frac{(ex+d)bdfn\log(ex+d)}{e^2} + \frac{(ex+d)^2agn\log(ex+d)}{2e^2} - \frac{(ex+d)adgn\log(ex+d)}{e^2} - \frac{(ex+d)^2bgn\log(c)}{2e^2} + \frac{2(ex+d)bdgn\log(c)}{e^2} + \frac{(ex+d)^2bg\log(c)^2}{2e^2} - \frac{(ex+d)bdg\log(c)^2}{e^2} - \frac{(ex+d)^2bfn}{4e^2} + \frac{(ex+d)bdfn}{e^2} - \frac{(ex+d)^2agn}{4e^2} + \frac{(ex+d)adgn}{e^2} + \frac{(ex+d)^2bf\log(c)}{2e^2} - \frac{(ex+d)bdf\log(c)}{e^2} + \frac{(ex+d)^2ag\log(c)}{2e^2} - \frac{(ex+d)adg\log(c)}{e^2} + \frac{(ex+d)^2af}{2e^2} - \frac{(ex+d)adf}{e^2}$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="gia
c")
```

```
[Out] 1/2*(e*x + d)^2*b*g*n^2*log(e*x + d)^2/e^2 - (e*x + d)*b*d*g*n^2*log(e*x +
d)^2/e^2 - 1/2*(e*x + d)^2*b*g*n^2*log(e*x + d)/e^2 + 2*(e*x + d)*b*d*g*n^2
*log(e*x + d)/e^2 + (e*x + d)^2*b*g*n*log(e*x + d)*log(c)/e^2 - 2*(e*x + d)
*b*d*g*n*log(e*x + d)*log(c)/e^2 + 1/4*(e*x + d)^2*b*g*n^2/e^2 - 2*(e*x + d
)*b*d*g*n^2/e^2 + 1/2*(e*x + d)^2*b*f*n*log(e*x + d)/e^2 - (e*x + d)*b*d*f*
n*log(e*x + d)/e^2 + 1/2*(e*x + d)^2*a*g*n*log(e*x + d)/e^2 - (e*x + d)*a*d
*g*n*log(e*x + d)/e^2 - 1/2*(e*x + d)^2*b*g*n*log(c)/e^2 + 2*(e*x + d)*b*d*
g*n*log(c)/e^2 + 1/2*(e*x + d)^2*b*g*log(c)^2/e^2 - (e*x + d)*b*d*g*log(c)^
2/e^2 - 1/4*(e*x + d)^2*b*f*n/e^2 + (e*x + d)*b*d*f*n/e^2 - 1/4*(e*x + d)^2
*a*g*n/e^2 + (e*x + d)*a*d*g*n/e^2 + 1/2*(e*x + d)^2*b*f*log(c)/e^2 - (e*x
```

```
+ d)*b*d*f*log(c)/e^2 + 1/2*(e*x + d)^2*a*g*log(c)/e^2 - (e*x + d)*a*d*g*lo
g(c)/e^2 + 1/2*(e*x + d)^2*a*f/e^2 - (e*x + d)*a*d*f/e^2
```

## Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.04

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= x\left(\frac{d\,(a\,f-b\,g\,n^2)}{e}-\frac{d\left(a\,f-\frac{a\,g\,n}{2}-\frac{b\,f\,n}{2}+\frac{b\,g\,n^2}{2}\right)}{e}\right) + \ln\left(c\,(d+e\,x)^n\right)\left(\left(\frac{a\,g}{2}+\frac{b\,f}{2}-\frac{b\,g\,n}{2}\right)x^2+\left(\frac{d\,(a\,g+b\,f)}{e}-\frac{d\,(a\,g+b\,f-b\,g\,n)}{e}\right)x\right) + \ln\left(c\,(d+e\,x)^n\right)^2\left(\frac{b\,g\,x^2}{2}-\frac{b\,d^2\,g}{2\,e^2}\right)+x^2\left(\frac{a\,f}{2}-\frac{a\,g\,n}{4}-\frac{b\,f\,n}{4}+\frac{b\,g\,n^2}{4}\right) - \frac{\ln\left(d+e\,x\right)\,\left(a\,d^2\,g\,n+b\,d^2\,f\,n-3\,b\,d^2\,g\,n^2\right)}{2\,e^2}$$

```
[In] int(x*(a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)),x)

[Out] x*((d*(a*f - b*g*n^2))/e - (d*(a*f - (a*g*n)/2 - (b*f*n)/2 + (b*g*n^2)/2))/
e) + log(c*(d + e*x)^n)*(x*((d*(a*g + b*f))/e - (d*(a*g + b*f - b*g*n))/e)
+ x^2*((a*g)/2 + (b*f)/2 - (b*g*n)/2)) + log(c*(d + e*x)^n)^2*((b*g*x^2)/2
- (b*d^2*g)/(2*e^2)) + x^2*((a*f)/2 - (a*g*n)/4 - (b*f*n)/4 + (b*g*n^2)/4)
- (log(d + e*x)*(a*d^2*g*n + b*d^2*f*n - 3*b*d^2*g*n^2))/(2*e^2)
```

# 3.381 $\int (a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 110

$$\begin{aligned}\int (a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx\\ &= -((bf+ag)nx)+2bgn^2x-\frac{2bgn(d+ex)\log(c(d+ex)^n)}{e}\\ &\quad +x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))+\frac{d(bf+ag+2bg\log(c(d+ex)^n))^2}{4beg}\end{aligned}$$

```
[Out] -(a*g+b*f)*n*x+2*b*g*n^2*x-2*b*g*n*(e*x+d)*ln(c*(e*x+d)^n)/e+x*(a+b*ln(c*(e
*x+d)^n))*(f+g*ln(c*(e*x+d)^n))+1/4*d*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))^2/b/e
/g
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2478, 2458, 2388, 2338, 2332}

$$\begin{aligned}\int (a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx\\ &= \frac{d(ag+2bg\log(c(d+ex)^n)+bf)^2}{4beg}+x(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)\\ &\quad -nx(ag+bf)-\frac{2bgn(d+ex)\log(c(d+ex)^n)}{e}+2bgn^2x\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]),x]
```

```
[Out] -((b*f + a*g)*n*x) + 2*b*g*n^2*x - (2*b*g*n*(d + e*x)*Log[c*(d + e*x)^n])/e
 + x*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]) + (d*(b*f + a*g
+ 2*b*g*Log[c*(d + e*x)^n])^2)/(4*b*e*g)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2478

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c*(d + e
*x)^n])*(f + g*Log[c*(d + e*x)^n]), x] - Dist[e*n, Int[(x*(b*f + a*g + 2*b*
g*Log[c*(d + e*x)^n]))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n},
 x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) \\
&\quad -(en)\int \frac{x(bf+ag+2bg\log(c(d+ex)^n))}{d+ex}\,dx \\
&= x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) \\
&\quad -n\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)(bf+ag+2bg\log(cx^n))}{x}\,dx,x,d+ex\right)
\end{aligned}$$

$$= x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) - \frac{n\text{Subst}\left(\int (bf+ag+2bg\log(cx^n))\,dx, x, d+ex\right)}{e} + \frac{(dn)\text{Subst}\left(\int \frac{bf+ag+2bg\log(cx^n)}{x}\,dx, x, d+ex\right)}{e}$$

$$= -((bf+ag)nx) + x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) + \frac{d(bf+ag+2bg\log(c(d+ex)^n))^2}{4beg} - \frac{(2bgn)\text{Subst}\left(\int \log(cx^n)\,dx, x, d+ex\right)}{e}$$

$$= -((bf+ag)nx) + 2bgn^2x - \frac{2bgn(d+ex)\log(c(d+ex)^n)}{e} + x(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) + \frac{d(bf+ag+2bg\log(c(d+ex)^n))^2}{4beg}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.69

$$\int (a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= \frac{e(a(f-gn)+bn(-f+2gn))x+(ag+b(f-2gn))(d+ex)\log(c(d+ex)^n)+bg(d+ex)\log^2(c(d+ex)^n)}{e}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]),x]
[Out] (e*(a*(f - g*n) + b*n*(-f + 2*g*n))*x + (a*g + b*(f - 2*g*n))*(d + e*x)*Log
[c*(d + e*x)^n] + b*g*(d + e*x)*Log[c*(d + e*x)^n]^2)/e
```

## Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

| method | result |
|---|---|
| norman | $(2bg\,n^2-agn-bfn+af)\,x+(-2bgn+ag+bf)\,x\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)+bgx\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)^2$ - |
| parts | $xaf+(ag+bf)\left(x\ln(c(ex+d)^n)-en\left(\frac{x}{e}-\frac{d\ln(ex+d)}{e^2}\right)\right)+bgx\ln\left(c\,\mathrm{e}^{n\ln(ex+d)}\right)^2+\frac{dgb\ln(c\,\mathrm{e}^{n\ln}}{e}$ |
| default | $xaf+xag\ln(c(ex+d)^n)-agnx+\frac{agnd\ln(ex+d)}{e}+xb\ln(c(ex+d)^n)f-bfnx+\frac{bfnd\ln(ex+d)}{e}$ |
| parallelrisch | $\frac{x\ln(c(ex+d)^n)^2begn-2x\ln(c(ex+d)^n)beg\,n^2+2xbeg\,n^3+x\ln(c(ex+d)^n)aegn+x\ln(c(ex+d)^n)befn-xaeg\,n^2-xbef\,n^2+\ln(c(e}{en}$ |
| risch | Expression too large to display |

```
[In] int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] (2*b*g*n^2-a*g*n-b*f*n+a*f)*x+(-2*b*g*n+a*g+b*f)*x*ln(c*exp(n*ln(e*x+d)))+b
*g*x*ln(c*exp(n*ln(e*x+d)))^2+n*(-2*b*d*g*n+a*d*g+b*d*f)/e*ln(e*x+d)+d*g*b/
e*ln(c*exp(n*ln(e*x+d)))^2
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.42

$$\int (a + b\log(c(d+ex)^n))(f + g\log(c(d+ex)^n))\,dx$$
$$= \frac{begx\log(c)^2 + (begn^2x + bdgn^2)\log(ex+d)^2 - (2\,begn - bef - aeg)x\log(c) + (2\,begn^2 + aef - (bef + \ldots}{}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="frica
s")
```

```
[Out] (b*e*g*x*log(c)^2 + (b*e*g*n^2*x + b*d*g*n^2)*log(e*x + d)^2 - (2*b*e*g*n -
 b*e*f - a*e*g)*x*log(c) + (2*b*e*g*n^2 + a*e*f - (b*e*f + a*e*g)*n)*x - (2
*b*d*g*n^2 - (b*d*f + a*d*g)*n + (2*b*e*g*n^2 - (b*e*f + a*e*g)*n)*x - 2*(b
*e*g*n*x + b*d*g*n)*log(c))*log(e*x + d))/e
```

## Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.72

$$\int (a + b\log(c(d+ex)^n))(f + g\log(c(d+ex)^n))\,dx$$
$$= \begin{cases} \frac{adg\log(c(d+ex)^n)}{e} + afx - agnx + agx\log(c(d+ex)^n) + \frac{bdf\log(c(d+ex)^n)}{e} - \frac{2bdgn\log(c(d+ex)^n)}{e} + \frac{bdg\log(c(d+ex)\ldots}{e} \\ x(a + b\log(cd^n))(f + g\log(cd^n)) \end{cases}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n)),x)
```

```
[Out] Piecewise((a*d*g*log(c*(d + e*x)**n)/e + a*f*x - a*g*n*x + a*g*x*log(c*(d +
 e*x)**n) + b*d*f*log(c*(d + e*x)**n)/e - 2*b*d*g*n*log(c*(d + e*x)**n)/e +
 b*d*g*log(c*(d + e*x)**n)**2/e - b*f*n*x + b*f*x*log(c*(d + e*x)**n) + 2*b
*g*n**2*x - 2*b*g*n*x*log(c*(d + e*x)**n) + b*g*x*log(c*(d + e*x)**n)**2, N
e(e, 0)), (x*(a + b*log(c*d**n))*(f + g*log(c*d**n)), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.50

$$\int (a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= -befn\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)-aegn\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right) + bgx\log((ex+d)^nc)^2+bfx\log((ex+d)^nc)+agx\log((ex+d)^nc) - \left(2\,en\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)\log((ex+d)^nc)+\frac{(d\log(ex+d)^2-2\,ex+2\,d\log(ex+d))n^2}{e}\right)bg + afx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="maxim
a")
```

```
[Out] -b*e*f*n*(x/e - d*log(e*x + d)/e^2) - a*e*g*n*(x/e - d*log(e*x + d)/e^2) +
b*g*x*log((e*x + d)^n*c)^2 + b*f*x*log((e*x + d)^n*c) + a*g*x*log((e*x + d)
^n*c) - (2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c) + (d*log(e*x +
 d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*b*g + a*f*x
```

## Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.90

$$\int (a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))\,dx$$
$$= \frac{(ex+d)bgn^2\log(ex+d)^2}{e}-\frac{2\,(ex+d)bgn^2\log(ex+d)}{e} + \frac{2\,(ex+d)bgn\log(ex+d)\log(c)}{e}+\frac{2\,(ex+d)bgn^2}{e}+\frac{(ex+d)bfn\log(ex+d)}{e} + \frac{(ex+d)agn\log(ex+d)}{e}-\frac{2\,(ex+d)bgn\log(c)}{e}+\frac{(ex+d)bg\log(c)^2}{e} - \frac{(ex+d)bfn}{e}-\frac{(ex+d)agn}{e}+\frac{(ex+d)bf\log(c)}{e}+\frac{(ex+d)ag\log(c)}{e}+\frac{(ex+d)af}{e}$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n)),x, algorithm="giac"
)
```

```
[Out] (e*x + d)*b*g*n^2*log(e*x + d)^2/e - 2*(e*x + d)*b*g*n^2*log(e*x + d)/e + 2
*(e*x + d)*b*g*n*log(e*x + d)*log(c)/e + 2*(e*x + d)*b*g*n^2/e + (e*x + d)*
b*f*n*log(e*x + d)/e + (e*x + d)*a*g*n*log(e*x + d)/e - 2*(e*x + d)*b*g*n*l
og(c)/e + (e*x + d)*b*g*log(c)^2/e - (e*x + d)*b*f*n/e - (e*x + d)*a*g*n/e
+ (e*x + d)*b*f*log(c)/e + (e*x + d)*a*g*log(c)/e + (e*x + d)*a*f/e
```

## Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int (a+b\log (c(d+ex)^n)) (f+g\log (c(d+ex)^n)) \, dx$$
$$= \ln \left(c\,{\left(d+e\,x\right)}^n\right)^2\,\left(b\,g\,x+\frac{b\,d\,g}{e}\right)+x\,\left(a\,f-a\,g\,n-b\,f\,n+2\,b\,g\,n^2\right)$$
$$+x\,\ln \left(c\,{\left(d+e\,x\right)}^n\right)\,\left(a\,g+b\,f-2\,b\,g\,n\right)+\frac{\ln \left(d+e\,x\right)\,\left(a\,d\,g\,n-2\,b\,d\,g\,n^2+b\,d\,f\,n\right)}{e}$$

```
[In] int((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)),x)
```

```
[Out] log(c*(d + e*x)^n)^2*(b*g*x + (b*d*g)/e) + x*(a*f - a*g*n - b*f*n + 2*b*g*n
^2) + x*log(c*(d + e*x)^n)*(a*g + b*f - 2*b*g*n) + (log(d + e*x)*(a*d*g*n -
 2*b*d*g*n^2 + b*d*f*n))/e
```

# 3.382 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 158

$$\begin{aligned}
&\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx \\
&= \log(x)(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n)) \\
&\quad -\frac{\log(x)(bf+ag+2bg\log(c(d+ex)^n))^2}{4bg}+\frac{\log\left(-\frac{ex}{d}\right)(bf+ag+2bg\log(c(d+ex)^n))^2}{4bg} \\
&\quad +n(bf+ag+2bg\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)-2bgn^2\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)
\end{aligned}$$

```
[Out] ln(x)*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))-1/4*ln(x)*(b*f+a*g+2*b*g*
ln(c*(e*x+d)^n))^2/b/g+1/4*ln(-e*x/d)*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))^2/b/g
+n*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))*polylog(2,1+e*x/d)-2*b*g*n^2*polylog(3,1
+e*x/d)
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2482, 2481, 2422, 2354, 2421, 6724}

$$\begin{aligned}
&\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx \\
&= n\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)(ag+2bg\log(c(d+ex)^n)+bf) \\
&\quad -\frac{\log(x)(ag+2bg\log(c(d+ex)^n)+bf)^2}{4bg}+\frac{\log\left(-\frac{ex}{d}\right)(ag+2bg\log(c(d+ex)^n)+bf)^2}{4bg} \\
&\quad +\log(x)(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)-2bgn^2\operatorname{PolyLog}\left(3,\frac{ex}{d}+1\right)
\end{aligned}$$

```
[In] Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x,x]
```

```
[Out] Log[x]*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]) - (Log[x]*(b*f
 + a*g + 2*b*g*Log[c*(d + e*x)^n])^2)/(4*b*g) + (Log[-((e*x)/d)]*(b*f + a*g
 + 2*b*g*Log[c*(d + e*x)^n])^2)/(4*b*g) + n*(b*f + a*g + 2*b*g*Log[c*(d + e
*x)^n])*PolyLog[2, 1 + (e*x)/d] - 2*b*g*n^2*PolyLog[3, 1 + (e*x)/d]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2482

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.
)*((d_) + (e_.)*(x_))^(n_.)]*(g_.)))/(x_), x_Symbol] :> Simp[Log[x]*(a + b*
Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]), x] - Dist[e*n, Int[(Log[x]*
(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d + e*x), x], x] /; FreeQ[{a, b, c
, d, e, f, g, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(f+g\log\left(c(d+ex)^n\right)\right)\\
&\quad-(en)\int\frac{\log(x)\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx\\
&= \log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(f+g\log\left(c(d+ex)^n\right)\right)\\
&\quad-n\text{Subst}\left(\int\frac{\left(bf+ag+2bg\log\left(cx^n\right)\right)\log\left(-\frac{d}{e}+\frac{x}{e}\right)}{x}\,dx,x,d+ex\right)\\
&= \log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(f+g\log\left(c(d+ex)^n\right)\right)\\
&\quad-\frac{\log(x)\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)^2}{4bg}\\
&\quad+\frac{\text{Subst}\left(\int\frac{\left(bf+ag+2bg\log\left(cx^n\right)\right)^2}{-\frac{d}{e}+\frac{x}{e}}\,dx,x,d+ex\right)}{4beg}\\
&= \log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(f+g\log\left(c(d+ex)^n\right)\right)\\
&\quad-\frac{\log(x)\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)^2}{4bg}\\
&\quad+\frac{\log\left(-\frac{ex}{d}\right)\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)^2}{4bg}\\
&\quad-n\text{Subst}\left(\int\frac{\left(bf+ag+2bg\log\left(cx^n\right)\right)\log\left(1-\frac{x}{d}\right)}{x}\,dx,x,d+ex\right)\\
&= \log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(f+g\log\left(c(d+ex)^n\right)\right)\\
&\quad-\frac{\log(x)\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)^2}{4bg}\\
&\quad+\frac{\log\left(-\frac{ex}{d}\right)\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)^2}{4bg}\\
&\quad+n\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)\text{Li}_2\left(1+\frac{ex}{d}\right)\\
&\quad-\left(2bgn^2\right)\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx,x,d+ex\right)
\end{aligned}$$

$$= \log(x)\left(a+b\log\left(c(d+ex)^n\right)\right)\left(f+g\log\left(c(d+ex)^n\right)\right) - \frac{\log(x)\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)^2}{4bg} + \frac{\log\left(-\frac{ex}{d}\right)\left(bf+ag+2bg\log\left(c(d+ex)^n\right)\right)^2}{4bg} + n(bf+ag+2bg\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(1+\frac{ex}{d}\right) - 2bgn^2\operatorname{Li}_3\left(1+\frac{ex}{d}\right)$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.44

$$\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\left(f+g\log\left(c(d+ex)^n\right)\right)}{x}\,dx = af\log(x)+bf\log\left(-\frac{ex}{d}\right)\log\left(c(d+ex)^n\right)+ag\log\left(-\frac{ex}{d}\right)\log\left(c(d+ex)^n\right) + bg\log(x)\left(-n\log(d+ex)+\log\left(c(d+ex)^n\right)\right)^2 + 2bgn(-n\log(d+ex) + \log\left(c(d+ex)^n\right))\left(\log(x)\left(\log(d+ex)-\log\left(1+\frac{ex}{d}\right)\right)-\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)\right) + bfn\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)+agn\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right) + 2bgn^2\left(\frac{1}{2}\log^2(d+ex)\log\left(1-\frac{d+ex}{d}\right)+\log(d+ex)\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right) - \operatorname{PolyLog}\left(3,\frac{d+ex}{d}\right)\right)$$

```
[In] Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x,x]
```

```
[Out] a*f*Log[x] + b*f*Log[-((e*x)/d)]*Log[c*(d + e*x)^n] + a*g*Log[-((e*x)/d)]*L
og[c*(d + e*x)^n] + b*g*Log[x]*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n])^2 +
 2*b*g*n*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n])*(Log[x]*(Log[d + e*x] - L
og[1 + (e*x)/d]) - PolyLog[2, -((e*x)/d)]) + b*f*n*PolyLog[2, (d + e*x)/d]
+ a*g*n*PolyLog[2, (d + e*x)/d] + 2*b*g*n^2*((Log[d + e*x]^2*Log[1 - (d + e
*x)/d])/2 + Log[d + e*x]*PolyLog[2, (d + e*x)/d] - PolyLog[3, (d + e*x)/d])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 626, normalized size of antiderivative = 3.96

| method | result |
|---|---|
| risch | $\left(-i\pi bg\operatorname{csgn}(ic)\operatorname{csgn}\left(i(ex+d)^n\right)\operatorname{csgn}\left(ic(ex+d)^n\right)+i\pi bg\operatorname{csgn}(ic)\operatorname{csgn}\left(ic(ex+d)^n\right)^2+i\pi bg\right.$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] (-I*Pi*b*g*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*Pi*b*g*csgn(I*
c)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b*g*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I
*Pi*b*g*csgn(I*c*(e*x+d)^n)^3+2*b*ln(c)*g+a*g+b*f)*(ln((e*x+d)^n)*ln(x)-e*n
*(dilog((e*x+d)/d)/e+ln(x)*ln((e*x+d)/d)/e))+ln(e*x+d)^2*ln(e*x)*b*g*n^2+ln
(e*x+d)^2*ln(1-(e*x+d)/d)*b*g*n^2-2*ln(e*x+d)^2*ln(-e*x/d)*b*g*n^2-2*ln(e*x
+d)*ln(e*x)*ln((e*x+d)^n)*b*g*n+2*ln(e*x+d)*polylog(2,(e*x+d)/d)*b*g*n^2-2*
ln(e*x+d)*dilog(-e*x/d)*b*g*n^2+2*ln(e*x+d)*ln(-e*x/d)*ln((e*x+d)^n)*b*g*n+
ln(e*x)*ln((e*x+d)^n)^2*b*g-2*polylog(3,(e*x+d)/d)*b*g*n^2+2*dilog(-e*x/d)*
ln((e*x+d)^n)*b*g*n+1/4*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+
d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*
c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*(-I*g*Pi*csgn(
I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*g*Pi*csgn(I*c)*csgn(I*c*(e*x+d
)^n)^2+I*g*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I*g*Pi*csgn(I*c*(e*x+
d)^n)^3+2*g*ln(c)+2*f)*ln(x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((ex+d)^n c)+f)}{x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x,x, algorithm="fri
cas")
```

```
[Out] integral((b*g*log((e*x + d)^n*c)^2 + a*f + (b*f + a*g)*log((e*x + d)^n*c))/
x, x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx = \int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x)**n))*(f + g*log(c*(d + e*x)**n))/x, x)
```

## Maxima [F]

$$\int \frac{(a + b\log(c(d+ex)^n))(f + g\log(c(d+ex)^n))}{x}\,dx = \int \frac{(b\log((ex+d)^n c) + a)(g\log((ex+d)^n c) + f)}{x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x,x, algorithm="max
ima")
```

```
[Out] a*f*log(x) + integrate((b*g*log((e*x + d)^n)^2 + a*g*log(c) + (g*log(c)^2 +
 f*log(c))*b + ((2*g*log(c) + f)*b + a*g)*log((e*x + d)^n))/x, x)
```

## Giac [F]

$$\int \frac{(a + b\log(c(d+ex)^n))(f + g\log(c(d+ex)^n))}{x}\,dx = \int \frac{(b\log((ex+d)^n c) + a)(g\log((ex+d)^n c) + f)}{x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x,x, algorithm="gia
c")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*(g*log((e*x + d)^n*c) + f)/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d+ex)^n))(f + g\log(c(d+ex)^n))}{x}\,dx = \int \frac{(a + b\,\ln(c\,(d+e\,x)^n))\,(f + g\,\ln(c\,(d+e\,x)^n))}{x}\,dx$$

```
[In] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x,x)
```

```
[Out] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x, x)
```

# 3.383 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 96

$$\int \frac{(a+b\log(c(d+ex)^n))\,(f+g\log(c(d+ex)^n))}{x^2}\,dx = -\frac{(a+b\log(c(d+ex)^n))\,(f+g\log(c(d+ex)^n))}{x} + \frac{en(bf+ag+2bg\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{d} - \frac{2begn^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d}$$

```
[Out] -(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x+e*n*(b*f+a*g+2*b*g*ln(c*(e*x
+d)^n))*ln(1-d/(e*x+d))/d-2*b*e*g*n^2*polylog(2,d/(e*x+d))/d
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2483, 2458, 2379, 2438}

$$\int \frac{(a+b\log(c(d+ex)^n))\,(f+g\log(c(d+ex)^n))}{x^2}\,dx = \frac{en\log\left(1-\frac{d}{d+ex}\right)(ag+2bg\log(c(d+ex)^n)+bf)}{d} - \frac{(a+b\log(c(d+ex)^n))\,(g\log(c(d+ex)^n)+f)}{x} - \frac{2begn^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d}$$

```
[In] Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^2,x]
```

```
[Out] -(((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x) + (e*n*(b*f +
a*g + 2*b*g*Log[c*(d + e*x)^n])*Log[1 - d/(d + e*x)])/d - (2*b*e*g*n^2*Poly
Log[2, d/(d + e*x)])/d
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Dist[
e*(n/(m + 1)), Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x} \\
&\quad+(en)\int\frac{bf+ag+2bg\log(c(d+ex)^n)}{x(d+ex)}\,dx \\
&= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x} \\
&\quad+n\text{Subst}\left(\int\frac{bf+ag+2bg\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx,x,d+ex\right) \\
&= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x} \\
&\quad+\frac{en(bf+ag+2bg\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{d} \\
&\quad-\frac{(2begn^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx,x,d+ex\right)}{d}
\end{aligned}$$

$$= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x} + \frac{en(bf+ag+2bg\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{d} - \frac{2begn^2\text{Li}_2\left(\frac{d}{d+ex}\right)}{d}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.88

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx$$
$$= -\frac{af}{x} + \frac{befn\log(x)}{d} + \frac{aegn\log(x)}{d} - \frac{befn\log(d+ex)}{d} - \frac{aegn\log(d+ex)}{d} - \frac{bf\log(c(d+ex)^n)}{x} - \frac{ag\log(c(d+ex)^n)}{x} + \frac{2begn\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{d} - \frac{beg\log^2(c(d+ex)^n)}{d} - \frac{bg\log^2(c(d+ex)^n)}{x} + \frac{2begn^2\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d}$$

```
[In] Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^2,x]
```

```
[Out] -((a*f)/x) + (b*e*f*n*Log[x])/d + (a*e*g*n*Log[x])/d - (b*e*f*n*Log[d + e*x
])/d - (a*e*g*n*Log[d + e*x])/d - (b*f*Log[c*(d + e*x)^n])/x - (a*g*Log[c*(
d + e*x)^n])/x + (2*b*e*g*n*Log[-((e*x)/d)]*Log[c*(d + e*x)^n])/d - (b*e*g*
Log[c*(d + e*x)^n]^2)/d - (b*g*Log[c*(d + e*x)^n]^2)/x + (2*b*e*g*n^2*PolyL
og[2, (d + e*x)/d])/d
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 517, normalized size of antiderivative = 5.39

| method | result |
|---|---|
| risch | $\Big(-i\pi bg\ \text{csgn}(ic)\ \text{csgn}(i(ex+d)^n)\ \text{csgn}(ic(ex+d)^n) + i\pi bg\ \text{csgn}(ic)\ \text{csgn}(ic(ex+d)^n)^2 + i\pi bg$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x^2,x,method=_RETURNVERBOSE
)
```

```
[Out] (-I*Pi*b*g*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*Pi*b*g*csgn(I*
c)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b*g*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I
*Pi*b*g*csgn(I*c*(e*x+d)^n)^3+2*b*ln(c)*g+a*g+b*f)*(-ln((e*x+d)^n)/x+e*n*(-
1/d*ln(e*x+d)+1/d*ln(x)))-1/x*ln((e*x+d)^n)^2*b*g-2*b*g*e*n*ln((e*x+d)^n)/d
*ln(e*x+d)+2*b*g*e*n*ln((e*x+d)^n)/d*ln(x)-2*b*g*e*n^2/d*dilog((e*x+d)/d)-2
*b*g*e*n^2/d*ln(x)*ln((e*x+d)/d)+b*g*e*n^2/d*ln(e*x+d)^2-1/4*(-I*b*Pi*csgn(
```

```
I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^
n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)
^n)^3*b+2*b*ln(c)+2*a)*(-I*g*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d
)^n)+I*g*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+I*g*Pi*csgn(I*(e*x+d)^n)*csgn(I
*c*(e*x+d)^n)^2-I*g*Pi*csgn(I*c*(e*x+d)^n)^3+2*g*ln(c)+2*f)/x
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((ex+d)^n c)+f)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^2,x, algorithm="f
ricas")
[Out] integral((b*g*log((e*x + d)^n*c)^2 + a*f + (b*f + a*g)*log((e*x + d)^n*c))/
x^2, x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n))/x**2,x)
[Out] Integral((a + b*log(c*(d + e*x)**n))*(f + g*log(c*(d + e*x)**n))/x**2, x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((ex+d)^n c)+f)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^2,x, algorithm="m
axima")
[Out] -b*e*f*n*(log(e*x + d)/d - log(x)/d) - a*e*g*n*(log(e*x + d)/d - log(x)/d)
- b*g*(log((e*x + d)^n)^2/x - integrate((e*x*log(c)^2 + d*log(c)^2 + 2*((e*
n + e*log(c))*x + d*log(c))*log((e*x + d)^n))/(e*x^3 + d*x^2), x)) - b*f*lo
g((e*x + d)^n*c)/x - a*g*log((e*x + d)^n*c)/x - a*f/x
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((ex+d)^n c)+f)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^2,x, algorithm="g
iac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*(g*log((e*x + d)^n*c) + f)/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^2}\,dx = \int \frac{(a+b\ln(c\,(d+e\,x)^n))\,(f+g\ln(c\,(d+e\,x)^n))}{x^2}\,dx$$

```
[In] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x^2,x)
```

```
[Out] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x^2, x)
```

# 3.384 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 156

$$\begin{aligned}
&\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx \\
&= \frac{be^2gn^2\log(x)}{d^2} - \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{2x^2} \\
&\quad - \frac{en(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{2d^2x} \\
&\quad - \frac{e^2n(bf+ag+2bg\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{2d^2} + \frac{be^2gn^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d^2}
\end{aligned}$$

```
[Out] b*e^2*g*n^2*ln(x)/d^2-1/2*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x^2-1
/2*e*n*(e*x+d)*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))/d^2/x-1/2*e^2*n*(b*f+a*g+2*b
*g*ln(c*(e*x+d)^n))*ln(1-d/(e*x+d))/d^2+b*e^2*g*n^2*polylog(2,d/(e*x+d))/d^
2
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2483, 2458, 2389, 2379, 2438, 2351, 31}

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx$$
$$= -\frac{e^2n\log\left(1-\frac{d}{d+ex}\right)(ag+2bg\log(c(d+ex)^n)+bf)}{2d^2} - \frac{en(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)}{2d^2x} - \frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{2x^2} + \frac{be^2gn^2\,\text{PolyLog}\left(2,\frac{d}{d+ex}\right)}{d^2} + \frac{be^2gn^2\log(x)}{d^2}$$

```
[In] Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^3,x]
```

```
[Out] (b*e^2*g*n^2*Log[x])/d^2 - ((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*
x)^n]))/(2*x^2) - (e*n*(d + e*x)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(2
*d^2*x) - (e^2*n*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n])*Log[1 - d/(d + e*x)
])/(2*d^2) + (b*e^2*g*n^2*PolyLog[2, d/(d + e*x)])/d^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
```

```
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Dist[
e*(n/(m + 1)), Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{2x^2} \\
&\quad +\frac{1}{2}(en)\int\frac{bf+ag+2bg\log(c(d+ex)^n)}{x^2(d+ex)}\,dx \\
&= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{2x^2} \\
&\quad +\frac{1}{2}n\text{Subst}\left(\int\frac{bf+ag+2bg\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+ex\right) \\
&= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{2x^2} \\
&\quad +\frac{n\text{Subst}\left(\int\frac{bf+ag+2bg\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+ex\right)}{2d} \\
&\quad -\frac{(en)\text{Subst}\left(\int\frac{bf+ag+2bg\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx,x,d+ex\right)}{2d}
\end{aligned}$$

$$= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{2x^2} - \frac{en(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{2d^2x} - \frac{e^2n(bf+ag+2bg\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{2d^2} + \frac{(begn^2)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex\right)}{d^2} + \frac{(be^2gn^2)\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx, x, d+ex\right)}{d^2}$$

$$= \frac{be^2gn^2\log(x)}{d^2} - \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{2x^2} - \frac{en(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{2d^2x} - \frac{e^2n(bf+ag+2bg\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{2d^2} + \frac{be^2gn^2\mathrm{Li}_2\left(\frac{d}{d+ex}\right)}{d^2}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.61

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx = -\frac{af}{2x^2} + \frac{1}{2}befn\left(-\frac{1}{dx} - \frac{e\log(x)}{d^2} + \frac{e\log(d+ex)}{d^2}\right) + \frac{1}{2}aegn\left(-\frac{1}{dx} - \frac{e\log(x)}{d^2} + \frac{e\log(d+ex)}{d^2}\right) - \frac{bf\log(c(d+ex)^n)}{2x^2} - \frac{ag\log(c(d+ex)^n)}{2x^2} - \frac{bg\log^2(c(d+ex)^n)}{2x^2} + begn\left(\frac{en\log(x)}{d^2} - \frac{en\log(d+ex)}{d^2} - \frac{\log(c(d+ex)^n)}{dx} - \frac{e\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{d^2} + \frac{e\log^2(c(d+ex)^n)}{2d^2n} - \frac{en\,\mathrm{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d^2}\right)$$

```
[In] Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^3,x]
```

```
[Out] -1/2*(a*f)/x^2 + (b*e*f*n*(-(1/(d*x)) - (e*Log[x])/d^2 + (e*Log[d + e*x])/d
^2))/2 + (a*e*g*n*(-(1/(d*x)) - (e*Log[x])/d^2 + (e*Log[d + e*x])/d^2))/2 -
 (b*f*Log[c*(d + e*x)^n])/(2*x^2) - (a*g*Log[c*(d + e*x)^n])/(2*x^2) - (b*g
*Log[c*(d + e*x)^n]^2)/(2*x^2) + b*e*g*n*((e*n*Log[x])/d^2 - (e*n*Log[d + e
*x])/d^2 - Log[c*(d + e*x)^n]/(d*x) - (e*Log[-((e*x)/d)]*Log[c*(d + e*x)^n]
)/d^2 + (e*Log[c*(d + e*x)^n]^2)/(2*d^2*n) - (e*n*PolyLog[2, (d + e*x)/d])/
d^2)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.78

| method | result |
|---|---|
| risch | $\left(-i\pi bg\ \mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i(ex+d)^{n}\right)\mathrm{csgn}\left(ic(ex+d)^{n}\right)+i\pi bg\ \mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic(ex+d)^{n}\right)^{2}+i\pi bg\right.$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x^3,x,method=_RETURNVERBOSE
)
```

```
[Out] (-I*Pi*b*g*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*Pi*b*g*csgn(I*
c)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b*g*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I
*Pi*b*g*csgn(I*c*(e*x+d)^n)^3+2*b*ln(c)*g+a*g+b*f)*(-1/2*ln((e*x+d)^n)/x^2+
1/2*e*n*(e/d^2*ln(e*x+d)-1/d/x-e/d^2*ln(x)))-1/2/x^2*ln((e*x+d)^n)^2*b*g+b*
g*e^2*n*ln((e*x+d)^n)/d^2*ln(e*x+d)-b*g*e*n*ln((e*x+d)^n)/d/x-b*g*e^2*n*ln(
(e*x+d)^n)/d^2*ln(x)-1/2*b*g*e^2*n^2/d^2*ln(e*x+d)^2-b*g*e^2*n^2/d^2*ln(e*x
+d)+b*e^2*g*n^2*ln(x)/d^2+b*g*e^2*n^2/d^2*dilog((e*x+d)/d)+b*g*e^2*n^2/d^2*
ln(x)*ln((e*x+d)/d)-1/8*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*csgn(I*c)*csgn(I*(e*x+
d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn(I*(e*x+d)^n)*csgn(I*
c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c)+2*a)*(-I*g*Pi*csgn(
I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*g*Pi*csgn(I*c)*csgn(I*c*(e*x+d
)^n)^2+I*g*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I*g*Pi*csgn(I*c*(e*x+
d)^n)^3+2*g*ln(c)+2*f)/x^2
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))\,(f+g\log(c(d+ex)^n))}{x^3}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((ex+d)^n c)+f)}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^3,x, algorithm="f
ricas")
```

```
[Out] integral((b*g*log((e*x + d)^n*c)^2 + a*f + (b*f + a*g)*log((e*x + d)^n*c))/
x^3, x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx$$
$$=\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n))/x**3,x)
[Out] Integral((a + b*log(c*(d + e*x)**n))*(f + g*log(c*(d + e*x)**n))/x**3, x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx$$
$$=\int \frac{(b\log((ex+d)^nc)+a)(g\log((ex+d)^nc)+f)}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^3,x, algorithm="m
axima")
[Out] 1/2*b*e*f*n*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + 1/2*a*e*g*n*(e*
log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) - 1/2*b*g*(log((e*x + d)^n)^2/x^
2 - 2*integrate((e*x*log(c)^2 + d*log(c)^2 + ((e*n + 2*e*log(c))*x + 2*d*lo
g(c))*log((e*x + d)^n))/(e*x^4 + d*x^3), x)) - 1/2*b*f*log((e*x + d)^n*c)/x
^2 - 1/2*a*g*log((e*x + d)^n*c)/x^2 - 1/2*a*f/x^2
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx$$
$$=\int \frac{(b\log((ex+d)^nc)+a)(g\log((ex+d)^nc)+f)}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^3,x, algorithm="g
iac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*(g*log((e*x + d)^n*c) + f)/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^3}\,dx$$
$$=\int \frac{(a+b\,\ln(c\,(d+e\,x)^n))\,\left(f+g\,\ln(c\,(d+e\,x)^n)\right)}{x^3}\,dx$$

```
[In] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x^3,x)
[Out] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x^3, x)
```

# 3.385 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 234

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx = -\frac{be^2gn^2}{3d^2x}-\frac{be^3gn^2\log(x)}{d^3}+\frac{be^3gn^2\log(d+ex)}{3d^3} - \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{3x^3} - \frac{en(bf+ag+2bg\log(c(d+ex)^n))}{6dx^2}+\frac{e^2n(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{3d^3x} + \frac{e^3n(bf+ag+2bg\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{3d^3}-\frac{2be^3gn^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex}\right)}{3d^3}$$

```
[Out] -1/3*b*e^2*g*n^2/d^2/x-b*e^3*g*n^2*ln(x)/d^3+1/3*b*e^3*g*n^2*ln(e*x+d)/d^3-
1/3*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x^3-1/6*e*n*(b*f+a*g+2*b*g*
ln(c*(e*x+d)^n))/d/x^2+1/3*e^2*n*(e*x+d)*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))/d^
3/x+1/3*e^3*n*(b*f+a*g+2*b*g*ln(c*(e*x+d)^n))*ln(1-d/(e*x+d))/d^3-2/3*b*e^3
*g*n^2*polylog(2,d/(e*x+d))/d^3
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used

$= \{2483, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46\}$

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx$$
$$= \frac{e^3n\log\left(1-\frac{d}{d+ex}\right)(ag+2bg\log(c(d+ex)^n)+bf)}{3d^3} + \frac{e^2n(d+ex)(ag+2bg\log(c(d+ex)^n)+bf)}{3d^3x} - \frac{(a+b\log(c(d+ex)^n))(g\log(c(d+ex)^n)+f)}{3x^3} - \frac{en(ag+2bg\log(c(d+ex)^n)+bf)}{6dx^2} - \frac{2be^3gn^2\,\mathrm{PolyLog}\left(2,\frac{d}{d+ex}\right)}{3d^3} - \frac{be^3gn^2\log(x)}{d^3} + \frac{be^3gn^2\log(d+ex)}{3d^3} - \frac{be^2gn^2}{3d^2x}$$

```
[In] Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^4,x]
```

```
[Out] -1/3*(b*e^2*g*n^2)/(d^2*x) - (b*e^3*g*n^2*Log[x])/d^3 + (b*e^3*g*n^2*Log[d
+ e*x])/(3*d^3) - ((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/(
3*x^3) - (e*n*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(6*d*x^2) + (e^2*n*(d
 + e*x)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(3*d^3*x) + (e^3*n*(b*f + a
*g + 2*b*g*Log[c*(d + e*x)^n])*Log[1 - d/(d + e*x)])/(3*d^3) - (2*b*e^3*g*n
^2*PolyLog[2, d/(d + e*x)])/(3*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
```

```
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Dist[
e*(n/(m + 1)), Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{3x^3}\\ &\quad+\frac{1}{3}(en)\int\frac{bf+ag+2bg\log(c(d+ex)^n)}{x^3(d+ex)}\,dx\end{aligned}$$

$$= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{3x^3} + \frac{1}{3}n\text{Subst}\left(\int \frac{bf+ag+2bg\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+ex\right)$$

$$= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{3x^3} + \frac{n\text{Subst}\left(\int \frac{bf+ag+2bg\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+ex\right)}{3d} - \frac{(en)\text{Subst}\left(\int \frac{bf+ag+2bg\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex\right)}{3d}$$

$$= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{3x^3} - \frac{en(bf+ag+2bg\log(c(d+ex)^n))}{6dx^2} - \frac{(en)\text{Subst}\left(\int \frac{bf+ag+2bg\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex\right)}{3d^2} + \frac{(e^2n)\,\text{Subst}\left(\int \frac{bf+ag+2bg\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+ex\right)}{3d^2} + \frac{(begn^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex\right)}{3d}$$

$$= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{3x^3} - \frac{en(bf+ag+2bg\log(c(d+ex)^n))}{6dx^2} + \frac{e^2n(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{3d^3x} + \frac{e^3n(bf+ag+2bg\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{3d^3} + \frac{(begn^2)\,\text{Subst}\left(\int \left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)dx, x, d+ex\right)}{3d} - \frac{(2be^2gn^2)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex\right)}{3d^3} - \frac{(2be^3gn^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx, x, d+ex\right)}{3d^3}$$

$$
\begin{aligned}
&= -\frac{be^2gn^2}{3d^2x} - \frac{be^3gn^2\log(x)}{d^3} + \frac{be^3gn^2\log(d+ex)}{3d^3} \\
&\quad - \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{3x^3} \\
&\quad - \frac{en(bf+ag+2bg\log(c(d+ex)^n))}{6dx^2} \\
&\quad + \frac{e^2n(d+ex)(bf+ag+2bg\log(c(d+ex)^n))}{3d^3x} \\
&\quad + \frac{e^3n(bf+ag+2bg\log(c(d+ex)^n))\log\left(1-\frac{d}{d+ex}\right)}{3d^3} - \frac{2be^3gn^2\text{Li}_2\left(\frac{d}{d+ex}\right)}{3d^3}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.40

$$
\begin{aligned}
&\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx \\
&= -\frac{af}{3x^3} + \frac{1}{3}befn\left(-\frac{1}{2dx^2} + \frac{e}{d^2x} + \frac{e^2\log(x)}{d^3} - \frac{e^2\log(d+ex)}{d^3}\right) \\
&\quad + \frac{1}{3}aegn\left(-\frac{1}{2dx^2} + \frac{e}{d^2x} + \frac{e^2\log(x)}{d^3} - \frac{e^2\log(d+ex)}{d^3}\right) \\
&\quad - \frac{bf\log(c(d+ex)^n)}{3x^3} - \frac{ag\log(c(d+ex)^n)}{3x^3} - \frac{bg\log^2(c(d+ex)^n)}{3x^3} \\
&\quad + \frac{2}{3}begn\left(-\frac{en}{2d^2x} - \frac{3e^2n\log(x)}{2d^3} + \frac{3e^2n\log(d+ex)}{2d^3} - \frac{\log(c(d+ex)^n)}{2dx^2} + \frac{e\log(c(d+ex)^n)}{d^2x}\right. \\
&\qquad \left. + \frac{e^2\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)}{d^3} - \frac{e^2\log^2(c(d+ex)^n)}{2d^3n} + \frac{e^2n\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d^3}\right)
\end{aligned}
$$

```
[In] Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[c*(d + e*x)^n]))/x^4,x]
```

```
[Out] -1/3*(a*f)/x^3 + (b*e*f*n*(-1/2*1/(d*x^2) + e/(d^2*x) + (e^2*Log[x])/d^3 -
(e^2*Log[d + e*x])/d^3))/3 + (a*e*g*n*(-1/2*1/(d*x^2) + e/(d^2*x) + (e^2*Lo
g[x])/d^3 - (e^2*Log[d + e*x])/d^3))/3 - (b*f*Log[c*(d + e*x)^n])/(3*x^3) -
 (a*g*Log[c*(d + e*x)^n])/(3*x^3) - (b*g*Log[c*(d + e*x)^n]^2)/(3*x^3) + (2
*b*e*g*n*(-1/2*(e*n)/(d^2*x) - (3*e^2*n*Log[x])/(2*d^3) + (3*e^2*n*Log[d +
e*x])/(2*d^3) - Log[c*(d + e*x)^n]/(2*d*x^2) + (e*Log[c*(d + e*x)^n])/(d^2*
x) + (e^2*Log[-((e*x)/d)]*Log[c*(d + e*x)^n])/d^3 - (e^2*Log[c*(d + e*x)^n]
^2)/(2*d^3*n) + (e^2*n*PolyLog[2, (d + e*x)/d])/d^3))/3
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.74

| method | result |
|---|---|
| risch | $\left(-i\pi bg\ \text{csgn}\left(ic\right)\text{csgn}\left(i(ex+d)^{n}\right)\text{csgn}\left(ic(ex+d)^{n}\right)+i\pi bg\ \text{csgn}\left(ic\right)\text{csgn}\left(ic(ex+d)^{n}\right)^{2}+i\pi bg$ |

```
[In] int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(c*(e*x+d)^n))/x^4,x,method=_RETURNVERBOSE
)
```

```
[Out] (-I*Pi*b*g*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*Pi*b*g*csgn(I*
c)*csgn(I*c*(e*x+d)^n)^2+I*Pi*b*g*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-I
*Pi*b*g*csgn(I*c*(e*x+d)^n)^3+2*b*ln(c)*g+a*g+b*f)*(-1/3*ln((e*x+d)^n)/x^3+
1/3*e*n*(-e^2/d^3*ln(e*x+d)-1/2/d/x^2+e^2/d^3*ln(x)+e/d^2/x))-1/3/x^3*ln((e
*x+d)^n)^2*b*g-2/3*b*g*e^3*n*ln((e*x+d)^n)/d^3*ln(e*x+d)-1/3*b*g*e*n*ln((e*
x+d)^n)/d/x^2+2/3*b*g*e^3*n*ln((e*x+d)^n)/d^3*ln(x)+2/3*b*g*e^2*n*ln((e*x+d
)^n)/d^2/x+b*e^3*g*n^2*ln(e*x+d)/d^3-1/3*b*e^2*g*n^2/d^2/x-b*e^3*g*n^2*ln(x
)/d^3-2/3*b*g*e^3*n^2/d^3*dilog((e*x+d)/d)-2/3*b*g*e^3*n^2/d^3*ln(x)*ln((e*
x+d)/d)+1/3*b*g*e^3*n^2/d^3*ln(e*x+d)^2-1/12*(-I*b*Pi*csgn(I*c*(e*x+d)^n)*c
sgn(I*c)*csgn(I*(e*x+d)^n)+I*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2*b+I*Pi*csgn
(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2*b-I*Pi*csgn(I*c*(e*x+d)^n)^3*b+2*b*ln(c
)+2*a)*(-I*g*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+I*g*Pi*csgn
(I*c)*csgn(I*c*(e*x+d)^n)^2+I*g*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-
I*g*Pi*csgn(I*c*(e*x+d)^n)^3+2*g*ln(c)+2*f)/x^3
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx = \int \frac{(b\log((ex+d)^nc)+a)(g\log((ex+d)^nc)+f)}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^4,x, algorithm="f
ricas")
```

```
[Out] integral((b*g*log((e*x + d)^n*c)^2 + a*f + (b*f + a*g)*log((e*x + d)^n*c))/
x^4, x)
```

## Sympy [F]

$$\int \frac{(a + b \log(c(d + ex)^n))(f + g \log(c(d + ex)^n))}{x^4} dx$$
$$= \int \frac{(a + b \log(c(d + ex)^n))(f + g \log(c(d + ex)^n))}{x^4} dx$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(c*(e*x+d)**n))/x**4,x)
[Out] Integral((a + b*log(c*(d + e*x)**n))*(f + g*log(c*(d + e*x)**n))/x**4, x)
```

## Maxima [F]

$$\int \frac{(a + b \log(c(d + ex)^n))(f + g \log(c(d + ex)^n))}{x^4} dx$$
$$= \int \frac{(b \log((ex + d)^n c) + a)(g \log((ex + d)^n c) + f)}{x^4} dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^4,x, algorithm="m
axima")
[Out] -1/6*b*e*f*n*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*
x^2)) - 1/6*a*e*g*n*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d
)/(d^2*x^2)) - 1/3*b*g*(log((e*x + d)^n)^2/x^3 - 3*integrate(1/3*(3*e*x*log
(c)^2 + 3*d*log(c)^2 + 2*((e*n + 3*e*log(c))*x + 3*d*log(c))*log((e*x + d)^
n))/(e*x^5 + d*x^4), x)) - 1/3*b*f*log((e*x + d)^n*c)/x^3 - 1/3*a*g*log((e*
x + d)^n*c)/x^3 - 1/3*a*f/x^3
```

## Giac [F]

$$\int \frac{(a + b \log(c(d + ex)^n))(f + g \log(c(d + ex)^n))}{x^4} dx$$
$$= \int \frac{(b \log((ex + d)^n c) + a)(g \log((ex + d)^n c) + f)}{x^4} dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(c*(e*x+d)^n))/x^4,x, algorithm="g
iac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*(g*log((e*x + d)^n*c) + f)/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(c(d+ex)^n))}{x^4}\,dx$$
$$=\int \frac{(a+b\,\ln(c\,(d+e\,x)^n))\,(f+g\,\ln(c\,(d+e\,x)^n))}{x^4}\,dx$$

```
[In] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x^4,x)
[Out] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(c*(d + e*x)^n)))/x^4, x)
```

# 3.386 $\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 742

$$\begin{aligned}
&\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx\\
&=\frac{agi^3mx}{4j^3}+\frac{bd^3fnx}{4e^3}-\frac{5bd^3gmnx}{16e^3}-\frac{5bgi^3mnx}{16j^3}-\frac{5bdgi^2mnx}{24ej^2}-\frac{5bd^2gimnx}{24e^2j}\\
&+\frac{3bd^2gmnx^2}{32e^2}+\frac{3bgi^2mnx^2}{32j^2}+\frac{bdgimnx^2}{12ej}-\frac{7bdgmnx^3}{144e}-\frac{7bgimnx^3}{144j}+\frac{1}{32}bgmnx^4\\
&+\frac{bd^4gmn\log(d+ex)}{16e^4}+\frac{bd^2gi^2mn\log(d+ex)}{8e^2j^2}+\frac{bd^3gimn\log(d+ex)}{12e^3j}\\
&+\frac{bgi^3m(d+ex)\log(c(d+ex)^n)}{4ej^3}-\frac{gi^2mx^2(a+b\log(c(d+ex)^n))}{8j^2}\\
&+\frac{gimx^3(a+b\log(c(d+ex)^n))}{12j}-\frac{1}{16}gmx^4(a+b\log(c(d+ex)^n))\\
&+\frac{bgi^4mn\log(i+jx)}{16j^4}+\frac{bdgi^3mn\log(i+jx)}{12ej^3}+\frac{bd^2gi^2mn\log(i+jx)}{8e^2j^2}\\
&-\frac{gi^4m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{4j^4}+\frac{bd^3gn(i+jx)\log(h(i+jx)^m)}{4e^3j}\\
&-\frac{bd^2nx^2(f+g\log(h(i+jx)^m))}{8e^2}+\frac{bdnx^3(f+g\log(h(i+jx)^m))}{12e}\\
&-\frac{1}{16}bnx^4(f+g\log(h(i+jx)^m))-\frac{bd^4n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{4e^4}\\
&+\frac{1}{4}x^4(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\\
&-\frac{bgi^4mn\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{4j^4}-\frac{bd^4gmn\operatorname{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{4e^4}
\end{aligned}$$

```
[Out] -1/4*b*g*i^4*m*n*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j^4-1/4*b*d^4*g*m*n*polyl
og(2,e*(j*x+i)/(-d*j+e*i))/e^4+1/16*b*d^4*g*m*n*ln(e*x+d)/e^4+1/16*b*g*i^4*
m*n*ln(j*x+i)/j^4+3/32*b*g*i^2*m*n*x^2/j^2-7/144*b*d*g*m*n*x^3/e-7/144*b*g*
i*m*n*x^3/j-5/16*b*d^3*g*m*n*x/e^3-5/16*b*g*i^3*m*n*x/j^3+3/32*b*d^2*g*m*n*
x^2/e^2+1/32*b*g*m*n*x^4+1/4*a*g*i^3*m*x/j^3+1/4*b*d^3*f*n*x/e^3-5/24*b*d*g
*i^2*m*n*x/e/j^2-5/24*b*d^2*g*i*m*n*x/e^2/j+1/12*b*d*g*i*m*n*x^2/e/j+1/8*b*
d^2*g*i^2*m*n*ln(e*x+d)/e^2/j^2+1/12*b*d^3*g*i*m*n*ln(e*x+d)/e^3/j+1/12*b*d
*g*i^3*m*n*ln(j*x+i)/e/j^3+1/8*b*d^2*g*i^2*m*n*ln(j*x+i)/e^2/j^2+1/4*b*g*i^
3*m*(e*x+d)*ln(c*(e*x+d)^n)/e/j^3+1/4*b*d^3*g*n*(j*x+i)*ln(h*(j*x+i)^m)/e^3
/j-1/16*g*m*x^4*(a+b*ln(c*(e*x+d)^n))-1/16*b*n*x^4*(f+g*ln(h*(j*x+i)^m))+1/
4*x^4*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))-1/8*g*i^2*m*x^2*(a+b*ln(c
*(e*x+d)^n))/j^2+1/12*g*i*m*x^3*(a+b*ln(c*(e*x+d)^n))/j-1/4*g*i^4*m*(a+b*ln
(c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/j^4-1/8*b*d^2*n*x^2*(f+g*ln(h*(j*x+
i)^m))/e^2+1/12*b*d*n*x^3*(f+g*ln(h*(j*x+i)^m))/e-1/4*b*d^4*n*ln(-j*(e*x+d)
/(-d*j+e*i))*(f+g*ln(h*(j*x+i)^m))/e^4
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used

$= \{2489, 45, 2463, 2436, 2332, 2442, 2441, 2440, 2438\}$

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$=\frac{1}{4}x^4(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$$
$$-\frac{gi^4m\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{4j^4}-\frac{gi^2mx^2(a+b\log(c(d+ex)^n))}{8j^2}$$
$$+\frac{gimx^3(a+b\log(c(d+ex)^n))}{12j}-\frac{1}{16}gmx^4(a+b\log(c(d+ex)^n))+\frac{agi^3mx}{4j^3}$$
$$+\frac{bgi^3m(d+ex)\log(c(d+ex)^n)}{4ej^3}-\frac{bd^4n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{4e^4}$$
$$-\frac{bd^4gmn\,\mathrm{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{4e^4}+\frac{bd^4gmn\log(d+ex)}{16e^4}+\frac{bd^3fnx}{4e^3}$$
$$+\frac{bd^3gn(i+jx)\log(h(i+jx)^m)}{4e^3j}+\frac{bd^3gimn\log(d+ex)}{12e^3j}-\frac{5bd^3gmnx}{16e^3}$$
$$-\frac{bd^2nx^2(f+g\log(h(i+jx)^m))}{8e^2}+\frac{bd^2gi^2mn\log(d+ex)}{8e^2j^2}+\frac{bd^2gi^2mn\log(i+jx)}{8e^2j^2}$$
$$-\frac{5bd^2gimnx}{24e^2j}+\frac{3bd^2gmnx^2}{32e^2}+\frac{bdnx^3(f+g\log(h(i+jx)^m))}{12e}$$
$$-\frac{bgi^4mn\,\mathrm{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{4j^4}+\frac{bdgi^3mn\log(i+jx)}{12ej^3}-\frac{5bdgi^2mnx}{24ej^2}$$
$$+\frac{bdgimnx^2}{12ej}-\frac{7bdgmnx^3}{144e}-\frac{1}{16}bnx^4(f+g\log(h(i+jx)^m))$$
$$+\frac{bgi^4mn\log(i+jx)}{16j^4}-\frac{5bgi^3mnx}{16j^3}+\frac{3bgi^2mnx^2}{32j^2}-\frac{7bgimnx^3}{144j}+\frac{1}{32}bgmnx^4$$

```
[In] Int[x^3*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] (a*g*i^3*m*x)/(4*j^3) + (b*d^3*f*n*x)/(4*e^3) - (5*b*d^3*g*m*n*x)/(16*e^3)
- (5*b*g*i^3*m*n*x)/(16*j^3) - (5*b*d*g*i^2*m*n*x)/(24*e*j^2) - (5*b*d^2*g*
i*m*n*x)/(24*e^2*j) + (3*b*d^2*g*m*n*x^2)/(32*e^2) + (3*b*g*i^2*m*n*x^2)/(3
2*j^2) + (b*d*g*i*m*n*x^2)/(12*e*j) - (7*b*d*g*m*n*x^3)/(144*e) - (7*b*g*i*
m*n*x^3)/(144*j) + (b*g*m*n*x^4)/32 + (b*d^4*g*m*n*Log[d + e*x])/(16*e^4) +
 (b*d^2*g*i^2*m*n*Log[d + e*x])/(8*e^2*j^2) + (b*d^3*g*i*m*n*Log[d + e*x])/
(12*e^3*j) + (b*g*i^3*m*(d + e*x)*Log[c*(d + e*x)^n])/(4*e*j^3) - (g*i^2*m*
x^2*(a + b*Log[c*(d + e*x)^n]))/(8*j^2) + (g*i*m*x^3*(a + b*Log[c*(d + e*x)
^n]))/(12*j) - (g*m*x^4*(a + b*Log[c*(d + e*x)^n]))/16 + (b*g*i^4*m*n*Log[i
 + j*x])/(16*j^4) + (b*d*g*i^3*m*n*Log[i + j*x])/(12*e*j^3) + (b*d^2*g*i^2*
m*n*Log[i + j*x])/(8*e^2*j^2) - (g*i^4*m*(a + b*Log[c*(d + e*x)^n])*Log[(e*
(i + j*x))/(e*i - d*j)])/(4*j^4) + (b*d^3*g*n*(i + j*x)*Log[h*(i + j*x)^m])
/(4*e^3*j) - (b*d^2*n*x^2*(f + g*Log[h*(i + j*x)^m]))/(8*e^2) + (b*d*n*x^3*
```

```
(f + g*Log[h*(i + j*x)^m]))/(12*e) - (b*n*x^4*(f + g*Log[h*(i + j*x)^m]))/1
6 - (b*d^4*n*Log[-((j*(d + e*x))/(e*i - d*j))]*(f + g*Log[h*(i + j*x)^m]))/
(4*e^4) + (x^4*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/4 - (
b*g*i^4*m*n*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/(4*j^4) - (b*d^4*g*m*
n*PolyLog[2, (e*(i + j*x))/(e*i - d*j)])/(4*e^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}x^4(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\ &\quad -\frac{1}{4}(gjm)\int\frac{x^4(a+b\log(c(d+ex)^n))}{i+jx}\,dx \\ &\quad -\frac{1}{4}(ben)\int\frac{x^4(f+g\log(h(i+jx)^m))}{d+ex}\,dx \\ &= \frac{1}{4}x^4(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\ &\quad -\frac{1}{4}(gjm)\int\left(-\frac{i^3(a+b\log(c(d+ex)^n))}{j^4}+\frac{i^2x(a+b\log(c(d+ex)^n))}{j^3}\right. \\ &\quad -\frac{ix^2(a+b\log(c(d+ex)^n))}{j^2}+\frac{x^3(a+b\log(c(d+ex)^n))}{j} \\ &\quad \left.+\frac{i^4(a+b\log(c(d+ex)^n))}{j^4(i+jx)}\right)dx-\frac{1}{4}(ben)\int\left(-\frac{d^3(f+g\log(h(i+jx)^m))}{e^4}\right. \\ &\quad +\frac{d^2x(f+g\log(h(i+jx)^m))}{e^3}-\frac{dx^2(f+g\log(h(i+jx)^m))}{e^2} \\ &\quad \left.+\frac{x^3(f+g\log(h(i+jx)^m))}{e}+\frac{d^4(f+g\log(h(i+jx)^m))}{e^4(d+ex)}\right)dx\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{4}x^4(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\
&\quad -\frac{1}{4}(gm)\int x^3(a+b\log(c(d+ex)^n))\,dx+\frac{(gi^3m)\int(a+b\log(c(d+ex)^n))\,dx}{4j^3} \\
&\quad -\frac{(gi^4m)\int\frac{a+b\log(c(d+ex)^n)}{i+jx}\,dx}{4j^3}-\frac{(gi^2m)\int x(a+b\log(c(d+ex)^n))\,dx}{4j^2} \\
&\quad +\frac{(gim)\int x^2(a+b\log(c(d+ex)^n))\,dx}{4j}-\frac{1}{4}(bn)\int x^3(f+g\log(h(i+jx)^m))\,dx \\
&\quad +\frac{(bd^3n)\int(f+g\log(h(i+jx)^m))\,dx}{4e^3}-\frac{(bd^4n)\int\frac{f+g\log(h(i+jx)^m)}{d+ex}\,dx}{4e^3} \\
&\quad -\frac{(bd^2n)\int x(f+g\log(h(i+jx)^m))\,dx}{4e^2}+\frac{(bdn)\int x^2(f+g\log(h(i+jx)^m))\,dx}{4e} \\
&= \frac{agi^3mx}{4j^3}+\frac{bd^3fnx}{4e^3}-\frac{gi^2mx^2(a+b\log(c(d+ex)^n))}{8j^2}+\frac{gimx^3(a+b\log(c(d+ex)^n))}{12j} \\
&\quad -\frac{1}{16}gmx^4(a+b\log(c(d+ex)^n))-\frac{gi^4m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{4j^4} \\
&\quad -\frac{bd^2nx^2(f+g\log(h(i+jx)^m))}{8e^2}+\frac{bdnx^3(f+g\log(h(i+jx)^m))}{12e} \\
&\quad -\frac{1}{16}bnx^4(f+g\log(h(i+jx)^m))-\frac{bd^4n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{4e^4} \\
&\quad +\frac{1}{4}x^4(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\
&\quad +\frac{(bgi^3m)\int\log(c(d+ex)^n)\,dx}{4j^3}+\frac{(bd^3gn)\int\log(h(i+jx)^m)\,dx}{4e^3} \\
&\quad +\frac{1}{16}(begmn)\int\frac{x^4}{d+ex}\,dx+\frac{(begi^4mn)\int\frac{\log\left(\frac{e(i+jx)}{ei-dj}\right)}{d+ex}\,dx}{4j^4} \\
&\quad +\frac{(begi^2mn)\int\frac{x^2}{d+ex}\,dx}{8j^2}-\frac{(begimn)\int\frac{x^3}{d+ex}\,dx}{12j}+\frac{1}{16}(bgjmn)\int\frac{x^4}{i+jx}\,dx \\
&\quad +\frac{(bd^4gjmn)\int\frac{\log\left(\frac{j(d+ex)}{-ei+dj}\right)}{i+jx}\,dx}{4e^4}+\frac{(bd^2gjmn)\int\frac{x^2}{i+jx}\,dx}{8e^2}-\frac{(bdgjmn)\int\frac{x^3}{i+jx}\,dx}{12e}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{agi^3mx}{4j^3}+\frac{bd^3fnx}{4e^3}-\frac{gi^2mx^2(a+b\log(c(d+ex)^n))}{8j^2}+\frac{gimx^3(a+b\log(c(d+ex)^n))}{12j} \\
&\quad-\frac{1}{16}gmx^4(a+b\log(c(d+ex)^n))-\frac{gi^4m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{4j^4} \\
&\quad-\frac{bd^2nx^2(f+g\log(h(i+jx)^m))}{8e^2}+\frac{bdnx^3(f+g\log(h(i+jx)^m))}{12e} \\
&\quad-\frac{1}{16}bnx^4(f+g\log(h(i+jx)^m))-\frac{bd^4n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{4e^4} \\
&\quad+\frac{1}{4}x^4(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\
&\quad+\frac{(bgi^3m)\,\text{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{4ej^3} \\
&\quad+\frac{(bd^3gn)\,\text{Subst}\left(\int\log(hx^m)\,dx,x,i+jx\right)}{4e^3j} \\
&\quad+\frac{(bd^4gmn)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{ex}{-ei+dj}\right)}{x}\,dx,x,i+jx\right)}{4e^4} \\
&\quad+\frac{1}{16}(begmn)\int\left(-\frac{d^3}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{e^2}+\frac{x^3}{e}+\frac{d^4}{e^4(d+ex)}\right)dx \\
&\quad+\frac{(bgi^4mn)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{jx}{ei-dj}\right)}{x}\,dx,x,d+ex\right)}{4j^4} \\
&\quad+\frac{(begi^2mn)\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx}{8j^2} \\
&\quad-\frac{(begimn)\int\left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)dx}{12j} \\
&\quad+\frac{1}{16}(bgjmn)\int\left(-\frac{i^3}{j^4}+\frac{i^2x}{j^3}-\frac{ix^2}{j^2}+\frac{x^3}{j}+\frac{i^4}{j^4(i+jx)}\right)dx \\
&\quad+\frac{(bd^2gjmn)\int\left(-\frac{i}{j^2}+\frac{x}{j}+\frac{i^2}{j^2(i+jx)}\right)dx}{8e^2} \\
&\quad-\frac{(bdgjmn)\int\left(\frac{i^2}{j^3}-\frac{ix}{j^2}+\frac{x^2}{j}-\frac{i^3}{j^3(i+jx)}\right)dx}{12e}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{agi^3mx}{4j^3} + \frac{bd^3fnx}{4e^3} - \frac{5bd^3gmnx}{16e^3} - \frac{5bgi^3mnx}{16j^3} - \frac{5bdgi^2mnx}{24ej^2} - \frac{5bd^2gimnx}{24e^2j} \\
&+ \frac{3bd^2gmnx^2}{32e^2} + \frac{3bgi^2mnx^2}{32j^2} + \frac{bdgimnx^2}{12ej} - \frac{7bdgmnx^3}{144e} - \frac{7bgimnx^3}{144j} + \frac{1}{32}bgmnx^4 \\
&+ \frac{bd^4gmn\log(d+ex)}{16e^4} + \frac{bd^2gi^2mn\log(d+ex)}{8e^2j^2} + \frac{bd^3gimn\log(d+ex)}{12e^3j} \\
&+ \frac{bgi^3m(d+ex)\log\left(c(d+ex)^n\right)}{4ej^3} - \frac{gi^2mx^2(a+b\log\left(c(d+ex)^n\right))}{8j^2} \\
&+ \frac{gimx^3(a+b\log\left(c(d+ex)^n\right))}{12j} - \frac{1}{16}gmx^4(a+b\log\left(c(d+ex)^n\right)) \\
&+ \frac{bgi^4mn\log(i+jx)}{16j^4} + \frac{bdgi^3mn\log(i+jx)}{12ej^3} + \frac{bd^2gi^2mn\log(i+jx)}{8e^2j^2} \\
&- \frac{gi^4m(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{4j^4} + \frac{bd^3gn(i+jx)\log\left(h(i+jx)^m\right)}{4e^3j} \\
&- \frac{bd^2nx^2(f+g\log\left(h(i+jx)^m\right))}{8e^2} + \frac{bdnx^3(f+g\log\left(h(i+jx)^m\right))}{12e} \\
&- \frac{1}{16}bnx^4(f+g\log\left(h(i+jx)^m\right)) - \frac{bd^4n\log\left(-\frac{j(d+ex)}{ei-dj}\right)\left(f+g\log\left(h(i+jx)^m\right)\right)}{4e^4} \\
&+ \frac{1}{4}x^4(a+b\log\left(c(d+ex)^n\right))\left(f+g\log\left(h(i+jx)^m\right)\right) \\
&- \frac{bgi^4mn\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{4j^4} - \frac{bd^4gmn\operatorname{Li}_2\left(\frac{e(i+jx)}{ei-dj}\right)}{4e^4}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 605, normalized size of antiderivative = 0.82

$$
\int x^3(a+b\log\left(c(d+ex)^n\right))\left(f+g\log\left(h(i+jx)^m\right)\right)\,dx
$$
$$
= \frac{6bn\log(d+ex)\left(12e^4gi^4m\log(i+jx) - 12g(e^4i^4-d^4j^4)\,m\log\left(\frac{e(i+jx)}{ei-dj}\right) + dj(12e^3gi^3m + 6de^2gi^2jm + \cdots\right.}{}
$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] (6*b*n*Log[d + e*x]*(12*e^4*g*i^4*m*Log[i + j*x] - 12*g*(e^4*i^4 - d^4*j^4)
*m*Log[(e*(i + j*x))/(e*i - d*j)] + d*j*(12*e^3*g*i^3*m + 6*d*e^2*g*i^2*j*m
 + 4*d^2*e*g*i*j^2*m + 3*d^3*j^3*(-4*f + g*m) - 12*d^3*g*j^3*Log[h*(i + j*x
)^m])) + e*(6*g*i*m*(-12*a*e^3*i^3 + b*(3*e^3*i^3 + 4*d*e^2*i^2*j + 6*d^2*e
*i*j^2 + 12*d^3*j^3)*n)*Log[i + j*x] - 6*b*e^3*Log[c*(d + e*x)^n]*(-12*f*j^
4*x^4 + g*j*m*x*(-12*i^3 + 6*i^2*j*x - 4*i*j^2*x^2 + 3*j^3*x^3) + 12*g*i^4*
m*Log[i + j*x] - 12*g*j^4*x^4*Log[h*(i + j*x)^m]) + j*(6*a*e^3*x*(12*f*j^3*
x^3 + g*m*(12*i^3 - 6*i^2*j*x + 4*i*j^2*x^2 - 3*j^3*x^3)) - b*n*(18*d^3*j^3
```

```
*(-4*f + 5*g*m)*x + 3*d^2*e*j^2*x*(12*f*j*x + g*m*(20*i - 9*j*x)) + e^3*x*(
18*f*j^3*x^3 + g*m*(90*i^3 - 27*i^2*j*x + 14*i*j^2*x^2 - 9*j^3*x^3)) + 2*d*
e^2*(-12*f*j^3*x^3 + g*m*(36*i^3 + 30*i^2*j*x - 12*i*j^2*x^2 + 7*j^3*x^3)))
 - 6*g*j^3*x*(-12*a*e^3*x^3 + b*n*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^
3*x^3))*Log[h*(i + j*x)^m])) - 72*b*g*(e^4*i^4 - d^4*j^4)*m*n*PolyLog[2, (j
*(d + e*x))/(-(e*i) + d*j)])/(288*e^4*j^4)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.06 (sec) , antiderivative size = 2094, normalized size of antiderivative = 2.82

Expression too large to display

```
[In] int(x^3*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m)),x)
```

```
[Out] 1/4/j^4*b*g*i^4*m*n*ln(j*x+i)*ln(((j*x+i)*e+d*j-e*i)/(d*j-e*i))-3/16/e^3/j*
b*d^3*g*i*m*n-11/96/e^2/j^2*b*d^2*g*i^2*m*n+1/4/e/j^3*ln(e*x+d)*b*d*g*i^3*m
*n+1/4/e^3/j*g*i*m*ln((e*x+d)*j-d*j+e*i)*b*d^3*n+1/8/e^2/j^2*g*i^2*m*ln((e*
x+d)*j-d*j+e*i)*b*d^2*n+1/12/e/j^3*g*i^3*m*ln((e*x+d)*j-d*j+e*i)*b*d*n+1/16
/j^4*g*i^4*m*ln((e*x+d)*j-d*j+e*i)*b*n+1/4/e^4*b*d^4*g*m*n*dilog(((e*x+d)*j
-d*j+e*i)/(-d*j+e*i))-1/16/e/j^3*b*d*g*i^3*m*n-205/576/e^4*b*d^4*g*m*n+1/4/
e^4*b*d^4*g*m*n*ln(e*x+d)*ln(((e*x+d)*j-d*j+e*i)/(-d*j+e*i))-1/16*ln(h)*x^4
*b*g*n-1/16*n*b*g*ln((j*x+i)^m)*x^4+(1/4*x^4*b*g*ln((j*x+i)^m)-1/48*b*(-6*I
*Pi*g*j^4*x^4*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2+6*I*Pi*g*j^4*x^4*csgn
(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+6*I*Pi*g*j^4*x^4*csgn(I*h*(j*x+
i)^m)^3-6*I*Pi*g*j^4*x^4*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)-12*ln(h)*g*j^4*x^4
+3*g*j^4*m*x^4-12*f*j^4*x^4-4*g*i*j^3*m*x^3+6*g*i^2*j^2*m*x^2+12*g*i^4*m*ln
(j*x+i)-12*g*i^3*j*m*x)/j^4)*ln((e*x+d)^n)+1/32*b*g*m*n*x^4+1/16*I/e^2*n*b*
Pi*x^2*d^2*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)-1/24*I/e*n*b*P
i*x^3*d*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+1/4/j^4*b*g*i^4*m
*n*dilog(((j*x+i)*e+d*j-e*i)/(d*j-e*i))+3/32*b*g*i^2*m*n*x^2/j^2-7/144*b*d*
g*m*n*x^3/e-7/144*b*g*i*m*n*x^3/j+1/16*b*d^4*g*m*n*ln(e*x+d)/e^4+1/12/e*ln(
h)*x^3*b*d*g*n-1/8/e^2*ln(h)*x^2*b*d^2*g*n+1/4/e^3*ln(h)*b*d^3*g*n*x+1/12/e
*x^3*b*d*f*n-1/8/e^2*x^2*b*d^2*f*n-1/4/e^4*ln(e*x+d)*b*d^4*f*n+1/4*b*d^3*f*
n*x/e^3-1/16*x^4*b*f*n-1/8*I/e^4*n*b*d^4*ln(e*x+d)*Pi*g*csgn(I*(j*x+i)^m)*c
sgn(I*h*(j*x+i)^m)^2-1/8*I/e^4*n*b*d^4*ln(e*x+d)*Pi*g*csgn(I*h*(j*x+i)^m)^2
*csgn(I*h)-1/16*I/e^2*n*b*Pi*x^2*d^2*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m
)^2-1/16*I/e^2*n*b*Pi*x^2*d^2*g*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+1/8*I/e^3*n
*b*x*Pi*d^3*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2+1/8*I/e^3*n*b*x*Pi*d^
3*g*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+1/24*I/e*n*b*Pi*x^3*d*g*csgn(I*(j*x+i)^
m)*csgn(I*h*(j*x+i)^m)^2+1/24*I/e*n*b*Pi*x^3*d*g*csgn(I*h*(j*x+i)^m)^2*csgn
(I*h)+1/8*I/e^4*n*b*d^4*ln(e*x+d)*Pi*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m
)*csgn(I*h)-1/8*I/e^3*n*b*x*Pi*d^3*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*
csgn(I*h)+1/12/e*n*b*g*ln((j*x+i)^m)*x^3*d-1/8/e^2*n*b*g*ln((j*x+i)^m)*x^2*
d^2+1/4/e^3*n*b*g*ln((j*x+i)^m)*x*d^3-1/4/e^4*n*b*g*ln((j*x+i)^m)*d^4*ln(e*
```

```
x+d)-1/4/e^4*n*b*d^4*ln(e*x+d)*ln(h)*g+1/32*I*n*b*Pi*x^4*g*csgn(I*h*(j*x+i)
^m)^3-5/24*b*d*g*i^2*m*n*x/e/j^2-5/24*b*d^2*g*i*m*n*x/e^2/j+1/12*b*d*g*i*m*
n*x^2/e/j+1/8*b*d^2*g*i^2*m*n*ln(e*x+d)/e^2/j^2+1/12*b*d^3*g*i*m*n*ln(e*x+d
)/e^3/j-5/16*b*d^3*g*m*n*x/e^3-5/16*b*g*i^3*m*n*x/j^3+3/32*b*d^2*g*m*n*x^2/
e^2-1/32*I*n*b*Pi*x^4*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2-1/32*I*n*b*
Pi*x^4*g*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+1/32*I*n*b*Pi*x^4*g*csgn(I*(j*x+i)
^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+1/8*I/e^4*n*b*d^4*ln(e*x+d)*Pi*g*csgn(I*h
*(j*x+i)^m)^3-1/24*I/e*n*b*Pi*x^3*d*g*csgn(I*h*(j*x+i)^m)^3+1/16*I/e^2*n*b*
Pi*x^2*d^2*g*csgn(I*h*(j*x+i)^m)^3-1/8*I/e^3*n*b*x*Pi*d^3*g*csgn(I*h*(j*x+i
)^m)^3+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/4*I*b
*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(
e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*a)*(1/4*(I*g*P
i*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2-I*g*Pi*csgn(I*(j*x+i)^m)*csgn(I*h
*(j*x+i)^m)*csgn(I*h)-I*g*Pi*csgn(I*h*(j*x+i)^m)^3+I*g*Pi*csgn(I*h*(j*x+i)^
m)^2*csgn(I*h)+2*g*ln(h)+2*f)*x^4+2*g*(1/4*ln((j*x+i)^m)*x^4-1/4*m*j*(1/j^4
*(1/4*x^4*j^3-1/3*x^3*i*j^2+1/2*j*i^2*x^2-i^3*x)+i^4/j^5*ln(j*x+i))))
```

## Fricas [F]

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="f
ricas")
```

```
[Out] integral(b*f*x^3*log((e*x + d)^n*c) + a*f*x^3 + (b*g*x^3*log((e*x + d)^n*c)
 + a*g*x^3)*log((j*x + i)^m*h), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m)),x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$=\int(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="m
axima")
```

```
[Out] 1/4*b*f*x^4*log((e*x + d)^n*c) + 1/4*a*g*x^4*log((j*x + i)^m*h) + 1/4*a*f*x
^4 - 1/48*b*e*f*n*(12*d^4*log(e*x + d)/e^5 + (3*e^3*x^4 - 4*d*e^2*x^3 + 6*d
^2*e*x^2 - 12*d^3*x)/e^4) - 1/48*a*g*j*m*(12*i^4*log(j*x + i)/j^5 + (3*j^3*
x^4 - 4*i*j^2*x^3 + 6*i^2*j*x^2 - 12*i^3*x)/j^4) + 1/48*b*g*((12*e^4*i^4*m*
n*log(e*x + d)*log(j*x + i) + (4*e^4*i*j^3*m*x^3 - 6*e^4*i^2*j^2*m*x^2 + 12
*e^4*i^3*j*m*x - 12*e^4*i^4*m*log(j*x + i) - 3*(j^4*m - 4*j^4*log(h))*e^4*x
^4)*log((e*x + d)^n) + (12*e^4*j^4*x^4*log((e*x + d)^n) + 4*d*e^3*j^4*n*x^3
 - 6*d^2*e^2*j^4*n*x^2 + 12*d^3*e*j^4*n*x - 12*d^4*j^4*n*log(e*x + d) - 3*(
e^4*j^4*n - 4*e^4*j^4*log(c))*x^4)*log((j*x + i)^m))/(e^4*j^4) + 48*integra
te(-1/48*(6*(2*(j^4*m - 4*j^4*log(h))*e^5*log(c) - (j^4*m*n - 2*j^4*n*log(h
))*e^5)*x^5 + (d*e^4*j^4*m*n + (i*j^3*m*n + 12*i*j^3*n*log(h))*e^5 - 12*(4*
e^5*i*j^3*log(h) - (j^4*m - 4*j^4*log(h))*d*e^4)*log(c))*x^4 - 2*(e^5*i^2*j
^2*m*n + d^2*e^3*j^4*m*n + 24*d*e^4*i*j^3*log(c)*log(h))*x^3 + 6*(e^5*i^3*j
*m*n + d^3*e^2*j^4*m*n)*x^2 + 12*(e^5*i^4*m*n + d^4*e*j^4*m*n)*x + 12*(d*e^
4*i^4*m*n - d^5*j^4*m*n + (e^5*i^4*m*n - d^4*e*j^4*m*n)*x)*log(e*x + d))/(e
^5*j^4*x^2 + d*e^4*i*j^3 + (e^5*i*j^3 + d*e^4*j^4)*x), x))
```

## Giac [F]

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$=\int(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="g
iac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f)*x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$=\int x^3\,(a+b\,\ln(c\,(d+e\,x)^n))\,(f+g\,\ln(h\,(i+j\,x)^m))\,dx$$

```
[In] int(x^3*(a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)),x)
[Out] int(x^3*(a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)), x)
```

# 3.387 $\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 558

$$\begin{aligned}
&\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx \\
&= -\frac{agi^2mx}{3j^2}-\frac{bd^2fnx}{3e^2}+\frac{4bd^2gmnx}{9e^2}+\frac{4bgi^2mnx}{9j^2}+\frac{bdgimnx}{3ej}-\frac{5bdgmnx^2}{36e} \\
&\quad-\frac{5bgimnx^2}{36j}+\frac{2}{27}bgmnx^3-\frac{bd^3gmn\log(d+ex)}{9e^3}-\frac{bd^2gimn\log(d+ex)}{6e^2j} \\
&\quad-\frac{bgi^2m(d+ex)\log(c(d+ex)^n)}{3ej^2}+\frac{gimx^2(a+b\log(c(d+ex)^n))}{6j} \\
&\quad-\frac{1}{9}gmx^3(a+b\log(c(d+ex)^n))-\frac{bgi^3mn\log(i+jx)}{9j^3} \\
&\quad-\frac{bdgi^2mn\log(i+jx)}{6ej^2}+\frac{gi^3m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{3j^3} \\
&\quad-\frac{bd^2gn(i+jx)\log(h(i+jx)^m)}{3e^2j}+\frac{bdnx^2(f+g\log(h(i+jx)^m))}{6e} \\
&\quad-\frac{1}{9}bnx^3(f+g\log(h(i+jx)^m))+\frac{bd^3n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{3e^3} \\
&\quad+\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\
&\quad+\frac{bgi^3mn\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{3j^3}+\frac{bd^3gmn\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{3e^3}
\end{aligned}$$

```
[Out] -1/3*a*g*i^2*m*x/j^2-1/3*b*d^2*f*n*x/e^2+4/9*b*d^2*g*m*n*x/e^2+4/9*b*g*i^2*
m*n*x/j^2+1/3*b*d*g*i*m*n*x/e/j-5/36*b*d*g*m*n*x^2/e-5/36*b*g*i*m*n*x^2/j+2
/27*b*g*m*n*x^3-1/9*b*d^3*g*m*n*ln(e*x+d)/e^3-1/6*b*d^2*g*i*m*n*ln(e*x+d)/e
^2/j-1/3*b*g*i^2*m*(e*x+d)*ln(c*(e*x+d)^n)/e/j^2+1/6*g*i*m*x^2*(a+b*ln(c*(e
```

```
*x+d)^n))/j-1/9*g*m*x^3*(a+b*ln(c*(e*x+d)^n))-1/9*b*g*i^3*m*n*ln(j*x+i)/j^3
-1/6*b*d*g*i^2*m*n*ln(j*x+i)/e/j^2+1/3*g*i^3*m*(a+b*ln(c*(e*x+d)^n))*ln(e*(
j*x+i)/(-d*j+e*i))/j^3-1/3*b*d^2*g*n*(j*x+i)*ln(h*(j*x+i)^m)/e^2/j+1/6*b*d*
n*x^2*(f+g*ln(h*(j*x+i)^m))/e-1/9*b*n*x^3*(f+g*ln(h*(j*x+i)^m))+1/3*b*d^3*n
*ln(-j*(e*x+d)/(-d*j+e*i))*(f+g*ln(h*(j*x+i)^m))/e^3+1/3*x^3*(a+b*ln(c*(e*x
+d)^n))*(f+g*ln(h*(j*x+i)^m))+1/3*b*g*i^3*m*n*polylog(2,-j*(e*x+d)/(-d*j+e*
i))/j^3+1/3*b*d^3*g*m*n*polylog(2,e*(j*x+i)/(-d*j+e*i))/e^3
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2489, 45, 2463, 2436, 2332, 2442, 2441, 2440, 2438}

$$\begin{aligned}
&\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx\\
&=\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\\
&+\frac{gi^3m\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{3j^3}+\frac{gimx^2(a+b\log(c(d+ex)^n))}{6j}\\
&-\frac{1}{9}gmx^3(a+b\log(c(d+ex)^n))-\frac{agi^2mx}{3j^2}-\frac{bgi^2m(d+ex)\log(c(d+ex)^n)}{3ej^2}\\
&+\frac{bd^3n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{3e^3}+\frac{bd^3gmn\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{3e^3}\\
&-\frac{bd^3gmn\log(d+ex)}{9e^3}-\frac{bd^2fnx}{3e^2}-\frac{bd^2gn(i+jx)\log(h(i+jx)^m)}{3e^2j}-\frac{bd^2gimn\log(d+ex)}{6e^2j}\\
&+\frac{4bd^2gmnx}{9e^2}+\frac{bdnx^2(f+g\log(h(i+jx)^m))}{6e}+\frac{bgi^3mn\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{3j^3}\\
&-\frac{bdgi^2mn\log(i+jx)}{6ej^2}+\frac{bdgimnx}{3ej}-\frac{5bdgmnx^2}{36e}-\frac{1}{9}bnx^3(f+g\log(h(i+jx)^m))\\
&-\frac{bgi^3mn\log(i+jx)}{9j^3}+\frac{4bgi^2mnx}{9j^2}-\frac{5bgimnx^2}{36j}+\frac{2}{27}bgmnx^3
\end{aligned}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] -1/3*(a*g*i^2*m*x)/j^2 - (b*d^2*f*n*x)/(3*e^2) + (4*b*d^2*g*m*n*x)/(9*e^2)
+ (4*b*g*i^2*m*n*x)/(9*j^2) + (b*d*g*i*m*n*x)/(3*e*j) - (5*b*d*g*m*n*x^2)/(
36*e) - (5*b*g*i*m*n*x^2)/(36*j) + (2*b*g*m*n*x^3)/27 - (b*d^3*g*m*n*Log[d
+ e*x])/(9*e^3) - (b*d^2*g*i*m*n*Log[d + e*x])/(6*e^2*j) - (b*g*i^2*m*(d +
e*x)*Log[c*(d + e*x)^n])/(3*e*j^2) + (g*i*m*x^2*(a + b*Log[c*(d + e*x)^n]))
/(6*j) - (g*m*x^3*(a + b*Log[c*(d + e*x)^n]))/9 - (b*g*i^3*m*n*Log[i + j*x]
)/(9*j^3) - (b*d*g*i^2*m*n*Log[i + j*x])/(6*e*j^2) + (g*i^3*m*(a + b*Log[c*
```

```
(d + e*x)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/(3*j^3) - (b*d^2*g*n*(i + j*x
)*Log[h*(i + j*x)^m])/(3*e^2*j) + (b*d*n*x^2*(f + g*Log[h*(i + j*x)^m]))/(6
*e) - (b*n*x^3*(f + g*Log[h*(i + j*x)^m]))/9 + (b*d^3*n*Log[-((j*(d + e*x))
/(e*i - d*j))]*(f + g*Log[h*(i + j*x)^m]))/(3*e^3) + (x^3*(a + b*Log[c*(d +
 e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/3 + (b*g*i^3*m*n*PolyLog[2, -((j*(d +
 e*x))/(e*i - d*j))])/(3*j^3) + (b*d^3*g*m*n*PolyLog[2, (e*(i + j*x))/(e*i
- d*j)])/(3*e^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\
&\quad -\frac{1}{3}(gjm)\int\frac{x^3(a+b\log(c(d+ex)^n))}{i+jx}\,dx \\
&\quad -\frac{1}{3}(ben)\int\frac{x^3(f+g\log(h(i+jx)^m))}{d+ex}\,dx \\
&= \frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\
&\quad -\frac{1}{3}(gjm)\int\left(\frac{i^2(a+b\log(c(d+ex)^n))}{j^3}-\frac{ix(a+b\log(c(d+ex)^n))}{j^2}\right. \\
&\qquad \left.+\frac{x^2(a+b\log(c(d+ex)^n))}{j}-\frac{i^3(a+b\log(c(d+ex)^n))}{j^3(i+jx)}\right)dx \\
&\quad -\frac{1}{3}(ben)\int\left(\frac{d^2(f+g\log(h(i+jx)^m))}{e^3}-\frac{dx(f+g\log(h(i+jx)^m))}{e^2}\right. \\
&\qquad \left.+\frac{x^2(f+g\log(h(i+jx)^m))}{e}-\frac{d^3(f+g\log(h(i+jx)^m))}{e^3(d+ex)}\right)dx
\end{aligned}$$

$$= \frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$$
$$-\frac{1}{3}(gm)\int x^2(a+b\log(c(d+ex)^n))\,dx - \frac{(gi^2m)\int(a+b\log(c(d+ex)^n))\,dx}{3j^2}$$
$$+\frac{(gi^3m)\int\frac{a+b\log(c(d+ex)^n)}{i+jx}\,dx}{3j^2}+\frac{(gim)\int x(a+b\log(c(d+ex)^n))\,dx}{3j}$$
$$-\frac{1}{3}(bn)\int x^2(f+g\log(h(i+jx)^m))\,dx-\frac{(bd^2n)\int(f+g\log(h(i+jx)^m))\,dx}{3e^2}$$
$$+\frac{(bd^3n)\int\frac{f+g\log(h(i+jx)^m)}{d+ex}\,dx}{3e^2}+\frac{(bdn)\int x(f+g\log(h(i+jx)^m))\,dx}{3e}$$

$$=-\frac{agi^2mx}{3j^2}-\frac{bd^2fnx}{3e^2}+\frac{gimx^2(a+b\log(c(d+ex)^n))}{6j}-\frac{1}{9}gmx^3(a+b\log(c(d+ex)^n))$$
$$+\frac{gi^3m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{3j^3}+\frac{bdnx^2(f+g\log(h(i+jx)^m))}{6e}$$
$$-\frac{1}{9}bnx^3(f+g\log(h(i+jx)^m))+\frac{bd^3n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{3e^3}$$
$$+\frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$$
$$-\frac{(bgi^2m)\int\log(c(d+ex)^n)\,dx}{3j^2}-\frac{(bd^2gn)\int\log(h(i+jx)^m)\,dx}{3e^2}$$
$$+\frac{1}{9}(begmn)\int\frac{x^3}{d+ex}\,dx-\frac{(begi^3mn)\int\frac{\log\left(\frac{e(i+jx)}{ei-dj}\right)}{d+ex}\,dx}{3j^3}-\frac{(begimn)\int\frac{x^2}{d+ex}\,dx}{6j}$$
$$+\frac{1}{9}(bgjmn)\int\frac{x^3}{i+jx}\,dx-\frac{(bd^3gjmn)\int\frac{\log\left(\frac{j(d+ex)}{-ei+dj}\right)}{i+jx}\,dx}{3e^3}-\frac{(bdgjmn)\int\frac{x^2}{i+jx}\,dx}{6e}$$

$$= -\frac{agi^2mx}{3j^2} - \frac{bd^2fnx}{3e^2} + \frac{gimx^2(a+b\log(c(d+ex)^n))}{6j} - \frac{1}{9}gmx^3(a+b\log(c(d+ex)^n))$$
$$+ \frac{gi^3m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{3j^3} + \frac{bdnx^2(f+g\log(h(i+jx)^m))}{6e}$$
$$- \frac{1}{9}bnx^3(f+g\log(h(i+jx)^m)) + \frac{bd^3n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{3e^3}$$
$$+ \frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$$
$$- \frac{(bgi^2m)\,\text{Subst}\left(\int\log(cx^n)\,dx, x, d+ex\right)}{3ej^2}$$
$$- \frac{(bd^2gn)\,\text{Subst}\left(\int\log(hx^m)\,dx, x, i+jx\right)}{3e^2j}$$
$$- \frac{(bd^3gmn)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{ex}{-ei+dj}\right)}{x}\,dx, x, i+jx\right)}{3e^3}$$
$$+ \frac{1}{9}(begmn)\int\left(\frac{d^2}{e^3} - \frac{dx}{e^2} + \frac{x^2}{e} - \frac{d^3}{e^3(d+ex)}\right)dx$$
$$- \frac{(bgi^3mn)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{jx}{ei-dj}\right)}{x}\,dx, x, d+ex\right)}{3j^3}$$
$$- \frac{(begimn)\int\left(-\frac{d}{e^2} + \frac{x}{e} + \frac{d^2}{e^2(d+ex)}\right)dx}{6j}$$
$$+ \frac{1}{9}(bgjmn)\int\left(\frac{i^2}{j^3} - \frac{ix}{j^2} + \frac{x^2}{j} - \frac{i^3}{j^3(i+jx)}\right)dx$$
$$- \frac{(bdgjmn)\int\left(-\frac{i}{j^2} + \frac{x}{j} + \frac{i^2}{j^2(i+jx)}\right)dx}{6e}$$

$$= -\frac{agi^2mx}{3j^2} - \frac{bd^2fnx}{3e^2} + \frac{4bd^2gmnx}{9e^2} + \frac{4bgi^2mnx}{9j^2} + \frac{bdgimnx}{3ej} - \frac{5bdgmnx^2}{36e} - \frac{5bgimnx^2}{36j} + \frac{2}{27}bgmnx^3 - \frac{bd^3gmn\log(d+ex)}{9e^3} - \frac{bd^2gimn\log(d+ex)}{6e^2j} - \frac{bgi^2m(d+ex)\log(c(d+ex)^n)}{3ej^2} + \frac{gimx^2(a+b\log(c(d+ex)^n))}{6j} - \frac{1}{9}gmx^3(a+b\log(c(d+ex)^n)) - \frac{bgi^3mn\log(i+jx)}{9j^3} - \frac{bdgi^2mn\log(i+jx)}{6ej^2} + \frac{gi^3m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{3j^3} - \frac{bd^2gn(i+jx)\log(h(i+jx)^m)}{3e^2j} + \frac{bdnx^2(f+g\log(h(i+jx)^m))}{6e} - \frac{1}{9}bnx^3(f+g\log(h(i+jx)^m)) + \frac{bd^3n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{3e^3} + \frac{1}{3}x^3(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) + \frac{bgi^3mn\text{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{3j^3} + \frac{bd^3gmn\text{Li}_2\left(\frac{e(i+jx)}{ei-dj}\right)}{3e^3}$$

## Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 492, normalized size of antiderivative = 0.88

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \frac{6bn\log(d+ex)\left(-6e^3gi^3m\log(i+jx)+6g(e^3i^3-d^3j^3)\,m\log\left(\frac{e(i+jx)}{ei-dj}\right)+dj(-6e^2gi^2m-3degijm+2d^2\right.}{}$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] (6*b*n*Log[d + e*x]*(-6*e^3*g*i^3*m*Log[i + j*x] + 6*g*(e^3*i^3 - d^3*j^3)*
m*Log[(e*(i + j*x))/(e*i - d*j)] + d*j*(-6*e^2*g*i^2*m - 3*d*e*g*i*j*m + 2*
d^2*j^2*(3*f - g*m) + 6*d^2*g*j^2*Log[h*(i + j*x)^m])) + e*(6*g*i*m*(6*a*e^
2*i^2 - b*(2*e^2*i^2 + 3*d*e*i*j + 6*d^2*j^2)*n)*Log[i + j*x] + 6*b*e^2*Log
[c*(d + e*x)^n]*(6*f*j^3*x^3 + g*j*m*x*(-6*i^2 + 3*i*j*x - 2*j^2*x^2) + 6*g
*i^3*m*Log[i + j*x] + 6*g*j^3*x^3*Log[h*(i + j*x)^m]) + j*(6*a*e^2*x*(6*f*j
^2*x^2 + g*m*(-6*i^2 + 3*i*j*x - 2*j^2*x^2)) + b*n*(12*d^2*j^2*(-3*f + 4*g*
m)*x + 3*d*e*(6*f*j^2*x^2 + g*m*(12*i^2 + 12*i*j*x - 5*j^2*x^2)) + e^2*x*(-
12*f*j^2*x^2 + g*m*(48*i^2 - 15*i*j*x + 8*j^2*x^2))) - 6*g*j^2*x*(-6*a*e^2*
x^2 + b*n*(6*d^2 - 3*d*e*x + 2*e^2*x^2))*Log[h*(i + j*x)^m])) + 36*b*g*(e^3
*i^3 - d^3*j^3)*m*n*PolyLog[2, (j*(d + e*x))/(-(e*i) + d*j)])/(108*e^3*j^3)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 234.84 (sec) , antiderivative size = 1724, normalized size of antiderivative = 3.09

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1724 |

```
[In] int(x^2*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m)),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/3/j^3*b*g*i^3*m*n*dilog(((j*x+i)*e+d*j-e*i)/(d*j-e*i))+49/108*b*d^3*g*m*
n/e^3-1/9*n*b*f*x^3+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+
d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)^
n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*a
)*(1/3*(I*g*Pi*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2-I*g*Pi*csgn(I*(j*x+i
)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)-I*g*Pi*csgn(I*h*(j*x+i)^m)^3+I*g*Pi*csgn
(I*h*(j*x+i)^m)^2*csgn(I*h)+2*g*ln(h)+2*f)*x^3+2/3*g*ln((j*x+i)^m)*x^3-2/9*
g*m*x^3+1/3*g*m/j*x^2*i-2/3*g*m/j^2*x*i^2+2/3*g*m/j^3*i^3*ln(j*x+i))-1/3/e^
3*b*d^3*n*g*m*dilog(((e*x+d)*j-d*j+e*i)/(-d*j+e*i))-1/9*n*b*g*ln((j*x+i)^m)
*x^3-1/3/j^3*b*g*i^3*m*n*ln(j*x+i)*ln(((j*x+i)*e+d*j-e*i)/(d*j-e*i))+1/9*b*
d*g*i^2*m*n/e/j^2+2/9*b*d^2*g*i*m*n/e^2/j-1/9/j^3*g*i^3*m*ln((e*x+d)*j-d*j+
e*i)*b*n-1/18*I*n*b*Pi*x^3*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2-1/18*I
*n*b*Pi*x^3*g*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+(1/3*x^3*b*g*ln((j*x+i)^m)+1/
18*b*(3*I*Pi*g*j^3*x^3*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2-3*I*Pi*g*j^3
*x^3*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)-3*I*Pi*g*j^3*x^3*csgn(
I*h*(j*x+i)^m)^3+3*I*Pi*g*j^3*x^3*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+6*j^3*x^3
*ln(h)*g-2*g*j^3*m*x^3+6*f*j^3*x^3+3*g*i*j^2*m*x^2+6*g*i^3*m*ln(j*x+i)-6*g*
i^2*j*m*x)/j^3)*ln((e*x+d)^n)+1/6/e*ln(h)*x^2*b*d*g*n-1/3/e^2*ln(h)*x*b*d^2
*g*n-1/3/e^3*b*d^3*n*g*m*ln(e*x+d)*ln(((e*x+d)*j-d*j+e*i)/(-d*j+e*i))-1/9*l
n(h)*x^3*b*g*n+2/27*b*g*m*n*x^3+1/6/e*n*b*g*ln((j*x+i)^m)*x^2*d-1/3/e^2*n*b
*g*ln((j*x+i)^m)*x*d^2-1/3/e^2/j*g*i*m*ln((e*x+d)*j-d*j+e*i)*b*d^2*n-1/3/e/
j^2*ln(e*x+d)*b*d*g*i^2*m*n-1/6/e/j^2*g*i^2*m*ln((e*x+d)*j-d*j+e*i)*b*d*n-1
/6*I/e^3*n*b*d^3*ln(e*x+d)*Pi*g*csgn(I*h*(j*x+i)^m)^3-1/12*I/e*n*b*Pi*x^2*d
*g*csgn(I*h*(j*x+i)^m)^3+1/6*I/e^2*n*b*x*Pi*d^2*g*csgn(I*h*(j*x+i)^m)^3+1/1
8*I*n*b*Pi*x^3*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+4/9*b*d^2*
g*m*n*x/e^2+4/9*b*g*i^2*m*n*x/j^2-5/36*b*d*g*m*n*x^2/e-5/36*b*g*i*m*n*x^2/j
-1/9*b*d^3*g*m*n*ln(e*x+d)/e^3-1/3*b*d^2*f*n*x/e^2+1/6/e*b*d*f*n*x^2+1/3/e^
3*ln(e*x+d)*b*d^3*f*n+1/3/e^3*n*b*g*ln((j*x+i)^m)*d^3*ln(e*x+d)+1/3/e^3*n*b
*d^3*ln(e*x+d)*ln(h)*g+1/18*I*n*b*Pi*x^3*g*csgn(I*h*(j*x+i)^m)^3+1/3*b*d*g*
i*m*n*x/e/j-1/6*b*d^2*g*i*m*n*ln(e*x+d)/e^2/j+1/6*I/e^2*n*b*x*Pi*d^2*g*csgn
(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)-1/12*I/e*n*b*Pi*x^2*d*g*csgn(I*
(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)-1/6*I/e^3*n*b*d^3*ln(e*x+d)*Pi*g*c
sgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+1/6*I/e^3*n*b*d^3*ln(e*x+d)*
Pi*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2+1/6*I/e^3*n*b*d^3*ln(e*x+d)*Pi
```

```
*g*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+1/12*I/e*n*b*Pi*x^2*d*g*csgn(I*(j*x+i)^m
)*csgn(I*h*(j*x+i)^m)^2-1/6*I/e^2*n*b*x*Pi*d^2*g*csgn(I*h*(j*x+i)^m)^2*csgn
(I*h)+1/12*I/e*n*b*Pi*x^2*d*g*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)-1/6*I/e^2*n*b
*x*Pi*d^2*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2
```

## Fricas [F]

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="f
ricas")
```

```
[Out] integral(b*f*x^2*log((e*x + d)^n*c) + a*f*x^2 + (b*g*x^2*log((e*x + d)^n*c)
 + a*g*x^2)*log((j*x + i)^m*h), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m)),x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="m
axima")
```

```
[Out] 1/3*b*f*x^3*log((e*x + d)^n*c) + 1/3*a*g*x^3*log((j*x + i)^m*h) + 1/3*a*f*x
^3 + 1/18*b*e*f*n*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*
x)/e^3) + 1/18*a*g*j*m*(6*i^3*log(j*x + i)/j^4 - (2*j^2*x^3 - 3*i*j*x^2 + 6
*i^2*x)/j^3) - 1/18*b*g*((6*e^3*i^3*m*n*log(e*x + d)*log(j*x + i) - (3*e^3*
```

```
i*j^2*m*x^2 - 6*e^3*i^2*j*m*x + 6*e^3*i^3*m*log(j*x + i) - 2*(j^3*m - 3*j^3
*log(h))*e^3*x^3)*log((e*x + d)^n) - (6*e^3*j^3*x^3*log((e*x + d)^n) + 3*d*
e^2*j^3*n*x^2 - 6*d^2*e*j^3*n*x + 6*d^3*j^3*n*log(e*x + d) - 2*(e^3*j^3*n -
 3*e^3*j^3*log(c))*x^3)*log((j*x + i)^m))/(e^3*j^3) + 18*integrate(1/18*(2*
(3*(j^3*m - 3*j^3*log(h))*e^4*log(c) - (2*j^3*m*n - 3*j^3*n*log(h))*e^4)*x^
4 + (d*e^3*j^3*m*n + (i*j^2*m*n + 6*i*j^2*n*log(h))*e^4 - 6*(3*e^4*i*j^2*lo
g(h) - (j^3*m - 3*j^3*log(h))*d*e^3)*log(c))*x^3 - 3*(e^4*i^2*j*m*n + d^2*e
^2*j^3*m*n + 6*d*e^3*i*j^2*log(c)*log(h))*x^2 - 6*(e^4*i^3*m*n + d^3*e*j^3*
m*n)*x - 6*(d*e^3*i^3*m*n - d^4*j^3*m*n + (e^4*i^3*m*n - d^3*e*j^3*m*n)*x)*
log(e*x + d))/(e^4*j^3*x^2 + d*e^3*i*j^2 + (e^4*i*j^2 + d*e^3*j^3)*x), x))
```

## Giac [F]

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^n c)+a)(g\log((jx+i)^m h)+f)x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="g
iac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f)*x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int x^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)\,\left(f+g\,\ln\left(h\,{\left(i+j\,x\right)}^m\right)\right)\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)),x)
[Out] int(x^2*(a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)), x)
```

# 3.388 $\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 397

$$\begin{aligned}\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx &= \frac{agimx}{2j}+\frac{bdfnx}{2e}-\frac{3bdgmnx}{4e}-\frac{3bgimnx}{4j}+\frac{1}{4}bgmnx^2+\frac{bd^2gmn\log(d+ex)}{4e^2}\\&+\frac{bgim(d+ex)\log(c(d+ex)^n)}{2ej}-\frac{1}{4}gmx^2(a+b\log(c(d+ex)^n))+\frac{bgi^2mn\log(i+jx)}{4j^2}\\&-\frac{gi^2m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2}+\frac{bdgn(i+jx)\log(h(i+jx)^m)}{2ej}\\&-\frac{1}{4}bnx^2(f+g\log(h(i+jx)^m))-\frac{bd^2n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{2e^2}\\&+\frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\\&-\frac{bgi^2mn\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{2j^2}-\frac{bd^2gmn\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{2e^2}\end{aligned}$$

```
[Out] 1/2*a*g*i*m*x/j+1/2*b*d*f*n*x/e-3/4*b*d*g*m*n*x/e-3/4*b*g*i*m*n*x/j+1/4*b*g
*m*n*x^2+1/4*b*d^2*g*m*n*ln(e*x+d)/e^2+1/2*b*g*i*m*(e*x+d)*ln(c*(e*x+d)^n)/
e/j-1/4*g*m*x^2*(a+b*ln(c*(e*x+d)^n))+1/4*b*g*i^2*m*n*ln(j*x+i)/j^2-1/2*g*i
^2*m*(a+b*ln(c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/j^2+1/2*b*d*g*n*(j*x+i)
*ln(h*(j*x+i)^m)/e/j-1/4*b*n*x^2*(f+g*ln(h*(j*x+i)^m))-1/2*b*d^2*n*ln(-j*(e
*x+d)/(-d*j+e*i))*(f+g*ln(h*(j*x+i)^m))/e^2+1/2*x^2*(a+b*ln(c*(e*x+d)^n))*(
f+g*ln(h*(j*x+i)^m))-1/2*b*g*i^2*m*n*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j^2-1
/2*b*d^2*g*m*n*polylog(2,e*(j*x+i)/(-d*j+e*i))/e^2
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2489, 45, 2463, 2436, 2332, 2442, 2441, 2440, 2438}

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$=\frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))$$
$$-\frac{gi^2m\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{2j^2}-\frac{1}{4}gmx^2(a+b\log(c(d+ex)^n))+\frac{agimx}{2j}$$
$$+\frac{bgim(d+ex)\log(c(d+ex)^n)}{2ej}-\frac{bd^2n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{2e^2}$$
$$-\frac{bd^2gmn\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{2e^2}+\frac{bd^2gmn\log(d+ex)}{4e^2}+\frac{bdfnx}{2e}$$
$$+\frac{bdgn(i+jx)\log(h(i+jx)^m)}{2ej}-\frac{bgi^2mn\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{2j^2}-\frac{3bdgmnx}{4e}$$
$$-\frac{1}{4}bnx^2(f+g\log(h(i+jx)^m))+\frac{bgi^2mn\log(i+jx)}{4j^2}-\frac{3bgimnx}{4j}+\frac{1}{4}bgmnx^2$$

```
[In] Int[x*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] (a*g*i*m*x)/(2*j) + (b*d*f*n*x)/(2*e) - (3*b*d*g*m*n*x)/(4*e) - (3*b*g*i*m*
n*x)/(4*j) + (b*g*m*n*x^2)/4 + (b*d^2*g*m*n*Log[d + e*x])/(4*e^2) + (b*g*i*
m*(d + e*x)*Log[c*(d + e*x)^n])/(2*e*j) - (g*m*x^2*(a + b*Log[c*(d + e*x)^n
]))/4 + (b*g*i^2*m*n*Log[i + j*x])/(4*j^2) - (g*i^2*m*(a + b*Log[c*(d + e*x
)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/(2*j^2) + (b*d*g*n*(i + j*x)*Log[h*(i
 + j*x)^m])/(2*e*j) - (b*n*x^2*(f + g*Log[h*(i + j*x)^m]))/4 - (b*d^2*n*Log
[-((j*(d + e*x))/(e*i - d*j))]*(f + g*Log[h*(i + j*x)^m]))/(2*e^2) + (x^2*(
a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/2 - (b*g*i^2*m*n*Poly
Log[2, -((j*(d + e*x))/(e*i - d*j))])/(2*j^2) - (b*d^2*g*m*n*PolyLog[2, (e*
(i + j*x))/(e*i - d*j)])/(2*e^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
```

```
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\
&\quad -\frac{1}{2}(gjm)\int\frac{x^2(a+b\log(c(d+ex)^n))}{i+jx}\,dx \\
&\quad -\frac{1}{2}(ben)\int\frac{x^2(f+g\log(h(i+jx)^m))}{d+ex}\,dx \\
&= \frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\
&\quad -\frac{1}{2}(gjm)\int\left(-\frac{i(a+b\log(c(d+ex)^n))}{j^2}+\frac{x(a+b\log(c(d+ex)^n))}{j}\right. \\
&\qquad \left.+\frac{i^2(a+b\log(c(d+ex)^n))}{j^2(i+jx)}\right)dx-\frac{1}{2}(ben)\int\left(-\frac{d(f+g\log(h(i+jx)^m))}{e^2}\right. \\
&\qquad \left.+\frac{x(f+g\log(h(i+jx)^m))}{e}+\frac{d^2(f+g\log(h(i+jx)^m))}{e^2(d+ex)}\right)dx \\
&= \frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\
&\quad -\frac{1}{2}(gm)\int x(a+b\log(c(d+ex)^n))\,dx+\frac{(gim)\int(a+b\log(c(d+ex)^n))\,dx}{2j} \\
&\quad -\frac{(gi^2m)\int\frac{a+b\log(c(d+ex)^n)}{i+jx}\,dx}{2j}-\frac{1}{2}(bn)\int x(f+g\log(h(i+jx)^m))\,dx \\
&\quad +\frac{(bdn)\int(f+g\log(h(i+jx)^m))\,dx}{2e}-\frac{(bd^2n)\int\frac{f+g\log(h(i+jx)^m)}{d+ex}\,dx}{2e}
\end{aligned}
$$

$$= \frac{agimx}{2j} + \frac{bdfnx}{2e} - \frac{1}{4}gmx^2(a + b\log(c(d+ex)^n)) - \frac{gi^2m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2} - \frac{1}{4}bnx^2(f+g\log(h(i+jx)^m)) - \frac{bd^2n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{2e^2} + \frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) + \frac{(bgim)\int\log(c(d+ex)^n)\,dx}{2j} + \frac{(bdgn)\int\log(h(i+jx)^m)\,dx}{2e} + \frac{1}{4}(begmn)\int\frac{x^2}{d+ex}\,dx + \frac{(begi^2mn)\int\frac{\log\left(\frac{e(i+jx)}{ei-dj}\right)}{d+ex}\,dx}{2j^2} + \frac{1}{4}(bgjmn)\int\frac{x^2}{i+jx}\,dx + \frac{(bd^2gjmn)\int\frac{\log\left(\frac{j(d+ex)}{-ei+dj}\right)}{i+jx}\,dx}{2e^2}$$

$$= \frac{agimx}{2j} + \frac{bdfnx}{2e} - \frac{1}{4}gmx^2(a + b\log(c(d+ex)^n)) - \frac{gi^2m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2} - \frac{1}{4}bnx^2(f+g\log(h(i+jx)^m)) - \frac{bd^2n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{2e^2} + \frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) + \frac{(bgim)\text{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{2ej} + \frac{(bdgn)\text{Subst}\left(\int\log(hx^m)\,dx,x,i+jx\right)}{2ej} + \frac{(bd^2gmn)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{ex}{-ei+dj}\right)}{x}\,dx,x,i+jx\right)}{2e^2} + \frac{1}{4}(begmn)\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx + \frac{(bgi^2mn)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{jx}{ei-dj}\right)}{x}\,dx,x,d+ex\right)}{2j^2} + \frac{1}{4}(bgjmn)\int\left(-\frac{i}{j^2}+\frac{x}{j}+\frac{i^2}{j^2(i+jx)}\right)dx$$

$$
\begin{aligned}
&= \frac{agimx}{2j} + \frac{bdfnx}{2e} - \frac{3bdgmnx}{4e} - \frac{3bgimnx}{4j} + \frac{1}{4}bgmnx^2 \\
&+ \frac{bd^2gmn\log(d+ex)}{4e^2} + \frac{bgim(d+ex)\log(c(d+ex)^n)}{2ej} \\
&- \frac{1}{4}gmx^2(a+b\log(c(d+ex)^n)) + \frac{bgi^2mn\log(i+jx)}{4j^2} \\
&- \frac{gi^2m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2} + \frac{bdgn(i+jx)\log(h(i+jx)^m)}{2ej} \\
&- \frac{1}{4}bnx^2(f+g\log(h(i+jx)^m)) - \frac{bd^2n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{2e^2} \\
&+ \frac{1}{2}x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\
&- \frac{bgi^2mn\text{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{2j^2} - \frac{bd^2gmn\text{Li}_2\left(\frac{e(i+jx)}{ei-dj}\right)}{2e^2}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.86

$$
\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx
$$

$$
= \frac{bn\log(d+ex)\left(2e^2gi^2m\log(i+jx) + 2g(-e^2i^2+d^2j^2)\,m\log\left(\frac{e(i+jx)}{ei-dj}\right) + dj(-2dfj+2egim+dgjm-2\right.}{}
$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] (b*n*Log[d + e*x]*(2*e^2*g*i^2*m*Log[i + j*x] + 2*g*(-(e^2*i^2) + d^2*j^2)*
m*Log[(e*(i + j*x))/(e*i - d*j)] + d*j*(-2*d*f*j + 2*e*g*i*m + d*g*j*m - 2*
d*g*j*Log[h*(i + j*x)^m])) + e*(g*i*m*(-2*a*e*i + b*(e*i + 2*d*j)*n)*Log[i
+ j*x] + j*(a*e*x*(2*f*j*x + g*m*(2*i - j*x)) - b*n*(e*x*(3*g*i*m + f*j*x -
 g*j*m*x) + d*(2*g*i*m - 2*f*j*x + 3*g*j*m*x)) + g*j*x*(2*a*e*x + b*n*(2*d
- e*x))*Log[h*(i + j*x)^m]) + b*e*Log[c*(d + e*x)^n]*(-2*g*i^2*m*Log[i + j*
x] + j*x*(2*g*i*m + 2*f*j*x - g*j*m*x + 2*g*j*x*Log[h*(i + j*x)^m]))) + 2*b
*g*(-(e^2*i^2) + d^2*j^2)*m*n*PolyLog[2, (j*(d + e*x))/(-(e*i) + d*j)])/(4*
e^2*j^2)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 111.26 (sec) , antiderivative size = 1372, normalized size of antiderivative = 3.46

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1372 |

```
[In] int(x*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e*n*b*g*ln((j*x+i)^m)*x*d-1/2*n*b*d^2*f/e^2*ln(e*x+d)-1/4*b*f*n*x^2-1/4
*ln(h)*x^2*b*g*n+(-1/4*I*b*Pi*csgn(I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^
n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)^n)*
csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*csgn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*a)*(
1/2*(I*g*Pi*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2-I*g*Pi*csgn(I*(j*x+i)^m
)*csgn(I*h*(j*x+i)^m)*csgn(I*h)-I*g*Pi*csgn(I*h*(j*x+i)^m)^3+I*g*Pi*csgn(I*
h*(j*x+i)^m)^2*csgn(I*h)+2*g*ln(h)+2*f)*x^2+g*ln((j*x+i)^m)*x^2-1/2*g*m*x^2
+g*m/j*x*i-g*m/j^2*i^2*ln(j*x+i))+1/2/e*n*b/j*d*ln(e*x+d)*g*i*m+1/2/e*n*b*g
*m/j*i*ln((e*x+d)*j-d*j+e*i)*d-1/4*I/e*n*b*x*Pi*d*g*csgn(I*h*(j*x+i)^m)^3+1
/4*I/e^2*n*b*d^2*ln(e*x+d)*Pi*g*csgn(I*h*(j*x+i)^m)^3+1/8*I*n*b*Pi*x^2*g*cs
gn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+(1/2*x^2*b*g*ln((j*x+i)^m)-1/
4*b*(-I*Pi*g*j^2*x^2*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2+I*Pi*g*j^2*x^2
*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+I*Pi*g*j^2*x^2*csgn(I*h*(j
*x+i)^m)^3-I*Pi*g*j^2*x^2*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)-2*j^2*x^2*ln(h)*g
+g*j^2*m*x^2+2*g*i^2*m*ln(j*x+i)-2*f*j^2*x^2-2*g*i*j*m*x)/j^2)*ln((e*x+d)^n
)-1/4*b*d*g*i*m*n/e/j+1/2/e^2*n*b*g*m*d^2*ln(e*x+d)*ln(((e*x+d)*j-d*j+e*i)/
(-d*j+e*i))+1/2*n*b*g*i^2*m/j^2*ln(j*x+i)*ln(((j*x+i)*e+d*j-e*i)/(d*j-e*i))
-1/8*I*n*b*Pi*x^2*g*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)-1/8*I*n*b*Pi*x^2*g*csgn
(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2-5/8*b*d^2*g*m*n/e^2-1/4*I/e*n*b*x*Pi*d*
g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+1/4*I/e^2*n*b*d^2*ln(e*x+
d)*Pi*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+1/2/e*ln(h)*x*b*d*g
*n+1/4*b*g*m*n*x^2+1/4*n*b*g*m/j^2*i^2*ln((e*x+d)*j-d*j+e*i)+1/2*n*b*g*i^2*
m/j^2*dilog(((j*x+i)*e+d*j-e*i)/(d*j-e*i))-1/2/e^2*n*b*g*ln((j*x+i)^m)*d^2*
ln(e*x+d)+1/2/e^2*n*b*g*m*d^2*dilog(((e*x+d)*j-d*j+e*i)/(-d*j+e*i))-1/2/e^2
*n*b*d^2*ln(e*x+d)*ln(h)*g+1/8*I*n*b*Pi*x^2*g*csgn(I*h*(j*x+i)^m)^3-1/4*n*b
*g*ln((j*x+i)^m)*x^2-3/4*b*d*g*m*n*x/e-3/4*b*g*i*m*n*x/j+1/4*b*d^2*g*m*n*ln
(e*x+d)/e^2+1/4*I/e*n*b*x*Pi*d*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2+1/
4*I/e*n*b*x*Pi*d*g*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)-1/4*I/e^2*n*b*d^2*ln(e*x
+d)*Pi*g*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2-1/4*I/e^2*n*b*d^2*ln(e*x+d
)*Pi*g*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+1/2*b*d*f*n*x/e
```

## Fricas [F]

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$=\int(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)x\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="fri
cas")
```

```
[Out] integral(b*f*x*log((e*x + d)^n*c) + a*f*x + (b*g*x*log((e*x + d)^n*c) + a*g
*x)*log((j*x + i)^m*h), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m)),x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$=\int(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)x\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="max
ima")
```

```
[Out] -1/4*b*e*f*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) - 1/4*a*g*j*m*(
2*i^2*log(j*x + i)/j^3 + (j*x^2 - 2*i*x)/j^2) + 1/2*b*f*x^2*log((e*x + d)^n
*c) + 1/2*a*g*x^2*log((j*x + i)^m*h) + 1/2*a*f*x^2 + 1/4*b*g*((2*e^2*i^2*m*
n*log(e*x + d)*log(j*x + i) + (2*e^2*i*j*m*x - 2*e^2*i^2*m*log(j*x + i) - (
j^2*m - 2*j^2*log(h))*e^2*x^2)*log((e*x + d)^n) + (2*e^2*j^2*x^2*log((e*x +
 d)^n) + 2*d*e*j^2*n*x - 2*d^2*j^2*n*log(e*x + d) - (e^2*j^2*n - 2*e^2*j^2*
log(c))*x^2)*log((j*x + i)^m))/(e^2*j^2) + 4*integrate(-1/4*(2*((j^2*m - 2*
j^2*log(h))*e^3*log(c) - (j^2*m*n - j^2*n*log(h))*e^3)*x^3 + (d*e^2*j^2*m*n
 + (i*j*m*n + 2*i*j*n*log(h))*e^3 - 2*(2*e^3*i*j*log(h) - (j^2*m - 2*j^2*lo
g(h))*d*e^2)*log(c))*x^2 + 2*(e^3*i^2*m*n + d^2*e*j^2*m*n - 2*d*e^2*i*j*log
(c)*log(h))*x + 2*(d*e^2*i^2*m*n - d^3*j^2*m*n + (e^3*i^2*m*n - d^2*e*j^2*m
*n)*x)*log(e*x + d))/(e^3*j^2*x^2 + d*e^2*i*j + (e^3*i*j + d*e^2*j^2)*x), x
))
```

## Giac [F]

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$=\int(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)x\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="gia
c")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f)*x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$=\int x\,(a+b\,\ln(c\,(d+e\,x)^n))\,(f+g\,\ln(h\,(i+j\,x)^m))\,dx$$

```
[In] int(x*(a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)),x)
```

```
[Out] int(x*(a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)), x)
```

# 3.389 $\int (a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 232

$$\begin{aligned}\int (a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx &= -agmx-bfnx+2bgmnx-\frac{bgm(d+ex)\log(c(d+ex)^n)}{e}\\ &\quad+\frac{gim(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}-\frac{bgn(i+jx)\log(h(i+jx)^m)}{j}\\ &\quad+\frac{bdn\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{e}\\ &\quad+x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\\ &\quad+\frac{bgimn\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j}+\frac{bdgmn\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{e}\end{aligned}$$

```
[Out] -a*g*m*x-b*f*n*x+2*b*g*m*n*x-b*g*m*(e*x+d)*ln(c*(e*x+d)^n)/e+g*i*m*(a+b*ln(
c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/j-b*g*n*(j*x+i)*ln(h*(j*x+i)^m)/j+b*
d*n*ln(-j*(e*x+d)/(-d*j+e*i))*(f+g*ln(h*(j*x+i)^m))/e+x*(a+b*ln(c*(e*x+d)^n
))*(f+g*ln(h*(j*x+i)^m))+b*g*i*m*n*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j+b*d*g
*m*n*polylog(2,e*(j*x+i)/(-d*j+e*i))/e
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2479, 45, 2463, 2436, 2332, 2441, 2440, 2438}

$$\begin{aligned}&\int (a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx\\&=x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\\&+\frac{gim\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{j}-agmx-\frac{bgm(d+ex)\log(c(d+ex)^n)}{e}\\&+\frac{bdn\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{e}+\frac{bgimn\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j}\\&+\frac{bdgmn\operatorname{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{e}-bfnx-\frac{bgn(i+jx)\log(h(i+jx)^m)}{j}+2bgmnx\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] -(a*g*m*x) - b*f*n*x + 2*b*g*m*n*x - (b*g*m*(d + e*x)*Log[c*(d + e*x)^n])/e
 + (g*i*m*(a + b*Log[c*(d + e*x)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/j - (b
*g*n*(i + j*x)*Log[h*(i + j*x)^m])/j + (b*d*n*Log[-((j*(d + e*x))/(e*i - d*
j))]*(f + g*Log[h*(i + j*x)^m]))/e + x*(a + b*Log[c*(d + e*x)^n])*(f + g*Lo
g[h*(i + j*x)^m]) + (b*g*i*m*n*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/j
+ (b*d*g*m*n*PolyLog[2, (e*(i + j*x))/(e*i - d*j)])/e
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2479

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[x*((a +
 b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Dist[b*e*n*p, Int[x*(a + b*Lo
g[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\
&\quad -(gjm)\int \frac{x(a+b\log(c(d+ex)^n))}{i+jx}\,dx-(ben)\int \frac{x(f+g\log(h(i+jx)^m))}{d+ex}\,dx \\
&= x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) \\
&\quad -(gjm)\int \left(\frac{a+b\log(c(d+ex)^n)}{j}-\frac{i(a+b\log(c(d+ex)^n))}{j(i+jx)}\right)dx \\
&\quad -(ben)\int \left(\frac{f+g\log(h(i+jx)^m)}{e}-\frac{d(f+g\log(h(i+jx)^m))}{e(d+ex)}\right)dx
\end{aligned}$$

$$= x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) - (gm)\int(a+b\log(c(d+ex)^n))\,dx + (gim)\int\frac{a+b\log(c(d+ex)^n)}{i+jx}\,dx - (bn)\int(f+g\log(h(i+jx)^m))\,dx + (bdn)\int\frac{f+g\log(h(i+jx)^m)}{d+ex}\,dx$$

$$= -agmx - bfnx + \frac{gim(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} + \frac{bdn\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{e} + x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) - (bgm)\int\log(c(d+ex)^n)\,dx - (bgn)\int\log(h(i+jx)^m)\,dx - \frac{(begimn)\int\frac{\log\left(\frac{e(i+jx)}{ei-dj}\right)}{d+ex}\,dx}{j} - \frac{(bdgjmn)\int\frac{\log\left(\frac{j(d+ex)}{-ei+dj}\right)}{i+jx}\,dx}{e}$$

$$= -agmx - bfnx + \frac{gim(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} + \frac{bdn\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{e} + x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) - \frac{(bgm)\text{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{e} - \frac{(bgn)\text{Subst}\left(\int\log(hx^m)\,dx,x,i+jx\right)}{j} - \frac{(bdgmn)\text{Subst}\left(\int\frac{\log\left(1+\frac{ex}{-ei+dj}\right)}{x}\,dx,x,i+jx\right)}{e} - \frac{(bgimn)\text{Subst}\left(\int\frac{\log\left(1+\frac{jx}{ei-dj}\right)}{x}\,dx,x,d+ex\right)}{j}$$

$$= -agmx - bfnx + 2bgmnx - \frac{bgm(d+ex)\log(c(d+ex)^n)}{e} + \frac{gim(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} - \frac{bgn(i+jx)\log(h(i+jx)^m)}{j} + \frac{bdn\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{e} + x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m)) + \frac{bgimn\text{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} + \frac{bdgmn\text{Li}_2\left(\frac{e(i+jx)}{ei-dj}\right)}{e}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.42

$$\int (a+b\log (c(d+ex)^n)) (f+g\log (h(i+jx)^m)) \, dx$$

$$= \frac{-bdfjn+bdgjmn+aefjx-aegjmx-befjnx+2begjmnx+befjx\log (c(d+ex)^n)-begjmx\log (c(}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] (-(b*d*f*j*n) + b*d*g*j*m*n + a*e*f*j*x - a*e*g*j*m*x - b*e*f*j*n*x + 2*b*e
*g*j*m*n*x + b*e*f*j*x*Log[c*(d + e*x)^n] - b*e*g*j*m*x*Log[c*(d + e*x)^n]
+ a*e*g*i*m*Log[i + j*x] - b*e*g*i*m*n*Log[i + j*x] + b*d*g*j*m*n*Log[i + j
*x] + b*e*g*i*m*Log[c*(d + e*x)^n]*Log[i + j*x] - b*d*g*j*n*Log[h*(i + j*x)
^m] + a*e*g*j*x*Log[h*(i + j*x)^m] - b*e*g*j*n*x*Log[h*(i + j*x)^m] + b*e*g
*j*x*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] + b*n*Log[d + e*x]*(-(e*g*i*m*Lo
g[i + j*x]) + g*(e*i - d*j)*m*Log[(e*(i + j*x))/(e*i - d*j)] + d*j*(f - g*m
 + g*Log[h*(i + j*x)^m])) + b*g*(e*i - d*j)*m*n*PolyLog[2, (j*(d + e*x))/(-
(e*i) + d*j)])/(e*j)
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 49.44 (sec) , antiderivative size = 1012, normalized size of antiderivative = 4.36

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1012 |

```
[In] int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m)),x,method=_RETURNVERBOSE)
```

```
[Out] (x*b*g*ln((j*x+i)^m)+1/2*b*(I*Pi*g*j*x*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m
)^2-I*Pi*g*j*x*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)-I*Pi*g*j*x*c
sgn(I*h*(j*x+i)^m)^3+I*Pi*g*j*x*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+2*j*x*ln(h)
*g+2*g*i*m*ln(j*x+i)-2*x*g*m*j+2*f*j*x)/j)*ln((e*x+d)^n)+(-1/4*I*b*Pi*csgn(
I*c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)+1/4*I*b*Pi*csgn(I*c)*csgn(I*c*(e
*x+d)^n)^2+1/4*I*b*Pi*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)^2-1/4*I*b*Pi*cs
gn(I*c*(e*x+d)^n)^3+1/2*b*ln(c)+1/2*a)*(I*Pi*g*x*csgn(I*(j*x+i)^m)*csgn(I*h
*(j*x+i)^m)^2+I*Pi*g*x*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+2*f*x+2*ln(h)*g*x+2*
g*ln((j*x+i)^m)*x-2*g*m*x+2*g*m/j*i*ln(j*x+i)-I*Pi*g*x*csgn(I*h*(j*x+i)^m)^
3-I*Pi*g*x*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h))+1/2*I*n*b*x*g*P
i*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+1/2*I*n*b*x*g*Pi*csgn(I*h
*(j*x+i)^m)^3-1/2*I*n*b*x*g*Pi*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)+1/2*I/e*n*b*
d*ln(e*x+d)*g*Pi*csgn(I*h*(j*x+i)^m)^2*csgn(I*h)-ln(h)*x*b*g*n+2*b*g*m*n*x-
```

```
b*f*n*x+1/2*I/e*n*b*d*ln(e*x+d)*g*Pi*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^
2-1/2*I*n*b*x*g*Pi*csgn(I*(j*x+i)^m)*csgn(I*h*(j*x+i)^m)^2-1/2*I/e*n*b*d*ln
(e*x+d)*g*Pi*csgn(I*h*(j*x+i)^m)^3-1/2*I/e*n*b*d*ln(e*x+d)*g*Pi*csgn(I*(j*x
+i)^m)*csgn(I*h*(j*x+i)^m)*csgn(I*h)+1/e*n*b*d*ln(e*x+d)*g*ln(h)-1/e*n*b*d*
ln(e*x+d)*g*m+b*f/e*n*d*ln(e*x+d)-n*b*g*ln((j*x+i)^m)*x+1/e*n*b*g*ln((j*x+i
)^m)*d*ln(e*x+d)+b*d*g*m*n/e-n*b*g*m/j*i*ln((e*x+d)*j-d*j+e*i)-1/e*n*b*g*m*
d*dilog(((e*x+d)*j-d*j+e*i)/(-d*j+e*i))-1/e*n*b*g*m*d*ln(e*x+d)*ln(((e*x+d)
*j-d*j+e*i)/(-d*j+e*i))-n*b*g*i*m/j*dilog(((j*x+i)*e+d*j-e*i)/(d*j-e*i))-n*
b*g*i*m/j*ln(j*x+i)*ln(((j*x+i)*e+d*j-e*i)/(d*j-e*i))
```

## Fricas [F]

$$\int (a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="frica
s")
```

```
[Out] integral(b*f*log((e*x + d)^n*c) + a*f + (b*g*log((e*x + d)^n*c) + a*g)*log(
(j*x + i)^m*h), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m)),x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int (a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="maxim
a")
```

```
[Out] -b*e*f*n*(x/e - d*log(e*x + d)/e^2) - a*g*j*m*(x/j - i*log(j*x + i)/j^2) +
b*f*x*log((e*x + d)^n*c) + a*g*x*log((j*x + i)^m*h) + a*f*x - b*g*((e*i*m*n
*log(e*x + d)*log(j*x + i) - (e*i*m*log(j*x + i) - (j*m - j*log(h))*e*x)*lo
g((e*x + d)^n) - (d*j*n*log(e*x + d) + e*j*x*log((e*x + d)^n) - (e*j*n - e*
j*log(c))*x)*log((j*x + i)^m))/(e*j) + integrate(-(d*e*i*log(c)*log(h) - ((
j*m - j*log(h))*e^2*log(c) - (2*j*m*n - j*n*log(h))*e^2)*x^2 + (d*e*j*m*n +
 (i*m*n - i*n*log(h))*e^2 + (e^2*i*log(h) - (j*m - j*log(h))*d*e)*log(c))*x
 + (d*e*i*m*n - d^2*j*m*n + (e^2*i*m*n - d*e*j*m*n)*x)*log(e*x + d))/(e^2*j
*x^2 + d*e*i + (e^2*i + d*e*j)*x), x))
```

## Giac [F]

$$\int (a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$=\int (b\log((ex+d)^n c)+a)(g\log((jx+i)^m h)+f)\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m)),x, algorithm="giac"
)
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))\,dx$$
$$=\int (a+b\,\ln(c\,(d+e\,x)^n))\,(f+g\,\ln(h\,(i+j\,x)^m))\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)), x)
```

# 3.390 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx$

## Optimal result

Integrand size = 32, antiderivative size = 637

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx$$
$$= f\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))+bgmn\log\left(-\frac{ex}{d}\right)\log(d+ex)\log(i+jx)$$
$$-bgm\log\left(-\frac{jx}{i}\right)(n\log(d+ex)-\log(c(d+ex)^n))\log(i+jx)$$
$$+\frac{1}{2}bgmn\left(\log\left(-\frac{ex}{d}\right)+\log\left(\frac{ei-dj}{e(i+jx)}\right)-\log\left(-\frac{(ei-dj)x}{d(i+jx)}\right)\right)\log^2\left(\frac{d(i+jx)}{i(d+ex)}\right)$$
$$-\frac{1}{2}bgmn\left(\log\left(-\frac{ex}{d}\right)-\log\left(-\frac{jx}{i}\right)\right)\left(\log(d+ex)+\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)^2$$
$$-bg\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)(m\log(i+jx)-\log(h(i+jx)^m))$$
$$+ag\log\left(-\frac{jx}{i}\right)\log(h(i+jx)^m)+bfn\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)$$
$$+bgmn\left(\log(i+jx)-\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)$$
$$-bgn(m\log(i+jx)-\log(h(i+jx)^m))\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)$$
$$+bgmn\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\operatorname{PolyLog}\left(2,\frac{i(d+ex)}{d(i+jx)}\right)$$
$$-bgmn\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\operatorname{PolyLog}\left(2,\frac{j(d+ex)}{e(i+jx)}\right)+agm\operatorname{PolyLog}\left(2,1+\frac{jx}{i}\right)$$
$$-bgm(n\log(d+ex)-\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,1+\frac{jx}{i}\right)$$
$$+bgmn\left(\log(d+ex)+\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)\operatorname{PolyLog}\left(2,1+\frac{jx}{i}\right)$$
$$-bgmn\operatorname{PolyLog}\left(3,1+\frac{ex}{d}\right)+bgmn\operatorname{PolyLog}\left(3,\frac{i(d+ex)}{d(i+jx)}\right)$$
$$-bgmn\operatorname{PolyLog}\left(3,\frac{j(d+ex)}{e(i+jx)}\right)-bgmn\operatorname{PolyLog}\left(3,1+\frac{jx}{i}\right)$$

```
[Out] f*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))+b*g*m*n*ln(-e*x/d)*ln(e*x+d)*ln(j*x+i)-b
*g*m*ln(-j*x/i)*(n*ln(e*x+d)-ln(c*(e*x+d)^n))*ln(j*x+i)+1/2*b*g*m*n*(ln(-e*
x/d)+ln((-d*j+e*i)/e/(j*x+i))-ln(-(-d*j+e*i)*x/d/(j*x+i)))*ln(d*(j*x+i)/i/(
e*x+d))^2-1/2*b*g*m*n*(ln(-e*x/d)-ln(-j*x/i))*(ln(e*x+d)+ln(d*(j*x+i)/i/(e*
x+d)))^2-b*g*ln(-e*x/d)*ln(c*(e*x+d)^n)*(m*ln(j*x+i)-ln(h*(j*x+i)^m))+a*g*l
n(-j*x/i)*ln(h*(j*x+i)^m)+b*f*n*polylog(2,1+e*x/d)+b*g*m*n*(ln(j*x+i)-ln(d*
(j*x+i)/i/(e*x+d)))*polylog(2,1+e*x/d)-b*g*n*(m*ln(j*x+i)-ln(h*(j*x+i)^m))*
polylog(2,1+e*x/d)+b*g*m*n*ln(d*(j*x+i)/i/(e*x+d))*polylog(2,i*(e*x+d)/d/(j
*x+i))-b*g*m*n*ln(d*(j*x+i)/i/(e*x+d))*polylog(2,j*(e*x+d)/e/(j*x+i))+a*g*m
```

```
*polylog(2,1+j*x/i)-b*g*m*(n*ln(e*x+d)-ln(c*(e*x+d)^n))*polylog(2,1+j*x/i)+
b*g*m*n*(ln(e*x+d)+ln(d*(j*x+i)/i/(e*x+d)))*polylog(2,1+j*x/i)-b*g*m*n*poly
log(3,1+e*x/d)+b*g*m*n*polylog(3,i*(e*x+d)/d/(j*x+i))-b*g*m*n*polylog(3,j*(
e*x+d)/e/(j*x+i))-b*g*m*n*polylog(3,1+j*x/i)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2488, 2441, 2352, 2487, 2485}

$$\begin{aligned}
&\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx \\
&= f\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n)) \\
&+ag\log\left(-\frac{jx}{i}\right)\log(h(i+jx)^m)+agm\,\text{PolyLog}\left(2,\frac{jx}{i}+1\right) \\
&-bg\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)(m\log(i+jx)-\log(h(i+jx)^m)) \\
&-bgm\,\text{PolyLog}\left(2,\frac{jx}{i}+1\right)(n\log(d+ex)-\log(c(d+ex)^n)) \\
&-bgm\log\left(-\frac{jx}{i}\right)\log(i+jx)(n\log(d+ex)-\log(c(d+ex)^n)) \\
&+bfn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)-bgn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)(m\log(i+jx)-\log(h(i+jx)^m)) \\
&+bgmn\,\text{PolyLog}\left(3,\frac{i(d+ex)}{d(i+jx)}\right)-bgmn\,\text{PolyLog}\left(3,\frac{j(d+ex)}{e(i+jx)}\right) \\
&+bgmn\,\text{PolyLog}\left(2,\frac{i(d+ex)}{d(i+jx)}\right)\log\left(\frac{d(i+jx)}{i(d+ex)}\right) \\
&-bgmn\,\text{PolyLog}\left(2,\frac{j(d+ex)}{e(i+jx)}\right)\log\left(\frac{d(i+jx)}{i(d+ex)}\right) \\
&+bgmn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)\left(\log(i+jx)-\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right) \\
&+bgmn\,\text{PolyLog}\left(2,\frac{jx}{i}+1\right)\left(\log\left(\frac{d(i+jx)}{i(d+ex)}\right)+\log(d+ex)\right) \\
&+\frac{1}{2}bgmn\left(\log\left(\frac{ei-dj}{e(i+jx)}\right)-\log\left(-\frac{x(ei-dj)}{d(i+jx)}\right)+\log\left(-\frac{ex}{d}\right)\right)\log^2\left(\frac{d(i+jx)}{i(d+ex)}\right) \\
&-\frac{1}{2}bgmn\left(\log\left(-\frac{ex}{d}\right)-\log\left(-\frac{jx}{i}\right)\right)\left(\log\left(\frac{d(i+jx)}{i(d+ex)}\right)+\log(d+ex)\right)^2 \\
&+bgmn\log\left(-\frac{ex}{d}\right)\log(d+ex)\log(i+jx) \\
&-bgmn\,\text{PolyLog}\left(3,\frac{ex}{d}+1\right)-bgmn\,\text{PolyLog}\left(3,\frac{jx}{i}+1\right)
\end{aligned}$$

```
[In] Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x,x]
```

```
[Out] f*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]) + b*g*m*n*Log[-((e*x)/d)]*Log[
d + e*x]*Log[i + j*x] - b*g*m*Log[-((j*x)/i)]*(n*Log[d + e*x] - Log[c*(d +
e*x)^n])*Log[i + j*x] + (b*g*m*n*(Log[-((e*x)/d)] + Log[(e*i - d*j)/(e*(i +
 j*x))] - Log[-(((e*i - d*j)*x)/(d*(i + j*x)))])*Log[(d*(i + j*x))/(i*(d +
e*x))]^2)/2 - (b*g*m*n*(Log[-((e*x)/d)] - Log[-((j*x)/i)])*(Log[d + e*x] +
Log[(d*(i + j*x))/(i*(d + e*x))])^2)/2 - b*g*Log[-((e*x)/d)]*Log[c*(d + e*x
)^n]*(m*Log[i + j*x] - Log[h*(i + j*x)^m]) + a*g*Log[-((j*x)/i)]*Log[h*(i +
 j*x)^m] + b*f*n*PolyLog[2, 1 + (e*x)/d] + b*g*m*n*(Log[i + j*x] - Log[(d*(
i + j*x))/(i*(d + e*x))])*PolyLog[2, 1 + (e*x)/d] - b*g*n*(m*Log[i + j*x] -
 Log[h*(i + j*x)^m])*PolyLog[2, 1 + (e*x)/d] + b*g*m*n*Log[(d*(i + j*x))/(i
*(d + e*x))]*PolyLog[2, (i*(d + e*x))/(d*(i + j*x))] - b*g*m*n*Log[(d*(i +
j*x))/(i*(d + e*x))]*PolyLog[2, (j*(d + e*x))/(e*(i + j*x))] + a*g*m*PolyLo
g[2, 1 + (j*x)/i] - b*g*m*(n*Log[d + e*x] - Log[c*(d + e*x)^n])*PolyLog[2,
1 + (j*x)/i] + b*g*m*n*(Log[d + e*x] + Log[(d*(i + j*x))/(i*(d + e*x))])*Po
lyLog[2, 1 + (j*x)/i] - b*g*m*n*PolyLog[3, 1 + (e*x)/d] + b*g*m*n*PolyLog[3
, (i*(d + e*x))/(d*(i + j*x))] - b*g*m*n*PolyLog[3, (j*(d + e*x))/(e*(i + j
*x))] - b*g*m*n*PolyLog[3, 1 + (j*x)/i]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2487

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[Log[i + j*x]*(Log[c*(d + e*x)^n]/x), x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2488

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(Log[(h_.)*((i_.)
 + (j_.)*(x_))^(m_.)]*(g_.) + (f_)))/(x_), x_Symbol] :> Dist[f, Int[(a + b*
Log[c*(d + e*x)^n])/x, x], x] + Dist[g, Int[Log[h*(i + j*x)^m]*((a + b*Log[
c*(d + e*x)^n])/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x
] && NeQ[e*i - d*j, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= f\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx+g\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(h(i+jx)^m\right)}{x}\,dx\\
&= f\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+(ag)\int \frac{\log\left(h(i+jx)^m\right)}{x}\,dx\\
&\quad+(bg)\int \frac{\log\left(c(d+ex)^n\right)\log\left(h(i+jx)^m\right)}{x}\,dx-(befn)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx\\
&= f\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)\\
&\quad+ag\log\left(-\frac{jx}{i}\right)\log\left(h(i+jx)^m\right)+bfn\text{Li}_2\left(1+\frac{ex}{d}\right)\\
&\quad+(bgm)\int \frac{\log\left(c(d+ex)^n\right)\log(i+jx)}{x}\,dx-(agjm)\int \frac{\log\left(-\frac{jx}{i}\right)}{i+jx}\,dx\\
&\quad-\left(bg\left(m\log(i+jx)-\log\left(h(i+jx)^m\right)\right)\right)\int \frac{\log\left(c(d+ex)^n\right)}{x}\,dx\\
&= f\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)\\
&\quad-bg\log\left(-\frac{ex}{d}\right)\log\left(c(d+ex)^n\right)\left(m\log(i+jx)-\log\left(h(i+jx)^m\right)\right)\\
&\quad+ag\log\left(-\frac{jx}{i}\right)\log\left(h(i+jx)^m\right)+bfn\text{Li}_2\left(1+\frac{ex}{d}\right)\\
&\quad+agm\text{Li}_2\left(1+\frac{jx}{i}\right)+(bgmn)\int \frac{\log(d+ex)\log(i+jx)}{x}\,dx\\
&\quad-\left(bgm\left(n\log(d+ex)-\log\left(c(d+ex)^n\right)\right)\right)\int \frac{\log(i+jx)}{x}\,dx\\
&\quad+\left(begn\left(m\log(i+jx)-\log\left(h(i+jx)^m\right)\right)\right)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= f\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n)) + bgmn\log\left(-\frac{ex}{d}\right)\log(d+ex)\log(i+jx) \\
&\quad - bgm\log\left(-\frac{jx}{i}\right)(n\log(d+ex)-\log(c(d+ex)^n))\log(i+jx) \\
&\quad + \frac{1}{2}bgmn\left(\log\left(-\frac{ex}{d}\right)+\log\left(\frac{ei-dj}{e(i+jx)}\right)-\log\left(-\frac{(ei-dj)x}{d(i+jx)}\right)\right)\log^2\left(\frac{d(i+jx)}{i(d+ex)}\right) \\
&\quad - \frac{1}{2}bgmn\left(\log\left(-\frac{ex}{d}\right)-\log\left(-\frac{jx}{i}\right)\right)\left(\log(d+ex)+\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)^2 \\
&\quad - bg\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^n)(m\log(i+jx)-\log(h(i+jx)^m)) \\
&\quad + ag\log\left(-\frac{jx}{i}\right)\log(h(i+jx)^m) + bfn\mathrm{Li}_2\left(1+\frac{ex}{d}\right) \\
&\quad + bgmn\left(\log(i+jx)-\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)\mathrm{Li}_2\left(1+\frac{ex}{d}\right) \\
&\quad - bgn(m\log(i+jx)-\log(h(i+jx)^m))\mathrm{Li}_2\left(1+\frac{ex}{d}\right) \\
&\quad + bgmn\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\mathrm{Li}_2\left(\frac{i(d+ex)}{d(i+jx)}\right) \\
&\quad - bgmn\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\mathrm{Li}_2\left(\frac{j(d+ex)}{e(i+jx)}\right) + agm\mathrm{Li}_2\left(1+\frac{jx}{i}\right) \\
&\quad + bgmn\left(\log(d+ex)+\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)\mathrm{Li}_2\left(1+\frac{jx}{i}\right) - bgmn\mathrm{Li}_3\left(1+\frac{ex}{d}\right) \\
&\quad + bgmn\mathrm{Li}_3\left(\frac{i(d+ex)}{d(i+jx)}\right) - bgmn\mathrm{Li}_3\left(\frac{j(d+ex)}{e(i+jx)}\right) - bgmn\mathrm{Li}_3\left(1+\frac{jx}{i}\right) \\
&\quad + (bgjm(n\log(d+ex)-\log(c(d+ex)^n)))\int\frac{\log\left(-\frac{jx}{i}\right)}{i+jx}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= f\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)+bgmn\log\left(-\frac{ex}{d}\right)\log(d+ex)\log(i+jx)\\
&\quad -bgm\log\left(-\frac{jx}{i}\right)\left(n\log(d+ex)-\log\left(c(d+ex)^n\right)\right)\log(i+jx)\\
&\quad +\frac{1}{2}bgmn\left(\log\left(-\frac{ex}{d}\right)+\log\left(\frac{ei-dj}{e(i+jx)}\right)-\log\left(-\frac{(ei-dj)x}{d(i+jx)}\right)\right)\log^2\left(\frac{d(i+jx)}{i(d+ex)}\right)\\
&\quad -\frac{1}{2}bgmn\left(\log\left(-\frac{ex}{d}\right)-\log\left(-\frac{jx}{i}\right)\right)\left(\log(d+ex)+\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)^2\\
&\quad -bg\log\left(-\frac{ex}{d}\right)\log\left(c(d+ex)^n\right)\left(m\log(i+jx)-\log\left(h(i+jx)^m\right)\right)\\
&\quad +ag\log\left(-\frac{jx}{i}\right)\log\left(h(i+jx)^m\right)+bfn\text{Li}_2\left(1+\frac{ex}{d}\right)\\
&\quad +bgmn\left(\log(i+jx)-\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)\text{Li}_2\left(1+\frac{ex}{d}\right)\\
&\quad -bgn(m\log(i+jx)-\log\left(h(i+jx)^m\right))\text{Li}_2\left(1+\frac{ex}{d}\right)\\
&\quad +bgmn\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\text{Li}_2\left(\frac{i(d+ex)}{d(i+jx)}\right)\\
&\quad -bgmn\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\text{Li}_2\left(\frac{j(d+ex)}{e(i+jx)}\right)+agm\text{Li}_2\left(1+\frac{jx}{i}\right)\\
&\quad -bgm(n\log(d+ex)-\log\left(c(d+ex)^n\right))\text{Li}_2\left(1+\frac{jx}{i}\right)\\
&\quad +bgmn\left(\log(d+ex)+\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)\text{Li}_2\left(1+\frac{jx}{i}\right)-bgmn\text{Li}_3\left(1+\frac{ex}{d}\right)\\
&\quad +bgmn\text{Li}_3\left(\frac{i(d+ex)}{d(i+jx)}\right)-bgmn\text{Li}_3\left(\frac{j(d+ex)}{e(i+jx)}\right)-bgmn\text{Li}_3\left(1+\frac{jx}{i}\right)
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 605, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx$$
$$= \log(x)(a-bn\log(d+ex)+b\log(c(d+ex)^n))(f-gm\log(i+jx)+g\log(h(i+jx)^m)) + bn(f-gm\log(i+jx)+g\log(h(i+jx)^m))\left(\log(x)\left(\log(d+ex)-\log\left(1+\frac{ex}{d}\right)\right) - \text{PolyLog}\left(2,-\frac{ex}{d}\right)\right) + agm\left(\log(x)\left(\log(i+jx)-\log\left(1+\frac{jx}{i}\right)\right)-\text{PolyLog}\left(2,-\frac{jx}{i}\right)\right) + bgm(-n\log(d+ex)+\log(c(d+ex)^n))\left(\log(x)\left(\log(i+jx)-\log\left(1+\frac{jx}{i}\right)\right) - \text{PolyLog}\left(2,-\frac{jx}{i}\right)\right) + bgmn\left(\log\left(-\frac{ex}{d}\right)\log(d+ex)\log(i+jx) + \frac{1}{2}\log^2\left(\frac{d(i+jx)}{i(d+ex)}\right)\left(\log\left(-\frac{ex}{d}\right)+\log\left(\frac{-ei+dj}{j(d+ex)}\right)-\log\left(\frac{eix-djx}{di+eix}\right)\right) + \left(-\log\left(-\frac{ex}{d}\right)+\log\left(-\frac{jx}{i}\right)\right)\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\log\left(1+\frac{jx}{i}\right) + \frac{1}{2}\left(\log\left(-\frac{ex}{d}\right)-\log\left(-\frac{jx}{i}\right)\right)\log\left(1+\frac{jx}{i}\right)\left(-2\log(d+ex)+\log\left(1+\frac{jx}{i}\right)\right) + \left(\log(i+jx)-\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)\text{PolyLog}\left(2,1+\frac{ex}{d}\right) + \log\left(\frac{d(i+jx)}{i(d+ex)}\right)\left(-\text{PolyLog}\left(2,\frac{d(i+jx)}{i(d+ex)}\right)+\text{PolyLog}\left(2,\frac{e(i+jx)}{j(d+ex)}\right)\right) + \left(\log(d+ex)+\log\left(\frac{d(i+jx)}{i(d+ex)}\right)\right)\text{PolyLog}\left(2,1+\frac{jx}{i}\right) - \text{PolyLog}\left(3,1+\frac{ex}{d}\right) + \text{PolyLog}\left(3,\frac{d(i+jx)}{i(d+ex)}\right) - \text{PolyLog}\left(3,\frac{e(i+jx)}{j(d+ex)}\right) - \text{PolyLog}\left(3,1+\frac{jx}{i}\right)\right)$$

```
[In] Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x,x]
```

```
[Out] Log[x]*(a - b*n*Log[d + e*x] + b*Log[c*(d + e*x)^n])*(f - g*m*Log[i + j*x]
+ g*Log[h*(i + j*x)^m]) + b*n*(f - g*m*Log[i + j*x] + g*Log[h*(i + j*x)^m])
*(Log[x]*(Log[d + e*x] - Log[1 + (e*x)/d]) - PolyLog[2, -((e*x)/d)]) + a*g*
m*(Log[x]*(Log[i + j*x] - Log[1 + (j*x)/i]) - PolyLog[2, -((j*x)/i)]) + b*g
*m*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n])*(Log[x]*(Log[i + j*x] - Log[1 +
 (j*x)/i]) - PolyLog[2, -((j*x)/i)]) + b*g*m*n*(Log[-((e*x)/d)]*Log[d + e*x
]*Log[i + j*x] + (Log[(d*(i + j*x))/(i*(d + e*x))]^2*(Log[-((e*x)/d)] + Log
[(-(e*i) + d*j)/(j*(d + e*x))] - Log[(e*i*x - d*j*x)/(d*i + e*i*x)]))/2 + (
-Log[-((e*x)/d)] + Log[-((j*x)/i)])*Log[(d*(i + j*x))/(i*(d + e*x))]*Log[1
+ (j*x)/i] + ((Log[-((e*x)/d)] - Log[-((j*x)/i)])*Log[1 + (j*x)/i]*(-2*Log[
```

```
d + e*x] + Log[1 + (j*x)/i]))/2 + (Log[i + j*x] - Log[(d*(i + j*x))/(i*(d +
 e*x))])*PolyLog[2, 1 + (e*x)/d] + Log[(d*(i + j*x))/(i*(d + e*x))]*(-PolyL
og[2, (d*(i + j*x))/(i*(d + e*x))] + PolyLog[2, (e*(i + j*x))/(j*(d + e*x))
]) + (Log[d + e*x] + Log[(d*(i + j*x))/(i*(d + e*x))])*PolyLog[2, 1 + (j*x)
/i] - PolyLog[3, 1 + (e*x)/d] + PolyLog[3, (d*(i + j*x))/(i*(d + e*x))] - P
olyLog[3, (e*(i + j*x))/(j*(d + e*x))] - PolyLog[3, 1 + (j*x)/i])
```

## Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))(f+g\ln(h(jx+i)^m))}{x}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x,x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x,x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^n c)+a)(g\log((jx+i)^m h)+f)}{x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="fri
cas")
```

```
[Out] integral((b*f*log((e*x + d)^n*c) + a*f + (b*g*log((e*x + d)^n*c) + a*g)*log
((j*x + i)^m*h))/x, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m))/x,x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)}{x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="max
ima")
```

```
[Out] a*f*log(x) + integrate((((g*log(h) + f)*b*log((e*x + d)^n) + (g*log(h) + f)*
b*log(c) + a*g*log(h) + (b*g*log((e*x + d)^n) + b*g*log(c) + a*g)*log((j*x
+ i)^m))/x, x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)}{x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="gia
c")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f)/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(a+b\ln(c(d+e\,x)^n))\,(f+g\ln(h\,(i+j\,x)^m))}{x}\,dx$$

```
[In] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)))/x,x)
```

```
[Out] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)))/x, x)
```

# 3.391 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 270

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^2}\,dx = \frac{gjm\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{i} - \frac{gjm(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{i} + \frac{ben\log\left(-\frac{jx}{i}\right)(f+g\log(h(i+jx)^m))}{d} - \frac{ben\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{d} - \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x} - \frac{bgjmn\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{i} + \frac{bgjmn\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{i} - \frac{begmn\operatorname{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{d} + \frac{begmn\operatorname{PolyLog}\left(2,1+\frac{jx}{i}\right)}{d}$$

```
[Out] g*j*m*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/i-g*j*m*(a+b*ln(c*(e*x+d)^n))*ln(e*(
j*x+i)/(-d*j+e*i))/i+b*e*n*ln(-j*x/i)*(f+g*ln(h*(j*x+i)^m))/d-b*e*n*ln(-j*(
e*x+d)/(-d*j+e*i))*(f+g*ln(h*(j*x+i)^m))/d-(a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*
(j*x+i)^m))/x-b*g*j*m*n*polylog(2,-j*(e*x+d)/(-d*j+e*i))/i+b*g*j*m*n*polylo
g(2,1+e*x/d)/i-b*e*g*m*n*polylog(2,e*(j*x+i)/(-d*j+e*i))/d+b*e*g*m*n*polylo
g(2,1+j*x/i)/d
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2489, 36, 29, 31, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^2}\,dx$$
$$= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x} + \frac{gjm\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{i} - \frac{gjm\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{i} + \frac{ben\log\left(-\frac{jx}{i}\right)(f+g\log(h(i+jx)^m))}{d} - \frac{ben\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{d} - \frac{bgjmn\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{i} + \frac{bgjmn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{i} - \frac{begmn\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{d} + \frac{begmn\,\text{PolyLog}\left(2,\frac{jx}{i}+1\right)}{d}$$

```
[In] Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x^2,x]
```

```
[Out] (g*j*m*Log[-((e*x)/d)]*(a + b*Log[c*(d + e*x)^n]))/i - (g*j*m*(a + b*Log[c*
(d + e*x)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/i + (b*e*n*Log[-((j*x)/i)]*(f
 + g*Log[h*(i + j*x)^m]))/d - (b*e*n*Log[-((j*(d + e*x))/(e*i - d*j))]*(f +
 g*Log[h*(i + j*x)^m]))/d - ((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j
*x)^m]))/x - (b*g*j*m*n*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/i + (b*g*
j*m*n*PolyLog[2, 1 + (e*x)/d])/i - (b*e*g*m*n*PolyLog[2, (e*(i + j*x))/(e*i
 - d*j)])/d + (b*e*g*m*n*PolyLog[2, 1 + (j*x)/i])/d
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x} \\
&\quad + (gjm)\int\frac{a+b\log(c(d+ex)^n)}{x(i+jx)}\,dx + (ben)\int\frac{f+g\log(h(i+jx)^m)}{x(d+ex)}\,dx
\end{aligned}
$$

$$= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x} + (gjm)\int\left(\frac{a+b\log(c(d+ex)^n)}{ix} - \frac{j(a+b\log(c(d+ex)^n))}{i(i+jx)}\right)dx + (ben)\int\left(\frac{f+g\log(h(i+jx)^m)}{dx} - \frac{e(f+g\log(h(i+jx)^m))}{d(d+ex)}\right)dx$$

$$= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x} + \frac{(gjm)\int\frac{a+b\log(c(d+ex)^n)}{x}dx}{i} - \frac{(gj^2m)\int\frac{a+b\log(c(d+ex)^n)}{i+jx}dx}{i} + \frac{(ben)\int\frac{f+g\log(h(i+jx)^m)}{x}dx}{d} - \frac{(be^2n)\int\frac{f+g\log(h(i+jx)^m)}{d+ex}dx}{d}$$

$$= \frac{gjm\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{i} - \frac{gjm(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{i} + \frac{ben\log\left(-\frac{jx}{i}\right)(f+g\log(h(i+jx)^m))}{d} - \frac{ben\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{d} - \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x} - \frac{(begjmn)\int\frac{\log\left(-\frac{jx}{i}\right)}{i+jx}dx}{d} + \frac{(begjmn)\int\frac{\log\left(\frac{j(d+ex)}{-ei+dj}\right)}{i+jx}dx}{d} - \frac{(begjmn)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}dx}{i} + \frac{(begjmn)\int\frac{\log\left(\frac{e(i+jx)}{ei-dj}\right)}{d+ex}dx}{i}$$

$$= \frac{gjm\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{i} - \frac{gjm(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{i} + \frac{ben\log\left(-\frac{jx}{i}\right)(f+g\log(h(i+jx)^m))}{d} - \frac{ben\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{d} - \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x} + \frac{bgjmn\text{Li}_2\left(1+\frac{ex}{d}\right)}{i} + \frac{begmn\text{Li}_2\left(1+\frac{jx}{i}\right)}{d} + \frac{(begmn)\text{Subst}\left(\int\frac{\log\left(1+\frac{ex}{-ei+dj}\right)}{x}dx, x, i+jx\right)}{d} + \frac{(bgjmn)\text{Subst}\left(\int\frac{\log\left(1+\frac{jx}{ei-dj}\right)}{x}dx, x, d+ex\right)}{i}$$

$$= \frac{gjm\log\left(-\frac{ex}{d}\right)(a+b\log\left(c(d+ex)^n\right))}{i} - \frac{gjm(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{i} + \frac{ben\log\left(-\frac{jx}{i}\right)(f+g\log\left(h(i+jx)^m\right))}{d} - \frac{ben\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log\left(h(i+jx)^m\right))}{d} - \frac{(a+b\log\left(c(d+ex)^n\right))(f+g\log\left(h(i+jx)^m\right))}{x} - \frac{bgjmn\text{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{i} + \frac{bgjmn\text{Li}_2\left(1+\frac{ex}{d}\right)}{i} - \frac{begmn\text{Li}_2\left(\frac{e(i+jx)}{ei-dj}\right)}{d} + \frac{begmn\text{Li}_2\left(1+\frac{jx}{i}\right)}{d}$$

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.76

$$\int \frac{(a+b\log\left(c(d+ex)^n\right))(f+g\log\left(h(i+jx)^m\right))}{x^2}\,dx = -\frac{adfi - befinx\log(x) - adgjmx\log\left(-\frac{jx}{i}\right) + befinx\log(d+ex) - bdgjmnx\log\left(-\frac{ex}{d}\right)\log(d+ex) + bdg}{}$$

```
[In] Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x^2,x]
```

```
[Out] -((a*d*f*i - b*e*f*i*n*x*Log[x] - a*d*g*j*m*x*Log[-((j*x)/i)] + b*e*f*i*n*x
*Log[d + e*x] - b*d*g*j*m*n*x*Log[-((e*x)/d)]*Log[d + e*x] + b*d*g*j*m*n*x*
Log[-((j*x)/i)]*Log[d + e*x] + b*d*f*i*Log[c*(d + e*x)^n] - b*d*g*j*m*x*Log
[-((j*x)/i)]*Log[c*(d + e*x)^n] + a*d*g*j*m*x*Log[i + j*x] - b*e*g*i*m*n*x*
Log[d + e*x]*Log[i + j*x] - b*d*g*j*m*n*x*Log[d + e*x]*Log[i + j*x] + b*e*g
*i*m*n*x*Log[(j*(d + e*x))/(-(e*i) + d*j)]*Log[i + j*x] + b*d*g*j*m*x*Log[c
*(d + e*x)^n]*Log[i + j*x] + b*d*g*j*m*n*x*Log[d + e*x]*Log[(e*(i + j*x))/(
e*i - d*j)] + a*d*g*i*Log[h*(i + j*x)^m] - b*e*g*i*n*x*Log[x]*Log[h*(i + j*
x)^m] + b*e*g*i*n*x*Log[d + e*x]*Log[h*(i + j*x)^m] + b*d*g*i*Log[c*(d + e*
x)^n]*Log[h*(i + j*x)^m] + b*e*g*i*m*n*x*Log[x]*Log[1 + (j*x)/i] + b*e*g*i*
m*n*x*PolyLog[2, -((j*x)/i)] + b*d*g*j*m*n*x*PolyLog[2, (j*(d + e*x))/(-(e*
i) + d*j)] - b*d*g*j*m*n*x*PolyLog[2, 1 + (e*x)/d] + b*e*g*i*m*n*x*PolyLog[
2, (e*(i + j*x))/(e*i - d*j)])/(d*i*x))
```

## Maple [F]

$$\int \frac{(a + b\ln{(c(ex + d)^n)})\,(f + g\ln{(h(jx + i)^m)})}{x^2}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

## Fricas [F]

$$\int \frac{(a + b\log{(c(d + ex)^n)})\,(f + g\log{(h(i + jx)^m)})}{x^2}\,dx$$
$$= \int \frac{(b\log{((ex + d)^n c)} + a)(g\log{((jx + i)^m h)} + f)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm="f
ricas")
```

```
[Out] integral((b*f*log((e*x + d)^n*c) + a*f + (b*g*log((e*x + d)^n*c) + a*g)*log
((j*x + i)^m*h))/x^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\log{(c(d + ex)^n)})\,(f + g\log{(h(i + jx)^m)})}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m))/x**2,x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(a + b\log{(c(d + ex)^n)})\,(f + g\log{(h(i + jx)^m)})}{x^2}\,dx$$
$$= \int \frac{(b\log{((ex + d)^n c)} + a)(g\log{((jx + i)^m h)} + f)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm="m
axima")
```

```
[Out] -b*e*f*n*(log(e*x + d)/d - log(x)/d) - a*g*j*m*(log(j*x + i)/i - log(x)/i)
+ b*g*integrate(((log((e*x + d)^n) + log(c))*log((j*x + i)^m) + log((e*x +
d)^n)*log(h) + log(c)*log(h))/x^2, x) - b*f*log((e*x + d)^n*c)/x - a*g*log(
(j*x + i)^m*h)/x - a*f/x
```

## Giac [F]

$$\int \frac{(a + b\log(c(d + ex)^n))(f + g\log(h(i + jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex + d)^n c) + a)(g\log((jx + i)^m h) + f)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm="g
iac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f)/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d + ex)^n))(f + g\log(h(i + jx)^m))}{x^2}\,dx = \int \frac{(a + b\,\ln(c\,(d + e\,x)^n))\,(f + g\,\ln(h\,(i + j\,x)^m))}{x^2}\,dx$$

```
[In] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)))/x^2,x)
```

```
[Out] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)))/x^2, x)
```

# 3.392 $\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 421

$$\begin{aligned}\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx &= \frac{begjmn\log(x)}{di}-\frac{begjmn\log(d+ex)}{2di}-\frac{gjm(a+b\log(c(d+ex)^n))}{2ix}\\
&-\frac{gj^2m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{2i^2}-\frac{begjmn\log(i+jx)}{2di}\\
&+\frac{gj^2m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2i^2}-\frac{ben(f+g\log(h(i+jx)^m))}{2dx}\\
&-\frac{be^2n\log\left(-\frac{jx}{i}\right)(f+g\log(h(i+jx)^m))}{2d^2}+\frac{be^2n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{2d^2}\\
&-\frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{2x^2}\\
&+\frac{bgj^2mn\,\mathrm{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{2i^2}-\frac{bgj^2mn\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{2i^2}\\
&+\frac{be^2gmn\,\mathrm{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{2d^2}-\frac{be^2gmn\,\mathrm{PolyLog}\left(2,1+\frac{jx}{i}\right)}{2d^2}\end{aligned}$$

```
[Out] b*e*g*j*m*n*ln(x)/d/i-1/2*b*e*g*j*m*n*ln(e*x+d)/d/i-1/2*g*j*m*(a+b*ln(c*(e*
x+d)^n))/i/x-1/2*g*j^2*m*ln(-e*x/d)*(a+b*ln(c*(e*x+d)^n))/i^2-1/2*b*e*g*j*m
*n*ln(j*x+i)/d/i+1/2*g*j^2*m*(a+b*ln(c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))
/i^2-1/2*b*e*n*(f+g*ln(h*(j*x+i)^m))/d/x-1/2*b*e^2*n*ln(-j*x/i)*(f+g*ln(h*(
j*x+i)^m))/d^2+1/2*b*e^2*n*ln(-j*(e*x+d)/(-d*j+e*i))*(f+g*ln(h*(j*x+i)^m))/
d^2-1/2*(a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x^2+1/2*b*g*j^2*m*n*pol
ylog(2,-j*(e*x+d)/(-d*j+e*i))/i^2-1/2*b*g*j^2*m*n*polylog(2,1+e*x/d)/i^2+1/
2*b*e^2*g*m*n*polylog(2,e*(j*x+i)/(-d*j+e*i))/d^2-1/2*b*e^2*g*m*n*polylog(2
,1+j*x/i)/d^2
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2489, 46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$
\begin{aligned}
&\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx \\
&= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{2x^2} \\
&\quad -\frac{gj^2m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{2i^2}+\frac{gj^2m\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{2i^2} \\
&\quad -\frac{gjm(a+b\log(c(d+ex)^n))}{2ix}-\frac{be^2n\log\left(-\frac{jx}{i}\right)(f+g\log(h(i+jx)^m))}{2d^2} \\
&\quad +\frac{be^2n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{2d^2}+\frac{be^2gmn\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{2d^2} \\
&\quad -\frac{be^2gmn\,\text{PolyLog}\left(2,\frac{jx}{i}+1\right)}{2d^2}-\frac{ben(f+g\log(h(i+jx)^m))}{2dx} \\
&\quad +\frac{bgj^2mn\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{2i^2}-\frac{bgj^2mn\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{2i^2} \\
&\quad +\frac{begjmn\log(x)}{di}-\frac{begjmn\log(d+ex)}{2di}-\frac{begjmn\log(i+jx)}{2di}
\end{aligned}
$$

```
[In] Int[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x^3,x]
```

```
[Out] (b*e*g*j*m*n*Log[x])/(d*i) - (b*e*g*j*m*n*Log[d + e*x])/(2*d*i) - (g*j*m*(a
 + b*Log[c*(d + e*x)^n]))/(2*i*x) - (g*j^2*m*Log[-((e*x)/d)]*(a + b*Log[c*(
d + e*x)^n]))/(2*i^2) - (b*e*g*j*m*n*Log[i + j*x])/(2*d*i) + (g*j^2*m*(a +
b*Log[c*(d + e*x)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/(2*i^2) - (b*e*n*(f +
 g*Log[h*(i + j*x)^m]))/(2*d*x) - (b*e^2*n*Log[-((j*x)/i)]*(f + g*Log[h*(i
+ j*x)^m]))/(2*d^2) + (b*e^2*n*Log[-((j*(d + e*x))/(e*i - d*j))]*(f + g*Log
[h*(i + j*x)^m]))/(2*d^2) - ((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j
*x)^m]))/(2*x^2) + (b*g*j^2*m*n*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/(
2*i^2) - (b*g*j^2*m*n*PolyLog[2, 1 + (e*x)/d])/(2*i^2) + (b*e^2*g*m*n*PolyL
og[2, (e*(i + j*x))/(e*i - d*j)])/(2*d^2) - (b*e^2*g*m*n*PolyLog[2, 1 + (j*
x)/i])/(2*d^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{2x^2} \\
&\quad+\frac{1}{2}(gjm)\int\frac{a+b\log(c(d+ex)^n)}{x^2(i+jx)}\,dx+\frac{1}{2}(ben)\int\frac{f+g\log(h(i+jx)^m)}{x^2(d+ex)}\,dx \\
&= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{2x^2} \\
&\quad+\frac{1}{2}(gjm)\int\left(\frac{a+b\log(c(d+ex)^n)}{ix^2}-\frac{j(a+b\log(c(d+ex)^n))}{i^2x}\right. \\
&\qquad\left.+\frac{j^2(a+b\log(c(d+ex)^n))}{i^2(i+jx)}\right)dx+\frac{1}{2}(ben)\int\left(\frac{f+g\log(h(i+jx)^m)}{dx^2}\right. \\
&\qquad\left.-\frac{e(f+g\log(h(i+jx)^m))}{d^2x}+\frac{e^2(f+g\log(h(i+jx)^m))}{d^2(d+ex)}\right)dx \\
&= -\frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{2x^2} \\
&\quad+\frac{(gjm)\int\frac{a+b\log(c(d+ex)^n)}{x^2}\,dx}{2i}-\frac{(gj^2m)\int\frac{a+b\log(c(d+ex)^n)}{x}\,dx}{2i^2} \\
&\quad+\frac{(gj^3m)\int\frac{a+b\log(c(d+ex)^n)}{i+jx}\,dx}{2i^2}+\frac{(ben)\int\frac{f+g\log(h(i+jx)^m)}{x^2}\,dx}{2d} \\
&\quad-\frac{(be^2n)\int\frac{f+g\log(h(i+jx)^m)}{x}\,dx}{2d^2}+\frac{(be^3n)\int\frac{f+g\log(h(i+jx)^m)}{d+ex}\,dx}{2d^2}
\end{aligned}$$

$$= -\frac{gjm(a+b\log(c(d+ex)^n))}{2ix} - \frac{gj^2m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{2i^2} + \frac{gj^2m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2i^2} - \frac{ben(f+g\log(h(i+jx)^m))}{2dx} - \frac{be^2n\log\left(-\frac{jx}{i}\right)(f+g\log(h(i+jx)^m))}{2d^2} + \frac{be^2n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{2d^2} - \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{2x^2} + \frac{(begjmn)\int\frac{1}{x(i+jx)}\,dx}{2d} + \frac{(be^2gjmn)\int\frac{\log\left(-\frac{jx}{i}\right)}{i+jx}\,dx}{2d^2} - \frac{(be^2gjmn)\int\frac{\log\left(\frac{j(d+ex)}{-ei+dj}\right)}{i+jx}\,dx}{2d^2} + \frac{(begjmn)\int\frac{1}{x(d+ex)}\,dx}{2i} + \frac{(begj^2mn)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{2i^2} - \frac{(begj^2mn)\int\frac{\log\left(\frac{e(i+jx)}{ei-dj}\right)}{d+ex}\,dx}{2i^2}$$

$$= -\frac{gjm(a+b\log(c(d+ex)^n))}{2ix} - \frac{gj^2m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{2i^2} + \frac{gj^2m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2i^2} - \frac{ben(f+g\log(h(i+jx)^m))}{2dx} - \frac{be^2n\log\left(-\frac{jx}{i}\right)(f+g\log(h(i+jx)^m))}{2d^2} + \frac{be^2n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{2d^2} - \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{2x^2} - \frac{bgj^2mn\text{Li}_2\left(1+\frac{ex}{d}\right)}{2i^2} - \frac{be^2gmn\text{Li}_2\left(1+\frac{jx}{i}\right)}{2d^2} - \frac{(be^2gmn)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{ex}{-ei+dj}\right)}{x}\,dx, x, i+jx\right)}{2d^2} + 2\frac{(begjmn)\int\frac{1}{x}\,dx}{2di} - \frac{(be^2gjmn)\int\frac{1}{d+ex}\,dx}{2di} - \frac{(bgj^2mn)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{jx}{ei-dj}\right)}{x}\,dx, x, d+ex\right)}{2i^2} - \frac{(begj^2mn)\int\frac{1}{i+jx}\,dx}{2di}$$

$$= \frac{begjmn\log(x)}{di} - \frac{begjmn\log(d+ex)}{2di} - \frac{gjm(a+b\log(c(d+ex)^n))}{2ix} - \frac{gj^2m\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{2i^2} - \frac{begjmn\log(i+jx)}{2di} + \frac{gj^2m(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2i^2} - \frac{ben(f+g\log(h(i+jx)^m))}{2dx} - \frac{be^2n\log\left(-\frac{jx}{i}\right)(f+g\log(h(i+jx)^m))}{2d^2} + \frac{be^2n\log\left(-\frac{j(d+ex)}{ei-dj}\right)(f+g\log(h(i+jx)^m))}{2d^2} - \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{2x^2} + \frac{bgj^2mn\text{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{2i^2} - \frac{bgj^2mn\text{Li}_2\left(1+\frac{ex}{d}\right)}{2i^2} + \frac{be^2gmn\text{Li}_2\left(\frac{e(i+jx)}{ei-dj}\right)}{2d^2} - \frac{be^2gmn\text{Li}_2\left(1+\frac{jx}{i}\right)}{2d^2}$$

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.82

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx$$
$$= -\frac{be^2n\log(x)(f+g(-m\log(i+jx)+\log(h(i+jx)^m)))}{2d^2} + \frac{be^2n\log(d+ex)(f+g(-m\log(i+jx)+\log(h(i+jx)^m)))}{2d^2} - \frac{bn\log(d+ex)(f+g(-m\log(i+jx)+\log(h(i+jx)^m)))}{2x^2} - \frac{(a+b(-n\log(d+ex)+\log(c(d+ex)^n)))(f+g(-m\log(i+jx)+\log(h(i+jx)^m)))}{2x^2} - \frac{e(bfn+bgn(-m\log(i+jx)+\log(h(i+jx)^m)))}{2dx} + \frac{1}{2}agm\left(\frac{j^2(i+jx)}{i^3\left(1-\frac{i+jx}{i}\right)} - \left(\frac{j^2(i+jx)^2}{i^4\left(1-\frac{i+jx}{i}\right)^2}+\frac{2j^2(i+jx)}{i^3\left(1-\frac{i+jx}{i}\right)}\right)\log(i+jx) - \frac{j^2\log\left(1-\frac{i+jx}{i}\right)}{i^2}\right) + \frac{1}{2}bgm(-n\log(d+ex)+\log(c(d+ex)^n))\left(\frac{j^2(i+jx)}{i^3\left(1-\frac{i+jx}{i}\right)} - \left(\frac{j^2(i+jx)^2}{i^4\left(1-\frac{i+jx}{i}\right)^2}+\frac{2j^2(i+jx)}{i^3\left(1-\frac{i+jx}{i}\right)}\right)\log(i+jx) - \frac{j^2\log\left(1-\frac{i+jx}{i}\right)}{i^2}\right) + \frac{1}{2}bgmn\left(-\frac{\log(d+ex)\log(i+jx)}{x^2} + j\left(\frac{\frac{e\log(x)}{d}-\frac{e\log(d+ex)}{d}-\frac{\log(d+ex)}{x}}{i} - \frac{j\left(\log\left(-\frac{ex}{d}\right)\log(d+ex)+\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)\right)}{i^2} + \frac{j^2\left(\frac{\log(d+ex)\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}+\frac{\operatorname{PolyLog}\left(2,\frac{j(d+ex)}{-ei+dj}\right)}{j}\right)}{i^2}\right) + e\left(\frac{\frac{j\log(x)}{i}-\frac{j\log(i+jx)}{i}-\frac{\log(i+jx)}{x}}{d} - \frac{e\left(\log(x)\left(\log(i+jx)-\log\left(1+\frac{jx}{i}\right)\right)-\operatorname{PolyLog}\left(2,-\frac{jx}{i}\right)\right)}{d^2} + \frac{e^2\left(\frac{\log\left(\frac{j(d+ex)}{-ei+dj}\right)\log(i+jx)}{e}+\frac{\operatorname{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{e}\right)}{d^2}\right)\right)$$

[In] Integrate[((a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]))/x^3,x]

[Out] -1/2*(b*e^2*n*Log[x]*(f + g*(-(m*Log[i + j*x]) + Log[h*(i + j*x)^m])))/d^2 + (b*e^2*n*Log[d + e*x]*(f + g*(-(m*Log[i + j*x]) + Log[h*(i + j*x)^m])))/(2*d^2) - (b*n*Log[d + e*x]*(f + g*(-(m*Log[i + j*x]) + Log[h*(i + j*x)^m])))/(2*x^2) - ((a + b*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n]))*(f + g*(-(m*Log[i + j*x]) + Log[h*(i + j*x)^m])))/(2*x^2) - (e*(b*f*n + b*g*n*(-(m*Log[i + j*x]) + Log[h*(i + j*x)^m])))/(2*d*x) + (a*g*m*((j^2*(i + j*x))/(i^3*(1 - (i + j*x)/i)) - ((j^2*(i + j*x)^2)/(i^4*(1 - (i + j*x)/i)^2) + (2*j^2*(i + j*x))/(i^3*(1 - (i + j*x)/i)))*Log[i + j*x] - (j^2*Log[1 - (i + j*x)/i])/i^2))/2 + (b*g*m*(-(n*Log[d + e*x]) + Log[c*(d + e*x)^n])*((j^2*(i + j*x))/(i^3*(1 - (i + j*x)/i)) - ((j^2*(i + j*x)^2)/(i^4*(1 - (i + j*x)/i)^2) + (2*j^2*(i + j*x))/(i^3*(1 - (i + j*x)/i)))*Log[i + j*x] - (j^2*Log[1 - (i + j*x)/i])/i^2))/2 + (b*g*m*n*(-((Log[d + e*x]*Log[i + j*x])/x^2) + j*(((e*Log[x])/d - (e*Log[d + e*x])/d - Log[d + e*x]/x)/i - (j*(Log[-((e*x)/d)]*Log[d + e*x] + PolyLog[2, (d + e*x)/d]))/i^2 + (j^2*((Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d*j)])/j + PolyLog[2, (j*(d + e*x))/(-(e*i) + d*j)]/j))/i^2) + e*(((j*Log[x])/i - (j*Log[i + j*x])/i - Log[i + j*x]/x)/d - (e*(Log[x]*(Log[i + j*x] - Log[1 + (j*x)/i]) - PolyLog[2, -((j*x)/i)]))/d^2 + (e^2*((Log[(j*(d + e*x))/(-(e*i) + d*j)]*Log[i + j*x])/e + PolyLog[2, (e*(i + j*x))/(e*i - d*j)]/e))/d^2)))/2

## Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))(f+g\ln(h(jx+i)^m))}{x^3}dx$$

[In] int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x^3,x)

[Out] int((a+b*ln(c*(e*x+d)^n))*(f+g*ln(h*(j*x+i)^m))/x^3,x)

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx = \int \frac{(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)}{x^3}\,dx$$

[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x^3,x, algorithm="fricas")

[Out] integral((b*f*log((e*x + d)^n*c) + a*f + (b*g*log((e*x + d)^n*c) + a*g)*log((j*x + i)^m*h))/x^3, x)

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))*(f+g*ln(h*(j*x+i)**m))/x**3,x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx = \int \frac{(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x^3,x, algorithm="m
axima")
[Out] 1/2*b*e*f*n*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + 1/2*a*g*j*m*(j*
log(j*x + i)/i^2 - j*log(x)/i^2 - 1/(i*x)) + b*g*integrate(((log((e*x + d)^
n) + log(c))*log((j*x + i)^m) + log((e*x + d)^n)*log(h) + log(c)*log(h))/x^
3, x) - 1/2*b*f*log((e*x + d)^n*c)/x^2 - 1/2*a*g*log((j*x + i)^m*h)/x^2 - 1
/2*a*f/x^2
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx = \int \frac{(b\log((ex+d)^nc)+a)(g\log((jx+i)^mh)+f)}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*(f+g*log(h*(j*x+i)^m))/x^3,x, algorithm="g
iac")
[Out] integrate((b*log((e*x + d)^n*c) + a)*(g*log((j*x + i)^m*h) + f)/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{x^3}\,dx$$
$$=\int \frac{(a+b\,\ln(c\,(d+e\,x)^n))\,\,(f+g\,\ln(h\,(i+j\,x)^m))}{x^3}\,dx$$

```
[In] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)))/x^3,x)
[Out] int(((a + b*log(c*(d + e*x)^n))*(f + g*log(h*(i + j*x)^m)))/x^3, x)
```

# 3.393 $\int x(a+b\log(c(d+ex)^n))^2\,(f+g\log(h(i+jx)^m))\,dx$

## Optimal result

Integrand size = 32, antiderivative size = 1210

$$
\begin{aligned}
&\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx \\
&= -\frac{2abdgmnx}{e}-\frac{3abgimnx}{2j}-\frac{2b^2dfn^2x}{e}+\frac{15b^2dgmn^2x}{4e}+\frac{7b^2gimn^2x}{4j}-\frac{1}{4}b^2gmn^2x^2 \\
&\quad+\frac{b^2fn^2(d+ex)^2}{4e^2}-\frac{b^2gmn^2(d+ex)^2}{8e^2}-\frac{b^2d^2gmn^2\log(d+ex)}{4e^2}+\frac{b^2d^2fn^2\log^2(d+ex)}{2e^2} \\
&\quad-\frac{2b^2dgmn(d+ex)\log(c(d+ex)^n)}{e^2}-\frac{3b^2gimn(d+ex)\log(c(d+ex)^n)}{2ej} \\
&\quad+\frac{1}{4}bgmnx^2(a+b\log(c(d+ex)^n))+\frac{2bdfn(d+ex)(a+b\log(c(d+ex)^n))}{e^2} \\
&\quad-\frac{bfn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2}+\frac{bgmn(d+ex)^2(a+b\log(c(d+ex)^n))}{4e^2} \\
&\quad-\frac{bd^2fn\log(d+ex)(a+b\log(c(d+ex)^n))}{e^2}+\frac{dgm(d+ex)(a+b\log(c(d+ex)^n))^2}{2e^2} \\
&\quad+\frac{gim(d+ex)(a+b\log(c(d+ex)^n))^2}{2ej}-\frac{gm(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2} \\
&\quad-\frac{b^2gi^2mn^2\log(i+jx)}{4j^2}+\frac{bgi^2mn(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2} \\
&\quad+\frac{bdgimn(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{ej} \\
&\quad+\frac{d^2gm(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2e^2} \\
&\quad-\frac{gi^2m(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2}+\frac{1}{4}b^2gn^2x^2\log(h(i+jx)^m) \\
&\quad-\frac{3b^2dgn^2(i+jx)\log(h(i+jx)^m)}{2ej}+\frac{3b^2d^2gn^2\log\left(-\frac{j(d+ex)}{ei-dj}\right)\log(h(i+jx)^m)}{2e^2} \\
&\quad+\frac{bdgnx(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{e} \\
&\quad-\frac{1}{2}bgnx^2(a+b\log(c(d+ex)^n))\log(h(i+jx)^m) \\
&\quad-\frac{d^2g(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{2e^2} \\
&\quad+\frac{1}{2}x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m)) \\
&\quad+\frac{b^2gi^2mn^2\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{2j^2}+\frac{b^2dgimn^2\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{ej} \\
&\quad+\frac{bd^2gmn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{e^2} \\
&\quad-\frac{bgi^2mn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j^2}
\end{aligned}
$$

```
[Out] -1/4*b^2*d^2*g*m*n^2*ln(e*x+d)/e^2+2*b*d*f*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))/
e^2+1/4*b*g*m*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2+1/2*g*i*m*(e*x+d)*(a+b*
ln(c*(e*x+d)^n))^2/e/j-1/4*b^2*g*i^2*m*n^2*ln(j*x+i)/j^2+3/2*b^2*d^2*g*n^2*
ln(-j*(e*x+d)/(-d*j+e*i))*ln(h*(j*x+i)^m)/e^2+15/4*b^2*d*g*m*n^2*x/e+7/4*b^
2*g*i*m*n^2*x/j+1/2*b^2*g*i^2*m*n^2*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j^2+3/
2*b^2*d^2*g*m*n^2*polylog(2,e*(j*x+i)/(-d*j+e*i))/e^2-3/2*b^2*g*i*m*n*(e*x+
d)*ln(c*(e*x+d)^n)/e/j-1/2*d^2*g*(a+b*ln(c*(e*x+d)^n))^2*ln(h*(j*x+i)^m)/e^
2-1/4*g*m*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2+1/4*b^2*g*n^2*x^2*ln(h*(j*x
+i)^m)-1/4*b^2*g*m*n^2*x^2+1/4*b^2*f*n^2*(e*x+d)^2/e^2+b*d*g*n*x*(a+b*ln(c*
(e*x+d)^n))*ln(h*(j*x+i)^m)/e+b*d^2*g*m*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-
j*(e*x+d)/(-d*j+e*i))/e^2-b*g*i^2*m*n*(a+b*ln(c*(e*x+d)^n))*polylog(2,-j*(e
*x+d)/(-d*j+e*i))/j^2-2*b^2*d*g*m*n*(e*x+d)*ln(c*(e*x+d)^n)/e^2+1/2*b*g*i^2
*m*n*(a+b*ln(c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/j^2-3/2*b^2*d*g*n^2*(j*
x+i)*ln(h*(j*x+i)^m)/e/j-2*a*b*d*g*m*n*x/e-3/2*a*b*g*i*m*n*x/j-b^2*d^2*g*m*
n^2*polylog(3,-j*(e*x+d)/(-d*j+e*i))/e^2+b^2*g*i^2*m*n^2*polylog(3,-j*(e*x+
d)/(-d*j+e*i))/j^2-b*d^2*f*n*ln(e*x+d)*(a+b*ln(c*(e*x+d)^n))/e^2+b^2*d*g*i*
m*n^2*polylog(2,-j*(e*x+d)/(-d*j+e*i))/e/j+b*d*g*i*m*n*(a+b*ln(c*(e*x+d)^n)
)*ln(e*(j*x+i)/(-d*j+e*i))/e/j-2*b^2*d*f*n^2*x/e-1/8*b^2*g*m*n^2*(e*x+d)^2/
e^2+1/2*b^2*d^2*f*n^2*ln(e*x+d)^2/e^2+1/4*b*g*m*n*x^2*(a+b*ln(c*(e*x+d)^n))
-1/2*b*f*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2+1/2*d*g*m*(e*x+d)*(a+b*ln(c*
(e*x+d)^n))^2/e^2+1/2*d^2*g*m*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(j*x+i)/(-d*j+e*
i))/e^2-1/2*g*i^2*m*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(j*x+i)/(-d*j+e*i))/j^2-1/
2*b*g*n*x^2*(a+b*ln(c*(e*x+d)^n))*ln(h*(j*x+i)^m)+1/2*x^2*(a+b*ln(c*(e*x+d)
^n))^2*(f+g*ln(h*(j*x+i)^m))
```

## Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 1210, normalized size of antiderivative = 1.00, number of steps used = 73, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.844$, Rules used = {2489, 2463, 2436, 2333, 2332, 2448, 2437, 2342, 2341, 2443, 2481, 2421, 6724, 6874,

2458, 45, 2372, 12, 14, 2338, 2479, 2441, 2440, 2438, 2442, 2422, 2354}

$$\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx$$

$$= -\frac{1}{4}gmn^2x^2b^2 + \frac{fn^2(d+ex)^2b^2}{4e^2} - \frac{gmn^2(d+ex)^2b^2}{8e^2} + \frac{d^2fn^2\log^2(d+ex)b^2}{2e^2} - \frac{2dfn^2xb^2}{e} + \frac{15dgmn^2xb^2}{4e} + \frac{7gimn^2xb^2}{4j} - \frac{d^2gmn^2\log(d+ex)b^2}{4e^2} - \frac{2dgmn(d+ex)\log(c(d+ex)^n)b^2}{e^2} - \frac{3gimn(d+ex)\log(c(d+ex)^n)b^2}{2ej} - \frac{gi^2mn^2\log(i+jx)b^2}{4j^2} + \frac{1}{4}gn^2x^2\log(h(i+jx)^m)b^2 - \frac{3dgn^2(i+jx)\log(h(i+jx)^m)b^2}{2ej} + \frac{3d^2gn^2\log\left(-\frac{j(d+ex)}{ei-dj}\right)\log(h(i+jx)^m)b^2}{2e^2} + \frac{dgimn^2\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)b^2}{ej} + \frac{gi^2mn^2\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)b^2}{2j^2} + \frac{3d^2gmn^2\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)b^2}{2e^2} - \frac{d^2gmn^2\,\text{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)b^2}{e^2} + \frac{gi^2mn^2\,\text{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)b^2}{j^2} - \frac{2adgmnxb}{e} - \frac{3agimnxb}{2j} + \frac{1}{4}gmnx^2(a+b\log(c(d+ex)^n))b - \frac{fn(d+ex)^2(a+b\log(c(d+ex)^n))b}{2e^2} + \frac{gmn(d+ex)^2(a+b\log(c(d+ex)^n))b}{4e^2} + \frac{2dfn(d+ex)(a+b\log(c(d+ex)^n))b}{e^2} - \frac{d^2fn\log(d+ex)(a+b\log(c(d+ex)^n))b}{e^2} + \frac{dgimn(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)b}{ej} + \frac{gi^2mn(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)b}{2j^2} - \frac{1}{2}gnx^2(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)b + \frac{dgnx(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)b}{e} + \frac{d^2gmn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)b}{e^2} - \frac{gi^2mn(a+b\log(c(d+ex)^n))\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)b}{j^2} - \frac{gm(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2} + \frac{dgm(d+ex)(a+b\log(c(d+ex)^n))^2}{2e^2} + \frac{gim(d+ex)(a+b\log(c(d+ex)^n))^2}{2ej} + \frac{d^2gm(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2e^2} - \frac{gi^2m(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2}$$

$$d^2g(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)$$

```
[In] Int[x*(a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] (-2*a*b*d*g*m*n*x)/e - (3*a*b*g*i*m*n*x)/(2*j) - (2*b^2*d*f*n^2*x)/e + (15*
b^2*d*g*m*n^2*x)/(4*e) + (7*b^2*g*i*m*n^2*x)/(4*j) - (b^2*g*m*n^2*x^2)/4 +
(b^2*f*n^2*(d + e*x)^2)/(4*e^2) - (b^2*g*m*n^2*(d + e*x)^2)/(8*e^2) - (b^2*
d^2*g*m*n^2*Log[d + e*x])/(4*e^2) + (b^2*d^2*f*n^2*Log[d + e*x]^2)/(2*e^2)
- (2*b^2*d*g*m*n*(d + e*x)*Log[c*(d + e*x)^n])/e^2 - (3*b^2*g*i*m*n*(d + e*
x)*Log[c*(d + e*x)^n])/(2*e*j) + (b*g*m*n*x^2*(a + b*Log[c*(d + e*x)^n]))/4
 + (2*b*d*f*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n]))/e^2 - (b*f*n*(d + e*x)^
2*(a + b*Log[c*(d + e*x)^n]))/(2*e^2) + (b*g*m*n*(d + e*x)^2*(a + b*Log[c*(
d + e*x)^n]))/(4*e^2) - (b*d^2*f*n*Log[d + e*x]*(a + b*Log[c*(d + e*x)^n]))
/e^2 + (d*g*m*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(2*e^2) + (g*i*m*(d +
 e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(2*e*j) - (g*m*(d + e*x)^2*(a + b*Log[c
*(d + e*x)^n])^2)/(4*e^2) - (b^2*g*i^2*m*n^2*Log[i + j*x])/(4*j^2) + (b*g*i
^2*m*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/(2*j^2) +
 (b*d*g*i*m*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/(e
*j) + (d^2*g*m*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(i + j*x))/(e*i - d*j)])
/(2*e^2) - (g*i^2*m*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(i + j*x))/(e*i - d
*j)])/(2*j^2) + (b^2*g*n^2*x^2*Log[h*(i + j*x)^m])/4 - (3*b^2*d*g*n^2*(i +
j*x)*Log[h*(i + j*x)^m])/(2*e*j) + (3*b^2*d^2*g*n^2*Log[-((j*(d + e*x))/(e*
i - d*j))]*Log[h*(i + j*x)^m])/(2*e^2) + (b*d*g*n*x*(a + b*Log[c*(d + e*x)^
n])*Log[h*(i + j*x)^m])/e - (b*g*n*x^2*(a + b*Log[c*(d + e*x)^n])*Log[h*(i
+ j*x)^m])/2 - (d^2*g*(a + b*Log[c*(d + e*x)^n])^2*Log[h*(i + j*x)^m])/(2*e
^2) + (x^2*(a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/2 + (b^
2*g*i^2*m*n^2*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/(2*j^2) + (b^2*d*g*
i*m*n^2*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/(e*j) + (b*d^2*g*m*n*(a +
 b*Log[c*(d + e*x)^n])*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/e^2 - (b*g
*i^2*m*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))
])/j^2 + (3*b^2*d^2*g*m*n^2*PolyLog[2, (e*(i + j*x))/(e*i - d*j)])/(2*e^2)
- (b^2*d^2*g*m*n^2*PolyLog[3, -((j*(d + e*x))/(e*i - d*j))])/e^2 + (b^2*g*i
^2*m*n^2*PolyLog[3, -((j*(d + e*x))/(e*i - d*j))])/j^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2479

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[x*((a +
 b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Dist[b*e*n*p, Int[x*(a + b*Lo
```

```
g[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\\ &\quad -\frac{1}{2}(gjm)\int\frac{x^2(a+b\log(c(d+ex)^n))^2}{i+jx}\,dx\\ &\quad -(ben)\int\frac{x^2(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{d+ex}\,dx\end{aligned}$$

$$= \frac{1}{2}x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m)) - \frac{1}{2}(gjm)\int\left(-\frac{i(a+b\log(c(d+ex)^n))^2}{j^2}+\frac{x(a+b\log(c(d+ex)^n))^2}{j} + \frac{i^2(a+b\log(c(d+ex)^n))^2}{j^2(i+jx)}\right)dx - (ben)\int\left(\frac{fx^2(a+b\log(c(d+ex)^n))}{d+ex} + \frac{gx^2(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{d+ex}\right)dx$$

$$= \frac{1}{2}x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m)) - \frac{1}{2}(gm)\int x(a+b\log(c(d+ex)^n))^2\,dx + \frac{(gim)\int(a+b\log(c(d+ex)^n))^2\,dx}{2j} - \frac{(gi^2m)\int\frac{(a+b\log(c(d+ex)^n))^2}{i+jx}\,dx}{2j} - (befn)\int\frac{x^2(a+b\log(c(d+ex)^n))}{d+ex}\,dx - (begn)\int\frac{x^2(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{d+ex}\,dx$$

$$= -\frac{gi^2m(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2} + \frac{1}{2}x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m)) - \frac{1}{2}(gm)\int\left(-\frac{d(a+b\log(c(d+ex)^n))^2}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{e}\right)dx + \frac{(gim)\text{Subst}\left(\int(a+b\log(cx^n))^2\,dx,x,d+ex\right)}{2ej} - (bfn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^2(a+b\log(cx^n))}{x}\,dx,x,d+ex\right) - (begn)\int\left(-\frac{d(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{e^2} + \frac{x(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{e} + \frac{d^2(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{e^2(d+ex)}\right)dx + \frac{(begi^2mn)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{d+ex}\,dx}{j^2}$$

$$
\begin{aligned}
&= \frac{2bdfn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^2}-\frac{bfn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2} \\
&\quad -\frac{bd^2fn\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^2} \\
&\quad +\frac{gim(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2ej} \\
&\quad -\frac{gi^2m\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2} \\
&\quad +\frac{1}{2}x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\left(f+g\log\left(h(i+jx)^m\right)\right) \\
&\quad -\frac{(gm)\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx}{2e} \\
&\quad +\frac{(dgm)\int\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx}{2e} \\
&\quad -(bgn)\int x\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(h(i+jx)^m\right)\,dx \\
&\quad +\frac{(bdgn)\int\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(h(i+jx)^m\right)\,dx}{e} \\
&\quad -\frac{(bd^2gn)\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(h(i+jx)^m\right)}{d+ex}\,dx}{e} \\
&\quad +\frac{(bgi^2mn)\,\mathrm{Subst}\left(\int\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(\frac{e\left(\frac{ei-dj}{e}+\frac{jx}{e}\right)}{ei-dj}\right)}{x}\,dx,x,d+ex\right)}{j^2} \\
&\quad -\frac{(bgimn)\mathrm{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)\,dx,x,d+ex\right)}{ej} \\
&\quad +\left(b^2fn^2\right)\mathrm{Subst}\left(\int\frac{x(-4d+x)+2d^2\log(x)}{2e^2x}\,dx,x,d+ex\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{abgimnx}{j} + \frac{2bdfn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^2} \\
&- \frac{bfn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2} \\
&- \frac{bd^2fn\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^2} \\
&+ \frac{gim(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2ej} \\
&- \frac{gi^2m(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2} \\
&+ \frac{bdgnx(a+b\log\left(c(d+ex)^n\right))\log\left(h(i+jx)^m\right)}{e} \\
&- \frac{1}{2}bgnx^2(a+b\log\left(c(d+ex)^n\right))\log\left(h(i+jx)^m\right) \\
&+ \frac{1}{2}x^2(a+b\log\left(c(d+ex)^n\right))^2\left(f+g\log\left(h(i+jx)^m\right)\right) \\
&- \frac{bgi^2mn(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j^2} \\
&- \frac{(gm)\text{Subst}\left(\int x(a+b\log\left(cx^n\right))^2\,dx,x,d+ex\right)}{2e^2} \\
&+ \frac{(dgm)\text{Subst}\left(\int (a+b\log\left(cx^n\right))^2\,dx,x,d+ex\right)}{2e^2} \\
&- \frac{(bd^2gn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(h\left(\frac{ei-dj}{e}+\frac{jx}{e}\right)^m\right)}{x}\,dx,x,d+ex\right)}{e^2} \\
&- \frac{(b^2gimn)\,\text{Subst}\left(\int \log\left(cx^n\right)\,dx,x,d+ex\right)}{ej} \\
&+ \frac{1}{2}(bgjmn)\int \frac{x^2(a+b\log\left(c(d+ex)^n\right))}{i+jx}\,dx - \frac{(bdgjmn)\int \frac{x(a+b\log(c(d+ex)^n))}{i+jx}\,dx}{e} \\
&+ \frac{(b^2fn^2)\,\text{Subst}\left(\int \frac{x(-4d+x)+2d^2\log(x)}{x}\,dx,x,d+ex\right)}{2e^2} \\
&- (b^2dgn^2)\int \frac{x\log\left(h(i+jx)^m\right)}{d+ex}\,dx + \frac{1}{2}(b^2egn^2)\int \frac{x^2\log\left(h(i+jx)^m\right)}{d+ex}\,dx \\
&+ \frac{(b^2gi^2mn^2)\,\text{Subst}\left(\int \frac{\operatorname{Li}_2\left(-\frac{jx}{ei-dj}\right)}{x}\,dx,x,d+ex\right)}{j^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{abgimnx}{j} + \frac{b^2gimn^2x}{j} - \frac{b^2gimn(d+ex)\log\left(c(d+ex)^n\right)}{ej} \\
&+ \frac{2bdfn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^2} - \frac{bfn(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)}{2e^2} \\
&- \frac{bd^2fn\log(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^2} \\
&+ \frac{dgm(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2e^2} \\
&+ \frac{gim(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2ej} - \frac{gm(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{4e^2} \\
&- \frac{gi^2m(a+b\log\left(c(d+ex)^n\right))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2} \\
&+ \frac{bdgnx(a+b\log\left(c(d+ex)^n\right))\log\left(h(i+jx)^m\right)}{e} \\
&- \frac{1}{2}bgnx^2(a+b\log\left(c(d+ex)^n\right))\log\left(h(i+jx)^m\right) \\
&- \frac{d^2g(a+b\log\left(c(d+ex)^n\right))^2\log\left(h(i+jx)^m\right)}{2e^2} \\
&+ \frac{1}{2}x^2(a+b\log\left(c(d+ex)^n\right))^2\left(f+g\log\left(h(i+jx)^m\right)\right) \\
&- \frac{bgi^2mn(a+b\log\left(c(d+ex)^n\right))\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j^2} \\
&+ \frac{b^2gi^2mn^2\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{j^2} + \frac{(d^2gjm)\operatorname{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\frac{ei-dj}{e}+\frac{jx}{e}}\,dx, x, d+ex\right)}{2e^3} \\
&+ \frac{(bgmn)\operatorname{Subst}\left(\int x(a+b\log\left(cx^n\right))\,dx, x, d+ex\right)}{2e^2} \\
&- \frac{(bdgmn)\operatorname{Subst}\left(\int (a+b\log\left(cx^n\right))\,dx, x, d+ex\right)}{e^2} \\
&+ \frac{1}{2}(bgjmn)\int \left(-\frac{i(a+b\log\left(c(d+ex)^n\right))}{j^2} + \frac{x(a+b\log\left(c(d+ex)^n\right))}{j}\right. \\
&\qquad\qquad \left. + \frac{i^2(a+b\log\left(c(d+ex)^n\right))}{j^2(i+jx)}\right)dx \\
&- \frac{(bdgjmn)\int \left(\frac{a+b\log(c(d+ex)^n)}{j} - \frac{i(a+b\log(c(d+ex)^n))}{j(i+jx)}\right)dx}{e} \\
&+ \frac{(b^2fn^2)\operatorname{Subst}\left(\int \left(-4d+x+\frac{2d^2\log(x)}{x}\right)dx, x, d+ex\right)}{2e^2} \\
&- (b^2dgn^2)\int \left(\frac{\log\left(h(i+jx)^m\right)}{e} - \frac{d\log\left(h(i+jx)^m\right)}{e(d+ex)}\right)dx \\
&+ \frac{1}{2}(b^2egn^2)\int \left(-\frac{d\log\left(h(i+jx)^m\right)}{e^2} + \frac{x\log\left(h(i+jx)^m\right)}{e}\right. \\
&\qquad\qquad \left. + \frac{d^2\log\left(h(i+jx)^m\right)}{e^2(d+ex)}\right)dx
\end{aligned}
$$

$$= -\frac{abdgmnx}{e} - \frac{abgimnx}{j} - \frac{2b^2dfn^2x}{e} + \frac{b^2gimn^2x}{j} + \frac{b^2fn^2(d+ex)^2}{4e^2}$$
$$- \frac{b^2gmn^2(d+ex)^2}{8e^2} - \frac{b^2gimn(d+ex)\log(c(d+ex)^n)}{ej}$$
$$+ \frac{2bdfn(d+ex)(a+b\log(c(d+ex)^n))}{e^2} - \frac{bfn(d+ex)^2(a+b\log(c(d+ex)^n))}{2e^2}$$
$$+ \frac{bgmn(d+ex)^2(a+b\log(c(d+ex)^n))}{4e^2}$$
$$- \frac{bd^2fn\log(d+ex)(a+b\log(c(d+ex)^n))}{e^2}$$
$$+ \frac{dgm(d+ex)(a+b\log(c(d+ex)^n))^2}{2e^2}$$
$$+ \frac{gim(d+ex)(a+b\log(c(d+ex)^n))^2}{2ej} - \frac{gm(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2}$$
$$+ \frac{d^2gm(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2e^2}$$
$$- \frac{gi^2m(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2}$$
$$+ \frac{bdgnx(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{e}$$
$$- \frac{1}{2}bgnx^2(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)$$
$$- \frac{d^2g(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{2e^2}$$
$$+ \frac{1}{2}x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))$$
$$- \frac{bgi^2mn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j^2}$$
$$+ \frac{b^2gi^2mn^2\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{j^2} + \frac{1}{2}(bgmn)\int x(a+b\log(c(d+ex)^n))\,dx$$
$$- \frac{(b^2dgmn)\operatorname{Subst}\left(\int\log(cx^n)\,dx, x, d+ex\right)}{e^2}$$
$$- \frac{(bd^2gmn)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(1+\frac{jx}{ei-dj}\right)}{x}\,dx, x, d+ex\right)}{e^2}$$
$$- \frac{(bdgmn)\int(a+b\log(c(d+ex)^n))\,dx}{e} + \frac{(bdgimn)\int\frac{a+b\log(c(d+ex)^n)}{i+jx}\,dx}{e}$$
$$- \frac{(bgimn)\int(a+b\log(c(d+ex)^n))\,dx}{2j} + \frac{(bgi^2mn)\int\frac{a+b\log(c(d+ex)^n)}{i+jx}\,dx}{2j}$$
$$+ \frac{(b^2d^2fn^2)\operatorname{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, d+ex\right)}{e^2} + \frac{1}{2}(b^2gn^2)\int x\log(h(i+jx)^m)\,dx$$
$$- \frac{(b^2dgn^2)\int\log(h(i+jx)^m)\,dx}{2e} - \frac{(b^2dgn^2)\int\log(h(i+jx)^m)\,dx}{e}$$
$$+ \frac{(b^2d^2gn^2)\int\frac{\log(h(i+jx)^m)}{d+ex}\,dx}{2e} + \frac{(b^2d^2gn^2)\int\frac{\log(h(i+jx)^m)}{d+ex}\,dx}{e}$$

= Too large to display

## Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 2067, normalized size of antiderivative = 1.71

$$\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx = \text{Result too large to show}$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] (-8*a*b*d*e*g*i*j*m*n + 4*b^2*d*e*g*i*j*m*n^2 + 8*b^2*d^2*g*j^2*m*n^2 + 4*a
^2*e^2*g*i*j*m*x + 8*a*b*d*e*f*j^2*n*x - 12*a*b*e^2*g*i*j*m*n*x - 12*a*b*d*
e*g*j^2*m*n*x - 12*b^2*d*e*f*j^2*n^2*x + 14*b^2*e^2*g*i*j*m*n^2*x + 28*b^2*
d*e*g*j^2*m*n^2*x + 4*a^2*e^2*f*j^2*x^2 - 2*a^2*e^2*g*j^2*m*x^2 - 4*a*b*e^2
*f*j^2*n*x^2 + 4*a*b*e^2*g*j^2*m*n*x^2 + 2*b^2*e^2*f*j^2*n^2*x^2 - 3*b^2*e^
2*g*j^2*m*n^2*x^2 - 8*a*b*d^2*f*j^2*n*Log[d + e*x] + 8*a*b*d*e*g*i*j*m*n*Lo
g[d + e*x] + 4*a*b*d^2*g*j^2*m*n*Log[d + e*x] + 12*b^2*d^2*f*j^2*n^2*Log[d
+ e*x] - 4*b^2*d*e*g*i*j*m*n^2*Log[d + e*x] - 16*b^2*d^2*g*j^2*m*n^2*Log[d
+ e*x] + 4*b^2*d^2*f*j^2*n^2*Log[d + e*x]^2 - 4*b^2*d*e*g*i*j*m*n^2*Log[d +
 e*x]^2 - 2*b^2*d^2*g*j^2*m*n^2*Log[d + e*x]^2 - 8*b^2*d*e*g*i*j*m*n*Log[c*
(d + e*x)^n] + 8*a*b*e^2*g*i*j*m*x*Log[c*(d + e*x)^n] + 8*b^2*d*e*f*j^2*n*x
*Log[c*(d + e*x)^n] - 12*b^2*e^2*g*i*j*m*n*x*Log[c*(d + e*x)^n] - 12*b^2*d*
e*g*j^2*m*n*x*Log[c*(d + e*x)^n] + 8*a*b*e^2*f*j^2*x^2*Log[c*(d + e*x)^n] -
 4*a*b*e^2*g*j^2*m*x^2*Log[c*(d + e*x)^n] - 4*b^2*e^2*f*j^2*n*x^2*Log[c*(d
+ e*x)^n] + 4*b^2*e^2*g*j^2*m*n*x^2*Log[c*(d + e*x)^n] - 8*b^2*d^2*f*j^2*n*
Log[d + e*x]*Log[c*(d + e*x)^n] + 8*b^2*d*e*g*i*j*m*n*Log[d + e*x]*Log[c*(d
 + e*x)^n] + 4*b^2*d^2*g*j^2*m*n*Log[d + e*x]*Log[c*(d + e*x)^n] + 4*b^2*e^
2*g*i*j*m*x*Log[c*(d + e*x)^n]^2 + 4*b^2*e^2*f*j^2*x^2*Log[c*(d + e*x)^n]^2
 - 2*b^2*e^2*g*j^2*m*x^2*Log[c*(d + e*x)^n]^2 - 4*a^2*e^2*g*i^2*m*Log[i + j
*x] + 4*a*b*e^2*g*i^2*m*n*Log[i + j*x] + 8*a*b*d*e*g*i*j*m*n*Log[i + j*x] -
 2*b^2*e^2*g*i^2*m*n^2*Log[i + j*x] - 12*b^2*d*e*g*i*j*m*n^2*Log[i + j*x] +
 8*a*b*e^2*g*i^2*m*n*Log[d + e*x]*Log[i + j*x] - 4*b^2*e^2*g*i^2*m*n^2*Log[
d + e*x]*Log[i + j*x] - 8*b^2*d*e*g*i*j*m*n^2*Log[d + e*x]*Log[i + j*x] - 4
*b^2*e^2*g*i^2*m*n^2*Log[d + e*x]^2*Log[i + j*x] - 8*a*b*e^2*g*i^2*m*Log[c*
(d + e*x)^n]*Log[i + j*x] + 4*b^2*e^2*g*i^2*m*n*Log[c*(d + e*x)^n]*Log[i +
j*x] + 8*b^2*d*e*g*i*j*m*n*Log[c*(d + e*x)^n]*Log[i + j*x] + 8*b^2*e^2*g*i^
2*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[i + j*x] - 4*b^2*e^2*g*i^2*m*Log[
c*(d + e*x)^n]^2*Log[i + j*x] - 8*a*b*e^2*g*i^2*m*n*Log[d + e*x]*Log[(e*(i
+ j*x))/(e*i - d*j)] + 8*a*b*d^2*g*j^2*m*n*Log[d + e*x]*Log[(e*(i + j*x))/(
e*i - d*j)] + 4*b^2*e^2*g*i^2*m*n^2*Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d
*j)] + 8*b^2*d*e*g*i*j*m*n^2*Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d*j)] -
12*b^2*d^2*g*j^2*m*n^2*Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d*j)] + 4*b^2*
e^2*g*i^2*m*n^2*Log[d + e*x]^2*Log[(e*(i + j*x))/(e*i - d*j)] - 4*b^2*d^2*g
*j^2*m*n^2*Log[d + e*x]^2*Log[(e*(i + j*x))/(e*i - d*j)] - 8*b^2*e^2*g*i^2*
```

```
m*n*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[(e*(i + j*x))/(e*i - d*j)] + 8*b^2*
d^2*g*j^2*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[(e*(i + j*x))/(e*i - d*j)
] + 8*a*b*d*e*g*j^2*n*x*Log[h*(i + j*x)^m] - 12*b^2*d*e*g*j^2*n^2*x*Log[h*(
i + j*x)^m] + 4*a^2*e^2*g*j^2*x^2*Log[h*(i + j*x)^m] - 4*a*b*e^2*g*j^2*n*x^
2*Log[h*(i + j*x)^m] + 2*b^2*e^2*g*j^2*n^2*x^2*Log[h*(i + j*x)^m] - 8*a*b*d
^2*g*j^2*n*Log[d + e*x]*Log[h*(i + j*x)^m] + 12*b^2*d^2*g*j^2*n^2*Log[d + e
*x]*Log[h*(i + j*x)^m] + 4*b^2*d^2*g*j^2*n^2*Log[d + e*x]^2*Log[h*(i + j*x)
^m] + 8*b^2*d*e*g*j^2*n*x*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] + 8*a*b*e^2
*g*j^2*x^2*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] - 4*b^2*e^2*g*j^2*n*x^2*Lo
g[c*(d + e*x)^n]*Log[h*(i + j*x)^m] - 8*b^2*d^2*g*j^2*n*Log[d + e*x]*Log[c*
(d + e*x)^n]*Log[h*(i + j*x)^m] + 4*b^2*e^2*g*j^2*x^2*Log[c*(d + e*x)^n]^2*
Log[h*(i + j*x)^m] - 4*b*g*(e*i - d*j)*m*n*(2*a*(e*i + d*j) - b*(e*i + 3*d*
j)*n + 2*b*(e*i + d*j)*Log[c*(d + e*x)^n])*PolyLog[2, (j*(d + e*x))/(-(e*i)
 + d*j)] + 8*b^2*g*(e^2*i^2 - d^2*j^2)*m*n^2*PolyLog[3, (j*(d + e*x))/(-(e*
i) + d*j)])/(8*e^2*j^2)
```

## Maple [F]

$$\int x(a+b\ln(c(ex+d)^n))^2(f+g\ln(h(jx+i)^m))\,dx$$

```
[In] int(x*(a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m)),x)
[Out] int(x*(a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m)),x)
```

## Fricas [F]

$$\int x(a+b\log(c(d+ex)^n))^2\,(f+g\log(h(i+jx)^m))\;dx = \int (b\log((ex+d)^n c)+a)^2(g\log((jx+i)^m h)+f)x\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m)),x, algorithm="f
ricas")
[Out] integral(b^2*f*x*log((e*x + d)^n*c)^2 + 2*a*b*f*x*log((e*x + d)^n*c) + a^2*
f*x + (b^2*g*x*log((e*x + d)^n*c)^2 + 2*a*b*g*x*log((e*x + d)^n*c) + a^2*g*
x)*log((j*x + i)^m*h), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(e*x+d)**n))**2*(f+g*ln(h*(j*x+i)**m)),x)
[Out] Timed out
```

## Maxima [F]

$$\int x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx$$
$$=\int(b\log((ex+d)^nc)+a)^2(g\log((jx+i)^mh)+f)x\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m)),x, algorithm="m
axima")
[Out] 1/2*b^2*f*x^2*log((e*x + d)^n*c)^2 - 1/2*a*b*e*f*n*(2*d^2*log(e*x + d)/e^3
+ (e*x^2 - 2*d*x)/e^2) - 1/4*a^2*g*j*m*(2*i^2*log(j*x + i)/j^3 + (j*x^2 - 2
*i*x)/j^2) + a*b*f*x^2*log((e*x + d)^n*c) + 1/2*a^2*g*x^2*log((j*x + i)^m*h
) + 1/2*a^2*f*x^2 - 1/4*(2*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^
2)*log((e*x + d)^n*c) - (e^2*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*l
og(e*x + d))*n^2/e^2)*b^2*f + 1/4*((2*b^2*e^2*g*i*j*m*x - 2*b^2*e^2*g*i^2*m
*log(j*x + i) - (j^2*m - 2*j^2*log(h))*b^2*e^2*g*x^2)*log((e*x + d)^n)^2 +
(2*b^2*d^2*g*j^2*n^2*log(e*x + d)^2 + 2*b^2*e^2*g*j^2*x^2*log((e*x + d)^n)^
2 - (2*(e^2*g*j^2*n - 2*e^2*g*j^2*log(c))*a*b - (e^2*g*j^2*n^2 - 2*e^2*g*j^
2*n*log(c) + 2*e^2*g*j^2*log(c)^2)*b^2)*x^2 + 2*(2*a*b*d*e*g*j^2*n - (3*d*e
*g*j^2*n^2 - 2*d*e*g*j^2*n*log(c))*b^2)*x - 2*(2*a*b*d^2*g*j^2*n - (3*d^2*g
*j^2*n^2 - 2*d^2*g*j^2*n*log(c))*b^2)*log(e*x + d) + 2*(2*b^2*d*e*g*j^2*n*x
 - 2*b^2*d^2*g*j^2*n*log(e*x + d) + (2*a*b*e^2*g*j^2 - (e^2*g*j^2*n - 2*e^2
*g*j^2*log(c))*b^2)*x^2)*log((e*x + d)^n))*log((j*x + i)^m))/(e^2*j^2) + in
tegrate(1/4*((2*(e^3*g*j^3*m*n - 2*(j^3*m - 2*j^3*log(h))*e^3*g*log(c))*a*b
 - (e^3*g*j^3*m*n^2 - 2*e^3*g*j^3*m*n*log(c) + 2*(j^3*m - 2*j^3*log(h))*e^3
*g*log(c)^2)*b^2)*x^3 - (2*(d*e^2*g*j^3*m*n - 2*(2*e^3*g*i*j^2*log(h) - (j^
3*m - 2*j^3*log(h))*d*e^2*g)*log(c))*a*b - (5*d*e^2*g*j^3*m*n^2 - 2*d*e^2*g
*j^3*m*n*log(c) + 2*(2*e^3*g*i*j^2*log(h) - (j^3*m - 2*j^3*log(h))*d*e^2*g)
*log(c)^2)*b^2)*x^2 - 2*(b^2*d^2*e*g*j^3*m*n^2*x + b^2*d^3*g*j^3*m*n^2)*log
(e*x + d)^2 - 2*(2*(d^2*e*g*j^3*m*n - 2*d*e^2*g*i*j^2*log(c)*log(h))*a*b -
(3*d^2*e*g*j^3*m*n^2 - 2*d^2*e*g*j^3*m*n*log(c) + 2*d*e^2*g*i*j^2*log(c)^2*
log(h))*b^2)*x + 2*(2*a*b*d^3*g*j^3*m*n - (3*d^3*g*j^3*m*n^2 - 2*d^3*g*j^3*
```

```
m*n*log(c))*b^2 + (2*a*b*d^2*e*g*j^3*m*n - (3*d^2*e*g*j^3*m*n^2 - 2*d^2*e*g
*j^3*m*n*log(c))*b^2)*x)*log(e*x + d) - 2*(2*((j^3*m - 2*j^3*log(h))*a*b*e^
3*g + ((j^3*m - 2*j^3*log(h))*e^3*g*log(c) - (j^3*m*n - j^3*n*log(h))*e^3*g
)*b^2)*x^3 - (2*(2*e^3*g*i*j^2*log(h) - (j^3*m - 2*j^3*log(h))*d*e^2*g)*a*b
 - (d*e^2*g*j^3*m*n + (i*j^2*m*n + 2*i*j^2*n*log(h))*e^3*g - 2*(2*e^3*g*i*j
^2*log(h) - (j^3*m - 2*j^3*log(h))*d*e^2*g)*log(c))*b^2)*x^2 - 2*(2*a*b*d*e
^2*g*i*j^2*log(h) - (e^3*g*i^2*j*m*n + d^2*e*g*j^3*m*n - 2*d*e^2*g*i*j^2*lo
g(c)*log(h))*b^2)*x - 2*(b^2*d^2*e*g*j^3*m*n*x + b^2*d^3*g*j^3*m*n)*log(e*x
 + d) - 2*(b^2*e^3*g*i^2*j*m*n*x + b^2*e^3*g*i^3*m*n)*log(j*x + i))*log((e*
x + d)^n))/(e^3*j^3*x^2 + d*e^2*i*j^2 + (e^3*i*j^2 + d*e^2*j^3)*x), x)
```

## Giac [F]

$$\int x(a+b\log\left(c(d+ex)^n\right))^2\left(f+g\log\left(h(i+jx)^m\right)\right)\,dx = \int (b\log\left((ex+d)^nc\right)+a)^2(g\log\left((jx+i)^mh\right)+f)x\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m)),x, algorithm="g
iac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*(g*log((j*x + i)^m*h) + f)*x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x(a+b\log\left(c(d+ex)^n\right))^2\left(f+g\log\left(h(i+jx)^m\right)\right)\,dx = \int x\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)^2\,\left(f+g\,\ln\left(h\,{\left(i+j\,x\right)}^m\right)\right)\,dx$$

```
[In] int(x*(a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)),x)
```

```
[Out] int(x*(a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)), x)
```

# 3.394 $\int (a+b\log(c(d+ex)^n))^2 (f+g\log(h(i+jx)^m))\,d$

## Optimal result

Integrand size = 31, antiderivative size = 649

$$\int (a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx$$
$$= -2abfnx+4abgmnx+2b^2fn^2x-6b^2gmn^2x-\frac{2b^2fn(d+ex)\log(c(d+ex)^n)}{e}$$
$$+\frac{4b^2gmn(d+ex)\log(c(d+ex)^n)}{e}+\frac{df(a+b\log(c(d+ex)^n))^2}{e}$$
$$-\frac{gm(d+ex)(a+b\log(c(d+ex)^n))^2}{e}-\frac{2bgimn(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}$$
$$-\frac{dgm(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{e}+\frac{gim(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}$$
$$+\frac{2b^2gn^2(i+jx)\log(h(i+jx)^m)}{j}-\frac{2b^2dgn^2\log\left(-\frac{j(d+ex)}{ei-dj}\right)\log(h(i+jx)^m)}{e}$$
$$-2bgnx(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)$$
$$+\frac{dg(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{e}$$
$$+x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))-\frac{2b^2gimn^2\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j}$$
$$-\frac{2bdgmn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{e}$$
$$+\frac{2bgimn(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j}$$
$$-\frac{2b^2dgmn^2\operatorname{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{e}+\frac{2b^2dgmn^2\operatorname{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)}{e}$$
$$-\frac{2b^2gimn^2\operatorname{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)}{j}$$

```
[Out] -2*a*b*f*n*x+4*a*b*g*m*n*x+2*b^2*f*n^2*x-6*b^2*g*m*n^2*x-2*b^2*f*n*(e*x+d)*
ln(c*(e*x+d)^n)/e+4*b^2*g*m*n*(e*x+d)*ln(c*(e*x+d)^n)/e+d*f*(a+b*ln(c*(e*x+
d)^n))^2/e-g*m*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e-2*b*g*i*m*n*(a+b*ln(c*(e*x
+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/j-d*g*m*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(j*x+
i)/(-d*j+e*i))/e+g*i*m*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(j*x+i)/(-d*j+e*i))/j+2
*b^2*g*n^2*(j*x+i)*ln(h*(j*x+i)^m)/j-2*b^2*d*g*n^2*ln(-j*(e*x+d)/(-d*j+e*i)
)*ln(h*(j*x+i)^m)/e-2*b*g*n*x*(a+b*ln(c*(e*x+d)^n))*ln(h*(j*x+i)^m)+d*g*(a+
b*ln(c*(e*x+d)^n))^2*ln(h*(j*x+i)^m)/e+x*(a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*
(j*x+i)^m))-2*b^2*g*i*m*n^2*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j-2*b*d*g*m*n*
```

```
(a+b*ln(c*(e*x+d)^n))*polylog(2,-j*(e*x+d)/(-d*j+e*i))/e+2*b*g*i*m*n*(a+b*l
n(c*(e*x+d)^n))*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j-2*b^2*d*g*m*n^2*polylog(
2,e*(j*x+i)/(-d*j+e*i))/e+2*b^2*d*g*m*n^2*polylog(3,-j*(e*x+d)/(-d*j+e*i))/
e-2*b^2*g*i*m*n^2*polylog(3,-j*(e*x+d)/(-d*j+e*i))/j
```

## Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.613$, Rules used = {2479, 2463, 2436, 2333, 2332, 2443, 2481, 2421, 6724, 6874, 2458, 2388, 2338, 45, 2441, 2440, 2438, 2422, 2354}

$$
\begin{aligned}
&\int (a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))\,dx \\
&= x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m)) \\
&+\frac{df(a+b\log(c(d+ex)^n))^2}{e}-2bgnx\log(h(i+jx)^m)(a+b\log(c(d+ex)^n)) \\
&+\frac{dg\log(h(i+jx)^m)(a+b\log(c(d+ex)^n))^2}{e} \\
&-\frac{2bdgmn\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{e} \\
&+\frac{2bgimn\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{j} \\
&-\frac{2bgimn\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))}{j} \\
&-\frac{dgm\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))^2}{e} \\
&+\frac{gim\log\left(\frac{e(i+jx)}{ei-dj}\right)(a+b\log(c(d+ex)^n))^2}{j}-\frac{gm(d+ex)(a+b\log(c(d+ex)^n))^2}{e} \\
&-2abfnx+4abgmnx-\frac{2b^2fn(d+ex)\log(c(d+ex)^n)}{e} \\
&+\frac{4b^2gmn(d+ex)\log(c(d+ex)^n)}{e}-\frac{2b^2dgn^2\log\left(-\frac{j(d+ex)}{ei-dj}\right)\log(h(i+jx)^m)}{e} \\
&-\frac{2b^2gimn^2\,\text{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j}-\frac{2b^2dgmn^2\,\text{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{e} \\
&+\frac{2b^2dgmn^2\,\text{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)}{e}-\frac{2b^2gimn^2\,\text{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)}{j} \\
&+2b^2fn^2x+\frac{2b^2gn^2(i+jx)\log(h(i+jx)^m)}{j}-6b^2gmn^2x
\end{aligned}
$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] -2*a*b*f*n*x + 4*a*b*g*m*n*x + 2*b^2*f*n^2*x - 6*b^2*g*m*n^2*x - (2*b^2*f*n
*(d + e*x)*Log[c*(d + e*x)^n])/e + (4*b^2*g*m*n*(d + e*x)*Log[c*(d + e*x)^n
])/e + (d*f*(a + b*Log[c*(d + e*x)^n])^2)/e - (g*m*(d + e*x)*(a + b*Log[c*(
d + e*x)^n])^2)/e - (2*b*g*i*m*n*(a + b*Log[c*(d + e*x)^n])*Log[(e*(i + j*x
))/(e*i - d*j)])/j - (d*g*m*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(i + j*x))/
(e*i - d*j)])/e + (g*i*m*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(i + j*x))/(e*
i - d*j)])/j + (2*b^2*g*n^2*(i + j*x)*Log[h*(i + j*x)^m])/j - (2*b^2*d*g*n^
2*Log[-((j*(d + e*x))/(e*i - d*j))]*Log[h*(i + j*x)^m])/e - 2*b*g*n*x*(a +
b*Log[c*(d + e*x)^n])*Log[h*(i + j*x)^m] + (d*g*(a + b*Log[c*(d + e*x)^n])^
2*Log[h*(i + j*x)^m])/e + x*(a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i +
j*x)^m]) - (2*b^2*g*i*m*n^2*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/j - (
2*b*d*g*m*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((j*(d + e*x))/(e*i - d*
j))])/e + (2*b*g*i*m*n*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((j*(d + e*x)
)/(e*i - d*j))])/j - (2*b^2*d*g*m*n^2*PolyLog[2, (e*(i + j*x))/(e*i - d*j)]
)/e + (2*b^2*d*g*m*n^2*PolyLog[3, -((j*(d + e*x))/(e*i - d*j))])/e - (2*b^2
*g*i*m*n^2*PolyLog[3, -((j*(d + e*x))/(e*i - d*j))])/j
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
```

```
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2479

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[x*((a +
 b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Dist[b*e*n*p, Int[x*(a + b*Lo
g[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m)) \\
&\quad -(gjm)\int \frac{x(a+b\log(c(d+ex)^n))^2}{i+jx}\,dx \\
&\quad -(2ben)\int \frac{x(a+b\log(c(d+ex)^n))(f+g\log(h(i+jx)^m))}{d+ex}\,dx \\
&= x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m)) \\
&\quad -(gjm)\int \left(\frac{(a+b\log(c(d+ex)^n))^2}{j}-\frac{i(a+b\log(c(d+ex)^n))^2}{j(i+jx)}\right)dx \\
&\quad -(2ben)\int \left(\frac{fx(a+b\log(c(d+ex)^n))}{d+ex}\right. \\
&\qquad \left.+\frac{gx(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{d+ex}\right)dx \\
&= x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))-(gm)\int (a+b\log(c(d+ex)^n))^2\,dx \\
&\quad +(gim)\int \frac{(a+b\log(c(d+ex)^n))^2}{i+jx}\,dx-(2befn)\int \frac{x(a+b\log(c(d+ex)^n))}{d+ex}\,dx \\
&\quad -(2begn)\int \frac{x(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{d+ex}\,dx
\end{aligned}$$

$$=\frac{gim(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}$$
$$+x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))$$
$$-\frac{(gm)\text{Subst}\left(\int(a+b\log(cx^n))^2\,dx,x,d+ex\right)}{e}$$
$$-(2bfn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)(a+b\log(cx^n))}{x}\,dx,x,d+ex\right)$$
$$-(2begn)\int\left(\frac{(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{e}-\frac{d(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{e(d+ex)}\right)dx$$
$$-\frac{(2begimn)\int\frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{d+ex}\,dx}{j}$$

$$=-\frac{gm(d+ex)(a+b\log(c(d+ex)^n))^2}{e}+\frac{gim(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}$$
$$+x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))$$
$$-\frac{(2bfn)\text{Subst}\left(\int(a+b\log(cx^n))\,dx,x,d+ex\right)}{e}$$
$$+\frac{(2bdfn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x}\,dx,x,d+ex\right)}{e}$$
$$-(2bgn)\int(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)\,dx$$
$$+(2bdgn)\int\frac{(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{d+ex}\,dx$$
$$+\frac{(2bgmn)\text{Subst}\left(\int(a+b\log(cx^n))\,dx,x,d+ex\right)}{e}$$
$$-\frac{(2bgimn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{ei-dj}{e}+\frac{jx}{e}\right)}{ei-dj}\right)}{x}\,dx,x,d+ex\right)}{j}$$

$$= -2abfnx + 2abgmnx + \frac{df(a + b\log(c(d + ex)^n))^2}{e} - \frac{gm(d + ex)(a + b\log(c(d + ex)^n))^2}{e} + \frac{gim(a + b\log(c(d + ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} - 2bgnx(a + b\log(c(d + ex)^n))\log(h(i + jx)^m) + x(a + b\log(c(d + ex)^n))^2(f + g\log(h(i + jx)^m)) + \frac{2bgimn(a + b\log(c(d + ex)^n))\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} - \frac{(2b^2fn)\operatorname{Subst}\left(\int\log(cx^n)\,dx, x, d + ex\right)}{e} + \frac{(2bdgn)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(h\left(\frac{ei-dj}{e}+\frac{jx}{e}\right)^m\right)}{x}\,dx, x, d + ex\right)}{e} + \frac{(2b^2gmn)\operatorname{Subst}\left(\int\log(cx^n)\,dx, x, d + ex\right)}{e} + (2bgjmn)\int\frac{x(a + b\log(c(d + ex)^n))}{i + jx}\,dx + \left(2b^2egn^2\right)\int\frac{x\log(h(i + jx)^m)}{d + ex}\,dx - \frac{(2b^2gimn^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{jx}{ei-dj}\right)}{x}\,dx, x, d + ex\right)}{j}$$

$$= -2abfnx + 2abgmnx + 2b^2fn^2x - 2b^2gmn^2x - \frac{2b^2fn(d + ex)\log(c(d + ex)^n)}{e} + \frac{2b^2gmn(d + ex)\log(c(d + ex)^n)}{e} + \frac{df(a + b\log(c(d + ex)^n))^2}{e} - \frac{gm(d + ex)(a + b\log(c(d + ex)^n))^2}{e} + \frac{gim(a + b\log(c(d + ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} - 2bgnx(a + b\log(c(d + ex)^n))\log(h(i + jx)^m) + \frac{dg(a + b\log(c(d + ex)^n))^2\log(h(i + jx)^m)}{e} + x(a + b\log(c(d + ex)^n))^2(f + g\log(h(i + jx)^m)) + \frac{2bgimn(a + b\log(c(d + ex)^n))\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} - \frac{2b^2gimn^2\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} - \frac{(dgjm)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{\frac{ei-dj}{e}+\frac{jx}{e}}\,dx, x, d + ex\right)}{e^2} + (2bgjmn)\int\left(\frac{a + b\log(c(d + ex)^n)}{j} - \frac{i(a + b\log(c(d + ex)^n))}{j(i + jx)}\right)dx + \left(2b^2egn^2\right)\int\left(\frac{\log(h(i + jx)^m)}{e} - \frac{d\log(h(i + jx)^m)}{e(d + ex)}\right)dx$$

$$
\begin{aligned}
&= -2abfnx + 2abgmnx + 2b^2fn^2x - 2b^2gmn^2x \\
&\quad - \frac{2b^2fn(d+ex)\log\left(c(d+ex)^n\right)}{e} + \frac{2b^2gmn(d+ex)\log\left(c(d+ex)^n\right)}{e} \\
&\quad + \frac{df\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} - \frac{gm(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} \\
&\quad - \frac{dgm\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{e} \\
&\quad + \frac{gim\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} \\
&\quad - 2bgnx\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(h(i+jx)^m\right) \\
&\quad + \frac{dg\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(h(i+jx)^m\right)}{e} \\
&\quad + x\left(a+b\log\left(c(d+ex)^n\right)\right)^2\left(f+g\log\left(h(i+jx)^m\right)\right) \\
&\quad + \frac{2bgimn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} \\
&\quad - \frac{2b^2gimn^2\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} + (2bgmn)\int\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx \\
&\quad + \frac{(2bdgmn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(1+\frac{jx}{ei-dj}\right)}{x}\,dx, x, d+ex\right)}{e} \\
&\quad - (2bgimn)\int\frac{a+b\log\left(c(d+ex)^n\right)}{i+jx}\,dx \\
&\quad + \left(2b^2gn^2\right)\int\log\left(h(i+jx)^m\right)\,dx - \left(2b^2dgn^2\right)\int\frac{\log\left(h(i+jx)^m\right)}{d+ex}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -2abfnx + 4abgmnx + 2b^2fn^2x - 2b^2gmn^2x \\
&\quad - \frac{2b^2fn(d+ex)\log\left(c(d+ex)^n\right)}{e} + \frac{2b^2gmn(d+ex)\log\left(c(d+ex)^n\right)}{e} \\
&\quad + \frac{df\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} - \frac{gm(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} \\
&\quad - \frac{2bgimn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} \\
&\quad - \frac{dgm\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{e} \\
&\quad + \frac{gim\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} \\
&\quad - \frac{2b^2dgn^2\log\left(-\frac{j(d+ex)}{ei-dj}\right)\log\left(h(i+jx)^m\right)}{e} \\
&\quad - 2bgnx\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(h(i+jx)^m\right) \\
&\quad + \frac{dg\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(h(i+jx)^m\right)}{e} \\
&\quad + x\left(a+b\log\left(c(d+ex)^n\right)\right)^2\left(f+g\log\left(h(i+jx)^m\right)\right) \\
&\quad - \frac{2bdgmn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{e} \\
&\quad + \frac{2bgimn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} - \frac{2b^2gimn^2\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} \\
&\quad + \left(2b^2gmn\right)\int \log\left(c(d+ex)^n\right)\,dx + \frac{\left(2b^2gn^2\right)\operatorname{Subst}\left(\int \log\left(hx^m\right)\,dx, x, i+jx\right)}{j} \\
&\quad + \frac{\left(2b^2dgmn^2\right)\operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(-\frac{jx}{ei-dj}\right)}{x}\,dx, x, d+ex\right)}{e} \\
&\quad + \frac{\left(2b^2egimn^2\right)\int \frac{\log\left(\frac{e(i+jx)}{ei-dj}\right)}{d+ex}\,dx}{j} + \frac{\left(2b^2dgjmn^2\right)\int \frac{\log\left(\frac{j(d+ex)}{-ei+dj}\right)}{i+jx}\,dx}{e}
\end{aligned}
$$

$$= -2abfnx + 4abgmnx + 2b^2fn^2x - 4b^2gmn^2x - \frac{2b^2fn(d+ex)\log(c(d+ex)^n)}{e} + \frac{2b^2gmn(d+ex)\log(c(d+ex)^n)}{e} + \frac{df(a+b\log(c(d+ex)^n))^2}{e} - \frac{gm(d+ex)(a+b\log(c(d+ex)^n))^2}{e} - \frac{2bgimn(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} - \frac{dgm(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{e} + \frac{gim(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} + \frac{2b^2gn^2(i+jx)\log(h(i+jx)^m)}{j} - \frac{2b^2dgn^2\log\left(-\frac{j(d+ex)}{ei-dj}\right)\log(h(i+jx)^m)}{e} - 2bgnx(a+b\log(c(d+ex)^n))\log(h(i+jx)^m) + \frac{dg(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{e} + x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m)) - \frac{2bdgmn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{e} + \frac{2bgimn(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} + \frac{2b^2dgmn^2\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{e} - \frac{2b^2gimn^2\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} + \frac{(2b^2gmn)\operatorname{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{e} + \frac{(2b^2dgmn^2)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{ex}{-ei+dj}\right)}{x}\,dx,x,i+jx\right)}{e} + \frac{(2b^2gimn^2)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{jx}{ei-dj}\right)}{x}\,dx,x,d+ex\right)}{j}$$

$$
\begin{aligned}
&= -2abfnx + 4abgmnx + 2b^2fn^2x - 6b^2gmn^2x \\
&\quad - \frac{2b^2fn(d+ex)\log\left(c(d+ex)^n\right)}{e} + \frac{4b^2gmn(d+ex)\log\left(c(d+ex)^n\right)}{e} \\
&\quad + \frac{df\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} - \frac{gm(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} \\
&\quad - \frac{2bgimn\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} \\
&\quad - \frac{dgm\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{e} \\
&\quad + \frac{gim\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} \\
&\quad + \frac{2b^2gn^2(i+jx)\log\left(h(i+jx)^m\right)}{j} - \frac{2b^2dgn^2\log\left(-\frac{j(d+ex)}{ei-dj}\right)\log\left(h(i+jx)^m\right)}{e} \\
&\quad - 2bgnx\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(h(i+jx)^m\right) \\
&\quad + \frac{dg\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(h(i+jx)^m\right)}{e} \\
&\quad + x\left(a+b\log\left(c(d+ex)^n\right)\right)^2\left(f+g\log\left(h(i+jx)^m\right)\right) \\
&\quad - \frac{2b^2gimn^2\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} - \frac{2bdgmn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{e} \\
&\quad + \frac{2bgimn\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} - \frac{2b^2dgmn^2\operatorname{Li}_2\left(\frac{e(i+jx)}{ei-dj}\right)}{e} \\
&\quad + \frac{2b^2dgmn^2\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{e} - \frac{2b^2gimn^2\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{j}
\end{aligned}
$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1355 vs. $2(649) = 1298$.

Time = 0.32 (sec) , antiderivative size = 1355, normalized size of antiderivative = 2.09

$$
\int \left(a+b\log\left(c(d+ex)^n\right)\right)^2\left(f+g\log\left(h(i+jx)^m\right)\right)\,dx
$$

$$
= \frac{-2abdfjn + 2abdgjmn - 2b^2dgjmn^2 + a^2efjx - a^2egjmx - 2abefjnx + 4abegjmnx + 2b^2efjn^2x - 6b
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] (-2*a*b*d*f*j*n + 2*a*b*d*g*j*m*n - 2*b^2*d*g*j*m*n^2 + a^2*e*f*j*x - a^2*e
*g*j*m*x - 2*a*b*e*f*j*n*x + 4*a*b*e*g*j*m*n*x + 2*b^2*e*f*j*n^2*x - 6*b^2*
```

```
e*g*j*m*n^2*x + 2*a*b*d*f*j*n*Log[d + e*x] - 2*a*b*d*g*j*m*n*Log[d + e*x] +
 2*b^2*d*g*j*m*n^2*Log[d + e*x] - b^2*d*f*j*n^2*Log[d + e*x]^2 + b^2*d*g*j*
m*n^2*Log[d + e*x]^2 - 2*b^2*d*f*j*n*Log[c*(d + e*x)^n] + 2*b^2*d*g*j*m*n*L
og[c*(d + e*x)^n] + 2*a*b*e*f*j*x*Log[c*(d + e*x)^n] - 2*a*b*e*g*j*m*x*Log[
c*(d + e*x)^n] - 2*b^2*e*f*j*n*x*Log[c*(d + e*x)^n] + 4*b^2*e*g*j*m*n*x*Log
[c*(d + e*x)^n] + 2*b^2*d*f*j*n*Log[d + e*x]*Log[c*(d + e*x)^n] - 2*b^2*d*g
*j*m*n*Log[d + e*x]*Log[c*(d + e*x)^n] + b^2*e*f*j*x*Log[c*(d + e*x)^n]^2 -
 b^2*e*g*j*m*x*Log[c*(d + e*x)^n]^2 + a^2*e*g*i*m*Log[i + j*x] - 2*a*b*e*g*
i*m*n*Log[i + j*x] + 2*a*b*d*g*j*m*n*Log[i + j*x] + 2*b^2*e*g*i*m*n^2*Log[i
 + j*x] - 2*a*b*e*g*i*m*n*Log[d + e*x]*Log[i + j*x] + 2*b^2*e*g*i*m*n^2*Log
[d + e*x]*Log[i + j*x] - 2*b^2*d*g*j*m*n^2*Log[d + e*x]*Log[i + j*x] + b^2*
e*g*i*m*n^2*Log[d + e*x]^2*Log[i + j*x] + 2*a*b*e*g*i*m*Log[c*(d + e*x)^n]*
Log[i + j*x] - 2*b^2*e*g*i*m*n*Log[c*(d + e*x)^n]*Log[i + j*x] + 2*b^2*d*g*
j*m*n*Log[c*(d + e*x)^n]*Log[i + j*x] - 2*b^2*e*g*i*m*n*Log[d + e*x]*Log[c*
(d + e*x)^n]*Log[i + j*x] + b^2*e*g*i*m*Log[c*(d + e*x)^n]^2*Log[i + j*x] +
 2*a*b*e*g*i*m*n*Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d*j)] - 2*a*b*d*g*j*
m*n*Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d*j)] - 2*b^2*e*g*i*m*n^2*Log[d +
 e*x]*Log[(e*(i + j*x))/(e*i - d*j)] + 2*b^2*d*g*j*m*n^2*Log[d + e*x]*Log[(
e*(i + j*x))/(e*i - d*j)] - b^2*e*g*i*m*n^2*Log[d + e*x]^2*Log[(e*(i + j*x)
)/(e*i - d*j)] + b^2*d*g*j*m*n^2*Log[d + e*x]^2*Log[(e*(i + j*x))/(e*i - d*
j)] + 2*b^2*e*g*i*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[(e*(i + j*x))/(e*
i - d*j)] - 2*b^2*d*g*j*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[(e*(i + j*x
))/(e*i - d*j)] - 2*a*b*d*g*j*n*Log[h*(i + j*x)^m] + a^2*e*g*j*x*Log[h*(i +
 j*x)^m] - 2*a*b*e*g*j*n*x*Log[h*(i + j*x)^m] + 2*b^2*e*g*j*n^2*x*Log[h*(i
+ j*x)^m] + 2*a*b*d*g*j*n*Log[d + e*x]*Log[h*(i + j*x)^m] - b^2*d*g*j*n^2*L
og[d + e*x]^2*Log[h*(i + j*x)^m] - 2*b^2*d*g*j*n*Log[c*(d + e*x)^n]*Log[h*(
i + j*x)^m] + 2*a*b*e*g*j*x*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] - 2*b^2*e
*g*j*n*x*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] + 2*b^2*d*g*j*n*Log[d + e*x]
*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] + b^2*e*g*j*x*Log[c*(d + e*x)^n]^2*L
og[h*(i + j*x)^m] + 2*b*g*(e*i - d*j)*m*n*(a - b*n + b*Log[c*(d + e*x)^n])*
PolyLog[2, (j*(d + e*x))/(-(e*i) + d*j)] + 2*b^2*g*(-(e*i) + d*j)*m*n^2*Pol
yLog[3, (j*(d + e*x))/(-(e*i) + d*j)])/(e*j)
```

## Maple [F]

$$\int (a+b\ln (c(ex+d)^n))^2(f+g\ln (h(jx+i)^m))\,dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m)),x)
[Out] int((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m)),x)
```

## Fricas [F]

$$\int (a + b \log(c(d + ex)^n))^2 (f + g \log(h(i + jx)^m)) \, dx = \int (b \log((ex + d)^n c) + a)^2 (g \log((jx + i)^m h) + f) \, dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m)),x, algorithm="fri
cas")
```

```
[Out] integral(b^2*f*log((e*x + d)^n*c)^2 + 2*a*b*f*log((e*x + d)^n*c) + a^2*f +
(b^2*g*log((e*x + d)^n*c)^2 + 2*a*b*g*log((e*x + d)^n*c) + a^2*g)*log((j*x
+ i)^m*h), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (a + b \log(c(d + ex)^n))^2 (f + g \log(h(i + jx)^m)) \, dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2*(f+g*ln(h*(j*x+i)**m)),x)
```

```
[Out] Timed out
```

## Maxima [F]

$$\int (a + b \log(c(d + ex)^n))^2 (f + g \log(h(i + jx)^m)) \, dx = \int (b \log((ex + d)^n c) + a)^2 (g \log((jx + i)^m h) + f) \, dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m)),x, algorithm="max
ima")
```

```
[Out] -2*a*b*e*f*n*(x/e - d*log(e*x + d)/e^2) - a^2*g*j*m*(x/j - i*log(j*x + i)/j
^2) + b^2*f*x*log((e*x + d)^n*c)^2 + 2*a*b*f*x*log((e*x + d)^n*c) + a^2*g*x
*log((j*x + i)^m*h) - (2*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x + d)^n*c)
+ (d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n^2/e)*b^2*f + a^2*f*x + ((
b^2*e*g*i*m*log(j*x + i) - (j*m - j*log(h))*b^2*e*g*x)*log((e*x + d)^n)^2 -
 (b^2*d*g*j*n^2*log(e*x + d)^2 - b^2*e*g*j*x*log((e*x + d)^n)^2 + (2*(e*g*j
*n - e*g*j*log(c))*a*b - (2*e*g*j*n^2 - 2*e*g*j*n*log(c) + e*g*j*log(c)^2)*
b^2)*x - 2*(a*b*d*g*j*n - (d*g*j*n^2 - d*g*j*n*log(c))*b^2)*log(e*x + d) -
```

```
2*(b^2*d*g*j*n*log(e*x + d) + (a*b*e*g*j - (e*g*j*n - e*g*j*log(c))*b^2)*x)
*log((e*x + d)^n))*log((j*x + i)^m))/(e*j) - integrate(-(b^2*d*e*g*i*j*log(
c)^2*log(h) + 2*a*b*d*e*g*i*j*log(c)*log(h) + (2*(e^2*g*j^2*m*n - (j^2*m -
j^2*log(h))*e^2*g*log(c))*a*b - (2*e^2*g*j^2*m*n^2 - 2*e^2*g*j^2*m*n*log(c)
 + (j^2*m - j^2*log(h))*e^2*g*log(c)^2)*b^2)*x^2 + (b^2*d*e*g*j^2*m*n^2*x +
 b^2*d^2*g*j^2*m*n^2)*log(e*x + d)^2 + (2*(d*e*g*j^2*m*n + (e^2*g*i*j*log(h
) - (j^2*m - j^2*log(h))*d*e*g)*log(c))*a*b - (2*d*e*g*j^2*m*n^2 - 2*d*e*g*
j^2*m*n*log(c) - (e^2*g*i*j*log(h) - (j^2*m - j^2*log(h))*d*e*g)*log(c)^2)*
b^2)*x - 2*(a*b*d^2*g*j^2*m*n - (d^2*g*j^2*m*n^2 - d^2*g*j^2*m*n*log(c))*b^
2 + (a*b*d*e*g*j^2*m*n - (d*e*g*j^2*m*n^2 - d*e*g*j^2*m*n*log(c))*b^2)*x)*l
og(e*x + d) + 2*(b^2*d*e*g*i*j*log(c)*log(h) + a*b*d*e*g*i*j*log(h) - ((j^2
*m - j^2*log(h))*a*b*e^2*g + ((j^2*m - j^2*log(h))*e^2*g*log(c) - (2*j^2*m*
n - j^2*n*log(h))*e^2*g)*b^2)*x^2 + ((e^2*g*i*j*log(h) - (j^2*m - j^2*log(h
))*d*e*g)*a*b + (d*e*g*j^2*m*n + (i*j*m*n - i*j*n*log(h))*e^2*g + (e^2*g*i*
j*log(h) - (j^2*m - j^2*log(h))*d*e*g)*log(c))*b^2)*x - (b^2*d*e*g*j^2*m*n*
x + b^2*d^2*g*j^2*m*n)*log(e*x + d) - (b^2*e^2*g*i*j*m*n*x + b^2*e^2*g*i^2*
m*n)*log(j*x + i))*log((e*x + d)^n))/(e^2*j^2*x^2 + d*e*i*j + (e^2*i*j + d*
e*j^2)*x), x)
```

## Giac [F]

$$\int (a+b\log(c(d+ex)^n))^2 (f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^n c)+a)^2(g\log((jx+i)^m h)+f)\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m)),x, algorithm="gia
c")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*(g*log((j*x + i)^m*h) + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d+ex)^n))^2 (f+g\log(h(i+jx)^m))\,dx = \int (a+b\,\ln(c\,(d+e\,x)^n))^2\,(f+g\,\ln(h\,(i+j\,x)^m))\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)),x)
```

```
[Out] int((a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)), x)
```

# 3.395 $\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx$



## Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx = \text{Int}\left(\frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m))/x,x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx$$

```
[In] Int[((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x,x]
[Out] Defer[Int][((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx$$
$$=\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx$$

```
[In] Integrate[((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x,x]
[Out] Integrate[((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x, x]
```

## Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(ex+d)^n))^2(f+g\ln(h(jx+i)^m))}{x}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m))/x,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m))/x,x)
```

## Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.74

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx$$
$$=\int \frac{(b\log((ex+d)^nc)+a)^2(g\log((jx+i)^mh)+f)}{x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="f
ricas")
[Out] integral((b^2*f*log((e*x + d)^n*c)^2 + 2*a*b*f*log((e*x + d)^n*c) + a^2*f +
 (b^2*g*log((e*x + d)^n*c)^2 + 2*a*b*g*log((e*x + d)^n*c) + a^2*g)*log((j*x
 + i)^m*h))/x, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2*(f+g*ln(h*(j*x+i)**m))/x,x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 4.94

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2(g\log((jx+i)^m h)+f)}{x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="m
axima")
[Out] a^2*f*log(x) + integrate(((g*log(h) + f)*b^2*log((e*x + d)^n)^2 + (g*log(h)
 + f)*b^2*log(c)^2 + 2*(g*log(h) + f)*a*b*log(c) + a^2*g*log(h) + 2*((g*log
(h) + f)*b^2*log(c) + (g*log(h) + f)*a*b)*log((e*x + d)^n) + (b^2*g*log((e*
x + d)^n)^2 + b^2*g*log(c)^2 + 2*a*b*g*log(c) + a^2*g + 2*(b^2*g*log(c) + a
*b*g)*log((e*x + d)^n))*log((j*x + i)^m))/x, x)
```

## Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2(g\log((jx+i)^m h)+f)}{x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="g
iac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*(g*log((j*x + i)^m*h) + f)/x, x)
```

## Mupad [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2\,(f+g\,\ln(h\,(i+j\,x)^m))}{x}\,dx$$

```
[In] int(((a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)))/x,x)
[Out] int(((a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)))/x, x)
```

# 3.396 $\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx$



## Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \text{Int}\left(\frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx$$

```
[In] Int[((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x^2,x]
[Out] Defer[Int][((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x^2, x
]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx$$

```
[In] Integrate[((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x^2,x]
[Out] Integrate[((a + b*Log[c*(d + e*x)^n])^2*(f + g*Log[h*(i + j*x)^m]))/x^2, x]
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(ex+d)^n))^2(f+g\ln(h(jx+i)^m))}{x^2}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m))/x^2,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^2*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.74

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2(g\log((jx+i)^m h)+f)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm=
"fricas")
[Out] integral((b^2*f*log((e*x + d)^n*c)^2 + 2*a*b*f*log((e*x + d)^n*c) + a^2*f +
 (b^2*g*log((e*x + d)^n*c)^2 + 2*a*b*g*log((e*x + d)^n*c) + a^2*g)*log((j*x
 + i)^m*h))/x^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**2*(f+g*ln(h*(j*x+i)**m))/x**2,x)
```

```
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 207, normalized size of antiderivative = 6.09

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2(g\log((jx+i)^m h)+f)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm=
"maxima")
```

```
[Out] -2*a*b*e*f*n*(log(e*x + d)/d - log(x)/d) - 2*a*b*f*log((e*x + d)^n*c)/x - a
^2*f/x + integrate(((g*log(h) + f)*b^2*log((e*x + d)^n)^2 + (g*log(h) + f)*
b^2*log(c)^2 + 2*a*b*g*log(c)*log(h) + a^2*g*log(h) + 2*((g*log(h) + f)*b^2
*log(c) + a*b*g*log(h))*log((e*x + d)^n) + (b^2*g*log((e*x + d)^n)^2 + b^2*
g*log(c)^2 + 2*a*b*g*log(c) + a^2*g + 2*(b^2*g*log(c) + a*b*g)*log((e*x + d
)^n))*log((j*x + i)^m))/x^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex+d)^n c)+a)^2(g\log((jx+i)^m h)+f)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^2*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm=
"giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^2*(g*log((j*x + i)^m*h) + f)/x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{x^2}\,dx$$
$$=\int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^2\,(f+g\,\ln(h\,(i+j\,x)^m))}{x^2}\,dx$$

```
[In] int(((a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)))/x^2,x)
[Out] int(((a + b*log(c*(d + e*x)^n))^2*(f + g*log(h*(i + j*x)^m)))/x^2, x)
```

# 3.397 $\int x(a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))$

## Optimal result

Integrand size = 32, antiderivative size = 2050

$$\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx$$

$$= -\frac{6ab^2dfn^2x}{e}+\frac{12ab^2dgmn^2x}{e}+\frac{21ab^2gimn^2x}{4j}+\frac{6b^3dfn^3x}{e}-\frac{141b^3dgmn^3x}{8e} -\frac{45b^3gimn^3x}{8j}+\frac{3}{8}b^3gmn^3x^2-\frac{3b^3fn^3(d+ex)^2}{8e^2}+\frac{3b^3gmn^3(d+ex)^2}{8e^2} +\frac{3b^3d^2gmn^3\log(d+ex)}{8e^2}-\frac{6b^3dfn^2(d+ex)\log(c(d+ex)^n)}{e^2} +\frac{12b^3dgmn^2(d+ex)\log(c(d+ex)^n)}{e^2}+\frac{21b^3gimn^2(d+ex)\log(c(d+ex)^n)}{4ej} -\frac{3}{8}b^2gmn^2x^2(a+b\log(c(d+ex)^n))+\frac{3b^2fn^2(d+ex)^2(a+b\log(c(d+ex)^n))}{4e^2} -\frac{3b^2gmn^2(d+ex)^2(a+b\log(c(d+ex)^n))}{4e^2}+\frac{3bdfn(d+ex)(a+b\log(c(d+ex)^n))^2}{e^2} -\frac{15bdgmn(d+ex)(a+b\log(c(d+ex)^n))^2}{4e^2}-\frac{9bgimn(d+ex)(a+b\log(c(d+ex)^n))^2}{4ej} -\frac{3bfn(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2}+\frac{3bgmn(d+ex)^2(a+b\log(c(d+ex)^n))^2}{4e^2} -\frac{d^2f(a+b\log(c(d+ex)^n))^3}{2e^2}+\frac{dgm(d+ex)(a+b\log(c(d+ex)^n))^3}{2e^2} +\frac{gim(d+ex)(a+b\log(c(d+ex)^n))^3}{2ej}-\frac{gm(d+ex)^2(a+b\log(c(d+ex)^n))^3}{4e^2} +\frac{3b^3gi^2mn^3\log(i+jx)}{8j^2}-\frac{3b^2gi^2mn^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{4j^2} -\frac{9b^2dgimn^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2ej} -\frac{9bd^2gmn(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{4e^2} +\frac{3bgi^2mn(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{4j^2} +\frac{3bdgimn(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2ej} +\frac{d^2gm(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2e^2} -\frac{gi^2m(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2}-\frac{3}{8}b^3gn^3x^2\log(h(i+jx)^m) +\frac{21b^3dgn^3(i+jx)\log(h(i+jx)^m)}{4ej}-\frac{21b^3d^2gn^3\log\left(-\frac{j(d+ex)}{ei-dj}\right)\log(h(i+jx)^m)}{4e^2} -\frac{9b^2dgn^2x(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{2e}$$

[Out] 3*b^2*d*g*i*m*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(2,-j*(e*x+d)/(-d*j+e*i))/e/j-9/2*b^2*d*g*i*m*n^2*(a+b*ln(c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/e/j+3/2*b*d*g*i*m*n*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(j*x+i)/(-d*j+e*i))/e/j-3/4*b^3*g*i^2*m*n^3*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j^2-21/4*b^3*d^2*g*m*n^3*polylog(2,e*(j*x+i)/(-d*j+e*i))/e^2+9/2*b^3*d^2*g*m*n^3*polylog(3,-j*(e*x+d)/(-d*j+e*i))/e^2-3/2*b^3*g*i^2*m*n^3*polylog(3,-j*(e*x+d)/(-d*j+e*i))/j^2+3*b^3*d^2*g*m*n^3*polylog(4,-j*(e*x+d)/(-d*j+e*i))/e^2-3*b^3*g*i^2*m*n^3*polylog(4,-j*(e*x+d)/(-d*j+e*i))/j^2+3*b*d*f*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^2+3/4*b*g*m*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2+1/2*g*i*m*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e/j+3/8*b^3*g*i^2*m*n^3*ln(j*x+i)/j^2-21/4*b^3*d^2*g*n^3*ln(-j*(e*x+d)/(-d*j+e*i))*ln(h*(j*x+i)^m)/e^2+9/4*b*d^2*g*n*(a+b*ln(c*(e*x+d)^n))^2*ln(h*(j*x+i)^m)/e^2+3/8*b^3*d^2*g*m*n^3*ln(e*x+d)/e^2-6*b^3*d*f*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e^2-3/4*b^2*g*m*n^2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2-6*a*b^2*d*f*n^2*x/e-141/8*b^3*d*g*m*n^3*x/e-45/8*b^3*g*i*m*n^3*x/j+21/4*b^3*g*i*m*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e/j-9/4*b*g*i*m*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e/j-9/2*b^3*d*g*i*m*n^3*polylog(2,-j*(e*x+d)/(-d*j+e*i))/e/j-3*b^3*d*g*i*m*n^3*polylog(3,-j*(e*x+d)/(-d*j+e*i))/e/j-9/2*b^2*d^2*g*m*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(2,-j*(e*x+d)/(-d*j+e*i))/e^2+3/2*b^2*g*i^2*m*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j^2+3/2*b*d^2*g*m*n*(a+b*ln(c*(e*x+d)^n))^2*polylog(2,-j*(e*x+d)/(-d*j+e*i))/e^2-3/2*b*g*i^2*m*n*(a+b*ln(c*(e*x+d)^n))^2*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j^2-3*b^2*d^2*g*m*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(3,-j*(e*x+d)/(-d*j+e*i))/e^2-1/4*g*m*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^3/e^2-3/4*b*g*n*x^2*(a+b*ln(c*(e*x+d)^n))^2*ln(h*(j*x+i)^m)-3/8*b^2*g*m*n^2*x^2*(a+b*ln(c*(e*x+d)^n))+3/4*b^2*f*n^2*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))/e^2-3/4*b*f*n*(e*x+d)^2*(a+b*ln(c*(e*x+d)^n))^2/e^2+3/4*b*g*i^2*m*n*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(j*x+i)/(-d*j+e*i))/j^2+21/4*b^3*d*g*n^3*(j*x+i)*ln(h*(j*x+i)^m)/e/j-9/2*b^2*d*g*n^2*x*(a+b*ln(c*(e*x+d)^n))*ln(h*(j*x+i)^m)/e+3/2*b*d*g*n*x*(a+b*ln(c*(e*x+d)^n))^2*ln(h*(j*x+i)^m)/e+3*b^2*g*i^2*m*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(3,-j*(e*x+d)/(-d*j+e*i))/j^2+12*a*b^2*d*g*m*n^2*x/e+21/4*a*b^2*g*i*m*n^2*x/j+12*b^3*d*g*m*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e^2-15/4*b*d*g*m*n*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^2/e^2-3/4*b^2*g*i^2*m*n^2*(a+b*ln(c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/j^2-9/4*b*d^2*g*m*n*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(j*x+i)/(-d*j+e*i))/e^2-3/8*b^3*g*n^3*x^2*ln(h*(j*x+i)^m)-1/2*d^2*g*(a+b*ln(c*(e*x+d)^n))^3*ln(h*(j*x+i)^m)/e^2+3/8*b^3*g*m*n^3*x^2-3/8*b^3*f*n^3*(e*x+d)^2/e^2-1/2*d^2*f*(a+b*ln(c*(e*x+d)^n))^3/e^2+6*b^3*d*f*n^3*x/e+3/8*b^3*g*m*n^3*(e*x+d)^2/e^2+1/2*d*g*m*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e^2+1/2*d^2*g*m*(a+b*ln(c*(e*x+d)^n))^3*ln(e*(j*x+i)/(-d*j+e*i))/e^2-1/2*g*i^2*m*(a+b*ln(c*(e*x+d)^n))^3*ln(e*(j*x+i)/(-d*j+e*i))/j^2+3/4*b^2*g*n^2*x^2*(a+b*ln(c*(e*x+d)^n))*ln(h*(j*x+i)^m)+1/2*x^2*(a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))

## Rubi [A] (verified)

Time = 4.77 (sec) , antiderivative size = 2050, normalized size of antiderivative = 1.00, number of steps used = 148, number of rules used = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2489, 2463, 2436, 2333, 2332, 2448, 2437, 2342, 2341, 2443, 2481, 2421, 2430, 6724, 6874, 2458, 2388, 2339, 30, 2367, 2479, 2338, 45, 2441, 2440, 2438, 2422, 2354, 2372, 12, 14, 2442}

$$\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx$$
$$=\frac{3}{8}gmn^3x^2b^3-\frac{3fn^3(d+ex)^2b^3}{8e^2}+\frac{3gmn^3(d+ex)^2b^3}{8e^2}+\frac{6dfn^3xb^3}{e}-\frac{141dgmn^3xb^3}{8e}$$
$$-\frac{45gimn^3xb^3}{8j}+\frac{3d^2gmn^3\log(d+ex)b^3}{8e^2}-\frac{6dfn^2(d+ex)\log(c(d+ex)^n)\,b^3}{e^2}$$
$$+\frac{12dgmn^2(d+ex)\log(c(d+ex)^n)\,b^3}{e^2}+\frac{21gimn^2(d+ex)\log(c(d+ex)^n)\,b^3}{4ej}$$
$$+\frac{3gi^2mn^3\log(i+jx)b^3}{8j^2}-\frac{3}{8}gn^3x^2\log(h(i+jx)^m)\,b^3$$
$$+\frac{21dgn^3(i+jx)\log(h(i+jx)^m)\,b^3}{4ej}-\frac{21d^2gn^3\log\left(-\frac{j(d+ex)}{ei-dj}\right)\log(h(i+jx)^m)\,b^3}{4e^2}$$
$$-\frac{9dgimn^3\,\mathrm{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)b^3}{2ej}-\frac{3gi^2mn^3\,\mathrm{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)b^3}{4j^2}$$
$$-\frac{21d^2gmn^3\,\mathrm{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)b^3}{4e^2}+\frac{9d^2gmn^3\,\mathrm{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)b^3}{2e^2}$$
$$-\frac{3dgimn^3\,\mathrm{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)b^3}{ej}-\frac{3gi^2mn^3\,\mathrm{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)b^3}{2j^2}$$
$$+\frac{3d^2gmn^3\,\mathrm{PolyLog}\left(4,-\frac{j(d+ex)}{ei-dj}\right)b^3}{e^2}-\frac{3gi^2mn^3\,\mathrm{PolyLog}\left(4,-\frac{j(d+ex)}{ei-dj}\right)b^3}{j^2}$$
$$-\frac{6adfn^2xb^2}{e}+\frac{12adgmn^2xb^2}{e}+\frac{21agimn^2xb^2}{4j}-\frac{3}{8}gmn^2x^2(a+b\log(c(d+ex)^n))\,b^2$$
$$+\frac{3fn^2(d+ex)^2\,(a+b\log(c(d+ex)^n))\,b^2}{4e^2}-\frac{3gmn^2(d+ex)^2\,(a+b\log(c(d+ex)^n))\,b^2}{4e^2}$$
$$-\frac{9dgimn^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)b^2}{2ej}$$
$$-\frac{3gi^2mn^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)b^2}{4j^2}$$
$$+\frac{3}{4}gn^2x^2(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)\,b^2$$
$$-\frac{9dgn^2x(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)\,b^2}{2e}$$
$$-\frac{9d^2gmn^2(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)b^2}{2e^2}$$
$$+\frac{3dgimn^2(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)b^2}{ej}$$
$$+\frac{3gi^2mn^2(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)b^2}{2j^2}$$
$$-\frac{3d^2gmn^2(a+b\log(c(d+ex)^n))\,\mathrm{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)b^2}{e^2}$$

```
[In] Int[x*(a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] (-6*a*b^2*d*f*n^2*x)/e + (12*a*b^2*d*g*m*n^2*x)/e + (21*a*b^2*g*i*m*n^2*x)/
(4*j) + (6*b^3*d*f*n^3*x)/e - (141*b^3*d*g*m*n^3*x)/(8*e) - (45*b^3*g*i*m*n
^3*x)/(8*j) + (3*b^3*g*m*n^3*x^2)/8 - (3*b^3*f*n^3*(d + e*x)^2)/(8*e^2) + (
3*b^3*g*m*n^3*(d + e*x)^2)/(8*e^2) + (3*b^3*d^2*g*m*n^3*Log[d + e*x])/(8*e^
2) - (6*b^3*d*f*n^2*(d + e*x)*Log[c*(d + e*x)^n])/e^2 + (12*b^3*d*g*m*n^2*(
d + e*x)*Log[c*(d + e*x)^n])/e^2 + (21*b^3*g*i*m*n^2*(d + e*x)*Log[c*(d + e
*x)^n])/(4*e*j) - (3*b^2*g*m*n^2*x^2*(a + b*Log[c*(d + e*x)^n]))/8 + (3*b^2
*f*n^2*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(4*e^2) - (3*b^2*g*m*n^2*(d
+ e*x)^2*(a + b*Log[c*(d + e*x)^n]))/(4*e^2) + (3*b*d*f*n*(d + e*x)*(a + b*
Log[c*(d + e*x)^n])^2)/e^2 - (15*b*d*g*m*n*(d + e*x)*(a + b*Log[c*(d + e*x)
^n])^2)/(4*e^2) - (9*b*g*i*m*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/(4*e
*j) - (3*b*f*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(4*e^2) + (3*b*g*m
*n*(d + e*x)^2*(a + b*Log[c*(d + e*x)^n])^2)/(4*e^2) - (d^2*f*(a + b*Log[c*
(d + e*x)^n])^3)/(2*e^2) + (d*g*m*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/(
2*e^2) + (g*i*m*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/(2*e*j) - (g*m*(d +
 e*x)^2*(a + b*Log[c*(d + e*x)^n])^3)/(4*e^2) + (3*b^3*g*i^2*m*n^3*Log[i +
j*x])/(8*j^2) - (3*b^2*g*i^2*m*n^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(i + j
*x))/(e*i - d*j)])/(4*j^2) - (9*b^2*d*g*i*m*n^2*(a + b*Log[c*(d + e*x)^n])*
Log[(e*(i + j*x))/(e*i - d*j)])/(2*e*j) - (9*b*d^2*g*m*n*(a + b*Log[c*(d +
e*x)^n])^2*Log[(e*(i + j*x))/(e*i - d*j)])/(4*e^2) + (3*b*g*i^2*m*n*(a + b*
Log[c*(d + e*x)^n])^2*Log[(e*(i + j*x))/(e*i - d*j)])/(4*j^2) + (3*b*d*g*i*
m*n*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(i + j*x))/(e*i - d*j)])/(2*e*j) +
(d^2*g*m*(a + b*Log[c*(d + e*x)^n])^3*Log[(e*(i + j*x))/(e*i - d*j)])/(2*e^
2) - (g*i^2*m*(a + b*Log[c*(d + e*x)^n])^3*Log[(e*(i + j*x))/(e*i - d*j)])/
(2*j^2) - (3*b^3*g*n^3*x^2*Log[h*(i + j*x)^m])/8 + (21*b^3*d*g*n^3*(i + j*x
)*Log[h*(i + j*x)^m])/(4*e*j) - (21*b^3*d^2*g*n^3*Log[-((j*(d + e*x))/(e*i
- d*j))]*Log[h*(i + j*x)^m])/(4*e^2) - (9*b^2*d*g*n^2*x*(a + b*Log[c*(d + e
*x)^n])*Log[h*(i + j*x)^m])/(2*e) + (3*b^2*g*n^2*x^2*(a + b*Log[c*(d + e*x)
^n])*Log[h*(i + j*x)^m])/4 + (9*b*d^2*g*n*(a + b*Log[c*(d + e*x)^n])^2*Log[
h*(i + j*x)^m])/(4*e^2) + (3*b*d*g*n*x*(a + b*Log[c*(d + e*x)^n])^2*Log[h*(
i + j*x)^m])/(2*e) - (3*b*g*n*x^2*(a + b*Log[c*(d + e*x)^n])^2*Log[h*(i + j
*x)^m])/4 - (d^2*g*(a + b*Log[c*(d + e*x)^n])^3*Log[h*(i + j*x)^m])/(2*e^2)
 + (x^2*(a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/2 - (3*b^3
*g*i^2*m*n^3*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/(4*j^2) - (9*b^3*d*g
*i*m*n^3*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/(2*e*j) - (9*b^2*d^2*g*m
*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/(
2*e^2) + (3*b^2*g*i^2*m*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((j*(d +
 e*x))/(e*i - d*j))])/(2*j^2) + (3*b^2*d*g*i*m*n^2*(a + b*Log[c*(d + e*x)^n
])*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/(e*j) + (3*b*d^2*g*m*n*(a + b*
Log[c*(d + e*x)^n])^2*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/(2*e^2) - (
3*b*g*i^2*m*n*(a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((j*(d + e*x))/(e*i
- d*j))])/(2*j^2) - (21*b^3*d^2*g*m*n^3*PolyLog[2, (e*(i + j*x))/(e*i - d*j
)])/(4*e^2) + (9*b^3*d^2*g*m*n^3*PolyLog[3, -((j*(d + e*x))/(e*i - d*j))])/
```

```
(2*e^2) - (3*b^3*g*i^2*m*n^3*PolyLog[3, -((j*(d + e*x))/(e*i - d*j))])/(2*j
^2) - (3*b^3*d*g*i*m*n^3*PolyLog[3, -((j*(d + e*x))/(e*i - d*j))])/(e*j) -
(3*b^2*d^2*g*m*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[3, -((j*(d + e*x))/(e
*i - d*j))])/e^2 + (3*b^2*g*i^2*m*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[3,
 -((j*(d + e*x))/(e*i - d*j))])/j^2 + (3*b^3*d^2*g*m*n^3*PolyLog[4, -((j*(d
 + e*x))/(e*i - d*j))])/e^2 - (3*b^3*g*i^2*m*n^3*PolyLog[4, -((j*(d + e*x))
/(e*i - d*j))])/j^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
```

eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2479

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[x*((a +
 b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Dist[b*e*n*p, Int[x*(a + b*Lo
g[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m)) \\
&\quad -\frac{1}{2}(gjm)\int\frac{x^2(a+b\log(c(d+ex)^n))^3}{i+jx}\,dx \\
&\quad -\frac{1}{2}(3ben)\int\frac{x^2(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{d+ex}\,dx \\
&= \frac{1}{2}x^2(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m)) \\
&\quad -\frac{1}{2}(gjm)\int\left(-\frac{i(a+b\log(c(d+ex)^n))^3}{j^2}+\frac{x(a+b\log(c(d+ex)^n))^3}{j}\right. \\
&\quad \left.+\frac{i^2(a+b\log(c(d+ex)^n))^3}{j^2(i+jx)}\right)dx-\frac{1}{2}(3ben)\int\left(\frac{fx^2(a+b\log(c(d+ex)^n))^2}{d+ex}\right. \\
&\quad \left.+\frac{gx^2(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{d+ex}\right)dx \\
&= \frac{1}{2}x^2(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m)) \\
&\quad -\frac{1}{2}(gm)\int x(a+b\log(c(d+ex)^n))^3\,dx+\frac{(gim)\int(a+b\log(c(d+ex)^n))^3\,dx}{2j} \\
&\quad -\frac{(gi^2m)\int\frac{(a+b\log(c(d+ex)^n))^3}{i+jx}\,dx}{2j}-\frac{1}{2}(3befn)\int\frac{x^2(a+b\log(c(d+ex)^n))^2}{d+ex}\,dx \\
&\quad -\frac{1}{2}(3begn)\int\frac{x^2(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{d+ex}\,dx
\end{aligned}$$

$$= -\frac{gi^2m(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2}$$
$$+\frac{1}{2}x^2(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))$$
$$-\frac{1}{2}(gm)\int\left(-\frac{d(a+b\log(c(d+ex)^n))^3}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{e}\right)dx$$
$$+\frac{(gim)\text{Subst}\left(\int(a+b\log(cx^n))^3\,dx,x,d+ex\right)}{2ej}$$
$$-\frac{1}{2}(3bfn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^2(a+b\log(cx^n))^2}{x}\,dx,x,d+ex\right)$$
$$-\frac{1}{2}(3begn)\int\left(-\frac{d(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{e^2}+\frac{x(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{e}+\frac{d^2(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{e^2(d+ex)}\right)dx$$
$$+\frac{(3begi^2mn)\int\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{d+ex}\,dx}{2j^2}$$

$$= \frac{gim(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2ej} - \frac{gi^2m\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2}$$
$$+ \frac{1}{2}x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\left(f+g\log\left(h(i+jx)^m\right)\right)$$
$$- \frac{(gm)\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx}{2e}$$
$$+ \frac{(dgm)\int\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx}{2e}$$
$$- \frac{(3bfn)\text{Subst}\left(\int\left(-\frac{d}{e}+\frac{x}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+ex\right)}{2e}$$
$$+ \frac{(3bdfn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx,x,d+ex\right)}{2e}$$
$$- \frac{1}{2}(3bgn)\int x\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(h(i+jx)^m\right)\,dx$$
$$+ \frac{(3bdgn)\int\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(h(i+jx)^m\right)\,dx}{2e}$$
$$- \frac{(3bd^2gn)\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(h(i+jx)^m\right)}{d+ex}\,dx}{2e}$$
$$+ \frac{(3bgi^2mn)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(\frac{e\left(\frac{ei-dj}{e}+\frac{jx}{e}\right)}{ei-dj}\right)}{x}\,dx,x,d+ex\right)}{2j^2}$$
$$- \frac{(3bgimn)\text{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+ex\right)}{2ej}$$

$$= -\frac{3bgimn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{2ej} + \frac{gim(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{2ej}$$
$$- \frac{gi^2m(a+b\log\left(c(d+ex)^n\right))^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{2j^2}$$
$$+ \frac{3bdgnx(a+b\log\left(c(d+ex)^n\right))^2\log\left(h(i+jx)^m\right)}{2e}$$
$$- \frac{3}{4}bgnx^2(a+b\log\left(c(d+ex)^n\right))^2\log\left(h(i+jx)^m\right)$$
$$+ \frac{1}{2}x^2(a+b\log\left(c(d+ex)^n\right))^3\left(f+g\log\left(h(i+jx)^m\right)\right)$$
$$- \frac{3bgi^2mn(a+b\log\left(c(d+ex)^n\right))^2\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{2j^2}$$
$$- \frac{(gm)\text{Subst}\left(\int x(a+b\log\left(cx^n\right))^3\,dx,x,d+ex\right)}{2e^2}$$
$$+ \frac{(dgm)\text{Subst}\left(\int (a+b\log\left(cx^n\right))^3\,dx,x,d+ex\right)}{2e^2}$$
$$+ \frac{(3bdfn)\text{Subst}\left(\int (a+b\log\left(cx^n\right))^2\,dx,x,d+ex\right)}{2e^2}$$
$$- \frac{(3bd^2fn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x}\,dx,x,d+ex\right)}{2e^2}$$
$$- \frac{(3bfn)\text{Subst}\left(\int \left(-\frac{d(a+b\log(cx^n))^2}{e}+\frac{x(a+b\log(cx^n))^2}{e}\right)dx,x,d+ex\right)}{2e}$$
$$- \frac{(3bd^2gn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\log\left(h\left(\frac{ei-dj}{e}+\frac{jx}{e}\right)^m\right)}{x}\,dx,x,d+ex\right)}{2e^2}$$
$$+ \frac{1}{4}(3bgjmn)\int \frac{x^2(a+b\log\left(c(d+ex)^n\right))^2}{i+jx}\,dx$$
$$- \frac{(3bdgjmn)\int \frac{x(a+b\log(c(d+ex)^n))^2}{i+jx}\,dx}{2e}$$
$$- \left(3b^2dgn^2\right)\int \frac{x(a+b\log\left(c(d+ex)^n\right))\log\left(h(i+jx)^m\right)}{d+ex}\,dx$$
$$+ \frac{1}{2}\left(3b^2egn^2\right)\int \frac{x^2(a+b\log\left(c(d+ex)^n\right))\log\left(h(i+jx)^m\right)}{d+ex}\,dx$$
$$+ \frac{(3b^2gi^2mn^2)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{jx}{ei-dj}\right)}{x}\,dx,x,d+ex\right)}{j^2}$$
$$+ \frac{(3b^2gimn^2)\,\text{Subst}\left(\int (a+b\log\left(cx^n\right))\,dx,x,d+ex\right)}{ej}$$

= Too large to display

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4971 vs. 2(2050) = 4100.

Time = 1.32 (sec) , antiderivative size = 4971, normalized size of antiderivative = 2.42

$$\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx = \text{Result too large to show}$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] (-12*a^2*b*d*e*g*i*j*m*n + 12*a*b^2*d*e*g*i*j*m*n^2 + 24*a*b^2*d^2*g*j^2*m*
n^2 - 6*b^3*d*e*g*i*j*m*n^3 - 36*b^3*d^2*g*j^2*m*n^3 + 4*a^3*e^2*g*i*j*m*x
+ 12*a^2*b*d*e*f*j^2*n*x - 18*a^2*b*e^2*g*i*j*m*n*x - 18*a^2*b*d*e*g*j^2*m*
n*x - 36*a*b^2*d*e*f*j^2*n^2*x + 42*a*b^2*e^2*g*i*j*m*n^2*x + 84*a*b^2*d*e*
g*j^2*m*n^2*x + 42*b^3*d*e*f*j^2*n^3*x - 45*b^3*e^2*g*i*j*m*n^3*x - 135*b^3
*d*e*g*j^2*m*n^3*x + 4*a^3*e^2*f*j^2*x^2 - 2*a^3*e^2*g*j^2*m*x^2 - 6*a^2*b*
e^2*f*j^2*n*x^2 + 6*a^2*b*e^2*g*j^2*m*n*x^2 + 6*a*b^2*e^2*f*j^2*n^2*x^2 - 9
*a*b^2*e^2*g*j^2*m*n^2*x^2 - 3*b^3*e^2*f*j^2*n^3*x^2 + 6*b^3*e^2*g*j^2*m*n^
3*x^2 - 12*a^2*b*d^2*f*j^2*n*Log[d + e*x] + 12*a^2*b*d*e*g*i*j*m*n*Log[d +
e*x] + 6*a^2*b*d^2*g*j^2*m*n*Log[d + e*x] + 36*a*b^2*d^2*f*j^2*n^2*Log[d +
e*x] - 12*a*b^2*d*e*g*i*j*m*n^2*Log[d + e*x] - 48*a*b^2*d^2*g*j^2*m*n^2*Log
[d + e*x] - 42*b^3*d^2*f*j^2*n^3*Log[d + e*x] + 30*b^3*d*e*g*i*j*m*n^3*Log[
d + e*x] + 69*b^3*d^2*g*j^2*m*n^3*Log[d + e*x] + 12*a*b^2*d^2*f*j^2*n^2*Log
[d + e*x]^2 - 12*a*b^2*d*e*g*i*j*m*n^2*Log[d + e*x]^2 - 6*a*b^2*d^2*g*j^2*m
*n^2*Log[d + e*x]^2 - 18*b^3*d^2*f*j^2*n^3*Log[d + e*x]^2 + 6*b^3*d*e*g*i*j
*m*n^3*Log[d + e*x]^2 + 24*b^3*d^2*g*j^2*m*n^3*Log[d + e*x]^2 - 4*b^3*d^2*f
*j^2*n^3*Log[d + e*x]^3 + 4*b^3*d*e*g*i*j*m*n^3*Log[d + e*x]^3 + 2*b^3*d^2*
g*j^2*m*n^3*Log[d + e*x]^3 - 24*a*b^2*d*e*g*i*j*m*n*Log[c*(d + e*x)^n] + 12
*b^3*d*e*g*i*j*m*n^2*Log[c*(d + e*x)^n] + 24*b^3*d^2*g*j^2*m*n^2*Log[c*(d +
 e*x)^n] + 12*a^2*b*e^2*g*i*j*m*x*Log[c*(d + e*x)^n] + 24*a*b^2*d*e*f*j^2*n
*x*Log[c*(d + e*x)^n] - 36*a*b^2*e^2*g*i*j*m*n*x*Log[c*(d + e*x)^n] - 36*a*
b^2*d*e*g*j^2*m*n*x*Log[c*(d + e*x)^n] - 36*b^3*d*e*f*j^2*n^2*x*Log[c*(d +
e*x)^n] + 42*b^3*e^2*g*i*j*m*n^2*x*Log[c*(d + e*x)^n] + 84*b^3*d*e*g*j^2*m*
n^2*x*Log[c*(d + e*x)^n] + 12*a^2*b*e^2*f*j^2*x^2*Log[c*(d + e*x)^n] - 6*a^
2*b*e^2*g*j^2*m*x^2*Log[c*(d + e*x)^n] - 12*a*b^2*e^2*f*j^2*n*x^2*Log[c*(d
+ e*x)^n] + 12*a*b^2*e^2*g*j^2*m*n*x^2*Log[c*(d + e*x)^n] + 6*b^3*e^2*f*j^2
*n^2*x^2*Log[c*(d + e*x)^n] - 9*b^3*e^2*g*j^2*m*n^2*x^2*Log[c*(d + e*x)^n]
- 24*a*b^2*d^2*f*j^2*n*Log[d + e*x]*Log[c*(d + e*x)^n] + 24*a*b^2*d*e*g*i*j
*m*n*Log[d + e*x]*Log[c*(d + e*x)^n] + 12*a*b^2*d^2*g*j^2*m*n*Log[d + e*x]*
Log[c*(d + e*x)^n] + 36*b^3*d^2*f*j^2*n^2*Log[d + e*x]*Log[c*(d + e*x)^n] -
 12*b^3*d*e*g*i*j*m*n^2*Log[d + e*x]*Log[c*(d + e*x)^n] - 48*b^3*d^2*g*j^2*
m*n^2*Log[d + e*x]*Log[c*(d + e*x)^n] + 12*b^3*d^2*f*j^2*n^2*Log[d + e*x]^2
*Log[c*(d + e*x)^n] - 12*b^3*d*e*g*i*j*m*n^2*Log[d + e*x]^2*Log[c*(d + e*x)
^n] - 6*b^3*d^2*g*j^2*m*n^2*Log[d + e*x]^2*Log[c*(d + e*x)^n] - 12*b^3*d*e*
g*i*j*m*n*Log[c*(d + e*x)^n]^2 + 12*a*b^2*e^2*g*i*j*m*x*Log[c*(d + e*x)^n]^
```

2 + 12*b^3*d*e*f*j^2*n*x*Log[c*(d + e*x)^n]^2 - 18*b^3*e^2*g*i*j*m*n*x*Log[
c*(d + e*x)^n]^2 - 18*b^3*d*e*g*j^2*m*n*x*Log[c*(d + e*x)^n]^2 + 12*a*b^2*e
^2*f*j^2*x^2*Log[c*(d + e*x)^n]^2 - 6*a*b^2*e^2*g*j^2*m*x^2*Log[c*(d + e*x)
^n]^2 - 6*b^3*e^2*f*j^2*n*x^2*Log[c*(d + e*x)^n]^2 + 6*b^3*e^2*g*j^2*m*n*x^
2*Log[c*(d + e*x)^n]^2 - 12*b^3*d^2*f*j^2*n*Log[d + e*x]*Log[c*(d + e*x)^n]
^2 + 12*b^3*d*e*g*i*j*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]^2 + 6*b^3*d^2*g*j
^2*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]^2 + 4*b^3*e^2*g*i*j*m*x*Log[c*(d + e
*x)^n]^3 + 4*b^3*e^2*f*j^2*x^2*Log[c*(d + e*x)^n]^3 - 2*b^3*e^2*g*j^2*m*x^2
*Log[c*(d + e*x)^n]^3 - 4*a^3*e^2*g*i^2*m*Log[i + j*x] + 6*a^2*b*e^2*g*i^2*
m*n*Log[i + j*x] + 12*a^2*b*d*e*g*i*j*m*n*Log[i + j*x] - 6*a*b^2*e^2*g*i^2*
m*n^2*Log[i + j*x] - 36*a*b^2*d*e*g*i*j*m*n^2*Log[i + j*x] + 3*b^3*e^2*g*i^
2*m*n^3*Log[i + j*x] + 42*b^3*d*e*g*i*j*m*n^3*Log[i + j*x] + 12*a^2*b*e^2*g
*i^2*m*n*Log[d + e*x]*Log[i + j*x] - 12*a*b^2*e^2*g*i^2*m*n^2*Log[d + e*x]*
Log[i + j*x] - 24*a*b^2*d*e*g*i*j*m*n^2*Log[d + e*x]*Log[i + j*x] + 6*b^3*e
^2*g*i^2*m*n^3*Log[d + e*x]*Log[i + j*x] + 36*b^3*d*e*g*i*j*m*n^3*Log[d + e
*x]*Log[i + j*x] - 12*a*b^2*e^2*g*i^2*m*n^2*Log[d + e*x]^2*Log[i + j*x] + 6
*b^3*e^2*g*i^2*m*n^3*Log[d + e*x]^2*Log[i + j*x] + 12*b^3*d*e*g*i*j*m*n^3*L
og[d + e*x]^2*Log[i + j*x] + 4*b^3*e^2*g*i^2*m*n^3*Log[d + e*x]^3*Log[i + j
*x] - 12*a^2*b*e^2*g*i^2*m*Log[c*(d + e*x)^n]*Log[i + j*x] + 12*a*b^2*e^2*g
*i^2*m*n*Log[c*(d + e*x)^n]*Log[i + j*x] + 24*a*b^2*d*e*g*i*j*m*n*Log[c*(d
+ e*x)^n]*Log[i + j*x] - 6*b^3*e^2*g*i^2*m*n^2*Log[c*(d + e*x)^n]*Log[i + j
*x] - 36*b^3*d*e*g*i*j*m*n^2*Log[c*(d + e*x)^n]*Log[i + j*x] + 24*a*b^2*e^2
*g*i^2*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[i + j*x] - 12*b^3*e^2*g*i^2*
m*n^2*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[i + j*x] - 24*b^3*d*e*g*i*j*m*n^2
*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[i + j*x] - 12*b^3*e^2*g*i^2*m*n^2*Log[
d + e*x]^2*Log[c*(d + e*x)^n]*Log[i + j*x] - 12*a*b^2*e^2*g*i^2*m*Log[c*(d
+ e*x)^n]^2*Log[i + j*x] + 6*b^3*e^2*g*i^2*m*n*Log[c*(d + e*x)^n]^2*Log[i +
 j*x] + 12*b^3*d*e*g*i*j*m*n*Log[c*(d + e*x)^n]^2*Log[i + j*x] + 12*b^3*e^2
*g*i^2*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]^2*Log[i + j*x] - 4*b^3*e^2*g*i^2
*m*Log[c*(d + e*x)^n]^3*Log[i + j*x] - 12*a^2*b*e^2*g*i^2*m*n*Log[d + e*x]*
Log[(e*(i + j*x))/(e*i - d*j)] + 12*a^2*b*d^2*g*j^2*m*n*Log[d + e*x]*Log[(e
*(i + j*x))/(e*i - d*j)] + 12*a*b^2*e^2*g*i^2*m*n^2*Log[d + e*x]*Log[(e*(i
+ j*x))/(e*i - d*j)] + 24*a*b^2*d*e*g*i*j*m*n^2*Log[d + e*x]*Log[(e*(i + j*
x))/(e*i - d*j)] - 36*a*b^2*d^2*g*j^2*m*n^2*Log[d + e*x]*Log[(e*(i + j*x))/
(e*i - d*j)] - 6*b^3*e^2*g*i^2*m*n^3*Log[d + e*x]*Log[(e*(i + j*x))/(e*i -
d*j)] - 36*b^3*d*e*g*i*j*m*n^3*Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d*j)]
+ 42*b^3*d^2*g*j^2*m*n^3*Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d*j)] + 12*a
*b^2*e^2*g*i^2*m*n^2*Log[d + e*x]^2*Log[(e*(i + j*x))/(e*i - d*j)] - 12*a*b
^2*d^2*g*j^2*m*n^2*Log[d + e*x]^2*Log[(e*(i + j*x))/(e*i - d*j)] - 6*b^3*e^
2*g*i^2*m*n^3*Log[d + e*x]^2*Log[(e*(i + j*x))/(e*i - d*j)] - 12*b^3*d*e*g*
i*j*m*n^3*Log[d + e*x]^2*Log[(e*(i + j*x))/(e*i - d*j)] + 18*b^3*d^2*g*j^2*
m*n^3*Log[d + e*x]^2*Log[(e*(i + j*x))/(e*i - d*j)] - 4*b^3*e^2*g*i^2*m*n^3
*Log[d + e*x]^3*Log[(e*(i + j*x))/(e*i - d*j)] + 4*b^3*d^2*g*j^2*m*n^3*Log[
d + e*x]^3*Log[(e*(i + j*x))/(e*i - d*j)] - 24*a*b^2*e^2*g*i^2*m*n*Log[d +
e*x]*Log[c*(d + e*x)^n]*Log[(e*(i + j*x))/(e*i - d*j)] + 24*a*b^2*d^2*g*j^2

*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[(e*(i + j*x))/(e*i - d*j)] + 12*b^3*e^2*g*i^2*m*n^2*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[(e*(i + j*x))/(e*i - d*j)] + 24*b^3*d*e*g*i*j*m*n^2*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[(e*(i + j*x))/(e*i - d*j)] - 36*b^3*d^2*g*j^2*m*n^2*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[(e*(i + j*x))/(e*i - d*j)] + 12*b^3*e^2*g*i^2*m*n^2*Log[d + e*x]^2*Log[c*(d + e*x)^n]*Log[(e*(i + j*x))/(e*i - d*j)] - 12*b^3*d^2*g*j^2*m*n^2*Log[d + e*x]^2*Log[c*(d + e*x)^n]*Log[(e*(i + j*x))/(e*i - d*j)] - 12*b^3*e^2*g*i^2*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]^2*Log[(e*(i + j*x))/(e*i - d*j)] + 12*b^3*d^2*g*j^2*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]^2*Log[(e*(i + j*x))/(e*i - d*j)] + 12*a^2*b*d*e*g*j^2*n*x*Log[h*(i + j*x)^m] - 36*a*b^2*d*e*g*j^2*n^2*x*Log[h*(i + j*x)^m] + 42*b^3*d*e*g*j^2*n^3*x*Log[h*(i + j*x)^m] + 4*a^3*e^2*g*j^2*x^2*Log[h*(i + j*x)^m] - 6*a^2*b*e^2*g*j^2*n*x^2*Log[h*(i + j*x)^m] + 6*a*b^2*e^2*g*j^2*n^2*x^2*Log[h*(i + j*x)^m] - 3*b^3*e^2*g*j^2*n^3*x^2*Log[h*(i + j*x)^m] - 12*a^2*b*d^2*g*j^2*n*Log[d + e*x]*Log[h*(i + j*x)^m] + 36*a*b^2*d^2*g*j^2*n^2*Log[d + e*x]*Log[h*(i + j*x)^m] - 42*b^3*d^2*g*j^2*n^3*Log[d + e*x]*Log[h*(i + j*x)^m] + 12*a*b^2*d^2*g*j^2*n^2*Log[d + e*x]^2*Log[h*(i + j*x)^m] - 18*b^3*d^2*g*j^2*n^3*Log[d + e*x]^2*Log[h*(i + j*x)^m] - 4*b^3*d^2*g*j^2*n^3*Log[d + e*x]^3*Log[h*(i + j*x)^m] + 24*a*b^2*d*e*g*j^2*n*x*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] - 36*b^3*d*e*g*j^2*n^2*x*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] + 12*a^2*b*e^2*g*j^2*x^2*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] - 12*a*b^2*e^2*g*j^2*n*x^2*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] + 6*b^3*e^2*g*j^2*n^2*x^2*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] - 24*a*b^2*d^2*g*j^2*n*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] + 36*b^3*d^2*g*j^2*n^2*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] + 12*b^3*d^2*g*j^2*n^2*Log[d + e*x]^2*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] + 12*b^3*d*e*g*j^2*n*x*Log[c*(d + e*x)^n]^2*Log[h*(i + j*x)^m] + 12*a*b^2*e^2*g*j^2*x^2*Log[c*(d + e*x)^n]^2*Log[h*(i + j*x)^m] - 6*b^3*e^2*g*j^2*n*x^2*Log[c*(d + e*x)^n]^2*Log[h*(i + j*x)^m] - 12*b^3*d^2*g*j^2*n*Log[d + e*x]*Log[c*(d + e*x)^n]^2*Log[h*(i + j*x)^m] + 4*b^3*e^2*g*j^2*x^2*Log[c*(d + e*x)^n]^3*Log[h*(i + j*x)^m] - 6*b*g*(e*i - d*j)*m*n*(2*a^2*(e*i + d*j) - 2*a*b*(e*i + 3*d*j)*n + b^2*(e*i + 7*d*j)*n^2 - 2*b*(-2*a*(e*i + d*j) + b*(e*i + 3*d*j)*n)*Log[c*(d + e*x)^n] + 2*b^2*(e*i + d*j)*Log[c*(d + e*x)^n]^2)*PolyLog[2, (j*(d + e*x))/(-(e*i) + d*j)] + 12*b^2*g*(e*i - d*j)*m*n^2*(2*a*(e*i + d*j) - b*(e*i + 3*d*j)*n + 2*b*(e*i + d*j)*Log[c*(d + e*x)^n])*PolyLog[3, (j*(d + e*x))/(-(e*i) + d*j)] - 24*b^3*e^2*g*i^2*m*n^3*PolyLog[4, (j*(d + e*x))/(-(e*i) + d*j)] + 24*b^3*d^2*g*j^2*m*n^3*PolyLog[4, (j*(d + e*x))/(-(e*i) + d*j)])/(8*e^2*j^2)

## Maple [F]

$$\int x(a+b\ln(c(ex+d)^n))^3(f+g\ln(h(jx+i)^m))\,dx$$

```
[In] int(x*(a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m)),x)
[Out] int(x*(a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m)),x)
```

## Fricas [F]

$$\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)x\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m)),x, algorithm="f
ricas")
[Out] integral(b^3*f*x*log((e*x + d)^n*c)^3 + 3*a*b^2*f*x*log((e*x + d)^n*c)^2 +
3*a^2*b*f*x*log((e*x + d)^n*c) + a^3*f*x + (b^3*g*x*log((e*x + d)^n*c)^3 +
3*a*b^2*g*x*log((e*x + d)^n*c)^2 + 3*a^2*b*g*x*log((e*x + d)^n*c) + a^3*g*x
)*log((j*x + i)^m*h), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(e*x+d)**n))**3*(f+g*ln(h*(j*x+i)**m)),x)
[Out] Timed out
```

## Maxima [F]

$$\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)x\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m)),x, algorithm="m
axima")
```

```
[Out] 1/2*b^3*f*x^2*log((e*x + d)^n*c)^3 + 3/2*a*b^2*f*x^2*log((e*x + d)^n*c)^2 -
 3/4*a^2*b*e*f*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) - 1/4*a^3*g
*j*m*(2*i^2*log(j*x + i)/j^3 + (j*x^2 - 2*i*x)/j^2) + 3/2*a^2*b*f*x^2*log((
e*x + d)^n*c) + 1/2*a^3*g*x^2*log((j*x + i)^m*h) + 1/2*a^3*f*x^2 - 3/4*(2*e
*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)^n*c) - (e^2
*x^2 + 2*d^2*log(e*x + d)^2 - 6*d*e*x + 6*d^2*log(e*x + d))*n^2/e^2)*a*b^2*
f - 1/8*(6*e*n*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2)*log((e*x + d)
^n*c)^2 + e*n*((4*d^2*log(e*x + d)^3 + 3*e^2*x^2 + 18*d^2*log(e*x + d)^2 -
42*d*e*x + 42*d^2*log(e*x + d))*n^2/e^3 - 6*(e^2*x^2 + 2*d^2*log(e*x + d)^2
 - 6*d*e*x + 6*d^2*log(e*x + d))*n*log((e*x + d)^n*c)/e^3))*b^3*f + 1/8*(2*
(2*b^3*e^2*g*i*j*m*x - 2*b^3*e^2*g*i^2*m*log(j*x + i) - (j^2*m - 2*j^2*log(
h))*b^3*e^2*g*x^2)*log((e*x + d)^n)^3 - (4*b^3*d^2*g*j^2*n^3*log(e*x + d)^3
 - 4*b^3*e^2*g*j^2*x^2*log((e*x + d)^n)^3 + (6*(e^2*g*j^2*n - 2*e^2*g*j^2*l
og(c))*a^2*b - 6*(e^2*g*j^2*n^2 - 2*e^2*g*j^2*n*log(c) + 2*e^2*g*j^2*log(c)
^2)*a*b^2 + (3*e^2*g*j^2*n^3 - 6*e^2*g*j^2*n^2*log(c) + 6*e^2*g*j^2*n*log(c
)^2 - 4*e^2*g*j^2*log(c)^3)*b^3)*x^2 - 6*(2*a*b^2*d^2*g*j^2*n^2 - (3*d^2*g*
j^2*n^3 - 2*d^2*g*j^2*n^2*log(c))*b^3)*log(e*x + d)^2 - 6*(2*b^3*d*e*g*j^2*
n*x - 2*b^3*d^2*g*j^2*n*log(e*x + d) + (2*a*b^2*e^2*g*j^2 - (e^2*g*j^2*n -
2*e^2*g*j^2*log(c))*b^3)*x^2)*log((e*x + d)^n)^2 - 6*(2*a^2*b*d*e*g*j^2*n -
 2*(3*d*e*g*j^2*n^2 - 2*d*e*g*j^2*n*log(c))*a*b^2 + (7*d*e*g*j^2*n^3 - 6*d*
e*g*j^2*n^2*log(c) + 2*d*e*g*j^2*n*log(c)^2)*b^3)*x + 6*(2*a^2*b*d^2*g*j^2*
n - 2*(3*d^2*g*j^2*n^2 - 2*d^2*g*j^2*n*log(c))*a*b^2 + (7*d^2*g*j^2*n^3 - 6
*d^2*g*j^2*n^2*log(c) + 2*d^2*g*j^2*n*log(c)^2)*b^3)*log(e*x + d) - 6*(2*b^
3*d^2*g*j^2*n^2*log(e*x + d)^2 + (2*a^2*b*e^2*g*j^2 - 2*(e^2*g*j^2*n - 2*e^
2*g*j^2*log(c))*a*b^2 + (e^2*g*j^2*n^2 - 2*e^2*g*j^2*n*log(c) + 2*e^2*g*j^2
*log(c)^2)*b^3)*x^2 + 2*(2*a*b^2*d*e*g*j^2*n - (3*d*e*g*j^2*n^2 - 2*d*e*g*j
^2*n*log(c))*b^3)*x - 2*(2*a*b^2*d^2*g*j^2*n - (3*d^2*g*j^2*n^2 - 2*d^2*g*j
^2*n*log(c))*b^3)*log(e*x + d))*log((e*x + d)^n))*log((j*x + i)^m))/(e^2*j^
2) + integrate(1/8*((6*(e^3*g*j^3*m*n - 2*(j^3*m - 2*j^3*log(h))*e^3*g*log(
c))*a^2*b - 6*(e^3*g*j^3*m*n^2 - 2*e^3*g*j^3*m*n*log(c) + 2*(j^3*m - 2*j^3*
log(h))*e^3*g*log(c)^2)*a*b^2 + (3*e^3*g*j^3*m*n^3 - 6*e^3*g*j^3*m*n^2*log(
c) + 6*e^3*g*j^3*m*n*log(c)^2 - 4*(j^3*m - 2*j^3*log(h))*e^3*g*log(c)^3)*b^
3)*x^3 + 4*(b^3*d^2*e*g*j^3*m*n^3*x + b^3*d^3*g*j^3*m*n^3)*log(e*x + d)^3 -
 (6*(d*e^2*g*j^3*m*n - 2*(2*e^3*g*i*j^2*log(h) - (j^3*m - 2*j^3*log(h))*d*e
^2*g)*log(c))*a^2*b - 6*(5*d*e^2*g*j^3*m*n^2 - 2*d*e^2*g*j^3*m*n*log(c) + 2
*(2*e^3*g*i*j^2*log(h) - (j^3*m - 2*j^3*log(h))*d*e^2*g)*log(c)^2)*a*b^2 +
(39*d*e^2*g*j^3*m*n^3 - 30*d*e^2*g*j^3*m*n^2*log(c) + 6*d*e^2*g*j^3*m*n*log
(c)^2 - 4*(2*e^3*g*i*j^2*log(h) - (j^3*m - 2*j^3*log(h))*d*e^2*g)*log(c)^3)
*b^3)*x^2 - 6*(2*a*b^2*d^3*g*j^3*m*n^2 - (3*d^3*g*j^3*m*n^3 - 2*d^3*g*j^3*m
*n^2*log(c))*b^3 + (2*a*b^2*d^2*e*g*j^3*m*n^2 - (3*d^2*e*g*j^3*m*n^3 - 2*d^
2*e*g*j^3*m*n^2*log(c))*b^3)*x)*log(e*x + d)^2 - 6*(2*((j^3*m - 2*j^3*log(h
))*a*b^2*e^3*g + ((j^3*m - 2*j^3*log(h))*e^3*g*log(c) - (j^3*m*n - j^3*n*lo
g(h))*e^3*g)*b^3)*x^3 - (2*(2*e^3*g*i*j^2*log(h) - (j^3*m - 2*j^3*log(h))*d
*e^2*g)*a*b^2 - (d*e^2*g*j^3*m*n + (i*j^2*m*n + 2*i*j^2*n*log(h))*e^3*g - 2
*(2*e^3*g*i*j^2*log(h) - (j^3*m - 2*j^3*log(h))*d*e^2*g)*log(c))*b^3)*x^2 -
```

```
 2*(2*a*b^2*d*e^2*g*i*j^2*log(h) - (e^3*g*i^2*j*m*n + d^2*e*g*j^3*m*n - 2*d
*e^2*g*i*j^2*log(c)*log(h))*b^3)*x - 2*(b^3*d^2*e*g*j^3*m*n*x + b^3*d^3*g*j
^3*m*n)*log(e*x + d) - 2*(b^3*e^3*g*i^2*j*m*n*x + b^3*e^3*g*i^3*m*n)*log(j*
x + i))*log((e*x + d)^n)^2 - 2*(6*(d^2*e*g*j^3*m*n - 2*d*e^2*g*i*j^2*log(c)
*log(h))*a^2*b - 6*(3*d^2*e*g*j^3*m*n^2 - 2*d^2*e*g*j^3*m*n*log(c) + 2*d*e^
2*g*i*j^2*log(c)^2*log(h))*a*b^2 + (21*d^2*e*g*j^3*m*n^3 - 18*d^2*e*g*j^3*m
*n^2*log(c) + 6*d^2*e*g*j^3*m*n*log(c)^2 - 4*d*e^2*g*i*j^2*log(c)^3*log(h))
*b^3)*x + 6*(2*a^2*b*d^3*g*j^3*m*n - 2*(3*d^3*g*j^3*m*n^2 - 2*d^3*g*j^3*m*n
*log(c))*a*b^2 + (7*d^3*g*j^3*m*n^3 - 6*d^3*g*j^3*m*n^2*log(c) + 2*d^3*g*j^
3*m*n*log(c)^2)*b^3 + (2*a^2*b*d^2*e*g*j^3*m*n - 2*(3*d^2*e*g*j^3*m*n^2 - 2
*d^2*e*g*j^3*m*n*log(c))*a*b^2 + (7*d^2*e*g*j^3*m*n^3 - 6*d^2*e*g*j^3*m*n^2
*log(c) + 2*d^2*e*g*j^3*m*n*log(c)^2)*b^3)*x)*log(e*x + d) - 6*((2*(j^3*m -
 2*j^3*log(h))*a^2*b*e^3*g - 2*(e^3*g*j^3*m*n - 2*(j^3*m - 2*j^3*log(h))*e^
3*g*log(c))*a*b^2 + (e^3*g*j^3*m*n^2 - 2*e^3*g*j^3*m*n*log(c) + 2*(j^3*m -
2*j^3*log(h))*e^3*g*log(c)^2)*b^3)*x^3 - (2*(2*e^3*g*i*j^2*log(h) - (j^3*m
- 2*j^3*log(h))*d*e^2*g)*a^2*b - 2*(d*e^2*g*j^3*m*n - 2*(2*e^3*g*i*j^2*log(
h) - (j^3*m - 2*j^3*log(h))*d*e^2*g)*log(c))*a*b^2 + (5*d*e^2*g*j^3*m*n^2 -
 2*d*e^2*g*j^3*m*n*log(c) + 2*(2*e^3*g*i*j^2*log(h) - (j^3*m - 2*j^3*log(h)
)*d*e^2*g)*log(c)^2)*b^3)*x^2 + 2*(b^3*d^2*e*g*j^3*m*n^2*x + b^3*d^3*g*j^3*
m*n^2)*log(e*x + d)^2 - 2*(2*a^2*b*d*e^2*g*i*j^2*log(h) - 2*(d^2*e*g*j^3*m*
n - 2*d*e^2*g*i*j^2*log(c)*log(h))*a*b^2 + (3*d^2*e*g*j^3*m*n^2 - 2*d^2*e*g
*j^3*m*n*log(c) + 2*d*e^2*g*i*j^2*log(c)^2*log(h))*b^3)*x - 2*(2*a*b^2*d^3*
g*j^3*m*n - (3*d^3*g*j^3*m*n^2 - 2*d^3*g*j^3*m*n*log(c))*b^3 + (2*a*b^2*d^2
*e*g*j^3*m*n - (3*d^2*e*g*j^3*m*n^2 - 2*d^2*e*g*j^3*m*n*log(c))*b^3)*x)*log
(e*x + d))*log((e*x + d)^n))/(e^3*j^3*x^2 + d*e^2*i*j^2 + (e^3*i*j^2 + d*e^
2*j^3)*x), x)
```

## Giac [F]

$$\int x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx$$
$$=\int (b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)x\,dx$$

```
[In] integrate(x*(a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m)),x, algorithm="g
iac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)^3*(g*log((j*x + i)^m*h) + f)*x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x(a+b\log \left(c(d+ex)^n\right))^3 \left(f+g\log \left(h(i+jx)^m\right)\right)\,dx$$
$$=\int x\,\left(a+b\,\ln \left(c\,{\left(d+e\,x\right)}^n\right)\right)^3\,\left(f+g\,\ln \left(h\,{\left(i+j\,x\right)}^m\right)\right)\,dx$$

```
[In] int(x*(a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)),x)
[Out] int(x*(a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)), x)
```

# 3.398 $\int (a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))\,d$

## Optimal result

Integrand size = 31, antiderivative size = 1147

$$\int (a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\,dx$$
$$= 6ab^2fn^2x-18ab^2gmn^2x-6b^3fn^3x+24b^3gmn^3x+\frac{6b^3fn^2(d+ex)\log(c(d+ex)^n)}{e}-\frac{18b^3gmn^2(d+ex)\log(c(d+ex)^n)}{e}-\frac{3bfn(d+ex)(a+b\log(c(d+ex)^n))^2}{e}+\frac{6bgmn(d+ex)(a+b\log(c(d+ex)^n))^2}{e}+\frac{df(a+b\log(c(d+ex)^n))^3}{e}-\frac{gm(d+ex)(a+b\log(c(d+ex)^n))^3}{e}+\frac{6b^2gimn^2(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}+\frac{3bdgmn(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{e}-\frac{3bgimn(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}-\frac{dgm(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{e}+\frac{gim(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}-\frac{6b^3gn^3(i+jx)\log(h(i+jx)^m)}{j}+\frac{6b^3dgn^3\log\left(-\frac{j(d+ex)}{ei-dj}\right)\log(h(i+jx)^m)}{e}+6b^2gn^2x(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)-\frac{3bdgn(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{e}-3bgnx(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)+\frac{dg(a+b\log(c(d+ex)^n))^3\log(h(i+jx)^m)}{e}+x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))+\frac{6b^3gimn^3\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j}+\frac{6b^2dgmn^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{e}-\frac{6b^2gimn^2(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j}-\frac{3bdgmn(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{e}+\frac{3bgimn(a+b\log(c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right)}{j}+\frac{6b^3dgmn^3\operatorname{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right)}{e}-\frac{6b^3dgmn^3\operatorname{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)}{e}+\frac{6b^3gimn^3\operatorname{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right)}{j}$$

```
[Out] -g*m*(e*x+d)*(a+b*ln(c*(e*x+d)^n))^3/e+d*g*(a+b*ln(c*(e*x+d)^n))^3*ln(h*(j*
x+i)^m)/e-d*g*m*(a+b*ln(c*(e*x+d)^n))^3*ln(e*(j*x+i)/(-d*j+e*i))/e+g*i*m*(a
+b*ln(c*(e*x+d)^n))^3*ln(e*(j*x+i)/(-d*j+e*i))/j+6*b^3*g*i*m*n^3*polylog(2,
-j*(e*x+d)/(-d*j+e*i))/j+6*b^3*d*g*m*n^3*polylog(2,e*(j*x+i)/(-d*j+e*i))/e-
6*b^3*d*g*m*n^3*polylog(3,-j*(e*x+d)/(-d*j+e*i))/e+6*b^3*g*i*m*n^3*polylog(
3,-j*(e*x+d)/(-d*j+e*i))/j-6*b^3*d*g*m*n^3*polylog(4,-j*(e*x+d)/(-d*j+e*i))
/e+6*b^3*g*i*m*n^3*polylog(4,-j*(e*x+d)/(-d*j+e*i))/j+6*b^3*d*g*n^3*ln(-j*(
e*x+d)/(-d*j+e*i))*ln(h*(j*x+i)^m)/e-3*b*d*g*n*(a+b*ln(c*(e*x+d)^n))^2*ln(h
*(j*x+i)^m)/e-18*b^3*g*m*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e+6*b*g*m*n*(e*x+d)*(a
+b*ln(c*(e*x+d)^n))^2/e+x*(a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))-3*b
*d*g*m*n*(a+b*ln(c*(e*x+d)^n))^2*polylog(2,-j*(e*x+d)/(-d*j+e*i))/e+3*b*g*i
*m*n*(a+b*ln(c*(e*x+d)^n))^2*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j+6*b^2*d*g*m
*n^2*(a+b*ln(c*(e*x+d)^n))*polylog(3,-j*(e*x+d)/(-d*j+e*i))/e-6*b^2*g*i*m*n
^2*(a+b*ln(c*(e*x+d)^n))*polylog(3,-j*(e*x+d)/(-d*j+e*i))/j-18*a*b^2*g*m*n^
2*x+6*b^3*f*n^2*(e*x+d)*ln(c*(e*x+d)^n)/e-3*b*f*n*(e*x+d)*(a+b*ln(c*(e*x+d)
^n))^2/e-6*b^3*g*n^3*(j*x+i)*ln(h*(j*x+i)^m)/j+6*b^2*g*n^2*x*(a+b*ln(c*(e*x
+d)^n))*ln(h*(j*x+i)^m)-3*b*g*n*x*(a+b*ln(c*(e*x+d)^n))^2*ln(h*(j*x+i)^m)+6
*b^2*g*i*m*n^2*(a+b*ln(c*(e*x+d)^n))*ln(e*(j*x+i)/(-d*j+e*i))/j+3*b*d*g*m*n
*(a+b*ln(c*(e*x+d)^n))^2*ln(e*(j*x+i)/(-d*j+e*i))/e-3*b*g*i*m*n*(a+b*ln(c*(
e*x+d)^n))^2*ln(e*(j*x+i)/(-d*j+e*i))/j+6*b^2*d*g*m*n^2*(a+b*ln(c*(e*x+d)^n
))*polylog(2,-j*(e*x+d)/(-d*j+e*i))/e-6*b^2*g*i*m*n^2*(a+b*ln(c*(e*x+d)^n))
*polylog(2,-j*(e*x+d)/(-d*j+e*i))/j+6*a*b^2*f*n^2*x+24*b^3*g*m*n^3*x-6*b^3*
f*n^3*x+d*f*(a+b*ln(c*(e*x+d)^n))^3/e
```

## Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 1147, normalized size of antiderivative = 1.00, number of steps used = 64, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.710$, Rules used = {2479, 2463, 2436, 2333, 2332, 2443, 2481, 2421, 2430, 6724, 6874, 2458, 2388, 2339,

30, 2338, 45, 2441, 2440, 2438, 2422, 2354}

$$\int (a+b\log (c(d+ex)^n))^3 (f+g\log (h(i+jx)^m))\ dx$$

$$= -6fn^3xb^3+24gmn^3xb^3+\frac{6fn^2(d+ex)\log (c(d+ex)^n)\, b^3}{e}$$
$$-\frac{18gmn^2(d+ex)\log (c(d+ex)^n)\, b^3}{e}-\frac{6gn^3(i+jx)\log (h(i+jx)^m)\, b^3}{j}$$
$$+\frac{6dgn^3\log \left(-\frac{j(d+ex)}{ei-dj}\right)\log (h(i+jx)^m)\, b^3}{e}+\frac{6gimn^3\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right) b^3}{j}$$
$$+\frac{6dgmn^3\operatorname{PolyLog}\left(2,\frac{e(i+jx)}{ei-dj}\right) b^3}{e}-\frac{6dgmn^3\operatorname{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right) b^3}{e}$$
$$+\frac{6gimn^3\operatorname{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right) b^3}{j}-\frac{6dgmn^3\operatorname{PolyLog}\left(4,-\frac{j(d+ex)}{ei-dj}\right) b^3}{e}$$
$$+\frac{6gimn^3\operatorname{PolyLog}\left(4,-\frac{j(d+ex)}{ei-dj}\right) b^3}{j}+6afn^2xb^2-18agmn^2xb^2$$
$$+\frac{6gimn^2(a+b\log (c(d+ex)^n))\log \left(\frac{e(i+jx)}{ei-dj}\right) b^2}{j}$$
$$+6gn^2x(a+b\log (c(d+ex)^n))\log (h(i+jx)^m)\, b^2$$
$$+\frac{6dgmn^2(a+b\log (c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right) b^2}{e}$$
$$-\frac{6gimn^2(a+b\log (c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right) b^2}{j}$$
$$+\frac{6dgmn^2(a+b\log (c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right) b^2}{e}$$
$$-\frac{6gimn^2(a+b\log (c(d+ex)^n))\operatorname{PolyLog}\left(3,-\frac{j(d+ex)}{ei-dj}\right) b^2}{j}$$
$$-\frac{3fn(d+ex)\,(a+b\log (c(d+ex)^n))^2\, b}{e}+\frac{6gmn(d+ex)\,(a+b\log (c(d+ex)^n))^2\, b}{e}$$
$$+\frac{3dgmn(a+b\log (c(d+ex)^n))^2\log \left(\frac{e(i+jx)}{ei-dj}\right) b}{e}$$
$$-\frac{3gimn(a+b\log (c(d+ex)^n))^2\log \left(\frac{e(i+jx)}{ei-dj}\right) b}{j}$$
$$-\frac{3dgn(a+b\log (c(d+ex)^n))^2\log (h(i+jx)^m)\, b}{e}$$
$$-3gnx(a+b\log (c(d+ex)^n))^2\log (h(i+jx)^m)\, b$$
$$-\frac{3dgmn(a+b\log (c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right) b}{e}$$
$$+\frac{3gimn(a+b\log (c(d+ex)^n))^2\operatorname{PolyLog}\left(2,-\frac{j(d+ex)}{ei-dj}\right) b}{j}$$
$$df(a+b\log (c(d+ex)^n))^3 \quad gm(d+ex)\,(a+b\log (c(d+ex)^n))^3$$

```
[In] Int[(a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]),x]
```

```
[Out] 6*a*b^2*f*n^2*x - 18*a*b^2*g*m*n^2*x - 6*b^3*f*n^3*x + 24*b^3*g*m*n^3*x + (
6*b^3*f*n^2*(d + e*x)*Log[c*(d + e*x)^n])/e - (18*b^3*g*m*n^2*(d + e*x)*Log
[c*(d + e*x)^n])/e - (3*b*f*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e + (
6*b*g*m*n*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^2)/e + (d*f*(a + b*Log[c*(d
+ e*x)^n])^3)/e - (g*m*(d + e*x)*(a + b*Log[c*(d + e*x)^n])^3)/e + (6*b^2*g
*i*m*n^2*(a + b*Log[c*(d + e*x)^n])*Log[(e*(i + j*x))/(e*i - d*j)])/j + (3*
b*d*g*m*n*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(i + j*x))/(e*i - d*j)])/e -
(3*b*g*i*m*n*(a + b*Log[c*(d + e*x)^n])^2*Log[(e*(i + j*x))/(e*i - d*j)])/j
 - (d*g*m*(a + b*Log[c*(d + e*x)^n])^3*Log[(e*(i + j*x))/(e*i - d*j)])/e +
(g*i*m*(a + b*Log[c*(d + e*x)^n])^3*Log[(e*(i + j*x))/(e*i - d*j)])/j - (6*
b^3*g*n^3*(i + j*x)*Log[h*(i + j*x)^m])/j + (6*b^3*d*g*n^3*Log[-((j*(d + e*
x))/(e*i - d*j))]*Log[h*(i + j*x)^m])/e + 6*b^2*g*n^2*x*(a + b*Log[c*(d + e
*x)^n])*Log[h*(i + j*x)^m] - (3*b*d*g*n*(a + b*Log[c*(d + e*x)^n])^2*Log[h*
(i + j*x)^m])/e - 3*b*g*n*x*(a + b*Log[c*(d + e*x)^n])^2*Log[h*(i + j*x)^m]
 + (d*g*(a + b*Log[c*(d + e*x)^n])^3*Log[h*(i + j*x)^m])/e + x*(a + b*Log[c
*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]) + (6*b^3*g*i*m*n^3*PolyLog[2, -
((j*(d + e*x))/(e*i - d*j))])/j + (6*b^2*d*g*m*n^2*(a + b*Log[c*(d + e*x)^n
])*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/e - (6*b^2*g*i*m*n^2*(a + b*Lo
g[c*(d + e*x)^n])*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/j - (3*b*d*g*m*
n*(a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((j*(d + e*x))/(e*i - d*j))])/e
+ (3*b*g*i*m*n*(a + b*Log[c*(d + e*x)^n])^2*PolyLog[2, -((j*(d + e*x))/(e*i
 - d*j))])/j + (6*b^3*d*g*m*n^3*PolyLog[2, (e*(i + j*x))/(e*i - d*j)])/e -
(6*b^3*d*g*m*n^3*PolyLog[3, -((j*(d + e*x))/(e*i - d*j))])/e + (6*b^3*g*i*m
*n^3*PolyLog[3, -((j*(d + e*x))/(e*i - d*j))])/j + (6*b^2*d*g*m*n^2*(a + b*
Log[c*(d + e*x)^n])*PolyLog[3, -((j*(d + e*x))/(e*i - d*j))])/e - (6*b^2*g*
i*m*n^2*(a + b*Log[c*(d + e*x)^n])*PolyLog[3, -((j*(d + e*x))/(e*i - d*j))]
)/j - (6*b^3*d*g*m*n^3*PolyLog[4, -((j*(d + e*x))/(e*i - d*j))])/e + (6*b^3
*g*i*m*n^3*PolyLog[4, -((j*(d + e*x))/(e*i - d*j))])/j
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
```

```
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2479

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[x*((a +
 b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Dist[b*e*n*p, Int[x*(a + b*Lo
g[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))\\ &\quad-(gjm)\int\frac{x(a+b\log(c(d+ex)^n))^3}{i+jx}\,dx\\ &\quad-(3ben)\int\frac{x(a+b\log(c(d+ex)^n))^2(f+g\log(h(i+jx)^m))}{d+ex}\,dx\end{aligned}$$

$$= x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m)) - (gjm)\int\left(\frac{(a+b\log(c(d+ex)^n))^3}{j} - \frac{i(a+b\log(c(d+ex)^n))^3}{j(i+jx)}\right)dx - (3ben)\int\left(\frac{fx(a+b\log(c(d+ex)^n))^2}{d+ex} + \frac{gx(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{d+ex}\right)dx$$

$$= x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m)) - (gm)\int(a+b\log(c(d+ex)^n))^3\,dx + (gim)\int\frac{(a+b\log(c(d+ex)^n))^3}{i+jx}dx - (3befn)\int\frac{x(a+b\log(c(d+ex)^n))^2}{d+ex}dx - (3begn)\int\frac{x(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{d+ex}dx$$

$$= \frac{gim(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} + x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m)) - \frac{(gm)\text{Subst}\left(\int(a+b\log(cx^n))^3\,dx, x, d+ex\right)}{e} - (3bfn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)(a+b\log(cx^n))^2}{x}dx, x, d+ex\right) - (3begn)\int\left(\frac{(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{e} - \frac{d(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{e(d+ex)}\right)dx - \frac{(3begimn)\int\frac{(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{d+ex}dx}{j}$$

$$= -\frac{gm(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e}+\frac{gim\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}$$
$$+x\left(a+b\log\left(c(d+ex)^n\right)\right)^3\left(f+g\log\left(h(i+jx)^m\right)\right)$$
$$-\frac{(3bfn)\mathrm{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+ex\right)}{e}$$
$$+\frac{(3bdfn)\mathrm{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{x}\,dx,x,d+ex\right)}{e}$$
$$-(3bgn)\int\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(h(i+jx)^m\right)\,dx$$
$$+(3bdgn)\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(h(i+jx)^m\right)}{d+ex}\,dx$$
$$+\frac{(3bgmn)\mathrm{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+ex\right)}{e}$$
$$-\frac{(3bgimn)\mathrm{Subst}\left(\int\frac{(a+b\log(cx^n))^2\log\left(\frac{e\left(\frac{ei-dj}{e}+\frac{jx}{e}\right)}{ei-dj}\right)}{x}\,dx,x,d+ex\right)}{j}$$

$$= -\frac{3bfn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}+\frac{3bgmn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}$$
$$-\frac{gm(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e}+\frac{gim\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j}$$
$$-3bgnx\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(h(i+jx)^m\right)$$
$$+x\left(a+b\log\left(c(d+ex)^n\right)\right)^3\left(f+g\log\left(h(i+jx)^m\right)\right)$$
$$+\frac{3bgimn\left(a+b\log\left(c(d+ex)^n\right)\right)^2\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j}$$
$$+\frac{(3df)\operatorname{Subst}\left(\int x^2\,dx,x,a+b\log\left(c(d+ex)^n\right)\right)}{e}$$
$$+\frac{(3bdgn)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))^2\log\left(h\left(\frac{ei-dj}{e}+\frac{jx}{e}\right)^m\right)}{x}\,dx,x,d+ex\right)}{e}$$
$$+(3bgjmn)\int\frac{x\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{i+jx}\,dx$$
$$+\frac{\left(6b^2fn^2\right)\operatorname{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)\,dx,x,d+ex\right)}{e}$$
$$+\left(6b^2egn^2\right)\int\frac{x\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(h(i+jx)^m\right)}{d+ex}\,dx$$
$$-\frac{\left(6b^2gmn^2\right)\operatorname{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)\,dx,x,d+ex\right)}{e}$$
$$-\frac{\left(6b^2gimn^2\right)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{jx}{ei-dj}\right)}{x}\,dx,x,d+ex\right)}{j}$$

$$= 6ab^2fn^2x - 6ab^2gmn^2x - \frac{3bfn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} + \frac{3bgmn(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e} + \frac{df\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e} - \frac{gm(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e} + \frac{gim\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} - 3bgnx\left(a+b\log\left(c(d+ex)^n\right)\right)^2\log\left(h(i+jx)^m\right) + \frac{dg\left(a+b\log\left(c(d+ex)^n\right)\right)^3\log\left(h(i+jx)^m\right)}{e} + x\left(a+b\log\left(c(d+ex)^n\right)\right)^3\left(f+g\log\left(h(i+jx)^m\right)\right) + \frac{3bgimn\left(a+b\log\left(c(d+ex)^n\right)\right)^2\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} - \frac{6b^2gimn^2\left(a+b\log\left(c(d+ex)^n\right)\right)\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} - \frac{(dgjm)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^3}{\frac{ei-dj}{e}+\frac{jx}{e}}\,dx,x,d+ex\right)}{e^2} + (3bgjmn)\int\left(\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{j} - \frac{i\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{j(i+jx)}\right)dx + \frac{(6b^3fn^2)\,\text{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{e} + (6b^2egn^2)\int\left(\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(h(i+jx)^m\right)}{e} - \frac{d\left(a+b\log\left(c(d+ex)^n\right)\right)\log\left(h(i+jx)^m\right)}{e(d+ex)}\right)dx - \frac{(6b^3gmn^2)\,\text{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex\right)}{e} + \frac{(6b^3gimn^3)\,\text{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{jx}{ei-dj}\right)}{x}\,dx,x,d+ex\right)}{j}$$

$$
\begin{aligned}
&= 6ab^2fn^2x - 6ab^2gmn^2x - 6b^3fn^3x + 6b^3gmn^3x + \frac{6b^3fn^2(d+ex)\log(c(d+ex)^n)}{e} \\
&- \frac{6b^3gmn^2(d+ex)\log(c(d+ex)^n)}{e} - \frac{3bfn(d+ex)(a+b\log(c(d+ex)^n))^2}{e} \\
&+ \frac{3bgmn(d+ex)(a+b\log(c(d+ex)^n))^2}{e} \\
&+ \frac{df(a+b\log(c(d+ex)^n))^3}{e} - \frac{gm(d+ex)(a+b\log(c(d+ex)^n))^3}{e} \\
&- \frac{dgm(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{e} \\
&+ \frac{gim(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} \\
&- 3bgnx(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m) \\
&+ \frac{dg(a+b\log(c(d+ex)^n))^3\log(h(i+jx)^m)}{e} \\
&+ x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m)) \\
&+ \frac{3bgimn(a+b\log(c(d+ex)^n))^2\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} \\
&- \frac{6b^2gimn^2(a+b\log(c(d+ex)^n))\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} \\
&+ \frac{6b^3gimn^3\operatorname{Li}_4\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} + (3bgmn)\int(a+b\log(c(d+ex)^n))^2\,dx \\
&+ \frac{(3bdgmn)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))^2\log\left(1+\frac{jx}{ei-dj}\right)}{x}\,dx, x, d+ex\right)}{e} \\
&- (3bgimn)\int\frac{(a+b\log(c(d+ex)^n))^2}{i+jx}\,dx \\
&+ \left(6b^2gn^2\right)\int(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)\,dx \\
&- \left(6b^2dgn^2\right)\int\frac{(a+b\log(c(d+ex)^n))\log(h(i+jx)^m)}{d+ex}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= 6ab^2fn^2x - 6ab^2gmn^2x - 6b^3fn^3x + 6b^3gmn^3x + \frac{6b^3fn^2(d+ex)\log(c(d+ex)^n)}{e} \\
&- \frac{6b^3gmn^2(d+ex)\log(c(d+ex)^n)}{e} - \frac{3bfn(d+ex)\left(a+b\log(c(d+ex)^n)\right)^2}{e} \\
&+ \frac{3bgmn(d+ex)\left(a+b\log(c(d+ex)^n)\right)^2}{e} \\
&+ \frac{df\left(a+b\log(c(d+ex)^n)\right)^3}{e} - \frac{gm(d+ex)\left(a+b\log(c(d+ex)^n)\right)^3}{e} \\
&- \frac{3bgimn\left(a+b\log(c(d+ex)^n)\right)^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} \\
&- \frac{dgm\left(a+b\log(c(d+ex)^n)\right)^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{e} \\
&+ \frac{gim\left(a+b\log(c(d+ex)^n)\right)^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} \\
&+ 6b^2gn^2x\left(a+b\log(c(d+ex)^n)\right)\log\left(h(i+jx)^m\right) \\
&- 3bgnx\left(a+b\log(c(d+ex)^n)\right)^2\log\left(h(i+jx)^m\right) \\
&+ \frac{dg\left(a+b\log(c(d+ex)^n)\right)^3\log\left(h(i+jx)^m\right)}{e} \\
&+ x\left(a+b\log(c(d+ex)^n)\right)^3\left(f+g\log\left(h(i+jx)^m\right)\right) \\
&- \frac{3bdgmn\left(a+b\log(c(d+ex)^n)\right)^2\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{e} \\
&+ \frac{3bgimn\left(a+b\log(c(d+ex)^n)\right)^2\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} \\
&- \frac{6b^2gimn^2\left(a+b\log(c(d+ex)^n)\right)\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} + \frac{6b^3gimn^3\operatorname{Li}_4\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} \\
&+ \frac{(3bgmn)\operatorname{Subst}\left(\int \left(a+b\log(cx^n)\right)^2\,dx, x, d+ex\right)}{e} \\
&- \frac{(6b^2dgn^2)\operatorname{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(h\left(\frac{ei-dj}{e}+\frac{jx}{e}\right)^m\right)}{x}\,dx, x, d+ex\right)}{e} \\
&+ \frac{(6b^2dgmn^2)\operatorname{Subst}\left(\int \frac{(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{jx}{ei-dj}\right)}{x}\,dx, x, d+ex\right)}{e} \\
&+ \frac{(6b^2egimn^2)\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(i+jx)}{ei-dj}\right)}{d+ex}\,dx}{j} \\
&- (6b^2gjmn^2)\int \frac{x\left(a+b\log(c(d+ex)^n)\right)}{i+jx}\,dx - (6b^3egn^3)\int \frac{x\log\left(h(i+jx)^m\right)}{d+ex}\,dx
\end{aligned}
$$

$$= 6ab^2fn^2x - 6ab^2gmn^2x - 6b^3fn^3x + 6b^3gmn^3x + \frac{6b^3fn^2(d+ex)\log(c(d+ex)^n)}{e} - \frac{6b^3gmn^2(d+ex)\log(c(d+ex)^n)}{e} - \frac{3bfn(d+ex)(a+b\log(c(d+ex)^n))^2}{e} + \frac{6bgmn(d+ex)(a+b\log(c(d+ex)^n))^2}{e} + \frac{df(a+b\log(c(d+ex)^n))^3}{e} - \frac{gm(d+ex)(a+b\log(c(d+ex)^n))^3}{e} - \frac{3bgimn(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} - \frac{dgm(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{e} + \frac{gim(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} + 6b^2gn^2x(a+b\log(c(d+ex)^n))\log(h(i+jx)^m) - \frac{3bdgn(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{e} - 3bgnx(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m) + \frac{dg(a+b\log(c(d+ex)^n))^3\log(h(i+jx)^m)}{e} + x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m)) - \frac{3bdgmn(a+b\log(c(d+ex)^n))^2\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{e} + \frac{3bgimn(a+b\log(c(d+ex)^n))^2\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} + \frac{6b^2dgmn^2(a+b\log(c(d+ex)^n))\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{e} - \frac{6b^2gimn^2(a+b\log(c(d+ex)^n))\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} + \frac{6b^3gimn^3\operatorname{Li}_4\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} + \frac{(3bdgjmn)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{\frac{ei-dj}{e}+\frac{jx}{e}}dx, x, d+ex\right)}{e^2} - \frac{(6b^2gmn^2)\operatorname{Subst}\left(\int(a+b\log(cx^n))\,dx, x, d+ex\right)}{e} + \frac{(6b^2gimn^2)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{ei-dj}{e}+\frac{jx}{e}\right)}{ei-dj}\right)}{x}dx, x, d+ex\right)}{j} - (6b^2gjmn^2)\int\left(\frac{a+b\log(c(d+ex)^n)}{j} - \frac{i(a+b\log(c(d+ex)^n))}{j(i+jx)}\right)dx - (6b^3egn^3)\int\left(\frac{\log(h(i+jx)^m)}{e} - \frac{d\log(h(i+jx)^m)}{e(d+ex)}\right)dx$$

$$(6b^3dgmn^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{jx}{ei-dj}\right)}{\ldots}dx, x, d+ex\right)$$

$$= 6ab^2fn^2x - 12ab^2gmn^2x - 6b^3fn^3x + 6b^3gmn^3x + \frac{6b^3fn^2(d+ex)\log(c(d+ex)^n)}{e} - \frac{6b^3gmn^2(d+ex)\log(c(d+ex)^n)}{e} - \frac{3bfn(d+ex)(a+b\log(c(d+ex)^n))^2}{e} + \frac{6bgmn(d+ex)(a+b\log(c(d+ex)^n))^2}{e} + \frac{df(a+b\log(c(d+ex)^n))^3}{e} - \frac{gm(d+ex)(a+b\log(c(d+ex)^n))^3}{e} + \frac{3bdgmn(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{e} - \frac{3bgimn(a+b\log(c(d+ex)^n))^2\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} - \frac{dgm(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{e} + \frac{gim(a+b\log(c(d+ex)^n))^3\log\left(\frac{e(i+jx)}{ei-dj}\right)}{j} + 6b^2gn^2x(a+b\log(c(d+ex)^n))\log(h(i+jx)^m) - \frac{3bdgn(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m)}{e} - 3bgnx(a+b\log(c(d+ex)^n))^2\log(h(i+jx)^m) + \frac{dg(a+b\log(c(d+ex)^n))^3\log(h(i+jx)^m)}{e} + x(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m)) - \frac{6b^2gimn^2(a+b\log(c(d+ex)^n))\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} - \frac{3bdgmn(a+b\log(c(d+ex)^n))^2\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{e} + \frac{3bgimn(a+b\log(c(d+ex)^n))^2\operatorname{Li}_2\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} + \frac{6b^2dgmn^2(a+b\log(c(d+ex)^n))\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{e} - \frac{6b^2gimn^2(a+b\log(c(d+ex)^n))\operatorname{Li}_3\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} - \frac{6b^3dgmn^3\operatorname{Li}_4\left(-\frac{j(d+ex)}{ei-dj}\right)}{e} + \frac{6b^3gimn^3\operatorname{Li}_4\left(-\frac{j(d+ex)}{ei-dj}\right)}{j} - (6b^2gmn^2)\int(a+b\log(c(d+ex)^n))\,dx - \frac{(6b^3gmn^2)\operatorname{Subst}\left(\int\log(cx^n)\,dx,x,d+ex\right)}{e} - \frac{(6b^2dgmn^2)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(1+\frac{jx}{ei-dj}\right)}{x}\,dx,x,d+ex\right)}{e} + (6b^2gimn^2)\int\frac{a+b\log(c(d+ex)^n)}{i+jx}\,dx$$

= Too large to display

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3163 vs. 2(1147) = 2294.

Time = 0.65 (sec) , antiderivative size = 3163, normalized size of antiderivative = 2.76

$$\int (a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m)) \, dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]),x]
[Out] (-3*a^2*b*d*f*j*n + 3*a^2*b*d*g*j*m*n - 6*a*b^2*d*g*j*m*n^2 + 6*b^3*d*g*j*m
*n^3 + a^3*e*f*j*x - a^3*e*g*j*m*x - 3*a^2*b*e*f*j*n*x + 6*a^2*b*e*g*j*m*n*
x + 6*a*b^2*e*f*j*n^2*x - 18*a*b^2*e*g*j*m*n^2*x - 6*b^3*e*f*j*n^3*x + 24*b
^3*e*g*j*m*n^3*x + 3*a^2*b*d*f*j*n*Log[d + e*x] - 3*a^2*b*d*g*j*m*n*Log[d +
 e*x] + 6*a*b^2*d*g*j*m*n^2*Log[d + e*x] + 6*b^3*d*f*j*n^3*Log[d + e*x] - 1
2*b^3*d*g*j*m*n^3*Log[d + e*x] - 3*a*b^2*d*f*j*n^2*Log[d + e*x]^2 + 3*a*b^2
*d*g*j*m*n^2*Log[d + e*x]^2 - 3*b^3*d*g*j*m*n^3*Log[d + e*x]^2 + b^3*d*f*j*
n^3*Log[d + e*x]^3 - b^3*d*g*j*m*n^3*Log[d + e*x]^3 - 6*a*b^2*d*f*j*n*Log[c
*(d + e*x)^n] + 6*a*b^2*d*g*j*m*n*Log[c*(d + e*x)^n] - 6*b^3*d*g*j*m*n^2*Lo
g[c*(d + e*x)^n] + 3*a^2*b*e*f*j*x*Log[c*(d + e*x)^n] - 3*a^2*b*e*g*j*m*x*L
og[c*(d + e*x)^n] - 6*a*b^2*e*f*j*n*x*Log[c*(d + e*x)^n] + 12*a*b^2*e*g*j*m
*n*x*Log[c*(d + e*x)^n] + 6*b^3*e*f*j*n^2*x*Log[c*(d + e*x)^n] - 18*b^3*e*g
*j*m*n^2*x*Log[c*(d + e*x)^n] + 6*a*b^2*d*f*j*n*Log[d + e*x]*Log[c*(d + e*x
)^n] - 6*a*b^2*d*g*j*m*n*Log[d + e*x]*Log[c*(d + e*x)^n] + 6*b^3*d*g*j*m*n^
2*Log[d + e*x]*Log[c*(d + e*x)^n] - 3*b^3*d*f*j*n^2*Log[d + e*x]^2*Log[c*(d
 + e*x)^n] + 3*b^3*d*g*j*m*n^2*Log[d + e*x]^2*Log[c*(d + e*x)^n] - 3*b^3*d*
f*j*n*Log[c*(d + e*x)^n]^2 + 3*b^3*d*g*j*m*n*Log[c*(d + e*x)^n]^2 + 3*a*b^2
*e*f*j*x*Log[c*(d + e*x)^n]^2 - 3*a*b^2*e*g*j*m*x*Log[c*(d + e*x)^n]^2 - 3*
b^3*e*f*j*n*x*Log[c*(d + e*x)^n]^2 + 6*b^3*e*g*j*m*n*x*Log[c*(d + e*x)^n]^2
 + 3*b^3*d*f*j*n*Log[d + e*x]*Log[c*(d + e*x)^n]^2 - 3*b^3*d*g*j*m*n*Log[d
+ e*x]*Log[c*(d + e*x)^n]^2 + b^3*e*f*j*x*Log[c*(d + e*x)^n]^3 - b^3*e*g*j*
m*x*Log[c*(d + e*x)^n]^3 + a^3*e*g*i*m*Log[i + j*x] - 3*a^2*b*e*g*i*m*n*Log
[i + j*x] + 3*a^2*b*d*g*j*m*n*Log[i + j*x] + 6*a*b^2*e*g*i*m*n^2*Log[i + j*
x] - 6*b^3*e*g*i*m*n^3*Log[i + j*x] - 3*a^2*b*e*g*i*m*n*Log[d + e*x]*Log[i
+ j*x] + 6*a*b^2*e*g*i*m*n^2*Log[d + e*x]*Log[i + j*x] - 6*a*b^2*d*g*j*m*n^
2*Log[d + e*x]*Log[i + j*x] - 6*b^3*e*g*i*m*n^3*Log[d + e*x]*Log[i + j*x] +
 3*a*b^2*e*g*i*m*n^2*Log[d + e*x]^2*Log[i + j*x] - 3*b^3*e*g*i*m*n^3*Log[d
+ e*x]^2*Log[i + j*x] + 3*b^3*d*g*j*m*n^3*Log[d + e*x]^2*Log[i + j*x] - b^3
*e*g*i*m*n^3*Log[d + e*x]^3*Log[i + j*x] + 3*a^2*b*e*g*i*m*Log[c*(d + e*x)^
n]*Log[i + j*x] - 6*a*b^2*e*g*i*m*n*Log[c*(d + e*x)^n]*Log[i + j*x] + 6*a*b
^2*d*g*j*m*n*Log[c*(d + e*x)^n]*Log[i + j*x] + 6*b^3*e*g*i*m*n^2*Log[c*(d +
 e*x)^n]*Log[i + j*x] - 6*a*b^2*e*g*i*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]*L
og[i + j*x] + 6*b^3*e*g*i*m*n^2*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[i + j*x
```

] - 6*b^3*d*g*j*m*n^2*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[i + j*x] + 3*b^3*
e*g*i*m*n^2*Log[d + e*x]^2*Log[c*(d + e*x)^n]*Log[i + j*x] + 3*a*b^2*e*g*i*
m*Log[c*(d + e*x)^n]^2*Log[i + j*x] - 3*b^3*e*g*i*m*n*Log[c*(d + e*x)^n]^2*
Log[i + j*x] + 3*b^3*d*g*j*m*n*Log[c*(d + e*x)^n]^2*Log[i + j*x] - 3*b^3*e*
g*i*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]^2*Log[i + j*x] + b^3*e*g*i*m*Log[c*
(d + e*x)^n]^3*Log[i + j*x] + 3*a^2*b*e*g*i*m*n*Log[d + e*x]*Log[(e*(i + j*
x))/(e*i - d*j)] - 3*a^2*b*d*g*j*m*n*Log[d + e*x]*Log[(e*(i + j*x))/(e*i -
d*j)] - 6*a*b^2*e*g*i*m*n^2*Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d*j)] + 6
*a*b^2*d*g*j*m*n^2*Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d*j)] + 6*b^3*e*g*
i*m*n^3*Log[d + e*x]*Log[(e*(i + j*x))/(e*i - d*j)] - 6*b^3*d*g*j*m*n^3*Log
[d + e*x]*Log[(e*(i + j*x))/(e*i - d*j)] - 3*a*b^2*e*g*i*m*n^2*Log[d + e*x]
^2*Log[(e*(i + j*x))/(e*i - d*j)] + 3*a*b^2*d*g*j*m*n^2*Log[d + e*x]^2*Log[
(e*(i + j*x))/(e*i - d*j)] + 3*b^3*e*g*i*m*n^3*Log[d + e*x]^2*Log[(e*(i + j
*x))/(e*i - d*j)] - 3*b^3*d*g*j*m*n^3*Log[d + e*x]^2*Log[(e*(i + j*x))/(e*i
 - d*j)] + b^3*e*g*i*m*n^3*Log[d + e*x]^3*Log[(e*(i + j*x))/(e*i - d*j)] -
b^3*d*g*j*m*n^3*Log[d + e*x]^3*Log[(e*(i + j*x))/(e*i - d*j)] + 6*a*b^2*e*g
*i*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[(e*(i + j*x))/(e*i - d*j)] - 6*a
*b^2*d*g*j*m*n*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[(e*(i + j*x))/(e*i - d*j
)] - 6*b^3*e*g*i*m*n^2*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[(e*(i + j*x))/(e
*i - d*j)] + 6*b^3*d*g*j*m*n^2*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[(e*(i +
j*x))/(e*i - d*j)] - 3*b^3*e*g*i*m*n^2*Log[d + e*x]^2*Log[c*(d + e*x)^n]*Lo
g[(e*(i + j*x))/(e*i - d*j)] + 3*b^3*d*g*j*m*n^2*Log[d + e*x]^2*Log[c*(d +
e*x)^n]*Log[(e*(i + j*x))/(e*i - d*j)] + 3*b^3*e*g*i*m*n*Log[d + e*x]*Log[c
*(d + e*x)^n]^2*Log[(e*(i + j*x))/(e*i - d*j)] - 3*b^3*d*g*j*m*n*Log[d + e*
x]*Log[c*(d + e*x)^n]^2*Log[(e*(i + j*x))/(e*i - d*j)] - 3*a^2*b*d*g*j*n*Lo
g[h*(i + j*x)^m] + a^3*e*g*j*x*Log[h*(i + j*x)^m] - 3*a^2*b*e*g*j*n*x*Log[h
*(i + j*x)^m] + 6*a*b^2*e*g*j*n^2*x*Log[h*(i + j*x)^m] - 6*b^3*e*g*j*n^3*x*
Log[h*(i + j*x)^m] + 3*a^2*b*d*g*j*n*Log[d + e*x]*Log[h*(i + j*x)^m] + 6*b^
3*d*g*j*n^3*Log[d + e*x]*Log[h*(i + j*x)^m] - 3*a*b^2*d*g*j*n^2*Log[d + e*x
]^2*Log[h*(i + j*x)^m] + b^3*d*g*j*n^3*Log[d + e*x]^3*Log[h*(i + j*x)^m] -
6*a*b^2*d*g*j*n*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] + 3*a^2*b*e*g*j*x*Log
[c*(d + e*x)^n]*Log[h*(i + j*x)^m] - 6*a*b^2*e*g*j*n*x*Log[c*(d + e*x)^n]*L
og[h*(i + j*x)^m] + 6*b^3*e*g*j*n^2*x*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m]
 + 6*a*b^2*d*g*j*n*Log[d + e*x]*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] - 3*b
^3*d*g*j*n^2*Log[d + e*x]^2*Log[c*(d + e*x)^n]*Log[h*(i + j*x)^m] - 3*b^3*d
*g*j*n*Log[c*(d + e*x)^n]^2*Log[h*(i + j*x)^m] + 3*a*b^2*e*g*j*x*Log[c*(d +
 e*x)^n]^2*Log[h*(i + j*x)^m] - 3*b^3*e*g*j*n*x*Log[c*(d + e*x)^n]^2*Log[h*
(i + j*x)^m] + 3*b^3*d*g*j*n*Log[d + e*x]*Log[c*(d + e*x)^n]^2*Log[h*(i + j
*x)^m] + b^3*e*g*j*x*Log[c*(d + e*x)^n]^3*Log[h*(i + j*x)^m] + 3*b*g*(e*i -
 d*j)*m*n*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b*(a - b*n)*Log[c*(d + e*x)^n] + b
^2*Log[c*(d + e*x)^n]^2)*PolyLog[2, (j*(d + e*x))/(-(e*i) + d*j)] - 6*b^2*g
*(e*i - d*j)*m*n^2*(a - b*n + b*Log[c*(d + e*x)^n])*PolyLog[3, (j*(d + e*x)
)/(-(e*i) + d*j)] + 6*b^3*e*g*i*m*n^3*PolyLog[4, (j*(d + e*x))/(-(e*i) + d*
j)] - 6*b^3*d*g*j*m*n^3*PolyLog[4, (j*(d + e*x))/(-(e*i) + d*j)])/(e*j)

## Maple [F]

$$\int (a+b\ln (c(ex+d)^n))^3 (f+g\ln (h(jx+i)^m))\, dx$$

[In] `int((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m)),x)`

[Out] `int((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m)),x)`

## Fricas [F]

$$\int (a+b\log (c(d+ex)^n))^3 \left(f+g\log \left(h(i+jx)^m\right)\right)\, dx = \int (b\log \left((ex+d)^n c\right)+a)^3 (g\log \left((jx+i)^m h\right)+f)\, dx$$

[In] `integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m)),x, algorithm="fricas")`

[Out] `integral(b^3*f*log((e*x + d)^n*c)^3 + 3*a*b^2*f*log((e*x + d)^n*c)^2 + 3*a^2*b*f*log((e*x + d)^n*c) + a^3*f + (b^3*g*log((e*x + d)^n*c)^3 + 3*a*b^2*g*log((e*x + d)^n*c)^2 + 3*a^2*b*g*log((e*x + d)^n*c) + a^3*g)*log((j*x + i)^m*h), x)`

## Sympy [F(-1)]

Timed out.

$$\int (a+b\log (c(d+ex)^n))^3 \left(f+g\log \left(h(i+jx)^m\right)\right)\, dx = \text{Timed out}$$

[In] `integrate((a+b*ln(c*(e*x+d)**n))**3*(f+g*ln(h*(j*x+i)**m)),x)`

[Out] `Timed out`

## Maxima [F]

$$\int (a+b\log (c(d+ex)^n))^3 \left(f+g\log \left(h(i+jx)^m\right)\right)\, dx = \int (b\log \left((ex+d)^n c\right)+a)^3 (g\log \left((jx+i)^m h\right)+f)\, dx$$

[In] `integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m)),x, algorithm="maxima")`

```
[Out] b^3*f*x*log((e*x + d)^n*c)^3 - 3*a^2*b*e*f*n*(x/e - d*log(e*x + d)/e^2) - a
^3*g*j*m*(x/j - i*log(j*x + i)/j^2) + 3*a*b^2*f*x*log((e*x + d)^n*c)^2 + 3*
a^2*b*f*x*log((e*x + d)^n*c) + a^3*g*x*log((j*x + i)^m*h) - 3*(2*e*n*(x/e -
 d*log(e*x + d)/e^2)*log((e*x + d)^n*c) + (d*log(e*x + d)^2 - 2*e*x + 2*d*l
og(e*x + d))*n^2/e)*a*b^2*f - (3*e*n*(x/e - d*log(e*x + d)/e^2)*log((e*x +
d)^n*c)^2 - e*n*((d*log(e*x + d)^3 + 3*d*log(e*x + d)^2 - 6*e*x + 6*d*log(e
*x + d))*n^2/e^2 - 3*(d*log(e*x + d)^2 - 2*e*x + 2*d*log(e*x + d))*n*log((e
*x + d)^n*c)/e^2))*b^3*f + a^3*f*x + ((b^3*e*g*i*m*log(j*x + i) - (j*m - j*
log(h))*b^3*e*g*x)*log((e*x + d)^n)^3 + (b^3*d*g*j*n^3*log(e*x + d)^3 + b^3
*e*g*j*x*log((e*x + d)^n)^3 - 3*(a*b^2*d*g*j*n^2 - (d*g*j*n^3 - d*g*j*n^2*l
og(c))*b^3)*log(e*x + d)^2 + 3*(b^3*d*g*j*n*log(e*x + d) + (a*b^2*e*g*j - (
e*g*j*n - e*g*j*log(c))*b^3)*x)*log((e*x + d)^n)^2 - (3*(e*g*j*n - e*g*j*lo
g(c))*a^2*b - 3*(2*e*g*j*n^2 - 2*e*g*j*n*log(c) + e*g*j*log(c)^2)*a*b^2 + (
6*e*g*j*n^3 - 6*e*g*j*n^2*log(c) + 3*e*g*j*n*log(c)^2 - e*g*j*log(c)^3)*b^3
)*x + 3*(a^2*b*d*g*j*n - 2*(d*g*j*n^2 - d*g*j*n*log(c))*a*b^2 + (2*d*g*j*n^
3 - 2*d*g*j*n^2*log(c) + d*g*j*n*log(c)^2)*b^3)*log(e*x + d) - 3*(b^3*d*g*j
*n^2*log(e*x + d)^2 - (a^2*b*e*g*j - 2*(e*g*j*n - e*g*j*log(c))*a*b^2 + (2*
e*g*j*n^2 - 2*e*g*j*n*log(c) + e*g*j*log(c)^2)*b^3)*x - 2*(a*b^2*d*g*j*n -
(d*g*j*n^2 - d*g*j*n*log(c))*b^3)*log(e*x + d))*log((e*x + d)^n))*log((j*x
+ i)^m))/(e*j) - integrate(-(b^3*d*e*g*i*j*log(c)^3*log(h) + 3*a*b^2*d*e*g*
i*j*log(c)^2*log(h) + 3*a^2*b*d*e*g*i*j*log(c)*log(h) - (b^3*d*e*g*j^2*m*n^
3*x + b^3*d^2*g*j^2*m*n^3)*log(e*x + d)^3 + (3*(e^2*g*j^2*m*n - (j^2*m - j^
2*log(h))*e^2*g*log(c))*a^2*b - 3*(2*e^2*g*j^2*m*n^2 - 2*e^2*g*j^2*m*n*log(
c) + (j^2*m - j^2*log(h))*e^2*g*log(c)^2)*a*b^2 + (6*e^2*g*j^2*m*n^3 - 6*e^
2*g*j^2*m*n^2*log(c) + 3*e^2*g*j^2*m*n*log(c)^2 - (j^2*m - j^2*log(h))*e^2*
g*log(c)^3)*b^3)*x^2 + 3*(a*b^2*d^2*g*j^2*m*n^2 - (d^2*g*j^2*m*n^3 - d^2*g*
j^2*m*n^2*log(c))*b^3 + (a*b^2*d*e*g*j^2*m*n^2 - (d*e*g*j^2*m*n^3 - d*e*g*j
^2*m*n^2*log(c))*b^3)*x)*log(e*x + d)^2 + 3*(b^3*d*e*g*i*j*log(c)*log(h) +
a*b^2*d*e*g*i*j*log(h) - ((j^2*m - j^2*log(h))*a*b^2*e^2*g + ((j^2*m - j^2*
log(h))*e^2*g*log(c) - (2*j^2*m*n - j^2*n*log(h))*e^2*g)*b^3)*x^2 + ((e^2*g
*i*j*log(h) - (j^2*m - j^2*log(h))*d*e*g)*a*b^2 + (d*e*g*j^2*m*n + (i*j*m*n
 - i*j*n*log(h))*e^2*g + (e^2*g*i*j*log(h) - (j^2*m - j^2*log(h))*d*e*g)*lo
g(c))*b^3)*x - (b^3*d*e*g*j^2*m*n*x + b^3*d^2*g*j^2*m*n)*log(e*x + d) - (b^
3*e^2*g*i*j*m*n*x + b^3*e^2*g*i^2*m*n)*log(j*x + i))*log((e*x + d)^n)^2 + (
3*(d*e*g*j^2*m*n + (e^2*g*i*j*log(h) - (j^2*m - j^2*log(h))*d*e*g)*log(c))*
a^2*b - 3*(2*d*e*g*j^2*m*n^2 - 2*d*e*g*j^2*m*n*log(c) - (e^2*g*i*j*log(h) -
 (j^2*m - j^2*log(h))*d*e*g)*log(c)^2)*a*b^2 + (6*d*e*g*j^2*m*n^3 - 6*d*e*g
*j^2*m*n^2*log(c) + 3*d*e*g*j^2*m*n*log(c)^2 + (e^2*g*i*j*log(h) - (j^2*m -
 j^2*log(h))*d*e*g)*log(c)^3)*b^3)*x - 3*(a^2*b*d^2*g*j^2*m*n - 2*(d^2*g*j^
2*m*n^2 - d^2*g*j^2*m*n*log(c))*a*b^2 + (2*d^2*g*j^2*m*n^3 - 2*d^2*g*j^2*m*
n^2*log(c) + d^2*g*j^2*m*n*log(c)^2)*b^3 + (a^2*b*d*e*g*j^2*m*n - 2*(d*e*g*
j^2*m*n^2 - d*e*g*j^2*m*n*log(c))*a*b^2 + (2*d*e*g*j^2*m*n^3 - 2*d*e*g*j^2*
m*n^2*log(c) + d*e*g*j^2*m*n*log(c)^2)*b^3)*x)*log(e*x + d) + 3*(b^3*d*e*g*
i*j*log(c)^2*log(h) + 2*a*b^2*d*e*g*i*j*log(c)*log(h) + a^2*b*d*e*g*i*j*log
(h) - ((j^2*m - j^2*log(h))*a^2*b*e^2*g - 2*(e^2*g*j^2*m*n - (j^2*m - j^2*l
```

```
og(h))*e^2*g*log(c))*a*b^2 + (2*e^2*g*j^2*m*n^2 - 2*e^2*g*j^2*m*n*log(c) +
(j^2*m - j^2*log(h))*e^2*g*log(c)^2)*b^3)*x^2 + (b^3*d*e*g*j^2*m*n^2*x + b^
3*d^2*g*j^2*m*n^2)*log(e*x + d)^2 + ((e^2*g*i*j*log(h) - (j^2*m - j^2*log(h
))*d*e*g)*a^2*b + 2*(d*e*g*j^2*m*n + (e^2*g*i*j*log(h) - (j^2*m - j^2*log(h
))*d*e*g)*log(c))*a*b^2 - (2*d*e*g*j^2*m*n^2 - 2*d*e*g*j^2*m*n*log(c) - (e^
2*g*i*j*log(h) - (j^2*m - j^2*log(h))*d*e*g)*log(c)^2)*b^3)*x - 2*(a*b^2*d^
2*g*j^2*m*n - (d^2*g*j^2*m*n^2 - d^2*g*j^2*m*n*log(c))*b^3 + (a*b^2*d*e*g*j
^2*m*n - (d*e*g*j^2*m*n^2 - d*e*g*j^2*m*n*log(c))*b^3)*x)*log(e*x + d))*log
((e*x + d)^n))/(e^2*j^2*x^2 + d*e*i*j + (e^2*i*j + d*e*j^2)*x), x)
```

## Giac [F]

$$\int (a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))\,dx = \int (b\log((ex+d)^n c)+a)^3 (g\log((jx+i)^m h)+f)\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m)),x, algorithm="gia
c")
[Out] integrate((b*log((e*x + d)^n*c) + a)^3*(g*log((j*x + i)^m*h) + f), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d+ex)^n))^3 (f+g\log(h(i+jx)^m))\,dx = \int (a+b\,\ln(c\,(d+e\,x)^n))^3\,(f+g\,\ln(h\,(i+j\,x)^m))\,dx$$

```
[In] int((a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)),x)
[Out] int((a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)), x)
```

# 3.399 $\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx$



## Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx = \mathrm{Int}\left(\frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))/x,x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx$$

```
[In] Int[((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x,x]
[Out] Defer[Int][((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx$$

```
[In] Integrate[((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x,x]
[Out] Integrate[((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x, x]
```

## Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(ex+d)^n))^3(f+g\ln(h(jx+i)^m))}{x}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))/x,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))/x,x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.97

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)}{x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="f
ricas")
[Out] integral((b^3*f*log((e*x + d)^n*c)^3 + 3*a*b^2*f*log((e*x + d)^n*c)^2 + 3*a
^2*b*f*log((e*x + d)^n*c) + a^3*f + (b^3*g*log((e*x + d)^n*c)^3 + 3*a*b^2*g
*log((e*x + d)^n*c)^2 + 3*a^2*b*g*log((e*x + d)^n*c) + a^3*g)*log((j*x + i)
^m*h))/x, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**3*(f+g*ln(h*(j*x+i)**m))/x,x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 292, normalized size of antiderivative = 8.59

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)}{x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="m
axima")
[Out] a^3*f*log(x) + integrate(((g*log(h) + f)*b^3*log((e*x + d)^n)^3 + (g*log(h)
 + f)*b^3*log(c)^3 + 3*(g*log(h) + f)*a*b^2*log(c)^2 + 3*(g*log(h) + f)*a^2
*b*log(c) + a^3*g*log(h) + 3*((g*log(h) + f)*b^3*log(c) + (g*log(h) + f)*a*
b^2)*log((e*x + d)^n)^2 + 3*((g*log(h) + f)*b^3*log(c)^2 + 2*(g*log(h) + f)
*a*b^2*log(c) + (g*log(h) + f)*a^2*b)*log((e*x + d)^n) + (b^3*g*log((e*x +
d)^n)^3 + b^3*g*log(c)^3 + 3*a*b^2*g*log(c)^2 + 3*a^2*b*g*log(c) + a^3*g +
3*(b^3*g*log(c) + a*b^2*g)*log((e*x + d)^n)^2 + 3*(b^3*g*log(c)^2 + 2*a*b^2
*g*log(c) + a^2*b*g)*log((e*x + d)^n))*log((j*x + i)^m))/x, x)
```

## Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)}{x}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m))/x,x, algorithm="g
iac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^3*(g*log((j*x + i)^m*h) + f)/x, x)
```

## Mupad [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x}\,dx = \int \frac{(a+b\ln(c\,(d+e\,x)^n))^3\,(f+g\ln(h\,(i+j\,x)^m))}{x}\,dx$$

```
[In] int(((a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)))/x,x)
[Out] int(((a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)))/x, x)
```

# 3.400 $\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx$



## Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx = \text{Int}\left(\frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx$$

```
[In] Int[((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x^2,x]
```

```
[Out] Defer[Int][((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x^2, x
]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx$$

```
[In] Integrate[((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x^2,x]
[Out] Integrate[((a + b*Log[c*(d + e*x)^n])^3*(f + g*Log[h*(i + j*x)^m]))/x^2, x]
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(ex+d)^n))^3(f+g\ln(h(jx+i)^m))}{x^2}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))/x^2,x)
[Out] int((a+b*ln(c*(e*x+d)^n))^3*(f+g*ln(h*(j*x+i)^m))/x^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.97

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm=
"fricas")
[Out] integral((b^3*f*log((e*x + d)^n*c)^3 + 3*a*b^2*f*log((e*x + d)^n*c)^2 + 3*a
^2*b*f*log((e*x + d)^n*c) + a^3*f + (b^3*g*log((e*x + d)^n*c)^3 + 3*a*b^2*g
*log((e*x + d)^n*c)^2 + 3*a^2*b*g*log((e*x + d)^n*c) + a^3*g)*log((j*x + i)
^m*h))/x^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))**3*(f+g*ln(h*(j*x+i)**m))/x**2,x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 335, normalized size of antiderivative = 9.85

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm=
"maxima")
[Out] -3*a^2*b*e*f*n*(log(e*x + d)/d - log(x)/d) - 3*a^2*b*f*log((e*x + d)^n*c)/x
 - a^3*f/x + integrate(((g*log(h) + f)*b^3*log((e*x + d)^n)^3 + (g*log(h) +
 f)*b^3*log(c)^3 + 3*(g*log(h) + f)*a*b^2*log(c)^2 + 3*a^2*b*g*log(c)*log(h
) + a^3*g*log(h) + 3*((g*log(h) + f)*b^3*log(c) + (g*log(h) + f)*a*b^2)*log
((e*x + d)^n)^2 + 3*((g*log(h) + f)*b^3*log(c)^2 + 2*(g*log(h) + f)*a*b^2*l
og(c) + a^2*b*g*log(h))*log((e*x + d)^n) + (b^3*g*log((e*x + d)^n)^3 + b^3*
g*log(c)^3 + 3*a*b^2*g*log(c)^2 + 3*a^2*b*g*log(c) + a^3*g + 3*(b^3*g*log(c
) + a*b^2*g)*log((e*x + d)^n)^2 + 3*(b^3*g*log(c)^2 + 2*a*b^2*g*log(c) + a^
2*b*g)*log((e*x + d)^n))*log((j*x + i)^m))/x^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx = \int \frac{(b\log((ex+d)^nc)+a)^3(g\log((jx+i)^mh)+f)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))^3*(f+g*log(h*(j*x+i)^m))/x^2,x, algorithm=
"giac")
[Out] integrate((b*log((e*x + d)^n*c) + a)^3*(g*log((j*x + i)^m*h) + f)/x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(c(d+ex)^n))^3(f+g\log(h(i+jx)^m))}{x^2}\,dx$$
$$= \int \frac{(a+b\,\ln(c\,(d+e\,x)^n))^3\,(f+g\,\ln(h\,(i+j\,x)^m))}{x^2}\,dx$$

```
[In] int(((a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)))/x^2,x)
[Out] int(((a + b*log(c*(d + e*x)^n))^3*(f + g*log(h*(i + j*x)^m)))/x^2, x)
```

# 3.401 $\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx$



## Optimal result

Integrand size = 40, antiderivative size = 66

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = -\frac{(a+b\log(c(d+ex)^n))\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)}{e}+\frac{bn\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{e}$$

```
[Out] -(a+b*ln(c*(e*x+d)^n))*polylog(2,-g*(e*x+d)/(-d*g+e*f))/e+b*n*polylog(3,-g*
(e*x+d)/(-d*g+e*f))/e
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2481, 2421, 6724}

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = \frac{bn\operatorname{PolyLog}\left(3,-\frac{g(d+ex)}{ef-dg}\right)}{e}-\frac{\operatorname{PolyLog}\left(2,-\frac{g(d+ex)}{ef-dg}\right)(a+b\log(c(d+ex)^n))}{e}$$

```
[In] Int[((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/(d + e*x),x
]
```

```
[Out] -(((a + b*Log[c*(d + e*x)^n])*PolyLog[2, -((g*(d + e*x))/(e*f - d*g))])/e)
+ (b*n*PolyLog[3, -((g*(d + e*x))/(e*f - d*g))])/e
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\frac{e\left(\frac{ef-dg}{e}+\frac{gx}{e}\right)}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{e}\\ &= -\frac{(a+b\log\left(c(d+ex)^n\right))\,\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{e}+\frac{(bn)\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex\right)}{e}\\ &= -\frac{(a+b\log\left(c(d+ex)^n\right))\,\text{Li}_2\left(-\frac{g(d+ex)}{ef-dg}\right)}{e}+\frac{bn\text{Li}_3\left(-\frac{g(d+ex)}{ef-dg}\right)}{e}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\log\left(c(d+ex)^n\right))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = \frac{-\left((a+b\log\left(c(d+ex)^n\right))\,\text{PolyLog}\left(2,\frac{g(d+ex)}{-ef+dg}\right)\right)+bn\,\text{PolyLog}\left(3,\frac{g(d+ex)}{-ef+dg}\right)}{e}$$

```
[In] Integrate[((a + b*Log[c*(d + e*x)^n])*Log[(e*(f + g*x))/(e*f - d*g)])/(d +
e*x),x]
```

```
[Out] (-((a + b*Log[c*(d + e*x)^n])*PolyLog[2, (g*(d + e*x))/(-(e*f) + d*g)]) + b
*n*PolyLog[3, (g*(d + e*x))/(-(e*f) + d*g)])/e
```

## Maple [F]

$$\int \frac{(a+b\ln(c(ex+d)^n))\ln\left(\frac{e(gx+f)}{-dg+ef}\right)}{ex+d}dx$$

```
[In] int((a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/(e*x+d),x)
```

```
[Out] int((a+b*ln(c*(e*x+d)^n))*ln(e*(g*x+f)/(-d*g+e*f))/(e*x+d),x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = \int \frac{(b\log((ex+d)^nc)+a)\log\left(\frac{(gx+f)e}{ef-dg}\right)}{ex+d}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*log(e*(g*x+f)/(-d*g+e*f))/(e*x+d),x, algor
ithm="fricas")
```

```
[Out] integral((b*log((e*x + d)^n*c)*log((e*g*x + e*f)/(e*f - d*g)) + a*log((e*g*
x + e*f)/(e*f - d*g)))/(e*x + d), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*ln(c*(e*x+d)**n))*ln(e*(g*x+f)/(-d*g+e*f))/(e*x+d),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = \int \frac{(b\log((ex+d)^n c)+a)\log\left(\frac{(gx+f)e}{ef-dg}\right)}{ex+d}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*log(e*(g*x+f)/(-d*g+e*f))/(e*x+d),x, algor
ithm="maxima")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*log((g*x + f)*e/(e*f - d*g))/(e*x + d)
, x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = \int \frac{(b\log((ex+d)^n c)+a)\log\left(\frac{(gx+f)e}{ef-dg}\right)}{ex+d}\,dx$$

```
[In] integrate((a+b*log(c*(e*x+d)^n))*log(e*(g*x+f)/(-d*g+e*f))/(e*x+d),x, algor
ithm="giac")
```

```
[Out] integrate((b*log((e*x + d)^n*c) + a)*log((g*x + f)*e/(e*f - d*g))/(e*x + d)
, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex)^n))\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx = \int \frac{\ln\left(-\frac{e\,(f+g\,x)}{d\,g-e\,f}\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)\right)}{d+e\,x}\,dx$$

```
[In] int((log(-(e*(f + g*x))/(d*g - e*f))*(a + b*log(c*(d + e*x)^n)))/(d + e*x),
x)
```

```
[Out] int((log(-(e*(f + g*x))/(d*g - e*f))*(a + b*log(c*(d + e*x)^n)))/(d + e*x),
 x)
```

# 3.402 $\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{b}{e(d+ex)} - \frac{b\log(c(d+ex))}{e(d+ex)} - \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{e(d+ex)} - \frac{a+b+b\log(c(d+ex))}{e(d+ex)}$$

```
[Out] -b/e/(e*x+d)-b*ln(c*(e*x+d))/e/(e*x+d)-ln(c*(e*x+d))*(a+b*ln(c*(e*x+d)))/e/
(e*x+d)+(-a-b-b*ln(c*(e*x+d)))/e/(e*x+d)
```

## Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2416, 12, 2341, 2413}

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{e(d+ex)} - \frac{a+b\log(c(d+ex))+b}{e(d+ex)} - \frac{b\log(c(d+ex))}{e(d+ex)} - \frac{b}{e(d+ex)}$$

```
[In] Int[(Log[c*(d + e*x)]*(a + b*Log[c*(d + e*x)]))/(d + e*x)^2,x]
```

```
[Out] -(b/(e*(d + e*x))) - (b*Log[c*(d + e*x)])/(e*(d + e*x)) - (Log[c*(d + e*x)]
*(a + b*Log[c*(d + e*x)]))/(e*(d + e*x)) - (a + b + b*Log[c*(d + e*x)])/(e*
(d + e*x))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2416

```
Int[((a_.) + Log[v_]*(b_.))^(p_.)*((c_.) + Log[v_]*(d_.))^(q_.)*(u_)^(m_.),
 x_Symbol] :> With[{e = Coeff[u, x, 0], f = Coeff[u, x, 1], g = Coeff[v, x,
 0], h = Coeff[v, x, 1]}, Dist[1/h, Subst[Int[(f*(x/h))^m*(a + b*Log[x])^p*
(c + d*Log[x])^q, x], x, v], x] /; EqQ[f*g - e*h, 0] && NeQ[g, 0]] /; FreeQ
[{a, b, c, d, m, p, q}, x] && LinearQ[{u, v}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{c^2\log(x)(a+b\log(x))}{x^2}\,dx, x, c(d+ex)\right)}{ce}\\
&= \frac{c\text{Subst}\left(\int \frac{\log(x)(a+b\log(x))}{x^2}\,dx, x, c(d+ex)\right)}{e}\\
&= -\frac{b\log(c(d+ex))}{e(d+ex)} - \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{e(d+ex)}\\
&\quad - \frac{c\text{Subst}\left(\int \frac{-a\left(1+\frac{b}{a}\right)-b\log(x)}{x^2}\,dx, x, c(d+ex)\right)}{e}
\end{aligned}$$

$$= -\frac{b}{e(d+ex)} - \frac{b\log(c(d+ex))}{e(d+ex)} - \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{e(d+ex)} - \frac{a+b+b\log(c(d+ex))}{e(d+ex)}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.47

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{a+2b+(a+2b)\log(c(d+ex))+b\log^2(c(d+ex))}{e(d+ex)}$$

```
[In] Integrate[(Log[c*(d + e*x)]*(a + b*Log[c*(d + e*x)]))/(d + e*x)^2,x]
[Out] -((a + 2*b + (a + 2*b)*Log[c*(d + e*x)] + b*Log[c*(d + e*x)]^2)/(e*(d + e*x
)))
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.59

| method | result | size |
|---|---|---|
| norman | $\frac{-\frac{a+2b}{e}-\frac{b\ln(c(ex+d))^2}{e}-\frac{(a+2b)\ln(c(ex+d))}{e}}{ex+d}$ | 54 |
| parallelrisch | $\frac{-\ln(c(ex+d))^2 b\,e^2-\ln(c(ex+d))a\,e^2-2\ln(c(ex+d))b\,e^2-a\,e^2-2e^2 b}{(ex+d)e^3}$ | 69 |
| risch | $-\frac{b\ln(c(ex+d))^2}{e(ex+d)}-\frac{(a+2b)\ln(c(ex+d))}{e(ex+d)}-\frac{a}{e(ex+d)}-\frac{2b}{e(ex+d)}$ | 76 |
| parts | $\frac{ac\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)}{e}+\frac{bc\left(-\frac{\ln(cex+cd)^2}{cex+cd}-\frac{2\ln(cex+cd)}{cex+cd}-\frac{2}{cex+cd}\right)}{e}$ | 105 |
| derivativedivides | $\frac{c^2 a\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)+c^2 b\left(-\frac{\ln(cex+cd)^2}{cex+cd}-\frac{2\ln(cex+cd)}{cex+cd}-\frac{2}{cex+cd}\right)}{ce}$ | 110 |
| default | $\frac{c^2 a\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)+c^2 b\left(-\frac{\ln(cex+cd)^2}{cex+cd}-\frac{2\ln(cex+cd)}{cex+cd}-\frac{2}{cex+cd}\right)}{ce}$ | 110 |

```
[In] int(ln(c*(e*x+d))*(a+b*ln(c*(e*x+d)))/(e*x+d)^2,x,method=_RETURNVERBOSE)
[Out] (-(a+2*b)/e-b/e*ln(c*(e*x+d))^2-(a+2*b)/e*ln(c*(e*x+d)))/(e*x+d)
```

## Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{b\log\left(cex+cd\right)^2+(a+2\,b)\log\left(cex+cd\right)+a+2\,b}{e^2x+de}$$

```
[In] integrate(log(c*(e*x+d))*(a+b*log(c*(e*x+d)))/(e*x+d)^2,x, algorithm="frica
s")
[Out] -(b*log(c*e*x + c*d)^2 + (a + 2*b)*log(c*e*x + c*d) + a + 2*b)/(e^2*x + d*e
)
```

## Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{b\log\left(c(d+ex)\right)^2}{de+e^2x} + \frac{(-a-2b)\log\left(c(d+ex)\right)}{de+e^2x} - \frac{a+2b}{de+e^2x}$$

```
[In] integrate(ln(c*(e*x+d))*(a+b*ln(c*(e*x+d)))/(e*x+d)**2,x)
[Out] -b*log(c*(d + e*x))**2/(d*e + e**2*x) + (-a - 2*b)*log(c*(d + e*x))/(d*e +
e**2*x) - (a + 2*b)/(d*e + e**2*x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx = -\left(b\left(\frac{ce}{ce^3x+cde^2}+\frac{\log\left(cex+cd\right)}{e^2x+de}\right)+\frac{a}{e^2x+de}\right)\log\left((ex+d)c\right) - \frac{(b(\log\left(c\right)+2)+b\log\left(ex+d\right)+a)e}{e^3x+de^2}$$

```
[In] integrate(log(c*(e*x+d))*(a+b*log(c*(e*x+d)))/(e*x+d)^2,x, algorithm="maxim
a")
[Out] -(b*(c*e/(c*e^3*x + c*d*e^2) + log(c*e*x + c*d)/(e^2*x + d*e)) + a/(e^2*x +
 d*e))*log((e*x + d)*c) - (b*(log(c) + 2) + b*log(e*x + d) + a)*e/(e^3*x +
d*e^2)
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{e\left(\frac{bc\log((ex+d)c)^2}{(ex+d)e^2} + \frac{(ac^2+2\,bc^2)\log((ex+d)c)}{(ex+d)ce^2} + \frac{ac^2+2\,bc^2}{(ex+d)ce^2}\right)}{c}$$

```
[In] integrate(log(c*(e*x+d))*(a+b*log(c*(e*x+d)))/(e*x+d)^2,x, algorithm="giac"
)
```

```
[Out] -e*(b*c*log((e*x + d)*c)^2/((e*x + d)*e^2) + (a*c^2 + 2*b*c^2)*log((e*x + d
)*c)/((e*x + d)*c*e^2) + (a*c^2 + 2*b*c^2)/((e*x + d)*c*e^2))/c
```

## Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int \frac{\log(c(d+ex))(a+b\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{d\left(b\ln\left(c\left(d+e\,x\right)\right)^2 + a\,\ln\left(c\left(d+e\,x\right)\right) + 2\,b\,\ln\left(c\left(d+e\,x\right)\right)\right) - e\left(a\,x+2\,b\,x\right)}{d\,e\,\left(d+e\,x\right)}$$

```
[In] int((log(c*(d + e*x))*(a + b*log(c*(d + e*x))))/(d + e*x)^2,x)
```

```
[Out] -(d*(b*log(c*(d + e*x))^2 + a*log(c*(d + e*x)) + 2*b*log(c*(d + e*x))) - e*
(a*x + 2*b*x))/(d*e*(d + e*x))
```

# 3.403 $\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2} dx$



## Optimal result

Integrand size = 32, antiderivative size = 102

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2} dx = -\frac{bg}{e(d+ex)} - \frac{g(a+b+b\log(c(d+ex)))}{e(d+ex)} - \frac{b(f+g\log(c(d+ex)))}{e(d+ex)} - \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{e(d+ex)}$$

```
[Out] -b*g/e/(e*x+d)-g*(a+b+b*ln(c*(e*x+d)))/e/(e*x+d)-b*(f+g*ln(c*(e*x+d)))/e/(e
*x+d)-(a+b*ln(c*(e*x+d)))*(f+g*ln(c*(e*x+d)))/e/(e*x+d)
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2416, 12, 2341, 2413}

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2} dx = -\frac{(a+b\log(c(d+ex)))(g\log(c(d+ex))+f)}{e(d+ex)} - \frac{g(a+b\log(c(d+ex))+b)}{e(d+ex)} - \frac{b(g\log(c(d+ex))+f)}{e(d+ex)} - \frac{bg}{e(d+ex)}$$

```
[In] Int[((a + b*Log[c*(d + e*x)])*(f + g*Log[c*(d + e*x)]))/(d + e*x)^2,x]
```

```
[Out] -((b*g)/(e*(d + e*x))) - (g*(a + b + b*Log[c*(d + e*x)]))/(e*(d + e*x)) - (
b*(f + g*Log[c*(d + e*x)]))/(e*(d + e*x)) - ((a + b*Log[c*(d + e*x)])*(f +
g*Log[c*(d + e*x)]))/(e*(d + e*x))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2416

```
Int[((a_.) + Log[v_]*(b_.))^(p_.)*((c_.) + Log[v_]*(d_.))^(q_.)*(u_)^(m_.),
 x_Symbol] :> With[{e = Coeff[u, x, 0], f = Coeff[u, x, 1], g = Coeff[v, x,
 0], h = Coeff[v, x, 1]}, Dist[1/h, Subst[Int[(f*(x/h))^m*(a + b*Log[x])^p*
(c + d*Log[x])^q, x], x, v], x] /; EqQ[f*g - e*h, 0] && NeQ[g, 0]] /; FreeQ
[{a, b, c, d, m, p, q}, x] && LinearQ[{u, v}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{c^2(a+b\log(x))(f+g\log(x))}{x^2}\,dx, x, c(d+ex)\right)}{ce}\\
&= \frac{c\text{Subst}\left(\int \frac{(a+b\log(x))(f+g\log(x))}{x^2}\,dx, x, c(d+ex)\right)}{e}\\
&= -\frac{b(f+g\log(c(d+ex)))}{e(d+ex)} - \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{e(d+ex)}\\
&\quad - \frac{(cg)\text{Subst}\left(\int \frac{-a\left(1+\frac{b}{a}\right)-b\log(x)}{x^2}\,dx, x, c(d+ex)\right)}{e}
\end{aligned}$$

$$= -\frac{bg}{e(d+ex)} - \frac{g(a+b+b\log(c(d+ex)))}{e(d+ex)} - \frac{b(f+g\log(c(d+ex)))}{e(d+ex)} - \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{e(d+ex)}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{a(f+g)+b(f+2g)+(ag+b(f+2g))\log(c(d+ex))+bg\log^2(c(d+ex))}{e(d+ex)}$$

```
[In] Integrate[((a + b*Log[c*(d + e*x)])*(f + g*Log[c*(d + e*x)]))/(d + e*x)^2,x
]
```

```
[Out] -((a*(f + g) + b*(f + 2*g) + (a*g + b*(f + 2*g))*Log[c*(d + e*x)] + b*g*Log
[c*(d + e*x)]^2)/(e*(d + e*x)))
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| norman | $\frac{-\frac{af+ag+bf+2bg}{e}-\frac{(ag+bf+2bg)\ln(c(ex+d))}{e}-\frac{bg\ln(c(ex+d))^2}{e}}{ex+d}$ |
| parallelrisch | $\frac{-\ln(c(ex+d))^2 b e^2 g-\ln(c(ex+d)) a e^2 g-\ln(c(ex+d)) b e^2 f-2\ln(c(ex+d)) b e^2 g-a e^2 f-a e^2 g-b e^2 f-2b e^2 g}{(ex+d)e^3}$ |
| risch | $-\frac{bg\ln(c(ex+d))^2}{e(ex+d)}-\frac{(ag+bf+2bg)\ln(c(ex+d))}{e(ex+d)}-\frac{af}{e(ex+d)}-\frac{ag}{e(ex+d)}-\frac{bf}{e(ex+d)}-\frac{2bg}{e(ex+d)}$ |
| parts | $-\frac{af}{e(ex+d)}+\frac{(ag+bf)c\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)}{e}+\frac{bgc\left(-\frac{\ln(cex+cd)^2}{cex+cd}-\frac{2\ln(cex+cd)}{cex+cd}-\frac{2}{cex+cd}\right)}{e}$ |
| derivativedivides | $\frac{-\frac{c^2af}{cex+cd}+c^2ag\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)+c^2bf\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)+c^2bg\left(-\frac{\ln(cex+cd)^2}{cex+cd}-\frac{2\ln(cex+cd)}{cex+cd}-\frac{2}{cex+cd}\right)}{ce}$ |
| default | $\frac{-\frac{c^2af}{cex+cd}+c^2ag\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)+c^2bf\left(-\frac{\ln(cex+cd)}{cex+cd}-\frac{1}{cex+cd}\right)+c^2bg\left(-\frac{\ln(cex+cd)^2}{cex+cd}-\frac{2\ln(cex+cd)}{cex+cd}-\frac{2}{cex+cd}\right)}{ce}$ |

```
[In] int((a+b*ln(c*(e*x+d)))*(f+g*ln(c*(e*x+d)))/(e*x+d)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] (-(a*f+a*g+b*f+2*b*g)/e-(a*g+b*f+2*b*g)/e*ln(c*(e*x+d))-b*g/e*ln(c*(e*x+d))
^2)/(e*x+d)
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{bg\log\left(cex+cd\right)^2+(a+b)f+(a+2\,b)g+(bf+(a+2\,b)g)\log\left(cex+cd\right)}{e^2x+de}$$

```
[In] integrate((a+b*log(c*(e*x+d)))*(f+g*log(c*(e*x+d)))/(e*x+d)^2,x, algorithm=
"fricas")
```

```
[Out] -(b*g*log(c*e*x + c*d)^2 + (a + b)*f + (a + 2*b)*g + (b*f + (a + 2*b)*g)*lo
g(c*e*x + c*d))/(e^2*x + d*e)
```

## Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{bg\log\left(c(d+ex)\right)^2}{de+e^2x}+\frac{(-ag-bf-2bg)\log\left(c(d+ex)\right)}{de+e^2x}-\frac{af+ag+bf+2bg}{de+e^2x}$$

```
[In] integrate((a+b*ln(c*(e*x+d)))*(f+g*ln(c*(e*x+d)))/(e*x+d)**2,x)
```

```
[Out] -b*g*log(c*(d + e*x))**2/(d*e + e**2*x) + (-a*g - b*f - 2*b*g)*log(c*(d + e
*x))/(d*e + e**2*x) - (a*f + a*g + b*f + 2*b*g)/(d*e + e**2*x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.56

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx = -b\left(\frac{ce}{ce^3x+cde^2}+\frac{\log\left(cex+cd\right)}{e^2x+de}\right)f-a\left(\frac{ce}{ce^3x+cde^2}+\frac{\log\left(cex+cd\right)}{e^2x+de}\right)g - \frac{af}{e^2x+de}-\frac{\left(c^2\log\left(cex+cd\right)^2+2\,c^2\log\left(cex+cd\right)+2\,c^2\right)bg}{(cex+cd)ce}$$

```
[In] integrate((a+b*log(c*(e*x+d)))*(f+g*log(c*(e*x+d)))/(e*x+d)^2,x, algorithm=
"maxima")
```

```
[Out] -b*(c*e/(c*e^3*x + c*d*e^2) + log(c*e*x + c*d)/(e^2*x + d*e))*f - a*(c*e/(c
*e^3*x + c*d*e^2) + log(c*e*x + c*d)/(e^2*x + d*e))*g - a*f/(e^2*x + d*e) -
 (c^2*log(c*e*x + c*d)^2 + 2*c^2*log(c*e*x + c*d) + 2*c^2)*b*g/((c*e*x + c*
d)*c*e)
```

## Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{\left(\frac{bcg\log((ex+d)c)^2}{(ex+d)e^2}+\frac{(bc^2f+ac^2g+2\,bc^2g)\log((ex+d)c)}{(ex+d)ce^2}+\frac{ac^2f+bc^2f+ac^2g+2\,bc^2g}{(ex+d)ce^2}\right)e}{c}$$

```
[In] integrate((a+b*log(c*(e*x+d)))*(f+g*log(c*(e*x+d)))/(e*x+d)^2,x, algorithm=
"giac")
```

```
[Out] -(b*c*g*log((e*x + d)*c)^2/((e*x + d)*e^2) + (b*c^2*f + a*c^2*g + 2*b*c^2*g
)*log((e*x + d)*c)/((e*x + d)*c*e^2) + (a*c^2*f + b*c^2*f + a*c^2*g + 2*b*c
^2*g)/((e*x + d)*c*e^2))*e/c
```

## Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\log(c(d+ex)))(f+g\log(c(d+ex)))}{(d+ex)^2}\,dx = -\frac{d\left(b\,g\ln\left(c\,d+c\,e\,x\right)^2+a\,g\,\ln\left(c\,d+c\,e\,x\right)+b\,f\,\ln\left(c\,d+c\,e\,x\right)+2\,b\,g\,\ln\left(c\,d+c\,e\,x\right)\right)-e\left(a\,f\,x+a\,g\,x\right.}{d^2\,e+x\,d\,e^2}$$

```
[In] int(((a + b*log(c*(d + e*x)))*(f + g*log(c*(d + e*x))))/(d + e*x)^2,x)
```

```
[Out] -(d*(b*g*log(c*d + c*e*x)^2 + a*g*log(c*d + c*e*x) + b*f*log(c*d + c*e*x) +
 2*b*g*log(c*d + c*e*x)) - e*(a*f*x + a*g*x + b*f*x + 2*b*g*x))/(d^2*e + d*
e^2*x)
```

# 3.404 $\int (a+b\log(c(d(e+fx)^m)^n))^4\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 160

$$\int (a+b\log(c(d(e+fx)^m)^n))^4\,dx = -24ab^3m^3n^3x+24b^4m^4n^4x - \frac{24b^4m^3n^3(e+fx)\log(c(d(e+fx)^m)^n)}{f} + \frac{12b^2m^2n^2(e+fx)(a+b\log(c(d(e+fx)^m)^n))^2}{f} - \frac{4bmn(e+fx)(a+b\log(c(d(e+fx)^m)^n))^3}{f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^4}{f}$$

```
[Out] -24*a*b^3*m^3*n^3*x+24*b^4*m^4*n^4*x-24*b^4*m^3*n^3*(f*x+e)*ln(c*(d*(f*x+e)
^m)^n)/f+12*b^2*m^2*n^2*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^2/f-4*b*m*n*(f*
x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^3/f+(f*x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^4/
f
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{2436, 2333, 2332, 2495\}$

$$\int (a + b\log(c(d(e+fx)^m)^n))^4\,dx = -24ab^3m^3n^3x + \frac{12b^2m^2n^2(e+fx)(a+b\log(c(d(e+fx)^m)^n))^2}{f} - \frac{4bmn(e+fx)(a+b\log(c(d(e+fx)^m)^n))^3}{f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^4}{f} - \frac{24b^4m^3n^3(e+fx)\log(c(d(e+fx)^m)^n)}{f} + 24b^4m^4n^4x$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^4,x]
```

```
[Out] -24*a*b^3*m^3*n^3*x + 24*b^4*m^4*n^4*x - (24*b^4*m^3*n^3*(e + f*x)*Log[c*(d
*(e + f*x)^m)^n])/f + (12*b^2*m^2*n^2*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^m
)^n])^2)/f - (4*b*m*n*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^3)/f + ((e
 + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^4)/f
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (a+b\log(cd^n(e+fx)^{mn}))^4\,dx, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int (a+b\log(cd^n x^{mn}))^4\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^4}{f} - \text{Subst}\left(\frac{(4bmn)\text{Subst}\left(\int (a+b\log(cd^n x^{mn}))^3\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{4bmn(e+fx)(a+b\log(c(d(e+fx)^m)^n))^3}{f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^4}{f} + \text{Subst}\left(\frac{(12b^2m^2n^2)\,\text{Subst}\left(\int (a+b\log(cd^n x^{mn}))^2\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \frac{12b^2m^2n^2(e+fx)(a+b\log(c(d(e+fx)^m)^n))^2}{f} - \frac{4bmn(e+fx)(a+b\log(c(d(e+fx)^m)^n))^3}{f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^4}{f} - \text{Subst}\left(\frac{(24b^3m^3n^3)\,\text{Subst}\left(\int (a+b\log(cd^n x^{mn}))\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -24ab^3m^3n^3x + \frac{12b^2m^2n^2(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^2}{f} - \frac{4bmn(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^3}{f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^4}{f} - \text{Subst}\left(\frac{(24b^4m^3n^3)\,\text{Subst}\left(\int \log\left(cd^nx^{mn}\right)\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -24ab^3m^3n^3x + 24b^4m^4n^4x - \frac{24b^4m^3n^3(e+fx)\log\left(c(d(e+fx)^m)^n\right)}{f} + \frac{12b^2m^2n^2(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^2}{f} - \frac{4bmn(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^3}{f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^4}{f}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int \left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^4\,dx = \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^4 - 4bmn\Big((e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^3 - 3bmn\Big((e+fx)}{f}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^4,x]
```

```
[Out] ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^4 - 4*b*m*n*((e + f*x)*(a + b*L
og[c*(d*(e + f*x)^m)^n])^3 - 3*b*m*n*((e + f*x)*(a + b*Log[c*(d*(e + f*x)^m
)^n])^2 - 2*b*m*n*(f*(a - b*m*n)*x + b*(e + f*x)*Log[c*(d*(e + f*x)^m)^n]))
))/f
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(160) = 320.

Time = 5.58 (sec) , antiderivative size = 642, normalized size of antiderivative = 4.01

| method | result |
|---|---|
| parallelrisch | $\frac{-12x\ln(c(d(fx+e)^m)^n)^2 a b^3 e f m n-12x\ln(c(d(fx+e)^m)^n)a^2b^2 e f m n+24x\ln(c(d(fx+e)^m)^n)a b^3 e f m^2n^2+4\ln(c(d(fx+e)^m)^n}{}$ |

```
[In] int((a+b*ln(c*(d*(f*x+e)^m)^n))^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-12*x*ln(c*(d*(f*x+e)^m)^n)^2*a*b^3*e*f*m*n-12*x*ln(c*(d*(f*x+e)^m)^n)*a^2
*b^2*e*f*m*n+24*x*ln(c*(d*(f*x+e)^m)^n)*a*b^3*e*f*m^2*n^2+4*ln(c*(d*(f*x+e)
^m)^n)^3*a*b^3*e^2+6*ln(c*(d*(f*x+e)^m)^n)^2*a^2*b^2*e^2+x*a^4*e*f-24*x*ln(
c*(d*(f*x+e)^m)^n)*b^4*e*f*m^3*n^3+12*x*ln(c*(d*(f*x+e)^m)^n)^2*b^4*e*f*m^2
*n^2+12*ln(c*(d*(f*x+e)^m)^n)^2*b^4*e^2*m^2*n^2-4*ln(c*(d*(f*x+e)^m)^n)^3*b
^4*e^2*m*n-24*ln(f*x+e)*b^4*e^2*m^4*n^4+x*ln(c*(d*(f*x+e)^m)^n)^4*b^4*e*f+2
4*x*b^4*e*f*m^4*n^4-12*ln(c*(d*(f*x+e)^m)^n)^2*a*b^3*e^2*m*n+24*ln(f*x+e)*a
*b^3*e^2*m^3*n^3-12*ln(f*x+e)*a^2*b^2*e^2*m^2*n^2-24*x*a*b^3*e*f*m^3*n^3-4*
x*ln(c*(d*(f*x+e)^m)^n)^3*b^4*e*f*m*n+12*x*a^2*b^2*e*f*m^2*n^2-4*x*a^3*b*e*
f*m*n+24*a*b^3*e^2*m^3*n^3-12*a^2*b^2*e^2*m^2*n^2+4*a^3*b*e^2*m*n+4*ln(f*x+
e)*a^3*b*e^2*m*n+4*x*ln(c*(d*(f*x+e)^m)^n)*a^3*b*e*f+4*x*ln(c*(d*(f*x+e)^m)
^n)^3*a*b^3*e*f+6*x*ln(c*(d*(f*x+e)^m)^n)^2*a^2*b^2*e*f-a^4*e^2-24*b^4*e^2*
m^4*n^4+ln(c*(d*(f*x+e)^m)^n)^4*b^4*e^2)/e/f
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1409 vs. 2(160) = 320.

Time = 0.32 (sec) , antiderivative size = 1409, normalized size of antiderivative = 8.81

$$\int (a+b\log(c(d(e+fx)^m)^n))^4\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^4,x, algorithm="fricas")
```

```
[Out] (b^4*f*n^4*x*log(d)^4 + b^4*f*x*log(c)^4 + (b^4*f*m^4*n^4*x + b^4*e*m^4*n^4
)*log(f*x + e)^4 - 4*(b^4*f*m*n - a*b^3*f)*x*log(c)^3 - 4*(b^4*e*m^4*n^4 -
a*b^3*e*m^3*n^3 + (b^4*f*m^4*n^4 - a*b^3*f*m^3*n^3)*x - (b^4*f*m^3*n^3*x +
b^4*e*m^3*n^3)*log(c) - (b^4*f*m^3*n^4*x + b^4*e*m^3*n^4)*log(d))*log(f*x +
 e)^3 + 6*(2*b^4*f*m^2*n^2 - 2*a*b^3*f*m*n + a^2*b^2*f)*x*log(c)^2 + 4*(b^4
*f*n^3*x*log(c) - (b^4*f*m*n^4 - a*b^3*f*n^3)*x)*log(d)^3 + 6*(2*b^4*e*m^4*
n^4 - 2*a*b^3*e*m^3*n^3 + a^2*b^2*e*m^2*n^2 + (b^4*f*m^2*n^2*x + b^4*e*m^2*
n^2)*log(c)^2 + (b^4*f*m^2*n^4*x + b^4*e*m^2*n^4)*log(d)^2 + (2*b^4*f*m^4*n
^4 - 2*a*b^3*f*m^3*n^3 + a^2*b^2*f*m^2*n^2)*x - 2*(b^4*e*m^3*n^3 - a*b^3*e*
m^2*n^2 + (b^4*f*m^3*n^3 - a*b^3*f*m^2*n^2)*x)*log(c) - 2*(b^4*e*m^3*n^4 -
```

```
a*b^3*e*m^2*n^3 + (b^4*f*m^3*n^4 - a*b^3*f*m^2*n^3)*x - (b^4*f*m^2*n^3*x +
b^4*e*m^2*n^3)*log(c))*log(d))*log(f*x + e)^2 - 4*(6*b^4*f*m^3*n^3 - 6*a*b^
3*f*m^2*n^2 + 3*a^2*b^2*f*m*n - a^3*b*f)*x*log(c) + 6*(b^4*f*n^2*x*log(c)^2
 - 2*(b^4*f*m*n^3 - a*b^3*f*n^2)*x*log(c) + (2*b^4*f*m^2*n^4 - 2*a*b^3*f*m*
n^3 + a^2*b^2*f*n^2)*x)*log(d)^2 + (24*b^4*f*m^4*n^4 - 24*a*b^3*f*m^3*n^3 +
 12*a^2*b^2*f*m^2*n^2 - 4*a^3*b*f*m*n + a^4*f)*x - 4*(6*b^4*e*m^4*n^4 - 6*a
*b^3*e*m^3*n^3 + 3*a^2*b^2*e*m^2*n^2 - a^3*b*e*m*n - (b^4*f*m*n*x + b^4*e*m
*n)*log(c)^3 - (b^4*f*m*n^4*x + b^4*e*m*n^4)*log(d)^3 + 3*(b^4*e*m^2*n^2 -
a*b^3*e*m*n + (b^4*f*m^2*n^2 - a*b^3*f*m*n)*x)*log(c)^2 + 3*(b^4*e*m^2*n^4
- a*b^3*e*m*n^3 + (b^4*f*m^2*n^4 - a*b^3*f*m*n^3)*x - (b^4*f*m*n^3*x + b^4*
e*m*n^3)*log(c))*log(d)^2 + (6*b^4*f*m^4*n^4 - 6*a*b^3*f*m^3*n^3 + 3*a^2*b^
2*f*m^2*n^2 - a^3*b*f*m*n)*x - 3*(2*b^4*e*m^3*n^3 - 2*a*b^3*e*m^2*n^2 + a^2
*b^2*e*m*n + (2*b^4*f*m^3*n^3 - 2*a*b^3*f*m^2*n^2 + a^2*b^2*f*m*n)*x)*log(c
) - 3*(2*b^4*e*m^3*n^4 - 2*a*b^3*e*m^2*n^3 + a^2*b^2*e*m*n^2 + (b^4*f*m*n^2
*x + b^4*e*m*n^2)*log(c)^2 + (2*b^4*f*m^3*n^4 - 2*a*b^3*f*m^2*n^3 + a^2*b^2
*f*m*n^2)*x - 2*(b^4*e*m^2*n^3 - a*b^3*e*m*n^2 + (b^4*f*m^2*n^3 - a*b^3*f*m
*n^2)*x)*log(c))*log(d))*log(f*x + e) + 4*(b^4*f*n*x*log(c)^3 - 3*(b^4*f*m*
n^2 - a*b^3*f*n)*x*log(c)^2 + 3*(2*b^4*f*m^2*n^3 - 2*a*b^3*f*m*n^2 + a^2*b^
2*f*n)*x*log(c) - (6*b^4*f*m^3*n^4 - 6*a*b^3*f*m^2*n^3 + 3*a^2*b^2*f*m*n^2
- a^3*b*f*n)*x)*log(d))/f
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(155) = 310$.

Time = 2.56 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.81

$$\int (a+b\log(c(d(e+fx)^m)^n))^4\,dx = \begin{cases} a^4x + \frac{4a^3be\log(c(d(e+fx)^m)^n)}{f} - 4a^3bmnx + 4a^3bx\log(c(d(e+fx)^m)^n) - \frac{12a^2b^2emn\log(c(d(e+fx)^m)^n)}{f} + \frac{6a^2b^2e\log \\ x(a+b\log(c(de^m)^n))^4 \end{cases}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*e*log(c*(d*(e + f*x)**m)**n)/f - 4*a**3*b*m*n*
x + 4*a**3*b*x*log(c*(d*(e + f*x)**m)**n) - 12*a**2*b**2*e*m*n*log(c*(d*(e
+ f*x)**m)**n)/f + 6*a**2*b**2*e*log(c*(d*(e + f*x)**m)**n)**2/f + 12*a**2*
b**2*m**2*n**2*x - 12*a**2*b**2*m*n*x*log(c*(d*(e + f*x)**m)**n) + 6*a**2*b
**2*x*log(c*(d*(e + f*x)**m)**n)**2 + 24*a*b**3*e*m**2*n**2*log(c*(d*(e + f
*x)**m)**n)/f - 12*a*b**3*e*m*n*log(c*(d*(e + f*x)**m)**n)**2/f + 4*a*b**3*
e*log(c*(d*(e + f*x)**m)**n)**3/f - 24*a*b**3*m**3*n**3*x + 24*a*b**3*m**2*
n**2*x*log(c*(d*(e + f*x)**m)**n) - 12*a*b**3*m*n*x*log(c*(d*(e + f*x)**m)*
*n)**2 + 4*a*b**3*x*log(c*(d*(e + f*x)**m)**n)**3 - 24*b**4*e*m**3*n**3*log
(c*(d*(e + f*x)**m)**n)/f + 12*b**4*e*m**2*n**2*log(c*(d*(e + f*x)**m)**n)*
```

```
*2/f - 4*b**4*e*m*n*log(c*(d*(e + f*x)**m)**n)**3/f + b**4*e*log(c*(d*(e +
f*x)**m)**n)**4/f + 24*b**4*m**4*n**4*x - 24*b**4*m**3*n**3*x*log(c*(d*(e +
 f*x)**m)**n) + 12*b**4*m**2*n**2*x*log(c*(d*(e + f*x)**m)**n)**2 - 4*b**4*
m*n*x*log(c*(d*(e + f*x)**m)**n)**3 + b**4*x*log(c*(d*(e + f*x)**m)**n)**4,
 Ne(f, 0)), (x*(a + b*log(c*(d*e**m)**n))**4, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(160) = 320$.

Time = 0.23 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.49

$$\int (a+b\log(c(d(e+fx)^m)^n))^4\,dx = b^4x\log\left(\left(\left((fx+e)^m d\right)^n c\right)^4\right.$$
$$-4a^3bfmn\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)+4ab^3x\log\left(\left((fx+e)^m d\right)^n c\right)^3$$
$$+6a^2b^2x\log\left(\left((fx+e)^m d\right)^n c\right)^2+4a^3bx\log\left(\left((fx+e)^m d\right)^n c\right)$$
$$-6\left(2fmn\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)\log\left(\left((fx+e)^m d\right)^n c\right)+\frac{\left(e\log(fx+e)^2-2fx+2e\log(fx+e)\right)m^2n^2}{f}\right.$$
$$-4\left(3fmn\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)\log\left(\left((fx+e)^m d\right)^n c\right)^2-\left(\frac{\left(e\log(fx+e)^3+3e\log(fx+e)^2-6fx+\right.}{f^2}\right.\right.$$
$$-\left(4fmn\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)\log\left(\left((fx+e)^m d\right)^n c\right)^3+\left(\left(\frac{\left(e\log(fx+e)^4+4e\log(fx+e)^3+12e\log\right.}{f}\right.\right.\right.$$
$$+a^4x$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(((f*x + e)^m*d)^n*c)^4 - 4*a^3*b*f*m*n*(x/f - e*log(f*x + e)/f^2)
 + 4*a*b^3*x*log(((f*x + e)^m*d)^n*c)^3 + 6*a^2*b^2*x*log(((f*x + e)^m*d)^n
*c)^2 + 4*a^3*b*x*log(((f*x + e)^m*d)^n*c) - 6*(2*f*m*n*(x/f - e*log(f*x +
e)/f^2)*log(((f*x + e)^m*d)^n*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x
+ e))*m^2*n^2/f)*a^2*b^2 - 4*(3*f*m*n*(x/f - e*log(f*x + e)/f^2)*log(((f*x
+ e)^m*d)^n*c)^2 - ((e*log(f*x + e)^3 + 3*e*log(f*x + e)^2 - 6*f*x + 6*e*lo
g(f*x + e))*m^2*n^2/f^2 - 3*(e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*m
*n*log(((f*x + e)^m*d)^n*c)/f^2)*f*m*n)*a*b^3 - (4*f*m*n*(x/f - e*log(f*x +
 e)/f^2)*log(((f*x + e)^m*d)^n*c)^3 + (((e*log(f*x + e)^4 + 4*e*log(f*x + e
)^3 + 12*e*log(f*x + e)^2 - 24*f*x + 24*e*log(f*x + e))*m^2*n^2/f^3 - 4*(e*
log(f*x + e)^3 + 3*e*log(f*x + e)^2 - 6*f*x + 6*e*log(f*x + e))*m*n*log(((f
*x + e)^m*d)^n*c)/f^3)*f*m*n + 6*(e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x +
e))*m*n*log(((f*x + e)^m*d)^n*c)^2/f^2)*f*m*n)*b^4 + a^4*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1697 vs. 2(160) = 320.

Time = 0.35 (sec) , antiderivative size = 1697, normalized size of antiderivative = 10.61

$$\int (a + b\log (c(d(e + fx)^m)^n))^4 \, dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^4,x, algorithm="giac")
```

```
[Out] (f*x + e)*b^4*m^4*n^4*log(f*x + e)^4/f - 4*(f*x + e)*b^4*m^4*n^4*log(f*x +
e)^3/f + 4*(f*x + e)*b^4*m^3*n^4*log(f*x + e)^3*log(d)/f + 12*(f*x + e)*b^4
*m^4*n^4*log(f*x + e)^2/f + 4*(f*x + e)*b^4*m^3*n^3*log(f*x + e)^3*log(c)/f
 - 12*(f*x + e)*b^4*m^3*n^4*log(f*x + e)^2*log(d)/f + 6*(f*x + e)*b^4*m^2*n
^4*log(f*x + e)^2*log(d)^2/f - 24*(f*x + e)*b^4*m^4*n^4*log(f*x + e)/f + 4*
(f*x + e)*a*b^3*m^3*n^3*log(f*x + e)^3/f - 12*(f*x + e)*b^4*m^3*n^3*log(f*x
 + e)^2*log(c)/f + 24*(f*x + e)*b^4*m^3*n^4*log(f*x + e)*log(d)/f + 12*(f*x
 + e)*b^4*m^2*n^3*log(f*x + e)^2*log(c)*log(d)/f - 12*(f*x + e)*b^4*m^2*n^4
*log(f*x + e)*log(d)^2/f + 4*(f*x + e)*b^4*m*n^4*log(f*x + e)*log(d)^3/f +
24*(f*x + e)*b^4*m^4*n^4/f - 12*(f*x + e)*a*b^3*m^3*n^3*log(f*x + e)^2/f +
24*(f*x + e)*b^4*m^3*n^3*log(f*x + e)*log(c)/f + 6*(f*x + e)*b^4*m^2*n^2*lo
g(f*x + e)^2*log(c)^2/f - 24*(f*x + e)*b^4*m^3*n^4*log(d)/f + 12*(f*x + e)*
a*b^3*m^2*n^3*log(f*x + e)^2*log(d)/f - 24*(f*x + e)*b^4*m^2*n^3*log(f*x +
e)*log(c)*log(d)/f + 12*(f*x + e)*b^4*m^2*n^4*log(d)^2/f + 12*(f*x + e)*b^4
*m*n^3*log(f*x + e)*log(c)*log(d)^2/f - 4*(f*x + e)*b^4*m*n^4*log(d)^3/f +
(f*x + e)*b^4*n^4*log(d)^4/f + 24*(f*x + e)*a*b^3*m^3*n^3*log(f*x + e)/f -
24*(f*x + e)*b^4*m^3*n^3*log(c)/f + 12*(f*x + e)*a*b^3*m^2*n^2*log(f*x + e)
^2*log(c)/f - 12*(f*x + e)*b^4*m^2*n^2*log(f*x + e)*log(c)^2/f - 24*(f*x +
e)*a*b^3*m^2*n^3*log(f*x + e)*log(d)/f + 24*(f*x + e)*b^4*m^2*n^3*log(c)*lo
g(d)/f + 12*(f*x + e)*b^4*m*n^2*log(f*x + e)*log(c)^2*log(d)/f + 12*(f*x +
e)*a*b^3*m*n^3*log(f*x + e)*log(d)^2/f - 12*(f*x + e)*b^4*m*n^3*log(c)*log(
d)^2/f + 4*(f*x + e)*b^4*n^3*log(c)*log(d)^3/f - 24*(f*x + e)*a*b^3*m^3*n^3
/f + 6*(f*x + e)*a^2*b^2*m^2*n^2*log(f*x + e)^2/f - 24*(f*x + e)*a*b^3*m^2*
n^2*log(f*x + e)*log(c)/f + 12*(f*x + e)*b^4*m^2*n^2*log(c)^2/f + 4*(f*x +
e)*b^4*m*n*log(f*x + e)*log(c)^3/f + 24*(f*x + e)*a*b^3*m^2*n^3*log(d)/f +
24*(f*x + e)*a*b^3*m*n^2*log(f*x + e)*log(c)*log(d)/f - 12*(f*x + e)*b^4*m*
n^2*log(c)^2*log(d)/f - 12*(f*x + e)*a*b^3*m*n^3*log(d)^2/f + 6*(f*x + e)*b
^4*n^2*log(c)^2*log(d)^2/f + 4*(f*x + e)*a*b^3*n^3*log(d)^3/f - 12*(f*x + e
)*a^2*b^2*m^2*n^2*log(f*x + e)/f + 24*(f*x + e)*a*b^3*m^2*n^2*log(c)/f + 12
*(f*x + e)*a*b^3*m*n*log(f*x + e)*log(c)^2/f - 4*(f*x + e)*b^4*m*n*log(c)^3
/f + 12*(f*x + e)*a^2*b^2*m*n^2*log(f*x + e)*log(d)/f - 24*(f*x + e)*a*b^3*
m*n^2*log(c)*log(d)/f + 4*(f*x + e)*b^4*n*log(c)^3*log(d)/f + 12*(f*x + e)*
a*b^3*n^2*log(c)*log(d)^2/f + 12*(f*x + e)*a^2*b^2*m^2*n^2/f + 12*(f*x + e)
*a^2*b^2*m*n*log(f*x + e)*log(c)/f - 12*(f*x + e)*a*b^3*m*n*log(c)^2/f + (f
*x + e)*b^4*log(c)^4/f - 12*(f*x + e)*a^2*b^2*m*n^2*log(d)/f + 12*(f*x + e)
```

```
*a*b^3*n*log(c)^2*log(d)/f + 6*(f*x + e)*a^2*b^2*n^2*log(d)^2/f + 4*(f*x +
e)*a^3*b*m*n*log(f*x + e)/f - 12*(f*x + e)*a^2*b^2*m*n*log(c)/f + 4*(f*x +
e)*a*b^3*log(c)^3/f + 12*(f*x + e)*a^2*b^2*n*log(c)*log(d)/f - 4*(f*x + e)*
a^3*b*m*n/f + 6*(f*x + e)*a^2*b^2*log(c)^2/f + 4*(f*x + e)*a^3*b*n*log(d)/f
 + 4*(f*x + e)*a^3*b*log(c)/f + (f*x + e)*a^4/f
```

## Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.38

$$\int (a+b\log(c(d(e+fx)^m)^n))^4\,dx$$
$$= \ln(c(d(e+fx)^m)^n)^3\left(\frac{4(ab^3e-b^4emn)}{f}+4b^3x(a-bmn)\right)$$
$$+\ln(c(d(e+fx)^m)^n)^4\left(b^4x+\frac{b^4e}{f}\right)$$
$$+x\left(a^4-4a^3bmn+12a^2b^2m^2n^2-24ab^3m^3n^3+24b^4m^4n^4\right)$$
$$+\ln(c(d(e+fx)^m)^n)^2\left(\frac{6(ea^2b^2-2eab^3mn+2eb^4m^2n^2)}{f}+6b^2x\left(a^2-2abmn+2b^2m^2n^2\right)\right)$$
$$-\frac{\ln(e+fx)\left(-4ea^3bmn+12ea^2b^2m^2n^2-24eab^3m^3n^3+24eb^4m^4n^4\right)}{f}$$
$$+\frac{\ln(c(d(e+fx)^m)^n)\left(4bf\left(a^3-3a^2bmn+6ab^2m^2n^2-6b^3m^3n^3\right)x^2+4be\left(a^3-3a^2bmn+6ab^2\right.\right.}{e+fx}$$

```
[In] int((a + b*log(c*(d*(e + f*x)^m)^n))^4,x)
```

```
[Out] log(c*(d*(e + f*x)^m)^n)^3*((4*(a*b^3*e - b^4*e*m*n))/f + 4*b^3*x*(a - b*m*
n)) + log(c*(d*(e + f*x)^m)^n)^4*(b^4*x + (b^4*e)/f) + x*(a^4 + 24*b^4*m^4*
n^4 - 24*a*b^3*m^3*n^3 - 4*a^3*b*m*n + 12*a^2*b^2*m^2*n^2) + log(c*(d*(e +
f*x)^m)^n)^2*((6*(a^2*b^2*e + 2*b^4*e*m^2*n^2 - 2*a*b^3*e*m*n))/f + 6*b^2*x
*(a^2 + 2*b^2*m^2*n^2 - 2*a*b*m*n)) - (log(e + f*x)*(24*b^4*e*m^4*n^4 - 24*
a*b^3*e*m^3*n^3 - 4*a^3*b*e*m*n + 12*a^2*b^2*e*m^2*n^2))/f + (log(c*(d*(e +
 f*x)^m)^n)*(4*b*f*x^2*(a^3 - 6*b^3*m^3*n^3 + 6*a*b^2*m^2*n^2 - 3*a^2*b*m*n
) + 4*b*e*x*(a^3 - 6*b^3*m^3*n^3 + 6*a*b^2*m^2*n^2 - 3*a^2*b*m*n)))/(e + f*
x)
```

# 3.405 $\int (a+b\log(c(d(e+fx)^m)^n))^3\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 121

$$\int (a+b\log(c(d(e+fx)^m)^n))^3\,dx = 6ab^2m^2n^2x-6b^3m^3n^3x+\frac{6b^3m^2n^2(e+fx)\log(c(d(e+fx)^m)^n)}{f}-\frac{3bmn(e+fx)(a+b\log(c(d(e+fx)^m)^n))^2}{f}+\frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^3}{f}$$

```
[Out] 6*a*b^2*m^2*n^2*x-6*b^3*m^3*n^3*x+6*b^3*m^2*n^2*(f*x+e)*ln(c*(d*(f*x+e)^m)^
n)/f-3*b*m*n*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^2/f+(f*x+e)*(a+b*ln(c*(d*(
f*x+e)^m)^n))^3/f
```

## Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2436, 2333, 2332, 2495}

$$\int (a+b\log(c(d(e+fx)^m)^n))^3\,dx = 6ab^2m^2n^2x-\frac{3bmn(e+fx)(a+b\log(c(d(e+fx)^m)^n))^2}{f}+\frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^3}{f}+\frac{6b^3m^2n^2(e+fx)\log(c(d(e+fx)^m)^n)}{f}-6b^3m^3n^3x$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^3,x]
```

```
[Out] 6*a*b^2*m^2*n^2*x - 6*b^3*m^3*n^3*x + (6*b^3*m^2*n^2*(e + f*x)*Log[c*(d*(e
+ f*x)^m)^n])/f - (3*b*m*n*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^2)/f
+ ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^3)/f
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^3\,dx, cd^n(e+fx)^{mn}, c\left(d(e+fx)^m\right)^n\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int \left(a+b\log\left(cd^nx^{mn}\right)\right)^3\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c\left(d(e+fx)^m\right)^n\right)\\
&= \frac{(e+fx)\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^3}{f}\\
&\quad - \text{Subst}\left(\frac{(3bmn)\text{Subst}\left(\int \left(a+b\log\left(cd^nx^{mn}\right)\right)^2\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c\left(d(e+fx)^m\right)^n\right)
\end{aligned}$$

$$= -\frac{3bmn(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^2}{f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^3}{f} + \text{Subst}\left(\frac{(6b^2m^2n^2)\,\text{Subst}\left(\int\left(a+b\log\left(cd^nx^{mn}\right)\right)\,dx,x,e+fx\right)}{f},cd^n(e+fx)^{mn},c(d(e+fx)^m)^n\right)$$

$$= 6ab^2m^2n^2x - \frac{3bmn(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^2}{f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^3}{f} + \text{Subst}\left(\frac{(6b^3m^2n^2)\,\text{Subst}\left(\int\log\left(cd^nx^{mn}\right)\,dx,x,e+fx\right)}{f},cd^n(e+fx)^{mn},c(d(e+fx)^m)^n\right)$$

$$= 6ab^2m^2n^2x - 6b^3m^3n^3x + \frac{6b^3m^2n^2(e+fx)\log\left(c(d(e+fx)^m)^n\right)}{f} - \frac{3bmn(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^2}{f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^3}{f}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^3\,dx = \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^3 - 3bmn\Big((e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^2 - 2bmn(f(a-bm}{f}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^3,x]
```

```
[Out] ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^3 - 3*b*m*n*((e + f*x)*(a + b*L
og[c*(d*(e + f*x)^m)^n])^2 - 2*b*m*n*(f*(a - b*m*n)*x + b*(e + f*x)*Log[c*(
d*(e + f*x)^m)^n]))))/f
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(121) = 242$.

Time = 1.51 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.19

| method | result |
|---|---|
| parallelrisch | $\frac{6\ln(fx+e)b^3e^2m^3n^3-6x\,b^3ef\,m^3n^3+6x\ln(c(d(fx+e)^m)^n)b^3ef\,m^2n^2+6b^3e^2m^3n^3-6\ln(fx+e)a\,b^2e^2m^2n^2-3x\ln(c(d(fx+e)^{m}}{}$ |

[In] int((a+b*ln(c*(d*(f*x+e)^m)^n))^3,x,method=_RETURNVERBOSE)

[Out] (6*ln(f*x+e)*b^3*e^2*m^3*n^3-6*x*b^3*e*f*m^3*n^3+6*x*ln(c*(d*(f*x+e)^m)^n)*b^3*e*f*m^2*n^2+6*b^3*e^2*m^3*n^3-6*ln(f*x+e)*a*b^2*e^2*m^2*n^2-3*x*ln(c*(d*(f*x+e)^m)^n)^2*b^3*e*f*m*n+6*x*a*b^2*e*f*m^2*n^2+x*ln(c*(d*(f*x+e)^m)^n)^3*b^3*e*f-6*x*ln(c*(d*(f*x+e)^m)^n)*a*b^2*e*f*m*n-3*ln(c*(d*(f*x+e)^m)^n)^2*b^3*e^2*m*n-6*a*b^2*e^2*m^2*n^2+3*ln(f*x+e)*a^2*b*e^2*m*n+3*x*ln(c*(d*(f*x+e)^m)^n)^2*a*b^2*e*f-3*x*a^2*b*e*f*m*n+ln(c*(d*(f*x+e)^m)^n)^3*b^3*e^2+3*x*ln(c*(d*(f*x+e)^m)^n)*a^2*b*e*f+3*ln(c*(d*(f*x+e)^m)^n)^2*a*b^2*e^2+3*a^2*b*e^2*m*n+x*a^3*e*f-a^3*e^2)/e/f

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(121) = 242$.

Time = 0.32 (sec) , antiderivative size = 639, normalized size of antiderivative = 5.28

$$\int (a + b\log(c(d(e+fx)^m)^n))^3\,dx = \frac{b^3fn^3x\log(d)^3 + b^3fx\log(c)^3 + (b^3fm^3n^3x + b^3em^3n^3)\log(fx+e)^3 - 3(b^3fmn - ab^2f)x\log(c)^2 - 3(b^3}{}$$

[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^3,x, algorithm="fricas")

[Out] (b^3*f*n^3*x*log(d)^3 + b^3*f*x*log(c)^3 + (b^3*f*m^3*n^3*x + b^3*e*m^3*n^3)*log(f*x + e)^3 - 3*(b^3*f*m*n - a*b^2*f)*x*log(c)^2 - 3*(b^3*e*m^3*n^3 - a*b^2*e*m^2*n^2 + (b^3*f*m^3*n^3 - a*b^2*f*m^2*n^2)*x - (b^3*f*m^2*n^2*x + b^3*e*m^2*n^2)*log(c) - (b^3*f*m^2*n^3*x + b^3*e*m^2*n^3)*log(d))*log(f*x + e)^2 + 3*(2*b^3*f*m^2*n^2 - 2*a*b^2*f*m*n + a^2*b*f)*x*log(c) + 3*(b^3*f*n^2*x*log(c) - (b^3*f*m*n^3 - a*b^2*f*n^2)*x)*log(d)^2 - (6*b^3*f*m^3*n^3 - 6*a*b^2*f*m^2*n^2 + 3*a^2*b*f*m*n - a^3*f)*x + 3*(2*b^3*e*m^3*n^3 - 2*a*b^2*e*m^2*n^2 + a^2*b*e*m*n + (b^3*f*m*n*x + b^3*e*m*n)*log(c)^2 + (b^3*f*m*n^3*x + b^3*e*m*n^3)*log(d)^2 + (2*b^3*f*m^3*n^3 - 2*a*b^2*f*m^2*n^2 + a^2*b*f*m*n)*x - 2*(b^3*e*m^2*n^2 - a*b^2*e*m*n + (b^3*f*m^2*n^2 - a*b^2*f*m*n)*x)*log(c) - 2*(b^3*e*m^2*n^3 - a*b^2*e*m*n^2 + (b^3*f*m^2*n^3 - a*b^2*f*m*n^2)*x - (b^3*f*m*n^2*x + b^3*e*m*n^2)*log(c))*log(d))*log(f*x + e) + 3*(b^3*f*n*x*log(c)^2 - 2*(b^3*f*m*n^2 - a*b^2*f*n)*x*log(c) + (2*b^3*f*m^2*n^3 - 2*a*b^2*f*m*n^2 + a^2*b*f*n)*x)*log(d))/f

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(117) = 234$.

Time = 1.14 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.98

$$\int (a+b\log (c(d(e+fx)^m)^n))^3\, dx = \begin{cases} a^3x + \frac{3a^2be\log (c(d(e+fx)^m)^n)}{f} - 3a^2bmnx + 3a^2bx\log (c(d(e+fx)^m)^n) - \frac{6ab^2emn\log (c(d(e+fx)^m)^n)}{f} + \frac{3ab^2e\log (\ldots}{} \\ x(a+b\log (c(de^m)^n))^3 \end{cases}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*e*log(c*(d*(e + f*x)**m)**n)/f - 3*a**2*b*m*n*
x + 3*a**2*b*x*log(c*(d*(e + f*x)**m)**n) - 6*a*b**2*e*m*n*log(c*(d*(e + f*
x)**m)**n)/f + 3*a*b**2*e*log(c*(d*(e + f*x)**m)**n)**2/f + 6*a*b**2*m**2*n
**2*x - 6*a*b**2*m*n*x*log(c*(d*(e + f*x)**m)**n) + 3*a*b**2*x*log(c*(d*(e
+ f*x)**m)**n)**2 + 6*b**3*e*m**2*n**2*log(c*(d*(e + f*x)**m)**n)/f - 3*b**
3*e*m*n*log(c*(d*(e + f*x)**m)**n)**2/f + b**3*e*log(c*(d*(e + f*x)**m)**n)
**3/f - 6*b**3*m**3*n**3*x + 6*b**3*m**2*n**2*x*log(c*(d*(e + f*x)**m)**n)
- 3*b**3*m*n*x*log(c*(d*(e + f*x)**m)**n)**2 + b**3*x*log(c*(d*(e + f*x)**m
)**n)**3, Ne(f, 0)), (x*(a + b*log(c*(d*e**m)**n))**3, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(121) = 242$.

Time = 0.24 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.62

$$\begin{aligned}\int (a+b\log (c(d(e+fx)^m)^n))^3\, dx &= -3\,a^2bfmn\left(\frac{x}{f} - \frac{e\log (fx+e)}{f^2}\right) \\ &\quad + b^3x\log \left(\left((fx+e)^m d\right)^n c\right)^3 + 3\,ab^2x\log \left(\left((fx+e)^m d\right)^n c\right)^2 + 3\,a^2bx\log \left(\left((fx+e)^m d\right)^n c\right) \\ &\quad -3\left(2\,fmn\left(\frac{x}{f} - \frac{e\log (fx+e)}{f^2}\right)\log \left(\left((fx+e)^m d\right)^n c\right) + \frac{(e\log (fx+e)^2 - 2\,fx + 2\,e\log (fx+e))m^2n\ldots}{f}\right. \\ &\quad -\left(3\,fmn\left(\frac{x}{f} - \frac{e\log (fx+e)}{f^2}\right)\log \left(\left((fx+e)^m d\right)^n c\right)^2 - \left(\frac{(e\log (fx+e)^3 + 3\,e\log (fx+e)^2 - 6\,fx + \ldots}{f^2}\right.\right. \\ &\quad + a^3x\end{aligned}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^3,x, algorithm="maxima")
```

```
[Out] -3*a^2*b*f*m*n*(x/f - e*log(f*x + e)/f^2) + b^3*x*log(((f*x + e)^m*d)^n*c)^
3 + 3*a*b^2*x*log(((f*x + e)^m*d)^n*c)^2 + 3*a^2*b*x*log(((f*x + e)^m*d)^n*
c) - 3*(2*f*m*n*(x/f - e*log(f*x + e)/f^2)*log(((f*x + e)^m*d)^n*c) + (e*lo
```

```
g(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*m^2*n^2/f)*a*b^2 - (3*f*m*n*(x/f -
 e*log(f*x + e)/f^2)*log(((f*x + e)^m*d)^n*c)^2 - ((e*log(f*x + e)^3 + 3*e*
log(f*x + e)^2 - 6*f*x + 6*e*log(f*x + e))*m^2*n^2/f^2 - 3*(e*log(f*x + e)^
2 - 2*f*x + 2*e*log(f*x + e))*m*n*log(((f*x + e)^m*d)^n*c)/f^2)*f*m*n)*b^3
+ a^3*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(121) = 242$.

Time = 0.32 (sec) , antiderivative size = 772, normalized size of antiderivative = 6.38

$$\int (a+b\log(c(d(e+fx)^m)^n))^3\,dx = \frac{(fx+e)b^3m^3n^3\log(fx+e)^3}{f} - \frac{3(fx+e)b^3m^3n^3\log(fx+e)^2}{f} + \frac{3(fx+e)b^3m^2n^3\log(fx+e)^2\log(d)}{f} + \frac{6(fx+e)b^3m^3n^3\log(fx+e)}{f} + \frac{3(fx+e)b^3m^2n^2\log(fx+e)^2\log(c)}{f} - \frac{6(fx+e)b^3m^2n^3\log(fx+e)\log(d)}{f} + \frac{3(fx+e)b^3mn^3\log(fx+e)\log(d)^2}{f} - \frac{6(fx+e)b^3m^3n^3}{f} + \frac{3(fx+e)ab^2m^2n^2\log(fx+e)^2}{f} - \frac{6(fx+e)b^3m^2n^2\log(fx+e)\log(c)}{f} + \frac{6(fx+e)b^3m^2n^3\log(d)}{f} + \frac{6(fx+e)b^3mn^2\log(fx+e)\log(c)\log(d)}{f} - \frac{3(fx+e)b^3mn^3\log(d)^2}{f} + \frac{(fx+e)b^3n^3\log(d)^3}{f} - \frac{6(fx+e)ab^2m^2n^2\log(fx+e)}{f} + \frac{6(fx+e)b^3m^2n^2\log(c)}{f} + \frac{3(fx+e)b^3mn\log(fx+e)\log(c)^2}{f} + \frac{6(fx+e)ab^2mn^2\log(fx+e)\log(d)}{f} - \frac{6(fx+e)b^3mn^2\log(c)\log(d)}{f} + \frac{3(fx+e)b^3n^2\log(c)\log(d)^2}{f} + \frac{6(fx+e)ab^2m^2n^2}{f} + \frac{6(fx+e)ab^2mn\log(fx+e)\log(c)}{f} - \frac{3(fx+e)b^3mn\log(c)^2}{f} - \frac{6(fx+e)ab^2mn^2\log(d)}{f} + \frac{3(fx+e)b^3n\log(c)^2\log(d)}{f}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^3,x, algorithm="giac")
```

```
[Out] (f*x + e)*b^3*m^3*n^3*log(f*x + e)^3/f - 3*(f*x + e)*b^3*m^3*n^3*log(f*x +
e)^2/f + 3*(f*x + e)*b^3*m^2*n^3*log(f*x + e)^2*log(d)/f + 6*(f*x + e)*b^3*
m^3*n^3*log(f*x + e)/f + 3*(f*x + e)*b^3*m^2*n^2*log(f*x + e)^2*log(c)/f -
6*(f*x + e)*b^3*m^2*n^3*log(f*x + e)*log(d)/f + 3*(f*x + e)*b^3*m*n^3*log(f
*x + e)*log(d)^2/f - 6*(f*x + e)*b^3*m^3*n^3/f + 3*(f*x + e)*a*b^2*m^2*n^2*
log(f*x + e)^2/f - 6*(f*x + e)*b^3*m^2*n^2*log(f*x + e)*log(c)/f + 6*(f*x +
 e)*b^3*m^2*n^3*log(d)/f + 6*(f*x + e)*b^3*m*n^2*log(f*x + e)*log(c)*log(d)
/f - 3*(f*x + e)*b^3*m*n^3*log(d)^2/f + (f*x + e)*b^3*n^3*log(d)^3/f - 6*(f
*x + e)*a*b^2*m^2*n^2*log(f*x + e)/f + 6*(f*x + e)*b^3*m^2*n^2*log(c)/f + 3
*(f*x + e)*b^3*m*n*log(f*x + e)*log(c)^2/f + 6*(f*x + e)*a*b^2*m*n^2*log(f*
x + e)*log(d)/f - 6*(f*x + e)*b^3*m*n^2*log(c)*log(d)/f + 3*(f*x + e)*b^3*n
^2*log(c)*log(d)^2/f + 6*(f*x + e)*a*b^2*m^2*n^2/f + 6*(f*x + e)*a*b^2*m*n*
log(f*x + e)*log(c)/f - 3*(f*x + e)*b^3*m*n*log(c)^2/f - 6*(f*x + e)*a*b^2*
m*n^2*log(d)/f + 3*(f*x + e)*b^3*n*log(c)^2*log(d)/f + 3*(f*x + e)*a*b^2*n^
2*log(d)^2/f + 3*(f*x + e)*a^2*b*m*n*log(f*x + e)/f - 6*(f*x + e)*a*b^2*m*n
*log(c)/f + (f*x + e)*b^3*log(c)^3/f + 6*(f*x + e)*a*b^2*n*log(c)*log(d)/f
- 3*(f*x + e)*a^2*b*m*n/f + 3*(f*x + e)*a*b^2*log(c)^2/f + 3*(f*x + e)*a^2*
b*n*log(d)/f + 3*(f*x + e)*a^2*b*log(c)/f + (f*x + e)*a^3/f
```

## Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.00

$$\int (a+b\log(c(d(e+fx)^m)^n))^3\,dx = x\left(a^3-3a^2bmn+6ab^2m^2n^2-6b^3m^3n^3\right) + \ln\left(c\left(d\left(e+fx\right)^m\right)^n\right)^2\left(\frac{3\left(ab^2e-b^3emn\right)}{f}+3b^2x\left(a-bmn\right)\right) + \ln\left(c\left(d\left(e+fx\right)^m\right)^n\right)^3\left(b^3x+\frac{b^3e}{f}\right) + \frac{\ln\left(e+fx\right)\left(3ea^2bmn-6eab^2m^2n^2+6eb^3m^3n^3\right)}{f} + \frac{\ln\left(c\left(d\left(e+fx\right)^m\right)^n\right)\left(3bf\left(a^2-2abmn+2b^2m^2n^2\right)x^2+3be\left(a^2-2abmn+2b^2m^2n^2\right)x\right)}{e+fx}$$

```
[In] int((a + b*log(c*(d*(e + f*x)^m)^n))^3,x)
```

```
[Out] x*(a^3 - 6*b^3*m^3*n^3 + 6*a*b^2*m^2*n^2 - 3*a^2*b*m*n) + log(c*(d*(e + f*x
)^m)^n)^2*((3*(a*b^2*e - b^3*e*m*n))/f + 3*b^2*x*(a - b*m*n)) + log(c*(d*(e
 + f*x)^m)^n)^3*(b^3*x + (b^3*e)/f) + (log(e + f*x)*(6*b^3*e*m^3*n^3 - 6*a*
b^2*e*m^2*n^2 + 3*a^2*b*e*m*n))/f + (log(c*(d*(e + f*x)^m)^n)*(3*b*e*x*(a^2
 + 2*b^2*m^2*n^2 - 2*a*b*m*n) + 3*b*f*x^2*(a^2 + 2*b^2*m^2*n^2 - 2*a*b*m*n)
))/(e + f*x)
```

# 3.406 $\int (a+b\log (c(d(e+fx)^m)^n))^2\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 78

$$\int (a+b\log (c(d(e+fx)^m)^n))^2\,dx=-2abmnx+2b^2m^2n^2x-\frac{2b^2mn(e+fx)\log (c(d(e+fx)^m)^n)}{f}+\frac{(e+fx)(a+b\log (c(d(e+fx)^m)^n))^2}{f}$$

```
[Out] -2*a*b*m*n*x+2*b^2*m^2*n^2*x-2*b^2*m*n*(f*x+e)*ln(c*(d*(f*x+e)^m)^n)/f+(f*x
+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^2/f
```

## Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2436, 2333, 2332, 2495}

$$\int (a+b\log (c(d(e+fx)^m)^n))^2\,dx=\frac{(e+fx)(a+b\log (c(d(e+fx)^m)^n))^2}{f}-2abmnx-\frac{2b^2mn(e+fx)\log (c(d(e+fx)^m)^n)}{f}+2b^2m^2n^2x$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^2,x]
```

```
[Out] -2*a*b*m*n*x + 2*b^2*m^2*n^2*x - (2*b^2*m*n*(e + f*x)*Log[c*(d*(e + f*x)^m)
^n])/f + ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^2)/f
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int (a+b\log(cd^n(e+fx)^{mn}))^2\,dx, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int (a+b\log(cd^nx^{mn}))^2\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)\\
&= \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^2}{f}\\
&\quad - \text{Subst}\left(\frac{(2bmn)\text{Subst}\left(\int (a+b\log(cd^nx^{mn}))\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)\\
&= -2abmnx + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^2}{f}\\
&\quad - \text{Subst}\left(\frac{(2b^2mn)\,\text{Subst}\left(\int \log(cd^nx^{mn})\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)
\end{aligned}$$

$$= -2abmnx + 2b^2m^2n^2x - \frac{2b^2mn(e+fx)\log\left(c(d(e+fx)^m)^n\right)}{f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^2}{f}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^2\,dx = \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^2}{f} - 2bmn\left(ax - bmnx + \frac{b(e+fx)\log\left(c(d(e+fx)^m)^n\right)}{f}\right)$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^2,x]
```

```
[Out] ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^2)/f - 2*b*m*n*(a*x - b*m*n*x +
 (b*(e + f*x)*Log[c*(d*(e + f*x)^m)^n])/f)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(78) = 156.

Time = 0.42 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.12

| method | result |
|---|---|
| parallelrisch | $\frac{-2\ln(fx+e)b^2e^2m^2n^2+2x\,b^2ef\,m^2n^2-2x\ln(c(d(fx+e)^m)^n)b^2efmn+2\ln(fx+e)ab\,e^2mn+x\ln(c(d(fx+e)^m)^n)^2b^2ef-2xab}{ef}$ |

```
[In] int((a+b*ln(c*(d*(f*x+e)^m)^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-2*ln(f*x+e)*b^2*e^2*m^2*n^2+2*x*b^2*e*f*m^2*n^2-2*x*ln(c*(d*(f*x+e)^m)^n)
*b^2*e*f*m*n+2*ln(f*x+e)*a*b*e^2*m*n+x*ln(c*(d*(f*x+e)^m)^n)^2*b^2*e*f-2*x*
a*b*e*f*m*n+2*x*ln(c*(d*(f*x+e)^m)^n)*a*b*e*f+ln(c*(d*(f*x+e)^m)^n)^2*b^2*e
^2+e*a^2*f*x)/e/f
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(78) = 156.

Time = 0.33 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.96

$$\int (a+b\log(c(d(e+fx)^m)^n))^2\,dx$$
$$=\frac{b^2fn^2x\log(d)^2+b^2fx\log(c)^2+(b^2fm^2n^2x+b^2em^2n^2)\log(fx+e)^2-2(b^2fmn-abf)x\log(c)+(2b^2f$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^2,x, algorithm="fricas")
```

```
[Out] (b^2*f*n^2*x*log(d)^2 + b^2*f*x*log(c)^2 + (b^2*f*m^2*n^2*x + b^2*e*m^2*n^2
)*log(f*x + e)^2 - 2*(b^2*f*m*n - a*b*f)*x*log(c) + (2*b^2*f*m^2*n^2 - 2*a*
b*f*m*n + a^2*f)*x - 2*(b^2*e*m^2*n^2 - a*b*e*m*n + (b^2*f*m^2*n^2 - a*b*f*
m*n)*x - (b^2*f*m*n*x + b^2*e*m*n)*log(c) - (b^2*f*m*n^2*x + b^2*e*m*n^2)*l
og(d))*log(f*x + e) + 2*(b^2*f*n*x*log(c) - (b^2*f*m*n^2 - a*b*f*n)*x)*log(
d))/f
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(76) = 152.

Time = 0.52 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.28

$$\int (a+b\log(c(d(e+fx)^m)^n))^2\,dx$$
$$=\begin{cases} a^2x+\frac{2abe\log(c(d(e+fx)^m)^n)}{f}-2abmnx+2abx\log(c(d(e+fx)^m)^n)-\frac{2b^2emn\log(c(d(e+fx)^m)^n)}{f}+\frac{b^2e\log(c(d(e+fx \\ x(a+b\log(c(de^m)^n))^2 \end{cases}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*e*log(c*(d*(e + f*x)**m)**n)/f - 2*a*b*m*n*x + 2*
a*b*x*log(c*(d*(e + f*x)**m)**n) - 2*b**2*e*m*n*log(c*(d*(e + f*x)**m)**n)/
f + b**2*e*log(c*(d*(e + f*x)**m)**n)**2/f + 2*b**2*m**2*n**2*x - 2*b**2*m*
n*x*log(c*(d*(e + f*x)**m)**n) + b**2*x*log(c*(d*(e + f*x)**m)**n)**2, Ne(f
, 0)), (x*(a + b*log(c*(d*e**m)**n))**2, True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int (a+b\log (c(d(e+fx)^m)^n))^2\,dx$$
$$= -2\,abfmn\left(\frac{x}{f}-\frac{e\log (fx+e)}{f^2}\right)+b^2x\log \left(\left(\left((fx+e)^m d\right)^n c\right)^2+2\,abx\log \left(\left((fx+e)^m d\right)^n c\right)$$
$$-\left(2\,fmn\left(\frac{x}{f}-\frac{e\log (fx+e)}{f^2}\right)\log \left(\left((fx+e)^m d\right)^n c\right)+\frac{\left(e\log (fx+e)^2-2\,fx+2\,e\log (fx+e)\right)m^2n^2}{f}\right.$$
$$+a^2x$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*f*m*n*(x/f - e*log(f*x + e)/f^2) + b^2*x*log(((f*x + e)^m*d)^n*c)^2
+ 2*a*b*x*log(((f*x + e)^m*d)^n*c) - (2*f*m*n*(x/f - e*log(f*x + e)/f^2)*lo
g(((f*x + e)^m*d)^n*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*m^2*
n^2/f)*b^2 + a^2*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(78) = 156.

Time = 0.33 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.63

$$\int (a+b\log(c(d(e+fx)^m)^n))^2\,dx = \frac{(fx+e)b^2m^2n^2\log(fx+e)^2}{f} - \frac{2(fx+e)b^2m^2n^2\log(fx+e)}{f} + \frac{2(fx+e)b^2mn^2\log(fx+e)\log(d)}{f} + \frac{2(fx+e)b^2m^2n^2}{f} + \frac{2(fx+e)b^2mn\log(fx+e)\log(c)}{f} - \frac{2(fx+e)b^2mn^2\log(d)}{f} + \frac{(fx+e)b^2n^2\log(d)^2}{f} + \frac{2(fx+e)abmn\log(fx+e)}{f} - \frac{2(fx+e)b^2mn\log(c)}{f} + \frac{2(fx+e)b^2n\log(c)\log(d)}{f} - \frac{2(fx+e)abmn}{f} + \frac{(fx+e)b^2\log(c)^2}{f} + \frac{2(fx+e)abn\log(d)}{f} + \frac{2(fx+e)ab\log(c)}{f} + \frac{(fx+e)a^2}{f}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^2,x, algorithm="giac")
```

```
[Out] (f*x + e)*b^2*m^2*n^2*log(f*x + e)^2/f - 2*(f*x + e)*b^2*m^2*n^2*log(f*x +
e)/f + 2*(f*x + e)*b^2*m*n^2*log(f*x + e)*log(d)/f + 2*(f*x + e)*b^2*m^2*n^
2/f + 2*(f*x + e)*b^2*m*n*log(f*x + e)*log(c)/f - 2*(f*x + e)*b^2*m*n^2*log
(d)/f + (f*x + e)*b^2*n^2*log(d)^2/f + 2*(f*x + e)*a*b*m*n*log(f*x + e)/f -
 2*(f*x + e)*b^2*m*n*log(c)/f + 2*(f*x + e)*b^2*n*log(c)*log(d)/f - 2*(f*x
+ e)*a*b*m*n/f + (f*x + e)*b^2*log(c)^2/f + 2*(f*x + e)*a*b*n*log(d)/f + 2*
(f*x + e)*a*b*log(c)/f + (f*x + e)*a^2/f
```

## Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int (a+b\log (c(d(e+fx)^m)^n))^2\, dx = \ln \left(c\left(d\left(e+f\,x\right)^m\right)^n\right)^2\left(b^2\,x+\frac{b^2\,e}{f}\right) + x\left(a^2-2\,a\,b\,m\,n+2\,b^2\,m^2\,n^2\right) - \frac{\ln\left(e+f\,x\right)\,\left(2\,b^2\,e\,m^2\,n^2-2\,a\,b\,e\,m\,n\right)}{f} + 2\,b\,x\,\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)\,\left(a-b\,m\,n\right)$$

```
[In] int((a + b*log(c*(d*(e + f*x)^m)^n))^2,x)
```

```
[Out] log(c*(d*(e + f*x)^m)^n)^2*(b^2*x + (b^2*e)/f) + x*(a^2 + 2*b^2*m^2*n^2 - 2
*a*b*m*n) - (log(e + f*x)*(2*b^2*e*m^2*n^2 - 2*a*b*e*m*n))/f + 2*b*x*log(c*
(d*(e + f*x)^m)^n)*(a - b*m*n)
```

# 3.407 $\int (a+b\log(c(d(e+fx)^m)^n))\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 34

$$\int (a+b\log(c(d(e+fx)^m)^n))\,dx = ax-bmnx+\frac{b(e+fx)\log(c(d(e+fx)^m)^n)}{f}$$

```
[Out] a*x-b*m*n*x+b*(f*x+e)*ln(c*(d*(f*x+e)^m)^n)/f
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2436, 2332, 2495}

$$\int (a+b\log(c(d(e+fx)^m)^n))\,dx = ax+\frac{b(e+fx)\log(c(d(e+fx)^m)^n)}{f}-bmnx$$

```
[In] Int[a + b*Log[c*(d*(e + f*x)^m)^n],x]
```

```
[Out] a*x - b*m*n*x + (b*(e + f*x)*Log[c*(d*(e + f*x)^m)^n])/f
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax + b\int \log\left(c(d(e+fx)^m)^n\right)\,dx\\ &= ax + b\text{Subst}\left(\int \log\left(cd^n(e+fx)^{mn}\right)\,dx, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)\\ &= ax + b\text{Subst}\left(\frac{\text{Subst}\left(\int \log\left(cd^n x^{mn}\right)\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)\\ &= ax - bmnx + \frac{b(e+fx)\log\left(c(d(e+fx)^m)^n\right)}{f}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \left(a + b\log\left(c(d(e+fx)^m)^n\right)\right)\,dx = ax - bmnx + \frac{b(e+fx)\log\left(c(d(e+fx)^m)^n\right)}{f}$$

```
[In] Integrate[a + b*Log[c*(d*(e + f*x)^m)^n],x]
[Out] a*x - b*m*n*x + (b*(e + f*x)*Log[c*(d*(e + f*x)^m)^n])/f
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

| method | result | size |
|---|---|---|
| default | $ax + b\ln\left(c(d(fx+e)^m)^n\right)x - bmnx + \frac{bnme\ln(fx+e)}{f}$ | 42 |
| parts | $ax + b\ln\left(c(d(fx+e)^m)^n\right)x - bmnx + \frac{bnme\ln(fx+e)}{f}$ | 42 |
| parallelrisch | $\frac{b\left(2\ln(fx+e)e^2mn - xefmn + x\ln(c(d(fx+e)^m)^n)ef - \ln(c(d(fx+e)^m)^n)e^2\right)}{ef} + ax$ | 71 |

```
[In] int(a+b*ln(c*(d*(f*x+e)^m)^n),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*ln(c*(d*(f*x+e)^m)^n)*x-b*m*n*x+b*n*m/f*e*ln(f*x+e)
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int (a+b\log(c(d(e+fx)^m)^n))\,dx = \frac{bfnx\log(d)+bfx\log(c)-(bfmn-af)x+(bfmnx+bemn)\log(fx+e)}{f}$$

```
[In] integrate(a+b*log(c*(d*(f*x+e)^m)^n),x, algorithm="fricas")
```

```
[Out] (b*f*n*x*log(d) + b*f*x*log(c) - (b*f*m*n - a*f)*x + (b*f*m*n*x + b*e*m*n)*
log(f*x + e))/f
```

## Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int (a+b\log(c(d(e+fx)^m)^n))\,dx = ax+b\left(\begin{cases}\frac{e\log(c(d(e+fx)^m)^n)}{f}-mnx+x\log(c(d(e+fx)^m)^n) & \text{for } f\neq 0\\ x\log(c(de^m)^n) & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(a+b*ln(c*(d*(f*x+e)**m)**n),x)
```

```
[Out] a*x + b*Piecewise((e*log(c*(d*(e + f*x)**m)**n)/f - m*n*x + x*log(c*(d*(e +
 f*x)**m)**n), Ne(f, 0)), (x*log(c*(d*e**m)**n), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int (a+b\log(c(d(e+fx)^m)^n))\,dx = -bfmn\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right) + bx\log\left(\left(((fx+e)^m d)^n c\right)\right)+ax$$

```
[In] integrate(a+b*log(c*(d*(f*x+e)^m)^n),x, algorithm="maxima")
```

```
[Out] -b*f*m*n*(x/f - e*log(f*x + e)/f^2) + b*x*log(((f*x + e)^m*d)^n*c) + a*x
```

## Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int (a + b \log (c(d(e + fx)^m)^n)) \, dx = \left( \frac{(fx + e)mn \log (fx + e)}{f} - \frac{(fx + e)mn}{f} + \frac{(fx + e)n \log (d)}{f} + \frac{(fx + e) \log (c)}{f} \right) b + ax$$

```
[In] integrate(a+b*log(c*(d*(f*x+e)^m)^n),x, algorithm="giac")
[Out] ((f*x + e)*m*n*log(f*x + e)/f - (f*x + e)*m*n/f + (f*x + e)*n*log(d)/f + (f
*x + e)*log(c)/f)*b + a*x
```

## Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int (a + b \log (c(d(e + fx)^m)^n)) \, dx = x (a - b m n) + b x \ln (c (d (e + f x)^m)^n) + \frac{b e m n \ln (e + f x)}{f}$$

```
[In] int(a + b*log(c*(d*(e + f*x)^m)^n),x)
[Out] x*(a - b*m*n) + b*x*log(c*(d*(e + f*x)^m)^n) + (b*e*m*n*log(e + f*x))/f
```

# 3.408 $\int \frac{1}{a+b\log(c(d(e+fx)^m)^n)}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 83

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)}{bfmn}$$

```
[Out] (f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^m)^n))/b/m/n)/b/exp(a/b/m/n)/f/m/n/((c*(d*(
f*x+e)^m)^n)^(1/m/n))
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2436, 2337, 2209, 2495}

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \frac{(e+fx)e^{-\frac{a}{bmn}}\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\,\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)}{bfmn}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-1),x]
```

```
[Out] ((e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)])/(b*E^(a
/(b*m*n))*f*m*n*(c*(d*(e + f*x)^m)^n)^(1/(m*n)))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{a+b\log\left(cd^n(e+fx)^{mn}\right)}\,dx, cd^n(e+fx)^{mn}, c\left(d(e+fx)^m\right)^n\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{a+b\log(cd^n x^{mn})}\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c\left(d(e+fx)^m\right)^n\right)$$

$$= \text{Subst}\left(\frac{\left((e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{mn}}}{a+bx}\,dx, x, \log\left(cd^n(e+fx)^{mn}\right)\right)}{fmn}, cd^n(e+fx)^{mn}, c\left(d(e+fx)^m\right)^n\right)$$

$$= \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c\left(d(e+fx)^m\right)^n\right)^{-\frac{1}{mn}}\text{Ei}\left(\frac{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}{bmn}\right)}{bfmn}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)}{bfmn}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-1),x]
```

```
[Out] ((e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)])/(b*E^(a
/(b*m*n))*f*m*n*(c*(d*(e + f*x)^m)^n)^(1/(m*n)))
```

## Maple [F]

$$\int \frac{1}{a+b\ln\left(c\left(d\left(fx+e\right)^m\right)^n\right)}dx$$

```
[In] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n)),x)
```

```
[Out] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n)),x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \frac{e^{\left(-\frac{bn\log(d)+b\log(c)+a}{bmn}\right)}\text{log\_integral}\left((fx+e)e^{\left(\frac{bn\log(d)+b\log(c)+a}{bmn}\right)}\right)}{bfmn}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n)),x, algorithm="fricas")
```

```
[Out] e^(-(b*n*log(d) + b*log(c) + a)/(b*m*n))*log_integral((f*x + e)*e^((b*n*log
(d) + b*log(c) + a)/(b*m*n)))/(b*f*m*n)
```

## Sympy [F]

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx=\int \frac{1}{a+b\log\left(c\left(d\left(e+fx\right)^m\right)^n\right)}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n)),x)
[Out] Integral(1/(a + b*log(c*(d*(e + f*x)**m)**n)), x)
```

## Maxima [F]

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx=\int \frac{1}{b\log\left(\left((fx+e)^m d\right)^n c\right)+a}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n)),x, algorithm="maxima")
[Out] integrate(1/(b*log(((f*x + e)^m*d)^n*c) + a), x)
```

## Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx=\frac{\operatorname{Ei}\left(\frac{\log(d)}{m}+\frac{\log(c)}{mn}+\frac{a}{bmn}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bmn}\right)}}{bc^{\frac{1}{mn}}d^{\left(\frac{1}{m}\right)}fmn}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n)),x, algorithm="giac")
[Out] Ei(log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))/(b*c^
(1/(m*n))*d^(1/m)*f*m*n)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx=\int \frac{1}{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)}\,dx$$

```
[In] int(1/(a + b*log(c*(d*(e + f*x)^m)^n)),x)
[Out] int(1/(a + b*log(c*(d*(e + f*x)^m)^n)), x)
```

# 3.409 $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 123

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^2}\,dx = \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)}{b^2fm^2n^2} - \frac{e+fx}{bfmn(a+b\log(c(d(e+fx)^m)^n))}$$

```
[Out] (f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^m)^n))/b/m/n)/b^2/exp(a/b/m/n)/f/m^2/n^2/((
c*(d*(f*x+e)^m)^n)^(1/m/n))+(-f*x-e)/b/f/m/n/(a+b*ln(c*(d*(f*x+e)^m)^n))
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2436, 2334, 2337, 2209, 2495}

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^2}\,dx = \frac{(e+fx)e^{-\frac{a}{bmn}}\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)}{b^2fm^2n^2} - \frac{e+fx}{bfmn(a+b\log(c(d(e+fx)^m)^n))}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-2),x]
```

```
[Out] ((e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)])/(b^2*E^
(a/(b*m*n))*f*m^2*n^2*(c*(d*(e + f*x)^m)^n)^(1/(m*n))) - (e + f*x)/(b*f*m*n
*(a + b*Log[c*(d*(e + f*x)^m)^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^2}\,dx, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cd^nx^{mn}))^2}\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)
\end{aligned}$$

$$= -\frac{e+fx}{bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)} + \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{a+b\log(cd^n x^{mn})}\,dx, x, e+fx\right)}{bfmn}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{e+fx}{bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)} + \text{Subst}\left(\frac{\left((e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{mn}}}{a+bx}\,dx, x, \log\left(cd^n(e+fx)^{mn}\right)\right)}{bfm^2n^2}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\text{Ei}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)}{b^2fm^2n^2} - \frac{e+fx}{bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^2}\,dx = -\frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\left(be^{\frac{a}{bmn}}mn(c(d(e+fx)^m)^n)^{\frac{1}{mn}} - \text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)\right)}{b^2fm^2n^2\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-2),x]
```

```
[Out] -(((e + f*x)*(b*E^(a/(b*m*n))*m*n*(c*(d*(e + f*x)^m)^n)^(1/(m*n)) - ExpInte
gralEi[(a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)]*(a + b*Log[c*(d*(e + f*x)^
m)^n])))/(b^2*E^(a/(b*m*n))*f*m^2*n^2*(c*(d*(e + f*x)^m)^n)^(1/(m*n))*(a +
b*Log[c*(d*(e + f*x)^m)^n])))
```

## Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{m}\right)^{n}\right)\right)^{2}}dx$$

[In] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^2,x)

[Out] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^2,x)

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^{m}\right)^{n}\right)\right)^{2}}\,dx = -\frac{\left((bfmnx+bemn)e^{\left(\frac{bn\log(d)+b\log(c)+a}{bmn}\right)}-(bmn\log\left(fx+e\right)+bn\log\left(d\right)+b\log\left(c\right)+a)\,\mathrm{log\_integral}\left(\left(f\right.\right.\right.}{b^{3}fm^{3}n^{3}\log\left(fx+e\right)+b^{3}fm^{2}n^{3}\log\left(d\right)+b^{3}fm^{2}n^{2}\log\left(c\right)+ab^{2}}$$

[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^2,x, algorithm="fricas")

[Out] -((b*f*m*n*x + b*e*m*n)*e^((b*n*log(d) + b*log(c) + a)/(b*m*n)) - (b*m*n*log(f*x + e) + b*n*log(d) + b*log(c) + a)*log_integral((f*x + e)*e^((b*n*log(d) + b*log(c) + a)/(b*m*n))))*e^(-(b*n*log(d) + b*log(c) + a)/(b*m*n))/(b^3*f*m^3*n^3*log(f*x + e) + b^3*f*m^2*n^3*log(d) + b^3*f*m^2*n^2*log(c) + a*b^2*f*m^2*n^2)

## Sympy [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^{m}\right)^{n}\right)\right)^{2}}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^{m}\right)^{n}\right)\right)^{2}}\,dx$$

[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n))**2,x)

[Out] Integral((a + b*log(c*(d*(e + f*x)**m)**n))**(-2), x)

## Maxima [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^2}\,dx = \int \frac{1}{\left(b\log\left(\left(\left(fx+e\right)^m d\right)^n c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^2,x, algorithm="maxima")
```

```
[Out] -(f*x + e)/(b^2*f*m*n*log(((f*x + e)^m)^n) + a*b*f*m*n + (f*m*n^2*log(d) +
f*m*n*log(c))*b^2) + integrate(1/(b^2*m*n*log(((f*x + e)^m)^n) + a*b*m*n +
(m*n^2*log(d) + m*n*log(c))*b^2), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. 2(123) = 246.

Time = 0.34 (sec) , antiderivative size = 582, normalized size of antiderivative = 4.73

$$\begin{aligned}
&\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^2}\,dx \\
&= -\frac{(fx+e)bmn}{b^3fm^3n^3\log(fx+e)+b^3fm^2n^3\log(d)+b^3fm^2n^2\log(c)+ab^2fm^2n^2} \\
&\quad+\frac{bmn\mathrm{Ei}\left(\frac{\log(d)}{m}+\frac{\log(c)}{mn}+\frac{a}{bmn}+\log(fx+e)\right)e^{\left(-\frac{a}{bmn}\right)}\log(fx+e)}{\left(b^3fm^3n^3\log(fx+e)+b^3fm^2n^3\log(d)+b^3fm^2n^2\log(c)+ab^2fm^2n^2\right)c^{\frac{1}{mn}}d^{\left(\frac{1}{m}\right)}} \\
&\quad+\frac{bn\mathrm{Ei}\left(\frac{\log(d)}{m}+\frac{\log(c)}{mn}+\frac{a}{bmn}+\log(fx+e)\right)e^{\left(-\frac{a}{bmn}\right)}\log(d)}{\left(b^3fm^3n^3\log(fx+e)+b^3fm^2n^3\log(d)+b^3fm^2n^2\log(c)+ab^2fm^2n^2\right)c^{\frac{1}{mn}}d^{\left(\frac{1}{m}\right)}} \\
&\quad+\frac{b\mathrm{Ei}\left(\frac{\log(d)}{m}+\frac{\log(c)}{mn}+\frac{a}{bmn}+\log(fx+e)\right)e^{\left(-\frac{a}{bmn}\right)}\log(c)}{\left(b^3fm^3n^3\log(fx+e)+b^3fm^2n^3\log(d)+b^3fm^2n^2\log(c)+ab^2fm^2n^2\right)c^{\frac{1}{mn}}d^{\left(\frac{1}{m}\right)}} \\
&\quad+\frac{a\mathrm{Ei}\left(\frac{\log(d)}{m}+\frac{\log(c)}{mn}+\frac{a}{bmn}+\log(fx+e)\right)e^{\left(-\frac{a}{bmn}\right)}}{\left(b^3fm^3n^3\log(fx+e)+b^3fm^2n^3\log(d)+b^3fm^2n^2\log(c)+ab^2fm^2n^2\right)c^{\frac{1}{mn}}d^{\left(\frac{1}{m}\right)}}
\end{aligned}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^2,x, algorithm="giac")
```

```
[Out] -(f*x + e)*b*m*n/(b^3*f*m^3*n^3*log(f*x + e) + b^3*f*m^2*n^3*log(d) + b^3*f
*m^2*n^2*log(c) + a*b^2*f*m^2*n^2) + b*m*n*Ei(log(d)/m + log(c)/(m*n) + a/(
b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(f*x + e)/((b^3*f*m^3*n^3*log(f*x
+ e) + b^3*f*m^2*n^3*log(d) + b^3*f*m^2*n^2*log(c) + a*b^2*f*m^2*n^2)*c^(1/
(m*n))*d^(1/m)) + b*n*Ei(log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e)
)*e^(-a/(b*m*n))*log(d)/((b^3*f*m^3*n^3*log(f*x + e) + b^3*f*m^2*n^3*log(d)
 + b^3*f*m^2*n^2*log(c) + a*b^2*f*m^2*n^2)*c^(1/(m*n))*d^(1/m)) + b*Ei(log(
d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(c)/((b^3
*f*m^3*n^3*log(f*x + e) + b^3*f*m^2*n^3*log(d) + b^3*f*m^2*n^2*log(c) + a*b
```

```
^2*f*m^2*n^2)*c^(1/(m*n))*d^(1/m)) + a*Ei(log(d)/m + log(c)/(m*n) + a/(b*m*
n) + log(f*x + e))*e^(-a/(b*m*n))/((b^3*f*m^3*n^3*log(f*x + e) + b^3*f*m^2*
n^3*log(d) + b^3*f*m^2*n^2*log(c) + a*b^2*f*m^2*n^2)*c^(1/(m*n))*d^(1/m))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^2}\,dx = \int \frac{1}{\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)\right)^2}\,dx$$

```
[In] int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^2,x)
```

```
[Out] int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^2, x)
```

# 3.410 $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx = \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)}{2b^3fm^3n^3} - \frac{e+fx}{2bfmn\,(a+b\log(c(d(e+fx)^m)^n))^2} - \frac{e+fx}{2b^2fm^2n^2\,(a+b\log(c(d(e+fx)^m)^n))}$$

```
[Out] 1/2*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^m)^n))/b/m/n)/b^3/exp(a/b/m/n)/f/m^3/n^
3/((c*(d*(f*x+e)^m)^n)^(1/m/n))+1/2*(-f*x-e)/b/f/m/n/(a+b*ln(c*(d*(f*x+e)^m
)^n))^2+1/2*(-f*x-e)/b^2/f/m^2/n^2/(a+b*ln(c*(d*(f*x+e)^m)^n))
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2436, 2334, 2337, 2209, 2495}

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx = \frac{(e+fx)e^{-\frac{a}{bmn}}\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)}{2b^3fm^3n^3} - \frac{e+fx}{2b^2fm^2n^2\,(a+b\log(c(d(e+fx)^m)^n))} - \frac{e+fx}{2bfmn\,(a+b\log(c(d(e+fx)^m)^n))^2}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-3),x]
```

```
[Out] ((e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)])/(2*b^3*
E^(a/(b*m*n))*f*m^3*n^3*(c*(d*(e + f*x)^m)^n)^(1/(m*n))) - (e + f*x)/(2*b*f
*m*n*(a + b*Log[c*(d*(e + f*x)^m)^n])^2) - (e + f*x)/(2*b^2*f*m^2*n^2*(a +
b*Log[c*(d*(e + f*x)^m)^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^3}\,dx, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cd^nx^{mn}))^3}\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{e+fx}{2bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^2} + \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cd^n x^{mn}))^2}\,dx, x, e+fx\right)}{2bfmn}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{e+fx}{2bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^2} - \frac{e+fx}{2b^2fm^2n^2\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)} + \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{a+b\log(cd^n x^{mn})}\,dx, x, e+fx\right)}{2b^2fm^2n^2}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{e+fx}{2bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^2} - \frac{e+fx}{2b^2fm^2n^2\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)} + \text{Subst}\left(\frac{\left((e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{mn}}}{a+bx}\,dx, x, \log\left(cd^n(e+fx)^{mn}\right)\right)}{2b^2fm^3n^3}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\text{Ei}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)}{2b^3fm^3n^3} - \frac{e+fx}{2bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^2} - \frac{e+fx}{2b^2fm^2n^2\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^3}\,dx = -\frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\left(-\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right.\right.}{2b^3fm^3n^3\left(a+b\log\left(c\left(d(e+fx)^m\right.\right.\right.}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-3),x]
```

```
[Out] -1/2*((e + f*x)*(-(ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)]*
(a + b*Log[c*(d*(e + f*x)^m)^n])^2) + b*E^(a/(b*m*n))*m*n*(c*(d*(e + f*x)^m
)^n)^(1/(m*n))*(a + b*m*n + b*Log[c*(d*(e + f*x)^m)^n])))/(b^3*E^(a/(b*m*n)
)*f*m^3*n^3*(c*(d*(e + f*x)^m)^n)^(1/(m*n))*(a + b*Log[c*(d*(e + f*x)^m)^n]
)^2)
```

## Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{m}\right)^{n}\right)\right)^{3}}dx$$

[In] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^3,x)

[Out] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^3,x)

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(163) = 326.

Time = 0.31 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.63

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^{m}\right)^{n}\right)\right)^{3}}\,dx = -\frac{\left((b^{2}em^{2}n^{2}+abemn+(b^{2}fm^{2}n^{2}+abfmn)x+(b^{2}fm^{2}n^{2}x+b^{2}em^{2}n^{2})\log\left(fx+e\right)+(b^{2}fmnx+b^{2}e\right.}{2\left(b^{5}fm^{5}n^{5}\log\right.}$$

[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^3,x, algorithm="fricas")

```
[Out] -1/2*((b^2*e*m^2*n^2 + a*b*e*m*n + (b^2*f*m^2*n^2 + a*b*f*m*n)*x + (b^2*f*m
^2*n^2*x + b^2*e*m^2*n^2)*log(f*x + e) + (b^2*f*m*n*x + b^2*e*m*n)*log(c) +
 (b^2*f*m*n^2*x + b^2*e*m*n^2)*log(d))*e^((b*n*log(d) + b*log(c) + a)/(b*m*
n)) - (b^2*m^2*n^2*log(f*x + e)^2 + b^2*n^2*log(d)^2 + b^2*log(c)^2 + 2*a*b
*log(c) + a^2 + 2*(b^2*m*n^2*log(d) + b^2*m*n*log(c) + a*b*m*n)*log(f*x + e
) + 2*(b^2*n*log(c) + a*b*n)*log(d))*log_integral((f*x + e)*e^((b*n*log(d)
+ b*log(c) + a)/(b*m*n))))*e^(-(b*n*log(d) + b*log(c) + a)/(b*m*n))/(b^5*f*
m^5*n^5*log(f*x + e)^2 + b^5*f*m^3*n^5*log(d)^2 + b^5*f*m^3*n^3*log(c)^2 +
2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3 + 2*(b^5*f*m^4*n^5*log(d) + b^
5*f*m^4*n^4*log(c) + a*b^4*f*m^4*n^4)*log(f*x + e) + 2*(b^5*f*m^3*n^4*log(c
) + a*b^4*f*m^3*n^4)*log(d))
```

## Sympy [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^{m}\right)^{n}\right)\right)^{3}}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^{m}\right)^{n}\right)\right)^{3}}\,dx$$

[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n))**3,x)

[Out] Integral((a + b*log(c*(d*(e + f*x)**m)**n))**(-3), x)

## Maxima [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx = \int \frac{1}{(b\log(((fx+e)^m d)^n c)+a)^3}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((e*m*n + e*n*log(d) + e*log(c))*b + a*e + ((f*m*n + f*n*log(d) + f*lo
g(c))*b + a*f)*x + (b*f*x + b*e)*log(((f*x + e)^m)^n))/(b^4*f*m^2*n^2*log((
(f*x + e)^m)^n)^2 + a^2*b^2*f*m^2*n^2 + 2*(f*m^2*n^3*log(d) + f*m^2*n^2*log
(c))*a*b^3 + (f*m^2*n^4*log(d)^2 + 2*f*m^2*n^3*log(c)*log(d) + f*m^2*n^2*lo
g(c)^2)*b^4 + 2*(a*b^3*f*m^2*n^2 + (f*m^2*n^3*log(d) + f*m^2*n^2*log(c))*b^
4)*log(((f*x + e)^m)^n)) + integrate(1/2/(b^3*m^2*n^2*log(((f*x + e)^m)^n)
+ a*b^2*m^2*n^2 + (m^2*n^3*log(d) + m^2*n^2*log(c))*b^3), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3401 vs. $2(163) = 326$.

Time = 0.38 (sec) , antiderivative size = 3401, normalized size of antiderivative = 20.12

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^3,x, algorithm="giac")
```

```
[Out] -1/2*(f*x + e)*b^2*m^2*n^2*log(f*x + e)/(b^5*f*m^5*n^5*log(f*x + e)^2 + 2*b
^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(c) + b^
5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^4*log
(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b^4*f*
m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3) + 1/2*b^2*m^2*n^2*Ei(log(d)/m + log(c)/
(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(f*x + e)^2/((b^5*f*m^5
*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4
*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x +
 e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^
3*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3)*c^(1/(m*n))*d^
(1/m)) - 1/2*(f*x + e)*b^2*m^2*n^2/(b^5*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*
m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(c) + b^5*f*m
^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^4*log(c)*l
og(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b^4*f*m^3*n
^3*log(c) + a^2*b^3*f*m^3*n^3) - 1/2*(f*x + e)*b^2*m*n^2*log(d)/(b^5*f*m^5*
n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*
log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x +
e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3
*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3) + b^2*m*n^2*Ei(
```

log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(f*x + e)*log(d)/((b^5*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3)*c^(1/(m*n))*d^(1/m)) - 1/2*(f*x + e)*b^2*m*n*log(c)/(b^5*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3) + b^2*m*n*Ei(log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(f*x + e)*log(c)/((b^5*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3)*c^(1/(m*n))*d^(1/m)) + 1/2*b^2*n^2*Ei(log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(d)^2/((b^5*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3)*c^(1/(m*n))*d^(1/m)) - 1/2*(f*x + e)*a*b*m*n/(b^5*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3) + a*b*m*n*Ei(log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(f*x + e)/((b^5*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3)*c^(1/(m*n))*d^(1/m)) + b^2*n*Ei(log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(c)*log(d)/((b^5*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3)*c^(1/(m*n))*d^(1/m)) + 1/2*b^2*Ei(log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(c)^2/((b^5*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3)*c^(1/(m*n))*d^(1/m)) + a*b*n*Ei(log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(d)/((b^5*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^

```
4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b
^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3)*c^(1/(m*n))*d^(1/m)) + a*b*Ei(log(
d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e))*e^(-a/(b*m*n))*log(c)/((b^5
*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x + e)*log(d) + 2*b^5*f*m
^4*n^4*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2 + 2*a*b^4*f*m^4*n^4*log
(f*x + e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^3*n^3*log(c)^2 + 2*a*b^
4*f*m^3*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*b^3*f*m^3*n^3)*c^(1/(m*
n))*d^(1/m)) + 1/2*a^2*Ei(log(d)/m + log(c)/(m*n) + a/(b*m*n) + log(f*x + e
))*e^(-a/(b*m*n))/((b^5*f*m^5*n^5*log(f*x + e)^2 + 2*b^5*f*m^4*n^5*log(f*x
+ e)*log(d) + 2*b^5*f*m^4*n^4*log(f*x + e)*log(c) + b^5*f*m^3*n^5*log(d)^2
+ 2*a*b^4*f*m^4*n^4*log(f*x + e) + 2*b^5*f*m^3*n^4*log(c)*log(d) + b^5*f*m^
3*n^3*log(c)^2 + 2*a*b^4*f*m^3*n^4*log(d) + 2*a*b^4*f*m^3*n^3*log(c) + a^2*
b^3*f*m^3*n^3)*c^(1/(m*n))*d^(1/m))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^3}\,dx=\int \frac{1}{\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)\right)^3}\,dx$$

```
[In] int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^3,x)
[Out] int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^3, x)
```

# 3.411 $\int (a+b\log(c(d(e+fx)^m)^n))^{5/2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 219

$$\int (a+b\log(c(d(e+fx)^m)^n))^{5/2}\,dx = -\frac{15b^{5/2}e^{-\frac{a}{bmn}}m^{5/2}n^{5/2}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{8f} + \frac{15b^2m^2n^2(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{4f} - \frac{5bmn(e+fx)\left(a+b\log(c(d(e+fx)^m)^n)\right)^{3/2}}{2f} + \frac{(e+fx)\left(a+b\log(c(d(e+fx)^m)^n)\right)^{5/2}}{f}$$

```
[Out] -5/2*b*m*n*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2)/f+(f*x+e)*(a+b*ln(c*(d
*(f*x+e)^m)^n))^(5/2)/f-15/8*b^(5/2)*m^(5/2)*n^(5/2)*(f*x+e)*erfi((a+b*ln(c
*(d*(f*x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2))*Pi^(1/2)/exp(a/b/m/n)/f/(
(c*(d*(f*x+e)^m)^n)^(1/m/n))+15/4*b^2*m^2*n^2*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^
m)^n))^(1/2)/f
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2436, 2333, 2337, 2211, 2235, 2495}

$$\int (a+b\log(c(d(e+fx)^m)^n))^{5/2}\,dx = -\frac{15\sqrt{\pi}b^{5/2}m^{5/2}n^{5/2}(e+fx)e^{-\frac{a}{bmn}}(c(d(e+fx)^m)^n)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{8f} + \frac{15b^2m^2n^2(e+fx)\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{4f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^{5/2}}{f} - \frac{5bmn(e+fx)(a+b\log(c(d(e+fx)^m)^n))^{3/2}}{2f}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(5/2),x]
```

```
[Out] (-15*b^(5/2)*m^(5/2)*n^(5/2)*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e
 + f*x)^m)^n]]/(Sqrt[b]*Sqrt[m]*Sqrt[n])])/(8*E^(a/(b*m*n))*f*(c*(d*(e + f*
x)^m)^n)^(1/(m*n))) + (15*b^2*m^2*n^2*(e + f*x)*Sqrt[a + b*Log[c*(d*(e + f*
x)^m)^n]])/(4*f) - (5*b*m*n*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^(3/2
))/(2*f) + ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^(5/2))/f
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{5/2}\,dx, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int \left(a+b\log\left(cd^n x^{mn}\right)\right)^{5/2}\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)\\
&= \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{5/2}}{f}\\
&\quad -\text{Subst}\left(\frac{(5bmn)\text{Subst}\left(\int \left(a+b\log\left(cd^n x^{mn}\right)\right)^{3/2}\,dx, x, e+fx\right)}{2f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)\\
&= -\frac{5bmn(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{3/2}}{2f}+\frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{5/2}}{f}\\
&\quad +\text{Subst}\left(\frac{\left(15b^2m^2n^2\right)\text{Subst}\left(\int \sqrt{a+b\log\left(cd^n x^{mn}\right)}\,dx, x, e+fx\right)}{4f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right.
\end{aligned}$$

$$= \frac{15b^2m^2n^2(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{4f} - \frac{5bmn(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{3/2}}{2f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{5/2}}{f} - \text{Subst}\left(\frac{(15b^3m^3n^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^nx^{mn})}}\,dx, x, e+fx\right)}{8f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \frac{15b^2m^2n^2(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{4f} - \frac{5bmn(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{3/2}}{2f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{5/2}}{f} - \text{Subst}\left(\frac{\left(15b^3m^2n^2(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{mn}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^n(e+fx)^{mn}\right)\right)}{8f}, c\right.$$

$$= \frac{15b^2m^2n^2(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{4f} - \frac{5bmn(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{3/2}}{2f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{5/2}}{f} - \text{Subst}\left(\frac{\left(15b^2m^2n^2(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int e^{-\frac{a}{bmn}+\frac{x^2}{bmn}}\,dx, x, \sqrt{a+b\log\left(cd^n(e+\right.}\right.}{4f}\right.$$

$$= -\frac{15b^{5/2}e^{-\frac{a}{bmn}}m^{5/2}n^{5/2}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{8f} + \frac{15b^2m^2n^2(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{4f} - \frac{5bmn(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{3/2}}{2f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^m)^n\right)\right)^{5/2}}{f}$$

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.87

$$\int (a+b\log(c(d(e+fx)^m)^n))^{5/2}\,dx = \frac{(e+fx)\left(8(a+b\log(c(d(e+fx)^m)^n))^{5/2}-5bmn\left(3b^{3/2}e^{-\frac{a}{bmn}}m^{3/2}n^{3/2}\sqrt{\pi}\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(5/2),x]
```

```
[Out] ((e + f*x)*(8*(a + b*Log[c*(d*(e + f*x)^m)^n])^(5/2) - 5*b*m*n*((3*b^(3/2)*
m^(3/2)*n^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]/(Sqrt[b]
*Sqrt[m]*Sqrt[n])]])/(E^(a/(b*m*n))*(c*(d*(e + f*x)^m)^n)^(1/(m*n))) + 2*Sqr
t[a + b*Log[c*(d*(e + f*x)^m)^n]]*(2*a - 3*b*m*n + 2*b*Log[c*(d*(e + f*x)^m
)^n]))))/(8*f)
```

## Maple [F]

$$\int (a+b\ln(c(d(fx+e)^m)^n))^{\frac{5}{2}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^m)^n))^(5/2),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^m)^n))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (a+b\log(c(d(e+fx)^m)^n))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \left(a + b\log\left(c(d(e+fx)^m)^n\right)\right)^{5/2} dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**(5/2),x)
[Out] Timed out
```

## Maxima [F]

$$\int \left(a + b\log\left(c(d(e+fx)^m)^n\right)\right)^{5/2} dx = \int \left(b\log\left(\left((fx+e)^m d\right)^n c\right) + a\right)^{\frac{5}{2}} dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(5/2),x, algorithm="maxima")
[Out] integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(5/2), x)
```

## Giac [F]

$$\int \left(a + b\log\left(c(d(e+fx)^m)^n\right)\right)^{5/2} dx = \int \left(b\log\left(\left((fx+e)^m d\right)^n c\right) + a\right)^{\frac{5}{2}} dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(5/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a + b\log\left(c(d(e+fx)^m)^n\right)\right)^{5/2} dx = \int \left(a + b\,\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)\right)^{5/2} dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^m)^n))^(5/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^m)^n))^(5/2), x)
```

# 3.412 $\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 176

$$\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx = \frac{3b^{3/2}e^{-\frac{a}{bmn}}m^{3/2}n^{3/2}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{4f} - \frac{3bmn(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{2f} + \frac{(e+fx)\left(a+b\log(c(d(e+fx)^m)^n)\right)^{3/2}}{f}$$

```
[Out] (f*x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2)/f+3/4*b^(3/2)*m^(3/2)*n^(3/2)*(f*
x+e)*erfi((a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2))*Pi^(1/
2)/exp(a/b/m/n)/f/((c*(d*(f*x+e)^m)^n)^(1/m/n))-3/2*b*m*n*(f*x+e)*(a+b*ln(c
*(d*(f*x+e)^m)^n))^(1/2)/f
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2436, 2333, 2337, 2211, 2235, 2495}

$$\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx = \frac{3\sqrt{\pi}b^{3/2}m^{3/2}n^{3/2}(e+fx)e^{-\frac{a}{bmn}}\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{4f} + \frac{(e+fx)\left(a+b\log(c(d(e+fx)^m)^n)\right)^{3/2}}{f} - \frac{3bmn(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{2f}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(3/2),x]
```

```
[Out] (3*b^(3/2)*m^(3/2)*n^(3/2)*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e +
 f*x)^m)^n]]/(Sqrt[b]*Sqrt[m]*Sqrt[n])])/(4*E^(a/(b*m*n))*f*(c*(d*(e + f*x)
^m)^n)^(1/(m*n))) - (3*b*m*n*(e + f*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]
)/(2*f) + ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^m)^n])^(3/2))/f
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (a+b\log(cd^n(e+fx)^{mn}))^{3/2}\,dx, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int (a+b\log(cd^n x^{mn}))^{3/2}\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^{3/2}}{f} - \text{Subst}\left(\frac{(3bmn)\text{Subst}\left(\int \sqrt{a+b\log(cd^n x^{mn})}\,dx, x, e+fx\right)}{2f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{3bmn(e+fx)\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{2f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^{3/2}}{f} + \text{Subst}\left(\frac{(3b^2m^2n^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^n x^{mn})}}\,dx, x, e+fx\right)}{4f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{3bmn(e+fx)\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{2f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^{3/2}}{f} + \text{Subst}\left(\frac{\left(3b^2mn(e+fx)(cd^n(e+fx)^{mn})^{-\frac{1}{mn}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{mn}}}{\sqrt{a+bx}}\,dx, x, \log(cd^n(e+fx)^{mn})\right)}{4f}, cd^n$$

$$= -\frac{3bmn(e+fx)\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{2f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^{3/2}}{f} + \text{Subst}\left(\frac{\left(3bmn(e+fx)(cd^n(e+fx)^{mn})^{-\frac{1}{mn}}\right)\text{Subst}\left(\int e^{-\frac{a}{bmn}+\frac{x^2}{bmn}}\,dx, x, \sqrt{a+b\log(cd^n(e+f}\right.}{2f}\right.$$

$$= \frac{3b^{3/2}e^{-\frac{a}{bmn}}m^{3/2}n^{3/2}\sqrt{\pi}(e+fx)(c(d(e+fx)^m)^n)^{-\frac{1}{mn}}\,\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{4f} - \frac{3bmn(e+fx)\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{2f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^m)^n))^{3/2}}{f}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.91

$$\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx = \frac{(e+fx)\left(3b^{3/2}e^{-\frac{a}{bmn}}m^{3/2}n^{3/2}\sqrt{\pi}(c(d(e+fx)^m)^n)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^n}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(3/2),x]
```

```
[Out] ((e + f*x)*((3*b^(3/2)*m^(3/2)*n^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d*(e
 + f*x)^m)^n]]/(Sqrt[b]*Sqrt[m]*Sqrt[n])])/(E^(a/(b*m*n))*(c*(d*(e + f*x)^m
)^n)^(1/(m*n))) + 2*Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]*(2*a - 3*b*m*n + 2
*b*Log[c*(d*(e + f*x)^m)^n])))/(4*f)
```

## Maple [F]

$$\int (a+b\ln(c(d(fx+e)^m)^n))^{\frac{3}{2}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx = \int (a+b\log(c(d(e+fx)^m)^n))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**(3/2),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**m)**n))**(3/2), x)
```

## Maxima [F]

$$\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx = \int (b\log(((fx+e)^m d)^n c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(3/2),x, algorithm="maxima")
[Out] integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(3/2), x)
```

## Giac [F]

$$\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx = \int (b\log(((fx+e)^m d)^n c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(3/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d(e+fx)^m)^n))^{3/2}\,dx = \int \left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)\right)^{3/2}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^m)^n))^(3/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^m)^n))^(3/2), x)
```

# 3.413 $\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = -\frac{\sqrt{b}e^{-\frac{a}{bmn}}\sqrt{m}\sqrt{n}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{2f} + \frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{f}$$

```
[Out] -1/2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2)
)*b^(1/2)*m^(1/2)*n^(1/2)*Pi^(1/2)/exp(a/b/m/n)/f/((c*(d*(f*x+e)^m)^n)^(1/m
/n))+(f*x+e)*(a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)/f
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2436, 2333, 2337, 2211, 2235, 2495}

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{f} - \frac{\sqrt{\pi}\sqrt{b}\sqrt{m}\sqrt{n}(e+fx)e^{-\frac{a}{bmn}}\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{2f}$$

```
[In] Int[Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]],x]
```

```
[Out] -1/2*(Sqrt[b]*Sqrt[m]*Sqrt[n]*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(
e + f*x)^m)^n]]/(Sqrt[b]*Sqrt[m]*Sqrt[n])])/(E^(a/(b*m*n))*f*(c*(d*(e + f*x
)^m)^n)^(1/(m*n))) + ((e + f*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]])/f
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}\,dx, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$=\text{Subst}\left(\frac{\text{Subst}\left(\int\sqrt{a+b\log\left(cd^n x^{mn}\right)}\,dx,x,e+fx\right)}{f},cd^n(e+fx)^{mn},c(d(e+fx)^m)^n\right)$$

$$=\frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{f}-\text{Subst}\left(\frac{(bmn)\text{Subst}\left(\int\frac{1}{\sqrt{a+b\log(cd^n x^{mn})}}\,dx,x,e+fx\right)}{2f},cd^n(e+fx)^{mn},c(d(e+fx)^m)^n\right)$$

$$=\frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{f}-\text{Subst}\left(\frac{\left(b(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{mn}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^n(e+fx)^{mn}\right)\right)}{2f},cd^n(e+fx)^{mn},c(d(e+fx)^m)^n\right)$$

$$=\frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{f}-\text{Subst}\left(\frac{\left((e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int e^{-\frac{a}{bmn}+\frac{x^2}{bmn}}\,dx,x,\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}\right)}{f},cd^n(e+fx)^{mn},c(d(e+fx)^m)^n\right)$$

$$=-\frac{\sqrt{b}e^{-\frac{a}{bmn}}\sqrt{m}\sqrt{n}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{2f}+\frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}{f}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \frac{(e+fx)\left(-\sqrt{b}e^{-\frac{a}{bmn}}\sqrt{m}\sqrt{n}\sqrt{\pi}\left(c\left(d(e+fx)^m\right)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)+2\sqrt{a+b\log\left(c\left(d(e+\right.\right.}}{2f}$$

```
[In] Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]],x]
[Out] ((e + f*x)*(-((Sqrt[b]*Sqrt[m]*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*(d*(e
 + f*x)^m)^n]]/(Sqrt[b]*Sqrt[m]*Sqrt[n])]])/(E^(a/(b*m*n))*(c*(d*(e + f*x)^m
)^n)^(1/(m*n)))) + 2*Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]))/(2*f)
```

## Maple [F]

$$\int \sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{m}\right)^{n}\right)}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2),x)
[Out] int((a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \int \sqrt{a+b\log\left(c\left(d\left(e+fx\right)^{m}\right)^{n}\right)}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**(1/2),x)
[Out] Integral(sqrt(a + b*log(c*(d*(e + f*x)**m)**n)), x)
```

## Maxima [F]

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \int \sqrt{b\log\left(\left((fx+e)^m d\right)^n c\right)+a}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*log(((f*x + e)^m*d)^n*c) + a), x)
```

## Giac [F]

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \int \sqrt{b\log\left(\left((fx+e)^m d\right)^n c\right)+a}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*log(((f*x + e)^m*d)^n*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}\,dx = \int \sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^m)^n))^(1/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^m)^n))^(1/2), x)
```

# 3.414 $\int \frac{1}{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}\,dx = \frac{e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{\sqrt{b}f\sqrt{m}\sqrt{n}}$$

```
[Out] (f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2))*Pi^
(1/2)/exp(a/b/m/n)/f/((c*(d*(f*x+e)^m)^n)^(1/m/n))/b^(1/2)/m^(1/2)/n^(1/2)
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2436, 2337, 2211, 2235, 2495}

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}\,dx = \frac{\sqrt{\pi}(e+fx)e^{-\frac{a}{bmn}}\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{\sqrt{b}f\sqrt{m}\sqrt{n}}$$

```
[In] Int[1/Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]],x]
```

```
[Out] (Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]/(Sqrt[b]*Sqrt
[m]*Sqrt[n])])/(Sqrt[b]*E^(a/(b*m*n))*f*Sqrt[m]*Sqrt[n]*(c*(d*(e + f*x)^m)^
n)^(1/(m*n)))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}}\,dx, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^nx^{mn})}}\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \text{Subst}\left(\frac{\left((e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{mn}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^n(e+fx)^{mn}\right)\right)}{fmn}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \text{Subst}\left(\frac{\left(2(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int e^{-\frac{a}{bmn}+\frac{x^2}{bmn}}\,dx, x, \sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}\right)}{bfmn}\right.$$

$$\left. + fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \frac{e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{\sqrt{b}f\sqrt{m}\sqrt{n}}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}\,dx$$
$$= \frac{e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{\sqrt{b}f\sqrt{m}\sqrt{n}}$$

```
[In] Integrate[1/Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]],x]
[Out] (Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]/(Sqrt[b]*Sqrt
[m]*Sqrt[n])])/(Sqrt[b]*E^(a/(b*m*n))*f*Sqrt[m]*Sqrt[n]*(c*(d*(e + f*x)^m)^
n)^(1/(m*n)))
```

## Maple [F]

$$\int \frac{1}{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^m\right)^n\right)}}dx$$

```
[In] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2),x)
[Out] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a + b\log\left(c\left(d(e+fx)^m\right)^n\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{a + b\log\left(c\left(d(e+fx)^m\right)^n\right)}}\,dx = \int \frac{1}{\sqrt{a + b\log\left(c\left(d\left(e+fx\right)^m\right)^n\right)}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n))**(1/2),x)
[Out] Integral(1/sqrt(a + b*log(c*(d*(e + f*x)**m)**n)), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{a + b\log\left(c\left(d(e+fx)^m\right)^n\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left(\left((fx+e)^m d\right)^n c\right) + a}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*log(((f*x + e)^m*d)^n*c) + a), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{a + b\log\left(c\left(d(e+fx)^m\right)^n\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left(\left((fx+e)^m d\right)^n c\right) + a}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*log(((f*x + e)^m*d)^n*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}\,dx = \int \frac{1}{\sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^m\right)^n\right)}}\,dx$$

```
[In] int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(1/2),x)
[Out] int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(1/2), x)
```

# 3.415 $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 147

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx=\frac{2e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{b^{3/2}fm^{3/2}n^{3/2}}-\frac{2(e+fx)}{bfmn\sqrt{a+b\log(c(d(e+fx)^m)^n)}}$$

```
[Out] 2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2))*P
i^(1/2)/b^(3/2)/exp(a/b/m/n)/f/m^(3/2)/n^(3/2)/((c*(d*(f*x+e)^m)^n)^(1/m/n)
)-2*(f*x+e)/b/f/m/n/(a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2436, 2334, 2337, 2211, 2235, 2495}

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx=\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bmn}}\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{b^{3/2}fm^{3/2}n^{3/2}}-\frac{2(e+fx)}{bfmn\sqrt{a+b\log(c(d(e+fx)^m)^n)}}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-3/2),x]
```

```
[Out] (2*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]/(Sqrt[b]*Sq
rt[m]*Sqrt[n])])/(b^(3/2)*E^(a/(b*m*n))*f*m^(3/2)*n^(3/2)*(c*(d*(e + f*x)^m
```

```
)^n)^(1/(m*n))) - (2*(e + f*x))/(b*f*m*n*Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n
]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\left(a+b\log\left(cd^n(e+fx)^{mn}\right)\right)^{3/2}}\,dx, cd^n(e+fx)^{mn}, c\left(d(e+fx)^m\right)^n\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cd^n x^{mn}))^{3/2}}\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{2(e+fx)}{bfmn\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}} + \text{Subst}\left(\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^n x^{mn})}}\,dx, x, e+fx\right)}{bfmn}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{2(e+fx)}{bfmn\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}} + \text{Subst}\left(\frac{\left(2(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{mn}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^n(e+fx)^{mn}\right)\right)}{bfm^2n^2}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{2(e+fx)}{bfmn\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}} + \text{Subst}\left(\frac{\left(4(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int e^{-\frac{a}{bmn}+\frac{x^2}{bmn}}\,dx, x, \sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right)}\right)}{b^2fm^2n^2}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \frac{2e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{b^{3/2}fm^{3/2}n^{3/2}} - \frac{2(e+fx)}{bfmn\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}$$

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.23

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx = -\frac{2e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\left(e^{\frac{a}{bmn}}\left(c(d(e+fx)^m)^n\right)^{\frac{1}{mn}}-\Gamma\left(\frac{1}{2},-\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)\sqrt{-\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}}\right)}{bfmn\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-3/2),x]
```

```
[Out] (-2*(e + f*x)*(E^(a/(b*m*n))*(c*(d*(e + f*x)^m)^n)^(1/(m*n)) - Gamma[1/2, -
((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n))]*Sqrt[-((a + b*Log[c*(d*(e + f*x
)^m)^n])/(b*m*n))]))/(b*E^(a/(b*m*n))*f*m*n*(c*(d*(e + f*x)^m)^n)^(1/(m*n))
*Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]])
```

## Maple [F]

$$\int \frac{1}{(a+b\ln(c(d(fx+e)^m)^n))^{\frac{3}{2}}}dx$$

```
[In] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2),x)
```

```
[Out] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx = \int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n))**(3/2),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**m)**n))**(-3/2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx = \int \frac{1}{(b\log(((fx+e)^m d)^n c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(3/2),x, algorithm="maxima")
[Out] integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(-3/2), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx = \int \frac{1}{(b\log(((fx+e)^m d)^n c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(3/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(-3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{3/2}}\,dx = \int \frac{1}{(a+b\,\ln(c(d(e+f\,x)^m)^n))^{3/2}}\,dx$$

```
[In] int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(3/2),x)
[Out] int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(3/2), x)
```

# 3.416 $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx=\frac{4e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{3b^{5/2}fm^{5/2}n^{5/2}}-\frac{2(e+fx)}{3bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}}-\frac{4(e+fx)}{3b^2fm^2n^2\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}$$

```
[Out] -2/3*(f*x+e)/b/f/m/n/(a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2)+4/3*(f*x+e)*erfi((a+
b*ln(c*(d*(f*x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2))*Pi^(1/2)/b^(5/2)/ex
p(a/b/m/n)/f/m^(5/2)/n^(5/2)/((c*(d*(f*x+e)^m)^n)^(1/m/n))-4/3*(f*x+e)/b^2/
f/m^2/n^2/(a+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2436, 2334, 2337, 2211, 2235, 2495}

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx=\frac{4\sqrt{\pi}(e+fx)e^{-\frac{a}{bmn}}\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{3b^{5/2}fm^{5/2}n^{5/2}}-\frac{4(e+fx)}{3b^2fm^2n^2\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}-\frac{2(e+fx)}{3bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-5/2),x]
```

```
[Out] (4*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]/(Sqrt[b]*Sq
rt[m]*Sqrt[n])])/(3*b^(5/2)*E^(a/(b*m*n))*f*m^(5/2)*n^(5/2)*(c*(d*(e + f*x)
```

```
^m)^n)^(1/(m*n))) - (2*(e + f*x))/(3*b*f*m*n*(a + b*Log[c*(d*(e + f*x)^m)^n
])^(3/2)) - (4*(e + f*x))/(3*b^2*f*m^2*n^2*Sqrt[a + b*Log[c*(d*(e + f*x)^m)
^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(a+b\log(cd^n(e+fx)^{mn}))^{5/2}}\,dx, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cd^n x^{mn}))^{5/2}}\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{2(e+fx)}{3bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}} + \text{Subst}\left(\frac{2\text{Subst}\left(\int \frac{1}{(a+b\log(cd^n x^{mn}))^{3/2}}\,dx, x, e+fx\right)}{3bfmn}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{2(e+fx)}{3bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}} - \frac{4(e+fx)}{3b^2fm^2n^2\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}} + \text{Subst}\left(\frac{4\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^n x^{mn})}}\,dx, x, e+fx\right)}{3b^2fm^2n^2}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{2(e+fx)}{3bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}} - \frac{4(e+fx)}{3b^2fm^2n^2\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}} + \text{Subst}\left(\frac{\left(4(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{mn}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^n(e+fx)^{mn}\right)\right)}{3b^2fm^3n^3}, cd^n(e+f$$

$$= -\frac{2(e+fx)}{3bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}} - \frac{4(e+fx)}{3b^2fm^2n^2\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}} + \text{Subst}\left(\frac{\left(8(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int e^{-\frac{a}{bmn}+\frac{x^2}{bmn}}\,dx, x, \sqrt{a+b\log\left(cd^n(e+fx)^{mn}\right.}\right.}{3b^3fm^3n^3}\right.$$

$$= \frac{4e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c\left(d(e+fx)^m\right)^n\right)^{-\frac{1}{mn}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{3b^{5/2}fm^{5/2}n^{5/2}} - \frac{2(e+fx)}{3bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}} - \frac{4(e+fx)}{3b^2fm^2n^2\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}$$

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{5/2}}\,dx = -\frac{2e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\left(2bmn\Gamma\left(\frac{1}{2}, -\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)\left(-\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)^{3/2} + e^{\frac{a}{bmn}}\right.}{3b^2fm^2n^2\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-5/2),x]
```

```
[Out] (-2*(e + f*x)*(2*b*m*n*Gamma[1/2, -((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n
))]*(-((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)))^(3/2) + E^(a/(b*m*n))*(c*
(d*(e + f*x)^m)^n)^(1/(m*n))*(2*a + b*m*n + 2*b*Log[c*(d*(e + f*x)^m)^n])))
/(3*b^2*E^(a/(b*m*n))*f*m^2*n^2*(c*(d*(e + f*x)^m)^n)^(1/(m*n))*(a + b*Log[
c*(d*(e + f*x)^m)^n])^(3/2))
```

## Maple [F]

$$\int \frac{1}{\left(a+b\ln \left(c\left(d\left(fx+e\right)^{m}\right)^{n}\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(5/2),x)
```

```
[Out] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\log \left(c\left(d(e+fx)^{m}\right)^{n}\right)\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{\left(a+b\log \left(c\left(d(e+fx)^{m}\right)^{n}\right)\right)^{5/2}}\,dx=\int \frac{1}{\left(a+b\log \left(c\left(d\left(e+fx\right)^{m}\right)^{n}\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n))**(5/2),x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**m)**n))**(-5/2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx = \int \frac{1}{(b\log(((fx+e)^m d)^n c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(5/2),x, algorithm="maxima")
[Out] integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(-5/2), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx = \int \frac{1}{(b\log(((fx+e)^m d)^n c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(5/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(-5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{5/2}}\,dx = \int \frac{1}{(a+b\,\ln(c\,(d\,(e+f\,x)^m)^n))^{5/2}}\,dx$$

```
[In] int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(5/2),x)
[Out] int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(5/2), x)
```

# 3.417 $\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{7/2}} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 237

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{7/2}} \, dx = \frac{8e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}} \operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{15b^{7/2}fm^{7/2}n^{7/2}} - \frac{2(e+fx)}{5bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{5/2}} - \frac{4(e+fx)}{15b^2fm^2n^2\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}} - \frac{8(e+fx)}{15b^3fm^3n^3\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}$$

```
[Out] -2/5*(f*x+e)/b/f/m/n/(a+b*ln(c*(d*(f*x+e)^m)^n))^(5/2)-4/15*(f*x+e)/b^2/f/m
^2/n^2/(a+b*ln(c*(d*(f*x+e)^m)^n))^(3/2)+8/15*(f*x+e)*erfi((a+b*ln(c*(d*(f*
x+e)^m)^n))^(1/2)/b^(1/2)/m^(1/2)/n^(1/2))*Pi^(1/2)/b^(7/2)/exp(a/b/m/n)/f/
m^(7/2)/n^(7/2)/((c*(d*(f*x+e)^m)^n)^(1/m/n))-8/15*(f*x+e)/b^3/f/m^3/n^3/(a
+b*ln(c*(d*(f*x+e)^m)^n))^(1/2)
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2436, 2334, 2337, 2211, 2235, 2495}

$$\int \frac{1}{(a+b\log(c(d(e+fx)^m)^n))^{7/2}} \, dx = \frac{8\sqrt{\pi}(e+fx)e^{-\frac{a}{bmn}}\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}} \operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{15b^{7/2}fm^{7/2}n^{7/2}} - \frac{8(e+fx)}{15b^3fm^3n^3\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}} - \frac{4(e+fx)}{15b^2fm^2n^2\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}} - \frac{2(e+fx)}{5bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{5/2}}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-7/2),x]
```

```
[Out] (8*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^m)^n]]/(Sqrt[b]*Sq
rt[m]*Sqrt[n])])/(15*b^(7/2)*E^(a/(b*m*n))*f*m^(7/2)*n^(7/2)*(c*(d*(e + f*x
)^m)^n)^(1/(m*n))) - (2*(e + f*x))/(5*b*f*m*n*(a + b*Log[c*(d*(e + f*x)^m)^
n])^(5/2)) - (4*(e + f*x))/(15*b^2*f*m^2*n^2*(a + b*Log[c*(d*(e + f*x)^m)^n
])^(3/2)) - (8*(e + f*x))/(15*b^3*f*m^3*n^3*Sqrt[a + b*Log[c*(d*(e + f*x)^m
)^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(a+b\log(cd^n(e+fx)^{mn}))^{7/2}}\,dx, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cd^nx^{mn}))^{7/2}}\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{2(e+fx)}{5bfmn\,(a+b\log(c\,(d(e+fx)^m)^n))^{5/2}} + \text{Subst}\left(\frac{2\text{Subst}\left(\int \frac{1}{(a+b\log(cd^nx^{mn}))^{5/2}}\,dx, x, e+fx\right)}{5bfmn}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{2(e+fx)}{5bfmn\,(a+b\log(c\,(d(e+fx)^m)^n))^{5/2}} - \frac{4(e+fx)}{15b^2fm^2n^2\,(a+b\log(c\,(d(e+fx)^m)^n))^{3/2}} + \text{Subst}\left(\frac{4\text{Subst}\left(\int \frac{1}{(a+b\log(cd^nx^{mn}))^{3/2}}\,dx, x, e+fx\right)}{15b^2fm^2n^2}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{2(e+fx)}{5bfmn\,(a+b\log(c\,(d(e+fx)^m)^n))^{5/2}} - \frac{4(e+fx)}{15b^2fm^2n^2\,(a+b\log(c\,(d(e+fx)^m)^n))^{3/2}} - \frac{8(e+fx)}{15b^3fm^3n^3\sqrt{a+b\log(c\,(d(e+fx)^m)^n)}} + \text{Subst}\left(\frac{8\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^nx^{mn})}}\,dx, x, e+fx\right)}{15b^3fm^3n^3}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= -\frac{2(e+fx)}{5bfmn\,(a+b\log(c\,(d(e+fx)^m)^n))^{5/2}} - \frac{4(e+fx)}{15b^2fm^2n^2\,(a+b\log(c\,(d(e+fx)^m)^n))^{3/2}} - \frac{8(e+fx)}{15b^3fm^3n^3\sqrt{a+b\log(c\,(d(e+fx)^m)^n)}} + \text{Subst}\left(\frac{\left(8(e+fx)\,(cd^n(e+fx)^{mn})^{-\frac{1}{mn}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{mn}}}{\sqrt{a+bx}}\,dx, x, \log(cd^n(e+fx)^{mn})\right)}{15b^3fm^4n^4}, cd^n(e+fx$$

$$= -\frac{2(e+fx)}{5bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{5/2}} - \frac{4(e+fx)}{15b^2fm^2n^2\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}} - \frac{8(e+fx)}{15b^3fm^3n^3\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}} + \text{Subst}\left(\frac{\left(16(e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int e^{-\frac{a}{bmn}+\frac{x^2}{bmn}}\,dx, x, \sqrt{a+b\log\left(cd^n(e+fx)^m\right.}\right.}{15b^4fm^4n^4}\right.$$

$$= \frac{8e^{-\frac{a}{bmn}}\sqrt{\pi}(e+fx)\left(c\left(d(e+fx)^m\right)^n\right)^{-\frac{1}{mn}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^m)^n)}}{\sqrt{b}\sqrt{m}\sqrt{n}}\right)}{15b^{7/2}fm^{7/2}n^{7/2}} - \frac{2(e+fx)}{5bfmn\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{5/2}} - \frac{4(e+fx)}{15b^2fm^2n^2\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{3/2}} - \frac{8(e+fx)}{15b^3fm^3n^3\sqrt{a+b\log\left(c\left(d(e+fx)^m\right)^n\right)}}$$

## Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{7/2}}\,dx = -\frac{2e^{-\frac{a}{bmn}}(e+fx)\left(c\left(d(e+fx)^m\right)^n\right)^{-\frac{1}{mn}}\left(-4\Gamma\left(\frac{1}{2}, -\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^2\sqrt{-\frac{a-}{}}\right.}{15b^3}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^(-7/2),x]
```

```
[Out] (-2*(e + f*x)*(-4*Gamma[1/2, -((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n))]*(
a + b*Log[c*(d*(e + f*x)^m)^n])^2*Sqrt[-((a + b*Log[c*(d*(e + f*x)^m)^n])/(
b*m*n))] + E^(a/(b*m*n))*(c*(d*(e + f*x)^m)^n)^(1/(m*n))*(4*a^2 + 2*a*b*m*n
 + 3*b^2*m^2*n^2 + 2*b*(4*a + b*m*n)*Log[c*(d*(e + f*x)^m)^n] + 4*b^2*Log[c
*(d*(e + f*x)^m)^n]^2)))/(15*b^3*E^(a/(b*m*n))*f*m^3*n^3*(c*(d*(e + f*x)^m)
^n)^(1/(m*n))*(a + b*Log[c*(d*(e + f*x)^m)^n])^(5/2))
```

## Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{m}\right)^{n}\right)\right)^{\frac{7}{2}}}dx$$

```
[In] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(7/2),x)
[Out] int(1/(a+b*ln(c*(d*(f*x+e)^m)^n))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^{m}\right)^{n}\right)\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^{m}\right)^{n}\right)\right)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**m)**n))**(7/2),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^{m}\right)^{n}\right)\right)^{7/2}}\,dx = \int \frac{1}{\left(b\log\left(\left(\left(fx+e\right)^{m}d\right)^{n}c\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(7/2),x, algorithm="maxima")
[Out] integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(-7/2), x)
```

## Giac [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{7/2}}\,dx=\int \frac{1}{\left(b\log\left(\left(\left(fx+e\right)^m d\right)^n c\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^m)^n))^(7/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^m*d)^n*c) + a)^(-7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^m\right)^n\right)\right)^{7/2}}\,dx=\int \frac{1}{\left(a+b\,\ln\left(c\,\left(d\,{\left(e+f\,x\right)}^m\right)^n\right)\right)^{7/2}}\,dx$$

```
[In] int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(7/2),x)
[Out] int(1/(a + b*log(c*(d*(e + f*x)^m)^n))^(7/2), x)
```

# 3.418 $\int (a+b\log(c(d(e+fx)^m)^n))^p \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 131

$$\int (a+b\log(c(d(e+fx)^m)^n))^p \, dx = \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\Gamma\left(1+p,-\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)(a+b\log(c(d(e+fx)^m)^n))^p\left(-\frac{a+b\log(\cdots}{\cdots}\right.}{f}$$

[Out] (f*x+e)*GAMMA(p+1,(-a-b*ln(c*(d*(f*x+e)^m)^n))/b/m/n)*(a+b*ln(c*(d*(f*x+e)^m)^n))^p/exp(a/b/m/n)/f/((c*(d*(f*x+e)^m)^n)^(1/m/n))/(((-a-b*ln(c*(d*(f*x+e)^m)^n))/b/m/n)^p)

## Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2436, 2337, 2212, 2495}

$$\int (a+b\log(c(d(e+fx)^m)^n))^p \, dx = \frac{(e+fx)e^{-\frac{a}{bmn}}\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}(a+b\log(c(d(e+fx)^m)^n))^p\left(-\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\cdots}{\cdots}\right.}{f}$$

[In] Int[(a + b*Log[c*(d*(e + f*x)^m)^n])^p,x]

[Out] ((e + f*x)*Gamma[1 + p, -((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n))]*(a + b*Log[c*(d*(e + f*x)^m)^n])^p)/(E^(a/(b*m*n))*f*(c*(d*(e + f*x)^m)^n)^(1/(m*n))*(-((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)))^p)

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (a+b\log(cd^n(e+fx)^{mn}))^p\,dx, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int (a+b\log(cd^nx^{mn}))^p\,dx, x, e+fx\right)}{f}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \text{Subst}\left(\frac{\left((e+fx)\left(cd^n(e+fx)^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int e^{\frac{x}{mn}}(a+bx)^p\,dx, x, \log(cd^n(e+fx)^{mn})\right)}{fmn}, cd^n(e+fx)^{mn}, c(d(e+fx)^m)^n\right)$$

$$= \frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\Gamma\left(1+p, -\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)(a+b\log(c(d(e+fx)^m)^n))^p}{f}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int (a+b\log (c(d(e+fx)^m)^n))^p\,dx$$
$$=\frac{e^{-\frac{a}{bmn}}(e+fx)\left(c(d(e+fx)^m)^n\right)^{-\frac{1}{mn}}\Gamma\left(1+p,-\frac{a+b\log(c(d(e+fx)^m)^n)}{bmn}\right)(a+b\log(c(d(e+fx)^m)^n))^p\left(-\frac{a+b\log(\ldots}{\ldots}\right.}{f}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^m)^n])^p,x]
```

```
[Out] ((e + f*x)*Gamma[1 + p, -((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n))]*(a + b
*Log[c*(d*(e + f*x)^m)^n])^p)/(E^(a/(b*m*n))*f*(c*(d*(e + f*x)^m)^n)^(1/(m*
n))*(-((a + b*Log[c*(d*(e + f*x)^m)^n])/(b*m*n)))^p)
```

## Maple [F]

$$\int (a+b\ln (c(d(fx+e)^m)^n))^p dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^m)^n))^p,x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^m)^n))^p,x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int (a+b\log (c(d(e+fx)^m)^n))^p\,dx$$
$$=\frac{e^{\left(-\frac{bmnp\log\left(-\frac{1}{bmn}\right)+bn\log(d)+b\log(c)+a}{bmn}\right)}\Gamma\left(p+1,-\frac{bmn\log(fx+e)+bn\log(d)+b\log(c)+a}{bmn}\right)}{f}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^p,x, algorithm="fricas")
```

```
[Out] e^(-(b*m*n*p*log(-1/(b*m*n)) + b*n*log(d) + b*log(c) + a)/(b*m*n))*gamma(p
+ 1, -(b*m*n*log(f*x + e) + b*n*log(d) + b*log(c) + a)/(b*m*n))/f
```

## Sympy [F]

$$\int (a+b\log (c(d(e+fx)^m)^n))^p \, dx = \int (a+b\log (c(d(e+fx)^m)^n))^p \, dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**m)**n))**p,x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**m)**n))**p, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (a+b\log (c(d(e+fx)^m)^n))^p \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^p,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

## Giac [F]

$$\int (a+b\log (c(d(e+fx)^m)^n))^p \, dx = \int (b\log (((fx+e)^m d)^n c)+a)^p \, dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^m)^n))^p,x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^m*d)^n*c) + a)^p, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\log (c(d(e+fx)^m)^n))^p \, dx = \int (a+b\,\ln (c\,(d\,(e+f\,x)^m)^n))^p \, dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^m)^n))^p,x)
[Out] int((a + b*log(c*(d*(e + f*x)^m)^n))^p, x)
```

# 3.419 $\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p dx$



## Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p dx = \frac{4^{-p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)}{b}\right)^{-p}}{c^4d^2f}$$

```
[Out] GAMMA(p+1,-4*(a+b*ln(c*(d*(f*x+e)^(1/2))^(1/2)))/b)*(a+b*ln(c*(d*(f*x+e)^(1
/2))^(1/2)))^p/(4^p)/c^4/d^2/exp(4*a/b)/f/(((-a-b*ln(c*(d*(f*x+e)^(1/2))^(1
/2)))/b)^p)
```

## Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2436, 2336, 2212, 2495}

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p dx = \frac{4^{-p}e^{-\frac{4a}{b}}\left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)}{b}\right)}{c^4d^2f}$$

```
[In] Int[(a + b*Log[c*Sqrt[d*Sqrt[e + f*x]]])^p,x]
```

```
[Out] (Gamma[1 + p, (-4*(a + b*Log[c*Sqrt[d*Sqrt[e + f*x]]]))/b]*(a + b*Log[c*Sqr
t[d*Sqrt[e + f*x]]])^p)/(4^p*c^4*d^2*E^((4*a)/b)*f*(-((a + b*Log[c*Sqrt[d*S
qrt[e + f*x]]])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \left(a+b\log\left(c\sqrt{d}\sqrt[4]{e+fx}\right)\right)^p\,dx, c\sqrt{d}\sqrt[4]{e+fx}, c\sqrt{d\sqrt{e+fx}}\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int \left(a+b\log\left(c\sqrt{d}\sqrt[4]{x}\right)\right)^p\,dx, x, e+fx\right)}{f}, c\sqrt{d}\sqrt[4]{e+fx}, c\sqrt{d\sqrt{e+fx}}\right)\\
&= \text{Subst}\left(\frac{4\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\sqrt{d}\sqrt[4]{e+fx}\right)\right)}{c^4d^2f}, c\sqrt{d}\sqrt[4]{e+fx}, c\sqrt{d\sqrt{e+fx}}\right)\\
&= \frac{4^{-p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)}{b}\right)^{-p}}{c^4d^2f}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx$$
$$=\frac{2^{-2p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)}{b}\right)^{-p}}{c^4d^2f}$$

```
[In] Integrate[(a + b*Log[c*Sqrt[d*Sqrt[e + f*x]]])^p,x]
[Out] (Gamma[1 + p, (-4*(a + b*Log[c*Sqrt[d*Sqrt[e + f*x]]]))/b]*(a + b*Log[c*Sqr
t[d*Sqrt[e + f*x]]])^p)/(2^(2*p)*c^4*d^2*E^((4*a)/b)*f*(-((a + b*Log[c*Sqrt
[d*Sqrt[e + f*x]]])/b))^p)
```

## Maple [F]

$$\int \left(a+b\ln\left(c\sqrt{d\sqrt{fx+e}}\right)\right)^p\,dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^(1/2))^(1/2)))^p,x)
[Out] int((a+b*ln(c*(d*(f*x+e)^(1/2))^(1/2)))^p,x)
```

## Fricas [F]

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx=\int \left(b\log\left(\sqrt{\sqrt{fx+e}dc}\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^(1/2))^(1/2)))^p,x, algorithm="fricas")
[Out] integral((b*log(sqrt(sqrt(f*x + e)*d)*c) + a)^p, x)
```

## Sympy [F]

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx=\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**(1/2))**(1/2)))**p,x)
[Out] Integral((a + b*log(c*sqrt(d*sqrt(e + f*x))))**p, x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx = -\frac{4\left(b\log\left(\sqrt{\sqrt{fx+e}dc}\right)+a\right)^{p+1}e^{\left(-\frac{4a}{b}\right)}E_{-p}\left(-\frac{4\left(b\log\left(\sqrt{\sqrt{fx+e}dc}\right)+a\right)}{b}\right)}{bc^4d^2f}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^(1/2))^(1/2)))^p,x, algorithm="maxima")
[Out] -4*(b*log(sqrt(sqrt(f*x + e)*d)*c) + a)^(p + 1)*e^(-4*a/b)*exp_integral_e(-
p, -4*(b*log(sqrt(sqrt(f*x + e)*d)*c) + a)/b)/(b*c^4*d^2*f)
```

## Giac [F]

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx = \int \left(b\log\left(\sqrt{\sqrt{fx+e}dc}\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^(1/2))^(1/2)))^p,x, algorithm="giac")
[Out] integrate((b*log(sqrt(sqrt(f*x + e)*d)*c) + a)^p, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\sqrt{d\sqrt{e+fx}}\right)\right)^p\,dx = \int \left(a+b\,\ln\left(c\,\sqrt{d\,\sqrt{e+f\,x}}\right)\right)^p\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^(1/2))^(1/2)))^p,x)
[Out] int((a + b*log(c*(d*(e + f*x)^(1/2))^(1/2)))^p, x)
```

# 3.420 $\int (g+hx)^3 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right) \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 158

$$\int (g+hx)^3 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right) \, dx = -\frac{b(fg-eh)^3pqx}{4f^3}-\frac{b(fg-eh)^2pq(g+hx)^2}{8f^2h}-\frac{b(fg-eh)pq(g+hx)^3}{12fh}-\frac{bpq(g+hx)^4}{16h}-\frac{b(fg-eh)^4pq\log(e+fx)}{4f^4h}+\frac{(g+hx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{4h}$$

```
[Out] -1/4*b*(-e*h+f*g)^3*p*q*x/f^3-1/8*b*(-e*h+f*g)^2*p*q*(h*x+g)^2/f^2/h-1/12*b
*(-e*h+f*g)*p*q*(h*x+g)^3/f/h-1/16*b*p*q*(h*x+g)^4/h-1/4*b*(-e*h+f*g)^4*p*q
*ln(f*x+e)/f^4/h+1/4*(h*x+g)^4*(a+b*ln(c*(d*(f*x+e)^p)^q))/h
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2442, 45, 2495}

$$\int (g+hx)^3 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right) \, dx = \frac{(g+hx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{4h}-\frac{bpq(fg-eh)^4\log(e+fx)}{4f^4h}-\frac{bpqx(fg-eh)^3}{4f^3}-\frac{bpq(g+hx)^2(fg-eh)^2}{8f^2h}-\frac{bpq(g+hx)^3(fg-eh)}{12fh}-\frac{bpq(g+hx)^4}{16h}$$

```
[In] Int[(g + h*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] -1/4*(b*(f*g - e*h)^3*p*q*x)/f^3 - (b*(f*g - e*h)^2*p*q*(g + h*x)^2)/(8*f^2
*h) - (b*(f*g - e*h)*p*q*(g + h*x)^3)/(12*f*h) - (b*p*q*(g + h*x)^4)/(16*h)
 - (b*(f*g - e*h)^4*p*q*Log[e + f*x])/(4*f^4*h) + ((g + h*x)^4*(a + b*Log[c
*(d*(e + f*x)^p)^q]))/(4*h)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int (g+hx)^3\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{(g+hx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{4h}\\
&\quad -\text{Subst}\left(\frac{(bfpq)\int\frac{(g+hx)^4}{e+fx}\,dx}{4h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{(g+hx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{4h}\\
&\quad -\text{Subst}\left(\frac{(bfpq)\int\left(\frac{h(fg-eh)^3}{f^4}+\frac{(fg-eh)^4}{f^4(e+fx)}+\frac{h(fg-eh)^2(g+hx)}{f^3}+\frac{h(fg-eh)(g+hx)^2}{f^2}+\frac{h(g+hx)^3}{f}\right)dx}{4h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}
$$

$$= -\frac{b(fg-eh)^3pqx}{4f^3} - \frac{b(fg-eh)^2pq(g+hx)^2}{8f^2h} - \frac{b(fg-eh)pq(g+hx)^3}{12fh} - \frac{bpq(g+hx)^4}{16h} - \frac{b(fg-eh)^4pq\log(e+fx)}{4f^4h} + \frac{(g+hx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{4h}$$

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.47

$$\int (g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx = \frac{fx(12af^3(4g^3+6g^2hx+4gh^2x^2+h^3x^3)-bpq(-12e^3h^3+6e^2fh^2(8g+hx)-4ef^2h(18g^2+6ghx+h^2x^2}{}$$

```
[In] Integrate[(g + h*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] (f*x*(12*a*f^3*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) - b*p*q*(-12*e^3
*h^3 + 6*e^2*f*h^2*(8*g + h*x) - 4*e*f^2*h*(18*g^2 + 6*g*h*x + h^2*x^2) + f
^3*(48*g^3 + 36*g^2*h*x + 16*g*h^2*x^2 + 3*h^3*x^3))) - 12*b*e^2*h*(6*f^2*g
^2 - 4*e*f*g*h + e^2*h^2)*p*q*Log[e + f*x] + 12*b*f^3*(4*e*g^3 + f*x*(4*g^3
 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3))*Log[c*(d*(e + f*x)^p)^q])/(48*f^4)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(146) = 292.

Time = 5.98 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.95

| method | result |
|---|---|
| parallelrisch | $-\frac{-48\ln(fx+e)be^3fgh^2pq+72\ln(fx+e)be^2f^2g^2hpq-24x^2bef^3gh^2pq+48xbe^2f^2gh^2pq-48x^3af^4gh^2-72x^2af^4g^2h-48x\ln}{}$ |

```
[In] int((h*x+g)^3*(a+b*ln(c*(d*(f*x+e)^p)^q)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(-48*ln(f*x+e)*b*e^3*f*g*h^2*p*q+72*ln(f*x+e)*b*e^2*f^2*g^2*h*p*q-24*
x^2*b*e*f^3*g*h^2*p*q+48*x*b*e^2*f^2*g*h^2*p*q-48*x^3*a*f^4*g*h^2-72*x^2*a*
f^4*g^2*h-48*x*ln(c*(d*(f*x+e)^p)^q)*b*f^4*g^3+48*ln(c*(d*(f*x+e)^p)^q)*b*e
*f^3*g^3-48*b*e*f^3*g^3*p*q+12*b*e^4*h^3*p*q-72*x*b*e*f^3*g^2*h*p*q+72*b*e^
2*g^2*h*p*q*f^2-48*b*e^3*f*g*h^2*p*q-12*x^4*ln(c*(d*(f*x+e)^p)^q)*b*f^4*h^3
+48*a*e*g^3*f^3-72*x^2*ln(c*(d*(f*x+e)^p)^q)*b*f^4*g^2*h+48*x*b*f^4*g^3*p*q
+12*ln(f*x+e)*b*e^4*h^3*p*q+3*x^4*b*f^4*h^3*p*q-48*x^3*ln(c*(d*(f*x+e)^p)^q
)*b*f^4*g*h^2-12*x^4*a*f^4*h^3-48*x*a*f^4*g^3+36*x^2*b*f^4*g^2*h*p*q-12*x*b
*e^3*f*h^3*p*q-96*ln(f*x+e)*b*e*f^3*g^3*p*q-4*x^3*b*e*f^3*h^3*p*q+16*x^3*b*
f^4*g*h^2*p*q+6*x^2*b*e^2*f^2*h^3*p*q)/f^4
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(146) = 292.

Time = 0.36 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.56

$$\int (g+hx)^3 (a+b\log(c(d(e+fx)^p)^q))\,dx = -\frac{3\,(bf^4h^3pq-4\,af^4h^3)x^4-4\,(12\,af^4gh^2-(4\,bf^4gh^2-bef^3h^3)pq)x^3-6\,(12\,af^4g^2h-(6\,bf^4g^2h-4\,be}{}$$

```
[In] integrate((h*x+g)^3*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

```
[Out] -1/48*(3*(b*f^4*h^3*p*q - 4*a*f^4*h^3)*x^4 - 4*(12*a*f^4*g*h^2 - (4*b*f^4*g
*h^2 - b*e*f^3*h^3)*p*q)*x^3 - 6*(12*a*f^4*g^2*h - (6*b*f^4*g^2*h - 4*b*e*f
^3*g*h^2 + b*e^2*f^2*h^3)*p*q)*x^2 - 12*(4*a*f^4*g^3 - (4*b*f^4*g^3 - 6*b*e
*f^3*g^2*h + 4*b*e^2*f^2*g*h^2 - b*e^3*f*h^3)*p*q)*x - 12*(b*f^4*h^3*p*q*x^
4 + 4*b*f^4*g*h^2*p*q*x^3 + 6*b*f^4*g^2*h*p*q*x^2 + 4*b*f^4*g^3*p*q*x + (4*
b*e*f^3*g^3 - 6*b*e^2*f^2*g^2*h + 4*b*e^3*f*g*h^2 - b*e^4*h^3)*p*q)*log(f*x
 + e) - 12*(b*f^4*h^3*x^4 + 4*b*f^4*g*h^2*x^3 + 6*b*f^4*g^2*h*x^2 + 4*b*f^4
*g^3*x)*log(c) - 12*(b*f^4*h^3*q*x^4 + 4*b*f^4*g*h^2*q*x^3 + 6*b*f^4*g^2*h*
q*x^2 + 4*b*f^4*g^3*q*x)*log(d))/f^4
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(139) = 278.

Time = 2.50 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.89

$$\int (g+hx)^3 (a+b\log(c(d(e+fx)^p)^q))\,dx = \begin{cases} ag^3x+\frac{3ag^2hx^2}{2}+agh^2x^3+\frac{ah^3x^4}{4}-\frac{be^4h^3\log(c(d(e+fx)^p)^q)}{4f^4}+\frac{be^3gh^2\log(c(d(e+fx)^p)^q)}{f^3}+\frac{be^3h^3pqx}{4f^3}-\frac{3be^2g^2h\log(c(d(e}{2f^2} \\ (a+b\log(c(de^p)^q))\left(g^3x+\frac{3g^2hx^2}{2}+gh^2x^3+\frac{h^3x^4}{4}\right) \end{cases}$$

```
[In] integrate((h*x+g)**3*(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

```
[Out] Piecewise((a*g**3*x + 3*a*g**2*h*x**2/2 + a*g*h**2*x**3 + a*h**3*x**4/4 - b
*e**4*h**3*log(c*(d*(e + f*x)**p)**q)/(4*f**4) + b*e**3*g*h**2*log(c*(d*(e
+ f*x)**p)**q)/f**3 + b*e**3*h**3*p*q*x/(4*f**3) - 3*b*e**2*g**2*h*log(c*(d
*(e + f*x)**p)**q)/(2*f**2) - b*e**2*g*h**2*p*q*x/f**2 - b*e**2*h**3*p*q*x*
*2/(8*f**2) + b*e*g**3*log(c*(d*(e + f*x)**p)**q)/f + 3*b*e*g**2*h*p*q*x/(2
*f) + b*e*g*h**2*p*q*x**2/(2*f) + b*e*h**3*p*q*x**3/(12*f) - b*g**3*p*q*x +
 b*g**3*x*log(c*(d*(e + f*x)**p)**q) - 3*b*g**2*h*p*q*x**2/4 + 3*b*g**2*h*x
**2*log(c*(d*(e + f*x)**p)**q)/2 - b*g*h**2*p*q*x**3/3 + b*g*h**2*x**3*log(
c*(d*(e + f*x)**p)**q) - b*h**3*p*q*x**4/16 + b*h**3*x**4*log(c*(d*(e + f*x
)**p)**q)/4, Ne(f, 0)), ((a + b*log(c*(d*e**p)**q))*(g**3*x + 3*g**2*h*x**2
/2 + g*h**2*x**3 + h**3*x**4/4), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(146) = 292.

Time = 0.20 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.92

$$
\begin{aligned}
&\int (g+hx)^3 (a+b\log(c(d(e+fx)^p)^q))\,dx \\
&= \frac{1}{4}bh^3x^4\log(((fx+e)^pd)^qc) + \frac{1}{4}ah^3x^4 - bfg^3pq\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right) \\
&\quad - \frac{1}{48}bfh^3pq\left(\frac{12\,e^4\log(fx+e)}{f^5} + \frac{3\,f^3x^4 - 4\,ef^2x^3 + 6\,e^2fx^2 - 12\,e^3x}{f^4}\right) \\
&\quad + \frac{1}{6}bfgh^2pq\left(\frac{6\,e^3\log(fx+e)}{f^4} - \frac{2\,f^2x^3 - 3\,efx^2 + 6\,e^2x}{f^3}\right) \\
&\quad - \frac{3}{4}bfg^2hpq\left(\frac{2\,e^2\log(fx+e)}{f^3} + \frac{fx^2 - 2\,ex}{f^2}\right) + bgh^2x^3\log(((fx+e)^pd)^qc) + agh^2x^3 \\
&\quad + \frac{3}{2}bg^2hx^2\log(((fx+e)^pd)^qc) + \frac{3}{2}ag^2hx^2 + bg^3x\log(((fx+e)^pd)^qc) + ag^3x
\end{aligned}
$$

```
[In] integrate((h*x+g)^3*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

```
[Out] 1/4*b*h^3*x^4*log(((f*x + e)^p*d)^q*c) + 1/4*a*h^3*x^4 - b*f*g^3*p*q*(x/f -
 e*log(f*x + e)/f^2) - 1/48*b*f*h^3*p*q*(12*e^4*log(f*x + e)/f^5 + (3*f^3*x
^4 - 4*e*f^2*x^3 + 6*e^2*f*x^2 - 12*e^3*x)/f^4) + 1/6*b*f*g*h^2*p*q*(6*e^3*
log(f*x + e)/f^4 - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/f^3) - 3/4*b*f*g^2*h*p
*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)/f^2) + b*g*h^2*x^3*log(((f*x +
 e)^p*d)^q*c) + a*g*h^2*x^3 + 3/2*b*g^2*h*x^2*log(((f*x + e)^p*d)^q*c) + 3/
2*a*g^2*h*x^2 + b*g^3*x*log(((f*x + e)^p*d)^q*c) + a*g^3*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 987 vs. 2(146) = 292.

Time = 0.34 (sec) , antiderivative size = 987, normalized size of antiderivative = 6.25

$$\int (g+hx)^3 (a+b\log(c(d(e+fx)^p)^q))\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^3*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

```
[Out] (f*x + e)*b*g^3*p*q*log(f*x + e)/f + 3/2*(f*x + e)^2*b*g^2*h*p*q*log(f*x +
e)/f^2 - 3*(f*x + e)*b*e*g^2*h*p*q*log(f*x + e)/f^2 + (f*x + e)^3*b*g*h^2*p
*q*log(f*x + e)/f^3 - 3*(f*x + e)^2*b*e*g*h^2*p*q*log(f*x + e)/f^3 + 3*(f*x
 + e)*b*e^2*g*h^2*p*q*log(f*x + e)/f^3 + 1/4*(f*x + e)^4*b*h^3*p*q*log(f*x
+ e)/f^4 - (f*x + e)^3*b*e*h^3*p*q*log(f*x + e)/f^4 + 3/2*(f*x + e)^2*b*e^2
*h^3*p*q*log(f*x + e)/f^4 - (f*x + e)*b*e^3*h^3*p*q*log(f*x + e)/f^4 - (f*x
```

```
 + e)*b*g^3*p*q/f - 3/4*(f*x + e)^2*b*g^2*h*p*q/f^2 + 3*(f*x + e)*b*e*g^2*h
*p*q/f^2 - 1/3*(f*x + e)^3*b*g*h^2*p*q/f^3 + 3/2*(f*x + e)^2*b*e*g*h^2*p*q/
f^3 - 3*(f*x + e)*b*e^2*g*h^2*p*q/f^3 - 1/16*(f*x + e)^4*b*h^3*p*q/f^4 + 1/
3*(f*x + e)^3*b*e*h^3*p*q/f^4 - 3/4*(f*x + e)^2*b*e^2*h^3*p*q/f^4 + (f*x +
e)*b*e^3*h^3*p*q/f^4 + (f*x + e)*b*g^3*q*log(d)/f + 3/2*(f*x + e)^2*b*g^2*h
*q*log(d)/f^2 - 3*(f*x + e)*b*e*g^2*h*q*log(d)/f^2 + (f*x + e)^3*b*g*h^2*q*
log(d)/f^3 - 3*(f*x + e)^2*b*e*g*h^2*q*log(d)/f^3 + 3*(f*x + e)*b*e^2*g*h^2
*q*log(d)/f^3 + 1/4*(f*x + e)^4*b*h^3*q*log(d)/f^4 - (f*x + e)^3*b*e*h^3*q*
log(d)/f^4 + 3/2*(f*x + e)^2*b*e^2*h^3*q*log(d)/f^4 - (f*x + e)*b*e^3*h^3*q
*log(d)/f^4 + (f*x + e)*b*g^3*log(c)/f + 3/2*(f*x + e)^2*b*g^2*h*log(c)/f^2
 - 3*(f*x + e)*b*e*g^2*h*log(c)/f^2 + (f*x + e)^3*b*g*h^2*log(c)/f^3 - 3*(f
*x + e)^2*b*e*g*h^2*log(c)/f^3 + 3*(f*x + e)*b*e^2*g*h^2*log(c)/f^3 + 1/4*(
f*x + e)^4*b*h^3*log(c)/f^4 - (f*x + e)^3*b*e*h^3*log(c)/f^4 + 3/2*(f*x + e
)^2*b*e^2*h^3*log(c)/f^4 - (f*x + e)*b*e^3*h^3*log(c)/f^4 + (f*x + e)*a*g^3
/f + 3/2*(f*x + e)^2*a*g^2*h/f^2 - 3*(f*x + e)*a*e*g^2*h/f^2 + (f*x + e)^3*
a*g*h^2/f^3 - 3*(f*x + e)^2*a*e*g*h^2/f^3 + 3*(f*x + e)*a*e^2*g*h^2/f^3 + 1
/4*(f*x + e)^4*a*h^3/f^4 - (f*x + e)^3*a*e*h^3/f^4 + 3/2*(f*x + e)^2*a*e^2*
h^3/f^4 - (f*x + e)*a*e^3*h^3/f^4
```

## Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.34

$$\int (g+hx)^3 (a+b\log (c(d(e+fx)^p)^q))\, dx$$
$$= \ln (c\,(d\,(e+f\,x)^p)^q)\,\left(b\,g^3\,x+\frac{3\,b\,g^2\,h\,x^2}{2}+b\,g\,h^2\,x^3+\frac{b\,h^3\,x^4}{4}\right)$$
$$-x^2\left(\frac{e\left(\frac{h^2\,(a\,e\,h+3\,a\,f\,g-b\,f\,g\,p\,q)}{f}-\frac{e\,h^3\,(4\,a-b\,p\,q)}{4\,f}\right)}{2\,f}-\frac{3\,g\,h\,(2\,a\,e\,h+2\,a\,f\,g-b\,f\,g\,p\,q)}{4\,f}\right)$$
$$+x\left(\frac{4\,a\,f\,g^3+12\,a\,e\,g^2\,h-4\,b\,f\,g^3\,p\,q}{4\,f}+\frac{e\left(\frac{e\left(\frac{h^2\,(a\,e\,h+3\,a\,f\,g-b\,f\,g\,p\,q)}{f}-\frac{e\,h^3\,(4\,a-b\,p\,q)}{4\,f}\right)}{f}-\frac{3\,g\,h\,(2\,a\,e\,h+2\,a\,f\,g-b\,f\,g\,p\,q)}{2\,f}\right)}{f}\right)$$
$$+x^3\left(\frac{h^2\,(a\,e\,h+3\,a\,f\,g-b\,f\,g\,p\,q)}{3\,f}-\frac{e\,h^3\,(4\,a-b\,p\,q)}{12\,f}\right)$$
$$-\frac{\ln (e+f\,x)\,\left(b\,p\,q\,e^4\,h^3-4\,b\,p\,q\,e^3\,f\,g\,h^2+6\,b\,p\,q\,e^2\,f^2\,g^2\,h-4\,b\,p\,q\,e\,f^3\,g^3\right)}{4\,f^4}$$
$$+\frac{h^3\,x^4\,(4\,a-b\,p\,q)}{16}$$

```
[In] int((g + h*x)^3*(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

```
[Out] log(c*(d*(e + f*x)^p)^q)*((b*h^3*x^4)/4 + b*g^3*x + (3*b*g^2*h*x^2)/2 + b*g
*h^2*x^3) - x^2*((e*((h^2*(a*e*h + 3*a*f*g - b*f*g*p*q))/f - (e*h^3*(4*a -
b*p*q))/(4*f)))/(2*f) - (3*g*h*(2*a*e*h + 2*a*f*g - b*f*g*p*q))/(4*f)) + x*
((4*a*f*g^3 + 12*a*e*g^2*h - 4*b*f*g^3*p*q)/(4*f) + (e*((e*((h^2*(a*e*h + 3
*a*f*g - b*f*g*p*q))/f - (e*h^3*(4*a - b*p*q))/(4*f)))/f - (3*g*h*(2*a*e*h
+ 2*a*f*g - b*f*g*p*q))/(2*f)))/f) + x^3*((h^2*(a*e*h + 3*a*f*g - b*f*g*p*q
))/(3*f) - (e*h^3*(4*a - b*p*q))/(12*f)) - (log(e + f*x)*(b*e^4*h^3*p*q - 4
*b*e*f^3*g^3*p*q + 6*b*e^2*f^2*g^2*h*p*q - 4*b*e^3*f*g*h^2*p*q))/(4*f^4) +
(h^3*x^4*(4*a - b*p*q))/16
```

# 3.421 $\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 128

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))\,dx = -\frac{b(fg-eh)^2pqx}{3f^2} - \frac{b(fg-eh)pq(g+hx)^2}{6fh} - \frac{bpq(g+hx)^3}{9h} - \frac{b(fg-eh)^3pq\log(e+fx)}{3f^3h} + \frac{(g+hx)^3(a+b\log(c(d(e+fx)^p)^q))}{3h}$$

```
[Out] -1/3*b*(-e*h+f*g)^2*p*q*x/f^2-1/6*b*(-e*h+f*g)*p*q*(h*x+g)^2/f/h-1/9*b*p*q*
(h*x+g)^3/h-1/3*b*(-e*h+f*g)^3*p*q*ln(f*x+e)/f^3/h+1/3*(h*x+g)^3*(a+b*ln(c*
(d*(f*x+e)^p)^q))/h
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2442, 45, 2495}

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))\,dx = \frac{(g+hx)^3(a+b\log(c(d(e+fx)^p)^q))}{3h} - \frac{bpq(fg-eh)^3\log(e+fx)}{3f^3h} - \frac{bpqx(fg-eh)^2}{3f^2} - \frac{bpq(g+hx)^2(fg-eh)}{6fh} - \frac{bpq(g+hx)^3}{9h}$$

```
[In] Int[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] -1/3*(b*(f*g - e*h)^2*p*q*x)/f^2 - (b*(f*g - e*h)*p*q*(g + h*x)^2)/(6*f*h)
- (b*p*q*(g + h*x)^3)/(9*h) - (b*(f*g - e*h)^3*p*q*Log[e + f*x])/(3*f^3*h)
+ ((g + h*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(3*h)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int (g+hx)^2 \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right) dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) \\
&= \frac{(g+hx)^3 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h} \\
&\quad - \text{Subst}\left(\frac{(bfpq)\int \frac{(g+hx)^3}{e+fx}\,dx}{3h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) \\
&= \frac{(g+hx)^3 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h} \\
&\quad - \text{Subst}\left(\frac{(bfpq)\int \left(\frac{h(fg-eh)^2}{f^3}+\frac{(fg-eh)^3}{f^3(e+fx)}+\frac{h(fg-eh)(g+hx)}{f^2}+\frac{h(g+hx)^2}{f}\right) dx}{3h}, cd^q(e\right. \\
&\quad \left. +fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$= -\frac{b(fg-eh)^2pqx}{3f^2} - \frac{b(fg-eh)pq(g+hx)^2}{6fh} - \frac{bpq(g+hx)^3}{9h} - \frac{b(fg-eh)^3pq\log(e+fx)}{3f^3h} + \frac{(g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h}$$

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.22

$$\int (g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx = \frac{6be^2h(-3fg+eh)pq\log(e+fx)+f(x(6af^2(3g^2+3ghx+h^2x^2)-bpq(6e^2h^2-3efh(6g+hx)+f^2(18}{18f^3}$$

```
[In] Integrate[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] (6*b*e^2*h*(-3*f*g + e*h)*p*q*Log[e + f*x] + f*(x*(6*a*f^2*(3*g^2 + 3*g*h*x
 + h^2*x^2) - b*p*q*(6*e^2*h^2 - 3*e*f*h*(6*g + h*x) + f^2*(18*g^2 + 9*g*h*
x + 2*h^2*x^2))) + 6*b*f*(3*e*g^2 + f*x*(3*g^2 + 3*g*h*x + h^2*x^2))*Log[c*
(d*(e + f*x)^p)^q]))/(18*f^3)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(118) = 236$.

Time = 1.97 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.27

| method | result |
|---|---|
| parallelrisch | $\frac{-2x^3bef^3h^2pq+6x^3\ln(c(d(fx+e)^p)^q)bef^3h^2+3x^2be^2f^2h^2pq-9x^2bef^3ghpq+6\ln(fx+e)be^4h^2pq-18\ln(fx+e)be^3fghpq+}{}$ |

```
[In] int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(-2*x^3*b*e*f^3*h^2*p*q+6*x^3*ln(c*(d*(f*x+e)^p)^q)*b*e*f^3*h^2+3*x^2*
b*e^2*f^2*h^2*p*q-9*x^2*b*e*f^3*g*h*p*q+6*ln(f*x+e)*b*e^4*h^2*p*q-18*ln(f*x
+e)*b*e^3*f*g*h*p*q+36*ln(f*x+e)*b*e^2*f^2*g^2*p*q+6*x^3*a*e*f^3*h^2+18*x^2
*ln(c*(d*(f*x+e)^p)^q)*b*e*f^3*g*h-6*x*b*e^3*f*h^2*p*q+18*x*b*e^2*f^2*g*h*p
*q-18*x*b*e*f^3*g^2*p*q+18*x^2*a*e*f^3*g*h+18*x*ln(c*(d*(f*x+e)^p)^q)*b*e*f
^3*g^2+18*x*a*e*f^3*g^2-18*ln(c*(d*(f*x+e)^p)^q)*b*e^2*f^2*g^2)/f^3/e
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(118) = 236$.

Time = 0.31 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.09

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))\,dx = -\frac{2\,(bf^3h^2pq-3\,af^3h^2)x^3-3\,(6\,af^3gh-(3\,bf^3gh-bef^2h^2)pq)x^2-6\,(3\,af^3g^2-(3\,bf^3g^2-3\,bef^2gh+b$$

[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")

[Out] -1/18*(2*(b*f^3*h^2*p*q - 3*a*f^3*h^2)*x^3 - 3*(6*a*f^3*g*h - (3*b*f^3*g*h - b*e*f^2*h^2)*p*q)*x^2 - 6*(3*a*f^3*g^2 - (3*b*f^3*g^2 - 3*b*e*f^2*g*h + b*e^2*f*h^2)*p*q)*x - 6*(b*f^3*h^2*p*q*x^3 + 3*b*f^3*g*h*p*q*x^2 + 3*b*f^3*g^2*p*q*x + (3*b*e*f^2*g^2 - 3*b*e^2*f*g*h + b*e^3*h^2)*p*q)*log(f*x + e) - 6*(b*f^3*h^2*x^3 + 3*b*f^3*g*h*x^2 + 3*b*f^3*g^2*x)*log(c) - 6*(b*f^3*h^2*q*x^3 + 3*b*f^3*g*h*q*x^2 + 3*b*f^3*g^2*q*x)*log(d))/f^3

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(112) = 224$.

Time = 1.23 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.23

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))\,dx = \begin{cases} ag^2x + aghx^2 + \frac{ah^2x^3}{3} + \frac{be^3h^2\log(c(d(e+fx)^p)^q)}{3f^3} - \frac{be^2gh\log(c(d(e+fx)^p)^q)}{f^2} - \frac{be^2h^2pqx}{3f^2} + \frac{beg^2\log(c(d(e+fx)^p)^q)}{f} + \frac{beghpq}{f} \\ (a+b\log(c(de^p)^q))\left(g^2x + ghx^2 + \frac{h^2x^3}{3}\right) \end{cases}$$

[In] integrate((h*x+g)**2*(a+b*ln(c*(d*(f*x+e)**p)**q)),x)

[Out] Piecewise((a*g**2*x + a*g*h*x**2 + a*h**2*x**3/3 + b*e**3*h**2*log(c*(d*(e + f*x)**p)**q)/(3*f**3) - b*e**2*g*h*log(c*(d*(e + f*x)**p)**q)/f**2 - b*e**2*h**2*p*q*x/(3*f**2) + b*e*g**2*log(c*(d*(e + f*x)**p)**q)/f + b*e*g*h*p*q*x/f + b*e*h**2*p*q*x**2/(6*f) - b*g**2*p*q*x + b*g**2*x*log(c*(d*(e + f*x)**p)**q) - b*g*h*p*q*x**2/2 + b*g*h*x**2*log(c*(d*(e + f*x)**p)**q) - b*h**2*p*q*x**3/9 + b*h**2*x**3*log(c*(d*(e + f*x)**p)**q)/3, Ne(f, 0)), ((a + b*log(c*(d*e**p)**q))*(g**2*x + g*h*x**2 + h**2*x**3/3), True))

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.58

$$\int (g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))\,dx = -bfg^2pq\left(\frac{x}{f} - \frac{e\log (fx+e)}{f^2}\right) + \frac{1}{18}\,bfh^2pq\left(\frac{6\,e^3\log (fx+e)}{f^4} - \frac{2\,f^2x^3 - 3\,efx^2 + 6\,e^2x}{f^3}\right) - \frac{1}{2}\,bfghpq\left(\frac{2\,e^2\log (fx+e)}{f^3} + \frac{fx^2 - 2\,ex}{f^2}\right) + \frac{1}{3}\,bh^2x^3\log \left(\left(\left(fx+e\right)^p d\right)^q c\right) + \frac{1}{3}\,ah^2x^3 + bghx^2\log \left(\left(\left(fx+e\right)^p d\right)^q c\right) + aghx^2 + bg^2x\log \left(\left(\left(fx+e\right)^p d\right)^q c\right) + ag^2x$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

```
[Out] -b*f*g^2*p*q*(x/f - e*log(f*x + e)/f^2) + 1/18*b*f*h^2*p*q*(6*e^3*log(f*x +
 e)/f^4 - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/f^3) - 1/2*b*f*g*h*p*q*(2*e^2*l
og(f*x + e)/f^3 + (f*x^2 - 2*e*x)/f^2) + 1/3*b*h^2*x^3*log(((f*x + e)^p*d)^
q*c) + 1/3*a*h^2*x^3 + b*g*h*x^2*log(((f*x + e)^p*d)^q*c) + a*g*h*x^2 + b*g
^2*x*log(((f*x + e)^p*d)^q*c) + a*g^2*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(118) = 236$.

Time = 0.36 (sec) , antiderivative size = 544, normalized size of antiderivative = 4.25

$$\int (g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx$$
$$=\frac{(fx+e)bg^2pq\log(fx+e)}{f}+\frac{(fx+e)^2bghpq\log(fx+e)}{f^2}-\frac{2(fx+e)beghpq\log(fx+e)}{f^2}+\frac{(fx+e)^3bh^2pq\log(fx+e)}{3f^3}-\frac{(fx+e)^2beh^2pq\log(fx+e)}{f^3}+\frac{(fx+e)be^2h^2pq\log(fx+e)}{f^3}-\frac{(fx+e)bg^2pq}{f}-\frac{(fx+e)^2bghpq}{2f^2}+\frac{2(fx+e)beghpq}{f^2}-\frac{(fx+e)^3bh^2pq}{9f^3}+\frac{(fx+e)^2beh^2pq}{2f^3}-\frac{(fx+e)be^2h^2pq}{f^3}+\frac{(fx+e)bg^2q\log(d)}{f}+\frac{(fx+e)^2bghq\log(d)}{f^2}-\frac{2(fx+e)beghq\log(d)}{f^2}+\frac{(fx+e)^3bh^2q\log(d)}{3f^3}-\frac{(fx+e)^2beh^2q\log(d)}{f^3}+\frac{(fx+e)be^2h^2q\log(d)}{f^3}+\frac{(fx+e)bg^2\log(c)}{f}+\frac{(fx+e)^2bgh\log(c)}{f^2}-\frac{2(fx+e)begh\log(c)}{f^2}+\frac{(fx+e)^3bh^2\log(c)}{3f^3}-\frac{(fx+e)^2beh^2\log(c)}{f^3}+\frac{(fx+e)be^2h^2\log(c)}{f^3}+\frac{(fx+e)ag^2}{f}+\frac{(fx+e)^2agh}{f^2}-\frac{2(fx+e)aegh}{f^2}+\frac{(fx+e)^3ah^2}{3f^3}-\frac{(fx+e)^2aeh^2}{f^3}+\frac{(fx+e)ae^2h^2}{f^3}$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

```
[Out] (f*x + e)*b*g^2*p*q*log(f*x + e)/f + (f*x + e)^2*b*g*h*p*q*log(f*x + e)/f^2
 - 2*(f*x + e)*b*e*g*h*p*q*log(f*x + e)/f^2 + 1/3*(f*x + e)^3*b*h^2*p*q*log
(f*x + e)/f^3 - (f*x + e)^2*b*e*h^2*p*q*log(f*x + e)/f^3 + (f*x + e)*b*e^2*
h^2*p*q*log(f*x + e)/f^3 - (f*x + e)*b*g^2*p*q/f - 1/2*(f*x + e)^2*b*g*h*p*
q/f^2 + 2*(f*x + e)*b*e*g*h*p*q/f^2 - 1/9*(f*x + e)^3*b*h^2*p*q/f^3 + 1/2*(
f*x + e)^2*b*e*h^2*p*q/f^3 - (f*x + e)*b*e^2*h^2*p*q/f^3 + (f*x + e)*b*g^2*
q*log(d)/f + (f*x + e)^2*b*g*h*q*log(d)/f^2 - 2*(f*x + e)*b*e*g*h*q*log(d)/
f^2 + 1/3*(f*x + e)^3*b*h^2*q*log(d)/f^3 - (f*x + e)^2*b*e*h^2*q*log(d)/f^3
 + (f*x + e)*b*e^2*h^2*q*log(d)/f^3 + (f*x + e)*b*g^2*log(c)/f + (f*x + e)^
2*b*g*h*log(c)/f^2 - 2*(f*x + e)*b*e*g*h*log(c)/f^2 + 1/3*(f*x + e)^3*b*h^2
*log(c)/f^3 - (f*x + e)^2*b*e*h^2*log(c)/f^3 + (f*x + e)*b*e^2*h^2*log(c)/f
^3 + (f*x + e)*a*g^2/f + (f*x + e)^2*a*g*h/f^2 - 2*(f*x + e)*a*e*g*h/f^2 +
1/3*(f*x + e)^3*a*h^2/f^3 - (f*x + e)^2*a*e*h^2/f^3 + (f*x + e)*a*e^2*h^2/f
^3
```

## Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.76

$$\int (g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))\, dx$$
$$= \ln (c\,(d\,(e+f\,x)^p)^q)\,\left(b\,g^2\,x+b\,g\,h\,x^2+\frac{b\,h^2\,x^3}{3}\right)$$
$$+x^2\left(\frac{h\,(a\,e\,h+2\,a\,f\,g-b\,f\,g\,p\,q)}{2\,f}-\frac{e\,h^2\,(3\,a-b\,p\,q)}{6\,f}\right)$$
$$+x\left(\frac{3\,a\,f\,g^2+6\,a\,e\,g\,h-3\,b\,f\,g^2\,p\,q}{3\,f}-\frac{e\left(\frac{h\,(a\,e\,h+2\,a\,f\,g-b\,f\,g\,p\,q)}{f}-\frac{e\,h^2\,(3\,a-b\,p\,q)}{3\,f}\right)}{f}\right)$$
$$+\frac{\ln (e+f\,x)\,\left(b\,p\,q\,e^3\,h^2-3\,b\,p\,q\,e^2\,f\,g\,h+3\,b\,p\,q\,e\,f^2\,g^2\right)}{3\,f^3}+\frac{h^2\,x^3\,(3\,a-b\,p\,q)}{9}$$

```
[In] int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

```
[Out] log(c*(d*(e + f*x)^p)^q)*((b*h^2*x^3)/3 + b*g^2*x + b*g*h*x^2) + x^2*((h*(a
*e*h + 2*a*f*g - b*f*g*p*q))/(2*f) - (e*h^2*(3*a - b*p*q))/(6*f)) + x*((3*a
*f*g^2 + 6*a*e*g*h - 3*b*f*g^2*p*q)/(3*f) - (e*((h*(a*e*h + 2*a*f*g - b*f*g
*p*q))/f - (e*h^2*(3*a - b*p*q))/(3*f)))/f) + (log(e + f*x)*(b*e^3*h^2*p*q
+ 3*b*e*f^2*g^2*p*q - 3*b*e^2*f*g*h*p*q))/(3*f^3) + (h^2*x^3*(3*a - b*p*q))
/9
```

# 3.422 $\int (g+hx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right) \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 98

$$\int (g+hx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right) \, dx = -\frac{b(fg-eh)pqx}{2f} - \frac{bpq(g+hx)^2}{4h} - \frac{b(fg-eh)^2pq\log(e+fx)}{2f^2h} + \frac{(g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)}{2h}$$

```
[Out] -1/2*b*(-e*h+f*g)*p*q*x/f-1/4*b*p*q*(h*x+g)^2/h-1/2*b*(-e*h+f*g)^2*p*q*ln(f
*x+e)/f^2/h+1/2*(h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/h
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2442, 45, 2495}

$$\int (g+hx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right) \, dx = \frac{(g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)}{2h} - \frac{bpq(fg-eh)^2\log(e+fx)}{2f^2h} - \frac{bpqx(fg-eh)}{2f} - \frac{bpq(g+hx)^2}{4h}$$

```
[In] Int[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] -1/2*(b*(f*g - e*h)*p*q*x)/f - (b*p*q*(g + h*x)^2)/(4*h) - (b*(f*g - e*h)^2
*p*q*Log[e + f*x])/(2*f^2*h) + ((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]
))/(2*h)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int (g+hx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{(g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2h}\\
&\quad -\text{Subst}\left(\frac{(bfpq)\int\frac{(g+hx)^2}{e+fx}\,dx}{2h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{(g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2h}\\
&\quad -\text{Subst}\left(\frac{(bfpq)\int\left(\frac{h(fg-eh)}{f^2}+\frac{(fg-eh)^2}{f^2(e+fx)}+\frac{h(g+hx)}{f}\right)dx}{2h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= -\frac{b(fg-eh)pqx}{2f}-\frac{bpq(g+hx)^2}{4h}-\frac{b(fg-eh)^2pq\log(e+fx)}{2f^2h}\\
&\quad +\frac{(g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2h}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx = agx-bgpqx+\frac{behpqx}{2f}+\frac{1}{2}ahx^2-\frac{1}{4}bhpqx^2-\frac{be^2hpq\log(e+fx)}{2f^2}+\frac{1}{2}bhx^2\log\left(c(d(e+fx)^p)^q\right)+\frac{bg(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f}$$

```
[In] Integrate[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] a*g*x - b*g*p*q*x + (b*e*h*p*q*x)/(2*f) + (a*h*x^2)/2 - (b*h*p*q*x^2)/4 - (
b*e^2*h*p*q*Log[e + f*x])/(2*f^2) + (b*h*x^2*Log[c*(d*(e + f*x)^p)^q])/2 +
(b*g*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/f
```

## Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.77

| method | result |
|---|---|
| parallelrisch | $-\frac{x^2bf^2hpq+2\ln(fx+e)be^2hpq-8\ln(fx+e)befgpq-2x^2\ln(c(d(fx+e)^p)^q)bf^2h-2xbefhpq+4xbf^2gpq-2x^2af^2h-4x\ln(c(d(}{4f^2}$ |

```
[In] int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(x^2*b*f^2*h*p*q+2*ln(f*x+e)*b*e^2*h*p*q-8*ln(f*x+e)*b*e*f*g*p*q-2*x^2
*ln(c*(d*(f*x+e)^p)^q)*b*f^2*h-2*x*b*e*f*h*p*q+4*x*b*f^2*g*p*q-2*x^2*a*f^2*
h-4*x*ln(c*(d*(f*x+e)^p)^q)*b*f^2*g+2*b*e^2*h*p*q-4*b*e*f*g*p*q-4*x*a*f^2*g
+4*ln(c*(d*(f*x+e)^p)^q)*b*e*f*g+4*a*e*f*g)/f^2
```

## Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.51

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx = -\frac{(bf^2hpq-2\,af^2h)x^2-2\,(2\,af^2g-(2\,bf^2g-befh)pq)x-2\,(bf^2hpqx^2+2\,bf^2gpqx+(2\,befg-be^2h)pq}{4\,f^2}$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

```
[Out] -1/4*((b*f^2*h*p*q - 2*a*f^2*h)*x^2 - 2*(2*a*f^2*g - (2*b*f^2*g - b*e*f*h)*
p*q)*x - 2*(b*f^2*h*p*q*x^2 + 2*b*f^2*g*p*q*x + (2*b*e*f*g - b*e^2*h)*p*q)*
log(f*x + e) - 2*(b*f^2*h*x^2 + 2*b*f^2*g*x)*log(c) - 2*(b*f^2*h*q*x^2 + 2*
b*f^2*g*q*x)*log(d))/f^2
```

## Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.59

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx$$
$$=\begin{cases} agx+\frac{ahx^2}{2}-\frac{be^2h\log\left(c(d(e+fx)^p)^q\right)}{2f^2}+\frac{beg\log\left(c(d(e+fx)^p)^q\right)}{f}+\frac{behpqx}{2f}-bgpqx+bgx\log\left(c(d(e+fx)^p)^q\right)-\frac{bhpqx}{4} \\ \left(a+b\log\left(c(de^p)^q\right)\right)\left(gx+\frac{hx^2}{2}\right) \end{cases}$$

```
[In] integrate((h*x+g)*(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

```
[Out] Piecewise((a*g*x + a*h*x**2/2 - b*e**2*h*log(c*(d*(e + f*x)**p)**q)/(2*f**2
) + b*e*g*log(c*(d*(e + f*x)**p)**q)/f + b*e*h*p*q*x/(2*f) - b*g*p*q*x + b*
g*x*log(c*(d*(e + f*x)**p)**q) - b*h*p*q*x**2/4 + b*h*x**2*log(c*(d*(e + f*
x)**p)**q)/2, Ne(f, 0)), ((a + b*log(c*(d*e**p)**q))*(g*x + h*x**2/2), True
))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx = -bfgpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right) - \frac{1}{4}bfhpq\left(\frac{2e^2\log(fx+e)}{f^3}+\frac{fx^2-2ex}{f^2}\right) + \frac{1}{2}bhx^2\log\left(((fx+e)^pd)^q c\right)+\frac{1}{2}ahx^2 + bgx\log\left(((fx+e)^pd)^q c\right)+agx$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

```
[Out] -b*f*g*p*q*(x/f - e*log(f*x + e)/f^2) - 1/4*b*f*h*p*q*(2*e^2*log(f*x + e)/f
^3 + (f*x^2 - 2*e*x)/f^2) + 1/2*b*h*x^2*log(((f*x + e)^p*d)^q*c) + 1/2*a*h*
x^2 + b*g*x*log(((f*x + e)^p*d)^q*c) + a*g*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(90) = 180.

Time = 0.32 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.41

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx = \frac{(fx+e)bgpq\log(fx+e)}{f} + \frac{(fx+e)^2bhpq\log(fx+e)}{2f^2} - \frac{(fx+e)behpq\log(fx+e)}{f^2} - \frac{(fx+e)bgpq}{f} - \frac{(fx+e)^2bhpq}{4f^2} + \frac{(fx+e)behpq}{f^2} + \frac{(fx+e)bgq\log(d)}{f} + \frac{(fx+e)^2bhq\log(d)}{2f^2} - \frac{(fx+e)behq\log(d)}{f^2} + \frac{(fx+e)bg\log(c)}{f} + \frac{(fx+e)^2bh\log(c)}{2f^2} - \frac{(fx+e)beh\log(c)}{f^2} + \frac{(fx+e)ag}{f} + \frac{(fx+e)^2ah}{2f^2} - \frac{(fx+e)aeh}{f^2}$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
[Out] (f*x + e)*b*g*p*q*log(f*x + e)/f + 1/2*(f*x + e)^2*b*h*p*q*log(f*x + e)/f^2
 - (f*x + e)*b*e*h*p*q*log(f*x + e)/f^2 - (f*x + e)*b*g*p*q/f - 1/4*(f*x +
e)^2*b*h*p*q/f^2 + (f*x + e)*b*e*h*p*q/f^2 + (f*x + e)*b*g*q*log(d)/f + 1/2
*(f*x + e)^2*b*h*q*log(d)/f^2 - (f*x + e)*b*e*h*q*log(d)/f^2 + (f*x + e)*b*
g*log(c)/f + 1/2*(f*x + e)^2*b*h*log(c)/f^2 - (f*x + e)*b*e*h*log(c)/f^2 +
(f*x + e)*a*g/f + 1/2*(f*x + e)^2*a*h/f^2 - (f*x + e)*a*e*h/f^2
```

## Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx = \ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\left(\frac{b\,h\,x^2}{2}+b\,g\,x\right) + x\left(\frac{2\,a\,e\,h+2\,a\,f\,g-2\,b\,f\,g\,p\,q}{2\,f} - \frac{e\,h\,(2\,a-b\,p\,q)}{2\,f}\right) + \frac{h\,x^2\,(2\,a-b\,p\,q)}{4} - \frac{\ln\left(e+f\,x\right)\,\left(b\,e^2\,h\,p\,q-2\,b\,e\,f\,g\,p\,q\right)}{2\,f^2}$$

```
[In] int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

```
[Out] log(c*(d*(e + f*x)^p)^q)*((b*h*x^2)/2 + b*g*x) + x*((2*a*e*h + 2*a*f*g - 2*
b*f*g*p*q)/(2*f) - (e*h*(2*a - b*p*q))/(2*f)) + (h*x^2*(2*a - b*p*q))/4 - (
log(e + f*x)*(b*e^2*h*p*q - 2*b*e*f*g*p*q))/(2*f^2)
```

# 3.423 $\int (a+b\log(c(d(e+fx)^p)^q))\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 34

$$\int (a+b\log(c(d(e+fx)^p)^q))\,dx = ax-bpqx+\frac{b(e+fx)\log(c(d(e+fx)^p)^q)}{f}$$

```
[Out] a*x-b*p*q*x+b*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2436, 2332, 2495}

$$\int (a+b\log(c(d(e+fx)^p)^q))\,dx = ax+\frac{b(e+fx)\log(c(d(e+fx)^p)^q)}{f}-bpqx$$

```
[In] Int[a + b*Log[c*(d*(e + f*x)^p)^q],x]
```

```
[Out] a*x - b*p*q*x + (b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/f
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \log\left(c(d(e+fx)^p)^q\right)\,dx\\
&= ax + b\text{Subst}\left(\int \log\left(cd^q(e+fx)^{pq}\right)\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= ax + b\text{Subst}\left(\frac{\text{Subst}\left(\int \log\left(cd^q x^{pq}\right)\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= ax - bpqx + \frac{b(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx = ax - bpqx + \frac{b(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f}$$

```
[In] Integrate[a + b*Log[c*(d*(e + f*x)^p)^q],x]
[Out] a*x - b*p*q*x + (b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/f
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

| method | result | size |
|---|---|---|
| default | $ax+b\ln\left(c(d(fx+e)^p)^q\right)x-bpqx+\frac{bqpe\ln(fx+e)}{f}$ | 42 |
| parts | $ax+b\ln\left(c(d(fx+e)^p)^q\right)x-bpqx+\frac{bqpe\ln(fx+e)}{f}$ | 42 |
| parallelrisch | $\frac{b(2\ln(fx+e)e^2pq-xefpq+x\ln(c(d(fx+e)^p)^q)ef-\ln(c(d(fx+e)^p)^q)e^2)}{ef}+ax$ | 71 |

```
[In] int(a+b*ln(c*(d*(f*x+e)^p)^q),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*ln(c*(d*(f*x+e)^p)^q)*x-b*p*q*x+b*q*p/f*e*ln(f*x+e)
```

## Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int (a+b\log(c(d(e+fx)^p)^q))\,dx = \frac{bfqx\log(d)+bfx\log(c)-(bfpq-af)x+(bfpqx+bepq)\log(fx+e)}{f}$$

```
[In] integrate(a+b*log(c*(d*(f*x+e)^p)^q),x, algorithm="fricas")
[Out] (b*f*q*x*log(d) + b*f*x*log(c) - (b*f*p*q - a*f)*x + (b*f*p*q*x + b*e*p*q)*
log(f*x + e))/f
```

## Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int (a+b\log(c(d(e+fx)^p)^q))\,dx = ax+b\left(\begin{cases}\frac{e\log(c(d(e+fx)^p)^q)}{f}-pqx+x\log(c(d(e+fx)^p)^q) & \text{for } f\neq 0\\ x\log(c(de^p)^q) & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(a+b*ln(c*(d*(f*x+e)**p)**q),x)
[Out] a*x + b*Piecewise((e*log(c*(d*(e + f*x)**p)**q)/f - p*q*x + x*log(c*(d*(e +
 f*x)**p)**q), Ne(f, 0)), (x*log(c*(d*e**p)**q), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int (a+b\log(c(d(e+fx)^p)^q))\,dx = -bfpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)+bx\log(((fx+e)^pd)^qc)+ax$$

```
[In] integrate(a+b*log(c*(d*(f*x+e)^p)^q),x, algorithm="maxima")
[Out] -b*f*p*q*(x/f - e*log(f*x + e)/f^2) + b*x*log(((f*x + e)^p*d)^q*c) + a*x
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx = \left(\frac{(fx+e)pq\log\left(fx+e\right)}{f} - \frac{(fx+e)pq}{f} + \frac{(fx+e)q\log\left(d\right)}{f} + \frac{(fx+e)\log\left(c\right)}{f}\right)b + ax$$

```
[In] integrate(a+b*log(c*(d*(f*x+e)^p)^q),x, algorithm="giac")
[Out] ((f*x + e)*p*q*log(f*x + e)/f - (f*x + e)*p*q/f + (f*x + e)*q*log(d)/f + (f
*x + e)*log(c)/f)*b + a*x
```

## Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx = x\,\left(a-b\,p\,q\right)+b\,x\,\ln\left(c\,\left(d\,\left(e+f\,x\right)^p\right)^q\right)+\frac{b\,e\,p\,q\,\ln\left(e+f\,x\right)}{f}$$

```
[In] int(a + b*log(c*(d*(e + f*x)^p)^q),x)
[Out] x*(a - b*p*q) + b*x*log(c*(d*(e + f*x)^p)^q) + (b*e*p*q*log(e + f*x))/f
```

# 3.424 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{g+hx}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 68

$$\int \frac{a+b\log(c(d(e+fx)^p)^q)}{g+hx}\,dx = \frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{bpq\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

```
[Out] (a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/h+b*p*q*polylog(2,-h*(
f*x+e)/(-e*h+f*g))/h
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2441, 2440, 2438, 2495}

$$\int \frac{a+b\log(c(d(e+fx)^p)^q)}{g+hx}\,dx = \frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h} + \frac{bpq\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x),x]
```

```
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/h + (b*p*
q*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}\\
&\quad -\text{Subst}\left(\frac{(bfpq)\int \frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}\\
&\quad -\text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int \frac{\log\left(1+\frac{hx}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}+\frac{bpq\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{g+hx} \, dx = \frac{\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)\log \left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{bpq\,\text{PolyLog}\left(2,\frac{h(e+fx)}{-fg+eh}\right)}{h}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x),x]
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/h + (b*p*
q*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)])/h
```

## Maple [F]

$$\int \frac{a+b\ln \left(c(d(fx+e)^p)^q\right)}{hx+g}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

## Fricas [F]

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{g+hx} \, dx = \int \frac{b\log \left(((fx+e)^p d)^q c\right)+a}{hx+g} \, dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="fricas")
[Out] integral((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

## Sympy [F]

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{g+hx} \, dx = \int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{g+hx} \, dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x), x)
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx = \int \frac{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="maxima")
```

```
[Out] b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(h*x + g), x) + a*lo
g(h*x + g)/h
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx = \int \frac{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{g+h\,x}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x),x)
```

```
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x), x)
```

# 3.425 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}\,dx = \frac{bfpq\log(e+fx)}{h(fg-eh)} - \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{h(g+hx)} - \frac{bfpq\log(g+hx)}{h(fg-eh)}$$

```
[Out] b*f*p*q*ln(f*x+e)/h/(-e*h+f*g)+(-a-b*ln(c*(d*(f*x+e)^p)^q))/h/(h*x+g)-b*f*p
*q*ln(h*x+g)/h/(-e*h+f*g)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2442, 36, 31, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}\,dx = -\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{h(g+hx)} + \frac{bfpq\log(e+fx)}{h(fg-eh)} - \frac{bfpq\log(g+hx)}{h(fg-eh)}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^2,x]
```

```
[Out] (b*f*p*q*Log[e + f*x])/(h*(f*g - e*h)) - (a + b*Log[c*(d*(e + f*x)^p)^q])/(
h*(g + h*x)) - (b*f*p*q*Log[g + h*x])/(h*(f*g - e*h))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(g+hx)^2}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= -\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{h(g+hx)} + \text{Subst}\left(\frac{(bfpq)\int \frac{1}{(e+fx)(g+hx)}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= -\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{h(g+hx)}\\
&\quad - \text{Subst}\left(\frac{(bfpq)\int \frac{1}{g+hx}\,dx}{fg-eh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad + \text{Subst}\left(\frac{(bf^2pq)\int \frac{1}{e+fx}\,dx}{h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{bfpq\log(e+fx)}{h(fg-eh)} - \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{h(g+hx)} - \frac{bfpq\log(g+hx)}{h(fg-eh)}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}\,dx=\frac{-\frac{a}{g+hx}-\frac{b\log(c(d(e+fx)^p)^q)}{g+hx}+\frac{bfpq(\log(e+fx)-\log(g+hx))}{fg-eh}}{h}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^2,x]
```

```
[Out] (-(a/(g + h*x)) - (b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x) + (b*f*p*q*(Log[e
+ f*x] - Log[g + h*x]))/(f*g - e*h))/h
```

## Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.74

| method | result |
|---|---|
| parallelrisch | $-\frac{\ln(fx+e)xb\,f^2hpq-\ln(hx+g)xb\,f^2hpq+\ln(fx+e)b\,f^2gpq-\ln(hx+g)b\,f^2gpq+\ln(c(d(fx+e)^p)^q)befh-\ln(c(d(fx+e)^p)^q)b\,f^2g}{(eh-fg)(hx+g)fh}$ |

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(ln(f*x+e)*x*b*f^2*h*p*q-ln(h*x+g)*x*b*f^2*h*p*q+ln(f*x+e)*b*f^2*g*p*q-ln(
h*x+g)*b*f^2*g*p*q+ln(c*(d*(f*x+e)^p)^q)*b*e*f*h-ln(c*(d*(f*x+e)^p)^q)*b*f^
2*g+a*e*f*h-a*f^2*g)/(e*h-f*g)/(h*x+g)/f/h
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}\,dx = -\frac{afg-aeh+(bfg-beh)q\log(d)-(bfhpqx+behpq)\log(fx+e)+(bfhpqx+bfgpq)\log(hx+g)+(}{fg^2h-egh^2+(fgh^2-eh^3)x}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] -(a*f*g - a*e*h + (b*f*g - b*e*h)*q*log(d) - (b*f*h*p*q*x + b*e*h*p*q)*log(
f*x + e) + (b*f*h*p*q*x + b*f*g*p*q)*log(h*x + g) + (b*f*g - b*e*h)*log(c))
/(f*g^2*h - e*g*h^2 + (f*g*h^2 - e*h^3)*x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(68) = 136$.

Time = 3.56 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.46

$$\int \frac{a + b\log \left(c(d(e + fx)^p)^q\right)}{(g + hx)^2}\,dx$$
$$= \begin{cases} \frac{ax + \frac{be\log\left(c(d(e+fx)^p)^q\right)}{f} - bpqx + bx\log\left(c(d(e+fx)^p)^q\right)}{g^2} \\ -\frac{a}{gh+h^2x} - \frac{bpq}{gh+h^2x} - \frac{b\log\left(c\left(d\left(\frac{fg}{h}+fx\right)^p\right)^q\right)}{gh+h^2x} \\ -\frac{aeh}{egh^2+eh^3x-fg^2h-fgh^2x} + \frac{afg}{egh^2+eh^3x-fg^2h-fgh^2x} - \frac{beh\log\left(c(d(e+fx)^p)^q\right)}{egh^2+eh^3x-fg^2h-fgh^2x} + \frac{bfgpq\log\left(\frac{g}{h}+x\right)}{egh^2+eh^3x-fg^2h-fgh^2x} + \frac{bfhpqx\log}{egh^2+eh^3x-} \end{cases}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**2,x)
```

```
[Out] Piecewise(((a*x + b*e*log(c*(d*(e + f*x)**p)**q)/f - b*p*q*x + b*x*log(c*(d
*(e + f*x)**p)**q))/g**2, Eq(h, 0)), (-a/(g*h + h**2*x) - b*p*q/(g*h + h**2
*x) - b*log(c*(d*(f*g/h + f*x)**p)**q)/(g*h + h**2*x), Eq(e, f*g/h)), (-a*e
*h/(e*g*h**2 + e*h**3*x - f*g**2*h - f*g*h**2*x) + a*f*g/(e*g*h**2 + e*h**3
*x - f*g**2*h - f*g*h**2*x) - b*e*h*log(c*(d*(e + f*x)**p)**q)/(e*g*h**2 +
e*h**3*x - f*g**2*h - f*g*h**2*x) + b*f*g*p*q*log(g/h + x)/(e*g*h**2 + e*h*
*3*x - f*g**2*h - f*g*h**2*x) + b*f*h*p*q*x*log(g/h + x)/(e*g*h**2 + e*h**3
*x - f*g**2*h - f*g*h**2*x) - b*f*h*x*log(c*(d*(e + f*x)**p)**q)/(e*g*h**2
+ e*h**3*x - f*g**2*h - f*g*h**2*x), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int \frac{a + b\log \left(c(d(e + fx)^p)^q\right)}{(g + hx)^2}\,dx = bfpq\left(\frac{\log(fx + e)}{fgh - eh^2} - \frac{\log(hx + g)}{fgh - eh^2}\right) - \frac{b\log\left(((fx + e)^p d)^q c\right)}{h^2x + gh} - \frac{a}{h^2x + gh}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] b*f*p*q*(log(f*x + e)/(f*g*h - e*h^2) - log(h*x + g)/(f*g*h - e*h^2)) - b*l
og(((f*x + e)^p*d)^q*c)/(h^2*x + g*h) - a/(h^2*x + g*h)
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}\,dx = \frac{bfpq\log\left(fx+e\right)}{fgh-eh^2} - \frac{bfpq\log\left(hx+g\right)}{fgh-eh^2} - \frac{bpq\log\left(fx+e\right)}{h^2x+gh} - \frac{bq\log\left(d\right)+b\log\left(c\right)+a}{h^2x+gh}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] b*f*p*q*log(f*x + e)/(f*g*h - e*h^2) - b*f*p*q*log(h*x + g)/(f*g*h - e*h^2)
 - b*p*q*log(f*x + e)/(h^2*x + g*h) - (b*q*log(d) + b*log(c) + a)/(h^2*x +
g*h)
```

## Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^2}\,dx = -\frac{a}{x\,h^2+g\,h} - \frac{b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{h\,\left(g+h\,x\right)} + \frac{b\,f\,p\,q\,\mathrm{atan}\left(\frac{f\,g\,2\mathrm{i}+f\,h\,x\,2\mathrm{i}}{e\,h-f\,g}+1\mathrm{i}\right)\,2\mathrm{i}}{h\,\left(e\,h-f\,g\right)}$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^2,x)
```

```
[Out] (b*f*p*q*atan((f*g*2i + f*h*x*2i)/(e*h - f*g) + 1i)*2i)/(h*(e*h - f*g)) - (
b*log(c*(d*(e + f*x)^p)^q))/(h*(g + h*x)) - a/(g*h + h^2*x)
```

# 3.426 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^3}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx = \frac{bfpq}{2h(fg-eh)(g+hx)} + \frac{bf^2pq\log(e+fx)}{2h(fg-eh)^2} - \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2h(g+hx)^2} - \frac{bf^2pq\log(g+hx)}{2h(fg-eh)^2}$$

```
[Out] 1/2*b*f*p*q/h/(-e*h+f*g)/(h*x+g)+1/2*b*f^2*p*q*ln(f*x+e)/h/(-e*h+f*g)^2+1/2
*(-a-b*ln(c*(d*(f*x+e)^p)^q))/h/(h*x+g)^2-1/2*b*f^2*p*q*ln(h*x+g)/h/(-e*h+f
*g)^2
```

## Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2442, 46, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx = -\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2h(g+hx)^2} + \frac{bf^2pq\log(e+fx)}{2h(fg-eh)^2} - \frac{bf^2pq\log(g+hx)}{2h(fg-eh)^2} + \frac{bfpq}{2h(g+hx)(fg-eh)}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^3,x]
```

```
[Out] (b*f*p*q)/(2*h*(f*g - e*h)*(g + h*x)) + (b*f^2*p*q*Log[e + f*x])/(2*h*(f*g
 - e*h)^2) - (a + b*Log[c*(d*(e + f*x)^p)^q])/(2*h*(g + h*x)^2) - (b*f^2*p*q
*Log[g + h*x])/(2*h*(f*g - e*h)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(g+hx)^3}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2h(g+hx)^2} + \text{Subst}\left(\frac{(bfpq)\int \frac{1}{(e+fx)(g+hx)^2}\,dx}{2h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2h(g+hx)^2} + \text{Subst}\left(\frac{(bfpq)\int \left(\frac{f^2}{(fg-eh)^2(e+fx)} - \frac{h}{(fg-eh)(g+hx)^2} - \frac{fh}{(fg-eh)^2(g+hx)}\right)dx}{2h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{bfpq}{2h(fg-eh)(g+hx)} + \frac{bf^2pq\log(e+fx)}{2h(fg-eh)^2} - \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2h(g+hx)^2} - \frac{bf^2pq\log(g+hx)}{2h(fg-eh)^2}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.74

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx = -\frac{a+b\log\left(c(d(e+fx)^p)^q\right)-\frac{bfpq(g+hx)(fg-eh+f(g+hx)\log(e+fx)-f(g+hx)\log(g+hx))}{(fg-eh)^2}}{2h(g+hx)^2}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^3,x]
```

```
[Out] -1/2*(a + b*Log[c*(d*(e + f*x)^p)^q] - (b*f*p*q*(g + h*x)*(f*g - e*h + f*(g
 + h*x)*Log[e + f*x] - f*(g + h*x)*Log[g + h*x]))/(f*g - e*h)^2)/(h*(g + h*
x)^2)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(114) = 228.

Time = 3.76 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.56

| method | result |
|---|---|
| parallelrisch | $\frac{2\ln(fx+e)xb\,f^3g\,h^2pq-a\,e^2f\,h^3-a\,f^3g^2h+\ln(fx+e)x^2b\,f^3h^3pq-\ln(hx+g)x^2b\,f^3h^3pq+\ln(fx+e)b\,f^3g^2hpq-\ln(hx+g)b\,f^3g^2}{}$ |

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*ln(f*x+e)*x*b*f^3*g*h^2*p*q-a*e^2*f*h^3-a*f^3*g^2*h+ln(f*x+e)*x^2*b*
f^3*h^3*p*q-ln(h*x+g)*x^2*b*f^3*h^3*p*q+ln(f*x+e)*b*f^3*g^2*h*p*q-ln(h*x+g)
*b*f^3*g^2*h*p*q-x*b*e*f^2*h^3*p*q+x*b*f^3*g*h^2*p*q-2*ln(h*x+g)*x*b*f^3*g*
h^2*p*q-b*e*f^2*g*h^2*p*q+b*f^3*g^2*h*p*q+2*a*e*f^2*g*h^2+2*ln(c*(d*(f*x+e)
^p)^q)*b*e*f^2*g*h^2-ln(c*(d*(f*x+e)^p)^q)*b*e^2*f*h^3-ln(c*(d*(f*x+e)^p)^q
)*b*f^3*g^2*h)/(e^2*h^2-2*e*f*g*h+f^2*g^2)/(h*x+g)^2/f/h^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(111) = 222.

Time = 0.31 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.61

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx = -\frac{af^2g^2-2\,aefgh+ae^2h^2-(bf^2gh-befh^2)pqx-(bf^2g^2-befgh)pq+(bf^2g^2-2\,befgh+be^2h^2)q\log}{2\,(f^2g^4h-2\,efg^3h^2}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*f^2*g^2 - 2*a*e*f*g*h + a*e^2*h^2 - (b*f^2*g*h - b*e*f*h^2)*p*q*x -
 (b*f^2*g^2 - b*e*f*g*h)*p*q + (b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*q*log(
d) - (b*f^2*h^2*p*q*x^2 + 2*b*f^2*g*h*p*q*x + (2*b*e*f*g*h - b*e^2*h^2)*p*q
)*log(f*x + e) + (b*f^2*h^2*p*q*x^2 + 2*b*f^2*g*h*p*q*x + b*f^2*g^2*p*q)*lo
g(h*x + g) + (b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*log(c))/(f^2*g^4*h - 2*e
*f*g^3*h^2 + e^2*g^2*h^3 + (f^2*g^2*h^3 - 2*e*f*g*h^4 + e^2*h^5)*x^2 + 2*(f
^2*g^3*h^2 - 2*e*f*g^2*h^3 + e^2*g*h^4)*x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1984 vs. $2(105) = 210$.

Time = 15.25 (sec) , antiderivative size = 1984, normalized size of antiderivative = 16.67

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**3,x)
```

```
[Out] Piecewise(((a*x + b*e*log(c*(d*(e + f*x)**p)**q)/f - b*p*q*x + b*x*log(c*(d
*(e + f*x)**p)**q))/g**3, Eq(h, 0)), (-2*a/(4*g**2*h + 8*g*h**2*x + 4*h**3*
x**2) - b*p*q/(4*g**2*h + 8*g*h**2*x + 4*h**3*x**2) - 2*b*log(c*(d*(f*g/h +
 f*x)**p)**q)/(4*g**2*h + 8*g*h**2*x + 4*h**3*x**2), Eq(e, f*g/h)), (-a*e**
2*h**2/(2*e**2*g**2*h**3 + 4*e**2*g*h**4*x + 2*e**2*h**5*x**2 - 4*e*f*g**3*
h**2 - 8*e*f*g**2*h**3*x - 4*e*f*g*h**4*x**2 + 2*f**2*g**4*h + 4*f**2*g**3*
h**2*x + 2*f**2*g**2*h**3*x**2) + 2*a*e*f*g*h/(2*e**2*g**2*h**3 + 4*e**2*g*
h**4*x + 2*e**2*h**5*x**2 - 4*e*f*g**3*h**2 - 8*e*f*g**2*h**3*x - 4*e*f*g*h
**4*x**2 + 2*f**2*g**4*h + 4*f**2*g**3*h**2*x + 2*f**2*g**2*h**3*x**2) - a*
f**2*g**2/(2*e**2*g**2*h**3 + 4*e**2*g*h**4*x + 2*e**2*h**5*x**2 - 4*e*f*g*
*3*h**2 - 8*e*f*g**2*h**3*x - 4*e*f*g*h**4*x**2 + 2*f**2*g**4*h + 4*f**2*g*
*3*h**2*x + 2*f**2*g**2*h**3*x**2) - b*e**2*h**2*log(c*(d*(e + f*x)**p)**q)
/(2*e**2*g**2*h**3 + 4*e**2*g*h**4*x + 2*e**2*h**5*x**2 - 4*e*f*g**3*h**2 -
 8*e*f*g**2*h**3*x - 4*e*f*g*h**4*x**2 + 2*f**2*g**4*h + 4*f**2*g**3*h**2*x
 + 2*f**2*g**2*h**3*x**2) - b*e*f*g*h*p*q/(2*e**2*g**2*h**3 + 4*e**2*g*h**4
*x + 2*e**2*h**5*x**2 - 4*e*f*g**3*h**2 - 8*e*f*g**2*h**3*x - 4*e*f*g*h**4*
x**2 + 2*f**2*g**4*h + 4*f**2*g**3*h**2*x + 2*f**2*g**2*h**3*x**2) + 2*b*e*
f*g*h*log(c*(d*(e + f*x)**p)**q)/(2*e**2*g**2*h**3 + 4*e**2*g*h**4*x + 2*e*
*2*h**5*x**2 - 4*e*f*g**3*h**2 - 8*e*f*g**2*h**3*x - 4*e*f*g*h**4*x**2 + 2*
f**2*g**4*h + 4*f**2*g**3*h**2*x + 2*f**2*g**2*h**3*x**2) - b*e*f*h**2*p*q*
x/(2*e**2*g**2*h**3 + 4*e**2*g*h**4*x + 2*e**2*h**5*x**2 - 4*e*f*g**3*h**2
- 8*e*f*g**2*h**3*x - 4*e*f*g*h**4*x**2 + 2*f**2*g**4*h + 4*f**2*g**3*h**2*
x + 2*f**2*g**2*h**3*x**2) - b*f**2*g**2*p*q*log(g/h + x)/(2*e**2*g**2*h**3
 + 4*e**2*g*h**4*x + 2*e**2*h**5*x**2 - 4*e*f*g**3*h**2 - 8*e*f*g**2*h**3*x
 - 4*e*f*g*h**4*x**2 + 2*f**2*g**4*h + 4*f**2*g**3*h**2*x + 2*f**2*g**2*h**
3*x**2) + b*f**2*g**2*p*q/(2*e**2*g**2*h**3 + 4*e**2*g*h**4*x + 2*e**2*h**5
*x**2 - 4*e*f*g**3*h**2 - 8*e*f*g**2*h**3*x - 4*e*f*g*h**4*x**2 + 2*f**2*g*
```

```
*4*h + 4*f**2*g**3*h**2*x + 2*f**2*g**2*h**3*x**2) - 2*b*f**2*g*h*p*q*x*log
(g/h + x)/(2*e**2*g**2*h**3 + 4*e**2*g*h**4*x + 2*e**2*h**5*x**2 - 4*e*f*g*
*3*h**2 - 8*e*f*g**2*h**3*x - 4*e*f*g*h**4*x**2 + 2*f**2*g**4*h + 4*f**2*g*
*3*h**2*x + 2*f**2*g**2*h**3*x**2) + b*f**2*g*h*p*q*x/(2*e**2*g**2*h**3 + 4
*e**2*g*h**4*x + 2*e**2*h**5*x**2 - 4*e*f*g**3*h**2 - 8*e*f*g**2*h**3*x - 4
*e*f*g*h**4*x**2 + 2*f**2*g**4*h + 4*f**2*g**3*h**2*x + 2*f**2*g**2*h**3*x*
*2) + 2*b*f**2*g*h*x*log(c*(d*(e + f*x)**p)**q)/(2*e**2*g**2*h**3 + 4*e**2*
g*h**4*x + 2*e**2*h**5*x**2 - 4*e*f*g**3*h**2 - 8*e*f*g**2*h**3*x - 4*e*f*g
*h**4*x**2 + 2*f**2*g**4*h + 4*f**2*g**3*h**2*x + 2*f**2*g**2*h**3*x**2) -
b*f**2*h**2*p*q*x**2*log(g/h + x)/(2*e**2*g**2*h**3 + 4*e**2*g*h**4*x + 2*e
**2*h**5*x**2 - 4*e*f*g**3*h**2 - 8*e*f*g**2*h**3*x - 4*e*f*g*h**4*x**2 + 2
*f**2*g**4*h + 4*f**2*g**3*h**2*x + 2*f**2*g**2*h**3*x**2) + b*f**2*h**2*x*
*2*log(c*(d*(e + f*x)**p)**q)/(2*e**2*g**2*h**3 + 4*e**2*g*h**4*x + 2*e**2*
h**5*x**2 - 4*e*f*g**3*h**2 - 8*e*f*g**2*h**3*x - 4*e*f*g*h**4*x**2 + 2*f**
2*g**4*h + 4*f**2*g**3*h**2*x + 2*f**2*g**2*h**3*x**2), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.45

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx$$
$$=\frac{1}{2}\,bfpq\left(\frac{f\log\left(fx+e\right)}{f^2g^2h-2\,efgh^2+e^2h^3}-\frac{f\log\left(hx+g\right)}{f^2g^2h-2\,efgh^2+e^2h^3}+\frac{1}{fg^2h-egh^2+(fgh^2-eh^3)x}\right)$$
$$-\frac{b\log\left(((fx+e)^pd)^q\,c\right)}{2\,(h^3x^2+2\,gh^2x+g^2h)}-\frac{a}{2\,(h^3x^2+2\,gh^2x+g^2h)}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*f*p*q*(f*log(f*x + e)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) - f*log(h*x
 + g)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) + 1/(f*g^2*h - e*g*h^2 + (f*g*h^2
 - e*h^3)*x)) - 1/2*b*log(((f*x + e)^p*d)^q*c)/(h^3*x^2 + 2*g*h^2*x + g^2*h
) - 1/2*a/(h^3*x^2 + 2*g*h^2*x + g^2*h)
```

## Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.86

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx = \frac{bf^2pq\log(fx+e)}{2\,(f^2g^2h-2\,efgh^2+e^2h^3)} - \frac{bf^2pq\log(hx+g)}{2\,(f^2g^2h-2\,efgh^2+e^2h^3)} - \frac{bpq\log(fx+e)}{2\,(h^3x^2+2\,gh^2x+g^2h)} + \frac{bfhpqx+bfgpq-bfgq\log(d)+behq\log(d)-bfg\log(c)+beh\log(c)-afg+aeh}{2\,(fgh^3x^2-eh^4x^2+2\,fg^2h^2x-2\,egh^3x+fg^3h-eg^2h^2)}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] 1/2*b*f^2*p*q*log(f*x + e)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) - 1/2*b*f^2*
p*q*log(h*x + g)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) - 1/2*b*p*q*log(f*x +
e)/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 1/2*(b*f*h*p*q*x + b*f*g*p*q - b*f*g*q*l
og(d) + b*e*h*q*log(d) - b*f*g*log(c) + b*e*h*log(c) - a*f*g + a*e*h)/(f*g*
h^3*x^2 - e*h^4*x^2 + 2*f*g^2*h^2*x - 2*e*g*h^3*x + f*g^3*h - e*g^2*h^2)
```

## Mupad [B] (verification not implemented)

Time = 3.49 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.51

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^3}\,dx = \frac{b\,f^2\,p\,q\,\mathrm{atanh}\left(\frac{2\,e^2\,h^3-2\,f^2\,g^2\,h}{2\,h\,(e\,h-f\,g)^2}+\frac{2\,f\,h\,x}{e\,h-f\,g}\right)}{h\,(e\,h-f\,g)^2} - \frac{b\,\ln\left(c\,(d\,(e+f\,x)^p)^q\right)}{2\,h\,(g^2+2\,g\,h\,x+h^2\,x^2)} - \frac{\frac{a\,e\,h-a\,f\,g+b\,f\,g\,p\,q}{e\,h-f\,g}+\frac{b\,f\,h\,p\,q\,x}{e\,h-f\,g}}{2\,g^2\,h+4\,g\,h^2\,x+2\,h^3\,x^2}$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^3,x)
```

```
[Out] (b*f^2*p*q*atanh((2*e^2*h^3 - 2*f^2*g^2*h)/(2*h*(e*h - f*g)^2) + (2*f*h*x)/
(e*h - f*g)))/(h*(e*h - f*g)^2) - (b*log(c*(d*(e + f*x)^p)^q))/(2*h*(g^2 +
h^2*x^2 + 2*g*h*x)) - ((a*e*h - a*f*g + b*f*g*p*q)/(e*h - f*g) + (b*f*h*p*q
*x)/(e*h - f*g))/(2*g^2*h + 2*h^3*x^2 + 4*g*h^2*x)
```

# 3.427 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^4} dx$



## Optimal result

Integrand size = 26, antiderivative size = 149

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^4} dx = \frac{bfpq}{6h(fg-eh)(g+hx)^2} + \frac{bf^2pq}{3h(fg-eh)^2(g+hx)} + \frac{bf^3pq\log(e+fx)}{3h(fg-eh)^3} - \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{3h(g+hx)^3} - \frac{bf^3pq\log(g+hx)}{3h(fg-eh)^3}$$

```
[Out] 1/6*b*f*p*q/h/(-e*h+f*g)/(h*x+g)^2+1/3*b*f^2*p*q/h/(-e*h+f*g)^2/(h*x+g)+1/3
*b*f^3*p*q*ln(f*x+e)/h/(-e*h+f*g)^3+1/3*(-a-b*ln(c*(d*(f*x+e)^p)^q))/h/(h*x
+g)^3-1/3*b*f^3*p*q*ln(h*x+g)/h/(-e*h+f*g)^3
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2442, 46, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^4} dx = -\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{3h(g+hx)^3} + \frac{bf^3pq\log(e+fx)}{3h(fg-eh)^3} - \frac{bf^3pq\log(g+hx)}{3h(fg-eh)^3} + \frac{bf^2pq}{3h(g+hx)(fg-eh)^2} + \frac{bfpq}{6h(g+hx)^2(fg-eh)}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^4,x]
```

```
[Out] (b*f*p*q)/(6*h*(f*g - e*h)*(g + h*x)^2) + (b*f^2*p*q)/(3*h*(f*g - e*h)^2*(g
 + h*x)) + (b*f^3*p*q*Log[e + f*x])/(3*h*(f*g - e*h)^3) - (a + b*Log[c*(d*(
e + f*x)^p)^q])/(3*h*(g + h*x)^3) - (b*f^3*p*q*Log[g + h*x])/(3*h*(f*g - e*
h)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(g+hx)^4}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= -\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{3h(g+hx)^3}\\
&\quad+\text{Subst}\left(\frac{(bfpq)\int\frac{1}{(e+fx)(g+hx)^3}\,dx}{3h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= -\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{3h(g+hx)^3}\\
&\quad+\text{Subst}\left(\frac{(bfpq)\int\left(\frac{f^3}{(fg-eh)^3(e+fx)}-\frac{h}{(fg-eh)(g+hx)^3}-\frac{fh}{(fg-eh)^2(g+hx)^2}-\frac{f^2h}{(fg-eh)^3(g+hx)}\right)dx}{3h}, cd^q(e\right.\\
&\qquad\left.+fx)^{pq}, c(d(e+fx)^p)^q\right)\end{aligned}$$

$$= \frac{bfpq}{6h(fg-eh)(g+hx)^2} + \frac{bf^2pq}{3h(fg-eh)^2(g+hx)} + \frac{bf^3pq\log(e+fx)}{3h(fg-eh)^3} - \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{3h(g+hx)^3} - \frac{bf^3pq\log(g+hx)}{3h(fg-eh)^3}$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^4}\,dx = \frac{-2a-2b\log\left(c(d(e+fx)^p)^q\right)+\frac{bfpq(g+hx)\left((fg-eh)(3fg-eh+2fhx)+2f^2(g+hx)^2\log(e+fx)-2f^2(g+hx)^2\log(g+hx)\right)}{(fg-eh)^3}}{6h(g+hx)^3}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^4,x]
```

```
[Out] (-2*a - 2*b*Log[c*(d*(e + f*x)^p)^q] + (b*f*p*q*(g + h*x)*((f*g - e*h)*(3*f
*g - e*h + 2*f*h*x) + 2*f^2*(g + h*x)^2*Log[e + f*x] - 2*f^2*(g + h*x)^2*Lo
g[g + h*x]))/(f*g - e*h)^3)/(6*h*(g + h*x)^3)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(142) = 284.

Time = 11.92 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.27

| method | result |
|---|---|
| parallelrisch | $-\frac{-6xbe f^3 g h^4 pq - 6\ln(c(d(fx+e)^p)^q) b e^2 f^2 g h^4 + 6\ln(c(d(fx+e)^p)^q) be f^3 g^2 h^3 + 2\ln(c(d(fx+e)^p)^q) b e^3 f h^5 - 2\ln(c(d(fx+e$ |

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-6*x*b*e*f^3*g*h^4*p*q-6*ln(c*(d*(f*x+e)^p)^q)*b*e^2*f^2*g*h^4+6*ln(c
*(d*(f*x+e)^p)^q)*b*e*f^3*g^2*h^3+2*ln(c*(d*(f*x+e)^p)^q)*b*e^3*f*h^5-2*ln(
c*(d*(f*x+e)^p)^q)*b*f^4*g^3*h^2+3*b*f^4*g^3*h^2*p*q+6*ln(f*x+e)*x^2*b*f^4*
g*h^4*p*q-6*ln(h*x+g)*x^2*b*f^4*g*h^4*p*q+6*ln(f*x+e)*x*b*f^4*g^2*h^3*p*q-6
*ln(h*x+g)*x*b*f^4*g^2*h^3*p*q+2*a*e^3*f*h^5-2*a*f^4*g^3*h^2-2*x^2*b*e*f^3*
h^5*p*q+2*x^2*b*f^4*g*h^4*p*q+2*ln(f*x+e)*x^3*b*f^4*h^5*p*q-2*ln(h*x+g)*x^3
*b*f^4*h^5*p*q+2*ln(f*x+e)*b*f^4*g^3*h^2*p*q-2*ln(h*x+g)*b*f^4*g^3*h^2*p*q-
6*a*e^2*f^2*g*h^4+6*a*e*f^3*g^2*h^3+b*e^2*f^2*g*h^4*p*q-4*b*e*f^3*g^2*h^3*p
*q+x*b*e^2*f^2*h^5*p*q+5*x*b*f^4*g^2*h^3*p*q)/(e^3*h^3-3*e^2*f*g*h^2+3*e*f^
2*g^2*h-f^3*g^3)/(h*x+g)^3/f/h^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(139) = 278$.

Time = 0.32 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.78

$$\int \frac{a + b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^4}\,dx = -\frac{2\,af^3g^3 - 6\,aef^2g^2h + 6\,ae^2fgh^2 - 2\,ae^3h^3 - 2\,(bf^3gh^2 - bef^2h^3)pqx^2 - (5\,bf^3g^2h - 6\,bef^2gh^2 + be^2fh}{}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*f^3*g^3 - 6*a*e*f^2*g^2*h + 6*a*e^2*f*g*h^2 - 2*a*e^3*h^3 - 2*(b*
f^3*g*h^2 - b*e*f^2*h^3)*p*q*x^2 - (5*b*f^3*g^2*h - 6*b*e*f^2*g*h^2 + b*e^2
*f*h^3)*p*q*x - (3*b*f^3*g^3 - 4*b*e*f^2*g^2*h + b*e^2*f*g*h^2)*p*q + 2*(b*
f^3*g^3 - 3*b*e*f^2*g^2*h + 3*b*e^2*f*g*h^2 - b*e^3*h^3)*q*log(d) - 2*(b*f^
3*h^3*p*q*x^3 + 3*b*f^3*g*h^2*p*q*x^2 + 3*b*f^3*g^2*h*p*q*x + (3*b*e*f^2*g^
2*h - 3*b*e^2*f*g*h^2 + b*e^3*h^3)*p*q)*log(f*x + e) + 2*(b*f^3*h^3*p*q*x^3
 + 3*b*f^3*g*h^2*p*q*x^2 + 3*b*f^3*g^2*h*p*q*x + b*f^3*g^3*p*q)*log(h*x + g
) + 2*(b*f^3*g^3 - 3*b*e*f^2*g^2*h + 3*b*e^2*f*g*h^2 - b*e^3*h^3)*log(c))/(
f^3*g^6*h - 3*e*f^2*g^5*h^2 + 3*e^2*f*g^4*h^3 - e^3*g^3*h^4 + (f^3*g^3*h^4
- 3*e*f^2*g^2*h^5 + 3*e^2*f*g*h^6 - e^3*h^7)*x^3 + 3*(f^3*g^4*h^3 - 3*e*f^2
*g^3*h^4 + 3*e^2*f*g^2*h^5 - e^3*g*h^6)*x^2 + 3*(f^3*g^5*h^2 - 3*e*f^2*g^4*
h^3 + 3*e^2*f*g^3*h^4 - e^3*g^2*h^5)*x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5673 vs. $2(133) = 266$.

Time = 63.37 (sec) , antiderivative size = 5673, normalized size of antiderivative = 38.07

$$\int \frac{a + b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**4,x)
```

```
[Out] Piecewise(((a*x + b*e*log(c*(d*(e + f*x)**p)**q)/f - b*p*q*x + b*x*log(c*(d
*(e + f*x)**p)**q))/g**4, Eq(h, 0)), (-3*a/(9*g**3*h + 27*g**2*h**2*x + 27*
g*h**3*x**2 + 9*h**4*x**3) - b*p*q/(9*g**3*h + 27*g**2*h**2*x + 27*g*h**3*x
**2 + 9*h**4*x**3) - 3*b*log(c*(d*(f*g/h + f*x)**p)**q)/(9*g**3*h + 27*g**2
*h**2*x + 27*g*h**3*x**2 + 9*h**4*x**3), Eq(e, f*g/h)), (-2*a*e**3*h**3/(6*
e**3*g**3*h**4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x*
*3 - 18*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2
 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54
*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3
```

*g**5*h**2*x - 18*f**3*g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3) + 6*a*e**2*f
*g*h**2/(6*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e
**3*h**7*x**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*g**
2*h**5*x**2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g**4*
h**3*x + 54*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*
h - 18*f**3*g**5*h**2*x - 18*f**3*g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3) -
6*a*e*f**2*g**2*h/(6*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6
*x**2 + 6*e**3*h**7*x**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 54
*e**2*f*g**2*h**5*x**2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54*e
*f**2*g**4*h**3*x + 54*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3 - 6
*f**3*g**6*h - 18*f**3*g**5*h**2*x - 18*f**3*g**4*h**3*x**2 - 6*f**3*g**3*h
**4*x**3) + 2*a*f**3*g**3/(6*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x + 18*e**3
*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**4
*x - 54*e**2*f*g**2*h**5*x**2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2
+ 54*e*f**2*g**4*h**3*x + 54*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*x
**3 - 6*f**3*g**6*h - 18*f**3*g**5*h**2*x - 18*f**3*g**4*h**3*x**2 - 6*f**3
*g**3*h**4*x**3) - 2*b*e**3*h**3*log(c*(d*(e + f*x)**p)**q)/(6*e**3*g**3*h*
*4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*e**2
*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e**2*f
*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2*g**3
*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h**2*x
- 18*f**3*g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3) - b*e**2*f*g*h**2*p*q/(6
*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x
**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**
2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 5
4*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**
3*g**5*h**2*x - 18*f**3*g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3) + 6*b*e**2*
f*g*h**2*log(c*(d*(e + f*x)**p)**q)/(6*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x
+ 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f
*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2
*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g
**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h**2*x - 18*f**3*g**4*h**3*x**
2 - 6*f**3*g**3*h**4*x**3) - b*e**2*f*h**3*p*q*x/(6*e**3*g**3*h**4 + 18*e**
3*g**2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*e**2*f*g**4*h**
3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e**2*f*g*h**6*x**
3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2*g**3*h**4*x**2
+ 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h**2*x - 18*f**3*
g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3) + 4*b*e*f**2*g**2*h*p*q/(6*e**3*g**
3*h**4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*
e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e*
*2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2*
g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h*
*2*x - 18*f**3*g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3) - 6*b*e*f**2*g**2*h*
log(c*(d*(e + f*x)**p)**q)/(6*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x + 18*e**
3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**

4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**
2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*
x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h**2*x - 18*f**3*g**4*h**3*x**2 - 6*f**
3*g**3*h**4*x**3) + 6*b*e*f**2*g*h**2*p*q*x/(6*e**3*g**3*h**4 + 18*e**3*g**
2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*e**2*f*g**4*h**3 - 5
4*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e**2*f*g*h**6*x**3 + 1
8*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2*g**3*h**4*x**2 + 18*
e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h**2*x - 18*f**3*g**4*
h**3*x**2 - 6*f**3*g**3*h**4*x**3) + 2*b*e*f**2*h**3*p*q*x**2/(6*e**3*g**3*
h**4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*e*
*2*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e**2
*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2*g*
*3*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h**2
*x - 18*f**3*g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3) + 2*b*f**3*g**3*p*q*lo
g(g/h + x)/(6*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6*x**2 +
6*e**3*h**7*x**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*
g**2*h**5*x**2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g*
*4*h**3*x + 54*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g*
*6*h - 18*f**3*g**5*h**2*x - 18*f**3*g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3
) - 3*b*f**3*g**3*p*q/(6*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h
**6*x**2 + 6*e**3*h**7*x**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x -
54*e**2*f*g**2*h**5*x**2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 5
4*e*f**2*g**4*h**3*x + 54*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3
- 6*f**3*g**6*h - 18*f**3*g**5*h**2*x - 18*f**3*g**4*h**3*x**2 - 6*f**3*g**
3*h**4*x**3) + 6*b*f**3*g**2*h*p*q*x*log(g/h + x)/(6*e**3*g**3*h**4 + 18*e*
*3*g**2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*e**2*f*g**4*h*
*3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e**2*f*g*h**6*x*
*3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2*g**3*h**4*x**2
+ 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h**2*x - 18*f**3
*g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3) - 5*b*f**3*g**2*h*p*q*x/(6*e**3*g*
*3*h**4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18
*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e
**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2
*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h
**2*x - 18*f**3*g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3) - 6*b*f**3*g**2*h*x
*log(c*(d*(e + f*x)**p)**q)/(6*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x + 18*e*
*3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h*
*4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h*
*2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5
*x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h**2*x - 18*f**3*g**4*h**3*x**2 - 6*f*
*3*g**3*h**4*x**3) + 6*b*f**3*g*h**2*p*q*x**2*log(g/h + x)/(6*e**3*g**3*h**
4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*e**2*
f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e**2*f*
g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2*g**3*
h**4*x**2 + 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h**2*x

```
- 18*f**3*g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3) - 2*b*f**3*g*h**2*p*q*x**
2/(6*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h*
*7*x**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h**5
*x**2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**3*x
 + 54*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h - 18
*f**3*g**5*h**2*x - 18*f**3*g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3) - 6*b*f
**3*g*h**2*x**2*log(c*(d*(e + f*x)**p)**q)/(6*e**3*g**3*h**4 + 18*e**3*g**2
*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*e**2*f*g**4*h**3 - 54
*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e**2*f*g*h**6*x**3 + 18
*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2*g**3*h**4*x**2 + 18*e
*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h**2*x - 18*f**3*g**4*h
**3*x**2 - 6*f**3*g**3*h**4*x**3) + 2*b*f**3*h**3*p*q*x**3*log(g/h + x)/(6*
e**3*g**3*h**4 + 18*e**3*g**2*h**5*x + 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x*
*3 - 18*e**2*f*g**4*h**3 - 54*e**2*f*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2
 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54
*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3
*g**5*h**2*x - 18*f**3*g**4*h**3*x**2 - 6*f**3*g**3*h**4*x**3) - 2*b*f**3*h
**3*x**3*log(c*(d*(e + f*x)**p)**q)/(6*e**3*g**3*h**4 + 18*e**3*g**2*h**5*x
 + 18*e**3*g*h**6*x**2 + 6*e**3*h**7*x**3 - 18*e**2*f*g**4*h**3 - 54*e**2*f
*g**3*h**4*x - 54*e**2*f*g**2*h**5*x**2 - 18*e**2*f*g*h**6*x**3 + 18*e*f**2
*g**5*h**2 + 54*e*f**2*g**4*h**3*x + 54*e*f**2*g**3*h**4*x**2 + 18*e*f**2*g
**2*h**5*x**3 - 6*f**3*g**6*h - 18*f**3*g**5*h**2*x - 18*f**3*g**4*h**3*x**
2 - 6*f**3*g**3*h**4*x**3), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(139) = 278$.

Time = 0.21 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.05

$$\int \frac{a + b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^4}\,dx$$
$$= \frac{1}{6}\left(\frac{2\,f^2\log\left(fx+e\right)}{f^3g^3h - 3\,ef^2g^2h^2 + 3\,e^2fgh^3 - e^3h^4} - \frac{2\,f^2\log\left(hx+g\right)}{f^3g^3h - 3\,ef^2g^2h^2 + 3\,e^2fgh^3 - e^3h^4} + \frac{}{f^2g^4h - 2\,efg^3h^2 +}\right.$$
$$- \frac{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)}{3\left(h^4x^3 + 3\,gh^3x^2 + 3\,g^2h^2x + g^3h\right)} - \frac{a}{3\left(h^4x^3 + 3\,gh^3x^2 + 3\,g^2h^2x + g^3h\right)}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*f^2*log(f*x + e)/(f^3*g^3*h - 3*e*f^2*g^2*h^2 + 3*e^2*f*g*h^3 - e^3*
h^4) - 2*f^2*log(h*x + g)/(f^3*g^3*h - 3*e*f^2*g^2*h^2 + 3*e^2*f*g*h^3 - e^
3*h^4) + (2*f*h*x + 3*f*g - e*h)/(f^2*g^4*h - 2*e*f*g^3*h^2 + e^2*g^2*h^3 +
 (f^2*g^2*h^3 - 2*e*f*g*h^4 + e^2*h^5)*x^2 + 2*(f^2*g^3*h^2 - 2*e*f*g^2*h^3
 + e^2*g*h^4)*x))*b*f*p*q - 1/3*b*log(((f*x + e)^p*d)^q*c)/(h^4*x^3 + 3*g*h
^3*x^2 + 3*g^2*h^2*x + g^3*h) - 1/3*a/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h^2*x
+ g^3*h)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(139) = 278$.

Time = 0.37 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.97

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)^4}\,dx = \frac{bf^3pq\log\left(fx + e\right)}{3\left(f^3g^3h - 3\,ef^2g^2h^2 + 3\,e^2fgh^3 - e^3h^4\right)} - \frac{bf^3pq\log\left(hx + g\right)}{3\left(f^3g^3h - 3\,ef^2g^2h^2 + 3\,e^2fgh^3 - e^3h^4\right)} - \frac{bpq\log\left(fx + e\right)}{3\left(h^4x^3 + 3\,gh^3x^2 + 3\,g^2h^2x + g^3h\right)} + \frac{2\,bf^2h^2pqx^2 + 5\,bf^2ghpqx - befh^2pqx + 3\,bf^2g^2pq - befghpq - 2\,bf^2g^2q\log\left(d\right) + 4\,befghq\log\left(d\right) - 2\,b}{6\left(f^2g^2h^4x^3 - 2\,efgh^5x^3 + e^2h^6x^3 + 3\,f^2g^3h^3x^2 - 6\,efg^2h^4x^2 + 3\,e^2gh^5x^2 +\right.}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/3*b*f^3*p*q*log(f*x + e)/(f^3*g^3*h - 3*e*f^2*g^2*h^2 + 3*e^2*f*g*h^3 - e
^3*h^4) - 1/3*b*f^3*p*q*log(h*x + g)/(f^3*g^3*h - 3*e*f^2*g^2*h^2 + 3*e^2*f
*g*h^3 - e^3*h^4) - 1/3*b*p*q*log(f*x + e)/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h
^2*x + g^3*h) + 1/6*(2*b*f^2*h^2*p*q*x^2 + 5*b*f^2*g*h*p*q*x - b*e*f*h^2*p*
q*x + 3*b*f^2*g^2*p*q - b*e*f*g*h*p*q - 2*b*f^2*g^2*q*log(d) + 4*b*e*f*g*h*
q*log(d) - 2*b*e^2*h^2*q*log(d) - 2*b*f^2*g^2*log(c) + 4*b*e*f*g*h*log(c) -
 2*b*e^2*h^2*log(c) - 2*a*f^2*g^2 + 4*a*e*f*g*h - 2*a*e^2*h^2)/(f^2*g^2*h^4
*x^3 - 2*e*f*g*h^5*x^3 + e^2*h^6*x^3 + 3*f^2*g^3*h^3*x^2 - 6*e*f*g^2*h^4*x^
2 + 3*e^2*g*h^5*x^2 + 3*f^2*g^4*h^2*x - 6*e*f*g^3*h^3*x + 3*e^2*g^2*h^4*x +
 f^2*g^5*h - 2*e*f*g^4*h^2 + e^2*g^3*h^3)
```

## Mupad [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.97

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)^4}\,dx = \frac{2\,a\,e\,f\,g}{3\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2} - \frac{a\,e^2\,h}{3\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2} - \frac{b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{3\,h\,{\left(g+h\,x\right)}^3} - \frac{a\,f^2\,g^2}{3\,h\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2} + \frac{b\,f^2\,h\,p\,q\,x^2}{3\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2} - \frac{b\,e\,f\,g\,p\,q}{6\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2} + \frac{b\,f^2\,g^2\,p\,q}{2\,h\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2} + \frac{5\,b\,f^2\,g\,p\,q\,x}{6\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2} - \frac{b\,e\,f\,h\,p\,q\,x}{6\,{\left(g+h\,x\right)}^3\,{\left(e\,h-f\,g\right)}^2} + \frac{b\,f^3\,p\,q\,\mathrm{atan}\left(\frac{e\,h\,1{}\mathrm{i}+f\,g\,1{}\mathrm{i}+f\,h\,x\,2{}\mathrm{i}}{e\,h-f\,g}\right)\,2{}\mathrm{i}}{3\,h\,{\left(e\,h-f\,g\right)}^3}$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^4,x)
```

```
[Out] (2*a*e*f*g)/(3*(g + h*x)^3*(e*h - f*g)^2) - (a*e^2*h)/(3*(g + h*x)^3*(e*h -
 f*g)^2) - (b*log(c*(d*(e + f*x)^p)^q))/(3*h*(g + h*x)^3) - (a*f^2*g^2)/(3*
h*(g + h*x)^3*(e*h - f*g)^2) + (b*f^3*p*q*atan((e*h*1i + f*g*1i + f*h*x*2i)
/(e*h - f*g))*2i)/(3*h*(e*h - f*g)^3) + (b*f^2*h*p*q*x^2)/(3*(g + h*x)^3*(e
*h - f*g)^2) - (b*e*f*g*p*q)/(6*(g + h*x)^3*(e*h - f*g)^2) + (b*f^2*g^2*p*q
)/(2*h*(g + h*x)^3*(e*h - f*g)^2) + (5*b*f^2*g*p*q*x)/(6*(g + h*x)^3*(e*h -
 f*g)^2) - (b*e*f*h*p*q*x)/(6*(g + h*x)^3*(e*h - f*g)^2)
```

# 3.428 $\int (g+hx)^3 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 409

$$\begin{aligned}
\int (g+hx)^3 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx
&= \frac{2b^2(fg-eh)^3p^2q^2x}{f^3} + \frac{3b^2h(fg-eh)^2p^2q^2(e+fx)^2}{4f^4} + \frac{2b^2h^2(fg-eh)p^2q^2(e+fx)^3}{9f^4} \\
&\quad + \frac{b^2h^3p^2q^2(e+fx)^4}{32f^4} + \frac{b^2(fg-eh)^4p^2q^2\log^2(e+fx)}{4f^4h} \\
&\quad - \frac{2b(fg-eh)^3pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^4} \\
&\quad - \frac{3bh(fg-eh)^2pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^4} \\
&\quad - \frac{2bh^2(fg-eh)pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3f^4} \\
&\quad - \frac{bh^3pq(e+fx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{8f^4} \\
&\quad - \frac{b(fg-eh)^4pq\log(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^4h} \\
&\quad + \frac{(g+hx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4h}
\end{aligned}$$

```
[Out] 2*b^2*(-e*h+f*g)^3*p^2*q^2*x/f^3+3/4*b^2*h*(-e*h+f*g)^2*p^2*q^2*(f*x+e)^2/f
^4+2/9*b^2*h^2*(-e*h+f*g)*p^2*q^2*(f*x+e)^3/f^4+1/32*b^2*h^3*p^2*q^2*(f*x+e
)^4/f^4+1/4*b^2*(-e*h+f*g)^4*p^2*q^2*ln(f*x+e)^2/f^4/h-2*b*(-e*h+f*g)^3*p*q
*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^4-3/2*b*h*(-e*h+f*g)^2*p*q*(f*x+e)^2
*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^4-2/3*b*h^2*(-e*h+f*g)*p*q*(f*x+e)^3*(a+b*ln
(c*(d*(f*x+e)^p)^q))/f^4-1/8*b*h^3*p*q*(f*x+e)^4*(a+b*ln(c*(d*(f*x+e)^p)^q)
)/f^4-1/2*b*(-e*h+f*g)^4*p*q*ln(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^4/h+1/
4*(h*x+g)^4*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/h
```

## Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2445, 2458, 45, 2372, 12, 2338, 2495}

$$\begin{aligned}&\int (g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2dx\\&=-\frac{2bh^2pq(e+fx)^3(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3f^4}\\&-\frac{bpq(fg-eh)^4\log(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^4h}\\&-\frac{2bpq(e+fx)(fg-eh)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^4}\\&-\frac{3bhpq(e+fx)^2(fg-eh)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^4}\\&-\frac{bh^3pq(e+fx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{8f^4}+\frac{(g+hx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4h}\\&+\frac{2b^2h^2p^2q^2(e+fx)^3(fg-eh)}{9f^4}+\frac{3b^2hp^2q^2(e+fx)^2(fg-eh)^2}{4f^4}\\&+\frac{b^2p^2q^2(fg-eh)^4\log^2(e+fx)}{4f^4h}+\frac{b^2h^3p^2q^2(e+fx)^4}{32f^4}+\frac{2b^2p^2q^2x(fg-eh)^3}{f^3}\end{aligned}$$

```
[In] Int[(g + h*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] (2*b^2*(f*g - e*h)^3*p^2*q^2*x)/f^3 + (3*b^2*h*(f*g - e*h)^2*p^2*q^2*(e + f
*x)^2)/(4*f^4) + (2*b^2*h^2*(f*g - e*h)*p^2*q^2*(e + f*x)^3)/(9*f^4) + (b^2
*h^3*p^2*q^2*(e + f*x)^4)/(32*f^4) + (b^2*(f*g - e*h)^4*p^2*q^2*Log[e + f*x
]^2)/(4*f^4*h) - (2*b*(f*g - e*h)^3*p*q*(e + f*x)*(a + b*Log[c*(d*(e + f*x)
^p)^q]))/f^4 - (3*b*h*(f*g - e*h)^2*p*q*(e + f*x)^2*(a + b*Log[c*(d*(e + f*
x)^p)^q]))/(2*f^4) - (2*b*h^2*(f*g - e*h)*p*q*(e + f*x)^3*(a + b*Log[c*(d*(
e + f*x)^p)^q]))/(3*f^4) - (b*h^3*p*q*(e + f*x)^4*(a + b*Log[c*(d*(e + f*x)
^p)^q]))/(8*f^4) - (b*(f*g - e*h)^4*p*q*Log[e + f*x]*(a + b*Log[c*(d*(e + f
*x)^p)^q]))/(2*f^4*h) + ((g + h*x)^4*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(4
*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (g+hx)^3 \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2 \,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(g+hx)^4 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4h} - \text{Subst}\left(\frac{(bfpq)\int \frac{(g+hx)^4(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}\,dx}{2h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(g+hx)^4 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4h} - \text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int \frac{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^4(a+b\log(cd^q x^{pq}))}{x}\,dx, x, e+fx\right)}{2h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2b(fg-eh)^3pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^4} - \frac{3bh(fg-eh)^2pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^4} - \frac{2bh^2(fg-eh)pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3f^4} - \frac{bh^3pq(e+fx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{8f^4} - \frac{b(fg-eh)^4pq\log(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^4h} + \frac{(g+hx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4h} + \text{Subst}\left(\frac{(b^2p^2q^2)\,\text{Subst}\left(\int \frac{48h(fg-eh)^3+36h^2(fg-eh)^2x+16h^3(fg-eh)x^2+3h^4x^3+\frac{12(fg-eh)^4\log(x)}{x}}{12f^4}\,dx, x, e+fx\right)}{2h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2b(fg-eh)^3pq(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{f^4} - \frac{3bh(fg-eh)^2pq(e+fx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{2f^4} - \frac{2bh^2(fg-eh)pq(e+fx)^3\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{3f^4} - \frac{bh^3pq(e+fx)^4\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{8f^4} - \frac{b(fg-eh)^4pq\log(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{2f^4h} + \frac{(g+hx)^4\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{4h} + \text{Subst}\left(\frac{(b^2p^2q^2)\,\text{Subst}\left(\int\left(48h(fg-eh)^3+36h^2(fg-eh)^2x+16h^3(fg-eh)x^2+3h^4x^3+\frac{12(f}{}\right.\right.}{24f^4h} + fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= \frac{2b^2(fg-eh)^3p^2q^2x}{f^3} + \frac{3b^2h(fg-eh)^2p^2q^2(e+fx)^2}{4f^4} + \frac{2b^2h^2(fg-eh)p^2q^2(e+fx)^3}{9f^4} + \frac{b^2h^3p^2q^2(e+fx)^4}{32f^4} - \frac{2b(fg-eh)^3pq(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{f^4} - \frac{3bh(fg-eh)^2pq(e+fx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{2f^4} - \frac{2bh^2(fg-eh)pq(e+fx)^3\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{3f^4} - \frac{bh^3pq(e+fx)^4\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{8f^4} - \frac{b(fg-eh)^4pq\log(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{2f^4h} + \frac{(g+hx)^4\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{4h} + \text{Subst}\left(\frac{(b^2(fg-eh)^4p^2q^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, e+fx\right)}{2f^4h}, cd^q(e + fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= \frac{2b^2(fg-eh)^3p^2q^2x}{f^3} + \frac{3b^2h(fg-eh)^2p^2q^2(e+fx)^2}{4f^4} + \frac{2b^2h^2(fg-eh)p^2q^2(e+fx)^3}{9f^4} + \frac{b^2h^3p^2q^2(e+fx)^4}{32f^4} + \frac{b^2(fg-eh)^4p^2q^2\log^2(e+fx)}{4f^4h} - \frac{2b(fg-eh)^3pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^4} - \frac{3bh(fg-eh)^2pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^4} - \frac{2bh^2(fg-eh)pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3f^4} - \frac{bh^3pq(e+fx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{8f^4} - \frac{b(fg-eh)^4pq\log(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^4h} + \frac{(g+hx)^4\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4h}$$

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.98

$$\int (g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx = \frac{288(fg-eh)^3(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2+432h(fg-eh)^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^{q}\right)\right)^{\cdots}}{}$$

```
[In] Integrate[(g + h*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] (288*(f*g - e*h)^3*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 432*h*(f*
g - e*h)^2*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 288*h^2*(f*g -
e*h)*(e + f*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 72*h^3*(e + f*x)^4*(a
 + b*Log[c*(d*(e + f*x)^p)^q])^2 - 576*b*(f*g - e*h)^3*p*q*(f*(a - b*p*q)*x
 + b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q]) + 216*b*h*(f*g - e*h)^2*p*q*(b*f*p
*q*x*(2*e + f*x) - 2*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])) + 64*b*h
^2*(f*g - e*h)*p*q*(b*f*p*q*x*(3*e^2 + 3*e*f*x + f^2*x^2) - 3*(e + f*x)^3*(
a + b*Log[c*(d*(e + f*x)^p)^q])) + 9*b*h^3*p*q*(b*f*p*q*x*(4*e^3 + 6*e^2*f*
x + 4*e*f^2*x^2 + f^3*x^3) - 4*(e + f*x)^4*(a + b*Log[c*(d*(e + f*x)^p)^q])
))/(288*f^4)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1536 vs. $2(391) = 782$.

Time = 16.24 (sec) , antiderivative size = 1537, normalized size of antiderivative = 3.76

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1537 |

```
[In] int((h*x+g)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/288*(1632*ln(f*x+e)*b^2*e^3*f*g*h^2*p^2*q^2-864*x*ln(c*(d*(f*x+e)^p)^q)*
b^2*e*f^3*g^2*h*p*q+576*x*a*b*e^2*f^2*g*h^2*p*q-864*x*a*b*e*f^3*g^2*h*p*q-5
76*ln(f*x+e)*a*b*e^3*f*g*h^2*p*q+864*ln(f*x+e)*a*b*e^2*f^2*g^2*h*p*q-288*x^
2*ln(c*(d*(f*x+e)^p)^q)*b^2*e*f^3*g*h^2*p*q-288*x^2*a*b*e*f^3*g*h^2*p*q+576
*x*ln(c*(d*(f*x+e)^p)^q)*b^2*e^2*f^2*g*h^2*p*q-2160*ln(f*x+e)*b^2*e^2*f^2*g
^2*h*p^2*q^2-1152*ln(f*x+e)*a*b*e*f^3*g^3*p*q-48*x^3*ln(c*(d*(f*x+e)^p)^q)*
b^2*e*f^3*h^3*p*q+192*x^3*ln(c*(d*(f*x+e)^p)^q)*b^2*f^4*g*h^2*p*q+240*x^2*b
^2*e*f^3*g*h^2*p^2*q^2-48*x^3*a*b*e*f^3*h^3*p*q+192*x^3*a*b*f^4*g*h^2*p*q+7
2*x^2*ln(c*(d*(f*x+e)^p)^q)*b^2*e^2*f^2*h^3*p*q+432*x^2*ln(c*(d*(f*x+e)^p)^
q)*b^2*f^4*g^2*h*p*q-1056*x*b^2*e^2*f^2*g*h^2*p^2*q^2+864*a*b*e^2*f^2*g^2*h
*p*q-576*a*b*e^3*f*g*h^2*p*q+1296*x*b^2*e*f^3*g^2*h*p^2*q^2+72*x^2*a*b*e^2*
f^2*h^3*p*q+432*x^2*a*b*f^4*g^2*h*p*q-144*x*ln(c*(d*(f*x+e)^p)^q)*b^2*e^3*f
*h^3*p*q-144*x*a*b*e^3*f*h^3*p*q-576*ln(c*(d*(f*x+e)^p)^q)*b^2*e^3*f*g*h^2*
p*q+864*ln(c*(d*(f*x+e)^p)^q)*b^2*e^2*f^2*g^2*h*p*q+36*x^4*ln(c*(d*(f*x+e)^
p)^q)*b^2*f^4*h^3*p*q+28*x^3*b^2*e*f^3*h^3*p^2*q^2-64*x^3*b^2*f^4*g*h^2*p^2
*q^2-576*a*b*e*f^3*g^3*p*q+1056*b^2*e^3*f*g*h^2*p^2*q^2-1296*b^2*e^2*f^2*g^
2*h*p^2*q^2+36*x^4*a*b*f^4*h^3*p*q-78*x^2*b^2*e^2*f^2*h^3*p^2*q^2-216*x^2*b
^2*f^4*g^2*h*p^2*q^2+300*x*b^2*e^3*f*h^3*p^2*q^2-576*x^3*ln(c*(d*(f*x+e)^p)
^q)*a*b*f^4*g*h^2+576*x*ln(c*(d*(f*x+e)^p)^q)*b^2*f^4*g^3*p*q-864*x^2*ln(c*
(d*(f*x+e)^p)^q)*a*b*f^4*g^2*h+576*x*a*b*f^4*g^3*p*q-576*ln(c*(d*(f*x+e)^p)
^q)*b^2*e*f^3*g^3*p*q+1152*ln(f*x+e)*b^2*e*f^3*g^3*p^2*q^2+144*ln(f*x+e)*a*
b*e^4*h^3*p*q-300*b^2*e^4*h^3*p^2*q^2+144*a*b*e^4*h^3*p*q+576*b^2*e*f^3*g^3
*p^2*q^2+288*a^2*e*f^3*g^3-9*x^4*b^2*f^4*h^3*p^2*q^2-144*x^4*ln(c*(d*(f*x+e
)^p)^q)*a*b*f^4*h^3-288*x^3*ln(c*(d*(f*x+e)^p)^q)^2*b^2*f^4*g*h^2-72*x^4*ln
(c*(d*(f*x+e)^p)^q)^2*b^2*f^4*h^3-288*x^3*a^2*f^4*g*h^2-288*x*ln(c*(d*(f*x+
e)^p)^q)^2*b^2*f^4*g^3-432*x^2*a^2*f^4*g^2*h-288*ln(c*(d*(f*x+e)^p)^q)^2*b^
2*e*f^3*g^3-288*x*a^2*f^4*g^3-72*x^4*a^2*f^4*h^3+72*ln(c*(d*(f*x+e)^p)^q)^2
*b^2*e^4*h^3-576*x*b^2*f^4*g^3*p^2*q^2-432*x^2*ln(c*(d*(f*x+e)^p)^q)^2*b^2*
f^4*g^2*h+144*ln(c*(d*(f*x+e)^p)^q)*b^2*e^4*h^3*p*q-576*x*ln(c*(d*(f*x+e)^p
)^q)*a*b*f^4*g^3-288*ln(c*(d*(f*x+e)^p)^q)^2*b^2*e^3*f*g*h^2+432*ln(c*(d*(f
*x+e)^p)^q)^2*b^2*e^2*f^2*g^2*h+576*ln(c*(d*(f*x+e)^p)^q)*a*b*e*f^3*g^3-444
*ln(f*x+e)*b^2*e^4*h^3*p^2*q^2)/f^4
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. 2(391) = 782.

Time = 0.35 (sec) , antiderivative size = 1742, normalized size of antiderivative = 4.26

$$\int (g+hx)^3 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^3*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

```
[Out] 1/288*(9*(b^2*f^4*h^3*p^2*q^2 - 4*a*b*f^4*h^3*p*q + 8*a^2*f^4*h^3)*x^4 + 4*
(72*a^2*f^4*g*h^2 + (16*b^2*f^4*g*h^2 - 7*b^2*e*f^3*h^3)*p^2*q^2 - 12*(4*a*
b*f^4*g*h^2 - a*b*e*f^3*h^3)*p*q)*x^3 + 6*(72*a^2*f^4*g^2*h + (36*b^2*f^4*g
^2*h - 40*b^2*e*f^3*g*h^2 + 13*b^2*e^2*f^2*h^3)*p^2*q^2 - 12*(6*a*b*f^4*g^2
*h - 4*a*b*e*f^3*g*h^2 + a*b*e^2*f^2*h^3)*p*q)*x^2 + 72*(b^2*f^4*h^3*p^2*q^
2*x^4 + 4*b^2*f^4*g*h^2*p^2*q^2*x^3 + 6*b^2*f^4*g^2*h*p^2*q^2*x^2 + 4*b^2*f
^4*g^3*p^2*q^2*x + (4*b^2*e*f^3*g^3 - 6*b^2*e^2*f^2*g^2*h + 4*b^2*e^3*f*g*h
^2 - b^2*e^4*h^3)*p^2*q^2)*log(f*x + e)^2 + 72*(b^2*f^4*h^3*x^4 + 4*b^2*f^4
*g*h^2*x^3 + 6*b^2*f^4*g^2*h*x^2 + 4*b^2*f^4*g^3*x)*log(c)^2 + 72*(b^2*f^4*
h^3*q^2*x^4 + 4*b^2*f^4*g*h^2*q^2*x^3 + 6*b^2*f^4*g^2*h*q^2*x^2 + 4*b^2*f^4
*g^3*q^2*x)*log(d)^2 + 12*(24*a^2*f^4*g^3 + (48*b^2*f^4*g^3 - 108*b^2*e*f^3
*g^2*h + 88*b^2*e^2*f^2*g*h^2 - 25*b^2*e^3*f*h^3)*p^2*q^2 - 12*(4*a*b*f^4*g
^3 - 6*a*b*e*f^3*g^2*h + 4*a*b*e^2*f^2*g*h^2 - a*b*e^3*f*h^3)*p*q)*x - 12*(
(48*b^2*e*f^3*g^3 - 108*b^2*e^2*f^2*g^2*h + 88*b^2*e^3*f*g*h^2 - 25*b^2*e^4
*h^3)*p^2*q^2 + 3*(b^2*f^4*h^3*p^2*q^2 - 4*a*b*f^4*h^3*p*q)*x^4 - 4*(12*a*b
*f^4*g*h^2*p*q - (4*b^2*f^4*g*h^2 - b^2*e*f^3*h^3)*p^2*q^2)*x^3 - 12*(4*a*b
*e*f^3*g^3 - 6*a*b*e^2*f^2*g^2*h + 4*a*b*e^3*f*g*h^2 - a*b*e^4*h^3)*p*q - 6
*(12*a*b*f^4*g^2*h*p*q - (6*b^2*f^4*g^2*h - 4*b^2*e*f^3*g*h^2 + b^2*e^2*f^2
*h^3)*p^2*q^2)*x^2 - 12*(4*a*b*f^4*g^3*p*q - (4*b^2*f^4*g^3 - 6*b^2*e*f^3*g
^2*h + 4*b^2*e^2*f^2*g*h^2 - b^2*e^3*f*h^3)*p^2*q^2)*x - 12*(b^2*f^4*h^3*p*
q*x^4 + 4*b^2*f^4*g*h^2*p*q*x^3 + 6*b^2*f^4*g^2*h*p*q*x^2 + 4*b^2*f^4*g^3*p
*q*x + (4*b^2*e*f^3*g^3 - 6*b^2*e^2*f^2*g^2*h + 4*b^2*e^3*f*g*h^2 - b^2*e^4
*h^3)*p*q)*log(c) - 12*(b^2*f^4*h^3*p*q^2*x^4 + 4*b^2*f^4*g*h^2*p*q^2*x^3 +
 6*b^2*f^4*g^2*h*p*q^2*x^2 + 4*b^2*f^4*g^3*p*q^2*x + (4*b^2*e*f^3*g^3 - 6*b
^2*e^2*f^2*g^2*h + 4*b^2*e^3*f*g*h^2 - b^2*e^4*h^3)*p*q^2)*log(d))*log(f*x
+ e) - 12*(3*(b^2*f^4*h^3*p*q - 4*a*b*f^4*h^3)*x^4 - 4*(12*a*b*f^4*g*h^2 -
(4*b^2*f^4*g*h^2 - b^2*e*f^3*h^3)*p*q)*x^3 - 6*(12*a*b*f^4*g^2*h - (6*b^2*f
^4*g^2*h - 4*b^2*e*f^3*g*h^2 + b^2*e^2*f^2*h^3)*p*q)*x^2 - 12*(4*a*b*f^4*g^
3 - (4*b^2*f^4*g^3 - 6*b^2*e*f^3*g^2*h + 4*b^2*e^2*f^2*g*h^2 - b^2*e^3*f*h^
3)*p*q)*x)*log(c) - 12*(3*(b^2*f^4*h^3*p*q^2 - 4*a*b*f^4*h^3*q)*x^4 - 4*(12
*a*b*f^4*g*h^2*q - (4*b^2*f^4*g*h^2 - b^2*e*f^3*h^3)*p*q^2)*x^3 - 6*(12*a*b
*f^4*g^2*h*q - (6*b^2*f^4*g^2*h - 4*b^2*e*f^3*g*h^2 + b^2*e^2*f^2*h^3)*p*q^
2)*x^2 - 12*(4*a*b*f^4*g^3*q - (4*b^2*f^4*g^3 - 6*b^2*e*f^3*g^2*h + 4*b^2*e
^2*f^2*g*h^2 - b^2*e^3*f*h^3)*p*q^2)*x - 12*(b^2*f^4*h^3*q*x^4 + 4*b^2*f^4*
g*h^2*q*x^3 + 6*b^2*f^4*g^2*h*q*x^2 + 4*b^2*f^4*g^3*q*x)*log(c))*log(d))/f^
4
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. $2(394) = 788$.

Time = 5.39 (sec) , antiderivative size = 1421, normalized size of antiderivative = 3.47

$$\int (g+hx)^3 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)**3*(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

```
[Out] Piecewise((a**2*g**3*x + 3*a**2*g**2*h*x**2/2 + a**2*g*h**2*x**3 + a**2*h**
3*x**4/4 - a*b*e**4*h**3*log(c*(d*(e + f*x)**p)**q)/(2*f**4) + 2*a*b*e**3*g
*h**2*log(c*(d*(e + f*x)**p)**q)/f**3 + a*b*e**3*h**3*p*q*x/(2*f**3) - 3*a*
b*e**2*g**2*h*log(c*(d*(e + f*x)**p)**q)/f**2 - 2*a*b*e**2*g*h**2*p*q*x/f**
2 - a*b*e**2*h**3*p*q*x**2/(4*f**2) + 2*a*b*e*g**3*log(c*(d*(e + f*x)**p)**
q)/f + 3*a*b*e*g**2*h*p*q*x/f + a*b*e*g*h**2*p*q*x**2/f + a*b*e*h**3*p*q*x*
*3/(6*f) - 2*a*b*g**3*p*q*x + 2*a*b*g**3*x*log(c*(d*(e + f*x)**p)**q) - 3*a
*b*g**2*h*p*q*x**2/2 + 3*a*b*g**2*h*x**2*log(c*(d*(e + f*x)**p)**q) - 2*a*b
*g*h**2*p*q*x**3/3 + 2*a*b*g*h**2*x**3*log(c*(d*(e + f*x)**p)**q) - a*b*h**
3*p*q*x**4/8 + a*b*h**3*x**4*log(c*(d*(e + f*x)**p)**q)/2 + 25*b**2*e**4*h*
*3*p*q*log(c*(d*(e + f*x)**p)**q)/(24*f**4) - b**2*e**4*h**3*log(c*(d*(e +
f*x)**p)**q)**2/(4*f**4) - 11*b**2*e**3*g*h**2*p*q*log(c*(d*(e + f*x)**p)**
q)/(3*f**3) + b**2*e**3*g*h**2*log(c*(d*(e + f*x)**p)**q)**2/f**3 - 25*b**2
*e**3*h**3*p**2*q**2*x/(24*f**3) + b**2*e**3*h**3*p*q*x*log(c*(d*(e + f*x)*
*p)**q)/(2*f**3) + 9*b**2*e**2*g**2*h*p*q*log(c*(d*(e + f*x)**p)**q)/(2*f**
2) - 3*b**2*e**2*g**2*h*log(c*(d*(e + f*x)**p)**q)**2/(2*f**2) + 11*b**2*e*
*2*g*h**2*p**2*q**2*x/(3*f**2) - 2*b**2*e**2*g*h**2*p*q*x*log(c*(d*(e + f*x
)**p)**q)/f**2 + 13*b**2*e**2*h**3*p**2*q**2*x**2/(48*f**2) - b**2*e**2*h**
3*p*q*x**2*log(c*(d*(e + f*x)**p)**q)/(4*f**2) - 2*b**2*e*g**3*p*q*log(c*(d
*(e + f*x)**p)**q)/f + b**2*e*g**3*log(c*(d*(e + f*x)**p)**q)**2/f - 9*b**2
*e*g**2*h*p**2*q**2*x/(2*f) + 3*b**2*e*g**2*h*p*q*x*log(c*(d*(e + f*x)**p)*
*q)/f - 5*b**2*e*g*h**2*p**2*q**2*x**2/(6*f) + b**2*e*g*h**2*p*q*x**2*log(c
*(d*(e + f*x)**p)**q)/f - 7*b**2*e*h**3*p**2*q**2*x**3/(72*f) + b**2*e*h**3
*p*q*x**3*log(c*(d*(e + f*x)**p)**q)/(6*f) + 2*b**2*g**3*p**2*q**2*x - 2*b*
*2*g**3*p*q*x*log(c*(d*(e + f*x)**p)**q) + b**2*g**3*x*log(c*(d*(e + f*x)**
p)**q)**2 + 3*b**2*g**2*h*p**2*q**2*x**2/4 - 3*b**2*g**2*h*p*q*x**2*log(c*(
d*(e + f*x)**p)**q)/2 + 3*b**2*g**2*h*x**2*log(c*(d*(e + f*x)**p)**q)**2/2
+ 2*b**2*g*h**2*p**2*q**2*x**3/9 - 2*b**2*g*h**2*p*q*x**3*log(c*(d*(e + f*x
)**p)**q)/3 + b**2*g*h**2*x**3*log(c*(d*(e + f*x)**p)**q)**2 + b**2*h**3*p*
*2*q**2*x**4/32 - b**2*h**3*p*q*x**4*log(c*(d*(e + f*x)**p)**q)/8 + b**2*h*
*3*x**4*log(c*(d*(e + f*x)**p)**q)**2/4, Ne(f, 0)), ((a + b*log(c*(d*e**p)*
*q))**2*(g**3*x + 3*g**2*h*x**2/2 + g*h**2*x**3 + h**3*x**4/4), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. 2(391) = 782.

Time = 0.23 (sec) , antiderivative size = 895, normalized size of antiderivative = 2.19

$$\int (g+hx)^3 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^3*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*h^3*x^4*log(((f*x + e)^p*d)^q*c)^2 + 1/2*a*b*h^3*x^4*log(((f*x + e)
^p*d)^q*c) + b^2*g*h^2*x^3*log(((f*x + e)^p*d)^q*c)^2 + 1/4*a^2*h^3*x^4 - 2
*a*b*f*g^3*p*q*(x/f - e*log(f*x + e)/f^2) - 1/24*a*b*f*h^3*p*q*(12*e^4*log(
f*x + e)/f^5 + (3*f^3*x^4 - 4*e*f^2*x^3 + 6*e^2*f*x^2 - 12*e^3*x)/f^4) + 1/
3*a*b*f*g*h^2*p*q*(6*e^3*log(f*x + e)/f^4 - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*
x)/f^3) - 3/2*a*b*f*g^2*h*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)/f^2
) + 2*a*b*g*h^2*x^3*log(((f*x + e)^p*d)^q*c) + 3/2*b^2*g^2*h*x^2*log(((f*x
+ e)^p*d)^q*c)^2 + a^2*g*h^2*x^3 + 3*a*b*g^2*h*x^2*log(((f*x + e)^p*d)^q*c)
 + b^2*g^3*x*log(((f*x + e)^p*d)^q*c)^2 + 3/2*a^2*g^2*h*x^2 + 2*a*b*g^3*x*l
og(((f*x + e)^p*d)^q*c) - (2*f*p*q*(x/f - e*log(f*x + e)/f^2)*log(((f*x + e
)^p*d)^q*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p^2*q^2/f)*b^2*
g^3 - 3/4*(2*f*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)/f^2)*log(((f*x
 + e)^p*d)^q*c) - (f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x + 6*e^2*log(f*x
 + e))*p^2*q^2/f^2)*b^2*g^2*h + 1/18*(6*f*p*q*(6*e^3*log(f*x + e)/f^4 - (2*
f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/f^3)*log(((f*x + e)^p*d)^q*c) + (4*f^3*x^3 -
 15*e*f^2*x^2 - 18*e^3*log(f*x + e)^2 + 66*e^2*f*x - 66*e^3*log(f*x + e))*p
^2*q^2/f^3)*b^2*g*h^2 - 1/288*(12*f*p*q*(12*e^4*log(f*x + e)/f^5 + (3*f^3*x
^4 - 4*e*f^2*x^3 + 6*e^2*f*x^2 - 12*e^3*x)/f^4)*log(((f*x + e)^p*d)^q*c) -
(9*f^4*x^4 - 28*e*f^3*x^3 + 78*e^2*f^2*x^2 + 72*e^4*log(f*x + e)^2 - 300*e^
3*f*x + 300*e^4*log(f*x + e))*p^2*q^2/f^4)*b^2*h^3 + a^2*g^3*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3738 vs. 2(391) = 782.

Time = 0.39 (sec) , antiderivative size = 3738, normalized size of antiderivative = 9.14

$$\int (g+hx)^3 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^3*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

```
[Out] (f*x + e)*b^2*g^3*p^2*q^2*log(f*x + e)^2/f + 3/2*(f*x + e)^2*b^2*g^2*h*p^2*
q^2*log(f*x + e)^2/f^2 - 3*(f*x + e)*b^2*e*g^2*h*p^2*q^2*log(f*x + e)^2/f^2
 + (f*x + e)^3*b^2*g*h^2*p^2*q^2*log(f*x + e)^2/f^3 - 3*(f*x + e)^2*b^2*e*g
*h^2*p^2*q^2*log(f*x + e)^2/f^3 + 3*(f*x + e)*b^2*e^2*g*h^2*p^2*q^2*log(f*x
```

+ e)^2/f^3 + 1/4*(f*x + e)^4*b^2*h^3*p^2*q^2*log(f*x + e)^2/f^4 - (f*x + e
)^3*b^2*e*h^3*p^2*q^2*log(f*x + e)^2/f^4 + 3/2*(f*x + e)^2*b^2*e^2*h^3*p^2*
q^2*log(f*x + e)^2/f^4 - (f*x + e)*b^2*e^3*h^3*p^2*q^2*log(f*x + e)^2/f^4 -
2*(f*x + e)*b^2*g^3*p^2*q^2*log(f*x + e)/f - 3/2*(f*x + e)^2*b^2*g^2*h*p^2
*q^2*log(f*x + e)/f^2 + 6*(f*x + e)*b^2*e*g^2*h*p^2*q^2*log(f*x + e)/f^2 -
2/3*(f*x + e)^3*b^2*g*h^2*p^2*q^2*log(f*x + e)/f^3 + 3*(f*x + e)^2*b^2*e*g*
h^2*p^2*q^2*log(f*x + e)/f^3 - 6*(f*x + e)*b^2*e^2*g*h^2*p^2*q^2*log(f*x +
e)/f^3 - 1/8*(f*x + e)^4*b^2*h^3*p^2*q^2*log(f*x + e)/f^4 + 2/3*(f*x + e)^3
*b^2*e*h^3*p^2*q^2*log(f*x + e)/f^4 - 3/2*(f*x + e)^2*b^2*e^2*h^3*p^2*q^2*l
og(f*x + e)/f^4 + 2*(f*x + e)*b^2*e^3*h^3*p^2*q^2*log(f*x + e)/f^4 + 2*(f*x
+ e)*b^2*g^3*p*q^2*log(f*x + e)*log(d)/f + 3*(f*x + e)^2*b^2*g^2*h*p*q^2*l
og(f*x + e)*log(d)/f^2 - 6*(f*x + e)*b^2*e*g^2*h*p*q^2*log(f*x + e)*log(d)/
f^2 + 2*(f*x + e)^3*b^2*g*h^2*p*q^2*log(f*x + e)*log(d)/f^3 - 6*(f*x + e)^2
*b^2*e*g*h^2*p*q^2*log(f*x + e)*log(d)/f^3 + 6*(f*x + e)*b^2*e^2*g*h^2*p*q^
2*log(f*x + e)*log(d)/f^3 + 1/2*(f*x + e)^4*b^2*h^3*p*q^2*log(f*x + e)*log(
d)/f^4 - 2*(f*x + e)^3*b^2*e*h^3*p*q^2*log(f*x + e)*log(d)/f^4 + 3*(f*x + e
)^2*b^2*e^2*h^3*p*q^2*log(f*x + e)*log(d)/f^4 - 2*(f*x + e)*b^2*e^3*h^3*p*q
^2*log(f*x + e)*log(d)/f^4 + 2*(f*x + e)*b^2*g^3*p^2*q^2/f + 3/4*(f*x + e)^
2*b^2*g^2*h*p^2*q^2/f^2 - 6*(f*x + e)*b^2*e*g^2*h*p^2*q^2/f^2 + 2/9*(f*x +
e)^3*b^2*g*h^2*p^2*q^2/f^3 - 3/2*(f*x + e)^2*b^2*e*g*h^2*p^2*q^2/f^3 + 6*(f
*x + e)*b^2*e^2*g*h^2*p^2*q^2/f^3 + 1/32*(f*x + e)^4*b^2*h^3*p^2*q^2/f^4 -
2/9*(f*x + e)^3*b^2*e*h^3*p^2*q^2/f^4 + 3/4*(f*x + e)^2*b^2*e^2*h^3*p^2*q^2
/f^4 - 2*(f*x + e)*b^2*e^3*h^3*p^2*q^2/f^4 + 2*(f*x + e)*b^2*g^3*p*q*log(f*
x + e)*log(c)/f + 3*(f*x + e)^2*b^2*g^2*h*p*q*log(f*x + e)*log(c)/f^2 - 6*(
f*x + e)*b^2*e*g^2*h*p*q*log(f*x + e)*log(c)/f^2 + 2*(f*x + e)^3*b^2*g*h^2*
p*q*log(f*x + e)*log(c)/f^3 - 6*(f*x + e)^2*b^2*e*g*h^2*p*q*log(f*x + e)*lo
g(c)/f^3 + 6*(f*x + e)*b^2*e^2*g*h^2*p*q*log(f*x + e)*log(c)/f^3 + 1/2*(f*x
+ e)^4*b^2*h^3*p*q*log(f*x + e)*log(c)/f^4 - 2*(f*x + e)^3*b^2*e*h^3*p*q*l
og(f*x + e)*log(c)/f^4 + 3*(f*x + e)^2*b^2*e^2*h^3*p*q*log(f*x + e)*log(c)/
f^4 - 2*(f*x + e)*b^2*e^3*h^3*p*q*log(f*x + e)*log(c)/f^4 - 2*(f*x + e)*b^2
*g^3*p*q^2*log(d)/f - 3/2*(f*x + e)^2*b^2*g^2*h*p*q^2*log(d)/f^2 + 6*(f*x +
e)*b^2*e*g^2*h*p*q^2*log(d)/f^2 - 2/3*(f*x + e)^3*b^2*g*h^2*p*q^2*log(d)/f
^3 + 3*(f*x + e)^2*b^2*e*g*h^2*p*q^2*log(d)/f^3 - 6*(f*x + e)*b^2*e^2*g*h^2
*p*q^2*log(d)/f^3 - 1/8*(f*x + e)^4*b^2*h^3*p*q^2*log(d)/f^4 + 2/3*(f*x + e
)^3*b^2*e*h^3*p*q^2*log(d)/f^4 - 3/2*(f*x + e)^2*b^2*e^2*h^3*p*q^2*log(d)/f
^4 + 2*(f*x + e)*b^2*e^3*h^3*p*q^2*log(d)/f^4 + (f*x + e)*b^2*g^3*q^2*log(d
)^2/f + 3/2*(f*x + e)^2*b^2*g^2*h*q^2*log(d)^2/f^2 - 3*(f*x + e)*b^2*e*g^2*
h*q^2*log(d)^2/f^2 + (f*x + e)^3*b^2*g*h^2*q^2*log(d)^2/f^3 - 3*(f*x + e)^2
*b^2*e*g*h^2*q^2*log(d)^2/f^3 + 3*(f*x + e)*b^2*e^2*g*h^2*q^2*log(d)^2/f^3
+ 1/4*(f*x + e)^4*b^2*h^3*q^2*log(d)^2/f^4 - (f*x + e)^3*b^2*e*h^3*q^2*log(
d)^2/f^4 + 3/2*(f*x + e)^2*b^2*e^2*h^3*q^2*log(d)^2/f^4 - (f*x + e)*b^2*e^3
*h^3*q^2*log(d)^2/f^4 + 2*(f*x + e)*a*b*g^3*p*q*log(f*x + e)/f + 3*(f*x + e
)^2*a*b*g^2*h*p*q*log(f*x + e)/f^2 - 6*(f*x + e)*a*b*e*g^2*h*p*q*log(f*x +
e)/f^2 + 2*(f*x + e)^3*a*b*g*h^2*p*q*log(f*x + e)/f^3 - 6*(f*x + e)^2*a*b*e
*g*h^2*p*q*log(f*x + e)/f^3 + 6*(f*x + e)*a*b*e^2*g*h^2*p*q*log(f*x + e)/f^

3 + 1/2*(f*x + e)^4*a*b*h^3*p*q*log(f*x + e)/f^4 - 2*(f*x + e)^3*a*b*e*h^3*
p*q*log(f*x + e)/f^4 + 3*(f*x + e)^2*a*b*e^2*h^3*p*q*log(f*x + e)/f^4 - 2*(
f*x + e)*a*b*e^3*h^3*p*q*log(f*x + e)/f^4 - 2*(f*x + e)*b^2*g^3*p*q*log(c)/
f - 3/2*(f*x + e)^2*b^2*g^2*h*p*q*log(c)/f^2 + 6*(f*x + e)*b^2*e*g^2*h*p*q*
log(c)/f^2 - 2/3*(f*x + e)^3*b^2*g*h^2*p*q*log(c)/f^3 + 3*(f*x + e)^2*b^2*e
*g*h^2*p*q*log(c)/f^3 - 6*(f*x + e)*b^2*e^2*g*h^2*p*q*log(c)/f^3 - 1/8*(f*x
 + e)^4*b^2*h^3*p*q*log(c)/f^4 + 2/3*(f*x + e)^3*b^2*e*h^3*p*q*log(c)/f^4 -
 3/2*(f*x + e)^2*b^2*e^2*h^3*p*q*log(c)/f^4 + 2*(f*x + e)*b^2*e^3*h^3*p*q*l
og(c)/f^4 + 2*(f*x + e)*b^2*g^3*q*log(c)*log(d)/f + 3*(f*x + e)^2*b^2*g^2*h
*q*log(c)*log(d)/f^2 - 6*(f*x + e)*b^2*e*g^2*h*q*log(c)*log(d)/f^2 + 2*(f*x
 + e)^3*b^2*g*h^2*q*log(c)*log(d)/f^3 - 6*(f*x + e)^2*b^2*e*g*h^2*q*log(c)*
log(d)/f^3 + 6*(f*x + e)*b^2*e^2*g*h^2*q*log(c)*log(d)/f^3 + 1/2*(f*x + e)^
4*b^2*h^3*q*log(c)*log(d)/f^4 - 2*(f*x + e)^3*b^2*e*h^3*q*log(c)*log(d)/f^4
 + 3*(f*x + e)^2*b^2*e^2*h^3*q*log(c)*log(d)/f^4 - 2*(f*x + e)*b^2*e^3*h^3*
q*log(c)*log(d)/f^4 - 2*(f*x + e)*a*b*g^3*p*q/f - 3/2*(f*x + e)^2*a*b*g^2*h
*p*q/f^2 + 6*(f*x + e)*a*b*e*g^2*h*p*q/f^2 - 2/3*(f*x + e)^3*a*b*g*h^2*p*q/
f^3 + 3*(f*x + e)^2*a*b*e*g*h^2*p*q/f^3 - 6*(f*x + e)*a*b*e^2*g*h^2*p*q/f^3
 - 1/8*(f*x + e)^4*a*b*h^3*p*q/f^4 + 2/3*(f*x + e)^3*a*b*e*h^3*p*q/f^4 - 3/
2*(f*x + e)^2*a*b*e^2*h^3*p*q/f^4 + 2*(f*x + e)*a*b*e^3*h^3*p*q/f^4 + (f*x
+ e)*b^2*g^3*log(c)^2/f + 3/2*(f*x + e)^2*b^2*g^2*h*log(c)^2/f^2 - 3*(f*x +
 e)*b^2*e*g^2*h*log(c)^2/f^2 + (f*x + e)^3*b^2*g*h^2*log(c)^2/f^3 - 3*(f*x
+ e)^2*b^2*e*g*h^2*log(c)^2/f^3 + 3*(f*x + e)*b^2*e^2*g*h^2*log(c)^2/f^3 +
1/4*(f*x + e)^4*b^2*h^3*log(c)^2/f^4 - (f*x + e)^3*b^2*e*h^3*log(c)^2/f^4 +
 3/2*(f*x + e)^2*b^2*e^2*h^3*log(c)^2/f^4 - (f*x + e)*b^2*e^3*h^3*log(c)^2/
f^4 + 2*(f*x + e)*a*b*g^3*q*log(d)/f + 3*(f*x + e)^2*a*b*g^2*h*q*log(d)/f^2
 - 6*(f*x + e)*a*b*e*g^2*h*q*log(d)/f^2 + 2*(f*x + e)^3*a*b*g*h^2*q*log(d)/
f^3 - 6*(f*x + e)^2*a*b*e*g*h^2*q*log(d)/f^3 + 6*(f*x + e)*a*b*e^2*g*h^2*q*
log(d)/f^3 + 1/2*(f*x + e)^4*a*b*h^3*q*log(d)/f^4 - 2*(f*x + e)^3*a*b*e*h^3
*q*log(d)/f^4 + 3*(f*x + e)^2*a*b*e^2*h^3*q*log(d)/f^4 - 2*(f*x + e)*a*b*e^
3*h^3*q*log(d)/f^4 + 2*(f*x + e)*a*b*g^3*log(c)/f + 3*(f*x + e)^2*a*b*g^2*h
*log(c)/f^2 - 6*(f*x + e)*a*b*e*g^2*h*log(c)/f^2 + 2*(f*x + e)^3*a*b*g*h^2*
log(c)/f^3 - 6*(f*x + e)^2*a*b*e*g*h^2*log(c)/f^3 + 6*(f*x + e)*a*b*e^2*g*h
^2*log(c)/f^3 + 1/2*(f*x + e)^4*a*b*h^3*log(c)/f^4 - 2*(f*x + e)^3*a*b*e*h^
3*log(c)/f^4 + 3*(f*x + e)^2*a*b*e^2*h^3*log(c)/f^4 - 2*(f*x + e)*a*b*e^3*h
^3*log(c)/f^4 + (f*x + e)*a^2*g^3/f + 3/2*(f*x + e)^2*a^2*g^2*h/f^2 - 3*(f*
x + e)*a^2*e*g^2*h/f^2 + (f*x + e)^3*a^2*g*h^2/f^3 - 3*(f*x + e)^2*a^2*e*g*
h^2/f^3 + 3*(f*x + e)*a^2*e^2*g*h^2/f^3 + 1/4*(f*x + e)^4*a^2*h^3/f^4 - (f*
x + e)^3*a^2*e*h^3/f^4 + 3/2*(f*x + e)^2*a^2*e^2*h^3/f^4 - (f*x + e)*a^2*e^
3*h^3/f^4

## Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 1154, normalized size of antiderivative = 2.82

$$\int (g+hx)^3 (a+b\log(c(d(e+fx)^p)^q))^2 \, dx$$

$$= x^3 \left( \frac{h^2 (6 a^2 e h + 18 a^2 f g - b^2 e h p^2 q^2 + 4 b^2 f g p^2 q^2 - 12 a b f g p q)}{18 f} - \frac{e h^3 (8 a^2 - 4 a b p q + b^2 p^2 q^2)}{24 f} \right)$$

$$+ \ln(c(d(e+fx)^p)^q) \left( \frac{x \left( \frac{e \left( \frac{e \left( \frac{4 b h^2 (a e h + 3 a f g - b f g p q)}{f} - \frac{b e h^3 (4 a - b p q)}{f} \right)}{f} - \frac{6 b g h (2 a e h + 2 a f g - b f g p q)}{f} \right)}{f} + \frac{4 b g^2 (3 a e h + a f \ldots}{f} \ldots}{2} + \frac{x^3 \left( \frac{4 b h^2 (a e h + 3 a f g - b f g p q)}{3 f} - \frac{b e h^3 (4 a - b p q)}{3 f} \right)}{2} - \frac{x^2 \left( \frac{e \left( \frac{4 b h^2 (a e h + 3 a f g - b f g p q)}{f} - \frac{b e h^3 (4 a - b p q)}{f} \right)}{2 f} - \frac{3 b g h (2 a e h + 2 a f g - b f g p q)}{f} \right)}{2} + \frac{b h^3 x^4 (4 a - b p q)}{8} \right)$$

$$+ \ln(c(d(e+fx)^p)^q)^2 \left( b^2 g^3 x - \frac{e (b^2 e^3 h^3 - 4 b^2 e^2 f g h^2 + 6 b^2 e f^2 g^2 h - 4 b^2 f^3 g^3)}{4 f^4} + \frac{b^2 h^3 x^4}{4} + \frac{3 b^2 g^2 h x^2}{2} + b^2 g h^2 x^3 \right)$$

$$+ x \left( \frac{72 a^2 e f^2 g^2 h + 24 a^2 f^3 g^3 - 48 a b f^3 g^3 p q - 12 b^2 e^3 h^3 p^2 q^2 + 48 b^2 e^2 f g h^2 p^2 q^2 - 72 b^2 e f^2 g^2 h p \ldots}{24 f^3} \ldots \right.$$

$$- \frac{e \left( \frac{e \left( \frac{h^2 (6 a^2 e h + 18 a^2 f g - b^2 e h p^2 q^2 + 4 b^2 f g p^2 q^2 - 12 a b f g p q)}{6 f} - \frac{e h^3 (8 a^2 - 4 a b p q + b^2 p^2 q^2)}{8 f} \right)}{\ldots} - \frac{h (12 a^2 e f g h + 12 a^2 f^2 g^2 - 12 a b f^2 g^2 \ldots}{\ldots}$$

```
[In] int((g + h*x)^3*(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)

[Out] x^3*((h^2*(6*a^2*e*h + 18*a^2*f*g - b^2*e*h*p^2*q^2 + 4*b^2*f*g*p^2*q^2 - 1
2*a*b*f*g*p*q))/(18*f) - (e*h^3*(8*a^2 + b^2*p^2*q^2 - 4*a*b*p*q))/(24*f))
+ log(c*(d*(e + f*x)^p)^q)*((x*((e*((e*((4*b*h^2*(a*e*h + 3*a*f*g - b*f*g*p
*q))/f - (b*e*h^3*(4*a - b*p*q))/f))/f - (6*b*g*h*(2*a*e*h + 2*a*f*g - b*f*
g*p*q))/f))/f + (4*b*g^2*(3*a*e*h + a*f*g - b*f*g*p*q))/f))/2 + (x^3*((4*b*
h^2*(a*e*h + 3*a*f*g - b*f*g*p*q))/(3*f) - (b*e*h^3*(4*a - b*p*q))/(3*f)))/
2 - (x^2*((e*((4*b*h^2*(a*e*h + 3*a*f*g - b*f*g*p*q))/f - (b*e*h^3*(4*a - b
*p*q))/f))/(2*f) - (3*b*g*h*(2*a*e*h + 2*a*f*g - b*f*g*p*q))/f))/2 + (b*h^3
*x^4*(4*a - b*p*q))/8) + log(c*(d*(e + f*x)^p)^q)^2*(b^2*g^3*x - (e*(b^2*e^
3*h^3 - 4*b^2*f^3*g^3 + 6*b^2*e*f^2*g^2*h - 4*b^2*e^2*f*g*h^2))/(4*f^4) + (
b^2*h^3*x^4)/4 + (3*b^2*g^2*h*x^2)/2 + b^2*g*h^2*x^3) + x*((24*a^2*f^3*g^3
- 12*b^2*e^3*h^3*p^2*q^2 + 48*b^2*f^3*g^3*p^2*q^2 + 72*a^2*e*f^2*g^2*h - 48
*a*b*f^3*g^3*p*q - 72*b^2*e*f^2*g^2*h*p^2*q^2 + 48*b^2*e^2*f*g*h^2*p^2*q^2)
/(24*f^3) + (e*((e*((h^2*(6*a^2*e*h + 18*a^2*f*g - b^2*e*h*p^2*q^2 + 4*b^2*
f*g*p^2*q^2 - 12*a*b*f*g*p*q))/(6*f) - (e*h^3*(8*a^2 + b^2*p^2*q^2 - 4*a*b*
p*q))/(8*f)))/f - (h*(12*a^2*f^2*g^2 + b^2*e^2*h^2*p^2*q^2 + 6*b^2*f^2*g^2*
p^2*q^2 + 12*a^2*e*f*g*h - 12*a*b*f^2*g^2*p*q - 4*b^2*e*f*g*h*p^2*q^2))/(4*
f^2)))/f) - x^2*((e*((h^2*(6*a^2*e*h + 18*a^2*f*g - b^2*e*h*p^2*q^2 + 4*b^2
*f*g*p^2*q^2 - 12*a*b*f*g*p*q))/(6*f) - (e*h^3*(8*a^2 + b^2*p^2*q^2 - 4*a*b
*p*q))/(8*f)))/(2*f) - (h*(12*a^2*f^2*g^2 + b^2*e^2*h^2*p^2*q^2 + 6*b^2*f^2
*g^2*p^2*q^2 + 12*a^2*e*f*g*h - 12*a*b*f^2*g^2*p*q - 4*b^2*e*f*g*h*p^2*q^2)
)/(8*f^2)) + (log(e + f*x)*(25*b^2*e^4*h^3*p^2*q^2 - 12*a*b*e^4*h^3*p*q - 4
8*b^2*e*f^3*g^3*p^2*q^2 - 88*b^2*e^3*f*g*h^2*p^2*q^2 + 108*b^2*e^2*f^2*g^2*
h*p^2*q^2 + 48*a*b*e*f^3*g^3*p*q + 48*a*b*e^3*f*g*h^2*p*q - 72*a*b*e^2*f^2*
g^2*h*p*q))/(24*f^4) + (h^3*x^4*(8*a^2 + b^2*p^2*q^2 - 4*a*b*p*q))/32
```

# 3.429 $\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$



## Optimal result

Integrand size = 28, antiderivative size = 323

$$\begin{aligned}
&\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx \\
&= \frac{2b^2(fg-eh)^2p^2q^2x}{f^2} + \frac{b^2h(fg-eh)p^2q^2(e+fx)^2}{2f^3} + \frac{2b^2h^2p^2q^2(e+fx)^3}{27f^3} \\
&\quad + \frac{b^2(fg-eh)^3p^2q^2\log^2(e+fx)}{3f^3h} - \frac{2b(fg-eh)^2pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^3} \\
&\quad - \frac{bh(fg-eh)pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^3} \\
&\quad - \frac{2bh^2pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9f^3} \\
&\quad - \frac{2b(fg-eh)^3pq\log(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3f^3h} \\
&\quad + \frac{(g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}
\end{aligned}$$

```
[Out] 2*b^2*(-e*h+f*g)^2*p^2*q^2*x/f^2+1/2*b^2*h*(-e*h+f*g)*p^2*q^2*(f*x+e)^2/f^3
+2/27*b^2*h^2*p^2*q^2*(f*x+e)^3/f^3+1/3*b^2*(-e*h+f*g)^3*p^2*q^2*ln(f*x+e)^
2/f^3/h-2*b*(-e*h+f*g)^2*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^3-b*h*(-
e*h+f*g)*p*q*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^3-2/9*b*h^2*p*q*(f*x+e
)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^3-2/3*b*(-e*h+f*g)^3*p*q*ln(f*x+e)*(a+b*l
n(c*(d*(f*x+e)^p)^q))/f^3/h+1/3*(h*x+g)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/h
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2445, 2458, 45, 2372, 12, 14, 2338, 2495}

$$\int (g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$$
$$=-\frac{2bpq(fg-eh)^3\log(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3f^3h}-\frac{2bpq(e+fx)(fg-eh)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^3}-\frac{bhpq(e+fx)^2(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^3}-\frac{2bh^2pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9f^3}+\frac{(g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}+\frac{b^2hp^2q^2(e+fx)^2(fg-eh)}{2f^3}+\frac{b^2p^2q^2(fg-eh)^3\log^2(e+fx)}{3f^3h}+\frac{2b^2h^2p^2q^2(e+fx)^3}{27f^3}+\frac{2b^2p^2q^2x(fg-eh)^2}{f^2}$$

```
[In] Int[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] (2*b^2*(f*g - e*h)^2*p^2*q^2*x)/f^2 + (b^2*h*(f*g - e*h)*p^2*q^2*(e + f*x)^
2)/(2*f^3) + (2*b^2*h^2*p^2*q^2*(e + f*x)^3)/(27*f^3) + (b^2*(f*g - e*h)^3*
p^2*q^2*Log[e + f*x]^2)/(3*f^3*h) - (2*b*(f*g - e*h)^2*p*q*(e + f*x)*(a + b
*Log[c*(d*(e + f*x)^p)^q]))/f^3 - (b*h*(f*g - e*h)*p*q*(e + f*x)^2*(a + b*L
og[c*(d*(e + f*x)^p)^q]))/f^3 - (2*b*h^2*p*q*(e + f*x)^3*(a + b*Log[c*(d*(e
 + f*x)^p)^q]))/(9*f^3) - (2*b*(f*g - e*h)^3*p*q*Log[e + f*x]*(a + b*Log[c*
(d*(e + f*x)^p)^q]))/(3*f^3*h) + ((g + h*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^
q])^2)/(3*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (g+hx)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(g+hx)^3 (a+b\log(c(d(e+fx)^p)^q))^2}{3h} - \text{Subst}\left(\frac{(2bfpq)\int \frac{(g+hx)^3(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}dx}{3h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(g+hx)^3 (a+b\log(c(d(e+fx)^p)^q))^2}{3h} - \text{Subst}\left(\frac{(2bpq)\text{Subst}\left(\int \frac{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^3(a+b\log(cd^q x^{pq}))}{x}dx, x, e+fx\right)}{3h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2b(fg-eh)^2pq(e+fx)(a+b\log(c(d(e+fx)^p)^q))}{f^3} - \frac{bh(fg-eh)pq(e+fx)^2(a+b\log(c(d(e+fx)^p)^q))}{f^3} - \frac{2bh^2pq(e+fx)^3(a+b\log(c(d(e+fx)^p)^q))}{9f^3} - \frac{2b(fg-eh)^3pq\log(e+fx)(a+b\log(c(d(e+fx)^p)^q))}{3f^3h} + \frac{(g+hx)^3(a+b\log(c(d(e+fx)^p)^q))^2}{3h} + \text{Subst}\left(\frac{(2b^2p^2q^2)\,\text{Subst}\left(\int \frac{hx(18f^2g^2+9fgh(-4e+x)+h^2(18e^2-9ex+2x^2))+6(fg-eh)^3\log(x)}{6f^3x}dx, x, e+fx\right)}{3h}, c\ldots\right.$$

$$\left. + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2b(fg-eh)^2pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^3}$$
$$- \frac{bh(fg-eh)pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^3}$$
$$- \frac{2bh^2pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9f^3}$$
$$- \frac{2b(fg-eh)^3pq\log(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3f^3h}$$
$$+ \frac{(g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}$$
$$+\text{Subst}\left(\frac{(b^2p^2q^2)\,\text{Subst}\left(\int \frac{hx(18f^2g^2+9fgh(-4e+x)+h^2(18e^2-9ex+2x^2))+6(fg-eh)^3\log(x)}{x}\,dx,x,e+fx\right)}{9f^3h}, cd^q(\right.$$
$$\left. + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2b(fg-eh)^2pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^3}$$
$$- \frac{bh(fg-eh)pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^3}$$
$$- \frac{2bh^2pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9f^3}$$
$$- \frac{2b(fg-eh)^3pq\log(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3f^3h}$$
$$+ \frac{(g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}$$
$$+\text{Subst}\left(\frac{(b^2p^2q^2)\,\text{Subst}\left(\int \left(h(18(fg-eh)^2+9h(fg-eh)x+2h^2x^2)+\frac{6(fg-eh)^3\log(x)}{x}\right)\,dx,x,e+\right.}{9f^3h}\right.$$
$$\left. + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2b(fg-eh)^2pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^3}$$
$$-\frac{bh(fg-eh)pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^3}$$
$$-\frac{2bh^2pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9f^3}$$
$$-\frac{2b(fg-eh)^3pq\log(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3f^3h}$$
$$+\frac{(g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}$$
$$+\text{Subst}\left(\frac{(b^2p^2q^2)\,\text{Subst}\left(\int\left(18(fg-eh)^2+9h(fg-eh)x+2h^2x^2\right)dx,x,e+fx\right)}{9f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$+\text{Subst}\left(\frac{(2b^2(fg-eh)^3p^2q^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}dx,x,e+fx\right)}{3f^3h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$=\frac{2b^2(fg-eh)^2p^2q^2x}{f^2}+\frac{b^2h(fg-eh)p^2q^2(e+fx)^2}{2f^3}$$
$$+\frac{2b^2h^2p^2q^2(e+fx)^3}{27f^3}+\frac{b^2(fg-eh)^3p^2q^2\log^2(e+fx)}{3f^3h}$$
$$-\frac{2b(fg-eh)^2pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^3}$$
$$-\frac{bh(fg-eh)pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{f^3}$$
$$-\frac{2bh^2pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9f^3}$$
$$-\frac{2b(fg-eh)^3pq\log(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3f^3h}$$
$$+\frac{(g+hx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.86

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^2 \, dx$$
$$= \frac{54(fg-eh)^2(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2 + 54h(fg-eh)(e+fx)^2(a+b\log(c(d(e+fx)^p)^q))^2 +}{}$$

```
[In] Integrate[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] (54*(f*g - e*h)^2*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 54*h*(f*g
- e*h)*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 18*h^2*(e + f*x)^3*
(a + b*Log[c*(d*(e + f*x)^p)^q])^2 - 108*b*(f*g - e*h)^2*p*q*(f*(a - b*p*q)
*x + b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q]) + 27*b*h*(f*g - e*h)*p*q*(b*f*p*
q*x*(2*e + f*x) - 2*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])) + 4*b*h^2
*p*q*(b*f*p*q*x*(3*e^2 + 3*e*f*x + f^2*x^2) - 3*(e + f*x)^3*(a + b*Log[c*(d
*(e + f*x)^p)^q])))/(54*f^3)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. 2(311) = 622.

Time = 5.68 (sec) , antiderivative size = 939, normalized size of antiderivative = 2.91

| method | result |
|---|---|
| parallelrisch | $\frac{162\ln(fx+e)b^2e^3fgh\,p^2q^2+108xab\,e^2f^2ghpq-108\ln(fx+e)ab\,e^3fghpq+108x\ln(c(d(fx+e)^p)^q)abe\,f^3g^2-108\ln(fx+e)b^2e^2f^2}{}$ |

```
[In] int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(162*ln(f*x+e)*b^2*e^3*f*g*h*p^2*q^2+108*x*a*b*e^2*f^2*g*h*p*q-108*ln(
f*x+e)*a*b*e^3*f*g*h*p*q+108*x*ln(c*(d*(f*x+e)^p)^q)*a*b*e*f^3*g^2-108*ln(f
*x+e)*b^2*e^2*f^2*g^2*p^2*q^2+36*ln(f*x+e)*a*b*e^4*h^2*p*q+4*x^3*b^2*e*f^3*
h^2*p^2*q^2-15*x^2*b^2*e^2*f^2*h^2*p^2*q^2+66*x*b^2*e^3*f*h^2*p^2*q^2+108*x
*b^2*e*f^3*g^2*p^2*q^2+36*x^3*ln(c*(d*(f*x+e)^p)^q)*a*b*e*f^3*h^2+54*x^2*ln
(c*(d*(f*x+e)^p)^q)^2*b^2*e*f^3*g*h+108*ln(f*x+e)*a*b*e^2*f^2*g^2*p*q-12*x^
3*ln(c*(d*(f*x+e)^p)^q)*b^2*e*f^3*h^2*p*q+27*x^2*b^2*e*f^3*g*h*p^2*q^2-12*x
^3*a*b*e*f^3*h^2*p*q+18*x^2*ln(c*(d*(f*x+e)^p)^q)*b^2*e^2*f^2*h^2*p*q-108*a
*b*e^3*f*g*h*p*q-54*x^2*ln(c*(d*(f*x+e)^p)^q)*b^2*e*f^3*g*h*p*q-54*x^2*a*b*
e*f^3*g*h*p*q+108*x*ln(c*(d*(f*x+e)^p)^q)*b^2*e^2*f^2*g*h*p*q-162*x*b^2*e^2
*f^2*g*h*p^2*q^2+18*x^2*a*b*e^2*f^2*h^2*p*q-36*x*ln(c*(d*(f*x+e)^p)^q)*b^2*
e^3*f*h^2*p*q-108*x*ln(c*(d*(f*x+e)^p)^q)*b^2*e*f^3*g^2*p*q+108*x^2*ln(c*(d
*(f*x+e)^p)^q)*a*b*e*f^3*g*h-36*x*a*b*e^3*f*h^2*p*q-108*x*a*b*e*f^3*g^2*p*q
-108*b^2*e^2*f^2*g^2*p^2*q^2+36*a*b*e^4*h^2*p*q+54*x*a^2*e*f^3*g^2+18*x^3*a
^2*e*f^3*h^2+54*ln(c*(d*(f*x+e)^p)^q)^2*b^2*e^2*f^2*g^2+108*a*b*e^2*f^2*g^2
```

```
*p*q+18*x^3*ln(c*(d*(f*x+e)^p)^q)^2*b^2*e*f^3*h^2+54*x*ln(c*(d*(f*x+e)^p)^q
)^2*b^2*e*f^3*g^2+54*x^2*a^2*e*f^3*g*h-54*ln(c*(d*(f*x+e)^p)^q)^2*b^2*e^3*f
*g*h-66*ln(f*x+e)*b^2*e^4*h^2*p^2*q^2-66*b^2*e^4*h^2*p^2*q^2+162*b^2*e^3*f*
g*h*p^2*q^2-54*a^2*e^2*f^2*g^2+18*ln(c*(d*(f*x+e)^p)^q)^2*b^2*e^4*h^2)/e/f^
3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(311) = 622$.

Time = 0.34 (sec) , antiderivative size = 1137, normalized size of antiderivative = 3.52

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

```
[Out] 1/54*(2*(2*b^2*f^3*h^2*p^2*q^2 - 6*a*b*f^3*h^2*p*q + 9*a^2*f^3*h^2)*x^3 + 3
*(18*a^2*f^3*g*h + (9*b^2*f^3*g*h - 5*b^2*e*f^2*h^2)*p^2*q^2 - 6*(3*a*b*f^3
*g*h - a*b*e*f^2*h^2)*p*q)*x^2 + 18*(b^2*f^3*h^2*p^2*q^2*x^3 + 3*b^2*f^3*g*
h*p^2*q^2*x^2 + 3*b^2*f^3*g^2*p^2*q^2*x + (3*b^2*e*f^2*g^2 - 3*b^2*e^2*f*g*
h + b^2*e^3*h^2)*p^2*q^2)*log(f*x + e)^2 + 18*(b^2*f^3*h^2*x^3 + 3*b^2*f^3*
g*h*x^2 + 3*b^2*f^3*g^2*x)*log(c)^2 + 18*(b^2*f^3*h^2*q^2*x^3 + 3*b^2*f^3*g
*h*q^2*x^2 + 3*b^2*f^3*g^2*q^2*x)*log(d)^2 + 6*(9*a^2*f^3*g^2 + (18*b^2*f^3
*g^2 - 27*b^2*e*f^2*g*h + 11*b^2*e^2*f*h^2)*p^2*q^2 - 6*(3*a*b*f^3*g^2 - 3*
a*b*e*f^2*g*h + a*b*e^2*f*h^2)*p*q)*x - 6*((18*b^2*e*f^2*g^2 - 27*b^2*e^2*f
*g*h + 11*b^2*e^3*h^2)*p^2*q^2 + 2*(b^2*f^3*h^2*p^2*q^2 - 3*a*b*f^3*h^2*p*q
)*x^3 - 6*(3*a*b*e*f^2*g^2 - 3*a*b*e^2*f*g*h + a*b*e^3*h^2)*p*q - 3*(6*a*b*
f^3*g*h*p*q - (3*b^2*f^3*g*h - b^2*e*f^2*h^2)*p^2*q^2)*x^2 - 6*(3*a*b*f^3*g
^2*p*q - (3*b^2*f^3*g^2 - 3*b^2*e*f^2*g*h + b^2*e^2*f*h^2)*p^2*q^2)*x - 6*(
b^2*f^3*h^2*p*q*x^3 + 3*b^2*f^3*g*h*p*q*x^2 + 3*b^2*f^3*g^2*p*q*x + (3*b^2*
e*f^2*g^2 - 3*b^2*e^2*f*g*h + b^2*e^3*h^2)*p*q)*log(c) - 6*(b^2*f^3*h^2*p*q
^2*x^3 + 3*b^2*f^3*g*h*p*q^2*x^2 + 3*b^2*f^3*g^2*p*q^2*x + (3*b^2*e*f^2*g^2
 - 3*b^2*e^2*f*g*h + b^2*e^3*h^2)*p*q^2)*log(d))*log(f*x + e) - 6*(2*(b^2*f
^3*h^2*p*q - 3*a*b*f^3*h^2)*x^3 - 3*(6*a*b*f^3*g*h - (3*b^2*f^3*g*h - b^2*e
*f^2*h^2)*p*q)*x^2 - 6*(3*a*b*f^3*g^2 - (3*b^2*f^3*g^2 - 3*b^2*e*f^2*g*h +
b^2*e^2*f*h^2)*p*q)*x)*log(c) - 6*(2*(b^2*f^3*h^2*p*q^2 - 3*a*b*f^3*h^2*q)*
x^3 - 3*(6*a*b*f^3*g*h*q - (3*b^2*f^3*g*h - b^2*e*f^2*h^2)*p*q^2)*x^2 - 6*(
3*a*b*f^3*g^2*q - (3*b^2*f^3*g^2 - 3*b^2*e*f^2*g*h + b^2*e^2*f*h^2)*p*q^2)*
x - 6*(b^2*f^3*h^2*q*x^3 + 3*b^2*f^3*g*h*q*x^2 + 3*b^2*f^3*g^2*q*x)*log(c))
*log(d))/f^3
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(311) = 622$.

Time = 2.59 (sec) , antiderivative size = 894, normalized size of antiderivative = 2.77

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 dx$$
$$= \begin{cases} a^2g^2x + a^2ghx^2 + \frac{a^2h^2x^3}{3} + \frac{2abe^3h^2\log\left(c(d(e+fx)^p)^q\right)}{3f^3} - \frac{2abe^2gh\log\left(c(d(e+fx)^p)^q\right)}{f^2} - \frac{2abe^2h^2pqx}{3f^2} + \frac{2abeg^2\log\left(c(d(e+fx)^p)^q\right)}{f} \\ \left(a+b\log\left(c(de^p)^q\right)\right)^2\left(g^2x+ghx^2+\frac{h^2x^3}{3}\right) \end{cases}$$

```
[In] integrate((h*x+g)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

```
[Out] Piecewise((a**2*g**2*x + a**2*g*h*x**2 + a**2*h**2*x**3/3 + 2*a*b*e**3*h**2
*log(c*(d*(e + f*x)**p)**q)/(3*f**3) - 2*a*b*e**2*g*h*log(c*(d*(e + f*x)**p
)**q)/f**2 - 2*a*b*e**2*h**2*p*q*x/(3*f**2) + 2*a*b*e*g**2*log(c*(d*(e + f*
x)**p)**q)/f + 2*a*b*e*g*h*p*q*x/f + a*b*e*h**2*p*q*x**2/(3*f) - 2*a*b*g**2
*p*q*x + 2*a*b*g**2*x*log(c*(d*(e + f*x)**p)**q) - a*b*g*h*p*q*x**2 + 2*a*b
*g*h*x**2*log(c*(d*(e + f*x)**p)**q) - 2*a*b*h**2*p*q*x**3/9 + 2*a*b*h**2*x
**3*log(c*(d*(e + f*x)**p)**q)/3 - 11*b**2*e**3*h**2*p*q*log(c*(d*(e + f*x)
**p)**q)/(9*f**3) + b**2*e**3*h**2*log(c*(d*(e + f*x)**p)**q)**2/(3*f**3) +
 3*b**2*e**2*g*h*p*q*log(c*(d*(e + f*x)**p)**q)/f**2 - b**2*e**2*g*h*log(c*
(d*(e + f*x)**p)**q)**2/f**2 + 11*b**2*e**2*h**2*p**2*q**2*x/(9*f**2) - 2*b
**2*e**2*h**2*p*q*x*log(c*(d*(e + f*x)**p)**q)/(3*f**2) - 2*b**2*e*g**2*p*q
*log(c*(d*(e + f*x)**p)**q)/f + b**2*e*g**2*log(c*(d*(e + f*x)**p)**q)**2/f
 - 3*b**2*e*g*h*p**2*q**2*x/f + 2*b**2*e*g*h*p*q*x*log(c*(d*(e + f*x)**p)**
q)/f - 5*b**2*e*h**2*p**2*q**2*x**2/(18*f) + b**2*e*h**2*p*q*x**2*log(c*(d*
(e + f*x)**p)**q)/(3*f) + 2*b**2*g**2*p**2*q**2*x - 2*b**2*g**2*p*q*x*log(c
*(d*(e + f*x)**p)**q) + b**2*g**2*x*log(c*(d*(e + f*x)**p)**q)**2 + b**2*g*
h*p**2*q**2*x**2/2 - b**2*g*h*p*q*x**2*log(c*(d*(e + f*x)**p)**q) + b**2*g*
h*x**2*log(c*(d*(e + f*x)**p)**q)**2 + 2*b**2*h**2*p**2*q**2*x**3/27 - 2*b*
*2*h**2*p*q*x**3*log(c*(d*(e + f*x)**p)**q)/9 + b**2*h**2*x**3*log(c*(d*(e
+ f*x)**p)**q)**2/3, Ne(f, 0)), ((a + b*log(c*(d*e**p)**q))**2*(g**2*x + g*
h*x**2 + h**2*x**3/3), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.87

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^2 \, dx$$
$$= \frac{1}{3} b^2h^2x^3 \log\left(((fx+e)^pd)^q c\right)^2 - 2\,abfg^2pq\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right)$$
$$+ \frac{1}{9} abfh^2pq\left(\frac{6\,e^3\log(fx+e)}{f^4} - \frac{2\,f^2x^3 - 3\,efx^2 + 6\,e^2x}{f^3}\right)$$
$$- abfghpq\left(\frac{2\,e^2\log(fx+e)}{f^3} + \frac{fx^2-2\,ex}{f^2}\right) + \frac{2}{3} abh^2x^3\log\left(((fx+e)^pd)^q c\right)$$
$$+ b^2ghx^2\log\left(((fx+e)^pd)^q c\right)^2 + \frac{1}{3} a^2h^2x^3 + 2\,abghx^2\log\left(((fx+e)^pd)^q c\right)$$
$$+ b^2g^2x\log\left(((fx+e)^pd)^q c\right)^2 + a^2ghx^2 + 2\,abg^2x\log\left(((fx+e)^pd)^q c\right)$$
$$- \left(2\,fpq\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right)\log\left(((fx+e)^pd)^q c\right) + \frac{(e\log(fx+e)^2 - 2\,fx + 2\,e\log(fx+e))p^2q^2}{f}\right)b$$
$$-\frac{1}{2}\left(2\,fpq\left(\frac{2\,e^2\log(fx+e)}{f^3} + \frac{fx^2-2\,ex}{f^2}\right)\log\left(((fx+e)^pd)^q c\right) - \frac{(f^2x^2 + 2\,e^2\log(fx+e)^2 - 6\,efx +}{f^2}\right.$$
$$+\frac{1}{54}\left(6\,fpq\left(\frac{6\,e^3\log(fx+e)}{f^4} - \frac{2\,f^2x^3 - 3\,efx^2 + 6\,e^2x}{f^3}\right)\log\left(((fx+e)^pd)^q c\right) + \frac{(4\,f^3x^3 - 15\,ef^2x^2 -}{}\right.$$
$$+ a^2g^2x$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*h^2*x^3*log(((f*x + e)^p*d)^q*c)^2 - 2*a*b*f*g^2*p*q*(x/f - e*log(f
*x + e)/f^2) + 1/9*a*b*f*h^2*p*q*(6*e^3*log(f*x + e)/f^4 - (2*f^2*x^3 - 3*e
*f*x^2 + 6*e^2*x)/f^3) - a*b*f*g*h*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2
*e*x)/f^2) + 2/3*a*b*h^2*x^3*log(((f*x + e)^p*d)^q*c) + b^2*g*h*x^2*log(((f
*x + e)^p*d)^q*c)^2 + 1/3*a^2*h^2*x^3 + 2*a*b*g*h*x^2*log(((f*x + e)^p*d)^q
*c) + b^2*g^2*x*log(((f*x + e)^p*d)^q*c)^2 + a^2*g*h*x^2 + 2*a*b*g^2*x*log(
((f*x + e)^p*d)^q*c) - (2*f*p*q*(x/f - e*log(f*x + e)/f^2)*log(((f*x + e)^p
*d)^q*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p^2*q^2/f)*b^2*g^2
 - 1/2*(2*f*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)/f^2)*log(((f*x +
e)^p*d)^q*c) - (f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x + 6*e^2*log(f*x +
e))*p^2*q^2/f^2)*b^2*g*h + 1/54*(6*f*p*q*(6*e^3*log(f*x + e)/f^4 - (2*f^2*x
^3 - 3*e*f*x^2 + 6*e^2*x)/f^3)*log(((f*x + e)^p*d)^q*c) + (4*f^3*x^3 - 15*e
*f^2*x^2 - 18*e^3*log(f*x + e)^2 + 66*e^2*f*x - 66*e^3*log(f*x + e))*p^2*q^
2/f^3)*b^2*h^2 + a^2*g^2*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2106 vs. $2(311) = 622$.

Time = 0.34 (sec) , antiderivative size = 2106, normalized size of antiderivative = 6.52

$$\int (g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^2 \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

```
[Out] (f*x + e)*b^2*g^2*p^2*q^2*log(f*x + e)^2/f + (f*x + e)^2*b^2*g*h*p^2*q^2*lo
g(f*x + e)^2/f^2 - 2*(f*x + e)*b^2*e*g*h*p^2*q^2*log(f*x + e)^2/f^2 + 1/3*(
f*x + e)^3*b^2*h^2*p^2*q^2*log(f*x + e)^2/f^3 - (f*x + e)^2*b^2*e*h^2*p^2*q
^2*log(f*x + e)^2/f^3 + (f*x + e)*b^2*e^2*h^2*p^2*q^2*log(f*x + e)^2/f^3 -
2*(f*x + e)*b^2*g^2*p^2*q^2*log(f*x + e)/f - (f*x + e)^2*b^2*g*h*p^2*q^2*lo
g(f*x + e)/f^2 + 4*(f*x + e)*b^2*e*g*h*p^2*q^2*log(f*x + e)/f^2 - 2/9*(f*x
+ e)^3*b^2*h^2*p^2*q^2*log(f*x + e)/f^3 + (f*x + e)^2*b^2*e*h^2*p^2*q^2*log
(f*x + e)/f^3 - 2*(f*x + e)*b^2*e^2*h^2*p^2*q^2*log(f*x + e)/f^3 + 2*(f*x +
 e)*b^2*g^2*p*q^2*log(f*x + e)*log(d)/f + 2*(f*x + e)^2*b^2*g*h*p*q^2*log(f
*x + e)*log(d)/f^2 - 4*(f*x + e)*b^2*e*g*h*p*q^2*log(f*x + e)*log(d)/f^2 +
2/3*(f*x + e)^3*b^2*h^2*p*q^2*log(f*x + e)*log(d)/f^3 - 2*(f*x + e)^2*b^2*e
*h^2*p*q^2*log(f*x + e)*log(d)/f^3 + 2*(f*x + e)*b^2*e^2*h^2*p*q^2*log(f*x
+ e)*log(d)/f^3 + 2*(f*x + e)*b^2*g^2*p^2*q^2/f + 1/2*(f*x + e)^2*b^2*g*h*p
^2*q^2/f^2 - 4*(f*x + e)*b^2*e*g*h*p^2*q^2/f^2 + 2/27*(f*x + e)^3*b^2*h^2*p
^2*q^2/f^3 - 1/2*(f*x + e)^2*b^2*e*h^2*p^2*q^2/f^3 + 2*(f*x + e)*b^2*e^2*h^
2*p^2*q^2/f^3 + 2*(f*x + e)*b^2*g^2*p*q*log(f*x + e)*log(c)/f + 2*(f*x + e)
^2*b^2*g*h*p*q*log(f*x + e)*log(c)/f^2 - 4*(f*x + e)*b^2*e*g*h*p*q*log(f*x
+ e)*log(c)/f^2 + 2/3*(f*x + e)^3*b^2*h^2*p*q*log(f*x + e)*log(c)/f^3 - 2*(
f*x + e)^2*b^2*e*h^2*p*q*log(f*x + e)*log(c)/f^3 + 2*(f*x + e)*b^2*e^2*h^2*
p*q*log(f*x + e)*log(c)/f^3 - 2*(f*x + e)*b^2*g^2*p*q^2*log(d)/f - (f*x + e
)^2*b^2*g*h*p*q^2*log(d)/f^2 + 4*(f*x + e)*b^2*e*g*h*p*q^2*log(d)/f^2 - 2/9
*(f*x + e)^3*b^2*h^2*p*q^2*log(d)/f^3 + (f*x + e)^2*b^2*e*h^2*p*q^2*log(d)/
f^3 - 2*(f*x + e)*b^2*e^2*h^2*p*q^2*log(d)/f^3 + (f*x + e)*b^2*g^2*q^2*log(
d)^2/f + (f*x + e)^2*b^2*g*h*q^2*log(d)^2/f^2 - 2*(f*x + e)*b^2*e*g*h*q^2*l
og(d)^2/f^2 + 1/3*(f*x + e)^3*b^2*h^2*q^2*log(d)^2/f^3 - (f*x + e)^2*b^2*e*
h^2*q^2*log(d)^2/f^3 + (f*x + e)*b^2*e^2*h^2*q^2*log(d)^2/f^3 + 2*(f*x + e)
*a*b*g^2*p*q*log(f*x + e)/f + 2*(f*x + e)^2*a*b*g*h*p*q*log(f*x + e)/f^2 -
4*(f*x + e)*a*b*e*g*h*p*q*log(f*x + e)/f^2 + 2/3*(f*x + e)^3*a*b*h^2*p*q*lo
g(f*x + e)/f^3 - 2*(f*x + e)^2*a*b*e*h^2*p*q*log(f*x + e)/f^3 + 2*(f*x + e)
*a*b*e^2*h^2*p*q*log(f*x + e)/f^3 - 2*(f*x + e)*b^2*g^2*p*q*log(c)/f - (f*x
 + e)^2*b^2*g*h*p*q*log(c)/f^2 + 4*(f*x + e)*b^2*e*g*h*p*q*log(c)/f^2 - 2/9
*(f*x + e)^3*b^2*h^2*p*q*log(c)/f^3 + (f*x + e)^2*b^2*e*h^2*p*q*log(c)/f^3
- 2*(f*x + e)*b^2*e^2*h^2*p*q*log(c)/f^3 + 2*(f*x + e)*b^2*g^2*q*log(c)*log
(d)/f + 2*(f*x + e)^2*b^2*g*h*q*log(c)*log(d)/f^2 - 4*(f*x + e)*b^2*e*g*h*q
*log(c)*log(d)/f^2 + 2/3*(f*x + e)^3*b^2*h^2*q*log(c)*log(d)/f^3 - 2*(f*x +
```

e)^2*b^2*e*h^2*q*log(c)*log(d)/f^3 + 2*(f*x + e)*b^2*e^2*h^2*q*log(c)*log(d)/f^3 - 2*(f*x + e)*a*b*g^2*p*q/f - (f*x + e)^2*a*b*g*h*p*q/f^2 + 4*(f*x + e)*a*b*e*g*h*p*q/f^2 - 2/9*(f*x + e)^3*a*b*h^2*p*q/f^3 + (f*x + e)^2*a*b*e*h^2*p*q/f^3 - 2*(f*x + e)*a*b*e^2*h^2*p*q/f^3 + (f*x + e)*b^2*g^2*log(c)^2/f + (f*x + e)^2*b^2*g*h*log(c)^2/f^2 - 2*(f*x + e)*b^2*e*g*h*log(c)^2/f^2 + 1/3*(f*x + e)^3*b^2*h^2*log(c)^2/f^3 - (f*x + e)^2*b^2*e*h^2*log(c)^2/f^3 + (f*x + e)*b^2*e^2*h^2*log(c)^2/f^3 + 2*(f*x + e)*a*b*g^2*q*log(d)/f + 2*(f*x + e)^2*a*b*g*h*q*log(d)/f^2 - 4*(f*x + e)*a*b*e*g*h*q*log(d)/f^2 + 2/3*(f*x + e)^3*a*b*h^2*q*log(d)/f^3 - 2*(f*x + e)^2*a*b*e*h^2*q*log(d)/f^3 + 2*(f*x + e)*a*b*e^2*h^2*q*log(d)/f^3 + 2*(f*x + e)*a*b*g^2*log(c)/f + 2*(f*x + e)^2*a*b*g*h*log(c)/f^2 - 4*(f*x + e)*a*b*e*g*h*log(c)/f^2 + 2/3*(f*x + e)^3*a*b*h^2*log(c)/f^3 - 2*(f*x + e)^2*a*b*e*h^2*log(c)/f^3 + 2*(f*x + e)*a*b*e^2*h^2*log(c)/f^3 + (f*x + e)*a^2*g^2/f + (f*x + e)^2*a^2*g*h/f^2 - 2*(f*x + e)*a^2*e*g*h/f^2 + 1/3*(f*x + e)^3*a^2*h^2/f^3 - (f*x + e)^2*a^2*e*h^2/f^3 + (f*x + e)*a^2*e^2*h^2/f^3

## Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 652, normalized size of antiderivative = 2.02

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$$
$$= \ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)^2 \left(b^2\,g^2\,x+\frac{b^2\,h^2\,x^3}{3}+\frac{e\,\left(b^2\,e^2\,h^2-3\,b^2\,e\,f\,g\,h+3\,b^2\,f^2\,g^2\right)}{3\,f^3}+b^2\,g\,h\,x^2\right)$$
$$+\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\left(\frac{x^2\,\left(\frac{3\,b\,h\,\left(a\,e\,h+2\,a\,f\,g-b\,f\,g\,p\,q\right)}{f}-\frac{b\,e\,h^2\,\left(3\,a-b\,p\,q\right)}{f}\right)}{3} - \frac{x\,\left(\frac{e\,\left(\frac{6\,b\,h\,\left(a\,e\,h+2\,a\,f\,g-b\,f\,g\,p\,q\right)}{f}-\frac{2\,b\,e\,h^2\,\left(3\,a-b\,p\,q\right)}{f}\right)}{f}-\frac{6\,b\,g\,\left(2\,a\,e\,h+a\,f\,g-b\,f\,g\,p\,q\right)}{f}\right)}{3} + \frac{2\,b\,h^2\,x^3\,\left(3\,a-b\,p\,q\right)}{9}\right)$$
$$+x\left(\frac{18\,a^2\,e\,f\,g\,h+9\,a^2\,f^2\,g^2-18\,a\,b\,f^2\,g^2\,p\,q+6\,b^2\,e^2\,h^2\,p^2\,q^2-18\,b^2\,e\,f\,g\,h\,p^2\,q^2+18\,b^2\,f^2\,g^2\,p^2\,q^2}{9\,f^2} - \frac{e\,\left(\frac{h\,\left(3\,a^2\,e\,h+6\,a^2\,f\,g-b^2\,e\,h\,p^2\,q^2+3\,b^2\,f\,g\,p^2\,q^2-6\,a\,b\,f\,g\,p\,q\right)}{3\,f}-\frac{e\,h^2\,\left(9\,a^2-6\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right)}{9\,f}\right)}{f}\right)$$
$$+x^2\left(\frac{h\,\left(3\,a^2\,e\,h+6\,a^2\,f\,g-b^2\,e\,h\,p^2\,q^2+3\,b^2\,f\,g\,p^2\,q^2-6\,a\,b\,f\,g\,p\,q\right)}{6\,f} - \frac{e\,h^2\,\left(9\,a^2-6\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right)}{18\,f}\right)$$
$$-\frac{\ln\left(e+f\,x\right)\,\left(11\,b^2\,e^3\,h^2\,p^2\,q^2-27\,b^2\,e^2\,f\,g\,h\,p^2\,q^2+18\,b^2\,e\,f^2\,g^2\,p^2\,q^2-6\,a\,b\,e^3\,h^2\,p\,q+18\,a\,b\,e^2\,f\,g\,h\,p\,q-\right.}{9\,f^3}$$
$$+\frac{h^2\,x^3\,\left(9\,a^2-6\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right)}{27}$$

```
[In] int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
```

```
[Out] log(c*(d*(e + f*x)^p)^q)^2*(b^2*g^2*x + (b^2*h^2*x^3)/3 + (e*(b^2*e^2*h^2 +
 3*b^2*f^2*g^2 - 3*b^2*e*f*g*h))/(3*f^3) + b^2*g*h*x^2) + log(c*(d*(e + f*x
)^p)^q)*((x^2*((3*b*h*(a*e*h + 2*a*f*g - b*f*g*p*q))/f - (b*e*h^2*(3*a - b*
p*q))/f))/3 - (x*((e*((6*b*h*(a*e*h + 2*a*f*g - b*f*g*p*q))/f - (2*b*e*h^2*
```

```
(3*a - b*p*q))/f))/f - (6*b*g*(2*a*e*h + a*f*g - b*f*g*p*q))/f))/3 + (2*b*h
^2*x^3*(3*a - b*p*q))/9) + x*((9*a^2*f^2*g^2 + 6*b^2*e^2*h^2*p^2*q^2 + 18*b
^2*f^2*g^2*p^2*q^2 + 18*a^2*e*f*g*h - 18*a*b*f^2*g^2*p*q - 18*b^2*e*f*g*h*p
^2*q^2)/(9*f^2) - (e*((h*(3*a^2*e*h + 6*a^2*f*g - b^2*e*h*p^2*q^2 + 3*b^2*f
*g*p^2*q^2 - 6*a*b*f*g*p*q))/(3*f) - (e*h^2*(9*a^2 + 2*b^2*p^2*q^2 - 6*a*b*
p*q))/(9*f)))/f) + x^2*((h*(3*a^2*e*h + 6*a^2*f*g - b^2*e*h*p^2*q^2 + 3*b^2
*f*g*p^2*q^2 - 6*a*b*f*g*p*q))/(6*f) - (e*h^2*(9*a^2 + 2*b^2*p^2*q^2 - 6*a*
b*p*q))/(18*f)) - (log(e + f*x)*(11*b^2*e^3*h^2*p^2*q^2 - 6*a*b*e^3*h^2*p*q
 + 18*b^2*e*f^2*g^2*p^2*q^2 - 27*b^2*e^2*f*g*h*p^2*q^2 - 18*a*b*e*f^2*g^2*p
*q + 18*a*b*e^2*f*g*h*p*q))/(9*f^3) + (h^2*x^3*(9*a^2 + 2*b^2*p^2*q^2 - 6*a
*b*p*q))/27
```

# 3.430 $\int (g+hx) \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 211

$$\begin{aligned}
&\int (g+hx) \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 \, dx \\
&= -\frac{2ab(fg-eh)pqx}{f}+\frac{2b^2(fg-eh)p^2q^2x}{f}+\frac{b^2hp^2q^2(e+fx)^2}{4f^2} \\
&\quad -\frac{2b^2(fg-eh)pq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f^2} \\
&\quad -\frac{bhpq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^2} \\
&\quad +\frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2} \\
&\quad +\frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^2}
\end{aligned}$$

```
[Out] -2*a*b*(-e*h+f*g)*p*q*x/f+2*b^2*(-e*h+f*g)*p^2*q^2*x/f+1/4*b^2*h*p^2*q^2*(f
*x+e)^2/f^2-2*b^2*(-e*h+f*g)*p*q*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f^2-1/2*b*h*
p*q*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^2+(-e*h+f*g)*(f*x+e)*(a+b*ln(c*
(d*(f*x+e)^p)^q))^2/f^2+1/2*h*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f^2
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2448, 2436, 2333, 2332, 2437, 2342, 2341, 2495}

$$\int (g+hx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2 dx = \frac{(e+fx)(fg-eh) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{f^2} - \frac{bhpq(e+fx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)}{2f^2} + \frac{h(e+fx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{2f^2} - \frac{2abpqx(fg-eh)}{f} - \frac{2b^2pq(e+fx)(fg-eh)\log \left(c(d(e+fx)^p)^q\right)}{f^2} + \frac{b^2hp^2q^2(e+fx)^2}{4f^2} + \frac{2b^2p^2q^2x(fg-eh)}{f}$$

```
[In] Int[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] (-2*a*b*(f*g - e*h)*p*q*x)/f + (2*b^2*(f*g - e*h)*p^2*q^2*x)/f + (b^2*h*p^2
*q^2*(e + f*x)^2)/(4*f^2) - (2*b^2*(f*g - e*h)*p*q*(e + f*x)*Log[c*(d*(e +
f*x)^p)^q])/f^2 - (b*h*p*q*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(2
*f^2) + ((f*g - e*h)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/f^2 + (h
*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(2*f^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (g+hx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\int\left(\frac{(fg-eh)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{f} + \frac{h(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{f}\right)dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{h\int (e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(fg-eh)\int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{h\text{Subst}\left(\int x\left(a+b\log\left(cd^q x^{pq}\right)\right)^2\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(fg-eh)\text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right)^2\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2} + \frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^2} - \text{Subst}\left(\frac{(bhpq)\text{Subst}\left(\int x\left(a+b\log\left(cd^q x^{pq}\right)\right)\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(2b(fg-eh)pq)\text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right)\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2ab(fg-eh)pqx}{f} + \frac{b^2hp^2q^2(e+fx)^2}{4f^2} - \frac{bhpq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^2} + \frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2} + \frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^2} - \text{Subst}\left(\frac{(2b^2(fg-eh)pq)\,\text{Subst}\left(\int \log\left(cd^q x^{pq}\right)\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$
\begin{aligned}
&= -\frac{2ab(fg-eh)pqx}{f} + \frac{2b^2(fg-eh)p^2q^2x}{f} + \frac{b^2hp^2q^2(e+fx)^2}{4f^2} \\
&\quad - \frac{2b^2(fg-eh)pq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f^2} \\
&\quad - \frac{bhpq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^2} \\
&\quad + \frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2} \\
&\quad + \frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^2}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.78

$$
\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx
$$
$$
= \frac{4(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 + 2h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 - 8b(fg-eh)}{}
$$

```
[In] Integrate[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] (4*(f*g - e*h)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 2*h*(e + f*x)
^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 - 8*b*(f*g - e*h)*p*q*(f*(a - b*p*q)*
x + b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q]) + b*h*p*q*(b*f*p*q*x*(2*e + f*x)
- 2*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])))/(4*f^2)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(205) = 410.

Time = 1.53 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.56

| method | result |
|---|---|
| parallelrisch | $-\frac{-4x\ln(c(d(fx+e)^p)^q)b^2efhpq-8\ln(c(d(fx+e)^p)^q)b^2efgpq+16\ln(fx+e)b^2efg\,p^2q^2-8abefgpq-6b^2e^2h\,p^2q^2-x^2b^2f^2h\,p^2q^2}{}$ |

```
[In] int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-4*x*ln(c*(d*(f*x+e)^p)^q)*b^2*e*f*h*p*q-8*ln(c*(d*(f*x+e)^p)^q)*b^2*
e*f*g*p*q+16*ln(f*x+e)*b^2*e*f*g*p^2*q^2-8*a*b*e*f*g*p*q-6*b^2*e^2*h*p^2*q^
2-x^2*b^2*f^2*h*p^2*q^2-8*x*b^2*f^2*g*p^2*q^2-4*x^2*ln(c*(d*(f*x+e)^p)^q)*a
*b*f^2*h+4*ln(c*(d*(f*x+e)^p)^q)*b^2*e^2*h*p*q-8*x*ln(c*(d*(f*x+e)^p)^q)*a*
b*f^2*g+8*ln(c*(d*(f*x+e)^p)^q)*a*b*e*f*g-10*ln(f*x+e)*b^2*e^2*h*p^2*q^2+4*
a^2*e*f*g+4*a*b*e^2*h*p*q+8*b^2*e*f*g*p^2*q^2+8*x*a*b*f^2*g*p*q+4*ln(f*x+e)
```

```
*a*b*e^2*h*p*q+2*x^2*ln(c*(d*(f*x+e)^p)^q)*b^2*f^2*h*p*q+6*x*b^2*e*f*h*p^2*
q^2+2*x^2*a*b*f^2*h*p*q-4*x*a*b*e*f*h*p*q-16*ln(f*x+e)*a*b*e*f*g*p*q+8*x*ln
(c*(d*(f*x+e)^p)^q)*b^2*f^2*g*p*q-2*x^2*ln(c*(d*(f*x+e)^p)^q)^2*b^2*f^2*h-4
*x*ln(c*(d*(f*x+e)^p)^q)^2*b^2*f^2*g-4*ln(c*(d*(f*x+e)^p)^q)^2*b^2*e*f*g-2*
x^2*a^2*f^2*h+2*ln(c*(d*(f*x+e)^p)^q)^2*b^2*e^2*h-4*x*a^2*f^2*g)/f^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(205) = 410$.

Time = 0.33 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.95

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$$
$$= \frac{(b^2f^2hp^2q^2 - 2\,abf^2hpq + 2\,a^2f^2h)x^2 + 2\,(b^2f^2hp^2q^2x^2 + 2\,b^2f^2gp^2q^2x + (2\,b^2efg - b^2e^2h)p^2q^2)\log\left(fx + \right.}{}$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*f^2*h*p^2*q^2 - 2*a*b*f^2*h*p*q + 2*a^2*f^2*h)*x^2 + 2*(b^2*f^2*h
*p^2*q^2*x^2 + 2*b^2*f^2*g*p^2*q^2*x + (2*b^2*e*f*g - b^2*e^2*h)*p^2*q^2)*l
og(f*x + e)^2 + 2*(b^2*f^2*h*x^2 + 2*b^2*f^2*g*x)*log(c)^2 + 2*(b^2*f^2*h*q
^2*x^2 + 2*b^2*f^2*g*q^2*x)*log(d)^2 + 2*(2*a^2*f^2*g + (4*b^2*f^2*g - 3*b^
2*e*f*h)*p^2*q^2 - 2*(2*a*b*f^2*g - a*b*e*f*h)*p*q)*x - 2*((4*b^2*e*f*g - 3
*b^2*e^2*h)*p^2*q^2 - 2*(2*a*b*e*f*g - a*b*e^2*h)*p*q + (b^2*f^2*h*p^2*q^2
- 2*a*b*f^2*h*p*q)*x^2 - 2*(2*a*b*f^2*g*p*q - (2*b^2*f^2*g - b^2*e*f*h)*p^2
*q^2)*x - 2*(b^2*f^2*h*p*q*x^2 + 2*b^2*f^2*g*p*q*x + (2*b^2*e*f*g - b^2*e^2
*h)*p*q)*log(c) - 2*(b^2*f^2*h*p*q^2*x^2 + 2*b^2*f^2*g*p*q^2*x + (2*b^2*e*f
*g - b^2*e^2*h)*p*q^2)*log(d))*log(f*x + e) - 2*((b^2*f^2*h*p*q - 2*a*b*f^2
*h)*x^2 - 2*(2*a*b*f^2*g - (2*b^2*f^2*g - b^2*e*f*h)*p*q)*x)*log(c) - 2*((b
^2*f^2*h*p*q^2 - 2*a*b*f^2*h*q)*x^2 - 2*(2*a*b*f^2*g*q - (2*b^2*f^2*g - b^2
*e*f*h)*p*q^2)*x - 2*(b^2*f^2*h*q*x^2 + 2*b^2*f^2*g*q*x)*log(c))*log(d))/f^
2
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(202) = 404$.

Time = 1.24 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.21

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx$$
$$= \begin{cases} a^2gx + \frac{a^2hx^2}{2} - \frac{abe^2h\log\left(c(d(e+fx)^p)^q\right)}{f^2} + \frac{2abeg\log\left(c(d(e+fx)^p)^q\right)}{f} + \frac{abehpqx}{f} - 2abgpqx + 2abgx\log\left(c(d(e+fx)^p\right. \\ \left(a+b\log\left(c(de^p)^q\right)\right)^2\left(gx+\frac{hx^2}{2}\right) \end{cases}$$

```
[In] integrate((h*x+g)*(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

```
[Out] Piecewise((a**2*g*x + a**2*h*x**2/2 - a*b*e**2*h*log(c*(d*(e + f*x)**p)**q)
/f**2 + 2*a*b*e*g*log(c*(d*(e + f*x)**p)**q)/f + a*b*e*h*p*q*x/f - 2*a*b*g*
p*q*x + 2*a*b*g*x*log(c*(d*(e + f*x)**p)**q) - a*b*h*p*q*x**2/2 + a*b*h*x**
2*log(c*(d*(e + f*x)**p)**q) + 3*b**2*e**2*h*p*q*log(c*(d*(e + f*x)**p)**q)
/(2*f**2) - b**2*e**2*h*log(c*(d*(e + f*x)**p)**q)**2/(2*f**2) - 2*b**2*e*g
*p*q*log(c*(d*(e + f*x)**p)**q)/f + b**2*e*g*log(c*(d*(e + f*x)**p)**q)**2/
f - 3*b**2*e*h*p**2*q**2*x/(2*f) + b**2*e*h*p*q*x*log(c*(d*(e + f*x)**p)**q
)/f + 2*b**2*g*p**2*q**2*x - 2*b**2*g*p*q*x*log(c*(d*(e + f*x)**p)**q) + b*
*2*g*x*log(c*(d*(e + f*x)**p)**q)**2 + b**2*h*p**2*q**2*x**2/4 - b**2*h*p*q
*x**2*log(c*(d*(e + f*x)**p)**q)/2 + b**2*h*x**2*log(c*(d*(e + f*x)**p)**q)
**2/2, Ne(f, 0)), ((a + b*log(c*(d*e**p)**q))**2*(g*x + h*x**2/2), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.65

$$\begin{aligned}
&\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx \\
&= -2\,abfgpq\left(\frac{x}{f}-\frac{e\log\left(fx+e\right)}{f^2}\right)-\frac{1}{2}\,abfhpq\left(\frac{2\,e^2\log\left(fx+e\right)}{f^3}+\frac{fx^2-2\,ex}{f^2}\right) \\
&\quad+\frac{1}{2}\,b^2hx^2\log\left(\left((fx+e)^pd\right)^qc\right)^2+abhx^2\log\left(\left((fx+e)^pd\right)^qc\right) \\
&\quad+b^2gx\log\left(\left((fx+e)^pd\right)^qc\right)^2+\frac{1}{2}\,a^2hx^2+2\,abgx\log\left(\left((fx+e)^pd\right)^qc\right) \\
&\quad-\left(2\,fpq\left(\frac{x}{f}-\frac{e\log\left(fx+e\right)}{f^2}\right)\log\left(\left((fx+e)^pd\right)^qc\right)+\frac{\left(e\log\left(fx+e\right)^2-2\,fx+2\,e\log\left(fx+e\right)\right)p^2q^2}{f}\right)b^2g \\
&\quad-\frac{1}{4}\left(2\,fpq\left(\frac{2\,e^2\log\left(fx+e\right)}{f^3}+\frac{fx^2-2\,ex}{f^2}\right)\log\left(\left((fx+e)^pd\right)^qc\right)-\frac{\left(f^2x^2+2\,e^2\log\left(fx+e\right)^2-6\,efx+\right.}{f^2}\right. \\
&\quad+a^2gx
\end{aligned}$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*f*g*p*q*(x/f - e*log(f*x + e)/f^2) - 1/2*a*b*f*h*p*q*(2*e^2*log(f*x
+ e)/f^3 + (f*x^2 - 2*e*x)/f^2) + 1/2*b^2*h*x^2*log(((f*x + e)^p*d)^q*c)^2
+ a*b*h*x^2*log(((f*x + e)^p*d)^q*c) + b^2*g*x*log(((f*x + e)^p*d)^q*c)^2 +
 1/2*a^2*h*x^2 + 2*a*b*g*x*log(((f*x + e)^p*d)^q*c) - (2*f*p*q*(x/f - e*log
(f*x + e)/f^2)*log(((f*x + e)^p*d)^q*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*l
og(f*x + e))*p^2*q^2/f)*b^2*g - 1/4*(2*f*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x
^2 - 2*e*x)/f^2)*log(((f*x + e)^p*d)^q*c) - (f^2*x^2 + 2*e^2*log(f*x + e)^2
 - 6*e*f*x + 6*e^2*log(f*x + e))*p^2*q^2/f^2)*b^2*h + a^2*g*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(205) = 410$.

Time = 0.32 (sec) , antiderivative size = 939, normalized size of antiderivative = 4.45

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

```
[Out] (f*x + e)*b^2*g*p^2*q^2*log(f*x + e)^2/f + 1/2*(f*x + e)^2*b^2*h*p^2*q^2*lo
g(f*x + e)^2/f^2 - (f*x + e)*b^2*e*h*p^2*q^2*log(f*x + e)^2/f^2 - 2*(f*x +
e)*b^2*g*p^2*q^2*log(f*x + e)/f - 1/2*(f*x + e)^2*b^2*h*p^2*q^2*log(f*x + e
)/f^2 + 2*(f*x + e)*b^2*e*h*p^2*q^2*log(f*x + e)/f^2 + 2*(f*x + e)*b^2*g*p*
q^2*log(f*x + e)*log(d)/f + (f*x + e)^2*b^2*h*p*q^2*log(f*x + e)*log(d)/f^2
 - 2*(f*x + e)*b^2*e*h*p*q^2*log(f*x + e)*log(d)/f^2 + 2*(f*x + e)*b^2*g*p^
2*q^2/f + 1/4*(f*x + e)^2*b^2*h*p^2*q^2/f^2 - 2*(f*x + e)*b^2*e*h*p^2*q^2/f
^2 + 2*(f*x + e)*b^2*g*p*q*log(f*x + e)*log(c)/f + (f*x + e)^2*b^2*h*p*q*lo
g(f*x + e)*log(c)/f^2 - 2*(f*x + e)*b^2*e*h*p*q*log(f*x + e)*log(c)/f^2 - 2
*(f*x + e)*b^2*g*p*q^2*log(d)/f - 1/2*(f*x + e)^2*b^2*h*p*q^2*log(d)/f^2 +
2*(f*x + e)*b^2*e*h*p*q^2*log(d)/f^2 + (f*x + e)*b^2*g*q^2*log(d)^2/f + 1/2
*(f*x + e)^2*b^2*h*q^2*log(d)^2/f^2 - (f*x + e)*b^2*e*h*q^2*log(d)^2/f^2 +
2*(f*x + e)*a*b*g*p*q*log(f*x + e)/f + (f*x + e)^2*a*b*h*p*q*log(f*x + e)/f
^2 - 2*(f*x + e)*a*b*e*h*p*q*log(f*x + e)/f^2 - 2*(f*x + e)*b^2*g*p*q*log(c
)/f - 1/2*(f*x + e)^2*b^2*h*p*q*log(c)/f^2 + 2*(f*x + e)*b^2*e*h*p*q*log(c)
/f^2 + 2*(f*x + e)*b^2*g*q*log(c)*log(d)/f + (f*x + e)^2*b^2*h*q*log(c)*log
(d)/f^2 - 2*(f*x + e)*b^2*e*h*q*log(c)*log(d)/f^2 - 2*(f*x + e)*a*b*g*p*q/f
 - 1/2*(f*x + e)^2*a*b*h*p*q/f^2 + 2*(f*x + e)*a*b*e*h*p*q/f^2 + (f*x + e)*
b^2*g*log(c)^2/f + 1/2*(f*x + e)^2*b^2*h*log(c)^2/f^2 - (f*x + e)*b^2*e*h*l
og(c)^2/f^2 + 2*(f*x + e)*a*b*g*q*log(d)/f + (f*x + e)^2*a*b*h*q*log(d)/f^2
 - 2*(f*x + e)*a*b*e*h*q*log(d)/f^2 + 2*(f*x + e)*a*b*g*log(c)/f + (f*x + e
)^2*a*b*h*log(c)/f^2 - 2*(f*x + e)*a*b*e*h*log(c)/f^2 + (f*x + e)*a^2*g/f +
 1/2*(f*x + e)^2*a^2*h/f^2 - (f*x + e)*a^2*e*h/f^2
```

## Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.43

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx = x\left(\frac{2a^2eh+2a^2fg-2b^2ehp^2q^2+4b^2fgp^2q^2-4abfgpq}{2f} - \frac{eh\left(2a^2-2abpq+b^2p^2q^2\right)}{2f}\right)+\ln\left(c\left(d\left(e+fx\right)^p\right)^q\right)\left(\frac{bh\left(2a-bpq\right)x^2}{2} + \left(\frac{2b\left(aeh+afg-bfgpq\right)}{f}-\frac{beh\left(2a-bpq\right)}{f}\right)x\right) + \ln\left(c\left(d\left(e+fx\right)^p\right)^q\right)^2\left(\frac{b^2hx^2}{2}-\frac{e\left(b^2eh-2b^2fg\right)}{2f^2}+b^2gx\right) + \frac{\ln\left(e+fx\right)\left(3hb^2e^2p^2q^2-4fgb^2ep^2q^2-2ahbe^2pq+4afgbepq\right)}{2f^2} + \frac{hx^2\left(2a^2-2abpq+b^2p^2q^2\right)}{4}$$

```
[In] int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
```

```
[Out] x*((2*a^2*e*h + 2*a^2*f*g - 2*b^2*e*h*p^2*q^2 + 4*b^2*f*g*p^2*q^2 - 4*a*b*f
*g*p*q)/(2*f) - (e*h*(2*a^2 + b^2*p^2*q^2 - 2*a*b*p*q))/(2*f)) + log(c*(d*(
e + f*x)^p)^q)*(x*((2*b*(a*e*h + a*f*g - b*f*g*p*q))/f - (b*e*h*(2*a - b*p*
q))/f) + (b*h*x^2*(2*a - b*p*q))/2) + log(c*(d*(e + f*x)^p)^q)^2*((b^2*h*x^
2)/2 - (e*(b^2*e*h - 2*b^2*f*g))/(2*f^2) + b^2*g*x) + (log(e + f*x)*(3*b^2*
e^2*h*p^2*q^2 - 2*a*b*e^2*h*p*q - 4*b^2*e*f*g*p^2*q^2 + 4*a*b*e*f*g*p*q))/(
2*f^2) + (h*x^2*(2*a^2 + b^2*p^2*q^2 - 2*a*b*p*q))/4
```

# 3.431 $\int (a+b\log(c(d(e+fx)^p)^q))^2\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 78

$$\int (a+b\log(c(d(e+fx)^p)^q))^2\,dx = -2abpqx+2b^2p^2q^2x-\frac{2b^2pq(e+fx)\log(c(d(e+fx)^p)^q)}{f} + \frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{f}$$

```
[Out] -2*a*b*p*q*x+2*b^2*p^2*q^2*x-2*b^2*p*q*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f+(f*x
+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2436, 2333, 2332, 2495}

$$\int (a+b\log(c(d(e+fx)^p)^q))^2\,dx = \frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{f}-2abpqx - \frac{2b^2pq(e+fx)\log(c(d(e+fx)^p)^q)}{f}+2b^2p^2q^2x$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] -2*a*b*p*q*x + 2*b^2*p^2*q^2*x - (2*b^2*p*q*(e + f*x)*Log[c*(d*(e + f*x)^p)
^q])/f + ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/f
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right)^2\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f}\\
&\quad - \text{Subst}\left(\frac{(2bpq)\text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right)\,dx, x, e+fx\right)}{f}, cd^q(e\right.\\
&\quad \left. + fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= -2abpqx + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f}\\
&\quad - \text{Subst}\left(\frac{(2b^2pq)\,\text{Subst}\left(\int \log\left(cd^q x^{pq}\right)\,dx, x, e+fx\right)}{f}, cd^q(e\right.\\
&\quad \left. + fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$= -2abpqx + 2b^2p^2q^2x - \frac{2b^2pq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\,dx = \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f} - 2bpq\left(ax - bpqx + \frac{b(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f}\right)$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/f - 2*b*p*q*(a*x - b*p*q*x +
 (b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/f)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(78) = 156.

Time = 0.42 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.12

| method | result |
|---|---|
| parallelrisch | $\frac{-2\ln(fx+e)b^2e^2p^2q^2+2x\,b^2ef\,p^2q^2-2x\ln(c(d(fx+e)^p)^q)b^2efpq+2\ln(fx+e)ab\,e^2pq+x\ln(c(d(fx+e)^p)^q)^2b^2ef-2xabefpq+}{ef}$ |

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-2*ln(f*x+e)*b^2*e^2*p^2*q^2+2*x*b^2*e*f*p^2*q^2-2*x*ln(c*(d*(f*x+e)^p)^q)
*b^2*e*f*p*q+2*ln(f*x+e)*a*b*e^2*p*q+x*ln(c*(d*(f*x+e)^p)^q)^2*b^2*e*f-2*x*
a*b*e*f*p*q+2*x*ln(c*(d*(f*x+e)^p)^q)*a*b*e*f+ln(c*(d*(f*x+e)^p)^q)^2*b^2*e
^2+e*a^2*f*x)/e/f
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(78) = 156.

Time = 0.29 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.96

$$\int (a+b\log(c(d(e+fx)^p)^q))^2\,dx$$
$$=\frac{b^2fq^2x\log(d)^2+b^2fx\log(c)^2+(b^2fp^2q^2x+b^2ep^2q^2)\log(fx+e)^2-2(b^2fpq-abf)x\log(c)+(2b^2fp^2q^2\cdots}{\cdots}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

```
[Out] (b^2*f*q^2*x*log(d)^2 + b^2*f*x*log(c)^2 + (b^2*f*p^2*q^2*x + b^2*e*p^2*q^2
)*log(f*x + e)^2 - 2*(b^2*f*p*q - a*b*f)*x*log(c) + (2*b^2*f*p^2*q^2 - 2*a*
b*f*p*q + a^2*f)*x - 2*(b^2*e*p^2*q^2 - a*b*e*p*q + (b^2*f*p^2*q^2 - a*b*f*
p*q)*x - (b^2*f*p*q*x + b^2*e*p*q)*log(c) - (b^2*f*p*q^2*x + b^2*e*p*q^2)*l
og(d))*log(f*x + e) + 2*(b^2*f*q*x*log(c) - (b^2*f*p*q^2 - a*b*f*q)*x)*log(
d))/f
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(76) = 152.

Time = 0.47 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.28

$$\int (a+b\log(c(d(e+fx)^p)^q))^2\,dx$$
$$=\begin{cases} a^2x+\frac{2abe\log(c(d(e+fx)^p)^q)}{f}-2abpqx+2abx\log(c(d(e+fx)^p)^q)-\frac{2b^2epq\log(c(d(e+fx)^p)^q)}{f}+\frac{b^2e\log(c(d(e+fx)^p)^q)}{f}\cdots \\ x(a+b\log(c(de^p)^q))^2 \end{cases}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*e*log(c*(d*(e + f*x)**p)**q)/f - 2*a*b*p*q*x + 2*
a*b*x*log(c*(d*(e + f*x)**p)**q) - 2*b**2*e*p*q*log(c*(d*(e + f*x)**p)**q)/
f + b**2*e*log(c*(d*(e + f*x)**p)**q)**2/f + 2*b**2*p**2*q**2*x - 2*b**2*p*
q*x*log(c*(d*(e + f*x)**p)**q) + b**2*x*log(c*(d*(e + f*x)**p)**q)**2, Ne(f
, 0)), (x*(a + b*log(c*(d*e**p)**q))**2, True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int (a+b\log (c(d(e+fx)^p)^q))^2 \, dx$$
$$= -2\,abfpq\left(\frac{x}{f}-\frac{e\log (fx+e)}{f^2}\right)+b^2x\log \left(\left(\left((fx+e)^pd\right)^q c\right)^2+2\,abx\log \left(\left((fx+e)^pd\right)^q c\right)$$
$$-\left(2\,fpq\left(\frac{x}{f}-\frac{e\log (fx+e)}{f^2}\right)\log \left(\left((fx+e)^pd\right)^q c\right)+\frac{(e\log (fx+e)^2-2\,fx+2\,e\log (fx+e))p^2q^2}{f}\right)b$$
$$+a^2x$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*f*p*q*(x/f - e*log(f*x + e)/f^2) + b^2*x*log(((f*x + e)^p*d)^q*c)^2
+ 2*a*b*x*log(((f*x + e)^p*d)^q*c) - (2*f*p*q*(x/f - e*log(f*x + e)/f^2)*lo
g(((f*x + e)^p*d)^q*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p^2*
q^2/f)*b^2 + a^2*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(78) = 156.

Time = 0.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.63

$$\int (a+b\log (c(d(e+fx)^p)^q))^2 \, dx = \frac{(fx+e)b^2p^2q^2\log (fx+e)^2}{f}$$
$$-\frac{2\,(fx+e)b^2p^2q^2\log (fx+e)}{f}$$
$$+\frac{2\,(fx+e)b^2pq^2\log (fx+e)\log (d)}{f}$$
$$+\frac{2\,(fx+e)b^2p^2q^2}{f}+\frac{2\,(fx+e)b^2pq\log (fx+e)\log (c)}{f}$$
$$-\frac{2\,(fx+e)b^2pq^2\log (d)}{f}+\frac{(fx+e)b^2q^2\log (d)^2}{f}$$
$$+\frac{2\,(fx+e)abpq\log (fx+e)}{f}-\frac{2\,(fx+e)b^2pq\log (c)}{f}$$
$$+\frac{2\,(fx+e)b^2q\log (c)\log (d)}{f}-\frac{2\,(fx+e)abpq}{f}$$
$$+\frac{(fx+e)b^2\log (c)^2}{f}+\frac{2\,(fx+e)abq\log (d)}{f}$$
$$+\frac{2\,(fx+e)ab\log (c)}{f}+\frac{(fx+e)a^2}{f}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

```
[Out] (f*x + e)*b^2*p^2*q^2*log(f*x + e)^2/f - 2*(f*x + e)*b^2*p^2*q^2*log(f*x +
e)/f + 2*(f*x + e)*b^2*p*q^2*log(f*x + e)*log(d)/f + 2*(f*x + e)*b^2*p^2*q^
2/f + 2*(f*x + e)*b^2*p*q*log(f*x + e)*log(c)/f - 2*(f*x + e)*b^2*p*q^2*log
(d)/f + (f*x + e)*b^2*q^2*log(d)^2/f + 2*(f*x + e)*a*b*p*q*log(f*x + e)/f -
 2*(f*x + e)*b^2*p*q*log(c)/f + 2*(f*x + e)*b^2*q*log(c)*log(d)/f - 2*(f*x
+ e)*a*b*p*q/f + (f*x + e)*b^2*log(c)^2/f + 2*(f*x + e)*a*b*q*log(d)/f + 2*
(f*x + e)*a*b*log(c)/f + (f*x + e)*a^2/f
```

## Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int (a+b\log(c(d(e+fx)^p)^q))^2\,dx = \ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)^2\left(b^2\,x+\frac{b^2\,e}{f}\right) + x\left(a^2-2\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right) - \frac{\ln\left(e+f\,x\right)\,\left(2\,b^2\,e\,p^2\,q^2-2\,a\,b\,e\,p\,q\right)}{f} + 2\,b\,x\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\,\left(a-b\,p\,q\right)$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
```

```
[Out] log(c*(d*(e + f*x)^p)^q)^2*(b^2*x + (b^2*e)/f) + x*(a^2 + 2*b^2*p^2*q^2 - 2
*a*b*p*q) - (log(e + f*x)*(2*b^2*e*p^2*q^2 - 2*a*b*e*p*q))/f + 2*b*x*log(c*
(d*(e + f*x)^p)^q)*(a - b*p*q)
```

# 3.432 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 123

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{2b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

```
[Out] (a+b*ln(c*(d*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/h+2*b*p*q*(a+b*ln(c*
(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h-2*b^2*p^2*q^2*polylog(
3,-h*(f*x+e)/(-e*h+f*g))/h
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used

$= \{2443, 2481, 2421, 6724, 2495\}$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \frac{2bpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h} + \frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{h} - \frac{2b^2p^2q^2\,\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x),x]
```

```
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(g + h*x))/(f*g - e*h)])/h + (2*
b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, -((h*(e + f*x))/(f*g - e*
h))])/h - (2*b^2*p^2*q^2*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))])/h
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
```

```
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} - \text{Subst}\left(\frac{(2bfpq)\int\frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} - \text{Subst}\left(\frac{(2bpq)\text{Subst}\left(\int\frac{\left(a+b\log\left(cd^qx^{pq}\right)\right)\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\,\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \text{Subst}\left(\frac{(2b^2p^2q^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\,\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{2b^2p^2q^2\text{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 324 vs. $2(123) = 246$.

Time = 0.14 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.63

$$\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$$

$$= \frac{a^2\log(g+hx) - 2abpq\log(e+fx)\log(g+hx) + b^2p^2q^2\log^2(e+fx)\log(g+hx) + 2ab\log(c(d(e+fx)^p)^{q}}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x),x]
```

```
[Out] (a^2*Log[g + h*x] - 2*a*b*p*q*Log[e + f*x]*Log[g + h*x] + b^2*p^2*q^2*Log[e
 + f*x]^2*Log[g + h*x] + 2*a*b*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 2*b^
2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + b^2*Log[c*(d*(e
+ f*x)^p)^q]^2*Log[g + h*x] + 2*a*b*p*q*Log[e + f*x]*Log[(f*(g + h*x))/(f*g
 - e*h)] - b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 2*b^
2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)]
+ 2*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, (h*(e + f*x))/(-(f*g)
 + e*h)] - 2*b^2*p^2*q^2*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)])/h
```

## Maple [F]

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^2}{hx+g}dx$$

[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)

[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)

## Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{hx+g}\,dx$$

[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="fricas")

[Out] integral((b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c) + a^2)/(h*x + g), x)

## Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$$

[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g),x)

[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/(g + h*x), x)

## Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{hx+g}\,dx$$

[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="maxima")

[Out] a^2*log(h*x + g)/h + integrate((b^2*log(((f*x + e)^p)^q)^2 + 2*(q*log(d) + log(c))*a*b + (q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*b^2 + 2*((q*log(d) + log(c))*b^2 + a*b)*log(((f*x + e)^p)^q))/(h*x + g), x)

## Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^pd)^qc)+a)^2}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \int \frac{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^2}{g+h\,x}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x), x)
```

# 3.433 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 144

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx = \frac{(e+fx)\,(a+b\log(c(d(e+fx)^p)^q))^2}{(fg-eh)(g+hx)} - \frac{2bfpq(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} - \frac{2b^2fp^2q^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)}$$

```
[Out] (f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(-e*h+f*g)/(h*x+g)-2*b*f*p*q*(a+b*ln(
c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/h/(-e*h+f*g)-2*b^2*f*p^2*q^2*p
olylog(2,-h*(f*x+e)/(-e*h+f*g))/h/(-e*h+f*g)
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2444, 2441, 2440, 2438, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx = -\frac{2bfpq\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h(fg-eh)} + \frac{(e+fx)\,(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(fg-eh)} - \frac{2b^2fp^2q^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^2,x]
```

```
[Out] ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/((f*g - e*h)*(g + h*x)) - (2
*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/(
h*(f*g - e*h)) - (2*b^2*f*p^2*q^2*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])
/(h*(f*g - e*h))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{(g+hx)^2}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(fg-eh)(g+hx)} - \text{Subst}\left(\frac{(2bfpq)\int\frac{a+b\log(cd^q(e+fx)^{pq})}{g+hx}\,dx}{fg-eh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(fg-eh)(g+hx)} - \frac{2bfpq(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} + \text{Subst}\left(\frac{(2b^2f^2p^2q^2)\int\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(fg-eh)(g+hx)} - \frac{2bfpq(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} + \text{Subst}\left(\frac{(2b^2fp^2q^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{hx}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(fg-eh)(g+hx)} - \frac{2bfpq(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} - \frac{2b^2fp^2q^2\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)}$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.39

$$\int\frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)^2}\,dx = \frac{b^2fp^2q^2(g+hx)\log^2(e+fx) - 2bfpq(g+hx)\log(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right) + \left(a+b\log\left(c(d(e\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^2,x]
```

```
[Out] (b^2*f*p^2*q^2*(g + h*x)*Log[e + f*x]^2 - 2*b*f*p*q*(g + h*x)*Log[e + f*x]*
(a + b*Log[c*(d*(e + f*x)^p)^q]) + (a + b*Log[c*(d*(e + f*x)^p)^q])*(a*(f*g
 - e*h) + b*(f*g - e*h)*Log[c*(d*(e + f*x)^p)^q] + 2*b*f*p*q*(g + h*x)*Log[
(f*(g + h*x))/(f*g - e*h)]) + 2*b^2*f*p^2*q^2*(g + h*x)*PolyLog[2, (h*(e +
f*x))/(-(f*g) + e*h)])/(h*(-(f*g) + e*h)*(g + h*x))
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c(d(fx+e)^{p})^{q}\right)\right)^{2}}{\left(hx+g\right)^{2}}dx$$

[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^2,x)

[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^2,x)

## Fricas [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^{p})^{q}\right))^{2}}{(g+hx)^{2}}\,dx=\int \frac{\left(b\log\left(\left((fx+e)^{p}d\right)^{q}c\right)+a\right)^{2}}{\left(hx+g\right)^{2}}\,dx$$

[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^2,x, algorithm="fricas")

[Out] integral((b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c) + a^2)/(h^2*x^2 + 2*g*h*x + g^2), x)

## Sympy [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^{p})^{q}\right))^{2}}{(g+hx)^{2}}\,dx=\int \frac{\left(a+b\log\left(c(d(e+fx)^{p})^{q}\right)\right)^{2}}{\left(g+hx\right)^{2}}\,dx$$

[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**2,x)

[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/(g + h*x)**2, x)

## Maxima [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^{p})^{q}\right))^{2}}{(g+hx)^{2}}\,dx=\int \frac{\left(b\log\left(\left((fx+e)^{p}d\right)^{q}c\right)+a\right)^{2}}{\left(hx+g\right)^{2}}\,dx$$

[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^2,x, algorithm="maxima")

[Out] 2*a*b*f*p*q*(log(f*x + e)/(f*g*h - e*h^2) - log(h*x + g)/(f*g*h - e*h^2)) - b^2*(log(((f*x + e)^p)^q)^2/(h^2*x + g*h) - integrate((e*h*q^2*log(d)^2 + 2*e*h*q*log(c)*log(d) + e*h*log(c)^2 + (f*h*q^2*log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*x + 2*(f*g*p*q + e*h*q*log(d) + e*h*log(c) + (f*h*p*q + f*h*q*log(d) + f*h*log(c))*x)*log(((f*x + e)^p)^q))/(f*h^3*x^3 + e*g^2*h + (2*f*g*h^2 + e*h^3)*x^2 + (f*g^2*h + 2*e*g*h^2)*x), x)) - 2*a*b*log(((f*x + e)^p*d)^q*c)/(h^2*x + g*h) - a^2/(h^2*x + g*h)

## Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^2,x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx = \int \frac{(a+b\ln(c(d(e+fx)^p)^q))^2}{(g+hx)^2}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^2,x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^2, x)
```

# 3.434 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 222

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx = -\frac{bfpq(e+fx)(a+b\log(c(d(e+fx)^p)^q))}{(fg-eh)^2(g+hx)} - \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{2h(g+hx)^2} + \frac{b^2f^2p^2q^2\log(g+hx)}{h(fg-eh)^2} - \frac{bf^2pq(a+b\log(c(d(e+fx)^p)^q))\log\left(1+\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2} + \frac{b^2f^2p^2q^2\,\mathrm{PolyLog}\left(2,-\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2}$$

```
[Out] -b*f*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))/(-e*h+f*g)^2/(h*x+g)-1/2*(a+b*
ln(c*(d*(f*x+e)^p)^q))^2/h/(h*x+g)^2+b^2*f^2*p^2*q^2*ln(h*x+g)/h/(-e*h+f*g)
^2-b*f^2*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(1+(-e*h+f*g)/h/(f*x+e))/h/(-e*h
+f*g)^2+b^2*f^2*p^2*q^2*polylog(2,(e*h-f*g)/h/(f*x+e))/h/(-e*h+f*g)^2
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{2445, 2458, 2389, 2379, 2438, 2351, 31, 2495\}$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx = -\frac{bf^2pq\log\left(\frac{fg-eh}{h(e+fx)}+1\right)(a+b\log(c(d(e+fx)^p)^q))}{h(fg-eh)^2} - \frac{bfpq(e+fx)(a+b\log(c(d(e+fx)^p)^q))}{(g+hx)(fg-eh)^2} - \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{2h(g+hx)^2} + \frac{b^2f^2p^2q^2\,\mathrm{PolyLog}\left(2,-\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2} + \frac{b^2f^2p^2q^2\log(g+hx)}{h(fg-eh)^2}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^3,x]
```

```
[Out] -((b*f*p*q*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/((f*g - e*h)^2*(g +
h*x))) - (a + b*Log[c*(d*(e + f*x)^p)^q])^2/(2*h*(g + h*x)^2) + (b^2*f^2*p^
2*q^2*Log[g + h*x])/(h*(f*g - e*h)^2) - (b*f^2*p*q*(a + b*Log[c*(d*(e + f*x
)^p)^q])*Log[1 + (f*g - e*h)/(h*(e + f*x))])/(h*(f*g - e*h)^2) + (b^2*f^2*p
^2*q^2*PolyLog[2, -((f*g - e*h)/(h*(e + f*x)))])/(h*(f*g - e*h)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
```

```
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{(g+hx)^3}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2h(g+hx)^2} + \text{Subst}\left(\frac{(bfpq)\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)(g+hx)^2}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{2h(g+hx)^2} + \text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int \frac{a+b\log(cd^q x^{pq})}{x\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^2}\,dx, x, e+fx\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{2h(g+hx)^2} - \text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int \frac{a+b\log(cd^q x^{pq})}{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^2}\,dx, x, e+fx\right)}{fg-eh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(bfpq)\text{Subst}\left(\int \frac{a+b\log(cd^q x^{pq})}{x\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}\,dx, x, e+fx\right)}{h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{bfpq(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{(fg-eh)^2(g+hx)} - \frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{2h(g+hx)^2} - \frac{bf^2pq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\log\left(1+\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2} + \text{Subst}\left(\frac{(b^2fp^2q^2)\,\text{Subst}\left(\int \frac{1}{\frac{fg-eh}{f}+\frac{hx}{f}}\,dx, x, e+fx\right)}{(fg-eh)^2}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right) + \text{Subst}\left(\frac{(b^2f^2p^2q^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{fg-eh}{hx}\right)}{x}\,dx, x, e+fx\right)}{h(fg-eh)^2}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= -\frac{bfpq(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{(fg-eh)^2(g+hx)} - \frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{2h(g+hx)^2} + \frac{b^2f^2p^2q^2\log(g+hx)}{h(fg-eh)^2} - \frac{bf^2pq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\log\left(1+\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2} + \frac{b^2f^2p^2q^2\text{Li}_2\left(-\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2}$$

## Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.42

$$\int \frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{(g+hx)^3}\,dx = -\frac{\left(a-bpq\log(e+fx)+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2+\frac{2bpq\left(a-bpq\log(e+fx)+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\left(h(e+fx)(eh-f(2g+hx))\log\right.}{(fg-eh)^2}}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^3,x]
```

```
[Out] -1/2*((a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])^2 + (2*b*p*q*(a
 - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])*(h*(e + f*x)*(e*h - f*(
```

```
2*g + h*x))*Log[e + f*x] + f*(g + h*x)*(h*(e + f*x) + f*(g + h*x)*Log[(f*(g
 + h*x))/(f*g - e*h)])))/(f*g - e*h)^2 + (b^2*p^2*q^2*(h*(e + f*x)*(e*h - f
*(2*g + h*x))*Log[e + f*x]^2 - 2*f^2*(g + h*x)^2*Log[(f*(g + h*x))/(f*g - e
*h)] + 2*f*(g + h*x)*Log[e + f*x]*(h*(e + f*x) + f*(g + h*x)*Log[(f*(g + h*
x))/(f*g - e*h)]) + 2*f^2*(g + h*x)^2*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*
h)]))/(f*g - e*h)^2)/(h*(g + h*x)^2)
```

## Maple [F]

$$\int \frac{(a + b\ln(c(d(fx+e)^p)^q))^2}{(hx+g)^3}dx$$

[In] `int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^3,x)`

[Out] `int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^3,x)`

## Fricas [F]

$$\int \frac{(a + b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c) + a)^2}{(hx+g)^3}\,dx$$

[In] `integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^3,x, algorithm="fricas")`

[Out] `integral((b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c) + a^2)/(h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3), x)`

## Sympy [F]

$$\int \frac{(a + b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx = \int \frac{(a + b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^3}\,dx$$

[In] `integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**3,x)`

[Out] `Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/(g + h*x)**3, x)`

## Maxima [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{(g+hx)^3}\,dx = \int \frac{\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)^2}{\left(hx+g\right)^3}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] a*b*f*p*q*(f*log(f*x + e)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) - f*log(h*x +
 g)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) + 1/(f*g^2*h - e*g*h^2 + (f*g*h^2 -
 e*h^3)*x)) - 1/2*b^2*(log(((f*x + e)^p)^q)^2/(h^3*x^2 + 2*g*h^2*x + g^2*h)
 - 2*integrate((e*h*q^2*log(d)^2 + 2*e*h*q*log(c)*log(d) + e*h*log(c)^2 + (
f*h*q^2*log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*x + (f*g*p*q + 2*e
*h*q*log(d) + 2*e*h*log(c) + (f*h*p*q + 2*f*h*q*log(d) + 2*f*h*log(c))*x)*l
og(((f*x + e)^p)^q))/(f*h^4*x^4 + e*g^3*h + (3*f*g*h^3 + e*h^4)*x^3 + 3*(f*
g^2*h^2 + e*g*h^3)*x^2 + (f*g^3*h + 3*e*g^2*h^2)*x), x)) - a*b*log(((f*x +
e)^p*d)^q*c)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*a^2/(h^3*x^2 + 2*g*h^2*x +
 g^2*h)
```

## Giac [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{(g+hx)^3}\,dx = \int \frac{\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)^2}{\left(hx+g\right)^3}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{(g+hx)^3}\,dx = \int \frac{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^2}{\left(g+h\,x\right)^3}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^3,x)
```

```
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^3, x)
```

# 3.435 $\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^3 \, dx$



## Optimal result

Integrand size = 28, antiderivative size = 492

$$\begin{aligned}
&\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^3 \, dx \\
&= \frac{6ab^2(fg-eh)^2p^2q^2x}{f^2} - \frac{6b^3(fg-eh)^2p^3q^3x}{f^2} - \frac{3b^3h(fg-eh)p^3q^3(e+fx)^2}{4f^3} \\
&\quad - \frac{2b^3h^2p^3q^3(e+fx)^3}{27f^3} + \frac{6b^3(fg-eh)^2p^2q^2(e+fx)\log(c(d(e+fx)^p)^q)}{f^3} \\
&\quad + \frac{3b^2h(fg-eh)p^2q^2(e+fx)^2(a+b\log(c(d(e+fx)^p)^q))}{2f^3} \\
&\quad + \frac{2b^2h^2p^2q^2(e+fx)^3(a+b\log(c(d(e+fx)^p)^q))}{9f^3} \\
&\quad - \frac{3b(fg-eh)^2pq(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{f^3} \\
&\quad - \frac{3bh(fg-eh)pq(e+fx)^2(a+b\log(c(d(e+fx)^p)^q))^2}{2f^3} \\
&\quad - \frac{bh^2pq(e+fx)^3(a+b\log(c(d(e+fx)^p)^q))^2}{3f^3} \\
&\quad + \frac{(fg-eh)^2(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3}{f^3} \\
&\quad + \frac{h(fg-eh)(e+fx)^2(a+b\log(c(d(e+fx)^p)^q))^3}{f^3} \\
&\quad + \frac{h^2(e+fx)^3(a+b\log(c(d(e+fx)^p)^q))^3}{3f^3}
\end{aligned}$$

```
[Out] 6*a*b^2*(-e*h+f*g)^2*p^2*q^2*x/f^2-6*b^3*(-e*h+f*g)^2*p^3*q^3*x/f^2-3/4*b^3
*h*(-e*h+f*g)*p^3*q^3*(f*x+e)^2/f^3-2/27*b^3*h^2*p^3*q^3*(f*x+e)^3/f^3+6*b^
```

```
3*(-e*h+f*g)^2*p^2*q^2*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f^3+3/2*b^2*h*(-e*h+f*
g)*p^2*q^2*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^3+2/9*b^2*h^2*p^2*q^2*(f
*x+e)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^3-3*b*(-e*h+f*g)^2*p*q*(f*x+e)*(a+b*l
n(c*(d*(f*x+e)^p)^q))^2/f^3-3/2*b*h*(-e*h+f*g)*p*q*(f*x+e)^2*(a+b*ln(c*(d*(
f*x+e)^p)^q))^2/f^3-1/3*b*h^2*p*q*(f*x+e)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f
^3+(-e*h+f*g)^2*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f^3+h*(-e*h+f*g)*(f*x
+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f^3+1/3*h^2*(f*x+e)^3*(a+b*ln(c*(d*(f*x
+e)^p)^q))^3/f^3
```

## Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2448, 2436, 2333, 2332, 2437, 2342, 2341, 2495}

$$\begin{aligned}
&\int (g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx \\
&= \frac{3b^2hp^2q^2(e+fx)^2(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^3} \\
&\quad+\frac{2b^2h^2p^2q^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9f^3}+\frac{6ab^2p^2q^2x(fg-eh)^2}{f^2} \\
&\quad-\frac{3bhpq(e+fx)^2(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^3} \\
&\quad-\frac{3bpq(e+fx)(fg-eh)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^3} \\
&\quad+\frac{h(e+fx)^2(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3} \\
&\quad+\frac{(e+fx)(fg-eh)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3} \\
&\quad-\frac{bh^2pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3f^3} \\
&\quad+\frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{3f^3} \\
&\quad+\frac{6b^3p^2q^2(e+fx)(fg-eh)^2\log\left(c(d(e+fx)^p)^q\right)}{f^3} \\
&\quad-\frac{3b^3hp^3q^3(e+fx)^2(fg-eh)}{4f^3}-\frac{2b^3h^2p^3q^3(e+fx)^3}{27f^3}-\frac{6b^3p^3q^3x(fg-eh)^2}{f^2}
\end{aligned}$$

```
[In] Int[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

```
[Out] (6*a*b^2*(f*g - e*h)^2*p^2*q^2*x)/f^2 - (6*b^3*(f*g - e*h)^2*p^3*q^3*x)/f^2
 - (3*b^3*h*(f*g - e*h)*p^3*q^3*(e + f*x)^2)/(4*f^3) - (2*b^3*h^2*p^3*q^3*(
e + f*x)^3)/(27*f^3) + (6*b^3*(f*g - e*h)^2*p^2*q^2*(e + f*x)*Log[c*(d*(e +
```

```
f*x)^p)^q])/f^3 + (3*b^2*h*(f*g - e*h)*p^2*q^2*(e + f*x)^2*(a + b*Log[c*(d
*(e + f*x)^p)^q]))/(2*f^3) + (2*b^2*h^2*p^2*q^2*(e + f*x)^3*(a + b*Log[c*(d
*(e + f*x)^p)^q]))/(9*f^3) - (3*b*(f*g - e*h)^2*p*q*(e + f*x)*(a + b*Log[c*
(d*(e + f*x)^p)^q])^2)/f^3 - (3*b*h*(f*g - e*h)*p*q*(e + f*x)^2*(a + b*Log[
c*(d*(e + f*x)^p)^q])^2)/(2*f^3) - (b*h^2*p*q*(e + f*x)^3*(a + b*Log[c*(d*(
e + f*x)^p)^q])^2)/(3*f^3) + ((f*g - e*h)^2*(e + f*x)*(a + b*Log[c*(d*(e +
f*x)^p)^q])^3)/f^3 + (h*(f*g - e*h)*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p
)^q])^3)/f^3 + (h^2*(e + f*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(3*f^3)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int (g+hx)^2 \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\int \left(\frac{(fg-eh)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{f^2} + \frac{2h(fg-eh)(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{f^2} + \frac{h^2(e+fx)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{f^2}\right)dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{h^2\int (e+fx)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3\,dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad + \text{Subst}\left(\frac{(2h(fg-eh))\int (e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3\,dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad + \text{Subst}\left(\frac{(fg-eh)^2\int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3\,dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$= \text{Subst}\left(\frac{h^2\text{Subst}\left(\int x^2(a+b\log\left(cd^qx^{pq}\right))^3\,dx,x,e+fx\right)}{f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(2h(fg-eh))\text{Subst}\left(\int x(a+b\log\left(cd^qx^{pq}\right))^3\,dx,x,e+fx\right)}{f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(fg-eh)^2\text{Subst}\left(\int (a+b\log\left(cd^qx^{pq}\right))^3\,dx,x,e+fx\right)}{f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(fg-eh)^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3} + \frac{h(fg-eh)(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3} + \frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{3f^3} - \text{Subst}\left(\frac{(bh^2pq)\,\text{Subst}\left(\int x^2(a+b\log\left(cd^qx^{pq}\right))^2\,dx,x,e+fx\right)}{f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(3bh(fg-eh)pq)\text{Subst}\left(\int x(a+b\log\left(cd^qx^{pq}\right))^2\,dx,x,e+fx\right)}{f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(3b(fg-eh)^2pq)\,\text{Subst}\left(\int (a+b\log\left(cd^qx^{pq}\right))^2\,dx,x,e+fx\right)}{f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$
\begin{aligned}
&= -\frac{3b(fg-eh)^2pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^3} \\
&\quad -\frac{3bh(fg-eh)pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^3} \\
&\quad -\frac{bh^2pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3f^3} \\
&\quad +\frac{(fg-eh)^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3} \\
&\quad +\frac{h(fg-eh)(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3} \\
&\quad +\frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{3f^3} \\
&\quad +\text{Subst}\left(\frac{(2b^2h^2p^2q^2)\,\text{Subst}\left(\int x^2(a+b\log(cd^qx^{pq}))\,dx,x,e+fx\right)}{3f^3},cd^q(e\right. \\
&\qquad \left. +fx)^{pq},c(d(e+fx)^p)^q\right) \\
&\quad +\text{Subst}\left(\frac{(3b^2h(fg-eh)p^2q^2)\,\text{Subst}\left(\int x(a+b\log(cd^qx^{pq}))\,dx,x,e+fx\right)}{f^3},cd^q(e\right. \\
&\qquad \left. +fx)^{pq},c(d(e+fx)^p)^q\right) \\
&\quad +\text{Subst}\left(\frac{(6b^2(fg-eh)^2p^2q^2)\,\text{Subst}\left(\int (a+b\log(cd^qx^{pq}))\,dx,x,e+fx\right)}{f^3},cd^q(e\right. \\
&\qquad \left. +fx)^{pq},c(d(e+fx)^p)^q\right)
\end{aligned}
$$

$$= \frac{6ab^2(fg-eh)^2p^2q^2x}{f^2} - \frac{3b^3h(fg-eh)p^3q^3(e+fx)^2}{4f^3} - \frac{2b^3h^2p^3q^3(e+fx)^3}{27f^3}$$
$$+ \frac{3b^2h(fg-eh)p^2q^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^3}$$
$$+ \frac{2b^2h^2p^2q^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9f^3}$$
$$- \frac{3b(fg-eh)^2pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^3}$$
$$- \frac{3bh(fg-eh)pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^3}$$
$$- \frac{bh^2pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3f^3}$$
$$+ \frac{(fg-eh)^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3}$$
$$+ \frac{h(fg-eh)(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3}$$
$$+ \frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{3f^3}$$
$$+ \text{Subst}\left(\frac{\left(6b^3(fg-eh)^2p^2q^2\right)\text{Subst}\left(\int\log\left(cd^qx^{pq}\right)\,dx,x,e+fx\right)}{f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{6ab^2(fg-eh)^2p^2q^2x}{f^2} - \frac{6b^3(fg-eh)^2p^3q^3x}{f^2} - \frac{3b^3h(fg-eh)p^3q^3(e+fx)^2}{4f^3} - \frac{2b^3h^2p^3q^3(e+fx)^3}{27f^3} + \frac{6b^3(fg-eh)^2p^2q^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f^3} + \frac{3b^2h(fg-eh)p^2q^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^3} + \frac{2b^2h^2p^2q^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9f^3} - \frac{3b(fg-eh)^2pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^3} - \frac{3bh(fg-eh)pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^3} - \frac{bh^2pq(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3f^3} + \frac{(fg-eh)^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3} + \frac{h(fg-eh)(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^3} + \frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{3f^3}$$

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.77

$$\int (g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx = \frac{108(fg-eh)^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3+108h(fg-eh)(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{}$$

```
[In] Integrate[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

```
[Out] (108*(f*g - e*h)^2*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3 + 108*h*(f*
g - e*h)*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3 + 36*h^2*(e + f*x)^
3*(a + b*Log[c*(d*(e + f*x)^p)^q])^3 - 324*b*(f*g - e*h)^2*p*q*((e + f*x)*(
a + b*Log[c*(d*(e + f*x)^p)^q])^2 - 2*b*p*q*(f*(a - b*p*q)*x + b*(e + f*x)*
Log[c*(d*(e + f*x)^p)^q])) - 81*b*h*(f*g - e*h)*p*q*(2*(e + f*x)^2*(a + b*L
og[c*(d*(e + f*x)^p)^q])^2 + b*p*q*(b*f*p*q*x*(2*e + f*x) - 2*(e + f*x)^2*(
a + b*Log[c*(d*(e + f*x)^p)^q]))) - 4*b*h^2*p*q*(9*(e + f*x)^3*(a + b*Log[c
*(d*(e + f*x)^p)^q])^2 + 2*b*p*q*(b*f*p*q*x*(3*e^2 + 3*e*f*x + f^2*x^2) - 3
*(e + f*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q]))))/(108*f^3)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2033 vs. 2(478) = 956.

Time = 16.19 (sec) , antiderivative size = 2034, normalized size of antiderivative = 4.13

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 2034 |

```
[In] int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/108*(648*x*ln(c*(d*(f*x+e)^p)^q)*a*b^2*e*f^2*g*h*p*q-972*x*ln(c*(d*(f*x+e
)^p)^q)*b^3*e*f^2*g*h*p^2*q^2+108*x^2*ln(c*(d*(f*x+e)^p)^q)*a*b^2*e*f^2*h^2
*p*q-2106*ln(f*x+e)*b^3*e^2*f*g*h*p^3*q^3-324*x^2*ln(c*(d*(f*x+e)^p)^q)*a*b
^2*f^3*g*h*p*q+324*x*ln(c*(d*(f*x+e)^p)^q)^2*b^3*e*f^2*g*h*p*q-972*x*a*b^2*
e*f^2*g*h*p^2*q^2-216*x*ln(c*(d*(f*x+e)^p)^q)*a*b^2*e^2*f*h^2*p*q+324*x*a^2
*b*e*f^2*g*h*p*q-648*ln(c*(d*(f*x+e)^p)^q)*a*b^2*e^2*f*g*h*p*q+1620*ln(f*x+
e)*a*b^2*e^2*f*g*h*p^2*q^2-324*ln(f*x+e)*a^2*b*e^2*f*g*h*p*q+972*a*b^2*e^2*
f*g*h*p^2*q^2-324*a^2*b*e^2*f*g*h*p*q-108*x*a^2*b*e^2*f*h^2*p*q+648*ln(c*(d
*(f*x+e)^p)^q)*a*b^2*e*f^2*g^2*p*q-90*x^2*ln(c*(d*(f*x+e)^p)^q)*b^3*e*f^2*h
^2*p^2*q^2+162*x^2*ln(c*(d*(f*x+e)^p)^q)*b^3*f^3*g*h*p^2*q^2+1134*x*b^3*e*f
^2*g*h*p^3*q^3-72*x^3*ln(c*(d*(f*x+e)^p)^q)*a*b^2*f^3*h^2*p*q+54*x^2*ln(c*(
d*(f*x+e)^p)^q)^2*b^3*e*f^2*h^2*p*q-1296*ln(f*x+e)*a*b^2*e*f^2*g^2*p^2*q^2+
648*ln(f*x+e)*a^2*b*e*f^2*g^2*p*q-162*x^2*ln(c*(d*(f*x+e)^p)^q)^2*b^3*f^3*g
*h*p*q-90*x^2*a*b^2*e*f^2*h^2*p^2*q^2+162*x^2*a*b^2*f^3*g*h*p^2*q^2+396*x*l
n(c*(d*(f*x+e)^p)^q)*b^3*e^2*f*h^2*p^2*q^2-108*x*ln(c*(d*(f*x+e)^p)^q)^2*b^
3*e^2*f*h^2*p*q+396*x*a*b^2*e^2*f*h^2*p^2*q^2+972*ln(c*(d*(f*x+e)^p)^q)*b^3
*e^2*f*g*h*p^2*q^2+54*x^2*a^2*b*e*f^2*h^2*p*q-162*x^2*a^2*b*f^3*g*h*p*q-648
*x*ln(c*(d*(f*x+e)^p)^q)*a*b^2*f^3*g^2*p*q+486*ln(c*(d*(f*x+e)^p)^q)^2*b^3*
e^2*f*g*h*p*q+324*a^2*b*e*f^2*g^2*p*q-1134*b^3*e^2*f*g*h*p^3*q^3-648*a*b^2*
e*f^2*g^2*p^2*q^2-81*x^2*b^3*f^3*g*h*p^3*q^3-36*x^3*ln(c*(d*(f*x+e)^p)^q)^2
*b^3*f^3*h^2*p*q+24*x^3*a*b^2*f^3*h^2*p^2*q^2-510*x*b^3*e^2*f*h^2*p^3*q^3+6
48*x*ln(c*(d*(f*x+e)^p)^q)*b^3*f^3*g^2*p^2*q^2-36*x^3*a^2*b*f^3*h^2*p*q-324
*x*ln(c*(d*(f*x+e)^p)^q)^2*b^3*f^3*g^2*p*q+648*x*a*b^2*f^3*g^2*p^2*q^2-648*
ln(c*(d*(f*x+e)^p)^q)*b^3*e*f^2*g^2*p^2*q^2+324*x^2*ln(c*(d*(f*x+e)^p)^q)^2
*a*b^2*f^3*g*h-324*ln(c*(d*(f*x+e)^p)^q)^2*b^3*e*f^2*g^2*p*q+324*x^2*ln(c*(
d*(f*x+e)^p)^q)*a^2*b*f^3*g*h-324*x*a^2*b*f^3*g^2*p*q+216*ln(c*(d*(f*x+e)^p
)^q)*a*b^2*e^3*h^2*p*q-324*ln(c*(d*(f*x+e)^p)^q)^2*a*b^2*e^2*f*g*h+1296*ln(
f*x+e)*b^3*e*f^2*g^2*p^3*q^3-612*ln(f*x+e)*a*b^2*e^3*h^2*p^2*q^2+108*ln(f*x
+e)*a^2*b*e^3*h^2*p*q+57*x^2*b^3*e*f^2*h^2*p^3*q^3+24*x^3*ln(c*(d*(f*x+e)^p
)^q)*b^3*f^3*h^2*p^2*q^2+510*b^3*e^3*h^2*p^3*q^3-108*a^3*e*f^2*g^2+36*x^3*l
n(c*(d*(f*x+e)^p)^q)^3*b^3*f^3*h^2+108*x*ln(c*(d*(f*x+e)^p)^q)^3*b^3*f^3*g^
2+108*ln(c*(d*(f*x+e)^p)^q)^3*b^3*e*f^2*g^2+108*x^2*a^3*f^3*g*h+108*ln(c*(d
*(f*x+e)^p)^q)^2*a*b^2*e^3*h^2+648*b^3*e*f^2*g^2*p^3*q^3-324*ln(c*(d*(f*x+e
)^p)^q)*a^2*b*e*f^2*g^2+906*ln(f*x+e)*b^3*e^3*h^2*p^3*q^3-8*x^3*b^3*f^3*h^2
*p^3*q^3-648*x*b^3*f^3*g^2*p^3*q^3+108*x^3*ln(c*(d*(f*x+e)^p)^q)^2*a*b^2*f^
```

```
3*h^2+108*x^2*ln(c*(d*(f*x+e)^p)^q)^3*b^3*f^3*g*h-396*ln(c*(d*(f*x+e)^p)^q)
*b^3*e^3*h^2*p^2*q^2+108*x^3*ln(c*(d*(f*x+e)^p)^q)*a^2*b*f^3*h^2-198*ln(c*(
d*(f*x+e)^p)^q)^2*b^3*e^3*h^2*p*q+324*x*ln(c*(d*(f*x+e)^p)^q)^2*a*b^2*f^3*g
^2-108*ln(c*(d*(f*x+e)^p)^q)^3*b^3*e^2*f*g*h+324*x*ln(c*(d*(f*x+e)^p)^q)*a^
2*b*f^3*g^2+324*ln(c*(d*(f*x+e)^p)^q)^2*a*b^2*e*f^2*g^2-396*a*b^2*e^3*h^2*p
^2*q^2+108*a^2*b*e^3*h^2*p*q+36*x^3*a^3*f^3*h^2+36*ln(c*(d*(f*x+e)^p)^q)^3*
b^3*e^3*h^2+108*x*a^3*f^3*g^2)/f^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3121 vs. $2(478) = 956$.

Time = 0.38 (sec) , antiderivative size = 3121, normalized size of antiderivative = 6.34

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")
[Out] -1/108*(4*(2*b^3*f^3*h^2*p^3*q^3 - 6*a*b^2*f^3*h^2*p^2*q^2 + 9*a^2*b*f^3*h^
2*p*q - 9*a^3*f^3*h^2)*x^3 - 36*(b^3*f^3*h^2*p^3*q^3*x^3 + 3*b^3*f^3*g*h*p^
3*q^3*x^2 + 3*b^3*f^3*g^2*p^3*q^3*x + (3*b^3*e*f^2*g^2 - 3*b^3*e^2*f*g*h +
b^3*e^3*h^2)*p^3*q^3)*log(f*x + e)^3 - 36*(b^3*f^3*h^2*x^3 + 3*b^3*f^3*g*h*
x^2 + 3*b^3*f^3*g^2*x)*log(c)^3 - 36*(b^3*f^3*h^2*q^3*x^3 + 3*b^3*f^3*g*h*q
^3*x^2 + 3*b^3*f^3*g^2*q^3*x)*log(d)^3 - 3*(36*a^3*f^3*g*h - (27*b^3*f^3*g*
h - 19*b^3*e*f^2*h^2)*p^3*q^3 + 6*(9*a*b^2*f^3*g*h - 5*a*b^2*e*f^2*h^2)*p^2
*q^2 - 18*(3*a^2*b*f^3*g*h - a^2*b*e*f^2*h^2)*p*q)*x^2 + 18*((18*b^3*e*f^2*
g^2 - 27*b^3*e^2*f*g*h + 11*b^3*e^3*h^2)*p^3*q^3 - 6*(3*a*b^2*e*f^2*g^2 - 3
*a*b^2*e^2*f*g*h + a*b^2*e^3*h^2)*p^2*q^2 + 2*(b^3*f^3*h^2*p^3*q^3 - 3*a*b^
2*f^3*h^2*p^2*q^2)*x^3 - 3*(6*a*b^2*f^3*g*h*p^2*q^2 - (3*b^3*f^3*g*h - b^3*
e*f^2*h^2)*p^3*q^3)*x^2 - 6*(3*a*b^2*f^3*g^2*p^2*q^2 - (3*b^3*f^3*g^2 - 3*b
^3*e*f^2*g*h + b^3*e^2*f*h^2)*p^3*q^3)*x - 6*(b^3*f^3*h^2*p^2*q^2*x^3 + 3*b
^3*f^3*g*h*p^2*q^2*x^2 + 3*b^3*f^3*g^2*p^2*q^2*x + (3*b^3*e*f^2*g^2 - 3*b^3
*e^2*f*g*h + b^3*e^3*h^2)*p^2*q^2)*log(c) - 6*(b^3*f^3*h^2*p^2*q^3*x^3 + 3*
b^3*f^3*g*h*p^2*q^3*x^2 + 3*b^3*f^3*g^2*p^2*q^3*x + (3*b^3*e*f^2*g^2 - 3*b^
3*e^2*f*g*h + b^3*e^3*h^2)*p^2*q^3)*log(d))*log(f*x + e)^2 + 18*(2*(b^3*f^3
*h^2*p*q - 3*a*b^2*f^3*h^2)*x^3 - 3*(6*a*b^2*f^3*g*h - (3*b^3*f^3*g*h - b^3
*e*f^2*h^2)*p*q)*x^2 - 6*(3*a*b^2*f^3*g^2 - (3*b^3*f^3*g^2 - 3*b^3*e*f^2*g*
h + b^3*e^2*f*h^2)*p*q)*x)*log(c)^2 + 18*(2*(b^3*f^3*h^2*p*q^3 - 3*a*b^2*f^
3*h^2*q^2)*x^3 - 3*(6*a*b^2*f^3*g*h*q^2 - (3*b^3*f^3*g*h - b^3*e*f^2*h^2)*p
*q^3)*x^2 - 6*(3*a*b^2*f^3*g^2*q^2 - (3*b^3*f^3*g^2 - 3*b^3*e*f^2*g*h + b^3
*e^2*f*h^2)*p*q^3)*x - 6*(b^3*f^3*h^2*q^2*x^3 + 3*b^3*f^3*g*h*q^2*x^2 + 3*b
^3*f^3*g^2*q^2*x)*log(c))*log(d)^2 - 6*(18*a^3*f^3*g^2 - (108*b^3*f^3*g^2 -
 189*b^3*e*f^2*g*h + 85*b^3*e^2*f*h^2)*p^3*q^3 + 6*(18*a*b^2*f^3*g^2 - 27*a
*b^2*e*f^2*g*h + 11*a*b^2*e^2*f*h^2)*p^2*q^2 - 18*(3*a^2*b*f^3*g^2 - 3*a^2*
b*e*f^2*g*h + a^2*b*e^2*f*h^2)*p*q)*x - 6*((108*b^3*e*f^2*g^2 - 189*b^3*e^2
```

```
*f*g*h + 85*b^3*e^3*h^2)*p^3*q^3 - 6*(18*a*b^2*e*f^2*g^2 - 27*a*b^2*e^2*f*g
*h + 11*a*b^2*e^3*h^2)*p^2*q^2 + 2*(2*b^3*f^3*h^2*p^3*q^3 - 6*a*b^2*f^3*h^2
*p^2*q^2 + 9*a^2*b*f^3*h^2*p*q)*x^3 + 18*(3*a^2*b*e*f^2*g^2 - 3*a^2*b*e^2*f
*g*h + a^2*b*e^3*h^2)*p*q + 3*(18*a^2*b*f^3*g*h*p*q + (9*b^3*f^3*g*h - 5*b^
3*e*f^2*h^2)*p^3*q^3 - 6*(3*a*b^2*f^3*g*h - a*b^2*e*f^2*h^2)*p^2*q^2)*x^2 +
 18*(b^3*f^3*h^2*p*q*x^3 + 3*b^3*f^3*g*h*p*q*x^2 + 3*b^3*f^3*g^2*p*q*x + (3
*b^3*e*f^2*g^2 - 3*b^3*e^2*f*g*h + b^3*e^3*h^2)*p*q)*log(c)^2 + 18*(b^3*f^3
*h^2*p*q^3*x^3 + 3*b^3*f^3*g*h*p*q^3*x^2 + 3*b^3*f^3*g^2*p*q^3*x + (3*b^3*e
*f^2*g^2 - 3*b^3*e^2*f*g*h + b^3*e^3*h^2)*p*q^3)*log(d)^2 + 6*(9*a^2*b*f^3*
g^2*p*q + (18*b^3*f^3*g^2 - 27*b^3*e*f^2*g*h + 11*b^3*e^2*f*h^2)*p^3*q^3 -
6*(3*a*b^2*f^3*g^2 - 3*a*b^2*e*f^2*g*h + a*b^2*e^2*f*h^2)*p^2*q^2)*x - 6*((
18*b^3*e*f^2*g^2 - 27*b^3*e^2*f*g*h + 11*b^3*e^3*h^2)*p^2*q^2 + 2*(b^3*f^3*
h^2*p^2*q^2 - 3*a*b^2*f^3*h^2*p*q)*x^3 - 6*(3*a*b^2*e*f^2*g^2 - 3*a*b^2*e^2
*f*g*h + a*b^2*e^3*h^2)*p*q - 3*(6*a*b^2*f^3*g*h*p*q - (3*b^3*f^3*g*h - b^3
*e*f^2*h^2)*p^2*q^2)*x^2 - 6*(3*a*b^2*f^3*g^2*p*q - (3*b^3*f^3*g^2 - 3*b^3*
e*f^2*g*h + b^3*e^2*f*h^2)*p^2*q^2)*x)*log(c) - 6*((18*b^3*e*f^2*g^2 - 27*b
^3*e^2*f*g*h + 11*b^3*e^3*h^2)*p^2*q^3 - 6*(3*a*b^2*e*f^2*g^2 - 3*a*b^2*e^2
*f*g*h + a*b^2*e^3*h^2)*p*q^2 + 2*(b^3*f^3*h^2*p^2*q^3 - 3*a*b^2*f^3*h^2*p*
q^2)*x^3 - 3*(6*a*b^2*f^3*g*h*p*q^2 - (3*b^3*f^3*g*h - b^3*e*f^2*h^2)*p^2*q
^3)*x^2 - 6*(3*a*b^2*f^3*g^2*p*q^2 - (3*b^3*f^3*g^2 - 3*b^3*e*f^2*g*h + b^3
*e^2*f*h^2)*p^2*q^3)*x - 6*(b^3*f^3*h^2*p*q^2*x^3 + 3*b^3*f^3*g*h*p*q^2*x^2
 + 3*b^3*f^3*g^2*p*q^2*x + (3*b^3*e*f^2*g^2 - 3*b^3*e^2*f*g*h + b^3*e^3*h^2
)*p*q^2)*log(c))*log(d))*log(f*x + e) - 6*(2*(2*b^3*f^3*h^2*p^2*q^2 - 6*a*b
^2*f^3*h^2*p*q + 9*a^2*b*f^3*h^2)*x^3 + 3*(18*a^2*b*f^3*g*h + (9*b^3*f^3*g*
h - 5*b^3*e*f^2*h^2)*p^2*q^2 - 6*(3*a*b^2*f^3*g*h - a*b^2*e*f^2*h^2)*p*q)*x
^2 + 6*(9*a^2*b*f^3*g^2 + (18*b^3*f^3*g^2 - 27*b^3*e*f^2*g*h + 11*b^3*e^2*f
*h^2)*p^2*q^2 - 6*(3*a*b^2*f^3*g^2 - 3*a*b^2*e*f^2*g*h + a*b^2*e^2*f*h^2)*p
*q)*x)*log(c) - 6*(2*(2*b^3*f^3*h^2*p^2*q^3 - 6*a*b^2*f^3*h^2*p*q^2 + 9*a^2
*b*f^3*h^2*q)*x^3 + 3*(18*a^2*b*f^3*g*h*q + (9*b^3*f^3*g*h - 5*b^3*e*f^2*h^
2)*p^2*q^3 - 6*(3*a*b^2*f^3*g*h - a*b^2*e*f^2*h^2)*p*q^2)*x^2 + 18*(b^3*f^3
*h^2*q*x^3 + 3*b^3*f^3*g*h*q*x^2 + 3*b^3*f^3*g^2*q*x)*log(c)^2 + 6*(9*a^2*b
*f^3*g^2*q + (18*b^3*f^3*g^2 - 27*b^3*e*f^2*g*h + 11*b^3*e^2*f*h^2)*p^2*q^3
 - 6*(3*a*b^2*f^3*g^2 - 3*a*b^2*e*f^2*g*h + a*b^2*e^2*f*h^2)*p*q^2)*x - 6*(
2*(b^3*f^3*h^2*p*q^2 - 3*a*b^2*f^3*h^2*q)*x^3 - 3*(6*a*b^2*f^3*g*h*q - (3*b
^3*f^3*g*h - b^3*e*f^2*h^2)*p*q^2)*x^2 - 6*(3*a*b^2*f^3*g^2*q - (3*b^3*f^3*
g^2 - 3*b^3*e*f^2*g*h + b^3*e^2*f*h^2)*p*q^2)*x)*log(c))*log(d))/f^3
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1846 vs. 2(481) = 962.

Time = 5.38 (sec) , antiderivative size = 1846, normalized size of antiderivative = 3.75

$$\int (g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^3 \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
```

```
[Out] Piecewise((a**3*g**2*x + a**3*g*h*x**2 + a**3*h**2*x**3/3 + a**2*b*e**3*h**
2*log(c*(d*(e + f*x)**p)**q)/f**3 - 3*a**2*b*e**2*g*h*log(c*(d*(e + f*x)**p
)**q)/f**2 - a**2*b*e**2*h**2*p*q*x/f**2 + 3*a**2*b*e*g**2*log(c*(d*(e + f*
x)**p)**q)/f + 3*a**2*b*e*g*h*p*q*x/f + a**2*b*e*h**2*p*q*x**2/(2*f) - 3*a*
*2*b*g**2*p*q*x + 3*a**2*b*g**2*x*log(c*(d*(e + f*x)**p)**q) - 3*a**2*b*g*h
*p*q*x**2/2 + 3*a**2*b*g*h*x**2*log(c*(d*(e + f*x)**p)**q) - a**2*b*h**2*p*
q*x**3/3 + a**2*b*h**2*x**3*log(c*(d*(e + f*x)**p)**q) - 11*a*b**2*e**3*h**
2*p*q*log(c*(d*(e + f*x)**p)**q)/(3*f**3) + a*b**2*e**3*h**2*log(c*(d*(e +
f*x)**p)**q)**2/f**3 + 9*a*b**2*e**2*g*h*p*q*log(c*(d*(e + f*x)**p)**q)/f**
2 - 3*a*b**2*e**2*g*h*log(c*(d*(e + f*x)**p)**q)**2/f**2 + 11*a*b**2*e**2*h
**2*p**2*q**2*x/(3*f**2) - 2*a*b**2*e**2*h**2*p*q*x*log(c*(d*(e + f*x)**p)*
*q)/f**2 - 6*a*b**2*e*g**2*p*q*log(c*(d*(e + f*x)**p)**q)/f + 3*a*b**2*e*g*
*2*log(c*(d*(e + f*x)**p)**q)**2/f - 9*a*b**2*e*g*h*p**2*q**2*x/f + 6*a*b**
2*e*g*h*p*q*x*log(c*(d*(e + f*x)**p)**q)/f - 5*a*b**2*e*h**2*p**2*q**2*x**2
/(6*f) + a*b**2*e*h**2*p*q*x**2*log(c*(d*(e + f*x)**p)**q)/f + 6*a*b**2*g**
2*p**2*q**2*x - 6*a*b**2*g**2*p*q*x*log(c*(d*(e + f*x)**p)**q) + 3*a*b**2*g
**2*x*log(c*(d*(e + f*x)**p)**q)**2 + 3*a*b**2*g*h*p**2*q**2*x**2/2 - 3*a*b
**2*g*h*p*q*x**2*log(c*(d*(e + f*x)**p)**q) + 3*a*b**2*g*h*x**2*log(c*(d*(e
 + f*x)**p)**q)**2 + 2*a*b**2*h**2*p**2*q**2*x**3/9 - 2*a*b**2*h**2*p*q*x**
3*log(c*(d*(e + f*x)**p)**q)/3 + a*b**2*h**2*x**3*log(c*(d*(e + f*x)**p)**q
)**2 + 85*b**3*e**3*h**2*p**2*q**2*log(c*(d*(e + f*x)**p)**q)/(18*f**3) - 1
1*b**3*e**3*h**2*p*q*log(c*(d*(e + f*x)**p)**q)**2/(6*f**3) + b**3*e**3*h**
2*log(c*(d*(e + f*x)**p)**q)**3/(3*f**3) - 21*b**3*e**2*g*h*p**2*q**2*log(c
*(d*(e + f*x)**p)**q)/(2*f**2) + 9*b**3*e**2*g*h*p*q*log(c*(d*(e + f*x)**p)
**q)**2/(2*f**2) - b**3*e**2*g*h*log(c*(d*(e + f*x)**p)**q)**3/f**2 - 85*b*
*3*e**2*h**2*p**3*q**3*x/(18*f**2) + 11*b**3*e**2*h**2*p**2*q**2*x*log(c*(d
*(e + f*x)**p)**q)/(3*f**2) - b**3*e**2*h**2*p*q*x*log(c*(d*(e + f*x)**p)**
q)**2/f**2 + 6*b**3*e*g**2*p**2*q**2*log(c*(d*(e + f*x)**p)**q)/f - 3*b**3*
e*g**2*p*q*log(c*(d*(e + f*x)**p)**q)**2/f + b**3*e*g**2*log(c*(d*(e + f*x)
**p)**q)**3/f + 21*b**3*e*g*h*p**3*q**3*x/(2*f) - 9*b**3*e*g*h*p**2*q**2*x*
log(c*(d*(e + f*x)**p)**q)/f + 3*b**3*e*g*h*p*q*x*log(c*(d*(e + f*x)**p)**q
)**2/f + 19*b**3*e*h**2*p**3*q**3*x**2/(36*f) - 5*b**3*e*h**2*p**2*q**2*x**
2*log(c*(d*(e + f*x)**p)**q)/(6*f) + b**3*e*h**2*p*q*x**2*log(c*(d*(e + f*x
)**p)**q)**2/(2*f) - 6*b**3*g**2*p**3*q**3*x + 6*b**3*g**2*p**2*q**2*x*log(
c*(d*(e + f*x)**p)**q) - 3*b**3*g**2*p*q*x*log(c*(d*(e + f*x)**p)**q)**2 +
```

```
b**3*g**2*x*log(c*(d*(e + f*x)**p)**q)**3 - 3*b**3*g*h*p**3*q**3*x**2/4 + 3
*b**3*g*h*p**2*q**2*x**2*log(c*(d*(e + f*x)**p)**q)/2 - 3*b**3*g*h*p*q*x**2
*log(c*(d*(e + f*x)**p)**q)**2/2 + b**3*g*h*x**2*log(c*(d*(e + f*x)**p)**q)
**3 - 2*b**3*h**2*p**3*q**3*x**3/27 + 2*b**3*h**2*p**2*q**2*x**3*log(c*(d*(
e + f*x)**p)**q)/9 - b**3*h**2*p*q*x**3*log(c*(d*(e + f*x)**p)**q)**2/3 + b
**3*h**2*x**3*log(c*(d*(e + f*x)**p)**q)**3/3, Ne(f, 0)), ((a + b*log(c*(d*
e**p)**q))**3*(g**2*x + g*h*x**2 + h**2*x**3/3), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1245 vs. $2(478) = 956$.

Time = 0.25 (sec) , antiderivative size = 1245, normalized size of antiderivative = 2.53

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*h^2*x^3*log(((f*x + e)^p*d)^q*c)^3 + a*b^2*h^2*x^3*log(((f*x + e)^p
*d)^q*c)^2 + b^3*g*h*x^2*log(((f*x + e)^p*d)^q*c)^3 - 3*a^2*b*f*g^2*p*q*(x/
f - e*log(f*x + e)/f^2) + 1/6*a^2*b*f*h^2*p*q*(6*e^3*log(f*x + e)/f^4 - (2*
f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/f^3) - 3/2*a^2*b*f*g*h*p*q*(2*e^2*log(f*x +
e)/f^3 + (f*x^2 - 2*e*x)/f^2) + a^2*b*h^2*x^3*log(((f*x + e)^p*d)^q*c) + 3*
a*b^2*g*h*x^2*log(((f*x + e)^p*d)^q*c)^2 + b^3*g^2*x*log(((f*x + e)^p*d)^q*
c)^3 + 1/3*a^3*h^2*x^3 + 3*a^2*b*g*h*x^2*log(((f*x + e)^p*d)^q*c) + 3*a*b^2
*g^2*x*log(((f*x + e)^p*d)^q*c)^2 + a^3*g*h*x^2 + 3*a^2*b*g^2*x*log(((f*x +
 e)^p*d)^q*c) - 3*(2*f*p*q*(x/f - e*log(f*x + e)/f^2)*log(((f*x + e)^p*d)^q
*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p^2*q^2/f)*a*b^2*g^2 -
(3*f*p*q*(x/f - e*log(f*x + e)/f^2)*log(((f*x + e)^p*d)^q*c)^2 - ((e*log(f*
x + e)^3 + 3*e*log(f*x + e)^2 - 6*f*x + 6*e*log(f*x + e))*p^2*q^2/f^2 - 3*(
e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p*q*log(((f*x + e)^p*d)^q*c)/f
^2)*f*p*q)*b^3*g^2 - 3/2*(2*f*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)
/f^2)*log(((f*x + e)^p*d)^q*c) - (f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x
+ 6*e^2*log(f*x + e))*p^2*q^2/f^2)*a*b^2*g*h - 1/4*(6*f*p*q*(2*e^2*log(f*x
+ e)/f^3 + (f*x^2 - 2*e*x)/f^2)*log(((f*x + e)^p*d)^q*c)^2 + ((4*e^2*log(f*
x + e)^3 + 3*f^2*x^2 + 18*e^2*log(f*x + e)^2 - 42*e*f*x + 42*e^2*log(f*x +
e))*p^2*q^2/f^3 - 6*(f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x + 6*e^2*log(f
*x + e))*p*q*log(((f*x + e)^p*d)^q*c)/f^3)*f*p*q)*b^3*g*h + 1/18*(6*f*p*q*(
6*e^3*log(f*x + e)/f^4 - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/f^3)*log(((f*x +
 e)^p*d)^q*c) + (4*f^3*x^3 - 15*e*f^2*x^2 - 18*e^3*log(f*x + e)^2 + 66*e^2*
f*x - 66*e^3*log(f*x + e))*p^2*q^2/f^3)*a*b^2*h^2 + 1/108*(18*f*p*q*(6*e^3*
log(f*x + e)/f^4 - (2*f^2*x^3 - 3*e*f*x^2 + 6*e^2*x)/f^3)*log(((f*x + e)^p*
d)^q*c)^2 - f*p*q*((8*f^3*x^3 - 36*e^3*log(f*x + e)^3 - 57*e*f^2*x^2 - 198*
e^3*log(f*x + e)^2 + 510*e^2*f*x - 510*e^3*log(f*x + e))*p^2*q^2/f^4 - 6*(4
*f^3*x^3 - 15*e*f^2*x^2 - 18*e^3*log(f*x + e)^2 + 66*e^2*f*x - 66*e^3*log(f
*x + e))*p*q*log(((f*x + e)^p*d)^q*c)/f^4))*b^3*h^2 + a^3*g^2*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5146 vs. $2(478) = 956$.

Time = 0.42 (sec) , antiderivative size = 5146, normalized size of antiderivative = 10.46

$$\int (g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^3 \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

```
[Out] 1/108*(108*(f*x + e)*b^3*f^2*g^2*p^3*q^3*log(f*x + e)^3 + 108*(f*x + e)^2*b
^3*f*g*h*p^3*q^3*log(f*x + e)^3 - 216*(f*x + e)*b^3*e*f*g*h*p^3*q^3*log(f*x
 + e)^3 + 36*(f*x + e)^3*b^3*h^2*p^3*q^3*log(f*x + e)^3 - 108*(f*x + e)^2*b
^3*e*h^2*p^3*q^3*log(f*x + e)^3 + 108*(f*x + e)*b^3*e^2*h^2*p^3*q^3*log(f*x
 + e)^3 - 324*(f*x + e)*b^3*f^2*g^2*p^3*q^3*log(f*x + e)^2 - 162*(f*x + e)^
2*b^3*f*g*h*p^3*q^3*log(f*x + e)^2 + 648*(f*x + e)*b^3*e*f*g*h*p^3*q^3*log(
f*x + e)^2 - 36*(f*x + e)^3*b^3*h^2*p^3*q^3*log(f*x + e)^2 + 162*(f*x + e)^
2*b^3*e*h^2*p^3*q^3*log(f*x + e)^2 - 324*(f*x + e)*b^3*e^2*h^2*p^3*q^3*log(
f*x + e)^2 + 324*(f*x + e)*b^3*f^2*g^2*p^2*q^3*log(f*x + e)^2*log(d) + 324*
(f*x + e)^2*b^3*f*g*h*p^2*q^3*log(f*x + e)^2*log(d) - 648*(f*x + e)*b^3*e*f
*g*h*p^2*q^3*log(f*x + e)^2*log(d) + 108*(f*x + e)^3*b^3*h^2*p^2*q^3*log(f*
x + e)^2*log(d) - 324*(f*x + e)^2*b^3*e*h^2*p^2*q^3*log(f*x + e)^2*log(d) +
 324*(f*x + e)*b^3*e^2*h^2*p^2*q^3*log(f*x + e)^2*log(d) + 648*(f*x + e)*b^
3*f^2*g^2*p^3*q^3*log(f*x + e) + 162*(f*x + e)^2*b^3*f*g*h*p^3*q^3*log(f*x
+ e) - 1296*(f*x + e)*b^3*e*f*g*h*p^3*q^3*log(f*x + e) + 24*(f*x + e)^3*b^3
*h^2*p^3*q^3*log(f*x + e) - 162*(f*x + e)^2*b^3*e*h^2*p^3*q^3*log(f*x + e)
+ 648*(f*x + e)*b^3*e^2*h^2*p^3*q^3*log(f*x + e) + 324*(f*x + e)*b^3*f^2*g^
2*p^2*q^2*log(f*x + e)^2*log(c) + 324*(f*x + e)^2*b^3*f*g*h*p^2*q^2*log(f*x
 + e)^2*log(c) - 648*(f*x + e)*b^3*e*f*g*h*p^2*q^2*log(f*x + e)^2*log(c) +
108*(f*x + e)^3*b^3*h^2*p^2*q^2*log(f*x + e)^2*log(c) - 324*(f*x + e)^2*b^3
*e*h^2*p^2*q^2*log(f*x + e)^2*log(c) + 324*(f*x + e)*b^3*e^2*h^2*p^2*q^2*lo
g(f*x + e)^2*log(c) - 648*(f*x + e)*b^3*f^2*g^2*p^2*q^3*log(f*x + e)*log(d)
 - 324*(f*x + e)^2*b^3*f*g*h*p^2*q^3*log(f*x + e)*log(d) + 1296*(f*x + e)*b
^3*e*f*g*h*p^2*q^3*log(f*x + e)*log(d) - 72*(f*x + e)^3*b^3*h^2*p^2*q^3*log
(f*x + e)*log(d) + 324*(f*x + e)^2*b^3*e*h^2*p^2*q^3*log(f*x + e)*log(d) -
648*(f*x + e)*b^3*e^2*h^2*p^2*q^3*log(f*x + e)*log(d) + 324*(f*x + e)*b^3*f
^2*g^2*p*q^3*log(f*x + e)*log(d)^2 + 324*(f*x + e)^2*b^3*f*g*h*p*q^3*log(f*
x + e)*log(d)^2 - 648*(f*x + e)*b^3*e*f*g*h*p*q^3*log(f*x + e)*log(d)^2 + 1
08*(f*x + e)^3*b^3*h^2*p*q^3*log(f*x + e)*log(d)^2 - 324*(f*x + e)^2*b^3*e*
h^2*p*q^3*log(f*x + e)*log(d)^2 + 324*(f*x + e)*b^3*e^2*h^2*p*q^3*log(f*x +
 e)*log(d)^2 - 648*(f*x + e)*b^3*f^2*g^2*p^3*q^3 - 81*(f*x + e)^2*b^3*f*g*h
*p^3*q^3 + 1296*(f*x + e)*b^3*e*f*g*h*p^3*q^3 - 8*(f*x + e)^3*b^3*h^2*p^3*q
^3 + 81*(f*x + e)^2*b^3*e*h^2*p^3*q^3 - 648*(f*x + e)*b^3*e^2*h^2*p^3*q^3 +
 324*(f*x + e)*a*b^2*f^2*g^2*p^2*q^2*log(f*x + e)^2 + 324*(f*x + e)^2*a*b^2
*f*g*h*p^2*q^2*log(f*x + e)^2 - 648*(f*x + e)*a*b^2*e*f*g*h*p^2*q^2*log(f*x
```

+ e)^2 + 108*(f*x + e)^3*a*b^2*h^2*p^2*q^2*log(f*x + e)^2 - 324*(f*x + e)^
2*a*b^2*e*h^2*p^2*q^2*log(f*x + e)^2 + 324*(f*x + e)*a*b^2*e^2*h^2*p^2*q^2*
log(f*x + e)^2 - 648*(f*x + e)*b^3*f^2*g^2*p^2*q^2*log(f*x + e)*log(c) - 32
4*(f*x + e)^2*b^3*f*g*h*p^2*q^2*log(f*x + e)*log(c) + 1296*(f*x + e)*b^3*e*
f*g*h*p^2*q^2*log(f*x + e)*log(c) - 72*(f*x + e)^3*b^3*h^2*p^2*q^2*log(f*x
+ e)*log(c) + 324*(f*x + e)^2*b^3*e*h^2*p^2*q^2*log(f*x + e)*log(c) - 648*(
f*x + e)*b^3*e^2*h^2*p^2*q^2*log(f*x + e)*log(c) + 648*(f*x + e)*b^3*f^2*g^
2*p^2*q^3*log(d) + 162*(f*x + e)^2*b^3*f*g*h*p^2*q^3*log(d) - 1296*(f*x + e
)*b^3*e*f*g*h*p^2*q^3*log(d) + 24*(f*x + e)^3*b^3*h^2*p^2*q^3*log(d) - 162*
(f*x + e)^2*b^3*e*h^2*p^2*q^3*log(d) + 648*(f*x + e)*b^3*e^2*h^2*p^2*q^3*lo
g(d) + 648*(f*x + e)*b^3*f^2*g^2*p*q^2*log(f*x + e)*log(c)*log(d) + 648*(f*
x + e)^2*b^3*f*g*h*p*q^2*log(f*x + e)*log(c)*log(d) - 1296*(f*x + e)*b^3*e*
f*g*h*p*q^2*log(f*x + e)*log(c)*log(d) + 216*(f*x + e)^3*b^3*h^2*p*q^2*log(
f*x + e)*log(c)*log(d) - 648*(f*x + e)^2*b^3*e*h^2*p*q^2*log(f*x + e)*log(c
)*log(d) + 648*(f*x + e)*b^3*e^2*h^2*p*q^2*log(f*x + e)*log(c)*log(d) - 324
*(f*x + e)*b^3*f^2*g^2*p*q^3*log(d)^2 - 162*(f*x + e)^2*b^3*f*g*h*p*q^3*log
(d)^2 + 648*(f*x + e)*b^3*e*f*g*h*p*q^3*log(d)^2 - 36*(f*x + e)^3*b^3*h^2*p
*q^3*log(d)^2 + 162*(f*x + e)^2*b^3*e*h^2*p*q^3*log(d)^2 - 324*(f*x + e)*b^
3*e^2*h^2*p*q^3*log(d)^2 + 108*(f*x + e)*b^3*f^2*g^2*q^3*log(d)^3 + 108*(f*
x + e)^2*b^3*f*g*h*q^3*log(d)^3 - 216*(f*x + e)*b^3*e*f*g*h*q^3*log(d)^3 +
36*(f*x + e)^3*b^3*h^2*q^3*log(d)^3 - 108*(f*x + e)^2*b^3*e*h^2*q^3*log(d)^
3 + 108*(f*x + e)*b^3*e^2*h^2*q^3*log(d)^3 - 648*(f*x + e)*a*b^2*f^2*g^2*p^
2*q^2*log(f*x + e) - 324*(f*x + e)^2*a*b^2*f*g*h*p^2*q^2*log(f*x + e) + 129
6*(f*x + e)*a*b^2*e*f*g*h*p^2*q^2*log(f*x + e) - 72*(f*x + e)^3*a*b^2*h^2*p
^2*q^2*log(f*x + e) + 324*(f*x + e)^2*a*b^2*e*h^2*p^2*q^2*log(f*x + e) - 64
8*(f*x + e)*a*b^2*e^2*h^2*p^2*q^2*log(f*x + e) + 648*(f*x + e)*b^3*f^2*g^2*
p^2*q^2*log(c) + 162*(f*x + e)^2*b^3*f*g*h*p^2*q^2*log(c) - 1296*(f*x + e)*
b^3*e*f*g*h*p^2*q^2*log(c) + 24*(f*x + e)^3*b^3*h^2*p^2*q^2*log(c) - 162*(f
*x + e)^2*b^3*e*h^2*p^2*q^2*log(c) + 648*(f*x + e)*b^3*e^2*h^2*p^2*q^2*log(
c) + 324*(f*x + e)*b^3*f^2*g^2*p*q*log(f*x + e)*log(c)^2 + 324*(f*x + e)^2*
b^3*f*g*h*p*q*log(f*x + e)*log(c)^2 - 648*(f*x + e)*b^3*e*f*g*h*p*q*log(f*x
+ e)*log(c)^2 + 108*(f*x + e)^3*b^3*h^2*p*q*log(f*x + e)*log(c)^2 - 324*(f
*x + e)^2*b^3*e*h^2*p*q*log(f*x + e)*log(c)^2 + 324*(f*x + e)*b^3*e^2*h^2*p
*q*log(f*x + e)*log(c)^2 + 648*(f*x + e)*a*b^2*f^2*g^2*p*q^2*log(f*x + e)*l
og(d) + 648*(f*x + e)^2*a*b^2*f*g*h*p*q^2*log(f*x + e)*log(d) - 1296*(f*x +
e)*a*b^2*e*f*g*h*p*q^2*log(f*x + e)*log(d) + 216*(f*x + e)^3*a*b^2*h^2*p*q
^2*log(f*x + e)*log(d) - 648*(f*x + e)^2*a*b^2*e*h^2*p*q^2*log(f*x + e)*log
(d) + 648*(f*x + e)*a*b^2*e^2*h^2*p*q^2*log(f*x + e)*log(d) - 648*(f*x + e)
*b^3*f^2*g^2*p*q^2*log(c)*log(d) - 324*(f*x + e)^2*b^3*f*g*h*p*q^2*log(c)*l
og(d) + 1296*(f*x + e)*b^3*e*f*g*h*p*q^2*log(c)*log(d) - 72*(f*x + e)^3*b^3
*h^2*p*q^2*log(c)*log(d) + 324*(f*x + e)^2*b^3*e*h^2*p*q^2*log(c)*log(d) -
648*(f*x + e)*b^3*e^2*h^2*p*q^2*log(c)*log(d) + 324*(f*x + e)*b^3*f^2*g^2*q
^2*log(c)*log(d)^2 + 324*(f*x + e)^2*b^3*f*g*h*q^2*log(c)*log(d)^2 - 648*(f
*x + e)*b^3*e*f*g*h*q^2*log(c)*log(d)^2 + 108*(f*x + e)^3*b^3*h^2*q^2*log(c
)*log(d)^2 - 324*(f*x + e)^2*b^3*e*h^2*q^2*log(c)*log(d)^2 + 324*(f*x + e)*

b^3*e^2*h^2*q^2*log(c)*log(d)^2 + 648*(f*x + e)*a*b^2*f^2*g^2*p^2*q^2 + 162
*(f*x + e)^2*a*b^2*f*g*h*p^2*q^2 - 1296*(f*x + e)*a*b^2*e*f*g*h*p^2*q^2 + 2
4*(f*x + e)^3*a*b^2*h^2*p^2*q^2 - 162*(f*x + e)^2*a*b^2*e*h^2*p^2*q^2 + 648
*(f*x + e)*a*b^2*e^2*h^2*p^2*q^2 + 648*(f*x + e)*a*b^2*f^2*g^2*p*q*log(f*x
+ e)*log(c) + 648*(f*x + e)^2*a*b^2*f*g*h*p*q*log(f*x + e)*log(c) - 1296*(f
*x + e)*a*b^2*e*f*g*h*p*q*log(f*x + e)*log(c) + 216*(f*x + e)^3*a*b^2*h^2*p
*q*log(f*x + e)*log(c) - 648*(f*x + e)^2*a*b^2*e*h^2*p*q*log(f*x + e)*log(c
) + 648*(f*x + e)*a*b^2*e^2*h^2*p*q*log(f*x + e)*log(c) - 324*(f*x + e)*b^3
*f^2*g^2*p*q*log(c)^2 - 162*(f*x + e)^2*b^3*f*g*h*p*q*log(c)^2 + 648*(f*x +
 e)*b^3*e*f*g*h*p*q*log(c)^2 - 36*(f*x + e)^3*b^3*h^2*p*q*log(c)^2 + 162*(f
*x + e)^2*b^3*e*h^2*p*q*log(c)^2 - 324*(f*x + e)*b^3*e^2*h^2*p*q*log(c)^2 -
 648*(f*x + e)*a*b^2*f^2*g^2*p*q^2*log(d) - 324*(f*x + e)^2*a*b^2*f*g*h*p*q
^2*log(d) + 1296*(f*x + e)*a*b^2*e*f*g*h*p*q^2*log(d) - 72*(f*x + e)^3*a*b^
2*h^2*p*q^2*log(d) + 324*(f*x + e)^2*a*b^2*e*h^2*p*q^2*log(d) - 648*(f*x +
e)*a*b^2*e^2*h^2*p*q^2*log(d) + 324*(f*x + e)*b^3*f^2*g^2*q*log(c)^2*log(d)
 + 324*(f*x + e)^2*b^3*f*g*h*q*log(c)^2*log(d) - 648*(f*x + e)*b^3*e*f*g*h*
q*log(c)^2*log(d) + 108*(f*x + e)^3*b^3*h^2*q*log(c)^2*log(d) - 324*(f*x +
e)^2*b^3*e*h^2*q*log(c)^2*log(d) + 324*(f*x + e)*b^3*e^2*h^2*q*log(c)^2*log
(d) + 324*(f*x + e)*a*b^2*f^2*g^2*q^2*log(d)^2 + 324*(f*x + e)^2*a*b^2*f*g*
h*q^2*log(d)^2 - 648*(f*x + e)*a*b^2*e*f*g*h*q^2*log(d)^2 + 108*(f*x + e)^3
*a*b^2*h^2*q^2*log(d)^2 - 324*(f*x + e)^2*a*b^2*e*h^2*q^2*log(d)^2 + 324*(f
*x + e)*a*b^2*e^2*h^2*q^2*log(d)^2 + 324*(f*x + e)*a^2*b*f^2*g^2*p*q*log(f*
x + e) + 324*(f*x + e)^2*a^2*b*f*g*h*p*q*log(f*x + e) - 648*(f*x + e)*a^2*b
*e*f*g*h*p*q*log(f*x + e) + 108*(f*x + e)^3*a^2*b*h^2*p*q*log(f*x + e) - 32
4*(f*x + e)^2*a^2*b*e*h^2*p*q*log(f*x + e) + 324*(f*x + e)*a^2*b*e^2*h^2*p*
q*log(f*x + e) - 648*(f*x + e)*a*b^2*f^2*g^2*p*q*log(c) - 324*(f*x + e)^2*a
*b^2*f*g*h*p*q*log(c) + 1296*(f*x + e)*a*b^2*e*f*g*h*p*q*log(c) - 72*(f*x +
 e)^3*a*b^2*h^2*p*q*log(c) + 324*(f*x + e)^2*a*b^2*e*h^2*p*q*log(c) - 648*(
f*x + e)*a*b^2*e^2*h^2*p*q*log(c) + 108*(f*x + e)*b^3*f^2*g^2*log(c)^3 + 10
8*(f*x + e)^2*b^3*f*g*h*log(c)^3 - 216*(f*x + e)*b^3*e*f*g*h*log(c)^3 + 36*
(f*x + e)^3*b^3*h^2*log(c)^3 - 108*(f*x + e)^2*b^3*e*h^2*log(c)^3 + 108*(f*
x + e)*b^3*e^2*h^2*log(c)^3 + 648*(f*x + e)*a*b^2*f^2*g^2*q*log(c)*log(d) +
 648*(f*x + e)^2*a*b^2*f*g*h*q*log(c)*log(d) - 1296*(f*x + e)*a*b^2*e*f*g*h
*q*log(c)*log(d) + 216*(f*x + e)^3*a*b^2*h^2*q*log(c)*log(d) - 648*(f*x + e
)^2*a*b^2*e*h^2*q*log(c)*log(d) + 648*(f*x + e)*a*b^2*e^2*h^2*q*log(c)*log(
d) - 324*(f*x + e)*a^2*b*f^2*g^2*p*q - 162*(f*x + e)^2*a^2*b*f*g*h*p*q + 64
8*(f*x + e)*a^2*b*e*f*g*h*p*q - 36*(f*x + e)^3*a^2*b*h^2*p*q + 162*(f*x + e
)^2*a^2*b*e*h^2*p*q - 324*(f*x + e)*a^2*b*e^2*h^2*p*q + 324*(f*x + e)*a*b^2
*f^2*g^2*log(c)^2 + 324*(f*x + e)^2*a*b^2*f*g*h*log(c)^2 - 648*(f*x + e)*a*
b^2*e*f*g*h*log(c)^2 + 108*(f*x + e)^3*a*b^2*h^2*log(c)^2 - 324*(f*x + e)^2
*a*b^2*e*h^2*log(c)^2 + 324*(f*x + e)*a*b^2*e^2*h^2*log(c)^2 + 324*(f*x + e
)*a^2*b*f^2*g^2*q*log(d) + 324*(f*x + e)^2*a^2*b*f*g*h*q*log(d) - 648*(f*x
+ e)*a^2*b*e*f*g*h*q*log(d) + 108*(f*x + e)^3*a^2*b*h^2*q*log(d) - 324*(f*x
 + e)^2*a^2*b*e*h^2*q*log(d) + 324*(f*x + e)*a^2*b*e^2*h^2*q*log(d) + 324*(
f*x + e)*a^2*b*f^2*g^2*log(c) + 324*(f*x + e)^2*a^2*b*f*g*h*log(c) - 648*(f

```
*x + e)*a^2*b*e*f*g*h*log(c) + 108*(f*x + e)^3*a^2*b*h^2*log(c) - 324*(f*x
+ e)^2*a^2*b*e*h^2*log(c) + 324*(f*x + e)*a^2*b*e^2*h^2*log(c) + 108*(f*x +
 e)*a^3*f^2*g^2 + 108*(f*x + e)^2*a^3*f*g*h - 216*(f*x + e)*a^3*e*f*g*h + 3
6*(f*x + e)^3*a^3*h^2 - 108*(f*x + e)^2*a^3*e*h^2 + 108*(f*x + e)*a^3*e^2*h
^2)/f^3
```

## Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 1400, normalized size of antiderivative = 2.85

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx = \text{Too large to display}$$

```
[In] int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^3,x)
[Out] x*((18*a^3*f^2*g^2 - 66*b^3*e^2*h^2*p^3*q^3 - 108*b^3*f^2*g^2*p^3*q^3 + 36*
a^3*e*f*g*h + 36*a*b^2*e^2*h^2*p^2*q^2 + 108*a*b^2*f^2*g^2*p^2*q^2 - 54*a^2
*b*f^2*g^2*p*q + 162*b^3*e*f*g*h*p^3*q^3 - 108*a*b^2*e*f*g*h*p^2*q^2)/(18*f
^2) - (e*((h*(6*a^3*e*h + 12*a^3*f*g + 5*b^3*e*h*p^3*q^3 - 9*b^3*f*g*p^3*q^
3 - 18*a^2*b*f*g*p*q - 6*a*b^2*e*h*p^2*q^2 + 18*a*b^2*f*g*p^2*q^2))/(6*f) -
 (e*h^2*(9*a^3 - 2*b^3*p^3*q^3 + 6*a*b^2*p^2*q^2 - 9*a^2*b*p*q))/(9*f)))/f)
 + log(c*(d*(e + f*x)^p)^q)^2*(x^2*((3*b^2*h*(a*e*h + 2*a*f*g - b*f*g*p*q))
/(2*f) - (b^2*e*h^2*(3*a - b*p*q))/(2*f)) - x*((e*((3*b^2*h*(a*e*h + 2*a*f*
g - b*f*g*p*q))/f - (b^2*e*h^2*(3*a - b*p*q))/f))/f - (3*b^2*g*(2*a*e*h + a
*f*g - b*f*g*p*q))/f) + (e*(6*a*b^2*e^2*h^2 + 18*a*b^2*f^2*g^2 - 11*b^3*e^2
*h^2*p*q - 18*b^3*f^2*g^2*p*q - 18*a*b^2*e*f*g*h + 27*b^3*e*f*g*h*p*q))/(6*
f^3) + (b^2*h^2*x^3*(3*a - b*p*q))/3) + log(c*(d*(e + f*x)^p)^q)^3*(b^3*g^2
*x + (b^3*h^2*x^3)/3 + (e*(b^3*e^2*h^2 + 3*b^3*f^2*g^2 - 3*b^3*e*f*g*h))/(3
*f^3) + b^3*g*h*x^2) + x^2*((h*(6*a^3*e*h + 12*a^3*f*g + 5*b^3*e*h*p^3*q^3
- 9*b^3*f*g*p^3*q^3 - 18*a^2*b*f*g*p*q - 6*a*b^2*e*h*p^2*q^2 + 18*a*b^2*f*g
*p^2*q^2))/(12*f) - (e*h^2*(9*a^3 - 2*b^3*p^3*q^3 + 6*a*b^2*p^2*q^2 - 9*a^2
*b*p*q))/(18*f)) + (log(e + f*x)*(85*b^3*e^3*h^2*p^3*q^3 - 66*a*b^2*e^3*h^2
*p^2*q^2 + 108*b^3*e*f^2*g^2*p^3*q^3 + 18*a^2*b*e^3*h^2*p*q - 108*a*b^2*e*f
^2*g^2*p^2*q^2 + 54*a^2*b*e*f^2*g^2*p*q - 189*b^3*e^2*f*g*h*p^3*q^3 + 162*a
*b^2*e^2*f*g*h*p^2*q^2 - 54*a^2*b*e^2*f*g*h*p*q))/(18*f^3) + (h^2*x^3*(9*a^
3 - 2*b^3*p^3*q^3 + 6*a*b^2*p^2*q^2 - 9*a^2*b*p*q))/27 + (log(c*(d*(e + f*x
)^p)^q)*(x^3*(f*(9*a^2*b*f*g*h - (5*b^3*e*h^2*p^2*q^2)/2 + 3*a*b^2*e*h^2*p*
q + (9*b^3*f*g*h*p^2*q^2)/2 - 9*a*b^2*f*g*h*p*q) + (b*e*f*h^2*(9*a^2 + 2*b^
2*p^2*q^2 - 6*a*b*p*q))/3) + x^2*(e*(9*a^2*b*f*g*h - (5*b^3*e*h^2*p^2*q^2)/
2 + 3*a*b^2*e*h^2*p*q + (9*b^3*f*g*h*p^2*q^2)/2 - 9*a*b^2*f*g*h*p*q) + 9*a^
2*b*f^2*g^2 + 11*b^3*e^2*h^2*p^2*q^2 + 18*b^3*f^2*g^2*p^2*q^2 - 6*a*b^2*e^2
*h^2*p*q - 18*a*b^2*f^2*g^2*p*q - 27*b^3*e*f*g*h*p^2*q^2 + 18*a*b^2*e*f*g*h
*p*q) + (e*x*(9*a^2*b*f^2*g^2 + 11*b^3*e^2*h^2*p^2*q^2 + 18*b^3*f^2*g^2*p^2
*q^2 - 6*a*b^2*e^2*h^2*p*q - 18*a*b^2*f^2*g^2*p*q - 27*b^3*e*f*g*h*p^2*q^2
+ 18*a*b^2*e*f*g*h*p*q))/f + (b*f^2*h^2*x^4*(9*a^2 + 2*b^2*p^2*q^2 - 6*a*b*
p*q))/3))/(3*f*(e + f*x))
```

# 3.436 $\int (g+hx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 306

$$\begin{aligned}
&\int (g+hx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3\,dx \\
&= \frac{6ab^2(fg-eh)p^2q^2x}{f}-\frac{6b^3(fg-eh)p^3q^3x}{f}-\frac{3b^3hp^3q^3(e+fx)^2}{8f^2} \\
&+\frac{6b^3(fg-eh)p^2q^2(e+fx)\log \left(c(d(e+fx)^p)^q\right)}{f^2} \\
&+\frac{3b^2hp^2q^2(e+fx)^2\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)}{4f^2} \\
&-\frac{3b(fg-eh)pq(e+fx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{f^2} \\
&-\frac{3bhpq(e+fx)^2\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{4f^2} \\
&+\frac{(fg-eh)(e+fx)\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3}{f^2} \\
&+\frac{h(e+fx)^2\left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3}{2f^2}
\end{aligned}$$

```
[Out] 6*a*b^2*(-e*h+f*g)*p^2*q^2*x/f-6*b^3*(-e*h+f*g)*p^3*q^3*x/f-3/8*b^3*h*p^3*q
^3*(f*x+e)^2/f^2+6*b^3*(-e*h+f*g)*p^2*q^2*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f^2
+3/4*b^2*h*p^2*q^2*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^2-3*b*(-e*h+f*g)
*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f^2-3/4*b*h*p*q*(f*x+e)^2*(a+b*l
n(c*(d*(f*x+e)^p)^q))^2/f^2+(-e*h+f*g)*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^
3/f^2+1/2*h*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f^2
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2448, 2436, 2333, 2332, 2437, 2342, 2341, 2495}

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx$$
$$=\frac{3b^2hp^2q^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{4f^2}+\frac{6ab^2p^2q^2x(fg-eh)}{f}$$
$$-\frac{3bpq(e+fx)(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2}$$
$$+\frac{(e+fx)(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^2}$$
$$-\frac{3bhpq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4f^2}+\frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{2f^2}$$
$$+\frac{6b^3p^2q^2(e+fx)(fg-eh)\log\left(c(d(e+fx)^p)^q\right)}{f^2}$$
$$-\frac{3b^3hp^3q^3(e+fx)^2}{8f^2}-\frac{6b^3p^3q^3x(fg-eh)}{f}$$

```
[In] Int[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

```
[Out] (6*a*b^2*(f*g - e*h)*p^2*q^2*x)/f - (6*b^3*(f*g - e*h)*p^3*q^3*x)/f - (3*b^
3*h*p^3*q^3*(e + f*x)^2)/(8*f^2) + (6*b^3*(f*g - e*h)*p^2*q^2*(e + f*x)*Log
[c*(d*(e + f*x)^p)^q])/f^2 + (3*b^2*h*p^2*q^2*(e + f*x)^2*(a + b*Log[c*(d*(
e + f*x)^p)^q]))/(4*f^2) - (3*b*(f*g - e*h)*p*q*(e + f*x)*(a + b*Log[c*(d*(
e + f*x)^p)^q])^2)/f^2 - (3*b*h*p*q*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p
)^q])^2)/(4*f^2) + ((f*g - e*h)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^
3)/f^2 + (h*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(2*f^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int (g+hx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\ &= \text{Subst}\left(\int\left(\frac{(fg-eh)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{f}\right.\right.\\ &\quad\left.\left.+\frac{h(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{f}\right)\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\end{aligned}$$

$$= \text{Subst}\left(\frac{h\int(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(fg-eh)\int\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{h\text{Subst}\left(\int x(a+b\log\left(cd^qx^{pq}\right))^3\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(fg-eh)\text{Subst}\left(\int\left(a+b\log\left(cd^qx^{pq}\right)\right)^3\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^2} + \frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{2f^2} - \text{Subst}\left(\frac{(3bhpq)\text{Subst}\left(\int x(a+b\log\left(cd^qx^{pq}\right))^2\,dx, x, e+fx\right)}{2f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(3b(fg-eh)pq)\text{Subst}\left(\int\left(a+b\log\left(cd^qx^{pq}\right)\right)^2\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{3b(fg-eh)pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2} - \frac{3bhpq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4f^2} + \frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^2} + \frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{2f^2} + \text{Subst}\left(\frac{(3b^2hp^2q^2)\,\text{Subst}\left(\int x(a+b\log\left(cd^qx^{pq}\right))\,dx,x,e+fx\right)}{2f^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(6b^2(fg-eh)p^2q^2)\,\text{Subst}\left(\int (a+b\log\left(cd^qx^{pq}\right))\,dx,x,e+fx\right)}{f^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{6ab^2(fg-eh)p^2q^2x}{f} - \frac{3b^3hp^3q^3(e+fx)^2}{8f^2} + \frac{3b^2hp^2q^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{4f^2} - \frac{3b(fg-eh)pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2} - \frac{3bhpq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4f^2} + \frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^2} + \frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{2f^2} + \text{Subst}\left(\frac{(6b^3(fg-eh)p^2q^2)\,\text{Subst}\left(\int \log\left(cd^qx^{pq}\right)\,dx,x,e+fx\right)}{f^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{6ab^2(fg-eh)p^2q^2x}{f} - \frac{6b^3(fg-eh)p^3q^3x}{f} - \frac{3b^3hp^3q^3(e+fx)^2}{8f^2} + \frac{6b^3(fg-eh)p^2q^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f^2} + \frac{3b^2hp^2q^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{4f^2} - \frac{3b(fg-eh)pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2} - \frac{3bhpq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4f^2} + \frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^2} + \frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{2f^2}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.75

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3 dx = \frac{8(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3 + 4h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3 - 24b(fg-e}{}$$

```
[In] Integrate[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

```
[Out] (8*(f*g - e*h)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3 + 4*h*(e + f*x)
^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3 - 24*b*(f*g - e*h)*p*q*((e + f*x)*(a
+ b*Log[c*(d*(e + f*x)^p)^q])^2 - 2*b*p*q*(f*(a - b*p*q)*x + b*(e + f*x)*Lo
g[c*(d*(e + f*x)^p)^q])) - 3*b*h*p*q*(2*(e + f*x)^2*(a + b*Log[c*(d*(e + f*
x)^p)^q])^2 + b*p*q*(b*f*p*q*x*(2*e + f*x) - 2*(e + f*x)^2*(a + b*Log[c*(d*
(e + f*x)^p)^q]))))/(8*f^2)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. 2(298) = 596.

Time = 5.22 (sec) , antiderivative size = 1084, normalized size of antiderivative = 3.54

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1084 |

```
[In] int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-12*x^2*ln(c*(d*(f*x+e)^p)^q)*a^2*b*f^2*h-24*x*ln(c*(d*(f*x+e)^p)^q)^
2*a*b^2*f^2*g-24*x*ln(c*(d*(f*x+e)^p)^q)*a^2*b*f^2*g-24*ln(c*(d*(f*x+e)^p)^
q)^2*a*b^2*e*f*g+24*ln(c*(d*(f*x+e)^p)^q)*a^2*b*e*f*g+78*ln(f*x+e)*b^3*e^2*
h*p^3*q^3-36*ln(c*(d*(f*x+e)^p)^q)*b^3*e^2*h*p^2*q^2-12*x^2*ln(c*(d*(f*x+e)
^p)^q)^2*a*b^2*f^2*h+36*x*ln(c*(d*(f*x+e)^p)^q)*b^3*e*f*h*p^2*q^2+24*ln(c*(
d*(f*x+e)^p)^q)*a*b^2*e^2*h*p*q-96*ln(f*x+e)*b^3*e*f*g*p^3*q^3-60*ln(f*x+e)
*a*b^2*e^2*h*p^2*q^2+12*ln(f*x+e)*a^2*b*e^2*h*p*q-24*a^2*b*e*f*g*p*q+48*x*l
n(c*(d*(f*x+e)^p)^q)*a*b^2*f^2*g*p*q-12*x*a^2*b*e*f*h*p*q-48*ln(c*(d*(f*x+e
)^p)^q)*a*b^2*e*f*g*p*q+96*ln(f*x+e)*a*b^2*e*f*g*p^2*q^2-48*ln(f*x+e)*a^2*b
*e*f*g*p*q-24*x*ln(c*(d*(f*x+e)^p)^q)*a*b^2*e*f*h*p*q+12*x^2*ln(c*(d*(f*x+e
)^p)^q)*a*b^2*f^2*h*p*q-12*x*ln(c*(d*(f*x+e)^p)^q)^2*b^3*e*f*h*p*q+36*x*a*b
^2*e*f*h*p^2*q^2+48*a*b^2*e*f*g*p^2*q^2+42*b^3*e^2*h*p^3*q^3+8*a^3*e*f*g-6*
x^2*ln(c*(d*(f*x+e)^p)^q)*b^3*f^2*h*p^2*q^2-42*x*b^3*e*f*h*p^3*q^3+6*x^2*ln
(c*(d*(f*x+e)^p)^q)^2*b^3*f^2*h*p*q-6*x^2*a*b^2*f^2*h*p^2*q^2-48*x*ln(c*(d*
(f*x+e)^p)^q)*b^3*f^2*g*p^2*q^2+24*x*ln(c*(d*(f*x+e)^p)^q)^2*b^3*f^2*g*p*q-
48*x*a*b^2*f^2*g*p^2*q^2+48*ln(c*(d*(f*x+e)^p)^q)*b^3*e*f*g*p^2*q^2+6*x^2*a
^2*b*f^2*h*p*q+24*ln(c*(d*(f*x+e)^p)^q)^2*b^3*e*f*g*p*q+24*x*a^2*b*f^2*g*p*
q+4*ln(c*(d*(f*x+e)^p)^q)^3*b^3*e^2*h-4*x^2*a^3*f^2*h-8*x*a^3*f^2*g+3*x^2*b
^3*f^2*h*p^3*q^3+48*x*b^3*f^2*g*p^3*q^3-18*ln(c*(d*(f*x+e)^p)^q)^2*b^3*e^2*
h*p*q-36*a*b^2*e^2*h*p^2*q^2+12*a^2*b*e^2*h*p*q-48*b^3*e*f*g*p^3*q^3-4*x^2*
ln(c*(d*(f*x+e)^p)^q)^3*b^3*f^2*h-8*x*ln(c*(d*(f*x+e)^p)^q)^3*b^3*f^2*g-8*l
n(c*(d*(f*x+e)^p)^q)^3*b^3*e*f*g+12*ln(c*(d*(f*x+e)^p)^q)^2*a*b^2*e^2*h)/f^
2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1692 vs. $2(298) = 596$.

Time = 0.39 (sec) , antiderivative size = 1692, normalized size of antiderivative = 5.53

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^3*f^2*h*p^3*q^3*x^2 + 2*b^3*f^2*g*p^3*q^3*x + (2*b^3*e*f*g - b^3*
e^2*h)*p^3*q^3)*log(f*x + e)^3 + 4*(b^3*f^2*h*x^2 + 2*b^3*f^2*g*x)*log(c)^3
 + 4*(b^3*f^2*h*q^3*x^2 + 2*b^3*f^2*g*q^3*x)*log(d)^3 - (3*b^3*f^2*h*p^3*q^
3 - 6*a*b^2*f^2*h*p^2*q^2 + 6*a^2*b*f^2*h*p*q - 4*a^3*f^2*h)*x^2 - 6*((4*b^
3*e*f*g - 3*b^3*e^2*h)*p^3*q^3 - 2*(2*a*b^2*e*f*g - a*b^2*e^2*h)*p^2*q^2 +
(b^3*f^2*h*p^3*q^3 - 2*a*b^2*f^2*h*p^2*q^2)*x^2 - 2*(2*a*b^2*f^2*g*p^2*q^2
- (2*b^3*f^2*g - b^3*e*f*h)*p^3*q^3)*x - 2*(b^3*f^2*h*p^2*q^2*x^2 + 2*b^3*f
^2*g*p^2*q^2*x + (2*b^3*e*f*g - b^3*e^2*h)*p^2*q^2)*log(c) - 2*(b^3*f^2*h*p
^2*q^3*x^2 + 2*b^3*f^2*g*p^2*q^3*x + (2*b^3*e*f*g - b^3*e^2*h)*p^2*q^3)*log
(d))*log(f*x + e)^2 - 6*((b^3*f^2*h*p*q - 2*a*b^2*f^2*h)*x^2 - 2*(2*a*b^2*f
^2*g - (2*b^3*f^2*g - b^3*e*f*h)*p*q)*x)*log(c)^2 - 6*((b^3*f^2*h*p*q^3 - 2
```

```
*a*b^2*f^2*h*q^2)*x^2 - 2*(2*a*b^2*f^2*g*q^2 - (2*b^3*f^2*g - b^3*e*f*h)*p*
q^3)*x - 2*(b^3*f^2*h*q^2*x^2 + 2*b^3*f^2*g*q^2*x)*log(c))*log(d)^2 - 2*(3*
(8*b^3*f^2*g - 7*b^3*e*f*h)*p^3*q^3 - 4*a^3*f^2*g - 6*(4*a*b^2*f^2*g - 3*a*
b^2*e*f*h)*p^2*q^2 + 6*(2*a^2*b*f^2*g - a^2*b*e*f*h)*p*q)*x + 6*((8*b^3*e*f
*g - 7*b^3*e^2*h)*p^3*q^3 - 2*(4*a*b^2*e*f*g - 3*a*b^2*e^2*h)*p^2*q^2 + 2*(
2*a^2*b*e*f*g - a^2*b*e^2*h)*p*q + (b^3*f^2*h*p^3*q^3 - 2*a*b^2*f^2*h*p^2*q
^2 + 2*a^2*b*f^2*h*p*q)*x^2 + 2*(b^3*f^2*h*p*q*x^2 + 2*b^3*f^2*g*p*q*x + (2
*b^3*e*f*g - b^3*e^2*h)*p*q)*log(c)^2 + 2*(b^3*f^2*h*p*q^3*x^2 + 2*b^3*f^2*
g*p*q^3*x + (2*b^3*e*f*g - b^3*e^2*h)*p*q^3)*log(d)^2 + 2*(2*a^2*b*f^2*g*p*
q + (4*b^3*f^2*g - 3*b^3*e*f*h)*p^3*q^3 - 2*(2*a*b^2*f^2*g - a*b^2*e*f*h)*p
^2*q^2)*x - 2*((4*b^3*e*f*g - 3*b^3*e^2*h)*p^2*q^2 - 2*(2*a*b^2*e*f*g - a*b
^2*e^2*h)*p*q + (b^3*f^2*h*p^2*q^2 - 2*a*b^2*f^2*h*p*q)*x^2 - 2*(2*a*b^2*f^
2*g*p*q - (2*b^3*f^2*g - b^3*e*f*h)*p^2*q^2)*x)*log(c) - 2*((4*b^3*e*f*g -
3*b^3*e^2*h)*p^2*q^3 - 2*(2*a*b^2*e*f*g - a*b^2*e^2*h)*p*q^2 + (b^3*f^2*h*p
^2*q^3 - 2*a*b^2*f^2*h*p*q^2)*x^2 - 2*(2*a*b^2*f^2*g*p*q^2 - (2*b^3*f^2*g -
 b^3*e*f*h)*p^2*q^3)*x - 2*(b^3*f^2*h*p*q^2*x^2 + 2*b^3*f^2*g*p*q^2*x + (2*
b^3*e*f*g - b^3*e^2*h)*p*q^2)*log(c))*log(d))*log(f*x + e) + 6*((b^3*f^2*h*
p^2*q^2 - 2*a*b^2*f^2*h*p*q + 2*a^2*b*f^2*h)*x^2 + 2*(2*a^2*b*f^2*g + (4*b^
3*f^2*g - 3*b^3*e*f*h)*p^2*q^2 - 2*(2*a*b^2*f^2*g - a*b^2*e*f*h)*p*q)*x)*lo
g(c) + 6*((b^3*f^2*h*p^2*q^3 - 2*a*b^2*f^2*h*p*q^2 + 2*a^2*b*f^2*h*q)*x^2 +
 2*(b^3*f^2*h*q*x^2 + 2*b^3*f^2*g*q*x)*log(c)^2 + 2*(2*a^2*b*f^2*g*q + (4*b
^3*f^2*g - 3*b^3*e*f*h)*p^2*q^3 - 2*(2*a*b^2*f^2*g - a*b^2*e*f*h)*p*q^2)*x
- 2*((b^3*f^2*h*p*q^2 - 2*a*b^2*f^2*h*q)*x^2 - 2*(2*a*b^2*f^2*g*q - (2*b^3*
f^2*g - b^3*e*f*h)*p*q^2)*x)*log(c))*log(d))/f^2
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(299) = 598$.

Time = 2.58 (sec) , antiderivative size = 991, normalized size of antiderivative = 3.24

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)*(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
```

```
[Out] Piecewise((a**3*g*x + a**3*h*x**2/2 - 3*a**2*b*e**2*h*log(c*(d*(e + f*x)**p
)**q)/(2*f**2) + 3*a**2*b*e*g*log(c*(d*(e + f*x)**p)**q)/f + 3*a**2*b*e*h*p
*q*x/(2*f) - 3*a**2*b*g*p*q*x + 3*a**2*b*g*x*log(c*(d*(e + f*x)**p)**q) - 3
*a**2*b*h*p*q*x**2/4 + 3*a**2*b*h*x**2*log(c*(d*(e + f*x)**p)**q)/2 + 9*a*b
**2*e**2*h*p*q*log(c*(d*(e + f*x)**p)**q)/(2*f**2) - 3*a*b**2*e**2*h*log(c*
(d*(e + f*x)**p)**q)**2/(2*f**2) - 6*a*b**2*e*g*p*q*log(c*(d*(e + f*x)**p)*
*q)/f + 3*a*b**2*e*g*log(c*(d*(e + f*x)**p)**q)**2/f - 9*a*b**2*e*h*p**2*q*
*2*x/(2*f) + 3*a*b**2*e*h*p*q*x*log(c*(d*(e + f*x)**p)**q)/f + 6*a*b**2*g*p
**2*q**2*x - 6*a*b**2*g*p*q*x*log(c*(d*(e + f*x)**p)**q) + 3*a*b**2*g*x*log
(c*(d*(e + f*x)**p)**q)**2 + 3*a*b**2*h*p**2*q**2*x**2/4 - 3*a*b**2*h*p*q*x
```

```
**2*log(c*(d*(e + f*x)**p)**q)/2 + 3*a*b**2*h*x**2*log(c*(d*(e + f*x)**p)**
q)**2/2 - 21*b**3*e**2*h*p**2*q**2*log(c*(d*(e + f*x)**p)**q)/(4*f**2) + 9*
b**3*e**2*h*p*q*log(c*(d*(e + f*x)**p)**q)**2/(4*f**2) - b**3*e**2*h*log(c*
(d*(e + f*x)**p)**q)**3/(2*f**2) + 6*b**3*e*g*p**2*q**2*log(c*(d*(e + f*x)*
*p)**q)/f - 3*b**3*e*g*p*q*log(c*(d*(e + f*x)**p)**q)**2/f + b**3*e*g*log(c
*(d*(e + f*x)**p)**q)**3/f + 21*b**3*e*h*p**3*q**3*x/(4*f) - 9*b**3*e*h*p**
2*q**2*x*log(c*(d*(e + f*x)**p)**q)/(2*f) + 3*b**3*e*h*p*q*x*log(c*(d*(e +
f*x)**p)**q)**2/(2*f) - 6*b**3*g*p**3*q**3*x + 6*b**3*g*p**2*q**2*x*log(c*(
d*(e + f*x)**p)**q) - 3*b**3*g*p*q*x*log(c*(d*(e + f*x)**p)**q)**2 + b**3*g
*x*log(c*(d*(e + f*x)**p)**q)**3 - 3*b**3*h*p**3*q**3*x**2/8 + 3*b**3*h*p**
2*q**2*x**2*log(c*(d*(e + f*x)**p)**q)/4 - 3*b**3*h*p*q*x**2*log(c*(d*(e +
f*x)**p)**q)**2/4 + b**3*h*x**2*log(c*(d*(e + f*x)**p)**q)**3/2, Ne(f, 0)),
 ((a + b*log(c*(d*e**p)**q))**3*(g*x + h*x**2/2), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(298) = 596$.

Time = 0.23 (sec) , antiderivative size = 732, normalized size of antiderivative = 2.39

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx$$

$$= \frac{1}{2}b^3hx^2\log\left(((fx+e)^pd)^qc\right)^3 - 3a^2bfgpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)$$

$$-\frac{3}{4}a^2bfhpq\left(\frac{2e^2\log(fx+e)}{f^3}+\frac{fx^2-2ex}{f^2}\right)+\frac{3}{2}ab^2hx^2\log\left(((fx+e)^pd)^qc\right)^2$$

$$+b^3gx\log\left(((fx+e)^pd)^qc\right)^3+\frac{3}{2}a^2bhx^2\log\left(((fx+e)^pd)^qc\right)$$

$$+3ab^2gx\log\left(((fx+e)^pd)^qc\right)^2+\frac{1}{2}a^3hx^2+3a^2bgx\log\left(((fx+e)^pd)^qc\right)$$

$$-3\left(2fpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)\log\left(((fx+e)^pd)^qc\right)+\frac{(e\log(fx+e)^2-2fx+2e\log(fx+e))p^2q^2}{f}\right)a$$

$$-\left(3fpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)\log\left(((fx+e)^pd)^qc\right)^2-\left(\frac{(e\log(fx+e)^3+3e\log(fx+e)^2-6fx+6e}{f^2}\right.\right.$$

$$-\frac{3}{4}\left(2fpq\left(\frac{2e^2\log(fx+e)}{f^3}+\frac{fx^2-2ex}{f^2}\right)\log\left(((fx+e)^pd)^qc\right)-\frac{(f^2x^2+2e^2\log(fx+e)^2-6efx+}{f^2}\right.$$

$$-\frac{1}{8}\left(6fpq\left(\frac{2e^2\log(fx+e)}{f^3}+\frac{fx^2-2ex}{f^2}\right)\log\left(((fx+e)^pd)^qc\right)^2+\left(\frac{(4e^2\log(fx+e)^3+3f^2x^2+18e}{}\right.\right.$$

$$+a^3gx$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*h*x^2*log(((f*x + e)^p*d)^q*c)^3 - 3*a^2*b*f*g*p*q*(x/f - e*log(f*x
 + e)/f^2) - 3/4*a^2*b*f*h*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)/f^
2) + 3/2*a*b^2*h*x^2*log(((f*x + e)^p*d)^q*c)^2 + b^3*g*x*log(((f*x + e)^p*
d)^q*c)^3 + 3/2*a^2*b*h*x^2*log(((f*x + e)^p*d)^q*c) + 3*a*b^2*g*x*log(((f*
x + e)^p*d)^q*c)^2 + 1/2*a^3*h*x^2 + 3*a^2*b*g*x*log(((f*x + e)^p*d)^q*c) -
 3*(2*f*p*q*(x/f - e*log(f*x + e)/f^2)*log(((f*x + e)^p*d)^q*c) + (e*log(f*
x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p^2*q^2/f)*a*b^2*g - (3*f*p*q*(x/f - e
*log(f*x + e)/f^2)*log(((f*x + e)^p*d)^q*c)^2 - ((e*log(f*x + e)^3 + 3*e*lo
g(f*x + e)^2 - 6*f*x + 6*e*log(f*x + e))*p^2*q^2/f^2 - 3*(e*log(f*x + e)^2
- 2*f*x + 2*e*log(f*x + e))*p*q*log(((f*x + e)^p*d)^q*c)/f^2)*f*p*q)*b^3*g
- 3/4*(2*f*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2*e*x)/f^2)*log(((f*x + e
)^p*d)^q*c) - (f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x + 6*e^2*log(f*x + e
))*p^2*q^2/f^2)*a*b^2*h - 1/8*(6*f*p*q*(2*e^2*log(f*x + e)/f^3 + (f*x^2 - 2
*e*x)/f^2)*log(((f*x + e)^p*d)^q*c)^2 + ((4*e^2*log(f*x + e)^3 + 3*f^2*x^2
+ 18*e^2*log(f*x + e)^2 - 42*e*f*x + 42*e^2*log(f*x + e))*p^2*q^2/f^3 - 6*(
f^2*x^2 + 2*e^2*log(f*x + e)^2 - 6*e*f*x + 6*e^2*log(f*x + e))*p*q*log(((f*
x + e)^p*d)^q*c)/f^3)*f*p*q)*b^3*h + a^3*g*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2532 vs. $2(298) = 596$.

Time = 0.38 (sec) , antiderivative size = 2532, normalized size of antiderivative = 8.27

$$\int (g+hx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

```
[Out] (f*x + e)*b^3*g*p^3*q^3*log(f*x + e)^3/f + 1/2*(f*x + e)^2*b^3*h*p^3*q^3*lo
g(f*x + e)^3/f^2 - (f*x + e)*b^3*e*h*p^3*q^3*log(f*x + e)^3/f^2 - 3*(f*x +
e)*b^3*g*p^3*q^3*log(f*x + e)^2/f - 3/4*(f*x + e)^2*b^3*h*p^3*q^3*log(f*x +
 e)^2/f^2 + 3*(f*x + e)*b^3*e*h*p^3*q^3*log(f*x + e)^2/f^2 + 3*(f*x + e)*b^
3*g*p^2*q^3*log(f*x + e)^2*log(d)/f + 3/2*(f*x + e)^2*b^3*h*p^2*q^3*log(f*x
 + e)^2*log(d)/f^2 - 3*(f*x + e)*b^3*e*h*p^2*q^3*log(f*x + e)^2*log(d)/f^2
+ 6*(f*x + e)*b^3*g*p^3*q^3*log(f*x + e)/f + 3/4*(f*x + e)^2*b^3*h*p^3*q^3*
log(f*x + e)/f^2 - 6*(f*x + e)*b^3*e*h*p^3*q^3*log(f*x + e)/f^2 + 3*(f*x +
e)*b^3*g*p^2*q^2*log(f*x + e)^2*log(c)/f + 3/2*(f*x + e)^2*b^3*h*p^2*q^2*lo
g(f*x + e)^2*log(c)/f^2 - 3*(f*x + e)*b^3*e*h*p^2*q^2*log(f*x + e)^2*log(c)
/f^2 - 6*(f*x + e)*b^3*g*p^2*q^3*log(f*x + e)*log(d)/f - 3/2*(f*x + e)^2*b^
3*h*p^2*q^3*log(f*x + e)*log(d)/f^2 + 6*(f*x + e)*b^3*e*h*p^2*q^3*log(f*x +
 e)*log(d)/f^2 + 3*(f*x + e)*b^3*g*p*q^3*log(f*x + e)*log(d)^2/f + 3/2*(f*x
 + e)^2*b^3*h*p*q^3*log(f*x + e)*log(d)^2/f^2 - 3*(f*x + e)*b^3*e*h*p*q^3*l
og(f*x + e)*log(d)^2/f^2 - 6*(f*x + e)*b^3*g*p^3*q^3/f - 3/8*(f*x + e)^2*b^
3*h*p^3*q^3/f^2 + 6*(f*x + e)*b^3*e*h*p^3*q^3/f^2 + 3*(f*x + e)*a*b^2*g*p^2
*q^2*log(f*x + e)^2/f + 3/2*(f*x + e)^2*a*b^2*h*p^2*q^2*log(f*x + e)^2/f^2
```

- 3*(f*x + e)*a*b^2*e*h*p^2*q^2*log(f*x + e)^2/f^2 - 6*(f*x + e)*b^3*g*p^2*q^2*log(f*x + e)*log(c)/f - 3/2*(f*x + e)^2*b^3*h*p^2*q^2*log(f*x + e)*log(c)/f^2 + 6*(f*x + e)*b^3*e*h*p^2*q^2*log(f*x + e)*log(c)/f^2 + 6*(f*x + e)*b^3*g*p^2*q^3*log(d)/f + 3/4*(f*x + e)^2*b^3*h*p^2*q^3*log(d)/f^2 - 6*(f*x + e)*b^3*e*h*p^2*q^3*log(d)/f^2 + 6*(f*x + e)*b^3*g*p*q^2*log(f*x + e)*log(c)*log(d)/f + 3*(f*x + e)^2*b^3*h*p*q^2*log(f*x + e)*log(c)*log(d)/f^2 - 6*(f*x + e)*b^3*e*h*p*q^2*log(f*x + e)*log(c)*log(d)/f^2 - 3*(f*x + e)*b^3*g*p*q^3*log(d)^2/f - 3/4*(f*x + e)^2*b^3*h*p*q^3*log(d)^2/f^2 + 3*(f*x + e)*b^3*e*h*p*q^3*log(d)^2/f^2 + (f*x + e)*b^3*g*q^3*log(d)^3/f + 1/2*(f*x + e)^2*b^3*h*q^3*log(d)^3/f^2 - (f*x + e)*b^3*e*h*q^3*log(d)^3/f^2 - 6*(f*x + e)*a*b^2*g*p^2*q^2*log(f*x + e)/f - 3/2*(f*x + e)^2*a*b^2*h*p^2*q^2*log(f*x + e)/f^2 + 6*(f*x + e)*a*b^2*e*h*p^2*q^2*log(f*x + e)/f^2 + 6*(f*x + e)*b^3*g*p^2*q^2*log(c)/f + 3/4*(f*x + e)^2*b^3*h*p^2*q^2*log(c)/f^2 - 6*(f*x + e)*b^3*e*h*p^2*q^2*log(c)/f^2 + 3*(f*x + e)*b^3*g*p*q*log(f*x + e)*log(c)^2/f + 3/2*(f*x + e)^2*b^3*h*p*q*log(f*x + e)*log(c)^2/f^2 - 3*(f*x + e)*b^3*e*h*p*q*log(f*x + e)*log(c)^2/f^2 + 6*(f*x + e)*a*b^2*g*p*q^2*log(f*x + e)*log(d)/f + 3*(f*x + e)^2*a*b^2*h*p*q^2*log(f*x + e)*log(d)/f^2 - 6*(f*x + e)*a*b^2*e*h*p*q^2*log(f*x + e)*log(d)/f^2 - 6*(f*x + e)*b^3*g*p*q^2*log(c)*log(d)/f - 3/2*(f*x + e)^2*b^3*h*p*q^2*log(c)*log(d)/f^2 + 6*(f*x + e)*b^3*e*h*p*q^2*log(c)*log(d)/f^2 + 3*(f*x + e)*b^3*g*q^2*log(c)*log(d)^2/f + 3/2*(f*x + e)^2*b^3*h*q^2*log(c)*log(d)^2/f^2 - 3*(f*x + e)*b^3*e*h*q^2*log(c)*log(d)^2/f^2 + 6*(f*x + e)*a*b^2*g*p^2*q^2/f + 3/4*(f*x + e)^2*a*b^2*h*p^2*q^2/f^2 - 6*(f*x + e)*a*b^2*e*h*p^2*q^2/f^2 + 6*(f*x + e)*a*b^2*g*p*q*log(f*x + e)*log(c)/f + 3*(f*x + e)^2*a*b^2*h*p*q*log(f*x + e)*log(c)/f^2 - 6*(f*x + e)*a*b^2*e*h*p*q*log(f*x + e)*log(c)/f^2 - 3*(f*x + e)*b^3*g*p*q*log(c)^2/f - 3/4*(f*x + e)^2*b^3*h*p*q*log(c)^2/f^2 + 3*(f*x + e)*b^3*e*h*p*q*log(c)^2/f^2 - 6*(f*x + e)*a*b^2*g*p*q^2*log(d)/f - 3/2*(f*x + e)^2*a*b^2*h*p*q^2*log(d)/f^2 + 6*(f*x + e)*a*b^2*e*h*p*q^2*log(d)/f^2 + 3*(f*x + e)*b^3*g*q*log(c)^2*log(d)/f + 3/2*(f*x + e)^2*b^3*h*q*log(c)^2*log(d)/f^2 - 3*(f*x + e)*b^3*e*h*q*log(c)^2*log(d)/f^2 + 3*(f*x + e)*a*b^2*g*q^2*log(d)^2/f + 3/2*(f*x + e)^2*a*b^2*h*q^2*log(d)^2/f^2 - 3*(f*x + e)*a*b^2*e*h*q^2*log(d)^2/f^2 + 3*(f*x + e)*a^2*b*g*p*q*log(f*x + e)/f + 3/2*(f*x + e)^2*a^2*b*h*p*q*log(f*x + e)/f^2 - 3*(f*x + e)*a^2*b*e*h*p*q*log(f*x + e)/f^2 - 6*(f*x + e)*a*b^2*g*p*q*log(c)/f - 3/2*(f*x + e)^2*a*b^2*h*p*q*log(c)/f^2 + 6*(f*x + e)*a*b^2*e*h*p*q*log(c)/f^2 + (f*x + e)*b^3*g*log(c)^3/f + 1/2*(f*x + e)^2*b^3*h*log(c)^3/f^2 - (f*x + e)*b^3*e*h*log(c)^3/f^2 + 6*(f*x + e)*a*b^2*g*q*log(c)*log(d)/f + 3*(f*x + e)^2*a*b^2*h*q*log(c)*log(d)/f^2 - 6*(f*x + e)*a*b^2*e*h*q*log(c)*log(d)/f^2 - 3*(f*x + e)*a^2*b*g*p*q/f - 3/4*(f*x + e)^2*a^2*b*h*p*q/f^2 + 3*(f*x + e)*a^2*b*e*h*p*q/f^2 + 3*(f*x + e)*a*b^2*g*log(c)^2/f + 3/2*(f*x + e)^2*a*b^2*h*log(c)^2/f^2 - 3*(f*x + e)*a*b^2*e*h*log(c)^2/f^2 + 3*(f*x + e)*a^2*b*g*q*log(d)/f + 3/2*(f*x + e)^2*a^2*b*h*q*log(d)/f^2 - 3*(f*x + e)*a^2*b*e*h*q*log(d)/f^2 + 3*(f*x + e)*a^2*b*g*log(c)/f + 3/2*(f*x + e)^2*a^2*b*h*log(c)/f^2 - 3*(f*x + e)*a^2*b*e*h*log(c)/f^2 + (f*x + e)*a^3*g/f + 1/2*(f*x + e)^2*a^3*h/f^2 - (f*x + e)*a^3*e*h/f^2

## Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.13

$$\int (g+hx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3 \, dx = x \left(\frac{4 a^3 e h + 4 a^3 f g + 18 b^3 e h p^3 q^3 - 24 b^3 f g p^3 q^3 - 12 a^2 b f g p q - 12 a b^2 e h p^2 q^2 + 24 a b^2 f g p^2 q^2}{4 f} - \frac{e h \left(4 a^3 - 6 a^2 b p q + 6 a b^2 p^2 q^2 - 3 b^3 p^3 q^3\right)}{4 f}\right) + \ln\left(c \left(d \left(e + f x\right)^p\right)^q\right)^2 \left(\frac{x \left(\frac{6 b^2 (a e h + a f g - b f g p q)}{f} - \frac{3 b^2 e h (2 a - b p q)}{f}\right)}{2} - \frac{3 e \left(2 a b^2 e h - 4 a b^2 f g - 3 b^3 e h p q + 4 b^3 f g p q\right)}{4 f^2} + \frac{3 b^2 h x^2 (2 a - b p q)}{4}\right) + \ln\left(c \left(d \left(e + f x\right)^p\right)^q\right)^3 \left(\frac{b^3 h x^2}{2} - \frac{e \left(b^3 e h - 2 b^3 f g\right)}{2 f^2} + b^3 g x\right) + \frac{\ln\left(c \left(d \left(e + f x\right)^p\right)^q\right) \left(x^2 \left(6 a^2 b f g + \frac{3 b e h \left(2 a^2 - 2 a b p q + b^2 p^2 q^2\right)}{2} - 9 b^3 e h p^2 q^2 + 12 b^3 f g p^2 q^2 + 6 a b^2 e h \cdots\right.\right.}{2 e \cdots} + \frac{h x^2 \left(4 a^3 - 6 a^2 b p q + 6 a b^2 p^2 q^2 - 3 b^3 p^3 q^3\right)}{8} - \frac{\ln\left(e + f x\right) \left(6 h a^2 b e^2 p q - 12 f g a^2 b e p q - 18 h a b^2 e^2 p^2 q^2 + 24 f g a b^2 e p^2 q^2 + 21 h b^3 e^2 p^3 q^3 - 24 \cdots\right)}{4 f^2}$$

```
[In] int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^3,x)
```

```
[Out] x*((4*a^3*e*h + 4*a^3*f*g + 18*b^3*e*h*p^3*q^3 - 24*b^3*f*g*p^3*q^3 - 12*a^
2*b*f*g*p*q - 12*a*b^2*e*h*p^2*q^2 + 24*a*b^2*f*g*p^2*q^2)/(4*f) - (e*h*(4*
a^3 - 3*b^3*p^3*q^3 + 6*a*b^2*p^2*q^2 - 6*a^2*b*p*q))/(4*f)) + log(c*(d*(e
+ f*x)^p)^q)^2*((x*((6*b^2*(a*e*h + a*f*g - b*f*g*p*q))/f - (3*b^2*e*h*(2*a
 - b*p*q))/f))/2 - (3*e*(2*a*b^2*e*h - 4*a*b^2*f*g - 3*b^3*e*h*p*q + 4*b^3*
f*g*p*q))/(4*f^2) + (3*b^2*h*x^2*(2*a - b*p*q))/4) + log(c*(d*(e + f*x)^p)^
q)^3*((b^3*h*x^2)/2 - (e*(b^3*e*h - 2*b^3*f*g))/(2*f^2) + b^3*g*x) + (log(c
*(d*(e + f*x)^p)^q)*(x^2*(6*a^2*b*f*g + (3*b*e*h*(2*a^2 + b^2*p^2*q^2 - 2*a
*b*p*q))/2 - 9*b^3*e*h*p^2*q^2 + 12*b^3*f*g*p^2*q^2 + 6*a*b^2*e*h*p*q - 12*
a*b^2*f*g*p*q) + (3*e*x*(2*a^2*b*f*g - 3*b^3*e*h*p^2*q^2 + 4*b^3*f*g*p^2*q^
2 + 2*a*b^2*e*h*p*q - 4*a*b^2*f*g*p*q))/f + (3*b*f*h*x^3*(2*a^2 + b^2*p^2*q
^2 - 2*a*b*p*q))/2))/(2*e + 2*f*x) + (h*x^2*(4*a^3 - 3*b^3*p^3*q^3 + 6*a*b^
2*p^2*q^2 - 6*a^2*b*p*q))/8 - (log(e + f*x)*(21*b^3*e^2*h*p^3*q^3 - 18*a*b^
2*e^2*h*p^2*q^2 + 6*a^2*b*e^2*h*p*q - 24*b^3*e*f*g*p^3*q^3 + 24*a*b^2*e*f*g
*p^2*q^2 - 12*a^2*b*e*f*g*p*q))/(4*f^2)
```

# 3.437 $\int (a+b\log(c(d(e+fx)^p)^q))^3\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 121

$$\int (a+b\log(c(d(e+fx)^p)^q))^3\,dx = 6ab^2p^2q^2x-6b^3p^3q^3x+\frac{6b^3p^2q^2(e+fx)\log(c(d(e+fx)^p)^q)}{f}-\frac{3bpq(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{f}+\frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3}{f}$$

```
[Out] 6*a*b^2*p^2*q^2*x-6*b^3*p^3*q^3*x+6*b^3*p^2*q^2*(f*x+e)*ln(c*(d*(f*x+e)^p)^
q)/f-3*b*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f+(f*x+e)*(a+b*ln(c*(d*(
f*x+e)^p)^q))^3/f
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2436, 2333, 2332, 2495}

$$\int (a+b\log(c(d(e+fx)^p)^q))^3\,dx = 6ab^2p^2q^2x-\frac{3bpq(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{f}+\frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3}{f}+\frac{6b^3p^2q^2(e+fx)\log(c(d(e+fx)^p)^q)}{f}-6b^3p^3q^3x$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

```
[Out] 6*a*b^2*p^2*q^2*x - 6*b^3*p^3*q^3*x + (6*b^3*p^2*q^2*(e + f*x)*Log[c*(d*(e
+ f*x)^p)^q])/f - (3*b*p*q*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/f
+ ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/f
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right)^3\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f} - \text{Subst}\left(\frac{(3bpq)\text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right)^2\,dx, x, e+fx\right)}{f}, cd^q(e + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{3bpq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f}+\frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f} + \text{Subst}\left(\frac{(6b^2p^2q^2)\,\text{Subst}\left(\int\left(a+b\log\left(cd^qx^{pq}\right)\right)\,dx,x,e+fx\right)}{f},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= 6ab^2p^2q^2x-\frac{3bpq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f}+\frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f} + \text{Subst}\left(\frac{(6b^3p^2q^2)\,\text{Subst}\left(\int\log\left(cd^qx^{pq}\right)\,dx,x,e+fx\right)}{f},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= 6ab^2p^2q^2x-6b^3p^3q^3x+\frac{6b^3p^2q^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f}-\frac{3bpq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f}+\frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\,dx = \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3-3bpq\left((e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2-2bpq(f(a-bpq)x+\right.}{f}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

```
[Out] ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3 - 3*b*p*q*((e + f*x)*(a + b*L
og[c*(d*(e + f*x)^p)^q])^2 - 2*b*p*q*(f*(a - b*p*q)*x + b*(e + f*x)*Log[c*(
d*(e + f*x)^p)^q])))/f
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(121) = 242$.

Time = 1.51 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.19

| method | result |
|---|---|
| parallelrisch | $\frac{6\ln(fx+e)b^3e^2p^3q^3-6x\,b^3ef\,p^3q^3+6x\ln(c(d(fx+e)^p)^q)b^3ef\,p^2q^2+6b^3e^2p^3q^3-6\ln(fx+e)a\,b^2e^2p^2q^2-3x\ln(c(d(fx+e)^p)^q)^2b}{}$ |

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (6*ln(f*x+e)*b^3*e^2*p^3*q^3-6*x*b^3*e*f*p^3*q^3+6*x*ln(c*(d*(f*x+e)^p)^q)*
b^3*e*f*p^2*q^2+6*b^3*e^2*p^3*q^3-6*ln(f*x+e)*a*b^2*e^2*p^2*q^2-3*x*ln(c*(d
*(f*x+e)^p)^q)^2*b^3*e*f*p*q+6*x*a*b^2*e*f*p^2*q^2+x*ln(c*(d*(f*x+e)^p)^q)^
3*b^3*e*f-6*x*ln(c*(d*(f*x+e)^p)^q)*a*b^2*e*f*p*q-3*ln(c*(d*(f*x+e)^p)^q)^2
*b^3*e^2*p*q-6*a*b^2*e^2*p^2*q^2+3*ln(f*x+e)*a^2*b*e^2*p*q+3*x*ln(c*(d*(f*x
+e)^p)^q)^2*a*b^2*e*f-3*x*a^2*b*e*f*p*q+ln(c*(d*(f*x+e)^p)^q)^3*b^3*e^2+3*x
*ln(c*(d*(f*x+e)^p)^q)*a^2*b*e*f+3*ln(c*(d*(f*x+e)^p)^q)^2*a*b^2*e^2+3*a^2*
b*e^2*p*q+x*a^3*e*f-a^3*e^2)/e/f
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(121) = 242$.

Time = 0.30 (sec) , antiderivative size = 639, normalized size of antiderivative = 5.28

$$\int (a+b\log(c(d(e+fx)^p)^q))^3\,dx$$
$$=\frac{b^3fq^3x\log(d)^3+b^3fx\log(c)^3+(b^3fp^3q^3x+b^3ep^3q^3)\log(fx+e)^3-3(b^3fpq-ab^2f)x\log(c)^2-3(b^3ep}{}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")
```

```
[Out] (b^3*f*q^3*x*log(d)^3 + b^3*f*x*log(c)^3 + (b^3*f*p^3*q^3*x + b^3*e*p^3*q^3
)*log(f*x + e)^3 - 3*(b^3*f*p*q - a*b^2*f)*x*log(c)^2 - 3*(b^3*e*p^3*q^3 -
a*b^2*e*p^2*q^2 + (b^3*f*p^3*q^3 - a*b^2*f*p^2*q^2)*x - (b^3*f*p^2*q^2*x +
b^3*e*p^2*q^2)*log(c) - (b^3*f*p^2*q^3*x + b^3*e*p^2*q^3)*log(d))*log(f*x +
 e)^2 + 3*(2*b^3*f*p^2*q^2 - 2*a*b^2*f*p*q + a^2*b*f)*x*log(c) + 3*(b^3*f*q
^2*x*log(c) - (b^3*f*p*q^3 - a*b^2*f*q^2)*x)*log(d)^2 - (6*b^3*f*p^3*q^3 -
6*a*b^2*f*p^2*q^2 + 3*a^2*b*f*p*q - a^3*f)*x + 3*(2*b^3*e*p^3*q^3 - 2*a*b^2
*e*p^2*q^2 + a^2*b*e*p*q + (b^3*f*p*q*x + b^3*e*p*q)*log(c)^2 + (b^3*f*p*q^
3*x + b^3*e*p*q^3)*log(d)^2 + (2*b^3*f*p^3*q^3 - 2*a*b^2*f*p^2*q^2 + a^2*b*
f*p*q)*x - 2*(b^3*e*p^2*q^2 - a*b^2*e*p*q + (b^3*f*p^2*q^2 - a*b^2*f*p*q)*x
)*log(c) - 2*(b^3*e*p^2*q^3 - a*b^2*e*p*q^2 + (b^3*f*p^2*q^3 - a*b^2*f*p*q^
2)*x - (b^3*f*p*q^2*x + b^3*e*p*q^2)*log(c))*log(d))*log(f*x + e) + 3*(b^3*
f*q*x*log(c)^2 - 2*(b^3*f*p*q^2 - a*b^2*f*q)*x*log(c) + (2*b^3*f*p^2*q^3 -
2*a*b^2*f*p*q^2 + a^2*b*f*q)*x)*log(d))/f
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(117) = 234.

Time = 1.02 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.98

$$\int (a+b\log(c(d(e+fx)^p)^q))^3\,dx$$
$$=\begin{cases} a^3x+\frac{3a^2be\log(c(d(e+fx)^p)^q)}{f}-3a^2bpqx+3a^2bx\log(c(d(e+fx)^p)^q)-\frac{6ab^2epq\log(c(d(e+fx)^p)^q)}{f}+\frac{3ab^2e\log(c(d(e+\dots}{f} \\ x(a+b\log(c(de^p)^q))^3 \end{cases}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*e*log(c*(d*(e + f*x)**p)**q)/f - 3*a**2*b*p*q*
x + 3*a**2*b*x*log(c*(d*(e + f*x)**p)**q) - 6*a*b**2*e*p*q*log(c*(d*(e + f*
x)**p)**q)/f + 3*a*b**2*e*log(c*(d*(e + f*x)**p)**q)**2/f + 6*a*b**2*p**2*q
**2*x - 6*a*b**2*p*q*x*log(c*(d*(e + f*x)**p)**q) + 3*a*b**2*x*log(c*(d*(e
+ f*x)**p)**q)**2 + 6*b**3*e*p**2*q**2*log(c*(d*(e + f*x)**p)**q)/f - 3*b**
3*e*p*q*log(c*(d*(e + f*x)**p)**q)**2/f + b**3*e*log(c*(d*(e + f*x)**p)**q)
**3/f - 6*b**3*p**3*q**3*x + 6*b**3*p**2*q**2*x*log(c*(d*(e + f*x)**p)**q)
- 3*b**3*p*q*x*log(c*(d*(e + f*x)**p)**q)**2 + b**3*x*log(c*(d*(e + f*x)**p
)**q)**3, Ne(f, 0)), (x*(a + b*log(c*(d*e**p)**q))**3, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(121) = 242.

Time = 0.20 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.62

$$\int (a+b\log(c(d(e+fx)^p)^q))^3\,dx=-3\,a^2bfpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)$$
$$+b^3x\log(((fx+e)^pd)^q\,c)^3+3\,ab^2x\log(((fx+e)^pd)^q\,c)^2+3\,a^2bx\log(((fx+e)^pd)^q\,c)$$
$$-3\left(2\,fpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)\log(((fx+e)^pd)^q\,c)+\frac{(e\log(fx+e)^2-2\,fx+2\,e\log(fx+e))p^2q^2}{f}\right)a\dots$$
$$-\left(3\,fpq\left(\frac{x}{f}-\frac{e\log(fx+e)}{f^2}\right)\log(((fx+e)^pd)^q\,c)^2-\left(\frac{(e\log(fx+e)^3+3\,e\log(fx+e)^2-6\,fx+6\,e\dots}{f^2}\right.\right.$$
$$+a^3x$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
```

```
[Out] -3*a^2*b*f*p*q*(x/f - e*log(f*x + e)/f^2) + b^3*x*log(((f*x + e)^p*d)^q*c)^
3 + 3*a*b^2*x*log(((f*x + e)^p*d)^q*c)^2 + 3*a^2*b*x*log(((f*x + e)^p*d)^q*
c) - 3*(2*f*p*q*(x/f - e*log(f*x + e)/f^2)*log(((f*x + e)^p*d)^q*c) + (e*lo
```

```
g(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p^2*q^2/f)*a*b^2 - (3*f*p*q*(x/f -
 e*log(f*x + e)/f^2)*log(((f*x + e)^p*d)^q*c)^2 - ((e*log(f*x + e)^3 + 3*e*
log(f*x + e)^2 - 6*f*x + 6*e*log(f*x + e))*p^2*q^2/f^2 - 3*(e*log(f*x + e)^
2 - 2*f*x + 2*e*log(f*x + e))*p*q*log(((f*x + e)^p*d)^q*c)/f^2)*f*p*q)*b^3
+ a^3*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(121) = 242$.

Time = 0.32 (sec) , antiderivative size = 772, normalized size of antiderivative = 6.38

$$\int (a+b\log(c(d(e+fx)^p)^q))^3\,dx = \frac{(fx+e)b^3p^3q^3\log(fx+e)^3}{f} - \frac{3(fx+e)b^3p^3q^3\log(fx+e)^2}{f} + \frac{3(fx+e)b^3p^2q^3\log(fx+e)^2\log(d)}{f} + \frac{6(fx+e)b^3p^3q^3\log(fx+e)}{f} + \frac{3(fx+e)b^3p^2q^2\log(fx+e)^2\log(c)}{f} - \frac{6(fx+e)b^3p^2q^3\log(fx+e)\log(d)}{f} + \frac{3(fx+e)b^3pq^3\log(fx+e)\log(d)^2}{f} - \frac{6(fx+e)b^3p^3q^3}{f} + \frac{3(fx+e)ab^2p^2q^2\log(fx+e)^2}{f} - \frac{6(fx+e)b^3p^2q^2\log(fx+e)\log(c)}{f} + \frac{6(fx+e)b^3p^2q^3\log(d)}{f} + \frac{6(fx+e)b^3pq^2\log(fx+e)\log(c)\log(d)}{f} - \frac{3(fx+e)b^3pq^3\log(d)^2}{f} + \frac{(fx+e)b^3q^3\log(d)^3}{f} - \frac{6(fx+e)ab^2p^2q^2\log(fx+e)}{f} + \frac{6(fx+e)b^3p^2q^2\log(c)}{f} + \frac{3(fx+e)b^3pq\log(fx+e)\log(c)^2}{f} + \frac{6(fx+e)ab^2pq^2\log(fx+e)\log(d)}{f} - \frac{6(fx+e)b^3pq^2\log(c)\log(d)}{f} + \frac{3(fx+e)b^3q^2\log(c)\log(d)^2}{f} + \frac{6(fx+e)ab^2p^2q^2}{f} + \frac{6(fx+e)ab^2pq\log(fx+e)\log(c)}{f} - \frac{3(fx+e)b^3pq\log(c)^2}{f} - \frac{6(fx+e)ab^2pq^2\log(d)}{f} + \frac{3(fx+e)b^3q\log(c)^2\log(d)}{f} + \frac{3(fx+e)ab^2q^2\log(d)^2}{f}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

```
[Out] (f*x + e)*b^3*p^3*q^3*log(f*x + e)^3/f - 3*(f*x + e)*b^3*p^3*q^3*log(f*x +
e)^2/f + 3*(f*x + e)*b^3*p^2*q^3*log(f*x + e)^2*log(d)/f + 6*(f*x + e)*b^3*
p^3*q^3*log(f*x + e)/f + 3*(f*x + e)*b^3*p^2*q^2*log(f*x + e)^2*log(c)/f -
6*(f*x + e)*b^3*p^2*q^3*log(f*x + e)*log(d)/f + 3*(f*x + e)*b^3*p*q^3*log(f
*x + e)*log(d)^2/f - 6*(f*x + e)*b^3*p^3*q^3/f + 3*(f*x + e)*a*b^2*p^2*q^2*
log(f*x + e)^2/f - 6*(f*x + e)*b^3*p^2*q^2*log(f*x + e)*log(c)/f + 6*(f*x +
 e)*b^3*p^2*q^3*log(d)/f + 6*(f*x + e)*b^3*p*q^2*log(f*x + e)*log(c)*log(d)
/f - 3*(f*x + e)*b^3*p*q^3*log(d)^2/f + (f*x + e)*b^3*q^3*log(d)^3/f - 6*(f
*x + e)*a*b^2*p^2*q^2*log(f*x + e)/f + 6*(f*x + e)*b^3*p^2*q^2*log(c)/f + 3
*(f*x + e)*b^3*p*q*log(f*x + e)*log(c)^2/f + 6*(f*x + e)*a*b^2*p*q^2*log(f*
x + e)*log(d)/f - 6*(f*x + e)*b^3*p*q^2*log(c)*log(d)/f + 3*(f*x + e)*b^3*q
^2*log(c)*log(d)^2/f + 6*(f*x + e)*a*b^2*p^2*q^2/f + 6*(f*x + e)*a*b^2*p*q*
log(f*x + e)*log(c)/f - 3*(f*x + e)*b^3*p*q*log(c)^2/f - 6*(f*x + e)*a*b^2*
p*q^2*log(d)/f + 3*(f*x + e)*b^3*q*log(c)^2*log(d)/f + 3*(f*x + e)*a*b^2*q^
2*log(d)^2/f + 3*(f*x + e)*a^2*b*p*q*log(f*x + e)/f - 6*(f*x + e)*a*b^2*p*q
*log(c)/f + (f*x + e)*b^3*log(c)^3/f + 6*(f*x + e)*a*b^2*q*log(c)*log(d)/f
- 3*(f*x + e)*a^2*b*p*q/f + 3*(f*x + e)*a*b^2*log(c)^2/f + 3*(f*x + e)*a^2*
b*q*log(d)/f + 3*(f*x + e)*a^2*b*log(c)/f + (f*x + e)*a^3/f
```

## Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.00

$$\int (a+b\log (c(d(e+fx)^p)^q))^3\,dx = x\left(a^3-3\,a^2\,b\,p\,q+6\,a\,b^2\,p^2\,q^2-6\,b^3\,p^3\,q^3\right) + \ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)^2\,\left(\frac{3\,\left(a\,b^2\,e-b^3\,e\,p\,q\right)}{f}+3\,b^2\,x\,\left(a-b\,p\,q\right)\right) + \ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)^3\,\left(b^3\,x+\frac{b^3\,e}{f}\right) + \frac{\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\,\left(3\,b\,f\,\left(a^2-2\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right)\,x^2+3\,b\,e\,\left(a^2-2\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right)\,x\right)}{e+f\,x} + \frac{\ln\left(e+f\,x\right)\,\left(3\,e\,a^2\,b\,p\,q-6\,e\,a\,b^2\,p^2\,q^2+6\,e\,b^3\,p^3\,q^3\right)}{f}$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^3,x)
```

```
[Out] x*(a^3 - 6*b^3*p^3*q^3 + 6*a*b^2*p^2*q^2 - 3*a^2*b*p*q) + log(c*(d*(e + f*x
)^p)^q)^2*((3*(a*b^2*e - b^3*e*p*q))/f + 3*b^2*x*(a - b*p*q)) + log(c*(d*(e
 + f*x)^p)^q)^3*(b^3*x + (b^3*e)/f) + (log(c*(d*(e + f*x)^p)^q)*(3*b*e*x*(a
^2 + 2*b^2*p^2*q^2 - 2*a*b*p*q) + 3*b*f*x^2*(a^2 + 2*b^2*p^2*q^2 - 2*a*b*p*
q)))/(e + f*x) + (log(e + f*x)*(6*b^3*e*p^3*q^3 - 6*a*b^2*e*p^2*q^2 + 3*a^2
*b*e*p*q))/f
```

# 3.438 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 177

$$\begin{aligned}\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx &= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}\\ &\quad+\frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h}\\ &\quad-\frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h}\\ &\quad+\frac{6b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h}\end{aligned}$$

```
[Out] (a+b*ln(c*(d*(f*x+e)^p)^q))^3*ln(f*(h*x+g)/(-e*h+f*g))/h+3*b*p*q*(a+b*ln(c*
(d*(f*x+e)^p)^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h-6*b^2*p^2*q^2*(a+b*l
n(c*(d*(f*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h+6*b^3*p^3*q^3*poly
log(4,-h*(f*x+e)/(-e*h+f*g))/h
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2443, 2481, 2421, 2430, 6724, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$$
$$=-\frac{6b^2p^2q^2\,\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h}$$
$$+\frac{3bpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{h}$$
$$+\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h}+\frac{6b^3p^3q^3\,\text{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x),x]
```

```
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(g + h*x))/(f*g - e*h)])/h + (3*
b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, -((h*(e + f*x))/(f*g -
e*h))])/h - (6*b^2*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[3, -((h
*(e + f*x))/(f*g - e*h))])/h + (6*b^3*p^3*q^3*PolyLog[4, -((h*(e + f*x))/(f
*g - e*h))])/h
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^3}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} - \text{Subst}\left(\frac{(3bfpq)\int\frac{(a+b\log(cd^q(e+fx)^{pq}))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} - \text{Subst}\left(\frac{(3bpq)\text{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))^2\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}dx,x,e+fx\right)}{h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \text{Subst}\left(\frac{(6b^2p^2q^2)\text{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\operatorname{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}dx,x,e+fx\right)}{h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} + \operatorname{Subst}\left(\frac{(6b^3p^3q^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{hx}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} + \frac{6b^3p^3q^3\operatorname{Li}_4\left(-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 646 vs. $2(177) = 354$.

Time = 0.23 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.65

$$\int\frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$$

$$= \frac{a^3\log(g+hx) - 3a^2bpq\log(e+fx)\log(g+hx) + 3ab^2p^2q^2\log^2(e+fx)\log(g+hx) - b^3p^3q^3\log^3(e+fx)}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x),x]
```

```
[Out] (a^3*Log[g + h*x] - 3*a^2*b*p*q*Log[e + f*x]*Log[g + h*x] + 3*a*b^2*p^2*q^2
*Log[e + f*x]^2*Log[g + h*x] - b^3*p^3*q^3*Log[e + f*x]^3*Log[g + h*x] + 3*
a^2*b*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 6*a*b^2*p*q*Log[e + f*x]*Log[
c*(d*(e + f*x)^p)^q]*Log[g + h*x] + 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(
e + f*x)^p)^q]*Log[g + h*x] + 3*a*b^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*
x] - 3*b^3*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + b^3*L
og[c*(d*(e + f*x)^p)^q]^3*Log[g + h*x] + 3*a^2*b*p*q*Log[e + f*x]*Log[(f*(g
```

```
 + h*x))/(f*g - e*h)] - 3*a*b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f
*g - e*h)] + b^3*p^3*q^3*Log[e + f*x]^3*Log[(f*(g + h*x))/(f*g - e*h)] + 6*
a*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*
h)] - 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x
))/(f*g - e*h)] + 3*b^3*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[(f*
(g + h*x))/(f*g - e*h)] + 3*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLo
g[2, (h*(e + f*x))/(-(f*g) + e*h)] - 6*b^2*p^2*q^2*(a + b*Log[c*(d*(e + f*x
)^p)^q])*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)] + 6*b^3*p^3*q^3*PolyLog[4
, (h*(e + f*x))/(-(f*g) + e*h)])/h
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{3}}{hx+g}dx$$

[In] `int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)`

[Out] `int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)`

## Fricas [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^{p})^{q}\right))^{3}}{g+hx}\,dx=\int \frac{\left(b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a\right)^{3}}{hx+g}\,dx$$

[In] `integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="fricas")`

[Out] `integral((b^3*log(((f*x + e)^p*d)^q*c)^3 + 3*a*b^2*log(((f*x + e)^p*d)^q*c)^2 + 3*a^2*b*log(((f*x + e)^p*d)^q*c) + a^3)/(h*x + g), x)`

## Sympy [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^{p})^{q}\right))^{3}}{g+hx}\,dx=\int \frac{\left(a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)\right)^{3}}{g+hx}\,dx$$

[In] `integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g),x)`

[Out] `Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3/(g + h*x), x)`

## Maxima [F]

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^3}{g + hx} \, dx = \int \frac{(b\log(((fx + e)^p d)^q c) + a)^3}{hx + g} \, dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="maxima")
```

```
[Out] a^3*log(h*x + g)/h + integrate((b^3*log(((f*x + e)^p)^q)^3 + 3*(q*log(d) +
log(c))*a^2*b + 3*(q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*a*b^2 + (q^
3*log(d)^3 + 3*q^2*log(c)*log(d)^2 + 3*q*log(c)^2*log(d) + log(c)^3)*b^3 +
3*((q*log(d) + log(c))*b^3 + a*b^2)*log(((f*x + e)^p)^q)^2 + 3*(2*(q*log(d)
 + log(c))*a*b^2 + (q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*b^3 + a^2*
b)*log(((f*x + e)^p)^q))/(h*x + g), x)
```

## Giac [F]

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^3}{g + hx} \, dx = \int \frac{(b\log(((fx + e)^p d)^q c) + a)^3}{hx + g} \, dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^3/(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^3}{g + hx} \, dx = \int \frac{(a + b \ln(c (d (e + f x)^p)^q))^3}{g + h x} \, dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x),x)
```

```
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x), x)
```

# 3.439 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 209

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx = \frac{(e+fx)\,(a+b\log(c(d(e+fx)^p)^q))^3}{(fg-eh)(g+hx)} - \frac{3bfpq(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} - \frac{6b^2fp^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)} + \frac{6b^3fp^3q^3\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)}$$

```
[Out] (f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/(-e*h+f*g)/(h*x+g)-3*b*f*p*q*(a+b*ln(
c*(d*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/h/(-e*h+f*g)-6*b^2*f*p^2*q^2
*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h/(-e*h+f*g)+
6*b^3*f*p^3*q^3*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h/(-e*h+f*g)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used

$= \{2444, 2443, 2481, 2421, 6724, 2495\}$

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx$$
$$= -\frac{6b^2fp^2q^2\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h(fg-eh)} - \frac{3bfpq\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{h(fg-eh)} + \frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(fg-eh)} + \frac{6b^3fp^3q^3\,\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x)^2,x]
```

```
[Out] ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/((f*g - e*h)*(g + h*x)) - (3
*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(g + h*x))/(f*g - e*h)])
/(h*(f*g - e*h)) - (6*b^2*f*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLo
g[2, -((h*(e + f*x))/(f*g - e*h))])/(h*(f*g - e*h)) + (6*b^3*f*p^3*q^3*Poly
Log[3, -((h*(e + f*x))/(f*g - e*h))])/(h*(f*g - e*h))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^3}{(g+hx)^2}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{(e+fx)\,(a+b\log(c(d(e+fx)^p)^q))^3}{(fg-eh)(g+hx)}\\
&\quad -\text{Subst}\left(\frac{(3bfpq)\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{g+hx}\,dx}{fg-eh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{(e+fx)\,(a+b\log(c(d(e+fx)^p)^q))^3}{(fg-eh)(g+hx)} - \frac{3bfpq(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)}\\
&\quad +\text{Subst}\left(\frac{(6b^2f^2p^2q^2)\int \frac{(a+b\log(cd^q(e+fx)^{pq}))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}
$$

$$= \frac{(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{(fg-eh)(g+hx)} - \frac{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} + \text{Subst}\left(\frac{(6b^2fp^2q^2)\,\text{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{h(fg-eh)}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{(fg-eh)(g+hx)} - \frac{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} - \frac{6b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)} + \text{Subst}\left(\frac{(6b^3fp^3q^3)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{h(fg-eh)}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{(fg-eh)(g+hx)} - \frac{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} - \frac{6b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)} + \frac{6b^3fp^3q^3\text{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 444 vs. 2(209) = 418.

Time = 0.34 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.12

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx$$
$$= \frac{-3b(fg-eh)pq\log(e+fx)\,(a-bpq\log(e+fx)+b\log(c(d(e+fx)^p)^q))^2+3bfpq(g+hx)\log(e+fx)}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x)^2,x]
```

```
[Out] (-3*b*(f*g - e*h)*p*q*Log[e + f*x]*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e
+ f*x)^p)^q])^2 + 3*b*f*p*q*(g + h*x)*Log[e + f*x]*(a - b*p*q*Log[e + f*x]
+ b*Log[c*(d*(e + f*x)^p)^q])^2 - (f*g - e*h)*(a - b*p*q*Log[e + f*x] + b*L
og[c*(d*(e + f*x)^p)^q])^3 - 3*b*f*p*q*(g + h*x)*(a - b*p*q*Log[e + f*x] +
b*Log[c*(d*(e + f*x)^p)^q])^2*Log[g + h*x] + 3*b^2*p^2*q^2*(a - b*p*q*Log[e
 + f*x] + b*Log[c*(d*(e + f*x)^p)^q])*(Log[e + f*x]*(h*(e + f*x)*Log[e + f*
x] - 2*f*(g + h*x)*Log[(f*(g + h*x))/(f*g - e*h)]) - 2*f*(g + h*x)*PolyLog[
2, (h*(e + f*x))/(-(f*g) + e*h)]) + b^3*p^3*q^3*(Log[e + f*x]^2*(h*(e + f*x
)*Log[e + f*x] - 3*f*(g + h*x)*Log[(f*(g + h*x))/(f*g - e*h)]) - 6*f*(g + h
*x)*Log[e + f*x]*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] + 6*f*(g + h*x)*P
olyLog[3, (h*(e + f*x))/(-(f*g) + e*h)]))/(h*(f*g - e*h)*(g + h*x))
```

## Maple [F]

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^3}{(hx+g)^2}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^2,x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{(hx+g)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(((f*x + e)^p*d)^q*c)^3 + 3*a*b^2*log(((f*x + e)^p*d)^q*c)
^2 + 3*a^2*b*log(((f*x + e)^p*d)^q*c) + a^3)/(h^2*x^2 + 2*g*h*x + g^2), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g)**2,x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3/(g + h*x)**2, x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{(hx+g)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^2,x, algorithm="maxima")
[Out] 3*a^2*b*f*p*q*(log(f*x + e)/(f*g*h - e*h^2) - log(h*x + g)/(f*g*h - e*h^2))
 - b^3*log(((f*x + e)^p)^q)^3/(h^2*x + g*h) - 3*a^2*b*log(((f*x + e)^p*d)^q
*c)/(h^2*x + g*h) - a^3/(h^2*x + g*h) + integrate((3*(e*h*q^2*log(d)^2 + 2*
e*h*q*log(c)*log(d) + e*h*log(c)^2)*a*b^2 + (e*h*q^3*log(d)^3 + 3*e*h*q^2*l
og(c)*log(d)^2 + 3*e*h*q*log(c)^2*log(d) + e*h*log(c)^3)*b^3 + 3*(a*b^2*e*h
 + (f*g*p*q + e*h*q*log(d) + e*h*log(c))*b^3 + (a*b^2*f*h + (f*h*p*q + f*h*
q*log(d) + f*h*log(c))*b^3)*x)*log(((f*x + e)^p)^q)^2 + (3*(f*h*q^2*log(d)^
2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*a*b^2 + (f*h*q^3*log(d)^3 + 3*f*h
*q^2*log(c)*log(d)^2 + 3*f*h*q*log(c)^2*log(d) + f*h*log(c)^3)*b^3)*x + 3*(
2*(e*h*q*log(d) + e*h*log(c))*a*b^2 + (e*h*q^2*log(d)^2 + 2*e*h*q*log(c)*lo
g(d) + e*h*log(c)^2)*b^3 + (2*(f*h*q*log(d) + f*h*log(c))*a*b^2 + (f*h*q^2*
log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*b^3)*x)*log(((f*x + e)^p)^
q))/(f*h^3*x^3 + e*g^2*h + (2*f*g*h^2 + e*h^3)*x^2 + (f*g^2*h + 2*e*g*h^2)*
x), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{(hx+g)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^2,x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^3/(h*x + g)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^3}{\left(g+h\,x\right)^2}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x)^2,x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x)^2, x)
```

# 3.440 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 376

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx = -\frac{3bfpq(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{2(fg-eh)^2(g+hx)} - \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2} + \frac{3b^2f^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)^2} - \frac{3bf^2pq(a+b\log(c(d(e+fx)^p)^q))^2\log\left(1+\frac{fg-eh}{h(e+fx)}\right)}{2h(fg-eh)^2} + \frac{3b^2f^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(2,-\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2} + \frac{3b^3f^2p^3q^3\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)^2} + \frac{3b^3f^2p^3q^3\operatorname{PolyLog}\left(3,-\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2}$$

```
[Out] -3/2*b*f*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(-e*h+f*g)^2/(h*x+g)-1/2
*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/h/(h*x+g)^2+3*b^2*f^2*p^2*q^2*(a+b*ln(c*(d*(
f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/h/(-e*h+f*g)^2-3/2*b*f^2*p*q*(a+b*ln
(c*(d*(f*x+e)^p)^q))^2*ln(1+(-e*h+f*g)/h/(f*x+e))/h/(-e*h+f*g)^2+3*b^2*f^2*
p^2*q^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(2,(e*h-f*g)/h/(f*x+e))/h/(-e*h+
f*g)^2+3*b^3*f^2*p^3*q^3*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h/(-e*h+f*g)^2+3*
b^3*f^2*p^3*q^3*polylog(3,(e*h-f*g)/h/(f*x+e))/h/(-e*h+f*g)^2
```

## Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx$$
$$= \frac{3b^2f^2p^2q^2\,\text{PolyLog}\left(2,-\frac{fg-eh}{h(e+fx)}\right)(a+b\log(c(d(e+fx)^p)^q))}{h(fg-eh)^2} + \frac{3b^2f^2p^2q^2\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h(fg-eh)^2} - \frac{3bf^2pq\log\left(\frac{fg-eh}{h(e+fx)}+1\right)(a+b\log(c(d(e+fx)^p)^q))^2}{2h(fg-eh)^2} - \frac{3bfpq(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{2(g+hx)(fg-eh)^2} - \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2} + \frac{3b^3f^2p^3q^3\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)^2} + \frac{3b^3f^2p^3q^3\,\text{PolyLog}\left(3,-\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x)^3,x]
```

```
[Out] (-3*b*f*p*q*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(2*(f*g - e*h)^2*
(g + h*x)) - (a + b*Log[c*(d*(e + f*x)^p)^q])^3/(2*h*(g + h*x)^2) + (3*b^2*
f^2*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)]
)/(h*(f*g - e*h)^2) - (3*b*f^2*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[1
 + (f*g - e*h)/(h*(e + f*x))])/(2*h*(f*g - e*h)^2) + (3*b^2*f^2*p^2*q^2*(a
+ b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, -((f*g - e*h)/(h*(e + f*x)))])/(h*
(f*g - e*h)^2) + (3*b^3*f^2*p^3*q^3*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))
])/(h*(f*g - e*h)^2) + (3*b^3*f^2*p^3*q^3*PolyLog[3, -((f*g - e*h)/(h*(e +
f*x)))])/(h*(f*g - e*h)^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
```

```
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{(g+hx)^3}\,dx, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&= -\frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{2h(g+hx)^2}\\
&\quad+\text{Subst}\left(\frac{(3bfpq)\int\frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{(e+fx)(g+hx)^2}\,dx}{2h}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&= -\frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{2h(g+hx)^2}\\
&\quad+\text{Subst}\left(\frac{(3bpq)\text{Subst}\left(\int\frac{\left(a+b\log\left(cd^qx^{pq}\right)\right)^2}{x\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^2}\,dx, x, e+fx\right)}{2h}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)
\end{aligned}$$

$$= -\frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2} - \text{Subst}\left(\frac{(3bpq)\text{Subst}\left(\int \frac{(a+b\log(cd^q x^{pq}))^2}{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^2}\,dx, x, e+fx\right)}{2(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(3bfpq)\text{Subst}\left(\int \frac{(a+b\log(cd^q x^{pq}))^2}{x\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}\,dx, x, e+fx\right)}{2h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{3bfpq(e+fx)\,(a+b\log(c(d(e+fx)^p)^q))^2}{2(fg-eh)^2(g+hx)} - \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{2h(g+hx)^2} - \frac{3bf^2pq(a+b\log(c(d(e+fx)^p)^q))^2\log\left(1+\frac{fg-eh}{h(e+fx)}\right)}{2h(fg-eh)^2} + \text{Subst}\left(\frac{(3b^2fp^2q^2)\,\text{Subst}\left(\int \frac{a+b\log(cd^q x^{pq})}{\frac{fg-eh}{f}+\frac{hx}{f}}\,dx, x, e+fx\right)}{(fg-eh)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(3b^2f^2p^2q^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{fg-eh}{hx}\right)(a+b\log(cd^q x^{pq}))}{x}\,dx, x, e+fx\right)}{h(fg-eh)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{3bfpq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2(fg-eh)^2(g+hx)} - \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{2h(g+hx)^2} + \frac{3b^2f^2p^2q^2(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)^2} - \frac{3bf^2pq(a+b\log\left(c(d(e+fx)^p)^q\right))^2\log\left(1+\frac{fg-eh}{h(e+fx)}\right)}{2h(fg-eh)^2} + \frac{3b^2f^2p^2q^2(a+b\log\left(c(d(e+fx)^p)^q\right))\operatorname{Li}_2\left(-\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2} - \operatorname{Subst}\left(\frac{(3b^3f^2p^3q^3)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{h(fg-eh)^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) - \operatorname{Subst}\left(\frac{(3b^3f^2p^3q^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{fg-eh}{hx}\right)}{x}\,dx,x,e+fx\right)}{h(fg-eh)^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= -\frac{3bfpq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2(fg-eh)^2(g+hx)} - \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{2h(g+hx)^2} + \frac{3b^2f^2p^2q^2(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)^2} - \frac{3bf^2pq(a+b\log\left(c(d(e+fx)^p)^q\right))^2\log\left(1+\frac{fg-eh}{h(e+fx)}\right)}{2h(fg-eh)^2} + \frac{3b^2f^2p^2q^2(a+b\log\left(c(d(e+fx)^p)^q\right))\operatorname{Li}_2\left(-\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2} + \frac{3b^3f^2p^3q^3\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)^2} + \frac{3b^3f^2p^3q^3\operatorname{Li}_3\left(-\frac{fg-eh}{h(e+fx)}\right)}{h(fg-eh)^2}$$

## Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.76

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx =$$
$$-\frac{-3bf(fg-eh)pq(g+hx)\left(a-bpq\log(e+fx)+b\log\left(c(d(e+fx)^p)^q\right)\right)^2+3b(fg-eh)^2pq\log(e+fx)}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x)^3,x]
```

```
[Out] -1/2*(-3*b*f*(f*g - e*h)*p*q*(g + h*x)*(a - b*p*q*Log[e + f*x] + b*Log[c*(d
*(e + f*x)^p)^q])^2 + 3*b*(f*g - e*h)^2*p*q*Log[e + f*x]*(a - b*p*q*Log[e +
 f*x] + b*Log[c*(d*(e + f*x)^p)^q])^2 - 3*b*f^2*p*q*(g + h*x)^2*Log[e + f*x
]*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])^2 + (f*g - e*h)^2*(
a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])^3 + 3*b*f^2*p*q*(g + h
*x)^2*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[g + h*x]
+ 3*b^2*p^2*q^2*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q])*(h*(e
 + f*x)*(e*h - f*(2*g + h*x))*Log[e + f*x]^2 - 2*f^2*(g + h*x)^2*Log[(f*(g
+ h*x))/(f*g - e*h)] + 2*f*(g + h*x)*Log[e + f*x]*(h*(e + f*x) + f*(g + h*x
)*Log[(f*(g + h*x))/(f*g - e*h)]) + 2*f^2*(g + h*x)^2*PolyLog[2, (h*(e + f*
x))/(-(f*g) + e*h)]) + b^3*p^3*q^3*(h*(e + f*x)*(e*h - f*(2*g + h*x))*Log[e
 + f*x]^3 + 3*f*(g + h*x)*Log[e + f*x]^2*(h*(e + f*x) + f*(g + h*x)*Log[(f*
(g + h*x))/(f*g - e*h)]) - 6*f^2*(g + h*x)^2*Log[e + f*x]*(Log[(f*(g + h*x)
)/(f*g - e*h)] - PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)]) - 6*f^2*(g + h*x
)^2*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] - 6*f^2*(g + h*x)^2*PolyLog[3,
 (h*(e + f*x))/(-(f*g) + e*h)]))/(h*(f*g - e*h)^2*(g + h*x)^2)
```

## Maple [F]

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^3}{(hx+g)^3}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^3,x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^3,x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx=\int \frac{(b\log(((fx+e)^pd)^q\,c)+a)^3}{(hx+g)^3}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^3,x, algorithm="fricas")
[Out] integral((b^3*log(((f*x + e)^p*d)^q*c)^3 + 3*a*b^2*log(((f*x + e)^p*d)^q*c)
^2 + 3*a^2*b*log(((f*x + e)^p*d)^q*c) + a^3)/(h^3*x^3 + 3*g*h^2*x^2 + 3*g^2
*h*x + g^3), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g)**3,x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3/(g + h*x)**3, x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)^3}\,dx=\int \frac{(b\log(((fx+e)^pd)^q\,c)+a)^3}{(hx+g)^3}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^3,x, algorithm="maxima")
[Out] 3/2*a^2*b*f*p*q*(f*log(f*x + e)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) - f*log
(h*x + g)/(f^2*g^2*h - 2*e*f*g*h^2 + e^2*h^3) + 1/(f*g^2*h - e*g*h^2 + (f*g
*h^2 - e*h^3)*x)) - 1/2*b^3*log(((f*x + e)^p)^q)^3/(h^3*x^2 + 2*g*h^2*x + g
^2*h) - 3/2*a^2*b*log(((f*x + e)^p*d)^q*c)/(h^3*x^2 + 2*g*h^2*x + g^2*h) -
1/2*a^3/(h^3*x^2 + 2*g*h^2*x + g^2*h) + integrate(1/2*(6*(e*h*q^2*log(d)^2
+ 2*e*h*q*log(c)*log(d) + e*h*log(c)^2)*a*b^2 + 2*(e*h*q^3*log(d)^3 + 3*e*h
*q^2*log(c)*log(d)^2 + 3*e*h*q*log(c)^2*log(d) + e*h*log(c)^3)*b^3 + 3*(2*a
*b^2*e*h + (f*g*p*q + 2*e*h*q*log(d) + 2*e*h*log(c))*b^3 + (2*a*b^2*f*h + (
f*h*p*q + 2*f*h*q*log(d) + 2*f*h*log(c))*b^3)*x)*log(((f*x + e)^p)^q)^2 + 2
*(3*(f*h*q^2*log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*a*b^2 + (f*h*
q^3*log(d)^3 + 3*f*h*q^2*log(c)*log(d)^2 + 3*f*h*q*log(c)^2*log(d) + f*h*lo
g(c)^3)*b^3)*x + 6*(2*(e*h*q*log(d) + e*h*log(c))*a*b^2 + (e*h*q^2*log(d)^2
 + 2*e*h*q*log(c)*log(d) + e*h*log(c)^2)*b^3 + (2*(f*h*q*log(d) + f*h*log(c
))*a*b^2 + (f*h*q^2*log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*b^3)*x
)*log(((f*x + e)^p)^q))/(f*h^4*x^4 + e*g^3*h + (3*f*g*h^3 + e*h^4)*x^3 + 3*
(f*g^2*h^2 + e*g*h^3)*x^2 + (f*g^3*h + 3*e*g^2*h^2)*x), x)
```

## Giac [F]

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^3}{(g + hx)^3}\,dx = \int \frac{(b\log(((fx + e)^p d)^q c) + a)^3}{(hx + g)^3}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)^3,x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^3/(h*x + g)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^3}{(g + hx)^3}\,dx = \int \frac{(a + b\,\ln(c\,(d\,(e + f\,x)^p)^q))^3}{(g + h\,x)^3}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x)^3,x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x)^3, x)
```

# 3.441 $\int (a+b\log(c(d(e+fx)^p)^q))^4\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 160

$$\int (a+b\log(c(d(e+fx)^p)^q))^4\,dx = -24ab^3p^3q^3x+24b^4p^4q^4x - \frac{24b^4p^3q^3(e+fx)\log(c(d(e+fx)^p)^q)}{f} + \frac{12b^2p^2q^2(e+fx)\left(a+b\log(c(d(e+fx)^p)^q)\right)^2}{f} - \frac{4bpq(e+fx)\left(a+b\log(c(d(e+fx)^p)^q)\right)^3}{f} + \frac{(e+fx)\left(a+b\log(c(d(e+fx)^p)^q)\right)^4}{f}$$

```
[Out] -24*a*b^3*p^3*q^3*x+24*b^4*p^4*q^4*x-24*b^4*p^3*q^3*(f*x+e)*ln(c*(d*(f*x+e)
^p)^q)/f+12*b^2*p^2*q^2*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f-4*b*p*q*(f*
x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f+(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^4/
f
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{2436, 2333, 2332, 2495\}$

$$\int (a+b\log(c(d(e+fx)^p)^q))^4\,dx = -24ab^3p^3q^3x + \frac{12b^2p^2q^2(e+fx)\left(a+b\log(c(d(e+fx)^p)^q)\right)^2}{f} - \frac{4bpq(e+fx)\left(a+b\log(c(d(e+fx)^p)^q)\right)^3}{f} + \frac{(e+fx)\left(a+b\log(c(d(e+fx)^p)^q)\right)^4}{f} - \frac{24b^4p^3q^3(e+fx)\log(c(d(e+fx)^p)^q)}{f} + 24b^4p^4q^4x$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^4,x]
```

```
[Out] -24*a*b^3*p^3*q^3*x + 24*b^4*p^4*q^4*x - (24*b^4*p^3*q^3*(e + f*x)*Log[c*(d
*(e + f*x)^p)^q])/f + (12*b^2*p^2*q^2*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p
)^q])^2)/f - (4*b*p*q*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/f + ((e
 + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^4)/f
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^4 \, dx, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right)^4 \, dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^4}{f} - \text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right)^3 \, dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= -\frac{4bpq(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{f} + \frac{(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^4}{f} + \text{Subst}\left(\frac{\left(12b^2p^2q^2\right)\text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right)^2 \, dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= \frac{12b^2p^2q^2(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{f} - \frac{4bpq(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{f} + \frac{(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^4}{f} - \text{Subst}\left(\frac{\left(24b^3p^3q^3\right)\text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right) \, dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= -24ab^3p^3q^3x + \frac{12b^2p^2q^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f} - \frac{4bpq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^4}{f} - \text{Subst}\left(\frac{(24b^4p^3q^3)\,\text{Subst}\left(\int\log\left(cd^qx^{pq}\right)\,dx,x,e+fx\right)}{f},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= -24ab^3p^3q^3x + 24b^4p^4q^4x - \frac{24b^4p^3q^3(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f} + \frac{12b^2p^2q^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f} - \frac{4bpq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^4}{f}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^4\,dx = \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^4 - 4bpq\Big((e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3 - 3bpq\Big((e+fx)(a+\cdots}{f}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^4,x]
```

```
[Out] ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^4 - 4*b*p*q*((e + f*x)*(a + b*L
og[c*(d*(e + f*x)^p)^q])^3 - 3*b*p*q*((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p
)^q])^2 - 2*b*p*q*(f*(a - b*p*q)*x + b*(e + f*x)*Log[c*(d*(e + f*x)^p)^q]))
))/f
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(160) = 320$.

Time = 5.17 (sec) , antiderivative size = 642, normalized size of antiderivative = 4.01

| method | result |
|---|---|
| parallelrisch | $\frac{24x\,b^4ef\,p^4q^4-12\ln(c(d(fx+e)^p)^q)^2a\,b^3e^2pq+24\ln(fx+e)a\,b^3e^2p^3q^3-24b^4e^2p^4q^4+12\ln(c(d(fx+e)^p)^q)^2b^4e^2p^2q^2-4\ln(c(d(}{}$ |

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^4,x,method=_RETURNVERBOSE)
```

```
[Out] (24*x*b^4*e*f*p^4*q^4-12*ln(c*(d*(f*x+e)^p)^q)^2*a*b^3*e^2*p*q+24*ln(f*x+e)
*a*b^3*e^2*p^3*q^3-24*b^4*e^2*p^4*q^4+12*ln(c*(d*(f*x+e)^p)^q)^2*b^4*e^2*p^
2*q^2-4*ln(c*(d*(f*x+e)^p)^q)^3*b^4*e^2*p*q-24*ln(f*x+e)*b^4*e^2*p^4*q^4+x*
ln(c*(d*(f*x+e)^p)^q)^4*b^4*e*f+24*x*ln(c*(d*(f*x+e)^p)^q)*a*b^3*e*f*p^2*q^
2+ln(c*(d*(f*x+e)^p)^q)^4*b^4*e^2-12*x*ln(c*(d*(f*x+e)^p)^q)^2*a*b^3*e*f*p*
q-12*x*ln(c*(d*(f*x+e)^p)^q)*a^2*b^2*e*f*p*q-a^4*e^2+4*ln(c*(d*(f*x+e)^p)^q
)^3*a*b^3*e^2+6*ln(c*(d*(f*x+e)^p)^q)^2*a^2*b^2*e^2+x*a^4*e*f+24*a*b^3*e^2*
p^3*q^3-12*a^2*b^2*e^2*p^2*q^2+4*a^3*b*e^2*p*q-24*x*ln(c*(d*(f*x+e)^p)^q)*b
^4*e*f*p^3*q^3+12*x*ln(c*(d*(f*x+e)^p)^q)^2*b^4*e*f*p^2*q^2-24*x*a*b^3*e*f*
p^3*q^3-4*x*ln(c*(d*(f*x+e)^p)^q)^3*b^4*e*f*p*q+12*x*a^2*b^2*e*f*p^2*q^2-4*
x*a^3*b*e*f*p*q-12*ln(f*x+e)*a^2*b^2*e^2*p^2*q^2+4*ln(f*x+e)*a^3*b*e^2*p*q+
4*x*ln(c*(d*(f*x+e)^p)^q)*a^3*b*e*f+4*x*ln(c*(d*(f*x+e)^p)^q)^3*a*b^3*e*f+6
*x*ln(c*(d*(f*x+e)^p)^q)^2*a^2*b^2*e*f)/e/f
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1409 vs. $2(160) = 320$.

Time = 0.31 (sec) , antiderivative size = 1409, normalized size of antiderivative = 8.81

$$\int (a + b\log(c(d(e+fx)^p)^q))^4\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4,x, algorithm="fricas")
```

```
[Out] (b^4*f*q^4*x*log(d)^4 + b^4*f*x*log(c)^4 + (b^4*f*p^4*q^4*x + b^4*e*p^4*q^4
)*log(f*x + e)^4 - 4*(b^4*f*p*q - a*b^3*f)*x*log(c)^3 - 4*(b^4*e*p^4*q^4 -
a*b^3*e*p^3*q^3 + (b^4*f*p^4*q^4 - a*b^3*f*p^3*q^3)*x - (b^4*f*p^3*q^3*x +
b^4*e*p^3*q^3)*log(c) - (b^4*f*p^3*q^4*x + b^4*e*p^3*q^4)*log(d))*log(f*x +
 e)^3 + 6*(2*b^4*f*p^2*q^2 - 2*a*b^3*f*p*q + a^2*b^2*f)*x*log(c)^2 + 4*(b^4
*f*q^3*x*log(c) - (b^4*f*p*q^4 - a*b^3*f*q^3)*x)*log(d)^3 + 6*(2*b^4*e*p^4*
q^4 - 2*a*b^3*e*p^3*q^3 + a^2*b^2*e*p^2*q^2 + (b^4*f*p^2*q^2*x + b^4*e*p^2*
q^2)*log(c)^2 + (b^4*f*p^2*q^4*x + b^4*e*p^2*q^4)*log(d)^2 + (2*b^4*f*p^4*q
^4 - 2*a*b^3*f*p^3*q^3 + a^2*b^2*f*p^2*q^2)*x - 2*(b^4*e*p^3*q^3 - a*b^3*e*
p^2*q^2 + (b^4*f*p^3*q^3 - a*b^3*f*p^2*q^2)*x)*log(c) - 2*(b^4*e*p^3*q^4 -
```

```
a*b^3*e*p^2*q^3 + (b^4*f*p^3*q^4 - a*b^3*f*p^2*q^3)*x - (b^4*f*p^2*q^3*x +
b^4*e*p^2*q^3)*log(c))*log(d))*log(f*x + e)^2 - 4*(6*b^4*f*p^3*q^3 - 6*a*b^
3*f*p^2*q^2 + 3*a^2*b^2*f*p*q - a^3*b*f)*x*log(c) + 6*(b^4*f*q^2*x*log(c)^2
 - 2*(b^4*f*p*q^3 - a*b^3*f*q^2)*x*log(c) + (2*b^4*f*p^2*q^4 - 2*a*b^3*f*p*
q^3 + a^2*b^2*f*q^2)*x)*log(d)^2 + (24*b^4*f*p^4*q^4 - 24*a*b^3*f*p^3*q^3 +
 12*a^2*b^2*f*p^2*q^2 - 4*a^3*b*f*p*q + a^4*f)*x - 4*(6*b^4*e*p^4*q^4 - 6*a
*b^3*e*p^3*q^3 + 3*a^2*b^2*e*p^2*q^2 - a^3*b*e*p*q - (b^4*f*p*q*x + b^4*e*p
*q)*log(c)^3 - (b^4*f*p*q^4*x + b^4*e*p*q^4)*log(d)^3 + 3*(b^4*e*p^2*q^2 -
a*b^3*e*p*q + (b^4*f*p^2*q^2 - a*b^3*f*p*q)*x)*log(c)^2 + 3*(b^4*e*p^2*q^4
- a*b^3*e*p*q^3 + (b^4*f*p^2*q^4 - a*b^3*f*p*q^3)*x - (b^4*f*p*q^3*x + b^4*
e*p*q^3)*log(c))*log(d)^2 + (6*b^4*f*p^4*q^4 - 6*a*b^3*f*p^3*q^3 + 3*a^2*b^
2*f*p^2*q^2 - a^3*b*f*p*q)*x - 3*(2*b^4*e*p^3*q^3 - 2*a*b^3*e*p^2*q^2 + a^2
*b^2*e*p*q + (2*b^4*f*p^3*q^3 - 2*a*b^3*f*p^2*q^2 + a^2*b^2*f*p*q)*x)*log(c
) - 3*(2*b^4*e*p^3*q^4 - 2*a*b^3*e*p^2*q^3 + a^2*b^2*e*p*q^2 + (b^4*f*p*q^2
*x + b^4*e*p*q^2)*log(c)^2 + (2*b^4*f*p^3*q^4 - 2*a*b^3*f*p^2*q^3 + a^2*b^2
*f*p*q^2)*x - 2*(b^4*e*p^2*q^3 - a*b^3*e*p*q^2 + (b^4*f*p^2*q^3 - a*b^3*f*p
*q^2)*x)*log(c))*log(d))*log(f*x + e) + 4*(b^4*f*q*x*log(c)^3 - 3*(b^4*f*p*
q^2 - a*b^3*f*q)*x*log(c)^2 + 3*(2*b^4*f*p^2*q^3 - 2*a*b^3*f*p*q^2 + a^2*b^
2*f*q)*x*log(c) - (6*b^4*f*p^3*q^4 - 6*a*b^3*f*p^2*q^3 + 3*a^2*b^2*f*p*q^2
- a^3*b*f*q)*x)*log(d))/f
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(155) = 310$.

Time = 2.07 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.81

$$\int (a + b\log(c(d(e+fx)^p)^q))^4 \, dx$$
$$= \begin{cases} a^4x + \frac{4a^3be\log(c(d(e+fx)^p)^q)}{f} - 4a^3bpqx + 4a^3bx\log(c(d(e+fx)^p)^q) - \frac{12a^2b^2epq\log(c(d(e+fx)^p)^q)}{f} + \frac{6a^2b^2e\log(c(d(}{f} \\ x(a+b\log(c(de^p)^q))^4 \end{cases}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*e*log(c*(d*(e + f*x)**p)**q)/f - 4*a**3*b*p*q*
x + 4*a**3*b*x*log(c*(d*(e + f*x)**p)**q) - 12*a**2*b**2*e*p*q*log(c*(d*(e
+ f*x)**p)**q)/f + 6*a**2*b**2*e*log(c*(d*(e + f*x)**p)**q)**2/f + 12*a**2*
b**2*p**2*q**2*x - 12*a**2*b**2*p*q*x*log(c*(d*(e + f*x)**p)**q) + 6*a**2*b
**2*x*log(c*(d*(e + f*x)**p)**q)**2 + 24*a*b**3*e*p**2*q**2*log(c*(d*(e + f
*x)**p)**q)/f - 12*a*b**3*e*p*q*log(c*(d*(e + f*x)**p)**q)**2/f + 4*a*b**3*
e*log(c*(d*(e + f*x)**p)**q)**3/f - 24*a*b**3*p**3*q**3*x + 24*a*b**3*p**2*
q**2*x*log(c*(d*(e + f*x)**p)**q) - 12*a*b**3*p*q*x*log(c*(d*(e + f*x)**p)*
*q)**2 + 4*a*b**3*x*log(c*(d*(e + f*x)**p)**q)**3 - 24*b**4*e*p**3*q**3*log
(c*(d*(e + f*x)**p)**q)/f + 12*b**4*e*p**2*q**2*log(c*(d*(e + f*x)**p)**q)*
```

```
*2/f - 4*b**4*e*p*q*log(c*(d*(e + f*x)**p)**q)**3/f + b**4*e*log(c*(d*(e +
f*x)**p)**q)**4/f + 24*b**4*p**4*q**4*x - 24*b**4*p**3*q**3*x*log(c*(d*(e +
 f*x)**p)**q) + 12*b**4*p**2*q**2*x*log(c*(d*(e + f*x)**p)**q)**2 - 4*b**4*
p*q*x*log(c*(d*(e + f*x)**p)**q)**3 + b**4*x*log(c*(d*(e + f*x)**p)**q)**4,
 Ne(f, 0)), (x*(a + b*log(c*(d*e**p)**q))**4, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(160) = 320$.

Time = 0.23 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.49

$$\int (a + b\log (c(d(e + fx)^p)^q))^4 \, dx$$
$$= b^4x\log \left(\left(\left(fx + e\right)^p d\right)^q c\right)^4 - 4\,a^3bfpq\left(\frac{x}{f} - \frac{e\log \left(fx + e\right)}{f^2}\right) + 4\,ab^3x\log \left(\left(\left(fx + e\right)^p d\right)^q c\right)^3 + 6\,a^2b^2x\log \left(\left(\left(fx + e\right)^p d\right)^q c\right)^2 + 4\,a^3bx\log \left(\left(\left(fx + e\right)^p d\right)^q c\right) - 6\left(2\,fpq\left(\frac{x}{f} - \frac{e\log \left(fx + e\right)}{f^2}\right)\log \left(\left(\left(fx + e\right)^p d\right)^q c\right) + \frac{\left(e\log \left(fx + e\right)^2 - 2\,fx + 2\,e\log \left(fx + e\right)\right)p^2q^2}{f}\right. - 4\left(3\,fpq\left(\frac{x}{f} - \frac{e\log \left(fx + e\right)}{f^2}\right)\log \left(\left(\left(fx + e\right)^p d\right)^q c\right)^2 - \left(\frac{\left(e\log \left(fx + e\right)^3 + 3\,e\log \left(fx + e\right)^2 - 6\,fx + \right.}{f^2}\right.\right. - \left(4\,fpq\left(\frac{x}{f} - \frac{e\log \left(fx + e\right)}{f^2}\right)\log \left(\left(\left(fx + e\right)^p d\right)^q c\right)^3 + \left(\left(\frac{\left(e\log \left(fx + e\right)^4 + 4\,e\log \left(fx + e\right)^3 + 12\,e\log\right.}{f^3}\right.\right.\right. + a^4x$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(((f*x + e)^p*d)^q*c)^4 - 4*a^3*b*f*p*q*(x/f - e*log(f*x + e)/f^2)
 + 4*a*b^3*x*log(((f*x + e)^p*d)^q*c)^3 + 6*a^2*b^2*x*log(((f*x + e)^p*d)^q
*c)^2 + 4*a^3*b*x*log(((f*x + e)^p*d)^q*c) - 6*(2*f*p*q*(x/f - e*log(f*x +
e)/f^2)*log(((f*x + e)^p*d)^q*c) + (e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x
+ e))*p^2*q^2/f)*a^2*b^2 - 4*(3*f*p*q*(x/f - e*log(f*x + e)/f^2)*log(((f*x
+ e)^p*d)^q*c)^2 - ((e*log(f*x + e)^3 + 3*e*log(f*x + e)^2 - 6*f*x + 6*e*lo
g(f*x + e))*p^2*q^2/f^2 - 3*(e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x + e))*p
*q*log(((f*x + e)^p*d)^q*c)/f^2)*f*p*q)*a*b^3 - (4*f*p*q*(x/f - e*log(f*x +
 e)/f^2)*log(((f*x + e)^p*d)^q*c)^3 + (((e*log(f*x + e)^4 + 4*e*log(f*x + e
)^3 + 12*e*log(f*x + e)^2 - 24*f*x + 24*e*log(f*x + e))*p^2*q^2/f^3 - 4*(e*
log(f*x + e)^3 + 3*e*log(f*x + e)^2 - 6*f*x + 6*e*log(f*x + e))*p*q*log(((f
*x + e)^p*d)^q*c)/f^3)*f*p*q + 6*(e*log(f*x + e)^2 - 2*f*x + 2*e*log(f*x +
e))*p*q*log(((f*x + e)^p*d)^q*c)^2/f^2)*f*p*q)*b^4 + a^4*x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1697 vs. $2(160) = 320$.

Time = 0.36 (sec) , antiderivative size = 1697, normalized size of antiderivative = 10.61

$$\int (a + b\log(c(d(e + fx)^p)^q))^4 \, dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4,x, algorithm="giac")
```

```
[Out] (f*x + e)*b^4*p^4*q^4*log(f*x + e)^4/f - 4*(f*x + e)*b^4*p^4*q^4*log(f*x +
e)^3/f + 4*(f*x + e)*b^4*p^3*q^4*log(f*x + e)^3*log(d)/f + 12*(f*x + e)*b^4
*p^4*q^4*log(f*x + e)^2/f + 4*(f*x + e)*b^4*p^3*q^3*log(f*x + e)^3*log(c)/f
 - 12*(f*x + e)*b^4*p^3*q^4*log(f*x + e)^2*log(d)/f + 6*(f*x + e)*b^4*p^2*q
^4*log(f*x + e)^2*log(d)^2/f - 24*(f*x + e)*b^4*p^4*q^4*log(f*x + e)/f + 4*
(f*x + e)*a*b^3*p^3*q^3*log(f*x + e)^3/f - 12*(f*x + e)*b^4*p^3*q^3*log(f*x
 + e)^2*log(c)/f + 24*(f*x + e)*b^4*p^3*q^4*log(f*x + e)*log(d)/f + 12*(f*x
 + e)*b^4*p^2*q^3*log(f*x + e)^2*log(c)*log(d)/f - 12*(f*x + e)*b^4*p^2*q^4
*log(f*x + e)*log(d)^2/f + 4*(f*x + e)*b^4*p*q^4*log(f*x + e)*log(d)^3/f +
24*(f*x + e)*b^4*p^4*q^4/f - 12*(f*x + e)*a*b^3*p^3*q^3*log(f*x + e)^2/f +
24*(f*x + e)*b^4*p^3*q^3*log(f*x + e)*log(c)/f + 6*(f*x + e)*b^4*p^2*q^2*lo
g(f*x + e)^2*log(c)^2/f - 24*(f*x + e)*b^4*p^3*q^4*log(d)/f + 12*(f*x + e)*
a*b^3*p^2*q^3*log(f*x + e)^2*log(d)/f - 24*(f*x + e)*b^4*p^2*q^3*log(f*x +
e)*log(c)*log(d)/f + 12*(f*x + e)*b^4*p^2*q^4*log(d)^2/f + 12*(f*x + e)*b^4
*p*q^3*log(f*x + e)*log(c)*log(d)^2/f - 4*(f*x + e)*b^4*p*q^4*log(d)^3/f +
(f*x + e)*b^4*q^4*log(d)^4/f + 24*(f*x + e)*a*b^3*p^3*q^3*log(f*x + e)/f -
24*(f*x + e)*b^4*p^3*q^3*log(c)/f + 12*(f*x + e)*a*b^3*p^2*q^2*log(f*x + e)
^2*log(c)/f - 12*(f*x + e)*b^4*p^2*q^2*log(f*x + e)*log(c)^2/f - 24*(f*x +
e)*a*b^3*p^2*q^3*log(f*x + e)*log(d)/f + 24*(f*x + e)*b^4*p^2*q^3*log(c)*lo
g(d)/f + 12*(f*x + e)*b^4*p*q^2*log(f*x + e)*log(c)^2*log(d)/f + 12*(f*x +
e)*a*b^3*p*q^3*log(f*x + e)*log(d)^2/f - 12*(f*x + e)*b^4*p*q^3*log(c)*log(
d)^2/f + 4*(f*x + e)*b^4*q^3*log(c)*log(d)^3/f - 24*(f*x + e)*a*b^3*p^3*q^3
/f + 6*(f*x + e)*a^2*b^2*p^2*q^2*log(f*x + e)^2/f - 24*(f*x + e)*a*b^3*p^2*
q^2*log(f*x + e)*log(c)/f + 12*(f*x + e)*b^4*p^2*q^2*log(c)^2/f + 4*(f*x +
e)*b^4*p*q*log(f*x + e)*log(c)^3/f + 24*(f*x + e)*a*b^3*p^2*q^3*log(d)/f +
24*(f*x + e)*a*b^3*p*q^2*log(f*x + e)*log(c)*log(d)/f - 12*(f*x + e)*b^4*p*
q^2*log(c)^2*log(d)/f - 12*(f*x + e)*a*b^3*p*q^3*log(d)^2/f + 6*(f*x + e)*b
^4*q^2*log(c)^2*log(d)^2/f + 4*(f*x + e)*a*b^3*q^3*log(d)^3/f - 12*(f*x + e
)*a^2*b^2*p^2*q^2*log(f*x + e)/f + 24*(f*x + e)*a*b^3*p^2*q^2*log(c)/f + 12
*(f*x + e)*a*b^3*p*q*log(f*x + e)*log(c)^2/f - 4*(f*x + e)*b^4*p*q*log(c)^3
/f + 12*(f*x + e)*a^2*b^2*p*q^2*log(f*x + e)*log(d)/f - 24*(f*x + e)*a*b^3*
p*q^2*log(c)*log(d)/f + 4*(f*x + e)*b^4*q*log(c)^3*log(d)/f + 12*(f*x + e)*
a*b^3*q^2*log(c)*log(d)^2/f + 12*(f*x + e)*a^2*b^2*p^2*q^2/f + 12*(f*x + e)
*a^2*b^2*p*q*log(f*x + e)*log(c)/f - 12*(f*x + e)*a*b^3*p*q*log(c)^2/f + (f
*x + e)*b^4*log(c)^4/f - 12*(f*x + e)*a^2*b^2*p*q^2*log(d)/f + 12*(f*x + e)
```

```
*a*b^3*q*log(c)^2*log(d)/f + 6*(f*x + e)*a^2*b^2*q^2*log(d)^2/f + 4*(f*x +
e)*a^3*b*p*q*log(f*x + e)/f - 12*(f*x + e)*a^2*b^2*p*q*log(c)/f + 4*(f*x +
e)*a*b^3*log(c)^3/f + 12*(f*x + e)*a^2*b^2*q*log(c)*log(d)/f - 4*(f*x + e)*
a^3*b*p*q/f + 6*(f*x + e)*a^2*b^2*log(c)^2/f + 4*(f*x + e)*a^3*b*q*log(d)/f
 + 4*(f*x + e)*a^3*b*log(c)/f + (f*x + e)*a^4/f
```

## Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.38

$$
\begin{aligned}
&\int (a+b\log (c(d(e+fx)^p)^q))^4 \, dx \\
&= \ln (c\,(d\,(e+f\,x)^p)^q)^3 \left(\frac{4\,(a\,b^3\,e-b^4\,e\,p\,q)}{f}+4\,b^3\,x\,(a-b\,p\,q)\right) \\
&\quad + \ln (c\,(d\,(e+f\,x)^p)^q)^4 \left(b^4\,x+\frac{b^4\,e}{f}\right) \\
&\quad + x\,\left(a^4-4\,a^3\,b\,p\,q+12\,a^2\,b^2\,p^2\,q^2-24\,a\,b^3\,p^3\,q^3+24\,b^4\,p^4\,q^4\right) \\
&\quad + \ln (c\,(d\,(e+f\,x)^p)^q)^2 \left(\frac{6\,(e\,a^2\,b^2-2\,e\,a\,b^3\,p\,q+2\,e\,b^4\,p^2\,q^2)}{f}\right. \\
&\qquad \left. + 6\,b^2\,x\,\left(a^2-2\,a\,b\,p\,q+2\,b^2\,p^2\,q^2\right)\right) \\
&\quad + \frac{\ln (c\,(d\,(e+f\,x)^p)^q)\,\left(4\,b\,f\,(a^3-3\,a^2\,b\,p\,q+6\,a\,b^2\,p^2\,q^2-6\,b^3\,p^3\,q^3)\,x^2+4\,b\,e\,(a^3-3\,a^2\,b\,p\,q+6\,a\,b^2\,p^2}{e+f\,x} \\
&\quad - \frac{\ln (e+f\,x)\,\left(-4\,e\,a^3\,b\,p\,q+12\,e\,a^2\,b^2\,p^2\,q^2-24\,e\,a\,b^3\,p^3\,q^3+24\,e\,b^4\,p^4\,q^4\right)}{f}
\end{aligned}
$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^4,x)
```

```
[Out] log(c*(d*(e + f*x)^p)^q)^3*((4*(a*b^3*e - b^4*e*p*q))/f + 4*b^3*x*(a - b*p*
q)) + log(c*(d*(e + f*x)^p)^q)^4*(b^4*x + (b^4*e)/f) + x*(a^4 + 24*b^4*p^4*
q^4 - 24*a*b^3*p^3*q^3 - 4*a^3*b*p*q + 12*a^2*b^2*p^2*q^2) + log(c*(d*(e +
f*x)^p)^q)^2*((6*(a^2*b^2*e + 2*b^4*e*p^2*q^2 - 2*a*b^3*e*p*q))/f + 6*b^2*x
*(a^2 + 2*b^2*p^2*q^2 - 2*a*b*p*q)) + (log(c*(d*(e + f*x)^p)^q)*(4*b*e*x*(a
^3 - 6*b^3*p^3*q^3 + 6*a*b^2*p^2*q^2 - 3*a^2*b*p*q) + 4*b*f*x^2*(a^3 - 6*b^
3*p^3*q^3 + 6*a*b^2*p^2*q^2 - 3*a^2*b*p*q)))/(e + f*x) - (log(e + f*x)*(24*
b^4*e*p^4*q^4 - 24*a*b^3*e*p^3*q^3 - 4*a^3*b*e*p*q + 12*a^2*b^2*e*p^2*q^2))
/f
```

# 3.442 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 231

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx = \frac{(a+b\log(c(d(e+fx)^p)^q))^4\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{4bpq(a+b\log(c(d(e+fx)^p)^q))^3\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{12b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h} + \frac{24b^3p^3q^3(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{24b^4p^4q^4\operatorname{PolyLog}\left(5,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

```
[Out] (a+b*ln(c*(d*(f*x+e)^p)^q))^4*ln(f*(h*x+g)/(-e*h+f*g))/h+4*b*p*q*(a+b*ln(c*
(d*(f*x+e)^p)^q))^3*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h-12*b^2*p^2*q^2*(a+b*
ln(c*(d*(f*x+e)^p)^q))^2*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h+24*b^3*p^3*q^3*
(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(4,-h*(f*x+e)/(-e*h+f*g))/h-24*b^4*p^4*q
^4*polylog(5,-h*(f*x+e)/(-e*h+f*g))/h
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2443, 2481, 2421, 2430, 6724, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx$$
$$= \frac{24b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h} - \frac{12b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{h} + \frac{4bpq\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h} + \frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^4}{h} - \frac{24b^4p^4q^4\operatorname{PolyLog}\left(5,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^4/(g + h*x),x]
```

```
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])^4*Log[(f*(g + h*x))/(f*g - e*h)])/h + (4*
b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^3*PolyLog[2, -((h*(e + f*x))/(f*g -
e*h))])/h - (12*b^2*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[3, -
((h*(e + f*x))/(f*g - e*h))])/h + (24*b^3*p^3*q^3*(a + b*Log[c*(d*(e + f*x)
^p)^q])*PolyLog[4, -((h*(e + f*x))/(f*g - e*h))])/h - (24*b^4*p^4*q^4*PolyL
og[5, -((h*(e + f*x))/(f*g - e*h))])/h
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
```

```
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^4}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^4\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}$$

$$- \text{Subst}\left(\frac{(4bfpq)\int\frac{(a+b\log(cd^q(e+fx)^{pq}))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h}, cd^q(e\right.$$

$$\left.+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^4\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} - \text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))^3\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}dx,x,e+fx\right)}{h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^4\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{4bpq(a+b\log(c(d(e+fx)^p)^q))^3\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \text{Subst}\left(\frac{(12b^2p^2q^2)\text{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))^2\operatorname{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}dx,x,e+fx\right)}{h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^4\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{4bpq(a+b\log(c(d(e+fx)^p)^q))^3\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{12b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} + \operatorname{Subst}\left(\frac{(24b^3p^3q^3)\operatorname{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\operatorname{Li}_3\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^4\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{4bpq(a+b\log(c(d(e+fx)^p)^q))^3\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{12b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} + \frac{24b^3p^3q^3(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_4\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \operatorname{Subst}\left(\frac{(24b^4p^4q^4)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_4\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^4\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{4bpq(a+b\log(c(d(e+fx)^p)^q))^3\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{12b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} + \frac{24b^3p^3q^3(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_4\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{24b^4p^4q^4\operatorname{Li}_5\left(-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1095 vs. $2(231) = 462$.

Time = 0.33 (sec) , antiderivative size = 1095, normalized size of antiderivative = 4.74

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx$$
$$= \frac{a^4\log(g+hx) - 4a^3bpq\log(e+fx)\log(g+hx) + 6a^2b^2p^2q^2\log^2(e+fx)\log(g+hx) - 4ab^3p^3q^3\log^3(e-$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^4/(g + h*x),x]
```

```
[Out] (a^4*Log[g + h*x] - 4*a^3*b*p*q*Log[e + f*x]*Log[g + h*x] + 6*a^2*b^2*p^2*q
^2*Log[e + f*x]^2*Log[g + h*x] - 4*a*b^3*p^3*q^3*Log[e + f*x]^3*Log[g + h*x
] + b^4*p^4*q^4*Log[e + f*x]^4*Log[g + h*x] + 4*a^3*b*Log[c*(d*(e + f*x)^p)
^q]*Log[g + h*x] - 12*a^2*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log
[g + h*x] + 12*a*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[g
+ h*x] - 4*b^4*p^3*q^3*Log[e + f*x]^3*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x]
 + 6*a^2*b^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] - 12*a*b^3*p*q*Log[e +
 f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + 6*b^4*p^2*q^2*Log[e + f*x]^
2*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + 4*a*b^3*Log[c*(d*(e + f*x)^p)^q
]^3*Log[g + h*x] - 4*b^4*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^3*Log[g
+ h*x] + b^4*Log[c*(d*(e + f*x)^p)^q]^4*Log[g + h*x] + 4*a^3*b*p*q*Log[e +
f*x]*Log[(f*(g + h*x))/(f*g - e*h)] - 6*a^2*b^2*p^2*q^2*Log[e + f*x]^2*Log[
(f*(g + h*x))/(f*g - e*h)] + 4*a*b^3*p^3*q^3*Log[e + f*x]^3*Log[(f*(g + h*x
))/(f*g - e*h)] - b^4*p^4*q^4*Log[e + f*x]^4*Log[(f*(g + h*x))/(f*g - e*h)]
 + 12*a^2*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(
f*g - e*h)] - 12*a*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[
(f*(g + h*x))/(f*g - e*h)] + 4*b^4*p^3*q^3*Log[e + f*x]^3*Log[c*(d*(e + f*x
)^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)] + 12*a*b^3*p*q*Log[e + f*x]*Log[c*(d
*(e + f*x)^p)^q]^2*Log[(f*(g + h*x))/(f*g - e*h)] - 6*b^4*p^2*q^2*Log[e + f
*x]^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 4*b^4*p*q
*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^3*Log[(f*(g + h*x))/(f*g - e*h)] + 4
```

```
*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^3*PolyLog[2, (h*(e + f*x))/(-(f*g)
+ e*h)] - 12*b^2*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[3, (h*(
e + f*x))/(-(f*g) + e*h)] + 24*a*b^3*p^3*q^3*PolyLog[4, (h*(e + f*x))/(-(f*
g) + e*h)] + 24*b^4*p^3*q^3*Log[c*(d*(e + f*x)^p)^q]*PolyLog[4, (h*(e + f*x
))/(-(f*g) + e*h)] - 24*b^4*p^4*q^4*PolyLog[5, (h*(e + f*x))/(-(f*g) + e*h)
])/h
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{4}}{hx+g}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^4/(h*x+g),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^4/(h*x+g),x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx=\int \frac{(b\log(((fx+e)^pd)^q\,c)+a)^4}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((b^4*log(((f*x + e)^p*d)^q*c)^4 + 4*a*b^3*log(((f*x + e)^p*d)^q*c)
^3 + 6*a^2*b^2*log(((f*x + e)^p*d)^q*c)^2 + 4*a^3*b*log(((f*x + e)^p*d)^q*c
) + a^4)/(h*x + g), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**4/(h*x+g),x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**4/(g + h*x), x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^pd)^qc)+a)^4}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4/(h*x+g),x, algorithm="maxima")
```

```
[Out] a^4*log(h*x + g)/h + integrate((b^4*log(((f*x + e)^p)^q)^4 + 4*(q*log(d) +
log(c))*a^3*b + 6*(q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*a^2*b^2 + 4
*(q^3*log(d)^3 + 3*q^2*log(c)*log(d)^2 + 3*q*log(c)^2*log(d) + log(c)^3)*a*
b^3 + (q^4*log(d)^4 + 4*q^3*log(c)*log(d)^3 + 6*q^2*log(c)^2*log(d)^2 + 4*q
*log(c)^3*log(d) + log(c)^4)*b^4 + 4*((q*log(d) + log(c))*b^4 + a*b^3)*log(
((f*x + e)^p)^q)^3 + 6*(2*(q*log(d) + log(c))*a*b^3 + (q^2*log(d)^2 + 2*q*l
og(c)*log(d) + log(c)^2)*b^4 + a^2*b^2)*log(((f*x + e)^p)^q)^2 + 4*(3*(q*lo
g(d) + log(c))*a^2*b^2 + 3*(q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*a*
b^3 + (q^3*log(d)^3 + 3*q^2*log(c)*log(d)^2 + 3*q*log(c)^2*log(d) + log(c)^
3)*b^4 + a^3*b)*log(((f*x + e)^p)^q))/(h*x + g), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^pd)^qc)+a)^4}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^4/(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{g+hx}\,dx = \int \frac{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^4}{g+h\,x}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^4/(g + h*x),x)
```

```
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^4/(g + h*x), x)
```

# 3.443 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2} dx$



## Optimal result

Integrand size = 28, antiderivative size = 274

$$\begin{aligned}
&\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2} dx \\
&= \frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^4}{(fg-eh)(g+hx)} - \frac{4bfpq(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} \\
&\quad - \frac{12b^2fp^2q^2(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)} \\
&\quad + \frac{24b^3fp^3q^3(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)} \\
&\quad - \frac{24b^4fp^4q^4\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)}
\end{aligned}$$

```
[Out] (f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^4/(-e*h+f*g)/(h*x+g)-4*b*f*p*q*(a+b*ln(
c*(d*(f*x+e)^p)^q))^3*ln(f*(h*x+g)/(-e*h+f*g))/h/(-e*h+f*g)-12*b^2*f*p^2*q^
2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h/(-e*h+f*
g)+24*b^3*f*p^3*q^3*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-e*h+
f*g))/h/(-e*h+f*g)-24*b^4*f*p^4*q^4*polylog(4,-h*(f*x+e)/(-e*h+f*g))/h/(-e*
h+f*g)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2444, 2443, 2481, 2421, 2430, 6724, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx$$
$$=\frac{24b^3fp^3q^3\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h(fg-eh)}$$
$$-\frac{12b^2fp^2q^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{h(fg-eh)}$$
$$-\frac{4bfpq\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h(fg-eh)}$$
$$+\frac{(e+fx)(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)(fg-eh)}-\frac{24b^4fp^4q^4\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^4/(g + h*x)^2,x]
```

```
[Out] ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^4)/((f*g - e*h)*(g + h*x)) - (4
*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(g + h*x))/(f*g - e*h)])
/(h*(f*g - e*h)) - (12*b^2*f*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*Pol
yLog[2, -((h*(e + f*x))/(f*g - e*h))])/(h*(f*g - e*h)) + (24*b^3*f*p^3*q^3*
(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))])/
(h*(f*g - e*h)) - (24*b^4*f*p^4*q^4*PolyLog[4, -((h*(e + f*x))/(f*g - e*h))
])/(h*(f*g - e*h))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^4}{(g+hx)^2}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\,(a+b\log(c(d(e+fx)^p)^q))^4}{(fg-eh)(g+hx)} - \text{Subst}\left(\frac{(4bfpq)\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^3}{g+hx}\,dx}{fg-eh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^4}{(fg-eh)(g+hx)} - \frac{4bfpq(a+b\log\left(c(d(e+fx)^p)^q\right))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} + \text{Subst}\left(\frac{(12b^2f^2p^2q^2)\int\frac{(a+b\log(cd^q(e+fx)^{pq}))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^4}{(fg-eh)(g+hx)} - \frac{4bfpq(a+b\log\left(c(d(e+fx)^p)^q\right))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} + \text{Subst}\left(\frac{(12b^2fp^2q^2)\,\text{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))^2\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}dx, x, e+fx\right)}{h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^4}{(fg-eh)(g+hx)} - \frac{4bfpq(a+b\log\left(c(d(e+fx)^p)^q\right))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} - \frac{12b^2fp^2q^2(a+b\log\left(c(d(e+fx)^p)^q\right))^2\,\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)} + \text{Subst}\left(\frac{(24b^3fp^3q^3)\,\text{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\text{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}dx, x, e+fx\right)}{h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^4}{(fg-eh)(g+hx)} - \frac{4bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} - \frac{12b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)} + \frac{24b^3fp^3q^3\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)} - \operatorname{Subst}\left(\frac{(24b^4fp^4q^4)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{hx}{fg-eh}\right)}{x}dx,x,e+fx\right)}{h(fg-eh)},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^4}{(fg-eh)(g+hx)} - \frac{4bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)} - \frac{12b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)} + \frac{24b^3fp^3q^3\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)} - \frac{24b^4fp^4q^4\operatorname{Li}_4\left(-\frac{h(e+fx)}{fg-eh}\right)}{h(fg-eh)}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1301 vs. $2(274) = 548$.

Time = 0.38 (sec) , antiderivative size = 1301, normalized size of antiderivative = 4.75

$$\int \frac{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^4}{(g+hx)^2}\,dx = \frac{a^4fg - a^4eh - 4a^3bfgpq\log(e+fx) - 4a^3bfhpqx\log(e+fx) + 6a^2b^2fgp^2q^2\log^2(e+fx) + 6a^2b^2fhp^2q^2x}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^4/(g + h*x)^2,x]
```

```
[Out] (a^4*f*g - a^4*e*h - 4*a^3*b*f*g*p*q*Log[e + f*x] - 4*a^3*b*f*h*p*q*x*Log[e
 + f*x] + 6*a^2*b^2*f*g*p^2*q^2*Log[e + f*x]^2 + 6*a^2*b^2*f*h*p^2*q^2*x*Lo
g[e + f*x]^2 - 4*a*b^3*f*g*p^3*q^3*Log[e + f*x]^3 - 4*a*b^3*f*h*p^3*q^3*x*L
og[e + f*x]^3 + b^4*f*g*p^4*q^4*Log[e + f*x]^4 + b^4*f*h*p^4*q^4*x*Log[e +
f*x]^4 + 4*a^3*b*f*g*Log[c*(d*(e + f*x)^p)^q] - 4*a^3*b*e*h*Log[c*(d*(e + f
*x)^p)^q] - 12*a^2*b^2*f*g*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q] - 12*a
^2*b^2*f*h*p*q*x*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q] + 12*a*b^3*f*g*p^2*q
^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q] + 12*a*b^3*f*h*p^2*q^2*x*Log[e +
 f*x]^2*Log[c*(d*(e + f*x)^p)^q] - 4*b^4*f*g*p^3*q^3*Log[e + f*x]^3*Log[c*(
d*(e + f*x)^p)^q] - 4*b^4*f*h*p^3*q^3*x*Log[e + f*x]^3*Log[c*(d*(e + f*x)^p
)^q] + 6*a^2*b^2*f*g*Log[c*(d*(e + f*x)^p)^q]^2 - 6*a^2*b^2*e*h*Log[c*(d*(e
 + f*x)^p)^q]^2 - 12*a*b^3*f*g*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2
- 12*a*b^3*f*h*p*q*x*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2 + 6*b^4*f*g*p^
2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]^2 + 6*b^4*f*h*p^2*q^2*x*Log[e
 + f*x]^2*Log[c*(d*(e + f*x)^p)^q]^2 + 4*a*b^3*f*g*Log[c*(d*(e + f*x)^p)^q]
^3 - 4*a*b^3*e*h*Log[c*(d*(e + f*x)^p)^q]^3 - 4*b^4*f*g*p*q*Log[e + f*x]*Lo
g[c*(d*(e + f*x)^p)^q]^3 - 4*b^4*f*h*p*q*x*Log[e + f*x]*Log[c*(d*(e + f*x)^
p)^q]^3 + b^4*f*g*Log[c*(d*(e + f*x)^p)^q]^4 - b^4*e*h*Log[c*(d*(e + f*x)^p
)^q]^4 + 4*a^3*b*f*g*p*q*Log[(f*(g + h*x))/(f*g - e*h)] + 4*a^3*b*f*h*p*q*x
*Log[(f*(g + h*x))/(f*g - e*h)] + 12*a^2*b^2*f*g*p*q*Log[c*(d*(e + f*x)^p)^
q]*Log[(f*(g + h*x))/(f*g - e*h)] + 12*a^2*b^2*f*h*p*q*x*Log[c*(d*(e + f*x)
^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)] + 12*a*b^3*f*g*p*q*Log[c*(d*(e + f*x)
^p)^q]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 12*a*b^3*f*h*p*q*x*Log[c*(d*(e +
f*x)^p)^q]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 4*b^4*f*g*p*q*Log[c*(d*(e + f
*x)^p)^q]^3*Log[(f*(g + h*x))/(f*g - e*h)] + 4*b^4*f*h*p*q*x*Log[c*(d*(e +
f*x)^p)^q]^3*Log[(f*(g + h*x))/(f*g - e*h)] + 12*b^2*f*p^2*q^2*(g + h*x)*(a
 + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] -
 24*b^3*f*p^3*q^3*(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[3, (h*
(e + f*x))/(-(f*g) + e*h)] + 24*b^4*f*g*p^4*q^4*PolyLog[4, (h*(e + f*x))/(-
(f*g) + e*h)] + 24*b^4*f*h*p^4*q^4*x*PolyLog[4, (h*(e + f*x))/(-(f*g) + e*h
)])/(h*(-(f*g) + e*h)*(g + h*x))
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{4}}{\left(hx+g\right)^{2}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^4/(h*x+g)^2,x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^4/(h*x+g)^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^4}{(hx+g)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] integral((b^4*log(((f*x + e)^p*d)^q*c)^4 + 4*a*b^3*log(((f*x + e)^p*d)^q*c)
^3 + 6*a^2*b^2*log(((f*x + e)^p*d)^q*c)^2 + 4*a^3*b*log(((f*x + e)^p*d)^q*c
) + a^4)/(h^2*x^2 + 2*g*h*x + g^2), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**4/(h*x+g)**2,x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**4/(g + h*x)**2, x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^4}{(hx+g)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] 4*a^3*b*f*p*q*(log(f*x + e)/(f*g*h - e*h^2) - log(h*x + g)/(f*g*h - e*h^2))
 - b^4*log(((f*x + e)^p)^q)^4/(h^2*x + g*h) - 4*a^3*b*log(((f*x + e)^p*d)^q
*c)/(h^2*x + g*h) - a^4/(h^2*x + g*h) + integrate((6*(e*h*q^2*log(d)^2 + 2*
e*h*q*log(c)*log(d) + e*h*log(c)^2)*a^2*b^2 + 4*(e*h*q^3*log(d)^3 + 3*e*h*q
^2*log(c)*log(d)^2 + 3*e*h*q*log(c)^2*log(d) + e*h*log(c)^3)*a*b^3 + (e*h*q
^4*log(d)^4 + 4*e*h*q^3*log(c)*log(d)^3 + 6*e*h*q^2*log(c)^2*log(d)^2 + 4*e
*h*q*log(c)^3*log(d) + e*h*log(c)^4)*b^4 + 4*(a*b^3*e*h + (f*g*p*q + e*h*q*
log(d) + e*h*log(c))*b^4 + (a*b^3*f*h + (f*h*p*q + f*h*q*log(d) + f*h*log(c
))*b^4)*x)*log(((f*x + e)^p)^q)^3 + 6*(a^2*b^2*e*h + 2*(e*h*q*log(d) + e*h*
log(c))*a*b^3 + (e*h*q^2*log(d)^2 + 2*e*h*q*log(c)*log(d) + e*h*log(c)^2)*b
^4 + (a^2*b^2*f*h + 2*(f*h*q*log(d) + f*h*log(c))*a*b^3 + (f*h*q^2*log(d)^2
 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*b^4)*x)*log(((f*x + e)^p)^q)^2 + (
6*(f*h*q^2*log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)*a^2*b^2 + 4*(f*
h*q^3*log(d)^3 + 3*f*h*q^2*log(c)*log(d)^2 + 3*f*h*q*log(c)^2*log(d) + f*h*
```

```
log(c)^3)*a*b^3 + (f*h*q^4*log(d)^4 + 4*f*h*q^3*log(c)*log(d)^3 + 6*f*h*q^2
*log(c)^2*log(d)^2 + 4*f*h*q*log(c)^3*log(d) + f*h*log(c)^4)*b^4)*x + 4*(3*
(e*h*q*log(d) + e*h*log(c))*a^2*b^2 + 3*(e*h*q^2*log(d)^2 + 2*e*h*q*log(c)*
log(d) + e*h*log(c)^2)*a*b^3 + (e*h*q^3*log(d)^3 + 3*e*h*q^2*log(c)*log(d)^
2 + 3*e*h*q*log(c)^2*log(d) + e*h*log(c)^3)*b^4 + (3*(f*h*q*log(d) + f*h*lo
g(c))*a^2*b^2 + 3*(f*h*q^2*log(d)^2 + 2*f*h*q*log(c)*log(d) + f*h*log(c)^2)
*a*b^3 + (f*h*q^3*log(d)^3 + 3*f*h*q^2*log(c)*log(d)^2 + 3*f*h*q*log(c)^2*l
og(d) + f*h*log(c)^3)*b^4)*x)*log(((f*x + e)^p)^q))/(f*h^3*x^3 + e*g^2*h +
(2*f*g*h^2 + e*h^3)*x^2 + (f*g^2*h + 2*e*g*h^2)*x), x)
```

## Giac [F]

$$\int \frac{(a + b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx = \int \frac{\left(b\log\left(\left((fx+e)^p d\right)^q c\right) + a\right)^4}{(hx+g)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^4/(h*x+g)^2,x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^4/(h*x + g)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d(e+fx)^p)^q))^4}{(g+hx)^2}\,dx = \int \frac{\left(a + b\,\ln\left(c\left(d\left(e + f\,x\right)^p\right)^q\right)\right)^4}{(g + h\,x)^2}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^4/(g + h*x)^2,x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^4/(g + h*x)^2, x)
```

# 3.444 $\int \log \left(c(d(e+fx)^p)^q\right) \, dx$



## Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \log \left(c(d(e+fx)^p)^q\right) \, dx = -pqx + \frac{(e+fx)\log \left(c(d(e+fx)^p)^q\right)}{f}$$

```
[Out] -p*q*x+(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2436, 2332, 2495}

$$\int \log \left(c(d(e+fx)^p)^q\right) \, dx = \frac{(e+fx)\log \left(c(d(e+fx)^p)^q\right)}{f} - pqx$$

```
[In] Int[Log[c*(d*(e + f*x)^p)^q],x]
```

```
[Out] -(p*q*x) + ((e + f*x)*Log[c*(d*(e + f*x)^p)^q])/f
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \log\left(cd^q(e+fx)^{pq}\right)\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\ &= \text{Subst}\left(\frac{\text{Subst}\left(\int \log\left(cd^q x^{pq}\right)\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\ &= -pqx + \frac{(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \log\left(c(d(e+fx)^p)^q\right)\,dx = -pqx + \frac{(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f}$$

```
[In] Integrate[Log[c*(d*(e + f*x)^p)^q],x]
[Out] -(p*q*x) + ((e + f*x)*Log[c*(d*(e + f*x)^p)^q])/f
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

| method | result | size |
|---|---|---|
| default | $\ln\left(c(d(fx+e)^p)^q\right)x - qpf\left(\frac{x}{f} - \frac{e\ln(fx+e)}{f^2}\right)$ | 41 |
| parts | $\ln\left(c(d(fx+e)^p)^q\right)x - qpf\left(\frac{x}{f} - \frac{e\ln(fx+e)}{f^2}\right)$ | 41 |
| parallelrisch | $\frac{2\ln(fx+e)e^2pq - xefpq + x\ln(c(d(fx+e)^p)^q)ef - \ln(c(d(fx+e)^p)^q)e^2}{ef}$ | 66 |

```
[In] int(ln(c*(d*(f*x+e)^p)^q),x,method=_RETURNVERBOSE)
[Out] ln(c*(d*(f*x+e)^p)^q)*x-q*p*f*(x/f-e/f^2*ln(f*x+e))
```

## Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \log\left(c(d(e+fx)^p)^q\right)\,dx = -\frac{fpqx - fqx\log(d) - fx\log(c) - (fpqx + epq)\log(fx+e)}{f}$$

```
[In] integrate(log(c*(d*(f*x+e)^p)^q),x, algorithm="fricas")
[Out] -(f*p*q*x - f*q*x*log(d) - f*x*log(c) - (f*p*q*x + e*p*q)*log(f*x + e))/f
```

## Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \log\left(c(d(e+fx)^p)^q\right)\,dx = \begin{cases} \frac{e\log\left(c(d(e+fx)^p)^q\right)}{f} - pqx + x\log\left(c(d(e+fx)^p)^q\right) & \text{for } f \neq 0 \\ x\log\left(c(de^p)^q\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(d*(f*x+e)**p)**q),x)
[Out] Piecewise((e*log(c*(d*(e + f*x)**p)**q)/f - p*q*x + x*log(c*(d*(e + f*x)**p
)**q), Ne(f, 0)), (x*log(c*(d*e**p)**q), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \log\left(c(d(e+fx)^p)^q\right)\,dx = -fpq\left(\frac{x}{f} - \frac{e\log(fx+e)}{f^2}\right) + x\log\left(((fx+e)^p d)^q\, c\right)$$

```
[In] integrate(log(c*(d*(f*x+e)^p)^q),x, algorithm="maxima")
[Out] -f*p*q*(x/f - e*log(f*x + e)/f^2) + x*log(((f*x + e)^p*d)^q*c)
```

## Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \log \left(c(d(e+fx)^p)^q\right) \, dx = \frac{(fx+e)pq\log\left(fx+e\right)}{f} - \frac{(fx+e)pq}{f} + \frac{(fx+e)q\log\left(d\right)}{f} + \frac{(fx+e)\log\left(c\right)}{f}$$

```
[In] integrate(log(c*(d*(f*x+e)^p)^q),x, algorithm="giac")
```

```
[Out] (f*x + e)*p*q*log(f*x + e)/f - (f*x + e)*p*q/f + (f*x + e)*q*log(d)/f + (f*
x + e)*log(c)/f
```

## Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \log \left(c(d(e+fx)^p)^q\right) \, dx = x\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)+\frac{p\,q\,\left(e\,\ln\left(e+f\,x\right)-f\,x\right)}{f}$$

```
[In] int(log(c*(d*(e + f*x)^p)^q),x)
```

```
[Out] x*log(c*(d*(e + f*x)^p)^q) + (p*q*(e*log(e + f*x) - f*x))/f
```

# 3.445 $\int \frac{(g+hx)^2}{a+b\log(c(d(e+fx)^p)^q)}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 279

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bf^3pq} + \frac{2e^{-\frac{2a}{bpq}}h(fg-eh)(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^3pq} + \frac{e^{-\frac{3a}{bpq}}h^2(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^3pq}$$

```
[Out] (-e*h+f*g)^2*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b/exp(a/b/p/q)/f
^3/p/q/((c*(d*(f*x+e)^p)^q)^(1/p/q))+2*h*(-e*h+f*g)*(f*x+e)^2*Ei(2*(a+b*ln(
c*(d*(f*x+e)^p)^q))/b/p/q)/b/exp(2*a/b/p/q)/f^3/p/q/((c*(d*(f*x+e)^p)^q)^(2
/p/q))+h^2*(f*x+e)^3*Ei(3*(a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b/exp(3*a/b/p/
q)/f^3/p/q/((c*(d*(f*x+e)^p)^q)^(3/p/q))
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{2446, 2436, 2337, 2209, 2437, 2347, 2495\}$

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$
$$= \frac{2h(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^3pq}$$
$$+ \frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bf^3pq}$$
$$+ \frac{h^2(e+fx)^3e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^3pq}$$

```
[In] Int[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] ((f*g - e*h)^2*(e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*
p*q)])/(b*E^(a/(b*p*q))*f^3*p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (2*h*(f*
g - e*h)*(e + f*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*
p*q)])/(b*E^((2*a)/(b*p*q))*f^3*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) + (h^2
*(e + f*x)^3*ExpIntegralEi[(3*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)])/(
b*E^((3*a)/(b*p*q))*f^3*p*q*(c*(d*(e + f*x)^p)^q)^(3/(p*q)))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(g+hx)^2}{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\Bigg(\int \Bigg(\frac{(fg-eh)^2}{f^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}+\frac{2h(fg-eh)(e+fx)}{f^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}\\
&\qquad +\frac{h^2(e+fx)^2}{f^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}\Bigg)\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\Bigg)\\
&= \text{Subst}\left(\frac{h^2\int \frac{(e+fx)^2}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad + \text{Subst}\left(\frac{(2h(fg-eh))\int \frac{e+fx}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad + \text{Subst}\left(\frac{(fg-eh)^2\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}
$$

$$= \text{Subst}\left(\frac{h^2\text{Subst}\left(\int \frac{x^2}{a+b\log(cd^q x^{pq})}\,dx, x, e+fx\right)}{f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$+ \text{Subst}\left(\frac{(2h(fg-eh))\text{Subst}\left(\int \frac{x}{a+b\log(cd^q x^{pq})}\,dx, x, e+fx\right)}{f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$+ \text{Subst}\left(\frac{(fg-eh)^2\text{Subst}\left(\int \frac{1}{a+b\log(cd^q x^{pq})}\,dx, x, e+fx\right)}{f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\left(h^2(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{pq}}}{a+bx}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{f^3pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$+\text{Subst}\left(\frac{\left(2h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{pq}}}{a+bx}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{f^3pq}, c\ldots + fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$+\text{Subst}\left(\frac{\left((fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{a+bx}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{f^3pq}, cd^q(\ldots + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bf^3pq} + \frac{2e^{-\frac{2a}{bpq}}h(fg-eh)(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{Ei}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^3pq} + \frac{e^{-\frac{3a}{bpq}}h^2(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{Ei}\left(\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^3pq}$$

## Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.90

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{e^{-\frac{3a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\left(e^{\frac{2a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)-\right.}{}$$

```
[In] Integrate[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] ((e + f*x)*(E^((2*a)/(b*p*q))*(f*g - e*h)^2*(c*(d*(e + f*x)^p)^q)^(2/(p*q))
*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)] - h*(e + f*x)*(-2*
E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*ExpIntegralEi[(2*
(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)] - h*(e + f*x)*ExpIntegralEi[(3*(
a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)])))/(b*E^((3*a)/(b*p*q))*f^3*p*q*(
c*(d*(e + f*x)^p)^q)^(3/(p*q)))
```

## Maple [F]

$$\int \frac{(hx+g)^2}{a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)}dx$$

```
[In] int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

```
[Out] int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.87

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{\left(h^2 \operatorname{log\_integral}\left((f^3x^3+3\,ef^2x^2+3\,e^2fx+e^3)e^{\left(\frac{3\,(bq\log(d)+b\log(c)+a)}{bpq}\right)}\right)+2\,(fgh-eh^2)e^{\left(\frac{bq\log(d)+b\log(c)+a}{bpq}\right)}\right.}{}$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

```
[Out] (h^2*log_integral((f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)*e^(3*(b*q*log(d
) + b*log(c) + a)/(b*p*q))) + 2*(f*g*h - e*h^2)*e^((b*q*log(d) + b*log(c) +
 a)/(b*p*q))*log_integral((f^2*x^2 + 2*e*f*x + e^2)*e^(2*(b*q*log(d) + b*lo
g(c) + a)/(b*p*q))) + (f^2*g^2 - 2*e*f*g*h + e^2*h^2)*e^(2*(b*q*log(d) + b*
log(c) + a)/(b*p*q))*log_integral((f*x + e)*e^((b*q*log(d) + b*log(c) + a)/
(b*p*q))))*e^(-3*(b*q*log(d) + b*log(c) + a)/(b*p*q))/(b*f^3*p*q)
```

## Sympy [F]

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \frac{(g+hx)^2}{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\,dx$$

```
[In] integrate((h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

```
[Out] Integral((g + h*x)**2/(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

## Maxima [F]

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \frac{(hx+g)^2}{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}\,dx$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

```
[Out] integrate((h*x + g)^2/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.85

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{g^2\mathrm{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bpq}\right)}}{bc^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}fpq} - \frac{2\,eghEi\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bpq}\right)}}{bc^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}f^2pq} + \frac{e^2h^2\mathrm{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bpq}\right)}}{bc^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}f^3pq} + \frac{2\,gh\mathrm{Ei}\left(\frac{2\,\log(d)}{p}+\frac{2\,\log(c)}{pq}+\frac{2\,a}{bpq}+2\,\log\left(fx+e\right)\right)e^{\left(-\frac{2\,a}{bpq}\right)}}{bc^{\frac{2}{pq}}d^{\frac{2}{p}}f^2pq} - \frac{2\,eh^2\mathrm{Ei}\left(\frac{2\,\log(d)}{p}+\frac{2\,\log(c)}{pq}+\frac{2\,a}{bpq}+2\,\log\left(fx+e\right)\right)e^{\left(-\frac{2\,a}{bpq}\right)}}{bc^{\frac{2}{pq}}d^{\frac{2}{p}}f^3pq} + \frac{h^2\mathrm{Ei}\left(\frac{3\,\log(d)}{p}+\frac{3\,\log(c)}{pq}+\frac{3\,a}{bpq}+3\,\log\left(fx+e\right)\right)e^{\left(-\frac{3\,a}{bpq}\right)}}{bc^{\frac{3}{pq}}d^{\frac{3}{p}}f^3pq}$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

```
[Out] g^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/(
b*c^(1/(p*q))*d^(1/p)*f*p*q) - 2*e*g*h*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*
q) + log(f*x + e))*e^(-a/(b*p*q))/(b*c^(1/(p*q))*d^(1/p)*f^2*p*q) + e^2*h^2
*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/(b*c
^(1/(p*q))*d^(1/p)*f^3*p*q) + 2*g*h*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b
*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))/(b*c^(2/(p*q))*d^(2/p)*f^2*p*q) -
2*e*h^2*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-
2*a/(b*p*q))/(b*c^(2/(p*q))*d^(2/p)*f^3*p*q) + h^2*Ei(3*log(d)/p + 3*log(c)
/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a/(b*p*q))/(b*c^(3/(p*q))*d^(3
/p)*f^3*p*q)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{{\left(g+h\,x\right)}^2}{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}\,dx$$

```
[In] int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q)),x)
[Out] int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

# 3.446 $\int \frac{g+hx}{a+b\log(c(d(e+fx)^p)^q)}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 179

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bf^2pq} + \frac{e^{-\frac{2a}{bpq}}h(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^2pq}$$

```
[Out] (-e*h+f*g)*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b/exp(a/b/p/q)/f^2
/p/q/((c*(d*(f*x+e)^p)^q)^(1/p/q))+h*(f*x+e)^2*Ei(2*(a+b*ln(c*(d*(f*x+e)^p)
^q))/b/p/q)/b/exp(2*a/b/p/q)/f^2/p/q/((c*(d*(f*x+e)^p)^q)^(2/p/q))
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2446, 2436, 2337, 2209, 2437, 2347, 2495}

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bf^2pq} + \frac{h(e+fx)^2e^{-\frac{2a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^2pq}$$

```
[In] Int[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] ((f*g - e*h)*(e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*
q)])/(b*E^(a/(b*p*q))*f^2*p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (h*(e + f*
x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)])/(b*E^((2*
a)/(b*p*q))*f^2*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
```

```
c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{g+hx}{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\int \left(\frac{fg-eh}{f\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}+\frac{h(e+fx)}{f\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}\right)dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{h\int \frac{e+fx}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(fg-eh)\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{h\text{Subst}\left(\int \frac{x}{a+b\log(cd^qx^{pq})}\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(fg-eh)\text{Subst}\left(\int \frac{1}{a+b\log(cd^qx^{pq})}\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\left(h(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{pq}}}{a+bx}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{f^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{\left((fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{a+bx}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{f^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bf^2pq} + \frac{e^{-\frac{2a}{bpq}}h(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{Ei}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{bf^2pq}$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.92

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(e^{\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right) \cdots\right.}{bf^2pq}$$

```
[In] Integrate[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] ((e + f*x)*(E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*ExpIn
tegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)] + h*(e + f*x)*ExpIntegra
lEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)]))/(b*E^((2*a)/(b*p*q))*f^
2*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))
```

## Maple [F]

$$\int \frac{hx+g}{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

```
[In] int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

```
[Out] int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.78

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{\left((fg-eh)e^{\left(\frac{bq\log(d)+b\log(c)+a}{bpq}\right)}\operatorname{log\_integral}\left((fx+e)e^{\left(\frac{bq\log(d)+b\log(c)+a}{bpq}\right)}\right)+h\operatorname{log\_integral}\left((f^2x^2+2\,efx \cdots\right.\right.}{bf^2pq}$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

```
[Out] ((f*g - e*h)*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))*log_integral((f*x + e)
*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))) + h*log_integral((f^2*x^2 + 2*e*f
*x + e^2)*e^(2*(b*q*log(d) + b*log(c) + a)/(b*p*q))))*e^(-2*(b*q*log(d) + b
*log(c) + a)/(b*p*q))/(b*f^2*p*q)
```

## Sympy [F]

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{g+hx}{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\,dx$$

```
[In] integrate((h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
[Out] Integral((g + h*x)/(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

## Maxima [F]

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{hx+g}{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}\,dx$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
[Out] integrate((h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.39

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\frac{g\mathrm{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bpq}\right)}}{bc^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}fpq}-\frac{eh\mathrm{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bpq}\right)}}{bc^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}f^2pq}+\frac{h\mathrm{Ei}\left(\frac{2\log(d)}{p}+\frac{2\log(c)}{pq}+\frac{2a}{bpq}+2\log\left(fx+e\right)\right)e^{\left(-\frac{2a}{bpq}\right)}}{bc^{\frac{2}{pq}}d^{\frac{2}{p}}f^2pq}$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
[Out] g*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/(b*
c^(1/(p*q))*d^(1/p)*f*p*q) - e*h*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + l
og(f*x + e))*e^(-a/(b*p*q))/(b*c^(1/(p*q))*d^(1/p)*f^2*p*q) + h*Ei(2*log(d)
/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))/(b*c^(
2/(p*q))*d^(2/p)*f^2*p*q)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{g+h\,x}{a+b\,\ln\left(c\left(d\,(e+f\,x)^p\right)^q\right)}\,dx$$

```
[In] int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q)),x)
[Out] int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

# 3.447 $\int \frac{1}{a+b\log(c(d(e+fx)^p)^q)}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 83

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bfpq}$$

```
[Out] (f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b/exp(a/b/p/q)/f/p/q/((c*(d*(
f*x+e)^p)^q)^(1/p/q))
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2436, 2337, 2209, 2495}

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{(e+fx)e^{-\frac{a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bfpq}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-1),x]
```

```
[Out] ((e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)])/(b*E^(a
/(b*p*q))*f*p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*q)))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{a+b\log(cd^q x^{pq})}\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{\left((e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{a+bx}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{fpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{bfpq}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c\left(d(e+fx)^p\right)^q\right)^{-\frac{1}{pq}}\operatorname{ExpIntegralEi}\left(\frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{bpq}\right)}{bfpq}$$

[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-1),x]

[Out] ((e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)])/(b*E^(a/(b*p*q))*f*p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*q)))

## Maple [F]

$$\int \frac{1}{a+b\ln \left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

[In] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)

[Out] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)

## Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{1}{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{e^{\left(-\frac{bq\log(d)+b\log(c)+a}{bpq}\right)}\operatorname{log\_integral}\left((fx+e)e^{\left(\frac{bq\log(d)+b\log(c)+a}{bpq}\right)}\right)}{bfpq}$$

[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")

[Out] e^(-(b*q*log(d) + b*log(c) + a)/(b*p*q))*log_integral((f*x + e)*e^((b*q*log(d) + b*log(c) + a)/(b*p*q)))/(b*f*p*q)

## Sympy [F]

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{1}{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
[Out] Integral(1/(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

## Maxima [F]

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{1}{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
[Out] integrate(1/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\frac{\operatorname{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log\left(fx+e\right)\right)e^{\left(-\frac{a}{bpq}\right)}}{bc^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}fpq}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
[Out] Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/(b*c^
(1/(p*q))*d^(1/p)*f*p*q)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{1}{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}\,dx$$

```
[In] int(1/(a + b*log(c*(d*(e + f*x)^p)^q)),x)
[Out] int(1/(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

# 3.448 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\text{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

```
[In] Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Defer[Int][1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

```
[In] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)}dx$$

```
[In] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
[Out] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)(b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a)}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
[Out] integral(1/(a*h*x + a*g + (b*h*x + b*g)*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)\right)(g+hx)}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)\left(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
[Out] integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)\left(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
[Out] integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(g+h\,x)\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}\,dx$$

```
[In] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
[Out] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

# 3.449 $\int \frac{1}{(g+hx)^2(a+b\log(c(d(e+fx)^p)^q))}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

```
[In] Int[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Defer[Int][1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

```
[In] Integrate[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Integrate[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)^2 \left(a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)\right)}dx$$

```
[In] int(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
[Out] int(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{(g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(hx+g)^2 (b\log\left(((fx+e)^p d)^q c\right)+a)}\,dx$$

```
[In] integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
[Out] integral(1/(a*h^2*x^2 + 2*a*g*h*x + a*g^2 + (b*h^2*x^2 + 2*b*g*h*x + b*g^2)
*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 3.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))} dx = \int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))(g+hx)^2} dx$$

```
[In] integrate(1/(h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))} dx = \int \frac{1}{(hx+g)^2(b\log(((fx+e)^p d)^q c)+a)} dx$$

```
[In] integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
[Out] integrate(1/((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))} dx = \int \frac{1}{(hx+g)^2(b\log(((fx+e)^p d)^q c)+a)} dx$$

```
[In] integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
[Out] integrate(1/((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{{\left(g+h\,x\right)}^2\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)} \,d x$$

```
[In] int(1/((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
[Out] int(1/((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

# 3.450 $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 326

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2f^3p^2q^2} + \frac{4e^{-\frac{2a}{bpq}}h(fg-eh)(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^2f^3p^2q^2} + \frac{3e^{-\frac{3a}{bpq}}h^2(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^2f^3p^2q^2} - \frac{(e+fx)(g+hx)^2}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

```
[Out] (-e*h+f*g)^2*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^2/exp(a/b/p/q)
/f^3/p^2/q^2/((c*(d*(f*x+e)^p)^q)^(1/p/q))+4*h*(-e*h+f*g)*(f*x+e)^2*Ei(2*(a
+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^2/exp(2*a/b/p/q)/f^3/p^2/q^2/((c*(d*(f*x
+e)^p)^q)^(2/p/q))+3*h^2*(f*x+e)^3*Ei(3*(a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/
b^2/exp(3*a/b/p/q)/f^3/p^2/q^2/((c*(d*(f*x+e)^p)^q)^(3/p/q))-(f*x+e)*(h*x+g
)^2/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))
```

## Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2447, 2446, 2436, 2337, 2209, 2437, 2347, 2495}

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$$
$$= \frac{4h(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^2f^3p^2q^2}$$
$$+\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2f^3p^2q^2}$$
$$+\frac{3h^2(e+fx)^3e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^2f^3p^2q^2}$$
$$-\frac{(e+fx)(g+hx)^2}{bfpq\,(a+b\log(c(d(e+fx)^p)^q))}$$

```
[In] Int[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] ((f*g - e*h)^2*(e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*
p*q)])/(b^2*E^(a/(b*p*q))*f^3*p^2*q^2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (4
*h*(f*g - e*h)*(e + f*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]
))/(b*p*q)])/(b^2*E^((2*a)/(b*p*q))*f^3*p^2*q^2*(c*(d*(e + f*x)^p)^q)^(2/(p
*q))) + (3*h^2*(e + f*x)^3*ExpIntegralEi[(3*(a + b*Log[c*(d*(e + f*x)^p)^q]
))/(b*p*q)])/(b^2*E^((3*a)/(b*p*q))*f^3*p^2*q^2*(c*(d*(e + f*x)^p)^q)^(3/(p
*q))) - ((e + f*x)*(g + h*x)^2)/(b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(g+hx)^2}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}\,dx, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= -\frac{(e+fx)(g+hx)^2}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{3\int \frac{(g+hx)^2}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{bpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(2(fg-eh))\int \frac{g+hx}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(e+fx)(g+hx)^2}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{3\int \left(\frac{(fg-eh)^2}{f^2(a+b\log(cd^q(e+fx)^{pq}))} + \frac{2h(fg-eh)(e+fx)}{f^2(a+b\log(cd^q(e+fx)^{pq}))} + \frac{h^2(e+fx)^2}{f^2(a+b\log(cd^q(e+fx)^{pq}))}\right)dx}{bpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(2(fg-eh))\int \left(\frac{fg-eh}{f(a+b\log(cd^q(e+fx)^{pq}))} + \frac{h(e+fx)}{f(a+b\log(cd^q(e+fx)^{pq}))}\right)dx}{bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(e+fx)(g+hx)^2}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{(3h^2)\int \frac{(e+fx)^2}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(2h(fg-eh))\int \frac{e+fx}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(6h(fg-eh))\int \frac{e+fx}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(2(fg-eh)^2)\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(3(fg-eh)^2)\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(e+fx)(g+hx)^2}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{(3h^2)\,\text{Subst}\left(\int \frac{x^2}{a+b\log(cd^qx^{pq})}\,dx, x, e+fx\right)}{bf^3pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(2h(fg-eh))\text{Subst}\left(\int \frac{x}{a+b\log(cd^qx^{pq})}\,dx, x, e+fx\right)}{bf^3pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(6h(fg-eh))\text{Subst}\left(\int \frac{x}{a+b\log(cd^qx^{pq})}\,dx, x, e+fx\right)}{bf^3pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(2(fg-eh)^2)\,\text{Subst}\left(\int \frac{1}{a+b\log(cd^qx^{pq})}\,dx, x, e+fx\right)}{bf^3pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(3(fg-eh)^2)\,\text{Subst}\left(\int \frac{1}{a+b\log(cd^qx^{pq})}\,dx, x, e+fx\right)}{bf^3pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(e+fx)(g+hx)^2}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

$$+\text{Subst}\left(\frac{\left(3h^2(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$-\text{Subst}\left(\frac{\left(2h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left(6h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$-\text{Subst}\left(\frac{\left(2(fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left(3(fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2f^3p^2q^2} + \frac{4e^{-\frac{2a}{bpq}}h(fg-eh)(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{Ei}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^2f^3p^2q^2} + \frac{3e^{-\frac{3a}{bpq}}h^2(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{Ei}\left(\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^2f^3p^2q^2} - \frac{(e+fx)(g+hx)^2}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1310 vs. $2(326) = 652$.

Time = 0.41 (sec) , antiderivative size = 1310, normalized size of antiderivative = 4.02

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \frac{e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\left(-bee^{\frac{3a}{bpq}}f^2g^2pq(c(d(e+fx)^p)^q)^{\frac{3}{pq}} - be^{\frac{3a}{bpq}}f^3g^2pqx(c(d(e+fx)^p)^q)^{\frac{3}{pq}} - 2bee^{\frac{3a}{bpq}}\right.}{}$$

```
[In] Integrate[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] (-(b*e*E^((3*a)/(b*p*q))*f^2*g^2*p*q*(c*(d*(e + f*x)^p)^q)^(3/(p*q))) - b*E
^((3*a)/(b*p*q))*f^3*g^2*p*q*x*(c*(d*(e + f*x)^p)^q)^(3/(p*q)) - 2*b*e*E^((
3*a)/(b*p*q))*f^2*g*h*p*q*x*(c*(d*(e + f*x)^p)^q)^(3/(p*q)) - 2*b*E^((3*a)/
(b*p*q))*f^3*g*h*p*q*x^2*(c*(d*(e + f*x)^p)^q)^(3/(p*q)) - b*e*E^((3*a)/(b*
p*q))*f^2*h^2*p*q*x^2*(c*(d*(e + f*x)^p)^q)^(3/(p*q)) - b*E^((3*a)/(b*p*q))
*f^3*h^2*p*q*x^3*(c*(d*(e + f*x)^p)^q)^(3/(p*q)) + a*E^((2*a)/(b*p*q))*f^2*
g^2*(e + f*x)*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*ExpIntegralEi[(a + b*Log[c*(d
*(e + f*x)^p)^q])/(b*p*q)] - 2*a*e*E^((2*a)/(b*p*q))*f*g*h*(e + f*x)*(c*(d*
(e + f*x)^p)^q)^(2/(p*q))*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b
*p*q)] + a*e^2*E^((2*a)/(b*p*q))*h^2*(e + f*x)*(c*(d*(e + f*x)^p)^q)^(2/(p*
q))*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)] + 4*a*E^(a/(b*p
*q))*f*g*h*(e + f*x)^2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*ExpIntegralEi[(2*(a
+ b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)] - 4*a*e*E^(a/(b*p*q))*h^2*(e + f*x)
^2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*
x)^p)^q]))/(b*p*q)] + 3*a*h^2*(e + f*x)^3*ExpIntegralEi[(3*(a + b*Log[c*(d*
(e + f*x)^p)^q]))/(b*p*q)] + b*E^((2*a)/(b*p*q))*f^2*g^2*(e + f*x)*(c*(d*(e
 + f*x)^p)^q)^(2/(p*q))*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p
*q)]*Log[c*(d*(e + f*x)^p)^q] - 2*b*e*E^((2*a)/(b*p*q))*f*g*h*(e + f*x)*(c*
(d*(e + f*x)^p)^q)^(2/(p*q))*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])
/(b*p*q)]*Log[c*(d*(e + f*x)^p)^q] + b*e^2*E^((2*a)/(b*p*q))*h^2*(e + f*x)*
```

(c*(d*(e + f*x)^p)^q)^(2/(p*q))*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)]*Log[c*(d*(e + f*x)^p)^q] + 4*b*E^(a/(b*p*q))*f*g*h*(e + f*x)^2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)]*Log[c*(d*(e + f*x)^p)^q] - 4*b*e*E^(a/(b*p*q))*h^2*(e + f*x)^2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)]*Log[c*(d*(e + f*x)^p)^q] + 3*b*h^2*(e + f*x)^3*ExpIntegralEi[(3*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)]*Log[c*(d*(e + f*x)^p)^q])/(b^2*E^((3*a)/(b*p*q))*f^3*p^2*q^2*(c*(d*(e + f*x)^p)^q)^(3/(p*q))*(a + b*Log[c*(d*(e + f*x)^p)^q]))

## Maple [F]

$$\int \frac{(hx+g)^2}{(a+b\ln(c(d(fx+e)^p)^q))^2}dx$$

[In] int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)

[Out] int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)

## Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.76

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \frac{\left(4\left(afgh-aeh^2+(bfgh-beh^2)pq\log(fx+e)+(bfgh-beh^2)q\log(d)+(bfgh-beh^2)\log(c)\right)e^{\left(\frac{bq\log(d)}{\cdots}\right.}\cdots\right.}{\cdots}$$

[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")

[Out] (4*(a*f*g*h - a*e*h^2 + (b*f*g*h - b*e*h^2)*p*q*log(f*x + e) + (b*f*g*h - b*e*h^2)*q*log(d) + (b*f*g*h - b*e*h^2)*log(c))*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))*log_integral((f^2*x^2 + 2*e*f*x + e^2)*e^(2*(b*q*log(d) + b*log(c) + a)/(b*p*q))) + (a*f^2*g^2 - 2*a*e*f*g*h + a*e^2*h^2 + (b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*p*q*log(f*x + e) + (b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*q*log(d) + (b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*log(c))*e^(2*(b*q*log(d) + b*log(c) + a)/(b*p*q))*log_integral((f*x + e)*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))) - (b*f^3*h^2*p*q*x^3 + b*e*f^2*g^2*p*q + (2*b*f^3*g*h + b*e*f^2*h^2)*p*q*x^2 + (b*f^3*g^2 + 2*b*e*f^2*g*h)*p*q*x)*e^(3*(b*q*log(d) + b*log(c) + a)/(b*p*q)) + 3*(b*h^2*p*q*log(f*x + e) + b*h^2*q*log(d) + b*h^2*log(c) + a*h^2)*log_integral((f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)*e^(3*(b*q*log(d) + b*log(c) + a)/(b*p*q))))*e^(-3*(b*q*log(d) + b*log(c) + a)/(b*p*q))/(b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)

## Sympy [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$$

```
[In] integrate((h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
[Out] Integral((g + h*x)**2/(a + b*log(c*(d*(e + f*x)**p)**q))**2, x)
```

## Maxima [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{(hx+g)^2}{(b\log(((fx+e)^p d)^q c)+a)^2}\,dx$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
[Out] -(f*h^2*x^3 + e*g^2 + (2*f*g*h + e*h^2)*x^2 + (f*g^2 + 2*e*g*h)*x)/(b^2*f*p
*q*log(((f*x + e)^p)^q) + a*b*f*p*q + (f*p*q^2*log(d) + f*p*q*log(c))*b^2)
+ integrate((3*f*h^2*x^2 + f*g^2 + 2*e*g*h + 2*(2*f*g*h + e*h^2)*x)/(b^2*f*
p*q*log(((f*x + e)^p)^q) + a*b*f*p*q + (f*p*q^2*log(d) + f*p*q*log(c))*b^2)
, x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3975 vs. $2(328) = 656$.

Time = 0.46 (sec) , antiderivative size = 3975, normalized size of antiderivative = 12.19

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
[Out] -(f*x + e)*b*f^2*g^2*p*q/(b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*lo
g(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2) - 2*(f*x + e)^2*b*f*g*h*
p*q/(b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^
2*log(c) + a*b^2*f^3*p^2*q^2) + 2*(f*x + e)*b*e*f*g*h*p*q/(b^3*f^3*p^3*q^3*
log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*
p^2*q^2) - (f*x + e)^3*b*h^2*p*q/(b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^
2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2) + 2*(f*x + e)^2*
b*e*h^2*p*q/(b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^
3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2) - (f*x + e)*b*e^2*h^2*p*q/(b^3*f^3*p^
3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^
```

2*f^3*p^2*q^2) + b*f^2*g^2*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log
(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)/((b^3*f^3*p^3*q^3*log(f*x + e) + b^3
*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c^(1/(p*q
))*d^(1/p)) - 2*b*e*f*g*h*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(
f*x + e))*e^(-a/(b*p*q))*log(f*x + e)/((b^3*f^3*p^3*q^3*log(f*x + e) + b^3*
f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c^(1/(p*q)
)*d^(1/p)) + b*e^2*h^2*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x
 + e))*e^(-a/(b*p*q))*log(f*x + e)/((b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3
*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c^(1/(p*q))*d
^(1/p)) + 4*b*f*g*h*p*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*lo
g(f*x + e))*e^(-2*a/(b*p*q))*log(f*x + e)/((b^3*f^3*p^3*q^3*log(f*x + e) +
b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c^(2/(
p*q))*d^(2/p)) - 4*b*e*h^2*p*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q)
 + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(f*x + e)/((b^3*f^3*p^3*q^3*log(f*x
+ e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)
*c^(2/(p*q))*d^(2/p)) + b*f^2*g^2*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q)
+ log(f*x + e))*e^(-a/(b*p*q))*log(d)/((b^3*f^3*p^3*q^3*log(f*x + e) + b^3*
f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c^(1/(p*q)
)*d^(1/p)) - 2*b*e*f*g*h*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x
 + e))*e^(-a/(b*p*q))*log(d)/((b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q
^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c^(1/(p*q))*d^(1/p)
) + b*e^2*h^2*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-
a/(b*p*q))*log(d)/((b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) +
 b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c^(1/(p*q))*d^(1/p)) + 3*b*h^2
*p*q*Ei(3*log(d)/p + 3*log(c)/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a
/(b*p*q))*log(f*x + e)/((b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log
(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c^(3/(p*q))*d^(3/p)) + b*
f^2*g^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q
))*log(c)/((b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3
*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c^(1/(p*q))*d^(1/p)) - 2*b*e*f*g*h*Ei(
log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)/(
(b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*lo
g(c) + a*b^2*f^3*p^2*q^2)*c^(1/(p*q))*d^(1/p)) + b*e^2*h^2*Ei(log(d)/p + lo
g(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)/((b^3*f^3*p^3*
q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*
f^3*p^2*q^2)*c^(1/(p*q))*d^(1/p)) + 4*b*f*g*h*q*Ei(2*log(d)/p + 2*log(c)/(p
*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(d)/((b^3*f^3*p^3*q
^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f
^3*p^2*q^2)*c^(2/(p*q))*d^(2/p)) - 4*b*e*h^2*q*Ei(2*log(d)/p + 2*log(c)/(p*
q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(d)/((b^3*f^3*p^3*q^
3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^
3*p^2*q^2)*c^(2/(p*q))*d^(2/p)) + a*f^2*g^2*Ei(log(d)/p + log(c)/(p*q) + a/
(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/((b^3*f^3*p^3*q^3*log(f*x + e) + b^3
*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c^(1/(p*q
))*d^(1/p)) - 2*a*e*f*g*h*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x

```
+ e))*e^(-a/(b*p*q))/((b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d
) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c^(1/(p*q))*d^(1/p)) + a*e^
2*h^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))
/((b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*
log(c) + a*b^2*f^3*p^2*q^2)*c^(1/(p*q))*d^(1/p)) + 4*b*f*g*h*Ei(2*log(d)/p
+ 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(c)/((
b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log
(c) + a*b^2*f^3*p^2*q^2)*c^(2/(p*q))*d^(2/p)) - 4*b*e*h^2*Ei(2*log(d)/p + 2
*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(c)/((b^3
*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c)
 + a*b^2*f^3*p^2*q^2)*c^(2/(p*q))*d^(2/p)) + 3*b*h^2*q*Ei(3*log(d)/p + 3*lo
g(c)/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a/(b*p*q))*log(d)/((b^3*f^
3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) +
a*b^2*f^3*p^2*q^2)*c^(3/(p*q))*d^(3/p)) + 4*a*f*g*h*Ei(2*log(d)/p + 2*log(c
)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))/((b^3*f^3*p^3*q^3*
log(f*x + e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*
p^2*q^2)*c^(2/(p*q))*d^(2/p)) - 4*a*e*h^2*Ei(2*log(d)/p + 2*log(c)/(p*q) +
2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))/((b^3*f^3*p^3*q^3*log(f*x +
e) + b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c
^(2/(p*q))*d^(2/p)) + 3*b*h^2*Ei(3*log(d)/p + 3*log(c)/(p*q) + 3*a/(b*p*q)
+ 3*log(f*x + e))*e^(-3*a/(b*p*q))*log(c)/((b^3*f^3*p^3*q^3*log(f*x + e) +
b^3*f^3*p^2*q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c^(3/(
p*q))*d^(3/p)) + 3*a*h^2*Ei(3*log(d)/p + 3*log(c)/(p*q) + 3*a/(b*p*q) + 3*l
og(f*x + e))*e^(-3*a/(b*p*q))/((b^3*f^3*p^3*q^3*log(f*x + e) + b^3*f^3*p^2*
q^3*log(d) + b^3*f^3*p^2*q^2*log(c) + a*b^2*f^3*p^2*q^2)*c^(3/(p*q))*d^(3/p
))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{{\left(g+h\,x\right)}^2}{{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^2}\,dx$$

```
[In] int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
[Out] int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^2, x)
```

# 3.451 $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2} dx$



## Optimal result

Integrand size = 26, antiderivative size = 224

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2} dx = \frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}} \text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2f^2p^2q^2} + \frac{2e^{-\frac{2a}{bpq}}h(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}} \text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^2f^2p^2q^2} - \frac{(e+fx)(g+hx)}{bfpq\,(a+b\log(c(d(e+fx)^p)^q))}$$

```
[Out] (-e*h+f*g)*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^2/exp(a/b/p/q)/f
^2/p^2/q^2/((c*(d*(f*x+e)^p)^q)^(1/p/q))+2*h*(f*x+e)^2*Ei(2*(a+b*ln(c*(d*(f
*x+e)^p)^q))/b/p/q)/b^2/exp(2*a/b/p/q)/f^2/p^2/q^2/((c*(d*(f*x+e)^p)^q)^(2/
p/q))-(f*x+e)*(h*x+g)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used

= {2447, 2446, 2436, 2337, 2209, 2437, 2347, 2495}

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$
$$=\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2f^2p^2q^2}$$
$$+\frac{2h(e+fx)^2e^{-\frac{2a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^2f^2p^2q^2}$$
$$-\frac{(e+fx)(g+hx)}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

```
[In] Int[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] ((f*g - e*h)*(e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*
q)])/(b^2*E^(a/(b*p*q))*f^2*p^2*q^2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (2*h
*(e + f*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)])/(
b^2*E^((2*a)/(b*p*q))*f^2*p^2*q^2*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) - ((e +
f*x)*(g + h*x))/(b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{g+hx}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= -\frac{(e+fx)(g+hx)}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\\
&\quad+\text{Subst}\left(\frac{2\int \frac{g+hx}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{bpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad-\text{Subst}\left(\frac{(fg-eh)\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}
$$

$$= -\frac{(e+fx)(g+hx)}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{2\int\left(\frac{fg-eh}{f(a+b\log(cd^q(e+fx)^{pq}))}+\frac{h(e+fx)}{f(a+b\log(cd^q(e+fx)^{pq}))}\right)dx}{bpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(fg-eh)\text{Subst}\left(\int\frac{1}{a+b\log(cd^qx^{pq})}dx, x, e+fx\right)}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(e+fx)(g+hx)}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{(2h)\int\frac{e+fx}{a+b\log(cd^q(e+fx)^{pq})}dx}{bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(2(fg-eh))\int\frac{1}{a+b\log(cd^q(e+fx)^{pq})}dx}{bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{\left((fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{a+bx}dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2f^2p^2q^2} - \frac{(e+fx)(g+hx)}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{(2h)\text{Subst}\left(\int\frac{x}{a+b\log(cd^qx^{pq})}dx, x, e+fx\right)}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(2(fg-eh))\text{Subst}\left(\int\frac{1}{a+b\log(cd^qx^{pq})}dx, x, e+fx\right)}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2f^2p^2q^2}$$
$$-\frac{(e+fx)(g+hx)}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$
$$+\text{Subst}\left(\frac{\left(2h(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^2p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$+\text{Subst}\left(\frac{\left(2(fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^2p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$=\frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2f^2p^2q^2}$$
$$+\frac{2e^{-\frac{2a}{bpq}}h(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{Ei}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^2f^2p^2q^2}$$
$$-\frac{(e+fx)(g+hx)}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.20

$$\int\frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=$$
$$-\frac{e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(be^{\frac{2a}{bpq}}fpq(c(d(e+fx)^p)^q)^{\frac{2}{pq}}(g+hx)-e^{\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right.\right.}{}$$

```
[In] Integrate[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] -(((e + f*x)*(b*E^((2*a)/(b*p*q))*f*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(g
+ h*x) - E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*ExpInteg
ralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)]*(a + b*Log[c*(d*(e + f*x)^p
)^q]) - 2*h*(e + f*x)*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b
*p*q)]*(a + b*Log[c*(d*(e + f*x)^p)^q])))/(b^2*E^((2*a)/(b*p*q))*f^2*p^2*q^
2*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(a + b*Log[c*(d*(e + f*x)^p)^q])))
```

## Maple [F]

$$\int \frac{hx+g}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}dx$$

[In] int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)

[Out] int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.46

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{2}}\,dx = \frac{\left(((bfg-beh)pq\log\left(fx+e\right)+afg-aeh+(bfg-beh)q\log\left(d\right)+(bfg-beh)\log\left(c\right))e^{\left(\frac{bq\log(d)+b\log(c)+a}{bpq}\right)}\right.}{}$$

[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")

[Out] (((b*f*g - b*e*h)*p*q*log(f*x + e) + a*f*g - a*e*h + (b*f*g - b*e*h)*q*log(
d) + (b*f*g - b*e*h)*log(c))*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))*log_in
tegral((f*x + e)*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))) - (b*f^2*h*p*q*x^
2 + b*e*f*g*p*q + (b*f^2*g + b*e*f*h)*p*q*x)*e^(2*(b*q*log(d) + b*log(c) +
a)/(b*p*q)) + 2*(b*h*p*q*log(f*x + e) + b*h*q*log(d) + b*h*log(c) + a*h)*lo
g_integral((f^2*x^2 + 2*e*f*x + e^2)*e^(2*(b*q*log(d) + b*log(c) + a)/(b*p*
q))))*e^(-2*(b*q*log(d) + b*log(c) + a)/(b*p*q))/(b^3*f^2*p^3*q^3*log(f*x +
 e) + b^3*f^2*p^2*q^3*log(d) + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2)

## Sympy [F]

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{2}}\,dx = \int \frac{g+hx}{\left(a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)\right)^{2}}\,dx$$

[In] integrate((h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)

[Out] Integral((g + h*x)/(a + b*log(c*(d*(e + f*x)**p)**q))**2, x)

## Maxima [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{hx+g}{(b\log(((fx+e)^pd)^q c)+a)^2}\,dx$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

```
[Out] -(f*h*x^2 + e*g + (f*g + e*h)*x)/(b^2*f*p*q*log(((f*x + e)^p)^q) + a*b*f*p*
q + (f*p*q^2*log(d) + f*p*q*log(c))*b^2) + integrate((2*f*h*x + f*g + e*h)/
(b^2*f*p*q*log(((f*x + e)^p)^q) + a*b*f*p*q + (f*p*q^2*log(d) + f*p*q*log(c
))*b^2), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1930 vs. $2(225) = 450$.

Time = 0.39 (sec) , antiderivative size = 1930, normalized size of antiderivative = 8.62

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

```
[Out] -(f*x + e)*b*f*g*p*q/(b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d)
 + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2) - (f*x + e)^2*b*h*p*q/(b^3*f
^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d) + b^3*f^2*p^2*q^2*log(c) +
 a*b^2*f^2*p^2*q^2) + (f*x + e)*b*e*h*p*q/(b^3*f^2*p^3*q^3*log(f*x + e) + b
^3*f^2*p^2*q^3*log(d) + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2) + b*f*g
*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*
log(f*x + e)/((b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d) + b^3*
f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2)*c^(1/(p*q))*d^(1/p)) - b*e*h*p*q*Ei
(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x
 + e)/((b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d) + b^3*f^2*p^2
*q^2*log(c) + a*b^2*f^2*p^2*q^2)*c^(1/(p*q))*d^(1/p)) + 2*b*h*p*q*Ei(2*log(
d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(
f*x + e)/((b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d) + b^3*f^2*
p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2)*c^(2/(p*q))*d^(2/p)) + b*f*g*q*Ei(log(d
)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)/((b^3*
f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d) + b^3*f^2*p^2*q^2*log(c)
+ a*b^2*f^2*p^2*q^2)*c^(1/(p*q))*d^(1/p)) - b*e*h*q*Ei(log(d)/p + log(c)/(p
*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)/((b^3*f^2*p^3*q^3*log
(f*x + e) + b^3*f^2*p^2*q^3*log(d) + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2
*q^2)*c^(1/(p*q))*d^(1/p)) + b*f*g*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) +
 log(f*x + e))*e^(-a/(b*p*q))*log(c)/((b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f
^2*p^2*q^3*log(d) + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2)*c^(1/(p*q))
```

```
*d^(1/p)) - b*e*h*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^
(-a/(b*p*q))*log(c)/((b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d)
 + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2)*c^(1/(p*q))*d^(1/p)) + 2*b*h
*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(
b*p*q))*log(d)/((b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d) + b^
3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2)*c^(2/(p*q))*d^(2/p)) + a*f*g*Ei(l
og(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/((b^3*f^2
*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d) + b^3*f^2*p^2*q^2*log(c) + a
*b^2*f^2*p^2*q^2)*c^(1/(p*q))*d^(1/p)) - a*e*h*Ei(log(d)/p + log(c)/(p*q) +
 a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/((b^3*f^2*p^3*q^3*log(f*x + e) +
b^3*f^2*p^2*q^3*log(d) + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2)*c^(1/(
p*q))*d^(1/p)) + 2*b*h*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log
(f*x + e))*e^(-2*a/(b*p*q))*log(c)/((b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2
*p^2*q^3*log(d) + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2)*c^(2/(p*q))*d
^(2/p)) + 2*a*h*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x +
e))*e^(-2*a/(b*p*q))/((b^3*f^2*p^3*q^3*log(f*x + e) + b^3*f^2*p^2*q^3*log(d
) + b^3*f^2*p^2*q^2*log(c) + a*b^2*f^2*p^2*q^2)*c^(2/(p*q))*d^(2/p))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{g+h\,x}{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^2\,dx$$

```
[In] int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
[Out] int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^2, x)
```

# 3.452 $\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 123

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2fp^2q^2} - \frac{e+fx}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

```
[Out] (f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^2/exp(a/b/p/q)/f/p^2/q^2/((
c*(d*(f*x+e)^p)^q)^(1/p/q))+(-f*x-e)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2436, 2334, 2337, 2209, 2495}

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \frac{(e+fx)e^{-\frac{a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2fp^2q^2} - \frac{e+fx}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-2),x]
```

```
[Out] ((e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)])/(b^2*E^
(a/(b*p*q))*f*p^2*q^2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) - (e + f*x)/(b*f*p*q
*(a + b*Log[c*(d*(e + f*x)^p)^q]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cd^qx^{pq}))^2}\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$= -\frac{e+fx}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{a+b\log(cd^q x^{pq})}\,dx, x, e+fx\right)}{bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{e+fx}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{\left((e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{a+bx}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{bfp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^2fp^2q^2} - \frac{e+fx}{bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = -\frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\left(be^{\frac{a}{bpq}}pq(c(d(e+fx)^p)^q)^{\frac{1}{pq}} - \text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)(a+\ldots\right)}{b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-2),x]
```

```
[Out] -(((e + f*x)*(b*E^(a/(b*p*q))*p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*q)) - ExpInte
gralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)]*(a + b*Log[c*(d*(e + f*x)^
p)^q])))/(b^2*E^(a/(b*p*q))*f*p^2*q^2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*(a +
b*Log[c*(d*(e + f*x)^p)^q])))
```

## Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)\right)^2}dx$$

```
[In] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

```
[Out] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = -\frac{\left((bfpqx+bepq)e^{\left(\frac{bq\log(d)+b\log(c)+a}{bpq}\right)} - (bpq\log(fx+e)+bq\log(d)+b\log(c)+a)\,\mathrm{log\_integral}\left((fx+\right.\right.}{b^3fp^3q^3\log(fx+e)+b^3fp^2q^3\log(d)+b^3fp^2q^2\log(c)+ab^2fp^2}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
```

```
[Out] -((b*f*p*q*x + b*e*p*q)*e^((b*q*log(d) + b*log(c) + a)/(b*p*q)) - (b*p*q*lo
g(f*x + e) + b*q*log(d) + b*log(c) + a)*log_integral((f*x + e)*e^((b*q*log(
d) + b*log(c) + a)/(b*p*q))))*e^(-(b*q*log(d) + b*log(c) + a)/(b*p*q))/(b^3
*f*p^3*q^3*log(f*x + e) + b^3*f*p^2*q^3*log(d) + b^3*f*p^2*q^2*log(c) + a*b
^2*f*p^2*q^2)
```

## Sympy [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^2}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(-2), x)
```

## Maxima [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
```

```
[Out] -(f*x + e)/(b^2*f*p*q*log(((f*x + e)^p)^q) + a*b*f*p*q + (f*p*q^2*log(d) +
f*p*q*log(c))*b^2) + integrate(1/(b^2*p*q*log(((f*x + e)^p)^q) + a*b*p*q +
(p*q^2*log(d) + p*q*log(c))*b^2), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(123) = 246$.

Time = 0.34 (sec) , antiderivative size = 582, normalized size of antiderivative = 4.73

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$$
$$= -\frac{(fx+e)bpq}{b^3fp^3q^3\log(fx+e)+b^3fp^2q^3\log(d)+b^3fp^2q^2\log(c)+ab^2fp^2q^2}$$
$$+\frac{bpq\text{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log(fx+e)\right)e^{\left(-\frac{a}{bpq}\right)}\log(fx+e)}{(b^3fp^3q^3\log(fx+e)+b^3fp^2q^3\log(d)+b^3fp^2q^2\log(c)+ab^2fp^2q^2)c^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}}$$
$$+\frac{bq\text{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log(fx+e)\right)e^{\left(-\frac{a}{bpq}\right)}\log(d)}{(b^3fp^3q^3\log(fx+e)+b^3fp^2q^3\log(d)+b^3fp^2q^2\log(c)+ab^2fp^2q^2)c^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}}$$
$$+\frac{b\text{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log(fx+e)\right)e^{\left(-\frac{a}{bpq}\right)}\log(c)}{(b^3fp^3q^3\log(fx+e)+b^3fp^2q^3\log(d)+b^3fp^2q^2\log(c)+ab^2fp^2q^2)c^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}}$$
$$+\frac{a\text{Ei}\left(\frac{\log(d)}{p}+\frac{\log(c)}{pq}+\frac{a}{bpq}+\log(fx+e)\right)e^{\left(-\frac{a}{bpq}\right)}}{(b^3fp^3q^3\log(fx+e)+b^3fp^2q^3\log(d)+b^3fp^2q^2\log(c)+ab^2fp^2q^2)c^{\frac{1}{pq}}d^{\left(\frac{1}{p}\right)}}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
```

```
[Out] -(f*x + e)*b*p*q/(b^3*f*p^3*q^3*log(f*x + e) + b^3*f*p^2*q^3*log(d) + b^3*f
*p^2*q^2*log(c) + a*b^2*f*p^2*q^2) + b*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(
b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)/((b^3*f*p^3*q^3*log(f*x
+ e) + b^3*f*p^2*q^3*log(d) + b^3*f*p^2*q^2*log(c) + a*b^2*f*p^2*q^2)*c^(1/
(p*q))*d^(1/p)) + b*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e)
)*e^(-a/(b*p*q))*log(d)/((b^3*f*p^3*q^3*log(f*x + e) + b^3*f*p^2*q^3*log(d)
 + b^3*f*p^2*q^2*log(c) + a*b^2*f*p^2*q^2)*c^(1/(p*q))*d^(1/p)) + b*Ei(log(
d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)/((b^3
*f*p^3*q^3*log(f*x + e) + b^3*f*p^2*q^3*log(d) + b^3*f*p^2*q^2*log(c) + a*b
^2*f*p^2*q^2)*c^(1/(p*q))*d^(1/p)) + a*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*
q) + log(f*x + e))*e^(-a/(b*p*q))/((b^3*f*p^3*q^3*log(f*x + e) + b^3*f*p^2*
q^3*log(d) + b^3*f*p^2*q^2*log(c) + a*b^2*f*p^2*q^2)*c^(1/(p*q))*d^(1/p))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^2}\,dx$$

```
[In] int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
[Out] int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^2, x)
```

# 3.453 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

```
[In] Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
[Out] Defer[Int][1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

```
[In] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
[Out] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)\right)^2}dx$$

```
[In] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
[Out] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(b\log\left(\left((fx+e)^p d\right)^q c\right)+a)^2}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*h*x + a^2*g + (b^2*h*x + b^2*g)*log(((f*x + e)^p*d)^q*c)^2
+ 2*(a*b*h*x + a*b*g)*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 5.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^2(g+hx)}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 267, normalized size of antiderivative = 9.54

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{(hx+g)(b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
[Out] (f*g - e*h)*integrate(1/(a*b*f*g^2*p*q + (f*g^2*p*q^2*log(d) + f*g^2*p*q*lo
g(c))*b^2 + (a*b*f*h^2*p*q + (f*h^2*p*q^2*log(d) + f*h^2*p*q*log(c))*b^2)*x
^2 + 2*(a*b*f*g*h*p*q + (f*g*h*p*q^2*log(d) + f*g*h*p*q*log(c))*b^2)*x + (b
^2*f*h^2*p*q*x^2 + 2*b^2*f*g*h*p*q*x + b^2*f*g^2*p*q)*log(((f*x + e)^p)^q))
, x) - (f*x + e)/(a*b*f*g*p*q + (f*g*p*q^2*log(d) + f*g*p*q*log(c))*b^2 + (
a*b*f*h*p*q + (f*h*p*q^2*log(d) + f*h*p*q*log(c))*b^2)*x + (b^2*f*h*p*q*x +
 b^2*f*g*p*q)*log(((f*x + e)^p)^q))
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{(hx+g)(b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
[Out] integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{1}{(g+h\,x)\,\left(a+b\,\ln\left(c\left(d\,(e+f\,x)^p\right)^q\right)\right)^2}\,dx$$

```
[In] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2),x)
[Out] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2), x)
```

# 3.454 $\int \frac{1}{(g+hx)^2(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)^2\,(a+b\log(c\,(d(e+fx)^p)^q))^2}\,dx = \mathrm{Int}\left(\frac{1}{(g+hx)^2\,(a+b\log(c\,(d(e+fx)^p)^q))^2},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(g+hx)^2\,(a+b\log(c\,(d(e+fx)^p)^q))^2}\,dx = \int \frac{1}{(g+hx)^2\,(a+b\log(c\,(d(e+fx)^p)^q))^2}\,dx$$

```
[In] Int[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

```
[Out] Defer[Int][1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(g+hx)^2\,(a+b\log(c\,(d(e+fx)^p)^q))^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 8.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^2}\,dx$$

```
[In] Integrate[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
[Out] Integrate[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)^2 (a+b\ln(c(d(fx+e)^p)^q))^2}dx$$

```
[In] int(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
[Out] int(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.00

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{1}{(hx+g)^2 (b\log(((fx+e)^p d)^q c)+a)^2}\,dx$$

```
[In] integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*h^2*x^2 + 2*a^2*g*h*x + a^2*g^2 + (b^2*h^2*x^2 + 2*b^2*g*h*
x + b^2*g^2)*log(((f*x + e)^p*d)^q*c)^2 + 2*(a*b*h^2*x^2 + 2*a*b*g*h*x + a*
b*g^2)*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 31.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^2 (g+hx)^2}\,dx$$

```
[In] integrate(1/(h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 406, normalized size of antiderivative = 14.50

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{1}{(hx+g)^2(b\log(((fx+e)^p d)^q c)+a)^2}\,dx$$

```
[In] integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
[Out] -(f*x + e)/(a*b*f*g^2*p*q + (f*g^2*p*q^2*log(d) + f*g^2*p*q*log(c))*b^2 + (
a*b*f*h^2*p*q + (f*h^2*p*q^2*log(d) + f*h^2*p*q*log(c))*b^2)*x^2 + 2*(a*b*f
*g*h*p*q + (f*g*h*p*q^2*log(d) + f*g*h*p*q*log(c))*b^2)*x + (b^2*f*h^2*p*q*
x^2 + 2*b^2*f*g*h*p*q*x + b^2*f*g^2*p*q)*log(((f*x + e)^p)^q)) - integrate(
(f*h*x - f*g + 2*e*h)/(a*b*f*g^3*p*q + (a*b*f*h^3*p*q + (f*h^3*p*q^2*log(d)
 + f*h^3*p*q*log(c))*b^2)*x^3 + (f*g^3*p*q^2*log(d) + f*g^3*p*q*log(c))*b^2
 + 3*(a*b*f*g*h^2*p*q + (f*g*h^2*p*q^2*log(d) + f*g*h^2*p*q*log(c))*b^2)*x^
2 + 3*(a*b*f*g^2*h*p*q + (f*g^2*h*p*q^2*log(d) + f*g^2*h*p*q*log(c))*b^2)*x
 + (b^2*f*h^3*p*q*x^3 + 3*b^2*f*g*h^2*p*q*x^2 + 3*b^2*f*g^2*h*p*q*x + b^2*f
*g^3*p*q)*log(((f*x + e)^p)^q)), x)
```

## Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^2}\,dx = \int \frac{1}{(hx+g)^2(b\log(((fx+e)^p d)^q c)+a)^2}\,dx$$

```
[In] integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
[Out] integrate(1/((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^2}\,dx=\int \frac{1}{{\left(g+h\,x\right)}^2\,{\left(a+b\,\ln \left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^2} \,d x$$

```
[In] int(1/((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2),x)
[Out] int(1/((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2), x)
```

# 3.455 $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 432

$$\begin{aligned}
\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx
&= \frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3f^3p^3q^3} \\
&\quad+\frac{4e^{-\frac{2a}{bpq}}h(fg-eh)(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^3f^3p^3q^3} \\
&\quad+\frac{9e^{-\frac{3a}{bpq}}h^2(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{2b^3f^3p^3q^3} \\
&\quad-\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}+\frac{(fg-eh)(e+fx)(g+hx)}{b^2f^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} \\
&\quad-\frac{3(e+fx)(g+hx)^2}{2b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}
\end{aligned}$$

```
[Out] 1/2*(-e*h+f*g)^2*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^3/exp(a/b/
p/q)/f^3/p^3/q^3/((c*(d*(f*x+e)^p)^q)^(1/p/q))+4*h*(-e*h+f*g)*(f*x+e)^2*Ei(
2*(a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^3/exp(2*a/b/p/q)/f^3/p^3/q^3/((c*(d*
(f*x+e)^p)^q)^(2/p/q))+9/2*h^2*(f*x+e)^3*Ei(3*(a+b*ln(c*(d*(f*x+e)^p)^q))/b
/p/q)/b^3/exp(3*a/b/p/q)/f^3/p^3/q^3/((c*(d*(f*x+e)^p)^q)^(3/p/q))-1/2*(f*x
+e)*(h*x+g)^2/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^2+(-e*h+f*g)*(f*x+e)*(h*x
+g)/b^2/f^2/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q))-3/2*(f*x+e)*(h*x+g)^2/b^2/f
/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q))
```

## Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2447, 2446, 2436, 2337, 2209, 2437, 2347, 2495}

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \frac{4h(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^3f^3p^3q^3} + \frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3f^3p^3q^3} + \frac{9h^2(e+fx)^3e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\text{ExpIntegralEi}\left(\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{2b^3f^3p^3q^3} + \frac{(e+fx)(g+hx)(fg-eh)}{b^2f^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))} - \frac{3(e+fx)(g+hx)^2}{2b^2fp^2q^2(a+b\log(c(d(e+fx)^p)^q))} - \frac{(e+fx)(g+hx)^2}{2bfpq(a+b\log(c(d(e+fx)^p)^q))^2}$$

```
[In] Int[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

```
[Out] ((f*g - e*h)^2*(e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*
p*q)])/(2*b^3*E^(a/(b*p*q))*f^3*p^3*q^3*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) +
(4*h*(f*g - e*h)*(e + f*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^
q]))/(b*p*q)])/(b^3*E^((2*a)/(b*p*q))*f^3*p^3*q^3*(c*(d*(e + f*x)^p)^q)^(2/
(p*q))) + (9*h^2*(e + f*x)^3*ExpIntegralEi[(3*(a + b*Log[c*(d*(e + f*x)^p)^
q]))/(b*p*q)])/(2*b^3*E^((3*a)/(b*p*q))*f^3*p^3*q^3*(c*(d*(e + f*x)^p)^q)^(
3/(p*q))) - ((e + f*x)*(g + h*x)^2)/(2*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)
^q])^2) + ((f*g - e*h)*(e + f*x)*(g + h*x))/(b^2*f^2*p^2*q^2*(a + b*Log[c*(
d*(e + f*x)^p)^q])) - (3*(e + f*x)*(g + h*x)^2)/(2*b^2*f*p^2*q^2*(a + b*Log
[c*(d*(e + f*x)^p)^q]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(g+hx)^2}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} + \text{Subst}\left(\frac{3\int \frac{(g+hx)^2}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}\,dx}{2bpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(fg-eh)\int \frac{g+hx}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}\,dx}{bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} + \frac{(fg-eh)(e+fx)(g+hx)}{b^2f^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} - \frac{3(e+fx)(g+hx)^2}{2b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{9\int \frac{(g+hx)^2}{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx}{2b^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(2(fg-eh))\int \frac{g+hx}{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx}{b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(3(fg-eh))\int \frac{g+hx}{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx}{b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(fg-eh)^2\int \frac{1}{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx}{b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} + \frac{(fg-eh)(e+fx)(g+hx)}{b^2f^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} - \frac{3(e+fx)(g+hx)^2}{2b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

$$+\text{Subst}\left(\frac{9\int\left(\frac{(fg-eh)^2}{f^2(a+b\log(cd^q(e+fx)^{pq}))}+\frac{2h(fg-eh)(e+fx)}{f^2(a+b\log(cd^q(e+fx)^{pq}))}+\frac{h^2(e+fx)^2}{f^2(a+b\log(cd^q(e+fx)^{pq}))}\right)dx}{2b^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$-\text{Subst}\left(\frac{(2(fg-eh))\int\left(\frac{fg-eh}{f(a+b\log(cd^q(e+fx)^{pq}))}+\frac{h(e+fx)}{f(a+b\log(cd^q(e+fx)^{pq}))}\right)dx}{b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$-\text{Subst}\left(\frac{(3(fg-eh))\int\left(\frac{fg-eh}{f(a+b\log(cd^q(e+fx)^{pq}))}+\frac{h(e+fx)}{f(a+b\log(cd^q(e+fx)^{pq}))}\right)dx}{b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{(fg-eh)^2\text{Subst}\left(\int\frac{1}{a+b\log(cd^qx^{pq})}dx, x, e+fx\right)}{b^2f^3p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} + \frac{(fg-eh)(e+fx)(g+hx)}{b^2f^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

$$- \frac{3(e+fx)(g+hx)^2}{2b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

$$+ \text{Subst}\left(\frac{(9h^2)\int \frac{(e+fx)^2}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{2b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$- \text{Subst}\left(\frac{(2h(fg-eh))\int \frac{e+fx}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$- \text{Subst}\left(\frac{(3h(fg-eh))\int \frac{e+fx}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$+ \text{Subst}\left(\frac{(9h(fg-eh))\int \frac{e+fx}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$- \text{Subst}\left(\frac{(2(fg-eh)^2)\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$- \text{Subst}\left(\frac{(3(fg-eh)^2)\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$+ \text{Subst}\left(\frac{(9(fg-eh)^2)\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{2b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left((fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{a+bx}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{b^2f^3p^3q^3}, cd^q(\right.$$

$$\left. + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$=\frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^3f^3p^3q^3}$$
$$-\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}+\frac{(fg-eh)(e+fx)(g+hx)}{b^2f^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$
$$-\frac{3(e+fx)(g+hx)^2}{2b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$
$$+\operatorname{Subst}\left(\frac{(9h^2)\operatorname{Subst}\left(\int\frac{x^2}{a+b\log(cd^qx^{pq})}\,dx,x,e+fx\right)}{2b^2f^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\operatorname{Subst}\left(\frac{(2h(fg-eh))\operatorname{Subst}\left(\int\frac{x}{a+b\log(cd^qx^{pq})}\,dx,x,e+fx\right)}{b^2f^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\operatorname{Subst}\left(\frac{(3h(fg-eh))\operatorname{Subst}\left(\int\frac{x}{a+b\log(cd^qx^{pq})}\,dx,x,e+fx\right)}{b^2f^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$+\operatorname{Subst}\left(\frac{(9h(fg-eh))\operatorname{Subst}\left(\int\frac{x}{a+b\log(cd^qx^{pq})}\,dx,x,e+fx\right)}{b^2f^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\operatorname{Subst}\left(\frac{(2(fg-eh)^2)\operatorname{Subst}\left(\int\frac{1}{a+b\log(cd^qx^{pq})}\,dx,x,e+fx\right)}{b^2f^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\operatorname{Subst}\left(\frac{(3(fg-eh)^2)\operatorname{Subst}\left(\int\frac{1}{a+b\log(cd^qx^{pq})}\,dx,x,e+fx\right)}{b^2f^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$+\operatorname{Subst}\left(\frac{(9(fg-eh)^2)\operatorname{Subst}\left(\int\frac{1}{a+b\log(cd^qx^{pq})}\,dx,x,e+fx\right)}{2b^2f^3p^2q^2},cd^q(e\right.$$

$$= \frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{b^3f^3p^3q^3}$$
$$-\frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$
$$+\frac{(fg-eh)(e+fx)(g+hx)}{b^2f^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}-\frac{3(e+fx)(g+hx)^2}{2b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$
$$+\text{Subst}\left(\frac{\left(9h^2(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{2b^2f^3p^3q^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\text{Subst}\left(\frac{\left(2h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{b^2f^3p^3q^3},\ldots+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\text{Subst}\left(\frac{\left(3h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{b^2f^3p^3q^3},\ldots+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$+\text{Subst}\left(\frac{\left(9h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{b^2f^3p^3q^3},\ldots+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\text{Subst}\left(\frac{\left(2(fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{b^2f^3p^3q^3},cd\ldots+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\text{Subst}\left(\frac{\left(3(fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{\ldots},cd\ldots\right.$$

$$= \frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3f^3p^3q^3} + \frac{4e^{-\frac{2a}{bpq}}h(fg-eh)(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{Ei}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^3f^3p^3q^3} + \frac{9e^{-\frac{3a}{bpq}}h^2(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{Ei}\left(\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{2b^3f^3p^3q^3} - \frac{(e+fx)(g+hx)^2}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} + \frac{(fg-eh)(e+fx)(g+hx)}{b^2f^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} - \frac{3(e+fx)(g+hx)^2}{2b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

## Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.01

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \frac{e^{-\frac{3a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\left(e^{\frac{2a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)(\ldots}{}$$

```
[In] Integrate[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

```
[Out] ((e + f*x)*(E^((2*a)/(b*p*q))*(f*g - e*h)^2*(c*(d*(e + f*x)^p)^q)^(2/(p*q))
*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)]*(a + b*Log[c*(d*(e
 + f*x)^p)^q])^2 - 8*E^(a/(b*p*q))*h*(-(f*g) + e*h)*(e + f*x)*(c*(d*(e + f*
x)^p)^q)^(1/(p*q))*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*
q)]*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + 9*h^2*(e + f*x)^2*ExpIntegralEi[(3
*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)]*(a + b*Log[c*(d*(e + f*x)^p)^q]
)^2 - b*E^((3*a)/(b*p*q))*f*p*q*(c*(d*(e + f*x)^p)^q)^(3/(p*q))*(g + h*x)*(
b*f*p*q*(g + h*x) + a*(f*g + 2*e*h + 3*f*h*x) + b*(2*e*h + f*(g + 3*h*x))*L
og[c*(d*(e + f*x)^p)^q])))/(2*b^3*E^((3*a)/(b*p*q))*f^3*p^3*q^3*(c*(d*(e +
f*x)^p)^q)^(3/(p*q))*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)
```

## Maple [F]

$$\int \frac{(hx+g)^2}{(a+b\ln(c(d(fx+e)^p)^q))^3}dx$$

[In] int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)

[Out] int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1682 vs. $2(426) = 852$.

Time = 0.37 (sec) , antiderivative size = 1682, normalized size of antiderivative = 3.89

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \text{Too large to display}$$

[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")

[Out] 1/2*(8*((b^2*f*g*h - b^2*e*h^2)*p^2*q^2*log(f*x + e)^2 + a^2*f*g*h - a^2*e*h^2 + (b^2*f*g*h - b^2*e*h^2)*q^2*log(d)^2 + (b^2*f*g*h - b^2*e*h^2)*log(c)^2 + 2*((b^2*f*g*h - b^2*e*h^2)*p*q^2*log(d) + (b^2*f*g*h - b^2*e*h^2)*p*q*log(c) + (a*b*f*g*h - a*b*e*h^2)*p*q)*log(f*x + e) + 2*(a*b*f*g*h - a*b*e*h^2)*log(c) + 2*((b^2*f*g*h - b^2*e*h^2)*q*log(c) + (a*b*f*g*h - a*b*e*h^2)*q)*log(d))*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))*log_integral((f^2*x^2 + 2*e*f*x + e^2)*e^(2*(b*q*log(d) + b*log(c) + a)/(b*p*q))) + ((b^2*f^2*g^2 - 2*b^2*e*f*g*h + b^2*e^2*h^2)*p^2*q^2*log(f*x + e)^2 + a^2*f^2*g^2 - 2*a^2*e*f*g*h + a^2*e^2*h^2 + (b^2*f^2*g^2 - 2*b^2*e*f*g*h + b^2*e^2*h^2)*q^2*log(d)^2 + (b^2*f^2*g^2 - 2*b^2*e*f*g*h + b^2*e^2*h^2)*log(c)^2 + 2*((b^2*f^2*g^2 - 2*b^2*e*f*g*h + b^2*e^2*h^2)*p*q^2*log(d) + (b^2*f^2*g^2 - 2*b^2*e*f*g*h + b^2*e^2*h^2)*p*q*log(c) + (a*b*f^2*g^2 - 2*a*b*e*f*g*h + a*b*e^2*h^2)*p*q)*log(f*x + e) + 2*(a*b*f^2*g^2 - 2*a*b*e*f*g*h + a*b*e^2*h^2)*log(c) + 2*((b^2*f^2*g^2 - 2*b^2*e*f*g*h + b^2*e^2*h^2)*q*log(c) + (a*b*f^2*g^2 - 2*a*b*e*f*g*h + a*b*e^2*h^2)*q)*log(d))*e^(2*(b*q*log(d) + b*log(c) + a)/(b*p*q))*log_integral((f*x + e)*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))) - (b^2*e*f^2*g^2*p^2*q^2 + (b^2*f^3*h^2*p^2*q^2 + 3*a*b*f^3*h^2*p*q)*x^3 + (a*b*e*f^2*g^2 + 2*a*b*e^2*f*g*h)*p*q + ((2*b^2*f^3*g*h + b^2*e*f^2*h^2)*p^2*q^2 + (4*a*b*f^3*g*h + 5*a*b*e*f^2*h^2)*p*q)*x^2 + ((b^2*f^3*g^2 + 2*b^2*e*f^2*g*h)*p^2*q^2 + (a*b*f^3*g^2 + 6*a*b*e*f^2*g*h + 2*a*b*e^2*f*h^2)*p*q)*x + (3*b^2*f^3*h^2*p^2*q^2*x^3 + (4*b^2*f^3*g*h + 5*b^2*e*f^2*h^2)*p^2*q^2*x^2 + (b^2*f^3*g^2 + 6*b^2*e*f^2*g*h + 2*b^2*e^2*f*h^2)*p^2*q^2*x + (b^2*e*f^2*g^2 + 2*b^2*e^2*f*g*h)*p^2*q^2)*log(f*x + e) + (3*b^2*f^3*h^2*p*q*x^3 + (4*b^2*f^3*g*h + 5*b^2*e*f^2*h^2)*p*q*x^2 + (b^2*f^3*g^2 + 6*b^2*e*f^2*g*h + 2*b^2*e^2*f*h^2)*p*q*x + (b^2*e*f^2*g^2 + 2*b^2*e^2*f*g*h)*p*q)*log(c) + (3*b^2*f^3*h^2*p*q^2*x^3 + (4*b^2*f^3*g*h + 5*b^2*e*f^2*h^2)*p*q^2*x^2 + (b^2*

```
f^3*g^2 + 6*b^2*e*f^2*g*h + 2*b^2*e^2*f*h^2)*p*q^2*x + (b^2*e*f^2*g^2 + 2*b
^2*e^2*f*g*h)*p*q^2)*log(d))*e^(3*(b*q*log(d) + b*log(c) + a)/(b*p*q)) + 9*
(b^2*h^2*p^2*q^2*log(f*x + e)^2 + b^2*h^2*q^2*log(d)^2 + b^2*h^2*log(c)^2 +
 2*a*b*h^2*log(c) + a^2*h^2 + 2*(b^2*h^2*p*q^2*log(d) + b^2*h^2*p*q*log(c)
+ a*b*h^2*p*q)*log(f*x + e) + 2*(b^2*h^2*q*log(c) + a*b*h^2*q)*log(d))*log_
integral((f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)*e^(3*(b*q*log(d) + b*log
(c) + a)/(b*p*q))))*e^(-3*(b*q*log(d) + b*log(c) + a)/(b*p*q))/(b^5*f^3*p^5
*q^5*log(f*x + e)^2 + b^5*f^3*p^3*q^5*log(d)^2 + b^5*f^3*p^3*q^3*log(c)^2 +
 2*a*b^4*f^3*p^3*q^3*log(c) + a^2*b^3*f^3*p^3*q^3 + 2*(b^5*f^3*p^4*q^5*log(
d) + b^5*f^3*p^4*q^4*log(c) + a*b^4*f^3*p^4*q^4)*log(f*x + e) + 2*(b^5*f^3*
p^3*q^4*log(c) + a*b^4*f^3*p^3*q^4)*log(d))
```

## Sympy [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx=\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$$

```
[In] integrate((h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
[Out] Integral((g + h*x)**2/(a + b*log(c*(d*(e + f*x)**p)**q))**3, x)
```

## Maxima [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx=\int \frac{(hx+g)^2}{(b\log(((fx+e)^p d)^q c)+a)^3}\,dx$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
[Out] -1/2*((3*a*f^2*h^2 + (f^2*h^2*p*q + 3*f^2*h^2*q*log(d) + 3*f^2*h^2*log(c))*
b)*x^3 + ((4*f^2*g*h + 5*e*f*h^2)*a + (2*f^2*g*h*p*q + e*f*h^2*p*q + (4*f^2
*g*h + 5*e*f*h^2)*log(c) + (4*f^2*g*h*q + 5*e*f*h^2*q)*log(d))*b)*x^2 + (e*
f*g^2 + 2*e^2*g*h)*a + (e*f*g^2*p*q + (e*f*g^2 + 2*e^2*g*h)*log(c) + (e*f*g
^2*q + 2*e^2*g*h*q)*log(d))*b + ((f^2*g^2 + 6*e*f*g*h + 2*e^2*h^2)*a + (f^2
*g^2*p*q + 2*e*f*g*h*p*q + (f^2*g^2 + 6*e*f*g*h + 2*e^2*h^2)*log(c) + (f^2*
g^2*q + 6*e*f*g*h*q + 2*e^2*h^2*q)*log(d))*b)*x + (3*b*f^2*h^2*x^3 + (4*f^2
*g*h + 5*e*f*h^2)*b*x^2 + (f^2*g^2 + 6*e*f*g*h + 2*e^2*h^2)*b*x + (e*f*g^2
+ 2*e^2*g*h)*b)*log(((f*x + e)^p)^q))/(b^4*f^2*p^2*q^2*log(((f*x + e)^p)^q)
^2 + a^2*b^2*f^2*p^2*q^2 + 2*(f^2*p^2*q^3*log(d) + f^2*p^2*q^2*log(c))*a*b^
3 + (f^2*p^2*q^4*log(d)^2 + 2*f^2*p^2*q^3*log(c)*log(d) + f^2*p^2*q^2*log(c
)^2)*b^4 + 2*(a*b^3*f^2*p^2*q^2 + (f^2*p^2*q^3*log(d) + f^2*p^2*q^2*log(c))
*b^4)*log(((f*x + e)^p)^q)) + integrate(1/2*(9*f^2*h^2*x^2 + f^2*g^2 + 6*e*
f*g*h + 2*e^2*h^2 + 2*(4*f^2*g*h + 5*e*f*h^2)*x)/(b^3*f^2*p^2*q^2*log(((f*x
 + e)^p)^q) + a*b^2*f^2*p^2*q^2 + (f^2*p^2*q^3*log(d) + f^2*p^2*q^2*log(c))
*b^3), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5889 vs. $2(426) = 852$.

Time = 0.54 (sec) , antiderivative size = 5889, normalized size of antiderivative = 13.63

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

```
[Out] -1/2*((f*x + e)*b^2*f^2*g^2*p^2*q^2*log(f*x + e) + 4*(f*x + e)^2*b^2*f*g*h*
p^2*q^2*log(f*x + e) - 2*(f*x + e)*b^2*e*f*g*h*p^2*q^2*log(f*x + e) + 3*(f*
x + e)^3*b^2*h^2*p^2*q^2*log(f*x + e) - 4*(f*x + e)^2*b^2*e*h^2*p^2*q^2*log
(f*x + e) + (f*x + e)*b^2*e^2*h^2*p^2*q^2*log(f*x + e) - b^2*f^2*g^2*p^2*q^
2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log
(f*x + e)^2/(c^(1/(p*q))*d^(1/p)) + 2*b^2*e*f*g*h*p^2*q^2*Ei(log(d)/p + log
(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)^2/(c^(1/(
p*q))*d^(1/p)) - b^2*e^2*h^2*p^2*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q)
 + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)^2/(c^(1/(p*q))*d^(1/p)) + (f*x
 + e)*b^2*f^2*g^2*p^2*q^2 + 2*(f*x + e)^2*b^2*f*g*h*p^2*q^2 - 2*(f*x + e)*b
^2*e*f*g*h*p^2*q^2 + (f*x + e)^3*b^2*h^2*p^2*q^2 - 2*(f*x + e)^2*b^2*e*h^2*
p^2*q^2 + (f*x + e)*b^2*e^2*h^2*p^2*q^2 - 8*b^2*f*g*h*p^2*q^2*Ei(2*log(d)/p
 + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(f*x
+ e)^2/(c^(2/(p*q))*d^(2/p)) + 8*b^2*e*h^2*p^2*q^2*Ei(2*log(d)/p + 2*log(c)
/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(f*x + e)^2/(c^(
2/(p*q))*d^(2/p)) + (f*x + e)*b^2*f^2*g^2*p*q^2*log(d) + 4*(f*x + e)^2*b^2*
f*g*h*p*q^2*log(d) - 2*(f*x + e)*b^2*e*f*g*h*p*q^2*log(d) + 3*(f*x + e)^3*b
^2*h^2*p*q^2*log(d) - 4*(f*x + e)^2*b^2*e*h^2*p*q^2*log(d) + (f*x + e)*b^2*
e^2*h^2*p*q^2*log(d) - 2*b^2*f^2*g^2*p*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(
b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)*log(d)/(c^(1/(p*q))*d^(1
/p)) + 4*b^2*e*f*g*h*p*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x
 + e))*e^(-a/(b*p*q))*log(f*x + e)*log(d)/(c^(1/(p*q))*d^(1/p)) - 2*b^2*e^2
*h^2*p*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*
p*q))*log(f*x + e)*log(d)/(c^(1/(p*q))*d^(1/p)) - 9*b^2*h^2*p^2*q^2*Ei(3*lo
g(d)/p + 3*log(c)/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a/(b*p*q))*lo
g(f*x + e)^2/(c^(3/(p*q))*d^(3/p)) + (f*x + e)*b^2*f^2*g^2*p*q*log(c) + 4*(
f*x + e)^2*b^2*f*g*h*p*q*log(c) - 2*(f*x + e)*b^2*e*f*g*h*p*q*log(c) + 3*(f
*x + e)^3*b^2*h^2*p*q*log(c) - 4*(f*x + e)^2*b^2*e*h^2*p*q*log(c) + (f*x +
e)*b^2*e^2*h^2*p*q*log(c) - 2*b^2*f^2*g^2*p*q*Ei(log(d)/p + log(c)/(p*q) +
a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)*log(c)/(c^(1/(p*q))*d
^(1/p)) + 4*b^2*e*f*g*h*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*
x + e))*e^(-a/(b*p*q))*log(f*x + e)*log(c)/(c^(1/(p*q))*d^(1/p)) - 2*b^2*e^
2*h^2*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p
*q))*log(f*x + e)*log(c)/(c^(1/(p*q))*d^(1/p)) - 16*b^2*f*g*h*p*q^2*Ei(2*lo
g(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*lo
```

g(f*x + e)*log(d)/(c^(2/(p*q))*d^(2/p)) + 16*b^2*e*h^2*p*q^2*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(f*x + e)*log(d)/(c^(2/(p*q))*d^(2/p)) - b^2*f^2*g^2*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)^2/(c^(1/(p*q))*d^(1/p)) + 2*b^2*e*f*g*h*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)^2/(c^(1/(p*q))*d^(1/p)) - b^2*e^2*h^2*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)^2/(c^(1/(p*q))*d^(1/p)) + (f*x + e)*a*b*f^2*g^2*p*q + 4*(f*x + e)^2*a*b*f*g*h*p*q - 2*(f*x + e)*a*b*e*f*g*h*p*q + 3*(f*x + e)^3*a*b*h^2*p*q - 4*(f*x + e)^2*a*b*e*h^2*p*q + (f*x + e)*a*b*e^2*h^2*p*q - 2*a*b*f^2*g^2*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)/(c^(1/(p*q))*d^(1/p)) + 4*a*b*e*f*g*h*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)/(c^(1/(p*q))*d^(1/p)) - 2*a*b*e^2*h^2*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)/(c^(1/(p*q))*d^(1/p)) - 16*b^2*f*g*h*p*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(f*x + e)*log(c)/(c^(2/(p*q))*d^(2/p)) + 16*b^2*e*h^2*p*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(f*x + e)*log(c)/(c^(2/(p*q))*d^(2/p)) - 18*b^2*h^2*p*q^2*Ei(3*log(d)/p + 3*log(c)/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a/(b*p*q))*log(f*x + e)*log(d)/(c^(3/(p*q))*d^(3/p)) - 2*b^2*f^2*g^2*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)*log(d)/(c^(1/(p*q))*d^(1/p)) + 4*b^2*e*f*g*h*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)*log(d)/(c^(1/(p*q))*d^(1/p)) - 2*b^2*e^2*h^2*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)*log(d)/(c^(1/(p*q))*d^(1/p)) - 8*b^2*f*g*h*q^2*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(d)^2/(c^(2/(p*q))*d^(2/p)) + 8*b^2*e*h^2*q^2*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(d)^2/(c^(2/(p*q))*d^(2/p)) - 16*a*b*f*g*h*p*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(f*x + e)/(c^(2/(p*q))*d^(2/p)) + 16*a*b*e*h^2*p*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(f*x + e)/(c^(2/(p*q))*d^(2/p)) - 18*b^2*h^2*p*q*Ei(3*log(d)/p + 3*log(c)/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a/(b*p*q))*log(f*x + e)*log(c)/(c^(3/(p*q))*d^(3/p)) - b^2*f^2*g^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)^2/(c^(1/(p*q))*d^(1/p)) + 2*b^2*e*f*g*h*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)^2/(c^(1/(p*q))*d^(1/p)) - b^2*e^2*h^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)^2/(c^(1/(p*q))*d^(1/p)) - 2*a*b*f^2*g^2*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)/(c^(1/(p*q))*d^(1/p)) + 4*a*b*e*f*g*h*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)/(c^(1/(p*q))*d^(1/p)) - 2*a*b*e^2*h^2*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)/(c^(1/(p*q))*d^(1/p)) - 16*b^2*f*g*h*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(c)*log(d)/(c^(2/(p*q))*d^

(2/p)) + 16*b^2*e*h^2*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(c)*log(d)/(c^(2/(p*q))*d^(2/p)) - 9*b^2*h^2*q^2*Ei(3*log(d)/p + 3*log(c)/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a/(b*p*q))*log(d)^2/(c^(3/(p*q))*d^(3/p)) - 18*a*b*h^2*p*q*Ei(3*log(d)/p + 3*log(c)/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a/(b*p*q))*log(f*x + e)/(c^(3/(p*q))*d^(3/p)) - 2*a*b*f^2*g^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)/(c^(1/(p*q))*d^(1/p)) + 4*a*b*e*f*g*h*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)/(c^(1/(p*q))*d^(1/p)) - 2*a*b*e^2*h^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)/(c^(1/(p*q))*d^(1/p)) - 8*b^2*f*g*h*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(c)^2/(c^(2/(p*q))*d^(2/p)) + 8*b^2*e*h^2*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(c)^2/(c^(2/(p*q))*d^(2/p)) - 16*a*b*f*g*h*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(d)/(c^(2/(p*q))*d^(2/p)) + 16*a*b*e*h^2*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(d)/(c^(2/(p*q))*d^(2/p)) - 18*b^2*h^2*q*Ei(3*log(d)/p + 3*log(c)/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a/(b*p*q))*log(c)*log(d)/(c^(3/(p*q))*d^(3/p)) - a^2*f^2*g^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/(c^(1/(p*q))*d^(1/p)) + 2*a^2*e*f*g*h*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/(c^(1/(p*q))*d^(1/p)) - a^2*e^2*h^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/(c^(1/(p*q))*d^(1/p)) - 16*a*b*f*g*h*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(c)/(c^(2/(p*q))*d^(2/p)) + 16*a*b*e*h^2*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(c)/(c^(2/(p*q))*d^(2/p)) - 9*b^2*h^2*Ei(3*log(d)/p + 3*log(c)/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a/(b*p*q))*log(c)^2/(c^(3/(p*q))*d^(3/p)) - 18*a*b*h^2*q*Ei(3*log(d)/p + 3*log(c)/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a/(b*p*q))*log(d)/(c^(3/(p*q))*d^(3/p)) - 8*a^2*f*g*h*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))/(c^(2/(p*q))*d^(2/p)) + 8*a^2*e*h^2*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))/(c^(2/(p*q))*d^(2/p)) - 18*a*b*h^2*Ei(3*log(d)/p + 3*log(c)/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a/(b*p*q))*log(c)/(c^(3/(p*q))*d^(3/p)) - 9*a^2*h^2*Ei(3*log(d)/p + 3*log(c)/(p*q) + 3*a/(b*p*q) + 3*log(f*x + e))*e^(-3*a/(b*p*q))/(c^(3/(p*q))*d^(3/p)))/(b^5*f^3*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^3*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^3*p^4*q^4*log(f*x + e)*log(c) + b^5*f^3*p^3*q^5*log(d)^2 + 2*a*b^4*f^3*p^4*q^4*log(f*x + e) + 2*b^5*f^3*p^3*q^4*log(c)*log(d) + b^5*f^3*p^3*q^3*log(c)^2 + 2*a*b^4*f^3*p^3*q^4*log(d) + 2*a*b^4*f^3*p^3*q^3*log(c) + a^2*b^3*f^3*p^3*q^3)

## Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \int \frac{\left(g+h\,x\right)^2}{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^3}\,dx$$

```
[In] int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^3,x)
[Out] int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^3, x)
```

# 3.456 $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 322

$$\begin{aligned}\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx &= \frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3f^2p^3q^3}\\ &\quad+\frac{2e^{-\frac{2a}{bpq}}h(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^3f^2p^3q^3}\\ &\quad-\frac{(e+fx)(g+hx)}{2bfpq\,(a+b\log(c(d(e+fx)^p)^q))^2}+\frac{(fg-eh)(e+fx)}{2b^2f^2p^2q^2\,(a+b\log(c(d(e+fx)^p)^q))}\\ &\quad-\frac{(e+fx)(g+hx)}{b^2fp^2q^2\,(a+b\log(c(d(e+fx)^p)^q))}\end{aligned}$$

```
[Out] 1/2*(-e*h+f*g)*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^3/exp(a/b/p/
q)/f^2/p^3/q^3/((c*(d*(f*x+e)^p)^q)^(1/p/q))+2*h*(f*x+e)^2*Ei(2*(a+b*ln(c*(
d*(f*x+e)^p)^q))/b/p/q)/b^3/exp(2*a/b/p/q)/f^2/p^3/q^3/((c*(d*(f*x+e)^p)^q)
^(2/p/q))-1/2*(f*x+e)*(h*x+g)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^2+1/2*(-e
*h+f*g)*(f*x+e)/b^2/f^2/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q))-(f*x+e)*(h*x+g)
/b^2/f/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q))
```

## Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2447, 2446, 2436, 2337, 2209, 2437, 2347, 2334, 2495}

$$\begin{aligned}&\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx\\&=\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3f^2p^3q^3}\\&+\frac{2h(e+fx)^2e^{-\frac{2a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{ExpIntegralEi}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^3f^2p^3q^3}\\&+\frac{(e+fx)(fg-eh)}{2b^2f^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))}\\&-\frac{(e+fx)(g+hx)}{b^2fp^2q^2(a+b\log(c(d(e+fx)^p)^q))}-\frac{(e+fx)(g+hx)}{2bfpq(a+b\log(c(d(e+fx)^p)^q))^2}\end{aligned}$$

```
[In] Int[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

```
[Out] ((f*g - e*h)*(e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*
q)])/(2*b^3*E^(a/(b*p*q))*f^2*p^3*q^3*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (2
*h*(e + f*x)^2*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(b*p*q)])
/(b^3*E^((2*a)/(b*p*q))*f^2*p^3*q^3*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) - ((e
+ f*x)*(g + h*x))/(2*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2) + ((f*g -
e*h)*(e + f*x))/(2*b^2*f^2*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])) - ((e
+ f*x)*(g + h*x))/(b^2*f*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{g+hx}{(a+b\log(cd^q(e+fx)^{pq}))^3}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(e+fx)(g+hx)}{2bfpq\,(a+b\log(c\,(d(e+fx)^p)^q))^2} + \text{Subst}\left(\frac{\int \frac{g+hx}{(a+b\log(cd^q(e+fx)^{pq}))^2}\,dx}{bpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(fg-eh)\int \frac{1}{(a+b\log(cd^q(e+fx)^{pq}))^2}\,dx}{2bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(e+fx)(g+hx)}{2bfpq\,(a+b\log(c\,(d(e+fx)^p)^q))^2} - \frac{(e+fx)(g+hx)}{b^2fp^2q^2\,(a+b\log(c\,(d(e+fx)^p)^q))} + \text{Subst}\left(\frac{2\int \frac{g+hx}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{b^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(fg-eh)\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(fg-eh)\text{Subst}\left(\int \frac{1}{(a+b\log(cd^qx^{pq}))^2}\,dx, x, e+fx\right)}{2bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} + \frac{(fg-eh)(e+fx)}{2b^2f^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$
$$- \frac{(e+fx)(g+hx)}{b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$
$$+ \text{Subst}\left(\frac{2\int\left(\frac{fg-eh}{f(a+b\log(cd^q(e+fx)^{pq}))} + \frac{h(e+fx)}{f(a+b\log(cd^q(e+fx)^{pq}))}\right)dx}{b^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$- \text{Subst}\left(\frac{(fg-eh)\text{Subst}\left(\int\frac{1}{a+b\log(cd^qx^{pq})}dx, x, e+fx\right)}{2b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$- \text{Subst}\left(\frac{(fg-eh)\text{Subst}\left(\int\frac{1}{a+b\log(cd^qx^{pq})}dx, x, e+fx\right)}{b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} + \frac{(fg-eh)(e+fx)}{2b^2f^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} - \frac{(e+fx)(g+hx)}{b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{(2h)\int \frac{e+fx}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(2(fg-eh))\int \frac{1}{a+b\log(cd^q(e+fx)^{pq})}\,dx}{b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{\left((fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{a+bx}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{2b^2f^2p^3q^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{\left((fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{a+bx}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{b^2f^2p^3q^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{3e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3f^2p^3q^3} - \frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} + \frac{(fg-eh)(e+fx)}{2b^2f^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} - \frac{(e+fx)(g+hx)}{b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{(2h)\text{Subst}\left(\int \frac{x}{a+b\log(cd^qx^{pq})}\,dx, x, e+fx\right)}{b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(2(fg-eh))\text{Subst}\left(\int \frac{1}{a+b\log(cd^qx^{pq})}\,dx, x, e+fx\right)}{b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{3e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3f^2p^3q^3}$$
$$-\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$
$$+\frac{(fg-eh)(e+fx)}{2b^2f^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}-\frac{(e+fx)(g+hx)}{b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$
$$+\text{Subst}\left(\frac{\left(2h(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{b^2f^2p^3q^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$+\text{Subst}\left(\frac{\left(2(fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{a+bx}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{b^2f^2p^3q^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$=\frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3f^2p^3q^3}$$
$$+\frac{2e^{-\frac{2a}{bpq}}h(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{Ei}\left(\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)}{b^3f^2p^3q^3}$$
$$-\frac{(e+fx)(g+hx)}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}+\frac{(fg-eh)(e+fx)}{2b^2f^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$
$$-\frac{(e+fx)(g+hx)}{b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00

$$\int\frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=$$
$$-\frac{e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(-e^{\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)}{bpq}\right.\right.}{}$$

```
[In] Integrate[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^3,x]
```

```
[Out] -1/2*((e + f*x)*(-(E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)^p)^q)^(1/(p*q)
)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)]*(a + b*Log[c*(d*(
e + f*x)^p)^q])^2) - 4*h*(e + f*x)*ExpIntegralEi[(2*(a + b*Log[c*(d*(e + f*
x)^p)^q]))/(b*p*q)]*(a + b*Log[c*(d*(e + f*x)^p)^q])^2 + b*E^((2*a)/(b*p*q)
)*p*q*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(b*f*p*q*(g + h*x) + a*(f*g + e*h + 2
*f*h*x) + b*(e*h + f*(g + 2*h*x))*Log[c*(d*(e + f*x)^p)^q])))/(b^3*E^((2*a)
/(b*p*q))*f^2*p^3*q^3*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(a + b*Log[c*(d*(e +
f*x)^p)^q])^2)
```

## Maple [F]

$$\int \frac{hx+g}{\left(a+b\ln \left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{3}}dx$$

```
[In] int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

```
[Out] int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(317) = 634$.

Time = 0.39 (sec) , antiderivative size = 931, normalized size of antiderivative = 2.89

$$\int \frac{g+hx}{\left(a+b\log \left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{3}}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(((b^2*f*g - b^2*e*h)*p^2*q^2*log(f*x + e)^2 + (b^2*f*g - b^2*e*h)*q^2*
log(d)^2 + a^2*f*g - a^2*e*h + (b^2*f*g - b^2*e*h)*log(c)^2 + 2*((b^2*f*g -
 b^2*e*h)*p*q^2*log(d) + (b^2*f*g - b^2*e*h)*p*q*log(c) + (a*b*f*g - a*b*e*
h)*p*q)*log(f*x + e) + 2*(a*b*f*g - a*b*e*h)*log(c) + 2*((b^2*f*g - b^2*e*h
)*q*log(c) + (a*b*f*g - a*b*e*h)*q)*log(d))*e^((b*q*log(d) + b*log(c) + a)/
(b*p*q))*log_integral((f*x + e)*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))) -
(b^2*e*f*g*p^2*q^2 + (a*b*e*f*g + a*b*e^2*h)*p*q + (b^2*f^2*h*p^2*q^2 + 2*a
*b*f^2*h*p*q)*x^2 + ((b^2*f^2*g + b^2*e*f*h)*p^2*q^2 + (a*b*f^2*g + 3*a*b*e
*f*h)*p*q)*x + (2*b^2*f^2*h*p^2*q^2*x^2 + (b^2*f^2*g + 3*b^2*e*f*h)*p^2*q^2
*x + (b^2*e*f*g + b^2*e^2*h)*p^2*q^2)*log(f*x + e) + (2*b^2*f^2*h*p*q*x^2 +
 (b^2*f^2*g + 3*b^2*e*f*h)*p*q*x + (b^2*e*f*g + b^2*e^2*h)*p*q)*log(c) + (2
*b^2*f^2*h*p*q^2*x^2 + (b^2*f^2*g + 3*b^2*e*f*h)*p*q^2*x + (b^2*e*f*g + b^2
*e^2*h)*p*q^2)*log(d))*e^(2*(b*q*log(d) + b*log(c) + a)/(b*p*q)) + 4*(b^2*h
*p^2*q^2*log(f*x + e)^2 + b^2*h*q^2*log(d)^2 + b^2*h*log(c)^2 + 2*a*b*h*log
(c) + a^2*h + 2*(b^2*h*p*q^2*log(d) + b^2*h*p*q*log(c) + a*b*h*p*q)*log(f*x
 + e) + 2*(b^2*h*q*log(c) + a*b*h*q)*log(d))*log_integral((f^2*x^2 + 2*e*f*
x + e^2)*e^(2*(b*q*log(d) + b*log(c) + a)/(b*p*q))))*e^(-2*(b*q*log(d) + b*
```

```
log(c) + a)/(b*p*q))/(b^5*f^2*p^5*q^5*log(f*x + e)^2 + b^5*f^2*p^3*q^5*log(
d)^2 + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*
p^3*q^3 + 2*(b^5*f^2*p^4*q^5*log(d) + b^5*f^2*p^4*q^4*log(c) + a*b^4*f^2*p^
4*q^4)*log(f*x + e) + 2*(b^5*f^2*p^3*q^4*log(c) + a*b^4*f^2*p^3*q^4)*log(d)
)
```

## Sympy [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$$

```
[In] integrate((h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
[Out] Integral((g + h*x)/(a + b*log(c*(d*(e + f*x)**p)**q))**3, x)
```

## Maxima [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \int \frac{hx+g}{(b\log(((fx+e)^p d)^q c)+a)^3}\,dx$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
[Out] -1/2*((2*a*f^2*h + (f^2*h*p*q + 2*f^2*h*q*log(d) + 2*f^2*h*log(c))*b)*x^2 +
 (e*f*g + e^2*h)*a + (e*f*g*p*q + (e*f*g + e^2*h)*log(c) + (e*f*g*q + e^2*h
*q)*log(d))*b + ((f^2*g + 3*e*f*h)*a + (f^2*g*p*q + e*f*h*p*q + (f^2*g + 3*
e*f*h)*log(c) + (f^2*g*q + 3*e*f*h*q)*log(d))*b)*x + (2*b*f^2*h*x^2 + (f^2*
g + 3*e*f*h)*b*x + (e*f*g + e^2*h)*b)*log(((f*x + e)^p)^q))/(b^4*f^2*p^2*q^
2*log(((f*x + e)^p)^q)^2 + a^2*b^2*f^2*p^2*q^2 + 2*(f^2*p^2*q^3*log(d) + f^
2*p^2*q^2*log(c))*a*b^3 + (f^2*p^2*q^4*log(d)^2 + 2*f^2*p^2*q^3*log(c)*log(
d) + f^2*p^2*q^2*log(c)^2)*b^4 + 2*(a*b^3*f^2*p^2*q^2 + (f^2*p^2*q^3*log(d)
 + f^2*p^2*q^2*log(c))*b^4)*log(((f*x + e)^p)^q)) + integrate(1/2*(4*f*h*x
+ f*g + 3*e*h)/(b^3*f*p^2*q^2*log(((f*x + e)^p)^q) + a*b^2*f*p^2*q^2 + (f*p
^2*q^3*log(d) + f*p^2*q^2*log(c))*b^3), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11278 vs. $2(317) = 634$.

Time = 0.55 (sec) , antiderivative size = 11278, normalized size of antiderivative = 35.02

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

```
[Out] -1/2*(f*x + e)*b^2*f*g*p^2*q^2*log(f*x + e)/(b^5*f^2*p^5*q^5*log(f*x + e)^2
 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*l
og(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5
*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4
*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) - (f*x + e)^2*b
^2*h*p^2*q^2*log(f*x + e)/(b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q
^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^
3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c
)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*
f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) + 1/2*(f*x + e)*b^2*e*h*p^2*q^2*l
og(f*x + e)/(b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e
)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2
 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5
*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*lo
g(c) + a^2*b^3*f^2*p^3*q^3) + 1/2*b^2*f*g*p^2*q^2*Ei(log(d)/p + log(c)/(p*q
) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)^2/((b^5*f^2*p^5*q
^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q
^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log
(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*
a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3
)*c^(1/(p*q))*d^(1/p)) - 1/2*b^2*e*h*p^2*q^2*Ei(log(d)/p + log(c)/(p*q) + a
/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)^2/((b^5*f^2*p^5*q^5*lo
g(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*lo
g(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x
+ e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4
*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(
1/(p*q))*d^(1/p)) - 1/2*(f*x + e)*b^2*f*g*p^2*q^2/(b^5*f^2*p^5*q^5*log(f*x
+ e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x
+ e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) +
 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p
^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) - 1/2*(f*
x + e)^2*b^2*h*p^2*q^2/(b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*
log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q
^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*l
og(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2
```

*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) + 1/2*(f*x + e)*b^2*e*h*p^2*q^2/(b^5
*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5
*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p
^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log
(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f
^2*p^3*q^3) + 2*b^2*h*p^2*q^2*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q)
+ 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(f*x + e)^2/((b^5*f^2*p^5*q^5*log(f*x
 + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x
 + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e)
+ 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*
p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(2/(p*
q))*d^(2/p)) - 1/2*(f*x + e)*b^2*f*g*p*q^2*log(d)/(b^5*f^2*p^5*q^5*log(f*x
+ e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x
+ e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) +
 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p
^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) - (f*x +
e)^2*b^2*h*p*q^2*log(d)/(b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5
*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*
q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*
log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^
2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) + 1/2*(f*x + e)*b^2*e*h*p*q^2*log(d
)/(b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) +
 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4
*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q
^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2
*b^3*f^2*p^3*q^3) + b^2*f*g*p*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) +
log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)*log(d)/((b^5*f^2*p^5*q^5*log(f*x
+ e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x
+ e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) +
 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p
^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q
))*d^(1/p)) - b^2*e*h*p*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*
x + e))*e^(-a/(b*p*q))*log(f*x + e)*log(d)/((b^5*f^2*p^5*q^5*log(f*x + e)^2
 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*l
og(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5
*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4
*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(
1/p)) - 1/2*(f*x + e)*b^2*f*g*p*q*log(c)/(b^5*f^2*p^5*q^5*log(f*x + e)^2 +
2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(
c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^
2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*lo
g(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) - (f*x + e)^2*b^2*
h*p*q*log(c)/(b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x +
e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^
2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^

5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) + 1/2*(f*x + e)*b^2*e*h*p*q*log(c)/(b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) + b^2*f*g*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)*log(c)/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p)) - b^2*e*h*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)*log(c)/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p)) + 4*b^2*h*p*q^2*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(f*x + e)*log(d)/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(2/(p*q))*d^(2/p)) + 1/2*b^2*f*g*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)^2/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p)) - 1/2*b^2*e*h*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)^2/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p)) - 1/2*(f*x + e)*a*b*f*g*p*q/(b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) - (f*x + e)^2*a*b*h*p*q/(b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3) + 1/2*(f*x + e)*a*b*e*h*p*q/

(b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2
*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f
^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3
*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b
^3*f^2*p^3*q^3) + a*b*f*g*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(
f*x + e))*e^(-a/(b*p*q))*log(f*x + e)/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*
b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c)
 + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*
p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(
d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p))
 - a*b*e*h*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a
/(b*p*q))*log(f*x + e)/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5
*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*
q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*
log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^
2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p)) + 4*b^2*h*p*q*
Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p
*q))*log(f*x + e)*log(c)/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q
^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^
3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c
)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*
f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(2/(p*q))*d^(2/p)) + b^2*f*g*q*
Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c
)*log(d)/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*
log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 +
 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f
^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(
c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p)) - b^2*e*h*q*Ei(log(d)/p + lo
g(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)*log(d)/((b^5*f
^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f
^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4
*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c
)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2
*p^3*q^3)*c^(1/(p*q))*d^(1/p)) + 2*b^2*h*q^2*Ei(2*log(d)/p + 2*log(c)/(p*q)
 + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(d)^2/((b^5*f^2*p^5*q^
5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^
4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(
f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a
*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)
*c^(2/(p*q))*d^(2/p)) + 4*a*b*h*p*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b
*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(f*x + e)/((b^5*f^2*p^5*q^5*log
(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log
(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x +
 e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*
f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(2

/(p*q))*d^(2/p)) + 1/2*b^2*f*g*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)^2/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p)) - 1/2*b^2*e*h*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)^2/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p)) + a*b*f*g*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p)) - a*b*e*h*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p)) + 4*b^2*h*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(c)*log(d)/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(2/(p*q))*d^(2/p)) + a*b*f*g*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p)) - a*b*e*h*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p)) + 2*b^2*h*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(c)^2/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2

```
*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(2/(p
*q))*d^(2/p)) + 4*a*b*h*q*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*
log(f*x + e))*e^(-2*a/(b*p*q))*log(d)/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*
b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c)
 + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*
p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(
d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(2/(p*q))*d^(2/p))
 + 1/2*a^2*f*g*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a
/(b*p*q))/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)
*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2
+ 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*
f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log
(c) + a^2*b^3*f^2*p^3*q^3)*c^(1/(p*q))*d^(1/p)) - 1/2*a^2*e*h*Ei(log(d)/p +
 log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))/((b^5*f^2*p^5*q^5*
log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*
log(f*x + e)*log(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*
x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b
^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c
^(1/(p*q))*d^(1/p)) + 4*a*b*h*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q)
+ 2*log(f*x + e))*e^(-2*a/(b*p*q))*log(c)/((b^5*f^2*p^5*q^5*log(f*x + e)^2
+ 2*b^5*f^2*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*lo
g(c) + b^5*f^2*p^3*q^5*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*
f^2*p^3*q^4*log(c)*log(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*
log(d) + 2*a*b^4*f^2*p^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(2/(p*q))*d^(2
/p)) + 2*a^2*h*Ei(2*log(d)/p + 2*log(c)/(p*q) + 2*a/(b*p*q) + 2*log(f*x + e
))*e^(-2*a/(b*p*q))/((b^5*f^2*p^5*q^5*log(f*x + e)^2 + 2*b^5*f^2*p^4*q^5*lo
g(f*x + e)*log(d) + 2*b^5*f^2*p^4*q^4*log(f*x + e)*log(c) + b^5*f^2*p^3*q^5
*log(d)^2 + 2*a*b^4*f^2*p^4*q^4*log(f*x + e) + 2*b^5*f^2*p^3*q^4*log(c)*log
(d) + b^5*f^2*p^3*q^3*log(c)^2 + 2*a*b^4*f^2*p^3*q^4*log(d) + 2*a*b^4*f^2*p
^3*q^3*log(c) + a^2*b^3*f^2*p^3*q^3)*c^(2/(p*q))*d^(2/p))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx=\int \frac{g+h\,x}{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^3}\,dx$$

```
[In] int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^3,x)
[Out] int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^3, x)
```

# 3.457 $\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3fp^3q^3} - \frac{e+fx}{2bfpq(a+b\log(c(d(e+fx)^p)^q))^2} - \frac{e+fx}{2b^2fp^2q^2(a+b\log(c(d(e+fx)^p)^q))}$$

```
[Out] 1/2*(f*x+e)*Ei((a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)/b^3/exp(a/b/p/q)/f/p^3/q^
3/((c*(d*(f*x+e)^p)^q)^(1/p/q))+1/2*(-f*x-e)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p
)^q))^2+1/2*(-f*x-e)/b^2/f/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q))
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2436, 2334, 2337, 2209, 2495}

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx = \frac{(e+fx)e^{-\frac{a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3fp^3q^3} - \frac{e+fx}{2b^2fp^2q^2(a+b\log(c(d(e+fx)^p)^q))} - \frac{e+fx}{2bfpq(a+b\log(c(d(e+fx)^p)^q))^2}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-3),x]
```

```
[Out] ((e + f*x)*ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)])/(2*b^3*
E^(a/(b*p*q))*f*p^3*q^3*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) - (e + f*x)/(2*b*f
*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2) - (e + f*x)/(2*b^2*f*p^2*q^2*(a +
b*Log[c*(d*(e + f*x)^p)^q]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}\,dx, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{\left(a+b\log\left(cd^q x^{pq}\right)\right)^3}\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)
\end{aligned}
$$

$$= -\frac{e+fx}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} + \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cd^qx^{pq}))^2}\,dx, x, e+fx\right)}{2bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{e+fx}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} - \frac{e+fx}{2b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{a+b\log(cd^qx^{pq})}\,dx, x, e+fx\right)}{2b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{e+fx}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} - \frac{e+fx}{2b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} + \text{Subst}\left(\frac{\left((e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{a+bx}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{2b^2fp^3q^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{Ei}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)}{2b^3fp^3q^3} - \frac{e+fx}{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} - \frac{e+fx}{2b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = -\frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\left(-\text{ExpIntegralEi}\left(\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 + \right.}{2b^3fp^3q^3\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-3),x]
```

```
[Out] -1/2*((e + f*x)*(-(ExpIntegralEi[(a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)]*
(a + b*Log[c*(d*(e + f*x)^p)^q])^2) + b*E^(a/(b*p*q))*p*q*(c*(d*(e + f*x)^p
)^q)^(1/(p*q))*(a + b*p*q + b*Log[c*(d*(e + f*x)^p)^q])))/(b^3*E^(a/(b*p*q)
)*f*p^3*q^3*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*(a + b*Log[c*(d*(e + f*x)^p)^q]
)^2)
```

## Maple [F]

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{3}}dx$$

[In] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)

[Out] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(163) = 326$.

Time = 0.32 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.63

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{3}}\,dx = -\frac{\left((b^2ep^2q^2+abepq+(b^2fp^2q^2+abfpq)x+(b^2fp^2q^2x+b^2ep^2q^2)\log\left(fx+e\right)+(b^2fpqx+b^2epq)\log\left(\right.\right.}{2\left(b^5fp^5q^5\log\left(fx\right.\right.}$$

[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")

[Out] -1/2*((b^2*e*p^2*q^2 + a*b*e*p*q + (b^2*f*p^2*q^2 + a*b*f*p*q)*x + (b^2*f*p^2*q^2*x + b^2*e*p^2*q^2)*log(f*x + e) + (b^2*f*p*q*x + b^2*e*p*q)*log(c) + (b^2*f*p*q^2*x + b^2*e*p*q^2)*log(d))*e^((b*q*log(d) + b*log(c) + a)/(b*p*q)) - (b^2*p^2*q^2*log(f*x + e)^2 + b^2*q^2*log(d)^2 + b^2*log(c)^2 + 2*a*b*log(c) + a^2 + 2*(b^2*p*q^2*log(d) + b^2*p*q*log(c) + a*b*p*q)*log(f*x + e) + 2*(b^2*q*log(c) + a*b*q)*log(d))*log_integral((f*x + e)*e^((b*q*log(d) + b*log(c) + a)/(b*p*q))))*e^(-(b*q*log(d) + b*log(c) + a)/(b*p*q))/(b^5*f*p^5*q^5*log(f*x + e)^2 + b^5*f*p^3*q^5*log(d)^2 + b^5*f*p^3*q^3*log(c)^2 + 2*a*b^4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q^3 + 2*(b^5*f*p^4*q^5*log(d) + b^5*f*p^4*q^4*log(c) + a*b^4*f*p^4*q^4)*log(f*x + e) + 2*(b^5*f*p^3*q^4*log(c) + a*b^4*f*p^3*q^4)*log(d))

## Sympy [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{3}}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)\right)^{3}}\,dx$$

[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)

[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(-3), x)

## Maxima [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \int \frac{1}{\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)^3}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((e*p*q + e*q*log(d) + e*log(c))*b + a*e + ((f*p*q + f*q*log(d) + f*lo
g(c))*b + a*f)*x + (b*f*x + b*e)*log(((f*x + e)^p)^q))/(b^4*f*p^2*q^2*log((
(f*x + e)^p)^q)^2 + a^2*b^2*f*p^2*q^2 + 2*(f*p^2*q^3*log(d) + f*p^2*q^2*log
(c))*a*b^3 + (f*p^2*q^4*log(d)^2 + 2*f*p^2*q^3*log(c)*log(d) + f*p^2*q^2*lo
g(c)^2)*b^4 + 2*(a*b^3*f*p^2*q^2 + (f*p^2*q^3*log(d) + f*p^2*q^2*log(c))*b^
4)*log(((f*x + e)^p)^q)) + integrate(1/2/(b^3*p^2*q^2*log(((f*x + e)^p)^q)
+ a*b^2*p^2*q^2 + (p^2*q^3*log(d) + p^2*q^2*log(c))*b^3), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3401 vs. $2(163) = 326$.

Time = 0.40 (sec) , antiderivative size = 3401, normalized size of antiderivative = 20.12

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

```
[Out] -1/2*(f*x + e)*b^2*p^2*q^2*log(f*x + e)/(b^5*f*p^5*q^5*log(f*x + e)^2 + 2*b
^5*f*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f*p^4*q^4*log(f*x + e)*log(c) + b^
5*f*p^3*q^5*log(d)^2 + 2*a*b^4*f*p^4*q^4*log(f*x + e) + 2*b^5*f*p^3*q^4*log
(c)*log(d) + b^5*f*p^3*q^3*log(c)^2 + 2*a*b^4*f*p^3*q^4*log(d) + 2*a*b^4*f*
p^3*q^3*log(c) + a^2*b^3*f*p^3*q^3) + 1/2*b^2*p^2*q^2*Ei(log(d)/p + log(c)/
(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)^2/((b^5*f*p^5
*q^5*log(f*x + e)^2 + 2*b^5*f*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f*p^4*q^4
*log(f*x + e)*log(c) + b^5*f*p^3*q^5*log(d)^2 + 2*a*b^4*f*p^4*q^4*log(f*x +
 e) + 2*b^5*f*p^3*q^4*log(c)*log(d) + b^5*f*p^3*q^3*log(c)^2 + 2*a*b^4*f*p^
3*q^4*log(d) + 2*a*b^4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q^3)*c^(1/(p*q))*d^
(1/p)) - 1/2*(f*x + e)*b^2*p^2*q^2/(b^5*f*p^5*q^5*log(f*x + e)^2 + 2*b^5*f*
p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f*p^4*q^4*log(f*x + e)*log(c) + b^5*f*p
^3*q^5*log(d)^2 + 2*a*b^4*f*p^4*q^4*log(f*x + e) + 2*b^5*f*p^3*q^4*log(c)*l
og(d) + b^5*f*p^3*q^3*log(c)^2 + 2*a*b^4*f*p^3*q^4*log(d) + 2*a*b^4*f*p^3*q
^3*log(c) + a^2*b^3*f*p^3*q^3) - 1/2*(f*x + e)*b^2*p*q^2*log(d)/(b^5*f*p^5*
q^5*log(f*x + e)^2 + 2*b^5*f*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f*p^4*q^4*
log(f*x + e)*log(c) + b^5*f*p^3*q^5*log(d)^2 + 2*a*b^4*f*p^4*q^4*log(f*x +
e) + 2*b^5*f*p^3*q^4*log(c)*log(d) + b^5*f*p^3*q^3*log(c)^2 + 2*a*b^4*f*p^3
*q^4*log(d) + 2*a*b^4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q^3) + b^2*p*q^2*Ei(
```

log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x
+ e)*log(d)/((b^5*f*p^5*q^5*log(f*x + e)^2 + 2*b^5*f*p^4*q^5*log(f*x + e)*l
og(d) + 2*b^5*f*p^4*q^4*log(f*x + e)*log(c) + b^5*f*p^3*q^5*log(d)^2 + 2*a*
b^4*f*p^4*q^4*log(f*x + e) + 2*b^5*f*p^3*q^4*log(c)*log(d) + b^5*f*p^3*q^3*
log(c)^2 + 2*a*b^4*f*p^3*q^4*log(d) + 2*a*b^4*f*p^3*q^3*log(c) + a^2*b^3*f*
p^3*q^3)*c^(1/(p*q))*d^(1/p)) - 1/2*(f*x + e)*b^2*p*q*log(c)/(b^5*f*p^5*q^5
*log(f*x + e)^2 + 2*b^5*f*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f*p^4*q^4*log
(f*x + e)*log(c) + b^5*f*p^3*q^5*log(d)^2 + 2*a*b^4*f*p^4*q^4*log(f*x + e)
+ 2*b^5*f*p^3*q^4*log(c)*log(d) + b^5*f*p^3*q^3*log(c)^2 + 2*a*b^4*f*p^3*q^
4*log(d) + 2*a*b^4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q^3) + b^2*p*q*Ei(log(d
)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(f*x + e)*
log(c)/((b^5*f*p^5*q^5*log(f*x + e)^2 + 2*b^5*f*p^4*q^5*log(f*x + e)*log(d)
+ 2*b^5*f*p^4*q^4*log(f*x + e)*log(c) + b^5*f*p^3*q^5*log(d)^2 + 2*a*b^4*f
*p^4*q^4*log(f*x + e) + 2*b^5*f*p^3*q^4*log(c)*log(d) + b^5*f*p^3*q^3*log(c
)^2 + 2*a*b^4*f*p^3*q^4*log(d) + 2*a*b^4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q
^3)*c^(1/(p*q))*d^(1/p)) + 1/2*b^2*q^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*
q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)^2/((b^5*f*p^5*q^5*log(f*x + e)^2 +
2*b^5*f*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f*p^4*q^4*log(f*x + e)*log(c)
+ b^5*f*p^3*q^5*log(d)^2 + 2*a*b^4*f*p^4*q^4*log(f*x + e) + 2*b^5*f*p^3*q^4
*log(c)*log(d) + b^5*f*p^3*q^3*log(c)^2 + 2*a*b^4*f*p^3*q^4*log(d) + 2*a*b^
4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q^3)*c^(1/(p*q))*d^(1/p)) - 1/2*(f*x + e
)*a*b*p*q/(b^5*f*p^5*q^5*log(f*x + e)^2 + 2*b^5*f*p^4*q^5*log(f*x + e)*log(
d) + 2*b^5*f*p^4*q^4*log(f*x + e)*log(c) + b^5*f*p^3*q^5*log(d)^2 + 2*a*b^4
*f*p^4*q^4*log(f*x + e) + 2*b^5*f*p^3*q^4*log(c)*log(d) + b^5*f*p^3*q^3*log
(c)^2 + 2*a*b^4*f*p^3*q^4*log(d) + 2*a*b^4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3
*q^3) + a*b*p*q*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-
a/(b*p*q))*log(f*x + e)/((b^5*f*p^5*q^5*log(f*x + e)^2 + 2*b^5*f*p^4*q^5*lo
g(f*x + e)*log(d) + 2*b^5*f*p^4*q^4*log(f*x + e)*log(c) + b^5*f*p^3*q^5*log
(d)^2 + 2*a*b^4*f*p^4*q^4*log(f*x + e) + 2*b^5*f*p^3*q^4*log(c)*log(d) + b^
5*f*p^3*q^3*log(c)^2 + 2*a*b^4*f*p^3*q^4*log(d) + 2*a*b^4*f*p^3*q^3*log(c)
+ a^2*b^3*f*p^3*q^3)*c^(1/(p*q))*d^(1/p)) + b^2*q*Ei(log(d)/p + log(c)/(p*q
) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)*log(d)/((b^5*f*p^5*q^5*
log(f*x + e)^2 + 2*b^5*f*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f*p^4*q^4*log(
f*x + e)*log(c) + b^5*f*p^3*q^5*log(d)^2 + 2*a*b^4*f*p^4*q^4*log(f*x + e) +
2*b^5*f*p^3*q^4*log(c)*log(d) + b^5*f*p^3*q^3*log(c)^2 + 2*a*b^4*f*p^3*q^4
*log(d) + 2*a*b^4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q^3)*c^(1/(p*q))*d^(1/p)
) + 1/2*b^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b
*p*q))*log(c)^2/((b^5*f*p^5*q^5*log(f*x + e)^2 + 2*b^5*f*p^4*q^5*log(f*x +
e)*log(d) + 2*b^5*f*p^4*q^4*log(f*x + e)*log(c) + b^5*f*p^3*q^5*log(d)^2 +
2*a*b^4*f*p^4*q^4*log(f*x + e) + 2*b^5*f*p^3*q^4*log(c)*log(d) + b^5*f*p^3*
q^3*log(c)^2 + 2*a*b^4*f*p^3*q^4*log(d) + 2*a*b^4*f*p^3*q^3*log(c) + a^2*b^
3*f*p^3*q^3)*c^(1/(p*q))*d^(1/p)) + a*b*q*Ei(log(d)/p + log(c)/(p*q) + a/(b
*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(d)/((b^5*f*p^5*q^5*log(f*x + e)^2
+ 2*b^5*f*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f*p^4*q^4*log(f*x + e)*log(c)
+ b^5*f*p^3*q^5*log(d)^2 + 2*a*b^4*f*p^4*q^4*log(f*x + e) + 2*b^5*f*p^3*q^

```
4*log(c)*log(d) + b^5*f*p^3*q^3*log(c)^2 + 2*a*b^4*f*p^3*q^4*log(d) + 2*a*b
^4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q^3)*c^(1/(p*q))*d^(1/p)) + a*b*Ei(log(
d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e))*e^(-a/(b*p*q))*log(c)/((b^5
*f*p^5*q^5*log(f*x + e)^2 + 2*b^5*f*p^4*q^5*log(f*x + e)*log(d) + 2*b^5*f*p
^4*q^4*log(f*x + e)*log(c) + b^5*f*p^3*q^5*log(d)^2 + 2*a*b^4*f*p^4*q^4*log
(f*x + e) + 2*b^5*f*p^3*q^4*log(c)*log(d) + b^5*f*p^3*q^3*log(c)^2 + 2*a*b^
4*f*p^3*q^4*log(d) + 2*a*b^4*f*p^3*q^3*log(c) + a^2*b^3*f*p^3*q^3)*c^(1/(p*
q))*d^(1/p)) + 1/2*a^2*Ei(log(d)/p + log(c)/(p*q) + a/(b*p*q) + log(f*x + e
))*e^(-a/(b*p*q))/((b^5*f*p^5*q^5*log(f*x + e)^2 + 2*b^5*f*p^4*q^5*log(f*x
+ e)*log(d) + 2*b^5*f*p^4*q^4*log(f*x + e)*log(c) + b^5*f*p^3*q^5*log(d)^2
+ 2*a*b^4*f*p^4*q^4*log(f*x + e) + 2*b^5*f*p^3*q^4*log(c)*log(d) + b^5*f*p^
3*q^3*log(c)^2 + 2*a*b^4*f*p^3*q^4*log(d) + 2*a*b^4*f*p^3*q^3*log(c) + a^2*
b^3*f*p^3*q^3)*c^(1/(p*q))*d^(1/p))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^3}\,dx=\int \frac{1}{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^3}\,dx$$

```
[In] int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^3,x)
[Out] int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^3, x)
```

# 3.458 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^3} dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3} dx = \text{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}, x\right)$$

```
[Out] Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

## Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3} dx = \int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3} dx$$

```
[In] Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3),x]
```

```
[Out] Defer[Int][1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3} dx$$

## Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx$$

```
[In] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3),x]
[Out] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3), x]
```

## Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)\right)^3}dx$$

```
[In] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
[Out] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.82

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \int \frac{1}{(hx+g)(b\log\left(((fx+e)^p d)^q\,c\right)+a)^3}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")
[Out] integral(1/(a^3*h*x + a^3*g + (b^3*h*x + b^3*g)*log(((f*x + e)^p*d)^q*c)^3
+ 3*(a*b^2*h*x + a*b^2*g)*log(((f*x + e)^p*d)^q*c)^2 + 3*(a^2*b*h*x + a^2*b
*g)*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 17.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^3(g+hx)}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**3*(g + h*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 1116, normalized size of antiderivative = 39.86

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{1}{(hx+g)\left(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a\right)^3}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
[Out] -1/2*(b*f^2*h*p*q*x^2 + (e*f*g - e^2*h)*a + (e*f*g*p*q + (e*f*g - e^2*h)*lo
g(c) + (e*f*g*q - e^2*h*q)*log(d))*b + ((f^2*g - e*f*h)*a + (f^2*g*p*q + e*
f*h*p*q + (f^2*g - e*f*h)*log(c) + (f^2*g*q - e*f*h*q)*log(d))*b)*x + ((f^2
*g - e*f*h)*b*x + (e*f*g - e^2*h)*b)*log(((f*x + e)^p)^q))/(a^2*b^2*f^2*g^2
*p^2*q^2 + 2*(f^2*g^2*p^2*q^3*log(d) + f^2*g^2*p^2*q^2*log(c))*a*b^3 + (f^2
*g^2*p^2*q^4*log(d)^2 + 2*f^2*g^2*p^2*q^3*log(c)*log(d) + f^2*g^2*p^2*q^2*l
og(c)^2)*b^4 + (a^2*b^2*f^2*h^2*p^2*q^2 + 2*(f^2*h^2*p^2*q^3*log(d) + f^2*h
^2*p^2*q^2*log(c))*a*b^3 + (f^2*h^2*p^2*q^4*log(d)^2 + 2*f^2*h^2*p^2*q^3*lo
g(c)*log(d) + f^2*h^2*p^2*q^2*log(c)^2)*b^4)*x^2 + (b^4*f^2*h^2*p^2*q^2*x^2
 + 2*b^4*f^2*g*h*p^2*q^2*x + b^4*f^2*g^2*p^2*q^2)*log(((f*x + e)^p)^q)^2 +
2*(a^2*b^2*f^2*g*h*p^2*q^2 + 2*(f^2*g*h*p^2*q^3*log(d) + f^2*g*h*p^2*q^2*lo
g(c))*a*b^3 + (f^2*g*h*p^2*q^4*log(d)^2 + 2*f^2*g*h*p^2*q^3*log(c)*log(d) +
 f^2*g*h*p^2*q^2*log(c)^2)*b^4)*x + 2*(a*b^3*f^2*g^2*p^2*q^2 + (f^2*g^2*p^2
*q^3*log(d) + f^2*g^2*p^2*q^2*log(c))*b^4 + (a*b^3*f^2*h^2*p^2*q^2 + (f^2*h
^2*p^2*q^3*log(d) + f^2*h^2*p^2*q^2*log(c))*b^4)*x^2 + 2*(a*b^3*f^2*g*h*p^2
*q^2 + (f^2*g*h*p^2*q^3*log(d) + f^2*g*h*p^2*q^2*log(c))*b^4)*x)*log(((f*x
+ e)^p)^q)) + integrate(1/2*(f^2*g^2 - 3*e*f*g*h + 2*e^2*h^2 - (f^2*g*h - e
*f*h^2)*x)/(a*b^2*f^2*g^3*p^2*q^2 + (f^2*g^3*p^2*q^3*log(d) + f^2*g^3*p^2*q
^2*log(c))*b^3 + (a*b^2*f^2*h^3*p^2*q^2 + (f^2*h^3*p^2*q^3*log(d) + f^2*h^3
*p^2*q^2*log(c))*b^3)*x^3 + 3*(a*b^2*f^2*g*h^2*p^2*q^2 + (f^2*g*h^2*p^2*q^3
*log(d) + f^2*g*h^2*p^2*q^2*log(c))*b^3)*x^2 + 3*(a*b^2*f^2*g^2*h*p^2*q^2 +
 (f^2*g^2*h*p^2*q^3*log(d) + f^2*g^2*h*p^2*q^2*log(c))*b^3)*x + (b^3*f^2*h^
3*p^2*q^2*x^3 + 3*b^3*f^2*g*h^2*p^2*q^2*x^2 + 3*b^3*f^2*g^2*h*p^2*q^2*x + b
^3*f^2*g^3*p^2*q^2)*log(((f*x + e)^p)^q)), x)
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{1}{(hx+g)(b\log\left(((fx+e)^p d)^q\,c\right)+a)^3}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
[Out] integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{1}{(g+h\,x)\,\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^3}\,dx$$

```
[In] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^3),x)
[Out] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^3), x)
```

# 3.459 $\int \frac{1}{(g+hx)^2(a+b\log(c(d(e+fx)^p)^q))^3}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \mathrm{Int}\left(\frac{1}{(g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx = \int \frac{1}{(g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx$$

```
[In] Int[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3),x]
```

```
[Out] Defer[Int][1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 23.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^3} dx = \int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^3} dx$$

```
[In] Integrate[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3),x]
[Out] Integrate[1/((g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3), x]
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)^2 (a+b\ln(c(d(fx+e)^p)^q))^3} dx$$

```
[In] int(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
[Out] int(1/(h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 5.89

$$\int \frac{1}{(g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^3} dx = \int \frac{1}{(hx+g)^2 (b\log(((fx+e)^p d)^q c)+a)^3} dx$$

```
[In] integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="fricas")
[Out] integral(1/(a^3*h^2*x^2 + 2*a^3*g*h*x + a^3*g^2 + (b^3*h^2*x^2 + 2*b^3*g*h*
x + b^3*g^2)*log(((f*x + e)^p*d)^q*c)^3 + 3*(a*b^2*h^2*x^2 + 2*a*b^2*g*h*x
+ a*b^2*g^2)*log(((f*x + e)^p*d)^q*c)^2 + 3*(a^2*b*h^2*x^2 + 2*a^2*b*g*h*x
+ a^2*b*g^2)*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 135.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{(g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^3} dx = \int \frac{1}{(a+b\log (c(d(e+fx)^p)^q))^3 (g+hx)^2} dx$$

```
[In] integrate(1/(h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**3,x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**3*(g + h*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 1486, normalized size of antiderivative = 53.07

$$\int \frac{1}{(g+hx)^2 (a+b\log (c(d(e+fx)^p)^q))^3} dx = \int \frac{1}{(hx+g)^2(b\log (((fx+e)^p d)^q c)+a)^3} dx$$

```
[In] integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="maxima")
[Out] 1/2*((a*f^2*h - (f^2*h*p*q - f^2*h*q*log(d) - f^2*h*log(c))*b)*x^2 - (e*f*g
 - 2*e^2*h)*a - (e*f*g*p*q + (e*f*g - 2*e^2*h)*log(c) + (e*f*g*q - 2*e^2*h*
q)*log(d))*b - ((f^2*g - 3*e*f*h)*a + (f^2*g*p*q + e*f*h*p*q + (f^2*g - 3*e
*f*h)*log(c) + (f^2*g*q - 3*e*f*h*q)*log(d))*b)*x + (b*f^2*h*x^2 - (f^2*g -
 3*e*f*h)*b*x - (e*f*g - 2*e^2*h)*b)*log(((f*x + e)^p)^q))/(a^2*b^2*f^2*g^3
*p^2*q^2 + 2*(f^2*g^3*p^2*q^3*log(d) + f^2*g^3*p^2*q^2*log(c))*a*b^3 + (f^2
*g^3*p^2*q^4*log(d)^2 + 2*f^2*g^3*p^2*q^3*log(c)*log(d) + f^2*g^3*p^2*q^2*l
og(c)^2)*b^4 + (a^2*b^2*f^2*h^3*p^2*q^2 + 2*(f^2*h^3*p^2*q^3*log(d) + f^2*h
^3*p^2*q^2*log(c))*a*b^3 + (f^2*h^3*p^2*q^4*log(d)^2 + 2*f^2*h^3*p^2*q^3*lo
g(c)*log(d) + f^2*h^3*p^2*q^2*log(c)^2)*b^4)*x^3 + 3*(a^2*b^2*f^2*g*h^2*p^2
*q^2 + 2*(f^2*g*h^2*p^2*q^3*log(d) + f^2*g*h^2*p^2*q^2*log(c))*a*b^3 + (f^2
*g*h^2*p^2*q^4*log(d)^2 + 2*f^2*g*h^2*p^2*q^3*log(c)*log(d) + f^2*g*h^2*p^2
*q^2*log(c)^2)*b^4)*x^2 + (b^4*f^2*h^3*p^2*q^2*x^3 + 3*b^4*f^2*g*h^2*p^2*q^
2*x^2 + 3*b^4*f^2*g^2*h*p^2*q^2*x + b^4*f^2*g^3*p^2*q^2)*log(((f*x + e)^p)^
q)^2 + 3*(a^2*b^2*f^2*g^2*h*p^2*q^2 + 2*(f^2*g^2*h*p^2*q^3*log(d) + f^2*g^2
*h*p^2*q^2*log(c))*a*b^3 + (f^2*g^2*h*p^2*q^4*log(d)^2 + 2*f^2*g^2*h*p^2*q^
3*log(c)*log(d) + f^2*g^2*h*p^2*q^2*log(c)^2)*b^4)*x + 2*(a*b^3*f^2*g^3*p^2
*q^2 + (f^2*g^3*p^2*q^3*log(d) + f^2*g^3*p^2*q^2*log(c))*b^4 + (a*b^3*f^2*h
^3*p^2*q^2 + (f^2*h^3*p^2*q^3*log(d) + f^2*h^3*p^2*q^2*log(c))*b^4)*x^3 + 3
*(a*b^3*f^2*g*h^2*p^2*q^2 + (f^2*g*h^2*p^2*q^3*log(d) + f^2*g*h^2*p^2*q^2*l
og(c))*b^4)*x^2 + 3*(a*b^3*f^2*g^2*h*p^2*q^2 + (f^2*g^2*h*p^2*q^3*log(d) +
f^2*g^2*h*p^2*q^2*log(c))*b^4)*x)*log(((f*x + e)^p)^q)) + integrate(1/2*(f^
```

```
2*h^2*x^2 + f^2*g^2 - 6*e*f*g*h + 6*e^2*h^2 - 2*(2*f^2*g*h - 3*e*f*h^2)*x)/
(a*b^2*f^2*g^4*p^2*q^2 + (a*b^2*f^2*h^4*p^2*q^2 + (f^2*h^4*p^2*q^3*log(d) +
 f^2*h^4*p^2*q^2*log(c))*b^3)*x^4 + (f^2*g^4*p^2*q^3*log(d) + f^2*g^4*p^2*q
^2*log(c))*b^3 + 4*(a*b^2*f^2*g*h^3*p^2*q^2 + (f^2*g*h^3*p^2*q^3*log(d) + f
^2*g*h^3*p^2*q^2*log(c))*b^3)*x^3 + 6*(a*b^2*f^2*g^2*h^2*p^2*q^2 + (f^2*g^2
*h^2*p^2*q^3*log(d) + f^2*g^2*h^2*p^2*q^2*log(c))*b^3)*x^2 + 4*(a*b^2*f^2*g
^3*h*p^2*q^2 + (f^2*g^3*h*p^2*q^3*log(d) + f^2*g^3*h*p^2*q^2*log(c))*b^3)*x
 + (b^3*f^2*h^4*p^2*q^2*x^4 + 4*b^3*f^2*g*h^3*p^2*q^2*x^3 + 6*b^3*f^2*g^2*h
^2*p^2*q^2*x^2 + 4*b^3*f^2*g^3*h*p^2*q^2*x + b^3*f^2*g^4*p^2*q^2)*log(((f*x
 + e)^p)^q)), x)
```

## Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{1}{(hx+g)^2\left(b\log\left(\left(((fx+e)^p d\right)^q c\right)+a\right)^3}\,dx$$

```
[In] integrate(1/(h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
[Out] integrate(1/((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}\,dx=\int \frac{1}{{\left(g+h\,x\right)}^2\,{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^3}\,dx$$

```
[In] int(1/((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^3),x)
[Out] int(1/((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^3), x)
```

# 3.460 $\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 488

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$
$$= -\frac{\sqrt{b}e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{p}\sqrt{\pi}\sqrt{q}(e+fx)\left(c\left(d(e+fx)^p\right)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^3}$$
$$-\frac{\sqrt{b}e^{-\frac{2a}{bpq}}h(fg-eh)\sqrt{p}\sqrt{\frac{\pi}{2}}\sqrt{q}(e+fx)^2\left(c\left(d(e+fx)^p\right)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^3}$$
$$-\frac{\sqrt{b}e^{-\frac{3a}{bpq}}h^2\sqrt{p}\sqrt{\frac{\pi}{3}}\sqrt{q}(e+fx)^3\left(c\left(d(e+fx)^p\right)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{6f^3}$$
$$+\frac{(fg-eh)^2(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3}$$
$$+\frac{h(fg-eh)(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3}$$
$$+\frac{h^2(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{3f^3}$$

```
[Out] -1/18*h^2*(f*x+e)^3*erfi(3^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/
p^(1/2)/q^(1/2))*b^(1/2)*p^(1/2)*3^(1/2)*Pi^(1/2)*q^(1/2)/exp(3*a/b/p/q)/f^
3/((c*(d*(f*x+e)^p)^q)^(3/p/q))-1/4*h*(-e*h+f*g)*(f*x+e)^2*erfi(2^(1/2)*(a+
b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*b^(1/2)*p^(1/2)*2^(
1/2)*Pi^(1/2)*q^(1/2)/exp(2*a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(2/p/q))-1/2*
(-e*h+f*g)^2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)
/q^(1/2))*b^(1/2)*p^(1/2)*Pi^(1/2)*q^(1/2)/exp(a/b/p/q)/f^3/((c*(d*(f*x+e)^
p)^q)^(1/p/q))+(-e*h+f*g)^2*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/f^3+h
*(-e*h+f*g)*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/f^3+1/3*h^2*(f*x+e)
^3*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/f^3
```

## Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2448, 2436, 2333, 2337, 2211, 2235, 2437, 2342, 2347, 2495}

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$
$$= -\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}h\sqrt{p}\sqrt{q}(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^3}$$
$$-\frac{\sqrt{\pi}\sqrt{b}\sqrt{p}\sqrt{q}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^3}$$
$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}h^2\sqrt{p}\sqrt{q}(e+fx)^3e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{6f^3}$$
$$+\frac{h(e+fx)^2(fg-eh)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3}$$
$$+\frac{(e+fx)(fg-eh)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3}$$
$$+\frac{h^2(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{3f^3}$$

```
[In] Int[(g + h*x)^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] -1/2*(Sqrt[b]*(f*g - e*h)^2*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*(e + f*x)*Erfi[Sqrt[a
+ b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^(a/(b*p*q))*f^
3*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) - (Sqrt[b]*h*(f*g - e*h)*Sqrt[p]*Sqrt[Pi
/2]*Sqrt[q]*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])
/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(2*E^((2*a)/(b*p*q))*f^3*(c*(d*(e + f*x)^p)^q)
^(2/(p*q))) - (Sqrt[b]*h^2*Sqrt[p]*Sqrt[Pi/3]*Sqrt[q]*(e + f*x)^3*Erfi[(Sqr
t[3]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(6*E
^((3*a)/(b*p*q))*f^3*(c*(d*(e + f*x)^p)^q)^(3/(p*q))) + ((f*g - e*h)^2*(e +
 f*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/f^3 + (h*(f*g - e*h)*(e + f*x)^
2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/f^3 + (h^2*(e + f*x)^3*Sqrt[a + b*L
og[c*(d*(e + f*x)^p)^q]])/(3*f^3)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
```

```
+ e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int (g+hx)^2\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\int \left(\frac{(fg-eh)^2\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{f^2} + \frac{2h(fg-eh)(e+fx)\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{f^2} + \frac{h^2(e+fx)^2\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{f^2}\right) dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{h^2\int (e+fx)^2\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad + \text{Subst}\left(\frac{(2h(fg-eh))\int (e+fx)\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad + \text{Subst}\left(\frac{(fg-eh)^2\int \sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$= \text{Subst}\left(\frac{h^2\text{Subst}\left(\int x^2\sqrt{a+b\log\left(cd^qx^{pq}\right)}\,dx,x,e+fx\right)}{f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{(2h(fg-eh))\text{Subst}\left(\int x\sqrt{a+b\log\left(cd^qx^{pq}\right)}\,dx,x,e+fx\right)}{f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{(fg-eh)^2\text{Subst}\left(\int\sqrt{a+b\log\left(cd^qx^{pq}\right)}\,dx,x,e+fx\right)}{f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(fg-eh)^2(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3} + \frac{h(fg-eh)(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3} + \frac{h^2(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{3f^3} - \text{Subst}\left(\frac{(bh^2pq)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx, x, e+fx\right)}{6f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(bh(fg-eh)pq)\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx, x, e+fx\right)}{2f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(b(fg-eh)^2pq)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx, x, e+fx\right)}{2f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(fg-eh)^2(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3}$$
$$+ \frac{h(fg-eh)(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3}$$
$$+ \frac{h^2(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{3f^3}$$
$$-\text{Subst}\left(\frac{\left(bh^2(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{6f^3},cd^q(e\right.$$
$$\left.+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\text{Subst}\left(\frac{\left(bh(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{2f^3}\right.$$
$$\left.+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\text{Subst}\left(\frac{\left(b(fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{2f^3},c\right.$$
$$\left.+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(fg-eh)^2(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3}$$
$$+ \frac{h(fg-eh)(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3}$$
$$+ \frac{h^2(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{3f^3}$$
$$-\text{Subst}\left(\frac{\left(h^2(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int e^{-\frac{3a}{bpq}+\frac{3x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\right)}{3f^3}\right.$$
$$\left. + fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$-\text{Subst}\left(\frac{\left(h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int e^{-\frac{2a}{bpq}+\frac{2x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+\right.}\right.}{f^3}\right.$$
$$\left. + fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$-\text{Subst}\left(\frac{\left((fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+f\right.}\right.}{f^3}\right.$$
$$\left. + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{\sqrt{b}e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{p}\sqrt{\pi}\sqrt{q}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^3} - \frac{\sqrt{b}e^{-\frac{2a}{bpq}}h(fg-eh)\sqrt{p}\sqrt{\frac{\pi}{2}}\sqrt{q}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^3} - \frac{\sqrt{b}e^{-\frac{3a}{bpq}}h^2\sqrt{p}\sqrt{\frac{\pi}{3}}\sqrt{q}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{6f^3} + \frac{(fg-eh)^2(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3} + \frac{h(fg-eh)(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^3} + \frac{h^2(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{3f^3}$$

## Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.94

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{(e+fx)\left(-18\sqrt{b}e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{p}\sqrt{\pi}\sqrt{q}(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)+9\sqrt{b}e^{-\frac{2a}{bpq}}h(-$$

```
[In] Integrate[(g + h*x)^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] ((e + f*x)*((-18*Sqrt[b]*(f*g - e*h)^2*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*Erfi[Sqrt[a
 + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^(a/(b*p*q))*(
c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (9*Sqrt[b]*h*(-(f*g) + e*h)*Sqrt[p]*Sqrt[
2*Pi]*Sqrt[q]*(e + f*x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])
/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^((2*a)/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(2/(p
*q))) - (2*Sqrt[b]*h^2*Sqrt[p]*Sqrt[3*Pi]*Sqrt[q]*(e + f*x)^2*Erfi[(Sqrt[3]
*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^((3*a
)/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(3/(p*q))) + 36*(f*g - e*h)^2*Sqrt[a + b*L
og[c*(d*(e + f*x)^p)^q]] + 36*h*(f*g - e*h)*(e + f*x)*Sqrt[a + b*Log[c*(d*(
e + f*x)^p)^q]] + 12*h^2*(e + f*x)^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]))
/(36*f^3)
```

## Maple [F]

$$\int (hx+g)^2 \sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)}dx$$

```
[In] int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
[Out] int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fricas
")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\left(g+hx\right)^2\,dx$$

```
[In] integrate((h*x+g)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
[Out] Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)**2, x)
```

## Maxima [F]

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int (hx+g)^2\sqrt{b\log\left(\left((fx+e)^pd\right)^qc\right)+a}\,dx$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxima
")
[Out] integrate((h*x + g)^2*sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [F]

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int (hx+g)^2\sqrt{b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a\right)}\,dx$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac")
[Out] integrate((h*x + g)^2*sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int (g+h\,x)^2\,\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}\,dx$$

```
[In] int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
[Out] int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

# 3.461 $\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 311

$$\begin{aligned}
&\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx\\
&= -\frac{\sqrt{b}e^{-\frac{a}{bpq}}(fg-eh)\sqrt{p}\sqrt{\pi}\sqrt{q}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^2}\\
&\quad-\frac{\sqrt{b}e^{-\frac{2a}{bpq}}h\sqrt{p}\sqrt{\frac{\pi}{2}}\sqrt{q}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f^2}\\
&\quad+\frac{(fg-eh)(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^2}\\
&\quad+\frac{h(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2}
\end{aligned}$$

```
[Out] -1/8*h*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(
1/2)/q^(1/2))*b^(1/2)*p^(1/2)*2^(1/2)*Pi^(1/2)*q^(1/2)/exp(2*a/b/p/q)/f^2/(
(c*(d*(f*x+e)^p)^q)^(2/p/q))-1/2*(-e*h+f*g)*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+
e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*b^(1/2)*p^(1/2)*Pi^(1/2)*q^(1/2)/e
xp(a/b/p/q)/f^2/((c*(d*(f*x+e)^p)^q)^(1/p/q))+(-e*h+f*g)*(f*x+e)*(a+b*ln(c*
(d*(f*x+e)^p)^q))^(1/2)/f^2+1/2*h*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/
2)/f^2
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2448, 2436, 2333, 2337, 2211, 2235, 2437, 2342, 2347, 2495}

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$
$$=-\frac{\sqrt{\pi}\sqrt{b}\sqrt{p}\sqrt{q}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^2}$$
$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}h\sqrt{p}\sqrt{q}(e+fx)^2e^{-\frac{2a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f^2}$$
$$+\frac{(e+fx)(fg-eh)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^2}$$
$$+\frac{h(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2}$$

```
[In] Int[(g + h*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] -1/2*(Sqrt[b]*(f*g - e*h)*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*(e + f*x)*Erfi[Sqrt[a +
b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])]])/(E^(a/(b*p*q))*f^2*
(c*(d*(e + f*x)^p)^q)^(1/(p*q))) - (Sqrt[b]*h*Sqrt[p]*Sqrt[Pi/2]*Sqrt[q]*(e
 + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt
[p]*Sqrt[q])]])/(4*E^((2*a)/(b*p*q))*f^2*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) +
((f*g - e*h)*(e + f*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/f^2 + (h*(e +
f*x)^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(2*f^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (g+hx)\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\int \left(\frac{(fg-eh)\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{f} + \frac{h(e+fx)\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}{f}\right) dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{h\int (e+fx)\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(fg-eh)\int \sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{h\text{Subst}\left(\int x\sqrt{a+b\log\left(cd^qx^{pq}\right)}\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(fg-eh)\text{Subst}\left(\int \sqrt{a+b\log\left(cd^qx^{pq}\right)}\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(fg-eh)(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^2} + \frac{h(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2} - \text{Subst}\left(\frac{(bhpq)\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx, x, e+fx\right)}{4f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(b(fg-eh)pq)\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx, x, e+fx\right)}{2f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(fg-eh)(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^2} + \frac{h(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2} - \text{Subst}\left(\frac{\left(bh(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{pq}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{4f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{\left(b(fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{2f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(fg-eh)(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^2} + \frac{h(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2} - \text{Subst}\left(\frac{\left(h(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int e^{-\frac{2a}{bpq}+\frac{2x^2}{bpq}}\,dx,x,\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\right)}{2f^2} \right.$$

$$\left. + fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{\left((fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx,x,\sqrt{a+b\log\left(cd^q(e+\right.}\right.}{f^2}\right.$$

$$\left. + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{\sqrt{b}e^{-\frac{a}{bpq}}(fg-eh)\sqrt{p}\sqrt{\pi}\sqrt{q}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f^2} - \frac{\sqrt{b}e^{-\frac{2a}{bpq}}h\sqrt{p}\sqrt{\frac{\pi}{2}}\sqrt{q}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f^2} + \frac{(fg-eh)(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f^2} + \frac{h(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2}$$

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.96

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = -\frac{e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(4\sqrt{b}e^{\frac{a}{bpq}}(fg-eh)\sqrt{p}\sqrt{\pi}\sqrt{q}\left(c(d(e+fx)^p)^q\right)^{\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx}{\sqrt{b}\sqrt{p}\sqrt{q}}\right.\right.}{}$$

```
[In] Integrate[(g + h*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] -1/8*((e + f*x)*(4*Sqrt[b]*E^(a/(b*p*q))*(f*g - e*h)*Sqrt[p]*Sqrt[Pi]*Sqrt[
q]*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]
]/(Sqrt[b]*Sqrt[p]*Sqrt[q])] + Sqrt[b]*h*Sqrt[p]*Sqrt[2*Pi]*Sqrt[q]*(e + f*
x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqr
t[q])] - 4*E^((2*a)/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(2*f*g - e*h +
 f*h*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]))/(E^((2*a)/(b*p*q))*f^2*(c*(d
*(e + f*x)^p)^q)^(2/(p*q)))
```

## Maple [F]

$$\int (hx+g)\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

```
[In] int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

```
[Out] int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}\,dx = \int \sqrt{a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}(g+hx)\,dx$$

```
[In] integrate((h*x+g)*(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x), x)
```

## Maxima [F]

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int (hx+g)\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}\,dx$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxima")
[Out] integrate((h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [F]

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int (hx+g)\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}\,dx$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac")
[Out] integrate((h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int (g+h\,x)\,\sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}\,dx$$

```
[In] int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
[Out] int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

# 3.462 $\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = -\frac{\sqrt{b}e^{-\frac{a}{bpq}}\sqrt{p}\sqrt{\pi}\sqrt{q}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f} + \frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f}$$

```
[Out] -1/2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2)
)*b^(1/2)*p^(1/2)*Pi^(1/2)*q^(1/2)/exp(a/b/p/q)/f/((c*(d*(f*x+e)^p)^q)^(1/p
/q))+(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/f
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2436, 2333, 2337, 2211, 2235, 2495}

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f} - \frac{\sqrt{\pi}\sqrt{b}\sqrt{p}\sqrt{q}(e+fx)e^{-\frac{a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f}$$

```
[In] Int[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] -1/2*(Sqrt[b]*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(
e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^(a/(b*p*q))*f*(c*(d*(e + f*x
)^p)^q)^(1/(p*q))) + ((e + f*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/f
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \sqrt{a+b\log\left(cd^q x^{pq}\right)}\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f} - \text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^q x^{pq})}}\,dx, x, e+fx\right)}{2f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f} - \text{Subst}\left(\frac{\left(b(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{2f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f} - \text{Subst}\left(\frac{\left((e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\right)}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{\sqrt{b}e^{-\frac{a}{bpq}}\sqrt{p}\sqrt{\pi}\sqrt{q}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{2f} + \frac{(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{f}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$
$$=\frac{(e+fx)\left(-\sqrt{b}e^{-\frac{a}{bpq}}\sqrt{p}\sqrt{\pi}\sqrt{q}\left(c\left(d(e+fx)^p\right)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)+2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right.\right.}\right.}{2f}$$

```
[In] Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] ((e + f*x)*(-((Sqrt[b]*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*Erfi[Sqrt[a + b*Log[c*(d*(e
 + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^(a/(b*p*q))*(c*(d*(e + f*x)^p
)^q)^(1/(p*q)))) + 2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]))/(2*f)
```

## Maple [F]

$$\int \sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \sqrt{a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

## Maxima [F]

$$\int \sqrt{a + b\log\left(c\left(d(e + fx)^p\right)^q\right)}\,dx = \int \sqrt{b\log\left(\left((fx + e)^p d\right)^q c\right) + a}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [F]

$$\int \sqrt{a + b\log\left(c\left(d(e + fx)^p\right)^q\right)}\,dx = \int \sqrt{b\log\left(\left((fx + e)^p d\right)^q c\right) + a}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b\log\left(c\left(d(e + fx)^p\right)^q\right)}\,dx = \int \sqrt{a + b\,\ln\left(c\left(d\left(e + f\,x\right)^p\right)^q\right)}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

# 3.463 $\int \frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{g+hx}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}\,dx=\mathrm{Int}\left(\frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g),x)
```

## Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}\,dx=\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}\,dx$$

```
[In] Int[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x),x]
[Out] Defer[Int][Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x), x]
```

Rubi steps

$$\text{integral}=\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 4.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}\,dx = \int \frac{\sqrt{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}\,dx$$

```
[In] Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x),x]
[Out] Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x), x]
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\ln \left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}{hx+g}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g),x)
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}}{g+hx}\,dx = \int \frac{\sqrt{a+b\log \left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}}{g+hx}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2)/(h*x+g),x)
[Out] Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x), x)
```

## Maxima [N/A]

Not integrable

Time = 11.96 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\log\left(c\left(d(e + fx)^p\right)^q\right)}}{g + hx}\,dx = \int \frac{\sqrt{b\log\left(\left(\left(fx + e\right)^p d\right)^q c\right) + a}}{hx + g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g),x, algorithm="maxima")
[Out] integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

## Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\log\left(c\left(d(e + fx)^p\right)^q\right)}}{g + hx}\,dx = \int \frac{\sqrt{b\log\left(\left(\left(fx + e\right)^p d\right)^q c\right) + a}}{hx + g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g),x, algorithm="giac")
[Out] integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

## Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\log\left(c\left(d(e + fx)^p\right)^q\right)}}{g + hx}\,dx = \int \frac{\sqrt{a + b\,\ln\left(c\left(d\left(e + f\,x\right)^p\right)^q\right)}}{g + h\,x}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x), x)
```

# 3.464 $\int \frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{(g+hx)^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx=\text{Int}\left(\frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx=\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx$$

```
[In] Int[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^2,x]
```

```
[Out] Defer[Int][Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx=\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx$$

```
[In] Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^2,x]
[Out] Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^2, x]
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}{\left(hx+g\right)^{2}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^2,x)
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^2,x, algorithm="fricas
")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx=\int \frac{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}}{\left(g+hx\right)^2}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2)/(h*x+g)**2,x)
[Out] Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x)**2, x)
```

## Maxima [N/A]

Not integrable

Time = 10.69 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx=\int \frac{\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}{\left(hx+g\right)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^2,x, algorithm="maxima
")
[Out] integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2}\,dx=\int \frac{\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}{\left(hx+g\right)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^2,x, algorithm="giac")
[Out] integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log \left(c \left(d(e+fx)^p\right)^q\right)}}{(g+hx)^2} \, dx = \int \frac{\sqrt{a+b\,\ln \left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}}{{\left(g+h\,x\right)}^2} \,d x$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x)^2,x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x)^2, x)
```

# 3.465 $\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2} dx$



## Optimal result

Integrand size = 30, antiderivative size = 625

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2} dx = \frac{3b^{3/2}e^{-\frac{a}{bpq}}(fg-eh)^2p^{3/2}\sqrt{\pi}q^{3/2}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f^3}$$
$$+\frac{3b^{3/2}e^{-\frac{2a}{bpq}}h(fg-eh)p^{3/2}\sqrt{\frac{\pi}{2}}q^{3/2}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{8f^3}$$
$$+\frac{b^{3/2}e^{-\frac{3a}{bpq}}h^2p^{3/2}\sqrt{\frac{\pi}{3}}q^{3/2}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{12f^3}$$
$$-\frac{3b(fg-eh)^2pq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^3}$$
$$-\frac{3bh(fg-eh)pq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{4f^3}$$
$$-\frac{bh^2pq(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{6f^3}$$
$$+\frac{(fg-eh)^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3}$$
$$+\frac{h(fg-eh)(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3}$$
$$+\frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{3f^3}$$

```
[Out] (-e*h+f*g)^2*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/f^3+h*(-e*h+f*g)*(f*
x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/f^3+1/3*h^2*(f*x+e)^3*(a+b*ln(c*(d
```

```
*(f*x+e)^p)^q))^(3/2)/f^3+1/36*b^(3/2)*h^2*p^(3/2)*q^(3/2)*(f*x+e)^3*erfi(3
^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*3^(1/2)*P
i^(1/2)/exp(3*a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(3/p/q))+3/16*b^(3/2)*h*(-e
*h+f*g)*p^(3/2)*q^(3/2)*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^
(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*2^(1/2)*Pi^(1/2)/exp(2*a/b/p/q)/f^3/((c*(d*(
f*x+e)^p)^q)^(2/p/q))+3/4*b^(3/2)*(-e*h+f*g)^2*p^(3/2)*q^(3/2)*(f*x+e)*erfi
((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*Pi^(1/2)/exp(a/
b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(1/p/q))-3/2*b*(-e*h+f*g)^2*p*q*(f*x+e)*(a+
b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/f^3-3/4*b*h*(-e*h+f*g)*p*q*(f*x+e)^2*(a+b*ln
(c*(d*(f*x+e)^p)^q))^(1/2)/f^3-1/6*b*h^2*p*q*(f*x+e)^3*(a+b*ln(c*(d*(f*x+e)
^p)^q))^(1/2)/f^3
```

## Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2448, 2436, 2333, 2337, 2211, 2235, 2437, 2342, 2347, 2495}

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^{3/2}\,dx = \frac{3\sqrt{\frac{\pi}{2}}b^{3/2}hp^{3/2}q^{3/2}(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\,l}}{\sqrt{}}\right.}{8f^3}$$
$$+\frac{3\sqrt{\pi}b^{3/2}p^{3/2}q^{3/2}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f^3}$$
$$+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}h^2p^{3/2}q^{3/2}(e+fx)^3e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{12f^3}$$
$$+\frac{h(e+fx)^2(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3}$$
$$+\frac{(e+fx)(fg-eh)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3}$$
$$-\frac{3bhpq(e+fx)^2(fg-eh)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{4f^3}$$
$$-\frac{3bpq(e+fx)(fg-eh)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^3}$$
$$+\frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{3f^3}-\frac{bh^2pq(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{6f^3}$$

```
[In] Int[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

```
[Out] (3*b^(3/2)*(f*g - e*h)^2*p^(3/2)*Sqrt[Pi]*q^(3/2)*(e + f*x)*Erfi[Sqrt[a + b
*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(4*E^(a/(b*p*q))*f^3
```

```
*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (3*b^(3/2)*h*(f*g - e*h)*p^(3/2)*Sqrt[P
i/2]*q^(3/2)*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]
)/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(8*E^((2*a)/(b*p*q))*f^3*(c*(d*(e + f*x)^p)^q
)^(2/(p*q))) + (b^(3/2)*h^2*p^(3/2)*Sqrt[Pi/3]*q^(3/2)*(e + f*x)^3*Erfi[(Sq
rt[3]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(12
*E^((3*a)/(b*p*q))*f^3*(c*(d*(e + f*x)^p)^q)^(3/(p*q))) - (3*b*(f*g - e*h)^
2*p*q*(e + f*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(2*f^3) - (3*b*h*(f*g
 - e*h)*p*q*(e + f*x)^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(4*f^3) - (b*
h^2*p*q*(e + f*x)^3*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(6*f^3) + ((f*g -
 e*h)^2*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))/f^3 + (h*(f*g - e
*h)*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))/f^3 + (h^2*(e + f*x
)^3*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))/(3*f^3)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
```

```
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int (g+hx)^2 \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\int \left(\frac{(fg-eh)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}{f^2} + \frac{2h(fg-eh)(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}{f^2} + \frac{h^2(e+fx)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}{f^2}\right)dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$= \text{Subst}\left(\frac{h^2 \int (e+fx)^2 \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2} \, dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$+ \text{Subst}\left(\frac{(2h(fg-eh)) \int (e+fx) \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2} \, dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$+ \text{Subst}\left(\frac{(fg-eh)^2 \int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2} \, dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{h^2 \text{Subst}\left(\int x^2 \left(a+b\log\left(cd^q x^{pq}\right)\right)^{3/2} \, dx, x, e+fx\right)}{f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$+ \text{Subst}\left(\frac{(2h(fg-eh)) \text{Subst}\left(\int x \left(a+b\log\left(cd^q x^{pq}\right)\right)^{3/2} \, dx, x, e+fx\right)}{f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p\right.$$
$$+ \text{Subst}\left(\frac{(fg-eh)^2 \text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right)^{3/2} \, dx, x, e+fx\right)}{f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(fg-eh)^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3}$$
$$+ \frac{h(fg-eh)(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3}$$
$$+ \frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{3f^3}$$
$$- \text{Subst}\left(\frac{(bh^2pq)\, \text{Subst}\left(\int x^2 \sqrt{a+b\log\left(cd^q x^{pq}\right)} \, dx, x, e+fx\right)}{2f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Su}$$

$$= -\frac{3b(fg-eh)^2pq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^3} - \frac{3bh(fg-eh)pq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{4f^3} - \frac{bh^2pq(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{6f^3} + \frac{(fg-eh)^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3} + \frac{h(fg-eh)(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3} + \frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{3f^3} + \text{Subst}\left(\frac{(b^2h^2p^2q^2)\,\text{Subst}\left(\int\frac{x^2}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{12f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) + \text{Subst}$$

$$= -\frac{3b(fg-eh)^2pq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^3} - \frac{3bh(fg-eh)pq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{4f^3} - \frac{bh^2pq(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{6f^3} + \frac{(fg-eh)^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3} + \frac{h(fg-eh)(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3} + \frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{3f^3} + \text{Subst}\left(\frac{\left(b^2h^2pq(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{12f^3},cd^q(e\right.$$

$$= -\frac{3b(fg-eh)^2pq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^3} - \frac{3bh(fg-eh)pq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{4f^3} - \frac{bh^2pq(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{6f^3} + \frac{(fg-eh)^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3} + \frac{h(fg-eh)(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3} + \frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{3f^3} + \text{Subst}\left(\frac{\left(bh^2pq(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int e^{-\frac{3a}{bpq}+\frac{3x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right.}\right.}{6f^3}\right.$$

$$= \frac{3b^{3/2}e^{-\frac{a}{bpq}}(fg-eh)^2p^{3/2}\sqrt{\pi}q^{3/2}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f^3} + \frac{3b^{3/2}e^{-\frac{2a}{bpq}}h(fg-eh)p^{3/2}\sqrt{\frac{\pi}{2}}q^{3/2}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{8f^3} + \frac{b^{3/2}e^{-\frac{3a}{bpq}}h^2p^{3/2}\sqrt{\frac{\pi}{3}}q^{3/2}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{12f^3} - \frac{3b(fg-eh)^2pq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^3} - \frac{3bh(fg-eh)pq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{4f^3} - \frac{bh^2pq(e+fx)^3\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{6f^3} + \frac{(fg-eh)^2(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3} + \frac{h(fg-eh)(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^3} + \frac{h^2(e+fx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{3f^3}$$

## Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.87

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^{3/2}\,dx = \frac{(e+fx)\left(144(fg-eh)^2(a+b\log(c(d(e+fx)^p)^q))^{3/2}+144h(fg-eh)(e\cdots\right.}{}$$

```
[In] Integrate[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

```
[Out] ((e + f*x)*(144*(f*g - e*h)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) + 144*
h*(f*g - e*h)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) + 48*h^2*(e
+ f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) + 4*b*h^2*p*q*(e + f*x)^2*(
(Sqrt[b]*Sqrt[p]*Sqrt[3*Pi]*Sqrt[q]*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d*(e +
f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]])/(E^((3*a)/(b*p*q))*(c*(d*(e + f*x)
^p)^q)^(3/(p*q))) - 6*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]) + 27*b*h*(f*g -
 e*h)*p*q*(e + f*x)*((Sqrt[b]*Sqrt[p]*Sqrt[2*Pi]*Sqrt[q]*Erfi[(Sqrt[2]*Sqrt
[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]])/(E^((2*a)/(b*
p*q))*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) - 4*Sqrt[a + b*Log[c*(d*(e + f*x)^p)
^q]]) + 108*b*(f*g - e*h)^2*p*q*((Sqrt[b]*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*Erfi[Sqr
t[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])]])/(E^(a/(b*p*q)
)*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) - 2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]
)))/(144*f^3)
```

## Maple [F]

$$\int (hx+g)^2 (a+b\ln(c(d(fx+e)^p)^q))^{\frac{3}{2}}dx$$

```
[In] int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

```
[Out] int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^{3/2} \, dx = \int \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^{\frac{3}{2}} (g+hx)^2 \, dx$$

```
[In] integrate((h*x+g)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(3/2)*(g + h*x)**2, x)
```

## Maxima [F]

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^{3/2} \, dx = \int (hx+g)^2 (b\log \left(((fx+e)^p d)^q c\right)+a)^{\frac{3}{2}} \, dx$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxima
")
[Out] integrate((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

## Giac [F]

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^{3/2} \, dx = \int (hx+g)^2 (b\log \left(((fx+e)^p d)^q c\right)+a)^{\frac{3}{2}} \, dx$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac")
[Out] integrate((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^{3/2} \, dx = \int (g+h\,x)^2\,\left(a+b\,\ln \left(c\,\left(d\,(e+f\,x)^p\right)^q\right)\right)^{3/2} \,d x$$

```
[In] int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
[Out] int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

# 3.466 $\int (g+hx) \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^{3/2} \, dx$



## Optimal result

Integrand size = 28, antiderivative size = 396

$$\int (g+hx) (a +b\log \left(c(d(e+fx)^p)^q\right))^{3/2} \, dx = \frac{3b^{3/2}e^{-\frac{a}{bpq}}(fg-eh)p^{3/2}\sqrt{\pi}q^{3/2}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(}}{\sqrt{b}\sqrt{p}}\right.}{4f^2}$$
$$+\frac{3b^{3/2}e^{-\frac{2a}{bpq}}hp^{3/2}\sqrt{\frac{\pi}{2}}q^{3/2}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{16f^2}$$
$$-\frac{3b(fg-eh)pq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2}$$
$$-\frac{3bhpq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{8f^2}$$
$$+\frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^2}$$
$$+\frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{2f^2}$$

```
[Out] (-e*h+f*g)*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/f^2+1/2*h*(f*x+e)^2*(a
+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/f^2+3/32*b^(3/2)*h*p^(3/2)*q^(3/2)*(f*x+e)^
2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*2
^(1/2)*Pi^(1/2)/exp(2*a/b/p/q)/f^2/((c*(d*(f*x+e)^p)^q)^(2/p/q))+3/4*b^(3/2
)*(-e*h+f*g)*p^(3/2)*q^(3/2)*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)
/b^(1/2)/p^(1/2)/q^(1/2))*Pi^(1/2)/exp(a/b/p/q)/f^2/((c*(d*(f*x+e)^p)^q)^(1
/p/q))-3/2*b*(-e*h+f*g)*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/f^2-3
/8*b*h*p*q*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/f^2
```

## Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2448, 2436, 2333, 2337, 2211, 2235, 2437, 2342, 2347, 2495}

$$\int (g+hx)(a+b\log(c(d(e+fx)^p)^q))^{3/2}\,dx=\frac{3\sqrt{\pi}b^{3/2}p^{3/2}q^{3/2}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f^2}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}hp^{3/2}q^{3/2}(e+fx)^2e^{-\frac{2a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{16f^2}+\frac{(e+fx)(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^2}-\frac{3bpq(e+fx)(fg-eh)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2}+\frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{2f^2}-\frac{3bhpq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{8f^2}$$

```
[In] Int[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

```
[Out] (3*b^(3/2)*(f*g - e*h)*p^(3/2)*Sqrt[Pi]*q^(3/2)*(e + f*x)*Erfi[Sqrt[a + b*L
og[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(4*E^(a/(b*p*q))*f^2*(
c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (3*b^(3/2)*h*p^(3/2)*Sqrt[Pi/2]*q^(3/2)*(
e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqr
t[p]*Sqrt[q])])/(16*E^((2*a)/(b*p*q))*f^2*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))
- (3*b*(f*g - e*h)*p*q*(e + f*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(2*f
^2) - (3*b*h*p*q*(e + f*x)^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(8*f^2)
+ ((f*g - e*h)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))/f^2 + (h*(
e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))/(2*f^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
```

```
c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int (g+hx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\int \left(\frac{(fg-eh)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}{f}\right.\right.\\
&\quad \left.\left.+\frac{h(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}{f}\right)\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{h\int (e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad +\text{Subst}\left(\frac{(fg-eh)\int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{h\text{Subst}\left(\int x\left(a+b\log\left(cd^qx^{pq}\right)\right)^{3/2}\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad +\text{Subst}\left(\frac{(fg-eh)\text{Subst}\left(\int \left(a+b\log\left(cd^qx^{pq}\right)\right)^{3/2}\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^2}\\
&\quad +\frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{2f^2}\\
&\quad -\text{Subst}\left(\frac{(3bhpq)\text{Subst}\left(\int x\sqrt{a+b\log\left(cd^qx^{pq}\right)}\,dx, x, e+fx\right)}{4f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad -\text{Subst}\left(\frac{(3b(fg-eh)pq)\text{Subst}\left(\int \sqrt{a+b\log\left(cd^qx^{pq}\right)}\,dx, x, e+fx\right)}{2f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right.
\end{aligned}$$

$$= -\frac{3b(fg-eh)pq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2}$$
$$-\frac{3bhpq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{8f^2}$$
$$+\frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^2}$$
$$+\frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{2f^2}$$
$$+\text{Subst}\left(\frac{(3b^2hp^2q^2)\,\text{Subst}\left(\int\frac{x}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{16f^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$+\text{Subst}\left(\frac{(3b^2(fg-eh)p^2q^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{4f^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= -\frac{3b(fg-eh)pq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2}$$
$$-\frac{3bhpq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{8f^2}$$
$$+\frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^2}$$
$$+\frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{2f^2}$$
$$+\text{Subst}\left(\frac{\left(3b^2hpq(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{16f^2},cd^q(e$$
$$+\text{Subst}\left(\frac{\left(3b^2(fg-eh)pq(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{4f^2}$$

$$= -\frac{3b(fg-eh)pq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2} - \frac{3bhpq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{8f^2} + \frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^2} + \frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{2f^2} + \text{Subst}\left(\frac{\left(3bhpq(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int e^{-\frac{2a}{bpq}+\frac{2x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+fx)^{p\cdots}\right.}\right.}{8f^2}\right. + \text{Subst}\left(\frac{\left(3b(fg-eh)pq(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e\cdots\right.}\right.}{2f^2}\right.$$

$$= \frac{3b^{3/2}e^{-\frac{a}{bpq}}(fg-eh)p^{3/2}\sqrt{\pi}q^{3/2}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f^2} + \frac{3b^{3/2}e^{-\frac{2a}{bpq}}hp^{3/2}\sqrt{\frac{\pi}{2}}q^{3/2}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{16f^2} - \frac{3b(fg-eh)pq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f^2} - \frac{3bhpq(e+fx)^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{8f^2} + \frac{(fg-eh)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f^2} + \frac{h(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{2f^2}$$

## Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.88

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx = \frac{(e+fx)\left(32(fg-eh)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}+16h(e+fx)\left(a+b\log\cdots\right.\right.}{\cdots}$$

```
[In] Integrate[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

```
[Out] ((e + f*x)*(32*(f*g - e*h)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) + 16*h*(e
 + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) + 3*b*h*p*q*(e + f*x)*((Sqrt
[b]*Sqrt[p]*Sqrt[2*Pi]*Sqrt[q]*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^
p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^((2*a)/(b*p*q))*(c*(d*(e + f*x)^p)^q
)^(2/(p*q))) - 4*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]) + 24*b*(f*g - e*h)*p
*q*((Sqrt[b]*Sqrt[p]*Sqrt[Pi]*Sqrt[q]*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)
^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^(a/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(1/(p
*q))) - 2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])))/(32*f^2)
```

## Maple [F]

$$\int (hx+g)\left(a+b\ln\left(c(d(fx+e)^{p})^{q}\right)\right)^{\frac{3}{2}}dx$$

```
[In] int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

```
[Out] int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^{p})^{q}\right)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^{p})^{q}\right)\right)^{3/2}\,dx=\int\left(a+b\log\left(c(d(e+fx)^{p})^{q}\right)\right)^{\frac{3}{2}}(g+hx)\,dx$$

```
[In] integrate((h*x+g)*(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(3/2)*(g + h*x), x)
```

## Maxima [F]

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx = \int (hx+g)(b\log\left(((fx+e)^p d)^q c\right)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxima")
[Out] integrate((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

## Giac [F]

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx = \int (hx+g)(b\log\left(((fx+e)^p d)^q c\right)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac")
[Out] integrate((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx = \int (g+h\,x)\,\left(a+b\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)\right)^{3/2}\,dx$$

```
[In] int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
[Out] int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

# 3.467 $\int (a+b\log (c(d(e+fx)^p)^q))^{3/2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 176

$$\int (a+b\log (c(d(e+fx)^p)^q))^{3/2}\,dx=\frac{3b^{3/2}e^{-\frac{a}{bpq}}p^{3/2}\sqrt{\pi}q^{3/2}(e+fx)\,(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f}-\frac{3bpq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f}+\frac{(e+fx)\,(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{f}$$

```
[Out] (f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/f+3/4*b^(3/2)*p^(3/2)*q^(3/2)*(f*
x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*Pi^(1/
2)/exp(a/b/p/q)/f/((c*(d*(f*x+e)^p)^q)^(1/p/q))-3/2*b*p*q*(f*x+e)*(a+b*ln(c
*(d*(f*x+e)^p)^q))^(1/2)/f
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2436, 2333, 2337, 2211, 2235, 2495}

$$\int (a+b\log (c(d(e+fx)^p)^q))^{3/2}\,dx=\frac{3\sqrt{\pi}b^{3/2}p^{3/2}q^{3/2}(e+fx)e^{-\frac{a}{bpq}}(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f}+\frac{(e+fx)\,(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{f}-\frac{3bpq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

```
[Out] (3*b^(3/2)*p^(3/2)*Sqrt[Pi]*q^(3/2)*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e +
 f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(4*E^(a/(b*p*q))*f*(c*(d*(e + f*x)
^p)^q)^(1/(p*q))) - (3*b*p*q*(e + f*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]
)/(2*f) + ((e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))/f
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (a+b\log(cd^q(e+fx)^{pq}))^{3/2}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int (a+b\log(cd^qx^{pq}))^{3/2}\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f} - \text{Subst}\left(\frac{(3bpq)\text{Subst}\left(\int \sqrt{a+b\log(cd^qx^{pq})}\,dx, x, e+fx\right)}{2f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{3bpq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f} + \text{Subst}\left(\frac{(3b^2p^2q^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx, x, e+fx\right)}{4f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{3bpq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f} + \text{Subst}\left(\frac{\left(3b^2pq(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{4f}, cd^q(e+$$

$$= -\frac{3bpq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f} + \text{Subst}\left(\frac{\left(3bpq(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\right.}{2f}\right.$$

$$= \frac{3b^{3/2}e^{-\frac{a}{bpq}}p^{3/2}\sqrt{\pi}q^{3/2}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{4f} - \frac{3bpq(e+fx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{2f} + \frac{(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}}{f}$$

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.91

$$\int (a+b\log(c(d(e+fx)^p)^q))^{3/2}\,dx=\frac{(e+fx)\left(3b^{3/2}e^{-\frac{a}{bpq}}p^{3/2}\sqrt{\pi}q^{3/2}(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)\right.}{4f}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

```
[Out] ((e + f*x)*((3*b^(3/2)*p^(3/2)*Sqrt[Pi]*q^(3/2)*Erfi[Sqrt[a + b*Log[c*(d*(e
 + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(E^(a/(b*p*q))*(c*(d*(e + f*x)^p
)^q)^(1/(p*q))) + 2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]*(2*a - 3*b*p*q + 2
*b*Log[c*(d*(e + f*x)^p)^q])))/(4*f)
```

## Maple [F]

$$\int (a+b\ln(c(d(fx+e)^p)^q))^{\frac{3}{2}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (a+b\log(c(d(e+fx)^p)^q))^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (a+b\log(c(d(e+fx)^p)^q))^{3/2}\,dx = \int (a+b\log(c(d(e+fx)^p)^q))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(3/2), x)
```

## Maxima [F]

$$\int (a+b\log(c(d(e+fx)^p)^q))^{3/2}\,dx = \int (b\log(((fx+e)^p d)^q c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxima")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

## Giac [F]

$$\int (a+b\log(c(d(e+fx)^p)^q))^{3/2}\,dx = \int (b\log(((fx+e)^p d)^q c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(d(e+fx)^p)^q))^{3/2}\,dx = \int (a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^{3/2}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

# 3.468 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx=\text{Int}\left(\frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g),x)
```

## Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x),x]
[Out] Defer[Int][(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x), x]
```

Rubi steps

$$\text{integral}=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x),x]
[Out] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x), x]
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^{\frac{3}{2}}}{hx+g}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g),x)
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 52.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{\frac{3}{2}}}{g+hx}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2)/(h*x+g),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(3/2)/(g + h*x), x)
```

## Maxima [N/A]

Not integrable

Time = 10.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^{\frac{3}{2}}}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g),x, algorithm="maxima")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)/(h*x + g), x)
```

## Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^{\frac{3}{2}}}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)/(h*x + g), x)
```

## Mupad [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{g+hx}\,dx = \int \frac{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^{3/2}}{g+h\,x}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^(3/2)/(g + h*x),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^(3/2)/(g + h*x), x)
```

# 3.469 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx = \text{Int}\left(\frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g)^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x)^2,x]
```

```
[Out] Defer[Int][(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x)^2,x]
[Out] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)/(g + h*x)^2, x]
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^{\frac{3}{2}}}{(hx+g)^2}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g)^2,x)
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g)^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g)^2,x, algorithm="fricas
")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2)/(h*x+g)**2,x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 10.70 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx = \int \frac{(b\log(((fx+e)^pd)^q\,c)+a)^{\frac{3}{2}}}{(hx+g)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g)^2,x, algorithm="maxima
")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)/(h*x + g)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx = \int \frac{(b\log(((fx+e)^pd)^q\,c)+a)^{\frac{3}{2}}}{(hx+g)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(3/2)/(h*x+g)^2,x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)/(h*x + g)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}{(g+hx)^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^{3/2}}{{\left(g+h\,x\right)}^2}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^(3/2)/(g + h*x)^2,x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^(3/2)/(g + h*x)^2, x)
```

# 3.470 $\int \frac{(g+hx)^2}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 355

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \frac{e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}} + \frac{e^{-\frac{2a}{bpq}}h(fg-eh)\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}} + \frac{e^{-\frac{3a}{bpq}}h^2\sqrt{\frac{\pi}{3}}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}}$$

```
[Out] 1/3*h^2*(f*x+e)^3*erfi(3^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^
(1/2)/q^(1/2))*3^(1/2)*Pi^(1/2)/exp(3*a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(3/
p/q))/b^(1/2)/p^(1/2)/q^(1/2)+(-e*h+f*g)^2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e
)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*Pi^(1/2)/exp(a/b/p/q)/f^3/((c*(d*(f
*x+e)^p)^q)^(1/p/q))/b^(1/2)/p^(1/2)/q^(1/2)+h*(-e*h+f*g)*(f*x+e)^2*erfi(2^
(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*2^(1/2)*Pi
^(1/2)/exp(2*a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(2/p/q))/b^(1/2)/p^(1/2)/q^(
1/2)
```

## Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2448, 2436, 2337, 2211, 2235, 2437, 2347, 2495}

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \frac{\sqrt{2\pi}h(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}} + \frac{\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}} + \frac{\sqrt{\frac{\pi}{3}}h^2(e+fx)^3e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}}$$

```
[In] Int[(g + h*x)^2/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] ((f*g - e*h)^2*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]
/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^(a/(b*p*q))*f^3*Sqrt[p]*Sqrt[q]*(c*
(d*(e + f*x)^p)^q)^(1/(p*q))) + (h*(f*g - e*h)*Sqrt[2*Pi]*(e + f*x)^2*Erfi[
(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/
(Sqrt[b]*E^((2*a)/(b*p*q))*f^3*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(2/(p*
q))) + (h^2*Sqrt[Pi/3]*(e + f*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d*(e + f
*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^((3*a)/(b*p*q))*f^3*Sqrt
[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(3/(p*q)))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\int \left(\frac{(fg-eh)^2}{f^2\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}} + \frac{2h(fg-eh)(e+fx)}{f^2\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}} + \frac{h^2(e+fx)^2}{f^2\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\right) dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$=\text{Subst}\left(\frac{h^2\int \frac{(e+fx)^2}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{f^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{(2h(fg-eh))\int \frac{e+fx}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{f^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{(fg-eh)^2\int \frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{f^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$=\text{Subst}\left(\frac{h^2\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{(2h(fg-eh))\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{(fg-eh)^2\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$=\text{Subst}\left(\frac{\left(h^2(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{f^3pq},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left(2h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{f^3pq}\right.$$

$$\left.+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left((fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{f^3pq},cd\right.$$

$$\left.+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$=\text{Subst}\left(\frac{\left(2h^2(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int e^{-\frac{3a}{bpq}+\frac{3x^2}{bpq}}\,dx,x,\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\right)}{bf^3pq},\right.$$

$$\left.+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left(4h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int e^{-\frac{2a}{bpq}+\frac{2x^2}{bpq}}\,dx,x,\sqrt{a+b\log\left(cd^q(e-\right.}\right.}{bf^3pq}\right.$$

$$\left.+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left(2(fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx,x,\sqrt{a+b\log\left(cd^q(e+\right.}\right.}{bf^3pq}\right.$$

$$\left.+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$=\frac{e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}}$$

$$+\frac{e^{-\frac{2a}{bpq}}h(fg-eh)\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}}$$

$$+\frac{e^{-\frac{3a}{bpq}}h^2\sqrt{\frac{\pi}{3}}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^3\sqrt{p}\sqrt{q}}$$

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.89

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \frac{e^{-\frac{3a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\left(3e^{\frac{2a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)+3\sqrt{\ldots}\right.}{\ldots}$$

```
[In] Integrate[(g + h*x)^2/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] (Sqrt[Pi]*(e + f*x)*(3*E^((2*a)/(b*p*q))*(f*g - e*h)^2*(c*(d*(e + f*x)^p)^q
)^(2/(p*q))*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt
[q])] + 3*Sqrt[2]*E^(a/(b*p*q))*h*(f*g - e*h)*(e + f*x)*(c*(d*(e + f*x)^p)^
q)^(1/(p*q))*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*S
qrt[p]*Sqrt[q])] + Sqrt[3]*h^2*(e + f*x)^2*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(
d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]))/(3*Sqrt[b]*E^((3*a)/(b*p*q
))*f^3*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(3/(p*q)))
```

## Maple [F]

$$\int \frac{(hx+g)^2}{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)}}dx$$

```
[In] int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

```
[Out] int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}}\,dx$$

```
[In] integrate((h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
[Out] Integral((g + h*x)**2/sqrt(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

## Maxima [F]

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{(hx+g)^2}{\sqrt{b\log\left(\left((fx+e)^pd\right)^qc\right)+a}}\,dx$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxima
")
[Out] integrate((h*x + g)^2/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [F]

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{(hx+g)^2}{\sqrt{b\log\left(\left((fx+e)^pd\right)^qc\right)+a}}\,dx$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac")
[Out] integrate((h*x + g)^2/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{(g+h\,x)^2}{\sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}}\,dx$$

```
[In] int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
[Out] int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

# 3.471 $\int \frac{g+hx}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 229

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \frac{e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^2\sqrt{p}\sqrt{q}} + \frac{e^{-\frac{2a}{bpq}}h\sqrt{\frac{\pi}{2}}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}$$

```
[Out] 1/2*h*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1
/2)/q^(1/2))*2^(1/2)*Pi^(1/2)/exp(2*a/b/p/q)/f^2/((c*(d*(f*x+e)^p)^q)^(2/p/
q))/b^(1/2)/p^(1/2)/q^(1/2)+(-e*h+f*g)*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)
^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*Pi^(1/2)/exp(a/b/p/q)/f^2/((c*(d*(f*x+e
)^p)^q)^(1/p/q))/b^(1/2)/p^(1/2)/q^(1/2)
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

= {2448, 2436, 2337, 2211, 2235, 2437, 2347, 2495}

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$
$$= \frac{\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}$$
$$+ \frac{\sqrt{\frac{\pi}{2}}h(e+fx)^2e^{-\frac{2a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}$$

```
[In] Int[(g + h*x)/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] ((f*g - e*h)*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(
Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^(a/(b*p*q))*f^2*Sqrt[p]*Sqrt[q]*(c*(d
*(e + f*x)^p)^q)^(1/(p*q))) + (h*Sqrt[Pi/2]*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[
a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(Sqrt[b]*E^((2
*a)/(b*p*q))*f^2*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{g+hx}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\int \left(\frac{fg-eh}{f\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\right.\right.\\
&\qquad\left.\left.+\frac{h(e+fx)}{f\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\right)dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{h\int \frac{e+fx}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad+\text{Subst}\left(\frac{(fg-eh)\int \frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$= \text{Subst}\left(\frac{h\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cd^q x^{pq})}}\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(fg-eh)\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^q x^{pq})}}\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\left(h(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{pq}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{f^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{\left((fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{f^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\left(2h(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int e^{-\frac{2a}{bpq}+\frac{2x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\right)}{bf^2pq}, c \ldots + fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{\left(2(fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+ \ldots\right.}}{bf^2pq} \ldots + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^2\sqrt{p}\sqrt{q}} + \frac{e^{-\frac{2a}{bpq}}h\sqrt{\frac{\pi}{2}}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f^2\sqrt{p}\sqrt{q}}$$

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.91

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \frac{e^{-\frac{2a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(2e^{\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)+\sqrt{2}\right.}{2\sqrt{b}f^2\sqrt{p}\sqrt{q}}$$

```
[In] Integrate[(g + h*x)/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] (Sqrt[Pi]*(e + f*x)*(2*E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)^p)^q)^(1/(
p*q))*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])]
+ Sqrt[2]*h*(e + f*x)*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(
Sqrt[b]*Sqrt[p]*Sqrt[q])]))/(2*Sqrt[b]*E^((2*a)/(b*p*q))*f^2*Sqrt[p]*Sqrt[q
]*(c*(d*(e + f*x)^p)^q)^(2/(p*q)))
```

## Maple [F]

$$\int \frac{hx+g}{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}dx$$

```
[In] int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

```
[Out] int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}}\,dx$$

```
[In] integrate((h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
[Out] Integral((g + h*x)/sqrt(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

## Maxima [F]

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{hx+g}{\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}\,dx$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxima")
[Out] integrate((h*x + g)/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [F]

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{hx+g}{\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}\,dx$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac")
[Out] integrate((h*x + g)/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{g+h\,x}{\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}}\,dx$$

```
[In] int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
[Out] int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

# 3.472 $\int \frac{1}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \frac{e^{-\frac{a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f\sqrt{p}\sqrt{q}}$$

```
[Out] (f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*Pi^
(1/2)/exp(a/b/p/q)/f/((c*(d*(f*x+e)^p)^q)^(1/p/q))/b^(1/2)/p^(1/2)/q^(1/2)
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2436, 2337, 2211, 2235, 2495}

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \frac{\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f\sqrt{p}\sqrt{q}}$$

```
[In] Int[1/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] (Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt
[p]*Sqrt[q])])/(Sqrt[b]*E^(a/(b*p*q))*f*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^
q)^(1/(p*q)))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^q x^{pq})}}\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\left((e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{fpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\left(2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\right)}{bfpq}, cd\right.$$

$$\left. + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{e^{-\frac{a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f\sqrt{p}\sqrt{q}}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \frac{e^{-\frac{a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{\sqrt{b}f\sqrt{p}\sqrt{q}}$$

```
[In] Integrate[1/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] (Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt
[p]*Sqrt[q])])/(Sqrt[b]*E^(a/(b*p*q))*f*Sqrt[p]*Sqrt[q]*(c*(d*(e + f*x)^p)^
q)^(1/(p*q)))
```

## Maple [F]

$$\int \frac{1}{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)}}dx$$

```
[In] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

```
[Out] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
[Out] Integral(1/sqrt(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{\sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}}\,dx$$

```
[In] int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
[Out] int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

# 3.473 $\int \frac{1}{(g+hx)\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\text{Int}\left(\frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

```
[In] Int[1/((g + h*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]),x]
[Out] Defer[Int][1/((g + h*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

```
[In] Integrate[1/((g + h*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]),x]
[Out] Integrate[1/((g + h*x)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]), x]
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{(hx+g)\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}dx$$

```
[In] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
[Out] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fricas
")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\,(g+hx)}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 10.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{(hx+g)\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxima
")
[Out] integrate(1/((h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{1}{(hx+g)\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac")
[Out] integrate(1/((h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{(g+h\,x)\ \sqrt{a+b\,\ln\left(c\left(d\,{\left(e+f\,x\right)}^p\right)^q\right)}}\,dx$$

```
[In] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)),x)
[Out] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)), x)
```

# 3.474 $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 404

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^3p^{3/2}q^{3/2}}$$
$$+\frac{4e^{-\frac{2a}{bpq}}h(fg-eh)\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^3p^{3/2}q^{3/2}}$$
$$+\frac{2e^{-\frac{3a}{bpq}}h^2\sqrt{3\pi}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^3p^{3/2}q^{3/2}}$$
$$-\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$

```
[Out] 2*(-e*h+f*g)^2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/
2)/q^(1/2))*Pi^(1/2)/b^(3/2)/exp(a/b/p/q)/f^3/p^(3/2)/q^(3/2)/((c*(d*(f*x+e
)^p)^q)^(1/p/q))+4*h*(-e*h+f*g)*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)
^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/exp(2*a/b/p
/q)/f^3/p^(3/2)/q^(3/2)/((c*(d*(f*x+e)^p)^q)^(2/p/q))+2*h^2*(f*x+e)^3*erfi(
3^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*3^(1/2)*
Pi^(1/2)/b^(3/2)/exp(3*a/b/p/q)/f^3/p^(3/2)/q^(3/2)/((c*(d*(f*x+e)^p)^q)^(3
/p/q))-2*(f*x+e)*(h*x+g)^2/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)
```

## Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2447, 2448, 2436, 2337, 2211, 2235, 2437, 2347, 2495}

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \frac{4\sqrt{2\pi}h(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c}}{\sqrt{b}\sqrt{}}\right.}{b^{3/2}f^3p^{3/2}q^{3/2}}$$

$$+\frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^3p^{3/2}q^{3/2}}$$

$$+\frac{2\sqrt{3\pi}h^2(e+fx)^3e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^3p^{3/2}q^{3/2}}$$

$$-\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$

```
[In] Int[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

```
[Out] (2*(f*g - e*h)^2*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q
]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(b^(3/2)*E^(a/(b*p*q))*f^3*p^(3/2)*q^(3/2)*(
c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (4*h*(f*g - e*h)*Sqrt[2*Pi]*(e + f*x)^2*E
rfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q]
)])/(b^(3/2)*E^((2*a)/(b*p*q))*f^3*p^(3/2)*q^(3/2)*(c*(d*(e + f*x)^p)^q)^(2
/(p*q))) + (2*h^2*Sqrt[3*Pi]*(e + f*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*(d*
(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(b^(3/2)*E^((3*a)/(b*p*q))*f^
3*p^(3/2)*q^(3/2)*(c*(d*(e + f*x)^p)^q)^(3/(p*q))) - (2*(e + f*x)*(g + h*x)
^2)/(b*f*p*q*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(g+hx)^2}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{6\int \frac{(g+hx)^2}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{bpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(4(fg-eh))\int \frac{g+hx}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{6\int \left(\frac{(fg-eh)^2}{f^2\sqrt{a+b\log(cd^q(e+fx)^{pq})}} + \frac{2h(fg-eh)(e+fx)}{f^2\sqrt{a+b\log(cd^q(e+fx)^{pq})}} + \frac{h^2(e+fx)^2}{f^2\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)dx}{bpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(4(fg-eh))\int \left(\frac{fg-eh}{f\sqrt{a+b\log(cd^q(e+fx)^{pq})}} + \frac{h(e+fx)}{f\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)dx}{bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$$+ \text{Subst}\left(\frac{(6h^2)\int \frac{(e+fx)^2}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$- \text{Subst}\left(\frac{(4h(fg-eh))\int \frac{e+fx}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$+ \text{Subst}\left(\frac{(12h(fg-eh))\int \frac{e+fx}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$- \text{Subst}\left(\frac{(4(fg-eh)^2)\int \frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$+ \text{Subst}\left(\frac{(6(fg-eh)^2)\int \frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{(6h^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx, x, e+fx\right)}{bf^3pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(4h(fg-eh))\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx, x, e+fx\right)}{bf^3pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(12h(fg-eh))\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx, x, e+fx\right)}{bf^3pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(4(fg-eh)^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx, x, e+fx\right)}{bf^3pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(6(fg-eh)^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx, x, e+fx\right)}{bf^3pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$$+\text{Subst}\left(\frac{\left(6h^2(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^3p^2q^2},cd^q(e\right.$$

$$\left.+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$-\text{Subst}\left(\frac{\left(4h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^3p^2q^2},\right.$$

$$\left.+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left(12h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^3p^2q^2}\right.$$

$$\left.+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$-\text{Subst}\left(\frac{\left(4(fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^3p^2q^2},cd\right.$$

$$\left.+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left(6(fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^3p^2q^2},cd\right.$$

$$\left.+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$$+\text{Subst}\left(\frac{\left(12h^2(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int e^{-\frac{3a}{bpq}+\frac{3x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+fx)^{p}\right.}\right.}{b^2f^3p^2q^2}\right.$$

$$\left.+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$-\text{Subst}\left(\frac{\left(8h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int e^{-\frac{2a}{bpq}+\frac{2x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e\right.}\right.}{b^2f^3p^2q^2}\right.$$

$$\left.+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left(24h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int e^{-\frac{2a}{bpq}+\frac{2x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(\right.}\right.}{b^2f^3p^2q^2}\right.$$

$$\left.+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$-\text{Subst}\left(\frac{\left(8(fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e-\right.}\right.}{b^2f^3p^2q^2}\right.$$

$$\left.+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left(12(fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e\right.}\right.}{b^2f^3p^2q^2}\right.$$

$$\left.+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{2e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^3p^{3/2}q^{3/2}} + \frac{4e^{-\frac{2a}{bpq}}h(fg-eh)\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^3p^{3/2}q^{3/2}} + \frac{2e^{-\frac{3a}{bpq}}h^2\sqrt{3\pi}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^3p^{3/2}q^{3/2}} - \frac{2(e+fx)(g+hx)^2}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1040 vs. 2(404) = 808.

Time = 1.52 (sec) , antiderivative size = 1040, normalized size of antiderivative = 2.57

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx = \frac{2\left(-\sqrt{b}ef^2g^2\sqrt{p}\sqrt{q} - \sqrt{b}f^3g^2\sqrt{p}\sqrt{q}x - 2\sqrt{b}ef^2gh\sqrt{p}\sqrt{q}x - 2\sqrt{b}f^3g\right.}{}$$

```
[In] Integrate[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

```
[Out] (2*(-(Sqrt[b]*e*f^2*g^2*Sqrt[p]*Sqrt[q]) - Sqrt[b]*f^3*g^2*Sqrt[p]*Sqrt[q]*
x - 2*Sqrt[b]*e*f^2*g*h*Sqrt[p]*Sqrt[q]*x - 2*Sqrt[b]*f^3*g*h*Sqrt[p]*Sqrt[
q]*x^2 - Sqrt[b]*e*f^2*h^2*Sqrt[p]*Sqrt[q]*x^2 - Sqrt[b]*f^3*h^2*Sqrt[p]*Sq
rt[q]*x^3 - (4*e*f*g*h*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x
)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(
E^(a/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (e^2*h^2*Sqrt[Pi]*(e + f*x
)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*Sqrt
[a + b*Log[c*(d*(e + f*x)^p)^q]])/(E^(a/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(1/(
p*q))) + (2*f*g*h*Sqrt[2*Pi]*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*
(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*Sqrt[a + b*Log[c*(d*(e + f*x)^
p)^q]])/(E^((2*a)/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) - (2*e*h^2*Sqrt
[2*Pi]*(e + f*x)^2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqr
t[b]*Sqrt[p]*Sqrt[q])]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(E^((2*a)/(b*p
*q))*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) + (h^2*Sqrt[3*Pi]*(e + f*x)^3*Erfi[(S
qrt[3]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*Sqr
t[a + b*Log[c*(d*(e + f*x)^p)^q]])/(E^((3*a)/(b*p*q))*(c*(d*(e + f*x)^p)^q)
^(3/(p*q))) + (Sqrt[b]*f^2*g^2*Sqrt[p]*Sqrt[q]*(e + f*x)*Gamma[1/2, -((a +
b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))]*Sqrt[-((a + b*Log[c*(d*(e + f*x)^p)^q
])/(b*p*q))])/(E^(a/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (2*Sqrt[b]*
e*f*g*h*Sqrt[p]*Sqrt[q]*(e + f*x)*Gamma[1/2, -((a + b*Log[c*(d*(e + f*x)^p)
^q])/(b*p*q))]*Sqrt[-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))])/(E^(a/(b*
p*q))*(c*(d*(e + f*x)^p)^q)^(1/(p*q)))))/(b^(3/2)*f^3*p^(3/2)*q^(3/2)*Sqrt[
a + b*Log[c*(d*(e + f*x)^p)^q]])
```

## Maple [F]

$$\int \frac{(hx+g)^2}{(a+b\ln(c(d(fx+e)^p)^q))^{\frac{3}{2}}}dx$$

```
[In] int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
[Out] int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="fricas
")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
[Out] Integral((g + h*x)**2/(a + b*log(c*(d*(e + f*x)**p)**q))**(3/2), x)
```

## Maxima [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{(hx+g)^2}{(b\log(((fx+e)^pd)^qc)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxima
")
[Out] integrate((h*x + g)^2/(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

## Giac [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{(hx+g)^2}{(b\log(((fx+e)^p d)^q c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac")
[Out] integrate((h*x + g)^2/(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{(g+h\,x)^2}{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^{3/2}}\,dx$$

```
[In] int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
[Out] int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

# 3.475 $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 275

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx=\frac{2e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}+\frac{2e^{-\frac{2a}{bpq}}h\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

```
[Out] 2*(-e*h+f*g)*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)
/q^(1/2))*Pi^(1/2)/b^(3/2)/exp(a/b/p/q)/f^2/p^(3/2)/q^(3/2)/((c*(d*(f*x+e)^
p)^q)^(1/p/q))+2*h*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)
/b^(1/2)/p^(1/2)/q^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/exp(2*a/b/p/q)/f^2/p^(3/
2)/q^(3/2)/((c*(d*(f*x+e)^p)^q)^(2/p/q))-2*(f*x+e)*(h*x+g)/b/f/p/q/(a+b*ln(
c*(d*(f*x+e)^p)^q))^(1/2)
```

## Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used

$= \{2447, 2448, 2436, 2337, 2211, 2235, 2437, 2347, 2495\}$

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\,(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}$$
$$+\frac{2\sqrt{2\pi}h(e+fx)^2e^{-\frac{2a}{bpq}}(c(d(e+fx)^p)^q)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}$$
$$-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$

```
[In] Int[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

```
[Out] (2*(f*g - e*h)*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]
/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(b^(3/2)*E^(a/(b*p*q))*f^2*p^(3/2)*q^(3/2)*(c*
(d*(e + f*x)^p)^q)^(1/(p*q))) + (2*h*Sqrt[2*Pi]*(e + f*x)^2*Erfi[(Sqrt[2]*S
qrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(b^(3/2)*E
^((2*a)/(b*p*q))*f^2*p^(3/2)*q^(3/2)*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) - (2*
(e + f*x)*(g + h*x))/(b*f*p*q*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{g+hx}{(a+b\log(cd^q(e+fx)^{pq}))^{3/2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{4\int \frac{g+hx}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{bpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(2(fg-eh))\int \frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{4\int \left(\frac{fg-eh}{f\sqrt{a+b\log(cd^q(e+fx)^{pq})}} + \frac{h(e+fx)}{f\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)dx}{bpq}, cd^q(e + fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(2(fg-eh))\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^q x^{pq})}}\,dx, x, e+fx\right)}{bf^2pq}, cd^q(e + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{(4h)\int \frac{e+fx}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(4(fg-eh))\int \frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{\left(2(fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^2p^2q^2}, cd^q(e + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$
$$+\text{Subst}\left(\frac{(4h)\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cd^q x^{pq})}}\,dx, x, e+fx\right)}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$+\text{Subst}\left(\frac{(4(fg-eh))\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^q x^{pq})}}\,dx, x, e+fx\right)}{bf^2pq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$-\text{Subst}\left(\frac{\left(4(fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+\right.}\right.}{b^2f^2p^2q^2} + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}$$
$$-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$
$$+\text{Subst}\left(\frac{\left(4h(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{pq}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{bf^2p^2q^2}, cd^q(e+f\right.$$

$$= -\frac{2e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}}$$
$$-\frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$
$$+\text{Subst}\left(\frac{\left(8h(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int e^{-\frac{2a}{bpq}+\frac{2x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\right.}{b^2f^2p^2q^2}\right.$$

$$= \frac{2e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}} + \frac{2e^{-\frac{2a}{bpq}}h\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}f^2p^{3/2}q^{3/2}} - \frac{2(e+fx)(g+hx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

## Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.58

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx = \frac{2e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(-2ee^{\frac{a}{bpq}}h\sqrt{\pi}(c(d(e+fx)^p)^q)^{\frac{1}{pq}}\,\mathrm{e}\right.}{}$$

```
[In] Integrate[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

```
[Out] (2*(e + f*x)*(-2*e*E^(a/(b*p*q))*h*Sqrt[Pi]*(c*(d*(e + f*x)^p)^q)^(1/(p*q))
*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*Sqrt[
a + b*Log[c*(d*(e + f*x)^p)^q]] + h*Sqrt[2*Pi]*(e + f*x)*Erfi[(Sqrt[2]*Sqrt
[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*Sqrt[a + b*Log
[c*(d*(e + f*x)^p)^q]] + Sqrt[b]*E^(a/(b*p*q))*Sqrt[p]*Sqrt[q]*(c*(d*(e + f
*x)^p)^q)^(1/(p*q))*(-(E^(a/(b*p*q))*f*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*(g +
 h*x)) + (f*g + e*h)*Gamma[1/2, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))
]*Sqrt[-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))])))/(b^(3/2)*E^((2*a)/(b
*p*q))*f^2*p^(3/2)*q^(3/2)*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*Sqrt[a + b*Log[c
*(d*(e + f*x)^p)^q]])
```

## Maple [F]

$$\int \frac{hx+g}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{\frac{3}{2}}}dx$$

```
[In] int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

```
[Out] int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
[Out] Integral((g + h*x)/(a + b*log(c*(d*(e + f*x)**p)**q))**(3/2), x)
```

## Maxima [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{hx+g}{(b\log(((fx+e)^pd)^qc)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxima")
[Out] integrate((h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

## Giac [F]

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{hx+g}{(b\log(((fx+e)^pd)^qc)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac")
[Out] integrate((h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx=\int \frac{g+h\,x}{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^{3/2}}\,dx$$

```
[In] int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
[Out] int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

# 3.476 $\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 147

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \frac{2e^{-\frac{a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}fp^{3/2}q^{3/2}} - \frac{2(e+fx)}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$

```
[Out] 2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*P
i^(1/2)/b^(3/2)/exp(a/b/p/q)/f/p^(3/2)/q^(3/2)/((c*(d*(f*x+e)^p)^q)^(1/p/q)
)-2*(f*x+e)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2436, 2334, 2337, 2211, 2235, 2495}

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \frac{2\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}fp^{3/2}q^{3/2}} - \frac{2(e+fx)}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-3/2),x]
```

```
[Out] (2*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sq
rt[p]*Sqrt[q])])/(b^(3/2)*E^(a/(b*p*q))*f*p^(3/2)*q^(3/2)*(c*(d*(e + f*x)^p
```

```
)^q)^(1/(p*q))) - (2*(e + f*x))/(b*f*p*q*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q
]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cd^q x^{pq}))^{3/2}}\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^q x^{pq})}}\,dx, x, e+fx\right)}{bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{\left(2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{bfp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{\left(4(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\right)}{b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{2e^{-\frac{a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{3/2}fp^{3/2}q^{3/2}} - \frac{2(e+fx)}{bfpq\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.23

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = -\frac{2e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\left(e^{\frac{a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{\frac{1}{pq}}-\Gamma\left(\frac{1}{2},-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)\sqrt{-\frac{a+b\log(c(d(e+fx)}{bpq}}}{bfpq\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-3/2),x]
```

```
[Out] (-2*(e + f*x)*(E^(a/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(1/(p*q)) - Gamma[1/2, -
((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))]*Sqrt[-((a + b*Log[c*(d*(e + f*x
)^p)^q])/(b*p*q))]))/(b*E^(a/(b*p*q))*f*p*q*(c*(d*(e + f*x)^p)^q)^(1/(p*q))
*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])
```

## Maple [F]

$$\int \frac{1}{(a+b\ln(c(d(fx+e)^p)^q))^{\frac{3}{2}}}dx$$

```
[In] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

```
[Out] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(-3/2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{1}{(b\log(((fx+e)^pd)^qc)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxima")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(-3/2), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{1}{(b\log(((fx+e)^pd)^qc)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(-3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{1}{(a+b\ln(c(d(e+f\,x)^p)^q))^{3/2}}\,dx$$

```
[In] int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
[Out] int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

# 3.477 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx=\text{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx$$

```
[In] Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)),x]
[Out] Defer[Int][1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx$$

```
[In] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)),x]
[Out] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)), x]
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
[Out] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="fricas
")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 24.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^{\frac{3}{2}}(g+hx)}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**(3/2)*(g + h*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 10.75 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx = \int \frac{1}{(hx+g)\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxima
")
[Out] integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)), x)
```

## Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx = \int \frac{1}{(hx+g)\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac")
[Out] integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))^{3/2}}\,dx=\int \frac{1}{(g+h\,x)\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^{3/2}}\,dx$$

```
[In] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2)),x)
[Out] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2)), x)
```

# 3.478 $\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 514

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \frac{4e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx}{\sqrt{b}\sqrt{p}\sqrt{q}}\right.}{3b^{5/2}f^3p^{5/2}q^{5/2}}$$
$$+\frac{16e^{-\frac{2a}{bpq}}h(fg-eh)\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^3p^{5/2}q^{5/2}}$$
$$+\frac{4e^{-\frac{3a}{bpq}}h^2\sqrt{3\pi}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{5/2}f^3p^{5/2}q^{5/2}}$$
$$-\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}+\frac{8(fg-eh)(e+fx)(g+hx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$
$$-\frac{4(e+fx)(g+hx)^2}{b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

```
[Out] -2/3*(f*x+e)*(h*x+g)^2/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)+4/3*(-e*h+
f*g)^2*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/
2))*Pi^(1/2)/b^(5/2)/exp(a/b/p/q)/f^3/p^(5/2)/q^(5/2)/((c*(d*(f*x+e)^p)^q)^
(1/p/q))+16/3*h*(-e*h+f*g)*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q
))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/exp(2*a/b/p/q)/f
^3/p^(5/2)/q^(5/2)/((c*(d*(f*x+e)^p)^q)^(2/p/q))+4*h^2*(f*x+e)^3*erfi(3^(1/
2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*3^(1/2)*Pi^(1
/2)/b^(5/2)/exp(3*a/b/p/q)/f^3/p^(5/2)/q^(5/2)/((c*(d*(f*x+e)^p)^q)^(3/p/q)
)+8/3*(-e*h+f*g)*(f*x+e)*(h*x+g)/b^2/f^2/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q)
)^(1/2)-4*(f*x+e)*(h*x+g)^2/b^2/f/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)
```

## Rubi [A] (verified)

Time = 2.52 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2447, 2448, 2436, 2337, 2211, 2235, 2437, 2347, 2495}

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx = \frac{16\sqrt{2\pi}h(e+fx)^2e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log}}{\sqrt{b}}\right.}{3b^{5/2}f^3p^{5/2}q^{5/2}}$$
$$+\frac{4\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^3p^{5/2}q^{5/2}}$$
$$+\frac{4\sqrt{3\pi}h^2(e+fx)^3e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{5/2}f^3p^{5/2}q^{5/2}}$$
$$+\frac{8(e+fx)(g+hx)(fg-eh)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}-\frac{4(e+fx)(g+hx)^2}{b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$
$$-\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

```
[In] Int[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2),x]
```

```
[Out] (4*(f*g - e*h)^2*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q
]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(3*b^(5/2)*E^(a/(b*p*q))*f^3*p^(5/2)*q^(5/2)
*(c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (16*h*(f*g - e*h)*Sqrt[2*Pi]*(e + f*x)^
2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt
[q])])/(3*b^(5/2)*E^((2*a)/(b*p*q))*f^3*p^(5/2)*q^(5/2)*(c*(d*(e + f*x)^p)^
q)^(2/(p*q))) + (4*h^2*Sqrt[3*Pi]*(e + f*x)^3*Erfi[(Sqrt[3]*Sqrt[a + b*Log[
c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(b^(5/2)*E^((3*a)/(b*p*q
))*f^3*p^(5/2)*q^(5/2)*(c*(d*(e + f*x)^p)^q)^(3/(p*q))) - (2*(e + f*x)*(g +
 h*x)^2)/(3*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2)) + (8*(f*g - e*h
)*(e + f*x)*(g + h*x))/(3*b^2*f^2*p^2*q^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^
q]]) - (4*(e + f*x)*(g + h*x)^2)/(b^2*f*p^2*q^2*Sqrt[a + b*Log[c*(d*(e + f*
x)^p)^q]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
```

```
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(g+hx)^2}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{5/2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} + \text{Subst}\left(\frac{2\int \frac{(g+hx)^2}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}\,dx}{bpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(4(fg-eh))\int \frac{g+hx}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}\,dx}{3bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} + \frac{8(fg-eh)(e+fx)(g+hx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} - \frac{4(e+fx)(g+hx)^2}{b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{12\int \frac{(g+hx)^2}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\,dx}{b^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(16(fg-eh))\int \frac{g+hx}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\,dx}{3b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(8(fg-eh))\int \frac{g+hx}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\,dx}{b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(8(fg-eh)^2)\int \frac{1}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\,dx}{3b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2} + \frac{8(fg-eh)(e+fx)(g+hx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} - \frac{4(e+fx)(g+hx)^2}{b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$
$$+\text{Subst}\left(\frac{12\int\left(\frac{(fg-eh)^2}{f^2\sqrt{a+b\log(cd^q(e+fx)^{pq})}}+\frac{2h(fg-eh)(e+fx)}{f^2\sqrt{a+b\log(cd^q(e+fx)^{pq})}}+\frac{h^2(e+fx)^2}{f^2\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)dx}{b^2p^2q^2}, cd^q(e+fx\right.$$
$$-\text{Subst}\left(\frac{(16(fg-eh))\int\left(\frac{fg-eh}{f\sqrt{a+b\log(cd^q(e+fx)^{pq})}}+\frac{h(e+fx)}{f\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)dx}{3b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+f\right.$$
$$-\text{Subst}\left(\frac{(8(fg-eh))\int\left(\frac{fg-eh}{f\sqrt{a+b\log(cd^q(e+fx)^{pq})}}+\frac{h(e+fx)}{f\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)dx}{b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx\right.$$
$$+\text{Subst}\left(\frac{(8(fg-eh)^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}dx, x, e+fx\right)}{3b^2f^3p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

$$+\frac{8(fg-eh)(e+fx)(g+hx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}-\frac{4(e+fx)(g+hx)^2}{b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$$+\operatorname{Subst}\left(\frac{(12h^2)\int\frac{(e+fx)^2}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{b^2f^2p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$-\operatorname{Subst}\left(\frac{(16h(fg-eh))\int\frac{e+fx}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{3b^2f^2p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$-\operatorname{Subst}\left(\frac{(8h(fg-eh))\int\frac{e+fx}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{b^2f^2p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\operatorname{Subst}\left(\frac{(24h(fg-eh))\int\frac{e+fx}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{b^2f^2p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$-\operatorname{Subst}\left(\frac{(16(fg-eh)^2)\int\frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{3b^2f^2p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$-\operatorname{Subst}\left(\frac{(8(fg-eh)^2)\int\frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{b^2f^2p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\operatorname{Subst}\left(\frac{(12(fg-eh)^2)\int\frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\,dx}{b^2f^2p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\operatorname{Subst}\left(\frac{\left(8(fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{3b^2f^3p^3q^3},c\right.$$

$$= -\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2} } + \frac{8(fg-eh)(e+fx)(g+hx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} - \frac{4(e+fx)(g+hx)^2}{b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$
$$+\text{Subst}\left(\frac{(12h^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{b^2f^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\text{Subst}\left(\frac{(16h(fg-eh))\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{3b^2f^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\text{Subst}\left(\frac{(8h(fg-eh))\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{b^2f^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$+\text{Subst}\left(\frac{(24h(fg-eh))\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{b^2f^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\text{Subst}\left(\frac{(16(fg-eh)^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{3b^2f^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\text{Subst}\left(\frac{(8(fg-eh)^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{b^2f^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$+\text{Subst}\left(\frac{(12(fg-eh)^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{b^2f^3p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$+\text{Subst}\left(\frac{\left(16(fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx,x,\sqrt{a+b\log\left(cd^q(e+\right.}\right.}{3b^3f^3p^3q^3}\right.$$

$$= \frac{8e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^3p^{5/2}q^{5/2}}$$
$$-\frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} + \frac{8(fg-eh)(e+fx)(g+hx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} - \frac{4(e+fx)(g+hx)^2}{b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$
$$+\text{Subst}\left(\frac{\left(12h^2(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{b^2f^3p^3q^3},cd^q(e+\right.$$

$$= \frac{8e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^3p^{5/2}q^{5/2}}$$
$$- \frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$
$$+ \frac{8(fg-eh)(e+fx)(g+hx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} - \frac{4(e+fx)(g+hx)^2}{b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$
$$+\text{Subst}\left(\frac{\left(24h^2(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int e^{-\frac{3a}{bpq}+\frac{3x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+fx)^p\right.}\right.}{b^3f^3p^3q^3}\right.$$

$$= \frac{4e^{-\frac{a}{bpq}}(fg-eh)^2\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^3p^{5/2}q^{5/2}}$$
$$+ \frac{16e^{-\frac{2a}{bpq}}h(fg-eh)\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^3p^{5/2}q^{5/2}}$$
$$+ \frac{4e^{-\frac{3a}{bpq}}h^2\sqrt{3\pi}(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{5/2}f^3p^{5/2}q^{5/2}}$$
$$- \frac{2(e+fx)(g+hx)^2}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} + \frac{8(fg-eh)(e+fx)(g+hx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$
$$- \frac{4(e+fx)(g+hx)^2}{b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

## Mathematica [A] (verified)

Time = 3.73 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.27

$$\int \frac{(g+hx)^2}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx =$$
$$-\frac{2e^{-\frac{3a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\left(2ee^{\frac{2a}{bpq}}h(8fg+eh)\sqrt{\pi}(c(d(e+fx)^p)^q)^{\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)(c\right.}{}$$

```
[In] Integrate[(g + h*x)^2/(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2),x]
```

```
[Out] (-2*(e + f*x)*(2*e*E^((2*a)/(b*p*q))*h*(8*f*g + e*h)*Sqrt[Pi]*(c*(d*(e + f*
x)^p)^q)^(2/(p*q))*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[
p]*Sqrt[q])]*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2) + 8*E^(a/(b*p*q))*h*(-(
f*g) + e*h)*Sqrt[2*Pi]*(e + f*x)*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*Erfi[(Sqrt
[2]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*(a + b
*Log[c*(d*(e + f*x)^p)^q])^(3/2) - 6*h^2*Sqrt[3*Pi]*(e + f*x)^2*Erfi[(Sqrt[
```

```
3]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*(a + b*
Log[c*(d*(e + f*x)^p)^q])^(3/2) + Sqrt[b]*E^((2*a)/(b*p*q))*Sqrt[p]*Sqrt[q]
*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(2*b*(f^2*g^2 + 6*e*f*g*h + 2*e^2*h^2)*p*q
*Gamma[1/2, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))]*(-((a + b*Log[c*(d
*(e + f*x)^p)^q])/(b*p*q)))^(3/2) + E^(a/(b*p*q))*f*(c*(d*(e + f*x)^p)^q)^(
1/(p*q))*(g + h*x)*(b*f*p*q*(g + h*x) + 2*a*(f*g + 2*e*h + 3*f*h*x) + 2*b*(
2*e*h + f*(g + 3*h*x))*Log[c*(d*(e + f*x)^p)^q]))))/(3*b^(5/2)*E^((3*a)/(b*
p*q))*f^3*p^(5/2)*q^(5/2)*(c*(d*(e + f*x)^p)^q)^(3/(p*q))*(a + b*Log[c*(d*(
e + f*x)^p)^q])^(3/2))
```

## Maple [F]

$$\int \frac{(hx+g)^2}{(a+b\ln(c(d(fx+e)^p)^q))^{\frac{5}{2}}}dx$$

```
[In] int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

```
[Out] int((h*x+g)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((h*x+g)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**(5/2),x)
```

```
[Out] Integral((g + h*x)**2/(a + b*log(c*(d*(e + f*x)**p)**q))**(5/2), x)
```

## Maxima [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \int \frac{(hx+g)^2}{(b\log(((fx+e)^p d)^q c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="maxima
")
[Out] integrate((h*x + g)^2/(b*log(((f*x + e)^p*d)^q*c) + a)^(5/2), x)
```

## Giac [F]

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \int \frac{(hx+g)^2}{(b\log(((fx+e)^p d)^q c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="giac")
[Out] integrate((h*x + g)^2/(b*log(((f*x + e)^p*d)^q*c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \int \frac{(g+h\,x)^2}{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^{5/2}}\,dx$$

```
[In] int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2),x)
[Out] int((g + h*x)^2/(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2), x)
```

# 3.479 $\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 380

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \frac{4e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\,(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^2p^{5/2}q^{5/2}}$$
$$+\frac{8e^{-\frac{2a}{bpq}}h\sqrt{2\pi}(e+fx)^2\,(c(d(e+fx)^p)^q)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^2p^{5/2}q^{5/2}}$$
$$-\frac{2(e+fx)(g+hx)}{3bfpq\,(a+b\log(c(d(e+fx)^p)^q))^{3/2}}+\frac{4(fg-eh)(e+fx)}{3b^2f^2p^2q^2\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$
$$-\frac{8(e+fx)(g+hx)}{3b^2fp^2q^2\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$

```
[Out] -2/3*(f*x+e)*(h*x+g)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)+4/3*(-e*h+f*
g)*(f*x+e)*erfi((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*
Pi^(1/2)/b^(5/2)/exp(a/b/p/q)/f^2/p^(5/2)/q^(5/2)/((c*(d*(f*x+e)^p)^q)^(1/p
/q))+8/3*h*(f*x+e)^2*erfi(2^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)
/p^(1/2)/q^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/exp(2*a/b/p/q)/f^2/p^(5/2)/q^(5/
2)/((c*(d*(f*x+e)^p)^q)^(2/p/q))+4/3*(-e*h+f*g)*(f*x+e)/b^2/f^2/p^2/q^2/(a+
b*ln(c*(d*(f*x+e)^p)^q))^(1/2)-8/3*(f*x+e)*(h*x+g)/b^2/f/p^2/q^2/(a+b*ln(c*
(d*(f*x+e)^p)^q))^(1/2)
```

## Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2447, 2448, 2436, 2337, 2211, 2235, 2437, 2347, 2334, 2495}

$$\int \frac{g+hx}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \frac{4\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+f}{\sqrt{b}\sqrt{p}\sqrt{q}}\right.}{3b^{5/2}f^2p^{5/2}q^{5/2}}$$
$$+\frac{8\sqrt{2\pi}h(e+fx)^2e^{-\frac{2a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^2p^{5/2}q^{5/2}}$$
$$+\frac{4(e+fx)(fg-eh)}{3b^2f^2p^2q^2\sqrt{a+b\log(c(d(e+fx)^p)^q)}}-\frac{8(e+fx)(g+hx)}{3b^2fp^2q^2\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$
$$-\frac{2(e+fx)(g+hx)}{3bfpq(a+b\log(c(d(e+fx)^p)^q))^{3/2}}$$

```
[In] Int[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2),x]
```

```
[Out] (4*(f*g - e*h)*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]
/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(3*b^(5/2)*E^(a/(b*p*q))*f^2*p^(5/2)*q^(5/2)*(
c*(d*(e + f*x)^p)^q)^(1/(p*q))) + (8*h*Sqrt[2*Pi]*(e + f*x)^2*Erfi[(Sqrt[2]
*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])])/(3*b^(5/
2)*E^((2*a)/(b*p*q))*f^2*p^(5/2)*q^(5/2)*(c*(d*(e + f*x)^p)^q)^(2/(p*q))) -
 (2*(e + f*x)*(g + h*x))/(3*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))
 + (4*(f*g - e*h)*(e + f*x))/(3*b^2*f^2*p^2*q^2*Sqrt[a + b*Log[c*(d*(e + f*
x)^p)^q]]) - (8*(e + f*x)*(g + h*x))/(3*b^2*f*p^2*q^2*Sqrt[a + b*Log[c*(d*(
e + f*x)^p)^q]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
```

```
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{g+hx}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{5/2}}\,dx, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&= -\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\\
&\quad+\text{Subst}\left(\frac{4\int \frac{g+hx}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}\,dx}{3bpq}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&\quad-\text{Subst}\left(\frac{(2(fg-eh))\int \frac{1}{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^{3/2}}\,dx}{3bfpq}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&= -\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}-\frac{8(e+fx)(g+hx)}{3b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\\
&\quad+\text{Subst}\left(\frac{16\int \frac{g+hx}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\,dx}{3b^2p^2q^2}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&\quad-\text{Subst}\left(\frac{(8(fg-eh))\int \frac{1}{\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}}\,dx}{3b^2fp^2q^2}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&\quad-\text{Subst}\left(\frac{(2(fg-eh))\text{Subst}\left(\int \frac{1}{\left(a+b\log\left(cd^qx^{pq}\right)\right)^{3/2}}\,dx, x, e+fx\right)}{3bf^2pq}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)
\end{aligned}
$$

$$= -\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} + \frac{4(fg-eh)(e+fx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} - \frac{8(e+fx)(g+hx)}{3b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{16\int\left(\frac{fg-eh}{f\sqrt{a+b\log(cd^q(e+fx)^{pq})}}+\frac{h(e+fx)}{f\sqrt{a+b\log(cd^q(e+fx)^{pq})}}\right)dx}{3b^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(4(fg-eh))\text{Subst}\left(\int\frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}dx, x, e+fx\right)}{3b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(8(fg-eh))\text{Subst}\left(\int\frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}dx, x, e+fx\right)}{3b^2f^2p^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} + \frac{4(fg-eh)(e+fx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} - \frac{8(e+fx)(g+hx)}{3b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{(16h)\int\frac{e+fx}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}dx}{3b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(16(fg-eh))\int\frac{1}{\sqrt{a+b\log(cd^q(e+fx)^{pq})}}dx}{3b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{\left(4(fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{3b^2f^2p^3q^3}, cd^q\right. - \text{Subst}\left(\frac{\left(8(fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{3b^2f^2p^3q^3}, cd^q\right.$$

$$= -\frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} + \frac{4(fg-eh)(e+fx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$$- \frac{8(e+fx)(g+hx)}{3b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$$+\text{Subst}\left(\frac{(16h)\text{Subst}\left(\int \frac{x}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{3b^2f^2p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{(16(fg-eh))\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^qx^{pq})}}\,dx,x,e+fx\right)}{3b^2f^2p^2q^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$-\text{Subst}\left(\frac{\left(8(fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx,x,\sqrt{a+b\log\left(cd^q(e+\right.}\right.}{3b^3f^2p^3q^3}\right.$$

$$-\text{Subst}\left(\frac{\left(16(fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx,x,\sqrt{a+b\log\left(cd^q(e\right.}\right.}{3b^3f^2p^3q^3}\right.$$

$$= -\frac{4e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{5/2}f^2p^{5/2}q^{5/2}}$$

$$- \frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} + \frac{4(fg-eh)(e+fx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$$- \frac{8(e+fx)(g+hx)}{3b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$$+\text{Subst}\left(\frac{\left(16h(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{pq}}}{\sqrt{a+bx}}\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{3b^2f^2p^3q^3},cd^q(e+\right.$$

$$= -\frac{4e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{b^{5/2}f^2p^{5/2}q^{5/2}}$$

$$- \frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} + \frac{4(fg-eh)(e+fx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$$- \frac{8(e+fx)(g+hx)}{3b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

$$+\text{Subst}\left(\frac{\left(32h(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int e^{-\frac{2a}{bpq}+\frac{2x^2}{bpq}}\,dx,x,\sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right.}\right.}{3b^3f^2p^3q^3}\right.$$

$$= \frac{4e^{-\frac{a}{bpq}}(fg-eh)\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^2p^{5/2}q^{5/2}} + \frac{8e^{-\frac{2a}{bpq}}h\sqrt{2\pi}(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}f^2p^{5/2}q^{5/2}} - \frac{2(e+fx)(g+hx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} + \frac{4(fg-eh)(e+fx)}{3b^2f^2p^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} - \frac{8(e+fx)(g+hx)}{3b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

## Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.29

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx = -\frac{2e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(8ee^{\frac{a}{bpq}}h\sqrt{\pi}\left(c(d(e+fx)^p)^q\right)^{\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)\left(a+b\log\left(c(d\right.\right.\right.}{}$$

```
[In] Integrate[(g + h*x)/(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2),x]
```

```
[Out] (-2*(e + f*x)*(8*e*E^(a/(b*p*q))*h*Sqrt[Pi]*(c*(d*(e + f*x)^p)^q)^(1/(p*q))
*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*(a +
b*Log[c*(d*(e + f*x)^p)^q])^(3/2) - 4*h*Sqrt[2*Pi]*(e + f*x)*Erfi[(Sqrt[2]*
Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]])/(Sqrt[b]*Sqrt[p]*Sqrt[q])]*(a + b*Log
[c*(d*(e + f*x)^p)^q])^(3/2) + Sqrt[b]*E^(a/(b*p*q))*Sqrt[p]*Sqrt[q]*(c*(d*
(e + f*x)^p)^q)^(1/(p*q))*(2*b*(f*g + 3*e*h)*p*q*Gamma[1/2, -((a + b*Log[c*
(d*(e + f*x)^p)^q])/(b*p*q))]*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))
^(3/2) + E^(a/(b*p*q))*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*(b*f*p*q*(g + h*x) +
 2*a*(f*g + e*h + 2*f*h*x) + 2*b*(e*h + f*(g + 2*h*x))*Log[c*(d*(e + f*x)^p
)^q]))))/(3*b^(5/2)*E^((2*a)/(b*p*q))*f^2*p^(5/2)*q^(5/2)*(c*(d*(e + f*x)^p
)^q)^(2/(p*q))*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2))
```

## Maple [F]

$$\int \frac{hx+g}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

```
[Out] int((h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{5/2}}\,dx = \int \frac{g+hx}{\left(a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(5/2),x)
[Out] Integral((g + h*x)/(a + b*log(c*(d*(e + f*x)**p)**q))**(5/2), x)
```

## Maxima [F]

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{5/2}}\,dx = \int \frac{hx+g}{\left(b\log\left(\left((fx+e)^{p}d\right)^{q}c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="maxima")
[Out] integrate((h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a)^(5/2), x)
```

## Giac [F]

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{5/2}}\,dx = \int \frac{hx+g}{\left(b\log\left(\left((fx+e)^{p}d\right)^{q}c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="giac")
[Out] integrate((h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{g+h\,x}{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^{5/2}}\,dx$$

```
[In] int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2),x)
[Out] int((g + h*x)/(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2), x)
```

# 3.480 $\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \frac{4e^{-\frac{a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}fp^{5/2}q^{5/2}} - \frac{2(e+fx)}{3bfpq(a+b\log(c(d(e+fx)^p)^q))^{3/2}} - \frac{4(e+fx)}{3b^2fp^2q^2\sqrt{a+b\log(c(d(e+fx)^p)^q)}}$$

```
[Out] -2/3*(f*x+e)/b/f/p/q/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2)+4/3*(f*x+e)*erfi((a+
b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/b^(1/2)/p^(1/2)/q^(1/2))*Pi^(1/2)/b^(5/2)/ex
p(a/b/p/q)/f/p^(5/2)/q^(5/2)/((c*(d*(f*x+e)^p)^q)^(1/p/q))-4/3*(f*x+e)/b^2/
f/p^2/q^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2436, 2334, 2337, 2211, 2235, 2495}

$$\int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx = \frac{4\sqrt{\pi}(e+fx)e^{-\frac{a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}fp^{5/2}q^{5/2}} - \frac{4(e+fx)}{3b^2fp^2q^2\sqrt{a+b\log(c(d(e+fx)^p)^q)}} - \frac{2(e+fx)}{3bfpq(a+b\log(c(d(e+fx)^p)^q))^{3/2}}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-5/2),x]
```

```
[Out] (4*Sqrt[Pi]*(e + f*x)*Erfi[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(Sqrt[b]*Sq
rt[p]*Sqrt[q])])/(3*b^(5/2)*E^(a/(b*p*q))*f*p^(5/2)*q^(5/2)*(c*(d*(e + f*x)
```

```
^p)^q)^(1/(p*q))) - (2*(e + f*x))/(3*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q
])^(3/2)) - (4*(e + f*x))/(3*b^2*f*p^2*q^2*Sqrt[a + b*Log[c*(d*(e + f*x)^p)
^q]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(a+b\log(cd^q(e+fx)^{pq}))^{5/2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{(a+b\log(cd^q x^{pq}))^{5/2}}\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} + \text{Subst}\left(\frac{2\text{Subst}\left(\int \frac{1}{(a+b\log(cd^q x^{pq}))^{3/2}}\,dx, x, e+fx\right)}{3bfpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} - \frac{4(e+fx)}{3b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{4\text{Subst}\left(\int \frac{1}{\sqrt{a+b\log(cd^q x^{pq})}}\,dx, x, e+fx\right)}{3b^2fp^2q^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(e+fx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} - \frac{4(e+fx)}{3b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{\left(4(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{pq}}}{\sqrt{a+bx}}\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{3b^2fp^3q^3}, cd^q(e+fx)^p$$

$$= -\frac{2(e+fx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} - \frac{4(e+fx)}{3b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}} + \text{Subst}\left(\frac{\left(8(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{-\frac{a}{bpq}+\frac{x^2}{bpq}}\,dx, x, \sqrt{a+b\log\left(cd^q(e+fx)^{pq}\right)}\right)}{3b^3fp^3q^3},\right.$$

$$= \frac{4e^{-\frac{a}{bpq}}\sqrt{\pi}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\text{erfi}\left(\frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{b}\sqrt{p}\sqrt{q}}\right)}{3b^{5/2}fp^{5/2}q^{5/2}} - \frac{2(e+fx)}{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}} - \frac{4(e+fx)}{3b^2fp^2q^2\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}$$

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx =$$

$$-\frac{2e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\left(2bpq\Gamma\left(\frac{1}{2}, -\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)\left(-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)^{3/2} + e^{\frac{a}{bpq}}(c(d(e\right.}{3b^2fp^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^(-5/2),x]
```

```
[Out] (-2*(e + f*x)*(2*b*p*q*Gamma[1/2, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q
))]*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^(3/2) + E^(a/(b*p*q))*(c*
(d*(e + f*x)^p)^q)^(1/(p*q))*(2*a + b*p*q + 2*b*Log[c*(d*(e + f*x)^p)^q])))
/(3*b^2*E^(a/(b*p*q))*f*p^2*q^2*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*(a + b*Log[
c*(d*(e + f*x)^p)^q])^(3/2))
```

## Maple [F]

$$\int \frac{1}{\left(a+b\ln \left(c \left(d \left(f x+e\right)^{p}\right)^{q}\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

```
[Out] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\log \left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{\left(a+b\log \left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{1}{\left(a+b\log \left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q))**(5/2),x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**(-5/2), x)
```

## Maxima [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="maxima")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(-5/2), x)
```

## Giac [F]

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(-5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{1}{\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^{5/2}}\,dx$$

```
[In] int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2),x)
[Out] int(1/(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2), x)
```

# 3.481 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^{5/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\text{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx$$

```
[In] Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2)),x]
[Out] Defer[Int][1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx$$

```
[In] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2)),x]
[Out] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^(5/2)), x]
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
[Out] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="fricas
")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(5/2),x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 12.52 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{1}{(hx+g)\left(b\log\left(\left((fx+e)^pd\right)^qc\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="maxima
")
[Out] integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^(5/2)), x)
```

## Giac [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{1}{(hx+g)\left(b\log\left(\left((fx+e)^pd\right)^qc\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^(5/2),x, algorithm="giac")
[Out] integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{5/2}}\,dx=\int \frac{1}{(g+h\,x)\,\left(a+b\,\ln\left(c\,{\left(d\,{(e+f\,x)}^p\right)}^q\right)\right)^{5/2}}\,dx$$

```
[In] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2)),x)
[Out] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^(5/2)), x)
```

# 3.482 $\int (g+hx)^{3/2} (a+b\log (c(d(e+fx)^p)^q)) \, dx$



## Optimal result

Integrand size = 28, antiderivative size = 171

$$\int (g+hx)^{3/2} (a+b\log (c(d(e+fx)^p)^q)) \, dx = -\frac{4b(fg-eh)^2pq\sqrt{g+hx}}{5f^2h} - \frac{4b(fg-eh)pq(g+hx)^{3/2}}{15fh} - \frac{4bpq(g+hx)^{5/2}}{25h} + \frac{4b(fg-eh)^{5/2}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{5f^{5/2}h} + \frac{2(g+hx)^{5/2}(a+b\log (c(d(e+fx)^p)^q))}{5h}$$

```
[Out] -4/15*b*(-e*h+f*g)*p*q*(h*x+g)^(3/2)/f/h-4/25*b*p*q*(h*x+g)^(5/2)/h+4/5*b*(
-e*h+f*g)^(5/2)*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/f^(5/2)
/h+2/5*(h*x+g)^(5/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))/h-4/5*b*(-e*h+f*g)^2*p*q*(
h*x+g)^(1/2)/f^2/h
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2442, 52, 65, 214, 2495}

$$\int (g+hx)^{3/2} (a+b\log (c(d(e+fx)^p)^q)) \, dx = \frac{2(g+hx)^{5/2}(a+b\log (c(d(e+fx)^p)^q))}{5h} + \frac{4bpq(fg-eh)^{5/2}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{5f^{5/2}h} - \frac{4bpq\sqrt{g+hx}(fg-eh)^2}{5f^2h} - \frac{4bpq(g+hx)^{3/2}(fg-eh)}{15fh} - \frac{4bpq(g+hx)^{5/2}}{25h}$$

```
[In] Int[(g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] (-4*b*(f*g - e*h)^2*p*q*Sqrt[g + h*x])/(5*f^2*h) - (4*b*(f*g - e*h)*p*q*(g
+ h*x)^(3/2))/(15*f*h) - (4*b*p*q*(g + h*x)^(5/2))/(25*h) + (4*b*(f*g - e*h
)^(5/2)*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/(5*f^(5/2)*h)
 + (2*(g + h*x)^(5/2)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(5*h)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (g+hx)^{3/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h} - \text{Subst}\left(\frac{(2bfpq)\int \frac{(g+hx)^{5/2}}{e+fx}\,dx}{5h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{4bpq(g+hx)^{5/2}}{25h} + \frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h} - \text{Subst}\left(\frac{(2b(fg-eh)pq)\int \frac{(g+hx)^{3/2}}{e+fx}\,dx}{5h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{4b(fg-eh)pq(g+hx)^{3/2}}{15fh} - \frac{4bpq(g+hx)^{5/2}}{25h} + \frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h} - \text{Subst}\left(\frac{(2b(fg-eh)^2pq)\int \frac{\sqrt{g+hx}}{e+fx}\,dx}{5fh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{4b(fg-eh)^2pq\sqrt{g+hx}}{5f^2h} - \frac{4b(fg-eh)pq(g+hx)^{3/2}}{15fh} - \frac{4bpq(g+hx)^{5/2}}{25h} + \frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h} - \text{Subst}\left(\frac{(2b(fg-eh)^3pq)\int \frac{1}{(e+fx)\sqrt{g+hx}}\,dx}{5f^2h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{4b(fg-eh)^2pq\sqrt{g+hx}}{5f^2h} - \frac{4b(fg-eh)pq(g+hx)^{3/2}}{15fh} - \frac{4bpq(g+hx)^{5/2}}{25h} + \frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h} - \text{Subst}\left(\frac{(4b(fg-eh)^3pq)\,\text{Subst}\left(\int \frac{1}{e-\frac{fg}{h}+\frac{fx^2}{h}}\,dx, x, \sqrt{g+hx}\right)}{5f^2h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{4b(fg-eh)^2pq\sqrt{g+hx}}{5f^2h} - \frac{4b(fg-eh)pq(g+hx)^{3/2}}{15fh} - \frac{4bpq(g+hx)^{5/2}}{25h} + \frac{4b(fg-eh)^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{5f^{5/2}h} + \frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h}$$

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int (g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))\,dx=\frac{2\left(\frac{1}{5}a(g+hx)^{5/2}-\frac{2}{75}bpq\left(3(g+hx)^{5/2}+\frac{5(fg-eh)\left(\sqrt{f}\sqrt{g+hx}(4fg-3eh+fhx)-3(fg-eh)^{3/2}\dots\right.}{f^{5/2}}\right.\right.}{h}$$

[In] Integrate[(g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]

[Out] (2*((a*(g + h*x)^(5/2))/5 - (2*b*p*q*(3*(g + h*x)^(5/2) + (5*(f*g - e*h)*(Sqrt[f]*Sqrt[g + h*x]*(4*f*g - 3*e*h + f*h*x) - 3*(f*g - e*h)^(3/2)*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]))/f^(5/2)))/75 + (b*(g + h*x)^(5/2)*Log[c*(d*(e + f*x)^p)^q])/5))/h

## Maple [F]

$$\int (hx+g)^{\frac{3}{2}}(a+b\ln(c(d(fx+e)^p)^q))\,dx$$

[In] int((h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q)),x)

[Out] int((h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q)),x)

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(143) = 286.

Time = 0.34 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.65

$$\int (g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))\,dx=\left[\frac{2\left(15(bf^2g^2-2befgh+be^2h^2)pq\sqrt{\frac{fg-eh}{f}}\log\left(\frac{fhx+2fg-eh+2\sqrt{hx+g}f\sqrt{\frac{fg-eh}{f}}}{fx+e}\right)+\dots\right.}{\dots}\right.$$

[In] integrate((h*x+g)^(3/2)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")

[Out] [2/75*(15*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*p*q*sqrt((f*g - e*h)/f)*log((f*h*x + 2*f*g - e*h + 2*sqrt(h*x + g)*f*sqrt((f*g - e*h)/f))/(f*x + e)) + (15*a*f^2*g^2 - 2*(23*b*f^2*g^2 - 35*b*e*f*g*h + 15*b*e^2*h^2)*p*q - 3*(2*b*f^2*h^2*p*q - 5*a*f^2*h^2)*x^2 + 2*(15*a*f^2*g*h - (11*b*f^2*g*h - 5*b*e*f*h^2)*p*q)*x + 15*(b*f^2*h^2*p*q*x^2 + 2*b*f^2*g*h*p*q*x + b*f^2*g^2*p*q)*

```
log(f*x + e) + 15*(b*f^2*h^2*x^2 + 2*b*f^2*g*h*x + b*f^2*g^2)*log(c) + 15*(
b*f^2*h^2*q*x^2 + 2*b*f^2*g*h*q*x + b*f^2*g^2*q)*log(d))*sqrt(h*x + g))/(f^
2*h), 2/75*(30*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*p*q*sqrt(-(f*g - e*h)/
f)*arctan(-sqrt(h*x + g)*f*sqrt(-(f*g - e*h)/f)/(f*g - e*h)) + (15*a*f^2*g^
2 - 2*(23*b*f^2*g^2 - 35*b*e*f*g*h + 15*b*e^2*h^2)*p*q - 3*(2*b*f^2*h^2*p*q
 - 5*a*f^2*h^2)*x^2 + 2*(15*a*f^2*g*h - (11*b*f^2*g*h - 5*b*e*f*h^2)*p*q)*x
 + 15*(b*f^2*h^2*p*q*x^2 + 2*b*f^2*g*h*p*q*x + b*f^2*g^2*p*q)*log(f*x + e)
+ 15*(b*f^2*h^2*x^2 + 2*b*f^2*g*h*x + b*f^2*g^2)*log(c) + 15*(b*f^2*h^2*q*x
^2 + 2*b*f^2*g*h*q*x + b*f^2*g^2*q)*log(d))*sqrt(h*x + g))/(f^2*h)]
```

## Sympy [F]

$$\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx=\int \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)(g+hx)^{\frac{3}{2}}\,dx$$

```
[In] integrate((h*x+g)**(3/2)*(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)^(3/2)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx=\int (hx+g)^{\frac{3}{2}}\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)dx$$

```
[In] integrate((h*x+g)^(3/2)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
[Out] integrate((h*x + g)^(3/2)*(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx=\int (g+h\,x)^{3/2}\left(a+b\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)\right)\,dx$$

```
[In] int((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q)),x)
[Out] int((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

# 3.483 $\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))\,dx = -\frac{4b(fg-eh)pq\sqrt{g+hx}}{3fh}-\frac{4bpq(g+hx)^{3/2}}{9h}+\frac{4b(fg-eh)^{3/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3f^{3/2}h}+\frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{3h}$$

```
[Out] -4/9*b*p*q*(h*x+g)^(3/2)/h+4/3*b*(-e*h+f*g)^(3/2)*p*q*arctanh(f^(1/2)*(h*x+
g)^(1/2)/(-e*h+f*g)^(1/2))/f^(3/2)/h+2/3*(h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)
^p)^q))/h-4/3*b*(-e*h+f*g)*p*q*(h*x+g)^(1/2)/f/h
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2442, 52, 65, 214, 2495}

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))\,dx = \frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{3h}+\frac{4bpq(fg-eh)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3f^{3/2}h}-\frac{4bpq\sqrt{g+hx}(fg-eh)}{3fh}-\frac{4bpq(g+hx)^{3/2}}{9h}$$

```
[In] Int[Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] (-4*b*(f*g - e*h)*p*q*Sqrt[g + h*x])/(3*f*h) - (4*b*p*q*(g + h*x)^(3/2))/(9
*h) + (4*b*(f*g - e*h)^(3/2)*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g -
 e*h]])/(3*f^(3/2)*h) + (2*(g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])
)/(3*h)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \sqrt{g+hx}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\,dx, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$
\begin{aligned}
&= \frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h} \\
&\quad - \text{Subst}\left(\frac{(2bfpq)\int \frac{(g+hx)^{3/2}}{e+fx}\,dx}{3h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) \\
&= -\frac{4bpq(g+hx)^{3/2}}{9h} + \frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h} \\
&\quad - \text{Subst}\left(\frac{(2b(fg-eh)pq)\int \frac{\sqrt{g+hx}}{e+fx}\,dx}{3h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) \\
&= -\frac{4b(fg-eh)pq\sqrt{g+hx}}{3fh} - \frac{4bpq(g+hx)^{3/2}}{9h} + \frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h} \\
&\quad - \text{Subst}\left(\frac{(2b(fg-eh)^2pq)\int \frac{1}{(e+fx)\sqrt{g+hx}}\,dx}{3fh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) \\
&= -\frac{4b(fg-eh)pq\sqrt{g+hx}}{3fh} - \frac{4bpq(g+hx)^{3/2}}{9h} + \frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h} \\
&\quad - \text{Subst}\left(\frac{(4b(fg-eh)^2pq)\,\text{Subst}\left(\int \frac{1}{e-\frac{fg}{h}+\frac{fx^2}{h}}\,dx, x, \sqrt{g+hx}\right)}{3fh^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) \\
&= -\frac{4b(fg-eh)pq\sqrt{g+hx}}{3fh} - \frac{4bpq(g+hx)^{3/2}}{9h} \\
&\quad + \frac{4b(fg-eh)^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3f^{3/2}h} + \frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$
\int \sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\,dx
$$

$$
= \frac{2\left(6b(fg-eh)^{3/2}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)+\sqrt{f}\sqrt{g+hx}(3af(g+hx)-2bpq(4fg-3eh+fhx)+3bf(g+\right.}{9f^{3/2}h}
$$

```
[In] Integrate[Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] (2*(6*b*(f*g - e*h)^(3/2)*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*
h]] + Sqrt[f]*Sqrt[g + h*x]*(3*a*f*(g + h*x) - 2*b*p*q*(4*f*g - 3*e*h + f*h
*x) + 3*b*f*(g + h*x)*Log[c*(d*(e + f*x)^p)^q])))/(9*f^(3/2)*h)
```

## Maple [F]

$$\int \sqrt{hx+g}\,(a+b\ln(c(d(fx+e)^p)^q))\,dx$$

```
[In] int((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

```
[Out] int((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.54

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))\,dx$$
$$= \left[-\frac{2\left(3(bfg-beh)pq\sqrt{\frac{fg-eh}{f}}\log\left(\frac{fhx+2fg-eh-2\sqrt{hx+g}f\sqrt{\frac{fg-eh}{f}}}{fx+e}\right)-(3afg-2(4bfg-3beh)pq-(2bfhpq\right.}{}\right.$$

```
[In] integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

```
[Out] [-2/9*(3*(b*f*g - b*e*h)*p*q*sqrt((f*g - e*h)/f)*log((f*h*x + 2*f*g - e*h -
 2*sqrt(h*x + g)*f*sqrt((f*g - e*h)/f))/(f*x + e)) - (3*a*f*g - 2*(4*b*f*g
- 3*b*e*h)*p*q - (2*b*f*h*p*q - 3*a*f*h)*x + 3*(b*f*h*p*q*x + b*f*g*p*q)*lo
g(f*x + e) + 3*(b*f*h*x + b*f*g)*log(c) + 3*(b*f*h*q*x + b*f*g*q)*log(d))*s
qrt(h*x + g))/(f*h), 2/9*(6*(b*f*g - b*e*h)*p*q*sqrt(-(f*g - e*h)/f)*arctan
(-sqrt(h*x + g)*f*sqrt(-(f*g - e*h)/f)/(f*g - e*h)) + (3*a*f*g - 2*(4*b*f*g
 - 3*b*e*h)*p*q - (2*b*f*h*p*q - 3*a*f*h)*x + 3*(b*f*h*p*q*x + b*f*g*p*q)*l
og(f*x + e) + 3*(b*f*h*x + b*f*g)*log(c) + 3*(b*f*h*q*x + b*f*g*q)*log(d))*
sqrt(h*x + g))/(f*h)]
```

## Sympy [F]

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))\,dx = \int (a+b\log(c(d(e+fx)^p)^q))\sqrt{g+hx}\,dx$$

```
[In] integrate((h*x+g)**(1/2)*(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))*sqrt(g + h*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \sqrt{g+hx}(a+b\log\left(c(d(e+fx)^p)^q\right))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int \sqrt{g+hx}(a+b\log\left(c(d(e+fx)^p)^q\right))\,dx = \int \sqrt{hx+g}(b\log\left(((fx+e)^p d)^q c\right)+a)\,dx$$

```
[In] integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{g+hx}(a+b\log\left(c(d(e+fx)^p)^q\right))\,dx = \int \sqrt{g+h\,x}\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)\,dx$$

```
[In] int((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q)),x)
```

```
[Out] int((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

# 3.484 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g+hx}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 103

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{g+hx}}\,dx=-\frac{4bpq\sqrt{g+hx}}{h}+\frac{4b\sqrt{fg-eh}\,pq\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}h}+\frac{2\sqrt{g+hx}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{h}$$

```
[Out] 4*b*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*(-e*h+f*g)^(1/2)/h/
f^(1/2)-4*b*p*q*(h*x+g)^(1/2)/h+2*(a+b*ln(c*(d*(f*x+e)^p)^q))*(h*x+g)^(1/2)
/h
```

## Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2442, 52, 65, 214, 2495}

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{g+hx}}\,dx=\frac{2\sqrt{g+hx}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{h}+\frac{4bpq\sqrt{fg-eh}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}h}-\frac{4bpq\sqrt{g+hx}}{h}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[g + h*x],x]
```

```
[Out] (-4*b*p*q*Sqrt[g + h*x])/h + (4*b*Sqrt[f*g - e*h]*p*q*ArcTanh[(Sqrt[f]*Sqrt
[g + h*x])/Sqrt[f*g - e*h]])/(Sqrt[f]*h) + (2*Sqrt[g + h*x]*(a + b*Log[c*(d
*(e + f*x)^p)^q]))/h
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{\sqrt{g+hx}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{2\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}\\
&\quad - \text{Subst}\left(\frac{(2bfpq)\int \frac{\sqrt{g+hx}}{e+fx}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$= -\frac{4bpq\sqrt{g+hx}}{h} + \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h} - \text{Subst}\left(\frac{(2b(fg-eh)pq)\int \frac{1}{(e+fx)\sqrt{g+hx}}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{4bpq\sqrt{g+hx}}{h} + \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h} - \text{Subst}\left(\frac{(4b(fg-eh)pq)\text{Subst}\left(\int \frac{1}{e-\frac{fg}{h}+\frac{fx^2}{h}}\,dx, x, \sqrt{g+hx}\right)}{h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{4bpq\sqrt{g+hx}}{h} + \frac{4b\sqrt{fg-eh}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}h} + \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h}$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g+hx}}\,dx = \frac{2\left(\frac{2b\sqrt{fg-eh}pq\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}} + \sqrt{g+hx}(a-2bpq+b\log(c(d(e+fx)^p)^q))\right)}{h}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[g + h*x],x]
[Out] (2*((2*b*Sqrt[f*g - e*h]*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h
]])/Sqrt[f] + Sqrt[g + h*x]*(a - 2*b*p*q + b*Log[c*(d*(e + f*x)^p)^q])))/h
```

## Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2\sqrt{hx+g}\,a+2b\left(\ln\left(c\left(d\left(\frac{f(hx+g)+eh-fg}{h}\right)^{p}\right)^{q}\right)\sqrt{hx+g}-2qpf\left(\frac{\sqrt{hx+g}}{f}+\frac{(-eh+fg)\arctan\left(\frac{f\sqrt{hx+g}}{\sqrt{(eh-fg)f}}\right)}{f\sqrt{(eh-fg)f}}\right)\right)}{h}$ | 118 |
| default | $\frac{2\sqrt{hx+g}\,a+2b\left(\ln\left(c\left(d\left(\frac{f(hx+g)+eh-fg}{h}\right)^{p}\right)^{q}\right)\sqrt{hx+g}-2qpf\left(\frac{\sqrt{hx+g}}{f}+\frac{(-eh+fg)\arctan\left(\frac{f\sqrt{hx+g}}{\sqrt{(eh-fg)f}}\right)}{f\sqrt{(eh-fg)f}}\right)\right)}{h}$ | 118 |
| parts | $\frac{2a\sqrt{hx+g}}{h}+\frac{2b\left(\ln\left(c\left(d\left(\frac{f(hx+g)+eh-fg}{h}\right)^{p}\right)^{q}\right)\sqrt{hx+g}-2qpf\left(\frac{\sqrt{hx+g}}{f}+\frac{(-eh+fg)\arctan\left(\frac{f\sqrt{hx+g}}{\sqrt{(eh-fg)f}}\right)}{f\sqrt{(eh-fg)f}}\right)\right)}{h}$ | 121 |

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/h*((h*x+g)^(1/2)*a+b*(ln(c*(d*((f*(h*x+g)+e*h-f*g)/h)^p)^q)*(h*x+g)^(1/2)
-2*q*p*f*((h*x+g)^(1/2)/f+(-e*h+f*g)/f/((e*h-f*g)*f)^(1/2)*arctan(f*(h*x+g)
^(1/2)/((e*h-f*g)*f)^(1/2)))))
```

## Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.95

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx$$
$$=\left[\frac{2\left(bpq\sqrt{\frac{fg-eh}{f}}\log\left(\frac{fhx+2\,fg-eh+2\,\sqrt{hx+g}f\sqrt{\frac{fg-eh}{f}}}{fx+e}\right)+(bpq\log\left(fx+e\right)-2\,bpq+bq\log\left(d\right)+b\log\left(c\right)+a)\right.}{h}\right.$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*(b*p*q*sqrt((f*g - e*h)/f)*log((f*h*x + 2*f*g - e*h + 2*sqrt(h*x + g)*f*
sqrt((f*g - e*h)/f))/(f*x + e)) + (b*p*q*log(f*x + e) - 2*b*p*q + b*q*log(d
) + b*log(c) + a)*sqrt(h*x + g))/h, 2*(2*b*p*q*sqrt(-(f*g - e*h)/f)*arctan(
-sqrt(h*x + g)*f*sqrt(-(f*g - e*h)/f)/(f*g - e*h)) + (b*p*q*log(f*x + e) -
2*b*p*q + b*q*log(d) + b*log(c) + a)*sqrt(h*x + g))/h]
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx = \int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**(1/2),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))/sqrt(g + h*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx = -\frac{2\Bigg(\Bigg(2f\Bigg(\frac{(fg-eh)\arctan\left(\frac{\sqrt{hx+g}f}{\sqrt{-f^2g+efh}}\right)}{\sqrt{-f^2g+efh}f}+\frac{\sqrt{hx+g}}{f}\Bigg)-\sqrt{hx+g}\log(fx+e)\Bigg)bpq-\sqrt{hx+g}bq\log(d)-\sqrt{hx}}{h}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x, algorithm="giac")
[Out] -2*((2*f*((f*g - e*h)*arctan(sqrt(h*x + g)*f/sqrt(-f^2*g + e*f*h))/(sqrt(-f
^2*g + e*f*h)*f) + sqrt(h*x + g)/f) - sqrt(h*x + g)*log(f*x + e))*b*p*q - s
qrt(h*x + g)*b*q*log(d) - sqrt(h*x + g)*b*log(c) - sqrt(h*x + g)*a)/h
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{\sqrt{g+h\,x}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(1/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(1/2), x)
```

# 3.485 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^{3/2}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 86

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx=-\frac{4b\sqrt{f}pq\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{h\sqrt{fg-eh}}-\frac{2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h\sqrt{g+hx}}$$

```
[Out] -4*b*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*f^(1/2)/h/(-e*h+f*
g)^(1/2)-2*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(h*x+g)^(1/2)
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2442, 65, 214, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx=-\frac{2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h\sqrt{g+hx}}-\frac{4b\sqrt{f}pq\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{h\sqrt{fg-eh}}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(3/2),x]
```

```
[Out] (-4*b*Sqrt[f]*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/(h*Sqrt
[f*g - e*h]) - (2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(h*Sqrt[g + h*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(g+hx)^{3/2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= -\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{h\sqrt{g+hx}}\\
&\quad+\text{Subst}\left(\frac{(2bfpq)\int\frac{1}{(e+fx)\sqrt{g+hx}}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= -\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{h\sqrt{g+hx}}\\
&\quad+\text{Subst}\left(\frac{(4bfpq)\text{Subst}\left(\int\frac{1}{e-\frac{fg}{h}+\frac{fx^2}{h}}\,dx, x, \sqrt{g+hx}\right)}{h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= -\frac{4b\sqrt{f}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{h\sqrt{fg-eh}}-\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{h\sqrt{g+hx}}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx=\frac{-\frac{4b\sqrt{f}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}}-\frac{2(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{g+hx}}}{h}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(3/2),x]
```

```
[Out] ((-4*b*Sqrt[f]*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/Sqrt[f
*g - e*h] - (2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/Sqrt[g + h*x])/h
```

## Maple [F]

$$\int \frac{a+b\ln\left(c(d(fx+e)^p)^q\right)}{(hx+g)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(3/2),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(3/2),x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.79

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx=\left[\frac{2\left((bhpqx+bgpq)\sqrt{\frac{f}{fg-eh}}\log\left(\frac{fhx+2\,fg-eh-2\,(fg-eh)\sqrt{hx+g}\sqrt{\frac{f}{fg-eh}}}{fx+e}\right)-(bpq\right.}{h^2x+gh}\right.$$

$$-\frac{2\left(2\,(bhpqx+bgpq)\sqrt{-\frac{f}{fg-eh}}\arctan\left(-\frac{(fg-eh)\sqrt{hx+g}\sqrt{-\frac{f}{fg-eh}}}{fhx+fg}\right)+(bpq\log(fx+e)+bq\log(d)+b\log(c)+\right.}{h^2x+gh}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(3/2),x, algorithm="fricas")
```

```
[Out] [2*((b*h*p*q*x + b*g*p*q)*sqrt(f/(f*g - e*h))*log((f*h*x + 2*f*g - e*h - 2*
(f*g - e*h)*sqrt(h*x + g)*sqrt(f/(f*g - e*h)))/(f*x + e)) - (b*p*q*log(f*x
+ e) + b*q*log(d) + b*log(c) + a)*sqrt(h*x + g))/(h^2*x + g*h), -2*(2*(b*h*
p*q*x + b*g*p*q)*sqrt(-f/(f*g - e*h))*arctan(-(f*g - e*h)*sqrt(h*x + g)*sqr
t(-f/(f*g - e*h))/(f*h*x + f*g)) + (b*p*q*log(f*x + e) + b*q*log(d) + b*log
(c) + a)*sqrt(h*x + g))/(h^2*x + g*h)]
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx = \int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**(3/2),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx = \frac{4\,bfpq\arctan\left(\frac{\sqrt{hx+g}f}{\sqrt{-f^2g+efh}}\right)}{\sqrt{-f^2g+efh}h} - \frac{2\,bpq\log\left((hx+g)f-fg+eh\right)}{\sqrt{hx+g}h} + \frac{2\,(bpq\log(h)-bq\log(d)-b\log(c)-a)}{\sqrt{hx+g}h}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(3/2),x, algorithm="giac")
[Out] 4*b*f*p*q*arctan(sqrt(h*x + g)*f/sqrt(-f^2*g + e*f*h))/(sqrt(-f^2*g + e*f*h
)*h) - 2*b*p*q*log((h*x + g)*f - f*g + e*h)/(sqrt(h*x + g)*h) + 2*(b*p*q*lo
g(h) - b*q*log(d) - b*log(c) - a)/(sqrt(h*x + g)*h)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{3/2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{{\left(g+h\,x\right)}^{3/2}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(3/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(3/2), x)
```

# 3.486 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^{5/2}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{5/2}}\,dx=\frac{4bfpq}{3h(fg-eh)\sqrt{g+hx}}-\frac{4bf^{3/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3h(fg-eh)^{3/2}}-\frac{2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h(g+hx)^{3/2}}$$

```
[Out] -4/3*b*f^(3/2)*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/h/(-e*h+
f*g)^(3/2)-2/3*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(h*x+g)^(3/2)+4/3*b*f*p*q/h/(-
e*h+f*g)/(h*x+g)^(1/2)
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2442, 53, 65, 214, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{5/2}}\,dx=-\frac{2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h(g+hx)^{3/2}}-\frac{4bf^{3/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3h(fg-eh)^{3/2}}+\frac{4bfpq}{3h\sqrt{g+hx}(fg-eh)}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(5/2),x]
```

```
[Out] (4*b*f*p*q)/(3*h*(f*g - e*h)*Sqrt[g + h*x]) - (4*b*f^(3/2)*p*q*ArcTanh[(Sqr
t[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/(3*h*(f*g - e*h)^(3/2)) - (2*(a + b*L
og[c*(d*(e + f*x)^p)^q]))/(3*h*(g + h*x)^(3/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(g+hx)^{5/2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{3h(g+hx)^{3/2}} + \text{Subst}\left(\frac{(2bfpq)\int\frac{1}{(e+fx)(g+hx)^{3/2}}\,dx}{3h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{4bfpq}{3h(fg-eh)\sqrt{g+hx}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))}{3h(g+hx)^{3/2}} + \text{Subst}\left(\frac{(2bf^2pq)\int \frac{1}{(e+fx)\sqrt{g+hx}}\,dx}{3h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{4bfpq}{3h(fg-eh)\sqrt{g+hx}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))}{3h(g+hx)^{3/2}} + \text{Subst}\left(\frac{(4bf^2pq)\,\text{Subst}\left(\int \frac{1}{e-\frac{fg}{h}+\frac{fx^2}{h}}\,dx, x, \sqrt{g+hx}\right)}{3h^2(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{4bfpq}{3h(fg-eh)\sqrt{g+hx}} - \frac{4bf^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3h(fg-eh)^{3/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))}{3h(g+hx)^{3/2}}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^{5/2}}\,dx = \frac{-4bfpq(g+hx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{f(g+hx)}{fg-eh}\right)+2(fg-eh)\,(a}{3h(-fg+eh)(g+hx)^{3/2}}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(5/2),x]
```

```
[Out] (-4*b*f*p*q*(g + h*x)*Hypergeometric2F1[-1/2, 1, 1/2, (f*(g + h*x))/(f*g -
e*h)] + 2*(f*g - e*h)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(3*h*(-(f*g) + e*h)
*(g + h*x)^(3/2))
```

## Maple [F]

$$\int \frac{a+b\ln(c(d(fx+e)^p)^q)}{(hx+g)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(5/2),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(5/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(100) = 200.

Time = 0.35 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.89

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{5/2}}\,dx = \left[-\frac{2\left(\left(bfh^2pqx^2+2\,bfghpqx+bfg^2pq\right)\sqrt{\frac{f}{fg-eh}}\log\left(\frac{fhx+2\,fg-eh+2\,(fg-eh)\sqrt{\cdots}}{fx+e}\right)\cdots\right.}{\cdots}\right.$$

$$-\frac{2\left(2\left(bfh^2pqx^2+2\,bfghpqx+bfg^2pq\right)\sqrt{-\frac{f}{fg-eh}}\arctan\left(-\frac{(fg-eh)\sqrt{hx+g}\sqrt{-\frac{f}{fg-eh}}}{fhx+fg}\right)-\left(2\,bfhpqx+2\,bfgpq\cdots\right.\right.}{3\left(fg^3h-eg^2h^2+\left(fgh^3-eh^4\right)x\cdots\right.}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(5/2),x, algorithm="fricas")
```

```
[Out] [-2/3*((b*f*h^2*p*q*x^2 + 2*b*f*g*h*p*q*x + b*f*g^2*p*q)*sqrt(f/(f*g - e*h)
)*log((f*h*x + 2*f*g - e*h + 2*(f*g - e*h)*sqrt(h*x + g)*sqrt(f/(f*g - e*h)
))/(f*x + e)) - (2*b*f*h*p*q*x + 2*b*f*g*p*q - (b*f*g - b*e*h)*p*q*log(f*x
+ e) - a*f*g + a*e*h - (b*f*g - b*e*h)*q*log(d) - (b*f*g - b*e*h)*log(c))*s
qrt(h*x + g))/(f*g^3*h - e*g^2*h^2 + (f*g*h^3 - e*h^4)*x^2 + 2*(f*g^2*h^2 -
 e*g*h^3)*x), -2/3*(2*(b*f*h^2*p*q*x^2 + 2*b*f*g*h*p*q*x + b*f*g^2*p*q)*sqr
t(-f/(f*g - e*h))*arctan(-(f*g - e*h)*sqrt(h*x + g)*sqrt(-f/(f*g - e*h))/(f
*h*x + f*g)) - (2*b*f*h*p*q*x + 2*b*f*g*p*q - (b*f*g - b*e*h)*p*q*log(f*x +
 e) - a*f*g + a*e*h - (b*f*g - b*e*h)*q*log(d) - (b*f*g - b*e*h)*log(c))*sq
rt(h*x + g))/(f*g^3*h - e*g^2*h^2 + (f*g*h^3 - e*h^4)*x^2 + 2*(f*g^2*h^2 -
e*g*h^3)*x)]
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{5/2}}\,dx = \int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**(5/2),x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x)**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.53

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{5/2}}\,dx = \frac{4\,bf^2pq\arctan\left(\frac{\sqrt{hx+g}f}{\sqrt{-f^2g+efh}}\right)}{3\sqrt{-f^2g+efh}(fgh-eh^2)} - \frac{2\,bpq\log\left((hx+g)f-fg+eh\right)}{3\,(hx+g)^{\frac{3}{2}}h} + \frac{2\,(bfgpq\log(h)-behpq\log(h)+2\,(hx+g)bfpq-bfgq\log(d)+behq\log(d)-bfg\log(c)+beh\log(c) -}{3\left((hx+g)^{\frac{3}{2}}fgh-(hx+g)^{\frac{3}{2}}eh^2\right)}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*b*f^2*p*q*arctan(sqrt(h*x + g)*f/sqrt(-f^2*g + e*f*h))/(sqrt(-f^2*g + e
*f*h)*(f*g*h - e*h^2)) - 2/3*b*p*q*log((h*x + g)*f - f*g + e*h)/((h*x + g)^
(3/2)*h) + 2/3*(b*f*g*p*q*log(h) - b*e*h*p*q*log(h) + 2*(h*x + g)*b*f*p*q -
 b*f*g*q*log(d) + b*e*h*q*log(d) - b*f*g*log(c) + b*e*h*log(c) - a*f*g + a*
e*h)/((h*x + g)^(3/2)*f*g*h - (h*x + g)^(3/2)*e*h^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{5/2}}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{{\left(g+h\,x\right)}^{5/2}} \,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(5/2),x)
```

```
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(5/2), x)
```

# 3.487 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^{7/2}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{7/2}}\,dx = \frac{4bfpq}{15h(fg-eh)(g+hx)^{3/2}} + \frac{4bf^2pq}{5h(fg-eh)^2\sqrt{g+hx}} - \frac{4bf^{5/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{5h(fg-eh)^{5/2}} - \frac{2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h(g+hx)^{5/2}}$$

```
[Out] 4/15*b*f*p*q/h/(-e*h+f*g)/(h*x+g)^(3/2)-4/5*b*f^(5/2)*p*q*arctanh(f^(1/2)*(
h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/h/(-e*h+f*g)^(5/2)-2/5*(a+b*ln(c*(d*(f*x+e)^
p)^q))/h/(h*x+g)^(5/2)+4/5*b*f^2*p*q/h/(-e*h+f*g)^2/(h*x+g)^(1/2)
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2442, 53, 65, 214, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{7/2}}\,dx = -\frac{2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5h(g+hx)^{5/2}} - \frac{4bf^{5/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{5h(fg-eh)^{5/2}} + \frac{4bf^2pq}{5h\sqrt{g+hx}(fg-eh)^2} + \frac{4bfpq}{15h(g+hx)^{3/2}(fg-eh)}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(7/2),x]
```

```
[Out] (4*b*f*p*q)/(15*h*(f*g - e*h)*(g + h*x)^(3/2)) + (4*b*f^2*p*q)/(5*h*(f*g -
e*h)^2*Sqrt[g + h*x]) - (4*b*f^(5/2)*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sq
rt[f*g - e*h]])/(5*h*(f*g - e*h)^(5/2)) - (2*(a + b*Log[c*(d*(e + f*x)^p)^q
]))/(5*h*(g + h*x)^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(g+hx)^{7/2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\ &= -\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{5h(g+hx)^{5/2}}\\ &\quad+\text{Subst}\left(\frac{(2bfpq)\int \frac{1}{(e+fx)(g+hx)^{5/2}}\,dx}{5h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\end{aligned}$$

$$= \frac{4bfpq}{15h(fg-eh)(g+hx)^{3/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))}{5h(g+hx)^{5/2}} + \text{Subst}\left(\frac{(2bf^2pq)\int\frac{1}{(e+fx)(g+hx)^{3/2}}\,dx}{5h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{4bfpq}{15h(fg-eh)(g+hx)^{3/2}} + \frac{4bf^2pq}{5h(fg-eh)^2\sqrt{g+hx}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))}{5h(g+hx)^{5/2}} + \text{Subst}\left(\frac{(2bf^3pq)\int\frac{1}{(e+fx)\sqrt{g+hx}}\,dx}{5h(fg-eh)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{4bfpq}{15h(fg-eh)(g+hx)^{3/2}} + \frac{4bf^2pq}{5h(fg-eh)^2\sqrt{g+hx}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))}{5h(g+hx)^{5/2}} + \text{Subst}\left(\frac{(4bf^3pq)\,\text{Subst}\left(\int\frac{1}{e-\frac{fg}{h}+\frac{fx^2}{h}}\,dx, x, \sqrt{g+hx}\right)}{5h^2(fg-eh)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{4bfpq}{15h(fg-eh)(g+hx)^{3/2}} + \frac{4bf^2pq}{5h(fg-eh)^2\sqrt{g+hx}} - \frac{4bf^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{5h(fg-eh)^{5/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))}{5h(g+hx)^{5/2}}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.60

$$\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^{7/2}}\,dx = \frac{-4bfpq(g+hx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{f(g+hx)}{fg-eh}\right)+6(fg-eh)\,(a}{15h(-fg+eh)(g+hx)^{5/2}}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(7/2),x]
```

```
[Out] (-4*b*f*p*q*(g + h*x)*Hypergeometric2F1[-3/2, 1, -1/2, (f*(g + h*x))/(f*g -
 e*h)] + 6*(f*g - e*h)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(15*h*(-(f*g) + e*
h)*(g + h*x)^(5/2))
```

## Maple [F]

$$\int \frac{a+b\ln\left(c\left(d(fx+e)^{p}\right)^{q}\right)}{(hx+g)^{\frac{7}{2}}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(7/2),x)
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(7/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(128) = 256.

Time = 0.37 (sec) , antiderivative size = 863, normalized size of antiderivative = 5.68

$$\int \frac{a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)}{(g+hx)^{7/2}}\,dx = \left[\frac{2\left(3\left(bf^2h^3pqx^3+3\,bf^2gh^2pqx^2+3\,bf^2g^2hpqx+bf^2g^3pq\right)\sqrt{\frac{f}{fg-eh}}\log\left(\right.\right.}{}\right.$$

$$-\frac{2\left(6\left(bf^2h^3pqx^3+3\,bf^2gh^2pqx^2+3\,bf^2g^2hpqx+bf^2g^3pq\right)\sqrt{-\frac{f}{fg-eh}}\arctan\left(-\frac{(fg-eh)\sqrt{hx+g}\sqrt{-\frac{f}{fg-eh}}}{fhx+fg}\right)-\left(\right.\right.}{15\left(f^2\right.}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(7/2),x, algorithm="fricas")
[Out] [2/15*(3*(b*f^2*h^3*p*q*x^3 + 3*b*f^2*g*h^2*p*q*x^2 + 3*b*f^2*g^2*h*p*q*x +
 b*f^2*g^3*p*q)*sqrt(f/(f*g - e*h))*log((f*h*x + 2*f*g - e*h - 2*(f*g - e*h
)*sqrt(h*x + g)*sqrt(f/(f*g - e*h)))/(f*x + e)) + (6*b*f^2*h^2*p*q*x^2 - 3*
a*f^2*g^2 + 6*a*e*f*g*h - 3*a*e^2*h^2 + 2*(7*b*f^2*g*h - b*e*f*h^2)*p*q*x -
 3*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*p*q*log(f*x + e) + 2*(4*b*f^2*g^2
- b*e*f*g*h)*p*q - 3*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*q*log(d) - 3*(b*
f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*log(c))*sqrt(h*x + g))/(f^2*g^5*h - 2*e*
f*g^4*h^2 + e^2*g^3*h^3 + (f^2*g^2*h^4 - 2*e*f*g*h^5 + e^2*h^6)*x^3 + 3*(f^
2*g^3*h^3 - 2*e*f*g^2*h^4 + e^2*g*h^5)*x^2 + 3*(f^2*g^4*h^2 - 2*e*f*g^3*h^3
 + e^2*g^2*h^4)*x), -2/15*(6*(b*f^2*h^3*p*q*x^3 + 3*b*f^2*g*h^2*p*q*x^2 + 3
*b*f^2*g^2*h*p*q*x + b*f^2*g^3*p*q)*sqrt(-f/(f*g - e*h))*arctan(-(f*g - e*h
)*sqrt(h*x + g)*sqrt(-f/(f*g - e*h))/(f*h*x + f*g)) - (6*b*f^2*h^2*p*q*x^2
- 3*a*f^2*g^2 + 6*a*e*f*g*h - 3*a*e^2*h^2 + 2*(7*b*f^2*g*h - b*e*f*h^2)*p*q
*x - 3*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*p*q*log(f*x + e) + 2*(4*b*f^2*
g^2 - b*e*f*g*h)*p*q - 3*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*q*log(d) - 3
*(b*f^2*g^2 - 2*b*e*f*g*h + b*e^2*h^2)*log(c))*sqrt(h*x + g))/(f^2*g^5*h -
2*e*f*g^4*h^2 + e^2*g^3*h^3 + (f^2*g^2*h^4 - 2*e*f*g*h^5 + e^2*h^6)*x^3 + 3
*(f^2*g^3*h^3 - 2*e*f*g^2*h^4 + e^2*g*h^5)*x^2 + 3*(f^2*g^4*h^2 - 2*e*f*g^3
*h^3 + e^2*g^2*h^4)*x)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(g+hx)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**(7/2),x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(g+hx)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(7/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(128) = 256.

Time = 0.34 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.13

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(g+hx)^{7/2}}\,dx = \frac{4\,bf^3hpq\arctan\left(\frac{\sqrt{hx+g}f}{\sqrt{-f^2g+efh}}\right)}{5\,(f^2g^2h^2-2\,efgh^3+e^2h^4)\sqrt{-f^2g+efh}} - \frac{2\,bpq\log\left((hx+g)f-fg+eh\right)}{5\,(hx+g)^{\frac{5}{2}}h} + \frac{2\,\left(3\,bf^2g^2pq\log\left(h\right)-6\,befghpq\log\left(h\right)+3\,be^2h^2pq\log\left(h\right)+6\,(hx+g)^2bf^2pq+2\,(hx+g)bf^2gpq-2\,(hx\right.}{15\,\left((hx\right.}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(7/2),x, algorithm="giac")
[Out] 4/5*b*f^3*h*p*q*arctan(sqrt(h*x + g)*f/sqrt(-f^2*g + e*f*h))/((f^2*g^2*h^2
 - 2*e*f*g*h^3 + e^2*h^4)*sqrt(-f^2*g + e*f*h)) - 2/5*b*p*q*log((h*x + g)*f
 - f*g + e*h)/((h*x + g)^(5/2)*h) + 2/15*(3*b*f^2*g^2*p*q*log(h) - 6*b*e*f*g
*h*p*q*log(h) + 3*b*e^2*h^2*p*q*log(h) + 6*(h*x + g)^2*b*f^2*p*q + 2*(h*x +
 g)*b*f^2*g*p*q - 2*(h*x + g)*b*e*f*h*p*q - 3*b*f^2*g^2*q*log(d) + 6*b*e*f*
g*h*q*log(d) - 3*b*e^2*h^2*q*log(d) - 3*b*f^2*g^2*log(c) + 6*b*e*f*g*h*log(
c) - 3*b*e^2*h^2*log(c) - 3*a*f^2*g^2 + 6*a*e*f*g*h - 3*a*e^2*h^2)/((h*x +
g)^(5/2)*f^2*g^2*h - 2*(h*x + g)^(5/2)*e*f*g*h^2 + (h*x + g)^(5/2)*e^2*h^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)^{7/2}}\,dx = \int \frac{a + b\,\ln\left(c\left(d\left(e + f\,x\right)^p\right)^q\right)}{\left(g + h\,x\right)^{7/2}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(7/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(7/2), x)
```

# 3.488 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)^{9/2}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 184

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{9/2}}\,dx = \frac{4bfpq}{35h(fg-eh)(g+hx)^{5/2}}+\frac{4bf^2pq}{21h(fg-eh)^2(g+hx)^{3/2}} + \frac{4bf^3pq}{7h(fg-eh)^3\sqrt{g+hx}}-\frac{4bf^{7/2}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{7h(fg-eh)^{7/2}}-\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{7h(g+hx)^{7/2}}$$

```
[Out] 4/35*b*f*p*q/h/(-e*h+f*g)/(h*x+g)^(5/2)+4/21*b*f^2*p*q/h/(-e*h+f*g)^2/(h*x+
g)^(3/2)-4/7*b*f^(7/2)*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/
h/(-e*h+f*g)^(7/2)-2/7*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(h*x+g)^(7/2)+4/7*b*f^
3*p*q/h/(-e*h+f*g)^3/(h*x+g)^(1/2)
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2442, 53, 65, 214, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{9/2}}\,dx = -\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))}{7h(g+hx)^{7/2}} -\frac{4bf^{7/2}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{7h(fg-eh)^{7/2}}+\frac{4bf^3pq}{7h\sqrt{g+hx}(fg-eh)^3} +\frac{4bf^2pq}{21h(g+hx)^{3/2}(fg-eh)^2}+\frac{4bfpq}{35h(g+hx)^{5/2}(fg-eh)}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(9/2),x]
```

```
[Out] (4*b*f*p*q)/(35*h*(f*g - e*h)*(g + h*x)^(5/2)) + (4*b*f^2*p*q)/(21*h*(f*g -
 e*h)^2*(g + h*x)^(3/2)) + (4*b*f^3*p*q)/(7*h*(f*g - e*h)^3*Sqrt[g + h*x])
- (4*b*f^(7/2)*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/(7*h*(
f*g - e*h)^(7/2)) - (2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(7*h*(g + h*x)^(7/
2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(g+hx)^{9/2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(a+b\log(c(d(e+fx)^p)^q))}{7h(g+hx)^{7/2}} + \text{Subst}\left(\frac{(2bfpq)\int\frac{1}{(e+fx)(g+hx)^{7/2}}\,dx}{7h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{4bfpq}{35h(fg-eh)(g+hx)^{5/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))}{7h(g+hx)^{7/2}} + \text{Subst}\left(\frac{(2bf^2pq)\int\frac{1}{(e+fx)(g+hx)^{5/2}}\,dx}{7h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{4bfpq}{35h(fg-eh)(g+hx)^{5/2}} + \frac{4bf^2pq}{21h(fg-eh)^2(g+hx)^{3/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))}{7h(g+hx)^{7/2}} + \text{Subst}\left(\frac{(2bf^3pq)\int\frac{1}{(e+fx)(g+hx)^{3/2}}\,dx}{7h(fg-eh)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{4bfpq}{35h(fg-eh)(g+hx)^{5/2}} + \frac{4bf^2pq}{21h(fg-eh)^2(g+hx)^{3/2}} + \frac{4bf^3pq}{7h(fg-eh)^3\sqrt{g+hx}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))}{7h(g+hx)^{7/2}} + \text{Subst}\left(\frac{(2bf^4pq)\int\frac{1}{(e+fx)\sqrt{g+hx}}\,dx}{7h(fg-eh)^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{4bfpq}{35h(fg-eh)(g+hx)^{5/2}} + \frac{4bf^2pq}{21h(fg-eh)^2(g+hx)^{3/2}} + \frac{4bf^3pq}{7h(fg-eh)^3\sqrt{g+hx}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))}{7h(g+hx)^{7/2}} + \text{Subst}\left(\frac{(4bf^4pq)\,\text{Subst}\left(\int\frac{1}{e-\frac{fg}{h}+\frac{fx^2}{h}}\,dx, x, \sqrt{g+hx}\right)}{7h^2(fg-eh)^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{4bfpq}{35h(fg-eh)(g+hx)^{5/2}} + \frac{4bf^2pq}{21h(fg-eh)^2(g+hx)^{3/2}} + \frac{4bf^3pq}{7h(fg-eh)^3\sqrt{g+hx}} - \frac{4bf^{7/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{7h(fg-eh)^{7/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))}{7h(g+hx)^{7/2}}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.49

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{9/2}}\,dx = \frac{-4bfpq(g+hx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{f(g+hx)}{fg-eh}\right)+10(fg-eh)}{35h(-fg+eh)(g+hx)^{7/2}}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x)^(9/2),x]
[Out] (-4*b*f*p*q*(g + h*x)*Hypergeometric2F1[-5/2, 1, -3/2, (f*(g + h*x))/(f*g -
 e*h)] + 10*(f*g - e*h)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(35*h*(-(f*g) + e
*h)*(g + h*x)^(7/2))
```

## Maple [F]

$$\int \frac{a+b\ln\left(c(d(fx+e)^p)^q\right)}{(hx+g)^{\frac{9}{2}}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(9/2),x)
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(9/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(156) = 312$.

Time = 0.41 (sec) , antiderivative size = 1362, normalized size of antiderivative = 7.40

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(9/2),x, algorithm="fricas")
[Out] [-2/105*(15*(b*f^3*h^4*p*q*x^4 + 4*b*f^3*g*h^3*p*q*x^3 + 6*b*f^3*g^2*h^2*p*
q*x^2 + 4*b*f^3*g^3*h*p*q*x + b*f^3*g^4*p*q)*sqrt(f/(f*g - e*h))*log((f*h*x
 + 2*f*g - e*h + 2*(f*g - e*h)*sqrt(h*x + g)*sqrt(f/(f*g - e*h)))/(f*x + e)
) - (30*b*f^3*h^3*p*q*x^3 - 15*a*f^3*g^3 + 45*a*e*f^2*g^2*h - 45*a*e^2*f*g*
h^2 + 15*a*e^3*h^3 + 10*(10*b*f^3*g*h^2 - b*e*f^2*h^3)*p*q*x^2 + 2*(58*b*f^
3*g^2*h - 16*b*e*f^2*g*h^2 + 3*b*e^2*f*h^3)*p*q*x - 15*(b*f^3*g^3 - 3*b*e*f
^2*g^2*h + 3*b*e^2*f*g*h^2 - b*e^3*h^3)*p*q*log(f*x + e) + 2*(23*b*f^3*g^3
- 11*b*e*f^2*g^2*h + 3*b*e^2*f*g*h^2)*p*q - 15*(b*f^3*g^3 - 3*b*e*f^2*g^2*h
 + 3*b*e^2*f*g*h^2 - b*e^3*h^3)*q*log(d) - 15*(b*f^3*g^3 - 3*b*e*f^2*g^2*h
+ 3*b*e^2*f*g*h^2 - b*e^3*h^3)*log(c))*sqrt(h*x + g))/(f^3*g^7*h - 3*e*f^2*
g^6*h^2 + 3*e^2*f*g^5*h^3 - e^3*g^4*h^4 + (f^3*g^3*h^5 - 3*e*f^2*g^2*h^6 +
```

```
3*e^2*f*g*h^7 - e^3*h^8)*x^4 + 4*(f^3*g^4*h^4 - 3*e*f^2*g^3*h^5 + 3*e^2*f*g
^2*h^6 - e^3*g*h^7)*x^3 + 6*(f^3*g^5*h^3 - 3*e*f^2*g^4*h^4 + 3*e^2*f*g^3*h^
5 - e^3*g^2*h^6)*x^2 + 4*(f^3*g^6*h^2 - 3*e*f^2*g^5*h^3 + 3*e^2*f*g^4*h^4 -
 e^3*g^3*h^5)*x), -2/105*(30*(b*f^3*h^4*p*q*x^4 + 4*b*f^3*g*h^3*p*q*x^3 + 6
*b*f^3*g^2*h^2*p*q*x^2 + 4*b*f^3*g^3*h*p*q*x + b*f^3*g^4*p*q)*sqrt(-f/(f*g
- e*h))*arctan(-(f*g - e*h)*sqrt(h*x + g)*sqrt(-f/(f*g - e*h))/(f*h*x + f*g
)) - (30*b*f^3*h^3*p*q*x^3 - 15*a*f^3*g^3 + 45*a*e*f^2*g^2*h - 45*a*e^2*f*g
*h^2 + 15*a*e^3*h^3 + 10*(10*b*f^3*g*h^2 - b*e*f^2*h^3)*p*q*x^2 + 2*(58*b*f
^3*g^2*h - 16*b*e*f^2*g*h^2 + 3*b*e^2*f*h^3)*p*q*x - 15*(b*f^3*g^3 - 3*b*e*
f^2*g^2*h + 3*b*e^2*f*g*h^2 - b*e^3*h^3)*p*q*log(f*x + e) + 2*(23*b*f^3*g^3
 - 11*b*e*f^2*g^2*h + 3*b*e^2*f*g*h^2)*p*q - 15*(b*f^3*g^3 - 3*b*e*f^2*g^2*
h + 3*b*e^2*f*g*h^2 - b*e^3*h^3)*q*log(d) - 15*(b*f^3*g^3 - 3*b*e*f^2*g^2*h
 + 3*b*e^2*f*g*h^2 - b*e^3*h^3)*log(c))*sqrt(h*x + g))/(f^3*g^7*h - 3*e*f^2
*g^6*h^2 + 3*e^2*f*g^5*h^3 - e^3*g^4*h^4 + (f^3*g^3*h^5 - 3*e*f^2*g^2*h^6 +
 3*e^2*f*g*h^7 - e^3*h^8)*x^4 + 4*(f^3*g^4*h^4 - 3*e*f^2*g^3*h^5 + 3*e^2*f*
g^2*h^6 - e^3*g*h^7)*x^3 + 6*(f^3*g^5*h^3 - 3*e*f^2*g^4*h^4 + 3*e^2*f*g^3*h
^5 - e^3*g^2*h^6)*x^2 + 4*(f^3*g^6*h^2 - 3*e*f^2*g^5*h^3 + 3*e^2*f*g^4*h^4
- e^3*g^3*h^5)*x)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{9/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**(9/2),x)
```

```
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)^{9/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)^{9/2}}\,dx = \int \frac{b\log\left(\left(\left((fx + e)^p d\right)^q c\right) + a\right)}{(hx + g)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(9/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g)^(9/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)^{9/2}}\,dx = \int \frac{a + b\,\ln\left(c\left(d\left(e + f\,x\right)^p\right)^q\right)}{\left(g + h\,x\right)^{9/2}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(9/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x)^(9/2), x)
```

# 3.489 $\int (g+hx)^{3/2} \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 dx$



## Optimal result

Integrand size = 30, antiderivative size = 635

$$\begin{aligned}
\int (g+hx)^{3/2} \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 dx &= \frac{368b^2(fg-eh)^2p^2q^2\sqrt{g+hx}}{75f^2h} \\
&+ \frac{128b^2(fg-eh)p^2q^2(g+hx)^{3/2}}{225fh} + \frac{16b^2p^2q^2(g+hx)^{5/2}}{125h} \\
&- \frac{368b^2(fg-eh)^{5/2}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{75f^{5/2}h} \\
&- \frac{8b^2(fg-eh)^{5/2}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5f^{5/2}h} \\
&- \frac{8b(fg-eh)^2pq\sqrt{g+hx}(a+b\log\left(c(d(e+fx)^p)^q\right))}{5f^2h} \\
&- \frac{8b(fg-eh)pq(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{15fh} \\
&- \frac{8bpq(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{25h} \\
&+ \frac{8b(fg-eh)^{5/2}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{5f^{5/2}h} \\
&+ \frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h} \\
&+ \frac{16b^2(fg-eh)^{5/2}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5f^{5/2}h} \\
&+ \frac{8b^2(fg-eh)^{5/2}p^2q^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5f^{5/2}h}
\end{aligned}$$

```
[Out] 128/225*b^2*(-e*h+f*g)*p^2*q^2*(h*x+g)^(3/2)/f/h+16/125*b^2*p^2*q^2*(h*x+g)
^(5/2)/h-368/75*b^2*(-e*h+f*g)^(5/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/
(-e*h+f*g)^(1/2))/f^(5/2)/h-8/5*b^2*(-e*h+f*g)^(5/2)*p^2*q^2*arctanh(f^(1/2
)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))^2/f^(5/2)/h-8/15*b*(-e*h+f*g)*p*q*(h*x+g)
^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))/f/h-8/25*b*p*q*(h*x+g)^(5/2)*(a+b*ln(c*(
d*(f*x+e)^p)^q))/h+8/5*b*(-e*h+f*g)^(5/2)*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)
/(-e*h+f*g)^(1/2))*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^(5/2)/h+2/5*(h*x+g)^(5/2)*
(a+b*ln(c*(d*(f*x+e)^p)^q))^2/h+16/5*b^2*(-e*h+f*g)^(5/2)*p^2*q^2*arctanh(f
^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*ln(2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h+
f*g)^(1/2)))/f^(5/2)/h+8/5*b^2*(-e*h+f*g)^(5/2)*p^2*q^2*polylog(2,1-2/(1-f^
(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))/f^(5/2)/h+368/75*b^2*(-e*h+f*g)^2*p^
2*q^2*(h*x+g)^(1/2)/f^2/h-8/5*b*(-e*h+f*g)^2*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q)
)*(h*x+g)^(1/2)/f^2/h
```

## Rubi [A] (verified)

Time = 3.08 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2445, 2458, 2388, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 52, 2495}

$$\begin{aligned}
&\int (g+hx)^{3/2} (a+b\log (c(d(e+fx)^p)^q))^2 \, dx = \frac{8bpq(fg-eh)^{5/2}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right) (a+b\log (c(d(e+fx)^p)^q))}{5f^{5/2}h} \\
&-\frac{8bpq\sqrt{g+hx}(fg-eh)^2 (a+b\log (c(d(e+fx)^p)^q))}{5f^2h} \\
&-\frac{8bpq(g+hx)^{3/2}(fg-eh) (a+b\log (c(d(e+fx)^p)^q))}{15fh} \\
&+\frac{2(g+hx)^{5/2} (a+b\log (c(d(e+fx)^p)^q))^2}{5h}-\frac{8bpq(g+hx)^{5/2} (a+b\log (c(d(e+fx)^p)^q))}{25h} \\
&-\frac{8b^2p^2q^2(fg-eh)^{5/2}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5f^{5/2}h}-\frac{368b^2p^2q^2(fg-eh)^{5/2}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{75f^{5/2}h} \\
&+\frac{16b^2p^2q^2(fg-eh)^{5/2}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5f^{5/2}h} \\
&+\frac{8b^2p^2q^2(fg-eh)^{5/2}\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5f^{5/2}h}+\frac{368b^2p^2q^2\sqrt{g+hx}(fg-eh)^2}{75f^2h} \\
&+\frac{128b^2p^2q^2(g+hx)^{3/2}(fg-eh)}{225fh}+\frac{16b^2p^2q^2(g+hx)^{5/2}}{125h}
\end{aligned}$$

```
[In] Int[(g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] (368*b^2*(f*g - e*h)^2*p^2*q^2*Sqrt[g + h*x])/(75*f^2*h) + (128*b^2*(f*g -
e*h)*p^2*q^2*(g + h*x)^(3/2))/(225*f*h) + (16*b^2*p^2*q^2*(g + h*x)^(5/2))/
(125*h) - (368*b^2*(f*g - e*h)^(5/2)*p^2*q^2*ArcTanh[(Sqrt[f]*Sqrt[g + h*x]
)/Sqrt[f*g - e*h]])/(75*f^(5/2)*h) - (8*b^2*(f*g - e*h)^(5/2)*p^2*q^2*ArcTa
nh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]^2)/(5*f^(5/2)*h) - (8*b*(f*g -
e*h)^2*p*q*Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(5*f^2*h) - (8*b
*(f*g - e*h)*p*q*(g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(15*f*h)
 - (8*b*p*q*(g + h*x)^(5/2)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(25*h) + (8*b
*(f*g - e*h)^(5/2)*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]*(a
+ b*Log[c*(d*(e + f*x)^p)^q]))/(5*f^(5/2)*h) + (2*(g + h*x)^(5/2)*(a + b*Lo
g[c*(d*(e + f*x)^p)^q])^2)/(5*h) + (16*b^2*(f*g - e*h)^(5/2)*p^2*q^2*ArcTan
h[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]*Log[2/(1 - (Sqrt[f]*Sqrt[g + h*x
])/Sqrt[f*g - e*h])])/(5*f^(5/2)*h) + (8*b^2*(f*g - e*h)^(5/2)*p^2*q^2*Poly
Log[2, 1 - 2/(1 - (Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h])])/(5*f^(5/2)*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
```

```
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (g+hx)^{3/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h} - \text{Subst}\left(\frac{(4bfpq)\int \frac{(g+hx)^{5/2}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{e+fx}\,dx}{5h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h} - \text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int \frac{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{5/2}(a+b\log(cd^qx^{pq}))}{x}\,dx,x,e+fx\right)}{5h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right.$$

$$= \frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h} - \text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int \left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{3/2}\left(a+b\log\left(cd^qx^{pq}\right)\right)\,dx,x,e+fx\right)}{5f},cd^q(e+fx)^{pq},c(d(e\right. - \text{Subst}\left(\frac{(4b(fg-eh)pq)\text{Subst}\left(\int \frac{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{3/2}(a+b\log(cd^qx^{pq}))}{x}\,dx,x,e+fx\right)}{5fh},cd^q(e+fx)^{pq},c(d(e\right.$$

$$= -\frac{8bpq(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{25h}+\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h} - \text{Subst}\left(\frac{(4b(fg-eh)pq)\text{Subst}\left(\int \sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}\left(a+b\log\left(cd^qx^{pq}\right)\right)\,dx,x,e+fx\right)}{5f^2},cd^q(e+fx)^{pq},\right. - \text{Subst}\left(\frac{(4b(fg-eh)^2pq)\,\text{Subst}\left(\int \frac{\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}(a+b\log(cd^qx^{pq}))}{x}\,dx,x,e+fx\right)}{5f^2h},cd^q(e+fx)^{pq},c(d(e-\right. + \text{Subst}\left(\frac{(8b^2p^2q^2)\,\text{Subst}\left(\int \frac{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{5/2}}{x}\,dx,x,e+fx\right)}{25h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{16b^2p^2q^2(g+hx)^{5/2}}{125h} - \frac{8b(fg-eh)pq(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{15fh} - \frac{8bpq(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{25h}+\frac{2(g+hx)^{5/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{5h} - \text{Subst}\left(\frac{(4b(fg-eh)^2pq)\,\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}}\,dx,x,e+fx\right)}{5f^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) -$$

$$= \frac{128b^2(fg-eh)p^2q^2(g+hx)^{3/2}}{225fh} + \frac{16b^2p^2q^2(g+hx)^{5/2}}{125h} - \frac{8b(fg-eh)^2pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{5f^2h} - \frac{8b(fg-eh)pq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{15fh} - \frac{8bpq(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))}{25h} + \frac{8b(fg-eh)^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5f^{5/2}h} + \frac{2(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))^2}{5h}$$

$$+\text{Subst}\left(\frac{(8b^2(fg-eh)^2p^2q^2)\,\text{Subst}\left(\int\frac{\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}}{x}\,dx,x,e+fx\right)}{25f^2h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)+$$

$$= \frac{368b^2(fg-eh)^2p^2q^2\sqrt{g+hx}}{75f^2h} + \frac{128b^2(fg-eh)p^2q^2(g+hx)^{3/2}}{225fh} + \frac{16b^2p^2q^2(g+hx)^{5/2}}{125h} - \frac{8b(fg-eh)^2pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{5f^2h} - \frac{8b(fg-eh)pq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{15fh} - \frac{8bpq(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))}{25h} + \frac{8b(fg-eh)^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5f^{5/2}h} + \frac{2(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))^2}{5h}$$

$$-\text{Subst}\left(\frac{(8b^2(fg-eh)^{5/2}p^2q^2)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{hx}{f}}}{\sqrt{fg-eh}}\right)}{x}\,dx,x,e+fx\right)}{5f^{5/2}h},cd^q(e+fx)^{pq},c(d(e$$

$$= \frac{368b^2(fg-eh)^2p^2q^2\sqrt{g+hx}}{75f^2h}+\frac{128b^2(fg-eh)p^2q^2(g+hx)^{3/2}}{225fh}+\frac{16b^2p^2q^2(g+hx)^{5/2}}{125h}-\frac{8b(fg-eh)^2pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{5f^2h}-\frac{8b(fg-eh)pq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{15fh}-\frac{8bpq(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))}{25h}+\frac{8b(fg-eh)^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5f^{5/2}h}+\frac{2(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))^2}{5h}-\text{Subst}\left(\frac{(16b^2(fg-eh)^{5/2}p^2q^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{eh+f(-g+x^2)}dx,x,\sqrt{g+hx}\right)}{5f^{3/2}h},cd^q(e+fx)^{pq},c(d(e+\right.$$

$$= \frac{368b^2(fg-eh)^2p^2q^2\sqrt{g+hx}}{75f^2h}+\frac{128b^2(fg-eh)p^2q^2(g+hx)^{3/2}}{225fh}+\frac{16b^2p^2q^2(g+hx)^{5/2}}{125h}-\frac{368b^2(fg-eh)^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{75f^{5/2}h}-\frac{8b(fg-eh)^2pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{5f^2h}-\frac{8b(fg-eh)pq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{15fh}-\frac{8bpq(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))}{25h}+\frac{8b(fg-eh)^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5f^{5/2}h}+\frac{2(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))^2}{5h}-\text{Subst}\left(\frac{(16b^2(fg-eh)^{5/2}p^2q^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{-fg+eh+fx^2}dx,x,\sqrt{g+hx}\right)}{5f^{3/2}h},cd^q(e+fx)^{pq},c(d(e+\right.$$

$$= \frac{368b^2(fg-eh)^2p^2q^2\sqrt{g+hx}}{75f^2h} + \frac{128b^2(fg-eh)p^2q^2(g+hx)^{3/2}}{225fh}$$
$$+ \frac{16b^2p^2q^2(g+hx)^{5/2}}{125h} - \frac{368b^2(fg-eh)^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{75f^{5/2}h}$$
$$- \frac{8b^2(fg-eh)^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5f^{5/2}h}$$
$$- \frac{8b(fg-eh)^2pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{5f^2h}$$
$$- \frac{8b(fg-eh)pq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{15fh}$$
$$- \frac{8bpq(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))}{25h}$$
$$+ \frac{8b(fg-eh)^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5f^{5/2}h}$$
$$+ \frac{2(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))^2}{5h}$$
$$+\text{Subst}\left(\frac{(16b^2(fg-eh)^2p^2q^2)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}\,dx, x, \sqrt{g+hx}\right)}{5f^2h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p$$

$$= \frac{368b^2(fg-eh)^2p^2q^2\sqrt{g+hx}}{75f^2h} + \frac{128b^2(fg-eh)p^2q^2(g+hx)^{3/2}}{225fh}$$
$$+ \frac{16b^2p^2q^2(g+hx)^{5/2}}{125h} - \frac{368b^2(fg-eh)^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{75f^{5/2}h}$$
$$- \frac{8b^2(fg-eh)^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5f^{5/2}h}$$
$$- \frac{8b(fg-eh)^2pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{5f^2h}$$
$$- \frac{8b(fg-eh)pq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{15fh}$$
$$- \frac{8bpq(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))}{25h}$$
$$+ \frac{8b(fg-eh)^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5f^{5/2}h}$$
$$+ \frac{2(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))^2}{5h}$$
$$+ \frac{16b^2(fg-eh)^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5f^{5/2}h}$$
$$-\text{Subst}\left(\frac{(16b^2(fg-eh)^2p^2q^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}\right)}{1-\frac{fx^2}{fg-eh}}\,dx, x, \sqrt{g+hx}\right)}{5f^2h}, cd^q(e+fx)^{pq}, c(d(e+f\right.$$

$$= \frac{368b^2(fg-eh)^2p^2q^2\sqrt{g+hx}}{75f^2h} + \frac{128b^2(fg-eh)p^2q^2(g+hx)^{3/2}}{225fh}$$
$$+ \frac{16b^2p^2q^2(g+hx)^{5/2}}{125h} - \frac{368b^2(fg-eh)^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{75f^{5/2}h}$$
$$- \frac{8b^2(fg-eh)^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5f^{5/2}h}$$
$$- \frac{8b(fg-eh)^2pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{5f^2h}$$
$$- \frac{8b(fg-eh)pq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{15fh}$$
$$- \frac{8bpq(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))}{25h}$$
$$+ \frac{8b(fg-eh)^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5f^{5/2}h}$$
$$+ \frac{2(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))^2}{5h}$$
$$+ \frac{16b^2(fg-eh)^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5f^{5/2}h}$$
$$+\text{Subst}\left(\frac{\left(16b^2(fg-eh)^{5/2}p^2q^2\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5f^{5/2}h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$
\begin{aligned}
&= \frac{368b^2(fg-eh)^2p^2q^2\sqrt{g+hx}}{75f^2h} + \frac{128b^2(fg-eh)p^2q^2(g+hx)^{3/2}}{225fh} \\
&+ \frac{16b^2p^2q^2(g+hx)^{5/2}}{125h} - \frac{368b^2(fg-eh)^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{75f^{5/2}h} \\
&- \frac{8b^2(fg-eh)^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5f^{5/2}h} \\
&- \frac{8b(fg-eh)^2pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{5f^2h} \\
&- \frac{8b(fg-eh)pq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{15fh} \\
&- \frac{8bpq(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))}{25h} \\
&+ \frac{8b(fg-eh)^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5f^{5/2}h} \\
&+ \frac{2(g+hx)^{5/2}(a+b\log(c(d(e+fx)^p)^q))^2}{5h} \\
&+ \frac{16b^2(fg-eh)^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5f^{5/2}h} \\
&+ \frac{8b^2(fg-eh)^{5/2}p^2q^2\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5f^{5/2}h}
\end{aligned}
$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4354 vs. 2(635) = 1270.

Time = 18.56 (sec) , antiderivative size = 4354, normalized size of antiderivative = 6.86

$$
\int (g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))^2\,dx = \text{Result too large to show}
$$

```
[In] Integrate[(g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] (4*b*g*p*q*((6*(f*g - e*h)^(3/2)*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + 
f*x))/f])/Sqrt[f*g - e*h]])/Sqrt[f] - Sqrt[(f*g - e*h + h*(e + f*x))/f]*(h*
(e + f*x)*(2 - 3*Log[e + f*x]) + (f*g - e*h)*(8 - 3*Log[e + f*x])))*(a + b*
q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]) + b*(-(q*(-(p*Log[e + f*x]) + Lo
g[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q - (q*(-(p*Log[e + f*x]) + Log[d*
(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E^(q*(-(p*Log[e + f*x]) + Log[d*
(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)
^p]))/Log[d*(e + f*x)^p])]))))/(9*f*h) - (4*b*p*q*(30*(f*g - e*h)^(3/2)*(2*f
```

```
*g + 3*e*h)*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f])/Sqrt[f*g -
e*h]] + Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f]*(9*h^2*(e + f*x)^2*(2 - 5
*Log[e + f*x]) + (f*g - e*h)*(3*e*h*(-46 + 15*Log[e + f*x]) + 2*f*g*(-31 +
15*Log[e + f*x])) + h*(e + f*x)*(f*g*(16 - 15*Log[e + f*x]) + 6*e*h*(-11 +
15*Log[e + f*x]))))*(a + b*q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]) + b*(
-(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q - (q*
(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E^(q*
(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*Log[
e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p])]))))/(225*f^(5/2)*h) +
Sqrt[g + h*x]*((2*g^2*(a + b*q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]) + b
*(-(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q - (
q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E^(
q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*Lo
g[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p])]))^2)/(5*h) + (4*g*x
*(a + b*q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]) + b*(-(q*(-(p*Log[e + f*
x]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q - (q*(-(p*Log[e + f*x])
+ Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E^(q*(-(p*Log[e + f*x])
+ Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*Log[e + f*x]) + Log[d*(
e + f*x)^p]))/Log[d*(e + f*x)^p])]))^2)/5 + (2*h*x^2*(a + b*q*(-(p*Log[e +
f*x]) + Log[d*(e + f*x)^p]) + b*(-(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p
])) - Log[d*(e + f*x)^p]*(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/
Log[d*(e + f*x)^p]) + Log[c*E^(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))*
(d*(e + f*x)^p)^(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e
+ f*x)^p])]))^2)/5) + (2*b^2*p^2*q^2*Sqrt[(f*g - e*h + h*(e + f*x))/f]*(372
0*f^3*g^3*Sqrt[h]*Sqrt[f*g - e*h]*Sqrt[e + f*x]*Sqrt[(f*g + f*h*x)/(h*(e +
f*x))]*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])] + 840*e*f^2*g^2*h
^(3/2)*Sqrt[f*g - e*h]*Sqrt[e + f*x]*Sqrt[(f*g + f*h*x)/(h*(e + f*x))]*ArcS
in[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])] - 12840*e^2*f*g*h^(5/2)*Sqrt
[f*g - e*h]*Sqrt[e + f*x]*Sqrt[(f*g + f*h*x)/(h*(e + f*x))]*ArcSin[Sqrt[-(f
*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])] + 8280*e^3*h^(7/2)*Sqrt[f*g - e*h]*Sqrt
[e + f*x]*Sqrt[(f*g + f*h*x)/(h*(e + f*x))]*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt
[h]*Sqrt[e + f*x])] + Sqrt[-(f*g - e*h)^2]*(f*g + f*h*x)*(27*h^2*(e + f*x)^
2*(8 - 20*Log[e + f*x] + 25*Log[e + f*x]^2) - (f*g - e*h)*(3*e*h*(3152 - 13
80*Log[e + f*x] + 225*Log[e + f*x]^2) + 2*f*g*(1772 - 930*Log[e + f*x] + 22
5*Log[e + f*x]^2)) + h*(e + f*x)*(f*g*(392 - 480*Log[e + f*x] + 225*Log[e +
 f*x]^2) - 6*e*h*(232 - 330*Log[e + f*x] + 225*Log[e + f*x]^2))) - 450*f^3*
g^3*Sqrt[-(f*g) + e*h]*(4*Sqrt[f*g + f*h*x]*ArcTanh[Sqrt[f*g - e*h + h*(e +
 f*x)]/Sqrt[f*g - e*h]]*(Log[e + f*x] - Log[(h*(e + f*x))/(-(f*g) + e*h)])
- Sqrt[f*g - e*h]*Sqrt[1 + (h*(e + f*x))/(f*g - e*h)]*(Log[(h*(e + f*x))/(-
(f*g) + e*h)]^2 - 4*Log[(h*(e + f*x))/(-(f*g) + e*h)]*Log[(1 + Sqrt[1 + (h*
(e + f*x))/(f*g - e*h)])/2] + 2*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)
])/2]^2 - 4*PolyLog[2, (1 - Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2])) + 225
*e*f^2*g^2*h*Sqrt[-(f*g) + e*h]*(4*Sqrt[f*g + f*h*x]*ArcTanh[Sqrt[f*g - e*h
 + h*(e + f*x)]/Sqrt[f*g - e*h]]*(Log[e + f*x] - Log[(h*(e + f*x))/(-(f*g)
+ e*h)]) - Sqrt[f*g - e*h]*Sqrt[1 + (h*(e + f*x))/(f*g - e*h)]*(Log[(h*(e +
```

f*x))/(-(f*g) + e*h)]^2 - 4*Log[(h*(e + f*x))/(-(f*g) + e*h)]*Log[(1 + Sqr
t[1 + (h*(e + f*x))/(f*g - e*h)])/2] + 2*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f
*g - e*h)])/2]^2 - 4*PolyLog[2, (1 - Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2
])) + 900*e^2*f*g*h^2*Sqrt[-(f*g) + e*h]*(4*Sqrt[f*g + f*h*x]*ArcTanh[Sqrt[
f*g - e*h + h*(e + f*x)]/Sqrt[f*g - e*h]]*(Log[e + f*x] - Log[(h*(e + f*x))
/(-(f*g) + e*h)]) - Sqrt[f*g - e*h]*Sqrt[1 + (h*(e + f*x))/(f*g - e*h)]*(Lo
g[(h*(e + f*x))/(-(f*g) + e*h)]^2 - 4*Log[(h*(e + f*x))/(-(f*g) + e*h)]*Log
[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2] + 2*Log[(1 + Sqrt[1 + (h*(e +
f*x))/(f*g - e*h)])/2]^2 - 4*PolyLog[2, (1 - Sqrt[1 + (h*(e + f*x))/(f*g -
e*h)])/2])) - 675*e^3*h^3*Sqrt[-(f*g) + e*h]*(4*Sqrt[f*g + f*h*x]*ArcTanh[
Sqrt[f*g - e*h + h*(e + f*x)]/Sqrt[f*g - e*h]]*(Log[e + f*x] - Log[(h*(e +
f*x))/(-(f*g) + e*h)]) - Sqrt[f*g - e*h]*Sqrt[1 + (h*(e + f*x))/(f*g - e*h)
]*(Log[(h*(e + f*x))/(-(f*g) + e*h)]^2 - 4*Log[(h*(e + f*x))/(-(f*g) + e*h)
]*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2] + 2*Log[(1 + Sqrt[1 + (h
*(e + f*x))/(f*g - e*h)])/2]^2 - 4*PolyLog[2, (1 - Sqrt[1 + (h*(e + f*x))/(
f*g - e*h)])/2]))))/(3375*f^2*h*Sqrt[-(f*g - e*h)^2]*(f*g + f*h*x)) + (b^2*
g*p^2*q^2*((-64*f^2*g^2*Sqrt[e + f*x]*Sqrt[1 + (f*g - e*h)/(h*(e + f*x))]*S
qrt[(f*g - e*h + h*(e + f*x))/f]*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e
+ f*x])])/(9*Sqrt[h]*Sqrt[-(f*g) + e*h]*(f*g - e*h + h*(e + f*x))) + (128*e
*f*g*Sqrt[h]*Sqrt[e + f*x]*Sqrt[1 + (f*g - e*h)/(h*(e + f*x))]*Sqrt[(f*g -
e*h + h*(e + f*x))/f]*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])])/(
9*Sqrt[-(f*g) + e*h]*(f*g - e*h + h*(e + f*x))) - (64*e^2*h^(3/2)*Sqrt[e +
f*x]*Sqrt[1 + (f*g - e*h)/(h*(e + f*x))]*Sqrt[(f*g - e*h + h*(e + f*x))/f]*
ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])])/(9*Sqrt[-(f*g) + e*h]*(
f*g - e*h + h*(e + f*x))) + Sqrt[(f*g - e*h + h*(e + f*x))/f]*((-2*(-(f*g)
+ e*h)*(104 - 48*Log[e + f*x] + 9*Log[e + f*x]^2))/(27*h) + (2*(e + f*x)*(8
- 12*Log[e + f*x] + 9*Log[e + f*x]^2))/27) + (4*e*f*g*Sqrt[(f*g - e*h + h*
(e + f*x))/f]*((-4*ArcTanh[Sqrt[f*g - e*h + h*(e + f*x)]/Sqrt[f*g - e*h]]*(
Log[e + f*x] - Log[-((h*(e + f*x))/(f*g - e*h))]))/Sqrt[f*g - e*h] + (Sqrt[
1 + (h*(e + f*x))/(f*g - e*h)]*(Log[-((h*(e + f*x))/(f*g - e*h))]^2 - 4*Log
[-((h*(e + f*x))/(f*g - e*h))]*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)]
)/2] + 2*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2]^2 - 4*PolyLog[2,
1 + (-1 - Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2]))/Sqrt[f*g - e*h + h*(e +
f*x)]))/(3*Sqrt[f*g - e*h + h*(e + f*x)]) - (2*f^2*g^2*Sqrt[(f*g - e*h + h
*(e + f*x))/f]*((-4*ArcTanh[Sqrt[f*g - e*h + h*(e + f*x)]/Sqrt[f*g - e*h]]*
(Log[e + f*x] - Log[-((h*(e + f*x))/(f*g - e*h))]))/Sqrt[f*g - e*h] + (Sqrt
[1 + (h*(e + f*x))/(f*g - e*h)]*(Log[-((h*(e + f*x))/(f*g - e*h))]^2 - 4*Lo
g[-((h*(e + f*x))/(f*g - e*h))]*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)
])/2] + 2*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2]^2 - 4*PolyLog[2,
1 + (-1 - Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2]))/Sqrt[f*g - e*h + h*(e
+ f*x)]))/(3*h*Sqrt[f*g - e*h + h*(e + f*x)]) - (2*e^2*h*Sqrt[(f*g - e*h +
h*(e + f*x))/f]*((-4*ArcTanh[Sqrt[f*g - e*h + h*(e + f*x)]/Sqrt[f*g - e*h]]
*(Log[e + f*x] - Log[-((h*(e + f*x))/(f*g - e*h))]))/Sqrt[f*g - e*h] + (Sqr
t[1 + (h*(e + f*x))/(f*g - e*h)]*(Log[-((h*(e + f*x))/(f*g - e*h))]^2 - 4*L
og[-((h*(e + f*x))/(f*g - e*h))]*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h

```
)])/2] + 2*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2]^2 - 4*PolyLog[2
, 1 + (-1 - Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2]))/Sqrt[f*g - e*h + h*(e
 + f*x)]))/(3*Sqrt[f*g - e*h + h*(e + f*x)])))/f
```

## Maple [F]

$$\int (hx+g)^{\frac{3}{2}} \left(a+b\ln \left(c(d(fx+e)^{p})^{q}\right)\right)^{2}dx$$

```
[In] int((h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
[Out] int((h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Fricas [F]

$$\int (g+hx)^{3/2} \left(a+b\log \left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{2} dx = \int (hx+g)^{\frac{3}{2}}\left(b\log \left(\left((fx+e)^{p}d\right)^{q}c\right)+a\right)^{2} dx$$

```
[In] integrate((h*x+g)^(3/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas
")
[Out] integral((b^2*h*x + b^2*g)*sqrt(h*x + g)*log(((f*x + e)^p*d)^q*c)^2 + 2*(a*
b*h*x + a*b*g)*sqrt(h*x + g)*log(((f*x + e)^p*d)^q*c) + (a^2*h*x + a^2*g)*s
qrt(h*x + g), x)
```

## Sympy [F]

$$\int (g+hx)^{3/2} \left(a+b\log \left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{2} dx = \int \left(a+b\log \left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{2} (g+hx)^{\frac{3}{2}} dx$$

```
[In] integrate((h*x+g)**(3/2)*(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^{3/2} \left(a+b\log \left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{2} dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)^(3/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima
")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int (g+hx)^{3/2} \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 dx = \int (hx+g)^{\frac{3}{2}} \left(b\log\left(\left(((fx+e)^p d\right)^q c\right)+a\right)^2 dx$$

```
[In] integrate((h*x+g)^(3/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
[Out] integrate((h*x + g)^(3/2)*(b*log(((f*x + e)^p*d)^q*c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (g+hx)^{3/2} \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2 dx = \int (g+h\,x)^{3/2} \left(a+b\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)\right)^2 dx$$

```
[In] int((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
[Out] int((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^2, x)
```

# 3.490 $\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 547

$$\begin{aligned}
&\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2\,dx \\
&= \frac{64b^2(fg-eh)p^2q^2\sqrt{g+hx}}{9fh}+\frac{16b^2p^2q^2(g+hx)^{3/2}}{27h} \\
&-\frac{64b^2(fg-eh)^{3/2}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{9f^{3/2}h}-\frac{8b^2(fg-eh)^{3/2}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3f^{3/2}h} \\
&-\frac{8b(fg-eh)pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{3fh} \\
&-\frac{8bpq(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9h} \\
&+\frac{8b(fg-eh)^{3/2}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3f^{3/2}h} \\
&+\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h} \\
&+\frac{16b^2(fg-eh)^{3/2}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3f^{3/2}h} \\
&+\frac{8b^2(fg-eh)^{3/2}p^2q^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3f^{3/2}h}
\end{aligned}$$

```
[Out] 16/27*b^2*p^2*q^2*(h*x+g)^(3/2)/h-64/9*b^2*(-e*h+f*g)^(3/2)*p^2*q^2*arctanh
(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/f^(3/2)/h-8/3*b^2*(-e*h+f*g)^(3/2)
*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))^2/f^(3/2)/h-8/9*b*
p*q*(h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))/h+8/3*b*(-e*h+f*g)^(3/2)*p*q*
```

```
arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*(a+b*ln(c*(d*(f*x+e)^p)^q))
/f^(3/2)/h+2/3*(h*x+g)^(3/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/h+16/3*b^2*(-e*h
+f*g)^(3/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*ln(2/(1
-f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))/f^(3/2)/h+8/3*b^2*(-e*h+f*g)^(3/2
)*p^2*q^2*polylog(2,1-2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))/f^(3/2)
/h+64/9*b^2*(-e*h+f*g)*p^2*q^2*(h*x+g)^(1/2)/f/h-8/3*b*(-e*h+f*g)*p*q*(a+b*
ln(c*(d*(f*x+e)^p)^q))*(h*x+g)^(1/2)/f/h
```

## Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2445, 2458, 2388, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 52, 2495}

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2\,dx$$
$$= \frac{8bpq(fg-eh)^{3/2}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3f^{3/2}h} - \frac{8bpq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{9h} - \frac{8bpq\sqrt{g+hx}(fg-eh)(a+b\log(c(d(e+fx)^p)^q))}{3fh} + \frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))^2}{3h} - \frac{8b^2p^2q^2(fg-eh)^{3/2}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3f^{3/2}h} - \frac{64b^2p^2q^2(fg-eh)^{3/2}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{9f^{3/2}h} + \frac{16b^2p^2q^2(fg-eh)^{3/2}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3f^{3/2}h} + \frac{8b^2p^2q^2(fg-eh)^{3/2}\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3f^{3/2}h} + \frac{64b^2p^2q^2\sqrt{g+hx}(fg-eh)}{9fh} + \frac{16b^2p^2q^2(g+hx)^{3/2}}{27h}$$

```
[In] Int[Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] (64*b^2*(f*g - e*h)*p^2*q^2*Sqrt[g + h*x])/(9*f*h) + (16*b^2*p^2*q^2*(g + h
*x)^(3/2))/(27*h) - (64*b^2*(f*g - e*h)^(3/2)*p^2*q^2*ArcTanh[(Sqrt[f]*Sqrt
[g + h*x])/Sqrt[f*g - e*h]])/(9*f^(3/2)*h) - (8*b^2*(f*g - e*h)^(3/2)*p^2*q
^2*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]^2)/(3*f^(3/2)*h) - (8*b
```

```
*(f*g - e*h)*p*q*Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(3*f*h) -
(8*b*p*q*(g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(9*h) + (8*b*(f*
g - e*h)^(3/2)*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]*(a + b*
Log[c*(d*(e + f*x)^p)^q]))/(3*f^(3/2)*h) + (2*(g + h*x)^(3/2)*(a + b*Log[c*
(d*(e + f*x)^p)^q])^2)/(3*h) + (16*b^2*(f*g - e*h)^(3/2)*p^2*q^2*ArcTanh[(S
qrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]*Log[2/(1 - (Sqrt[f]*Sqrt[g + h*x])/S
qrt[f*g - e*h])])/(3*f^(3/2)*h) + (8*b^2*(f*g - e*h)^(3/2)*p^2*q^2*PolyLog[
2, 1 - 2/(1 - (Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h])])/(3*f^(3/2)*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \sqrt{g+hx}\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h} - \text{Subst}\left(\frac{(4bfpq)\int \frac{(g+hx)^{3/2}(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}\,dx}{3h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h} - \text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int \frac{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{3/2}(a+b\log(cd^q x^{pq}))}{x}\,dx, x, e+fx\right)}{3h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}$$

$$-\text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}(a+b\log\left(cd^qx^{pq}\right))\,dx,x,e+fx\right)}{3f},cd^q(e+fx)^{pq},c(d(e+f$$

$$-\text{Subst}\left(\frac{(4b(fg-eh)pq)\text{Subst}\left(\int\frac{\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}(a+b\log(cd^qx^{pq}))}{x}\,dx,x,e+fx\right)}{3fh},cd^q(e+fx)^{pq},c(d(e+$$

$$= -\frac{8bpq(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9h}+\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}$$

$$-\text{Subst}\left(\frac{(4b(fg-eh)pq)\text{Subst}\left(\int\frac{a+b\log(cd^qx^{pq})}{\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}}\,dx,x,e+fx\right)}{3f^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$-\text{Subst}\left(\frac{(4b(fg-eh)^2pq)\,\text{Subst}\left(\int\frac{a+b\log(cd^qx^{pq})}{x\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}}\,dx,x,e+fx\right)}{3f^2h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{(8b^2p^2q^2)\,\text{Subst}\left(\int\frac{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{3/2}}{x}\,dx,x,e+fx\right)}{9h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{16b^2p^2q^2(g+hx)^{3/2}}{27h}-\frac{8b(fg-eh)pq\sqrt{g+hx}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3fh}$$

$$-\frac{8bpq(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{9h}$$

$$+\frac{8b(fg-eh)^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3f^{3/2}h}$$

$$+\frac{2(g+hx)^{3/2}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{3h}$$

$$+\text{Subst}\left(\frac{(8b^2(fg-eh)p^2q^2)\,\text{Subst}\left(\int\frac{\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}}{x}\,dx,x,e+fx\right)}{9fh},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)+$$

$$= \frac{64b^2(fg-eh)p^2q^2\sqrt{g+hx}}{9fh} + \frac{16b^2p^2q^2(g+hx)^{3/2}}{27h} - \frac{8b(fg-eh)pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{3fh} - \frac{8bpq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{9h} + \frac{8b(fg-eh)^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3f^{3/2}h} + \frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))^2}{3h} - \text{Subst}\left(\frac{(8b^2(fg-eh)^{3/2}p^2q^2)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{hx}{f}}}{\sqrt{fg-eh}}\right)}{x}dx, x, e+fx\right)}{3f^{3/2}h}, cd^q(e+fx)^{pq}, c(d(e\right.$$

$$= \frac{64b^2(fg-eh)p^2q^2\sqrt{g+hx}}{9fh} + \frac{16b^2p^2q^2(g+hx)^{3/2}}{27h} - \frac{8b(fg-eh)pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{3fh} - \frac{8bpq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{9h} + \frac{8b(fg-eh)^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3f^{3/2}h} + \frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))^2}{3h} - \text{Subst}\left(\frac{(16b^2(fg-eh)^{3/2}p^2q^2)\,\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{eh+f(-g+x^2)}dx, x, \sqrt{g+hx}\right)}{3\sqrt{f}h}, cd^q(e+fx)^{pq}, c(d(e+\right.$$

$$= \frac{64b^2(fg-eh)p^2q^2\sqrt{g+hx}}{9fh} + \frac{16b^2p^2q^2(g+hx)^{3/2}}{27h} - \frac{64b^2(fg-eh)^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{9f^{3/2}h} - \frac{8b(fg-eh)pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{3fh} - \frac{8bpq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{9h} + \frac{8b(fg-eh)^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3f^{3/2}h} + \frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))^2}{3h} - \text{Subst}\left(\frac{\left(16b^2(fg-eh)^{3/2}p^2q^2\right)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{-fg+eh+fx^2}dx,x,\sqrt{g+hx}\right)}{3\sqrt{f}h}, cd^q(e+fx)^{pq}, c(d(e+\right.$$

$$= \frac{64b^2(fg-eh)p^2q^2\sqrt{g+hx}}{9fh} + \frac{16b^2p^2q^2(g+hx)^{3/2}}{27h} - \frac{64b^2(fg-eh)^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{9f^{3/2}h} - \frac{8b^2(fg-eh)^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3f^{3/2}h} - \frac{8b(fg-eh)pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{3fh} - \frac{8bpq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{9h} + \frac{8b(fg-eh)^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3f^{3/2}h} + \frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))^2}{3h} + \text{Subst}\left(\frac{\left(16b^2(fg-eh)p^2q^2\right)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}dx,x,\sqrt{g+hx}\right)}{3fh}, cd^q(e+fx)^{pq}, c(d(e+fx)\right.$$

$$= \frac{64b^2(fg-eh)p^2q^2\sqrt{g+hx}}{9fh} + \frac{16b^2p^2q^2(g+hx)^{3/2}}{27h}$$

$$-\frac{64b^2(fg-eh)^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{9f^{3/2}h}$$

$$-\frac{8b^2(fg-eh)^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3f^{3/2}h}$$

$$-\frac{8b(fg-eh)pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{3fh}$$

$$-\frac{8bpq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{9h}$$

$$+\frac{8b(fg-eh)^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3f^{3/2}h}$$

$$+\frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))^2}{3h}$$

$$+\frac{16b^2(fg-eh)^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3f^{3/2}h}$$

$$-\text{Subst}\left(\frac{(16b^2(fg-eh)p^2q^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}\right)}{1-\frac{fx^2}{fg-eh}}\,dx, x, \sqrt{g+hx}\right)}{3fh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p$$

$$= \frac{64b^2(fg-eh)p^2q^2\sqrt{g+hx}}{9fh} + \frac{16b^2p^2q^2(g+hx)^{3/2}}{27h}$$
$$- \frac{64b^2(fg-eh)^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{9f^{3/2}h}$$
$$- \frac{8b^2(fg-eh)^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3f^{3/2}h}$$
$$- \frac{8b(fg-eh)pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{3fh}$$
$$- \frac{8bpq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{9h}$$
$$+ \frac{8b(fg-eh)^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3f^{3/2}h}$$
$$+ \frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))^2}{3h}$$
$$+ \frac{16b^2(fg-eh)^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3f^{3/2}h}$$
$$+ \text{Subst}\left(\frac{(16b^2(fg-eh)^{3/2}p^2q^2)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3f^{3/2}h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{64b^2(fg-eh)p^2q^2\sqrt{g+hx}}{9fh} + \frac{16b^2p^2q^2(g+hx)^{3/2}}{27h} - \frac{64b^2(fg-eh)^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{9f^{3/2}h} - \frac{8b^2(fg-eh)^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3f^{3/2}h} - \frac{8b(fg-eh)pq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{3fh} - \frac{8bpq(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}{9h} + \frac{8b(fg-eh)^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3f^{3/2}h} + \frac{2(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))^2}{3h} + \frac{16b^2(fg-eh)^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3f^{3/2}h} + \frac{8b^2(fg-eh)^{3/2}p^2q^2\mathrm{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3f^{3/2}h}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1323 vs. $2(547) = 1094$.

Time = 11.77 (sec) , antiderivative size = 1323, normalized size of antiderivative = 2.42

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2\,dx$$

$$= \frac{2\left(-\frac{6bpq\left(6(fg-eh)^{3/2}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)+\sqrt{f}\sqrt{g+hx}(6eh-2f(4g+hx)+3f(g+hx)\log(e+fx))\right)(-a+bpq\log(e+fx)-b\log(c(d(e+fx)^p)^q}{f^{3/2}}\right.}{}$$

```
[In] Integrate[Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] (2*((-6*b*p*q*(6*(f*g - e*h)^(3/2)*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g
 - e*h]] + Sqrt[f]*Sqrt[g + h*x]*(6*e*h - 2*f*(4*g + h*x) + 3*f*(g + h*x)*L
og[e + f*x]))*(-a + b*p*q*Log[e + f*x] - b*Log[c*(d*(e + f*x)^p)^q]))/f^(3/
2) + 9*(g + h*x)^(3/2)*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q]
)^2 - (b^2*p^2*q^2*(96*f^2*g^2*h^(3/2)*Sqrt[f*g - e*h]*(e + f*x)^(3/2)*((f*
(g + h*x))/(h*(e + f*x)))^(3/2)*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e +
 f*x])] - 192*e*f*g*h^(5/2)*Sqrt[f*g - e*h]*(e + f*x)^(3/2)*((f*(g + h*x))/
```

```
(h*(e + f*x)))^(3/2)*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])] + 9
6*e^2*h^(7/2)*Sqrt[f*g - e*h]*(e + f*x)^(3/2)*((f*(g + h*x))/(h*(e + f*x)))
^(3/2)*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])] - f^2*Sqrt[-(f*g
- e*h)^2]*(g + h*x)^2*(8*(13*f*g - 12*e*h + f*h*x) - 12*(4*f*g - 3*e*h + f*
h*x)*Log[e + f*x] + 9*f*(g + h*x)*Log[e + f*x]^2) - 9*f^3*g^2*Sqrt[-(f*g) +
 e*h]*(g + h*x)*(4*Sqrt[f*(g + h*x)]*ArcTanh[Sqrt[f*(g + h*x)]/Sqrt[f*g - e
*h]]*(Log[e + f*x] - Log[(h*(e + f*x))/(-(f*g) + e*h)]) - Sqrt[f*g - e*h]*S
qrt[(f*(g + h*x))/(f*g - e*h)]*(Log[(h*(e + f*x))/(-(f*g) + e*h)]^2 - 4*Log
[(h*(e + f*x))/(-(f*g) + e*h)]*Log[(1 + Sqrt[(f*(g + h*x))/(f*g - e*h)])/2]
 + 2*Log[(1 + Sqrt[(f*(g + h*x))/(f*g - e*h)])/2]^2 - 4*PolyLog[2, 1/2 - Sq
rt[(f*(g + h*x))/(f*g - e*h)]/2])) + 18*e*f^2*g*h*Sqrt[-(f*g) + e*h]*(g + h
*x)*(4*Sqrt[f*(g + h*x)]*ArcTanh[Sqrt[f*(g + h*x)]/Sqrt[f*g - e*h]]*(Log[e
+ f*x] - Log[(h*(e + f*x))/(-(f*g) + e*h)]) - Sqrt[f*g - e*h]*Sqrt[(f*(g +
h*x))/(f*g - e*h)]*(Log[(h*(e + f*x))/(-(f*g) + e*h)]^2 - 4*Log[(h*(e + f*x
))/(-(f*g) + e*h)]*Log[(1 + Sqrt[(f*(g + h*x))/(f*g - e*h)])/2] + 2*Log[(1
+ Sqrt[(f*(g + h*x))/(f*g - e*h)])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[(f*(g + h
*x))/(f*g - e*h)]/2])) - 9*e^2*f*h^2*Sqrt[-(f*g) + e*h]*(g + h*x)*(4*Sqrt[f
*(g + h*x)]*ArcTanh[Sqrt[f*(g + h*x)]/Sqrt[f*g - e*h]]*(Log[e + f*x] - Log[
(h*(e + f*x))/(-(f*g) + e*h)]) - Sqrt[f*g - e*h]*Sqrt[(f*(g + h*x))/(f*g -
e*h)]*(Log[(h*(e + f*x))/(-(f*g) + e*h)]^2 - 4*Log[(h*(e + f*x))/(-(f*g) +
e*h)]*Log[(1 + Sqrt[(f*(g + h*x))/(f*g - e*h)])/2] + 2*Log[(1 + Sqrt[(f*(g
+ h*x))/(f*g - e*h)])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[(f*(g + h*x))/(f*g - e
*h)]/2]))))/(f^3*Sqrt[-(f*g - e*h)^2]*(g + h*x)^(3/2))))/(27*h)
```

## Maple [F]

$$\int \sqrt{hx+g}\,(a+b\ln(c(d(fx+e)^p)^q))^2dx$$

[In] int((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)

[Out] int((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)

## Fricas [F]

$$\int \sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2\,dx=\int \sqrt{hx+g}(b\log(((fx+e)^pd)^q\,c)+a)^2\,dx$$

[In] integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas
")

[Out] integral(sqrt(h*x + g)*b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*sqrt(h*x + g)*a*b
*log(((f*x + e)^p*d)^q*c) + sqrt(h*x + g)*a^2, x)

## Sympy [F]

$$\int \sqrt{g+hx}(a+b\log\left(c(d(e+fx)^p)^q\right))^2\,dx = \int \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\sqrt{g+hx}\,dx$$

```
[In] integrate((h*x+g)**(1/2)*(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2*sqrt(g + h*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \sqrt{g+hx}(a+b\log\left(c(d(e+fx)^p)^q\right))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima
")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int \sqrt{g+hx}(a+b\log\left(c(d(e+fx)^p)^q\right))^2\,dx = \int \sqrt{hx+g}(b\log\left(((fx+e)^p d)^q c\right)+a)^2\,dx$$

```
[In] integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
[Out] integrate(sqrt(h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{g+hx}(a+b\log\left(c(d(e+fx)^p)^q\right))^2\,dx = \int \sqrt{g+h\,x}\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^2\,dx$$

```
[In] int((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
[Out] int((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^2, x)
```

# 3.491 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 447

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx = \frac{16b^2p^2q^2\sqrt{g+hx}}{h} - \frac{16b^2\sqrt{fg-eh}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}h} - \frac{8b^2\sqrt{fg-eh}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{\sqrt{f}h} - \frac{8bpq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h} + \frac{8b\sqrt{fg-eh}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{f}h} + \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h} + \frac{16b^2\sqrt{fg-eh}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}h} + \frac{8b^2\sqrt{fg-eh}p^2q^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}h}$$

```
[Out] -16*b^2*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*(-e*h+f*g)^
(1/2)/h/f^(1/2)-8*b^2*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2
))^2*(-e*h+f*g)^(1/2)/h/f^(1/2)+8*b*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h
+f*g)^(1/2))*(a+b*ln(c*(d*(f*x+e)^p)^q))*(-e*h+f*g)^(1/2)/h/f^(1/2)+16*b^2*
p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*ln(2/(1-f^(1/2)*(h*
x+g)^(1/2)/(-e*h+f*g)^(1/2)))*(-e*h+f*g)^(1/2)/h/f^(1/2)+8*b^2*p^2*q^2*poly
```

```
log(2,1-2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))*(-e*h+f*g)^(1/2)/h/f^
(1/2)+16*b^2*p^2*q^2*(h*x+g)^(1/2)/h-8*b*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))*(h
*x+g)^(1/2)/h+2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2*(h*x+g)^(1/2)/h
```

## Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2445, 2458, 2388, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 52, 2495}

$$
\begin{aligned}
&\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx \\
&= \frac{8bpq\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{f}h} \\
&\quad -\frac{8bpq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h}+\frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h} \\
&\quad -\frac{8b^2p^2q^2\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{\sqrt{f}h}-\frac{16b^2p^2q^2\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}h} \\
&\quad +\frac{16b^2p^2q^2\sqrt{fg-eh}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}h} \\
&\quad +\frac{8b^2p^2q^2\sqrt{fg-eh}\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}h}+\frac{16b^2p^2q^2\sqrt{g+hx}}{h}
\end{aligned}
$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/Sqrt[g + h*x],x]
```

```
[Out] (16*b^2*p^2*q^2*Sqrt[g + h*x])/h - (16*b^2*Sqrt[f*g - e*h]*p^2*q^2*ArcTanh[
(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/(Sqrt[f]*h) - (8*b^2*Sqrt[f*g - e
*h]*p^2*q^2*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]^2)/(Sqrt[f]*h)
 - (8*b*p*q*Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q]))/h + (8*b*Sqrt[f
*g - e*h]*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]*(a + b*Log[c
*(d*(e + f*x)^p)^q]))/(Sqrt[f]*h) + (2*Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f
*x)^p)^q])^2)/h + (16*b^2*Sqrt[f*g - e*h]*p^2*q^2*ArcTanh[(Sqrt[f]*Sqrt[g +
 h*x])/Sqrt[f*g - e*h]]*Log[2/(1 - (Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h])
])/(Sqrt[f]*h) + (8*b^2*Sqrt[f*g - e*h]*p^2*q^2*PolyLog[2, 1 - 2/(1 - (Sqrt
[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h])])/(Sqrt[f]*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
```

```
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
```

```
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{\sqrt{g+hx}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h} - \text{Subst}\left(\frac{(4bfpq)\int \frac{\sqrt{g+hx}(a+b\log(cd^q(e+fx)^{pq}))}{e+fx}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h} - \text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int \frac{\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}(a+b\log(cd^qx^{pq}))}{x}\,dx, x, e+fx\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{2\sqrt{g+hx}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{h} - \text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}}\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right) - \text{Subst}\left(\frac{(4b(fg-eh)pq)\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{x\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}}\,dx, x, e+fx\right)}{fh}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= -\frac{8bpq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h} + \frac{8b\sqrt{fg-eh}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{f}h} + \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h} + \text{Subst}\left(\frac{(8b^2p^2q^2)\,\text{Subst}\left(\int \frac{\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}}{x}\,dx, x, e+fx\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(4b^2(fg-eh)p^2q^2)\,\text{Subst}\left(\int -\frac{2\sqrt{f}\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{hx}{f}}}{\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}x}\,dx, x, e+fx\right)}{fh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{16b^2p^2q^2\sqrt{g+hx}}{h} - \frac{8bpq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h} + \frac{8b\sqrt{fg-eh}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{f}h} + \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h} - \text{Subst}\left(\frac{\left(8b^2\sqrt{fg-eh}p^2q^2\right)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{hx}{f}}}{\sqrt{fg-eh}}\right)}{x}dx,x,e+fx\right)}{\sqrt{f}h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{\left(8b^2(fg-eh)p^2q^2\right)\text{Subst}\left(\int\frac{1}{x\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}}dx,x,e+fx\right)}{fh},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{16b^2p^2q^2\sqrt{g+hx}}{h} - \frac{8bpq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h} + \frac{8b\sqrt{fg-eh}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{f}h} + \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h} - \text{Subst}\left(\frac{(16b^2\sqrt{f}\sqrt{fg-eh}p^2q^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{eh+f(-g+x^2)}\,dx, x, \sqrt{g+hx}\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(16b^2(fg-eh)p^2q^2)\,\text{Subst}\left(\int\frac{1}{-\frac{fg-eh}{h}+\frac{fx^2}{h}}\,dx, x, \sqrt{g+hx}\right)}{h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{16b^2p^2q^2\sqrt{g+hx}}{h} - \frac{16b^2\sqrt{fg-eh}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}h} - \frac{8bpq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h} + \frac{8b\sqrt{fg-eh}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{f}h} + \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h} - \text{Subst}\left(\frac{(16b^2\sqrt{f}\sqrt{fg-eh}p^2q^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{-fg+eh+fx^2}\,dx, x, \sqrt{g+hx}\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{16b^2p^2q^2\sqrt{g+hx}}{h} - \frac{16b^2\sqrt{fg-eh}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}h}$$

$$- \frac{8b^2\sqrt{fg-eh}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{\sqrt{f}h}$$

$$- \frac{8bpq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h}$$

$$+ \frac{8b\sqrt{fg-eh}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{f}h}$$

$$+ \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h}$$

$$+ \text{Subst}\left(\frac{(16b^2p^2q^2)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}\,dx, x, \sqrt{g+hx}\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{16b^2p^2q^2\sqrt{g+hx}}{h} - \frac{16b^2\sqrt{fg-eh}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}h}$$

$$- \frac{8b^2\sqrt{fg-eh}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{\sqrt{f}h}$$

$$- \frac{8bpq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h}$$

$$+ \frac{8b\sqrt{fg-eh}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{f}h}$$

$$+ \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h}$$

$$+ \frac{16b^2\sqrt{fg-eh}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}h}$$

$$- \text{Subst}\left(\frac{(16b^2p^2q^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}\right)}{1-\frac{fx^2}{fg-eh}}dx, x, \sqrt{g+hx}\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{16b^2p^2q^2\sqrt{g+hx}}{h} - \frac{16b^2\sqrt{fg-eh}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}h} - \frac{8b^2\sqrt{fg-eh}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{\sqrt{f}h} - \frac{8bpq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h} + \frac{8b\sqrt{fg-eh}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{f}h} + \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h} + \frac{16b^2\sqrt{fg-eh}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}h} + \text{Subst}\left(\frac{\left(16b^2\sqrt{fg-eh}p^2q^2\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{16b^2p^2q^2\sqrt{g+hx}}{h} - \frac{16b^2\sqrt{fg-eh}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{\sqrt{f}h} - \frac{8b^2\sqrt{fg-eh}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{\sqrt{f}h} - \frac{8bpq\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}{h} + \frac{8b\sqrt{fg-eh}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{f}h} + \frac{2\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))^2}{h} + \frac{16b^2\sqrt{fg-eh}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}h} + \frac{8b^2\sqrt{fg-eh}p^2q^2\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{\sqrt{f}h}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1407 vs. $2(447) = 894$.

Time = 12.63 (sec) , antiderivative size = 1407, normalized size of antiderivative = 3.15

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx$$

$$= \frac{2bpq\left(\frac{4\sqrt{f}\sqrt{fg-eh}\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{h}+\frac{2f\sqrt{\frac{fg-eh+h(e+fx)}{f}}(-2+\log(e+fx))}{h}\right)\left(a+bq(-p\log(e+fx)+\text{lo}\right.}{}$$

$$+\frac{2\sqrt{g+hx}\left(a+bq(-p\log(e+fx)+\log(d(e+fx)^p))+b\left(-q(-p\log(e+fx)+\log(d(e+fx)^p))-\text{lo}\right.\right.}{}$$

$$+\frac{b^2p^2q^2\left(-\frac{16f^2g\sqrt{e+fx}\sqrt{1+\frac{fg-eh}{h(e+fx)}}\sqrt{\frac{fg-eh+h(e+fx)}{f}}\arcsin\left(\frac{\sqrt{-fg+eh}}{\sqrt{h}\sqrt{e+fx}}\right)}{\sqrt{h}\sqrt{-fg+eh}(fg-eh+h(e+fx))}+\frac{16ef\sqrt{h}\sqrt{e+fx}\sqrt{1+\frac{fg-eh}{h(e+fx)}}\sqrt{\frac{fg-eh+h(e+fx)}{f}}\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.}{\sqrt{-fg+eh}(fg-eh+h(e+fx))}\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/Sqrt[g + h*x],x]
```

```
[Out] (2*b*p*q*((4*Sqrt[f]*Sqrt[f*g - e*h]*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(
e + f*x))/f])/Sqrt[f*g - e*h]])/h + (2*f*Sqrt[(f*g - e*h + h*(e + f*x))/f]*
(-2 + Log[e + f*x]))/h)*(a + b*q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]) +
 b*(-(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q -
 (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E
^(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*
Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p])])))/f + (2*Sqrt[g
+ h*x]*(a + b*q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]) + b*(-(q*(-(p*Log[
e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q - (q*(-(p*Log[e +
f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E^(q*(-(p*Log[e +
f*x]) + Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*Log[e + f*x]) + L
og[d*(e + f*x)^p]))/Log[d*(e + f*x)^p])]))^2)/h + (b^2*p^2*q^2*((-16*f^2*g*
Sqrt[e + f*x]*Sqrt[1 + (f*g - e*h)/(h*(e + f*x))]*Sqrt[(f*g - e*h + h*(e +
f*x))/f]*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])])/(Sqrt[h]*Sqrt[
-(f*g) + e*h]*(f*g - e*h + h*(e + f*x))) + (16*e*f*Sqrt[h]*Sqrt[e + f*x]*Sq
rt[1 + (f*g - e*h)/(h*(e + f*x))]*Sqrt[(f*g - e*h + h*(e + f*x))/f]*ArcSin[
Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])])/(Sqrt[-(f*g) + e*h]*(f*g - e*h
 + h*(e + f*x))) + (2*f*Sqrt[(f*g - e*h + h*(e + f*x))/f]*(8 - 4*Log[e + f*
x] + Log[e + f*x]^2))/h + (2*e*f*Sqrt[(f*g - e*h + h*(e + f*x))/f]*((-4*Arc
Tanh[Sqrt[f*g - e*h + h*(e + f*x)]/Sqrt[f*g - e*h]]*(Log[e + f*x] - Log[-((
```

```
h*(e + f*x))/(f*g - e*h))]))/Sqrt[f*g - e*h] + (Sqrt[1 + (h*(e + f*x))/(f*g
 - e*h)]*(Log[-((h*(e + f*x))/(f*g - e*h))]^2 - 4*Log[-((h*(e + f*x))/(f*g
- e*h))]*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2] + 2*Log[(1 + Sqrt
[1 + (h*(e + f*x))/(f*g - e*h)])/2]^2 - 4*PolyLog[2, 1 + (-1 - Sqrt[1 + (h*
(e + f*x))/(f*g - e*h)])/2]))/Sqrt[f*g - e*h + h*(e + f*x)]))/Sqrt[f*g - e*
h + h*(e + f*x)] - (2*f^2*g*Sqrt[(f*g - e*h + h*(e + f*x))/f]*((-4*ArcTanh[
Sqrt[f*g - e*h + h*(e + f*x)]/Sqrt[f*g - e*h]]*(Log[e + f*x] - Log[-((h*(e
+ f*x))/(f*g - e*h))]))/Sqrt[f*g - e*h] + (Sqrt[1 + (h*(e + f*x))/(f*g - e*
h)]*(Log[-((h*(e + f*x))/(f*g - e*h))]^2 - 4*Log[-((h*(e + f*x))/(f*g - e*h
))]*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2] + 2*Log[(1 + Sqrt[1 +
(h*(e + f*x))/(f*g - e*h)])/2]^2 - 4*PolyLog[2, 1 + (-1 - Sqrt[1 + (h*(e +
f*x))/(f*g - e*h)])/2]))/Sqrt[f*g - e*h + h*(e + f*x)]))/(h*Sqrt[f*g - e*h
+ h*(e + f*x)])))/f
```

## Maple [F]

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^2}{\sqrt{hx+g}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(1/2),x)
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(1/2),x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{\sqrt{hx+g}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(1/2),x, algorithm="fricas
")
[Out] integral((sqrt(h*x + g)*b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*sqrt(h*x + g)*a*
b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x + g)*a^2)/(h*x + g), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{\sqrt{g+hx}}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**(1/2),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/sqrt(g + h*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^2}{\sqrt{g + hx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^2}{\sqrt{g + hx}}\,dx = \int \frac{(b\log(((fx + e)^p d)^q c) + a)^2}{\sqrt{hx + g}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/sqrt(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d(e + fx)^p)^q))^2}{\sqrt{g + hx}}\,dx = \int \frac{(a + b\,\ln(c\,(d\,(e + f\,x)^p)^q))^2}{\sqrt{g + h\,x}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(1/2),x)
```

```
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(1/2), x)
```

# 3.492 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 330

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx = \frac{8b^2\sqrt{f}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{h\sqrt{fg-eh}} - \frac{8b\sqrt{f}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{fg-eh}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}} - \frac{16b^2\sqrt{f}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{h\sqrt{fg-eh}} - \frac{8b^2\sqrt{f}p^2q^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{h\sqrt{fg-eh}}$$

```
[Out] 8*b^2*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))^2*f^(1/2)/h/(
-e*h+f*g)^(1/2)-8*b*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*(a+
b*ln(c*(d*(f*x+e)^p)^q))*f^(1/2)/h/(-e*h+f*g)^(1/2)-16*b^2*p^2*q^2*arctanh(
f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*ln(2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h
+f*g)^(1/2)))*f^(1/2)/h/(-e*h+f*g)^(1/2)-8*b^2*p^2*q^2*polylog(2,1-2/(1-f^(
1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))*f^(1/2)/h/(-e*h+f*g)^(1/2)-2*(a+b*ln(
c*(d*(f*x+e)^p)^q))^2/h/(h*x+g)^(1/2)
```

## Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {2445, 2458, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx = -\frac{8b\sqrt{f}pq\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{fg-eh}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}} + \frac{8b^2\sqrt{f}p^2q^2\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{h\sqrt{fg-eh}} - \frac{16b^2\sqrt{f}p^2q^2\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{h\sqrt{fg-eh}} - \frac{8b^2\sqrt{f}p^2q^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{h\sqrt{fg-eh}}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(3/2),x]
```

```
[Out] (8*b^2*Sqrt[f]*p^2*q^2*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]^2)/
(h*Sqrt[f*g - e*h]) - (8*b*Sqrt[f]*p*q*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt
[f*g - e*h]]*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(h*Sqrt[f*g - e*h]) - (2*(a
+ b*Log[c*(d*(e + f*x)^p)^q])^2)/(h*Sqrt[g + h*x]) - (16*b^2*Sqrt[f]*p^2*q^
2*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]*Log[2/(1 - (Sqrt[f]*Sqrt
[g + h*x])/Sqrt[f*g - e*h])])/(h*Sqrt[f*g - e*h]) - (8*b^2*Sqrt[f]*p^2*q^2*
PolyLog[2, 1 - 2/(1 - (Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h])])/(h*Sqrt[f*
g - e*h])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{(g+hx)^{3/2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h\sqrt{g+hx}} + \text{Subst}\left(\frac{(4bfpq)\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)\sqrt{g+hx}}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}} + \text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int \frac{a+b\log(cd^q x^{pq})}{x\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}}\,dx, x, e+fx\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{8b\sqrt{f}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{fg-eh}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}} - \text{Subst}\left(\frac{(4b^2p^2q^2)\,\text{Subst}\left(\int -\frac{2\sqrt{f}\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{hx}{f}}}{\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}x}\,dx, x, e+fx\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{8b\sqrt{f}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{fg-eh}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}} + \text{Subst}\left(\frac{(8b^2\sqrt{f}p^2q^2)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{hx}{f}}}{\sqrt{fg-eh}}\right)}{x}dx, x, e+fx\right)}{h\sqrt{fg-eh}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{8b\sqrt{f}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{fg-eh}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}} + \text{Subst}\left(\frac{(16b^2f^{3/2}p^2q^2)\,\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{eh+f(-g+x^2)}dx, x, \sqrt{g+hx}\right)}{h\sqrt{fg-eh}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{8b\sqrt{f}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{fg-eh}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}} + \text{Subst}\left(\frac{(16b^2f^{3/2}p^2q^2)\,\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{-fg+eh+fx^2}dx, x, \sqrt{g+hx}\right)}{h\sqrt{fg-eh}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{8b^2\sqrt{f}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{h\sqrt{fg-eh}} - \frac{8b\sqrt{f}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{fg-eh}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}} - \text{Subst}\left(\frac{(16b^2fp^2q^2)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}dx,x,\sqrt{g+hx}\right)}{h(fg-eh)},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{8b^2\sqrt{f}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{h\sqrt{fg-eh}} - \frac{8b\sqrt{f}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{fg-eh}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}} - \frac{16b^2\sqrt{f}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{h\sqrt{fg-eh}} + \text{Subst}\left(\frac{(16b^2fp^2q^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}\right)}{1-\frac{fx^2}{fg-eh}}dx,x,\sqrt{g+hx}\right)}{h(fg-eh)},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{8b^2\sqrt{f}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{h\sqrt{fg-eh}} - \frac{8b\sqrt{f}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{fg-eh}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}} - \frac{16b^2\sqrt{f}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{h\sqrt{fg-eh}} - \text{Subst}\left(\frac{(16b^2\sqrt{f}p^2q^2)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{h\sqrt{fg-eh}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{8b^2\sqrt{f}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{h\sqrt{fg-eh}} - \frac{8b\sqrt{f}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{fg-eh}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{h\sqrt{g+hx}} - \frac{16b^2\sqrt{f}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{h\sqrt{fg-eh}} - \frac{8b^2\sqrt{f}p^2q^2\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{h\sqrt{fg-eh}}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 785 vs. $2(330) = 660$.

Time = 2.54 (sec) , antiderivative size = 785, normalized size of antiderivative = 2.38

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx = -\frac{2\left(a^2\sqrt{fg-eh}\sqrt{f(g+hx)} + 4b^2fgp^2q^2\text{arctanh}\left(\frac{\sqrt{f(g+hx)}}{\sqrt{fg-eh}}\right)\log(e+fx) + 4b^2fhp^2q^2x\,\text{arctanh}\left(\frac{\sqrt{f(g+hx)}}{\sqrt{fg-eh}}\right)\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(3/2),x]
```

```
[Out] (-2*(a^2*Sqrt[f*g - e*h]*Sqrt[f*(g + h*x)] + 4*b^2*f*g*p^2*q^2*ArcTanh[Sqrt
[f*(g + h*x)]/Sqrt[f*g - e*h]]*Log[e + f*x] + 4*b^2*f*h*p^2*q^2*x*ArcTanh[S
qrt[f*(g + h*x)]/Sqrt[f*g - e*h]]*Log[e + f*x] - 4*b^2*f*g*p^2*q^2*ArcTanh[
Sqrt[f*(g + h*x)]/Sqrt[f*g - e*h]]*Log[(h*(e + f*x))/(-(f*g) + e*h)] - 4*b^
2*f*h*p^2*q^2*x*ArcTanh[Sqrt[f*(g + h*x)]/Sqrt[f*g - e*h]]*Log[(h*(e + f*x)
```

```
)/(-(f*g) + e*h)] - b^2*Sqrt[f*g - e*h]*p^2*q^2*Sqrt[f*(g + h*x)]*Sqrt[(f*(
g + h*x))/(f*g - e*h)]*Log[(h*(e + f*x))/(-(f*g) + e*h)]^2 + 2*a*b*Sqrt[f*g
 - e*h]*Sqrt[f*(g + h*x)]*Log[c*(d*(e + f*x)^p)^q] + b^2*Sqrt[f*g - e*h]*Sq
rt[f*(g + h*x)]*Log[c*(d*(e + f*x)^p)^q]^2 + 4*b*Sqrt[f]*p*q*Sqrt[g + h*x]*
Sqrt[f*(g + h*x)]*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]*(a - b*p
*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^q]) + 4*b^2*Sqrt[f*g - e*h]*p^2*q
^2*Sqrt[f*(g + h*x)]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Log[(h*(e + f*x))/(-(f
*g) + e*h)]*Log[(1 + Sqrt[(f*(g + h*x))/(f*g - e*h)])/2] - 2*b^2*Sqrt[f*g -
 e*h]*p^2*q^2*Sqrt[f*(g + h*x)]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Log[(1 + Sq
rt[(f*(g + h*x))/(f*g - e*h)])/2]^2 + 4*b^2*Sqrt[f*g - e*h]*p^2*q^2*Sqrt[f*
(g + h*x)]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*PolyLog[2, 1/2 - Sqrt[(f*(g + h*
x))/(f*g - e*h)]/2]))/(h*Sqrt[f*g - e*h]*Sqrt[g + h*x]*Sqrt[f*(g + h*x)])
```

## Maple [F]

$$\int \frac{(a + b\ln(c(d(fx+e)^p)^q))^2}{(hx+g)^{\frac{3}{2}}}dx$$

[In] `int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(3/2),x)`

[Out] `int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(3/2),x)`

## Fricas [F]

$$\int \frac{(a + b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c) + a)^2}{(hx+g)^{\frac{3}{2}}}\,dx$$

[In] `integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(3/2),x, algorithm="fricas")`

[Out] `integral((sqrt(h*x + g)*b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*sqrt(h*x + g)*a*b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x + g)*a^2)/(h^2*x^2 + 2*g*h*x + g^2), x)`

## Sympy [F]

$$\int \frac{(a + b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{3/2}}\,dx = \int \frac{(a + b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{\frac{3}{2}}}\,dx$$

[In] `integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**(3/2),x)`

[Out] `Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/(g + h*x)**(3/2), x)`

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(3/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)^{3/2}}\,dx = \int \frac{\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}{(hx+g)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)^{3/2}}\,dx = \int \frac{\left(a+b\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)\right)^2}{(g+h\,x)^{3/2}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(3/2),x)
```

```
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(3/2), x)
```

# 3.493 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 449

$$\begin{aligned}
\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx &= \frac{16b^2f^{3/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3h(fg-eh)^{3/2}} \\
&+ \frac{8b^2f^{3/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3h(fg-eh)^{3/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)\sqrt{g+hx}} \\
&- \frac{8bf^{3/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)^{3/2}} \\
&- \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} - \frac{16b^2f^{3/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3h(fg-eh)^{3/2}} \\
&- \frac{8b^2f^{3/2}p^2q^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3h(fg-eh)^{3/2}}
\end{aligned}$$

```
[Out] 16/3*b^2*f^(3/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/h/
(-e*h+f*g)^(3/2)+8/3*b^2*f^(3/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*
h+f*g)^(1/2))^2/h/(-e*h+f*g)^(3/2)-8/3*b*f^(3/2)*p*q*arctanh(f^(1/2)*(h*x+g
)^(1/2)/(-e*h+f*g)^(1/2))*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h+f*g)^(3/2)-2/
3*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/h/(h*x+g)^(3/2)-16/3*b^2*f^(3/2)*p^2*q^2*ar
ctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*ln(2/(1-f^(1/2)*(h*x+g)^(1/2)
/(-e*h+f*g)^(1/2)))/h/(-e*h+f*g)^(3/2)-8/3*b^2*f^(3/2)*p^2*q^2*polylog(2,1-
2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))/h/(-e*h+f*g)^(3/2)+8/3*b*f*p*
q*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h+f*g)/(h*x+g)^(1/2)
```

## Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2445, 2458, 2389, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx = -\frac{8bf^{3/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)^{3/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{3h\sqrt{g+hx}(fg-eh)} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} + \frac{8b^2f^{3/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3h(fg-eh)^{3/2}} + \frac{16b^2f^{3/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3h(fg-eh)^{3/2}} - \frac{16b^2f^{3/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3h(fg-eh)^{3/2}} - \frac{8b^2f^{3/2}p^2q^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3h(fg-eh)^{3/2}}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(5/2),x]
```

```
[Out] (16*b^2*f^(3/2)*p^2*q^2*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/(
3*h*(f*g - e*h)^(3/2)) + (8*b^2*f^(3/2)*p^2*q^2*ArcTanh[(Sqrt[f]*Sqrt[g + h
*x])/Sqrt[f*g - e*h]]^2)/(3*h*(f*g - e*h)^(3/2)) + (8*b*f*p*q*(a + b*Log[c*
(d*(e + f*x)^p)^q]))/(3*h*(f*g - e*h)*Sqrt[g + h*x]) - (8*b*f^(3/2)*p*q*Arc
Tanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]*(a + b*Log[c*(d*(e + f*x)^p)^
q]))/(3*h*(f*g - e*h)^(3/2)) - (2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(3*h*
(g + h*x)^(3/2)) - (16*b^2*f^(3/2)*p^2*q^2*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/
Sqrt[f*g - e*h]]*Log[2/(1 - (Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h])])/(3*h
*(f*g - e*h)^(3/2)) - (8*b^2*f^(3/2)*p^2*q^2*PolyLog[2, 1 - 2/(1 - (Sqrt[f]
*Sqrt[g + h*x])/Sqrt[f*g - e*h])])/(3*h*(f*g - e*h)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
```

```
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{(g+hx)^{5/2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} + \text{Subst}\left(\frac{(4bfpq)\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)(g+hx)^{3/2}}\,dx}{3h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} + \text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{x\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{3/2}}\,dx, x, e+fx\right)}{3h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} - \text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{3/2}}\,dx, x, e+fx\right)}{3(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(4bfpq)\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{x\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}}\,dx, x, e+fx\right)}{3h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)\sqrt{g+hx}} - \frac{8bf^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)^{3/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} - \text{Subst}\left(\frac{(4b^2fp^2q^2)\,\text{Subst}\left(\int -\frac{2\sqrt{f}\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{hx}{f}}}{\sqrt{fg-eh}}\right)}{\sqrt{fg-eh}x}dx, x, e+fx\right)}{3h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx$$

$$= \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)\sqrt{g+hx}} - \frac{8bf^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)^{3/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} + \text{Subst}\left(\frac{(8b^2f^{3/2}p^2q^2)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{hx}{f}}}{\sqrt{fg-eh}}\right)}{x}dx, x, e+fx\right)}{3h(fg-eh)^{3/2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)$$

$$= \frac{16b^2f^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3h(fg-eh)^{3/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)\sqrt{g+hx}} - \frac{8bf^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)^{3/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} + \text{Subst}\left(\frac{(16b^2f^{5/2}p^2q^2)\,\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{eh+f(-g+x^2)}dx, x, \sqrt{g+hx}\right)}{3h(fg-eh)^{3/2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{16b^2f^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3h(fg-eh)^{3/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)\sqrt{g+hx}} - \frac{8bf^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)^{3/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} + \text{Subst}\left(\frac{(16b^2f^{5/2}p^2q^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{-fg+eh+fx^2}dx, x, \sqrt{g+hx}\right)}{3h(fg-eh)^{3/2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{16b^2f^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3h(fg-eh)^{3/2}} + \frac{8b^2f^{3/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3h(fg-eh)^{3/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)\sqrt{g+hx}} - \frac{8bf^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)^{3/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} - \text{Subst}\left(\frac{(16b^2f^2p^2q^2)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}dx, x, \sqrt{g+hx}\right)}{3h(fg-eh)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{16b^2 f^{3/2} p^2 q^2 \tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3h(fg-eh)^{3/2}} + \frac{8b^2 f^{3/2} p^2 q^2 \tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3h(fg-eh)^{3/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)\sqrt{g+hx}} - \frac{8bf^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)^{3/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} - \frac{16b^2 f^{3/2} p^2 q^2 \tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3h(fg-eh)^{3/2}} + \text{Subst}\left(\frac{(16b^2f^2p^2q^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}\right)}{1-\frac{fx^2}{fg-eh}}\,dx, x, \sqrt{g+hx}\right)}{3h(fg-eh)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{16b^2 f^{3/2} p^2 q^2 \tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3h(fg-eh)^{3/2}} + \frac{8b^2 f^{3/2} p^2 q^2 \tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3h(fg-eh)^{3/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)\sqrt{g+hx}} - \frac{8bf^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)^{3/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} - \frac{16b^2 f^{3/2} p^2 q^2 \tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3h(fg-eh)^{3/2}} - \text{Subst}\left(\frac{(16b^2f^{3/2}p^2q^2)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3h(fg-eh)^{3/2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$
\begin{aligned}
&= \frac{16b^2 f^{3/2} p^2 q^2 \tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{3h(fg-eh)^{3/2}} + \frac{8b^2 f^{3/2} p^2 q^2 \tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{3h(fg-eh)^{3/2}} \\
&\quad + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)\sqrt{g+hx}} \\
&\quad - \frac{8bf^{3/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{3h(fg-eh)^{3/2}} \\
&\quad - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{3h(g+hx)^{3/2}} \\
&\quad - \frac{16b^2 f^{3/2} p^2 q^2 \tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3h(fg-eh)^{3/2}} \\
&\quad - \frac{8b^2 f^{3/2} p^2 q^2 \operatorname{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{3h(fg-eh)^{3/2}}
\end{aligned}
$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1289 vs. $2(449) = 898$.

Time = 12.81 (sec) , antiderivative size = 1289, normalized size of antiderivative = 2.87

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx = \frac{4abf^{3/2}pq\left(-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{3/2}}+\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}(2h(e+fx)-f\ldots}{(fg-e\ldots}\right.}{3h}$$

$$+\frac{4b^2f^{3/2}pq^2\left(-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{3/2}}+\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}(2h(e+fx)-fg(-2+\log(e+fx))+eh(-2+\log(e+fx)))}{(fg-eh)(fg+fhx)^2}\right)(-\ldots}{3h}$$

$$+\frac{4b^2f^{3/2}pq\left(-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{3/2}}+\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}(2h(e+fx)-fg(-2+\log(e+fx))+eh(-2+\log(e+fx)))}{(fg-eh)(fg+fhx)^2}\right)\left(-\ldots\right.}{\ldots}$$

$$-\frac{2\left(a+bq(-p\log(e+fx)+\log(d(e+fx)^p))+b\left(-q(-p\log(e+fx)+\log(d(e+fx)^p))-\log(d(e+f\ldots\right.\right.}{3h\ldots}$$

$$+\frac{2b^2f^2p^2q^2\sqrt{\frac{fg-eh+h(e+fx)}{f}}\left(-\frac{8\arcsin\left(\frac{\sqrt{-fg+eh}}{\sqrt{h}\sqrt{e+fx}}\right)}{(-fg+eh)^{3/2}\sqrt{e+fx}\sqrt{\frac{fg+fhx}{h(e+fx)}}}-\frac{\sqrt{h}(4h(e+fx)-fg(-4+\log(e+fx))+eh(-4+\log(e+fx)))\log(e+f\ldots}{(-fg+eh)(fg+fhx)^2}\right.}{\ldots}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(5/2),x]
```

```
[Out] (4*a*b*f^(3/2)*p*q*((-2*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f])
/Sqrt[f*g - e*h]])/(f*g - e*h)^(3/2) + (Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*
x))/f]*(2*h*(e + f*x) - f*g*(-2 + Log[e + f*x]) + e*h*(-2 + Log[e + f*x])))
/((f*g - e*h)*(f*g + f*h*x)^2)))/(3*h) + (4*b^2*f^(3/2)*p*q^2*((-2*ArcTanh[
(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f])/Sqrt[f*g - e*h]])/(f*g - e*h)^(
3/2) + (Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f]*(2*h*(e + f*x) - f*g*(-2
+ Log[e + f*x]) + e*h*(-2 + Log[e + f*x])))/((f*g - e*h)*(f*g + f*h*x)^2))*
(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/(3*h) + (4*b^2*f^(3/2)*p*q*((-2*A
rcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f])/Sqrt[f*g - e*h]])/(f*g -
 e*h)^(3/2) + (Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f]*(2*h*(e + f*x) - f
*g*(-2 + Log[e + f*x]) + e*h*(-2 + Log[e + f*x])))/((f*g - e*h)*(f*g + f*h*
x)^2))*(-(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*
(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log
[c*E^(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(
-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p])]))/(3*h) - (2*
```

```
(a + b*q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]) + b*(-(q*(-(p*Log[e + f*x
]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q - (q*(-(p*Log[e + f*x]) +
 Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E^(q*(-(p*Log[e + f*x]) +
 Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*Log[e + f*x]) + Log[d*(e
 + f*x)^p]))/Log[d*(e + f*x)^p])]))^2)/(3*h*(g + h*x)^(3/2)) + (2*b^2*f^2*p
^2*q^2*Sqrt[(f*g - e*h + h*(e + f*x))/f]*((-8*ArcSin[Sqrt[-(f*g) + e*h]/(Sq
rt[h]*Sqrt[e + f*x])])/((-(f*g) + e*h)^(3/2)*Sqrt[e + f*x]*Sqrt[(f*g + f*h*
x)/(h*(e + f*x))]) - (Sqrt[h]*(4*h*(e + f*x) - f*g*(-4 + Log[e + f*x]) + e*
h*(-4 + Log[e + f*x]))*Log[e + f*x])/((-(f*g) + e*h)*(f*g + f*h*x)^2) + (Sq
rt[h]*((-4*ArcTanh[Sqrt[f*g - e*h + h*(e + f*x)]/Sqrt[f*g - e*h]]*(Log[e +
f*x] - Log[(h*(e + f*x))/(-(f*g) + e*h)]))/Sqrt[f*g - e*h] + (Sqrt[1 + (h*(
e + f*x))/(f*g - e*h)]*(Log[(h*(e + f*x))/(-(f*g) + e*h)]^2 - 4*Log[(h*(e +
 f*x))/(-(f*g) + e*h)]*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2] + 2
*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2]^2 - 4*PolyLog[2, (1 - Sqr
t[1 + (h*(e + f*x))/(f*g - e*h)])/2]))/Sqrt[f*g + f*h*x]))/((f*g - e*h)*Sqr
t[f*g + f*h*x])))/(3*h^(3/2))
```

## Maple [F]

$$\int \frac{(a + b\ln(c(d(fx+e)^p)^q))^2}{(hx+g)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(5/2),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(5/2),x)
```

## Fricas [F]

$$\int \frac{(a + b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{5/2}}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c) + a)^2}{(hx+g)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(h*x + g)*b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*sqrt(h*x + g)*a*
b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x + g)*a^2)/(h^3*x^3 + 3*g*h^2*x^2 + 3*
g^2*h*x + g^3), x)
```

## Sympy [F]

$$\int \frac{(a+b\log{(c(d(e+fx)^p)^q)})^2}{(g+hx)^{5/2}}\,dx = \int \frac{(a+b\log{(c(d(e+fx)^p)^q)})^2}{(g+hx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**(5/2),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/(g + h*x)**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log{(c(d(e+fx)^p)^q)})^2}{(g+hx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(5/2),x, algorithm="maxima
")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int \frac{(a+b\log{(c(d(e+fx)^p)^q)})^2}{(g+hx)^{5/2}}\,dx = \int \frac{(b\log{(((fx+e)^pd)^q c)}+a)^2}{(hx+g)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(5/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(c(d(e+fx)^p)^q)})^2}{(g+hx)^{5/2}}\,dx = \int \frac{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^2}{{\left(g+h\,x\right)}^{5/2}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(5/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(5/2), x)
```

# 3.494 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 537

$$\begin{aligned}\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx &= -\frac{16b^2f^2p^2q^2}{15h(fg-eh)^2\sqrt{g+hx}} \\ &+ \frac{64b^2f^{5/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{15h(fg-eh)^{5/2}} + \frac{8b^2f^{5/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5h(fg-eh)^{5/2}} \\ &+ \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{15h(fg-eh)(g+hx)^{3/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^2\sqrt{g+hx}} \\ &- \frac{8bf^{5/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^{5/2}} \\ &- \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}} \\ &- \frac{16b^2f^{5/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5h(fg-eh)^{5/2}} \\ &- \frac{8b^2f^{5/2}p^2q^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5h(fg-eh)^{5/2}}\end{aligned}$$

```
[Out] 64/15*b^2*f^(5/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/h
/(-e*h+f*g)^(5/2)+8/5*b^2*f^(5/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e
*h+f*g)^(1/2))^2/h/(-e*h+f*g)^(5/2)+8/15*b*f*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q)
)/h/(-e*h+f*g)/(h*x+g)^(3/2)-8/5*b*f^(5/2)*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2
)/(-e*h+f*g)^(1/2))*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h+f*g)^(5/2)-2/5*(a+b
*ln(c*(d*(f*x+e)^p)^q))^2/h/(h*x+g)^(5/2)-16/5*b^2*f^(5/2)*p^2*q^2*arctanh(
f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*ln(2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h
```

```
+f*g)^(1/2)))/h/(-e*h+f*g)^(5/2)-8/5*b^2*f^(5/2)*p^2*q^2*polylog(2,1-2/(1-f
^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))/h/(-e*h+f*g)^(5/2)-16/15*b^2*f^2*p^
2*q^2/h/(-e*h+f*g)^2/(h*x+g)^(1/2)+8/5*b*f^2*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q)
)/h/(-e*h+f*g)^2/(h*x+g)^(1/2)
```

## Rubi [A] (verified)

Time = 2.14 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2445, 2458, 2389, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 53, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx = -\frac{8bf^{5/2}pq\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^{5/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{5h\sqrt{g+hx}(fg-eh)^2} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{15h(g+hx)^{3/2}(fg-eh)} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}} + \frac{8b^2f^{5/2}p^2q^2\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5h(fg-eh)^{5/2}} + \frac{64b^2f^{5/2}p^2q^2\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{15h(fg-eh)^{5/2}} - \frac{16b^2f^{5/2}p^2q^2\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5h(fg-eh)^{5/2}} - \frac{8b^2f^{5/2}p^2q^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5h(fg-eh)^{5/2}} - \frac{16b^2f^2p^2q^2}{15h\sqrt{g+hx}(fg-eh)^2}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(7/2),x]
```

```
[Out] (-16*b^2*f^2*p^2*q^2)/(15*h*(f*g - e*h)^2*Sqrt[g + h*x]) + (64*b^2*f^(5/2)*
p^2*q^2*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/(15*h*(f*g - e*h)
^(5/2)) + (8*b^2*f^(5/2)*p^2*q^2*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g -
 e*h]]^2)/(5*h*(f*g - e*h)^(5/2)) + (8*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)
^q]))/(15*h*(f*g - e*h)*(g + h*x)^(3/2)) + (8*b*f^2*p*q*(a + b*Log[c*(d*(e
+ f*x)^p)^q]))/(5*h*(f*g - e*h)^2*Sqrt[g + h*x]) - (8*b*f^(5/2)*p*q*ArcTanh
[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]*(a + b*Log[c*(d*(e + f*x)^p)^q]))
/(5*h*(f*g - e*h)^(5/2)) - (2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(5*h*(g +
 h*x)^(5/2)) - (16*b^2*f^(5/2)*p^2*q^2*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt
[f*g - e*h]]*Log[2/(1 - (Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h])])/(5*h*(f*
```

```
g - e*h)^(5/2)) - (8*b^2*f^(5/2)*p^2*q^2*PolyLog[2, 1 - 2/(1 - (Sqrt[f]*Sqr
t[g + h*x])/Sqrt[f*g - e*h])])/(5*h*(f*g - e*h)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
```

```
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
```

```
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{(g+hx)^{7/2}}\,dx, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&= -\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{5h(g+hx)^{5/2}}\\
&\quad+\text{Subst}\left(\frac{(4bfpq)\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(e+fx)(g+hx)^{5/2}}\,dx}{5h}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&= -\frac{2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{5h(g+hx)^{5/2}}\\
&\quad+\text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int \frac{a+b\log\left(cd^qx^{pq}\right)}{x\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{5/2}}\,dx, x, e+fx\right)}{5h}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)
\end{aligned}$$

$$= -\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}} - \text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{5/2}}\,dx, x, e+fx\right)}{5(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(4bfpq)\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{x\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{3/2}}\,dx, x, e+fx\right)}{5h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{15h(fg-eh)(g+hx)^{3/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}} - \text{Subst}\left(\frac{(4bfpq)\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{3/2}}\,dx, x, e+fx\right)}{5(fg-eh)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\Bigg($$

$$= -\frac{16b^2f^2p^2q^2}{15h(fg-eh)^2\sqrt{g+hx}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{15h(fg-eh)(g+hx)^{3/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^2\sqrt{g+hx}} - \frac{8bf^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^{5/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}}$$

$$- \text{Subst}\left(\frac{(8b^2f^2p^2q^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}}\,dx, x, e+fx\right)}{15h(fg-eh)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}$$

$$= -\frac{16b^2f^2p^2q^2}{15h(fg-eh)^2\sqrt{g+hx}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{15h(fg-eh)(g+hx)^{3/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^2\sqrt{g+hx}} - \frac{8bf^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^{5/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}} + \text{Subst}\left(\frac{(8b^2f^{5/2}p^2q^2)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{hx}{f}}}{\sqrt{fg-eh}}\right)}{x}dx,x,e+fx\right)}{5h(fg-eh)^{5/2}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= -\frac{16b^2f^2p^2q^2}{15h(fg-eh)^2\sqrt{g+hx}} + \frac{64b^2f^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{15h(fg-eh)^{5/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{15h(fg-eh)(g+hx)^{3/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^2\sqrt{g+hx}} - \frac{8bf^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^{5/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}} + \text{Subst}\left(\frac{(16b^2f^{7/2}p^2q^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{eh+f(-g+x^2)}dx,x,\sqrt{g+hx}\right)}{5h(fg-eh)^{5/2}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= -\frac{16b^2f^2p^2q^2}{15h(fg-eh)^2\sqrt{g+hx}} + \frac{64b^2f^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{15h(fg-eh)^{5/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{15h(fg-eh)(g+hx)^{3/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^2\sqrt{g+hx}} - \frac{8bf^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^{5/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}} + \text{Subst}\left(\frac{(16b^2f^{7/2}p^2q^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{-fg+eh+fx^2}\,dx, x, \sqrt{g+hx}\right)}{5h(fg-eh)^{5/2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{16b^2f^2p^2q^2}{15h(fg-eh)^2\sqrt{g+hx}} + \frac{64b^2f^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{15h(fg-eh)^{5/2}} + \frac{8b^2f^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5h(fg-eh)^{5/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{15h(fg-eh)(g+hx)^{3/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^2\sqrt{g+hx}} - \frac{8bf^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^{5/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}} - \text{Subst}\left(\frac{(16b^2f^3p^2q^2)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}\,dx, x, \sqrt{g+hx}\right)}{5h(fg-eh)^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$
\begin{aligned}
&= -\frac{16b^2f^2p^2q^2}{15h(fg-eh)^2\sqrt{g+hx}}+\frac{64b^2f^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{15h(fg-eh)^{5/2}} \\
&+\frac{8b^2f^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5h(fg-eh)^{5/2}}+\frac{8bfpq(a+b\log\left(c(d(e+fx)^p)^q\right))}{15h(fg-eh)(g+hx)^{3/2}} \\
&+\frac{8bf^2pq(a+b\log\left(c(d(e+fx)^p)^q\right))}{5h(fg-eh)^2\sqrt{g+hx}} \\
&-\frac{8bf^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log\left(c(d(e+fx)^p)^q\right))}{5h(fg-eh)^{5/2}} \\
&-\frac{2(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{5h(g+hx)^{5/2}} \\
&-\frac{16b^2f^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5h(fg-eh)^{5/2}} \\
&+\text{Subst}\left(\frac{(16b^2f^3p^2q^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}\right)}{1-\frac{fx^2}{fg-eh}}\,dx,x,\sqrt{g+hx}\right)}{5h(fg-eh)^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)
\end{aligned}
$$

$$= -\frac{16b^2f^2p^2q^2}{15h(fg-eh)^2\sqrt{g+hx}} + \frac{64b^2f^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{15h(fg-eh)^{5/2}}$$
$$+ \frac{8b^2f^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5h(fg-eh)^{5/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{15h(fg-eh)(g+hx)^{3/2}}$$
$$+ \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^2\sqrt{g+hx}}$$
$$- \frac{8bf^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^{5/2}}$$
$$- \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}}$$
$$- \frac{16b^2f^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5h(fg-eh)^{5/2}}$$
$$-\text{Subst}\left(\frac{(16b^2f^{5/2}p^2q^2)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5h(fg-eh)^{5/2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{16b^2f^2p^2q^2}{15h(fg-eh)^2\sqrt{g+hx}} + \frac{64b^2f^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{15h(fg-eh)^{5/2}}$$
$$+ \frac{8b^2f^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{5h(fg-eh)^{5/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{15h(fg-eh)(g+hx)^{3/2}}$$
$$+ \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^2\sqrt{g+hx}}$$
$$- \frac{8bf^{5/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{5h(fg-eh)^{5/2}}$$
$$- \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{5h(g+hx)^{5/2}}$$
$$- \frac{16b^2f^{5/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5h(fg-eh)^{5/2}}$$
$$- \frac{8b^2f^{5/2}p^2q^2\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{5h(fg-eh)^{5/2}}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1349 vs. 2(537) = 1074.

Time = 9.70 (sec) , antiderivative size = 1349, normalized size of antiderivative = 2.51

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx = \frac{4abf^{5/2}pq\left(-\frac{6\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{5/2}}+\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}(2(fg-eh)(fg+\cdots}{(fg-e\cdots}\right.}{15h}$$
$$+\frac{4b^2f^{5/2}pq^2\left(-\frac{6\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{5/2}}+\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}(2(fg-eh)(fg+fhx)+6(fg+fhx)^2-3(fg-eh)^2\log(e+fx))}{(fg-eh)^2(fg+fhx)^3}\right)}{15h}$$
$$+\frac{4b^2f^{5/2}pq\left(-\frac{6\text{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{5/2}}+\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}(2(fg-eh)(fg+fhx)+6(fg+fhx)^2-3(fg-eh)^2\log(e+fx))}{(fg-eh)^2(fg+fhx)^3}\right)(\cdots}{\cdots}$$
$$-\frac{2\left(a+bq(-p\log(e+fx)+\log(d(e+fx)^p))+b\left(-q(-p\log(e+fx)+\log(d(e+fx)^p))-\log(d(e+fx\cdots\right.\right.}{5h(g\cdots}$$
$$+\frac{2b^2f^3p^2q^2\sqrt{\frac{fg-eh+h(e+fx)}{f}}\left(\frac{32h^{5/2}(e+fx)^{5/2}\left(\frac{fg+fhx}{h(e+fx)}\right)^{5/2}\arcsin\left(\frac{\sqrt{-fg+eh}}{\sqrt{h}\sqrt{e+fx}}\right)}{(-fg+eh)^{5/2}}+4\left(1+\frac{h(e+fx)}{fg-eh}\right)\log(e+fx)-3\log^2(e+\cdots\right.}{\cdots}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(7/2),x]
```

```
[Out] (4*a*b*f^(5/2)*p*q*((-6*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f])
/Sqrt[f*g - e*h]])/(f*g - e*h)^(5/2) + (Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*
x))/f]*(2*(f*g - e*h)*(f*g + f*h*x) + 6*(f*g + f*h*x)^2 - 3*(f*g - e*h)^2*L
og[e + f*x]))/((f*g - e*h)^2*(f*g + f*h*x)^3)))/(15*h) + (4*b^2*f^(5/2)*p*q
^2*((-6*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f])/Sqrt[f*g - e*h]
])/(f*g - e*h)^(5/2) + (Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f]*(2*(f*g -
 e*h)*(f*g + f*h*x) + 6*(f*g + f*h*x)^2 - 3*(f*g - e*h)^2*Log[e + f*x]))/((
f*g - e*h)^2*(f*g + f*h*x)^3))*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/(1
5*h) + (4*b^2*f^(5/2)*p*q*((-6*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*
x))/f])/Sqrt[f*g - e*h]])/(f*g - e*h)^(5/2) + (Sqrt[f]*Sqrt[(f*g - e*h + h*
(e + f*x))/f]*(2*(f*g - e*h)*(f*g + f*h*x) + 6*(f*g + f*h*x)^2 - 3*(f*g - e
*h)^2*Log[e + f*x]))/((f*g - e*h)^2*(f*g + f*h*x)^3))*(-(q*(-(p*Log[e + f*x
]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q - (q*(-(p*Log[e + f*x]) +
 Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E^(q*(-(p*Log[e + f*x]) +
```

```
Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*Log[e + f*x]) + Log[d*(e
 + f*x)^p]))/Log[d*(e + f*x)^p])]))/(15*h) - (2*(a + b*q*(-(p*Log[e + f*x])
 + Log[d*(e + f*x)^p]) + b*(-(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) -
 Log[d*(e + f*x)^p]*(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d
*(e + f*x)^p]) + Log[c*E^(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))*(d*(e
 + f*x)^p)^(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x
)^p])]))^2)/(5*h*(g + h*x)^(5/2)) + (2*b^2*f^3*p^2*q^2*Sqrt[(f*g - e*h + h*
(e + f*x))/f]*((32*h^(5/2)*(e + f*x)^(5/2)*((f*g + f*h*x)/(h*(e + f*x)))^(5
/2)*ArcSin[Sqrt[-(f*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])])/(-(f*g) + e*h)^(5/2
) + 4*(1 + (h*(e + f*x))/(f*g - e*h))*Log[e + f*x] - 3*Log[e + f*x]^2 + (4*
(f*g + f*h*x)^2*(-2 + 3*Log[e + f*x]))/(f*g - e*h)^2 - (12*(f*g + f*h*x)^(5
/2)*ArcTanh[Sqrt[f*g - e*h + h*(e + f*x)]/Sqrt[f*g - e*h]]*(Log[e + f*x] -
Log[(h*(e + f*x))/(-(f*g) + e*h)]))/(f*g - e*h)^(5/2) + (3*(f*g + f*h*x)^2*
Sqrt[1 + (h*(e + f*x))/(f*g - e*h)]*(Log[(h*(e + f*x))/(-(f*g) + e*h)]^2 -
4*Log[(h*(e + f*x))/(-(f*g) + e*h)]*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g -
e*h)])/2] + 2*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2]^2 - 4*PolyLo
g[2, (1 - Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2]))/(f*g - e*h)^2))/(15*h*(
f*g + f*h*x)^3)
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c(d(fx+e)^p)^q\right)\right)^2}{(hx+g)^{\frac{7}{2}}}dx$$

[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(7/2),x)

[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(7/2),x)

## Fricas [F]

$$\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)^{7/2}}\,dx=\int \frac{\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}{(hx+g)^{\frac{7}{2}}}\,dx$$

[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(7/2),x, algorithm="fricas
")

[Out] integral((sqrt(h*x + g)*b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*sqrt(h*x + g)*a*
b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x + g)*a^2)/(h^4*x^4 + 4*g*h^3*x^3 + 6*
g^2*h^2*x^2 + 4*g^3*h*x + g^4), x)

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**(7/2),x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(7/2),x, algorithm="maxima
")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(7/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{7/2}}\,dx = \int \frac{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^2}{(g+h\,x)^{7/2}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(7/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(7/2), x)
```

# 3.495 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 625

$$\begin{aligned}
\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx = &-\frac{16b^2f^2p^2q^2}{105h(fg-eh)^2(g+hx)^{3/2}} \\
&-\frac{128b^2f^3p^2q^2}{105h(fg-eh)^3\sqrt{g+hx}}+\frac{368b^2f^{7/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{105h(fg-eh)^{7/2}} \\
&+\frac{8b^2f^{7/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{7h(fg-eh)^{7/2}}+\frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{35h(fg-eh)(g+hx)^{5/2}} \\
&+\frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{21h(fg-eh)^2(g+hx)^{3/2}}+\frac{8bf^3pq(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^3\sqrt{g+hx}} \\
&-\frac{8bf^{7/2}pq\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^{7/2}} \\
&-\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} \\
&-\frac{16b^2f^{7/2}p^2q^2\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{7h(fg-eh)^{7/2}} \\
&-\frac{8b^2f^{7/2}p^2q^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{7h(fg-eh)^{7/2}}
\end{aligned}$$

```
[Out] -16/105*b^2*f^2*p^2*q^2/h/(-e*h+f*g)^2/(h*x+g)^(3/2)+368/105*b^2*f^(7/2)*p^
2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))/h/(-e*h+f*g)^(7/2)+8/
7*b^2*f^(7/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))^2/h/(
-e*h+f*g)^(7/2)+8/35*b*f*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h+f*g)/(h*x+
```

```
g)^(5/2)+8/21*b*f^2*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h+f*g)^2/(h*x+g)^
(3/2)-8/7*b*f^(7/2)*p*q*arctanh(f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2))*(a+
b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h+f*g)^(7/2)-2/7*(a+b*ln(c*(d*(f*x+e)^p)^q))
^2/h/(h*x+g)^(7/2)-16/7*b^2*f^(7/2)*p^2*q^2*arctanh(f^(1/2)*(h*x+g)^(1/2)/(
-e*h+f*g)^(1/2))*ln(2/(1-f^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)))/h/(-e*h+f
*g)^(7/2)-8/7*b^2*f^(7/2)*p^2*q^2*polylog(2,1-2/(1-f^(1/2)*(h*x+g)^(1/2)/(-
e*h+f*g)^(1/2)))/h/(-e*h+f*g)^(7/2)-128/105*b^2*f^3*p^2*q^2/h/(-e*h+f*g)^3/
(h*x+g)^(1/2)+8/7*b*f^3*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(-e*h+f*g)^3/(h*x
+g)^(1/2)
```

## Rubi [A] (verified)

Time = 2.67 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2445, 2458, 2389, 65, 214, 2390, 12, 1601, 6873, 6131, 6055, 2449, 2352, 2356, 53, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx = -\frac{8bf^{7/2}pq\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^{7/2}} + \frac{8bf^3pq(a+b\log(c(d(e+fx)^p)^q))}{7h\sqrt{g+hx}(fg-eh)^3} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{21h(g+hx)^{3/2}(fg-eh)^2} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{35h(g+hx)^{5/2}(fg-eh)} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} + \frac{8b^2f^{7/2}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{7h(fg-eh)^{7/2}} + \frac{368b^2f^{7/2}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{105h(fg-eh)^{7/2}} - \frac{16b^2f^{7/2}p^2q^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{7h(fg-eh)^{7/2}} - \frac{8b^2f^{7/2}p^2q^2\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{7h(fg-eh)^{7/2}} - \frac{128b^2f^3p^2q^2}{105h\sqrt{g+hx}(fg-eh)^3} - \frac{16b^2f^2p^2q^2}{105h(g+hx)^{3/2}(fg-eh)^2}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(9/2),x]
```

```
[Out] (-16*b^2*f^2*p^2*q^2)/(105*h*(f*g - e*h)^2*(g + h*x)^(3/2)) - (128*b^2*f^3*
p^2*q^2)/(105*h*(f*g - e*h)^3*Sqrt[g + h*x]) + (368*b^2*f^(7/2)*p^2*q^2*Arc
Tanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]])/(105*h*(f*g - e*h)^(7/2)) +
(8*b^2*f^(7/2)*p^2*q^2*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]^2)/
```

```
(7*h*(f*g - e*h)^(7/2)) + (8*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(35*
h*(f*g - e*h)*(g + h*x)^(5/2)) + (8*b*f^2*p*q*(a + b*Log[c*(d*(e + f*x)^p)^
q]))/(21*h*(f*g - e*h)^2*(g + h*x)^(3/2)) + (8*b*f^3*p*q*(a + b*Log[c*(d*(e
 + f*x)^p)^q]))/(7*h*(f*g - e*h)^3*Sqrt[g + h*x]) - (8*b*f^(7/2)*p*q*ArcTan
h[(Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h]]*(a + b*Log[c*(d*(e + f*x)^p)^q])
)/(7*h*(f*g - e*h)^(7/2)) - (2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(7*h*(g
+ h*x)^(7/2)) - (16*b^2*f^(7/2)*p^2*q^2*ArcTanh[(Sqrt[f]*Sqrt[g + h*x])/Sqr
t[f*g - e*h]]*Log[2/(1 - (Sqrt[f]*Sqrt[g + h*x])/Sqrt[f*g - e*h])])/(7*h*(f
*g - e*h)^(7/2)) - (8*b^2*f^(7/2)*p^2*q^2*PolyLog[2, 1 - 2/(1 - (Sqrt[f]*Sq
rt[g + h*x])/Sqrt[f*g - e*h])])/(7*h*(f*g - e*h)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
```

```
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{(g+hx)^{9/2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} + \text{Subst}\left(\frac{(4bfpq)\int \frac{a+b\log(cd^q(e+fx)^{pq})}{(e+fx)(g+hx)^{7/2}}\,dx}{7h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} + \text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{x\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{7/2}}dx, x, e+fx\right)}{7h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} - \text{Subst}\left(\frac{(4bpq)\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{7/2}}dx, x, e+fx\right)}{7(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(4bfpq)\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{x\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{5/2}}dx, x, e+fx\right)}{7h(fg-eh)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{35h(fg-eh)(g+hx)^{5/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} - \text{Subst}\left(\frac{(4bfpq)\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{5/2}}dx, x, e+fx\right)}{7(fg-eh)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left( (4 \right.$$

$$= -\frac{16b^2f^2p^2q^2}{105h(fg-eh)^2(g+hx)^{3/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{35h(fg-eh)(g+hx)^{5/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{21h(fg-eh)^2(g+hx)^{3/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} - \text{Subst}\left(\frac{(4bf^2pq)\,\text{Subst}\left(\int \frac{a+b\log(cd^qx^{pq})}{\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)^{3/2}}dx, x, e+fx\right)}{7(fg-eh)^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left( \right.$$

$$= -\frac{16b^2f^2p^2q^2}{105h(fg-eh)^2(g+hx)^{3/2}} - \frac{128b^2f^3p^2q^2}{105h(fg-eh)^3\sqrt{g+hx}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{35h(fg-eh)(g+hx)^{5/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{21h(fg-eh)^2(g+hx)^{3/2}} + \frac{8bf^3pq(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^3\sqrt{g+hx}} - \frac{8bf^{7/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^{7/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}}$$

$$-\text{Subst}\left(\frac{(8b^2f^3p^2q^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{fg-eh}{f}+\frac{hx}{f}}}\,dx, x, e+fx\right)}{35h(fg-eh)^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}$$

$$= -\frac{16b^2f^2p^2q^2}{105h(fg-eh)^2(g+hx)^{3/2}} - \frac{128b^2f^3p^2q^2}{105h(fg-eh)^3\sqrt{g+hx}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{35h(fg-eh)(g+hx)^{5/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{21h(fg-eh)^2(g+hx)^{3/2}} + \frac{8bf^3pq(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^3\sqrt{g+hx}} - \frac{8bf^{7/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^{7/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}}$$

$$+\text{Subst}\left(\frac{(8b^2f^{7/2}p^2q^2)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g-\frac{eh}{f}+\frac{hx}{f}}}{\sqrt{fg-eh}}\right)}{x}\,dx, x, e+fx\right)}{7h(fg-eh)^{7/2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)\right.$$

$$= -\frac{16b^2f^2p^2q^2}{105h(fg-eh)^2(g+hx)^{3/2}} - \frac{128b^2f^3p^2q^2}{105h(fg-eh)^3\sqrt{g+hx}} + \frac{368b^2f^{7/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{105h(fg-eh)^{7/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{35h(fg-eh)(g+hx)^{5/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{21h(fg-eh)^2(g+hx)^{3/2}} + \frac{8bf^3pq(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^3\sqrt{g+hx}} - \frac{8bf^{7/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^{7/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} + \text{Subst}\left(\frac{(16b^2f^{9/2}p^2q^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{eh+f(-g+x^2)}\,dx, x, \sqrt{g+hx}\right)}{7h(fg-eh)^{7/2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{16b^2f^2p^2q^2}{105h(fg-eh)^2(g+hx)^{3/2}} - \frac{128b^2f^3p^2q^2}{105h(fg-eh)^3\sqrt{g+hx}} + \frac{368b^2f^{7/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{105h(fg-eh)^{7/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{35h(fg-eh)(g+hx)^{5/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{21h(fg-eh)^2(g+hx)^{3/2}} + \frac{8bf^3pq(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^3\sqrt{g+hx}} - \frac{8bf^{7/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^{7/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} + \text{Subst}\left(\frac{(16b^2f^{9/2}p^2q^2)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{-fg+eh+fx^2}\,dx, x, \sqrt{g+hx}\right)}{7h(fg-eh)^{7/2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{16b^2f^2p^2q^2}{105h(fg-eh)^2(g+hx)^{3/2}} - \frac{128b^2f^3p^2q^2}{105h(fg-eh)^3\sqrt{g+hx}} + \frac{368b^2f^{7/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{105h(fg-eh)^{7/2}} + \frac{8b^2f^{7/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{7h(fg-eh)^{7/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{35h(fg-eh)(g+hx)^{5/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{21h(fg-eh)^2(g+hx)^{3/2}} + \frac{8bf^3pq(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^3\sqrt{g+hx}} - \frac{8bf^{7/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^{7/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} - \text{Subst}\left(\frac{(16b^2f^4p^2q^2)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{fg-eh}}\right)}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}\,dx, x, \sqrt{g+hx}\right)}{7h(fg-eh)^4}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{16b^2f^2p^2q^2}{105h(fg-eh)^2(g+hx)^{3/2}} - \frac{128b^2f^3p^2q^2}{105h(fg-eh)^3\sqrt{g+hx}} + \frac{368b^2f^{7/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{105h(fg-eh)^{7/2}} + \frac{8b^2f^{7/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{7h(fg-eh)^{7/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{35h(fg-eh)(g+hx)^{5/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{21h(fg-eh)^2(g+hx)^{3/2}} + \frac{8bf^3pq(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^3\sqrt{g+hx}} - \frac{8bf^{7/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^{7/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} - \frac{16b^2f^{7/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{7h(fg-eh)^{7/2}} + \text{Subst}\left(\frac{(16b^2f^4p^2q^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{\sqrt{f}x}{\sqrt{fg-eh}}}\right)}{1-\frac{fx^2}{fg-eh}}\,dx, x, \sqrt{g+hx}\right)}{7h(fg-eh)^4}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{16b^2f^2p^2q^2}{105h(fg-eh)^2(g+hx)^{3/2}} - \frac{128b^2f^3p^2q^2}{105h(fg-eh)^3\sqrt{g+hx}} + \frac{368b^2f^{7/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{105h(fg-eh)^{7/2}} + \frac{8b^2f^{7/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{7h(fg-eh)^{7/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{35h(fg-eh)(g+hx)^{5/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{21h(fg-eh)^2(g+hx)^{3/2}} + \frac{8bf^3pq(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^3\sqrt{g+hx}} - \frac{8bf^{7/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^{7/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} - \frac{16b^2f^{7/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{7h(fg-eh)^{7/2}} - \text{Subst}\left(\frac{(16b^2f^{7/2}p^2q^2)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{7h(fg-eh)^{7/2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{16b^2f^2p^2q^2}{105h(fg-eh)^2(g+hx)^{3/2}} - \frac{128b^2f^3p^2q^2}{105h(fg-eh)^3\sqrt{g+hx}} + \frac{368b^2f^{7/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)}{105h(fg-eh)^{7/2}} + \frac{8b^2f^{7/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)^2}{7h(fg-eh)^{7/2}} + \frac{8bfpq(a+b\log(c(d(e+fx)^p)^q))}{35h(fg-eh)(g+hx)^{5/2}} + \frac{8bf^2pq(a+b\log(c(d(e+fx)^p)^q))}{21h(fg-eh)^2(g+hx)^{3/2}} + \frac{8bf^3pq(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^3\sqrt{g+hx}} - \frac{8bf^{7/2}pq\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)(a+b\log(c(d(e+fx)^p)^q))}{7h(fg-eh)^{7/2}} - \frac{2(a+b\log(c(d(e+fx)^p)^q))^2}{7h(g+hx)^{7/2}} - \frac{16b^2f^{7/2}p^2q^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}\right)\log\left(\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{7h(fg-eh)^{7/2}} - \frac{8b^2f^{7/2}p^2q^2\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{f}\sqrt{g+hx}}{\sqrt{fg-eh}}}\right)}{7h(fg-eh)^{7/2}}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1457 vs. 2(625) = 1250.

Time = 11.04 (sec) , antiderivative size = 1457, normalized size of antiderivative = 2.33

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx = \frac{4abf^{7/2}pq\left(-\frac{30\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{7/2}}+\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}(6(fg-eh)^2(f}{}\right.}{105h}$$

$$+\frac{4b^2f^{7/2}pq^2\left(-\frac{30\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{7/2}}+\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}(6(fg-eh)^2(fg+fhx)+10(fg-eh)(fg+fhx)^2+30(fg+fhx)^3}{(fg-eh)^3(fg+fhx)^4}\right.}{105h}$$

$$+\frac{4b^2f^{7/2}pq\left(-\frac{30\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}}{\sqrt{fg-eh}}\right)}{(fg-eh)^{7/2}}+\frac{\sqrt{f}\sqrt{\frac{fg-eh+h(e+fx)}{f}}(6(fg-eh)^2(fg+fhx)+10(fg-eh)(fg+fhx)^2+30(fg+fhx)^3-}{(fg-eh)^3(fg+fhx)^4}\right.}{}$$

$$-\frac{2\left(a+bq(-p\log(e+fx)+\log(d(e+fx)^p))+b\left(-q(-p\log(e+fx)+\log(d(e+fx)^p))-\log(d(e+fx)\right.\right.}{7h(g}$$

$$+\frac{2b^2f^4p^2q^2\sqrt{\frac{fg-eh+h(e+fx)}{f}}\left(-\frac{184\sqrt{h}\sqrt{e+fx}(fg+fhx)^3\sqrt{\frac{fg+fhx}{h(e+fx)}}\arcsin\left(\frac{\sqrt{-fg+eh}}{\sqrt{h}\sqrt{e+fx}}\right)}{(-fg+eh)^{7/2}}+12\left(1+\frac{h(e+fx)}{fg-eh}\right)\log(e+fx)-15\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x)^(9/2),x]
```

```
[Out] (4*a*b*f^(7/2)*p*q*((-30*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f*x))/f]
)/Sqrt[f*g - e*h]])/(f*g - e*h)^(7/2) + (Sqrt[f]*Sqrt[(f*g - e*h + h*(e + f
*x))/f]*(6*(f*g - e*h)^2*(f*g + f*h*x) + 10*(f*g - e*h)*(f*g + f*h*x)^2 + 3
0*(f*g + f*h*x)^3 - 15*(f*g - e*h)^3*Log[e + f*x]))/((f*g - e*h)^3*(f*g + f
*h*x)^4)))/(105*h) + (4*b^2*f^(7/2)*p*q^2*((-30*ArcTanh[(Sqrt[f]*Sqrt[(f*g
- e*h + h*(e + f*x))/f])/Sqrt[f*g - e*h]])/(f*g - e*h)^(7/2) + (Sqrt[f]*Sqr
t[(f*g - e*h + h*(e + f*x))/f]*(6*(f*g - e*h)^2*(f*g + f*h*x) + 10*(f*g - e
*h)*(f*g + f*h*x)^2 + 30*(f*g + f*h*x)^3 - 15*(f*g - e*h)^3*Log[e + f*x]))/
((f*g - e*h)^3*(f*g + f*h*x)^4))*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/
(105*h) + (4*b^2*f^(7/2)*p*q*((-30*ArcTanh[(Sqrt[f]*Sqrt[(f*g - e*h + h*(e
+ f*x))/f])/Sqrt[f*g - e*h]])/(f*g - e*h)^(7/2) + (Sqrt[f]*Sqrt[(f*g - e*h
+ h*(e + f*x))/f]*(6*(f*g - e*h)^2*(f*g + f*h*x) + 10*(f*g - e*h)*(f*g + f*
h*x)^2 + 30*(f*g + f*h*x)^3 - 15*(f*g - e*h)^3*Log[e + f*x]))/((f*g - e*h)^
3*(f*g + f*h*x)^4))*(-(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*
```

```
(e + f*x)^p]*(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f
*x)^p]) + Log[c*E^(q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))*(d*(e + f*x)
^p)^(q - (q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p])])
)/(105*h) - (2*(a + b*q*(-(p*Log[e + f*x]) + Log[d*(e + f*x)^p]) + b*(-(q*(
-(p*Log[e + f*x]) + Log[d*(e + f*x)^p])) - Log[d*(e + f*x)^p]*(q - (q*(-(p*
Log[e + f*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p]) + Log[c*E^(q*(-(p*
Log[e + f*x]) + Log[d*(e + f*x)^p]))*(d*(e + f*x)^p)^(q - (q*(-(p*Log[e + f
*x]) + Log[d*(e + f*x)^p]))/Log[d*(e + f*x)^p])]))^2)/(7*h*(g + h*x)^(7/2))
 + (2*b^2*f^4*p^2*q^2*Sqrt[(f*g - e*h + h*(e + f*x))/f]*((-184*Sqrt[h]*Sqrt
[e + f*x]*(f*g + f*h*x)^3*Sqrt[(f*g + f*h*x)/(h*(e + f*x))]*ArcSin[Sqrt[-(f
*g) + e*h]/(Sqrt[h]*Sqrt[e + f*x])])/(-(f*g) + e*h)^(7/2) + 12*(1 + (h*(e +
 f*x))/(f*g - e*h))*Log[e + f*x] - 15*Log[e + f*x]^2 + (4*(f*g + f*h*x)^2*(
-2 + 5*Log[e + f*x]))/(f*g - e*h)^2 + (4*(f*g + f*h*x)^3*(-16 + 15*Log[e +
f*x]))/(f*g - e*h)^3 - (60*(f*g + f*h*x)^(7/2)*ArcTanh[Sqrt[f*g - e*h + h*(
e + f*x)]/Sqrt[f*g - e*h]]*(Log[e + f*x] - Log[(h*(e + f*x))/(-(f*g) + e*h)
]))/(f*g - e*h)^(7/2) + (15*(f*g + f*h*x)^3*Sqrt[1 + (h*(e + f*x))/(f*g - e
*h)]*(Log[(h*(e + f*x))/(-(f*g) + e*h)]^2 - 4*Log[(h*(e + f*x))/(-(f*g) + e
*h)]*Log[(1 + Sqrt[1 + (h*(e + f*x))/(f*g - e*h)])/2] + 2*Log[(1 + Sqrt[1 +
 (h*(e + f*x))/(f*g - e*h)])/2]^2 - 4*PolyLog[2, (1 - Sqrt[1 + (h*(e + f*x)
)/(f*g - e*h)])/2]))/(f*g - e*h)^3))/(105*h*(f*g + f*h*x)^4)
```

## Maple [F]

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^2}{(hx+g)^{\frac{9}{2}}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(9/2),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(9/2),x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)^{9/2}}\,dx=\int \frac{(b\log(((fx+e)^pd)^qc)+a)^2}{(hx+g)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(9/2),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(h*x + g)*b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*sqrt(h*x + g)*a*
b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x + g)*a^2)/(h^5*x^5 + 5*g*h^4*x^4 + 10
*g^2*h^3*x^3 + 10*g^3*h^2*x^2 + 5*g^4*h*x + g^5), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(c(d(e+fx)^p)^q)})^2}{(g+hx)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)**(9/2),x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log{(c(d(e+fx)^p)^q)})^2}{(g+hx)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(9/2),x, algorithm="maxima
")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*h-f*g>0)', see `assume?` for more
 detail
```

## Giac [F]

$$\int \frac{(a+b\log{(c(d(e+fx)^p)^q)})^2}{(g+hx)^{9/2}}\,dx = \int \frac{(b\log{(((fx+e)^pd)^q\,c)}+a)^2}{(hx+g)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)^(9/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g)^(9/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(c(d(e+fx)^p)^q)})^2}{(g+hx)^{9/2}}\,dx = \int \frac{(a+b\,\ln{(c\,(d\,(e+f\,x)^p)^q)})^2}{(g+h\,x)^{9/2}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(9/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x)^(9/2), x)
```

# 3.496 $\int \frac{(g+hx)^{3/2}}{a+b\log(c(d(e+fx)^p)^q)}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \text{Int}\left(\frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)},x\right)$$

```
[Out] Unintegrable((h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

```
[In] Int[(g + h*x)^(3/2)/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] Defer[Int][(g + h*x)^(3/2)/(a + b*Log[c*(d*(e + f*x)^p)^q]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

```
[In] Integrate[(g + h*x)^(3/2)/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
[Out] Integrate[(g + h*x)^(3/2)/(a + b*Log[c*(d*(e + f*x)^p)^q]), x]
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(hx+g)^{\frac{3}{2}}}{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

```
[In] int((h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
[Out] int((h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(hx+g)^{\frac{3}{2}}}{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}\,dx$$

```
[In] integrate((h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
[Out] integral((h*x + g)^(3/2)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Sympy [N/A]

Not integrable

Time = 33.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(g+hx)^{\frac{3}{2}}}{a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}\,dx$$

```
[In] integrate((h*x+g)**(3/2)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
[Out] Integral((g + h*x)**(3/2)/(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(hx+g)^{\frac{3}{2}}}{b\log\left((((fx+e)^p d)^q c\right)+a}\,dx$$

```
[In] integrate((h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
[Out] integrate((h*x + g)^(3/2)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(hx+g)^{\frac{3}{2}}}{b\log\left((((fx+e)^p d)^q c\right)+a}\,dx$$

```
[In] integrate((h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
[Out] integrate((h*x + g)^(3/2)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^{3/2}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(g+h\,x)^{3/2}}{a+b\,\ln\left(c\,{\left(d\,{(e+f\,x)}^p\right)}^q\right)}\,dx$$

```
[In] int((g + h*x)^(3/2)/(a + b*log(c*(d*(e + f*x)^p)^q)),x)
[Out] int((g + h*x)^(3/2)/(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

# 3.497 $\int \frac{\sqrt{g+hx}}{a+b\log(c(d(e+fx)^p)^q)}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\operatorname{Int}\left(\frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)},x\right)$$

```
[Out] Unintegrable((h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

```
[In] Int[Sqrt[g + h*x]/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
[Out] Defer[Int][Sqrt[g + h*x]/(a + b*Log[c*(d*(e + f*x)^p)^q]), x]
```

Rubi steps

$$\text{integral}=\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

```
[In] Integrate[Sqrt[g + h*x]/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
[Out] Integrate[Sqrt[g + h*x]/(a + b*Log[c*(d*(e + f*x)^p)^q]), x]
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{hx+g}}{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

```
[In] int((h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
[Out] int((h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{\sqrt{hx+g}}{b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a}\,dx$$

```
[In] integrate((h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
[Out] integral(sqrt(h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}\,dx$$

```
[In] integrate((h*x+g)**(1/2)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
[Out] Integral(sqrt(g + h*x)/(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{\sqrt{hx+g}}{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)}\,dx$$

```
[In] integrate((h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
[Out] integrate(sqrt(h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{\sqrt{hx+g}}{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)}\,dx$$

```
[In] integrate((h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
[Out] integrate(sqrt(h*x + g)/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{g+hx}}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{\sqrt{g+h\,x}}{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}\,dx$$

```
[In] int((g + h*x)^(1/2)/(a + b*log(c*(d*(e + f*x)^p)^q)),x)
[Out] int((g + h*x)^(1/2)/(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

# 3.498 $\int \frac{1}{\sqrt{g+hx}(a+b\log(c(d(e+fx)^p)^q))}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log(c\,(d(e+fx)^p)^q))}\,dx = \text{Int}\left(\frac{1}{\sqrt{g+hx}\,(a+b\log(c\,(d(e+fx)^p)^q))},x\right)$$

```
[Out] Unintegrable(1/(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log(c\,(d(e+fx)^p)^q))}\,dx = \int \frac{1}{\sqrt{g+hx}\,(a+b\log(c\,(d(e+fx)^p)^q))}\,dx$$

```
[In] Int[1/(Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

```
[Out] Defer[Int][1/(Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{g+hx}\,(a+b\log(c\,(d(e+fx)^p)^q))}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))}\,dx=\int \frac{1}{\sqrt{g+hx}\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))}\,dx$$

```
[In] Integrate[1/(Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Integrate[1/(Sqrt[g + h*x]*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)\sqrt{hx+g}}dx$$

```
[In] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x)
[Out] int(1/(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))}\,dx=\int \frac{1}{\sqrt{hx+g}(b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a)}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x, algorithm="fricas
")
[Out] integral(sqrt(h*x + g)/(a*h*x + a*g + (b*h*x + b*g)*log(((f*x + e)^p*d)^q*c
)), x)
```

## Sympy [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)\sqrt{g+hx}}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)**(1/2),x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*sqrt(g + h*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))}\,dx = \int \frac{1}{\sqrt{hx+g}(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a)}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x, algorithm="maxima
")
[Out] integrate(1/(sqrt(h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))}\,dx = \int \frac{1}{\sqrt{hx+g}(b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a)}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{g+hx}\,(a+b\log(c\,(d(e+fx)^p)^q))}\,dx = \int \frac{1}{\sqrt{g+h\,x}\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}\,dx$$

```
[In] int(1/((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
[Out] int(1/((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

# 3.499 $\int \frac{1}{(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{(g+hx)^{3/2}\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))}\,dx=\operatorname{Int}\left(\frac{1}{(g+hx)^{3/2}\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(g+hx)^{3/2}\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))}\,dx=\int \frac{1}{(g+hx)^{3/2}\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))}\,dx$$

```
[In] Int[1/((g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Defer[Int][1/((g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(g+hx)^{3/2}\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

```
[In] Integrate[1/((g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Integrate[1/((g + h*x)^(3/2)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{(hx+g)^{\frac{3}{2}}\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)}dx$$

```
[In] int(1/(h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
[Out] int(1/(h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.23

$$\int \frac{1}{(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)^{\frac{3}{2}}\left(b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a\right)}\,dx$$

```
[In] integrate(1/(h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas
")
[Out] integral(sqrt(h*x + g)/(a*h^2*x^2 + 2*a*g*h*x + a*g^2 + (b*h^2*x^2 + 2*b*g*
h*x + b*g^2)*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 15.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}\,dx = \int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))(g+hx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(h*x+g)**(3/2)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)**(3/2)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}\,dx = \int \frac{1}{(hx+g)^{\frac{3}{2}}(b\log(((fx+e)^pd)^qc)+a)}\,dx$$

```
[In] integrate(1/(h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima
")
[Out] integrate(1/((h*x + g)^(3/2)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)^{3/2}(a+b\log(c(d(e+fx)^p)^q))}\,dx = \int \frac{1}{(hx+g)^{\frac{3}{2}}(b\log(((fx+e)^pd)^qc)+a)}\,dx$$

```
[In] integrate(1/(h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
[Out] integrate(1/((h*x + g)^(3/2)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g+hx)^{3/2}\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{{\left(g+h\,x\right)}^{3/2}\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)} \,d x$$

```
[In] int(1/((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
[Out] int(1/((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

# 3.500 $\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\text{Int}\left(\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)},x\right)$$

```
[Out] Unintegrable((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

```
[In] Int[Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
[Out] Defer[Int][Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]], x]
```

Rubi steps

$$\text{integral}=\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

```
[In] Integrate[Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
[Out] Integrate[Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]], x]
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{hx+g}\,\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}dx$$

```
[In] int((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
[Out] int((h*x+g)^(1/2)*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fr
icas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \sqrt{a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}\sqrt{g+hx}\,dx$$

```
[In] integrate((h*x+g)**(1/2)*(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
[Out] Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))*sqrt(g + h*x), x)
```

## Maxima [N/A]

Not integrable

Time = 11.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \sqrt{hx+g}\sqrt{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right.}\,dx$$

```
[In] integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \sqrt{hx+g}\sqrt{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right.}\,dx$$

```
[In] integrate((h*x+g)^(1/2)*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int \sqrt{g+h\,x}\,\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}\,dx$$

```
[In] int((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
```

```
[Out] int((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

# 3.501 $\int \frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{\sqrt{g+hx}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx = \operatorname{Int}\left(\frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx$$

```
[In] Int[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/Sqrt[g + h*x],x]
[Out] Defer[Int][Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/Sqrt[g + h*x], x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx=\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx$$

```
[In] Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/Sqrt[g + h*x],x]
[Out] Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/Sqrt[g + h*x], x]
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}{\sqrt{hx+g}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(1/2),x)
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(1/2),x, algorithm="fr
icas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))/sqrt(g + h*x), x)
```

## Maxima [N/A]

Not integrable

Time = 11.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx = \int \frac{\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}{\sqrt{hx+g}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(1/2),x, algorithm="ma
xima")
[Out] integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/sqrt(h*x + g), x)
```

## Giac [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx = \int \frac{\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}{\sqrt{hx+g}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(1/2),x, algorithm="gi
ac")
[Out] integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/sqrt(h*x + g), x)
```

## Mupad [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{\sqrt{g+hx}}\,dx=\int \frac{\sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}}{\sqrt{g+h\,x}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x)^(1/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x)^(1/2), x)
```

# 3.502 $\int \frac{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}{(g+hx)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx = \text{Int}\left(\frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(3/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx = \int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx$$

```
[In] Int[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^(3/2),x]
[Out] Defer[Int][Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx=\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx$$

```
[In] Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^(3/2),x]
[Out] Integrate[Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]/(g + h*x)^(3/2), x]
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}{\left(hx+g\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(3/2),x)
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(3/2),x, algorithm="fr
icas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 17.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx=\int \frac{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}}{(g+hx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2)/(h*x+g)**(3/2),x)
[Out] Integral(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x)**(3/2), x)
```

## Maxima [N/A]

Not integrable

Time = 11.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx=\int \frac{\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}{(hx+g)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(3/2),x, algorithm="ma
xima")
[Out] integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g)^(3/2), x)
```

## Giac [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx=\int \frac{\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}{(hx+g)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^(1/2)/(h*x+g)^(3/2),x, algorithm="gi
ac")
[Out] integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}}{(g+hx)^{3/2}}\,dx=\int \frac{\sqrt{a+b\,\ln \left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}}{{\left(g+h\,x\right)}^{3/2}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x)^(3/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)/(g + h*x)^(3/2), x)
```

# 3.503 $\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \text{Int}\left(\frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}},x\right)$$

```
[Out] Unintegrable((h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx = \int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

```
[In] Int[Sqrt[g + h*x]/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] Defer[Int][Sqrt[g + h*x]/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]], x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 4.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

```
[In] Integrate[Sqrt[g + h*x]/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
[Out] Integrate[Sqrt[g + h*x]/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]], x]
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{hx+g}}{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}dx$$

```
[In] int((h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
[Out] int((h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fr
icas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log \left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)}}\,dx$$

```
[In] integrate((h*x+g)**(1/2)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
[Out] Integral(sqrt(g + h*x)/sqrt(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

## Maxima [N/A]

Not integrable

Time = 11.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{\sqrt{hx+g}}{\sqrt{b\log \left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a}}\,dx$$

```
[In] integrate((h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="ma
xima")
[Out] integrate(sqrt(h*x + g)/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{\sqrt{hx+g}}{\sqrt{b\log \left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a}}\,dx$$

```
[In] integrate((h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="gi
ac")
[Out] integrate(sqrt(h*x + g)/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{g+hx}}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{\sqrt{g+h\,x}}{\sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}}\,dx$$

```
[In] int((g + h*x)^(1/2)/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
[Out] int((g + h*x)^(1/2)/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

# 3.504 $\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\text{Int}\left(\frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

```
[In] Int[1/(Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]),x]
[Out] Defer[Int][1/(Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

```
[In] Integrate[1/(Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]),x]
```

```
[Out] Integrate[1/(Sqrt[g + h*x]*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]), x]
```

## Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{hx+g}\,\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}dx$$

```
[In] int(1/(h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

```
[Out] int(1/(h*x+g)^(1/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="
fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\sqrt{g+hx}}\,dx$$

```
[In] integrate(1/(h*x+g)**(1/2)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))*sqrt(g + h*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 11.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{\sqrt{hx+g}\sqrt{b\log\left(\left(\left(fx+e\right)^pd\right)^qc\right)+a}}\,dx$$

```
[In] integrate(1/(h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="
maxima")
[Out] integrate(1/(sqrt(h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{\sqrt{hx+g}\sqrt{b\log\left(\left(\left(fx+e\right)^pd\right)^qc\right)+a}}\,dx$$

```
[In] integrate(1/(h*x+g)^(1/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="
giac")
[Out] integrate(1/(sqrt(h*x + g)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{g+hx}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{\sqrt{g+h\,x}\,\sqrt{a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}}\,dx$$

```
[In] int(1/((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)),x)
[Out] int(1/((g + h*x)^(1/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)), x)
```

# 3.505 $\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\mathrm{Int}\left(\frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

```
[In] Int[1/((g + h*x)^(3/2)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]),x]
[Out] Defer[Int][1/((g + h*x)^(3/2)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

```
[In] Integrate[1/((g + h*x)^(3/2)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]),x]
[Out] Integrate[1/((g + h*x)^(3/2)*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]]), x]
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{(hx+g)^{\frac{3}{2}}\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)}}dx$$

```
[In] int(1/(h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
[Out] int(1/(h*x+g)^(3/2)/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="
fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 51.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\left(g+hx\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(h*x+g)**(3/2)/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)**(3/2)), x)
```

## Maxima [N/A]

Not integrable

Time = 11.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{(hx+g)^{\frac{3}{2}}\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

```
[In] integrate(1/(h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="
maxima")
[Out] integrate(1/((h*x + g)^(3/2)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{(hx+g)^{\frac{3}{2}}\sqrt{b\log\left(\left(\left(fx+e\right)^p d\right)^q c\right)+a}}\,dx$$

```
[In] integrate(1/(h*x+g)^(3/2)/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="
giac")
[Out] integrate(1/((h*x + g)^(3/2)*sqrt(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{(g+hx)^{3/2}\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{1}{{\left(g+h\,x\right)}^{3/2}\,\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}}\,dx$$

```
[In] int(1/((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)),x)
[Out] int(1/((g + h*x)^(3/2)*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2)), x)
```

# 3.506 $\int (g+hx)^m (a+b\log (c(d(e+fx)^p)^q))\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 99

$$\int (g+hx)^m (a+b\log (c(d(e+fx)^p)^q))\,dx = \frac{bfpq(g+hx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)(1+m)(2+m)} + \frac{(g+hx)^{1+m}(a+b\log (c(d(e+fx)^p)^q))}{h(1+m)}$$

```
[Out] b*f*p*q*(h*x+g)^(2+m)*hypergeom([1, 2+m],[3+m],f*(h*x+g)/(-e*h+f*g))/h/(-e*
h+f*g)/(1+m)/(2+m)+(h*x+g)^(1+m)*(a+b*ln(c*(d*(f*x+e)^p)^q))/h/(1+m)
```

## Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2442, 70, 2495}

$$\int (g+hx)^m (a+b\log (c(d(e+fx)^p)^q))\,dx = \frac{(g+hx)^{m+1}(a+b\log (c(d(e+fx)^p)^q))}{h(m+1)} + \frac{bfpq(g+hx)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{f(g+hx)}{fg-eh}\right)}{h(m+1)(m+2)(fg-eh)}$$

```
[In] Int[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] (b*f*p*q*(g + h*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (f*(g + h*x))
/(f*g - e*h)])/(h*(f*g - e*h)*(1 + m)*(2 + m)) + ((g + h*x)^(1 + m)*(a + b*
Log[c*(d*(e + f*x)^p)^q]))/(h*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int (g+hx)^m\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{(g+hx)^{1+m}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h(1+m)}\\
&\quad-\text{Subst}\left(\frac{(bfpq)\int\frac{(g+hx)^{1+m}}{e+fx}\,dx}{h(1+m)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{bfpq(g+hx)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{f(g+hx)}{fg-eh}\right)}{h(fg-eh)(1+m)(2+m)}+\frac{(g+hx)^{1+m}\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h(1+m)}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

$$\int (g+hx)^m (a+b\log (c(d(e+fx)^p)^q))\ dx = \frac{(g+hx)^{1+m}\left(a+\frac{bfpq(g+hx)\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{f(g+hx)}{fg-eh}\right)}{(fg-eh)(2+m)}+b\log (c(d(e+fx)^p)^q)\right)}{h(1+m)}$$

```
[In] Integrate[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] ((g + h*x)^(1 + m)*(a + (b*f*p*q*(g + h*x)*Hypergeometric2F1[1, 2 + m, 3 +
m, (f*(g + h*x))/(f*g - e*h)])/((f*g - e*h)*(2 + m)) + b*Log[c*(d*(e + f*x)
^p)^q]))/(h*(1 + m))
```

## Maple [F]

$$\int (hx+g)^m (a+b\ln (c(d(fx+e)^p)^q))\, dx$$

```
[In] int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

```
[Out] int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [F]

$$\int (g+hx)^m (a+b\log (c(d(e+fx)^p)^q))\ dx = \int (b\log (((fx+e)^p d)^q c)+a)(hx+g)^m\, dx$$

```
[In] integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
```

```
[Out] integral((h*x + g)^m*b*log(((f*x + e)^p*d)^q*c) + (h*x + g)^m*a, x)
```

## Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^m (a+b\log (c(d(e+fx)^p)^q))\ dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x+g)**m*(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [F]

$$\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))\,dx = \int (b\log(((fx+e)^pd)^q c)+a)(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
[Out] b*((h*x + g)*(h*x + g)^m*log(((f*x + e)^p)^q)/(h*(m + 1)) + integrate(-(f*g
*p*q - e*h*(m + 1)*log(c) - (m*q + q)*e*h*log(d) + (f*h*p*q - f*h*(m + 1)*l
og(c) - (m*q + q)*f*h*log(d))*x)*(h*x + g)^m/(f*h*(m + 1)*x + e*h*(m + 1)),
 x)) + (h*x + g)^(m + 1)*a/(h*(m + 1))
```

## Giac [F]

$$\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))\,dx = \int (b\log(((fx+e)^pd)^q c)+a)(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)*(h*x + g)^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))\,dx = \int (g+h\,x)^m\,(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))\,dx$$

```
[In] int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q)),x)
[Out] int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

# 3.507 $\int \frac{(g+hx)^m}{a+b\log(c(d(e+fx)^p)^q)}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\text{Int}\left(\frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)},x\right)$$

```
[Out] Unintegrable((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

```
[In] Int[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
```

```
[Out] Defer[Int][(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q]), x]
```

Rubi steps

$$\text{integral}=\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

```
[In] Integrate[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q]),x]
[Out] Integrate[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q]), x]
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(hx+g)^m}{a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)}dx$$

```
[In] int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
[Out] int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(hx+g)^m}{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}\,dx$$

```
[In] integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
[Out] integral((h*x + g)^m/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Sympy [N/A]

Not integrable

Time = 13.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(g+hx)^m}{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}\,dx$$

```
[In] integrate((h*x+g)**m/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
[Out] Integral((g + h*x)**m/(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(hx+g)^m}{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a}\,dx$$

```
[In] integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
[Out] integrate((h*x + g)^m/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(hx+g)^m}{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a}\,dx$$

```
[In] integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
[Out] integrate((h*x + g)^m/(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \frac{(g+h\,x)^m}{a+b\,\ln\left(c\,\left(d\,{\left(e+f\,x\right)}^p\right)^q\right)}\,dx$$

```
[In] int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q)),x)
[Out] int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q)), x)
```

# 3.508 $\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2} dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2} dx = \text{Int}\left(\frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}, x\right)$$

```
[Out] Unintegrable((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2} dx = \int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2} dx$$

```
[In] Int[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
```

```
[Out] Defer[Int][(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2} dx$$

## Mathematica [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx=\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx$$

```
[In] Integrate[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^2,x]
[Out] Integrate[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^2, x]
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(hx+g)^m}{(a+b\ln(c(d(fx+e)^p)^q))^2}dx$$

```
[In] int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
[Out] int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx=\int \frac{(hx+g)^m}{(b\log(((fx+e)^p d)^q c)+a)^2}\,dx$$

```
[In] integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
[Out] integral((h*x + g)^m/(b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)
^p*d)^q*c) + a^2), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^m}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x+g)**m/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 6.29

$$\int \frac{(g+hx)^m}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{(hx+g)^m}{\left(b\log\left(\left((fx+e)^pd\right)^q c\right)+a\right)^2}\,dx$$

```
[In] integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
[Out] -(f*x + e)*(h*x + g)^m/(b^2*f*p*q*log(((f*x + e)^p)^q) + a*b*f*p*q + (f*p*q
^2*log(d) + f*p*q*log(c))*b^2) + integrate((f*h*(m + 1)*x + e*h*m + f*g)*(h
*x + g)^m/(a*b*f*g*p*q + (f*g*p*q^2*log(d) + f*g*p*q*log(c))*b^2 + (a*b*f*h
*p*q + (f*h*p*q^2*log(d) + f*h*p*q*log(c))*b^2)*x + (b^2*f*h*p*q*x + b^2*f*
g*p*q)*log(((f*x + e)^p)^q)), x)
```

## Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{(hx+g)^m}{\left(b\log\left(\left((fx+e)^pd\right)^q c\right)+a\right)^2}\,dx$$

```
[In] integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
[Out] integrate((h*x + g)^m/(b*log(((f*x + e)^p*d)^q*c) + a)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^2}\,dx=\int \frac{(g+h\,x)^m}{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^2}\,dx$$

```
[In] int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q))^2,x)
[Out] int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q))^2, x)
```

# 3.509 $\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2} dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int (g+hx)^m (a + b\log(c(d(e+fx)^p)^q))^{3/2} dx = \text{Int}\left((g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^{3/2}, x\right)$$

```
[Out] Unintegrable((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^{3/2} dx = \int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^{3/2} dx$$

```
[In] Int[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
[Out] Defer[Int][(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2), x]
```

Rubi steps

$$\text{integral} = \int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^{3/2} dx$$

## Mathematica [N/A]

Not integrable

Time = 15.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2} dx = \int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2} dx$$

```
[In] Integrate[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
[Out] Integrate[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2), x]
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (hx+g)^m \left(a+b\ln\left(c(d(fx+e)^p)^q\right)\right)^{\frac{3}{2}} dx$$

```
[In] int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
[Out] int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2} dx = \int \left(b\log\left(((fx+e)^p d)^q c\right)+a\right)^{\frac{3}{2}} (hx+g)^m dx$$

```
[In] integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="fricas
")
[Out] integral(((h*x + g)^m*b*log(((f*x + e)^p*d)^q*c) + (h*x + g)^m*a)*sqrt(b*lo
g(((f*x + e)^p*d)^q*c) + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((h*x+g)**m*(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 11.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx=\int \left(b\log\left(\left((fx+e)^pd\right)^qc\right)+a\right)^{\frac{3}{2}}(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxima
")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)*(h*x + g)^m, x)
```

## Giac [N/A]

Not integrable

Time = 3.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx=\int \left(b\log\left(\left((fx+e)^pd\right)^qc\right)+a\right)^{\frac{3}{2}}(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^(3/2)*(h*x + g)^m, x)
```

## Mupad [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^{3/2}\,dx=\int (g+h\,x)^m\,\left(a+b\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)\right)^{3/2}\,dx$$

```
[In] int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
[Out] int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

# 3.510 $\int (g+hx)^m \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int (g+hx)^m \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \text{Int}\left((g+hx)^m \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)},x\right)$$

```
[Out] Unintegrable((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (g+hx)^m \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx = \int (g+hx)^m \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

```
[In] Int[(g + h*x)^m*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] Defer[Int][(g + h*x)^m*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]], x]
```

Rubi steps

$$\text{integral} = \int (g+hx)^m \sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int (g+hx)^m\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int (g+hx)^m\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx$$

```
[In] Integrate[(g + h*x)^m*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
[Out] Integrate[(g + h*x)^m*Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]], x]
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (hx+g)^m\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)}dx$$

```
[In] int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
[Out] int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int \sqrt{b\log\left(\left((fx+e)^pd\right)^qc\right)+a}(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fricas
")
[Out] integral(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)*(h*x + g)^m, x)
```

## Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^m\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x+g)**m*(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [N/A]

Not integrable

Time = 11.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int\sqrt{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right)}(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxima
")
[Out] integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)*(h*x + g)^m, x)
```

## Giac [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int\sqrt{b\log\left(\left(\left((fx+e)^pd\right)^q c\right)+a\right)}(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)*(h*x + g)^m, x)
```

## Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (g+hx)^m\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}\,dx=\int (g+h\,x)^m\,\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}\,dx$$

```
[In] int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
[Out] int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

# 3.511 $\int \frac{(g+hx)^m}{\sqrt{a+b\log(c(d(e+fx)^p)^q)}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\text{Int}\left(\frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}},x\right)$$

```
[Out] Unintegrable((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

```
[In] Int[(g + h*x)^m/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
```

```
[Out] Defer[Int][(g + h*x)^m/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]], x]
```

Rubi steps

$$\text{integral}=\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 10.61 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx$$

```
[In] Integrate[(g + h*x)^m/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]],x]
[Out] Integrate[(g + h*x)^m/Sqrt[a + b*Log[c*(d*(e + f*x)^p)^q]], x]
```

## Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(hx+g)^m}{\sqrt{a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)}}dx$$

```
[In] int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
[Out] int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{(hx+g)^m}{\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}\,dx$$

```
[In] integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="fricas
")
[Out] integral((h*x + g)^m/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Sympy [N/A]

Not integrable

Time = 5.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)}}\,dx$$

```
[In] integrate((h*x+g)**m/(a+b*ln(c*(d*(f*x+e)**p)**q))**(1/2),x)
[Out] Integral((g + h*x)**m/sqrt(a + b*log(c*(d*(e + f*x)**p)**q)), x)
```

## Maxima [N/A]

Not integrable

Time = 10.98 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{(hx+g)^m}{\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}\,dx$$

```
[In] integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="maxima
")
[Out] integrate((h*x + g)^m/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{(hx+g)^m}{\sqrt{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}}\,dx$$

```
[In] integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^(1/2),x, algorithm="giac")
[Out] integrate((h*x + g)^m/sqrt(b*log(((f*x + e)^p*d)^q*c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{\sqrt{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}}\,dx=\int \frac{(g+h\,x)^m}{\sqrt{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}}\,dx$$

```
[In] int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2),x)
[Out] int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q))^(1/2), x)
```

# 3.512 $\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \text{Int}\left(\frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}},x\right)$$

```
[Out] Unintegrable((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx = \int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$$

```
[In] Int[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
```

```
[Out] Defer[Int][(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 13.56 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx=\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx$$

```
[In] Integrate[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2),x]
[Out] Integrate[(g + h*x)^m/(a + b*Log[c*(d*(e + f*x)^p)^q])^(3/2), x]
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(hx+g)^m}{(a+b\ln(c(d(fx+e)^p)^q))^{\frac{3}{2}}}dx$$

```
[In] int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
[Out] int((h*x+g)^m/(a+b*ln(c*(d*(f*x+e)^p)^q))^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.47

$$\int \frac{(g+hx)^m}{(a+b\log(c(d(e+fx)^p)^q))^{3/2}}\,dx=\int \frac{(hx+g)^m}{(b\log(((fx+e)^pd)^q c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="fricas
")
[Out] integral(sqrt(b*log(((f*x + e)^p*d)^q*c) + a)*(h*x + g)^m/(b^2*log(((f*x +
e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c) + a^2), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^m}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x+g)**m/(a+b*ln(c*(d*(f*x+e)**p)**q))**(3/2),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [N/A]

Not integrable

Time = 11.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx = \int \frac{(hx+g)^m}{\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="maxima
")
[Out] integrate((h*x + g)^m/(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

## Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx = \int \frac{(hx+g)^m}{\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)^m/(a+b*log(c*(d*(f*x+e)^p)^q))^(3/2),x, algorithm="giac")
[Out] integrate((h*x + g)^m/(b*log(((f*x + e)^p*d)^q*c) + a)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(g+hx)^m}{\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^{3/2}}\,dx=\int \frac{(g+h\,x)^m}{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^{3/2}}\,dx$$

```
[In] int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2),x)
[Out] int((g + h*x)^m/(a + b*log(c*(d*(e + f*x)^p)^q))^(3/2), x)
```

# 3.513 $\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^n\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^n\,dx = \mathrm{Int}\left((g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^n, x\right)$$

```
[Out] Unintegrable((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^n\,dx = \int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^n\,dx$$

```
[In] Int[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
[Out] Defer[Int][(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^n, x]
```

Rubi steps

$$\text{integral} = \int (g+hx)^m (a+b\log(c(d(e+fx)^p)^q))^n\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (g+hx)^m\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\,dx=\int (g+hx)^m\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\,dx$$

```
[In] Integrate[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
[Out] Integrate[(g + h*x)^m*(a + b*Log[c*(d*(e + f*x)^p)^q])^n, x]
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (hx+g)^m\left(a+b\ln\left(c(d(fx+e)^p)^q\right)\right)^n dx$$

```
[In] int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
[Out] int((h*x+g)^m*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
```

## Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (g+hx)^m\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\,dx=\int (hx+g)^m(b\log\left(((fx+e)^p d)^q\,c\right)+a)^n\,dx$$

```
[In] integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="fricas")
[Out] integral((h*x + g)^m*(b*log(((f*x + e)^p*d)^q*c) + a)^n, x)
```

## Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^m\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x+g)**m*(a+b*ln(c*(d*(f*x+e)**p)**q))**n,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

## Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

## Giac [F(-2)]

Exception generated.

$$\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((h*x+g)^m*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,0
,7,4,0,5,0,3,3,3,0,2,0,0,0]%%%}+%%%{5,[0,0,6,4,0,4,1,3,3,3,0,2,0,0,0]%%%}+%
%%{2,[0,0
```

## Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (g+hx)^m \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n \, dx = \int (g+h\,x)^m\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^n \,d x$$

```
[In] int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q))^n,x)
```

```
[Out] int((g + h*x)^m*(a + b*log(c*(d*(e + f*x)^p)^q))^n, x)
```

# 3.514 $\int (g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^n\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 432

$$\int (g+hx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^n\,dx$$
$$= \frac{3^{-1-n}e^{-\frac{3a}{bpq}}h^2(e+fx)^3\left(c\left(d(e+fx)^p\right)^q\right)^{-\frac{3}{pq}}\Gamma\left(1+n,-\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^n \Big(}{f^3}$$
$$+\frac{2^{-n}e^{-\frac{2a}{bpq}}h(fg-eh)(e+fx)^2\left(c\left(d(e+fx)^p\right)^q\right)^{-\frac{2}{pq}}\Gamma\left(1+n,-\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)\left(a+b\log\left(c\left(d(e+fx\right.\right.\right.}{f^3}$$
$$+\frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c\left(d(e+fx)^p\right)^q\right)^{-\frac{1}{pq}}\Gamma\left(1+n,-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^n}{f^3}$$

```
[Out] 3^(-1-n)*h^2*(f*x+e)^3*GAMMA(1+n,-3*(a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)*(a+b
*ln(c*(d*(f*x+e)^p)^q))^n/exp(3*a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(3/p/q))/
(((-a-b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)^n)+h*(-e*h+f*g)*(f*x+e)^2*GAMMA(1+n,-
2*(a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)*(a+b*ln(c*(d*(f*x+e)^p)^q))^n/(2^n)/ex
p(2*a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(2/p/q))/(((-a-b*ln(c*(d*(f*x+e)^p)^q
))/b/p/q)^n)+(-e*h+f*g)^2*(f*x+e)*GAMMA(1+n,(-a-b*ln(c*(d*(f*x+e)^p)^q))/b/
p/q)*(a+b*ln(c*(d*(f*x+e)^p)^q))^n/exp(a/b/p/q)/f^3/((c*(d*(f*x+e)^p)^q)^(1
/p/q))/(((-a-b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)^n)
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2448, 2436, 2337, 2212, 2437, 2347, 2495}

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^n \, dx$$
$$= \frac{h2^{-n}(e+fx)^2 e^{-\frac{2a}{bpq}}(fg-eh)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}(a+b\log(c(d(e+fx)^p)^q))^n\left(-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)^{-n}\Gamma}{f^3}$$
$$+\frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}(a+b\log(c(d(e+fx)^p)^q))^n\left(-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)^{-n}\Gamma\Big(n}{f^3}$$
$$+\frac{h^2 3^{-n-1}(e+fx)^3 e^{-\frac{3a}{bpq}}\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}(a+b\log(c(d(e+fx)^p)^q))^n\left(-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)^{-n}\Gamma\Big(n+}{f^3}$$

```
[In] Int[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
```

```
[Out] (3^(-1 - n)*h^2*(e + f*x)^3*Gamma[1 + n, (-3*(a + b*Log[c*(d*(e + f*x)^p)^q
]))/(b*p*q)]*(a + b*Log[c*(d*(e + f*x)^p)^q])^n)/(E^((3*a)/(b*p*q))*f^3*(c*
(d*(e + f*x)^p)^q)^(3/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^
n) + (h*(f*g - e*h)*(e + f*x)^2*Gamma[1 + n, (-2*(a + b*Log[c*(d*(e + f*x)^
p)^q]))/(b*p*q)]*(a + b*Log[c*(d*(e + f*x)^p)^q])^n)/(2^n*E^((2*a)/(b*p*q))
*f^3*(c*(d*(e + f*x)^p)^q)^(2/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b
*p*q)))^n) + ((f*g - e*h)^2*(e + f*x)*Gamma[1 + n, -((a + b*Log[c*(d*(e + f
*x)^p)^q])/(b*p*q))]*(a + b*Log[c*(d*(e + f*x)^p)^q])^n)/(E^(a/(b*p*q))*f^3
*(c*(d*(e + f*x)^p)^q)^(1/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q
)))^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (g+hx)^2 \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^n \, dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\int \left(\frac{(fg-eh)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^n}{f^2} + \frac{2h(fg-eh)(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^n}{f^2} + \frac{h^2(e+fx)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^n}{f^2}\right) dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{h^2 \int (e+fx)^2 \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^n\,dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(2h(fg-eh)) \int (e+fx) \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^n\,dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(fg-eh)^2 \int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^n\,dx}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{h^2 \text{Subst}\left(\int x^2(a+b\log\left(cd^q x^{pq}\right))^n\,dx, x, e+fx\right)}{f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(2h(fg-eh)) \text{Subst}\left(\int x(a+b\log\left(cd^q x^{pq}\right))^n\,dx, x, e+fx\right)}{f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(fg-eh)^2 \text{Subst}\left(\int (a+b\log\left(cd^q x^{pq}\right))^n\,dx, x, e+fx\right)}{f^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$=\text{Subst}\left(\frac{\left(h^2(e+fx)^3\left(cd^q(e+fx)^{pq}\right)^{-\frac{3}{pq}}\right)\text{Subst}\left(\int e^{\frac{3x}{pq}}(a+bx)^n\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{f^3pq},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left(2h(fg-eh)(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int e^{\frac{2x}{pq}}(a+bx)^n\,dx,x,\log\left(cd^q(e+fx\right.\right.}{f^3pq}+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left((fg-eh)^2(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{\frac{x}{pq}}(a+bx)^n\,dx,x,\log\left(cd^q(e+fx)^{pq}\right.\right.}{f^3pq}+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$=\frac{3^{-1-n}e^{-\frac{3a}{bpq}}h^2(e+fx)^3\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\Gamma\left(1+n,-\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)\left(a+b\log\left(c(d(e+fx\right.\right.}{f^3}$$

$$+\frac{2^{-n}e^{-\frac{2a}{bpq}}h(fg-eh)(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\Gamma\left(1+n,-\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)\left(a+b\log\left(c(c\right.\right.}{f^3}$$

$$+\frac{e^{-\frac{a}{bpq}}(fg-eh)^2(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\Gamma\left(1+n,-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)\left(a+b\log\left(c(d(e+f\right.\right.}{f^3}$$

## Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.75

$$\int(g+hx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\,dx$$

$$=\frac{2^{-n}3^{-1-n}e^{-\frac{3a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{3}{pq}}\left(2^nh^2(e+fx)^2\Gamma\left(1+n,-\frac{3(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)+3^{1+n}e^{\frac{a}{bpq}}(fg\right.}{}$$

```
[In] Integrate[(g + h*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
```

```
[Out] (3^(-1 - n)*(e + f*x)*(2^n*h^2*(e + f*x)^2*Gamma[1 + n, (-3*(a + b*Log[c*(d
*(e + f*x)^p)^q]))/(b*p*q)] + 3^(1 + n)*E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e
+ f*x)^p)^q)^(1/(p*q))*(h*(e + f*x)*Gamma[1 + n, (-2*(a + b*Log[c*(d*(e + f
```

```
*x)^p)^q]))/(b*p*q)] + 2^n*E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)^p)^q)^
(1/(p*q))*Gamma[1 + n, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))]))*(a +
b*Log[c*(d*(e + f*x)^p)^q])^n)/(2^n*E^((3*a)/(b*p*q))*f^3*(c*(d*(e + f*x)^p
)^q)^(3/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^n)
```

## Maple [F]

$$\int (hx+g)^2 (a+b\ln(c(d(fx+e)^p)^q))^n dx$$

```
[In] int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
[Out] int((h*x+g)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)
```

## Fricas [F]

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^n \, dx = \int (hx+g)^2 (b\log(((fx+e)^p d)^q c)+a)^n \, dx$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="fricas")
[Out] integral((h^2*x^2 + 2*g*h*x + g^2)*(b*log(((f*x + e)^p*d)^q*c) + a)^n, x)
```

## Sympy [F(-1)]

Timed out.

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^n \, dx = \text{Timed out}$$

```
[In] integrate((h*x+g)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**n,x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^2 (a+b\log(c(d(e+fx)^p)^q))^n \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

## Giac [F]

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n \, dx = \int (hx+g)^2 \left(b\log\left(((fx+e)^p d)^q c\right)+a\right)^n dx$$

```
[In] integrate((h*x+g)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="giac")
[Out] integrate((h*x + g)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n \, dx = \int (g+h\,x)^2 \left(a+b\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)\right)^n dx$$

```
[In] int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^n,x)
[Out] int((g + h*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^n, x)
```

# 3.515 $\int (g+hx) (a+b\log (c(d(e+fx)^p)^q))^n \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 281

$$\int (g+hx) (a+b\log (c(d(e+fx)^p)^q))^n \, dx = \frac{2^{-1-n}e^{-\frac{2a}{bpq}}h(e+fx)^2 (c(d(e+fx)^p)^q)^{-\frac{2}{pq}}\Gamma\left(1+n,-\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)(a+b\log (c(d(e+fx)^p)^q))^n \left(\right.}{f^2} + \frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx) (c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\Gamma\left(1+n,-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)(a+b\log (c(d(e+fx)^p)^q))^n}{f^2}$$

```
[Out] 2^(-1-n)*h*(f*x+e)^2*GAMMA(1+n,-2*(a+b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)*(a+b*l
n(c*(d*(f*x+e)^p)^q))^n/exp(2*a/b/p/q)/f^2/((c*(d*(f*x+e)^p)^q)^(2/p/q))/((
(-a-b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)^n)+(-e*h+f*g)*(f*x+e)*GAMMA(1+n,(-a-b*l
n(c*(d*(f*x+e)^p)^q))/b/p/q)*(a+b*ln(c*(d*(f*x+e)^p)^q))^n/exp(a/b/p/q)/f^2
/((c*(d*(f*x+e)^p)^q)^(1/p/q))/(((-a-b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)^n)
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2448, 2436, 2337, 2212, 2437, 2347, 2495}

$$\int (g+hx) (a+b\log (c(d(e+fx)^p)^q))^n \, dx = \frac{(e+fx)e^{-\frac{a}{bpq}}(fg-eh) (c(d(e+fx)^p)^q)^{-\frac{1}{pq}} (a+b\log (c(d(e+fx)^p)^q))^n \left(-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)^{-n}\Gamma\left(n+\right.}{f^2} + \frac{h2^{-n-1}(e+fx)^2e^{-\frac{2a}{bpq}} (c(d(e+fx)^p)^q)^{-\frac{2}{pq}} (a+b\log (c(d(e+fx)^p)^q))^n \left(-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)^{-n}\Gamma\left(n+\right.}{f^2}$$

```
[In] Int[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
```

```
[Out] (2^(-1 - n)*h*(e + f*x)^2*Gamma[1 + n, (-2*(a + b*Log[c*(d*(e + f*x)^p)^q])
)/(b*p*q)]*(a + b*Log[c*(d*(e + f*x)^p)^q])^n)/(E^((2*a)/(b*p*q))*f^2*(c*(d
*(e + f*x)^p)^q)^(2/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^n)
 + ((f*g - e*h)*(e + f*x)*Gamma[1 + n, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(
b*p*q))]*(a + b*Log[c*(d*(e + f*x)^p)^q])^n)/(E^(a/(b*p*q))*f^2*(c*(d*(e +
f*x)^p)^q)^(1/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
```

```
d*g, 0] && IGtQ[q, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int (g+hx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^n\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\int \left(\frac{(fg-eh)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^n}{f} + \frac{h(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^n}{f}\right)dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{h\int (e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^n\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad+\text{Subst}\left(\frac{(fg-eh)\int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^n\,dx}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{h\text{Subst}\left(\int x\left(a+b\log\left(cd^q x^{pq}\right)\right)^n\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad+\text{Subst}\left(\frac{(fg-eh)\text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right)^n\,dx, x, e+fx\right)}{f^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$=\text{Subst}\left(\frac{\left(h(e+fx)^2\left(cd^q(e+fx)^{pq}\right)^{-\frac{2}{pq}}\right)\text{Subst}\left(\int e^{\frac{2x}{pq}}(a+bx)^n\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{f^2pq},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left((fg-eh)(e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{\frac{x}{pq}}(a+bx)^n\,dx,x,\log\left(cd^q(e+fx)^{pq}\right)\right)}{f^2pq},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$=\frac{2^{-1-n}e^{-\frac{2a}{bpq}}h(e+fx)^2\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\Gamma\left(1+n,-\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)(a+b\log(c(d(e+fx)^p$$

$$+\frac{e^{-\frac{a}{bpq}}(fg-eh)(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\Gamma\left(1+n,-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)(a+b\log(c(d(e+fx}{f^2}$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.81

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\,dx$$

$$=\frac{2^{-1-n}e^{-\frac{2a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{2}{pq}}\left(h(e+fx)\Gamma\left(1+n,-\frac{2(a+b\log(c(d(e+fx)^p)^q))}{bpq}\right)+2^{1+n}e^{\frac{a}{bpq}}(fg-eh)\right.}{f^2}$$

```
[In] Integrate[(g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
```

```
[Out] (2^(-1 - n)*(e + f*x)*(h*(e + f*x)*Gamma[1 + n, (-2*(a + b*Log[c*(d*(e + f*
x)^p)^q]))/(b*p*q)] + 2^(1 + n)*E^(a/(b*p*q))*(f*g - e*h)*(c*(d*(e + f*x)^p
)^q)^(1/(p*q))*Gamma[1 + n, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))])*(
a + b*Log[c*(d*(e + f*x)^p)^q])^n)/(E^((2*a)/(b*p*q))*f^2*(c*(d*(e + f*x)^p
)^q)^(2/(p*q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^n)
```

## Maple [F]

$$\int (hx + g) (a + b \ln (c(d(fx + e)^p)^q))^n dx$$

[In] int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)

[Out] int((h*x+g)*(a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)

## Fricas [F]

$$\int (g + hx) (a + b \log (c(d(e + fx)^p)^q))^n \, dx = \int (hx + g)(b \log (((fx + e)^p d)^q \, c) + a)^n \, dx$$

[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="fricas")

[Out] integral((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^n, x)

## Sympy [F]

$$\int (g + hx) (a + b \log (c(d(e + fx)^p)^q))^n \, dx = \int (a + b \log (c(d(e + fx)^p)^q))^n (g + hx) \, dx$$

[In] integrate((h*x+g)*(a+b*ln(c*(d*(f*x+e)**p)**q))**n,x)

[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**n*(g + h*x), x)

## Maxima [F(-2)]

Exception generated.

$$\int (g + hx) (a + b \log (c(d(e + fx)^p)^q))^n \, dx = \text{Exception raised: RuntimeError}$$

[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST

## Giac [F]

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\,dx=\int (hx+g)\left(b\log\left(\left((fx+e)^pd\right)^qc\right)+a\right)^n\,dx$$

```
[In] integrate((h*x+g)*(a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="giac")
[Out] integrate((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\,dx=\int (g+h\,x)\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^n\,dx$$

```
[In] int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^n,x)
[Out] int((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^n, x)
```

# 3.516 $\int (a+b\log(c(d(e+fx)^p)^q))^n\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 131

$$\int (a+b\log(c(d(e+fx)^p)^q))^n\,dx = \frac{e^{-\frac{a}{bpq}}(e+fx)(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}\Gamma\left(1+n,-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)(a+b\log(c(d(e+fx)^p)^q))^n\left(-\frac{a+b\log(c(\cdots}{b\cdots}\right.}{f}$$

```
[Out] (f*x+e)*GAMMA(1+n,(-a-b*ln(c*(d*(f*x+e)^p)^q))/b/p/q)*(a+b*ln(c*(d*(f*x+e)^
p)^q))^n/exp(a/b/p/q)/f/((c*(d*(f*x+e)^p)^q)^(1/p/q))/(((-a-b*ln(c*(d*(f*x+
e)^p)^q))/b/p/q)^n)
```

## Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2436, 2337, 2212, 2495}

$$\int (a+b\log(c(d(e+fx)^p)^q))^n\,dx = \frac{(e+fx)e^{-\frac{a}{bpq}}(c(d(e+fx)^p)^q)^{-\frac{1}{pq}}(a+b\log(c(d(e+fx)^p)^q))^n\left(-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\log(\cdots}{\cdots}\right.}{f}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]
```

```
[Out] ((e + f*x)*Gamma[1 + n, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))]*(a + b
*Log[c*(d*(e + f*x)^p)^q])^n)/(E^(a/(b*p*q))*f*(c*(d*(e + f*x)^p)^q)^(1/(p*
q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^n\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right)^n\,dx, x, e+fx\right)}{f}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\left((e+fx)\left(cd^q(e+fx)^{pq}\right)^{-\frac{1}{pq}}\right)\text{Subst}\left(\int e^{\frac{x}{pq}}(a+bx)^n\,dx, x, \log\left(cd^q(e+fx)^{pq}\right)\right)}{fpq}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\Gamma\left(1+n,-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^n\left(-\frac{a\cdots}{\cdots}\right.}{f}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int (a+b\log (c(d(e+fx)^p)^q))^n \, dx = \frac{e^{-\frac{a}{bpq}}(e+fx)\left(c(d(e+fx)^p)^q\right)^{-\frac{1}{pq}}\Gamma\left(1+n,-\frac{a+b\log(c(d(e+fx)^p)^q)}{bpq}\right)(a+b\log (c(d(e+fx)^p)^q))^n\left(-\frac{a+b\log(c(}{b}\right.}{f}$$

[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^n,x]

[Out] ((e + f*x)*Gamma[1 + n, -((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q))]*(a + b *Log[c*(d*(e + f*x)^p)^q])^n)/(E^(a/(b*p*q))*f*(c*(d*(e + f*x)^p)^q)^(1/(p* q))*(-((a + b*Log[c*(d*(e + f*x)^p)^q])/(b*p*q)))^n)

## Maple [F]

$$\int (a+b\ln (c(d(fx+e)^p)^q))^n dx$$

[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)

[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^n,x)

## Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int (a+b\log (c(d(e+fx)^p)^q))^n \, dx = \frac{e^{\left(-\frac{bnpq\log\left(-\frac{1}{bpq}\right)+bq\log(d)+b\log(c)+a}{bpq}\right)}\Gamma\left(n+1,-\frac{bpq\log(fx+e)+bq\log(d)+b\log(c)+a}{bpq}\right)}{f}$$

[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="fricas")

[Out] e^(-(b*n*p*q*log(-1/(b*p*q)) + b*q*log(d) + b*log(c) + a)/(b*p*q))*gamma(n + 1, -(b*p*q*log(f*x + e) + b*q*log(d) + b*log(c) + a)/(b*p*q))/f

## Sympy [F]

$$\int (a+b\log (c(d(e+fx)^p)^q))^n\,dx=\int (a+b\log (c(d(e+fx)^p)^q))^n\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**n,x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**n, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (a+b\log (c(d(e+fx)^p)^q))^n\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

## Giac [F]

$$\int (a+b\log (c(d(e+fx)^p)^q))^n\,dx=\int (b\log (((fx+e)^pd)^qc)+a)^n\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^n,x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\log (c(d(e+fx)^p)^q))^n\,dx=\int (a+b\,\ln (c\,(d\,(e+f\,x)^p)^q))^n\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^n,x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^n, x)
```

# 3.517 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx} \, dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx} \, dx = \text{Int}\left(\frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d*(f*x+e)^p)^q))^n/(h*x+g),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx} \, dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx} \, dx$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^n/(g + h*x),x]
[Out] Defer[Int][(a + b*Log[c*(d*(e + f*x)^p)^q])^n/(g + h*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx} \, dx$$

## Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^n/(g + h*x),x]
[Out] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^n/(g + h*x), x]
```

## Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^n}{hx+g}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^n/(h*x+g),x)
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^n/(h*x+g),x)
```

## Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx=\int \frac{(b\log(((fx+e)^pd)^q\,c)+a)^n}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^n/(h*x+g),x, algorithm="fricas")
[Out] integral((b*log(((f*x + e)^p*d)^q*c) + a)^n/(h*x + g), x)
```

## Sympy [N/A]

Not integrable

Time = 6.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx=\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**n/(h*x+g),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**n/(g + h*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^n/(h*x+g),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

## Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^n}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^n/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^n/(h*x + g), x)
```

## Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^n}{g+hx}\,dx = \int \frac{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^n}{g+h\,x}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^n/(g + h*x),x)
```

```
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^n/(g + h*x), x)
```

# 3.518 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{g+hx^2}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 249

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx^2}\,dx = \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f\left(\sqrt{-g}-\sqrt{h}x\right)}{f\sqrt{-g}+e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}} - \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f\left(\sqrt{-g}+\sqrt{h}x\right)}{f\sqrt{-g}-e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}} - \frac{bpq\,\text{PolyLog}\left(2,-\frac{\sqrt{h}(e+fx)}{f\sqrt{-g}-e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}} + \frac{bpq\,\text{PolyLog}\left(2,\frac{\sqrt{h}(e+fx)}{f\sqrt{-g}+e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}}$$

```
[Out] 1/2*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*((-g)^(1/2)-x*h^(1/2))/(f*(-g)^(1/2)+e
*h^(1/2)))/(-g)^(1/2)/h^(1/2)-1/2*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*((-g)^(1
/2)+x*h^(1/2))/(f*(-g)^(1/2)-e*h^(1/2)))/(-g)^(1/2)/h^(1/2)-1/2*b*p*q*polyl
og(2,-(f*x+e)*h^(1/2)/(f*(-g)^(1/2)-e*h^(1/2)))/(-g)^(1/2)/h^(1/2)+1/2*b*p*
q*polylog(2,(f*x+e)*h^(1/2)/(f*(-g)^(1/2)+e*h^(1/2)))/(-g)^(1/2)/h^(1/2)
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2456, 2441, 2440, 2438, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx^2}\,dx = \frac{\log\left(\frac{f\left(\sqrt{-g}-\sqrt{h}x\right)}{e\sqrt{h}+f\sqrt{-g}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2\sqrt{-g}\sqrt{h}} - \frac{\log\left(\frac{f\left(\sqrt{-g}+\sqrt{h}x\right)}{f\sqrt{-g}-e\sqrt{h}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2\sqrt{-g}\sqrt{h}} - \frac{bpq\operatorname{PolyLog}\left(2,-\frac{\sqrt{h}(e+fx)}{f\sqrt{-g}-e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}} + \frac{bpq\operatorname{PolyLog}\left(2,\frac{\sqrt{h}(e+fx)}{\sqrt{h}e+f\sqrt{-g}}\right)}{2\sqrt{-g}\sqrt{h}}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x^2),x]
```

```
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(Sqrt[-g] - Sqrt[h]*x))/(f*Sqrt[-g
] + e*Sqrt[h])])/(2*Sqrt[-g]*Sqrt[h]) - ((a + b*Log[c*(d*(e + f*x)^p)^q])*L
og[(f*(Sqrt[-g] + Sqrt[h]*x))/(f*Sqrt[-g] - e*Sqrt[h])])/(2*Sqrt[-g]*Sqrt[h
]) - (b*p*q*PolyLog[2, -((Sqrt[h]*(e + f*x))/(f*Sqrt[-g] - e*Sqrt[h]))])/(2
*Sqrt[-g]*Sqrt[h]) + (b*p*q*PolyLog[2, (Sqrt[h]*(e + f*x))/(f*Sqrt[-g] + e*
Sqrt[h])])/(2*Sqrt[-g]*Sqrt[h])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{g+hx^2}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\int \left(\frac{\sqrt{-g}(a+b\log\left(cd^q(e+fx)^{pq}\right))}{2g\left(\sqrt{-g}-\sqrt{h}x\right)}\right.\right.\\
&\qquad \left.\left.+\frac{\sqrt{-g}(a+b\log\left(cd^q(e+fx)^{pq}\right))}{2g\left(\sqrt{-g}+\sqrt{h}x\right)}\right)\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= -\text{Subst}\left(\frac{\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{\sqrt{-g}-\sqrt{h}x}\,dx}{2\sqrt{-g}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad -\text{Subst}\left(\frac{\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{\sqrt{-g}+\sqrt{h}x}\,dx}{2\sqrt{-g}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}
$$

$$=\frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(\sqrt{-g}-\sqrt{h}x)}{f\sqrt{-g}+e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}}-\frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(\sqrt{-g}+\sqrt{h}x)}{f\sqrt{-g}-e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}}-\text{Subst}\left(\frac{(bfpq)\int\frac{\log\left(\frac{f(\sqrt{-g}-\sqrt{h}x)}{f\sqrt{-g}+e\sqrt{h}}\right)}{e+fx}dx}{2\sqrt{-g}\sqrt{h}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)+\text{Subst}\left(\frac{(bfpq)\int\frac{\log\left(\frac{f(\sqrt{-g}+\sqrt{h}x)}{f\sqrt{-g}-e\sqrt{h}}\right)}{e+fx}dx}{2\sqrt{-g}\sqrt{h}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$=\frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(\sqrt{-g}-\sqrt{h}x)}{f\sqrt{-g}+e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}}-\frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(\sqrt{-g}+\sqrt{h}x)}{f\sqrt{-g}-e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}}+\text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{h}x}{f\sqrt{-g}-e\sqrt{h}}\right)}{x}dx,x,e+fx\right)}{2\sqrt{-g}\sqrt{h}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)-\text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{h}x}{f\sqrt{-g}+e\sqrt{h}}\right)}{x}dx,x,e+fx\right)}{2\sqrt{-g}\sqrt{h}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(\sqrt{-g}-\sqrt{h}x)}{f\sqrt{-g}+e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}} - \frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(\sqrt{-g}+\sqrt{h}x)}{f\sqrt{-g}-e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}} - \frac{bpq\text{Li}_2\left(-\frac{\sqrt{h}(e+fx)}{f\sqrt{-g}-e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}} + \frac{bpq\text{Li}_2\left(\frac{\sqrt{h}(e+fx)}{f\sqrt{-g}+e\sqrt{h}}\right)}{2\sqrt{-g}\sqrt{h}}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.76

$$\int \frac{a+b\log(c(d(e+fx)^p)^q)}{g+hx^2}\,dx = \frac{(a+b\log(c(d(e+fx)^p)^q))\left(\log\left(\frac{f(\sqrt{-g}-\sqrt{h}x)}{f\sqrt{-g}+e\sqrt{h}}\right)-\log\left(\frac{f(\sqrt{-g}+\sqrt{h}x)}{f\sqrt{-g}-e\sqrt{h}}\right)\right)-bpq\,\text{PolyLog}\left(2,-\frac{\sqrt{h}(e+fx)}{f\sqrt{-g}-e\sqrt{h}}\right)+}{2\sqrt{-g}\sqrt{h}}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x^2),x]
```

```
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])*(Log[(f*(Sqrt[-g] - Sqrt[h]*x))/(f*Sqrt[-
g] + e*Sqrt[h])] - Log[(f*(Sqrt[-g] + Sqrt[h]*x))/(f*Sqrt[-g] - e*Sqrt[h])]
) - b*p*q*PolyLog[2, -((Sqrt[h]*(e + f*x))/(f*Sqrt[-g] - e*Sqrt[h]))] + b*p
*q*PolyLog[2, (Sqrt[h]*(e + f*x))/(f*Sqrt[-g] + e*Sqrt[h])])/(2*Sqrt[-g]*Sq
rt[h])
```

## Maple [F]

$$\int \frac{a+b\ln(c(d(fx+e)^p)^q)}{h\,x^2+g}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x^2+g),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x^2+g),x)
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx^2}\,dx=\int \frac{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}{hx^2+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+g),x, algorithm="fricas")
[Out] integral((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x^2 + g), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x**2+g),x)
[Out] Timed out
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx^2}\,dx=\int \frac{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}{hx^2+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+g),x, algorithm="maxima")
[Out] b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(h*x^2 + g), x) + a*
arctan(h*x/sqrt(g*h))/sqrt(g*h)
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx^2}\,dx=\int \frac{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}{hx^2+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+g),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x^2 + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{g+hx^2}\,dx=\int \frac{a+b\,\ln \left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{h\,x^2+g} \,d x$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x^2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x^2), x)
```

# 3.519 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{2+hx^2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 335

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2+hx^2}}\,dx = \frac{bpq\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)^2}{2\sqrt{h}} - \frac{bpq\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}} - \frac{bpq\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}+\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}} + \frac{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}} - \frac{bpq\operatorname{PolyLog}\left(2,-\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}} - \frac{bpq\operatorname{PolyLog}\left(2,-\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}+\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}}$$

```
[Out] 1/2*b*p*q*arcsinh(1/2*x*h^(1/2)*2^(1/2))^2/h^(1/2)+arcsinh(1/2*x*h^(1/2)*2^
(1/2))*(a+b*ln(c*(d*(f*x+e)^p)^q))/h^(1/2)-b*p*q*arcsinh(1/2*x*h^(1/2)*2^(1
/2))*ln(1+(1/2*x*h^(1/2)*2^(1/2)+1/2*(2*h*x^2+4)^(1/2))*f*2^(1/2)/(e*h^(1/2
)-(e^2*h+2*f^2)^(1/2)))/h^(1/2)-b*p*q*arcsinh(1/2*x*h^(1/2)*2^(1/2))*ln(1+(
1/2*x*h^(1/2)*2^(1/2)+1/2*(2*h*x^2+4)^(1/2))*f*2^(1/2)/(e*h^(1/2)+(e^2*h+2*
```

```
f^2)^(1/2)))/h^(1/2)-b*p*q*polylog(2,-(1/2*x*h^(1/2)*2^(1/2)+1/2*(2*h*x^2+4
)^(1/2))*f*2^(1/2)/(e*h^(1/2)-(e^2*h+2*f^2)^(1/2)))/h^(1/2)-b*p*q*polylog(2
,-(1/2*x*h^(1/2)*2^(1/2)+1/2*(2*h*x^2+4)^(1/2))*f*2^(1/2)/(e*h^(1/2)+(e^2*h
+2*f^2)^(1/2)))/h^(1/2)
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {221, 2451, 12, 5827, 5680, 2221, 2317, 2438, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2+hx^2}}\,dx=\frac{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}}-\frac{bpq\,\text{PolyLog}\left(2,-\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{he^2+2f^2}}\right)}{\sqrt{h}}-\frac{bpq\,\text{PolyLog}\left(2,-\frac{\sqrt{2}e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{\sqrt{h}e+\sqrt{he^2+2f^2}}\right)}{\sqrt{h}}-\frac{bpq\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\log\left(\frac{\sqrt{2}fe^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}}{e\sqrt{h}-\sqrt{e^2h+2f^2}}+1\right)}{\sqrt{h}}-\frac{bpq\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\log\left(\frac{\sqrt{2}fe^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}}{\sqrt{e^2h+2f^2}+e\sqrt{h}}+1\right)}{\sqrt{h}}+\frac{bpq\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)^2}{2\sqrt{h}}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[2 + h*x^2],x]
```

```
[Out] (b*p*q*ArcSinh[(Sqrt[h]*x)/Sqrt[2]]^2)/(2*Sqrt[h]) - (b*p*q*ArcSinh[(Sqrt[h
]*x)/Sqrt[2]]*Log[1 + (Sqrt[2]*E^ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*f)/(e*Sqrt[h]
 - Sqrt[2*f^2 + e^2*h])])/Sqrt[h] - (b*p*q*ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*Log
[1 + (Sqrt[2]*E^ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*f)/(e*Sqrt[h] + Sqrt[2*f^2 + e
^2*h])])/Sqrt[h] + (ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*(a + b*Log[c*(d*(e + f*x)^
p)^q]))/Sqrt[h] - (b*p*q*PolyLog[2, -((Sqrt[2]*E^ArcSinh[(Sqrt[h]*x)/Sqrt[2
]]*f)/(e*Sqrt[h] - Sqrt[2*f^2 + e^2*h]))])/Sqrt[h] - (b*p*q*PolyLog[2, -((S
qrt[2]*E^ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*f)/(e*Sqrt[h] + Sqrt[2*f^2 + e^2*h]))
])/Sqrt[h]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
```

```
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{\sqrt{2+hx^2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}} - \text{Subst}\left((bfpq)\int \frac{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}{\sqrt{h}(e+fx)}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}} - \text{Subst}\left(\frac{(bfpq)\int \frac{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}{e+fx}\,dx}{\sqrt{h}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}} - \text{Subst}\left(\frac{(bfpq)\text{Subst}\left(\int \frac{x\cosh(x)}{\frac{e\sqrt{h}}{\sqrt{2}}+f\sinh(x)}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\right)}{\sqrt{h}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{bpq \sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)^2}{2\sqrt{h}} + \frac{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)(a + b\log(c(d(e + fx)^p)^q))}{\sqrt{h}}$$

$$- \text{Subst}\left(\frac{(bfpq)\text{Subst}\left(\int \frac{e^x x}{e^x f + \frac{e\sqrt{h}}{\sqrt{2}} - \frac{\sqrt{2f^2+e^2h}}{\sqrt{2}}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\right)}{\sqrt{h}}, cd^q(e + fx)^{pq}, c(d(e + fx)^p)^q\right)$$

$$- \text{Subst}\left(\frac{(bfpq)\text{Subst}\left(\int \frac{e^x x}{e^x f + \frac{e\sqrt{h}}{\sqrt{2}} + \frac{\sqrt{2f^2+e^2h}}{\sqrt{2}}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\right)}{\sqrt{h}}, cd^q(e + fx)^{pq}, c(d(e + fx)^p)^q\right)$$

$$=\frac{bpq\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)^2}{2\sqrt{h}}-\frac{bpq\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}}$$

$$-\frac{bpq\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}+\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}}$$

$$+\frac{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}}$$

$$+\text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int\log\left(1+\frac{e^xf}{\frac{e\sqrt{h}}{\sqrt{2}}-\frac{\sqrt{2f^2+e^2h}}{\sqrt{2}}}\right)dx,x,\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\right)}{\sqrt{h}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int\log\left(1+\frac{e^xf}{\frac{e\sqrt{h}}{\sqrt{2}}+\frac{\sqrt{2f^2+e^2h}}{\sqrt{2}}}\right)dx,x,\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\right)}{\sqrt{h}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{bpq \sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)^2}{2\sqrt{h}} - \frac{bpq \sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right) \log\left(1 + \frac{\sqrt{2}e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}}$$

$$- \frac{bpq \sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right) \log\left(1 + \frac{\sqrt{2}e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}+\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}}$$

$$+ \frac{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)(a + b\log(c(d(e+fx)^p)^q))}{\sqrt{h}}$$

$$+ \text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int \frac{\log\left(1+\frac{fx}{\frac{e\sqrt{h}}{\sqrt{2}}-\frac{\sqrt{2f^2+e^2h}}{\sqrt{2}}}\right)}{x}\,dx, x, e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}\right)}{\sqrt{h}}, cd^q(e + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$+ \text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int \frac{\log\left(1+\frac{fx}{\frac{e\sqrt{h}}{\sqrt{2}}+\frac{\sqrt{2f^2+e^2h}}{\sqrt{2}}}\right)}{x}\,dx, x, e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}\right)}{\sqrt{h}}, cd^q(e + fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{bpq\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)^2}{2\sqrt{h}} - \frac{bpq\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}}$$
$$- \frac{bpq\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\log\left(1+\frac{\sqrt{2}e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}+\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}}$$
$$+ \frac{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}}$$
$$- \frac{bpq\text{Li}_2\left(-\frac{\sqrt{2}e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}} - \frac{bpq\text{Li}_2\left(-\frac{\sqrt{2}e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}+\sqrt{2f^2+e^2h}}\right)}{\sqrt{h}}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{2+hx^2}}\,dx$$
$$= \frac{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)\left(2a+bpq\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)-2bpq\log\left(1+\frac{\sqrt{2}e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}-\sqrt{2f^2+e^2h}}\right)-2bpq\log\left(1+\frac{\sqrt{2}e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{2}}\right)}f}{e\sqrt{h}+\sqrt{2f^2+e^2h}}\right.\right.}{2}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[2 + h*x^2],x]
```

```
[Out] (ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*(2*a + b*p*q*ArcSinh[(Sqrt[h]*x)/Sqrt[2]] - 2
*b*p*q*Log[1 + (Sqrt[2]*E^ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*f)/(e*Sqrt[h] - Sqrt
[2*f^2 + e^2*h])] - 2*b*p*q*Log[1 + (Sqrt[2]*E^ArcSinh[(Sqrt[h]*x)/Sqrt[2]]
*f)/(e*Sqrt[h] + Sqrt[2*f^2 + e^2*h])] + 2*b*Log[c*(d*(e + f*x)^p)^q]) - 2*
b*p*q*PolyLog[2, (Sqrt[2]*E^ArcSinh[(Sqrt[h]*x)/Sqrt[2]]*f)/(-(e*Sqrt[h]) +
 Sqrt[2*f^2 + e^2*h])] - 2*b*p*q*PolyLog[2, -((Sqrt[2]*E^ArcSinh[(Sqrt[h]*x
)/Sqrt[2]]*f)/(e*Sqrt[h] + Sqrt[2*f^2 + e^2*h]))])/(2*Sqrt[h])
```

## Maple [F]

$$\int \frac{a+b\ln(c(d(fx+e)^p)^q)}{\sqrt{h\,x^2+2}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x^2+2)^(1/2),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x^2+2)^(1/2),x)
```

## Fricas [F]

$$\int \frac{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{2+hx^2}}\,dx=\int \frac{b\log \left(\left((fx+e)^pd\right)^qc\right)+a}{\sqrt{hx^2+2}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+2)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(h*x^2 + 2)*b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x^2 + 2)*a)/(
h*x^2 + 2), x)
```

## Sympy [F]

$$\int \frac{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{2+hx^2}}\,dx=\int \frac{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{hx^2+2}}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x**2+2)**(1/2),x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))/sqrt(h*x**2 + 2), x)
```

## Maxima [F]

$$\int \frac{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{2+hx^2}}\,dx=\int \frac{b\log \left(\left((fx+e)^pd\right)^qc\right)+a}{\sqrt{hx^2+2}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+2)^(1/2),x, algorithm="maxima
")
```

```
[Out] b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/sqrt(h*x^2 + 2), x)
+ a*arcsinh(1/2*sqrt(2)*sqrt(h)*x)/sqrt(h)
```

## Giac [F]

$$\int \frac{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{2+hx^2}}\,dx=\int \frac{b\log \left(\left((fx+e)^pd\right)^qc\right)+a}{\sqrt{hx^2+2}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)/sqrt(h*x^2 + 2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2+hx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,\left(d\,\left(e+f\,x\right)^p\right)^q\right)}{\sqrt{h\,x^2+2}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/(h*x^2 + 2)^(1/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/(h*x^2 + 2)^(1/2), x)
```

# 3.520 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g+hx^2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 515

$$\begin{aligned}\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}\,dx &= \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)^2}{2\sqrt{h}\sqrt{g+hx^2}} \\ &- \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\log\left(1+\frac{e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}-\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}} \\ &- \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\log\left(1+\frac{e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}+\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}} \\ &+ \frac{\sqrt{g}\sqrt{1+\frac{hx^2}{g}}\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}\sqrt{g+hx^2}} \\ &- \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}-\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}} \\ &- \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}+\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}}\end{aligned}$$

```
[Out] 1/2*b*p*q*arcsinh(x*h^(1/2)/g^(1/2))^2*g^(1/2)*(1+h*x^2/g)^(1/2)/h^(1/2)/(h
*x^2+g)^(1/2)+arcsinh(x*h^(1/2)/g^(1/2))*(a+b*ln(c*(d*(f*x+e)^p)^q))*g^(1/2
```

```
)*(1+h*x^2/g)^(1/2)/h^(1/2)/(h*x^2+g)^(1/2)-b*p*q*arcsinh(x*h^(1/2)/g^(1/2)
)*ln(1+(x*h^(1/2)/g^(1/2)+(1+h*x^2/g)^(1/2))*f*g^(1/2)/(e*h^(1/2)-(e^2*h+f^
2*g)^(1/2)))*g^(1/2)*(1+h*x^2/g)^(1/2)/h^(1/2)/(h*x^2+g)^(1/2)-b*p*q*arcsin
h(x*h^(1/2)/g^(1/2))*ln(1+(x*h^(1/2)/g^(1/2)+(1+h*x^2/g)^(1/2))*f*g^(1/2)/(
e*h^(1/2)+(e^2*h+f^2*g)^(1/2)))*g^(1/2)*(1+h*x^2/g)^(1/2)/h^(1/2)/(h*x^2+g)
^(1/2)-b*p*q*polylog(2,-(x*h^(1/2)/g^(1/2)+(1+h*x^2/g)^(1/2))*f*g^(1/2)/(e*
h^(1/2)-(e^2*h+f^2*g)^(1/2)))*g^(1/2)*(1+h*x^2/g)^(1/2)/h^(1/2)/(h*x^2+g)^(
1/2)-b*p*q*polylog(2,-(x*h^(1/2)/g^(1/2)+(1+h*x^2/g)^(1/2))*f*g^(1/2)/(e*h^
(1/2)+(e^2*h+f^2*g)^(1/2)))*g^(1/2)*(1+h*x^2/g)^(1/2)/h^(1/2)/(h*x^2+g)^(1/
2)
```

## Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2453, 221, 2451, 12, 5827, 5680, 2221, 2317, 2438, 2495}

$$\begin{aligned}
&\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}\,dx \\
&= \frac{\sqrt{g}\sqrt{\frac{hx^2}{g}+1}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}\sqrt{g+hx^2}} \\
&\quad -\frac{b\sqrt{g}pq\sqrt{\frac{hx^2}{g}+1}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}-\sqrt{he^2+f^2g}}\right)}{\sqrt{h}\sqrt{g+hx^2}} \\
&\quad -\frac{b\sqrt{g}pq\sqrt{\frac{hx^2}{g}+1}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{\sqrt{h}e+\sqrt{he^2+f^2g}}\right)}{\sqrt{h}\sqrt{g+hx^2}} \\
&\quad -\frac{b\sqrt{g}pq\sqrt{\frac{hx^2}{g}+1}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\log\left(\frac{f\sqrt{g}e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}}{e\sqrt{h}-\sqrt{e^2h+f^2g}}+1\right)}{\sqrt{h}\sqrt{g+hx^2}} \\
&\quad -\frac{b\sqrt{g}pq\sqrt{\frac{hx^2}{g}+1}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\log\left(\frac{f\sqrt{g}e^{\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}}{\sqrt{e^2h+f^2g}+e\sqrt{h}}+1\right)}{\sqrt{h}\sqrt{g+hx^2}} \\
&\quad +\frac{b\sqrt{g}pq\sqrt{\frac{hx^2}{g}+1}\text{arcsinh}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)^2}{2\sqrt{h}\sqrt{g+hx^2}}
\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[g + h*x^2],x]
```

```
[Out] (b*Sqrt[g]*p*q*Sqrt[1 + (h*x^2)/g]*ArcSinh[(Sqrt[h]*x)/Sqrt[g]]^2)/(2*Sqrt[
h]*Sqrt[g + h*x^2]) - (b*Sqrt[g]*p*q*Sqrt[1 + (h*x^2)/g]*ArcSinh[(Sqrt[h]*x
```

```
)/Sqrt[g]]*Log[1 + (E^ArcSinh[(Sqrt[h]*x)/Sqrt[g]]*f*Sqrt[g])/(e*Sqrt[h] -
Sqrt[f^2*g + e^2*h])])/(Sqrt[h]*Sqrt[g + h*x^2]) - (b*Sqrt[g]*p*q*Sqrt[1 +
(h*x^2)/g]*ArcSinh[(Sqrt[h]*x)/Sqrt[g]]*Log[1 + (E^ArcSinh[(Sqrt[h]*x)/Sqrt
[g]]*f*Sqrt[g])/(e*Sqrt[h] + Sqrt[f^2*g + e^2*h])])/(Sqrt[h]*Sqrt[g + h*x^2
]) + (Sqrt[g]*Sqrt[1 + (h*x^2)/g]*ArcSinh[(Sqrt[h]*x)/Sqrt[g]]*(a + b*Log[c
*(d*(e + f*x)^p)^q]))/(Sqrt[h]*Sqrt[g + h*x^2]) - (b*Sqrt[g]*p*q*Sqrt[1 + (
h*x^2)/g]*PolyLog[2, -((E^ArcSinh[(Sqrt[h]*x)/Sqrt[g]]*f*Sqrt[g])/(e*Sqrt[h
] - Sqrt[f^2*g + e^2*h]))])/(Sqrt[h]*Sqrt[g + h*x^2]) - (b*Sqrt[g]*p*q*Sqrt
[1 + (h*x^2)/g]*PolyLog[2, -((E^ArcSinh[(Sqrt[h]*x)/Sqrt[g]]*f*Sqrt[g])/(e*
Sqrt[h] + Sqrt[f^2*g + e^2*h]))])/(Sqrt[h]*Sqrt[g + h*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 2453

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (g/f)*x^2]/Sqrt[f + g*x^2], Int[(a + b*
Log[c*(d + e*x)^n])/Sqrt[1 + (g/f)*x^2], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, n}, x] && !GtQ[f, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{\sqrt{g+hx^2}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{\sqrt{1+\frac{hx^2}{g}}\int\frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{\sqrt{1+\frac{hx^2}{g}}}\,dx}{\sqrt{g+hx^2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{\sqrt{g}\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{\sqrt{h}\sqrt{g+hx^2}}\\
&\quad-\text{Subst}\left(\frac{\left(bfpq\sqrt{1+\frac{hx^2}{g}}\right)\int\frac{\sqrt{g}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}{\sqrt{h}(e+fx)}\,dx}{\sqrt{g+hx^2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$= \frac{\sqrt{g}\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)(a+b\log\left(c(d(e+fx)^p)^q\right))}{\sqrt{h}\sqrt{g+hx^2}} - \text{Subst}\left(\frac{\left(bf\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\right)\int\frac{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}{e+fx}\,dx}{\sqrt{h}\sqrt{g+hx^2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{\sqrt{g}\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)(a+b\log\left(c(d(e+fx)^p)^q\right))}{\sqrt{h}\sqrt{g+hx^2}} - \text{Subst}\left(\frac{\left(bf\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\right)\text{Subst}\left(\int\frac{x\cosh(x)}{\frac{e\sqrt{h}}{\sqrt{g}}+f\sinh(x)}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\right)}{\sqrt{h}\sqrt{g+hx^2}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)^2}{2\sqrt{h}\sqrt{g+hx^2}} + \frac{\sqrt{g}\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}\sqrt{g+hx^2}}$$

$$-\text{Subst}\left(\frac{\left(bf\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\right)\text{Subst}\left(\int\frac{e^x x}{e^x f+\frac{e\sqrt{h}}{\sqrt{g}}-\frac{\sqrt{f^2g+e^2h}}{\sqrt{g}}}\,dx,x,\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\right)}{\sqrt{h}\sqrt{g+hx^2}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$-\text{Subst}\left(\frac{\left(bf\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\right)\text{Subst}\left(\int\frac{e^x x}{e^x f+\frac{e\sqrt{h}}{\sqrt{g}}+\frac{\sqrt{f^2g+e^2h}}{\sqrt{g}}}\,dx,x,\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\right)}{\sqrt{h}\sqrt{g+hx^2}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)^2}{2\sqrt{h}\sqrt{g+hx^2}}$$

$$- \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\log\left(1+\frac{e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}-\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}}$$

$$- \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\log\left(1+\frac{e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}+\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}}$$

$$+ \frac{\sqrt{g}\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}\sqrt{g+hx^2}}$$

$$+\text{Subst}\left(\frac{\left(b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\right)\text{Subst}\left(\int\log\left(1+\frac{e^xf}{\frac{e\sqrt{h}}{\sqrt{g}}-\frac{\sqrt{f^2g+e^2h}}{\sqrt{g}}}\right)dx,x,\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\right)}{\sqrt{h}\sqrt{g+hx^2}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left(b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\right)\text{Subst}\left(\int\log\left(1+\frac{e^xf}{\frac{e\sqrt{h}}{\sqrt{g}}+\frac{\sqrt{f^2g+e^2h}}{\sqrt{g}}}\right)dx,x,\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\right)}{\sqrt{h}\sqrt{g+hx^2}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)^2}{2\sqrt{h}\sqrt{g+hx^2}} - \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\log\left(1+\frac{e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}-\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}} - \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\log\left(1+\frac{e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}+\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}} + \frac{\sqrt{g}\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}\sqrt{g+hx^2}}$$

$$+\text{Subst}\left(\frac{\left(b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{fx}{\frac{e\sqrt{h}}{\sqrt{g}}-\frac{\sqrt{f^2g+e^2h}}{\sqrt{g}}}\right)}{x}dx,x,e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}\right)}{\sqrt{h}\sqrt{g+hx^2}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left(b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{fx}{\frac{e\sqrt{h}}{\sqrt{g}}+\frac{\sqrt{f^2g+e^2h}}{\sqrt{g}}}\right)}{x}dx,x,e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}\right)}{\sqrt{h}\sqrt{g+hx^2}},cd^q(e\right.$$

$$= \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)^2}{2\sqrt{h}\sqrt{g+hx^2}} - \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\log\left(1+\frac{e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}-\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}} - \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)\log\left(1+\frac{e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}+\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}} + \frac{\sqrt{g}\sqrt{1+\frac{hx^2}{g}}\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)(a+b\log(c(d(e+fx)^p)^q))}{\sqrt{h}\sqrt{g+hx^2}} - \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\text{Li}_2\left(-\frac{e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}-\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}} - \frac{b\sqrt{g}pq\sqrt{1+\frac{hx^2}{g}}\text{Li}_2\left(-\frac{e^{\sinh^{-1}\left(\frac{\sqrt{h}x}{\sqrt{g}}\right)}f\sqrt{g}}{e\sqrt{h}+\sqrt{f^2g+e^2h}}\right)}{\sqrt{h}\sqrt{g+hx^2}}$$

## Mathematica [F]

$$\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g+hx^2}}\,dx = \int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g+hx^2}}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[g + h*x^2],x]
[Out] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/Sqrt[g + h*x^2], x]
```

## Maple [F]

$$\int \frac{a+b\ln(c(d(fx+e)^p)^q)}{\sqrt{h\,x^2+g}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x^2+g)^(1/2),x)
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x^2+g)^(1/2),x)
```

## Fricas [F]

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{g+hx^2}}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a\right.}{\sqrt{hx^2+g}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+g)^(1/2),x, algorithm="fricas
")
[Out] integral((sqrt(h*x^2 + g)*b*log(((f*x + e)^p*d)^q*c) + sqrt(h*x^2 + g)*a)/(
h*x^2 + g), x)
```

## Sympy [F]

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{g+hx^2}}\,dx=\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{g+hx^2}}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x**2+g)**(1/2),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))/sqrt(g + h*x**2), x)
```

## Maxima [F]

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{g+hx^2}}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a\right.}{\sqrt{hx^2+g}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+g)^(1/2),x, algorithm="maxima
")
[Out] b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/sqrt(h*x^2 + g), x)
+ a*arcsinh(h*x/sqrt(g*h))/sqrt(h)
```

## Giac [F]

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{g+hx^2}}\,dx=\int \frac{b\log\left(\left(\left((fx+e)^pd\right)^qc\right)+a\right.}{\sqrt{hx^2+g}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x^2+g)^(1/2),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)/sqrt(h*x^2 + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{\sqrt{g+hx^2}}\,dx=\int \frac{a+b\,\ln \left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{\sqrt{h\,x^2+g}} \,d x$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x^2)^(1/2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x^2)^(1/2), x)
```

# 3.521 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx$



## Optimal result

Integrand size = 38, antiderivative size = 287

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx = \frac{ibpq\arcsin\left(\frac{hx}{2}\right)^2}{2h} - \frac{bpq\arcsin\left(\frac{hx}{2}\right)\log\left(1+\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh-\sqrt{4f^2-e^2h^2}}\right)}{h} - \frac{bpq\arcsin\left(\frac{hx}{2}\right)\log\left(1+\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh+\sqrt{4f^2-e^2h^2}}\right)}{h} + \frac{\arcsin\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h} + \frac{ibpq\,\mathrm{PolyLog}\left(2,-\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh-\sqrt{4f^2-e^2h^2}}\right)}{h} + \frac{ibpq\,\mathrm{PolyLog}\left(2,-\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh+\sqrt{4f^2-e^2h^2}}\right)}{h}$$

```
[Out] 1/2*I*b*p*q*arcsin(1/2*h*x)^2/h+arcsin(1/2*h*x)*(a+b*ln(c*(d*(f*x+e)^p)^q))
/h-b*p*q*arcsin(1/2*h*x)*ln(1+2*(1/2*I*h*x+1/2*(-h^2*x^2+4)^(1/2))*f/(I*e*h
-(-e^2*h^2+4*f^2)^(1/2)))/h-b*p*q*arcsin(1/2*h*x)*ln(1+2*(1/2*I*h*x+1/2*(-h
^2*x^2+4)^(1/2))*f/(I*e*h+(-e^2*h^2+4*f^2)^(1/2)))/h+I*b*p*q*polylog(2,-2*(
1/2*I*h*x+1/2*(-h^2*x^2+4)^(1/2))*f/(I*e*h-(-e^2*h^2+4*f^2)^(1/2)))/h+I*b*p
*q*polylog(2,-2*(1/2*I*h*x+1/2*(-h^2*x^2+4)^(1/2))*f/(I*e*h+(-e^2*h^2+4*f^2
)^(1/2)))/h
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {222, 2452, 4825, 4617, 2221, 2317, 2438, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx=\frac{\arcsin\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}+\frac{ibpq\,\text{PolyLog}\left(2,-\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh-\sqrt{4f^2-e^2h^2}}\right)}{h}+\frac{ibpq\,\text{PolyLog}\left(2,-\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh+\sqrt{4f^2-e^2h^2}}\right)}{h}-\frac{bpq\arcsin\left(\frac{hx}{2}\right)\log\left(1+\frac{2fe^{i\arcsin\left(\frac{hx}{2}\right)}}{-\sqrt{4f^2-e^2h^2}+ieh}\right)}{h}-\frac{bpq\arcsin\left(\frac{hx}{2}\right)\log\left(1+\frac{2fe^{i\arcsin\left(\frac{hx}{2}\right)}}{\sqrt{4f^2-e^2h^2}+ieh}\right)}{h}+\frac{ibpq\arcsin\left(\frac{hx}{2}\right)^2}{2h}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(Sqrt[2 - h*x]*Sqrt[2 + h*x]),x]
```

```
[Out] ((I/2)*b*p*q*ArcSin[(h*x)/2]^2)/h - (b*p*q*ArcSin[(h*x)/2]*Log[1 + (2*E^(I*
ArcSin[(h*x)/2])*f)/(I*e*h - Sqrt[4*f^2 - e^2*h^2])])/h - (b*p*q*ArcSin[(h*
x)/2]*Log[1 + (2*E^(I*ArcSin[(h*x)/2])*f)/(I*e*h + Sqrt[4*f^2 - e^2*h^2])])
/h + (ArcSin[(h*x)/2]*(a + b*Log[c*(d*(e + f*x)^p)^q]))/h + (I*b*p*q*PolyLo
g[2, (-2*E^(I*ArcSin[(h*x)/2])*f)/(I*e*h - Sqrt[4*f^2 - e^2*h^2])])/h + (I*
b*p*q*PolyLog[2, (-2*E^(I*ArcSin[(h*x)/2])*f)/(I*e*h + Sqrt[4*f^2 - e^2*h^2
])])/h
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2452

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/(Sqrt[(f1_) + (g1_
.)*(x_)]*Sqrt[(f2_) + (g2_.)*(x_)]), x_Symbol] :> With[{u = IntHide[1/Sqrt[
f1*f2 + g1*g2*x^2], x]}, Simp[u*(a + b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n
, Int[SimplifyIntegrand[u/(d + e*x), x], x], x]] /; FreeQ[{a, b, c, d, e, f
1, g1, f2, g2, n}, x] && EqQ[f2*g1 + f1*g2, 0] && GtQ[f1, 0] && GtQ[f2, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 4617

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2
] + b*E^(I*(c + d*x)))), x], x] + Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/
(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{a + b\log\left(cd^q(e+fx)^{pq}\right)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= \frac{\sin^{-1}\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h} - \text{Subst}\left((bfpq)\int \frac{\sin^{-1}\left(\frac{hx}{2}\right)}{eh+fhx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{\sin^{-1}\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h} - \text{Subst}\left((bfpq)\text{Subst}\left(\int \frac{x\cos(x)}{\frac{eh^2}{2}+fh\sin(x)}\,dx, x, \sin^{-1}\left(\frac{hx}{2}\right)\right), cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{ibpq\sin^{-1}\left(\frac{hx}{2}\right)^2}{2h} + \frac{\sin^{-1}\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h} - \text{Subst}\left((ibfpq)\text{Subst}\left(\int \frac{e^{ix}x}{e^{ix}fh+\frac{1}{2}ieh^2-\frac{1}{2}h\sqrt{4f^2-e^2h^2}}\,dx, x, \sin^{-1}\left(\frac{hx}{2}\right)\right), cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left((ibfpq)\text{Subst}\left(\int \frac{e^{ix}x}{e^{ix}fh+\frac{1}{2}ieh^2+\frac{1}{2}h\sqrt{4f^2-e^2h^2}}\,dx, x, \sin^{-1}\left(\frac{hx}{2}\right)\right), cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{ibpq\sin^{-1}\left(\frac{hx}{2}\right)^2}{2h} - \frac{bpq\sin^{-1}\left(\frac{hx}{2}\right)\log\left(1+\frac{2e^{i\sin^{-1}\left(\frac{hx}{2}\right)}f}{ieh-\sqrt{4f^2-e^2h^2}}\right)}{h} - \frac{bpq\sin^{-1}\left(\frac{hx}{2}\right)\log\left(1+\frac{2e^{i\sin^{-1}\left(\frac{hx}{2}\right)}f}{ieh+\sqrt{4f^2-e^2h^2}}\right)}{h} + \frac{\sin^{-1}\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h} + \text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int\log\left(1+\frac{e^{ix}fh}{\frac{1}{2}ieh^2-\frac{1}{2}h\sqrt{4f^2-e^2h^2}}\right)dx, x, \sin^{-1}\left(\frac{hx}{2}\right)\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int\log\left(1+\frac{e^{ix}fh}{\frac{1}{2}ieh^2+\frac{1}{2}h\sqrt{4f^2-e^2h^2}}\right)dx, x, \sin^{-1}\left(\frac{hx}{2}\right)\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{ibpq\sin^{-1}\left(\frac{hx}{2}\right)^2}{2h} - \frac{bpq\sin^{-1}\left(\frac{hx}{2}\right)\log\left(1+\frac{2e^{i\sin^{-1}\left(\frac{hx}{2}\right)}f}{ieh-\sqrt{4f^2-e^2h^2}}\right)}{h} - \frac{bpq\sin^{-1}\left(\frac{hx}{2}\right)\log\left(1+\frac{2e^{i\sin^{-1}\left(\frac{hx}{2}\right)}f}{ieh+\sqrt{4f^2-e^2h^2}}\right)}{h} + \frac{\sin^{-1}\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h} - \text{Subst}\left(\frac{(ibpq)\text{Subst}\left(\int\frac{\log\left(1+\frac{fhx}{\frac{1}{2}ieh^2-\frac{1}{2}h\sqrt{4f^2-e^2h^2}}\right)}{x}dx, x, e^{i\sin^{-1}\left(\frac{hx}{2}\right)}\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(ibpq)\text{Subst}\left(\int\frac{\log\left(1+\frac{fhx}{\frac{1}{2}ieh^2+\frac{1}{2}h\sqrt{4f^2-e^2h^2}}\right)}{x}dx, x, e^{i\sin^{-1}\left(\frac{hx}{2}\right)}\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{ibpq\sin^{-1}\left(\frac{hx}{2}\right)^2}{2h} - \frac{bpq\sin^{-1}\left(\frac{hx}{2}\right)\log\left(1+\frac{2e^{i\sin^{-1}\left(\frac{hx}{2}\right)}f}{ieh-\sqrt{4f^2-e^2h^2}}\right)}{h} - \frac{bpq\sin^{-1}\left(\frac{hx}{2}\right)\log\left(1+\frac{2e^{i\sin^{-1}\left(\frac{hx}{2}\right)}f}{ieh+\sqrt{4f^2-e^2h^2}}\right)}{h} + \frac{\sin^{-1}\left(\frac{hx}{2}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h} + \frac{ibpq\text{Li}_2\left(-\frac{2e^{i\sin^{-1}\left(\frac{hx}{2}\right)}f}{ieh-\sqrt{4f^2-e^2h^2}}\right)}{h} + \frac{ibpq\text{Li}_2\left(-\frac{2e^{i\sin^{-1}\left(\frac{hx}{2}\right)}f}{ieh+\sqrt{4f^2-e^2h^2}}\right)}{h}$$

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.90

$$\int \frac{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx = \frac{\arcsin\left(\frac{hx}{2}\right)\left(2a+ibpq\arcsin\left(\frac{hx}{2}\right)-2bpq\log\left(1-\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{-ieh+\sqrt{4f^2-e^2h^2}}\right)-2bpq\log\left(1+\frac{2e^{i\arcsin\left(\frac{hx}{2}\right)}f}{ieh+\sqrt{4f^2-e^2h^2}}\right)+2b\log\right.}{2h}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(Sqrt[2 - h*x]*Sqrt[2 + h*x]),x]
```

```
[Out] (ArcSin[(h*x)/2]*(2*a + I*b*p*q*ArcSin[(h*x)/2] - 2*b*p*q*Log[1 - (2*E^(I*A
rcSin[(h*x)/2])*f)/((-I)*e*h + Sqrt[4*f^2 - e^2*h^2])] - 2*b*p*q*Log[1 + (2
*E^(I*ArcSin[(h*x)/2])*f)/(I*e*h + Sqrt[4*f^2 - e^2*h^2])] + 2*b*Log[c*(d*(
e + f*x)^p)^q]) + (2*I)*b*p*q*PolyLog[2, (2*E^(I*ArcSin[(h*x)/2])*f)/((-I)*
e*h + Sqrt[4*f^2 - e^2*h^2])] + (2*I)*b*p*q*PolyLog[2, (-2*E^(I*ArcSin[(h*x
)/2])*f)/(I*e*h + Sqrt[4*f^2 - e^2*h^2])])/(2*h)
```

## Maple [F]

$$\int \frac{a+b\ln\left(c\left(d(fx+e)^p\right)^q\right)}{\sqrt{-hx+2}\,\sqrt{hx+2}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(-h*x+2)^(1/2)/(h*x+2)^(1/2),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(-h*x+2)^(1/2)/(h*x+2)^(1/2),x)
```

## Fricas [F]

$$\int \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx = \int \frac{b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right.}{\sqrt{hx+2}\sqrt{-hx+2}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(-h*x+2)^(1/2)/(h*x+2)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(-(sqrt(h*x + 2)*sqrt(-h*x + 2)*b*log(((f*x + e)^p*d)^q*c) + sqrt(h
*x + 2)*sqrt(-h*x + 2)*a)/(h^2*x^2 - 4), x)
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx=\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{-hx+2}\sqrt{hx+2}}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(-h*x+2)**(1/2)/(h*x+2)**(1/2),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))/(sqrt(-h*x + 2)*sqrt(h*x + 2)),
 x)
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx=\int \frac{b\log\left(((fx+e)^pd)^qc\right)+a}{\sqrt{hx+2}\sqrt{-hx+2}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(-h*x+2)^(1/2)/(h*x+2)^(1/2),x, algo
rithm="maxima")
[Out] b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(sqrt(h*x + 2)*sqrt(
-h*x + 2)), x) + a*arcsin(1/2*h*x)/h
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx=\int \frac{b\log\left(((fx+e)^pd)^qc\right)+a}{\sqrt{hx+2}\sqrt{-hx+2}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(-h*x+2)^(1/2)/(h*x+2)^(1/2),x, algo
rithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)/(sqrt(h*x + 2)*sqrt(-h*x + 2)),
x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{2-hx}\sqrt{2+hx}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{\sqrt{2-h\,x}\,\sqrt{h\,x+2}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/((2 - h*x)^(1/2)*(h*x + 2)^(1/2)),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/((2 - h*x)^(1/2)*(h*x + 2)^(1/2)), x)
```

# 3.522 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx$



## Optimal result

Integrand size = 38, antiderivative size = 519

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx = \frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\arcsin\left(\frac{hx}{g}\right)^2}{2h\sqrt{g-hx}\sqrt{g+hx}} - \frac{bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\arcsin\left(\frac{hx}{g}\right)\log\left(1+\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}fg}{ieh-\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}} - \frac{bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\arcsin\left(\frac{hx}{g}\right)\log\left(1+\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}fg}{ieh+\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}} + \frac{g\sqrt{1-\frac{h^2x^2}{g^2}}\arcsin\left(\frac{hx}{g}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h\sqrt{g-hx}\sqrt{g+hx}} + \frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\operatorname{PolyLog}\left(2,-\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}fg}{ieh-\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}} + \frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\operatorname{PolyLog}\left(2,-\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}fg}{ieh+\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}}$$

```
[Out] 1/2*I*b*g*p*q*arcsin(h*x/g)^2*(1-h^2*x^2/g^2)^(1/2)/h/(-h*x+g)^(1/2)/(h*x+g
)^(1/2)+g*arcsin(h*x/g)*(a+b*ln(c*(d*(f*x+e)^p)^q))*(1-h^2*x^2/g^2)^(1/2)/h
/(-h*x+g)^(1/2)/(h*x+g)^(1/2)-b*g*p*q*arcsin(h*x/g)*ln(1+(I*h*x/g+(1-h^2*x^
2/g^2)^(1/2))*f*g/(I*e*h-(-e^2*h^2+f^2*g^2)^(1/2)))*(1-h^2*x^2/g^2)^(1/2)/h
/(-h*x+g)^(1/2)/(h*x+g)^(1/2)-b*g*p*q*arcsin(h*x/g)*ln(1+(I*h*x/g+(1-h^2*x^
2/g^2)^(1/2))*f*g/(I*e*h+(-e^2*h^2+f^2*g^2)^(1/2)))*(1-h^2*x^2/g^2)^(1/2)/h
/(-h*x+g)^(1/2)/(h*x+g)^(1/2)+I*b*g*p*q*polylog(2,-(I*h*x/g+(1-h^2*x^2/g^2)
```

```
^(1/2))*f*g/(I*e*h-(-e^2*h^2+f^2*g^2)^(1/2)))*(1-h^2*x^2/g^2)^(1/2)/h/(-h*x
+g)^(1/2)/(h*x+g)^(1/2)+I*b*g*p*q*polylog(2,-(I*h*x/g+(1-h^2*x^2/g^2)^(1/2)
)*f*g/(I*e*h+(-e^2*h^2+f^2*g^2)^(1/2)))*(1-h^2*x^2/g^2)^(1/2)/h/(-h*x+g)^(1
/2)/(h*x+g)^(1/2)
```

## Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2454, 222, 2451, 12, 4825, 4617, 2221, 2317, 2438, 2495}

$$\begin{aligned}\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx = &\frac{g\sqrt{1-\frac{h^2x^2}{g^2}}\arcsin\left(\frac{hx}{g}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h\sqrt{g-hx}\sqrt{g+hx}}\\ &+\frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\operatorname{PolyLog}\left(2,-\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}fg}{ieh-\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}}\\ &+\frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\operatorname{PolyLog}\left(2,-\frac{e^{i\arcsin\left(\frac{hx}{g}\right)}fg}{ieh+\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}}\\ &-\frac{bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\arcsin\left(\frac{hx}{g}\right)\log\left(1+\frac{fge^{i\arcsin\left(\frac{hx}{g}\right)}}{-\sqrt{f^2g^2-e^2h^2}+ieh}\right)}{h\sqrt{g-hx}\sqrt{g+hx}}\\ &-\frac{bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\arcsin\left(\frac{hx}{g}\right)\log\left(1+\frac{fge^{i\arcsin\left(\frac{hx}{g}\right)}}{\sqrt{f^2g^2-e^2h^2}+ieh}\right)}{h\sqrt{g-hx}\sqrt{g+hx}}\\ &+\frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\arcsin\left(\frac{hx}{g}\right)^2}{2h\sqrt{g-hx}\sqrt{g+hx}}\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(Sqrt[g - h*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((I/2)*b*g*p*q*Sqrt[1 - (h^2*x^2)/g^2]*ArcSin[(h*x)/g]^2)/(h*Sqrt[g - h*x]*
Sqrt[g + h*x]) - (b*g*p*q*Sqrt[1 - (h^2*x^2)/g^2]*ArcSin[(h*x)/g]*Log[1 + (
E^(I*ArcSin[(h*x)/g])*f*g)/(I*e*h - Sqrt[f^2*g^2 - e^2*h^2])]])/(h*Sqrt[g -
h*x]*Sqrt[g + h*x]) - (b*g*p*q*Sqrt[1 - (h^2*x^2)/g^2]*ArcSin[(h*x)/g]*Log[
1 + (E^(I*ArcSin[(h*x)/g])*f*g)/(I*e*h + Sqrt[f^2*g^2 - e^2*h^2])]])/(h*Sqrt
[g - h*x]*Sqrt[g + h*x]) + (g*Sqrt[1 - (h^2*x^2)/g^2]*ArcSin[(h*x)/g]*(a +
b*Log[c*(d*(e + f*x)^p)^q]))/(h*Sqrt[g - h*x]*Sqrt[g + h*x]) + (I*b*g*p*q*S
qrt[1 - (h^2*x^2)/g^2]*PolyLog[2, -((E^(I*ArcSin[(h*x)/g])*f*g)/(I*e*h - Sq
rt[f^2*g^2 - e^2*h^2]))]])/(h*Sqrt[g - h*x]*Sqrt[g + h*x]) + (I*b*g*p*q*Sqrt
[1 - (h^2*x^2)/g^2]*PolyLog[2, -((E^(I*ArcSin[(h*x)/g])*f*g)/(I*e*h + Sqrt[
f^2*g^2 - e^2*h^2]))]])/(h*Sqrt[g - h*x]*Sqrt[g + h*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/(Sqrt[(f1_) + (g1_
.)*(x_)]*Sqrt[(f2_) + (g2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + g1*(g2/(f1*
f2))*x^2]/(Sqrt[f1 + g1*x]*Sqrt[f2 + g2*x]), Int[(a + b*Log[c*(d + e*x)^n])
/Sqrt[1 + g1*(g2/(f1*f2))*x^2], x], x] /; FreeQ[{a, b, c, d, e, f1, g1, f2,
 g2, n}, x] && EqQ[f2*g1 + f1*g2, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
```

```
c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 4617

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2
] + b*E^(I*(c + d*x)))), x], x] + Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/
(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{\sqrt{1-\frac{h^2x^2}{g^2}}\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{\sqrt{1-\frac{h^2x^2}{g^2}}}\,dx}{\sqrt{g-hx}\sqrt{g+hx}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{g\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h\sqrt{g-hx}\sqrt{g+hx}} - \text{Subst}\left(\frac{\left(bfpq\sqrt{1-\frac{h^2x^2}{g^2}}\right)\int \frac{g\sin^{-1}\left(\frac{hx}{g}\right)}{eh+fhx}\,dx}{\sqrt{g-hx}\sqrt{g+hx}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{g\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h\sqrt{g-hx}\sqrt{g+hx}} - \text{Subst}\left(\frac{\left(bfgpq\sqrt{1-\frac{h^2x^2}{g^2}}\right)\int \frac{\sin^{-1}\left(\frac{hx}{g}\right)}{eh+fhx}\,dx}{\sqrt{g-hx}\sqrt{g+hx}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{g\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h\sqrt{g-hx}\sqrt{g+hx}} - \text{Subst}\left(\frac{\left(bfgpq\sqrt{1-\frac{h^2x^2}{g^2}}\right)\text{Subst}\left(\int\frac{x\cos(x)}{\frac{eh^2}{g}+fh\sin(x)}\,dx,x,\sin^{-1}\left(\frac{hx}{g}\right)\right)}{\sqrt{g-hx}\sqrt{g+hx}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)^2}{2h\sqrt{g-hx}\sqrt{g+hx}} + \frac{g\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h\sqrt{g-hx}\sqrt{g+hx}} - \text{Subst}\left(\frac{\left(ibfgpq\sqrt{1-\frac{h^2x^2}{g^2}}\right)\text{Subst}\left(\int\frac{e^{ix}x}{e^{ix}fh+\frac{ieh^2}{g}-\frac{h\sqrt{f^2g^2-e^2h^2}}{g}}\,dx,x,\sin^{-1}\left(\frac{hx}{g}\right)\right)}{\sqrt{g-hx}\sqrt{g+hx}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{\left(ibfgpq\sqrt{1-\frac{h^2x^2}{g^2}}\right)\text{Subst}\left(\int\frac{e^{ix}x}{e^{ix}fh+\frac{ieh^2}{g}+\frac{h\sqrt{f^2g^2-e^2h^2}}{g}}\,dx,x,\sin^{-1}\left(\frac{hx}{g}\right)\right)}{\sqrt{g-hx}\sqrt{g+hx}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)^2}{2h\sqrt{g-hx}\sqrt{g+hx}} - \frac{bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)\log\left(1+\frac{e^{i\sin^{-1}\left(\frac{hx}{g}\right)}fg}{ieh-\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}}$$

$$- \frac{bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)\log\left(1+\frac{e^{i\sin^{-1}\left(\frac{hx}{g}\right)}fg}{ieh+\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}}$$

$$+ \frac{g\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{g-hx}\sqrt{g+hx}}$$

$$+\text{Subst}\left(\frac{\left(bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\right)\text{Subst}\left(\int\log\left(1+\frac{e^{ix}fh}{\frac{ieh^2}{g}-\frac{h\sqrt{f^2g^2-e^2h^2}}{g}}\right)dx,x,\sin^{-1}\left(\frac{hx}{g}\right)\right)}{h\sqrt{g-hx}\sqrt{g+hx}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\text{Subst}\left(\frac{\left(bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\right)\text{Subst}\left(\int\log\left(1+\frac{e^{ix}fh}{\frac{ieh^2}{g}+\frac{h\sqrt{f^2g^2-e^2h^2}}{g}}\right)dx,x,\sin^{-1}\left(\frac{hx}{g}\right)\right)}{h\sqrt{g-hx}\sqrt{g+hx}},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)^2}{2h\sqrt{g-hx}\sqrt{g+hx}} - \frac{bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)\log\left(1+\frac{e^{i\sin^{-1}\left(\frac{hx}{g}\right)}fg}{ieh-\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}}$$

$$- \frac{bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)\log\left(1+\frac{e^{i\sin^{-1}\left(\frac{hx}{g}\right)}fg}{ieh+\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}}$$

$$+ \frac{g\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{g-hx}\sqrt{g+hx}}$$

$$-\text{Subst}\left(\frac{\left(ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{fhx}{\frac{ieh^2}{g}-\frac{h\sqrt{f^2g^2-e^2h^2}}{g}}\right)}{x}dx, x, e^{i\sin^{-1}\left(\frac{hx}{g}\right)}\right)}{h\sqrt{g-hx}\sqrt{g+hx}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$-\text{Subst}\left(\frac{\left(ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{fhx}{\frac{ieh^2}{g}+\frac{h\sqrt{f^2g^2-e^2h^2}}{g}}\right)}{x}dx, x, e^{i\sin^{-1}\left(\frac{hx}{g}\right)}\right)}{h\sqrt{g-hx}\sqrt{g+hx}}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)^2}{2h\sqrt{g-hx}\sqrt{g+hx}} - \frac{bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)\log\left(1+\frac{e^{i\sin^{-1}\left(\frac{hx}{g}\right)}fg}{ieh-\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}} - \frac{bgpq\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)\log\left(1+\frac{e^{i\sin^{-1}\left(\frac{hx}{g}\right)}fg}{ieh+\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}} + \frac{g\sqrt{1-\frac{h^2x^2}{g^2}}\sin^{-1}\left(\frac{hx}{g}\right)(a+b\log(c(d(e+fx)^p)^q))}{h\sqrt{g-hx}\sqrt{g+hx}} + \frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\mathrm{Li}_2\left(-\frac{e^{i\sin^{-1}\left(\frac{hx}{g}\right)}fg}{ieh-\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}} + \frac{ibgpq\sqrt{1-\frac{h^2x^2}{g^2}}\mathrm{Li}_2\left(-\frac{e^{i\sin^{-1}\left(\frac{hx}{g}\right)}fg}{ieh+\sqrt{f^2g^2-e^2h^2}}\right)}{h\sqrt{g-hx}\sqrt{g+hx}}$$

## Mathematica [A] (warning: unable to verify)

Time = 10.98 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.23

$$\int \frac{a+b\log(c(d(e+fx)^p)^q)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx = \frac{\arctan\left(\frac{hx}{\sqrt{g-hx}\sqrt{g+hx}}\right)(a-bpq\log(e+fx)+b\log(c(d(e+fx)^p)^q))}{h} - \frac{bpq\sqrt{g-hx}\left(2gh(e+fx)\sqrt{\frac{g+hx}{g-hx}}\arctan\left(\frac{1}{\sqrt{\frac{g+hx}{g-hx}}}\right)\log(e+fx)+(g+hx)\left(eh+fg\cos\left(2\arctan\left(\frac{}{\sqrt{}}\right.\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(Sqrt[g - h*x]*Sqrt[g + h*x]),x]
```

```
[Out] (ArcTan[(h*x)/(Sqrt[g - h*x]*Sqrt[g + h*x])]*(a - b*p*q*Log[e + f*x] + b*Lo
g[c*(d*(e + f*x)^p)^q]))/h - (b*p*q*Sqrt[g - h*x]*(2*g*h*(e + f*x)*Sqrt[(g
+ h*x)/(g - h*x)]*ArcTan[1/Sqrt[(g + h*x)/(g - h*x)]]*Log[e + f*x] + (g + h
*x)*(e*h + f*g*Cos[2*ArcTan[1/Sqrt[(g + h*x)/(g - h*x)]]])*Csc[2*ArcTan[1/S
qrt[(g + h*x)/(g - h*x)]]]*((2*I)*ArcTan[1/Sqrt[(g + h*x)/(g - h*x)]]^2 - (
4*I)*ArcSin[Sqrt[1 + (e*h)/(f*g)]/Sqrt[2]]*ArcTan[(-(f*g) + e*h)/(Sqrt[-(f^
2*g^2) + e^2*h^2]*Sqrt[(g + h*x)/(g - h*x)])] - 2*(ArcSin[Sqrt[1 + (e*h)/(f
*g)]/Sqrt[2]] + ArcTan[1/Sqrt[(g + h*x)/(g - h*x)]])*Log[1 + (E^((2*I)*ArcT
an[1/Sqrt[(g + h*x)/(g - h*x)]])*(e*h - Sqrt[-(f^2*g^2) + e^2*h^2]))/(f*g)]
 + 2*(ArcSin[Sqrt[1 + (e*h)/(f*g)]/Sqrt[2]] - ArcTan[1/Sqrt[(g + h*x)/(g -
h*x)]])*Log[1 + (E^((2*I)*ArcTan[1/Sqrt[(g + h*x)/(g - h*x)]])*(e*h + Sqrt[
-(f^2*g^2) + e^2*h^2]))/(f*g)] + I*(PolyLog[2, (E^((2*I)*ArcTan[1/Sqrt[(g +
 h*x)/(g - h*x)]])*(-(e*h) + Sqrt[-(f^2*g^2) + e^2*h^2]))/(f*g)] + PolyLog[
2, -((E^((2*I)*ArcTan[1/Sqrt[(g + h*x)/(g - h*x)]])*(e*h + Sqrt[-(f^2*g^2)
+ e^2*h^2]))/(f*g))]))))/(g*h^2*(e + f*x)*Sqrt[g + h*x])
```

## Maple [F]

$$\int \frac{a+b\ln\left(c(d(fx+e)^{p})^{q}\right)}{\sqrt{-hx+g}\,\sqrt{hx+g}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(-h*x+g)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(-h*x+g)^(1/2)/(h*x+g)^(1/2),x)
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^{p})^{q}\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx = \int \frac{b\log\left(((fx+e)^{p}d)^{q}\,c\right)+a}{\sqrt{hx+g}\sqrt{-hx+g}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(-h*x+g)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(-(sqrt(h*x + g)*sqrt(-h*x + g)*b*log(((f*x + e)^p*d)^q*c) + sqrt(h
*x + g)*sqrt(-h*x + g)*a)/(h^2*x^2 - g^2), x)
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^{p})^{q}\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx = \int \frac{a+b\log\left(c(d(e+fx)^{p})^{q}\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(-h*x+g)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))/(sqrt(g - h*x)*sqrt(g + h*x)),
x)
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^{p})^{q}\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx = \int \frac{b\log\left(((fx+e)^{p}d)^{q}\,c\right)+a}{\sqrt{hx+g}\sqrt{-hx+g}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(-h*x+g)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
```

```
[Out] b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(sqrt(h*x + g)*sqrt(
-h*x + g)), x) + a*arcsin(h*x/g)/h
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx = \int \frac{b\log\left(((fx+e)^p d)^q c\right)+a}{\sqrt{hx+g}\sqrt{-hx+g}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(-h*x+g)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)/(sqrt(h*x + g)*sqrt(-h*x + g)),
x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{\sqrt{g-hx}\sqrt{g+hx}}\,dx = \int \frac{a+b\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)}{\sqrt{g+h\,x}\,\sqrt{g-h\,x}}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)^(1/2)*(g - h*x)^(1/2)),x)
```

```
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)^(1/2)*(g - h*x)^(1/2)), x)
```

# 3.523 $\int \frac{(i+jx)^3(a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx$



## Optimal result

Integrand size = 33, antiderivative size = 427

$$
\begin{aligned}
\int \frac{(i+jx)^3\,(a+b\log\left(c(d(e+fx)^p)^q\right))}{g+hx}\,dx
&= \frac{aj(hi-gj)^2x}{h^3}-\frac{bj(fi-ej)^2pqx}{3f^2h}-\frac{bj(fi-ej)(hi-gj)pqx}{2fh^2}-\frac{bj(hi-gj)^2pqx}{h^3}\\
&-\frac{b(fi-ej)pq(i+jx)^2}{6fh}-\frac{b(hi-gj)pq(i+jx)^2}{4h^2}-\frac{bpq(i+jx)^3}{9h}\\
&-\frac{b(fi-ej)^3pq\log(e+fx)}{3f^3h}-\frac{b(fi-ej)^2(hi-gj)pq\log(e+fx)}{2f^2h^2}\\
&+\frac{bj(hi-gj)^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh^3}\\
&+\frac{(hi-gj)(i+jx)^2\,(a+b\log\left(c(d(e+fx)^p)^q\right))}{2h^2}+\frac{(i+jx)^3\,(a+b\log\left(c(d(e+fx)^p)^q\right))}{3h}\\
&+\frac{(hi-gj)^3\,(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^4}\\
&+\frac{b(hi-gj)^3pq\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^4}
\end{aligned}
$$

```
[Out] a*j*(-g*j+h*i)^2*x/h^3-1/3*b*j*(-e*j+f*i)^2*p*q*x/f^2/h-1/2*b*j*(-e*j+f*i)*
(-g*j+h*i)*p*q*x/f/h^2-b*j*(-g*j+h*i)^2*p*q*x/h^3-1/6*b*(-e*j+f*i)*p*q*(j*x
+i)^2/f/h-1/4*b*(-g*j+h*i)*p*q*(j*x+i)^2/h^2-1/9*b*p*q*(j*x+i)^3/h-1/3*b*(-
e*j+f*i)^3*p*q*ln(f*x+e)/f^3/h-1/2*b*(-e*j+f*i)^2*(-g*j+h*i)*p*q*ln(f*x+e)/
f^2/h^2+b*j*(-g*j+h*i)^2*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f/h^3+1/2*(-g*j+h*i)
*(j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/h^2+1/3*(j*x+i)^3*(a+b*ln(c*(d*(f*x+
e)^p)^q))/h+(-g*j+h*i)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g
))/h^4+b*(-g*j+h*i)^3*p*q*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h^4
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2465, 2436, 2332, 2441, 2440, 2438, 2442, 45, 2495}

$$\begin{aligned}
&\int \frac{(i+jx)^3 (a+b\log (c(d(e+fx)^p)^q))}{g+hx} \, dx \\
&= \frac{(hi-gj)^3 \log \left(\frac{f(g+hx)}{fg-eh}\right) (a+b\log (c(d(e+fx)^p)^q))}{h^4} \\
&\quad + \frac{(i+jx)^2(hi-gj) (a+b\log (c(d(e+fx)^p)^q))}{2h^2} + \frac{(i+jx)^3 (a+b\log (c(d(e+fx)^p)^q))}{3h} \\
&\quad + \frac{ajx(hi-gj)^2}{h^3} + \frac{bj(e+fx)(hi-gj)^2 \log (c(d(e+fx)^p)^q)}{fh^3} \\
&\quad - \frac{bpq(fi-ej)^3 \log(e+fx)}{3f^3h} - \frac{bpq(fi-ej)^2 \log(e+fx)(hi-gj)}{2f^2h^2} - \frac{bjpqx(fi-ej)^2}{3f^2h} \\
&\quad + \frac{bpq(hi-gj)^3 \operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^4} - \frac{bjpqx(fi-ej)(hi-gj)}{2fh^2} \\
&\quad - \frac{bpq(i+jx)^2(fi-ej)}{6fh} - \frac{bjpqx(hi-gj)^2}{h^3} - \frac{bpq(i+jx)^2(hi-gj)}{4h^2} - \frac{bpq(i+jx)^3}{9h}
\end{aligned}$$

```
[In] Int[((i + j*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(g + h*x),x]
```

```
[Out] (a*j*(h*i - g*j)^2*x)/h^3 - (b*j*(f*i - e*j)^2*p*q*x)/(3*f^2*h) - (b*j*(f*i
 - e*j)*(h*i - g*j)*p*q*x)/(2*f*h^2) - (b*j*(h*i - g*j)^2*p*q*x)/h^3 - (b*(
f*i - e*j)*p*q*(i + j*x)^2)/(6*f*h) - (b*(h*i - g*j)*p*q*(i + j*x)^2)/(4*h^
2) - (b*p*q*(i + j*x)^3)/(9*h) - (b*(f*i - e*j)^3*p*q*Log[e + f*x])/(3*f^3*
h) - (b*(f*i - e*j)^2*(h*i - g*j)*p*q*Log[e + f*x])/(2*f^2*h^2) + (b*j*(h*i
 - g*j)^2*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/(f*h^3) + ((h*i - g*j)*(i + j
*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(2*h^2) + ((i + j*x)^3*(a + b*Log[c
*(d*(e + f*x)^p)^q]))/(3*h) + ((h*i - g*j)^3*(a + b*Log[c*(d*(e + f*x)^p)^q
])*Log[(f*(g + h*x))/(f*g - e*h)])/h^4 + (b*(h*i - g*j)^3*p*q*PolyLog[2, -(
(h*(e + f*x))/(f*g - e*h))])/h^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
```

```
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(i+jx)^3\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\int \left(\frac{j(hi-gj)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h^3} + \frac{(hi-gj)^3\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h^3(g+hx)} + \frac{j(hi-gj)(i+jx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h^2} + \frac{j(i+jx)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h}\right) dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{j\int(i+jx)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(j(hi-gj))\int(i+jx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\,dx}{h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(j(hi-gj)^2)\int\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\,dx}{h^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(hi-gj)^3\int\frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{g+hx}\,dx}{h^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$
\begin{aligned}
&= \frac{aj(hi-gj)^2x}{h^3} + \frac{(hi-gj)(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2h^2} \\
&\quad + \frac{(i+jx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h} \\
&\quad + \frac{(hi-gj)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^4} \\
&\quad + \text{Subst}\left(\frac{\left(bj(hi-gj)^2\right)\int\log\left(cd^q(e+fx)^{pq}\right)\,dx}{h^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) \\
&\quad - \text{Subst}\left(\frac{(bfpq)\int\frac{(i+jx)^3}{e+fx}\,dx}{3h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) \\
&\quad - \text{Subst}\left(\frac{(bf(hi-gj)pq)\int\frac{(i+jx)^2}{e+fx}\,dx}{2h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) \\
&\quad - \text{Subst}\left(\frac{\left(bf(hi-gj)^3pq\right)\int\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h^4}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{aj(hi-gj)^2x}{h^3} + \frac{(hi-gj)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{2h^2} \\
&+ \frac{(i+jx)^3\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{3h} \\
&+ \frac{(hi-gj)^3\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^4} \\
&+ \text{Subst}\left(\frac{\left(bj(hi-gj)^2\right)\text{Subst}\left(\int\log\left(cd^qx^{pq}\right)\,dx,x,e+fx\right)}{fh^3},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right) \\
&- \text{Subst}\left(\frac{(bfpq)\int\left(\frac{j(fi-ej)^2}{f^3}+\frac{(fi-ej)^3}{f^3(e+fx)}+\frac{j(fi-ej)(i+jx)}{f^2}+\frac{j(i+jx)^2}{f}\right)dx}{3h},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right) \\
&- \text{Subst}\left(\frac{(bf(hi-gj)pq)\int\left(\frac{j(fi-ej)}{f^2}+\frac{(fi-ej)^2}{f^2(e+fx)}+\frac{j(i+jx)}{f}\right)dx}{2h^2},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right) \\
&- \text{Subst}\left(\frac{\left(b(hi-gj)^3pq\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{h^4},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right)
\end{aligned}
$$

$$= \frac{aj(hi-gj)^2x}{h^3} - \frac{bj(fi-ej)^2pqx}{3f^2h} - \frac{bj(fi-ej)(hi-gj)pqx}{2fh^2} - \frac{bj(hi-gj)^2pqx}{h^3}$$
$$- \frac{b(fi-ej)pq(i+jx)^2}{6fh} - \frac{b(hi-gj)pq(i+jx)^2}{4h^2} - \frac{bpq(i+jx)^3}{9h}$$
$$- \frac{b(fi-ej)^3pq\log(e+fx)}{3f^3h} - \frac{b(fi-ej)^2(hi-gj)pq\log(e+fx)}{2f^2h^2}$$
$$+ \frac{bj(hi-gj)^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh^3}$$
$$+ \frac{(hi-gj)(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2h^2}$$
$$+ \frac{(i+jx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{3h}$$
$$+ \frac{(hi-gj)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^4}$$
$$+ \frac{b(hi-gj)^3pq\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^4}$$

## Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.90

$$\int \frac{(i+jx)^3\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}\,dx$$
$$= \frac{6be^2h^2j^2(-9fhi+3fgj+2ehj)pq\log(e+fx)+f\Big(hjx(6af^2(6g^2j^2-3ghj(6i+jx)+h^2(18i^2+9ijx+2}{}$$

```
[In] Integrate[((i + j*x)^3*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(g + h*x),x]
```

```
[Out] (6*b*e^2*h^2*j^2*(-9*f*h*i + 3*f*g*j + 2*e*h*j)*p*q*Log[e + f*x] + f*(h*j*x
*(6*a*f^2*(6*g^2*j^2 - 3*g*h*j*(6*i + j*x) + h^2*(18*i^2 + 9*i*j*x + 2*j^2*
x^2)) - b*p*q*(12*e^2*h^2*j^2 - 6*e*f*h*j*(9*h*i - 3*g*j + h*j*x) + f^2*(36
*g^2*j^2 - 9*g*h*j*(12*i + j*x) + h^2*(108*i^2 + 27*i*j*x + 4*j^2*x^2)))) +
 36*a*f^2*(h*i - g*j)^3*Log[(f*(g + h*x))/(f*g - e*h)] + 6*b*f*Log[c*(d*(e
+ f*x)^p)^q]*(h*j*(6*e*(3*h^2*i^2 - 3*g*h*i*j + g^2*j^2) + f*x*(6*g^2*j^2 -
 3*g*h*j*(6*i + j*x) + h^2*(18*i^2 + 9*i*j*x + 2*j^2*x^2))) + 6*f*(h*i - g*
j)^3*Log[(f*(g + h*x))/(f*g - e*h)])) + 36*b*f^3*(h*i - g*j)^3*p*q*PolyLog[
2, (h*(e + f*x))/(-(f*g) + e*h)])/(36*f^3*h^4)
```

## Maple [F]

$$\int \frac{(jx+i)^3 (a+b\ln(c(d(fx+e)^p)^q))}{hx+g}dx$$

```
[In] int((j*x+i)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
[Out] int((j*x+i)^3*(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

## Fricas [F]

$$\int \frac{(i+jx)^3 (a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(jx+i)^3(b\log(((fx+e)^pd)^q c)+a)}{hx+g}\,dx$$

```
[In] integrate((j*x+i)^3*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="fric
as")
[Out] integral((a*j^3*x^3 + 3*a*i*j^2*x^2 + 3*a*i^2*j*x + a*i^3 + (b*j^3*x^3 + 3*
b*i*j^2*x^2 + 3*b*i^2*j*x + b*i^3)*log(((f*x + e)^p*d)^q*c))/(h*x + g), x)
```

## Sympy [F]

$$\int \frac{(i+jx)^3 (a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))(i+jx)^3}{g+hx}\,dx$$

```
[In] integrate((j*x+i)**3*(a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))*(i + j*x)**3/(g + h*x), x)
```

## Maxima [F]

$$\int \frac{(i+jx)^3 (a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(jx+i)^3(b\log(((fx+e)^pd)^q c)+a)}{hx+g}\,dx$$

```
[In] integrate((j*x+i)^3*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="maxi
ma")
[Out] 3*a*i^2*j*(x/h - g*log(h*x + g)/h^2) - 1/6*a*j^3*(6*g^3*log(h*x + g)/h^4 -
(2*h^2*x^3 - 3*g*h*x^2 + 6*g^2*x)/h^3) + 3/2*a*i*j^2*(2*g^2*log(h*x + g)/h^
3 + (h*x^2 - 2*g*x)/h^2) + a*i^3*log(h*x + g)/h + integrate(((j^3*q*log(d)
+ j^3*log(c))*b*x^3 + 3*(i*j^2*q*log(d) + i*j^2*log(c))*b*x^2 + 3*(i^2*j*q*
log(d) + i^2*j*log(c))*b*x + (i^3*q*log(d) + i^3*log(c))*b + (b*j^3*x^3 + 3
*b*i*j^2*x^2 + 3*b*i^2*j*x + b*i^3)*log(((f*x + e)^p)^q))/(h*x + g), x)
```

## Giac [F]

$$\int \frac{(i+jx)^3 (a+b\log(c(d(e+fx)^p)^q))}{g+hx} dx = \int \frac{(jx+i)^3(b\log(((fx+e)^p d)^q c)+a)}{hx+g} dx$$

```
[In] integrate((j*x+i)^3*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="giac
")
```

```
[Out] integrate((j*x + i)^3*(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)^3 (a+b\log(c(d(e+fx)^p)^q))}{g+hx} dx = \int \frac{(i+j\,x)^3\,(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))}{g+h\,x} dx$$

```
[In] int(((i + j*x)^3*(a + b*log(c*(d*(e + f*x)^p)^q)))/(g + h*x),x)
```

```
[Out] int(((i + j*x)^3*(a + b*log(c*(d*(e + f*x)^p)^q)))/(g + h*x), x)
```

# 3.524 $\int \frac{(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))}{g+hx} dx$



## Optimal result

Integrand size = 33, antiderivative size = 258

$$\begin{aligned}\int \frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}\,dx &= \frac{aj(hi-gj)x}{h^2}-\frac{bj(fi-ej)pqx}{2fh}-\frac{bj(hi-gj)pqx}{h^2}-\frac{bpq(i+jx)^2}{4h}\\ &\quad -\frac{b(fi-ej)^2pq\log(e+fx)}{2f^2h}+\frac{bj(hi-gj)(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh^2}\\ &\quad +\frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2h}\\ &\quad +\frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3}\\ &\quad +\frac{b(hi-gj)^2pq\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^3}\end{aligned}$$

```
[Out] a*j*(-g*j+h*i)*x/h^2-1/2*b*j*(-e*j+f*i)*p*q*x/f/h-b*j*(-g*j+h*i)*p*q*x/h^2-
1/4*b*p*q*(j*x+i)^2/h-1/2*b*(-e*j+f*i)^2*p*q*ln(f*x+e)/f^2/h+b*j*(-g*j+h*i)
*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f/h^2+1/2*(j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^
q))/h+(-g*j+h*i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/h^3
+b*(-g*j+h*i)^2*p*q*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h^3
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2465, 2436, 2332, 2441, 2440, 2438, 2442, 45, 2495}

$$\int \frac{(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx$$
$$=\frac{(hi-gj)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h^3}+\frac{(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))}{2h}+\frac{ajx(hi-gj)}{h^2}+\frac{bj(e+fx)(hi-gj)\log(c(d(e+fx)^p)^q)}{fh^2}-\frac{bpq(fi-ej)^2\log(e+fx)}{2f^2h}+\frac{bpq(hi-gj)^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^3}-\frac{bjpqx(fi-ej)}{2fh}-\frac{bjpqx(hi-gj)}{h^2}-\frac{bpq(i+jx)^2}{4h}$$

```
[In] Int[((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(g + h*x),x]
```

```
[Out] (a*j*(h*i - g*j)*x)/h^2 - (b*j*(f*i - e*j)*p*q*x)/(2*f*h) - (b*j*(h*i - g*j
)*p*q*x)/h^2 - (b*p*q*(i + j*x)^2)/(4*h) - (b*(f*i - e*j)^2*p*q*Log[e + f*x
])/(2*f^2*h) + (b*j*(h*i - g*j)*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/(f*h^2)
 + ((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(2*h) + ((h*i - g*j)^2*(a
 + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/h^3 + (b*(h*
i - g*j)^2*p*q*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(i+jx)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\int \left(\frac{j(hi-gj)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h^2}+\frac{(hi-gj)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h^2(g+hx)} + \frac{j(i+jx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h}\right)dx, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= \text{Subst}\left(\frac{j\int (i+jx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)dx}{h}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right) + \text{Subst}\left(\frac{(j(hi-gj))\int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)dx}{h^2}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right) + \text{Subst}\left(\frac{(hi-gj)^2\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{g+hx}dx}{h^2}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= \frac{aj(hi-gj)x}{h^2}+\frac{(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{2h} + \frac{(hi-gj)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} + \text{Subst}\left(\frac{(bj(hi-gj))\int \log\left(cd^q(e+fx)^{pq}\right)dx}{h^2}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right) - \text{Subst}\left(\frac{(bfpq)\int \frac{(i+jx)^2}{e+fx}dx}{2h}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right) - \text{Subst}\left(\frac{(bf(hi-gj)^2pq)\int \frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{h^3}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= \frac{aj(hi-gj)x}{h^2} + \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))}{2h} + \frac{(hi-gj)^2 (a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} + \text{Subst}\left(\frac{(bj(hi-gj))\text{Subst}\left(\int\log\left(cd^q x^{pq}\right)dx, x, e+fx\right)}{fh^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(bfpq)\int\left(\frac{j(fi-ej)}{f^2}+\frac{(fi-ej)^2}{f^2(e+fx)}+\frac{j(i+jx)}{f}\right)dx}{2h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(b(hi-gj)^2pq)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{hx}{fg-eh}\right)}{x}dx, x, e+fx\right)}{h^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{aj(hi-gj)x}{h^2} - \frac{bj(fi-ej)pqx}{2fh} - \frac{bj(hi-gj)pqx}{h^2} - \frac{bpq(i+jx)^2}{4h} - \frac{b(fi-ej)^2pq\log(e+fx)}{2f^2h} + \frac{bj(hi-gj)(e+fx)\log(c(d(e+fx)^p)^q)}{fh^2} + \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))}{2h} + \frac{(hi-gj)^2 (a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} + \frac{b(hi-gj)^2pq\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3}$$

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.90

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx$$
$$= \frac{-2be^2h^2j^2pq\log(e+fx)+f\Big(hjx(2af(4hi-2gj+hjx)+bpq(2ehj-f(8hi-4gj+hjx)))+4af(hi-$$

[In] Integrate[((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(g + h*x),x]

[Out] (-2*b*e^2*h^2*j^2*p*q*Log[e + f*x] + f*(h*j*x*(2*a*f*(4*h*i - 2*g*j + h*j*x
) + b*p*q*(2*e*h*j - f*(8*h*i - 4*g*j + h*j*x))) + 4*a*f*(h*i - g*j)^2*Log[
(f*(g + h*x))/(f*g - e*h)] + 2*b*Log[c*(d*(e + f*x)^p)^q]*(h*j*(e*(4*h*i -
2*g*j) + f*x*(4*h*i - 2*g*j + h*j*x)) + 2*f*(h*i - g*j)^2*Log[(f*(g + h*x))
/(f*g - e*h)])) + 4*b*f^2*(h*i - g*j)^2*p*q*PolyLog[2, (h*(e + f*x))/(-(f*g
) + e*h)])/(4*f^2*h^3)

## Maple [F]

$$\int \frac{(jx+i)^2 (a+b\ln(c(d(fx+e)^p)^q))}{hx+g}dx$$

[In] int((j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)

[Out] int((j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)

## Fricas [F]

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx = \int \frac{(jx+i)^2(b\log(((fx+e)^pd)^q c)+a)}{hx+g}\,dx$$

[In] integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="fric
as")

[Out] integral((a*j^2*x^2 + 2*a*i*j*x + a*i^2 + (b*j^2*x^2 + 2*b*i*j*x + b*i^2)*l
og(((f*x + e)^p*d)^q*c))/(h*x + g), x)

## Sympy [F]

$$\int \frac{(i+jx)^2\,(a+b\log{(c(d(e+fx)^p)^q)})}{g+hx}\,dx = \int \frac{(a+b\log{(c(d(e+fx)^p)^q)})\,(i+jx)^2}{g+hx}\,dx$$

```
[In] integrate((j*x+i)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))*(i + j*x)**2/(g + h*x), x)
```

## Maxima [F]

$$\int \frac{(i+jx)^2\,(a+b\log{(c(d(e+fx)^p)^q)})}{g+hx}\,dx = \int \frac{(jx+i)^2(b\log{(((fx+e)^p d)^q\,c)}+a)}{hx+g}\,dx$$

```
[In] integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="maxi
ma")
[Out] 2*a*i*j*(x/h - g*log(h*x + g)/h^2) + 1/2*a*j^2*(2*g^2*log(h*x + g)/h^3 + (h
*x^2 - 2*g*x)/h^2) + a*i^2*log(h*x + g)/h + integrate(((j^2*q*log(d) + j^2*
log(c))*b*x^2 + 2*(i*j*q*log(d) + i*j*log(c))*b*x + (i^2*q*log(d) + i^2*log
(c))*b + (b*j^2*x^2 + 2*b*i*j*x + b*i^2)*log(((f*x + e)^p)^q))/(h*x + g), x
)
```

## Giac [F]

$$\int \frac{(i+jx)^2\,(a+b\log{(c(d(e+fx)^p)^q)})}{g+hx}\,dx = \int \frac{(jx+i)^2(b\log{(((fx+e)^p d)^q\,c)}+a)}{hx+g}\,dx$$

```
[In] integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="giac
")
[Out] integrate((j*x + i)^2*(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)^2\,(a+b\log{(c(d(e+fx)^p)^q)})}{g+hx}\,dx = \int \frac{{\left(i+j\,x\right)}^2\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}{g+h\,x}\,dx$$

```
[In] int(((i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q)))/(g + h*x),x)
[Out] int(((i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q)))/(g + h*x), x)
```

# 3.525 $\int \frac{(i+jx)(a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 129

$$\int \frac{(i+jx)\,(a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx = \frac{ajx}{h}-\frac{bjpqx}{h}+\frac{bj(e+fx)\log(c(d(e+fx)^p)^q)}{fh} + \frac{(hi-gj)\,(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} + \frac{b(hi-gj)pq\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}$$

```
[Out] a*j*x/h-b*j*p*q*x/h+b*j*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f/h+(-g*j+h*i)*(a+b*l
n(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/h^2+b*(-g*j+h*i)*p*q*polylog
(2,-h*(f*x+e)/(-e*h+f*g))/h^2
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2465, 2436, 2332, 2441, 2440, 2438, 2495}

$$\int \frac{(i+jx)\,(a+b\log(c(d(e+fx)^p)^q))}{g+hx}\,dx = \frac{(hi-gj)\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h^2}+\frac{ajx}{h} + \frac{bj(e+fx)\log(c(d(e+fx)^p)^q)}{fh}+\frac{bpq(hi-gj)\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}-\frac{bjpqx}{h}$$

```
[In] Int[((i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(g + h*x),x]
```

```
[Out] (a*j*x)/h - (b*j*p*q*x)/h + (b*j*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/(f*h)
+ ((h*i - g*j)*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*
h)])/h^2 + (b*(h*i - g*j)*p*q*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(i+jx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\int \left(\frac{j\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h} + \frac{(hi-gj)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{h(g+hx)}\right)dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{j\int\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(hi-gj)\int\frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{g+hx}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{ajx}{h} + \frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} + \text{Subst}\left(\frac{(bj)\int\log\left(cd^q(e+fx)^{pq}\right)dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(bf(hi-gj)pq)\int\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{ajx}{h} + \frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} + \text{Subst}\left(\frac{(bj)\text{Subst}\left(\int\log\left(cd^qx^{pq}\right)dx, x, e+fx\right)}{fh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(b(hi-gj)pq)\text{Subst}\left(\int\frac{\log\left(1+\frac{hx}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{ajx}{h} - \frac{bjpqx}{h} + \frac{bj(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh} + \frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} + \frac{b(hi-gj)pq\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^2}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.93

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}\,dx = \frac{ahjx - bhjpqx + \frac{bhj(e+fx)\log(c(d(e+fx)^p)^q)}{f} + (hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right) + b(hi-g}{h^2}$$

```
[In] Integrate[((i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(g + h*x),x]
```

```
[Out] (a*h*j*x - b*h*j*p*q*x + (b*h*j*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/f + (h*
i - g*j)*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)] +
b*(h*i - g*j)*p*q*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)])/h^2
```

## Maple [F]

$$\int \frac{(jx+i)\left(a+b\ln\left(c(d(fx+e)^p)^q\right)\right)}{hx+g}dx$$

```
[In] int((j*x+i)*(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

```
[Out] int((j*x+i)*(a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

## Fricas [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}\,dx = \int \frac{(jx+i)(b\log\left(((fx+e)^p d)^q c\right)+a)}{hx+g}\,dx$$

```
[In] integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="fricas
")
```

```
[Out] integral((a*j*x + a*i + (b*j*x + b*i)*log(((f*x + e)^p*d)^q*c))/(h*x + g),
x)
```

## Sympy [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}\,dx=\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)(i+jx)}{g+hx}\,dx$$

```
[In] integrate((j*x+i)*(a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))*(i + j*x)/(g + h*x), x)
```

## Maxima [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}\,dx=\int \frac{(jx+i)(b\log\left(((fx+e)^pd)^q\,c\right)+a)}{hx+g}\,dx$$

```
[In] integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="maxima
")
[Out] a*j*(x/h - g*log(h*x + g)/h^2) + a*i*log(h*x + g)/h + integrate(((j*q*log(d
) + j*log(c))*b*x + (i*q*log(d) + i*log(c))*b + (b*j*x + b*i)*log(((f*x + e
)^p)^q))/(h*x + g), x)
```

## Giac [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}\,dx=\int \frac{(jx+i)(b\log\left(((fx+e)^pd)^q\,c\right)+a)}{hx+g}\,dx$$

```
[In] integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="giac")
[Out] integrate((j*x + i)*(b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{g+hx}\,dx=\int \frac{(i+j\,x)\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}{g+h\,x}\,dx$$

```
[In] int(((i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q)))/(g + h*x),x)
[Out] int(((i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q)))/(g + h*x), x)
```

# 3.526 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{g+hx}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 68

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}+\frac{bpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

```
[Out] (a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/h+b*p*q*polylog(2,-h*(
f*x+e)/(-e*h+f*g))/h
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2441, 2440, 2438, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h}+\frac{bpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x),x]
```

```
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/h + (b*p*
q*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}\\
&\quad-\text{Subst}\left(\frac{(bfpq)\int\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}\\
&\quad-\text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int\frac{\log\left(1+\frac{hx}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h}+\frac{bpq\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int \frac{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}{g+hx}\,dx = \frac{\left(a+b\log \left(c\left(d(e+fx)^p\right)^q\right)\right)\log \left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{bpq\,\text{PolyLog}\left(2,\frac{h(e+fx)}{-fg+eh}\right)}{h}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/(g + h*x),x]
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/h + (b*p*
q*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)])/h
```

## Maple [F]

$$\int \frac{a+b\ln \left(c\left(d(fx+e)^p\right)^q\right)}{hx+g}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g),x)
```

## Fricas [F]

$$\int \frac{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}{g+hx}\,dx = \int \frac{b\log \left(\left((fx+e)^p d\right)^q c\right)+a}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="fricas")
[Out] integral((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

## Sympy [F]

$$\int \frac{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}{g+hx}\,dx = \int \frac{a+b\log \left(c\left(d(e+fx)^p\right)^q\right)}{g+hx}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))/(g + h*x), x)
```

## Maxima [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\int \frac{b\log\left(\left(((fx+e)^pd)^qc\right)\right)+a}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="maxima")
```

```
[Out] b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(h*x + g), x) + a*lo
g(h*x + g)/h
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\int \frac{b\log\left(\left(((fx+e)^pd)^qc\right)\right)+a}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)/(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{g+hx}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{g+h\,x}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x),x)
```

```
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/(g + h*x), x)
```

# 3.527 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)(i+jx)}\,dx$



## Optimal result

Integrand size = 33, antiderivative size = 165

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)}\,dx = \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} + \frac{bpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} - \frac{bpq\,\text{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}$$

```
[Out] (a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/(-g*j+h*i)-(a+b*ln(c*(
d*(f*x+e)^p)^q))*ln(f*(j*x+i)/(-e*j+f*i))/(-g*j+h*i)+b*p*q*polylog(2,-h*(f*
x+e)/(-e*h+f*g))/(-g*j+h*i)-b*p*q*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*
i)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2465, 2441, 2440, 2438, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)}\,dx = \frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{hi-gj} - \frac{\log\left(\frac{f(i+jx)}{fi-ej}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{hi-gj} + \frac{bpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} - \frac{bpq\,\text{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/((g + h*x)*(i + j*x)),x]
```

```
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/(h*i - g*
j) - ((a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(i + j*x))/(f*i - e*j)])/(h*i
 - g*j) + (b*p*q*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/(h*i - g*j) - (b
*p*q*PolyLog[2, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{a + b\log\left(cd^q(e+fx)^{pq}\right)}{(g+hx)(i+jx)}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\int \left(\frac{h(a+b\log(cd^q(e+fx)^{pq}))}{(hi-gj)(g+hx)} - \frac{j(a+b\log(cd^q(e+fx)^{pq}))}{(hi-gj)(i+jx)}\right) dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{h\int \frac{a+b\log(cd^q(e+fx)^{pq})}{g+hx}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{j\int \frac{a+b\log(cd^q(e+fx)^{pq})}{i+jx}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} - \text{Subst}\left(\frac{(bfpq)\int \frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(bfpq)\int \frac{\log\left(\frac{f(i+jx)}{fi-ej}\right)}{e+fx}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} - \text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int \frac{\log\left(1+\frac{hx}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(bpq)\text{Subst}\left(\int \frac{\log\left(1+\frac{jx}{fi-ej}\right)}{x}\,dx, x, e+fx\right)}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} + \frac{bpq\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} - \frac{bpq\text{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.71

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)}\,dx$$
$$=\frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\left(\log\left(\frac{f(g+hx)}{fg-eh}\right)-\log\left(\frac{f(i+jx)}{fi-ej}\right)\right)+bpq\,\text{PolyLog}\left(2,\frac{h(e+fx)}{-fg+eh}\right)-bpq\,\text{PolyLog}\left(2,\right.}{hi-gj}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/((g + h*x)*(i + j*x)),x]
```

```
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])*(Log[(f*(g + h*x))/(f*g - e*h)] - Log[(f*
(i + j*x))/(f*i - e*j)]) + b*p*q*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] -
 b*p*q*PolyLog[2, (j*(e + f*x))/(-(f*i) + e*j)])/(h*i - g*j)
```

## Maple [F]

$$\int \frac{a+b\ln\left(c(d(fx+e)^p)^q\right)}{(hx+g)\,(jx+i)}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i),x)
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)}\,dx=\int \frac{b\log\left(((fx+e)^pd)^q\,c\right)+a}{(hx+g)(jx+i)}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i),x, algorithm="fricas
")
```

```
[Out] integral((b*log(((f*x + e)^p*d)^q*c) + a)/(h*j*x^2 + g*i + (h*i + g*j)*x),
x)
```

## Sympy [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)}\,dx=\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)\,(i+jx)}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)/(j*x+i),x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))/((g + h*x)*(i + j*x)), x)
```

## Maxima [F]

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)(i + jx)}\,dx = \int \frac{b\log\left(\left(\left(fx + e\right)^p d\right)^q c\right) + a}{(hx + g)(jx + i)}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i),x, algorithm="maxima
")
```

```
[Out] a*(log(h*x + g)/(h*i - g*j) - log(j*x + i)/(h*i - g*j)) + b*integrate((q*lo
g(d) + log(((f*x + e)^p)^q) + log(c))/(h*j*x^2 + g*i + (h*i + g*j)*x), x)
```

## Giac [F]

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)(i + jx)}\,dx = \int \frac{b\log\left(\left(\left(fx + e\right)^p d\right)^q c\right) + a}{(hx + g)(jx + i)}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i),x, algorithm="giac")
```

```
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)/((h*x + g)*(j*x + i)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d(e + fx)^p)^q\right)}{(g + hx)(i + jx)}\,dx = \int \frac{a + b\,\ln\left(c\left(d\left(e + f\,x\right)^p\right)^q\right)}{\left(g + h\,x\right)\,\left(i + j\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)*(i + j*x)),x)
```

```
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)*(i + j*x)), x)
```

# 3.528 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)(i+jx)^2}\,dx$



## Optimal result

Integrand size = 33, antiderivative size = 268

$$\begin{aligned}\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx = &-\frac{bfpq\log(e+fx)}{(fi-ej)(hi-gj)}+\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(hi-gj)(i+jx)}\\&+\frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2}\\&+\frac{bfpq\log(i+jx)}{(fi-ej)(hi-gj)}\\&-\frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2}\\&+\frac{bhpq\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2}\\&-\frac{bhpq\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}\end{aligned}$$

```
[Out] -b*f*p*q*ln(f*x+e)/(-e*j+f*i)/(-g*j+h*i)+(a+b*ln(c*(d*(f*x+e)^p)^q))/(-g*j+
h*i)/(j*x+i)+h*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/(-g*j+h
*i)^2+b*f*p*q*ln(j*x+i)/(-e*j+f*i)/(-g*j+h*i)-h*(a+b*ln(c*(d*(f*x+e)^p)^q))
*ln(f*(j*x+i)/(-e*j+f*i))/(-g*j+h*i)^2+b*h*p*q*polylog(2,-h*(f*x+e)/(-e*h+f
*g))/(-g*j+h*i)^2-b*h*p*q*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)^2
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2465, 2441, 2440, 2438, 2442, 36, 31, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx=\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(i+jx)(hi-gj)}+\frac{h\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{(hi-gj)^2}-\frac{h\log\left(\frac{f(i+jx)}{fi-ej}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{(hi-gj)^2}+\frac{bhpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2}-\frac{bhpq\,\text{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}-\frac{bfpq\log(e+fx)}{(fi-ej)(hi-gj)}+\frac{bfpq\log(i+jx)}{(fi-ej)(hi-gj)}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/((g + h*x)*(i + j*x)^2),x]
```

```
[Out] -((b*f*p*q*Log[e + f*x])/((f*i - e*j)*(h*i - g*j))) + (a + b*Log[c*(d*(e +
f*x)^p)^q])/((h*i - g*j)*(i + j*x)) + (h*(a + b*Log[c*(d*(e + f*x)^p)^q])*L
og[(f*(g + h*x))/(f*g - e*h)])/(h*i - g*j)^2 + (b*f*p*q*Log[i + j*x])/((f*i
 - e*j)*(h*i - g*j)) - (h*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(i + j*x)
)/(f*i - e*j)])/(h*i - g*j)^2 + (b*h*p*q*PolyLog[2, -((h*(e + f*x))/(f*g -
e*h))])/(h*i - g*j)^2 - (b*h*p*q*PolyLog[2, -((j*(e + f*x))/(f*i - e*j))])/
(h*i - g*j)^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(g+hx)(i+jx)^2}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\ &= \text{Subst}\left(\int \left(\frac{h^2(a+b\log\left(cd^q(e+fx)^{pq}\right))}{(hi-gj)^2(g+hx)}-\frac{j(a+b\log\left(cd^q(e+fx)^{pq}\right))}{(hi-gj)(i+jx)^2}\right.\right.\\ &\qquad \left.\left.-\frac{hj(a+b\log\left(cd^q(e+fx)^{pq}\right))}{(hi-gj)^2(i+jx)}\right)\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\end{aligned}$$

$$= \text{Subst}\left(\frac{h^2 \int \frac{a+b\log(cd^q(e+fx)^{pq})}{g+hx}\,dx}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$- \text{Subst}\left(\frac{(hj) \int \frac{a+b\log(cd^q(e+fx)^{pq})}{i+jx}\,dx}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$- \text{Subst}\left(\frac{j \int \frac{a+b\log(cd^q(e+fx)^{pq})}{(i+jx)^2}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(hi-gj)(i+jx)} + \frac{h(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2}$$
$$- \frac{h(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2}$$
$$- \text{Subst}\left(\frac{(bfhpq) \int \frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$+ \text{Subst}\left(\frac{(bfhpq) \int \frac{\log\left(\frac{f(i+jx)}{fi-ej}\right)}{e+fx}\,dx}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$- \text{Subst}\left(\frac{(bfpq) \int \frac{1}{(e+fx)(i+jx)}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(hi-gj)(i+jx)} + \frac{h\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2} - \frac{h\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2} - \text{Subst}\left(\frac{(bhpq)\text{Subst}\left(\int \frac{\log\left(1+\frac{hx}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right) + \text{Subst}\left(\frac{(bhpq)\text{Subst}\left(\int \frac{\log\left(1+\frac{jx}{fi-ej}\right)}{x}\,dx, x, e+fx\right)}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right) - \text{Subst}\left(\frac{(bf^2pq)\int \frac{1}{e+fx}\,dx}{(fi-ej)(hi-gj)}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right) + \text{Subst}\left(\frac{(bfjpq)\int \frac{1}{i+jx}\,dx}{(fi-ej)(hi-gj)}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= -\frac{bfpq\log(e+fx)}{(fi-ej)(hi-gj)} + \frac{a+b\log\left(c\left(d(e+fx)^p\right)^q\right)}{(hi-gj)(i+jx)} + \frac{h\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2} + \frac{bfpq\log(i+jx)}{(fi-ej)(hi-gj)} - \frac{h\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2} + \frac{bhpq\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} - \frac{bhpq\text{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.84

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx$$
$$= \frac{\frac{a(hi-gj)}{i+jx}+\frac{b(hi-gj)\log(c(d(e+fx)^p)^q)}{i+jx}+h(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)-\frac{bf(hi-gj)pq(\log(e+fx)-\log(i+jx}{fi-ej}}{(hi-}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/((g + h*x)*(i + j*x)^2),x]
```

```
[Out] ((a*(h*i - g*j))/(i + j*x) + (b*(h*i - g*j)*Log[c*(d*(e + f*x)^p)^q])/(i +
j*x) + h*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)] -
(b*f*(h*i - g*j)*p*q*(Log[e + f*x] - Log[i + j*x]))/(f*i - e*j) - h*(a + b*
Log[c*(d*(e + f*x)^p)^q])*Log[(f*(i + j*x))/(f*i - e*j)] + b*h*p*q*PolyLog[
2, (h*(e + f*x))/(-(f*g) + e*h)] - b*h*p*q*PolyLog[2, (j*(e + f*x))/(-(f*i)
 + e*j)])/(h*i - g*j)^2
```

## Maple [F]

$$\int \frac{a+b\ln\left(c\left(d(fx+e)^p\right)^q\right)}{(hx+g)\left(jx+i\right)^2}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^2,x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^2,x)
```

## Fricas [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx = \int \frac{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}{(hx+g)(jx+i)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^2,x, algorithm="fric
as")
```

```
[Out] integral((b*log(((f*x + e)^p*d)^q*c) + a)/(h*j^2*x^3 + g*i^2 + (2*h*i*j + g
*j^2)*x^2 + (h*i^2 + 2*g*i*j)*x), x)
```

## Sympy [F]

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx=\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{(g+hx)\,(i+jx)^2}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)/(j*x+i)**2,x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))/((g + h*x)*(i + j*x)**2), x)
```

## Maxima [F]

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx=\int \frac{b\log \left(\left((fx+e)^pd\right)^qc\right)+a}{(hx+g)(jx+i)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^2,x, algorithm="maxi
ma")
[Out] a*(h*log(h*x + g)/(h^2*i^2 - 2*g*h*i*j + g^2*j^2) - h*log(j*x + i)/(h^2*i^2
 - 2*g*h*i*j + g^2*j^2) + 1/(h*i^2 - g*i*j + (h*i*j - g*j^2)*x)) + b*integr
ate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(h*j^2*x^3 + g*i^2 + (2*h*i*
j + g*j^2)*x^2 + (h*i^2 + 2*g*i*j)*x), x)
```

## Giac [F]

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx=\int \frac{b\log \left(\left((fx+e)^pd\right)^qc\right)+a}{(hx+g)(jx+i)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^2,x, algorithm="giac
")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)/((h*x + g)*(j*x + i)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^2}\,dx=\int \frac{a+b\,\ln \left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{\left(g+h\,x\right)\,{\left(i+j\,x\right)}^2}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)*(i + j*x)^2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)*(i + j*x)^2), x)
```

# 3.529 $\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)(i+jx)^3}\,dx$



## Optimal result

Integrand size = 33, antiderivative size = 425

$$\begin{aligned}\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^3}\,dx =& -\frac{bfpq}{2(fi-ej)(hi-gj)(i+jx)}-\frac{bfhpq\log(e+fx)}{(fi-ej)(hi-gj)^2}\\&-\frac{bf^2pq\log(e+fx)}{2(fi-ej)^2(hi-gj)}+\frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2(hi-gj)(i+jx)^2}\\&+\frac{h(a+b\log\left(c(d(e+fx)^p)^q\right))}{(hi-gj)^2(i+jx)}\\&+\frac{h^2(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^3}\\&+\frac{bfhpq\log(i+jx)}{(fi-ej)(hi-gj)^2}+\frac{bf^2pq\log(i+jx)}{2(fi-ej)^2(hi-gj)}\\&-\frac{h^2(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^3}\\&+\frac{bh^2pq\,\mathrm{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^3}\\&-\frac{bh^2pq\,\mathrm{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^3}\end{aligned}$$

```
[Out] -1/2*b*f*p*q/(-e*j+f*i)/(-g*j+h*i)/(j*x+i)-b*f*h*p*q*ln(f*x+e)/(-e*j+f*i)/(
-g*j+h*i)^2-1/2*b*f^2*p*q*ln(f*x+e)/(-e*j+f*i)^2/(-g*j+h*i)+1/2*(a+b*ln(c*(
d*(f*x+e)^p)^q))/(-g*j+h*i)/(j*x+i)^2+h*(a+b*ln(c*(d*(f*x+e)^p)^q))/(-g*j+h
*i)^2/(j*x+i)+h^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(h*x+g)/(-e*h+f*g))/(-g*
j+h*i)^3+b*f*h*p*q*ln(j*x+i)/(-e*j+f*i)/(-g*j+h*i)^2+1/2*b*f^2*p*q*ln(j*x+i
)/(-e*j+f*i)^2/(-g*j+h*i)-h^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(j*x+i)/(-e*
```

```
j+f*i))/(-g*j+h*i)^3+b*h^2*p*q*polylog(2,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*i)^
3-b*h^2*p*q*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)^3
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2465, 2441, 2440, 2438, 2442, 46, 36, 31, 2495}

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^3}\,dx = \frac{h^2\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{(hi-gj)^3} - \frac{h^2\log\left(\frac{f(i+jx)}{fi-ej}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{(hi-gj)^3} + \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{(i+jx)(hi-gj)^2} + \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2(i+jx)^2(hi-gj)} - \frac{bf^2pq\log(e+fx)}{2(fi-ej)^2(hi-gj)} + \frac{bf^2pq\log(i+jx)}{2(fi-ej)^2(hi-gj)} + \frac{bh^2pq\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^3} - \frac{bh^2pq\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^3} - \frac{bfpq}{2(i+jx)(fi-ej)(hi-gj)} - \frac{bfhpq\log(e+fx)}{(fi-ej)(hi-gj)^2} + \frac{bfhpq\log(i+jx)}{(fi-ej)(hi-gj)^2}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])/((g + h*x)*(i + j*x)^3),x]
```

```
[Out] -1/2*(b*f*p*q)/((f*i - e*j)*(h*i - g*j)*(i + j*x)) - (b*f*h*p*q*Log[e + f*x
])/((f*i - e*j)*(h*i - g*j)^2) - (b*f^2*p*q*Log[e + f*x])/(2*(f*i - e*j)^2*
(h*i - g*j)) + (a + b*Log[c*(d*(e + f*x)^p)^q])/(2*(h*i - g*j)*(i + j*x)^2)
 + (h*(a + b*Log[c*(d*(e + f*x)^p)^q]))/((h*i - g*j)^2*(i + j*x)) + (h^2*(a
 + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(g + h*x))/(f*g - e*h)])/(h*i - g*j)^
3 + (b*f*h*p*q*Log[i + j*x])/((f*i - e*j)*(h*i - g*j)^2) + (b*f^2*p*q*Log[i
 + j*x])/(2*(f*i - e*j)^2*(h*i - g*j)) - (h^2*(a + b*Log[c*(d*(e + f*x)^p)^
q])*Log[(f*(i + j*x))/(f*i - e*j)])/(h*i - g*j)^3 + (b*h^2*p*q*PolyLog[2, -
((h*(e + f*x))/(f*g - e*h))])/(h*i - g*j)^3 - (b*h^2*p*q*PolyLog[2, -((j*(e
 + f*x))/(f*i - e*j))])/(h*i - g*j)^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
```

```
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(g+hx)(i+jx)^3}\,dx, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&= \text{Subst}\Bigg(\int \Bigg(\frac{h^3\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{(hi-gj)^3(g+hx)} - \frac{j\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{(hi-gj)(i+jx)^3}\\
&\quad - \frac{hj\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{(hi-gj)^2(i+jx)^2} - \frac{h^2j\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)}{(hi-gj)^3(i+jx)}\Bigg)\,dx, cd^q(e\\
&\quad + fx)^{pq}, c\left(d(e+fx)^p\right)^q\Bigg)\\
&= \text{Subst}\left(\frac{h^3\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{g+hx}\,dx}{(hi-gj)^3}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&\quad - \text{Subst}\left(\frac{\left(h^2j\right)\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{i+jx}\,dx}{(hi-gj)^3}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&\quad - \text{Subst}\left(\frac{(hj)\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(i+jx)^2}\,dx}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)\\
&\quad - \text{Subst}\left(\frac{j\int \frac{a+b\log\left(cd^q(e+fx)^{pq}\right)}{(i+jx)^3}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2(hi-gj)(i+jx)^2}+\frac{h(a+b\log\left(c(d(e+fx)^p)^q\right))}{(hi-gj)^2(i+jx)} \\
&+ \frac{h^2(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^3} \\
&- \frac{h^2(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^3} \\
&- \text{Subst}\left(\frac{(bfh^2pq)\int\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{(hi-gj)^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) \\
&+ \text{Subst}\left(\frac{(bfh^2pq)\int\frac{\log\left(\frac{f(i+jx)}{fi-ej}\right)}{e+fx}\,dx}{(hi-gj)^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) \\
&- \text{Subst}\left(\frac{(bfhpq)\int\frac{1}{(e+fx)(i+jx)}\,dx}{(hi-gj)^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) \\
&- \text{Subst}\left(\frac{(bfpq)\int\frac{1}{(e+fx)(i+jx)^2}\,dx}{2(hi-gj)},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)
\end{aligned}
$$

$$= \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2(hi-gj)(i+jx)^2}+\frac{h(a+b\log\left(c(d(e+fx)^p)^q\right))}{(hi-gj)^2(i+jx)}$$

$$+\frac{h^2(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^3}$$

$$-\frac{h^2(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^3}$$

$$-\operatorname{Subst}\left(\frac{(bh^2pq)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{(hi-gj)^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\operatorname{Subst}\left(\frac{(bh^2pq)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{jx}{fi-ej}\right)}{x}\,dx,x,e+fx\right)}{(hi-gj)^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$-\operatorname{Subst}\left(\frac{(bf^2hpq)\int\frac{1}{e+fx}\,dx}{(fi-ej)(hi-gj)^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+\operatorname{Subst}\left(\frac{(bfhjpq)\int\frac{1}{i+jx}\,dx}{(fi-ej)(hi-gj)^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$-\operatorname{Subst}\left(\frac{(bfpq)\int\left(\frac{f^2}{(fi-ej)^2(e+fx)}-\frac{j}{(fi-ej)(i+jx)^2}-\frac{fj}{(fi-ej)^2(i+jx)}\right)dx}{2(hi-gj)},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= -\frac{bfpq}{2(fi-ej)(hi-gj)(i+jx)} - \frac{bfhpq\log(e+fx)}{(fi-ej)(hi-gj)^2} - \frac{bf^2pq\log(e+fx)}{2(fi-ej)^2(hi-gj)} + \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{2(hi-gj)(i+jx)^2} + \frac{h(a+b\log\left(c(d(e+fx)^p)^q\right))}{(hi-gj)^2(i+jx)} + \frac{h^2(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^3} + \frac{bfhpq\log(i+jx)}{(fi-ej)(hi-gj)^2} + \frac{bf^2pq\log(i+jx)}{2(fi-ej)^2(hi-gj)} - \frac{h^2(a+b\log\left(c(d(e+fx)^p)^q\right))\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^3} + \frac{bh^2pq\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^3} - \frac{bh^2pq\text{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^3}$$

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^3}\,dx = \frac{\frac{a(hi-gj)^2}{(i+jx)^2} + \frac{2ah(hi-gj)}{i+jx} + \frac{b(hi-gj)^2\log(c(d(e+fx)^p)^q)}{(i+jx)^2} + \frac{2bh(hi-gj)\log(c(d(e+fx)^p)^q)}{i+jx} + 2h^2(a+b\log\left(c(d(e+fx)^p)^q\right))\,\mathrm{l}}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])/((g + h*x)*(i + j*x)^3),x]
```

```
[Out] ((a*(h*i - g*j)^2)/(i + j*x)^2 + (2*a*h*(h*i - g*j))/(i + j*x) + (b*(h*i -
g*j)^2*Log[c*(d*(e + f*x)^p)^q])/(i + j*x)^2 + (2*b*h*(h*i - g*j)*Log[c*(d*
(e + f*x)^p)^q])/(i + j*x) + 2*h^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*
(g + h*x))/(f*g - e*h)] - (2*b*f*h*(h*i - g*j)*p*q*(Log[e + f*x] - Log[i +
j*x]))/(f*i - e*j) - (b*f*(h*i - g*j)^2*p*q*(f*i - e*j + f*(i + j*x)*Log[e
+ f*x] - f*(i + j*x)*Log[i + j*x]))/((f*i - e*j)^2*(i + j*x)) - 2*h^2*(a +
b*Log[c*(d*(e + f*x)^p)^q])*Log[(f*(i + j*x))/(f*i - e*j)] + 2*b*h^2*p*q*Po
lyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] - 2*b*h^2*p*q*PolyLog[2, (j*(e + f*x
))/(-(f*i) + e*j)])/(2*(h*i - g*j)^3)
```

## Maple [F]

$$\int \frac{a+b\ln\left(c(d(fx+e)^p)^q\right)}{(hx+g)\,(jx+i)^3}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^3,x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^3,x)
```

## Fricas [F]

$$\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)(i+jx)^3}\,dx = \int \frac{b\log\left(((fx+e)^p d)^q c\right)+a}{(hx+g)(jx+i)^3}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^3,x, algorithm="fric
as")
```

```
[Out] integral((b*log(((f*x + e)^p*d)^q*c) + a)/(h*j^3*x^4 + g*i^3 + (3*h*i*j^2 +
 g*j^3)*x^3 + 3*(h*i^2*j + g*i*j^2)*x^2 + (h*i^3 + 3*g*i^2*j)*x), x)
```

## Sympy [F]

$$\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)(i+jx)^3}\,dx = \int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)(i+jx)^3}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))/(h*x+g)/(j*x+i)**3,x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))/((g + h*x)*(i + j*x)**3), x)
```

## Maxima [F]

$$\int \frac{a+b\log(c(d(e+fx)^p)^q)}{(g+hx)(i+jx)^3}\,dx = \int \frac{b\log\left(((fx+e)^p d)^q c\right)+a}{(hx+g)(jx+i)^3}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*h^2*log(h*x + g)/(h^3*i^3 - 3*g*h^2*i^2*j + 3*g^2*h*i*j^2 - g^3*j^3)
 - 2*h^2*log(j*x + i)/(h^3*i^3 - 3*g*h^2*i^2*j + 3*g^2*h*i*j^2 - g^3*j^3) +
 (2*h*j*x + 3*h*i - g*j)/(h^2*i^4 - 2*g*h*i^3*j + g^2*i^2*j^2 + (h^2*i^2*j^
2 - 2*g*h*i*j^3 + g^2*j^4)*x^2 + 2*(h^2*i^3*j - 2*g*h*i^2*j^2 + g^2*i*j^3)*
x))*a + b*integrate((q*log(d) + log(((f*x + e)^p)^q) + log(c))/(h*j^3*x^4 +
 g*i^3 + (3*h*i*j^2 + g*j^3)*x^3 + 3*(h*i^2*j + g*i*j^2)*x^2 + (h*i^3 + 3*g
*i^2*j)*x), x)
```

## Giac [F]

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^3}\,dx=\int \frac{b\log\left(\left((fx+e)^p d\right)^q c\right)+a}{(hx+g)(jx+i)^3}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))/(h*x+g)/(j*x+i)^3,x, algorithm="giac
")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)/((h*x + g)*(j*x + i)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d(e+fx)^p)^q\right)}{(g+hx)(i+jx)^3}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)}{\left(g+h\,x\right)\,{\left(i+j\,x\right)}^3}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)*(i + j*x)^3),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))/((g + h*x)*(i + j*x)^3), x)
```

# 3.530 $\int \frac{(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$



## Optimal result

Integrand size = 35, antiderivative size = 519

$$\begin{aligned}
&\int \frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx \\
&= -\frac{2abj(fi-ej)pqx}{fh}-\frac{2abj(hi-gj)pqx}{h^2}+\frac{2b^2j(fi-ej)p^2q^2x}{fh}+\frac{2b^2j(hi-gj)p^2q^2x}{h^2} \\
&\quad+\frac{b^2j^2p^2q^2(e+fx)^2}{4f^2h}-\frac{2b^2j(fi-ej)pq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f^2h} \\
&\quad-\frac{2b^2j(hi-gj)pq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh^2} \\
&\quad-\frac{bj^2pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^2h} \\
&\quad+\frac{j(fi-ej)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2h} \\
&\quad+\frac{j(hi-gj)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh^2} \\
&\quad+\frac{j^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^2h} \\
&\quad+\frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} \\
&\quad+\frac{2b(hi-gj)^2pq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^3} \\
&\quad-\frac{2b^2(hi-gj)^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h^3}
\end{aligned}$$

```
[Out] -2*a*b*j*(-e*j+f*i)*p*q*x/f/h-2*a*b*j*(-g*j+h*i)*p*q*x/h^2+2*b^2*j*(-e*j+f*
i)*p^2*q^2*x/f/h+2*b^2*j*(-g*j+h*i)*p^2*q^2*x/h^2+1/4*b^2*j^2*p^2*q^2*(f*x+
e)^2/f^2/h-2*b^2*j*(-e*j+f*i)*p*q*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f^2/h-2*b^2
*j*(-g*j+h*i)*p*q*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f/h^2-1/2*b*j^2*p*q*(f*x+e)
^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^2/h+j*(-e*j+f*i)*(f*x+e)*(a+b*ln(c*(d*(f*x
+e)^p)^q))^2/f^2/h+j*(-g*j+h*i)*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f/h^2
+1/2*j^2*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f^2/h+(-g*j+h*i)^2*(a+b*ln
(c*(d*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/h^3+2*b*(-g*j+h*i)^2*p*q*(a
+b*ln(c*(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h^3-2*b^2*(-g*j+
h*i)^2*p^2*q^2*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h^3
```

## Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2465, 2436, 2333, 2332, 2443, 2481, 2421, 6724, 2448, 2437, 2342, 2341, 2495}

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))^2}{g+hx} \, dx$$
$$= \frac{j(e+fx)(fi-ej)(a+b\log(c(d(e+fx)^p)^q))^2}{f^2h} - \frac{bj^2pq(e+fx)^2(a+b\log(c(d(e+fx)^p)^q))}{2f^2h} + \frac{j^2(e+fx)^2(a+b\log(c(d(e+fx)^p)^q))^2}{2f^2h} + \frac{2bpq(hi-gj)^2 \operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h^3} + \frac{(hi-gj)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{h^3} + \frac{j(e+fx)(hi-gj)(a+b\log(c(d(e+fx)^p)^q))^2}{fh^2} - \frac{2abjpqx(fi-ej)}{fh} - \frac{2abjpqx(hi-gj)}{h^2} - \frac{2b^2jpq(e+fx)(fi-ej)\log(c(d(e+fx)^p)^q)}{f^2h} - \frac{2b^2jpq(e+fx)(hi-gj)\log(c(d(e+fx)^p)^q)}{fh^2} + \frac{b^2j^2p^2q^2(e+fx)^2}{4f^2h} - \frac{2b^2p^2q^2(hi-gj)^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h^3} + \frac{2b^2jp^2q^2x(fi-ej)}{fh} + \frac{2b^2jp^2q^2x(hi-gj)}{h^2}$$

```
[In] Int[((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(g + h*x),x]
```

```
[Out] (-2*a*b*j*(f*i - e*j)*p*q*x)/(f*h) - (2*a*b*j*(h*i - g*j)*p*q*x)/h^2 + (2*b
^2*j*(f*i - e*j)*p^2*q^2*x)/(f*h) + (2*b^2*j*(h*i - g*j)*p^2*q^2*x)/h^2 + (
b^2*j^2*p^2*q^2*(e + f*x)^2)/(4*f^2*h) - (2*b^2*j*(f*i - e*j)*p*q*(e + f*x)
*Log[c*(d*(e + f*x)^p)^q])/(f^2*h) - (2*b^2*j*(h*i - g*j)*p*q*(e + f*x)*Log
```

```
[c*(d*(e + f*x)^p)^q])/(f*h^2) - (b*j^2*p*q*(e + f*x)^2*(a + b*Log[c*(d*(e
+ f*x)^p)^q]))/(2*f^2*h) + (j*(f*i - e*j)*(e + f*x)*(a + b*Log[c*(d*(e + f*
x)^p)^q])^2)/(f^2*h) + (j*(h*i - g*j)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p
)^q])^2)/(f*h^2) + (j^2*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(2*
f^2*h) + ((h*i - g*j)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(g + h*x)
)/(f*g - e*h)])/h^3 + (2*b*(h*i - g*j)^2*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q
])*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h^3 - (2*b^2*(h*i - g*j)^2*p^2
*q^2*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))])/h^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(i+jx)^2 \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\int \left(\frac{j(hi-gj)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{h^2} + \frac{(hi-gj)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{h^2(g+hx)} + \frac{j(i+jx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{h}\right) dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{j\int (i+jx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(j(hi-gj))\int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2\,dx}{h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(hi-gj)^2\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{g+hx}\,dx}{h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} + \text{Subst}\left(\frac{j\int \left(\frac{(fi-ej)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{f} + \frac{j(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{f}\right)dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(j(hi-gj))\text{Subst}\left(\int \left(a+b\log\left(cd^q x^{pq}\right)\right)^2\,dx, x, e+fx\right)}{fh^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(2bf(hi-gj)^2pq)\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{j(hi-gj)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh^2} + \frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} + \operatorname{Subst}\left(\frac{j^2\int(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2\,dx}{fh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \operatorname{Subst}\left(\frac{(j(fi-ej))\int\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2\,dx}{fh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \operatorname{Subst}\left(\frac{(2bj(hi-gj)pq)\operatorname{Subst}\left(\int\left(a+b\log\left(cd^qx^{pq}\right)\right)\,dx, x, e+fx\right)}{fh^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \operatorname{Subst}\left(\frac{(2b(hi-gj)^2pq)\operatorname{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{h^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{2abj(hi-gj)pqx}{h^2} + \frac{j(hi-gj)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh^2}$$

$$+ \frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3}$$

$$+ \frac{2b(hi-gj)^2pq(a+b\log\left(c(d(e+fx)^p)^q\right))\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3}$$

$$+ \operatorname{Subst}\left(\frac{j^2\operatorname{Subst}\left(\int x(a+b\log\left(cd^qx^{pq}\right))^2\,dx,x,e+fx\right)}{f^2h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$+ \operatorname{Subst}\left(\frac{(j(fi-ej))\operatorname{Subst}\left(\int (a+b\log\left(cd^qx^{pq}\right))^2\,dx,x,e+fx\right)}{f^2h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$- \operatorname{Subst}\left(\frac{(2b^2j(hi-gj)pq)\operatorname{Subst}\left(\int \log\left(cd^qx^{pq}\right)\,dx,x,e+fx\right)}{fh^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$- \operatorname{Subst}\left(\frac{(2b^2(hi-gj)^2p^2q^2)\operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{h^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= -\frac{2abj(hi-gj)pqx}{h^2} + \frac{2b^2j(hi-gj)p^2q^2x}{h^2} - \frac{2b^2j(hi-gj)pq(e+fx)\log\left(c\left(d(e+fx)^p\right)^q\right)}{fh^2} + \frac{j(fi-ej)(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{f^2h} + \frac{j(hi-gj)(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{fh^2} + \frac{j^2(e+fx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{2f^2h} + \frac{(hi-gj)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} + \frac{2b(hi-gj)^2pq(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} - \frac{2b^2(hi-gj)^2p^2q^2\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} - \operatorname{Subst}\left(\frac{(bj^2pq)\operatorname{Subst}\left(\int x(a+b\log\left(cd^qx^{pq}\right))\,dx,x,e+fx\right)}{f^2h},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right) - \operatorname{Subst}\left(\frac{(2bj(fi-ej)pq)\operatorname{Subst}\left(\int(a+b\log\left(cd^qx^{pq}\right))\,dx,x,e+fx\right)}{f^2h},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right)$$

$$
\begin{aligned}
&= -\frac{2abj(fi-ej)pqx}{fh} - \frac{2abj(hi-gj)pqx}{h^2} + \frac{2b^2j(hi-gj)p^2q^2x}{h^2} \\
&+ \frac{b^2j^2p^2q^2(e+fx)^2}{4f^2h} - \frac{2b^2j(hi-gj)pq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh^2} \\
&- \frac{bj^2pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^2h} \\
&+ \frac{j(fi-ej)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2h} \\
&+ \frac{j(hi-gj)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh^2} \\
&+ \frac{j^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^2h} \\
&+ \frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} \\
&+ \frac{2b(hi-gj)^2pq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} \\
&- \frac{2b^2(hi-gj)^2p^2q^2\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} \\
&- \operatorname{Subst}\left(\frac{(2b^2j(fi-ej)pq)\operatorname{Subst}\left(\int\log\left(cd^qx^{pq}\right)\,dx,x,e+fx\right)}{f^2h},cd^q(e\right. \\
&\qquad \left.+fx)^{pq},c(d(e+fx)^p)^q\right)
\end{aligned}
$$

$$= -\frac{2abj(fi-ej)pqx}{fh} - \frac{2abj(hi-gj)pqx}{h^2} + \frac{2b^2j(fi-ej)p^2q^2x}{fh} + \frac{2b^2j(hi-gj)p^2q^2x}{h^2} + \frac{b^2j^2p^2q^2(e+fx)^2}{4f^2h} - \frac{2b^2j(fi-ej)pq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f^2h} - \frac{2b^2j(hi-gj)pq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh^2} - \frac{bj^2pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{2f^2h} + \frac{j(fi-ej)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2h} + \frac{j(hi-gj)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh^2} + \frac{j^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{2f^2h} + \frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} + \frac{2b(hi-gj)^2pq(a+b\log\left(c(d(e+fx)^p)^q\right))\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} - \frac{2b^2(hi-gj)^2p^2q^2\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3}$$

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 927, normalized size of antiderivative = 1.79

$$\int \frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx$$

$$= \frac{4f^2hj(2hi-gj)x(a-bpq\log(e+fx)+b\log\left(c(d(e+fx)^p)^q\right))^2 + 2f^2h^2j^2x^2(a-bpq\log(e+fx)+b\log}{}$$

```
[In] Integrate[((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(g + h*x),x]

[Out] (4*f^2*h*j*(2*h*i - g*j)*x*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p
)^q])^2 + 2*f^2*h^2*j^2*x^2*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^
p)^q])^2 + 4*f^2*(h*i - g*j)^2*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*
x)^p)^q])^2*Log[g + h*x] - 8*b*f^2*h^2*i^2*p*q*(-a + b*p*q*Log[e + f*x] - b
*Log[c*(d*(e + f*x)^p)^q])*(Log[e + f*x]*Log[(f*(g + h*x))/(f*g - e*h)] + P
olyLog[2, (h*(e + f*x))/(-(f*g) + e*h)]) - 16*b*f*h*i*j*p*q*(-a + b*p*q*Log
[e + f*x] - b*Log[c*(d*(e + f*x)^p)^q])*(-(h*(e + f*x)) + Log[e + f*x]*(e*h
 + f*h*x - f*g*Log[(f*(g + h*x))/(f*g - e*h)]) - f*g*PolyLog[2, (h*(e + f*x
))/(-(f*g) + e*h)]) + 2*b*j^2*p*q*(-a + b*p*q*Log[e + f*x] - b*Log[c*(d*(e
```

```
+ f*x)^p)^q])*(f*h*(f*x*(-4*g + h*x) - 2*e*(2*g + h*x)) + 2*Log[e + f*x]*(h
*(e + f*x)*(2*f*g + e*h - f*h*x) - 2*f^2*g^2*Log[(f*(g + h*x))/(f*g - e*h)]
) - 4*f^2*g^2*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)]) + 8*b^2*f*h*i*j*p^2
*q^2*(h*(2*f*x - 2*(e + f*x)*Log[e + f*x] + (e + f*x)*Log[e + f*x]^2) - f*g
*(Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 2*Log[e + f*x]*PolyLog[2,
 (h*(e + f*x))/(-(f*g) + e*h)] - 2*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)]
)) - b^2*j^2*p^2*q^2*(4*f*g*h*(2*f*x - 2*(e + f*x)*Log[e + f*x] + (e + f*x)
*Log[e + f*x]^2) + h^2*(f*x*(6*e - f*x) + (-6*e^2 - 4*e*f*x + 2*f^2*x^2)*Lo
g[e + f*x] + 2*(e^2 - f^2*x^2)*Log[e + f*x]^2) - 4*f^2*g^2*(Log[e + f*x]^2*
Log[(f*(g + h*x))/(f*g - e*h)] + 2*Log[e + f*x]*PolyLog[2, (h*(e + f*x))/(-
(f*g) + e*h)] - 2*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)])) + 4*b^2*f^2*h^
2*i^2*p^2*q^2*(Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 2*Log[e + f*
x]*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] - 2*PolyLog[3, (h*(e + f*x))/(-
(f*g) + e*h)]))/(4*f^2*h^3)
```

## Maple [F]

$$\int \frac{(jx+i)^2 \left(a+b\ln \left(c(d(fx+e)^p)^q\right)\right)^2}{hx+g}dx$$

```
[In] int((j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

```
[Out] int((j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

## Fricas [F]

$$\int \frac{(i+jx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx = \int \frac{(jx+i)^2(b\log \left(((fx+e)^pd)^q\,c\right)+a)^2}{hx+g}\,dx$$

```
[In] integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="fr
icas")
```

```
[Out] integral((a^2*j^2*x^2 + 2*a^2*i*j*x + a^2*i^2 + (b^2*j^2*x^2 + 2*b^2*i*j*x
+ b^2*i^2)*log(((f*x + e)^p*d)^q*c)^2 + 2*(a*b*j^2*x^2 + 2*a*b*i*j*x + a*b*
i^2)*log(((f*x + e)^p*d)^q*c))/(h*x + g), x)
```

## Sympy [F]

$$\int \frac{(i+jx)^2\,(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{g+hx}\,dx = \int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2(i+jx)^2}{g+hx}\,dx$$

```
[In] integrate((j*x+i)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2*(i + j*x)**2/(g + h*x), x)
```

## Maxima [F]

$$\int \frac{(i+jx)^2\,(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{g+hx}\,dx = \int \frac{(jx+i)^2\left(b\log\left(((fx+e)^pd)^q\,c\right)+a\right)^2}{hx+g}\,dx$$

```
[In] integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="ma
xima")
[Out] 2*a^2*i*j*(x/h - g*log(h*x + g)/h^2) + 1/2*a^2*j^2*(2*g^2*log(h*x + g)/h^3
+ (h*x^2 - 2*g*x)/h^2) + a^2*i^2*log(h*x + g)/h + integrate((2*(i^2*q*log(d
) + i^2*log(c))*a*b + (i^2*q^2*log(d)^2 + 2*i^2*q*log(c)*log(d) + i^2*log(c
)^2)*b^2 + (2*(j^2*q*log(d) + j^2*log(c))*a*b + (j^2*q^2*log(d)^2 + 2*j^2*q
*log(c)*log(d) + j^2*log(c)^2)*b^2)*x^2 + (b^2*j^2*x^2 + 2*b^2*i*j*x + b^2*
i^2)*log(((f*x + e)^p)^q)^2 + 2*(2*(i*j*q*log(d) + i*j*log(c))*a*b + (i*j*q
^2*log(d)^2 + 2*i*j*q*log(c)*log(d) + i*j*log(c)^2)*b^2)*x + 2*(a*b*i^2 + (
i^2*q*log(d) + i^2*log(c))*b^2 + (a*b*j^2 + (j^2*q*log(d) + j^2*log(c))*b^2
)*x^2 + 2*(a*b*i*j + (i*j*q*log(d) + i*j*log(c))*b^2)*x)*log(((f*x + e)^p)^
q))/(h*x + g), x)
```

## Giac [F]

$$\int \frac{(i+jx)^2\,(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{g+hx}\,dx = \int \frac{(jx+i)^2\left(b\log\left(((fx+e)^pd)^q\,c\right)+a\right)^2}{hx+g}\,dx$$

```
[In] integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="gi
ac")
[Out] integrate((j*x + i)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx=\int \frac{{\left(i+j\,x\right)}^2\,{\left(a+b\,\ln \left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^2}{g+h\,x}\,dx$$

```
[In] int(((i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2)/(g + h*x),x)
[Out] int(((i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2)/(g + h*x), x)
```

# 3.531 $\int \frac{(i+jx)(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$



## Optimal result

Integrand size = 33, antiderivative size = 240

$$\begin{aligned}\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx &= -\frac{2abjpqx}{h}+\frac{2b^2jp^2q^2x}{h}-\frac{2b^2jpq(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh}\\&+\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh}\\&+\frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2}\\&+\frac{2b(hi-gj)pq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}\\&-\frac{2b^2(hi-gj)p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}\end{aligned}$$

```
[Out] -2*a*b*j*p*q*x/h+2*b^2*j*p^2*q^2*x/h-2*b^2*j*p*q*(f*x+e)*ln(c*(d*(f*x+e)^p)
^q)/f/h+j*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f/h+(-g*j+h*i)*(a+b*ln(c*(d
*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/h^2+2*b*(-g*j+h*i)*p*q*(a+b*ln(c
*(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h^2-2*b^2*(-g*j+h*i)*p^
2*q^2*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h^2
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2465, 2436, 2333, 2332, 2443, 2481, 2421, 6724, 2495}

$$\int \frac{(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{g+hx}\,dx$$
$$=\frac{2bpq(hi-gj)\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{h^2}$$
$$+\frac{(hi-gj)\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{h^2}$$
$$+\frac{j(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{fh}$$
$$-\frac{2abjpqx}{h}-\frac{2b^2jpq(e+fx)\log\left(c\left(d(e+fx)^p\right)^q\right)}{fh}$$
$$-\frac{2b^2p^2q^2(hi-gj)\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}+\frac{2b^2jp^2q^2x}{h}$$

```
[In] Int[((i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(g + h*x),x]
```

```
[Out] (-2*a*b*j*p*q*x)/h + (2*b^2*j*p^2*q^2*x)/h - (2*b^2*j*p*q*(e + f*x)*Log[c*(
d*(e + f*x)^p)^q])/(f*h) + (j*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)
/(f*h) + ((h*i - g*j)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(g + h*x))/
(f*g - e*h)])/h^2 + (2*b*(h*i - g*j)*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*P
olyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h^2 - (2*b^2*(h*i - g*j)*p^2*q^2*P
olyLog[3, -((h*(e + f*x))/(f*g - e*h))])/h^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
```

```
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(i+jx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\int \left(\frac{j\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{h} + \frac{(hi-gj)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{h(g+hx)}\right)dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{j\int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad + \text{Subst}\left(\frac{(hi-gj)\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{g+hx}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2}\\
&\quad + \text{Subst}\left(\frac{j\text{Subst}\left(\int \left(a+b\log\left(cd^qx^{pq}\right)\right)^2\,dx, x, e+fx\right)}{fh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad - \text{Subst}\left(\frac{(2bf(hi-gj)pq)\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}
$$

$$= \frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh} + \frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} - \text{Subst}\left(\frac{(2bjpq)\text{Subst}\left(\int\left(a+b\log\left(cd^qx^{pq}\right)\right)dx,x,e+fx\right)}{fh},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(2b(hi-gj)pq)\text{Subst}\left(\int\frac{\left(a+b\log\left(cd^qx^{pq}\right)\right)\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}dx,x,e+fx\right)}{h^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= -\frac{2abjpqx}{h} + \frac{j(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{fh} + \frac{(hi-gj)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} + \frac{2b(hi-gj)pq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^2} - \text{Subst}\left(\frac{(2b^2jpq)\,\text{Subst}\left(\int\log\left(cd^qx^{pq}\right)\,dx,x,e+fx\right)}{fh},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right) - \text{Subst}\left(\frac{(2b^2(hi-gj)p^2q^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{h^2},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right)$$

$$= -\frac{2abjpqx}{h} + \frac{2b^2jp^2q^2x}{h} - \frac{2b^2jpq(e+fx)\log\left(c\left(d(e+fx)^p\right)^q\right)}{fh} + \frac{j(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{fh} + \frac{(hi-gj)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} + \frac{2b(hi-gj)pq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\text{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^2} - \frac{2b^2(hi-gj)p^2q^2\text{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^2}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 852 vs. $2(240) = 480$.

Time = 0.23 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.55

$$\int \frac{(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{g+hx}\,dx$$

$$= \frac{-2abehjpq + a^2fhjx - 2abfhjpqx + 2b^2fhjp^2q^2x + 2abehjpq\log(e+fx) - b^2ehjp^2q^2\log^2(e+fx) - 2b^2}{}$$

```
[In] Integrate[((i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(g + h*x),x]
```

```
[Out] (-2*a*b*e*h*j*p*q + a^2*f*h*j*x - 2*a*b*f*h*j*p*q*x + 2*b^2*f*h*j*p^2*q^2*x
 + 2*a*b*e*h*j*p*q*Log[e + f*x] - b^2*e*h*j*p^2*q^2*Log[e + f*x]^2 - 2*b^2*
e*h*j*p*q*Log[c*(d*(e + f*x)^p)^q] + 2*a*b*f*h*j*x*Log[c*(d*(e + f*x)^p)^q]
 - 2*b^2*f*h*j*p*q*x*Log[c*(d*(e + f*x)^p)^q] + 2*b^2*e*h*j*p*q*Log[e + f*x
]*Log[c*(d*(e + f*x)^p)^q] + b^2*f*h*j*x*Log[c*(d*(e + f*x)^p)^q]^2 + a^2*f
*h*i*Log[g + h*x] - a^2*f*g*j*Log[g + h*x] - 2*a*b*f*h*i*p*q*Log[e + f*x]*L
og[g + h*x] + 2*a*b*f*g*j*p*q*Log[e + f*x]*Log[g + h*x] + b^2*f*h*i*p^2*q^2
*Log[e + f*x]^2*Log[g + h*x] - b^2*f*g*j*p^2*q^2*Log[e + f*x]^2*Log[g + h*x
] + 2*a*b*f*h*i*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 2*a*b*f*g*j*Log[c*(
d*(e + f*x)^p)^q]*Log[g + h*x] - 2*b^2*f*h*i*p*q*Log[e + f*x]*Log[c*(d*(e +
 f*x)^p)^q]*Log[g + h*x] + 2*b^2*f*g*j*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^
p)^q]*Log[g + h*x] + b^2*f*h*i*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] - b^
2*f*g*j*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + 2*a*b*f*h*i*p*q*Log[e + f
*x]*Log[(f*(g + h*x))/(f*g - e*h)] - 2*a*b*f*g*j*p*q*Log[e + f*x]*Log[(f*(g
 + h*x))/(f*g - e*h)] - b^2*f*h*i*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/
(f*g - e*h)] + b^2*f*g*j*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*
h)] + 2*b^2*f*h*i*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x
))/(f*g - e*h)] - 2*b^2*f*g*j*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log
[(f*(g + h*x))/(f*g - e*h)] + 2*b*f*(h*i - g*j)*p*q*(a + b*Log[c*(d*(e + f*
x)^p)^q])*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] + 2*b^2*f*(-(h*i) + g*j)
*p^2*q^2*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)])/(f*h^2)
```

## Maple [F]

$$\int \frac{(jx+i)\left(a+b\ln\left(c\left(d(fx+e)^{p}\right)^{q}\right)\right)^{2}}{hx+g}\,dx$$

```
[In] int((j*x+i)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

```
[Out] int((j*x+i)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

## Fricas [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right)\right)^{2}}{g+hx}\,dx=\int \frac{(jx+i)\left(b\log\left(\left((fx+e)^{p}d\right)^{q}c\right)+a\right)^{2}}{hx+g}\,dx$$

```
[In] integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="fric
as")
```

```
[Out] integral((a^2*j*x + a^2*i + (b^2*j*x + b^2*i)*log(((f*x + e)^p*d)^q*c)^2 +
2*(a*b*j*x + a*b*i)*log(((f*x + e)^p*d)^q*c))/(h*x + g), x)
```

## Sympy [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx = \int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2(i+jx)}{g+hx}\,dx$$

```
[In] integrate((j*x+i)*(a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2*(i + j*x)/(g + h*x), x)
```

## Maxima [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx = \int \frac{(jx+i)\left(b\log\left(((fx+e)^pd)^qc\right)+a\right)^2}{hx+g}\,dx$$

```
[In] integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="maxi
ma")
[Out] a^2*j*(x/h - g*log(h*x + g)/h^2) + a^2*i*log(h*x + g)/h + integrate((2*(i*q
*log(d) + i*log(c))*a*b + (i*q^2*log(d)^2 + 2*i*q*log(c)*log(d) + i*log(c)^
2)*b^2 + (b^2*j*x + b^2*i)*log(((f*x + e)^p)^q)^2 + (2*(j*q*log(d) + j*log(
c))*a*b + (j*q^2*log(d)^2 + 2*j*q*log(c)*log(d) + j*log(c)^2)*b^2)*x + 2*((
i*q*log(d) + i*log(c))*b^2 + a*b*i + ((j*q*log(d) + j*log(c))*b^2 + a*b*j)*
x)*log(((f*x + e)^p)^q))/(h*x + g), x)
```

## Giac [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{g+hx}\,dx = \int \frac{(jx+i)\left(b\log\left(((fx+e)^pd)^qc\right)+a\right)^2}{hx+g}\,dx$$

```
[In] integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="giac
")
[Out] integrate((j*x + i)*(b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{g+hx}\,dx=\int \frac{(i+j\,x)\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^2}{g+h\,x}\,dx$$

```
[In] int(((i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2)/(g + h*x),x)
[Out] int(((i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2)/(g + h*x), x)
```

# 3.532 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 123

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{2b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

```
[Out] (a+b*ln(c*(d*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/h+2*b*p*q*(a+b*ln(c*
(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h-2*b^2*p^2*q^2*polylog(
3,-h*(f*x+e)/(-e*h+f*g))/h
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

= {2443, 2481, 2421, 6724, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \frac{2bpq\,\text{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h} + \frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{h} - \frac{2b^2p^2q^2\,\text{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x),x]
```

```
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(g + h*x))/(f*g - e*h)])/h + (2*
b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, -((h*(e + f*x))/(f*g - e*
h))])/h - (2*b^2*p^2*q^2*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))])/h
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
```

```
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} - \text{Subst}\left(\frac{(2bfpq)\int \frac{(a+b\log(cd^q(e+fx)^{pq}))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} - \text{Subst}\left(\frac{(2bpq)\text{Subst}\left(\int \frac{(a+b\log(cd^q x^{pq}))\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \operatorname{Subst}\left(\frac{(2b^2p^2q^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{2b^2p^2q^2\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 324 vs. 2(123) = 246.

Time = 0.05 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.63

$$\int\frac{(a+b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx$$

$$= \frac{a^2\log(g+hx)-2abpq\log(e+fx)\log(g+hx)+b^2p^2q^2\log^2(e+fx)\log(g+hx)+2ab\log(c(d(e+fx)^p}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/(g + h*x),x]
```

```
[Out] (a^2*Log[g + h*x] - 2*a*b*p*q*Log[e + f*x]*Log[g + h*x] + b^2*p^2*q^2*Log[e
 + f*x]^2*Log[g + h*x] + 2*a*b*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 2*b^
2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + b^2*Log[c*(d*(e
+ f*x)^p)^q]^2*Log[g + h*x] + 2*a*b*p*q*Log[e + f*x]*Log[(f*(g + h*x))/(f*g
 - e*h)] - b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 2*b^
2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)]
+ 2*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, (h*(e + f*x))/(-(f*g)
 + e*h)] - 2*b^2*p^2*q^2*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)])/h
```

## Maple [F]

$$\int \frac{(a + b \ln{(c(d(fx + e)^{p})^{q})})^{2}}{hx + g}\,dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x)
```

## Fricas [F]

$$\int \frac{(a + b\log{(c(d(e + fx)^{p})^{q})})^{2}}{g + hx}\,dx = \int \frac{(b\log{(((fx + e)^{p}d)^{q}\,c)} + a)^{2}}{hx + g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c) +
 a^2)/(h*x + g), x)
```

## Sympy [F]

$$\int \frac{(a + b\log{(c(d(e + fx)^{p})^{q})})^{2}}{g + hx}\,dx = \int \frac{(a + b\log{(c(d(e + fx)^{p})^{q})})^{2}}{g + hx}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g),x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/(g + h*x), x)
```

## Maxima [F]

$$\int \frac{(a + b\log{(c(d(e + fx)^{p})^{q})})^{2}}{g + hx}\,dx = \int \frac{(b\log{(((fx + e)^{p}d)^{q}\,c)} + a)^{2}}{hx + g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="maxima")
```

```
[Out] a^2*log(h*x + g)/h + integrate((b^2*log(((f*x + e)^p)^q)^2 + 2*(q*log(d) +
log(c))*a*b + (q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*b^2 + 2*((q*log
(d) + log(c))*b^2 + a*b)*log(((f*x + e)^p)^q))/(h*x + g), x)
```

## Giac [F]

$$\int \frac{(a + b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^p d)^q\,c) + a)^2}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g),x, algorithm="giac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(c(d(e+fx)^p)^q))^2}{g+hx}\,dx = \int \frac{(a + b\,\ln(c\,(d\,(e+f\,x)^p)^q))^2}{g+h\,x}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/(g + h*x), x)
```

# 3.533 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx$



## Optimal result

Integrand size = 35, antiderivative size = 288

$$\begin{aligned}\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx &= \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} \\ &+ \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} \\ &- \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj} \\ &- \frac{2b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} + \frac{2b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}\end{aligned}$$

```
[Out] (a+b*ln(c*(d*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/(-g*j+h*i)-(a+b*ln(c
*(d*(f*x+e)^p)^q))^2*ln(f*(j*x+i)/(-e*j+f*i))/(-g*j+h*i)+2*b*p*q*(a+b*ln(c*
(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*i)-2*b*p*q*(a+b*
ln(c*(d*(f*x+e)^p)^q))*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)-2*b^2*p^
2*q^2*polylog(3,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*i)+2*b^2*p^2*q^2*polylog(3,-
j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2465, 2443, 2481, 2421, 6724, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx$$
$$=\frac{2bpq\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{hi-gj}$$
$$-\frac{2bpq\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))}{hi-gj}$$
$$+\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{hi-gj}-\frac{\log\left(\frac{f(i+jx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{hi-gj}$$
$$-\frac{2b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj}+\frac{2b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/((g + h*x)*(i + j*x)),x]
```

```
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(g + h*x))/(f*g - e*h)])/(h*i -
g*j) - ((a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(i + j*x))/(f*i - e*j)])/
(h*i - g*j) + (2*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, -((h*(e
+ f*x))/(f*g - e*h))])/(h*i - g*j) - (2*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^
q])*PolyLog[2, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j) - (2*b^2*p^2*q^2*
PolyLog[3, -((h*(e + f*x))/(f*g - e*h))])/(h*i - g*j) + (2*b^2*p^2*q^2*Poly
Log[3, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j)
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{(g+hx)(i+jx)}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\int \left(\frac{h\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{(hi-gj)(g+hx)} - \frac{j\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{(hi-gj)(i+jx)}\right)dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{h\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{g+hx}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad - \text{Subst}\left(\frac{j\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2}{i+jx}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} - \text{Subst}\left(\frac{(2bfpq)\int\frac{(a+b\log(cd^q(e+fx)^{pq}))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(2bfpq)\int\frac{(a+b\log(cd^q(e+fx)^{pq}))\log\left(\frac{f(i+jx)}{fi-ej}\right)}{e+fx}dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} - \text{Subst}\left(\frac{(2bpq)\text{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}dx, x, e+fx\right)}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(2bpq)\text{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\log\left(\frac{f\left(\frac{fi-ej}{f}+\frac{jx}{f}\right)}{fi-ej}\right)}{x}dx, x, e+fx\right)}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} + \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} - \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj} - \operatorname{Subst}\left(\frac{(2b^2p^2q^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{hi-gj},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) + \operatorname{Subst}\left(\frac{(2b^2p^2q^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{jx}{fi-ej}\right)}{x}\,dx,x,e+fx\right)}{hi-gj},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} + \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} - \frac{2bpq(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj} - \frac{2b^2p^2q^2\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} + \frac{2b^2p^2q^2\operatorname{Li}_3\left(-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 652 vs. 2(288) = 576.

Time = 0.18 (sec) , antiderivative size = 652, normalized size of antiderivative = 2.26

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx$$
$$= \frac{a^2\log(g+hx)-2abpq\log(e+fx)\log(g+hx)+b^2p^2q^2\log^2(e+fx)\log(g+hx)+2ab\log(c(d(e+fx)^p}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/((g + h*x)*(i + j*x)),x]
```

```
[Out] (a^2*Log[g + h*x] - 2*a*b*p*q*Log[e + f*x]*Log[g + h*x] + b^2*p^2*q^2*Log[e
 + f*x]^2*Log[g + h*x] + 2*a*b*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 2*b^
2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + b^2*Log[c*(d*(e
+ f*x)^p)^q]^2*Log[g + h*x] + 2*a*b*p*q*Log[e + f*x]*Log[(f*(g + h*x))/(f*g
 - e*h)] - b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 2*b^
2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)]
- a^2*Log[i + j*x] + 2*a*b*p*q*Log[e + f*x]*Log[i + j*x] - b^2*p^2*q^2*Log[
e + f*x]^2*Log[i + j*x] - 2*a*b*Log[c*(d*(e + f*x)^p)^q]*Log[i + j*x] + 2*b
^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[i + j*x] - b^2*Log[c*(d*(e
 + f*x)^p)^q]^2*Log[i + j*x] - 2*a*b*p*q*Log[e + f*x]*Log[(f*(i + j*x))/(f*
i - e*j)] + b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(i + j*x))/(f*i - e*j)] - 2*b
^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(i + j*x))/(f*i - e*j)]
 + 2*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, (h*(e + f*x))/(-(f*g
) + e*h)] - 2*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, (j*(e + f*x
))/(-(f*i) + e*j)] - 2*b^2*p^2*q^2*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)]
 + 2*b^2*p^2*q^2*PolyLog[3, (j*(e + f*x))/(-(f*i) + e*j)])/(h*i - g*j)
```

## Maple [F]

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^2}{(hx+g)(jx+i)}\,dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i),x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)(jx+i)}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i),x, algorithm="fric
as")
```

```
[Out] integral((b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c) +
 a^2)/(h*j*x^2 + g*i + (h*i + g*j)*x), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)/(j*x+i),x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/((g + h*x)*(i + j*x)), x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)(jx+i)}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i),x, algorithm="maxi
ma")
```

```
[Out] a^2*(log(h*x + g)/(h*i - g*j) - log(j*x + i)/(h*i - g*j)) + integrate((b^2*
log(((f*x + e)^p)^q)^2 + 2*(q*log(d) + log(c))*a*b + (q^2*log(d)^2 + 2*q*lo
g(c)*log(d) + log(c)^2)*b^2 + 2*((q*log(d) + log(c))*b^2 + a*b)*log(((f*x +
 e)^p)^q))/(h*j*x^2 + g*i + (h*i + g*j)*x), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)(jx+i)}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i),x, algorithm="giac
")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/((h*x + g)*(j*x + i)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)}\,dx = \int \frac{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^2}{(g+h\,x)\,(i+j\,x)}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/((g + h*x)*(i + j*x)),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/((g + h*x)*(i + j*x)), x)
```

# 3.534 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx$



## Optimal result

Integrand size = 35, antiderivative size = 463

$$\begin{aligned}
&\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx \\
&= -\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(fi-ej)(hi-gj)(i+jx)} + \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2} \\
&\quad + \frac{2bfpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} \\
&\quad - \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2} \\
&\quad + \frac{2bhpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} \\
&\quad + \frac{2b^2fp^2q^2\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} \\
&\quad - \frac{2bhpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2} \\
&\quad - \frac{2b^2hp^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} + \frac{2b^2hp^2q^2\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}
\end{aligned}$$

```
[Out] -j*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/(-e*j+f*i)/(-g*j+h*i)/(j*x+i)+h*(a
+b*ln(c*(d*(f*x+e)^p)^q))^2*ln(f*(h*x+g)/(-e*h+f*g))/(-g*j+h*i)^2+2*b*f*p*q
*(a+b*ln(c*(d*(f*x+e)^p)^q))*ln(f*(j*x+i)/(-e*j+f*i))/(-e*j+f*i)/(-g*j+h*i)
-h*(a+b*ln(c*(d*(f*x+e)^p)^q))^2*ln(f*(j*x+i)/(-e*j+f*i))/(-g*j+h*i)^2+2*b*
```

```
h*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(2,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*
i)^2+2*b^2*f*p^2*q^2*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-e*j+f*i)/(-g*j+h*i)
-2*b*h*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-g
*j+h*i)^2-2*b^2*h*p^2*q^2*polylog(3,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*i)^2+2*b
^2*h*p^2*q^2*polylog(3,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)^2
```

## Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2465, 2443, 2481, 2421, 6724, 2444, 2441, 2440, 2438, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx$$
$$= \frac{2bhpq\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{(hi-gj)^2} - \frac{2bhpq\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))}{(hi-gj)^2} + \frac{2bfpq\log\left(\frac{f(i+jx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))}{(fi-ej)(hi-gj)} - \frac{j(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{(i+jx)(fi-ej)(hi-gj)} + \frac{h\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{(hi-gj)^2} - \frac{h\log\left(\frac{f(i+jx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{(hi-gj)^2} + \frac{2b^2fp^2q^2\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} - \frac{2b^2hp^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} + \frac{2b^2hp^2q^2\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/((g + h*x)*(i + j*x)^2),x]
```

```
[Out] -((j*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/((f*i - e*j)*(h*i - g*j)
*(i + j*x))) + (h*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[(f*(g + h*x))/(f*g
 - e*h)])/(h*i - g*j)^2 + (2*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*Log[(
f*(i + j*x))/(f*i - e*j)])/((f*i - e*j)*(h*i - g*j)) - (h*(a + b*Log[c*(d*(
e + f*x)^p)^q])^2*Log[(f*(i + j*x))/(f*i - e*j)])/(h*i - g*j)^2 + (2*b*h*p*
q*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[2, -((h*(e + f*x))/(f*g - e*h))]
)/(h*i - g*j)^2 + (2*b^2*f*p^2*q^2*PolyLog[2, -((j*(e + f*x))/(f*i - e*j))]
)/((f*i - e*j)*(h*i - g*j)) - (2*b*h*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])*P
olyLog[2, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j)^2 - (2*b^2*h*p^2*q^2*P
olyLog[3, -((h*(e + f*x))/(f*g - e*h))])/(h*i - g*j)^2 + (2*b^2*h*p^2*q^2*P
olyLog[3, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j)^2
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
```

```
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{(g+hx)(i+jx)^2}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\int \left(\frac{h^2(a+b\log(cd^q(e+fx)^{pq}))^2}{(hi-gj)^2(g+hx)} - \frac{j(a+b\log(cd^q(e+fx)^{pq}))^2}{(hi-gj)(i+jx)^2} \right.\right.\\
&\qquad \left.\left. - \frac{hj(a+b\log(cd^q(e+fx)^{pq}))^2}{(hi-gj)^2(i+jx)}\right) dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{h^2\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{g+hx}\,dx}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad - \text{Subst}\left(\frac{(hj)\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{i+jx}\,dx}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad - \text{Subst}\left(\frac{j\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{(i+jx)^2}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$= -\frac{j(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{(fi-ej)(hi-gj)(i+jx)} + \frac{h\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2} - \frac{h\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2} - \text{Subst}\left(\frac{(2bfhpq)\int\frac{(a+b\log(cd^q(e+fx)^{pq}))\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right) + \text{Subst}\left(\frac{(2bfhpq)\int\frac{(a+b\log(cd^q(e+fx)^{pq}))\log\left(\frac{f(i+jx)}{fi-ej}\right)}{e+fx}\,dx}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right) + \text{Subst}\left(\frac{(2bfjpq)\int\frac{a+b\log(cd^q(e+fx)^{pq})}{i+jx}\,dx}{(fi-ej)(hi-gj)}, cd^q(e+fx)^{pq}, c\left(d(e+fx)^p\right)^q\right)$$

$$= -\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(fi-ej)(hi-gj)(i+jx)} + \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2} + \frac{2bfpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} - \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2} - \text{Subst}\left(\frac{(2bhpq)\text{Subst}\left(\int \frac{(a+b\log(cd^qx^{pq}))\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}dx, x, e+fx\right)}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(2bhpq)\text{Subst}\left(\int \frac{(a+b\log(cd^qx^{pq}))\log\left(\frac{f\left(\frac{fi-ej}{f}+\frac{jx}{f}\right)}{fi-ej}\right)}{x}dx, x, e+fx\right)}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(2b^2f^2p^2q^2)\int \frac{\log\left(\frac{f(i+jx)}{fi-ej}\right)}{e+fx}dx}{(fi-ej)(hi-gj)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$
\begin{aligned}
&= -\frac{j(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{(fi-ej)(hi-gj)(i+jx)} \\
&+ \frac{h\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2} \\
&+ \frac{2bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} \\
&- \frac{h\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2} \\
&+ \frac{2bhpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} \\
&- \frac{2bhpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\operatorname{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2} \\
&- \operatorname{Subst}\left(\frac{(2b^2hp^2q^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{(hi-gj)^2},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right) \\
&+ \operatorname{Subst}\left(\frac{(2b^2hp^2q^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{jx}{fi-ej}\right)}{x}\,dx,x,e+fx\right)}{(hi-gj)^2},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right) \\
&- \operatorname{Subst}\left(\frac{(2b^2fp^2q^2)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{jx}{fi-ej}\right)}{x}\,dx,x,e+fx\right)}{(fi-ej)(hi-gj)},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(fi-ej)(hi-gj)(i+jx)} \\
&+ \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2} \\
&+ \frac{2bfpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} \\
&- \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2} \\
&+ \frac{2bhpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} \\
&+ \frac{2b^2fp^2q^2\operatorname{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} - \frac{2bhpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2} \\
&- \frac{2b^2hp^2q^2\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} + \frac{2b^2hp^2q^2\operatorname{Li}_3\left(-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.41

$$
\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{(g+hx)(i+jx)^2}\,dx
= \frac{(fi-ej)(hi-gj)\left(a-bpq\log(e+fx)+b\log\left(c(d(e+fx)^p)^q\right)\right)^2+h(fi-ej)(i+jx)\left(a-bpq\log(e+f\right.}{}
$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^2/((g + h*x)*(i + j*x)^2),x]
```

```
[Out] ((f*i - e*j)*(h*i - g*j)*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^
q])^2 + h*(f*i - e*j)*(i + j*x)*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f
*x)^p)^q])^2*Log[g + h*x] - h*(f*i - e*j)*(i + j*x)*(a - b*p*q*Log[e + f*x]
 + b*Log[c*(d*(e + f*x)^p)^q])^2*Log[i + j*x] - 2*b*p*q*(a - b*p*q*Log[e +
f*x] + b*Log[c*(d*(e + f*x)^p)^q])*((h*i - g*j)*(j*(e + f*x)*Log[e + f*x] -
 f*(i + j*x)*Log[i + j*x]) - h*(f*i - e*j)*(i + j*x)*(Log[e + f*x]*Log[(f*(
g + h*x))/(f*g - e*h)] + PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)]) + h*(f*i
 - e*j)*(i + j*x)*(Log[e + f*x]*Log[(f*(i + j*x))/(f*i - e*j)] + PolyLog[2,
 (j*(e + f*x))/(-(f*i) + e*j)])) - b^2*p^2*q^2*((h*i - g*j)*(Log[e + f*x]*(
j*(e + f*x)*Log[e + f*x] - 2*f*(i + j*x)*Log[(f*(i + j*x))/(f*i - e*j)]) -
2*f*(i + j*x)*PolyLog[2, (j*(e + f*x))/(-(f*i) + e*j)]) - h*(f*i - e*j)*(i
+ j*x)*(Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 2*Log[e + f*x]*Poly
Log[2, (h*(e + f*x))/(-(f*g) + e*h)] - 2*PolyLog[3, (h*(e + f*x))/(-(f*g) +
```

```
 e*h)]) + h*(f*i - e*j)*(i + j*x)*(Log[e + f*x]^2*Log[(f*(i + j*x))/(f*i -
e*j)] + 2*Log[e + f*x]*PolyLog[2, (j*(e + f*x))/(-(f*i) + e*j)] - 2*PolyLog
[3, (j*(e + f*x))/(-(f*i) + e*j)])))/((f*i - e*j)*(h*i - g*j)^2*(i + j*x))
```

## Maple [F]

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^2}{(hx+g)(jx+i)^2}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i)^2,x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i)^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)(jx+i)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i)^2,x, algorithm="fr
icas")
```

```
[Out] integral((b^2*log(((f*x + e)^p*d)^q*c)^2 + 2*a*b*log(((f*x + e)^p*d)^q*c) +
 a^2)/(h*j^2*x^3 + g*i^2 + (2*h*i*j + g*j^2)*x^2 + (h*i^2 + 2*g*i*j)*x), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**2/(h*x+g)/(j*x+i)**2,x)
```

```
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**2/((g + h*x)*(i + j*x)**2), x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^2}{(g+hx)(i+jx)^2}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^2}{(hx+g)(jx+i)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i)^2,x, algorithm="ma
xima")
```

```
[Out] a^2*(h*log(h*x + g)/(h^2*i^2 - 2*g*h*i*j + g^2*j^2) - h*log(j*x + i)/(h^2*i
^2 - 2*g*h*i*j + g^2*j^2) + 1/(h*i^2 - g*i*j + (h*i*j - g*j^2)*x)) + integr
ate((b^2*log(((f*x + e)^p)^q)^2 + 2*(q*log(d) + log(c))*a*b + (q^2*log(d)^2
 + 2*q*log(c)*log(d) + log(c)^2)*b^2 + 2*((q*log(d) + log(c))*b^2 + a*b)*lo
g(((f*x + e)^p)^q))/(h*j^2*x^3 + g*i^2 + (2*h*i*j + g*j^2)*x^2 + (h*i^2 + 2
*g*i*j)*x), x)
```

## Giac [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{(g+hx)(i+jx)^2}\,dx=\int \frac{\left(b\log\left(((fx+e)^p d)^q c\right)+a\right)^2}{(hx+g)(jx+i)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^2/(h*x+g)/(j*x+i)^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^2/((h*x + g)*(j*x + i)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log\left(c(d(e+fx)^p)^q\right))^2}{(g+hx)(i+jx)^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^2}{\left(g+h\,x\right)\,{\left(i+j\,x\right)}^2}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/((g + h*x)*(i + j*x)^2),x)
```

```
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^2/((g + h*x)*(i + j*x)^2), x)
```

# 3.535 $\int \frac{(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$

## Optimal result

Integrand size = 35, antiderivative size = 742

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \frac{6ab^2j(fi-ej)p^2q^2x}{fh}+\frac{6ab^2j(hi-gj)p^2q^2x}{h^2}-\frac{6b^3j(fi-ej)p^3q^3x}{fh}-\frac{6b^3j(hi-gj)p^3q^3x}{h^2} - \frac{3b^3j^2p^3q^3(e+fx)^2}{8f^2h}+\frac{6b^3j(fi-ej)p^2q^2(e+fx)\log(c(d(e+fx)^p)^q)}{f^2h} + \frac{6b^3j(hi-gj)p^2q^2(e+fx)\log(c(d(e+fx)^p)^q)}{fh^2} + \frac{3b^2j^2p^2q^2(e+fx)^2(a+b\log(c(d(e+fx)^p)^q))}{4f^2h} - \frac{3bj(fi-ej)pq(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{f^2h} - \frac{3bj(hi-gj)pq(e+fx)(a+b\log(c(d(e+fx)^p)^q))^2}{fh^2} - \frac{3bj^2pq(e+fx)^2(a+b\log(c(d(e+fx)^p)^q))^2}{4f^2h} + \frac{j(fi-ej)(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3}{f^2h} + \frac{j(hi-gj)(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3}{fh^2} + \frac{j^2(e+fx)^2(a+b\log(c(d(e+fx)^p)^q))^3}{2f^2h} + \frac{(hi-gj)^2(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} + \frac{3b(hi-gj)^2pq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^3} - \frac{6b^2(hi-gj)^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h^3} + \frac{6b^3(hi-gj)^2p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h^3}$$

```
[Out] 6*a*b^2*j*(-e*j+f*i)*p^2*q^2*x/f/h+6*a*b^2*j*(-g*j+h*i)*p^2*q^2*x/h^2-6*b^3
*j*(-e*j+f*i)*p^3*q^3*x/f/h-6*b^3*j*(-g*j+h*i)*p^3*q^3*x/h^2-3/8*b^3*j^2*p^
3*q^3*(f*x+e)^2/f^2/h+6*b^3*j*(-e*j+f*i)*p^2*q^2*(f*x+e)*ln(c*(d*(f*x+e)^p)
^q)/f^2/h+6*b^3*j*(-g*j+h*i)*p^2*q^2*(f*x+e)*ln(c*(d*(f*x+e)^p)^q)/f/h^2+3/
4*b^2*j^2*p^2*q^2*(f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))/f^2/h-3*b*j*(-e*j+f
```

```
*i)*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f^2/h-3*b*j*(-g*j+h*i)*p*q*(f
*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f/h^2-3/4*b*j^2*p*q*(f*x+e)^2*(a+b*ln(c
*(d*(f*x+e)^p)^q))^2/f^2/h+j*(-e*j+f*i)*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))
^3/f^2/h+j*(-g*j+h*i)*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f/h^2+1/2*j^2*(
f*x+e)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f^2/h+(-g*j+h*i)^2*(a+b*ln(c*(d*(f*x
+e)^p)^q))^3*ln(f*(h*x+g)/(-e*h+f*g))/h^3+3*b*(-g*j+h*i)^2*p*q*(a+b*ln(c*(d
*(f*x+e)^p)^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h^3-6*b^2*(-g*j+h*i)^2*p
^2*q^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h^3+6*b
^3*(-g*j+h*i)^2*p^3*q^3*polylog(4,-h*(f*x+e)/(-e*h+f*g))/h^3
```

## Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules

used = {2465, 2436, 2333, 2332, 2443, 2481, 2421, 2430, 6724, 2448, 2437, 2342, 2341, 2495}

$$\int \frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx$$
$$= \frac{3b^2j^2p^2q^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{4f^2h}$$
$$-\frac{6b^2p^2q^2(hi-gj)^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h^3}$$
$$+\frac{6ab^2jp^2q^2x(fi-ej)}{fh}+\frac{6ab^2jp^2q^2x(hi-gj)}{h^2}$$
$$-\frac{3bjpq(e+fx)(fi-ej)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2h}$$
$$+\frac{j(e+fx)(fi-ej)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^2h}$$
$$-\frac{3bj^2pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4f^2h}$$
$$+\frac{j^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{2f^2h}$$
$$+\frac{3bpq(hi-gj)^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h^3}$$
$$+\frac{(hi-gj)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{h^3}$$
$$-\frac{3bjpq(e+fx)(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh^2}$$
$$+\frac{j(e+fx)(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{fh^2}$$
$$+\frac{6b^3jp^2q^2(e+fx)(fi-ej)\log\left(c(d(e+fx)^p)^q\right)}{f^2h}$$
$$+\frac{6b^3jp^2q^2(e+fx)(hi-gj)\log\left(c(d(e+fx)^p)^q\right)}{fh^2}-\frac{3b^3j^2p^3q^3(e+fx)^2}{8f^2h}$$
$$+\frac{6b^3p^3q^3(hi-gj)^2\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h^3}-\frac{6b^3jp^3q^3x(fi-ej)}{fh}-\frac{6b^3jp^3q^3x(hi-gj)}{h^2}$$

```
[In] Int[((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(g + h*x),x]
```

```
[Out] (6*a*b^2*j*(f*i - e*j)*p^2*q^2*x)/(f*h) + (6*a*b^2*j*(h*i - g*j)*p^2*q^2*x)
/h^2 - (6*b^3*j*(f*i - e*j)*p^3*q^3*x)/(f*h) - (6*b^3*j*(h*i - g*j)*p^3*q^3
*x)/h^2 - (3*b^3*j^2*p^3*q^3*(e + f*x)^2)/(8*f^2*h) + (6*b^3*j*(f*i - e*j)*
p^2*q^2*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/(f^2*h) + (6*b^3*j*(h*i - g*j)*
p^2*q^2*(e + f*x)*Log[c*(d*(e + f*x)^p)^q])/(f*h^2) + (3*b^2*j^2*p^2*q^2*(e
```

```
 + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q]))/(4*f^2*h) - (3*b*j*(f*i - e*j)*
p*q*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(f^2*h) - (3*b*j*(h*i - g
*j)*p*q*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(f*h^2) - (3*b*j^2*p*
q*(e + f*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2)/(4*f^2*h) + (j*(f*i - e*j
)*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(f^2*h) + (j*(h*i - g*j)*(e
 + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(f*h^2) + (j^2*(e + f*x)^2*(a +
 b*Log[c*(d*(e + f*x)^p)^q])^3)/(2*f^2*h) + ((h*i - g*j)^2*(a + b*Log[c*(d*
(e + f*x)^p)^q])^3*Log[(f*(g + h*x))/(f*g - e*h)])/h^3 + (3*b*(h*i - g*j)^2
*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, -((h*(e + f*x))/(f*g - e
*h))])/h^3 - (6*b^2*(h*i - g*j)^2*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*
PolyLog[3, -((h*(e + f*x))/(f*g - e*h))])/h^3 + (6*b^3*(h*i - g*j)^2*p^3*q^
3*PolyLog[4, -((h*(e + f*x))/(f*g - e*h))])/h^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(i+jx)^2 \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\int \left(\frac{j(hi-gj)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{h^2}+\frac{(hi-gj)^2\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{h^2(g+hx)}\right.\right.\\
&\qquad\left.\left.+\frac{j(i+jx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{h}\right)\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&= \text{Subst}\left(\frac{j\int (i+jx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad+\text{Subst}\left(\frac{(j(hi-gj))\int \left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3\,dx}{h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)\\
&\quad+\text{Subst}\left(\frac{(hi-gj)^2\int \frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{g+hx}\,dx}{h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$= \frac{(hi-gj)^2 \left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3 \log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} + \text{Subst}\left(\frac{j\int\left(\frac{(fi-ej)(a+b\log(cd^q(e+fx)^{pq}))^3}{f}+\frac{j(e+fx)(a+b\log(cd^q(e+fx)^{pq}))^3}{f}\right)dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(j(hi-gj))\text{Subst}\left(\int\left(a+b\log\left(cd^q x^{pq}\right)\right)^3 dx, x, e+fx\right)}{fh^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(3bf(hi-gj)^2pq)\int\frac{(a+b\log(cd^q(e+fx)^{pq}))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{h^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{j(hi-gj)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{fh^2}$$

$$+ \frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3}$$

$$+ \text{Subst}\left(\frac{j^2\int(e+fx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3\,dx}{fh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$+ \text{Subst}\left(\frac{(j(fi-ej))\int\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3\,dx}{fh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$- \text{Subst}\left(\frac{(3bj(hi-gj)pq)\text{Subst}\left(\int\left(a+b\log\left(cd^qx^{pq}\right)\right)^2\,dx, x, e+fx\right)}{fh^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$- \text{Subst}\left(\frac{(3b(hi-gj)^2pq)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(cd^qx^{pq}\right)\right)^2\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{h^3}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{3bj(hi-gj)pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh^2} + \frac{j(hi-gj)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{fh^2} + \frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} + \frac{3b(hi-gj)^2pq(a+b\log\left(c(d(e+fx)^p)^q\right))^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} + \operatorname{Subst}\left(\frac{j^2\operatorname{Subst}\left(\int x(a+b\log\left(cd^qx^{pq}\right))^3\,dx,x,e+fx\right)}{f^2h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) + \operatorname{Subst}\left(\frac{(j(fi-ej))\operatorname{Subst}\left(\int\left(a+b\log\left(cd^qx^{pq}\right)\right)^3\,dx,x,e+fx\right)}{f^2h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) + \operatorname{Subst}\left(\frac{(6b^2j(hi-gj)p^2q^2)\operatorname{Subst}\left(\int\left(a+b\log\left(cd^qx^{pq}\right)\right)\,dx,x,e+fx\right)}{fh^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) - \operatorname{Subst}\left(\frac{(6b^2(hi-gj)^2p^2q^2)\operatorname{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\operatorname{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{h^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{6ab^2j(hi-gj)p^2q^2x}{h^2} - \frac{3bj(hi-gj)pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh^2}$$
$$+ \frac{j(fi-ej)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^2h}$$
$$+ \frac{j(hi-gj)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{fh^2}$$
$$+ \frac{j^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{2f^2h}$$
$$+ \frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3}$$
$$+ \frac{3b(hi-gj)^2pq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3}$$
$$- \frac{6b^2(hi-gj)^2p^2q^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3}$$
$$- \operatorname{Subst}\left(\frac{(3bj^2pq)\operatorname{Subst}\left(\int x\left(a+b\log\left(cd^qx^{pq}\right)\right)^2\,dx,x,e+fx\right)}{2f^2h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$- \operatorname{Subst}\left(\frac{(3bj(fi-ej)pq)\operatorname{Subst}\left(\int\left(a+b\log\left(cd^qx^{pq}\right)\right)^2\,dx,x,e+fx\right)}{f^2h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$+ \operatorname{Subst}\left(\frac{(6b^3j(hi-gj)p^2q^2)\operatorname{Subst}\left(\int\log\left(cd^qx^{pq}\right)\,dx,x,e+fx\right)}{fh^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$+ \operatorname{Subst}\left(\frac{(6b^3(hi-gj)^2p^3q^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{h^3},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$
\begin{aligned}
&= \frac{6ab^2j(hi-gj)p^2q^2x}{h^2} - \frac{6b^3j(hi-gj)p^3q^3x}{h^2} \\
&\quad + \frac{6b^3j(hi-gj)p^2q^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh^2} \\
&\quad - \frac{3bj(fi-ej)pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2h} \\
&\quad - \frac{3bj(hi-gj)pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh^2} \\
&\quad - \frac{3bj^2pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4f^2h} \\
&\quad + \frac{j(fi-ej)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^2h} \\
&\quad + \frac{j(hi-gj)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{fh^2} \\
&\quad + \frac{j^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{2f^2h} \\
&\quad + \frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} \\
&\quad + \frac{3b(hi-gj)^2pq(a+b\log\left(c(d(e+fx)^p)^q\right))^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} \\
&\quad - \frac{6b^2(hi-gj)^2p^2q^2(a+b\log\left(c(d(e+fx)^p)^q\right))\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} \\
&\quad + \frac{6b^3(hi-gj)^2p^3q^3\operatorname{Li}_4\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} \\
&\quad + \operatorname{Subst}\left(\frac{(3b^2j^2p^2q^2)\operatorname{Subst}\left(\int x(a+b\log\left(cd^qx^{pq}\right))\,dx,x,e+fx\right)}{2f^2h},cd^q(e\right. \\
&\qquad \left. +fx)^{pq},c(d(e+fx)^p)^q\right) \\
&\quad + \operatorname{Subst}\left(\frac{(6b^2j(fi-ej)p^2q^2)\operatorname{Subst}\left(\int (a+b\log\left(cd^qx^{pq}\right))\,dx,x,e+fx\right)}{f^2h},cd^q(e\right. \\
&\qquad \left. +fx)^{pq},c(d(e+fx)^p)^q\right)
\end{aligned}
$$

$$= \frac{6ab^2j(fi-ej)p^2q^2x}{fh} + \frac{6ab^2j(hi-gj)p^2q^2x}{h^2} - \frac{6b^3j(hi-gj)p^3q^3x}{h^2} - \frac{3b^3j^2p^3q^3(e+fx)^2}{8f^2h} + \frac{6b^3j(hi-gj)p^2q^2(e+fx)\log\left(c\left(d(e+fx)^p\right)^q\right)}{fh^2} + \frac{3b^2j^2p^2q^2(e+fx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}{4f^2h} - \frac{3bj(fi-ej)pq(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{f^2h} - \frac{3bj(hi-gj)pq(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{fh^2} - \frac{3bj^2pq(e+fx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{4f^2h} + \frac{j(fi-ej)(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{f^2h} + \frac{j(hi-gj)(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{fh^2} + \frac{j^2(e+fx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{2f^2h} + \frac{(hi-gj)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} + \frac{3b(hi-gj)^2pq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} - \frac{6b^2(hi-gj)^2p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} + \frac{6b^3(hi-gj)^2p^3q^3\operatorname{Li}_4\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} + \operatorname{Subst}\left(\frac{(6b^3j(fi-ej)p^2q^2)\operatorname{Subst}\left(\int\log\left(cd^qx^{pq}\right)\,dx,x,e+fx\right)}{f^2h},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right)$$

$$= \frac{6ab^2j(fi-ej)p^2q^2x}{fh} + \frac{6ab^2j(hi-gj)p^2q^2x}{h^2} - \frac{6b^3j(fi-ej)p^3q^3x}{fh} - \frac{6b^3j(hi-gj)p^3q^3x}{h^2} - \frac{3b^3j^2p^3q^3(e+fx)^2}{8f^2h} + \frac{6b^3j(fi-ej)p^2q^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{f^2h} + \frac{6b^3j(hi-gj)p^2q^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh^2} + \frac{3b^2j^2p^2q^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{4f^2h} - \frac{3bj(fi-ej)pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{f^2h} - \frac{3bj(hi-gj)pq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh^2} - \frac{3bj^2pq(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{4f^2h} + \frac{j(fi-ej)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{f^2h} + \frac{j(hi-gj)(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{fh^2} + \frac{j^2(e+fx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{2f^2h} + \frac{(hi-gj)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^3} + \frac{3b(hi-gj)^2pq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} - \frac{6b^2(hi-gj)^2p^2q^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3} + \frac{6b^3(hi-gj)^2p^3q^3\operatorname{Li}_4\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^3}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4056 vs. 2(742) = 1484.

Time = 0.89 (sec) , antiderivative size = 4056, normalized size of antiderivative = 5.47

$$\int \frac{(i+jx)^2 (a+b\log (c(d(e+fx)^p)^q))^3}{g+hx} dx = \text{Result too large to show}$$

```
[In] Integrate[((i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(g + h*x),x]
[Out] (-48*a^2*b*e*f*h^2*i*j*p*q + 24*a^2*b*e*f*g*h*j^2*p*q + 16*a^3*f^2*h^2*i*j*
x - 8*a^3*f^2*g*h*j^2*x - 48*a^2*b*f^2*h^2*i*j*p*q*x + 24*a^2*b*f^2*g*h*j^2
*p*q*x + 12*a^2*b*e*f*h^2*j^2*p*q*x + 96*a*b^2*f^2*h^2*i*j*p^2*q^2*x - 48*a
*b^2*f^2*g*h*j^2*p^2*q^2*x - 36*a*b^2*e*f*h^2*j^2*p^2*q^2*x - 96*b^3*f^2*h^
2*i*j*p^3*q^3*x + 48*b^3*f^2*g*h*j^2*p^3*q^3*x + 42*b^3*e*f*h^2*j^2*p^3*q^3
*x + 4*a^3*f^2*h^2*j^2*x^2 - 6*a^2*b*f^2*h^2*j^2*p*q*x^2 + 6*a*b^2*f^2*h^2*
j^2*p^2*q^2*x^2 - 3*b^3*f^2*h^2*j^2*p^3*q^3*x^2 + 48*a^2*b*e*f*h^2*i*j*p*q*
Log[e + f*x] - 24*a^2*b*e*f*g*h*j^2*p*q*Log[e + f*x] - 12*a^2*b*e^2*h^2*j^2
*p*q*Log[e + f*x] + 36*a*b^2*e^2*h^2*j^2*p^2*q^2*Log[e + f*x] + 96*b^3*e*f*
h^2*i*j*p^3*q^3*Log[e + f*x] - 48*b^3*e*f*g*h*j^2*p^3*q^3*Log[e + f*x] - 42
*b^3*e^2*h^2*j^2*p^3*q^3*Log[e + f*x] - 48*a*b^2*e*f*h^2*i*j*p^2*q^2*Log[e
+ f*x]^2 + 24*a*b^2*e*f*g*h*j^2*p^2*q^2*Log[e + f*x]^2 + 12*a*b^2*e^2*h^2*j
^2*p^2*q^2*Log[e + f*x]^2 - 18*b^3*e^2*h^2*j^2*p^3*q^3*Log[e + f*x]^2 + 16*
b^3*e*f*h^2*i*j*p^3*q^3*Log[e + f*x]^3 - 8*b^3*e*f*g*h*j^2*p^3*q^3*Log[e +
f*x]^3 - 4*b^3*e^2*h^2*j^2*p^3*q^3*Log[e + f*x]^3 - 96*a*b^2*e*f*h^2*i*j*p*
q*Log[c*(d*(e + f*x)^p)^q] + 48*a*b^2*e*f*g*h*j^2*p*q*Log[c*(d*(e + f*x)^p)
^q] + 48*a^2*b*f^2*h^2*i*j*x*Log[c*(d*(e + f*x)^p)^q] - 24*a^2*b*f^2*g*h*j^
2*x*Log[c*(d*(e + f*x)^p)^q] - 96*a*b^2*f^2*h^2*i*j*p*q*x*Log[c*(d*(e + f*x
)^p)^q] + 48*a*b^2*f^2*g*h*j^2*p*q*x*Log[c*(d*(e + f*x)^p)^q] + 24*a*b^2*e*
f*h^2*j^2*p*q*x*Log[c*(d*(e + f*x)^p)^q] + 96*b^3*f^2*h^2*i*j*p^2*q^2*x*Log
[c*(d*(e + f*x)^p)^q] - 48*b^3*f^2*g*h*j^2*p^2*q^2*x*Log[c*(d*(e + f*x)^p)^
q] - 36*b^3*e*f*h^2*j^2*p^2*q^2*x*Log[c*(d*(e + f*x)^p)^q] + 12*a^2*b*f^2*h
^2*j^2*x^2*Log[c*(d*(e + f*x)^p)^q] - 12*a*b^2*f^2*h^2*j^2*p*q*x^2*Log[c*(d
*(e + f*x)^p)^q] + 6*b^3*f^2*h^2*j^2*p^2*q^2*x^2*Log[c*(d*(e + f*x)^p)^q] +
 96*a*b^2*e*f*h^2*i*j*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q] - 48*a*b^2*
e*f*g*h*j^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q] - 24*a*b^2*e^2*h^2*j^
2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q] + 36*b^3*e^2*h^2*j^2*p^2*q^2*Lo
g[e + f*x]*Log[c*(d*(e + f*x)^p)^q] - 48*b^3*e*f*h^2*i*j*p^2*q^2*Log[e + f*
x]^2*Log[c*(d*(e + f*x)^p)^q] + 24*b^3*e*f*g*h*j^2*p^2*q^2*Log[e + f*x]^2*L
og[c*(d*(e + f*x)^p)^q] + 12*b^3*e^2*h^2*j^2*p^2*q^2*Log[e + f*x]^2*Log[c*(
d*(e + f*x)^p)^q] - 48*b^3*e*f*h^2*i*j*p*q*Log[c*(d*(e + f*x)^p)^q]^2 + 24*
b^3*e*f*g*h*j^2*p*q*Log[c*(d*(e + f*x)^p)^q]^2 + 48*a*b^2*f^2*h^2*i*j*x*Log
[c*(d*(e + f*x)^p)^q]^2 - 24*a*b^2*f^2*g*h*j^2*x*Log[c*(d*(e + f*x)^p)^q]^2
 - 48*b^3*f^2*h^2*i*j*p*q*x*Log[c*(d*(e + f*x)^p)^q]^2 + 24*b^3*f^2*g*h*j^2
*p*q*x*Log[c*(d*(e + f*x)^p)^q]^2 + 12*b^3*e*f*h^2*j^2*p*q*x*Log[c*(d*(e +
```

f*x)^p)^q]^2 + 12*a*b^2*f^2*h^2*j^2*x^2*Log[c*(d*(e + f*x)^p)^q]^2 - 6*b^3*
f^2*h^2*j^2*p*q*x^2*Log[c*(d*(e + f*x)^p)^q]^2 + 48*b^3*e*f*h^2*i*j*p*q*Log
[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2 - 24*b^3*e*f*g*h*j^2*p*q*Log[e + f*x]*
Log[c*(d*(e + f*x)^p)^q]^2 - 12*b^3*e^2*h^2*j^2*p*q*Log[e + f*x]*Log[c*(d*(
e + f*x)^p)^q]^2 + 16*b^3*f^2*h^2*i*j*x*Log[c*(d*(e + f*x)^p)^q]^3 - 8*b^3*
f^2*g*h*j^2*x*Log[c*(d*(e + f*x)^p)^q]^3 + 4*b^3*f^2*h^2*j^2*x^2*Log[c*(d*(
e + f*x)^p)^q]^3 + 8*a^3*f^2*h^2*i^2*Log[g + h*x] - 16*a^3*f^2*g*h*i*j*Log[
g + h*x] + 8*a^3*f^2*g^2*j^2*Log[g + h*x] - 24*a^2*b*f^2*h^2*i^2*p*q*Log[e
+ f*x]*Log[g + h*x] + 48*a^2*b*f^2*g*h*i*j*p*q*Log[e + f*x]*Log[g + h*x] -
24*a^2*b*f^2*g^2*j^2*p*q*Log[e + f*x]*Log[g + h*x] + 24*a*b^2*f^2*h^2*i^2*p
^2*q^2*Log[e + f*x]^2*Log[g + h*x] - 48*a*b^2*f^2*g*h*i*j*p^2*q^2*Log[e + f
*x]^2*Log[g + h*x] + 24*a*b^2*f^2*g^2*j^2*p^2*q^2*Log[e + f*x]^2*Log[g + h*
x] - 8*b^3*f^2*h^2*i^2*p^3*q^3*Log[e + f*x]^3*Log[g + h*x] + 16*b^3*f^2*g*h
*i*j*p^3*q^3*Log[e + f*x]^3*Log[g + h*x] - 8*b^3*f^2*g^2*j^2*p^3*q^3*Log[e
+ f*x]^3*Log[g + h*x] + 24*a^2*b*f^2*h^2*i^2*Log[c*(d*(e + f*x)^p)^q]*Log[g
+ h*x] - 48*a^2*b*f^2*g*h*i*j*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + 24*a
^2*b*f^2*g^2*j^2*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 48*a*b^2*f^2*h^2*i
^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + 96*a*b^2*f^2*g*
h*i*j*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 48*a*b^2*f^2
*g^2*j^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] + 24*b^3*f^
2*h^2*i^2*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 48
*b^3*f^2*g*h*i*j*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*
x] + 24*b^3*f^2*g^2*j^2*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log
[g + h*x] + 24*a*b^2*f^2*h^2*i^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] -
48*a*b^2*f^2*g*h*i*j*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + 24*a*b^2*f^2
*g^2*j^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] - 24*b^3*f^2*h^2*i^2*p*q*L
og[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + 48*b^3*f^2*g*h*i*j*p*
q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] - 24*b^3*f^2*g^2*j^2
*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + 8*b^3*f^2*h^2*i
^2*Log[c*(d*(e + f*x)^p)^q]^3*Log[g + h*x] - 16*b^3*f^2*g*h*i*j*Log[c*(d*(e
+ f*x)^p)^q]^3*Log[g + h*x] + 8*b^3*f^2*g^2*j^2*Log[c*(d*(e + f*x)^p)^q]^3
*Log[g + h*x] + 24*a^2*b*f^2*h^2*i^2*p*q*Log[e + f*x]*Log[(f*(g + h*x))/(f*
g - e*h)] - 48*a^2*b*f^2*g*h*i*j*p*q*Log[e + f*x]*Log[(f*(g + h*x))/(f*g -
e*h)] + 24*a^2*b*f^2*g^2*j^2*p*q*Log[e + f*x]*Log[(f*(g + h*x))/(f*g - e*h)
] - 24*a*b^2*f^2*h^2*i^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*
h)] + 48*a*b^2*f^2*g*h*i*j*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g -
e*h)] - 24*a*b^2*f^2*g^2*j^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g
- e*h)] + 8*b^3*f^2*h^2*i^2*p^3*q^3*Log[e + f*x]^3*Log[(f*(g + h*x))/(f*g -
e*h)] - 16*b^3*f^2*g*h*i*j*p^3*q^3*Log[e + f*x]^3*Log[(f*(g + h*x))/(f*g -
e*h)] + 8*b^3*f^2*g^2*j^2*p^3*q^3*Log[e + f*x]^3*Log[(f*(g + h*x))/(f*g -
e*h)] + 48*a*b^2*f^2*h^2*i^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[
(f*(g + h*x))/(f*g - e*h)] - 96*a*b^2*f^2*g*h*i*j*p*q*Log[e + f*x]*Log[c*(d
*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)] + 48*a*b^2*f^2*g^2*j^2*p*q*
Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)] - 24*b
^3*f^2*h^2*i^2*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g +

```
h*x))/(f*g - e*h)] + 48*b^3*f^2*g*h*i*j*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e
+ f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)] - 24*b^3*f^2*g^2*j^2*p^2*q^2*Lo
g[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)] + 24*b
^3*f^2*h^2*i^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[(f*(g + h*x)
)/(f*g - e*h)] - 48*b^3*f^2*g*h*i*j*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^
q]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 24*b^3*f^2*g^2*j^2*p*q*Log[e + f*x]*L
og[c*(d*(e + f*x)^p)^q]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 24*b*f^2*(h*i -
g*j)^2*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, (h*(e + f*x))/(-(f
*g) + e*h)] - 48*b^2*f^2*(h*i - g*j)^2*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)
^q])*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)] + 48*b^3*f^2*h^2*i^2*p^3*q^3*
PolyLog[4, (h*(e + f*x))/(-(f*g) + e*h)] - 96*b^3*f^2*g*h*i*j*p^3*q^3*PolyL
og[4, (h*(e + f*x))/(-(f*g) + e*h)] + 48*b^3*f^2*g^2*j^2*p^3*q^3*PolyLog[4,
 (h*(e + f*x))/(-(f*g) + e*h)])/(8*f^2*h^3)
```

## Maple [F]

$$\int \frac{(jx+i)^2 \left(a+b\ln \left(c(d(fx+e)^p)^q\right)\right)^3}{hx+g}dx$$

```
[In] int((j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)
```

```
[Out] int((j*x+i)^2*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)
```

## Fricas [F]

$$\int \frac{(i+jx)^2 \left(a+b\log \left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx = \int \frac{(jx+i)^2(b\log \left(((fx+e)^p d)^q\, c\right)+a)^3}{hx+g}\,dx$$

```
[In] integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="fr
icas")
```

```
[Out] integral((a^3*j^2*x^2 + 2*a^3*i*j*x + a^3*i^2 + (b^3*j^2*x^2 + 2*b^3*i*j*x
+ b^3*i^2)*log(((f*x + e)^p*d)^q*c)^3 + 3*(a*b^2*j^2*x^2 + 2*a*b^2*i*j*x +
a*b^2*i^2)*log(((f*x + e)^p*d)^q*c)^2 + 3*(a^2*b*j^2*x^2 + 2*a^2*b*i*j*x +
a^2*b*i^2)*log(((f*x + e)^p*d)^q*c))/(h*x + g), x)
```

## Sympy [F]

$$\int \frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx = \int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3(i+jx)^2}{g+hx}\,dx$$

```
[In] integrate((j*x+i)**2*(a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3*(i + j*x)**2/(g + h*x), x)
```

## Maxima [F]

$$\int \frac{(i+jx)^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx = \int \frac{(jx+i)^2\left(b\log\left(((fx+e)^pd)^q\,c\right)+a\right)^3}{hx+g}\,dx$$

```
[In] integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="ma
xima")
[Out] 2*a^3*i*j*(x/h - g*log(h*x + g)/h^2) + 1/2*a^3*j^2*(2*g^2*log(h*x + g)/h^3
+ (h*x^2 - 2*g*x)/h^2) + a^3*i^2*log(h*x + g)/h + integrate((3*(i^2*q*log(d
) + i^2*log(c))*a^2*b + 3*(i^2*q^2*log(d)^2 + 2*i^2*q*log(c)*log(d) + i^2*l
og(c)^2)*a*b^2 + (i^2*q^3*log(d)^3 + 3*i^2*q^2*log(c)*log(d)^2 + 3*i^2*q*lo
g(c)^2*log(d) + i^2*log(c)^3)*b^3 + (b^3*j^2*x^2 + 2*b^3*i*j*x + b^3*i^2)*l
og(((f*x + e)^p)^q)^3 + (3*(j^2*q*log(d) + j^2*log(c))*a^2*b + 3*(j^2*q^2*l
og(d)^2 + 2*j^2*q*log(c)*log(d) + j^2*log(c)^2)*a*b^2 + (j^2*q^3*log(d)^3 +
 3*j^2*q^2*log(c)*log(d)^2 + 3*j^2*q*log(c)^2*log(d) + j^2*log(c)^3)*b^3)*x
^2 + 3*(a*b^2*i^2 + (i^2*q*log(d) + i^2*log(c))*b^3 + (a*b^2*j^2 + (j^2*q*l
og(d) + j^2*log(c))*b^3)*x^2 + 2*(a*b^2*i*j + (i*j*q*log(d) + i*j*log(c))*b
^3)*x)*log(((f*x + e)^p)^q)^2 + 2*(3*(i*j*q*log(d) + i*j*log(c))*a^2*b + 3*
(i*j*q^2*log(d)^2 + 2*i*j*q*log(c)*log(d) + i*j*log(c)^2)*a*b^2 + (i*j*q^3*
log(d)^3 + 3*i*j*q^2*log(c)*log(d)^2 + 3*i*j*q*log(c)^2*log(d) + i*j*log(c)
^3)*b^3)*x + 3*(a^2*b*i^2 + 2*(i^2*q*log(d) + i^2*log(c))*a*b^2 + (i^2*q^2*
log(d)^2 + 2*i^2*q*log(c)*log(d) + i^2*log(c)^2)*b^3 + (a^2*b*j^2 + 2*(j^2*
q*log(d) + j^2*log(c))*a*b^2 + (j^2*q^2*log(d)^2 + 2*j^2*q*log(c)*log(d) +
j^2*log(c)^2)*b^3)*x^2 + 2*(a^2*b*i*j + 2*(i*j*q*log(d) + i*j*log(c))*a*b^2
 + (i*j*q^2*log(d)^2 + 2*i*j*q*log(c)*log(d) + i*j*log(c)^2)*b^3)*x)*log(((
f*x + e)^p)^q))/(h*x + g), x)
```

## Giac [F]

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(jx+i)^2 (b\log(((fx+e)^p d)^q c)+a)^3}{hx+g}\,dx$$

[In] integrate((j*x+i)^2*(a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="giac")

[Out] integrate((j*x + i)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^3/(h*x + g), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(i+j\,x)^2 \left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^3}{g+h\,x}\,dx$$

[In] int(((i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^3)/(g + h*x),x)

[Out] int(((i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^3)/(g + h*x), x)

# 3.536 $\int \frac{(i+jx)(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$



## Optimal result

Integrand size = 33, antiderivative size = 349

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx = \frac{6ab^2jp^2q^2x}{h} - \frac{6b^3jp^3q^3x}{h} + \frac{6b^3jp^2q^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh} - \frac{3bjpq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh} + \frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{fh} + \frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} + \frac{3b(hi-gj)pq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h^2} - \frac{6b^2(hi-gj)p^2q^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h^2} + \frac{6b^3(hi-gj)p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}$$

```
[Out] 6*a*b^2*j*p^2*q^2*x/h-6*b^3*j*p^3*q^3*x/h+6*b^3*j*p^2*q^2*(f*x+e)*ln(c*(d*(
f*x+e)^p)^q)/f/h-3*b*j*p*q*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^2/f/h+j*(f*x
+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/f/h+(-g*j+h*i)*(a+b*ln(c*(d*(f*x+e)^p)^q)
)^3*ln(f*(h*x+g)/(-e*h+f*g))/h^2+3*b*(-g*j+h*i)*p*q*(a+b*ln(c*(d*(f*x+e)^p)
^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h^2-6*b^2*(-g*j+h*i)*p^2*q^2*(a+b*l
n(c*(d*(f*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h^2+6*b^3*(-g*j+h*i)
*p^3*q^3*polylog(4,-h*(f*x+e)/(-e*h+f*g))/h^2
```

## Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2465, 2436, 2333, 2332, 2443, 2481, 2421, 2430, 6724, 2495}

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx$$
$$= -\frac{6b^2p^2q^2(hi-gj)\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)}{h^2}$$
$$+\frac{6ab^2jp^2q^2x}{h}+\frac{3bpq(hi-gj)\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{h^2}$$
$$+\frac{(hi-gj)\log\left(\frac{f(g+hx)}{fg-eh}\right)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{h^2}$$
$$-\frac{3bjpq(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2}{fh}$$
$$+\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{fh}+\frac{6b^3jp^2q^2(e+fx)\log\left(c(d(e+fx)^p)^q\right)}{fh}$$
$$+\frac{6b^3p^3q^3(hi-gj)\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h^2}-\frac{6b^3jp^3q^3x}{h}$$

```
[In] Int[((i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(g + h*x),x]
```

```
[Out] (6*a*b^2*j*p^2*q^2*x)/h - (6*b^3*j*p^3*q^3*x)/h + (6*b^3*j*p^2*q^2*(e + f*x
)*Log[c*(d*(e + f*x)^p)^q])/(f*h) - (3*b*j*p*q*(e + f*x)*(a + b*Log[c*(d*(e
 + f*x)^p)^q])^2)/(f*h) + (j*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/
(f*h) + ((h*i - g*j)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(g + h*x))/(
f*g - e*h)])/h^2 + (3*b*(h*i - g*j)*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*
PolyLog[2, -((h*(e + f*x))/(f*g - e*h))])/h^2 - (6*b^2*(h*i - g*j)*p^2*q^2*
(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[3, -((h*(e + f*x))/(f*g - e*h))])/
h^2 + (6*b^3*(h*i - g*j)*p^3*q^3*PolyLog[4, -((h*(e + f*x))/(f*g - e*h))])/
h^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(i+jx)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\int \left(\frac{j\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{h} + \frac{(hi-gj)\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{h(g+hx)}\right)dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{j\int\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(hi-gj)\int\frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^3}{g+hx}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} + \text{Subst}\left(\frac{j\text{Subst}\left(\int\left(a+b\log\left(cd^qx^{pq}\right)\right)^3\,dx, x, e+fx\right)}{fh}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(3bf(hi-gj)pq)\int\frac{\left(a+b\log\left(cd^q(e+fx)^{pq}\right)\right)^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{fh} + \frac{(hi-gj)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} - \text{Subst}\left(\frac{(3bjpq)\text{Subst}\left(\int\left(a+b\log\left(cd^qx^{pq}\right)\right)^2\,dx,x,e+fx\right)}{fh},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(3b(hi-gj)pq)\text{Subst}\left(\int\frac{\left(a+b\log\left(cd^qx^{pq}\right)\right)^2\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{h^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= -\frac{3bjpq(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{fh}$$
$$+\frac{j(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{fh}$$
$$+\frac{(hi-gj)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2}$$
$$+\frac{3b(hi-gj)pq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^2}$$
$$+\operatorname{Subst}\left(\frac{(6b^2jp^2q^2)\operatorname{Subst}\left(\int\left(a+b\log\left(cd^qx^{pq}\right)\right)\,dx,x,e+fx\right)}{fh},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right)$$
$$-\operatorname{Subst}\left(\frac{(6b^2(hi-gj)p^2q^2)\operatorname{Subst}\left(\int\frac{\left(a+b\log\left(cd^qx^{pq}\right)\right)\operatorname{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{h^2},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right)$$

$$= \frac{6ab^2jp^2q^2x}{h} - \frac{3bjpq(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{fh} + \frac{j(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{fh} + \frac{(hi-gj)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} + \frac{3b(hi-gj)pq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^2} - \frac{6b^2(hi-gj)p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^2} + \operatorname{Subst}\left(\frac{(6b^3jp^2q^2)\operatorname{Subst}\left(\int\log\left(cd^qx^{pq}\right)\,dx,x,e+fx\right)}{fh},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right) + \operatorname{Subst}\left(\frac{(6b^3(hi-gj)p^3q^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{h^2},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right)$$

$$= \frac{6ab^2jp^2q^2x}{h} - \frac{6b^3jp^3q^3x}{h} + \frac{6b^3jp^2q^2(e+fx)\log\left(c\left(d(e+fx)^p\right)^q\right)}{fh} - \frac{3bjpq(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}{fh} + \frac{j(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{fh} + \frac{(hi-gj)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h^2} + \frac{3b(hi-gj)pq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^2} - \frac{6b^2(hi-gj)p^2q^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^2} + \frac{6b^3(hi-gj)p^3q^3\operatorname{Li}_4\left(-\frac{h(e+fx)}{fg-eh}\right)}{h^2}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1769 vs. 2(349) = 698.

Time = 0.48 (sec) , antiderivative size = 1769, normalized size of antiderivative = 5.07

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx$$
$$= \frac{-3a^2behjpq + a^3fhjx - 3a^2bfhjpqx + 6ab^2fhjp^2q^2x - 6b^3fhjp^3q^3x + 3a^2behjpq\log(e+fx) + 6b^3ehjp^3}{}$$

```
[In] Integrate[((i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/(g + h*x),x]
```

```
[Out] (-3*a^2*b*e*h*j*p*q + a^3*f*h*j*x - 3*a^2*b*f*h*j*p*q*x + 6*a*b^2*f*h*j*p^2
*q^2*x - 6*b^3*f*h*j*p^3*q^3*x + 3*a^2*b*e*h*j*p*q*Log[e + f*x] + 6*b^3*e*h
*j*p^3*q^3*Log[e + f*x] - 3*a*b^2*e*h*j*p^2*q^2*Log[e + f*x]^2 + b^3*e*h*j*
p^3*q^3*Log[e + f*x]^3 - 6*a*b^2*e*h*j*p*q*Log[c*(d*(e + f*x)^p)^q] + 3*a^2
*b*f*h*j*x*Log[c*(d*(e + f*x)^p)^q] - 6*a*b^2*f*h*j*p*q*x*Log[c*(d*(e + f*x
)^p)^q] + 6*b^3*f*h*j*p^2*q^2*x*Log[c*(d*(e + f*x)^p)^q] + 6*a*b^2*e*h*j*p*
q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q] - 3*b^3*e*h*j*p^2*q^2*Log[e + f*x]^
2*Log[c*(d*(e + f*x)^p)^q] - 3*b^3*e*h*j*p*q*Log[c*(d*(e + f*x)^p)^q]^2 + 3
*a*b^2*f*h*j*x*Log[c*(d*(e + f*x)^p)^q]^2 - 3*b^3*f*h*j*p*q*x*Log[c*(d*(e +
 f*x)^p)^q]^2 + 3*b^3*e*h*j*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2 + b
^3*f*h*j*x*Log[c*(d*(e + f*x)^p)^q]^3 + a^3*f*h*i*Log[g + h*x] - a^3*f*g*j*
Log[g + h*x] - 3*a^2*b*f*h*i*p*q*Log[e + f*x]*Log[g + h*x] + 3*a^2*b*f*g*j*
p*q*Log[e + f*x]*Log[g + h*x] + 3*a*b^2*f*h*i*p^2*q^2*Log[e + f*x]^2*Log[g
+ h*x] - 3*a*b^2*f*g*j*p^2*q^2*Log[e + f*x]^2*Log[g + h*x] - b^3*f*h*i*p^3*
q^3*Log[e + f*x]^3*Log[g + h*x] + b^3*f*g*j*p^3*q^3*Log[e + f*x]^3*Log[g +
h*x] + 3*a^2*b*f*h*i*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 3*a^2*b*f*g*j*
Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 6*a*b^2*f*h*i*p*q*Log[e + f*x]*Log[
c*(d*(e + f*x)^p)^q]*Log[g + h*x] + 6*a*b^2*f*g*j*p*q*Log[e + f*x]*Log[c*(d
*(e + f*x)^p)^q]*Log[g + h*x] + 3*b^3*f*h*i*p^2*q^2*Log[e + f*x]^2*Log[c*(d
*(e + f*x)^p)^q]*Log[g + h*x] - 3*b^3*f*g*j*p^2*q^2*Log[e + f*x]^2*Log[c*(d
*(e + f*x)^p)^q]*Log[g + h*x] + 3*a*b^2*f*h*i*Log[c*(d*(e + f*x)^p)^q]^2*Lo
g[g + h*x] - 3*a*b^2*f*g*j*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] - 3*b^3*
f*h*i*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + 3*b^3*f*g*
j*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + b^3*f*h*i*Log[
c*(d*(e + f*x)^p)^q]^3*Log[g + h*x] - b^3*f*g*j*Log[c*(d*(e + f*x)^p)^q]^3*
Log[g + h*x] + 3*a^2*b*f*h*i*p*q*Log[e + f*x]*Log[(f*(g + h*x))/(f*g - e*h)
] - 3*a^2*b*f*g*j*p*q*Log[e + f*x]*Log[(f*(g + h*x))/(f*g - e*h)] - 3*a*b^2
*f*h*i*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 3*a*b^2*f*g*
j*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + b^3*f*h*i*p^3*q^3
*Log[e + f*x]^3*Log[(f*(g + h*x))/(f*g - e*h)] - b^3*f*g*j*p^3*q^3*Log[e +
f*x]^3*Log[(f*(g + h*x))/(f*g - e*h)] + 6*a*b^2*f*h*i*p*q*Log[e + f*x]*Log[
c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)] - 6*a*b^2*f*g*j*p*q*Log
```

```
[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*h)] - 3*b^3*f
*h*i*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g
 - e*h)] + 3*b^3*f*g*j*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[
(f*(g + h*x))/(f*g - e*h)] + 3*b^3*f*h*i*p*q*Log[e + f*x]*Log[c*(d*(e + f*x
)^p)^q]^2*Log[(f*(g + h*x))/(f*g - e*h)] - 3*b^3*f*g*j*p*q*Log[e + f*x]*Log
[c*(d*(e + f*x)^p)^q]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 3*b*f*(h*i - g*j)*
p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, (h*(e + f*x))/(-(f*g) + e
*h)] - 6*b^2*f*(h*i - g*j)*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog
[3, (h*(e + f*x))/(-(f*g) + e*h)] + 6*b^3*f*h*i*p^3*q^3*PolyLog[4, (h*(e +
f*x))/(-(f*g) + e*h)] - 6*b^3*f*g*j*p^3*q^3*PolyLog[4, (h*(e + f*x))/(-(f*g
) + e*h)])/(f*h^2)
```

## Maple [F]

$$\int \frac{(jx+i)\,(a+b\ln(c(d(fx+e)^{p})^{q}))^{3}}{hx+g}dx$$

```
[In] int((j*x+i)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)
[Out] int((j*x+i)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)
```

## Fricas [F]

$$\int \frac{(i+jx)\,(a+b\log(c(d(e+fx)^{p})^{q}))^{3}}{g+hx}\,dx = \int \frac{(jx+i)(b\log(((fx+e)^{p}d)^{q}\,c)+a)^{3}}{hx+g}\,dx$$

```
[In] integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="fric
as")
[Out] integral((a^3*j*x + a^3*i + (b^3*j*x + b^3*i)*log(((f*x + e)^p*d)^q*c)^3 +
3*(a*b^2*j*x + a*b^2*i)*log(((f*x + e)^p*d)^q*c)^2 + 3*(a^2*b*j*x + a^2*b*i
)*log(((f*x + e)^p*d)^q*c))/(h*x + g), x)
```

## Sympy [F]

$$\int \frac{(i+jx)\,(a+b\log(c(d(e+fx)^{p})^{q}))^{3}}{g+hx}\,dx = \int \frac{(a+b\log(c(d(e+fx)^{p})^{q}))^{3}\,(i+jx)}{g+hx}\,dx$$

```
[In] integrate((j*x+i)*(a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g),x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3*(i + j*x)/(g + h*x), x)
```

## Maxima [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx = \int \frac{(jx+i)\left(b\log\left(((fx+e)^pd)^q\,c\right)+a\right)^3}{hx+g}\,dx$$

```
[In] integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="maxi
ma")
```

```
[Out] a^3*j*(x/h - g*log(h*x + g)/h^2) + a^3*i*log(h*x + g)/h + integrate((3*(i*q
*log(d) + i*log(c))*a^2*b + 3*(i*q^2*log(d)^2 + 2*i*q*log(c)*log(d) + i*log
(c)^2)*a*b^2 + (i*q^3*log(d)^3 + 3*i*q^2*log(c)*log(d)^2 + 3*i*q*log(c)^2*l
og(d) + i*log(c)^3)*b^3 + (b^3*j*x + b^3*i)*log(((f*x + e)^p)^q)^3 + 3*((i*
q*log(d) + i*log(c))*b^3 + a*b^2*i + ((j*q*log(d) + j*log(c))*b^3 + a*b^2*j
)*x)*log(((f*x + e)^p)^q)^2 + (3*(j*q*log(d) + j*log(c))*a^2*b + 3*(j*q^2*l
og(d)^2 + 2*j*q*log(c)*log(d) + j*log(c)^2)*a*b^2 + (j*q^3*log(d)^3 + 3*j*q
^2*log(c)*log(d)^2 + 3*j*q*log(c)^2*log(d) + j*log(c)^3)*b^3)*x + 3*(2*(i*q
*log(d) + i*log(c))*a*b^2 + (i*q^2*log(d)^2 + 2*i*q*log(c)*log(d) + i*log(c
)^2)*b^3 + a^2*b*i + (2*(j*q*log(d) + j*log(c))*a*b^2 + (j*q^2*log(d)^2 + 2
*j*q*log(c)*log(d) + j*log(c)^2)*b^3 + a^2*b*j)*x)*log(((f*x + e)^p)^q))/(h
*x + g), x)
```

## Giac [F]

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx = \int \frac{(jx+i)\left(b\log\left(((fx+e)^pd)^q\,c\right)+a\right)^3}{hx+g}\,dx$$

```
[In] integrate((j*x+i)*(a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="giac
")
```

```
[Out] integrate((j*x + i)*(b*log(((f*x + e)^p*d)^q*c) + a)^3/(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(i+jx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{g+hx}\,dx = \int \frac{(i+j\,x)\,\left(a+b\,\ln\left(c\,\left(d\,(e+f\,x)^p\right)^q\right)\right)^3}{g+h\,x}\,dx$$

```
[In] int(((i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^3)/(g + h*x),x)
```

```
[Out] int(((i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^3)/(g + h*x), x)
```

# 3.537 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 177

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{h} + \frac{6b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

```
[Out] (a+b*ln(c*(d*(f*x+e)^p)^q))^3*ln(f*(h*x+g)/(-e*h+f*g))/h+3*b*p*q*(a+b*ln(c*
(d*(f*x+e)^p)^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))/h-6*b^2*p^2*q^2*(a+b*l
n(c*(d*(f*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-e*h+f*g))/h+6*b^3*p^3*q^3*poly
log(4,-h*(f*x+e)/(-e*h+f*g))/h
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2443, 2481, 2421, 2430, 6724, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = -\frac{6b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{h} + \frac{3bpq\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{h} + \frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{h} + \frac{6b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x),x]
```

```
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(g + h*x))/(f*g - e*h)])/h + (3*
b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, -((h*(e + f*x))/(f*g -
e*h))])/h - (6*b^2*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[3, -((h
*(e + f*x))/(f*g - e*h))])/h + (6*b^3*p^3*q^3*PolyLog[4, -((h*(e + f*x))/(f
*g - e*h))])/h
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^3}{g+hx}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} - \text{Subst}\left(\frac{(3bfpq)\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} - \text{Subst}\left(\frac{(3bpq)\text{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))^2\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}dx,x,e+fx\right)}{h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \text{Subst}\left(\frac{(6b^2p^2q^2)\text{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\operatorname{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}dx,x,e+fx\right)}{h},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3 \log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2 \operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q)) \operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} + \text{Subst}\left(\frac{(6b^3p^3q^3)\,\text{Subst}\left(\int \frac{\operatorname{Li}_3\left(-\frac{hx}{fg-eh}\right)}{x}\,dx, x, e+fx\right)}{h}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3 \log\left(\frac{f(g+hx)}{fg-eh}\right)}{h} + \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2 \operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} - \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q)) \operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{h} + \frac{6b^3p^3q^3\operatorname{Li}_4\left(-\frac{h(e+fx)}{fg-eh}\right)}{h}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 646 vs. $2(177) = 354$.

Time = 0.10 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.65

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \frac{a^3\log(g+hx) - 3a^2bpq\log(e+fx)\log(g+hx) + 3ab^2p^2q^2\log^2(e+fx)\log(g+hx) - b^3p^3q^3\log^3(e+f}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/(g + h*x),x]
```

```
[Out] (a^3*Log[g + h*x] - 3*a^2*b*p*q*Log[e + f*x]*Log[g + h*x] + 3*a*b^2*p^2*q^2
*Log[e + f*x]^2*Log[g + h*x] - b^3*p^3*q^3*Log[e + f*x]^3*Log[g + h*x] + 3*
a^2*b*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 6*a*b^2*p*q*Log[e + f*x]*Log[
c*(d*(e + f*x)^p)^q]*Log[g + h*x] + 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(
e + f*x)^p)^q]*Log[g + h*x] + 3*a*b^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*
x] - 3*b^3*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + b^3*L
og[c*(d*(e + f*x)^p)^q]^3*Log[g + h*x] + 3*a^2*b*p*q*Log[e + f*x]*Log[(f*(g
```

```
 + h*x))/(f*g - e*h)] - 3*a*b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f
*g - e*h)] + b^3*p^3*q^3*Log[e + f*x]^3*Log[(f*(g + h*x))/(f*g - e*h)] + 6*
a*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*
h)] - 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x
))/(f*g - e*h)] + 3*b^3*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[(f*
(g + h*x))/(f*g - e*h)] + 3*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLo
g[2, (h*(e + f*x))/(-(f*g) + e*h)] - 6*b^2*p^2*q^2*(a + b*Log[c*(d*(e + f*x
)^p)^q])*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)] + 6*b^3*p^3*q^3*PolyLog[4
, (h*(e + f*x))/(-(f*g) + e*h)])/h
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{3}}{hx+g}dx$$

[In] `int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)`

[Out] `int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x)`

## Fricas [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^{p})^{q}\right))^{3}}{g+hx}\,dx=\int \frac{\left(b\log\left(\left((fx+e)^{p}d\right)^{q}c\right)+a\right)^{3}}{hx+g}\,dx$$

[In] `integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="fricas")`

[Out]
```
integral((b^3*log(((f*x + e)^p*d)^q*c)^3 + 3*a*b^2*log(((f*x + e)^p*d)^q*c)
^2 + 3*a^2*b*log(((f*x + e)^p*d)^q*c) + a^3)/(h*x + g), x)
```

## Sympy [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^{p})^{q}\right))^{3}}{g+hx}\,dx=\int \frac{\left(a+b\log\left(c(d(e+fx)^{p})^{q}\right)\right)^{3}}{g+hx}\,dx$$

[In] `integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g),x)`

[Out] `Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3/(g + h*x), x)`

## Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="maxima")
```

```
[Out] a^3*log(h*x + g)/h + integrate((b^3*log(((f*x + e)^p)^q)^3 + 3*(q*log(d) +
log(c))*a^2*b + 3*(q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*a*b^2 + (q^
3*log(d)^3 + 3*q^2*log(c)*log(d)^2 + 3*q*log(c)^2*log(d) + log(c)^3)*b^3 +
3*((q*log(d) + log(c))*b^3 + a*b^2)*log(((f*x + e)^p)^q)^2 + 3*(2*(q*log(d)
 + log(c))*a*b^2 + (q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*b^3 + a^2*
b)*log(((f*x + e)^p)^q))/(h*x + g), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{hx+g}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^3/(h*x + g), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{g+hx}\,dx = \int \frac{(a+b\,\ln(c\,(d\,(e+f\,x)^p)^q))^3}{g+h\,x}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x),x)
```

```
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^3/(g + h*x), x)
```

# 3.538 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx$



## Optimal result

Integrand size = 35, antiderivative size = 410

$$\begin{aligned}\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx &= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj}\\ &+ \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj}\\ &- \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}\\ &- \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj}\\ &+ \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}\\ &+ \frac{6b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} - \frac{6b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}\end{aligned}$$

```
[Out] (a+b*ln(c*(d*(f*x+e)^p)^q))^3*ln(f*(h*x+g)/(-e*h+f*g))/(-g*j+h*i)-(a+b*ln(c
*(d*(f*x+e)^p)^q))^3*ln(f*(j*x+i)/(-e*j+f*i))/(-g*j+h*i)+3*b*p*q*(a+b*ln(c*
(d*(f*x+e)^p)^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*i)-3*b*p*q*(a+
b*ln(c*(d*(f*x+e)^p)^q))^2*polylog(2,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)-6*b^
2*p^2*q^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-e*h+f*g))/(-g*
j+h*i)+6*b^2*p^2*q^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(3,-j*(f*x+e)/(-e*j
+f*i))/(-g*j+h*i)+6*b^3*p^3*q^3*polylog(4,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*i)
-6*b^3*p^3*q^3*polylog(4,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)
```

## Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2465, 2443, 2481, 2421, 2430, 6724, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx$$
$$= -\frac{6b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{hi-gj}$$
$$+\frac{6b^2p^2q^2\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))}{hi-gj}$$
$$+\frac{3bpq\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{hi-gj}$$
$$-\frac{3bpq\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{hi-gj}$$
$$+\frac{\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{hi-gj}-\frac{\log\left(\frac{f(i+jx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{hi-gj}$$
$$+\frac{6b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj}-\frac{6b^3p^3q^3\operatorname{PolyLog}\left(4,-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/((g + h*x)*(i + j*x)),x]
```

```
[Out] ((a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(g + h*x))/(f*g - e*h)])/(h*i -
g*j) - ((a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(i + j*x))/(f*i - e*j)])/
(h*i - g*j) + (3*b*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, -((h*(
e + f*x))/(f*g - e*h))])/(h*i - g*j) - (3*b*p*q*(a + b*Log[c*(d*(e + f*x)^p
)^q])^2*PolyLog[2, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j) - (6*b^2*p^2*
q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[3, -((h*(e + f*x))/(f*g - e*h)
)])/(h*i - g*j) + (6*b^2*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[3
, -((j*(e + f*x))/(f*i - e*j))])/(h*i - g*j) + (6*b^3*p^3*q^3*PolyLog[4, -(
(h*(e + f*x))/(f*g - e*h))])/(h*i - g*j) - (6*b^3*p^3*q^3*PolyLog[4, -((j*(
e + f*x))/(f*i - e*j))])/(h*i - g*j)
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^3}{(g+hx)(i+jx)}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\int \left(\frac{h(a+b\log(cd^q(e+fx)^{pq}))^3}{(hi-gj)(g+hx)} - \frac{j(a+b\log(cd^q(e+fx)^{pq}))^3}{(hi-gj)(i+jx)}\right)dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \text{Subst}\left(\frac{h\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^3}{g+hx}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{j\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^3}{i+jx}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj}$$
$$-\text{Subst}\left(\frac{(3bfpq)\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$
$$+\text{Subst}\left(\frac{(3bfpq)\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{e+fx}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} - \text{Subst}\left(\frac{(3bpq)\text{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))^2\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}dx,x,e+fx\right)}{hi-gj},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(3bpq)\text{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))^2\log\left(\frac{f\left(\frac{fi-ej}{f}+\frac{jx}{f}\right)}{fi-ej}\right)}{x}dx,x,e+fx\right)}{hi-gj},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$
\begin{aligned}
&= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} \\
&- \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} \\
&+ \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} \\
&- \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj} \\
&- \operatorname{Subst}\left(\frac{(6b^2p^2q^2)\operatorname{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\operatorname{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{hi-gj},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) \\
&+ \operatorname{Subst}\left(\frac{(6b^2p^2q^2)\operatorname{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\operatorname{Li}_2\left(-\frac{jx}{fi-ej}\right)}{x}\,dx,x,e+fx\right)}{hi-gj},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)
\end{aligned}
$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} + \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} - \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj} - \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} + \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_3\left(-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj} + \operatorname{Subst}\left(\frac{(6b^3p^3q^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{hi-gj},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right) - \operatorname{Subst}\left(\frac{(6b^3p^3q^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{jx}{fi-ej}\right)}{x}\,dx,x,e+fx\right)}{hi-gj},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{hi-gj} - \frac{(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{hi-gj} + \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} - \frac{3bpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj} - \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} + \frac{6b^2p^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{Li}_3\left(-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj} + \frac{6b^3p^3q^3\operatorname{Li}_4\left(-\frac{h(e+fx)}{fg-eh}\right)}{hi-gj} - \frac{6b^3p^3q^3\operatorname{Li}_4\left(-\frac{j(e+fx)}{fi-ej}\right)}{hi-gj}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1350 vs. 2(410) = 820.

Time = 0.31 (sec) , antiderivative size = 1350, normalized size of antiderivative = 3.29

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx$$

$$= \frac{a^3\log(g+hx) - 3a^2bpq\log(e+fx)\log(g+hx) + 3ab^2p^2q^2\log^2(e+fx)\log(g+hx) - b^3p^3q^3\log^3(e+f}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/((g + h*x)*(i + j*x)),x]
```

```
[Out] (a^3*Log[g + h*x] - 3*a^2*b*p*q*Log[e + f*x]*Log[g + h*x] + 3*a*b^2*p^2*q^2
*Log[e + f*x]^2*Log[g + h*x] - b^3*p^3*q^3*Log[e + f*x]^3*Log[g + h*x] + 3*
a^2*b*Log[c*(d*(e + f*x)^p)^q]*Log[g + h*x] - 6*a*b^2*p*q*Log[e + f*x]*Log[
c*(d*(e + f*x)^p)^q]*Log[g + h*x] + 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(
e + f*x)^p)^q]*Log[g + h*x] + 3*a*b^2*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*
x] - 3*b^3*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[g + h*x] + b^3*L
og[c*(d*(e + f*x)^p)^q]^3*Log[g + h*x] + 3*a^2*b*p*q*Log[e + f*x]*Log[(f*(g
 + h*x))/(f*g - e*h)] - 3*a*b^2*p^2*q^2*Log[e + f*x]^2*Log[(f*(g + h*x))/(f
*g - e*h)] + b^3*p^3*q^3*Log[e + f*x]^3*Log[(f*(g + h*x))/(f*g - e*h)] + 6*
a*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x))/(f*g - e*
h)] - 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[(f*(g + h*x
))/(f*g - e*h)] + 3*b^3*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]^2*Log[(f*
```

```
(g + h*x))/(f*g - e*h)] - a^3*Log[i + j*x] + 3*a^2*b*p*q*Log[e + f*x]*Log[i
 + j*x] - 3*a*b^2*p^2*q^2*Log[e + f*x]^2*Log[i + j*x] + b^3*p^3*q^3*Log[e +
 f*x]^3*Log[i + j*x] - 3*a^2*b*Log[c*(d*(e + f*x)^p)^q]*Log[i + j*x] + 6*a*
b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Log[i + j*x] - 3*b^3*p^2*q^2*
Log[e + f*x]^2*Log[c*(d*(e + f*x)^p)^q]*Log[i + j*x] - 3*a*b^2*Log[c*(d*(e
+ f*x)^p)^q]^2*Log[i + j*x] + 3*b^3*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^
q]^2*Log[i + j*x] - b^3*Log[c*(d*(e + f*x)^p)^q]^3*Log[i + j*x] - 3*a^2*b*p
*q*Log[e + f*x]*Log[(f*(i + j*x))/(f*i - e*j)] + 3*a*b^2*p^2*q^2*Log[e + f*
x]^2*Log[(f*(i + j*x))/(f*i - e*j)] - b^3*p^3*q^3*Log[e + f*x]^3*Log[(f*(i
+ j*x))/(f*i - e*j)] - 6*a*b^2*p*q*Log[e + f*x]*Log[c*(d*(e + f*x)^p)^q]*Lo
g[(f*(i + j*x))/(f*i - e*j)] + 3*b^3*p^2*q^2*Log[e + f*x]^2*Log[c*(d*(e + f
*x)^p)^q]*Log[(f*(i + j*x))/(f*i - e*j)] - 3*b^3*p*q*Log[e + f*x]*Log[c*(d*
(e + f*x)^p)^q]^2*Log[(f*(i + j*x))/(f*i - e*j)] + 3*b*p*q*(a + b*Log[c*(d*
(e + f*x)^p)^q])^2*PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)] - 3*b*p*q*(a +
b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, (j*(e + f*x))/(-(f*i) + e*j)] - 6*
a*b^2*p^2*q^2*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)] - 6*b^3*p^2*q^2*Log[
c*(d*(e + f*x)^p)^q]*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)] + 6*a*b^2*p^2
*q^2*PolyLog[3, (j*(e + f*x))/(-(f*i) + e*j)] + 6*b^3*p^2*q^2*Log[c*(d*(e +
 f*x)^p)^q]*PolyLog[3, (j*(e + f*x))/(-(f*i) + e*j)] + 6*b^3*p^3*q^3*PolyLo
g[4, (h*(e + f*x))/(-(f*g) + e*h)] - 6*b^3*p^3*q^3*PolyLog[4, (j*(e + f*x))
/(-(f*i) + e*j)])/(h*i - g*j)
```

## Maple [F]

$$\int \frac{(a+b\ln(c(d(fx+e)^p)^q))^3}{(hx+g)(jx+i)}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i),x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i),x)
```

## Fricas [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{(hx+g)(jx+i)}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i),x, algorithm="fric
as")
```

```
[Out] integral((b^3*log(((f*x + e)^p*d)^q*c)^3 + 3*a*b^2*log(((f*x + e)^p*d)^q*c)
^2 + 3*a^2*b*log(((f*x + e)^p*d)^q*c) + a^3)/(h*j*x^2 + g*i + (h*i + g*j)*x
), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)\,(i+jx)}\,dx$$

[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g)/(j*x+i),x)

[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3/((g + h*x)*(i + j*x)), x)

## Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{(hx+g)(jx+i)}\,dx$$

[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i),x, algorithm="maxima")

[Out] a^3*(log(h*x + g)/(h*i - g*j) - log(j*x + i)/(h*i - g*j)) + integrate((b^3*log(((f*x + e)^p)^q)^3 + 3*(q*log(d) + log(c))*a^2*b + 3*(q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*a*b^2 + (q^3*log(d)^3 + 3*q^2*log(c)*log(d)^2 + 3*q*log(c)^2*log(d) + log(c)^3)*b^3 + 3*((q*log(d) + log(c))*b^3 + a*b^2)*log(((f*x + e)^p)^q)^2 + 3*(2*(q*log(d) + log(c))*a*b^2 + (q^2*log(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*b^3 + a^2*b)*log(((f*x + e)^p)^q))/(h*j*x^2 + g*i + (h*i + g*j)*x), x)

## Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)}\,dx = \int \frac{(b\log(((fx+e)^p d)^q c)+a)^3}{(hx+g)(jx+i)}\,dx$$

[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i),x, algorithm="giac")

[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^3/((h*x + g)*(j*x + i)), x)

## Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{(g+hx)(i+jx)}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d\,{\left(e+f\,x\right)}^p\right)^q\right)\right)^3}{\left(g+h\,x\right)\,\left(i+j\,x\right)}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^3/((g + h*x)*(i + j*x)),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^3/((g + h*x)*(i + j*x)), x)
```

# 3.539 $\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx$

## Optimal result

Integrand size = 35, antiderivative size = 659

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx$$
$$= -\frac{j(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3}{(fi-ej)(hi-gj)(i+jx)} + \frac{h(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2}$$
$$+ \frac{3bfpq(a+b\log(c(d(e+fx)^p)^q))^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(fi-ej)(hi-gj)}$$
$$- \frac{h(a+b\log(c(d(e+fx)^p)^q))^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2}$$
$$+ \frac{3bhpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2}$$
$$+ \frac{6b^2fp^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)}$$
$$- \frac{3bhpq(a+b\log(c(d(e+fx)^p)^q))^2\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$
$$- \frac{6b^2hp^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2}$$
$$- \frac{6b^3fp^3q^3\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)}$$
$$+ \frac{6b^2hp^2q^2(a+b\log(c(d(e+fx)^p)^q))\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$
$$+ \frac{6b^3hp^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} - \frac{6b^3hp^3q^3\operatorname{PolyLog}\left(4,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$

```
[Out] -j*(f*x+e)*(a+b*ln(c*(d*(f*x+e)^p)^q))^3/(-e*j+f*i)/(-g*j+h*i)/(j*x+i)+h*(a
+b*ln(c*(d*(f*x+e)^p)^q))^3*ln(f*(h*x+g)/(-e*h+f*g))/(-g*j+h*i)^2+3*b*f*p*q
*(a+b*ln(c*(d*(f*x+e)^p)^q))^2*ln(f*(j*x+i)/(-e*j+f*i))/(-e*j+f*i)/(-g*j+h*
i)-h*(a+b*ln(c*(d*(f*x+e)^p)^q))^3*ln(f*(j*x+i)/(-e*j+f*i))/(-g*j+h*i)^2+3*
b*h*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))^2*polylog(2,-h*(f*x+e)/(-e*h+f*g))/(-g*
j+h*i)^2+6*b^2*f*p^2*q^2*(a+b*ln(c*(d*(f*x+e)^p)^q))*polylog(2,-j*(f*x+e)/(
-e*j+f*i))/(-e*j+f*i)/(-g*j+h*i)-3*b*h*p*q*(a+b*ln(c*(d*(f*x+e)^p)^q))^2*po
lylog(2,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)^2-6*b^2*h*p^2*q^2*(a+b*ln(c*(d*(f
*x+e)^p)^q))*polylog(3,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*i)^2-6*b^3*f*p^3*q^3*
polylog(3,-j*(f*x+e)/(-e*j+f*i))/(-e*j+f*i)/(-g*j+h*i)+6*b^2*h*p^2*q^2*(a+b
```

```
*ln(c*(d*(f*x+e)^p)^q))*polylog(3,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)^2+6*b^3
*h*p^3*q^3*polylog(4,-h*(f*x+e)/(-e*h+f*g))/(-g*j+h*i)^2-6*b^3*h*p^3*q^3*po
lylog(4,-j*(f*x+e)/(-e*j+f*i))/(-g*j+h*i)^2
```

## Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2465, 2443, 2481, 2421, 2430, 6724, 2444, 2495}

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx$$
$$=\frac{6b^2fp^2q^2\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))}{(fi-ej)(hi-gj)}$$
$$-\frac{6b^2hp^2q^2\operatorname{PolyLog}\left(3,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))}{(hi-gj)^2}$$
$$+\frac{6b^2hp^2q^2\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))}{(hi-gj)^2}$$
$$+\frac{3bhpq\operatorname{PolyLog}\left(2,-\frac{h(e+fx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{(hi-gj)^2}$$
$$-\frac{3bhpq\operatorname{PolyLog}\left(2,-\frac{j(e+fx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{(hi-gj)^2}$$
$$+\frac{3bfpq\log\left(\frac{f(i+jx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^2}{(fi-ej)(hi-gj)}$$
$$-\frac{j(e+fx)(a+b\log(c(d(e+fx)^p)^q))^3}{(i+jx)(fi-ej)(hi-gj)}+\frac{h\log\left(\frac{f(g+hx)}{fg-eh}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{(hi-gj)^2}$$
$$-\frac{h\log\left(\frac{f(i+jx)}{fi-ej}\right)(a+b\log(c(d(e+fx)^p)^q))^3}{(hi-gj)^2}-\frac{6b^3fp^3q^3\operatorname{PolyLog}\left(3,-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)}$$
$$+\frac{6b^3hp^3q^3\operatorname{PolyLog}\left(4,-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2}-\frac{6b^3hp^3q^3\operatorname{PolyLog}\left(4,-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$

```
[In] Int[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/((g + h*x)*(i + j*x)^2),x]
```

```
[Out] -((j*(e + f*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^3)/((f*i - e*j)*(h*i - g*j)
*(i + j*x))) + (h*(a + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[(f*(g + h*x))/(f*g
 - e*h)])/(h*i - g*j)^2 + (3*b*f*p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*Log
[(f*(i + j*x))/(f*i - e*j)])/((f*i - e*j)*(h*i - g*j)) - (h*(a + b*Log[c*(d
*(e + f*x)^p)^q])^3*Log[(f*(i + j*x))/(f*i - e*j)])/(h*i - g*j)^2 + (3*b*h*
```

```
p*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, -((h*(e + f*x))/(f*g - e*
h))])/(h*i - g*j)^2 + (6*b^2*f*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*Pol
yLog[2, -((j*(e + f*x))/(f*i - e*j))])/((f*i - e*j)*(h*i - g*j)) - (3*b*h*p
*q*(a + b*Log[c*(d*(e + f*x)^p)^q])^2*PolyLog[2, -((j*(e + f*x))/(f*i - e*j
))])/(h*i - g*j)^2 - (6*b^2*h*p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*Poly
Log[3, -((h*(e + f*x))/(f*g - e*h))])/(h*i - g*j)^2 - (6*b^3*f*p^3*q^3*Poly
Log[3, -((j*(e + f*x))/(f*i - e*j))])/((f*i - e*j)*(h*i - g*j)) + (6*b^2*h*
p^2*q^2*(a + b*Log[c*(d*(e + f*x)^p)^q])*PolyLog[3, -((j*(e + f*x))/(f*i -
e*j))])/(h*i - g*j)^2 + (6*b^3*h*p^3*q^3*PolyLog[4, -((h*(e + f*x))/(f*g -
e*h))])/(h*i - g*j)^2 - (6*b^3*h*p^3*q^3*PolyLog[4, -((j*(e + f*x))/(f*i -
e*j))])/(h*i - g*j)^2
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
```

```
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^3}{(g+hx)(i+jx)^2}\,dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) \\
&= \text{Subst}\left(\int \left(\frac{h^2(a+b\log(cd^q(e+fx)^{pq}))^3}{(hi-gj)^2(g+hx)} - \frac{j(a+b\log(cd^q(e+fx)^{pq}))^3}{(hi-gj)(i+jx)^2} \right.\right. \\
&\qquad \left.\left. - \frac{hj(a+b\log(cd^q(e+fx)^{pq}))^3}{(hi-gj)^2(i+jx)}\right) dx, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) \\
&= \text{Subst}\left(\frac{h^2\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^3}{g+hx}\,dx}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) \\
&\quad - \text{Subst}\left(\frac{(hj)\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^3}{i+jx}\,dx}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) \\
&\quad - \text{Subst}\left(\frac{j\int \frac{(a+b\log(cd^q(e+fx)^{pq}))^3}{(i+jx)^2}\,dx}{hi-gj}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)
\end{aligned}$$

$$= -\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{(fi-ej)(hi-gj)(i+jx)} + \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2} - \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2} - \text{Subst}\left(\frac{(3bfhpq)\int\frac{(a+b\log(cd^q(e+fx)^{pq}))^2\log\left(\frac{f(g+hx)}{fg-eh}\right)}{e+fx}dx}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(3bfhpq)\int\frac{(a+b\log(cd^q(e+fx)^{pq}))^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{e+fx}dx}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(3bfjpq)\int\frac{(a+b\log(cd^q(e+fx)^{pq}))^2}{i+jx}dx}{(fi-ej)(hi-gj)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{(fi-ej)(hi-gj)(i+jx)} + \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2} + \frac{3bfpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} - \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2} - \text{Subst}\left(\frac{(3bhpq)\text{Subst}\left(\int \frac{(a+b\log(cd^q x^{pq}))^2\log\left(\frac{f\left(\frac{fg-eh}{f}+\frac{hx}{f}\right)}{fg-eh}\right)}{x}dx, x, e+fx\right)}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) + \text{Subst}\left(\frac{(3bhpq)\text{Subst}\left(\int \frac{(a+b\log(cd^q x^{pq}))^2\log\left(\frac{f\left(\frac{fi-ej}{f}+\frac{jx}{f}\right)}{fi-ej}\right)}{x}dx, x, e+fx\right)}{(hi-gj)^2}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right) - \text{Subst}\left(\frac{(6b^2f^2p^2q^2)\int \frac{(a+b\log(cd^q(e+fx)^{pq}))\log\left(\frac{f(i+jx)}{fi-ej}\right)}{e+fx}dx}{(fi-ej)(hi-gj)}, cd^q(e+fx)^{pq}, c(d(e+fx)^p)^q\right)$$

$$= -\frac{j(e+fx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3}{(fi-ej)(hi-gj)(i+jx)} + \frac{h\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2} + \frac{3bfpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} - \frac{h\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2} + \frac{3bhpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} - \frac{3bhpq\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2} - \operatorname{Subst}\left(\frac{(6b^2hp^2q^2)\operatorname{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\operatorname{Li}_2\left(-\frac{hx}{fg-eh}\right)}{x}dx,x,e+fx\right)}{(hi-gj)^2},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right) + \operatorname{Subst}\left(\frac{(6b^2hp^2q^2)\operatorname{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\operatorname{Li}_2\left(-\frac{jx}{fi-ej}\right)}{x}dx,x,e+fx\right)}{(hi-gj)^2},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right) - \operatorname{Subst}\left(\frac{(6b^2fp^2q^2)\operatorname{Subst}\left(\int\frac{(a+b\log(cd^qx^{pq}))\log\left(\frac{f\left(\frac{fi-ej}{f}+\frac{jx}{f}\right)}{fi-ej}\right)}{x}dx,x,e+fx\right)}{(fi-ej)(hi-gj)},cd^q(e+fx)^{pq},c\left(d(e+fx)^p\right)^q\right)$$

$$= -\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{(fi-ej)(hi-gj)(i+jx)}$$
$$+\frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2}$$
$$+\frac{3bfpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(fi-ej)(hi-gj)}$$
$$-\frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2}$$
$$+\frac{3bhpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2}$$
$$+\frac{6b^2fp^2q^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)}$$
$$-\frac{3bhpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$
$$-\frac{6b^2hp^2q^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2}$$
$$+\frac{6b^2hp^2q^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{Li}_3\left(-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$
$$+\operatorname{Subst}\left(\frac{(6b^3hp^3q^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{hx}{fg-eh}\right)}{x}\,dx,x,e+fx\right)}{(hi-gj)^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\operatorname{Subst}\left(\frac{(6b^3hp^3q^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{jx}{fi-ej}\right)}{x}\,dx,x,e+fx\right)}{(hi-gj)^2},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$
$$-\operatorname{Subst}\left(\frac{(6b^3fp^3q^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{jx}{fi-ej}\right)}{x}\,dx,x,e+fx\right)}{(fi-ej)(hi-gj)},cd^q(e+fx)^{pq},c(d(e+fx)^p)^q\right)$$

$$= -\frac{j(e+fx)\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{(fi-ej)(hi-gj)(i+jx)} + \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(g+hx)}{fg-eh}\right)}{(hi-gj)^2} + \frac{3bfpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} - \frac{h\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3\log\left(\frac{f(i+jx)}{fi-ej}\right)}{(hi-gj)^2} + \frac{3bhpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} + \frac{6b^2fp^2q^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} - \frac{3bhpq\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^2\operatorname{Li}_2\left(-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2} - \frac{6b^2hp^2q^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{Li}_3\left(-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} - \frac{6b^3fp^3q^3\operatorname{Li}_3\left(-\frac{j(e+fx)}{fi-ej}\right)}{(fi-ej)(hi-gj)} + \frac{6b^2hp^2q^2\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)\operatorname{Li}_3\left(-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2} + \frac{6b^3hp^3q^3\operatorname{Li}_4\left(-\frac{h(e+fx)}{fg-eh}\right)}{(hi-gj)^2} - \frac{6b^3hp^3q^3\operatorname{Li}_4\left(-\frac{j(e+fx)}{fi-ej}\right)}{(hi-gj)^2}$$

## Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 1057, normalized size of antiderivative = 1.60

$$\int \frac{\left(a+b\log\left(c(d(e+fx)^p)^q\right)\right)^3}{(g+hx)(i+jx)^2}\,dx$$

$$= \frac{(fi-ej)(hi-gj)\left(a-bpq\log(e+fx)+b\log\left(c(d(e+fx)^p)^q\right)\right)^3 + h(fi-ej)(i+jx)\left(a-bpq\log(e+fx\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d*(e + f*x)^p)^q])^3/((g + h*x)*(i + j*x)^2),x]
```

```
[Out] ((f*i - e*j)*(h*i - g*j)*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f*x)^p)^
q])^3 + h*(f*i - e*j)*(i + j*x)*(a - b*p*q*Log[e + f*x] + b*Log[c*(d*(e + f
*x)^p)^q])^3*Log[g + h*x] - h*(f*i - e*j)*(i + j*x)*(a - b*p*q*Log[e + f*x]
 + b*Log[c*(d*(e + f*x)^p)^q])^3*Log[i + j*x] - 3*b*p*q*(a - b*p*q*Log[e +
f*x] + b*Log[c*(d*(e + f*x)^p)^q])^2*((h*i - g*j)*(j*(e + f*x)*Log[e + f*x]
```

```
 - f*(i + j*x)*Log[i + j*x]) - h*(f*i - e*j)*(i + j*x)*(Log[e + f*x]*Log[(f
*(g + h*x))/(f*g - e*h)] + PolyLog[2, (h*(e + f*x))/(-(f*g) + e*h)]) + h*(f
*i - e*j)*(i + j*x)*(Log[e + f*x]*Log[(f*(i + j*x))/(f*i - e*j)] + PolyLog[
2, (j*(e + f*x))/(-(f*i) + e*j)])) - 3*b^2*p^2*q^2*(a - b*p*q*Log[e + f*x]
+ b*Log[c*(d*(e + f*x)^p)^q])*((h*i - g*j)*(Log[e + f*x]*(j*(e + f*x)*Log[e
 + f*x] - 2*f*(i + j*x)*Log[(f*(i + j*x))/(f*i - e*j)]) - 2*f*(i + j*x)*Pol
yLog[2, (j*(e + f*x))/(-(f*i) + e*j)]) - h*(f*i - e*j)*(i + j*x)*(Log[e + f
*x]^2*Log[(f*(g + h*x))/(f*g - e*h)] + 2*Log[e + f*x]*PolyLog[2, (h*(e + f*
x))/(-(f*g) + e*h)] - 2*PolyLog[3, (h*(e + f*x))/(-(f*g) + e*h)]) + h*(f*i
- e*j)*(i + j*x)*(Log[e + f*x]^2*Log[(f*(i + j*x))/(f*i - e*j)] + 2*Log[e +
 f*x]*PolyLog[2, (j*(e + f*x))/(-(f*i) + e*j)] - 2*PolyLog[3, (j*(e + f*x))
/(-(f*i) + e*j)])) - b^3*p^3*q^3*((h*i - g*j)*(Log[e + f*x]^2*(j*(e + f*x)*
Log[e + f*x] - 3*f*(i + j*x)*Log[(f*(i + j*x))/(f*i - e*j)]) - 6*f*(i + j*x
)*Log[e + f*x]*PolyLog[2, (j*(e + f*x))/(-(f*i) + e*j)] + 6*f*(i + j*x)*Pol
yLog[3, (j*(e + f*x))/(-(f*i) + e*j)]) - h*(f*i - e*j)*(i + j*x)*(Log[e + f
*x]^3*Log[(f*(g + h*x))/(f*g - e*h)] + 3*Log[e + f*x]^2*PolyLog[2, (h*(e +
f*x))/(-(f*g) + e*h)] - 6*Log[e + f*x]*PolyLog[3, (h*(e + f*x))/(-(f*g) + e
*h)] + 6*PolyLog[4, (h*(e + f*x))/(-(f*g) + e*h)]) + h*(f*i - e*j)*(i + j*x
)*(Log[e + f*x]^3*Log[(f*(i + j*x))/(f*i - e*j)] + 3*Log[e + f*x]^2*PolyLog
[2, (j*(e + f*x))/(-(f*i) + e*j)] - 6*Log[e + f*x]*PolyLog[3, (j*(e + f*x))
/(-(f*i) + e*j)] + 6*PolyLog[4, (j*(e + f*x))/(-(f*i) + e*j)])))/((f*i - e*
j)*(h*i - g*j)^2*(i + j*x))
```

## Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{3}}{\left(hx+g\right)\left(jx+i\right)^{2}}dx$$

```
[In] int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i)^2,x)
```

```
[Out] int((a+b*ln(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i)^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\log\left(c(d(e+fx)^{p})^{q}\right))^{3}}{(g+hx)(i+jx)^{2}}\,dx=\int \frac{\left(b\log\left(\left(\left(fx+e\right)^{p}d\right)^{q}c\right)+a\right)^{3}}{(hx+g)(jx+i)^{2}}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i)^2,x, algorithm="fr
icas")
```

```
[Out] integral((b^3*log(((f*x + e)^p*d)^q*c)^3 + 3*a*b^2*log(((f*x + e)^p*d)^q*c)
^2 + 3*a^2*b*log(((f*x + e)^p*d)^q*c) + a^3)/(h*j^2*x^3 + g*i^2 + (2*h*i*j
+ g*j^2)*x^2 + (h*i^2 + 2*g*i*j)*x), x)
```

## Sympy [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx = \int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx$$

```
[In] integrate((a+b*ln(c*(d*(f*x+e)**p)**q))**3/(h*x+g)/(j*x+i)**2,x)
[Out] Integral((a + b*log(c*(d*(e + f*x)**p)**q))**3/((g + h*x)*(i + j*x)**2), x)
```

## Maxima [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx = \int \frac{(b\log(((fx+e)^pd)^qc)+a)^3}{(hx+g)(jx+i)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i)^2,x, algorithm="ma
xima")
[Out] a^3*(h*log(h*x + g)/(h^2*i^2 - 2*g*h*i*j + g^2*j^2) - h*log(j*x + i)/(h^2*i
^2 - 2*g*h*i*j + g^2*j^2) + 1/(h*i^2 - g*i*j + (h*i*j - g*j^2)*x)) + integr
ate((b^3*log(((f*x + e)^p)^q)^3 + 3*(q*log(d) + log(c))*a^2*b + 3*(q^2*log(
d)^2 + 2*q*log(c)*log(d) + log(c)^2)*a*b^2 + (q^3*log(d)^3 + 3*q^2*log(c)*l
og(d)^2 + 3*q*log(c)^2*log(d) + log(c)^3)*b^3 + 3*((q*log(d) + log(c))*b^3
+ a*b^2)*log(((f*x + e)^p)^q)^2 + 3*(2*(q*log(d) + log(c))*a*b^2 + (q^2*log
(d)^2 + 2*q*log(c)*log(d) + log(c)^2)*b^3 + a^2*b)*log(((f*x + e)^p)^q))/(h
*j^2*x^3 + g*i^2 + (2*h*i*j + g*j^2)*x^2 + (h*i^2 + 2*g*i*j)*x), x)
```

## Giac [F]

$$\int \frac{(a+b\log(c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx = \int \frac{(b\log(((fx+e)^pd)^qc)+a)^3}{(hx+g)(jx+i)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d*(f*x+e)^p)^q))^3/(h*x+g)/(j*x+i)^2,x, algorithm="gi
ac")
[Out] integrate((b*log(((f*x + e)^p*d)^q*c) + a)^3/((h*x + g)*(j*x + i)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log (c(d(e+fx)^p)^q))^3}{(g+hx)(i+jx)^2}\,dx=\int \frac{{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^3}{\left(g+h\,x\right)\,{\left(i+j\,x\right)}^2}\,dx$$

```
[In] int((a + b*log(c*(d*(e + f*x)^p)^q))^3/((g + h*x)*(i + j*x)^2),x)
[Out] int((a + b*log(c*(d*(e + f*x)^p)^q))^3/((g + h*x)*(i + j*x)^2), x)
```

# 3.540 $\int \frac{i+jx}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))} dx$



## Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} dx = \text{Int}\left(\frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)},x\right)$$

```
[Out] Unintegrable((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} dx = \int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} dx$$

```
[In] Int[(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Defer[Int][(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

Rubi steps

$$\text{integral} = \int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)} dx$$

## Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

```
[In] Integrate[(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Integrate[(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{jx+i}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)}dx$$

```
[In] int((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
[Out] int((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{jx+i}{(hx+g)(b\log\left(\left((fx+e)^{p}d\right)^{q}c\right)+a)}\,dx$$

```
[In] integrate((j*x+i)/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas
")
[Out] integral((j*x + i)/(a*h*x + a*g + (b*h*x + b*g)*log(((f*x + e)^p*d)^q*c)),
x)
```

## Sympy [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{i+jx}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)\left(g+hx\right)}\,dx$$

```
[In] integrate((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
[Out] Integral((i + j*x)/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{jx+i}{(hx+g)\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)}\,dx$$

```
[In] integrate((j*x+i)/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima
")
[Out] integrate((j*x + i)/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{jx+i}{(hx+g)\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)}\,dx$$

```
[In] integrate((j*x+i)/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
[Out] integrate((j*x + i)/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{i+j\,x}{(g+h\,x)\ \left(a+b\,\ln\left(c\,\left(d\,{\left(e+f\,x\right)}^p\right)^q\right)\right)}\,dx$$

```
[In] int((i + j*x)/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
[Out] int((i + j*x)/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

# 3.541 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\text{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

```
[In] Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Defer[Int][1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

```
[In] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

## Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)}dx$$

```
[In] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
[Out] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(hx+g)(b\log\left(\left((fx+e)^p d\right)^q c\right)+a)}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fricas")
[Out] integral(1/(a*h*x + a*g + (b*h*x + b*g)*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right)\right)(g+hx)}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)(b\log\left(\left(\left(fx+e\right)^pd\right)^qc\right)+a)}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxima")
[Out] integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(hx+g)(b\log\left(\left(\left(fx+e\right)^pd\right)^qc\right)+a)}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac")
[Out] integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx=\int \frac{1}{(g+h\,x)\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}\,dx$$

```
[In] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
[Out] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

# 3.542 $\int \frac{1}{(g+hx)(i+jx)(a+b\log(c(d(e+fx)^p)^q))}\,dx$



## Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \operatorname{Int}\left(\frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

```
[In] Int[1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Defer[Int][1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$
$$=\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

```
[In] Integrate[1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Integrate[1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

## Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\,(jx+i)\,\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)}dx$$

```
[In] int(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
[Out] int(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.86

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$
$$=\int \frac{1}{(hx+g)(jx+i)(b\log\left(((fx+e)^p d)^q\,c\right)+a)}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fric
as")
[Out] integral(1/(a*h*j*x^2 + a*g*i + (a*h*i + a*g*j)*x + (b*h*j*x^2 + b*g*i + (b
*h*i + b*g*j)*x)*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 3.73 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)\left(g+hx\right)\left(i+jx\right)}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)*(i + j*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(hx+g)(jx+i)(b\log\left(\left((fx+e)^p d\right)^q c\right)+a)}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="maxi
ma")
[Out] integrate(1/((h*x + g)*(j*x + i)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(hx+g)(jx+i)(b\log\left(\left((fx+e)^p d\right)^q c\right)+a)}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="giac
")
[Out] integrate(1/((h*x + g)*(j*x + i)*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(g+h\,x)\,\left(i+j\,x\right)\,\left(a+b\,\ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)}\,dx$$

```
[In] int(1/((g + h*x)*(i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
[Out] int(1/((g + h*x)*(i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

# 3.543 $\int \frac{1}{(g+hx)(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))}\,dx$



## Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \mathrm{Int}\left(\frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)/(j*x+i)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

```
[In] Int[1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
[Out] Defer[Int][1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$
$$=\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$

```
[In] Integrate[1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])),x]
```

```
[Out] Integrate[1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])), x]
```

## Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\,(jx+i)^2\,\left(a+b\ln\left(c\left(d\,(fx+e)^p\right)^q\right)\right)}dx$$

```
[In] int(1/(h*x+g)/(j*x+i)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

```
[Out] int(1/(h*x+g)/(j*x+i)^2/(a+b*ln(c*(d*(f*x+e)^p)^q)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.29

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx$$
$$=\int \frac{1}{(hx+g)(jx+i)^2\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="fr
icas")
```

```
[Out] integral(1/(a*h*j^2*x^3 + a*g*i^2 + (2*a*h*i*j + a*g*j^2)*x^2 + (a*h*i^2 +
2*a*g*i*j)*x + (b*h*j^2*x^3 + b*g*i^2 + (2*b*h*i*j + b*g*j^2)*x^2 + (b*h*i^
2 + 2*b*g*i*j)*x)*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 12.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{(g+hx)(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))}\,dx = \int \frac{1}{(a+b\log(c(d(e+fx)^p)^q))(g+hx)(i+jx)^2}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)**2/(a+b*ln(c*(d*(f*x+e)**p)**q)),x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))*(g + h*x)*(i + j*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))}\,dx = \int \frac{1}{(hx+g)(jx+i)^2(b\log(((fx+e)^pd)^q c)+a)}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="ma
xima")
[Out] integrate(1/((h*x + g)*(j*x + i)^2*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2 (a+b\log(c(d(e+fx)^p)^q))}\,dx = \int \frac{1}{(hx+g)(jx+i)^2(b\log(((fx+e)^pd)^q c)+a)}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)^2/(a+b*log(c*(d*(f*x+e)^p)^q)),x, algorithm="gi
ac")
[Out] integrate(1/((h*x + g)*(j*x + i)^2*(b*log(((f*x + e)^p*d)^q*c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)}\,dx = \int \frac{1}{(g+h\,x)\,\left(i+j\,x\right)^2\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}\,dx$$

```
[In] int(1/((g + h*x)*(i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))),x)
[Out] int(1/((g + h*x)*(i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))), x)
```

# 3.544 $\int \frac{i+jx}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^2} dx$



## Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{i+jx}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^2} dx = \text{Int}\left(\frac{i+jx}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^2}, x\right)$$

```
[Out] Unintegrable((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{i+jx}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^2} dx = \int \frac{i+jx}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^2} dx$$

```
[In] Int[(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
[Out] Defer[Int][(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{i+jx}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^2} dx$$

## Mathematica [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

```
[In] Integrate[(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
[Out] Integrate[(i + j*x)/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

## Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{jx+i}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^p\right)^q\right)\right)^2}dx$$

```
[In] int((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
[Out] int((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{jx+i}{(hx+g)(b\log\left(\left((fx+e)^pd\right)^q c\right)+a)^2}\,dx$$

```
[In] integrate((j*x+i)/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fric
as")
[Out] integral((j*x + i)/(a^2*h*x + a^2*g + (b^2*h*x + b^2*g)*log(((f*x + e)^p*d)
^q*c)^2 + 2*(a*b*h*x + a*b*g)*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 16.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{i+jx}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^2(g+hx)}\,dx$$

```
[In] integrate((j*x+i)/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
[Out] Integral((i + j*x)/((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 302, normalized size of antiderivative = 9.15

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{jx+i}{(hx+g)(b\log\left(\left(((fx+e)^pd)^q\,c\right)+a\right)^2}\,dx$$

```
[In] integrate((j*x+i)/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxi
ma")
[Out] -(f*j*x^2 + e*i + (f*i + e*j)*x)/(a*b*f*g*p*q + (f*g*p*q^2*log(d) + f*g*p*q
*log(c))*b^2 + (a*b*f*h*p*q + (f*h*p*q^2*log(d) + f*h*p*q*log(c))*b^2)*x +
(b^2*f*h*p*q*x + b^2*f*g*p*q)*log(((f*x + e)^p)^q)) + integrate((f*h*j*x^2
+ 2*f*g*j*x + f*g*i - (h*i - g*j)*e)/(a*b*f*g^2*p*q + (f*g^2*p*q^2*log(d) +
 f*g^2*p*q*log(c))*b^2 + (a*b*f*h^2*p*q + (f*h^2*p*q^2*log(d) + f*h^2*p*q*l
og(c))*b^2)*x^2 + 2*(a*b*f*g*h*p*q + (f*g*h*p*q^2*log(d) + f*g*h*p*q*log(c)
)*b^2)*x + (b^2*f*h^2*p*q*x^2 + 2*b^2*f*g*h*p*q*x + b^2*f*g^2*p*q)*log(((f*
x + e)^p)^q)), x)
```

## Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx=\int \frac{jx+i}{(hx+g)(b\log\left(\left(((fx+e)^pd)^q\,c\right)+a\right)^2}\,dx$$

```
[In] integrate((j*x+i)/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac
")
[Out] integrate((j*x + i)/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{i+jx}{(g+hx)\,(a+b\log\left(c\left(d(e+fx)^p\right)^q\right))^2}\,dx = \int \frac{i+j\,x}{(g+h\,x)\,\left(a+b\,\ln\left(c\left(d\,(e+f\,x)^p\right)^q\right)\right)^2}\,dx$$

```
[In] int((i + j*x)/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2),x)
[Out] int((i + j*x)/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2), x)
```

# 3.545 $\int \frac{1}{(g+hx)(a+b\log(c(d(e+fx)^p)^q))^2} dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \operatorname{Int}\left(\frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

```
[In] Int[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

```
[Out] Defer[Int][1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

```
[In] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
[Out] Integrate[1/((g + h*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

## Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}dx$$

```
[In] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
[Out] int(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*h*x + a^2*g + (b^2*h*x + b^2*g)*log(((f*x + e)^p*d)^q*c)^2
+ 2*(a*b*h*x + a*b*g)*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 5.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(g+hx)\,(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right))^{2}}\,dx = \int \frac{1}{(a+b\log\left(c\left(d\left(e+fx\right)^{p}\right)^{q}\right))^{2}\,(g+hx)}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 267, normalized size of antiderivative = 9.54

$$\int \frac{1}{(g+hx)\,(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right))^{2}}\,dx = \int \frac{1}{(hx+g)(b\log\left(\left((fx+e)^{p}d\right)^{q}c\right)+a)^{2}}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="maxima")
[Out] (f*g - e*h)*integrate(1/(a*b*f*g^2*p*q + (f*g^2*p*q^2*log(d) + f*g^2*p*q*lo
g(c))*b^2 + (a*b*f*h^2*p*q + (f*h^2*p*q^2*log(d) + f*h^2*p*q*log(c))*b^2)*x
^2 + 2*(a*b*f*g*h*p*q + (f*g*h*p*q^2*log(d) + f*g*h*p*q*log(c))*b^2)*x + (b
^2*f*h^2*p*q*x^2 + 2*b^2*f*g*h*p*q*x + b^2*f*g^2*p*q)*log(((f*x + e)^p)^q))
, x) - (f*x + e)/(a*b*f*g*p*q + (f*g*p*q^2*log(d) + f*g*p*q*log(c))*b^2 + (
a*b*f*h*p*q + (f*h*p*q^2*log(d) + f*h*p*q*log(c))*b^2)*x + (b^2*f*h*p*q*x +
 b^2*f*g*p*q)*log(((f*x + e)^p)^q))
```

## Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\,(a+b\log\left(c\left(d(e+fx)^{p}\right)^{q}\right))^{2}}\,dx = \int \frac{1}{(hx+g)(b\log\left(\left((fx+e)^{p}d\right)^{q}c\right)+a)^{2}}\,dx$$

```
[In] integrate(1/(h*x+g)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="giac")
[Out] integrate(1/((h*x + g)*(b*log(((f*x + e)^p*d)^q*c) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{(g+hx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(g+h\,x)\ \left(a+b\ \ln\left(c\left(d\left(e+f\,x\right)^p\right)^q\right)\right)^2}\,dx$$

```
[In] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2),x)
[Out] int(1/((g + h*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2), x)
```

# 3.546 $\int \frac{1}{(g+hx)(i+jx)(a+b\log(c(d(e+fx)^p)^q))^2} dx$



## Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} dx = \text{Int}\left(\frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} dx = \int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} dx$$

```
[In] Int[1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
```

```
[Out] Defer[Int][1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2} dx$$

## Mathematica [N/A]

Not integrable

Time = 13.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

```
[In] Integrate[1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
[Out] Integrate[1/((g + h*x)*(i + j*x)*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

## Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)(jx+i)\left(a+b\ln\left(c\left(d\left(fx+e\right)^{p}\right)^{q}\right)\right)^{2}}dx$$

```
[In] int(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
[Out] int(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.60

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(jx+i)(b\log\left(((fx+e)^p d)^q c\right)+a)^2}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="fr
icas")
[Out] integral(1/(a^2*h*j*x^2 + a^2*g*i + (b^2*h*j*x^2 + b^2*g*i + (b^2*h*i + b^2
*g*j)*x)*log(((f*x + e)^p*d)^q*c)^2 + (a^2*h*i + a^2*g*j)*x + 2*(a*b*h*j*x^
2 + a*b*g*i + (a*b*h*i + a*b*g*j)*x)*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 16.73 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^2\left(g+hx\right)\left(i+jx\right)}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)*(i + j*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 701, normalized size of antiderivative = 20.03

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(jx+i)(b\log\left(\left(\left(fx+e\right)^pd\right)^qc\right)+a)^2}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="ma
xima")
[Out] -(f*x + e)/(a*b*f*g*i*p*q + (f*g*i*p*q^2*log(d) + f*g*i*p*q*log(c))*b^2 + (
a*b*f*h*j*p*q + (f*h*j*p*q^2*log(d) + f*h*j*p*q*log(c))*b^2)*x^2 + ((h*i*p*
q + g*j*p*q)*a*b*f + ((h*i*p*q + g*j*p*q)*f*log(c) + (h*i*p*q^2 + g*j*p*q^2
)*f*log(d))*b^2)*x + (b^2*f*h*j*p*q*x^2 + b^2*f*g*i*p*q + (h*i*p*q + g*j*p*
q)*b^2*f*x)*log(((f*x + e)^p)^q)) - integrate((f*h*j*x^2 + 2*e*h*j*x - f*g*
i + (h*i + g*j)*e)/(a*b*f*g^2*i^2*p*q + (a*b*f*h^2*j^2*p*q + (f*h^2*j^2*p*q
^2*log(d) + f*h^2*j^2*p*q*log(c))*b^2)*x^4 + 2*((h^2*i*j*p*q + g*h*j^2*p*q)
*a*b*f + ((h^2*i*j*p*q + g*h*j^2*p*q)*f*log(c) + (h^2*i*j*p*q^2 + g*h*j^2*p
*q^2)*f*log(d))*b^2)*x^3 + (f*g^2*i^2*p*q^2*log(d) + f*g^2*i^2*p*q*log(c))*
b^2 + ((h^2*i^2*p*q + 4*g*h*i*j*p*q + g^2*j^2*p*q)*a*b*f + ((h^2*i^2*p*q +
4*g*h*i*j*p*q + g^2*j^2*p*q)*f*log(c) + (h^2*i^2*p*q^2 + 4*g*h*i*j*p*q^2 +
g^2*j^2*p*q^2)*f*log(d))*b^2)*x^2 + 2*((g*h*i^2*p*q + g^2*i*j*p*q)*a*b*f +
((g*h*i^2*p*q + g^2*i*j*p*q)*f*log(c) + (g*h*i^2*p*q^2 + g^2*i*j*p*q^2)*f*l
og(d))*b^2)*x + (b^2*f*h^2*j^2*p*q*x^4 + b^2*f*g^2*i^2*p*q + 2*(h^2*i*j*p*q
 + g*h*j^2*p*q)*b^2*f*x^3 + (h^2*i^2*p*q + 4*g*h*i*j*p*q + g^2*j^2*p*q)*b^2
*f*x^2 + 2*(g*h*i^2*p*q + g^2*i*j*p*q)*b^2*f*x)*log(((f*x + e)^p)^q)), x)
```

## Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(jx+i)(b\log\left(((fx+e)^pd)^q\,c\right)+a)^2}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((h*x + g)*(j*x + i)*(b*log(((f*x + e)^p*d)^q*c) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(g+h\,x)\,\left(i+j\,x\right)\,\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)^2}\,dx$$

```
[In] int(1/((g + h*x)*(i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2),x)
```

```
[Out] int(1/((g + h*x)*(i + j*x)*(a + b*log(c*(d*(e + f*x)^p)^q))^2), x)
```

## 3.547 $\int \frac{1}{(g+hx)(i+jx)^2(a+b\log(c(d(e+fx)^p)^q))^2} dx$



### Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(h*x+g)/(j*x+i)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

### Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

```
[In] Int[1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
[Out] Defer[Int][1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 19.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(g+hx)(i+jx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx$$

```
[In] Integrate[1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2),x]
[Out] Integrate[1/((g + h*x)*(i + j*x)^2*(a + b*Log[c*(d*(e + f*x)^p)^q])^2), x]
```

## Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(hx+g)\,(jx+i)^2\,\left(a+b\ln\left(c\left(d\,(fx+e)^p\right)^q\right)\right)^2}dx$$

```
[In] int(1/(h*x+g)/(j*x+i)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
[Out] int(1/(h*x+g)/(j*x+i)^2/(a+b*ln(c*(d*(f*x+e)^p)^q))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 6.03

$$\int \frac{1}{(g+hx)(i+jx)^2 \left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(jx+i)^2(b\log\left(\left((fx+e)^p d\right)^q c\right)+a)^2}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="
fricas")
[Out] integral(1/(a^2*h*j^2*x^3 + a^2*g*i^2 + (2*a^2*h*i*j + a^2*g*j^2)*x^2 + (b^
2*h*j^2*x^3 + b^2*g*i^2 + (2*b^2*h*i*j + b^2*g*j^2)*x^2 + (b^2*h*i^2 + 2*b^
2*g*i*j)*x)*log(((f*x + e)^p*d)^q*c)^2 + (a^2*h*i^2 + 2*a^2*g*i*j)*x + 2*(a
*b*h*j^2*x^3 + a*b*g*i^2 + (2*a*b*h*i*j + a*b*g*j^2)*x^2 + (a*b*h*i^2 + 2*a
*b*g*i*j)*x)*log(((f*x + e)^p*d)^q*c)), x)
```

## Sympy [N/A]

Not integrable

Time = 112.69 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d\left(e+fx\right)^p\right)^q\right)\right)^2\left(g+hx\right)\left(i+jx\right)^2}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)**2/(a+b*ln(c*(d*(f*x+e)**p)**q))**2,x)
[Out] Integral(1/((a + b*log(c*(d*(e + f*x)**p)**q))**2*(g + h*x)*(i + j*x)**2),
x)
```

## Maxima [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 1039, normalized size of antiderivative = 29.69

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(jx+i)^2\left(b\log\left(\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="
maxima")
[Out] -(f*x + e)/(a*b*f*g*i^2*p*q + (a*b*f*h*j^2*p*q + (f*h*j^2*p*q^2*log(d) + f*
h*j^2*p*q*log(c))*b^2)*x^3 + (f*g*i^2*p*q^2*log(d) + f*g*i^2*p*q*log(c))*b^
2 + ((2*h*i*j*p*q + g*j^2*p*q)*a*b*f + ((2*h*i*j*p*q + g*j^2*p*q)*f*log(c)
+ (2*h*i*j*p*q^2 + g*j^2*p*q^2)*f*log(d))*b^2)*x^2 + ((h*i^2*p*q + 2*g*i*j*
p*q)*a*b*f + ((h*i^2*p*q + 2*g*i*j*p*q)*f*log(c) + (h*i^2*p*q^2 + 2*g*i*j*p
*q^2)*f*log(d))*b^2)*x + (b^2*f*h*j^2*p*q*x^3 + b^2*f*g*i^2*p*q + (2*h*i*j*
p*q + g*j^2*p*q)*b^2*f*x^2 + (h*i^2*p*q + 2*g*i*j*p*q)*b^2*f*x)*log(((f*x +
 e)^p)^q)) - integrate((2*f*h*j*x^2 - f*g*i + (h*i + 2*g*j)*e + (f*g*j + 3*
e*h*j)*x)/(a*b*f*g^2*i^3*p*q + (a*b*f*h^2*j^3*p*q + (f*h^2*j^3*p*q^2*log(d)
 + f*h^2*j^3*p*q*log(c))*b^2)*x^5 + ((3*h^2*i*j^2*p*q + 2*g*h*j^3*p*q)*a*b*
f + ((3*h^2*i*j^2*p*q + 2*g*h*j^3*p*q)*f*log(c) + (3*h^2*i*j^2*p*q^2 + 2*g*
h*j^3*p*q^2)*f*log(d))*b^2)*x^4 + ((3*h^2*i^2*j*p*q + 6*g*h*i*j^2*p*q + g^2
*j^3*p*q)*a*b*f + ((3*h^2*i^2*j*p*q + 6*g*h*i*j^2*p*q + g^2*j^3*p*q)*f*log(
c) + (3*h^2*i^2*j*p*q^2 + 6*g*h*i*j^2*p*q^2 + g^2*j^3*p*q^2)*f*log(d))*b^2)
*x^3 + (f*g^2*i^3*p*q^2*log(d) + f*g^2*i^3*p*q*log(c))*b^2 + ((h^2*i^3*p*q
+ 6*g*h*i^2*j*p*q + 3*g^2*i*j^2*p*q)*a*b*f + ((h^2*i^3*p*q + 6*g*h*i^2*j*p*
```

```
q + 3*g^2*i*j^2*p*q)*f*log(c) + (h^2*i^3*p*q^2 + 6*g*h*i^2*j*p*q^2 + 3*g^2*
i*j^2*p*q^2)*f*log(d))*b^2)*x^2 + ((2*g*h*i^3*p*q + 3*g^2*i^2*j*p*q)*a*b*f
+ ((2*g*h*i^3*p*q + 3*g^2*i^2*j*p*q)*f*log(c) + (2*g*h*i^3*p*q^2 + 3*g^2*i^
2*j*p*q^2)*f*log(d))*b^2)*x + (b^2*f*h^2*j^3*p*q*x^5 + b^2*f*g^2*i^3*p*q +
(3*h^2*i*j^2*p*q + 2*g*h*j^3*p*q)*b^2*f*x^4 + (3*h^2*i^2*j*p*q + 6*g*h*i*j^
2*p*q + g^2*j^3*p*q)*b^2*f*x^3 + (h^2*i^3*p*q + 6*g*h*i^2*j*p*q + 3*g^2*i*j
^2*p*q)*b^2*f*x^2 + (2*g*h*i^3*p*q + 3*g^2*i^2*j*p*q)*b^2*f*x)*log(((f*x +
e)^p)^q)), x)
```

## Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{(hx+g)(jx+i)^2\left(b\log\left(\left((fx+e)^p d\right)^q c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(h*x+g)/(j*x+i)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((h*x + g)*(j*x + i)^2*(b*log(((f*x + e)^p*d)^q*c) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(g+hx)(i+jx)^2\left(a+b\log\left(c\left(d(e+fx)^p\right)^q\right)\right)^2}\,dx = \int \frac{1}{\left(g+h\,x\right)\,{\left(i+j\,x\right)}^2\,{\left(a+b\,\ln\left(c\,{\left(d\,{\left(e+f\,x\right)}^p\right)}^q\right)\right)}^2} \,d x$$

```
[In] int(1/((g + h*x)*(i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2),x)
```

```
[Out] int(1/((g + h*x)*(i + j*x)^2*(a + b*log(c*(d*(e + f*x)^p)^q))^2), x)
```

# CHAPTER 4

# APPENDIX



## 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                 end if
             else #result contains complex but optimal is not
                 if debug then
                         print("result contains complex but optimal is not");
                 fi;
                 return "C","Result contains complex when optimal does not.";
             fi;
         else # result do not contain complex
              # this assumes optimal do not as well. No check is needed here.
             if debug then
                    print("result do not contain complex, this assumes optimal do not as well")
             fi;
             if leaf_count_result<=2*leaf_count_optimal then
                 if debug then
                     print("leaf_count_result<=2*leaf_count_optimal");
                 fi;
                 return "A"," ";
             else
                 if debug then
                     print("leaf_count_result>2*leaf_count_optimal");
                 fi;
                 return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
             fi;
         fi;
     else #ExpnType(result) > ExpnType(optimal)
         if debug then
             print("ExpnType(result) > ExpnType(optimal)");
         fi;
         return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
     fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

## Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

## SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```